

Exam on Dynamical Systems, June 7, 2021

1. (1p) Find the general solution of the differential system

$$x' = -3x + y, \quad y' = 2x - 4y,$$

using the reduction (to a second order differential equation) method.

2. (0.5p) Study the solvability of the following problem

$$x_{k+2} = x_{k+1}^2 \sin(x_k) + 7, \quad k \geq 0, \quad x_0 = 0, \quad x_1 = 0, \quad x_2 = 8.$$

Here, as usual, the unknown is the sequence $k \in \mathbb{N} \mapsto x_k \in \mathbb{R}$.

3. We consider the nonlinear planar system

$$\dot{x} = y + 1, \quad \dot{y} = -3x - xy.$$

- a) (0.5p) Find its equilibrium point and the linearized system around it.
- b) (2p) Find a first integral.
- c) (0.5p) If the linearized system around the equilibrium point has a global first integral, find it.
- d) (0.5p) Represent the phase portrait of the linearized system around the equilibrium point.

4. (1p) Study the solvability of the boundary value problem

$$x'' + 4x = 3, \quad x(0) = x(4\pi) = 0.$$

Here, as usual, the unknown is the function $t \in \mathbb{R} \mapsto x(t) \in \mathbb{R}$.