Seminar 13

1. Consider a $(63, 56)$-code.
(i) What is the number of digits in the message before coding?
(ii) What is the number of check digits?
(iii) What is the information rate?
(iv) How many different syndromes are there?

2. Using the parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

and the syndromes and coset leaders

Syndrome	000	001	010	011
Coset leader	000000	001000	010000	000010

Syndrome	100	101	110	111
Coset leader	100000	000110	000100	000001

decode the following words: 101110, 011000, 001011, 111111, 110011.

3. A (7,4)-code is defined by the equations $u_1 = u_4 + u_5 + u_7$, $u_2 = u_4 + u_6 + u_7$, $u_3 = u_4 + u_5 + u_6$, where u_4, u_5, u_6, u_7 are the message digits and u_1, u_2, u_3 are the check digits. Write its generator matrix and parity check matrix. Decode the received words 0000111 and 0001111.

4. Find the syndromes of all the received words in the (3,2)-parity check code and in the (3,1)-repeating code.

5. Construct a table of coset leaders and syndromes for the (7,4)-code with parity check matrix

$$H = \begin{pmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{pmatrix}.$$

6. Determine the parity check matrix and all syndromes and coset leaders of the (5,3)-code with generator matrix $G = \begin{pmatrix} P \\ I_3 \end{pmatrix} \in M_{5,3}(\mathbb{Z}_2)$, where:

$$P = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

7. Construct a table of coset leaders and syndromes for the (3,1)-code generated by $p = 1 + X + X^2 \in \mathbb{Z}_2[X]$.

8. Construct a table of coset leaders and syndromes for the (7,3)-code generated by $p = 1 + X^2 + X^3 + X^4 \in \mathbb{Z}_2[X]$.