

Seminar 8

1. Let $A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 3 & 1 \\ 3 & 0 & -1 \end{pmatrix}$, $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$. Show that A is invertible, determine A^{-1} and solve the linear system $AX = B$.

2. Using the Kronecker-Capelli theorem, decide if the following linear systems are compatible and then solve the compatible ones:

$$(i) \begin{cases} x_1 + x_2 + x_3 - 2x_4 = 5 \\ 2x_1 + x_2 - 2x_3 + x_4 = 1 \\ 2x_1 - 3x_2 + x_3 + 2x_4 = 3 \end{cases} \quad (ii) \begin{cases} x_1 - 2x_2 + x_3 + x_4 = 1 \\ x_1 - 2x_2 + x_3 - x_4 = -1 \\ x_1 - 2x_2 + x_3 + 5x_4 = 5 \end{cases}$$

$$(iii) \begin{cases} x + y + z = 3 \\ x - y + z = 1 \\ 2x - y + 2z = 3 \\ x + z = 4 \end{cases}$$

3. Using the Rouché theorem, decide if the systems from 2. are compatible and then solve the compatible ones.

4. Decide when the following linear system is compatible determinate and in that case solve it by using Cramer's method:

$$\begin{cases} ay + bx = c \\ cx + az = b \\ bz + cy = a \end{cases} \quad (a, b, c \in \mathbb{R}).$$

Solve the following linear systems by the Gauss and Gauss-Jordan methods:

$$5. \quad (i) \begin{cases} 2x + 2y + 3z = 3 \\ x - y = 1 \\ -x + 2y + z = 2 \end{cases} \quad (ii) \begin{cases} 2x + 5y + z = 7 \\ x + 2y - z = 3 \\ x + y - 4z = 2 \end{cases} \quad (iii) \begin{cases} x + y + z = 3 \\ x - y + z = 1 \\ 2x - y + 2z = 3 \\ x + z = 4 \end{cases}$$

$$6. \quad \begin{cases} 2x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 + 2x_2 - x_3 + 4x_4 = 2 \\ x_1 + 5x_2 - 4x_3 + 11x_4 = \lambda \end{cases} \quad (\lambda \in \mathbb{R})$$

$$7. \quad \begin{cases} ax + y + z = 1 \\ x + ay + z = a \\ x + y + az = a^2 \end{cases} \quad (a \in \mathbb{R})$$

8. Determine the positive solutions of the following non-linear system:

$$\begin{cases} xyz = 1 \\ x^3 y^2 z^2 = 27 \\ \frac{z}{xy} = 81 \end{cases}$$