

Lab 8, Numerical Calculus

Newton-Cotes Quadratures; Adaptive Quadratures; Romberg's Algorithm

1. Implement the composite rectangle, trapezoidal and Simpson's rules in the form

$$I = \text{Name}(f, a, b, n),$$

where

f is the integrand,

$[a, b]$ is the interval of integration,

n is the number of subintervals used,

I is the approximate value of $\int_a^b f(x)dx$.

2. Implement adaptive quadratures based on composite rectangle, trapezoidal and Simpson's rules.

3. Implement Romberg's method for the composite trapezoidal rule.

Applications

1. Approximate $\ln 2$ with 3 correct decimals, using the composite rectangle, trapezoidal and Simpson's rule, with the appropriate number of subintervals.

2. Approximate $\int_0^1 e^{-x^2} dx$ using adaptive quadratures.

3. Approximate $\int_0^{2\pi} \frac{dx}{2 + \cos x}$ (whose exact value is $\frac{2\pi}{\sqrt{3}}$) using Romberg's method for the composite trapezoidal rule.