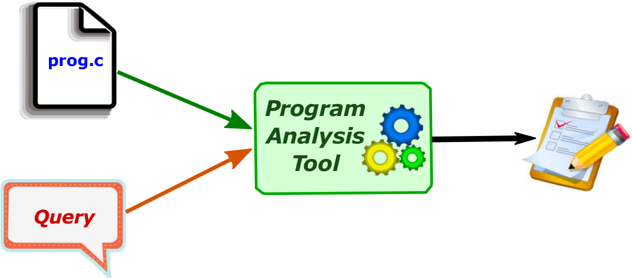
Advanced Programming Methods
Lecture 12
RECAP
Introduction in program analysis

Final Exam Organization

Introduction in program analysis

What is Program Analysis?

- Very broad topic, but generally speaking, automated analysis of program behavior
- Program analysis is about developing algorithms and tools that can analyze other programs



Applications of Program Analysis

- Bug finding. e.g., expose as many assertion failures as possible
- Security. e.g., does an app leak private user data?
- Verification. e.g., does the program always behave according to its specification?
- Compiler optimizations. e.g., which variables should be kept in registers for fastest memory access?
- Automatic parallelization. e.g., is it safe to execute different loop iterations on parallel?

Dynamic vs. Static Program Analysis

- Two flavors of program analysis:
  - Dynamic analysis: Analyzes program while it is running
  - Static analysis: Analyzes source code of the program

Static

+ reasons about all executions

- less precise

Dynamic

+ more precise

- results limited to observed executions

Testing /Formal Verification

A very crude dichotomy:

Testing	Formal Verification
Correct with respect to the *set of test inputs*, and reference system	Correct with respect to *all inputs*, with respect to a *formal specification*
Easy to perform	Decidability problems, Computational problems,
Dynamic	Static

Static Program Analysis

- Typical static analysis question: "Given source code of program P and desired property Q, does P exhibit Q in all possible executions?"

But this question is undecidable! This means

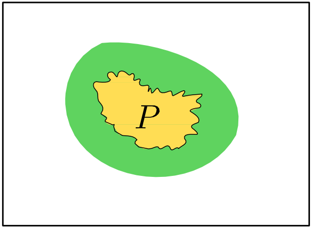
static analyses are either:

- Unsound: May say program is safe even though it is unsafe
- Sound, but incomplete: May say program is unsafe even though it is safe
- Non-terminating: Always gives correct answer when it terminates, but may run forever

Many static analysis techniques are sound but incomplete.

How to design Sound Static Analyses

Key idea: Overapproximate (i.e., abstract) program behavior



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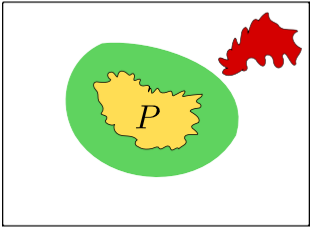
- Bad states outside over-approximation
  $\Rightarrow$ Program safe



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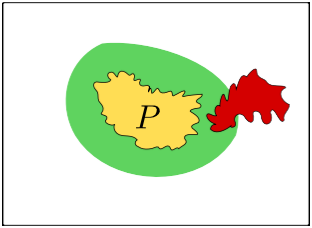

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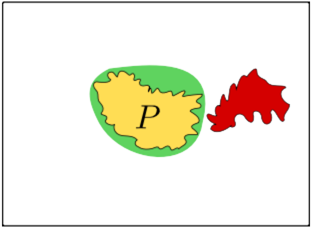

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⇒ Goal: Construct abstractions that are precise enough (i.e., few false alarms) and that scale to real programs

Examples of Abstractions

There is no "one size fits all" abstraction

- What information is useful depends on what you want to prove about the program!

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Application	Useful abstraction
No division-by-zero errors	zero vs. non-zero
Data structure verification	list, tree, graph, . . .
No out-of-bounds array accesses	ranges of integer variables

How to create Sound Abstractions

- Useful theory for understanding how to design sound static analyses is abstract interpretation
  - Seminal '77 paper by Patrick & Radhia Cousot
- Not a specific analysis, but rather a framework for designing sound-by-construction static analyses
- Let's look at an example: A static analysis that tracks the sign of each integer variable (e.g., positive, non-negative, zero etc.)

First Step: Design An Abstract Domain

- An abstract domain is just a set of abstract values we want to track in our analysis

- For our example, let's fix the following abstract domain:
  - pos: $\{x \mid x \in \mathbb{Z} \wedge x > 0\}$
  - zero: $\{0\}$
  - neg: $\{x \mid x \in \mathbb{Z} \wedge x < 0\}$
  - non-neg: $\{x \mid x \in \mathbb{Z} \wedge x \geq 0\}$

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- In addition, every abstract domain contains:
  - $\top$ (top): "Don't know", represents any value
  - $\perp$ (bottom): Represents empty-set

- Abstraction function (α) maps sets of concrete elements to the most precise value in the abstract domain

Second Step: Abstraction and Concretization Function

- Abstraction function (α) maps sets of concrete elements to the most precise value in the abstract domain
  - $\alpha(\{2, 10, 0\}) =$ non-neg
  - $\alpha(\{3, 99\}) =$ pos
  - $\alpha(\{-3, 2\}) = \top$

Second Step: Abstraction and Concretization Function

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  - $\alpha(\{2, 10, 0\}) = \text{non-neg}$
  - $\alpha(\{3, 99\}) = \text{pos}$
  - $\alpha(\{-3, 2\}) = \top$
- Concretization function (γ) maps each abstract value to sets of concrete elements
  - $\gamma(\text{pos}) = \{x \mid x \in \mathbb{Z} \wedge x > 0\}$

Lattices and Abstract Domains

- Concretization function defines partial order on abstract values:

$$A_1 \leq A_2 \text{ iff } \gamma(A_1) \subseteq \gamma(A_2)$$

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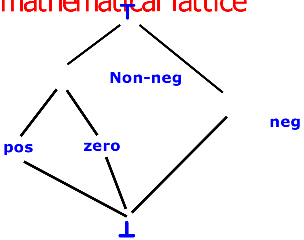
- Furthermore, in an abstract domain, every pair of elements has a lub and glb $\Rightarrow$ mathematical lattice



- Least upper bound of two elements is called their join – useful for reasoning about control flow in programs

Almost Inverses

- Important property of the abstraction and concretization function is that they are almost inverses:

$$\alpha(\gamma(A)) = A$$

$$\begin{tikzcd} A \arrow[r, bend left, "\gamma"] & C \arrow[l, bend left, "\alpha"] \end{tikzcd}$$

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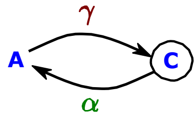

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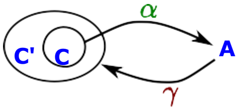

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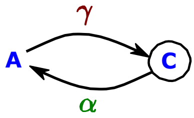

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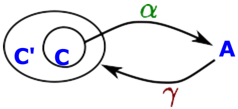

- This is called a Galois insertion and captures the soundness of the abstraction

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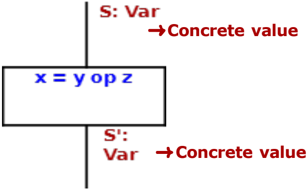
Operational Semantics



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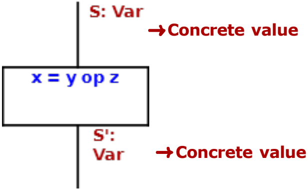

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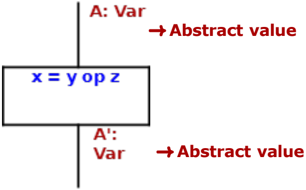


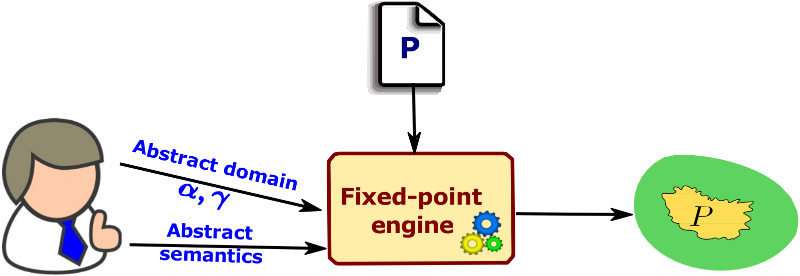
Back to Our Example

- For our sign analysis, we can define abstract transformer for x = y + z as follows:

	pos	neg	zero	non-neg	$\top$	$\bot$
pos	pos	$\top$	pos	pos	$\top$	$\bot$
neg	$\top$	neg	neg	$\top$	$\top$	$\bot$
zero	pos	neg	zero	non-neg	$\top$	$\bot$
non-neg	pos	$\top$	non-neg	non-neg	$\top$	$\bot$
$\top$	$\top$	$\top$	$\top$	$\top$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$	$\bot$

- To ensure soundness of static analysis, must prove that abstract semantics faithfully models concrete semantics

Putting It All Together



Fixed-point Computations

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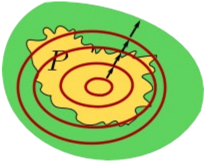

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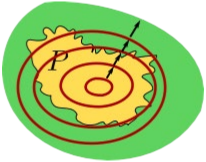
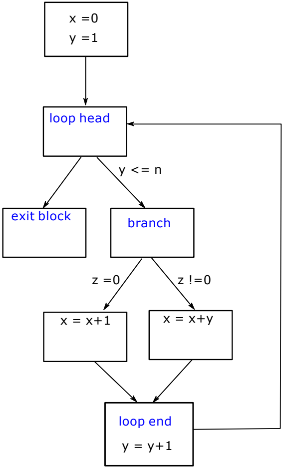


- Assuming correctness of your abstract semantics, the least fixed point is an overapproximation of the program!

Performing Least Fixed Point Computation

- Represent program as a control-flow graph



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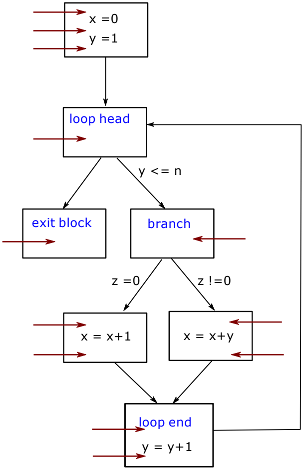

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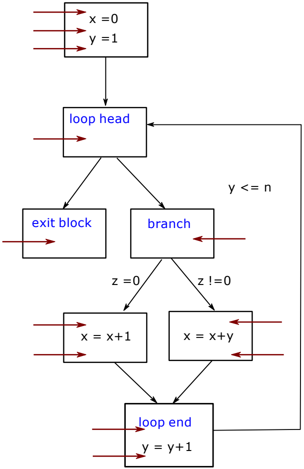

Performing Least Fixed Point Computation

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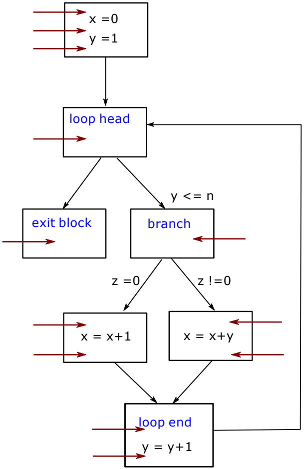

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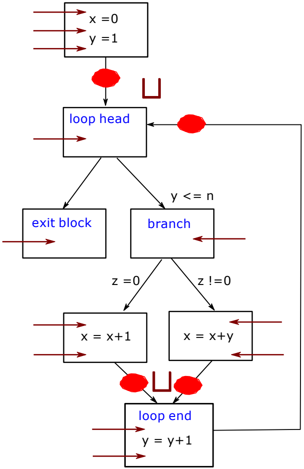

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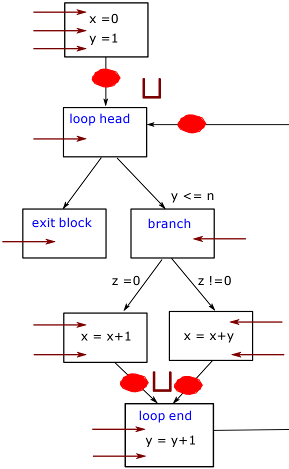
    - Symbolically execute each basic block using abstract semantics



An Example

```
x = 0;
y =0;

while(y <= n)
{
  if (z == 0)
    { x = x+1;
  }
  else {
    x = x + y;
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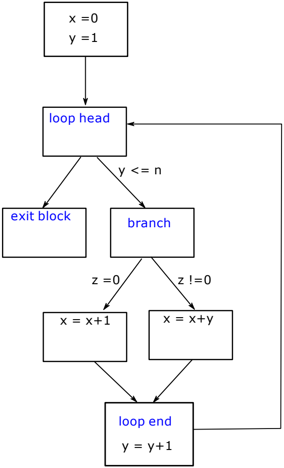

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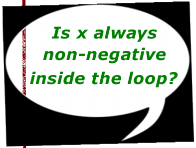

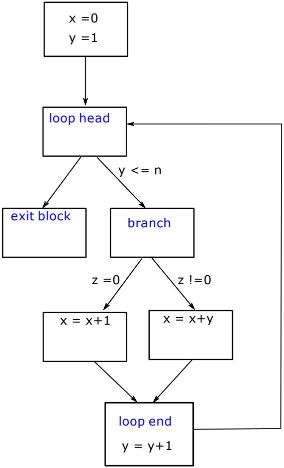

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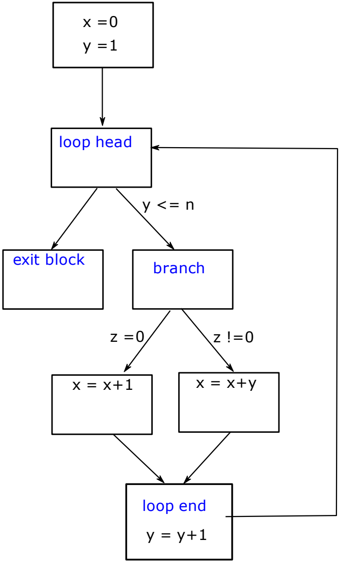

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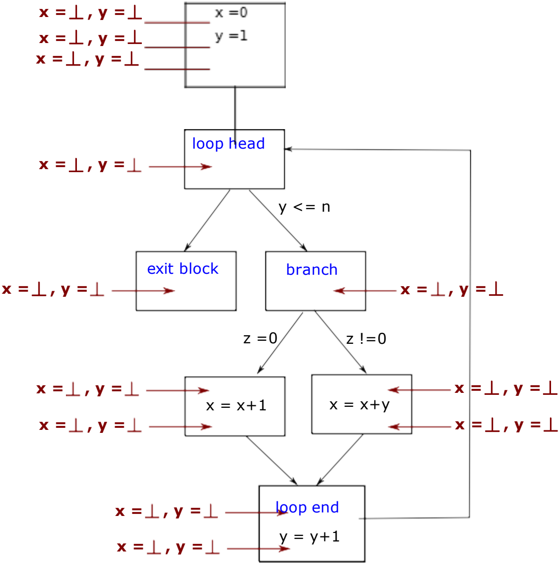

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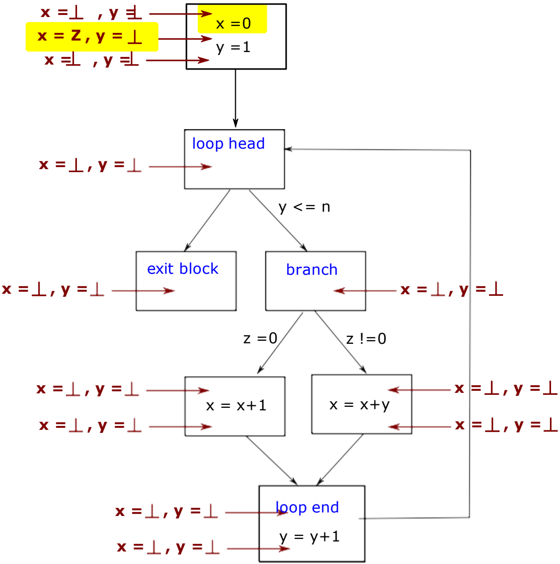

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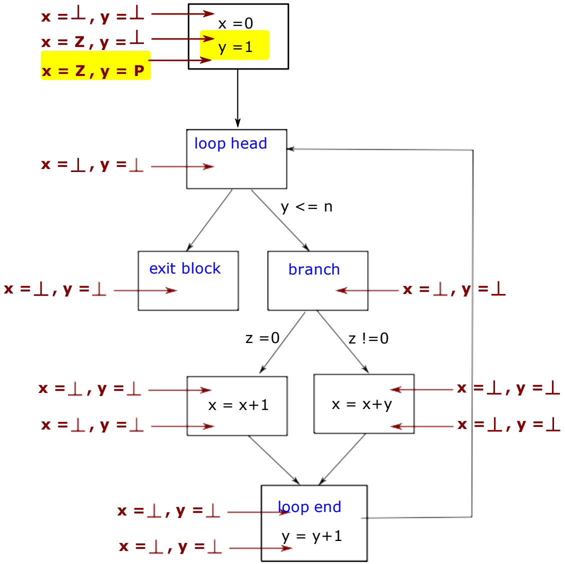

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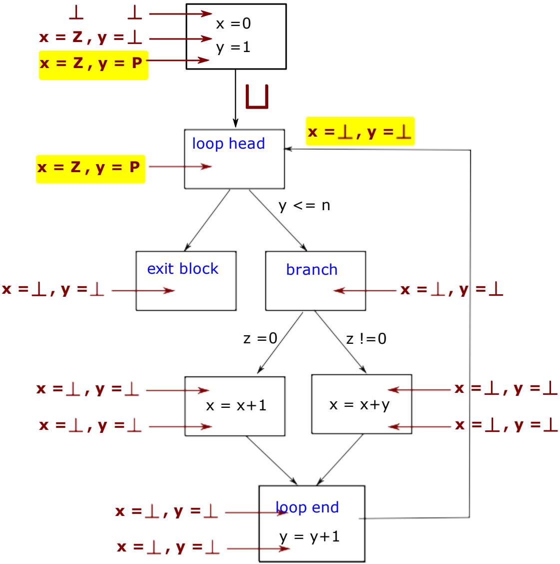

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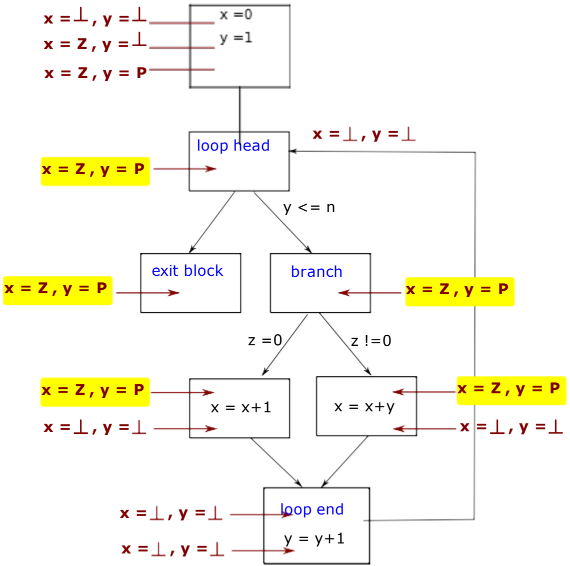

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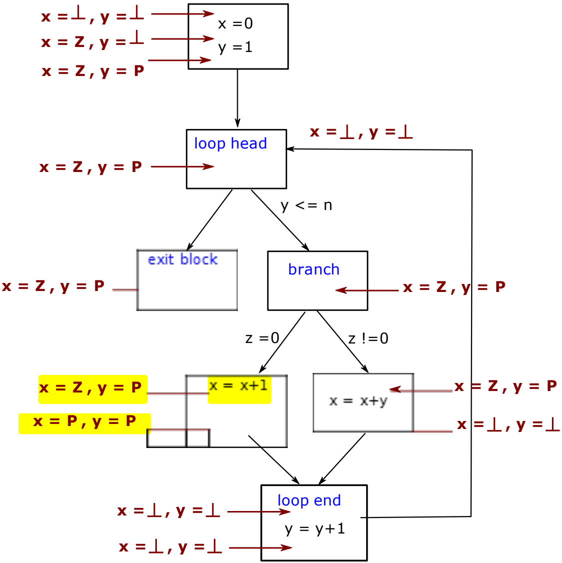

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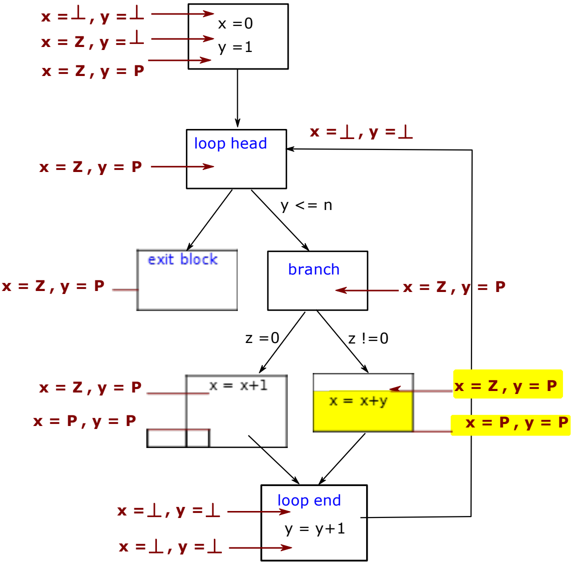

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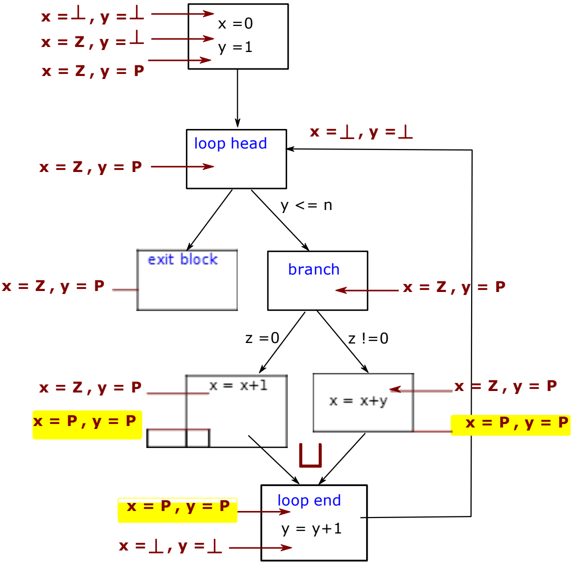

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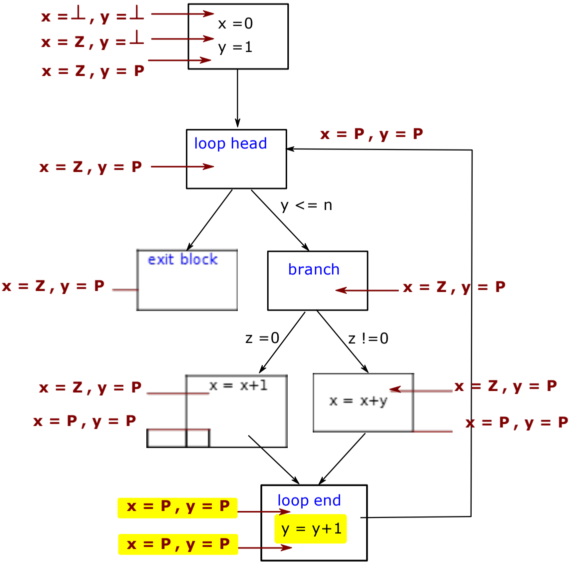

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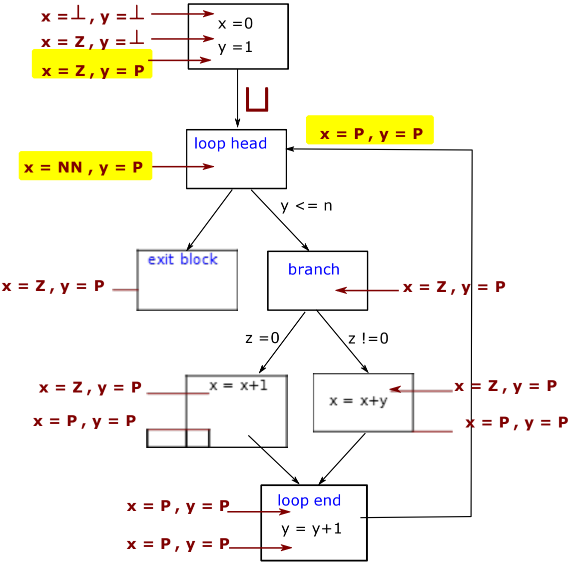

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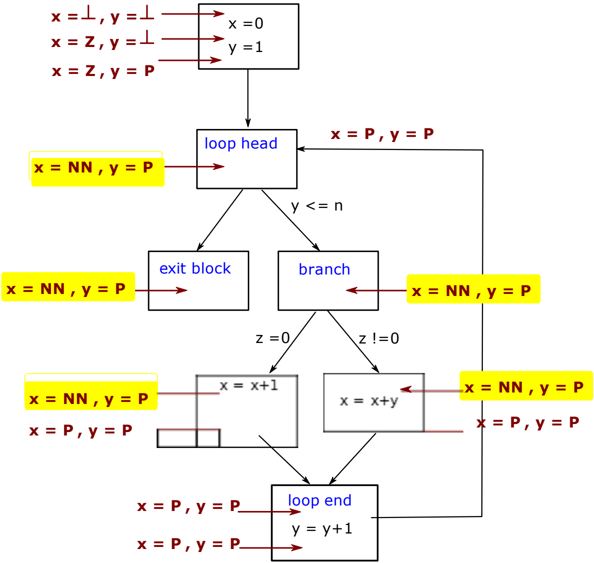

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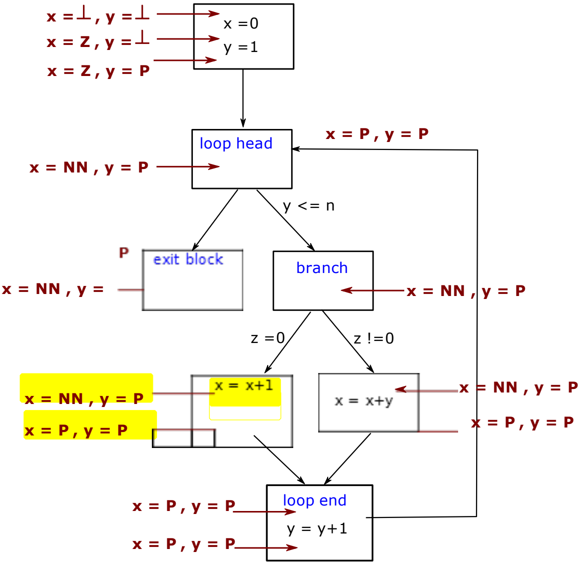

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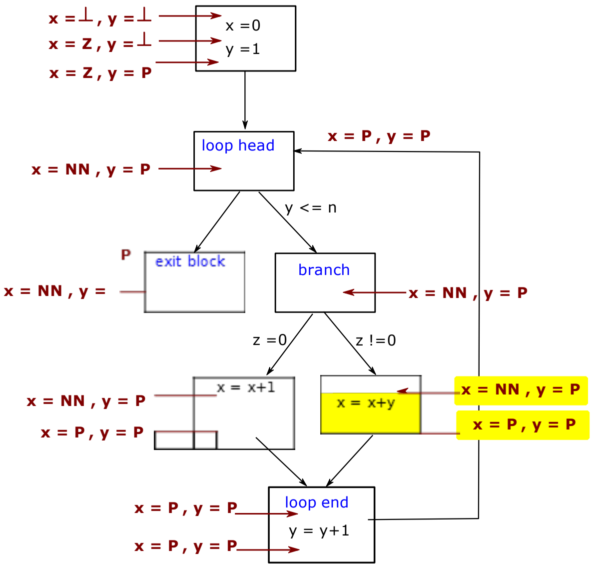

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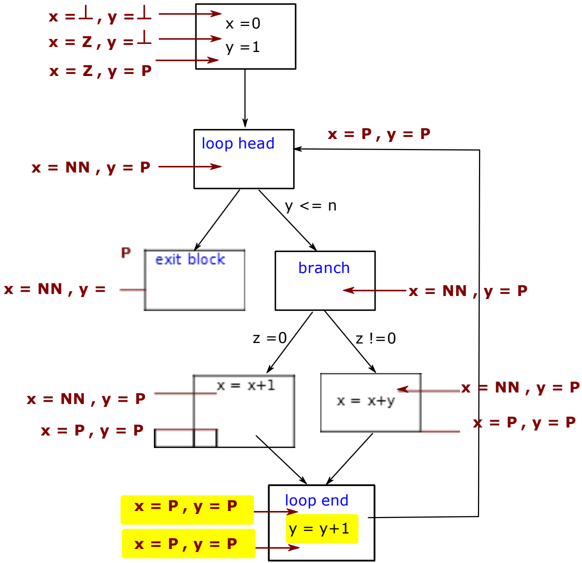

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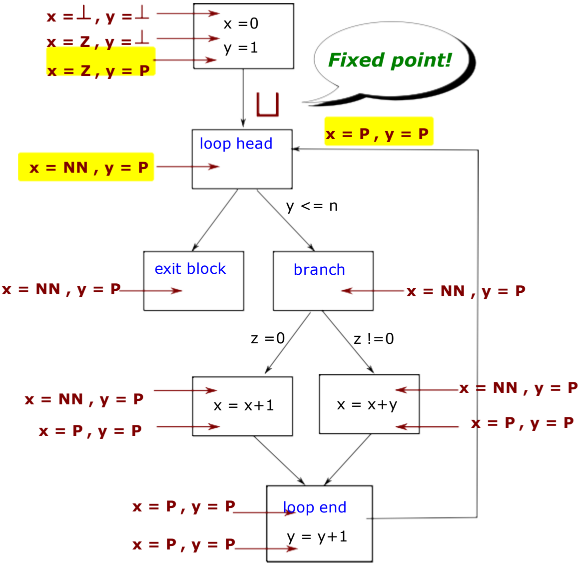

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- In this example, we quickly reached least fixed point – but does this computation always terminate?
  - Yes, assuming abstract domain forms complete lattice
  - This means every subset of elements (including infinite subsets) have a LUB
- Unfortunately, many interesting domains do not have this property, so we need widening operators for convergence.

Lessons To Take Away

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  - Fixed-point computation: Allows us to obtain sound overapproximation of the program
- But many static analyses also differ in several ways:
  - Flow (in)sensitivity: Some analyses only compute facts for the whole program, not for every program point
  - Path sensitivity: More precise analyses compute different facts for different program paths

Challenges and Open Problems

Many open problems

- Precise and scalable heap reasoning
- Concurrency
- Dealing with open programs
- Modular program analysis