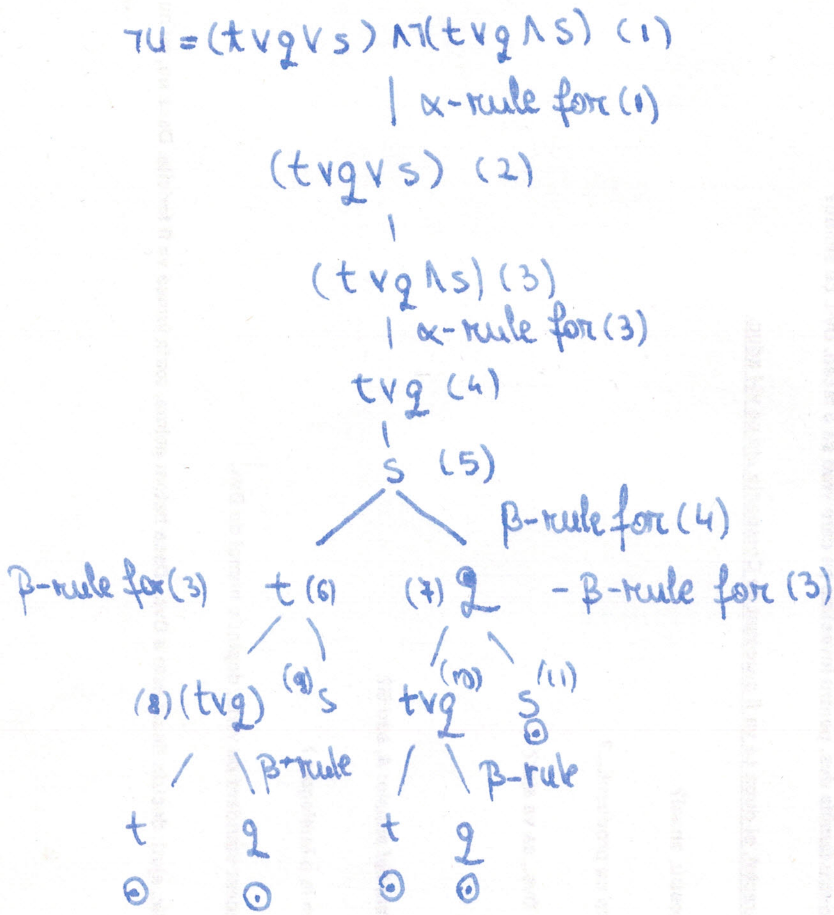
$u = (\neg t \to q \vee s) \to (\neg t \to q \wedge s)$; prove that u is not valid & build the anti-models using sem. tableaux method; is u inconsistent?

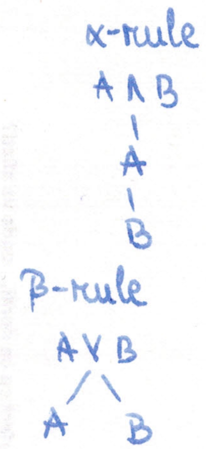
to prove that u is not valid (not a tautology), we have to build the semantic tableau of $\neg u$ first. If $\neg u$ has a closed semantic tableau, then u is a tautology. Otherwise, it is not valid

$(u \to v \equiv \neg u \vee v)$

$\cancel{\neg u \equiv}\ u = (\neg t \to q \vee s) \to (\neg t \to q \wedge s) \equiv \neg(\neg t \to q \vee s) \vee (\neg t \to q \wedge s) \equiv$

$u \equiv \neg(t \vee q \vee s) \vee (t \vee q \wedge s) \Longrightarrow \neg u = (t \vee q \vee s) \wedge (t \vee q \wedge s)$





⊙ - symbolises an open branch (does not contain a formula and its negation)

because the semantic tableau of $\neg u$ is not closed (closed = all the branches are closed), the theorem of soundness & completeness does not apply, therefore u is not a tautology, which means it's not valid

th. of soundness & completeness: u is a tautology $\Leftrightarrow$ $\neg u$ has a closed semantic tableau

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Now, to find anti-models & to prove inconsistency, we build the sem. tableau of U

$U = \neg(t \vee q \vee s) \vee (t \vee q \wedge s)$ (1)

β-rule for (1)

$\neg t \wedge \neg q \wedge \neg s \equiv \neg(t \vee q \vee s)$ (2) $\qquad$ $t \vee q \wedge s$ (3)

α-rule for (2) $\qquad$ α-rule for (3)

$\neg t$ $\qquad$ $t \vee q$ (4)

$\neg q$ $\qquad$ s

β-rule for (4)

$\neg s$ $\qquad$ t $\quad$ q

⊙ $\qquad$ ⊙ $\quad$ ⊙

Again, we have no closed branches, so the tableau is not closed $\Rightarrow$ U is not inconsistent

The semantic tableau is complete & open $\Rightarrow$ U is consistent

$DNF(U) = (\neg t \wedge \neg q \wedge \neg s) \vee (t \wedge s) \vee (s \wedge q)$

the branch provided by the cube $(\neg t \wedge \neg q \wedge \neg s)$ provides the anti-models:

$i_1, i_2, i_3, i_4, i_5, i_6 : \{t, q, s\} \to \{T, F\}$

$i_1(q) = F$, $i_1(t) = T$, $i_1(s) = T$

$i_2(q) = F$, $i_2(t) = F$, $i_2(s) = T$

$i_3(q) = F$, $i_3(t) = T$, $i_3(s) = F$

$i_4(q) = T$, $i_4(t) = F$, $i_4(s) = T$

$i_5(q) = T$, $i_5(t) = F$, $i_5(s) = F$

$i_6(q) = T$, $i_6(t) = T$, $i_6(s) = F$

the branch $t \wedge s$ provides the anti-models:

$i_7, i_8, i_9, i_{10} : \{t, q, s\} \to \{T, F\}$

$i_7(t) = T$, $i_7(s) = F$, $i_7(q) = T$

$i_8(t) = T$, $i_8(s) = F$, $i_8(q) = F$

$i_9(t) = F$, $i_9(s) = T$, $i_9(q) = T$

$i_{10}(t) = F$, $i_{10}(s) = T$, $i_{10}(q) = T$

$s \wedge q$: $i_{11}, i_{12}, i_{13}, i_{14}$

$i_{11}(t) = T$, $i_{11}(s) = T$, $i_{11}(q) = F$

$i_{12}(t) = F$, $i_{12}(s) = T$, $i_{12}(q) = F$

$i_{13}(t) = T$, $i_{13}(s) = F$, $i_{13}(q) = T$

$i_{14}(t) = F$, $i_{14}(s) = F$, $i_{14}(q) = T$

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Hypotheses

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H1: Every child loves candies
H2: Anyone who loves candies is not a nutrition fanatic
H3: Anyone who eats pumpkin is a nutrition fanatic
H4: Anyone who buys a pumpkin carves it or eats it
H5: John is a child & buys a pumpkin

Conclusion: John carves the pumpkin

John - constant
Variables: x
Predicate symbols (all are unary)

$ic(x): D \to \{T, F\}$, $ic(x) = T$ if x is a child
$lc(x): D \to \{T, F\}$, $lc(x) = T$ if x loves candies
$nf(x): D \to \{T, F\}$, $nf(x) = T$ if x is a nutrition fanatic
$ep(x): D \to \{T, F\}$, $ep(x) = T$ if x eats pumpkin
$bp(x): D \to \{T, F\}$, $bp(x) = T$ if x buys a pumpkin
$cp(x): D \to \{T, F\}$, $cp(x) = T$ if x carves a pumpkin

D is the domain: the universe of people

H1: $(\forall x)(ic(x) \to lc(x))$
H2: $(\forall x)(lc(x) \to \neg nf(x))$
H3: $(\forall x)(ep(x) \to nf(x))$
H4: $(\forall x)(bp(x) \to (cp(x) \vee ep(x)))$
H5: $ic(John) \wedge bp(John)$
C: $cp(John)$

$H_1 \vdash_{\text{univ inst, John}} ic(John) \to lc(John) : f_6$

$H_2 \vdash_{\text{univ\_inst, John}} lc(John) \to \neg np(John) : f_7$

$H_4 \vdash_{\text{univ\_inst, John}} bp(John) \to (cp(John) \vee ep(John)) : \cancel{f(8)}\ f_8$

$H_3 \vdash_{mt} \neg nf(x) \to \neg ep(x) : \cancel{f(8)}\ f_9$

$f_9 \vdash_{\text{univ\_inst, John}} \neg nf(John) \to \neg(ep(John)) : f_{10}$

$H_1, H_2 \vdash_{\text{syllogism}} ic(x) \to \neg nf(x) : f_{11}$

$f_{11}, f_9 \vdash_{syllogism} ic(John) \to \neg ep(John) : f_{13}$

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$f_8: \quad bp(John) \to (cp(John) \vee ep(John))$

$f_{13}: \quad ic(John) \to \neg ep(John)$

$H5: \quad ic(John) \wedge bp(John) = f_{14} \wedge f_{15}$

$f_{14}, f_{13} \vdash_{mp} \neg ep(John) : f_{16}$

$f_{15}, f_8 \vdash_{mp} cp(John) \vee ep(John) : f_{17}$

$f_{16}, f_{17} \Rightarrow cp(John) = C$

$(H_1, H_2, H_3, H_4, H_5, f_1, ---, f_{17})$ is the proof of $C \Rightarrow$ John carves the pumpkin

inference rules used:

$(\forall x) U(x) \vdash_{univ\_inst} U(t)$, t is a variable or constant

$U \to V \vdash_{mt} \neg V \to \neg U$ - modus tollens

$U \to V, V \to Z \vdash_{syllogism} U \to Z$

$U, U \to V \vdash_{mp} V$ - modus ponens

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$f(x,y,z) = x(y \otimes z) \vee y(x \downarrow z) \vee \bar{x}(\bar{y} \oplus z)$

$f(x,y,z) = x(\bar{y} \vee z)(y \vee \bar{z}) \vee y\bar{x}\bar{z} \vee \bar{x}(yz \vee \bar{y}\bar{z})$

$\equiv (x\bar{y} \vee xz)(y \vee \bar{z}) \vee y\bar{x}\bar{z} \vee \bar{x}yz \vee \bar{x}\bar{y}\bar{z}$

$\equiv x \vee x\bar{y}\bar{z} \vee xyz \vee \bar{x}y\bar{z} \vee \bar{x}yz \vee \bar{x}\bar{y}\bar{z} \equiv$

$\equiv x\bar{y}z \vee xy\bar{z} \vee x\bar{y}\bar{z} \vee xyz \vee \bar{x}y\bar{z} \vee \bar{x}yz \vee \bar{x}\bar{y}\bar{z}$

$f(x,y,z) = m_5 \vee m_6 \vee m_4 \vee m_7 \vee m_2 \vee m_3 \vee m_0$ - DCF

$S_f = \{\underset{m_0}{(0,0,0)}, \underset{m_2}{(0,1,0)}, \underset{m_3}{(0,1,1)}, \underset{m_4}{(1,0,0)}, \underset{m_5}{(1,0,1)}, \underset{m_6}{(1,1,0)}, \underset{m_7}{(1,1,1)}\}$

support set of f

$M(f) = \{max_1, \dots, max_8\}$

set of maximal monoms

I	0	0	0	m_0 ✓	
	0	1	0	m_2 ✓	
II	1	0	0	m_4 ✓	
	0	1	1	m_3 ✓	
III	1	0	1	m_5 ✓	
	1	1	0	m_6 ✓	
IV	1	1	1	m_7 ✓	
V = I + II	0	-	0	$m_0 \vee m_2 = max_1$	
	-	0	0	$m_0 \vee m_4 = max_2$	
VI = II + III	0	1	-	$m_2 \vee m_3 = max_3$	
	1	0	-	$m_4 \vee m_5 = max_4$	
	1	-	0	$m_4 \vee m_6 = max_5$	
VII = III + IV	-	1	1	$m_3 \vee m_7 = max_6$	
	1	-	1	$m_5 \vee m_7 = max_7$	
	1	1	-	$m_6 \vee m_7 = max_8$	

m_0 is covered by max_1 or max_2 : $p_1 \vee p_2 \equiv T$

m_2 - by max_1 or max_3 : $p_1 \vee p_3 \equiv T$

m_3 - max_3 or max_6 : $p_3 \vee p_6 \equiv T$

m_4 - max_2 or max_4 or max_5 : $p_2 \vee p_4 \vee p_5 \equiv T$

m_5 - max_4 or max_7 : $p_4 \vee p_7 \equiv T$

m_6 - max_5 or max_8 : $p_5 \vee p_8 \equiv T$

m_7 - max_6 or max_8 or max_7 : $p_6 \vee p_7 \vee p_8 \equiv T$

$(p_1 \vee p_2) \wedge (p_1 \vee p_3) \wedge (p_3 \vee p_6) \wedge (p_2 \vee p_4 \vee p_5) \wedge (p_4 \vee p_7) \wedge (p_5 \vee p_8) \wedge (p_6 \vee p_7 \vee p_8) \equiv T$

simplify using Moisil

+

circuit for the function

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