Lab 6

Least squares approximation

1. The following table list the temperatures of a room recorded during the time interval $[1:00, 7:00]$. Find the best liniar least squares function $\varphi(x) = ax + b$ that approximates the table, using the normal equations. Use your result to predict the temperature of the room at $8:00$. Find the minimum value $E(a, b)$, for the obtained a and b. In the same figure, plot the points (Time, Temperature) and the least squares function.

Time	$1:00$	$2:00$	$3:00$	$4:00$	$5:00$	$6:00$	$7:00$
Temperature	13	15	20	14	15	13	10

2. The vapor pressure P of the water (in bars) as a function of temperature T (in $^\circ C$) is:

T (temperature)	0	10	20	30	40	60	80	100
P (pressure)	0.0061	0.0123	0.0234	0.0424	0.0738	0.1992	0.4736	1.0133

a) Obtain two least squares approximations for the given data, using $polyfit$ for 2 different degrees of the polynomials. Find their values for $T = 45$ using $polyval$. Compute the approximation errors, knowing that the exact value is $P(45) = 0.095848$.

b) Plot the interpolation points, the least squares approximants and the interpolation polynomial, in the same figure.

3. Find the least squares polynomial of 4th degree that fit the data given by the vectors $x = -3 : 0.4 : 3$ and $y = \sin(x)$. Plot the points and the least squares polynomial in the same figure. (Use $polyfit$ and $polyval$.)

4. Consider 10 random points in the plane $[0, 3] \times [0, 5]$ using Matlab function *ginput*. Plot the points and the least squares polynomial of 2nd degree that best fits these points.

Facultative:

5. Consider 12 random points in the interval $[0, 10]$. Find the discret least squares approximant of n-th degree for the function $f(x) = x^2$ using the least square approximation method with weight function $w(x) = 1$ and the basis $1, x, x^2, ..., x^n$. (The least squares approximant is of the form $\varphi(x) = \sum_{i=1}^{n} a_i g_i(x)$, where $\{g_i,\ i = 1, ..., n\}$ is a basis of the space and the coefficients a_i are obtained solving the normal equations: $\sum_{i=1}^{n} a_i \langle g_i, g_k \rangle = \langle f, g_k \rangle, \quad k = 1, ..., n.$). Plot the obtained approximant.