DATA STRUCTURES AND ALGORITHMS
LECTURE 3

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In Lecture 2...

- Dynamic Array
- Iterator

Today

1. Linked Lists
   - Singly Linked Lists
   - Doubly Linked Lists

Linked Lists

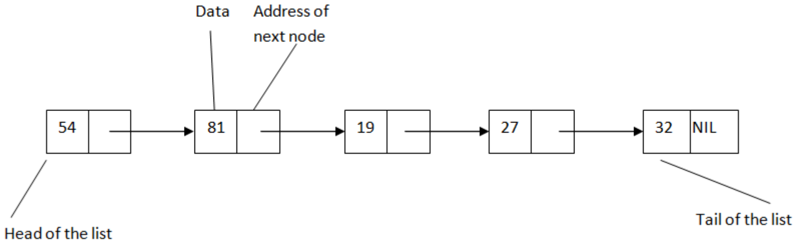
- A *linked list* is a linear data structure, but the order of the elements is determined not by indexes, but by a pointer which is placed in each element.
- A linked list is a structure that consists of *nodes* (sometimes called *links*) and each node contains, besides the data (that we store in the linked list), a pointer to the address of the next node (and possibly a pointer to the address of the previous node).
- The nodes of a linked list are not necessarily adjacent in the memory, this is why we need to keep the address of the successor in each node.

Linked Lists

- Elements from a linked list are accessed based on the pointers stored in the nodes.
- We can directly access only the first element (and maybe the last one) of the list.

Linked Lists

- Example of a linked list with 5 nodes:



Singly Linked Lists - SLL

- The linked list from the previous slide is actually a *singly linked list - SLL*.
- In a SLL each node from the list contains the data and the address of the next node.
- The first node of the list is called *head* of the list and the last node is called *tail* of the list.
- The tail of the list contains the special value *NIL* as the address of the next node (which does not exist).
- If the head of the SLL is *NIL*, the list is considered empty.

Singly Linked Lists - Representation

- For the representation of a SLL we need two structures: one structure for the node and one for the list itself.

<u>SLLNode:</u>
  info: TElem *//the actual information*
  next: ↑ SLLNode *//address of the next node*

<u>SLL:</u>
  head: ↑ SLLNode *//address of the first node*

- Usually, for a SLL, we only memorize the address of the head. However, there might be situations when we memorize the address of the tail as well (if the application requires it).

SLL - Operations

- Possible operations for a singly linked list:
  - search for an element with a given value
  - add an element (to the beginning, to the end, to a given position, after a given value)
  - delete an element (from the beginning, from the end, from a given position, with a given value)
  - get an element from a position
- These are *possible* operations; usually we need only part of them, depending on the container that we implement using a SLL.

SLL - Search

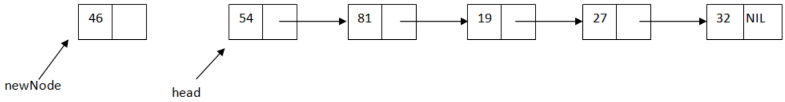
function search (sll, elem) **is:**
//pre: sll is a SLL - singly linked list; elem is a TElem
//post: returns the node which contains elem as info or NIL
   current ← sll.head
   while current $\neq$ NIL **and** [current].info $\neq$ elem **execute**
      current ← [current].next
   end-while
   search ← current
end-function

- Complexity: $O(n)$ - we can find the element in the first node, or we may need to verify every node.

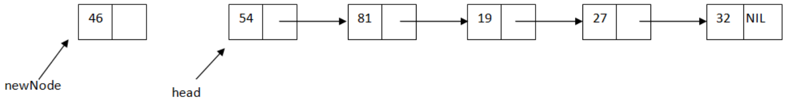
SLL - Walking through a linked list

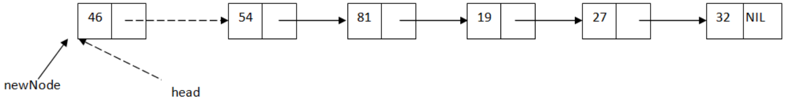
- In the *search* function we have seen how we can walk through the elements of a linked list:
  - we need an auxiliary node (called *current*), which starts at the head of the list
  - at each step, the value of the *current* node becomes the address of the successor node (through the *current* $\leftarrow$ *[current].next* instruction)
  - we stop when the current node becomes *NIL*

SLL - Insert at the beginning



SLL - Insert at the beginning





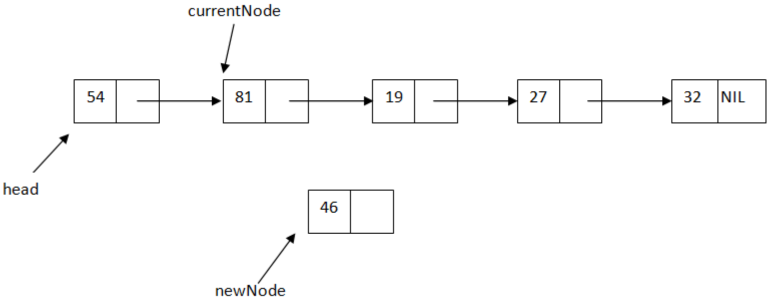
SLL - Insert at the beginning

subalgorithm insertFirst (sll, elem) **is:**
//pre: sll is a SLL; elem is a TElem
//post: the element elem will be inserted at the beginning of sll
    newNode ← allocate() *//allocate a new SLLNode*
    [newNode].info ← elem
    [newNode].next ← sll.head
    sll.head ← newNode
sf-subalgorithm

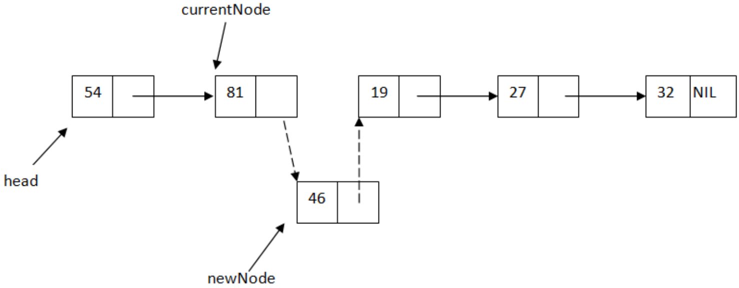
- Complexity: $\Theta(1)$

SLL - Insert after a node

- Suppose that we have the address of a node from the SLL and we want to insert a new element after that node.



SLL - Insert after a node



SLL - Insert after a node

subalgorithm insertAfter(sll, currentNode, elem) **is:**
//pre: sll is a SLL; currentNode is an SLLNode from sll;
//elem is a TElem
//post: a node with elem will be inserted after node currentNode
  newNode ← allocate() *//allocate a new SLLNode*
  [newNode].info ← elem
  [newNode].next ←[currentNode].next
  [currentNode].next ← newNode
sf-subalgorithm

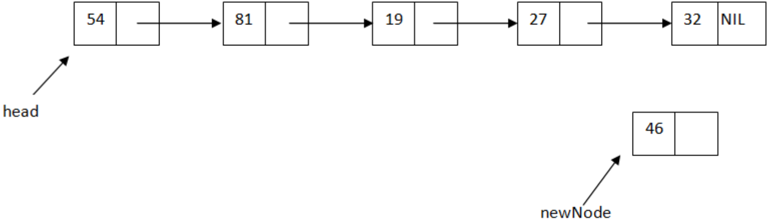
- Complexity: $\Theta(1)$

SLL - Insert at a position

- We usually do not have the node after which we want to insert an element: we either know the position to which we want to insert or know the element (but not the node containing it) after which we want to insert an element.
- Suppose we want to insert a new element at position p (after insertion the new element will be at position p). Since we only have access to the *head* of the list we first need to find the position *after* which we insert the element.

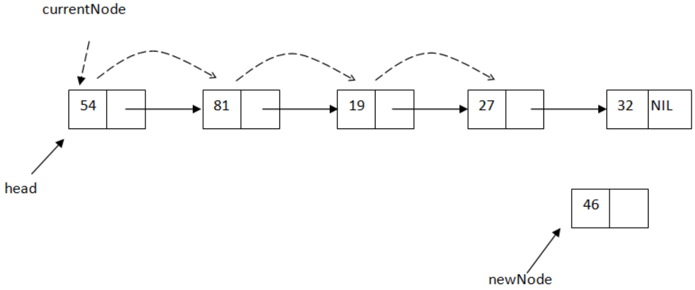
SLL - Insert at a position

- We want to insert element 46 at position 5.



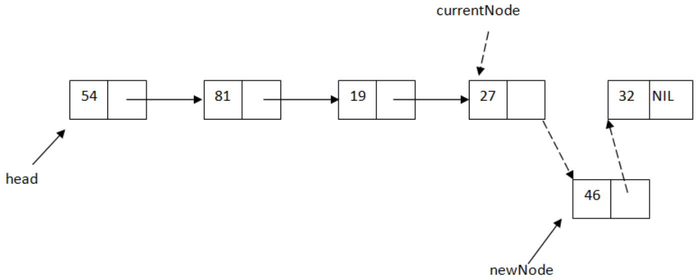
SLL - Insert at a position

- We need the 4^{th} node (to insert element 46 after it), but we have direct access only to the first one, so we have to take an auxiliary node (*currentNode*) to get to the position.



SLL - Insert at a position

- Now we insert after node *currentNode* (like in the previous case, when we already had the node).



SLL - Insert at a position

subalgorithm insertPosition(sll, pos, elem) **is:**
//pre: sll is a SLL; pos is an integer number; elem is a TElem
//post: a node with TElem will be inserted at position pos
    if pos $<$ 1 **then**
        @error, invalid position
    else if pos $=$ 1 **then** *//we want to insert at the beginning*
        insertFirst(sll, elem)
    else
        currentNode $\leftarrow$ sll.head
        currentPos $\leftarrow$ 1
        while currentPos $<$ pos - 1 **and** currentNode $\neq$ NIL **execute**
            currentNode $\leftarrow$ [currentNode].next
            currentPos $\leftarrow$ currentPos $+$ 1
        end-while
//continued on the next slide...

```
    if currentNode ≠ NIL then
        insertAfter(sll, currentNode, elem)
    else
        @error, invalid position
    end-if
  end-if
end-subalgorithm
```

- Complexity: $O(n)$

SLL - Delete from the beginning

- Deleting a node from the beginning simply means setting the head of the list to the next element

function deleteFirst(sll) **is:**
//pre: sll is a SLL
//post: the first node from sll is deleted and returned
  deletedNode $\leftarrow$ NIL
  if sll.head $\neq$ NIL **then**
    deletedNode $\leftarrow$ sll.head
    sll.head $\leftarrow$ [sll.head].next
    [deletedNode].next $\leftarrow$ NIL
  end-if
  deleteFirst $\leftarrow$ deletedNode
end-function

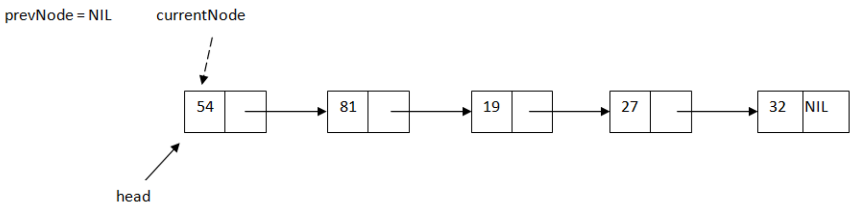
- Complexity: $\Theta(1)$

SLL - Delete a given element

- When we want to delete a node from the middle of the list (either a node with a given element, or a node from a position), we need to find the node *before* the one we want to delete.
- The simplest way to do this, is to walk through the list using two pointers: *currentNode* and *prevNode* (the node before *currentNode*). We will stop when *currentNode* points to the node we want to delete.

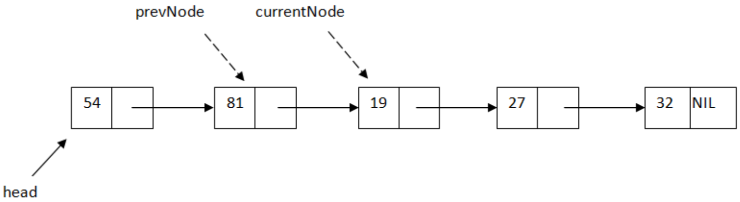
SLL - Delete a given element

- Suppose we want to delete the node with information 19.



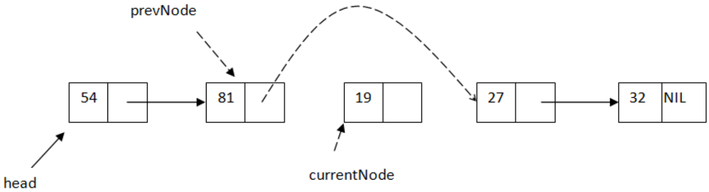
SLL - Delete a given element

- Move with the two pointers until *currentNode* is the node we want to delete.



SLL - Delete a given element

- Delete *currentNode* by *jumping over it*



SLL - Delete a given element

```
function deleteElement(sll, elem) is:
//pre: sll is a SLL, elem is a TElem
//post: the node with elem is removed from sll and returned
   currentNode ← sll.head
   prevNode ← NIL
   while currentNode ≠ NIL and [currentNode].info ≠ elem execute
      prevNode ← currentNode
      currentNode ← [currentNode].next
   end-while
   if prevNode = NIL then //we delete the head
      currentNode ← deleteFirst(sll)
   else if currentNode ≠ NIL then
      [prevNode].next ← [currentNode].next
      [currentNode].next ← NIL
   end-if
   deleteElement ← currentNode
end-function
```

SLL - Delete a given element

- Complexity of *deleteElement* function: $O(n)$

SLL - Other operations

- Insert element at the end of the list - walk through the list until we find the last node, add a new node after it.
- Delete element from the end of the list - walk through the list (with two nodes) until we find the last node and delete it by setting the *next* field of the previous node to *NIL*.
- Get element from a position - walk through the list until we get to the element from that position and return it.

SLL - Iterator

- How can we define an iterator for a SLL?
- Remember, an iterator needs a reference to a *current element* from the data structure it iterates over. How can we denote a *current element* for a SLL?

SLL - Iterator

- In case of a SLL, the current element from the iterator is actually a node of the list.

SLLIterator:
list: SLL
currentElement: ↑ SLLNode

SLL - Iterator - init operation

- What should the *init* operation do?

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- What should the *init* operation do?

subalgorithm init(it, sll) **is:**
//pre: sll is a SLL
//post: it is a SLLIterator over sll
  it.sll ← sll
  it.currentElement ← sll.head
end-subalgorithm

- Complexity: $\Theta(1)$

SLL - Iterator - getCurrent operation

- What should the *getCurrent* operation do?

SLL - Iterator - getCurrent operation

- What should the *getCurrent* operation do?

subalgorithm getCurrent(it, e) **is:**
//pre: it is a SLLIterator, it is valid
//post: e is TElem, e is the current element from it
  e $\leftarrow$ [it.currentElement].info
end-subalgorithm

- Complexity: $\Theta(1)$

SLL - Iterator - next operation

- What should the *next* operation do?

SLL - Iterator - next operation

- What should the *next* operation do?

subalgorithm next(it) **is:**
//pre: it is a SLLIterator, it is valid
//post: it' is a SLLIterator, the current element from it'
//refers to the next element
  it.currentElement $\leftarrow$ [it.currentElement].next
end-subalgorithm

- Complexity: $\Theta(1)$

SLL - Iterator - valid operation

- What should the *valid* operation do?

SLL - Iterator - valid operation

- What should the *valid* operation do?

function valid(it) **is:**
//pre: it is a SLLIterator
//post: true if it is valid, false otherwise
  if it.currentElement $\neq$ NIL **then**
    valid $\leftarrow$ True
  else
    valid $\leftarrow$ False
  end-if
end-function

- Complexity: $\Theta(1)$

Doubly Linked Lists - DLL

- A doubly linked list is similar to a singly linked list, but the nodes have references to the address of the previous node as well (besides the *next* link, we have a *prev* link as well).
- If we have a node from a DLL, we can go the next node or to the previous one: we can walk through the elements of the list in both directions.
- The *prev* link of the first element is set to *NIL* (just like the *next* link of the last element).

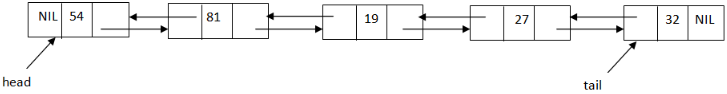
Doubly Linked List - Representation

- For the representation of a DLL we need two structures: one structure for the node and one for the list itself.

DLLNode:
  info: TElem
  next: ↑ DLLNode
  prev: ↑ DLLNode

DLL:
  head: ↑ DLLNode
  tail: ↑ DLLNode

Example of a Doubly Linked List



- Example of a doubly linked list with 5 nodes.

DLL - Operations

- We can have the same operations on a DLL that we had on a SLL:
  - search for an element with a given value
  - add an element (to the beginning, to the end, to a given position, etc.)
  - delete an element (from the beginning, from the end, from a given positions, etc.)
  - get an element from a position
- Some of the operations have the exact same implementation as for SLL (e.g. search, get element), others have similar implementations. In general, we need to modify more links and have to pay attention to the *tail* node.

DLL - Insert at the end

- Inserting a new element at the end of a DLL is simple, because we have the *tail* of the list, we no longer have to walk through all the elements.

```
subalgorithm insertLast(dll, elem) is:
//pre: dll is a DLL, elem is TElem
//post: elem is added to the end of dll
   newNode ← allocate() //allocate a new DLLNode
   [newNode].info ← elem
   [newNode].next ← NIL
   [newNode].prev ← dll.tail
   if dll.head = NIL then //the list is empty
      dll.head ← newNode
      dll.tail ← newNode
   else
      [dll.tail].next ← newNode
      dll.tail ← newNode
   end-if
end-subalgorithm
```

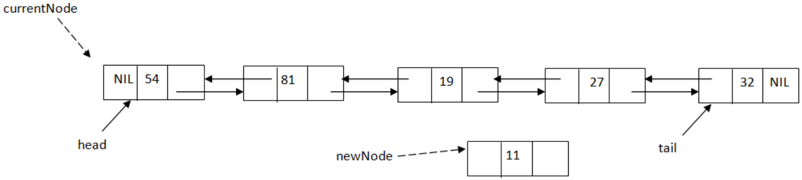
- Complexity: $\Theta(1)$

DLL - Insert at a position

- The basic principle of inserting a new element at a given position is the same as in case of SLL.
- The main difference is that we need to set more links (we have the *prev* links as well) and we have to check whether we modify the tail of the list.
- In case of SLL we *had to* stop at the node after which we wanted to insert an element, in case of DLL we can stop before or after the node (but we have to decide in advance, because this decision influences the special cases we need to test).

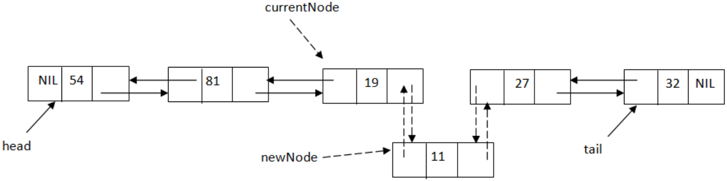
DLL - Insert at a position

- Let's insert value 11 at the 4^{th} position in the following list:



DLL - Insert at a position

- We move with the *currentNode* to position 3, and set the 4 links.



DLL - Insert at a position

subalgorithm insertPosition(dll, pos, elem) **is:**
//pre: dll is a DLL; pos is an integer number; elem is a TElem
//post: elem will be inserted on position pos in dll
  if pos $<$ 1 **then**
    @ error, invalid position
  else if pos $=$ 1 **then**
    insertFirst(dll, elem)
  else
    currentNode $\leftarrow$ dll.head
    currentPos $\leftarrow$ 1
    while currentNode $\neq$ NIL **and** currentPos $<$ pos - 1 **execute**
      currentNode $\leftarrow$ [currentNode].next
      currentPos $\leftarrow$ currentPos $+$ 1
    end-while
//continued on the next slide...

DLL - Insert at position

```
    if currentNode = NIL then
        @error, invalid position
    else if currentNode = dll.tail then
        insertLast(dll, elem)
    else
        newNode ← alocate()
        [newNode].info ← elem
        [newNode].next ← [currentNode].next
        [newNode].prev ← currentNode
        [[currentNode].next].prev ← newNode
        [currentNode].next ← newNode
    end-if
  end-if
end-subalgorithm
```

- Complexitate: $O(n)$

DLL - Insert at a position

- Observations regarding the *insertPosition* subalgorithm:
  - We did not implement the *insertFirst* subalgorithm, but we suppose it exists.
  - The order in which we set the links is important: reversing the setting of the last two links will lead to a problem with the list.
  - It is possible to use two *currentNodes*: after we found the node after which we insert a new element, we can do the following:

```
nodeAfter ← currentNode
nodeBefore ← [currentNode].next
//now we insert between nodeAfter and nodeBefore
[newNode].next ← nodeBefore
[newNode].prev ← nodeAfter
[nodeBefore].prev ← newNode
[nodeAfter].next ← newNode
```

DLL - Delete from the beginning

function deleteFirst(dll) **is:**
//pre: dll is a DLL
//post: the first node is removed and returned
    deletedNode ← NIL
    if dll.head ≠ NIL **then**
        deletedNode ← dll.head
        if dll.head = dll.tail **then**
            dll.head ← NIL
            dll.tail ← NIL
        else
            dll.head ← [dll.head].next
            [dll.head].prev ← NIL
        end-if
    @set links of deletedNode to NIL
    deleteFirst ← deletedNode
end-function

DLL - Delete from the beginning

- Complexity of *deleteFirst*: $\Theta(1)$

DLL - Delete a given element

- If we want to delete a node with a given element, we first have to find the node:
  - we can use the *search* function (discussed at SLL, but it is the same here as well)
  - we can walk through the elements of the list until we find the node with the element (this is implemented below)

DLL - Delete a given element

function deleteElement(dll, elem) **is:**
//pre: dll is a DLL, elem is a TElem
//post: the node with element elem will be removed and returned
    currentNode ← dll.head
    while currentNode ≠ NIL **and** [currentNode].info ≠ elem **execute**
        currentNode ← [currentNode].next
    end-while
    deletedNode ← currentNode
    if currentNode ≠ NIL **then**
        if currentNode = dll.head **then**
            deleteElement ← deleteFirst(dll)
        else if currentNode = dll.tail **then**
            deleteElement ← deleteLast(dll)
        else
//continued on the next slide...

DLL - Delete a given element

```
        [[currentNode].next].prev ← [currentNode].prev
        [[currentNode].prev].next ← [currentNode].next
        @set links of deletedNode to NIL
    end-if
  end-if
  deleteElement ← deletedNode
end-function
```

- Complexity: $O(n)$
- If we used the *search* algorithm to find the node to delete, the complexity would still be $O(n)$ - *deleteElement* would be $\Theta(1)$, but searching is $O(n)$

DLL - Iterator

- The iterator for a DLL is identical to the iterator for the SLL (but *currentNode* is *DLLNode* not *SLLNode*).
- In case of a DLL it is easy to define a bi-directional iterator:
    - Besides the operations for the unidirectional iterator, we need another operation: *previous*.
    - It would be useful to define two operations: *first* and *last* to set the iterator to the head/tail of the list.

Think about it

- How could we define a bi-directional iterator for a SLL? What would be the complexity of the *previous* operation?
- How could we define a bi-directional iterator for a SLL if we know that the *previous* operation will never be called twice consecutively (two consecutive calls for the *previous* operation will always be divided by at least one call to the *next* operation)? What would be the complexity of the operations?

Dynamic Array vs. Linked Lists

- Dynamic Arrays and Linked Lists support the same general operations, but they can have different time complexities

Algorithm	**DA**	**SLL**	**DLL**
search	$O(n)$	$O(n)$	$O(n)$
get element from position	$\Theta(1)$	$O(n)$*	$O(n)$*
insert first position	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$
insert last position	$\Theta(1)$	$O(n)$**	$\Theta(1)$
insert position	$O(n)$	$O(n)$	$O(n)$
delete first position	$\Theta(n)$	$\Theta(1)$	$\Theta(1)$
delete last position	$\Theta(1)$	$\Theta(n)$	$\Theta(1)$
delete position	$O(n)$	$O(n)$	$O(n)$

Dynamic Array vs. Linked Lists

- Observations regarding the previous table:
  - * - getting the element from a position i for a linked list has complexity $\Theta(i)$ - we need exactly i steps to get to the i^{th} node, but since $i \leq n$ we usually use $O(n)$.
  - ** - can be done in $\Theta(1)$ if we keep the address of the tail node as well.

Dynamic Array vs. Linked Lists

- Advantages of Linked Lists
  - No memory used for non-existing elements.
  - Constant time operations at the beginning of the list.
  - Elements are never *moved* (important if copying an element takes a lot of time).
- Disadvantages of Linked Lists
  - We have no direct access to an element from a given position (however, iterating through all elements of the list using an iterator has $\Theta(n)$ time complexity).
  - Extra space is used up by the addresses stored in the nodes.
  - Nodes are not stored at consecutive memory locations (no benefit from modern CPU caching methods).

Algorithmic problems using Linked Lists

- Find the n^{th} node from the end of a SLL, passing through the list only once.
- We need to use two auxiliary variables, two nodes, both set to the first node of the list. At the beginning of the algorithm we will go forward $n-1$ times with one of the nodes. Once the first node is at the n^{th} position, we move with both nodes in parallel. When the first node gets to the end of the list, the second one is at the n^{th} element from the end of the list.

N-th node from the end of the list

function findNthFromEnd (sll, n) **is:**
//pre: sll is a SLL, n is an integer number
//post: the n-th node from the end of the list or NIL
  oneNode ← sll.head
  secondNode ← sll.head
  position ← 1
  while position < n **and** oneNode ≠ NIL **execute**
    oneNode ← [oneNode].next
    position ← position + 1
  end-while
  if oneNode = NIL **then**
    findNthFromEnd ← NIL
  else
  //continued on the next slide...

N-th node from the end of the list

while [oneNode].next $\neq$ NIL **execute**
oneNode $\leftarrow$ [oneNode].next
secondNode $\leftarrow$ [secondNode].next
end-while
findNthFromEnd $\leftarrow$ secondNode
end-if
end-function

Think about it

- Given the first node of a SLL, determine whether the list ends with a node that has NIL as *next* or whether it ends with a cycle (the *last* node contains the address of a previous node as *next*).
- If the list from the previous problems contains a cycle, find the length of the cycle.
- Find if a SLL has an even or an odd number of elements, without counting the number of nodes in any way.
- Reverse a SLL in linear time using $\Theta(1)$ extra storage.