Seminar 12

1. (*i*) Which of the following received words contain detectable errors when using the (3,2)-parity check code: 110, 010, 001, 111, 101, 000?

(*ii*) Decode the following words using the (3,1)-repeating code to correct errors: 111, 011, 101, 010, 000, 001. Which of them contain detectable errors?

2. Are $1 + X^3 + X^4 + X^6 + X^7$ and $X + X^2 + X^3 + X^6$ code words in the polynomial (8,4)-code generated by $p = 1 + X^2 + X^3 + X^4 \in \mathbb{Z}_2[X]$?

3. Write down all the words in the (6,3)-code generated by $p = 1 + X^2 + X^3 \in \mathbb{Z}_2[X]$.

4. A code is defined by the generator matrix $G = \left(\frac{P}{I_3}\right) \in M_{5,3}(\mathbb{Z}_2)$, where:

$$P = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Write down the parity check matrix and all the code words.

5. Determine the minimum Hamming distance between the code words of the code with generator matrix $G = \left(\frac{P}{I_4}\right) \in M_{9,4}(\mathbb{Z}_2)$, where:

$$P = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}.$$

Discuss the error-detecting and error-correcting capabilities of this code, and write down the parity check matrix.

6. Encode the following messages using the generator matrix of the (9,4)-code of Exercise **5.**: 1101, 0111, 0000, 1000.

Determine the generator matrix and the parity check matrix for:

7. The (4,1)-code generated by $p = 1 + X + X^2 + X^3 \in \mathbb{Z}_2[X]$.

8. The (7,3)-code generated by $p = 1 + X^2 + X^3 + X^4 \in \mathbb{Z}_2[X]$.