

Lecture 1 - Introduction in declarative programming.

Recursion

Official web site: www.cs.ubbcluj.ro/~hfpop/pfl

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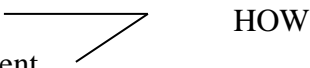
Examples of recursion

References

Czibula, G., Pop, H.F., Elemente avansate de programare în Lisp și Prolog. Aplicații în Inteligența Artificială., Ed. Albastră, Cluj-Napoca, 2012

Programming and programming languages

➤ LANGUAGES

- **Procedural (imperative) - high level languages**
 - Fortran, Cobol, Algol, Pascal, C, ...
 - program - sequence of instructions
 - the assignment statement, control structures - for the control of sequential execution, branching and cycling
 - the role of the programmer - “what” and “how”
 1. to describe what is to be calculated
 2. to organize the calculation
 3. to organize memory management
 - !!! it is argued that the assignment instruction is dangerous in high-level languages, just as the GO TO instruction was considered dangerous for structured programming in the '68s.
- **Declarative (descriptive, applied) - very high level languages**
 - based on expressions
 - expressive, easy to understand (have a simple basis), extensible

- programs can be seen as descriptions that state information about values, rather than instructions to determine values or effects.
 - they give up instructions
 1. thus they protects users from making too many mistakes
 2. they are generated from mathematical principles - analysis, design, specification, implementation, abstraction and reasoning (deductions of consequences and properties) become more and more formal activities.
 - the role of the programmer - “what” (not “how”)
 - two classes of declarative languages
 1. **functional languages** (eg Lisp, ML, Scheme, Haskell, Erlang)
 - focus on values of data described by expressions (built through applications of functions and definitions of functions), with automatic evaluation of expressions
 2. **logical languages** (e.g. Prolog, Datalog, Parlog), which focus on logical assertions that describe the relationships between data values and automatic derivations of answers to questions, starting from these assertions.
 - applications in Artificial Intelligence – automated proofs, natural language processing and speech understanding, expert systems, machine learning, intelligent agents, etc.
- Multiparadigm languages: **F#, Python, Scala** (imperative, functional, object oriented)
 - Interactions between declarative and imperative languages - declarative languages that provide interfaces with imperative languages (eg C, Java): SWI Prolog, GNU Prolog, etc.
 - **Logtalk** – integrates logic and object-oriented programming
 - Logic programming in **Python**:
 - **Karen**
 - **SymPy** – library for symbolic computations

Recursion

- general mechanism to elaborate programs
- recursion arose from practical necessities (direct transcription of recursive mathematical formulas; see Ackermann's function)
- recursion is the mechanism by which a subprogram (function, procedure) calls itself
 - two types of recursion: **direct** or **indirect**
- **!!! Result**
 - any calculable function can therefore be expressed and programmed) in terms of recursive functions
- two things to consider in describing a recursive algorithm: **the recursive rule** and **the termination condition**
- **advantage** of recursion: source text that is extremely short and very clear.

- **disadvantage** of recursion: filling the stack segment if the number of recursive calls, respectively of the formal and local parameters of the recursive subprograms is high enough.
 - declarative languages have specific mechanisms to optimize the recursion (see the mechanism of tail recursion in Prolog).

Examples of recursion

Remarks

- a list is a sequence of items ($l_1 l_2 \dots l_n$)
- the empty list (with 0 elements) is denoted by \emptyset
- adding an item to a list is denoted by \oplus

1. Create list (1,2,3, ... n)

a) directly recursive

$$createLista(n) = \begin{cases} \emptyset & \text{daca } n = 0 \\ createLista(n-1) \oplus n & \text{altfel} \end{cases}$$

b) using a recursive auxiliary function to create the sublist (i, i + 1, ..., n)

// create the list consisting of the elements i, i + 1, ..., n

Recursive mathematical model

$$create(i, n) = \begin{cases} \emptyset & \text{daca } i > n \\ i \oplus create(i+1, n) & \text{altfel} \end{cases}$$

// create the list consisting of elements 1, 2, ..., n

$$createLista(n) = create(1, n)$$

Pseudocode

Data representation : singly linked list with dynamic allocation of nodes.

NodeLSI

e: TElement // useful information of node

urm: ^NodeLSI // address the following node is stored

LSI

prim: ^NodeLSI // address of the first node in the list

Function createNodeLSI(e)

{pre: e: TElement }

{post: return a ^NodeLSI having e as useful information }

{ allocates a storing space for a NodeLSI }

{p: ^NodLSI }

allocate(p)

[p].e \leftarrow e

[p].urm \leftarrow NIL

{ result returned by the function }

createNodeLSI \leftarrow p

EndFunction

Function create(i, n)

{post: return a ^NodLSI, pointer towards the head of the linked list formed by }

{ elements i, i+1, ..., n }

If i > n **then**

create \leftarrow NIL

else

{ allocate a storage space for a NodeLSI with usefun information e }

q \leftarrow **createNodeLSI**(i)

{ create the link between node q and the head of the linked list formed }

{ by elements i+1, ..., n }

[q].urm \leftarrow **create**(i+1, n)

create \leftarrow q

EndIf

EndFunction

Function createList(n)

{post: return a ^NodeLSI, pointer towards the head of the linked list formed by }

{ elements 1, 2,..., n }

createList \leftarrow **create**(1, n)

EndFunction

2. Given a natural number n, calculate the sum $1 + 2 + 3 + \dots + n$.

a) directly recursive

$$suma(n) = \begin{cases} 0 & \text{daca } n = 0 \\ n + suma(n - 1) & \text{altfel} \end{cases}$$

b) using a recursive auxiliary function for calculating the amount $and + (i + 1) + \dots + n$

$$suma_aux(n, i) = \begin{cases} 0 & \text{daca } i > n \\ i + suma(n, i + 1) & \text{altfel} \end{cases}$$

$$suma(n) = suma_aux(n, 0)$$

3. Add an item at the end of a list.

// build the list (l1, l2,..., ln, e)

$$adaug(e, l_1 l_2 \dots l_n) = \begin{cases} (e) & \text{daca } l \text{ e vida} \\ l_1 \oplus adaug(e, l_2 \dots l_n) & \text{altfel} \end{cases}$$

4. Search for an element in a list.

$$apare(E, l_1 l_2 \dots l_n) = \begin{cases} fals & \text{daca } l \text{ e vida} \\ adevarat & \text{daca } l_1 = E \\ apare(E, l_2 \dots l_n) & \text{altfel} \end{cases}$$

5. Count the number of occurrences of an item in the list.

$$nrap(E, l_1 l_2 \dots l_n) = \begin{cases} 0 & \text{daca } l \text{ e vida} \\ 1 + nrap(E, l_2 \dots l_n) & \text{daca } l_1 = E \\ nrap(E, l_2 \dots l_n) & \text{altfel} \end{cases}$$

6. Check if a numeric list is set.

$$eMultime(l_1 l_2 \dots l_n) = \begin{cases} \text{adevarat} & \text{daca } l \text{ e vida} \\ \text{fals} & \text{daca } l_1 \in (l_2 \dots l_n) \\ eMultime(l_2 \dots l_n) & \text{altfel} \end{cases}$$

7. Transform a numeric list into a set.

$$multime(l_1 l_2 \dots l_n) = \begin{cases} \phi & \text{daca } l \text{ e vida} \\ multime(l_2 \dots l_n) & \text{daca } l_1 \in (l_2 \dots l_n) \\ l_1 \oplus multime(l_2 \dots l_n) & \text{altfel} \end{cases}$$

8. Return the inverse of a list.

$$invers(l_1 l_2 \dots l_n) = \begin{cases} \phi & \text{daca } l \text{ e vida} \\ invers(l_2 \dots l_n) \oplus l_1 & \text{altfel} \end{cases}$$

9. Remove all occurrences of an item from a list.

$$stergera(E, l_1 l_2 \dots l_n) = \begin{cases} \phi & \text{daca } l \text{ e vida} \\ l_1 \oplus stergera(E, l_2 \dots l_n) & \text{daca } l_1 \neq E \\ stergera(E, l_2 \dots l_n) & \text{altfel} \end{cases}$$

10. Return the k-th element of a list (k >= 1).

$$element(l_1 l_2 \dots l_n, k) = \begin{cases} \phi & \text{daca } l \text{ e vida} \\ l_1 & \text{daca } k = 1 \\ element(l_2, \dots, l_n, k-1) & \text{altfel} \end{cases}$$

11. Return the difference between two sets represented as lists.

$$diferenta(l_1 l_2 \dots l_n, p_1 p_2 \dots p_m) = \begin{cases} \phi & \text{daca } l \text{ e vida} \\ diferenta(l_2 \dots l_n, p_1 p_2 \dots p_m) & \text{daca } l_1 \in (p_1 p_2 \dots p_m) \\ l_1 \oplus diferenta(l_2 \dots l_n, p_1 p_2 \dots p_m) & \text{altfel} \end{cases}$$

Homework

1. Verify whether a natural number is prime.
2. Calculate the sum of the first k elements in a numeric list ($l_1 l_2 \dots l_n$)
3. Remove the first k even numbers from a numeric list.