

$U = (\neg t \rightarrow q \vee s) \rightarrow (\neg t \rightarrow q \wedge s)$; prove that U is not valid & build the anti-models using sem. tableaux method; is U inconsistent?

to prove that U is not valid (not a tautology), we have to build the semantic tableau of $\neg U$ first. If $\neg U$ has a closed semantic tableau, then U is a tautology. Otherwise, it is not valid

$$(U \rightarrow V \equiv \neg U \vee V)$$

$$\neg U = U = (\neg t \rightarrow q \vee s) \rightarrow (\neg t \rightarrow q \wedge s) \equiv \neg(\neg t \rightarrow q \vee s) \vee (\neg t \rightarrow q \wedge s) \equiv$$

$$U \equiv \neg(t \vee q \vee s) \vee (t \vee q \wedge s) \Rightarrow \neg U = (t \vee q \vee s) \wedge (t \vee q \wedge s)$$

$$\neg U = (t \vee q \vee s) \wedge (t \vee q \wedge s) \quad (1)$$

| α -rule for (1)

$$(t \vee q \vee s) \quad (2)$$

|

$$(t \vee q \wedge s) \quad (3)$$

| α -rule for (3)

$$t \vee q \quad (4)$$

|

$$s \quad (5)$$

| β -rule for (4)

| β -rule for (3)

$$t \quad (6)$$

$$(7) \quad q$$

| β -rule for (3)

$$(8) (t \vee q)$$

$$(9) s$$

$$(10) t \vee q$$

$$(11) s$$

| β -rule

| β -rule

$$t$$

$$q$$

$$\odot$$

$$\odot$$

$$t$$

$$q$$

$$\odot$$

$$\odot$$

α -rule

$$A \wedge B$$

$$|$$

$$A$$

$$|$$

$$B$$

β -rule

$$A \vee B$$

$$|$$

$$A$$

$$|$$

$$B$$

\odot - symbolises an open branch (does not contain a formula and its negation)

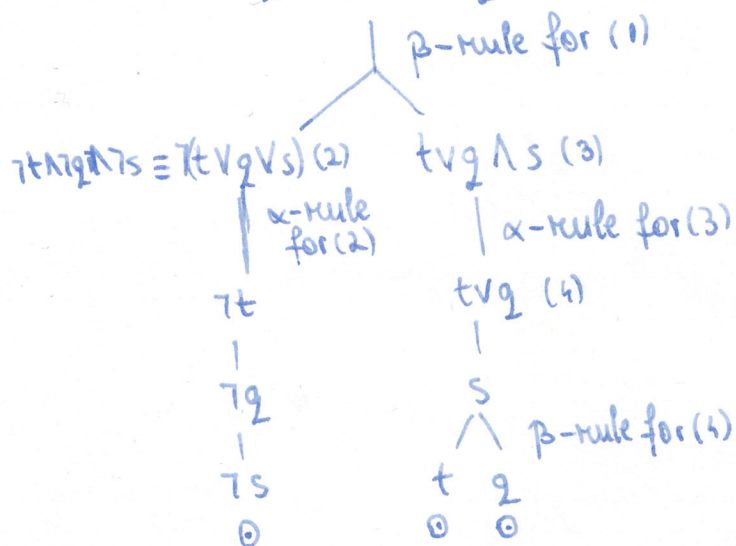
Because the semantic tableau of $\neg U$ is not closed (closed = all the branches are closed), the theorem of soundness & completeness does not apply, therefore U is not a tautology, which means it's not valid

th. of soundness & completeness: U is a tautology $\Rightarrow \neg U$ has a closed semantic tableau

Astalus, Aduiom, 9/11

Now, to find anti-models & to prove inconsistency, we build the sem. tableau of U

$$U = \neg(t \vee q \vee s) \vee (t \vee q \wedge s) \quad (1)$$



Again, we have no closed branches, so the tableau is not closed $\Rightarrow U$ is not inconsistent

The semantic tableau is complete & open $\Rightarrow U$ is consistent

$$DNF(U) = (\neg t \wedge \neg q \wedge \neg s) \vee (t \wedge s) \vee (s \wedge q)$$

the branch provided by the cube $(\neg t \wedge \neg q \wedge \neg s)$ provides the anti-models:

$$i_1, i_2, i_3, i_4, i_5, i_6 : \{t, q, s\} \rightarrow \{T, F\}$$

$$i_1(q) = F, i_1(t) = T, i_1(s) = T$$

$$i_2(q) = F, i_2(t) = F, i_2(s) = T$$

$$i_3(q) = F, i_3(t) = T, i_3(s) = F$$

$$i_4(q) = T, i_4(t) = F, i_4(s) = T$$

$$i_5(q) = T, i_5(t) = F, i_5(s) = F$$

$$i_6(q) = T, i_6(t) = T, i_6(s) = F$$

the branch $t \wedge s$ provides the anti-models:

$$s \wedge q : i_{11}, i_{12}, i_{13}, i_{14}$$

$$i_{11}(t) = T, i_{11}(s) = T, i_{11}(q) = F$$

$$i_{12}(t) = F, i_{12}(s) = T, i_{12}(q) = F$$

$$i_{13}(t) = T, i_{13}(s) = F, i_{13}(q) = T$$

$$i_{14}(t) = F, i_{14}(s) = F, i_{14}(q) = T$$

$$i_{15}(t) = T, i_{15}(s) = F, i_{15}(q) = F$$

$$i_{16}(t) = F, i_{16}(s) = F, i_{16}(q) = F$$

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Hypotheses

Astolus Adrium, 911
~~Astolus~~

- H1: Every child loves candies
- H2: Anyone who loves candies is not a nutrition fanatic
- H3: Anyone who eats pumpkin is a nutrition fanatic
- H4: Anyone who buys a pumpkin carves it or eats it
- H5: John is a child & buys a pumpkin

Conclusion: John carves the pumpkin

John - constant

Variables: x

Predicate symbols (all are unary)

$ic(x): D \rightarrow \{T, F\}$, $ic(x) = T$ if x is a child

$lc(x): D \rightarrow \{T, F\}$, $lc(x) = T$ if x loves candies

$mf(x): D \rightarrow \{T, F\}$, $mf(x) = T$ if x is a nutrition fanatic

$ep(x): D \rightarrow \{T, F\}$, $ep(x) = T$ if x eats pumpkin

$bp(x): D \rightarrow \{T, F\}$, $bp(x) = T$ if x buys a pumpkin

$cp(x): D \rightarrow \{T, F\}$, $cp(x) = T$ if x carves a pumpkin

D is the domain: the universe of people

H1: $(\forall x) (ic(x) \rightarrow lc(x))$

H2: $(\forall x) (lc(x) \rightarrow \neg mf(x))$

H3: $(\forall x) (ep(x) \rightarrow mf(x))$

H4: $(\forall x) (bp(x) \rightarrow (cp(x) \vee ep(x)))$

H5: $ic(\text{John}) \wedge bp(\text{John})$

C: $cp(\text{John})$

$H1 \vdash_{\text{univ. inst., John}} ic(\text{John}) \rightarrow lc(\text{John}) : \#6$

$H2 \vdash_{\text{univ. inst., John}} lc(\text{John}) \rightarrow \neg mp(\text{John}) : \#7$

$H4 \vdash_{\text{univ. inst., John}} bp(\text{John}) \rightarrow (cp(\text{John}) \vee ep(\text{John})) : \#8$

$H3 \vdash_{\text{mt}} \neg mf(x) \rightarrow \neg ep(x) : \#9$

$\#9 \vdash_{\text{univ. inst., John}} \neg mf(\text{John}) \rightarrow \neg ep(\text{John}) : \#10$

$H1, H2 \vdash_{\text{sylogism}} ic(x) \rightarrow \neg mf(x) : \#11$

$f_{11}, f_9 \vdash \text{syll} \text{ic}(\text{John}) \rightarrow \neg \text{ep}(\text{John}) : f_{13}$

Astolus Adusom, 311
~~Astolus~~

$f_8 : \text{bp}(\text{John}) \rightarrow (\text{cp}(\text{John}) \vee \text{ep}(\text{John}))$

$f_{13} : \text{ic}(\text{John}) \rightarrow \neg \text{ep}(\text{John})$

$H_5 : \text{ic}(\text{John}) \wedge \text{bp}(\text{John}) = f_{14} \wedge f_{15}$

$f_{14}, f_{13} \vdash \text{mp} \neg \text{ep}(\text{John}) : f_{16}$

$f_{15}, f_8 \vdash \text{mp} \text{cp}(\text{John}) \vee \text{ep}(\text{John}) : f_{17}$

$f_{16}, f_{17} \Rightarrow \text{cp}(\text{John}) = C$

$(H_1, H_2, H_3, H_4, H_5, f_1, \dots, f_{17})$ is the proof of $C \Rightarrow \text{John carries the pumpkin}$

inference rules used:

$(\forall x)U(x) \vdash \text{univ-inst } U(t)$, t is a variable or constant

$U \rightarrow V \vdash \text{mt} \neg V \rightarrow \neg U$ - modus tollens

$U \rightarrow V, V \rightarrow Z \vdash \text{syll} U \rightarrow Z$

$U, U \rightarrow V \vdash \text{mp} V$ - modus ponens

Assalms

$$f(x,y,z) = x(y \oplus z) \vee y(x \downarrow z) \vee \bar{x}(\bar{y} \oplus z)$$

$$f(x,y,z) = x(\bar{y} \vee z)(y \vee \bar{z}) \vee y \bar{x} \bar{z} \vee \bar{x}(y \vee \bar{y} \bar{z})$$

$$\equiv (x\bar{y} \vee xz)(y \vee \bar{z}) \vee y \bar{x} \bar{z} \vee \bar{x} y \bar{z} \vee \bar{x} \bar{y} \bar{z}$$

$$\equiv x \vee x\bar{y} \bar{z} \vee x y \bar{z} \vee \bar{x} y \bar{z} \vee \bar{x} y \bar{z} \vee \bar{x} \bar{y} \bar{z} \equiv$$

$$\equiv x\bar{y} \bar{z} \vee x y \bar{z} \vee x \bar{y} \bar{z} \vee x y \bar{z} \vee \bar{x} y \bar{z} \vee \bar{x} y \bar{z} \vee \bar{x} \bar{y} \bar{z}$$

$$f(x,y,z) = m_5 \vee m_6 \vee m_4 \vee m_7 \vee m_2 \vee m_3 \vee m_0 - \text{DCF}$$

$$S_f = \{(0,0,0), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$$

support
set of f

$$\text{I} \quad \begin{array}{ccc|c} 0 & 0 & 0 & m_0 \checkmark \\ 0 & 1 & 0 & m_2 \checkmark \end{array}$$

$$\text{II} \quad \begin{array}{ccc|c} 1 & 0 & 0 & m_4 \checkmark \\ 0 & 1 & 1 & m_3 \checkmark \end{array}$$

$$\text{III} \quad \begin{array}{ccc|c} 1 & 0 & 1 & m_5 \checkmark \\ 1 & 1 & 0 & m_6 \checkmark \end{array}$$

$$\text{IV} \quad \begin{array}{ccc|c} 1 & 1 & 1 & m_7 \checkmark \end{array}$$

$$\text{V} = \text{I} + \text{II} \quad \begin{array}{ccc|c} 0 & - & 0 & m_0 \vee m_2 = m_{\text{max}_1} \\ - & 0 & 0 & m_0 \vee m_3 = m_{\text{max}_2} \end{array}$$

$$\text{VI} = \text{II} + \text{III} \quad \begin{array}{ccc|c} 0 & 1 & - & m_2 \vee m_3 = m_{\text{max}_3} \\ 1 & 0 & - & m_4 \vee m_5 = m_{\text{max}_4} \\ 1 & - & 0 & m_4 \vee m_6 = m_{\text{max}_5} \end{array}$$

$$\text{VII} = \text{III} + \text{IV} \quad \begin{array}{ccc|c} - & 1 & 1 & m_3 \vee m_7 = m_{\text{max}_6} \\ 1 & - & 1 & m_5 \vee m_7 = m_{\text{max}_7} \\ 1 & 1 & - & m_6 \vee m_7 = m_{\text{max}_8} \end{array}$$

$$M(f) = \{m_{\text{max}_1}, \dots, m_{\text{max}_8}\}$$

set of
maximal
monoms

$$m_0 \text{ is covered by } m_{\text{max}_1} \text{ or } m_{\text{max}_2} : p_1 \vee p_2 \equiv T$$

$$m_2 - \text{by } m_{\text{max}_1} \text{ or } m_{\text{max}_3} : p_1 \vee p_3 \equiv T$$

$$m_3 - m_{\text{max}_3} \text{ or } m_{\text{max}_6} : p_3 \vee p_6 \equiv T$$

$$m_4 - m_{\text{max}_2} \text{ or } m_{\text{max}_4} \text{ or } m_{\text{max}_5} : p_2 \vee p_4 \vee p_5 \equiv T$$

$$m_5 - m_{\text{max}_4} \text{ or } m_{\text{max}_7} : p_4 \vee p_7 \equiv T$$

$$m_6 - m_{\text{max}_5} \text{ or } m_{\text{max}_8} : p_5 \vee p_8 \equiv T$$

$$m_7 - m_{\text{max}_6} \text{ or } m_{\text{max}_8} \text{ or } m_{\text{max}_7} : p_6 \vee p_7 \vee p_8 \equiv T$$

$$(p_1 \vee p_2) \wedge (p_1 \vee p_3) \wedge (p_3 \vee p_6) \wedge (p_2 \vee p_4 \vee p_5) \wedge (p_4 \vee p_7) \wedge (p_5 \vee p_8) \wedge (p_6 \vee p_7 \vee p_8) \equiv T$$

simplify using Moasil

circuit for the function

