Astălus Adrian Claudiu
~~Astalu~~

1. $u \wedge s$ derivable from $\{u \vee r, \neg t, t \vee q \wedge s, r \to \neg q\}$

~~from the theorem of deduction~~ :

$u \vee r, \neg t, t \vee q \wedge s, r \to \neg q \vdash u \wedge s$ ~~if and only if~~

we prove this using the definition of deduction

$$\boxed{\begin{array}{l} U \to V \equiv \neg U \vee V \\ U, U \to V \vdash_{mp} V \end{array}}$$

$f_1: u \vee r \equiv \neg u \to r$

$f_2: \neg t$

$f_3: t \vee q \wedge s$

$f_4: r \to \neg q \equiv \neg r \vee \neg q$

$f_5: (\neg u \to r) \to (u \to \neg r)$ - axiom A3 (modus tollens)

$f_6: (r \to \neg q) \to (\neg r \to q)$ - axiom A3 (m.t.)

$f_1, f_5 \vdash_{mp} u \to \neg r$

$f_6: u \to \neg r \equiv \neg u \vee \neg r$

$f_4, f_6 \vdash_{mp} \neg r \to q$

$f_7: \neg r \to q \equiv r \vee q$

~~$f_8: f_1 \wedge f_4 =$~~

there are no other formulas we can apply, therefore $u \wedge s$ is not derivable from the given set of propositional formulas

$(u \vee r, \neg t, t \vee q \wedge s, r \to \neg q \nvdash u \wedge s)$

Propositional logic is decidable, and it can be proved by constructing the truth table for any given propositional formula.

Astalus Adrian Claudiu
~~Astalus~~

②

H1: Anyone who makes an 'A' at logic ex. studies or is brill. or lucky
H2: No CS student is lucky
H3: Mary is a CS stud. and made an 'A' at logic ex.
H4: Mary likes to party and does not study
C: Mary is brilliant

Syntactic proof methods: def. of deduction, th. of deduction & its reverse, resolution method

Mary - constant
variables: x
Predicate symbols: $ALE(x): D \to \{T, F\}$, $ALE(x) = T$ if a student makes an 'A' at logic exam
$ST(x): D \to \{T, F\}$, $ST(x) = T$ if a student studies
$BR(x): D \to \{T, F\}$, $BR(x) = T$ if a student is brilliant
$IL(x): D \to \{T, F\}$, $IL(x) = T$ if a stud. is lucky
$LP(x): D \to \{T, F\}$, $LP(x) = T$ if a stud. likes to party
$CS(x): D \to \{T, F\}$, $CS(x) = T$ if a stud. is a CS stud.

D - the domain: the universe of students

H1: $(\forall x)\ ALE(x) \to (ST(x) \vee BR(x) \vee IL(x))$
H2: $(\forall x)\ CS(x) \to \neg IL(x)$
H3: $CS(Mary) \wedge ALE(Mary) - f_5 \wedge f_6$
H4: $LP(Mary) \wedge \neg ST(Mary) - f_7 \wedge f_8$
C: $BR(Mary)$

$H2 \vdash_{\substack{univ.inst,\\ Mary}} CS(Mary) \to \neg IL(MARY) : f_9$

$H1 \vdash_{\substack{u.i.\\ Mary}} ALE(Mary) \to (ST(Mary) \vee BR(Mary) \vee IL(Mary)) : f_{10}$

$f_6, f_{10} \vdash_{mp} ST(Mary) \vee BR(Mary) \vee IL(Mary) : f_{11}$

$f_5, f_9 \vdash_{mp} \neg IL(Mary) : f_{12}$

$f_{11} \wedge f_8 \vdash_{conj.} BR(Mary) \vee IL(Mary) : f_{13}$

$f_{13} \wedge f_{12} \vdash_{conj.} BR(Mary) = C$

Astăluș Adrian Claudiu
~~Astalo~~

(H1, H2, H3, H4, f5, f6, f7, f8, f9, f10, f11, f12, f13) is the proof of C. Therefore, Mary is brilliant

inference rules used:

universal inst.: $(\forall x) U(x) \vdash U(t)$, t is a term

modus ponens: $U, U \to V \vdash V$

Astăluş Adrian Claudiu

③ Boolean function S_7

digit	x_1	x_2	x_3	x_4	S_7
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	1
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	0
8	1	0	0	0	1
9	1	0	0	1	1
-	1	0	1	0	1
-	1	0	1	1	1
-	1	1	0	0	1
-	1	1	0	1	1
-	1	1	1	0	1
-	1	1	1	1	1

$$S_7 = (\bar{x}_1\bar{x}_2x_3\bar{x}_4) \vee (\bar{x}_1\bar{x}_2x_3x_4) \vee (\bar{x}_1x_2\bar{x}_3\bar{x}_4) \vee$$
$$\vee (\bar{x}_1x_2\bar{x}_3x_4) \vee (\bar{x}_1x_2x_3\bar{x}_4) \vee (x_1\bar{x}_2\bar{x}_3\bar{x}_4) \vee$$
$$\vee (x_1\bar{x}_2\bar{x}_3x_4) \vee (x_1\bar{x}_2x_3\bar{x}_4) \vee (x_1\bar{x}_2x_3x_4) \vee$$
$$\vee (x_1x_2\bar{x}_3\bar{x}_4) \vee (x_1x_2\bar{x}_3x_4) \vee (x_1x_2x_3\bar{x}_4) \vee$$
$$\vee (x_1x_2x_3x_4) =$$
$$= m_2 \vee m_3 \vee m_4 \vee m_5 \vee m_6 \vee m_8 \vee m_9 \vee m_{10} \vee$$
$$\vee m_{11} \vee m_{12} \vee m_{13} \vee m_{14} \vee m_{15} \text{ - DCF}$$

$x_1x_2 \backslash x_3x_4$	00	01	11	10
00			m_3	m_2
01	m_4	m_5		m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

Karnaugh diagram for our DCF

$max_1 = m_8 \vee m_9 \vee m_{10} \vee m_{11} \vee m_{12} \vee m_{13} \vee m_{14} \vee m_{15} = x_1$ - triple factorization

$max_2 = m_4 \vee m_5 \vee m_{12} \vee m_{13} = x_2\bar{x}_3$ - double factorization

$max_3 = m_2 \vee m_6 \vee m_{14} \vee m_{10} = x_3\bar{x}_4$ - double fact.

$max_4 = m_3 \vee m_2 = \bar{x}_1\bar{x}_2x_3$ - simple fact.

$max_5 = m_3 \vee m_{11} = \bar{x}_2x_3x_4$ - simple fact.

$max_6 = m_4 \vee m_6 = \bar{x}_1x_2\bar{x}_4$ - simple fact.

$M_f(S_7) = \{max_1, max_2, max_3, max_4, max_5, max_6\}$ - maximal monoms

$C_f(S_7) = \{max_1, max_2\}$ are central monoms, because $m_5, m_{15}, \dots,$ are circled only once

$C_f(S_7) \neq \emptyset$, $C_f(S_7) \not\equiv S_f(S_7) \Rightarrow g = max_1 \vee max_2$

$f_1^{S}(S_7) = g \vee max_3 \vee max_4$

$f_2^{S}(S_7) = g \vee max_3 \vee max_5$

$f_3^{S}(S_7) = g \vee max_6 \vee max_4$

Astaluş Adrian Claudiu
Astalus

$f_1^S(S_7) = g \vee max_3 \vee max_4 =$

$= x_1 \vee x_2\bar{x}_3 \vee x_3\bar{x}_4 \vee \bar{x}_1\bar{x}_2 x_3$