Databases

Lecture 7

Relational Algebra

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- query languages in the relational model
  - relational algebra and calculus - formal query languages with a significant influence on SQL
    - relational algebra
      - queries are specified in an operational manner
    - relational calculus
      - queries describe the desired answer, without specifying how it will be computed (declarative)
  - not expected to be Turing complete
  - not intended for complex calculations
  - provide efficient access to large datasets
  - allow optimizations

- relational algebra
  - used by DBMSs to represent query execution plans
  - a relational algebra query:
    - is built using a collection of operators
    - describes a step-by-step procedure for computing the result set
    - is evaluated on the input relations' instances
    - produces an instance of the output relation
  - every operation returns a relation, so operators can be composed; the algebra is closed
  - the result of an algebra expression is a relation, and a relation is a set of tuples
- relational algebra on bags (multisets) - duplicates are not eliminated

Conditions

- conditions that can be used in several algebraic operators
- similar to the SELECT filter conditions

1. *attribute_name relational_operator value*

- *value* - attribute name, expression

2. *attribute_name IS [NOT] IN single_column_relation*

- a relation with one column can be considered a set
- the condition tests whether a value belongs to a set

3. *relation {IS [NOT] IN | = | <>} relation*

- the relations in the condition must be union-compatible

Conditions

4. *(condition)*
NOT condition
$condition_1$ *AND* $condition_2$
$condition_1$ *OR* $condition_2$,

where *condition*, $condition_1$, $condition_2$ are conditions of type 1-4.

Operators in the Algebra

- equivalent SELECT statements can be specified for the relational algebra expressions
- *selection*
  - notation: $\sigma_C(R)$
  - resulting relation:
    - schema: R's schema
    - tuples: records in R that satisfy condition C
  - equivalent SELECT statement

```
SELECT *
FROM R
WHERE C
```

- *projection*
  - notation: $\pi_\alpha(R)$
  - resulting relation:
    - schema: attributes in α
    - tuples: every record in R is projected on α
  - α can be extended to a set of expressions, specifying the columns of the relation being computed
  - equivalent SELECT statement

```
SELECT DISTINCT α
FROM R

SELECT α
FROM R              -- algebra on bags
```

- *cross-product*
  - notation: $R_1 \times R_2$
  - resulting relation:
    - schema: the attributes of R_1 followed by the attributes of R_2
    - tuples: every tuple r_1 in R_1 is concatenated with every tuple r_2 in R_2
  - equivalent SELECT statement

```
SELECT *
FROM R1 CROSS JOIN R2
```

- *union, set-difference, intersection*
  - notation: $R_1 \cup R_2$, $R_1 - R_2$, $R_1 \cap R_2$
  - R_1 and R_2 must be union-compatible:
    - same number of columns
    - corresponding columns, taken in order from left to right, have the same domains
  - equivalent SELECT statements

```
SELECT *          SELECT *         SELECT *
FROM R1           FROM R1          FROM R1
UNION             EXCEPT           INTERSECT
SELECT *          SELECT *         SELECT *
FROM R2           FROM R2          FROM R2


-- algebra on bags: SELECT statements that don't eliminate
duplicates (e.g., UNION ALL)
```

- join operators
  - *condition join* (or *theta join*)
    - notation: $R_1 \otimes_{\Theta} R_2$
    - result: the records in the cross-product of R_1 and R_2 that satisfy a certain condition
  - definition $\Rightarrow R_1 \otimes_{\Theta} R_2 = \sigma_{\Theta}(R_1 \times R_2)$
  - equivalent SELECT statement

```
SELECT *
FROM R1 INNER JOIN R2 ON Θ
```

- join operators
  - *natural join*
    - notation: $R_1 * R_2$
    - resulting relation:
      - schema: the union of the attributes of the two relations (attributes with the same name in R_1 and R_2 appear once in the result)
      - tuples: obtained from tuples <r_1, r_2>, where r_1 in R_1, r_2 in R_2, and r_1 and r_2 agree on the common attributes of R_1 and R_2
    - let $R_1[\alpha]$, $R_2[\beta]$, $\alpha \cap \beta = \{A_1, A_2, \ldots, A_m\}$; then:

$$R_1 * R_2 = \pi_{\alpha \cup \beta}(R_1 \otimes_{R_1.A_1 = R_2.A_1\ AND\ \ldots\ AND\ R_1.A_m = R_2.A_m} R_2)$$

    - equivalent SELECT statement

```
SELECT *
FROM R1 NATURAL JOIN R2
```

- join operators
  - *left outer join*
    - notation (in these notes): $R_1 \ltimes_{\mathrm{C}} R_2$
    - resulting relation:
      - schema: the attributes of R_1 followed by the attributes of R_2
      - tuples: tuples from the condition join $R_1 \otimes_{\mathrm{c}} R_2$ + the tuples in R_1 that were not used in $R_1 \otimes_{\mathrm{c}} R_2$ combined with the *null* value for the attributes of R_2
    - equivalent SELECT statement

```
SELECT *
FROM R1 LEFT OUTER JOIN R2 ON C
```

- join operators
  - *right outer join*
    - notation: $R_1 \rtimes_{\mathrm{C}} R_2$
    - resulting relation:
      - schema: the attributes of R_1 followed by the attributes of R_2
      - tuples: tuples from the condition join $R_1 \otimes_{\mathrm{c}} R_2$ + the tuples in R_2 that were not used in $R_1 \otimes_{\mathrm{c}} R_2$ combined with the *null* value for the attributes of R_1
    - equivalent SELECT statement

```
SELECT *
FROM R1 RIGHT OUTER JOIN R2 ON C
```

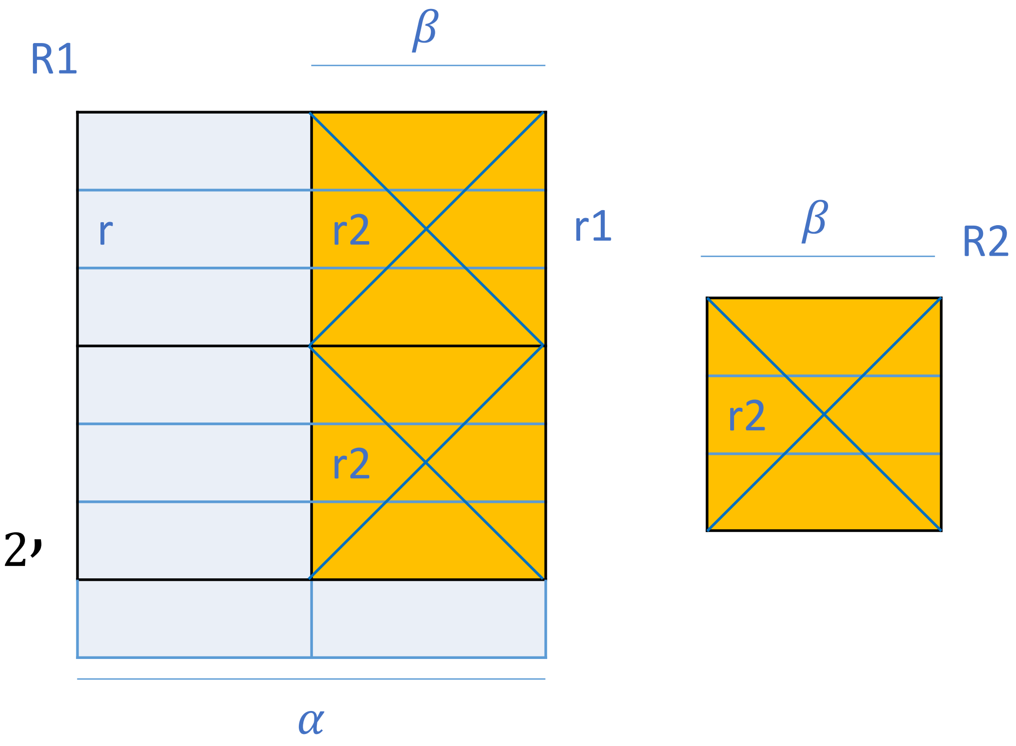
- join operators
  - *full outer join*
    - notation: $R_1 ⟗_{\mathrm{C}} R_2$
    - resulting relation:
      - schema: the attributes of R_1 followed by the attributes of R_2
      - tuples:
        - tuples from the condition join $R_1 \otimes_{\mathrm{c}} R_2$ +
        - the tuples in R_1 that were not used in $R_1 \otimes_{\mathrm{c}} R_2$ combined with the *null* value for the attributes of R_2 +
        - the tuples in R_2 that were not used in $R_1 \otimes_{\mathrm{c}} R_2$ combined with the *null* value for the attributes of R_1
    - equivalent SELECT statement

```
SELECT *
FROM R1 FULL OUTER JOIN R2 ON C
```

- join operators
  - *left semi join*
    - notation: $R_1 \rhd R_2$
    - resulting relation:
      - schema: R_1's schema
      - tuples: the tuples in R_1 that are used in the natural join $R_1 * R_2$

- join operators
  - *right semi join*
    - notation: $R_1 \lhd R_2$
    - resulting relation:
      - schema: R_2's schema
      - tuples: the tuples in R_2 that are used in the natural join $R_1 * R_2$

- *division*
  - notation: $R_1 \div R_2$
  - $R_1[\alpha], R_2[\beta], \beta \subset \alpha$
  - resulting relation:
    - schema: $\alpha - \beta$
    - tuples: a record $r \in R_1 \div R_2$ iff $\forall r_2 \in R_2$, $\exists r_1 \in R_1$ such that:
      - $\pi_{\alpha-\beta}(r_1) = r$
      - $\pi_{\beta}(r_1) = r_2$
      - i.e., a record r belongs to the result if in R_1 r is concatenated with every record in R_2



- see lecture examples (at the board) with algebra queries:

- selection

- projection

- division

- selection, projection

- natural join, selection, projection

- set-difference, natural join, selection, projection

- different algebra expressions producing the same result (optimization - reducing the size of intermediate relations)

An Independent Subset of Operators

- independent set of operators M:
  - eliminating any operator *op* from M: there will be a relation that can be obtained using M's operators, but cannot be obtained with the operators in M-{*op*}
- for the previously described query language, with operators:

  $\{\sigma, \pi, \times, \cup, -, \cap, \otimes, *, \ltimes, \rtimes, \bowtie, \rhd, \lhd, \div\}$

an independent set of operators is $\{\sigma, \pi, \times, \cup, -\}$

- the other operators are obtained as follows (some expressions have already been introduced):
  - $R_1 \cap R_2 = R_1 - (R_1 - R_2)$
  - $R_1 \otimes_{\mathrm{C}} R_2 = \sigma_C(R_1 \times R_2)$

- the other operators are obtained as follows (some expressions have already been introduced):
  - $R_1[\alpha], R_2[\beta], \alpha \cap \beta = \{A_1, A_2, \dots, A_m\}$, then:

    $$R_1 * R_2 = \pi_{\alpha \cup \beta}(R_1 \otimes_{R_1.A_1 = R_2.A_1 \ AND \ \dots \ AND \ R_1.A_m = R_2.A_m} R_2)$$

  - $R_1[\alpha], R_2[\beta], R_3[\beta] = \{(null, \dots, null)\}, R_4[\alpha] = \{(null, \dots, null)\}$

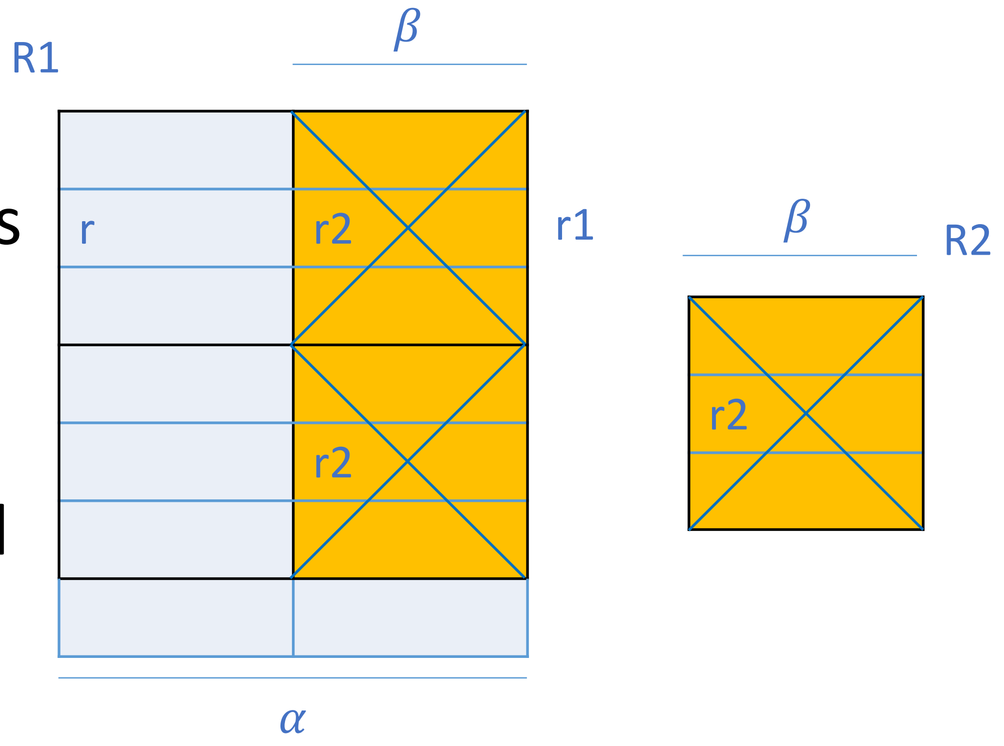
    $$R_1 \ltimes_C R_2 = (R_1 \otimes_c R_2) \cup (R_1 - \pi_\alpha(R_1 \otimes_c R_2)) \times R_3$$
    $$R_1 \rtimes_C R_2 = (R_1 \otimes_c R_2) \cup R_4 \times (R_2 - \pi_\beta(R_1 \otimes_c R_2))$$
    $$R_1 \bowtie_C R_2 = (R_1 \ltimes_C R_2) \cup (R_1 \rtimes_C R_2)$$

  - $R_1[\alpha], R_2[\beta]$

    $$R_1 \rhd R_2 = \pi_\alpha(R_1 * R_2)$$
    $$R_1 \lhd R_2 = \pi_\beta(R_1 * R_2)$$

- the other operators are obtained as follows (some expressions have already been introduced):
  - if $R_1[\alpha]$, $R_2[\beta]$, $\beta \subset \alpha$, then $r \in R_1 \div R_2$ iff $\forall r_2 \in R_2, \exists r_1 \in R_1$ such that: $\pi_{\alpha-\beta}(r_1) = r$ and $\pi_\beta(r_1) = r_2$

    => r is in $\pi_{\alpha-\beta}(R_1)$, but not all the elements in $\pi_{\alpha-\beta}(R_1)$ are in the result



  - $(\pi_{\alpha-\beta}(R_1)) \times R_2$ contains all the elements with one part in $\pi_{\alpha-\beta}(R_1)$ and the second part in R_2
  - to obtain values that are disqualified, R_1 is subtracted from the obtained relation, and the result is projected on $\alpha - \beta$
  - the final expression:

$$R_1 \div R_2 = \pi_{\alpha-\beta}(R_1) - \pi_{\alpha-\beta}((\pi_{\alpha-\beta}(R_1)) \times R_2 - R_1)$$

- the *renaming* operator

$\rho(\mathrm{R}'(\mathrm{A}_1 \to \mathrm{A}_1{}', \mathrm{A}_2 \to \mathrm{A}_2{}', \mathrm{A}_3 \to \mathrm{A}_3{}'), \mathrm{E})$

  - E - relational algebra expression
  - the result, relation R', has the same tuples as the result of E
  - attributes A_1, A_2, and A_3 are renamed to $\mathrm{A}_1{}'$, $\mathrm{A}_2{}'$, and $\mathrm{A}_3{}'$, respectively

* the next examples use the statements below:

- assignment

  $\mathrm{R[list]} \coloneqq \mathrm{expression}$

  - the expression's result (a relation) is assigned to a variable (R[list]), specifying the name of the relation [and the names of its columns]

- eliminating duplicates from a relation

  $\delta(\mathrm{R})$

- sorting records in a relation

  $\mathrm{S}_{\{\mathrm{list}\}}(\mathrm{R})$

- grouping

  $\gamma_{\{\mathrm{list1}\}\ \mathrm{group\ by}\ \{\mathrm{list2}\}}(\mathrm{R})$

  - R's records are grouped by the columns in *list2*
  - *list1* (that can contain aggregate functions) is evaluated for each group of records

```
students [id, name, sgroup, gpa, dob]
groups [id, year, program]
schedule [day, starthour, endhour, activtype, room, sgroup, facultym_id]
faculty_members [id, name]
```

1. The names of students in a given group:

$$R \coloneqq \pi_{\{name\}}\left(\sigma_{sgroup='222'}(students)\right)$$

```
SELECT name
FROM students
WHERE sgroup='222'
```

2. The students in a given program (alphabetical list, by groups):

$$G \coloneqq \pi_{\{id\}}\left(\sigma_{program='IG'}(groups)\right)$$
$$R \coloneqq S_{\{sgroup,name\}}\left(\sigma_{sgroup\ is\ in\ G}(students)\right)$$

```
SELECT *
FROM students
WHERE sgroup IN
   (SELECT id
   FROM groups
   WHERE program='IG')
ORDER BY sgroup, name
```

```
students [id, name, sgroup, gpa, dob]
groups [id, year, program]
schedule [day, starthour, endhour, activtype, room,
  sgroup, facultym_id]
faculty_members [id, name]
```

3. The number of students in every group of a given program:

$$ST \coloneqq \sigma_{sgroup\ is\ in\left(\pi_{\{id\}}\left(\sigma_{program='IG'}(groups)\right)\right)}(students)$$

$$NR \coloneqq \gamma_{\{sgroup,\, count(*)\}\ group\ by\ \{sgroup\}}(ST)$$

```
SELECT sgroup, COUNT(*)
FROM (SELECT *
      FROM students
      WHERE sgroup IN
          (SELECT id
           FROM groups
           WHERE program='IG')
       ) t
GROUP BY sgroup
```

```
students [id, name, sgroup, gpa, dob]
groups [id, year, program]
schedule [day, starthour, endhour, activtype, room,
  sgroup, facultym_id]
faculty_members [id, name]
```

4. A student's schedule (the student is given by name):

$$T \coloneqq \sigma_{sgroup\ is\ in\left(\pi_{\{sgroup\}}\left(\sigma_{name='Ionescu\ M.\ Razvan'}(students)\right)\right)}(schedule)$$

5. The number of hours per week for every group:

$$F(no, sgroup) \coloneqq \pi_{\{endhour-starthour, sgroup\}}(schedule)$$

$$NoHours(sgroup, nohours) \coloneqq \gamma_{\{sgroup, sum(no)\}\ group\ by\ \{sgroup\}}(F)$$

```
students [id, name, sgroup, gpa, dob]
groups [id, year, program]
schedule [day, starthour, endhour, activtype, room, sgroup, facultym_id]
faculty_members [id, name]
```

6. The faculty members (their names) who teach a given student:

$A \coloneqq (\sigma_{name='Ionescu\ M.\ Razvan'}(students)) \otimes_{students.sgroup=schedule.sgroup} schedule$

$$B \coloneqq \pi_{\{facultym\_id\}}(A)$$

$$C \coloneqq faculty\_members \otimes_{faculty\_members.id=B.facultym\_id} B$$

$$D \coloneqq \pi_{\{name\}}(C)$$

```
students [id, name, sgroup, gpa, dob]
groups [id, year, program]
schedule [day, starthour, endhour, activtype, room, sgroup, facultym_id]
faculty_members [id, name]
```

7. The faculty members with no teaching assignments (i.e., not on the schedule):

$$C \coloneqq \pi_{\{name\}}(faculty\_members) - \pi_{\{name\}}(schedule \otimes_{schedule.facultym\_id=faculty\_members.id} faculty\_members)$$

* Is there a problem if two different faculty members have the same name?

```
students [id, name, sgroup, gpa, dob]
groups [id, year, program]
schedule [day, starthour, endhour, activtype, room, sgroup, facultym_id]
faculty_members [id, name]
```

8. Students with school activities on every day of the week (all days with school activities considered):

$$A \coloneqq \delta\left(\pi_{\{day\}}(schedule)\right)$$

$$B \coloneqq students \otimes_{students.sgroup=schedule.sgroup} schedule$$

$$C \coloneqq \delta\left(\pi_{\{name,day\}}(B)\right)$$

$$D \coloneqq C \div A$$

* Is there a problem if two different students have the same name?

```
students [id, name, sgroup, gpa, dob]
groups [id, year, program]
schedule [day, starthour, endhour, activtype, room, sgroup, facultym_id]
faculty_members [id, name]
```

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