

Let $S = \{\text{tl}: \forall 'a, 'a \text{ list} \rightarrow 'a \text{ list}, (::): \forall 'a, 'a \rightarrow 'a \text{ list} \rightarrow 'a \text{ list}, (=): \forall 'a, 'a \rightarrow 'a \rightarrow \text{bool}\}$

Let $R = S \cup \{length: \forall 'a, 'a \text{ list} \rightarrow list\}$

IF_THEN_ELSE	R - “a”: string	R - []: list	
$R \cup \{lst: \forall' a, 'a \text{ list}\} \text{ - if lst = [] then 0 else 1 + length (tl(lst)): int}$	R - (“a”::[]) : list		R - []: list
R - fun lst -> if lst = [] then 0 else 1 + length (tl(lst)) : ‘a list -> int	R - (“a”::[])::[] : list		
S - (let rec length = fun lst -> if lst = [] then 0 else 1 + length (tl(lst)) in length ((“a”::[])::[]) : int			

$$\text{Let } T = R \cup \{lst: \forall' a, \text{ 'a list}\}$$

If_THEN_ELSE:

					$T - \text{tl}: \forall' a, 'a \text{ list} \rightarrow 'a \text{ list}$	$T - \text{lst}: \forall' a, 'a \text{ list}$
					$T - \text{tl}(\text{lst}): \forall' a, 'a \text{ list}$	
$T - \text{lst}: \forall' a, 'a \text{ list}$	$T - (=): \forall' a, 'a \rightarrow 'a \rightarrow \text{bool}$	$T - []: \text{list}$		$T - 1: \text{int}$	$T - \text{length}(\text{tl}(\text{lst})): \text{int}$	
$T - \text{lst} = [] : \text{bool}$			$T - 0: \text{int}$	$T - 1 + \text{length}(\text{tl}(\text{lst})): \text{int}$		
$R \cup \{\text{lst}: \forall' a, 'a \text{ list}\} - \text{if } \text{lst} = [] \text{ then } 0 \text{ else } 1 + \text{length}(\text{tl}(\text{lst})): \text{int}$						