Exercise 5.1 - Boolean functions

Simplify the following Boolean functions given by their values 0 (use Quine's method)

$f_1(0,1,0) = f_1(0,1,1) = f_1(1,0,1) = 0$

x_1	x_2	x_3	$f_1(x_1,x_2,x_3)$	
0	0	0	1	m_0
0	0	1	1	m_1
0	1	0	0	m_2
0	1	1	0	m_3
1	0	0	1	m_4
1	0	1	0	m_5
1	1	0	1	m_6
1	1	1	1	m_7

$f_1(0,1,0) = f_1(0,1,1) = f_1(1,0,1) = 0 \Rightarrow$

$\Rightarrow f_1(x_1,x_2,x_3) = \bar{x}_1\bar{x}_2\bar{x}_3 \vee \bar{x}_1\bar{x}_2x_3 \vee x_1\bar{x}_2\bar{x}_3 \vee x_1x_2\bar{x}_3 \vee x_1x_2x_3$

$= m_0 \vee m_1 \vee m_4 \vee m_6 \vee m_7 \Rightarrow$

$\Rightarrow S_f = \{(0,0,0),(0,0,1),(1,0,0),(1,1,0),(1,1,1)\}$

(this is the ascending order) of the monoms

Group	x_1	x_2	x_3	
I	0	0	0	m_0 ✓
II	1	0	0	m_4 ✓
	0	0	1	m_1 ✓
III	1	1	0	m_6 ✓
IV	1	1	1	m_7 ✓
V = I + II	-	0	0	$m_0 \vee m_4 = \bar{x}_2\bar{x}_3$
	0	0	-	$m_0 \vee m_1 = \bar{x}_1\bar{x}_2$
VI = II + III	1	-	0	$m_4 \vee m_6 = x_1\bar{x}_3$
VII = III + IV	1	1	-	$m_6 \vee m_7 = x_1x_2$

- for the groups V, VI and VII we use the simple factorization
- m_1 and m_6 cannot be factorized because they are not neighbours
- we cannot apply double factorization because there are no two neighbouring monoms (from groups V & VI or VI & VII) that have a „-" on the same column

$$\Rightarrow \begin{cases} m_0 \vee m_4 = \bar{x}_2\bar{x}_3 = max_1 \\ m_0 \vee m_1 = \bar{x}_1\bar{x}_2 = max_2 \\ m_4 \vee m_6 = x_1\bar{x}_3 = max_3 \\ m_6 \vee m_7 = x_1x_2 = max_4 \end{cases}$$

$\Rightarrow M(f) = \{max_1, max_2, max_3, max_4\}$

the maximal monoms

	max_1	max_2	max_3	max_4
m_0	*	*		
m_1		*		
m_4	*		*	
m_6			*	*
m_7				*

→ m_0 covered by max_1 & max_2

→ m_1 covered by max_2

→ m_4 covered by max_1 & max_3

→ m_6 covered by max_3 & max_4

→ m_7 covered by max_4

because m_1 and m_7 are covered by only one maximal monom each, these two maximal monoms will be the central monoms

$C(f) = \{max_2, max_4\}$

$M(f) \neq C(f) \neq 0 \Rightarrow$ 2nd simplification method

$g = max_2 \vee max_4$

g covers m_0, m_1, m_6 and m_7 $\Rightarrow \begin{cases} f_1^S = g \vee max_1 = \bar{x}_1\bar{x}_2 \vee x_1x_2 \vee \bar{x}_2\bar{x}_3 \\ f_2^S = g \vee max_3 = \bar{x}_1\bar{x}_2 \vee x_1x_2 \vee \bar{x}_1\bar{x}_3 \end{cases}$

m_4 is not covered by g, so our final function(s) consists of g and a maximal monom that covers m_4 (in this case, either max_1 or ~~max_3~~ max_3)