Homework - teams - team 12

Student 1 : Ciama Andreea Elena

Student 2 : Cibotariu Andrei

Subject 1 : operations

- Students will work with two bases : b1 and b2, one of them is less than 10 and the other one is 16, $b1 > 2$, $b2 > 2$.

- Student 1 chooses b1, b2, x, y, z, f and performs the operations:

$x(b_1) + y(b_1) = s(b_1)$, x has 6 digits and y has 5 digits

$z(b_2) * f(b_2) = p(b_2)$, z has 6 digits, f is a digit, $f \neq 1$, $f \neq 0$

- Student 2 receives s, y, p, f from Student 1 and performs the following operations to verify the correctness of the results obtained by Student 1:

$s(b_1) - y(b_1) = ?(b_1)$

$p b_2 : f(b_2) = ?(b_2)$

Student 1 - Ciama Andreea Elena

$b_1 = 8$; $b_2 = 16$

$x = 123456$ $\quad$ $y = 12345$

$x(b_1) + y(b_1) = 123456(8) + 12345(8)$

$$\begin{array}{r} 123456(8) + \\ 012345(8) \\ \hline 136023(8) \end{array}$$

$\Rightarrow s(b_1) = 136023(8)$

Calculations

$5+6=11$; $11/8=1$; $11\%8=3$

$1+5+4=10$; $10/8=1$; $10\%8=2$

$1+4+3=8$; $8/8=1$; $8\%8=0$

$3+2+1=6$; $6/8=0$; $6\%8=6$

$2+1=3$; $3/8=0$; $3\%8=3$

$1+0=1$; $1/8=0$; $1\%8=1$

$z = 235461$ $\qquad\qquad$ $f = 2$

$z(b_2) * f(b_2) = 235461(16) * 2(16)$

$$
\begin{array}{r}
235461(16)\, * \\
2(16) \\
\hline
46A8C2
\end{array}
$$

$\Rightarrow p(b_2) = 46A8C2(16)$

Calculations

$1 \cdot 2 = 2$; $2 / 16 = 0$; $2 \% 16 = 2$

$6 \cdot 2 = 12$; $12_{(10)} = C_{(16)}$

$4 \cdot 2 = 8$,

$5 \cdot 2 = 10$; $10_{(10)} = A_{(16)}$

$3 \cdot 2 = 6$

$2 \cdot 2 = 4$

• Cibotariu Andrei

$x = 123456_{(8)}$

$y = 12345_{(8)}$

$s = 136023_{(8)}$

$$\begin{array}{lcccccccl} s: & 1 & 3 & \overset{-1}{6} & \overset{-1}{0} & \overset{-1}{2} & 3 & - & (8) \\ y: & & 1 & 2 & 3 & 4 & 5 & & (8) \\ \hline x: & 1 & 2 & 3 & 4 & 5 & 6 & & (8) \end{array}$$

So $\underline{s - y = x}$ => The calculus is correct!

$p = 46A8C2_{(16)}$

$f = 2_{(16)}$

$z = 235461_{(16)}$

$\underline{p : f} = 46A8C2_{(16)} : 2_{(16)} =$
$= 235461_{(16)} \underline{= z}$

So $\underline{p : f = z}$ => The calculus is correct!

$$\begin{array}{l|l} 46A8C2_{(16)} & 2_{(16)} \\ \hline \underline{4} & 235461_{(16)} \\ =6 & \\ \underline{6} & \\ =10_{(10)} & \\ \underline{10_{(10)}} & \\ =8 & \\ \underline{8} & \\ =12_{(10)} & \\ \underline{12_{(10)}} & \\ =2 & \\ \underline{2} & \\ = & \end{array}$$

$4_{(16)} = 4_{(10)}$; $4 \bmod 2 = 0$; $4 \operatorname{div} 2 = 2$

$6_{(16)} = 6_{(10)}$; $6 \bmod 2 = 0$; $6 \operatorname{div} 2 = 3$

$A_{(16)} = 10_{(10)}$; $10 \bmod 2 = 0$; $10 \operatorname{div} 2 = 5$

$8_{(16)} = 8_{(10)}$; $8 \bmod 2 = 0$; $8 \operatorname{div} 2 = 4$

$C_{(16)} = 12_{(10)}$; $12 \bmod 2 = 0$; $12 \operatorname{div} 2 = 6$

$2_{(16)} = 2_{(10)}$; $2 \bmod 2 = 0$; $2 \operatorname{div} 2 = 1$

Subject 2: conversions of real numbers choosing the appropriate methods

- Student 2: (Cibotariu Andrei)
  - chooses b (source base) and h (destination base) such that $b, h \neq 10$ and $b < h$
  - chooses the initial real number $x_{(b)}$ having 5 digits at the integer part and 3 digits at the fractional part
  - converts $x_{(b)}$ into base h, with a precision of 3 digits, obtaining $Y_{(h)}$

- Student 1:
  - receives $Y_{(h)}$ from Student 2 and converts $Y_{(h)}$ into base b, with a precision of 3 digits to verify the correctness of the result obtained by Student 2

- Don't use rapid conversions!
- Don't use base 10 as an intermediate base

• Cibotariu Andrei

$b=2,\ h=4$

$x_{(2)} = 10110,011_{(2)}$

We will convert $x_{(2)}$ to $y_{(4)}$ using the Substitution method:

a) First, we convert all digits from $x_{(2)}$ to the destination base 4:

$1_{(2)} = 1_{(4)}$ $\qquad$ $0_{(2)} = 0_{(4)}$

b) After that, base 2 is converted into base 4:

$2 = 2_{(4)}$

c) In the end, we calculate in base 4 the result:

$y_{(4)} = 2^{4}_{(4)} + 2^{2}_{(4)} + 2^{1}_{(4)} + 2^{-2}_{(4)} + 2^{-3}_{(4)} =$

$= 10^{2}_{(4)} + 10_{(4)} + 2_{(4)} + 0,2_{(4)}/2_{(4)} + 0,2_{(4)}/10_{(4)} =$

$= 100_{(4)} + 10_{(4)} + 2_{(4)} + 0,1_{(4)} + 0,02_{(4)} =$

$= 112,12_{(4)}$

So $x_{(2)} = 10110,011_{(2)} = 112,12_{(4)} = y_{(4)}$

Student 1 - Ciana Andreea Elena

$y(4) = 112,12$

The method of successive divisions and multiplications

$112 \mid 2(4)$, 112, $r\,0$, quotient 23

$23 \mid 2(4)$, 22, $r\,1$, quotient 11

$11 \mid 2(4)$, 10, $r\,1$, quotient 2

$2 \mid 2(4)$, 2, $r\,0$, quotient 1

$1 \mid 2(4)$, 0, $r\,1$, quotient 0

$112(4) = 010110(2)$

$0,12(4) \cdot 2(4) = 0,30(4)$

$0,30(4) \cdot 2(4) = 1,20(4)$

$0,20(4) \cdot 2(4) = 1,00(4)$

$0,12(4) = 011(4)$

$\Rightarrow 112,12(4) = 10110,011(2)$

Subject 3: representations

Option 2: addition and ~~subst~~ subtraction of subunitary numbers in complementary code

- Student 1
  - choose three subunitary positive decimal numbers (at least 3 digits at the fractional part): x, y, z, such that x < y < z.
  - represents in direct, invers an complementary codes on 16 bits, x, -x, y, -y, z, -z

- Student 2
  - receives from Student 1: [x] compl, [-x] compl, [y] compl [-y] compl, [z] compl, [-z] compl
  - performs in complementary code the following operations: [x+y] compl, [x-y] compl, [z-x] compl, [-z-x] compl
  - from the complementary codes obtained in the previous step calculates the corresponding decimal values

Student 1 - Ciama Andreea Elena

$x = 0,123$

$y = 0,345$

$z = 0,561$

$r = 16$ bits

$x = 0,123 = 0,0001111101111100_{(2)}$

$0,123 \cdot 2 = 0,246$

$0,246 \cdot 2 = 0,492$

$0,492 \cdot 2 = 0,984$

$0,984 \cdot 2 = 1,968$

$0,968 \cdot 2 = 1,936$

$0,936 \cdot 2 = 1,872$

$0,872 \cdot 2 = 1,744$

$0,744 \cdot 2 = 1,488$

$0,488 \cdot 2 = 0,976$

$0,976 \cdot 2 = 1,952$

$0,952 \cdot 2 = 1,904$

$0,904 \cdot 2 = 1,808$

$0,808 \cdot 2 = 1,616$

$0,616 \cdot 2 = 1,232$

$0,232 \cdot 2 = 0,464$

$0,464 \cdot 2 = 0,928$

positions	S15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
$[0{,}123]_{dir} = [0{,}123]_{inv} = [0{,}123]_{compl}$	0	0	0	0	1	1	1	1	1	0	1	1	1	1	1	0
$[-0{,}123]_{dir}$	1	0	0	0	1	1	1	1	1	0	1	1	1	1	1	0
$[-0{,}123]_{inv}$	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	1
$[-0{,}123]_{compl}$	1	1	1	1	0	0	0	0	0	1	0	0	0	0	1	0

$y = 0{,}345 = 0{,}010110000101000_{(2)}$

$0{,}345 \cdot 2 = 0{,}691$

$0{,}691 \cdot 2 = 1{,}38$

$0{,}38 \cdot 2 = 0{,}76$

$0{,}76 \cdot 2 = 1{,}52$

$0{,}52 \cdot 2 = 1{,}04$

$0{,}04 \cdot 2 = 0{,}08$

$0{,}08 \cdot 2 = 0{,}16$

$0{,}16 \cdot 2 = 0{,}32$

$0{,}32 \cdot 2 = 0{,}64$

$0{,}64 \cdot 2 = 1{,}28$

$0{,}28 \cdot 2 = 0{,}56$

$0{,}56 \cdot 2 = 1{,}12$

$0{,}12 \cdot 2 = 0{,}24$

$0{,}24 \cdot 2 = 0{,}48$

$0{,}48 \cdot 2 = 0{,}96$

positions	S15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
$[0,345]_{dir} = [0,345]_{inv} = [0,345]_{compl}$	0	0	1	0	1	1	0	0	0	0	1	0	1	0	0	0
$[-0,345]_{dir}$	1	0	1	0	1	1	0	0	0	0	1	0	1	0	0	0
$[-0,345]_{inv}$	1	1	0	1	0	0	1	1	1	1	0	1	0	1	1	1
$[-0,345]_{compl}$	1	1	0	1	0	0	1	1	1	1	0	1	1	0	0	0

$z = 0,561 = 0,1000\ 11111\ 00\ 1110_{(2)}$

$0,561 \cdot 2 = 1,122$

$0,122 \cdot 2 = 0,244$

$0,244 \cdot 2 = 0,488$

$0,488 \cdot 2 = 0,976$

$0,976 \cdot 2 = 1,952$

$0,952 \cdot 2 = 1,904$

$0,904 \cdot 2 = 1,808$

$0,808 \cdot 2 = 1,616$

$0,616 \cdot 2 = 1,232$

$0,232 \cdot 2 = 0,464$

$0,464 \cdot 2 = 0,928$

$0,928 \cdot 2 = 1,856$

$0,856 \cdot 2 = 1,712$

$0,712 \cdot 2 = 1,424$

$0,424 \cdot 2 = 0,848$

positions	S15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0
$[0,561]_{dir} = [0,561]_{inv} = [0,561]_{compl}$	0	1	0	0	0	1	1	1	1	1	0	0	1	1	1	0
$[-0,561]_{dir}$	1	1	0	0	0	1	1	1	1	1	0	0	1	1	1	0
$[-0,561]_{inv}$	1	0	1	1	1	0	0	0	0	0	1	1	0	0	0	1
$[-0,561]_{compl}$	1	0	1	1	1	0	0	0	0	0	1	1	0	0	1	0

Student 2 - Cibotariu Andrei

- $[x+y]_{compl.} = [x]_{compl.} \oplus [y]_{compl.}$

		S_1	
$[x]_{compl.}$		0000 1111 1011 1110	$\oplus$
$[y]_{compl.}$		0010 1100 0010 1000	
		0011 1011 1110 0110	OK (result 1)

Since the result is positive, we can compute its value in base 10 right away:

$$\underline{r_1} = 2^{-2} + 2^{-3} + 2^{-4} + 2^{-6} + 2^{-7} + 2^{-8} + 2^{-9} + 2^{-10} + 2^{-13} + 2^{-14} =$$

$$= \underline{0{,}46795654296875}_{(10)}$$

$[x-y]_{compl.} = [x]_{compl.} \oplus [-y]_{compl.}$

		S_1	
$[x]_{compl.}$		0000 1111 1011 1110	$\oplus$
$[y]_{compl.}$		1101 0011 1101 1000	
		1110 0011 1001 0110	OK (result 2)

Since the result is negative, we have to compute its complement in order to determine its value in base 10.

1110 0011 1001 0110
0001 1100 0110 1010

$$\underline{r_2} = -(2^{-3} + 2^{-4} + 2^{-5} + 2^{-9} + 2^{-10} + 2^{-12} + 2^{-14}) =$$

$$= \underline{-0{,}2219848632 8125}_{(10)}$$

$[z-x]_{compl.} = [z]_{compl.} \oplus [-x]_{compl.}$

		S_1	
$[z]_{compl.}$		0100 0111 1100 1110	$\oplus$
$[-x]_{compl.}$		1111 00000 1000010	
	1	0011100000010000	OK (result 3)

Since the result is positive, we can compute its value in base 10 right away:

$$\underline{r_3} = 2^{-2} + 2^{-3} + 2^{-4} + 2^{-11} =$$
$$= \underline{0,43798828125}_{(10)}$$

$$[-z-x]_{compl.} = [-z]_{compl.} \oplus [-x]_{compl.}$$

		s,	
$[-z]_{compl.}$		1011 1000 0011 0010 ⊕	
$[-x]_{compl.}$		1111 0000 0100 0010	
	+	1010 1000 0111 0100	OK (result 4)

Since the result is negative, we have to compute its complement in order to determine its value in base 10.

1010 1000 0111 0100
0101 0111 1000 1100

$$\underline{r_4} = -(2^{-1} + 2^{-3} + 2^{-5} + 2^{-6} + 2^{-7} + 2^{-8} + 2^{-12} + 2^{-13}) =$$
$$= \underline{-0,6839599609375}_{(10)}$$

We could determine the sign of each number by looking at the sign bit (the most semnificative bit): 0 → positive, 1 → negative (in base 2).

To compute the value in base 10 of a negative number represented in base 2, we first calculate its complement (using the method „Two's complement"). After that, we calculate its value and, in the end, negate it.