

## Dynamical Systems 2020/21 Lab Test

1. Find a periodic solution of the differential equation

$$u'' + 5u' - 7u = 5 \cos x - 7 \sin x,$$

then plot its graph on the interval  $[-11, 5]$ , and finally compute, for it and for its first order derivative, approximate values in  $\pi/2$ . Note that the unknown is the function denoted by  $u(x)$ .

2. a) Plot the planar curve of parametric equations  $x = \cos(2t) + 3 \sin(2t)$ ,  $y = \sin(2t)$  for  $t \in [0, 4]$ .  
b)\* Can  $\varphi(t) = (\cos(2t) + 3 \sin(2t), \sin(2t))$ ,  $t \in \mathbb{R}$ , be a solution of a linear planar system  $\dot{X} = AX$ ?
3. Introduce the matrix  $A$  corresponding to the linear system  $x' = -x - 7y$ ,  $y' = 7x - y$ . Compute its determinant and eigenvalues. Compute  $e^{tA}$ . Specify the type and stability of the linear system.
4. We consider the nonlinear system  $x' = -49x - 7y + x^2$ ,  $y' = 7x + y$ . Is  $(0, 0)$  the unique equilibrium point? Is  $(0, 0)$  a hyperbolic equilibrium point?
5. We consider the map  $f : \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^2 - 0.5$ . Find its fixed points, preferably as decimals (*hint*: all of them are in the interval  $(-2, 2)$ ). Describe your opinion on the behavior of the sequences of iterations starting with 0, 1.2 and, respectively,  $-1.1$ .