Seminar Nr.3, Probabilistic Models

Theory Review

Binomial Model: The probability of k successes in n Bernoulli trials, with probability of success p ($q = 1 - p$), is

$P(n, k) = C_n^k p^k q^{n-k}, \ k = \overline{0, n}$

Hypergeometric Model: The probability that in n trials, we get k white balls out of n_1 and $n - k$ black balls out of $N - n_1$ ($0 \le k \le n_1$, $0 \le n - k \le N - n_1$), is

$P(n; k) = \dfrac{C_{n_1}^k C_{N-n_1}^{n-k}}{C_N^n}$

Poisson Model: The probability of k successes ($0 \le k \le n$) in n trials, with probability of success p_i in
the i^{th} trial ($q_i = 1 - p_i$), $i = \overline{1, n}$, is

$$P(n; k) = \sum_{1 \le i_1 < \ldots < i_k \le n} p_{i_1} \ldots p_{i_k} q_{i_{k+1}} \ldots q_{i_n}, \qquad i_{k+1}, \ldots, i_n \in \{1, \ldots, n\} \setminus \{i_1, \ldots, i_k\}$$
$$= \text{the coefficient of } x^k \text{ in the expansion } (p_1 x + q_1)(p_2 x + q_2) \ldots (p_n x + q_n)$$

Pascal (Negative Binomial) Model: The probability of the n^{th} success occurring after k failures in a sequence of Bernoulli trials with probability of success p ($q = 1 - p$), is

$P(n; k) = C_{n+k-1}^{n-1} p^n q^k = C_{n+k-1}^k p^n q^k$

Geometric Model: The probability of the 1^{st} success occurring after k failures in a sequence of Bernoulli trials with probability of success p ($q = 1 - p$), is

$p_k = pq^k$

---

1. Five percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 16 parts contains
a) exactly 3 defective parts (ev. A)?
b) more than 3 defective parts? (ev. B)?
c) at least one defective part (ev. C)?
d) less than 3 defective parts (ev. D)?

2. There are 200 seats in a theater, 10 of which are reserved for the press. 150 people come to the show one night, and are seated randomly. What is the probability of all the seats reserved for the press to be occupied (ev. A)?

3. Among 10 laptop computers, five are good and five have defects. Unaware of this, a customer buys 6 laptops.
a) What is the probability of exactly 2 defective ones among them (ev. A)?
b) Knowing that *at least* 2 purchased laptops are defective, what is the probability that *exactly* 2 are defective (ev. B)?

4. A computer program is tested by 5 independent tests. If there is an error, these tests will detect it with probabilities 0.1, 0.2, 0.3, 0.4 and 0.5, respectively. Suppose that the program contains an error. What is the probability that it will be found by
a) at least one test (ev. A)?
b) more than two tests (ev. B)?
c) all five tests (ev. C)?

5. An engineer tests the quality of produced computers. Suppose that 5% of computers have defects and defects occur independently of each other. Find the probability
a) of exactly 3 defective computers in a shipment of 20 (ev. A);
b) that the engineer has to test at least 5 computers in order to find 2 defective ones (ev. B).

6. (Banach's Problem). A person buys 2 boxes of aspirin, each containing n pills. He takes one aspirin at a time, randomly from one of the two boxes. After a while, he realizes that one box is empty.
a) Find the probability of event A: when he notices that one box is empty, there are k ($k \leq n$) pills left in the other box.
b) Use part a) to find a formula for $S_n = C_{2n}^n + 2 \cdot C_{2n-1}^n + \ldots + 2^n \cdot C_n^n$.

Bonus Problems:

7. Three contestants participate in a trivia game show. Their probability of answering a question correctly are 0.8, 0.9 and 0.75, respectively. If 10 questions are asked and every contestant answers every question, find the probability that one contestant (any one) answers exactly 7 questions correctly, while the other two give any *other* number of correct answers. (event A)?

8. In a department store at the mall, black and brown gloves are on sale. There are N (identical) pairs of black gloves and N (identical) pairs of brown gloves. If N customers come in, one at a time and randomly choose and buy 2 pairs each, find the probability of event A: each customer buys 2 pairs of different colors (one black and one brown).