

Databases

Lectures 9-10*

Indexes. Tree-Structured Indexing. Hash-Based Indexing

*01.12.2021 - National Day

Indexes

- motivating example
 - file of students records sorted by name
 - good file organization
 - retrieve students in alphabetical order
 - not a good file organization
 - retrieve students whose age is in a given range
 - retrieve students who live in Timișoara
- index
 - auxiliary data structure that speeds up operations which can't be efficiently carried out given the file's organization
 - enables the retrieval of the rids of records that meet a selection condition (e.g., the rids of records describing students who live in Timișoara)

Indexes

- *search key*
 - set of one or more attributes of the indexed file (different from the *key* that identifies records)
- an index speeds up queries with equality / range selection conditions on the search key
- *entries*
 - records in the index (e.g., <search key, rid>)
 - enable the retrieval of records with a given search key value

Indexes

- example
 - files with students records
 - index built on attribute *city*
 - entries: $\langle \text{city}, \text{rid} \rangle$, where rid identifies a student record
 - such an index would speed up queries about students living in a given city:
 - find entries in the index with $\text{city} = \text{'Timișoara'}$
 - follow rids from obtained entries to retrieve records describing students who live in Timișoara

Indexes

- an index can improve the efficiency of certain types of queries, not of all queries (analogy - when searching for a book at the library, index cards sorted on author name cannot be used to efficiently locate a book given its title)
- organization techniques (access methods) - examples
 - B+ trees
 - hash-based structures
- changing the data in the file => update the indexes associated with the file (e.g., inserting records, updating search key columns, updating columns that are not part of the key, but are included in the index)
- index size
 - as small as possible, as indexes are brought into main memory for searches

Indexes - Data Entries

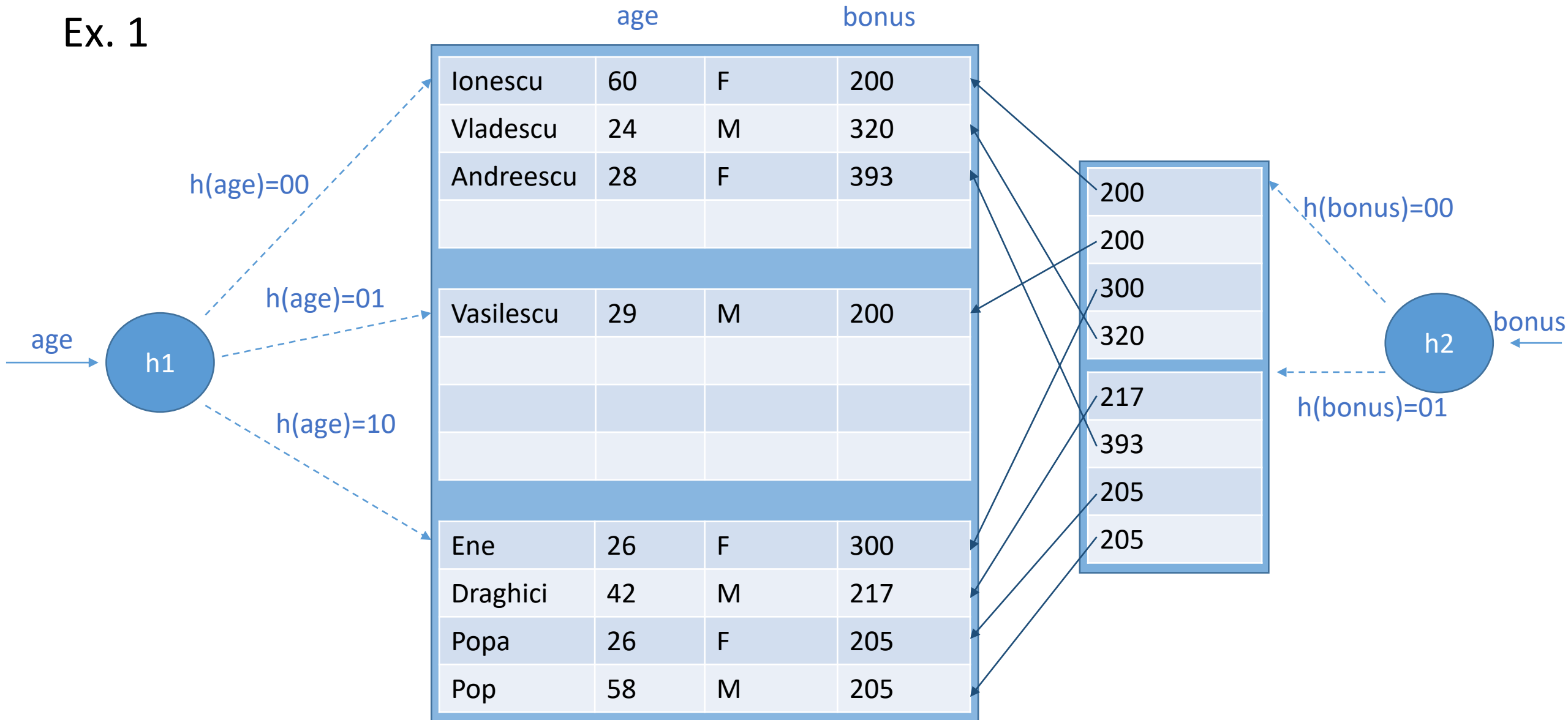
- problems
 - what does a data entry contain?
 - how are the entries of an index organized?
- let k^* be a data entry in an index; the data entry:
 - alternative 1
 - is an actual data record with search key value = k
 - alternative 2
 - is a pair $\langle k, \text{rid} \rangle$ (rid – id of a data record with search key value = k)
 - alternative 3
 - is a pair $\langle k, \text{rid_list} \rangle$ (rid_list – list of ids of data records with search key value = k)

Indexes - Data Entries

- a1
 - the file of data records needn't be stored in addition to the index
 - the index is seen as a special file organization
 - at most 1 index / collection of records should use alternative a1 (to avoid redundancy)
- a2, a3
 - data entries point to corresponding data records
 - in general, the size of an entry is much smaller than the size of a data record
 - a3 is more compact than a2, but can contain variable-length records
 - can be used by several indexes on a collection of records
 - independent of the file organization

Indexes - Data Entries

Ex. 1



Indexes - Data Entries

Ex. 1

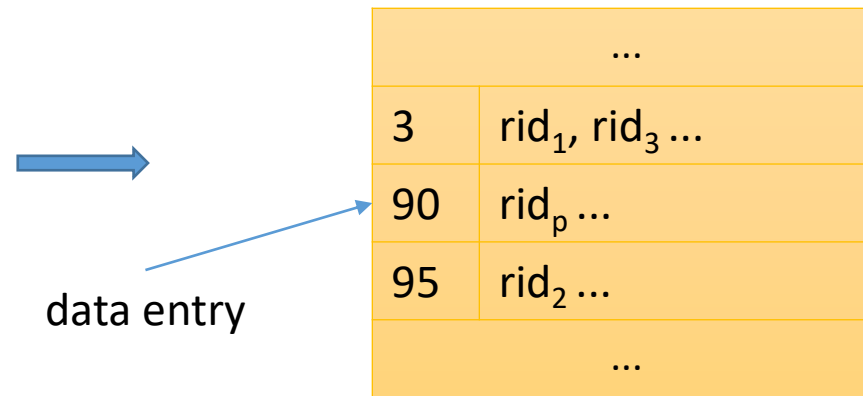
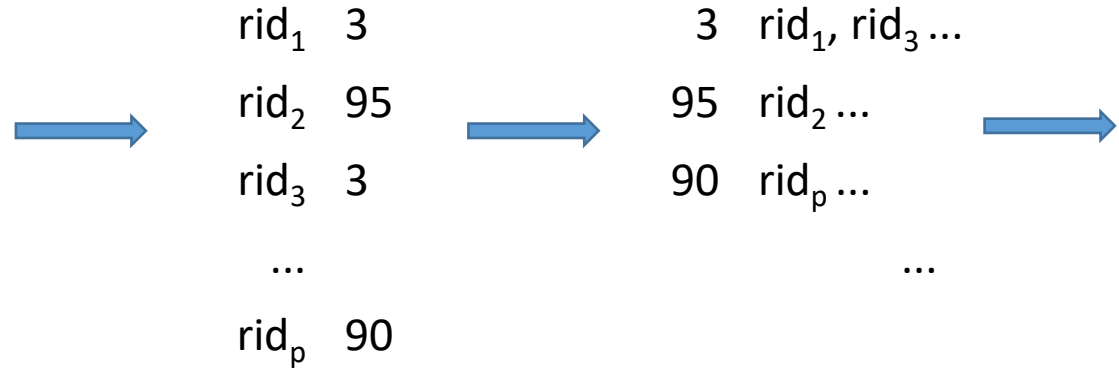
- file with Employee records hashed on *age*
 - record <Ionescu, 60, F, 200>:
 - apply hash function to *age*: convert 60 to its binary representation, take the 2 least significant bits as the bucket identifier for the record
- index file that uses alternative 1 (data entries are the actual data records), search key *age*
- index that uses alternative 2 (data entries have the form <search key, rid>), search key *bonus*
- both indexes use hashing to locate data entries

Indexes - Data Entries

Ex. 2

	Name	Score	Age
rid ₁	Popescu	3	44
rid ₂	Ionescu	95	80
rid ₃	Vladescu	3	45
...		...	
rid _p	Xulescu	90	14

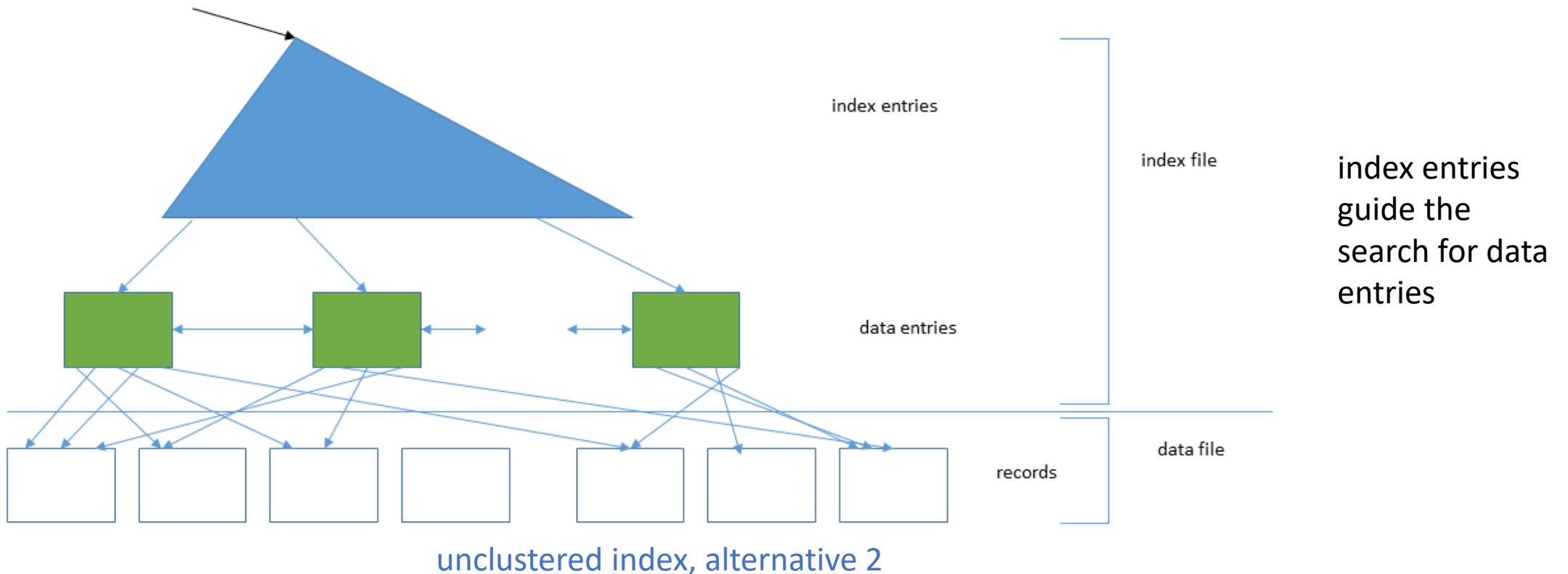
search key



index file

Clustered / Unclustered Indexes

- clustered index: the order of the data records is close to / the same as the order of the data entries
- unclustered index: index that is not clustered



Clustered / Unclustered Indexes

- index that uses alternative 1 - clustered (by definition, since the data entries are the actual data records)
- indexes using alternatives 2 / 3 are clustered only if the data records are ordered on the search key
- in practice:
 - expensive to maintain the sort order for files, so they are rarely kept sorted
 - a clustered index is an index that uses alternative 1 for data entries
 - an index that uses alternative 2 or 3 for data entries is unclustered
- on a collection of records:
 - there can be at most 1 clustered index
 - and several unclustered indexes

Clustered / Unclustered Indexes

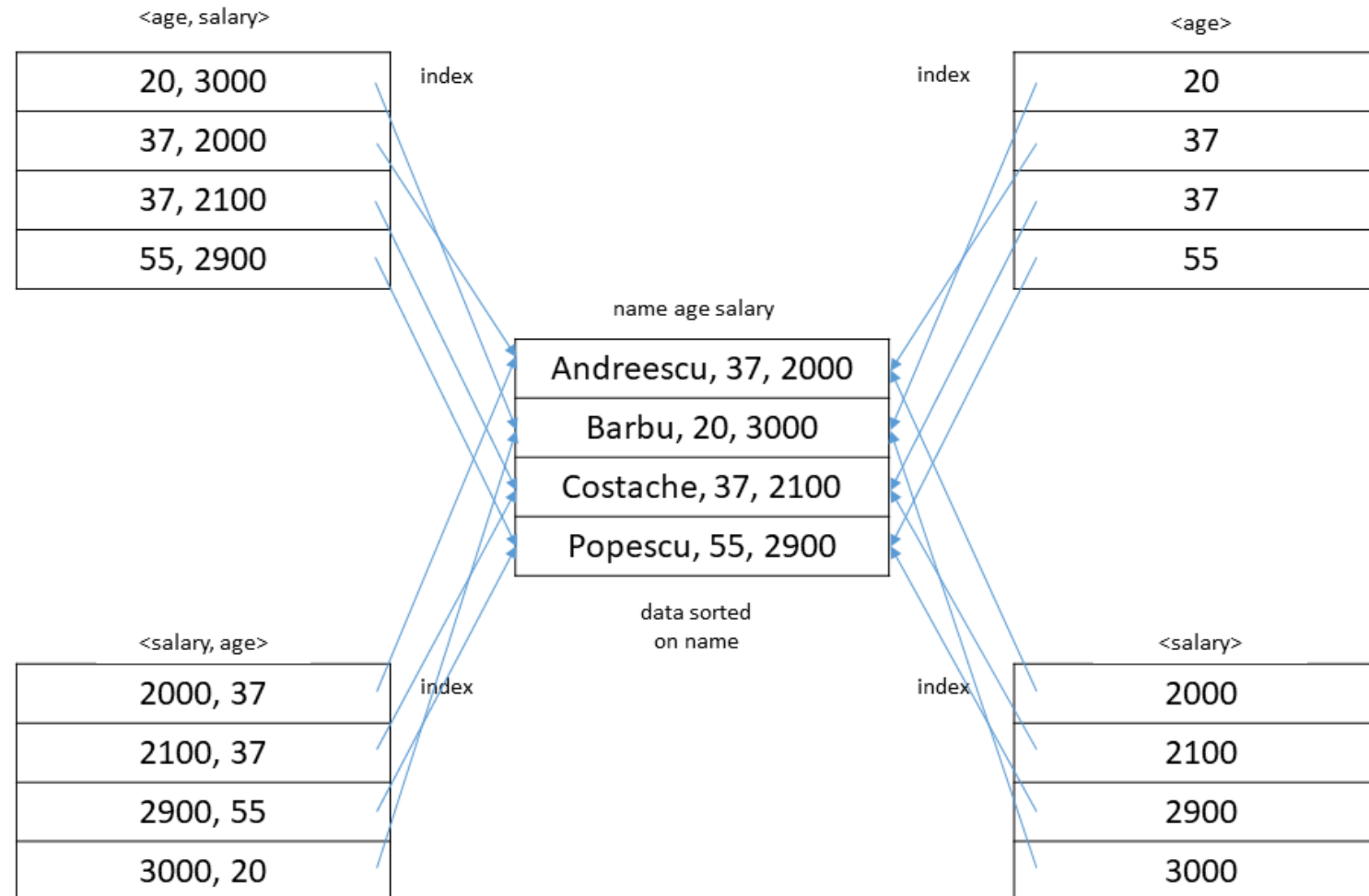
- range search query (e.g., *where age between 20 and 30*)
 - cost of using an unclustered index
 - each data entry that meets the condition in the query could contain a rid pointing to a distinct page
 - the number of I/O operations could be equal to the number of data entries that satisfy the query's condition

Primary / Secondary Indexes

- primary index
 - the search key includes the primary key
- secondary index
 - index that is not primary
- unique index
 - the search key contains a candidate key
- duplicates
 - data entries with the same search key value
- primary indexes, unique indexes cannot contain duplicates
- secondary indexes can contain duplicates

Composite Search Keys

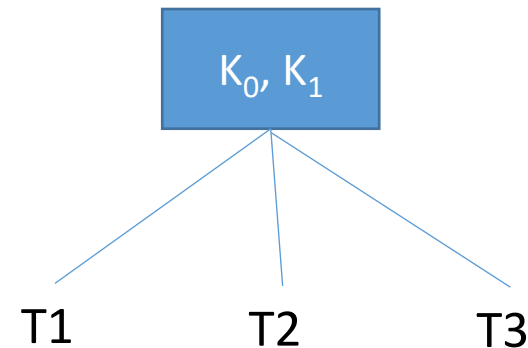
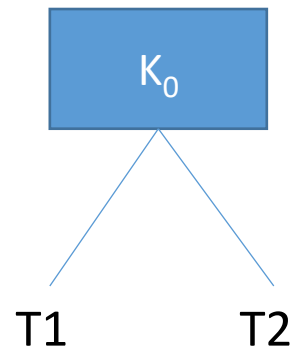
- composite (concatenated) search key - search key that contains several fields
 - examples



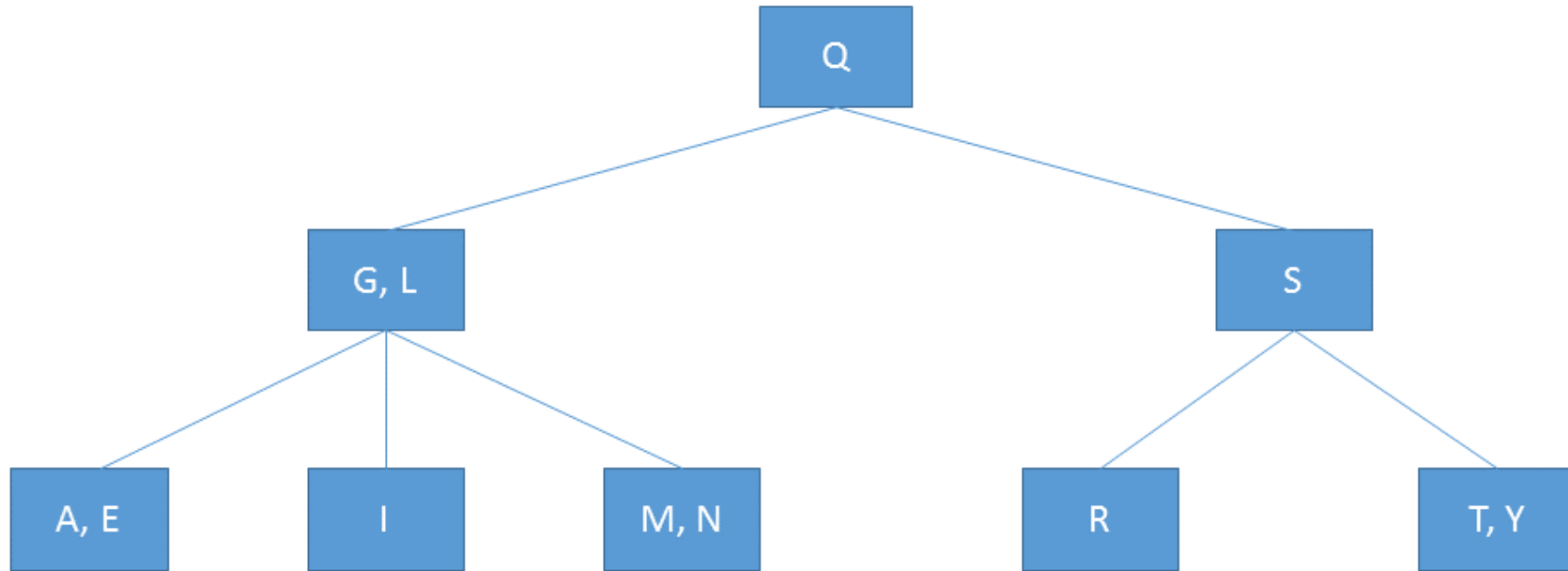
2-3 trees

2-3 tree storing key values (collection of distinct values)

- all the terminal nodes are on the same level
- every node has 1 or 2 key values
 - a non-terminal node with one value K_0 has 2 subtrees: one with values less than K_0 , and one with values greater than K_0
 - a non-terminal node with 2 values K_0 and K_1 , $K_0 < K_1$, has 3 subtrees: one with values less than K_0 , a subtree with values between K_0 and K_1 , and a subtree with values greater than K_1



* Example (key values are letters)



- storing a 2-3 tree
 - 2-3 tree index storing the values of a key
 - tree - key value + address of record (file / DB address of record with corresponding key value)

- 2 options
 1. transform 2-3 tree into a binary tree
 - nodes with 2 values are transformed (see figure below)
 - nodes with 1 value - unchanged



- the structure of a node



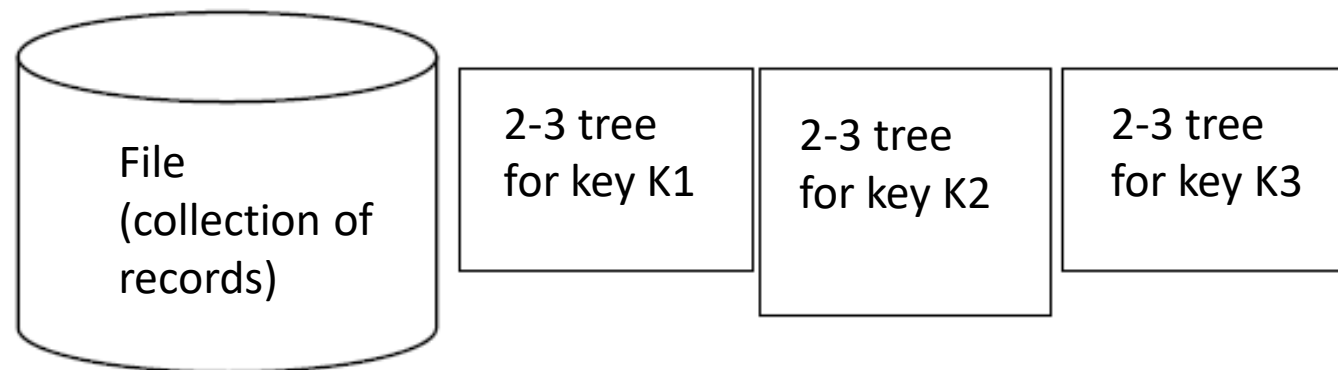
- K - key value
- ADDR - address of the record with the current key value (address in the file)
- PointerL, PointerR - the 2 subtrees' addresses (address in the tree)

- IND - indicator that specifies the type of the link to the right (the 2 possible values can be seen in the previous figure)

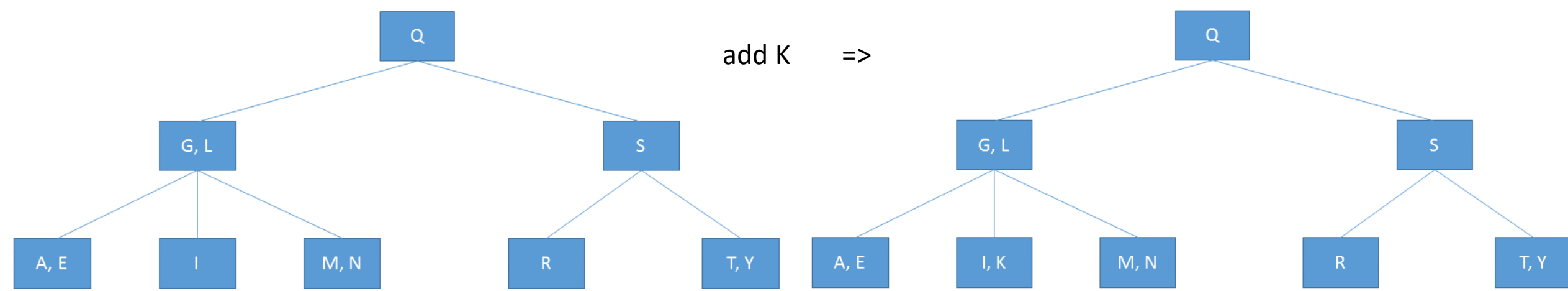
2. the memory area allocated for a node can store 2 values and 3 subtree addresses

NV	K_1	ADDR ₁	K_2	ADDR ₂	Pointer ₁	Pointer ₂	Pointer ₃
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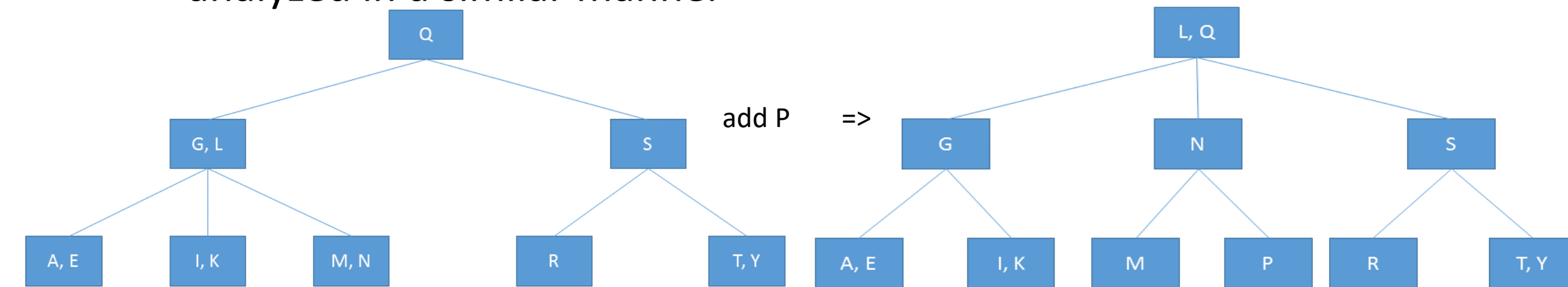
- NV – number of values in the node (1 or 2)
- K_1, K_2 – key values
- ADDR₁, ADDR₂ – the records' addresses (corresponding to K_1 and K_2)
- Pointer₁, Pointer₂, Pointer₃ – the 3 subtrees' addresses
- obs. a file (a relation in a relational DB) can have several associated 2-3 trees (one tree / key)



- operations in a 2-3 tree
 - searching for a record with key value K_0
 - inserting a record - description
 - removing a record - description
 - tree traversal (partial, total)
- add a new value
 - values in the tree must be distinct (the new value should not exist in the tree)
 - perform a test: search for the value in the tree; if the new value can be added, the search ends in a terminal node
 - if the reached terminal node has 1 value, the new value can be stored in the node



- if the reached terminal node has 2 values, the new value is added to the node, the 3 values are sorted, the node is split into 2 nodes: one node will contain the smallest value, the 2nd node - the largest value, and the middle value is attached to the parent node; the parent is then analyzed in a similar manner

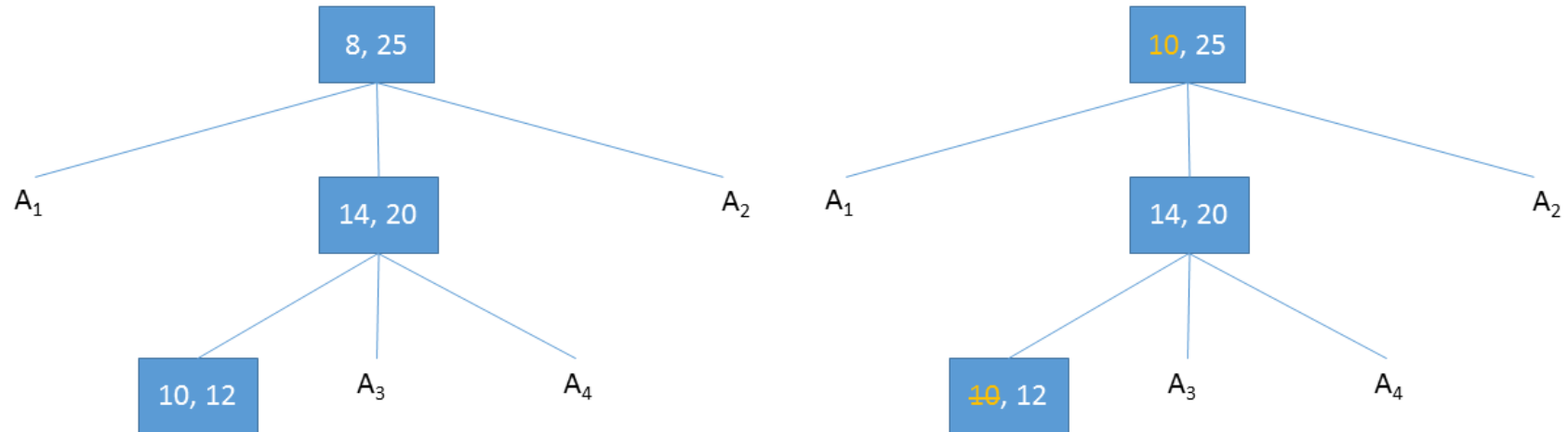


- delete a value K_0

1. search for K_0 ; if K_0 appears in an inner node, change it with a neighbor value K_1 from a terminal node (there is no other value between K_0 and K_1)

- K_1 's previous position (in the terminal node) is eliminated

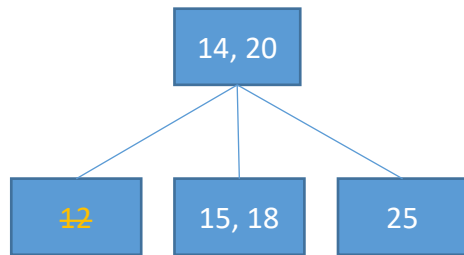
- e.g., remove 8:



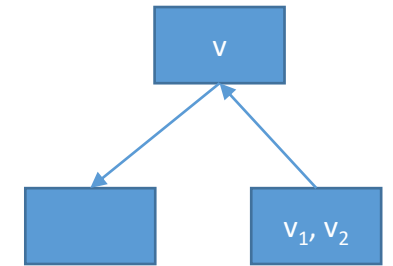
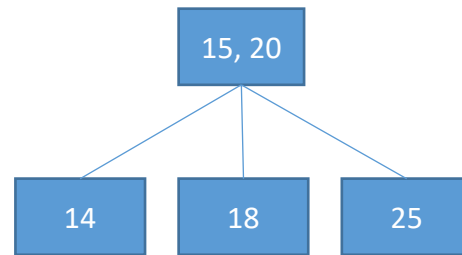
2. perform this step until case a / b occurs

a. if the current node (from which a value is removed) is the root or a node with 1 remaining value, the value is eliminated; the algorithm ends

b. if the delete operation empties the current node, but 2 values exist in one of the sibling nodes (left / right), 1 of the sibling's values is transferred to the parent, 1 of the parent's values is transferred to the current node; the algorithm ends



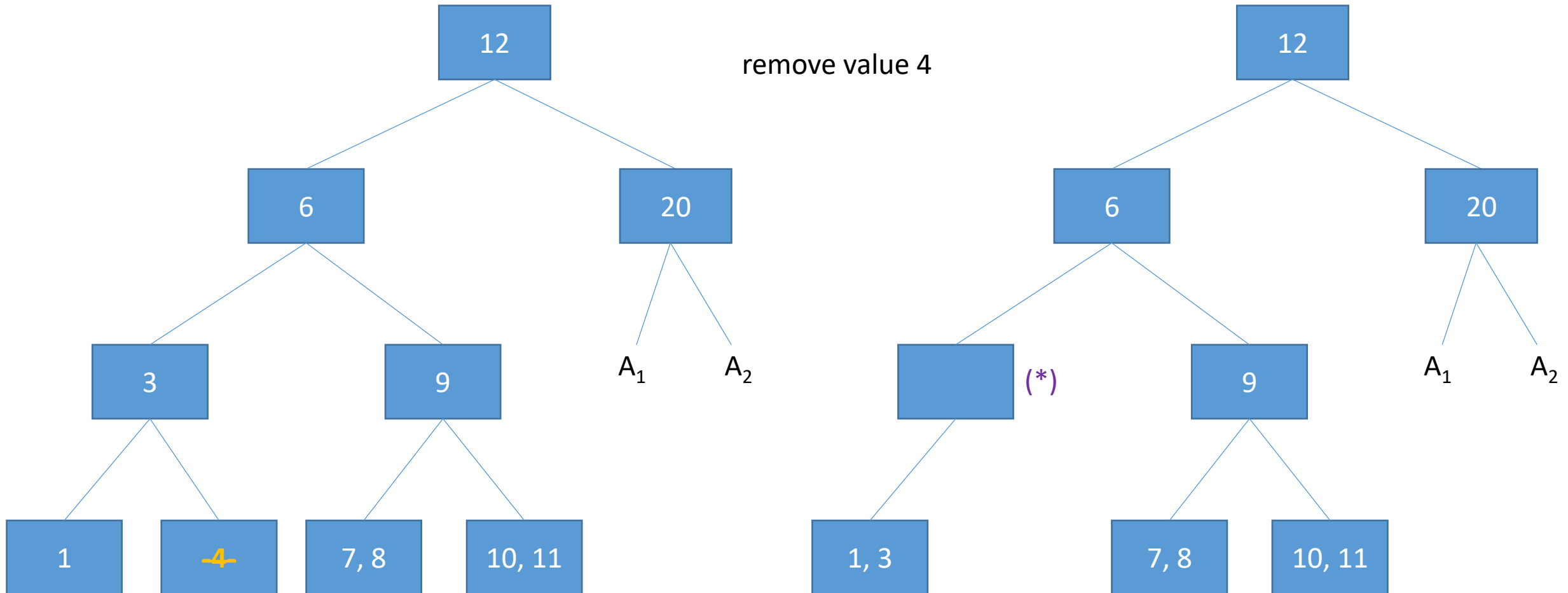
delete value 12

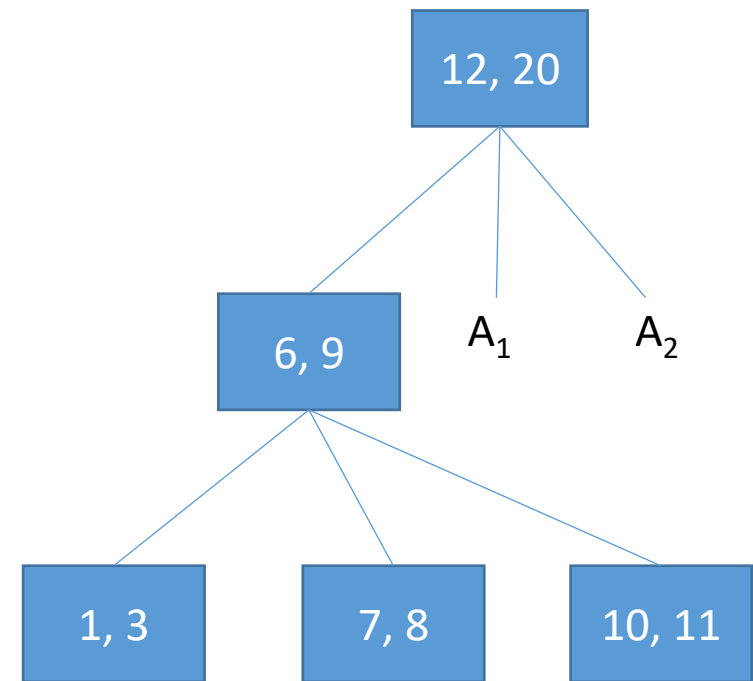
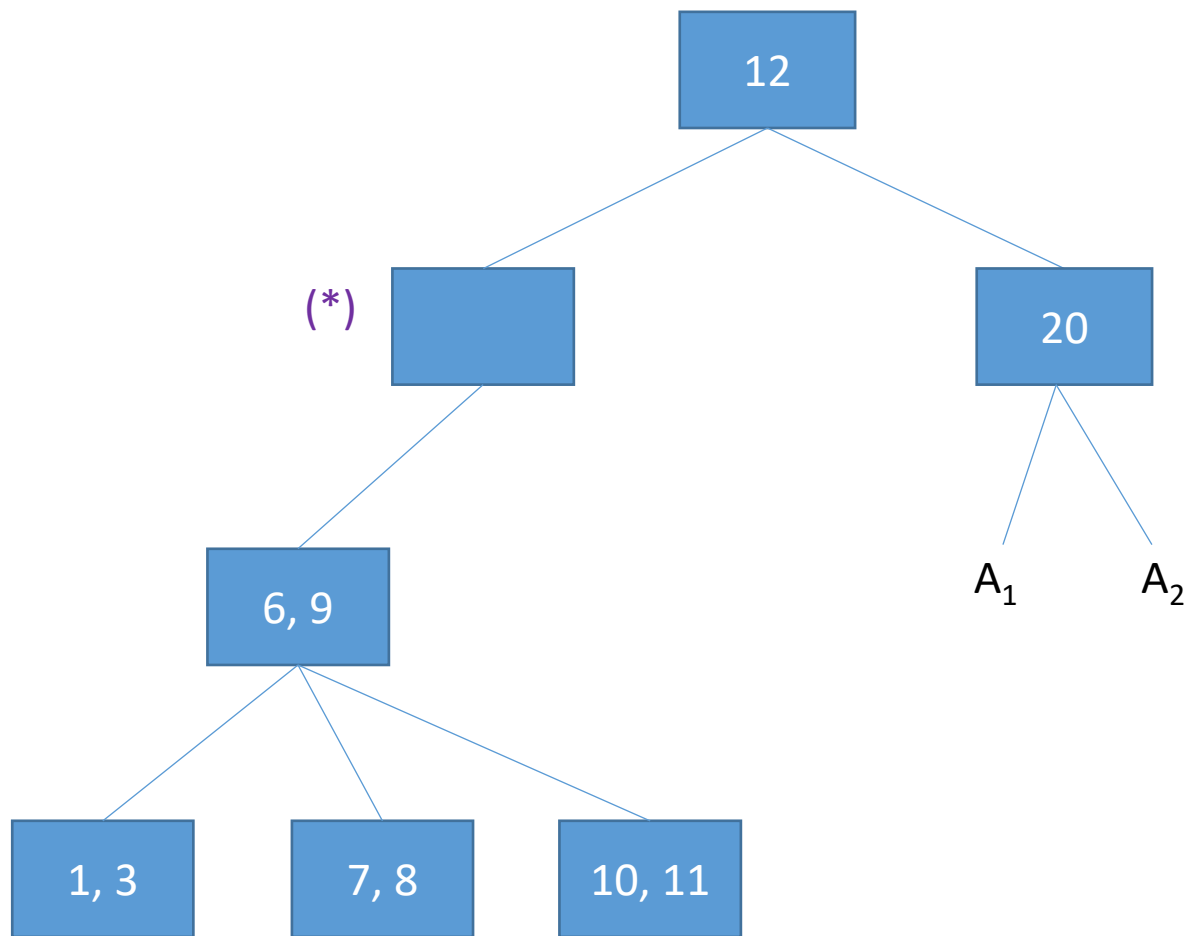


c. if the previous cases do not occur (current node has no values, sibling nodes have 1 value each), then the current node is merged with a sibling and a value from the parent node; case 2 is then analyzed for the parent

- if the root is reached and it has no values, it is eliminated and the current node becomes the root

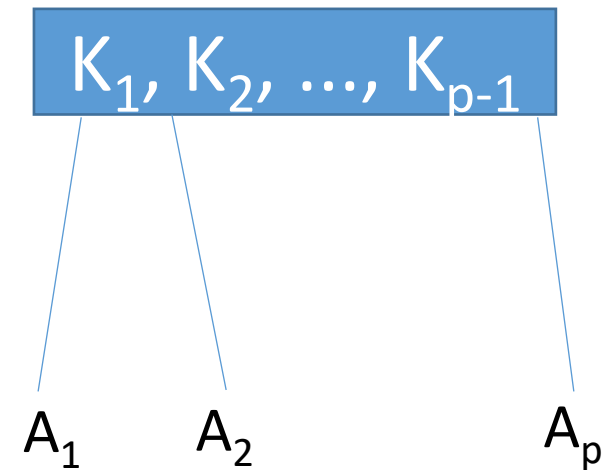
- example: case c for the node marked with (*)





B-trees

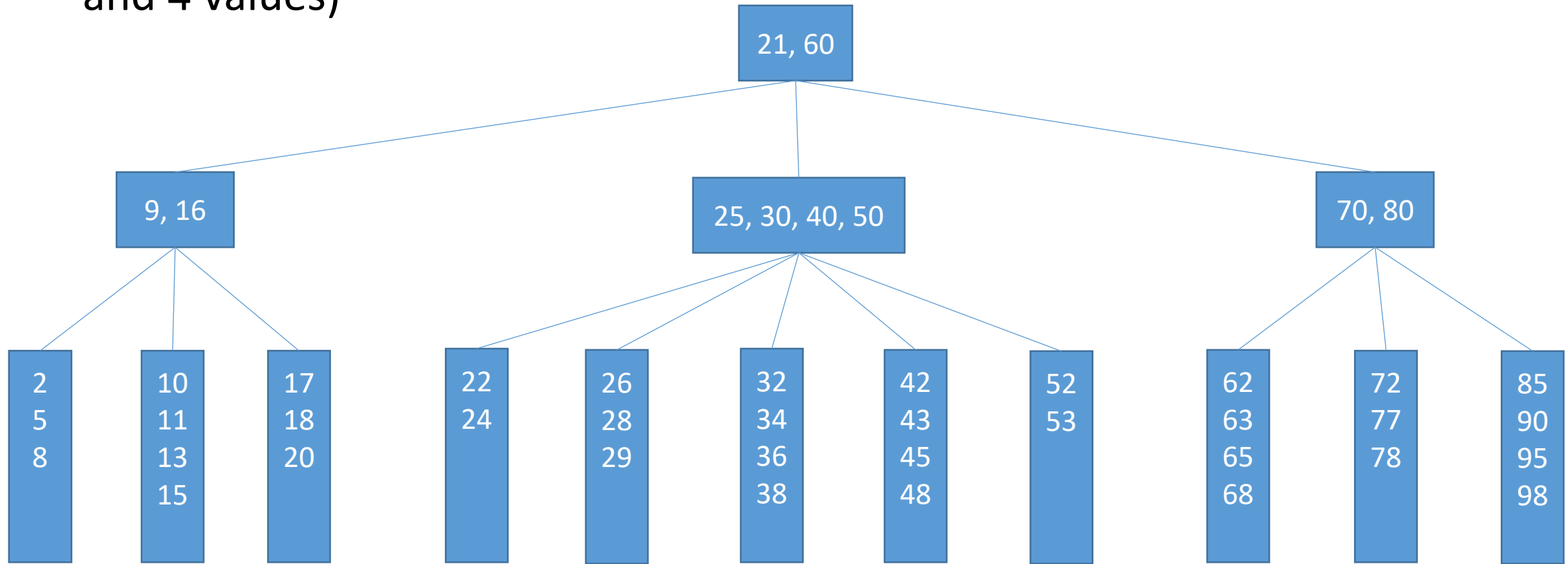
- generalization of 2-3 trees
- B-tree of order m
 1. if the root is not a terminal, it has at least 2 subtrees
 2. all terminal nodes – same level
 3. every non-terminal node – at most m subtrees
 4. a node with p subtrees has p-1 ordered values (ascending order): $K_1 < K_2 < \dots < K_{p-1}$
 - A_1 : values less than K_1
 - A_i : values between K_{i-1} and K_i , $i=2, \dots, p-1$
 - A_p : values greater than K_{p-1}



5. every non-terminal node – at least $\left\lceil \frac{m}{2} \right\rceil$ subtrees
- obs. limits on number of subtrees (and values) / node result from the manner in which inserts / deletes are performed such that the second requirement in the definition is met

* Example - B-tree of order 5

- non-terminal, non-root node – at most 5, at least 3 subtrees (between 2 and 4 values)



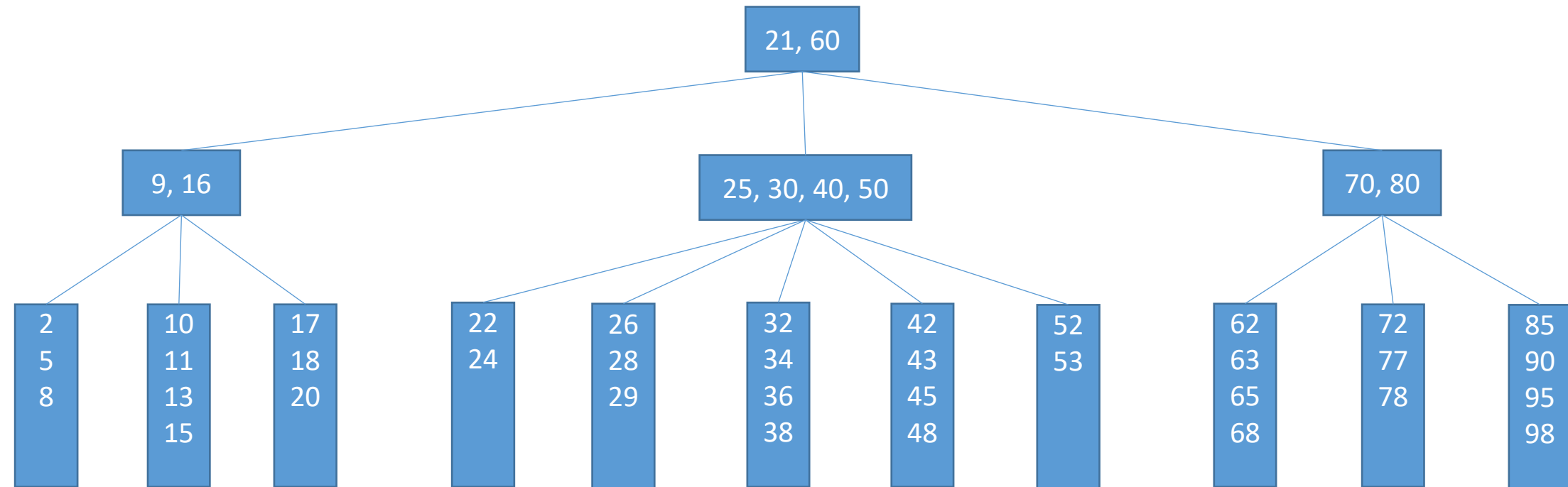
- B-tree of order m
 - storing the values of a key (a database index)
 - tree
 - key value + address of record
- 1. transformed into a binary tree
 - 2-3 tree method
- 2. the memory area allocated for a node can store the maximum number of values and subtree addresses

NV	K_1	$ADDR_1$...	K_{m-1}	$ADDR_{m-1}$	$Pointer_1$...	$Pointer_m$
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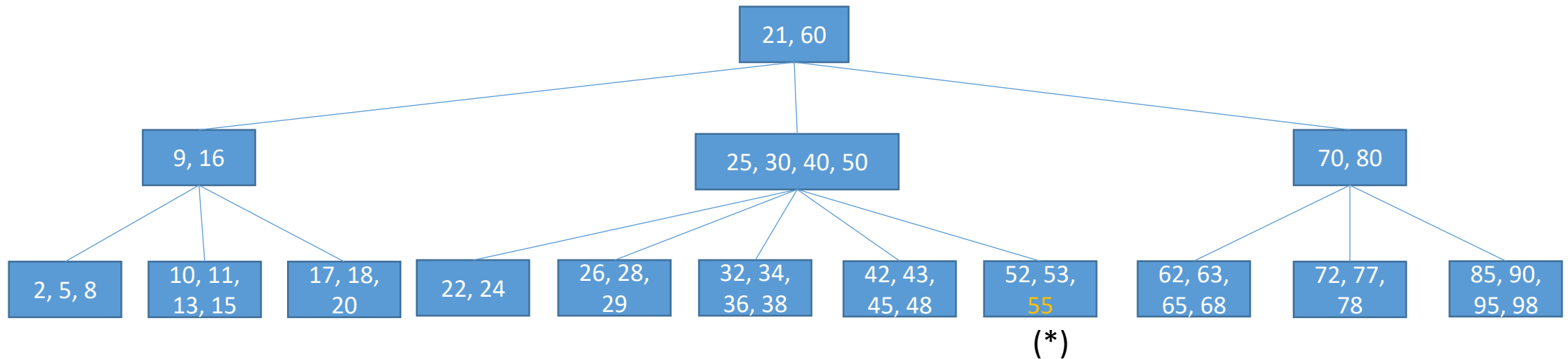
- NV - number of values in the node
- K_1, \dots, K_{m-1} - key values
- $ADDR_1, \dots, ADDR_{m-1}$ - the records' addresses (corresponding to the key's values)
- $Pointer_1, \dots, Pointer_m$ – subtree addresses

- B-tree of order m
 - useful operations in a B-tree
 - searching for a value
 - adding a value - description
 - removing a value- description
 - tree traversal (partial, total)

- B-tree of order m
 - adding a new value
 1. values in the tree must be distinct (the new value should not exist in the tree); perform a test (search for the value in the tree)
 - if the new value can be added, the search ends in a terminal node
 2. if the reached terminal node has less than $m-1$ values, the new value can be stored in the node, e.g., 55 is added to the tree below:

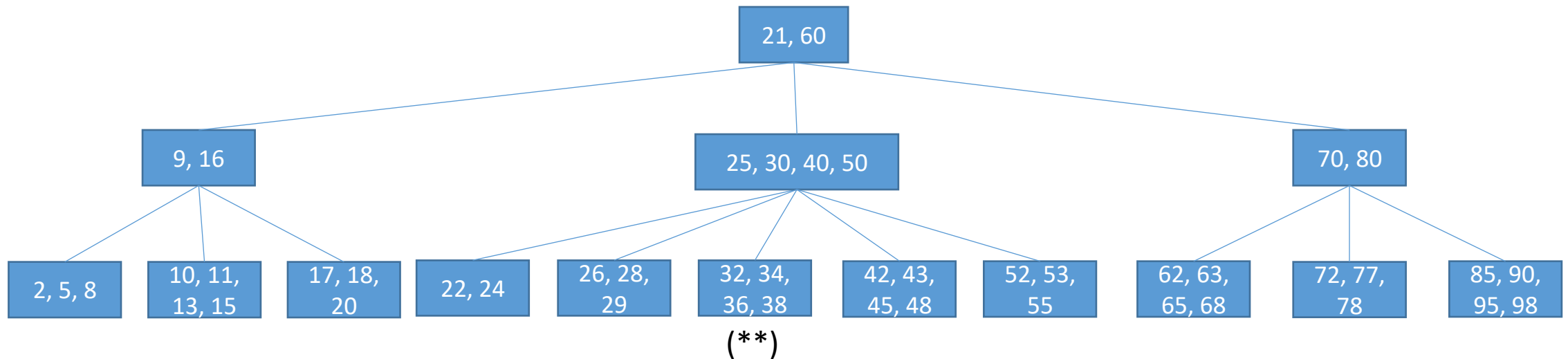


- B-tree of order m
 - adding a new value
 - the resulting tree is shown below:



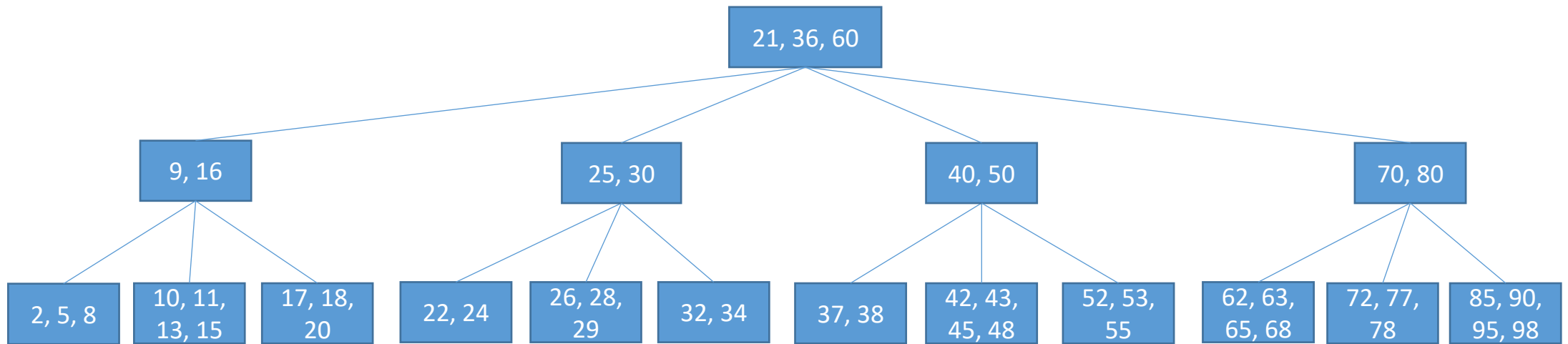
- 55 belongs to the node marked with (*), which can store at most 4 values

- B-tree of order m
 - adding a new value
 3. if the terminal node already has $m-1$ values, the new value is attached to the node, the m values are sorted, the node is split into 2 nodes, and the middle value (median) is attached to the parent node; the parent is then analyzed in a similar manner
 - e.g., add 37 to the tree below

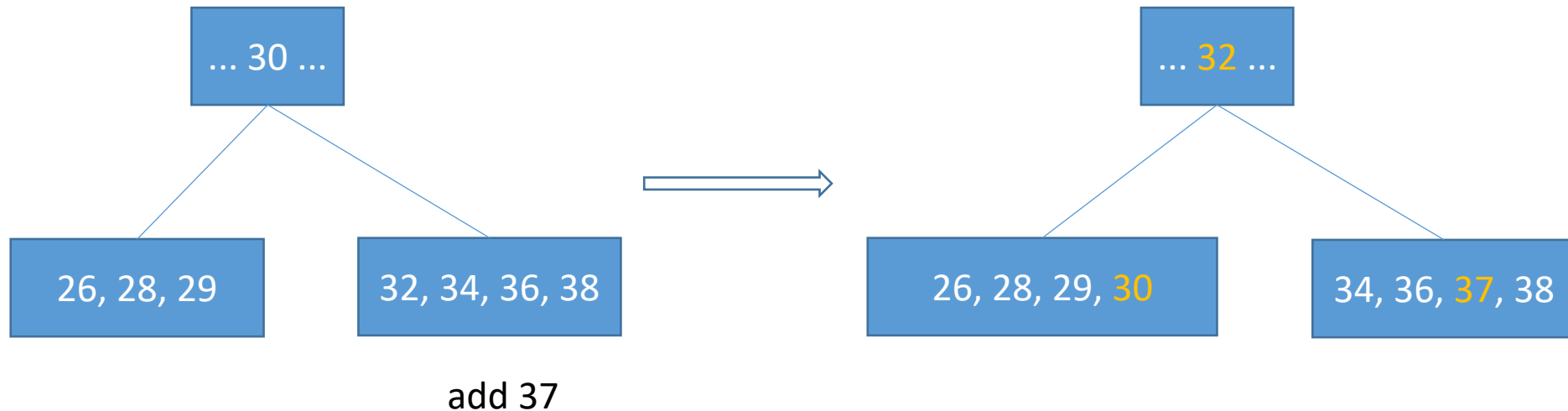


- the node marked with (**) should contain values 32, 34, 36, 37, 38

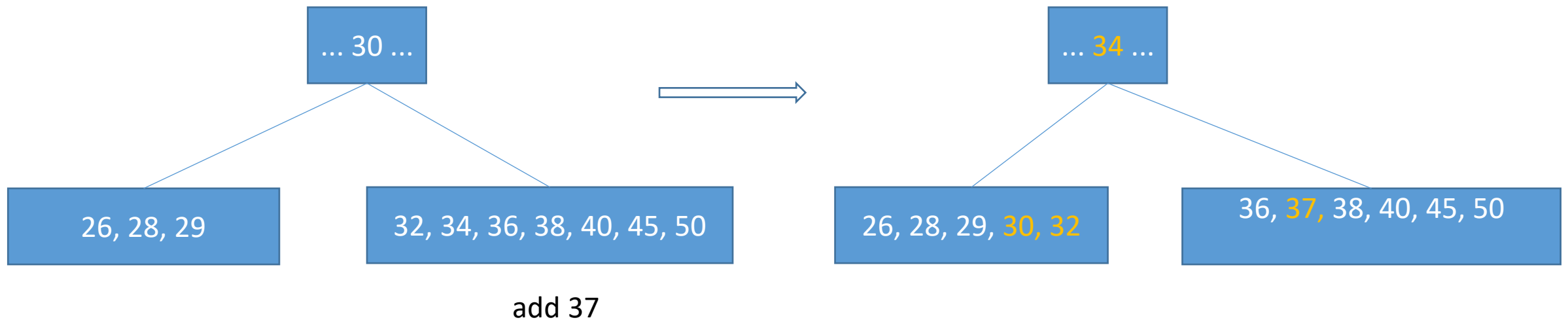
- B-tree of order m
 - adding a new value
 - since the node's capacity is exceeded, it is split into nodes 32, 34, and 37, 38, and 36 is attached to the parent node (with values 25, 30, 40, 50)
 - in turn, the parent must be split into 2 nodes (values 25, 30, and 40, 50), and 36 is attached to its parent



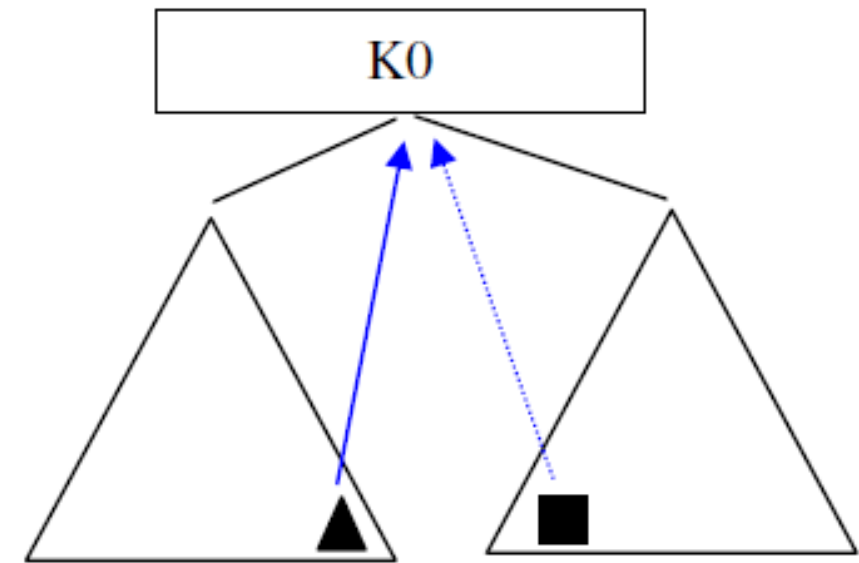
- B-tree of order m
 - adding a new value
 - optimizations
 - before performing a split - analyze whether one or more values can be transferred from the current node (with $m-1$ values) to a sibling node
 - e.g., B-tree of order 5 (non-terminal node - between 2 and 4 values, i.e., between 3 and 5 subtrees):



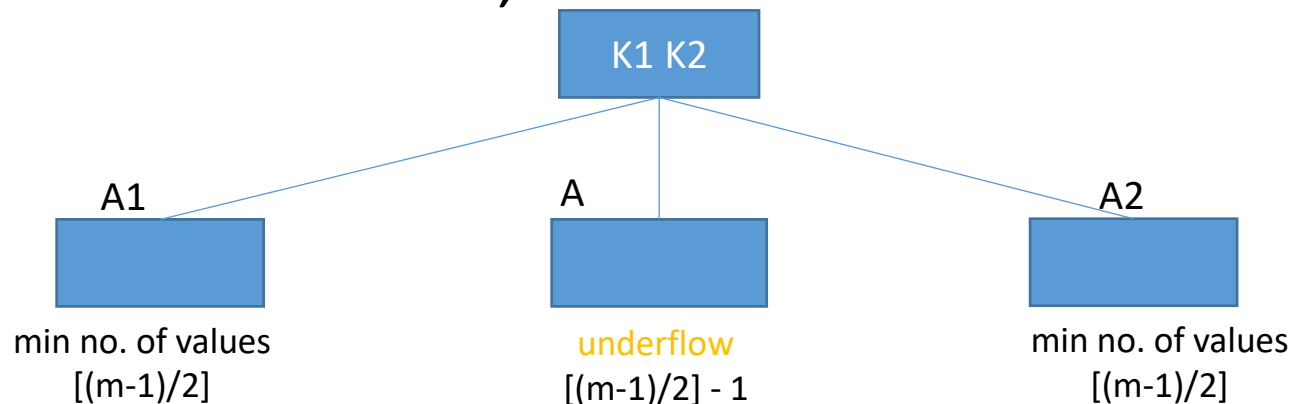
- B-tree of order m
 - adding a new value
 - optimizations
 - e.g., B-tree of order 8 (non-terminal node - between 3 and 7 values, i.e., between 4 and 8 subtrees):



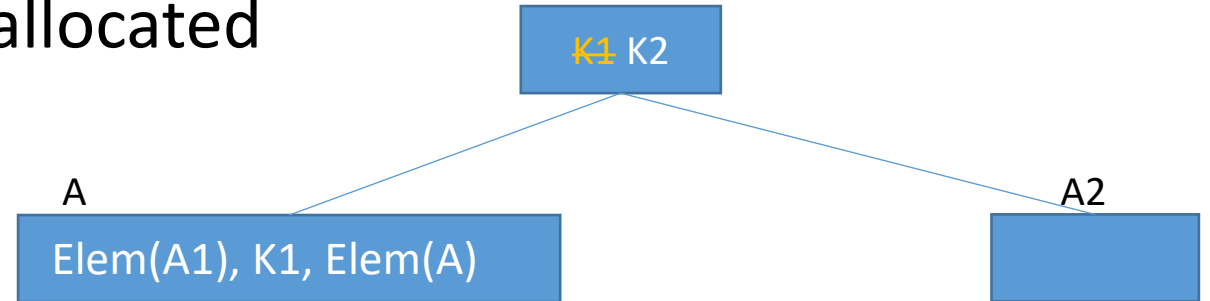
- B-tree of order m
 - removing a value
 - a node can have at most m subtrees, i.e., a maximum of m-1 values, and at least $\lceil \frac{m}{2} \rceil$ subtrees, i.e., at least $\lceil \frac{m}{2} \rceil - 1 = \lceil \frac{m-1}{2} \rceil$ values
 - when eliminating a value from a node, an underflow can occur (the node can end up with less values than the required minimum)
 - eliminate value K_0
 1. search for K_0 ; if it doesn't exist, the algorithm ends
 2. if K_0 is found in a non-terminal node (like in the figure on the right), K_0 is replaced with a *neighbor value* from a terminal node (this value can be chosen between 2 values from the trees separated by K_0)



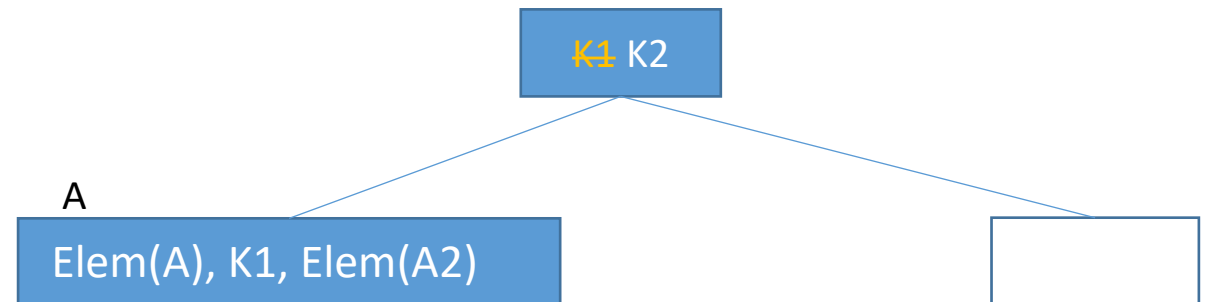
- B-tree of order m
 - removing a value
 3. perform this step until case a / b occurs
 - a. if the current node (from which a value is removed) is the root or underflow doesn't occur, the value is eliminated; the algorithm ends
 - b. if the delete operation causes an underflow in the current node (A), but one of the sibling nodes (left / right - B) has at least 1 extra value, values are transferred between A and B via the parent node; the algorithm ends
 - c. if there is an underflow in A, and sibling nodes A1 and A2 have the minimum number of values, nodes must be concatenated:



- B-tree of order m
 - removing a value
 - if A1 exists, A1 is merged with A and value K1 (separating A1 from A); the node at address A1 is deallocated



- if there is no A1 (A is the first subtree for its parent), A is merged with A2 and K1 (separating A from A2); the node at address A2 is deallocated



- case 3 is then analyzed for the parent node
- if the root is reached and has no values, it is removed and the current node becomes the root

- B-tree of order m
 - obs. a block stores a node from a B-tree
- e.g.:
 - key size: 10b
 - record address / node address: 10b
 - NV value (number of values in the node): 2b
 - block size: 1024b (10b for the header)
- then: $2 + (m-1) * (10+10) + m * 10 = 1024 - 10 \Rightarrow m = 34$
- if the size of a block is 2048b and the other values are unchanged, then the order of the tree is $m = 68$, i.e., a node can have between 33 and 67 values

- B-tree of order m
- the maximum number of required blocks (from the file that stores the B-tree) when searching for a value - the maximum number of levels in the tree; for $m=68$, if the number of values is 1.000.000, then:
 - the root node (on level 0) contains at least 1 value (2 subtrees)
 - on the next level (level 1) - at least 2 nodes * 33 values/node = 66 values
 - level 2 – at least $2 \cdot 34$ nodes * 33 values/node = 2.244 values
 - level 3 – at least $2 \cdot 34 \cdot 34$ nodes * 33 values/node = 76.296 values
 - level 4 – at least $2 \cdot 34 \cdot 34 \cdot 34$ nodes * 33 values/node = 2.594.064 values, which is greater than the number of existing values => this level does not appear in the tree

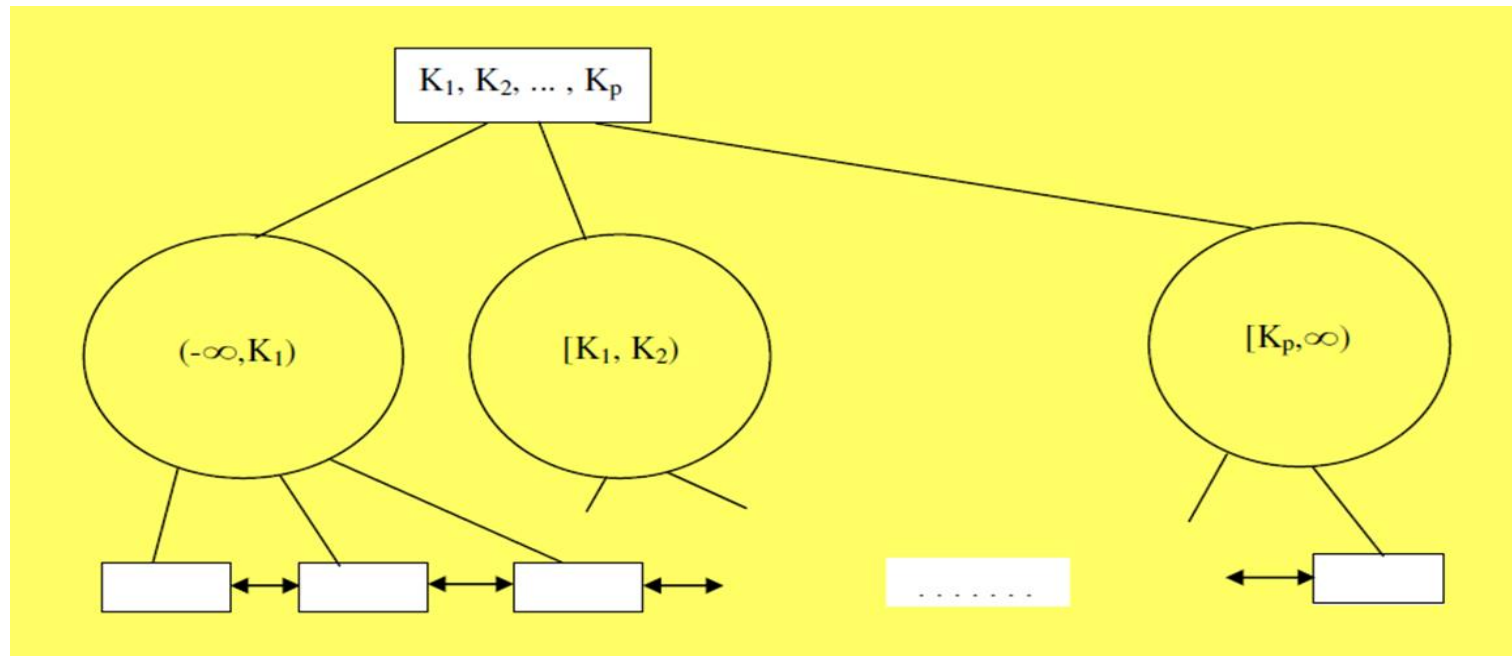
=> at most 4 levels in the tree

- after at most 4 block reads and a number of comparisons in main memory, it can be determined whether the value exists (the corresponding record's address can then be retrieved) or the search was unsuccessful

B+ trees

B+ tree

- B-tree variant
- last level contains all values (key values and the records' addresses)
- some key values can also appear in non-terminal nodes, without the records' addresses; their purpose is to separate values from terminal nodes (guide the search)
- terminal nodes are maintained in a doubly linked list (data can be easily scanned)



- example



B+ tree

- storing a B+ tree
 - B-tree methods
- operations (algorithms)
 - B-tree

B+ tree - in practice

- concept of *order* - relaxed, replaced by a physical space criterion (for instance, nodes should be at least half-full)
- terminal / non-terminal nodes - different numbers of entries; usually, inner nodes can store more entries than terminal ones
- variable-length search key \Rightarrow variable-length entries \Rightarrow variable number of entries / page
- if alternative 3 is used ($\langle k, \text{rid_list} \rangle$) \Rightarrow variable-length entries (in the presence of duplicates), even if attributes are of fixed length

B+ tree - in practice

* prefix key compression

- larger key size => less index entries fit on a page, i.e., less children / index page => larger B+ tree height
- keys in index entries - just direct the search => often, they can be compressed
- adjacent index entries with search key values: *Meteiut*, *Mircqkjt*, *Morqwkj*
- compress key values: *Me*, *Mi*, etc
- what if the subtree also contains *Micfgjh*? => need to store *Mir* (instead of *Mi*)
- it's not enough to analyze neighbor index entries *Meteiut* and *Morqwkj*; the largest key value in *Mircqkjt*'s left subtree and the smallest key value in its right subtree must also be examined
- inserts / deletes - modified correspondingly

B+ tree - in practice

- values found in practice
 - order – 200
 - fill factor (node) – 67%
 - fan-out – 133
 - capacity
 - height 4: $133^4 = 312,900,721$
 - height 3: $133^3 = 2,352,637$
- top levels can often be kept in the BP
 - 1st level – 1 page (8KB)
 - 2nd level – 133 pages (approx. 1MB)
 - 3rd level – $133^2 = 17689$ pages (approx. 133 MB)

B+ tree - benefits

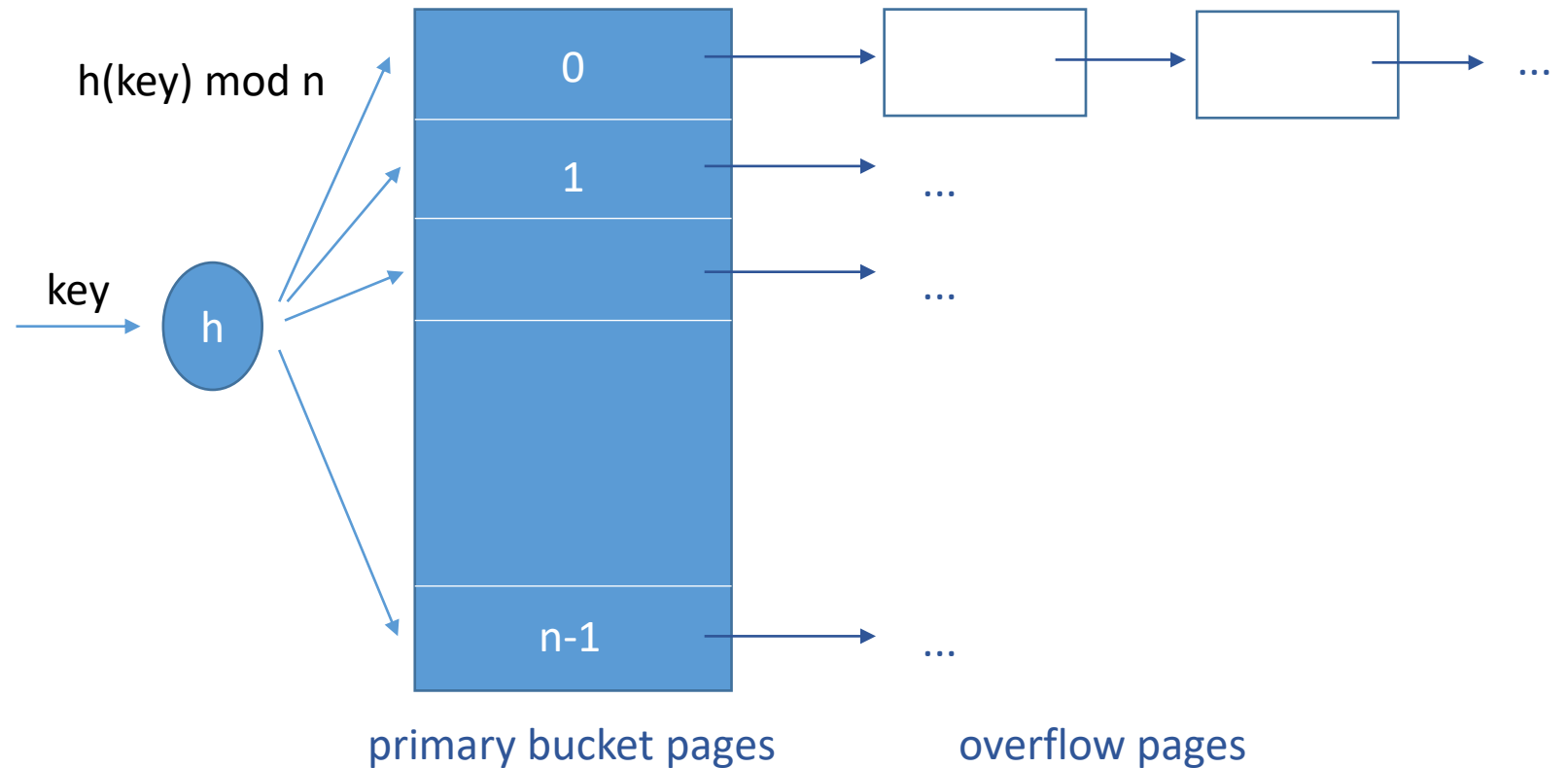
- balanced index => uniform search time
- rarely more than 3-5 levels, the top levels can be kept in main memory => only a few I/O operations are needed to search for a record
- widely used in DBMSs
- ideal for range selections, good for equality selections as well

Hash-Based Indexing

- hashing function
 - maps search key values into a range of bucket numbers
- hashed file
 - search key (field(s) of the file)
 - records grouped into *buckets*
 - determine record r's bucket
 - apply hash function to search key
 - quick location of records with given search key value
 - example: file hashed on *EmployeeName*
 - Find employee *Popescu*.
- ideal for equality selections

Static Hashing

- * static hashing
 - buckets 0 to $n-1$
 - bucket
 - one primary page
 - possibly extra overflow pages
 - data entries in buckets
 - $a_1/a_2/a_3$



- * static hashing
 - search for a data entry
 - apply hashing function to identify the bucket
 - search the bucket
 - possible optimization
 - entries sorted by search key

- * static hashing
 - add a data entry
 - apply hashing function to identify the bucket
 - add the entry to the bucket
 - if there is no space in the bucket:
 - allocate an overflow page
 - add the data entry to the page
 - add the overflow page to the bucket's overflow chain

- * static hashing
- delete a data entry
 - apply hashing function to identify the bucket
 - search the bucket to locate the data entry
 - remove the entry from the bucket
 - if the data entry is the last one on its overflow page:
 - remove the overflow page from its overflow chain
 - add the page to a free pages list

- * static hashing
- good hashing function
 - few empty buckets
 - few records in the same bucket
 - i.e., key values are uniformly distributed over the set of buckets
 - good function in practice
 - $h(val) = a * val + b$
 - $h(val) \bmod n$ to identify bucket, for buckets numbered 0..n-1

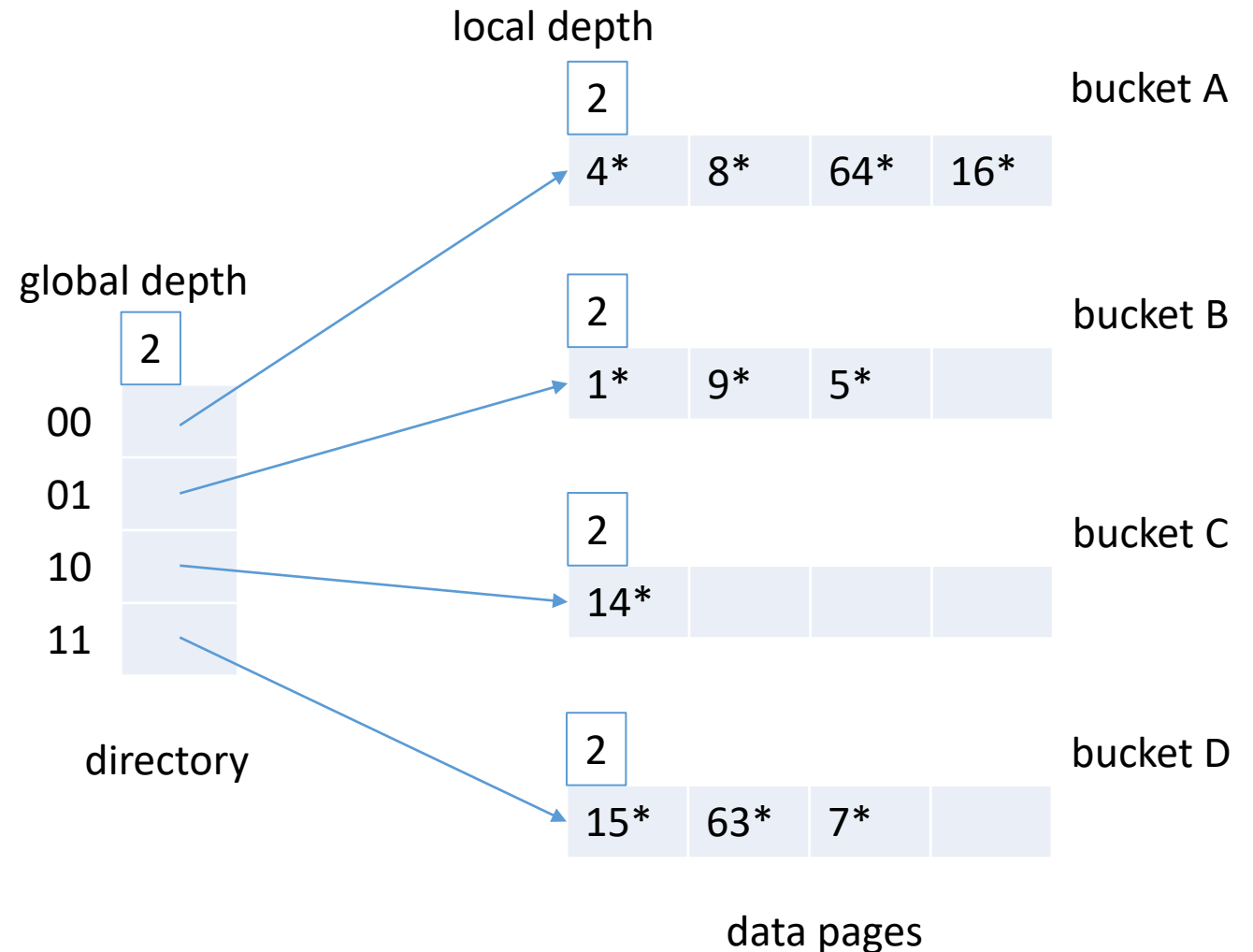
- * static hashing
 - number of buckets known when the file is created
 - ideally
 - search: 1 I/O
 - insert / delete: 2 I/Os
 - file grows a lot => overflow chains; long chains can significantly affect performance
 - tackle overflow chains
 - initially, pages - 80% full
 - create a new file with more buckets
 - file shrinks => wasted space

- * static hashing
 - main problem
 - fixed number of buckets
 - solutions
 - periodic rehash
 - dynamic hashing

Extendible Hashing

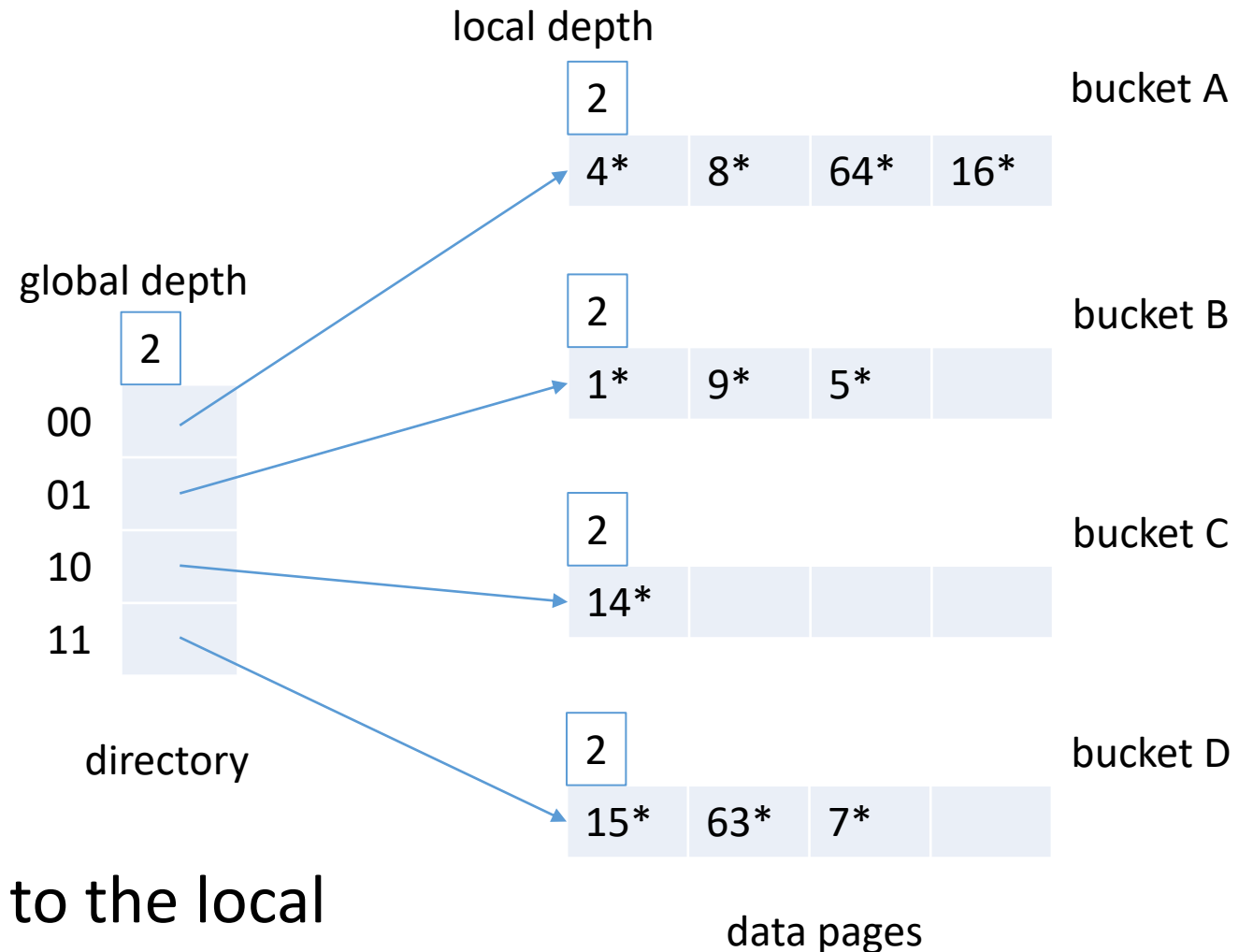
* extendible hashing

- dynamic hashing technique
- directory of pointers to buckets
- double the size of the number of buckets
 - double the directory
 - split overflowing bucket
- directory: array of 4 elements
- directory element: pointer to bucket
- entry r with key value K
- $h(K) = (... a_2 a_1 a_0)_2$
- $nr = a_1 a_0$, i.e., last 2 bits in $(... a_2 a_1 a_0)_2$, nr between 0 and 3
- $directory[nr]$: pointer to desired bucket



* extendible hashing

- global depth gd of hashed file
 - number of bits at the end of hashed value interpreted as an offset into the directory
 - kept in the header
 - depends on the size of the directory
 - 4 buckets $\Rightarrow gd = 2$
 - 8 buckets $\Rightarrow gd = 3$
- initially, the global depth is equal to the local depth of every bucket



* extendible hashing

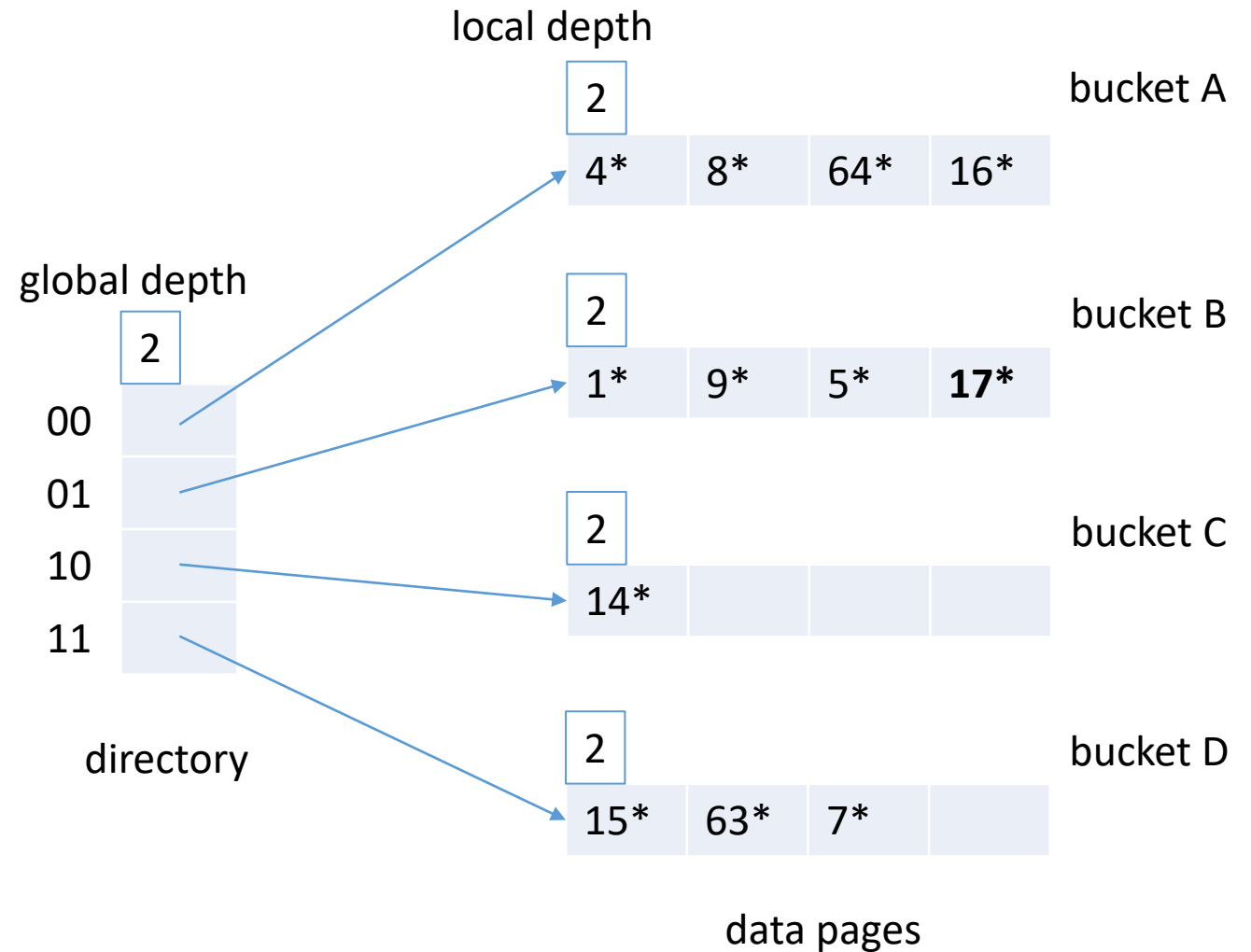
- insert entry

- find bucket

- a. bucket has free space => the new value can be added

- example: add data entry with hash value 17 to bucket B

obs. data entry with hash value 17 is denoted as 17*



* extendible hashing

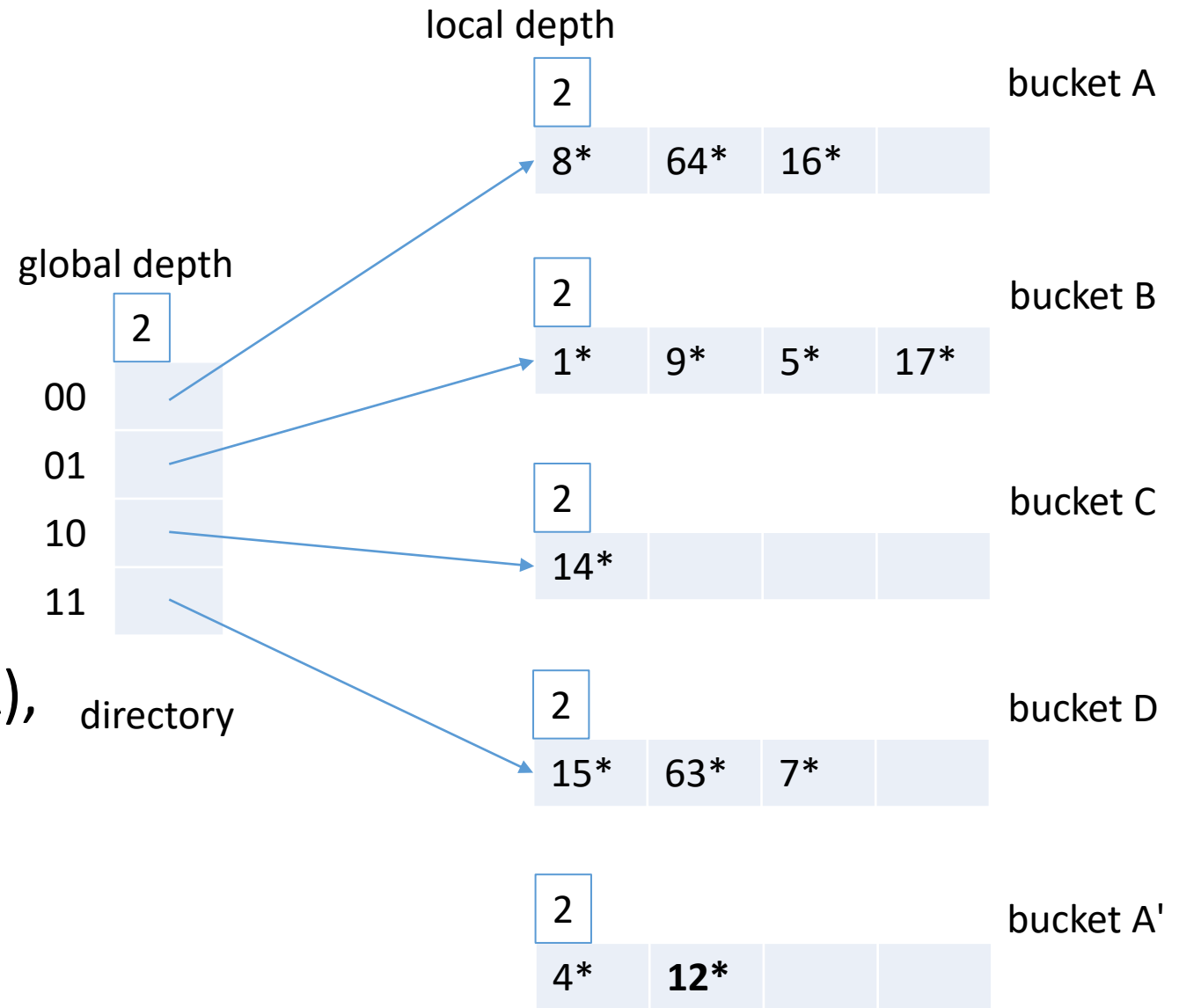
- insert entry

- b. bucket is full

- example: add entry 12^* , bucket A full

- split bucket A

- allocate new bucket A'
 - redistribute entries across A & A' (the split image of A), by taking into account the last 3 bits of $h(K)$

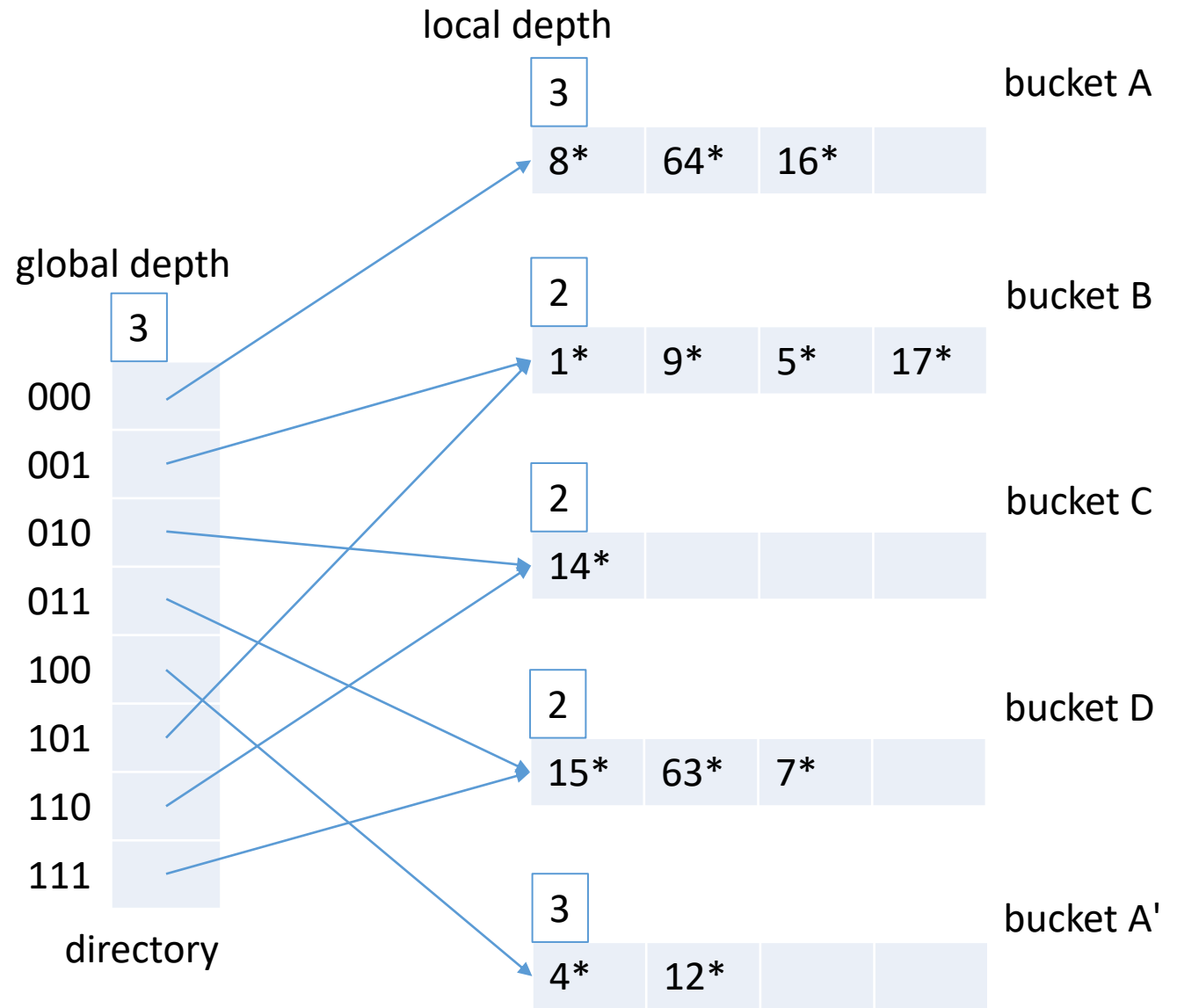


* extendible hashing

- insert entry

- b. bucket is full

- if $gd = \text{local depth of bucket being split} \Rightarrow \text{double the directory, } gd++$
 - 3 bits are needed to discriminate between A & A', but the directory has only enough space to store numbers that can be represented on 2 bits, so it is doubled
 - increment local depth of bucket: $LD(A) = 3$
 - assign new local depth to bucket's split image: $LD(A') = 3$



* extendible hashing

- insert entry

- b. bucket is full

- *corresponding elements*

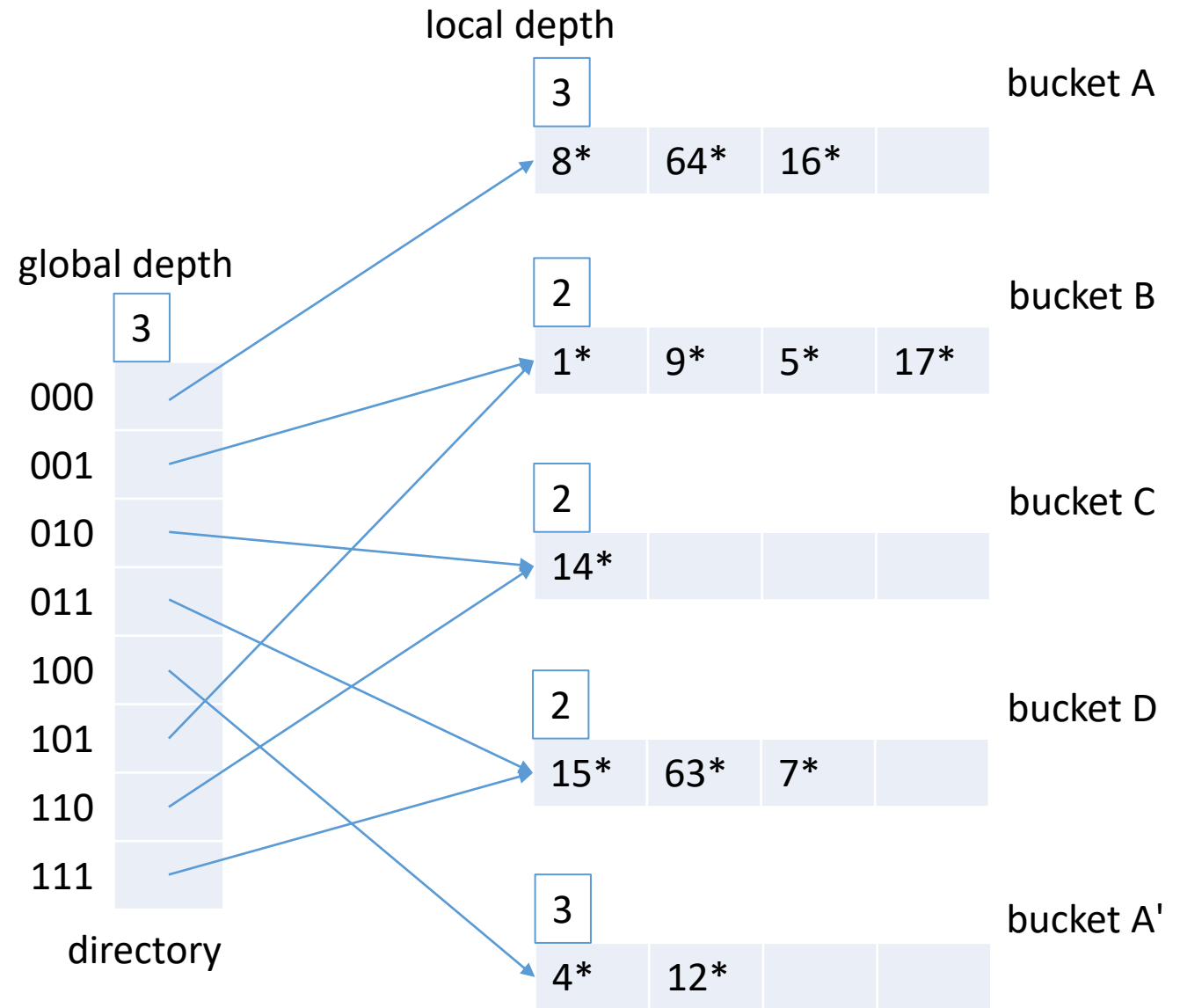
- 000, 100

- 001, 101

- 010, 110

- 011, 111

- point to the same bucket,
except for 000 and 100,
which point to A and
split image A', respectively



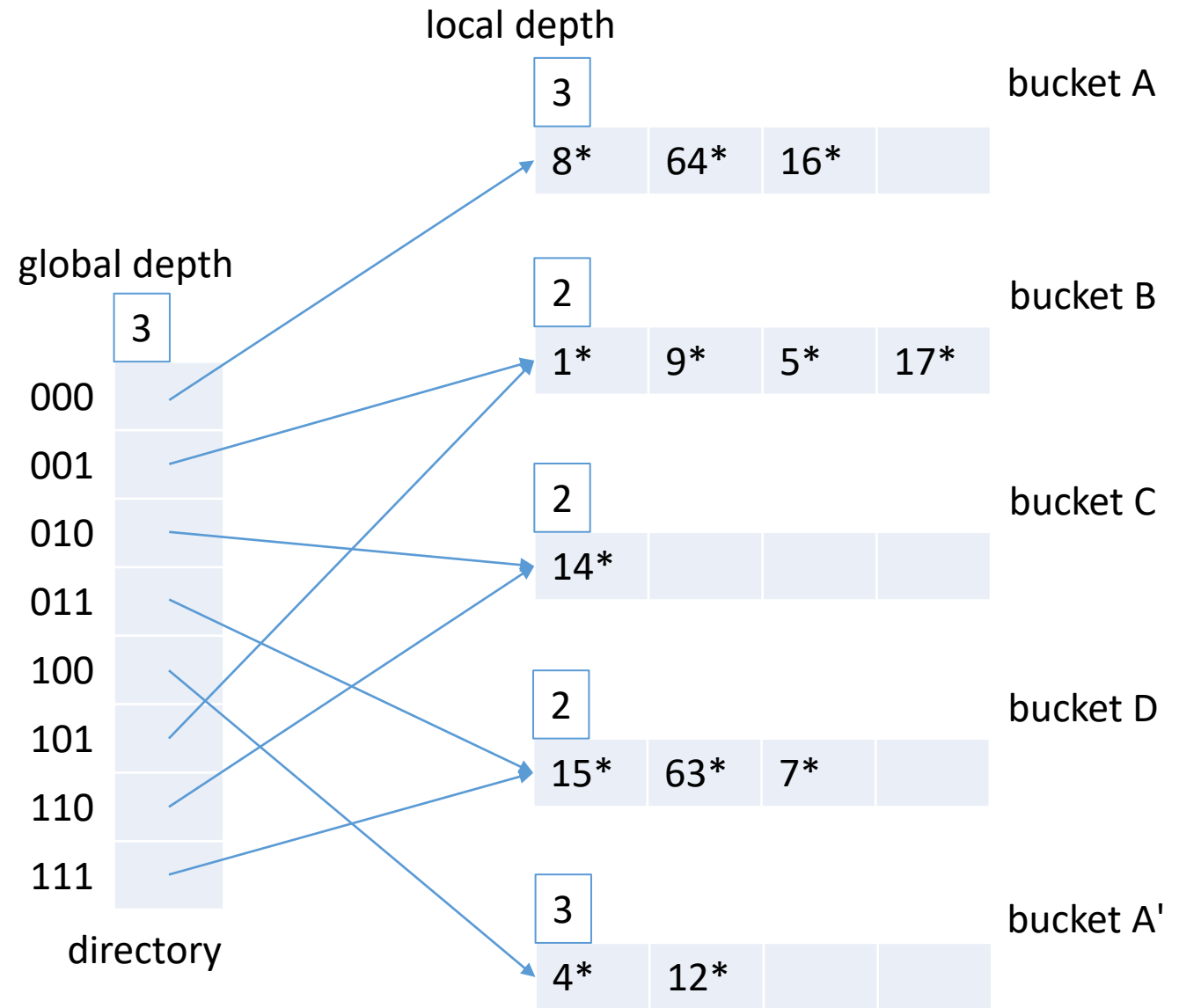
* extendible hashing

- insert entry

- b. bucket is full

- example: add 21^*

- it belongs to bucket B, which is already full, but its local depth is 2 and $gd = 3$



* extendible hashing

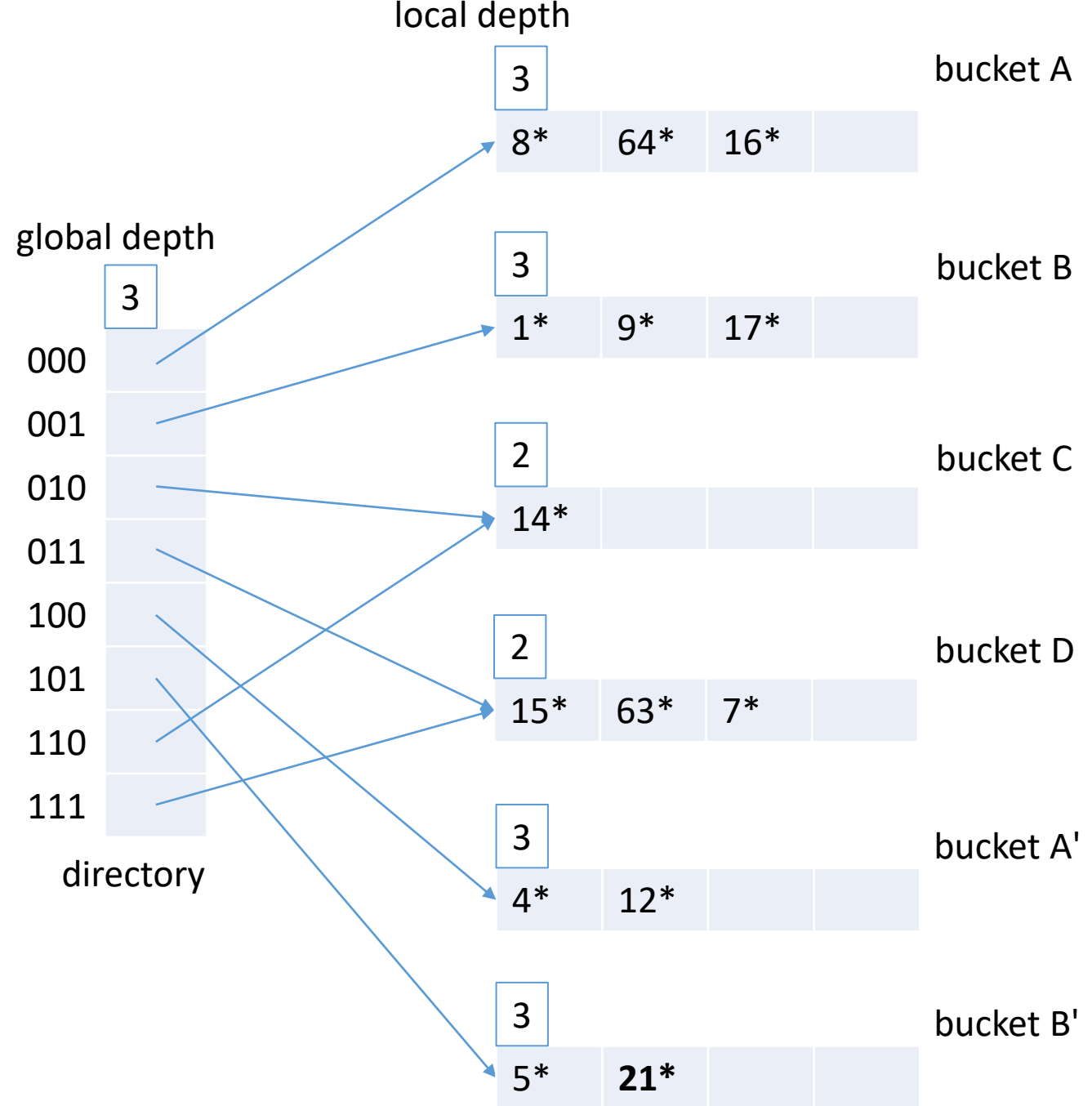
- insert entry

- b. bucket is full

- example: add 21^*

- it belongs to bucket B, which is already full, but its local depth is 2 and $gd = 3$

=> split B, redistribute entries, increase local depth for B and its split image; directory isn't doubled, gd doesn't change



- * extendible hashing
 - search for entry with key value K_0
 - compute $h(K_0)$
 - take last gd bits to identify directory element
 - search corresponding bucket
 - delete entry
 - locate & remove entry
 - if bucket is empty:
 - merge bucket with its split image, decrement local depth
 - if every directory element points to the same bucket as its split image:
 - halve the directory
 - decrement global depth

* extendible hashing

- obs 1. 2^{gd-l_d} elements point to a bucket Bk with local depth l_d
 - if $gd=l_d$ and bucket Bk is split \Rightarrow double directory
- obs 2. manage collisions - overflow pages
- bucket split accompanied by directory doubling
 - allocate new bucket page nBk
 - write nBk and bucket being split
 - double directory array (which should be much smaller than file, since it has 1 page-id / element)
 - if using *least significant bits* (last gd bits) \Rightarrow efficient operation:
 - copy directory over
 - adjust split buckets' elements

- * extendible hashing
 - equality selection
 - if directory fits in memory:
 - => 1 I/O (as for Static Hashing with no overflow chains)
 - otherwise
 - 2 I/Os
- example: 100 MB file, entry = 50 bytes => 2.000.000 entries
- page size = 8 KB => approx. 160 entries / bucket
- => need $2.000.000 / 160 = 12.500$ directory elements

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