Seminar Nr.2, Classical Probability; Rules of Probability; Conditional Probability; Independent Events

<u>**Theory Review**</u>

Classical Probability: $P(A) = \dfrac{\text{nr. of favorable outcomes}}{\text{total nr. of possible outcomes}} = \dfrac{N_f}{N_t}$.

Mutually Exclusive Events: $A,\ B$ **m. e. (disjoint, incompatible)** $<=> P(A \cap B) = 0$.

Rules of Probability:

$P(\overline{A}) = 1 - P(A)$;
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$;
$P(A \setminus B) = P(A) - P(A \cap B)$.

Conditional Probability: $P(A|B) = \dfrac{P(A \cap B)}{P(B)},\ P(B) \neq 0$.

Independent Events: $A,\ B$ **ind.** $<=> P(A \cap B) = P(A)P(B) <=> P(A|B) = P(A)$.

Total Probability Rule: $\{A_i\}_{i \in I}$ a partition of S, then $P(E) = \sum\limits_{i \in I} P(A_i)P(E|A_i)$.

Multiplication Rule: $P\left(\bigcap\limits_{i=1}^{n} A_i\right) = P(A_1)\, P(A_2|A_1)\, P(A_3|A_1 \cap A_2)\ \ldots\ P\left(A_n | \bigcap\limits_{i=1}^{n-1} A_i\right)$.

1.The faces of a cube are painted each in a different color (the cube is transparent on the inside). Then the cube is broken into 1000 smaller, equally-sized cubes and one such cube is randomly picked. Find the probability of the following events:
a) A: the cube picked has exactly three colored faces;
b) B: the cube picked has exactly two colored faces;
c) C: the cube picked has exactly one colored face;
d) D: the cube picked has no colored faces.

2. (Pigeonhole Principle) A postman distributes n letters in N mailboxes. What is the probability of the event A: there are m letters in a given (fixed) mailbox ($0 \leq m \leq n$)?

3. (Breaking Passwords) An account uses 8-character passwords, consisting of letters (distinguishing between lower-case and capital letters) and digits. A spy program can check about 1 million passwords per second.
a) On the average, how long will it take the spy program to guess your password?
b) What is the probability that the spy program will break your password within a week?
c) Same questions, if capital letters are not used.

4. (System Reliability) Compute the reliability of the system in Figure 1 if each of the five components is operable with probability 0.92, independently of each other.

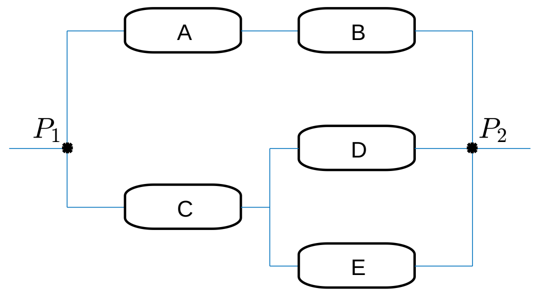

Figure 1: System Reliability

5. Among employees of a certain firm, 70% know C/C++, 60% know Fortran and 50% know both. What portion of programmers
a) does not know Fortran?
b) does not know C/C++ and does not know Fortran?
c) knows C/C++, but not Fortran?
d) Are "knowing C/C++" and "knowing Fortran" independent of each other?
e) What is the probability that someone who knows Fortran, also knows C/C++?
f) What is the probability that someone who knows C/C++, does not also know Fortran?

6. Three shooters aim at a target. The probabilities that they hit the target are 0.4, 0.5 and 0.7, respectively. Find the probability that the target is hit exactly once.

7. Under good weather conditions, 80% of flights arrive on time. During bad weather, only 50% of flights arrive on time. Tomorrow, the chance of good weather is 60%. What is the probability that your flight will arrive on time?

Bonus Problems:

8. There are N people in a room, each wearing a different hat. They all take their hats off, put them together and then each randomly picks one up. What is the probability of no person getting their own hat back (denote this event by A)? What does this probability become as $N \to \infty$?

9. A computer program consists of two blocks written independently by two different programmers. The first block has an error with probability 0.2, the second block has an error with probability 0.3. If the program returns an error, what is the probability that there is an error in both blocks?