

# Differentiation

October 30, 2023

1. The slope of the normal to the curve  $y = 2x^2 + 3 \sin x$  at  $x = 0$  is \_\_\_\_\_.
2. The total revenue (in ₹) received from sale of  $x$  units of a product is  $R(x) = 3x^2 + 36x + 5$ . The marginal revenue, when  $x = 12$  is \_\_\_\_\_.
3. If  $\sin y = x \sin(a + y)$ , then prove that  $\frac{dy}{dx} = \frac{\sin(a+y)}{\sin a}$ .
4. Find the equation of tangent to the curve  $y = x^2 + 4x + 1$  at the point (3,22).
5. If  $Y = \tan^{-1} \left( \frac{3x-x^3}{1-3x^2} \right)$ ,  $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$  then find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ .
6. If  $\sec^{-1} \left( \frac{1+x}{1-y} \right) = a$ , then  $\frac{dy}{dx}$  is equal to
7. The order and degree of the differential equation of the family of parabolas having at origin and axis along positive x-axis is
8. If  $y = \log x$ , then  $\frac{d^2y}{dx^2} =$
9. Find the intervals in which the function  $f$  defined as  $f(x) = \sin(x) + \cos(x)$ ,  $0 \leq x \leq 2\pi$  is strictly increasing or decreasing.
10. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone.
11. If  $y = x^{\sin x} + \sin^{-1}(\sqrt{x})$ , then find  $\frac{dy}{dx}$ .
12. The supply function of a commodity is  $100p = (x + 20)^2$ . Find the Producer's Surplus (PS), when the market price is ₹25.

13. Find:  $\int \frac{2x^2+1}{x^2-3x+2} dx$