Lab 11

Jacobi, Gauss-Siedel and relaxation iterative methods for solving linear systems

Consider the system $Ax = b$, with $A = (a_{ij})_{i,j=\overline{1,n}}$, $x = (x_1, ..., x_n)'$ and $b = (b_1, ..., b_n)'$.

Input: A -matrix of coefficients; b -vector of free terms; $x^{(0)}$ -the initial approximation of the solution; ε -precision, N - maximum number of iterations; $(\omega-$parameter for relaxation method);

Output: x -vector of the solutions or *a message* - in case that the maximum number of iterations is exceeded.

Problem:

1. Solve the following system using *Jacobi, Gauss-Seidel* and *relaxation* iterative methods, for $\varepsilon = 10^{-3}$:

$$
\begin{pmatrix}
3 & -1 & 0 & 0 & 0 & 0 \\
-1 & 3 & -1 & 0 & 0 & 0 \\
0 & -1 & 3 & -1 & 0 & 0 \\
0 & 0 & -1 & 3 & -1 & 0 \\
0 & 0 & 0 & -1 & 3 & -1 \\
0 & 0 & 0 & 0 & -1 & 3
\end{pmatrix}
x =
\begin{pmatrix}
2 \\
1 \\
1 \\
1 \\
1 \\
2
\end{pmatrix}.
$$

Return the solution of the system and the number of iterations needed for finding the solution.

2. Solve the following system using the matriceal forms of Jacobi, Gauss-Seidel and relaxation methods, for $\varepsilon = 10^{-5}$:

$$
\begin{pmatrix}
3 & 1 & 1 \\
-2 & 4 & 0 \\
-1 & 2 & -6
\end{pmatrix}
x =
\begin{pmatrix}
12 \\
2 \\
-5
\end{pmatrix}.
$$