Seminar 4

1. Study the convergence of the following series:

   (a) $\sum_{n\geq 1} \frac{1}{\ln n}$. (b) $\sum_{n\geq 1} \frac{\ln\left(1+\frac{1}{n}\right)}{n}$. (c) $\sum_{n\geq 1} \frac{1}{n(\ln n)^p}$. (d) ★ $\sum_{n\geq 1} \frac{1}{(\ln n)^{\ln n}}$.

2. Study the convergence and the absolute convergence of the following series:

   (a) $\sum_{n\geq 1} \frac{(-1)^{n+1}}{\sqrt{n(n+1)}}$.

   (b) $\sum_{n\geq 1} (-1)^n \sin\frac{1}{n}$.

   (c) $\sum_{n\geq 1} \frac{\sin n}{n^2}$.

   (d) ★ $\sum_{n\geq 1} \sin\left(\pi\sqrt{n^2+1}\right)$.

3. Study if the following series are convergent or divergent:

   (a) $\sum_{n\geq 1} \frac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot \ldots \cdot 2n}$.

   (b) ★ $\sum_{n\geq 1} \frac{1\cdot 3\cdot \ldots \cdot (2n-1)}{2\cdot 4\cdot \ldots \cdot 2n} \cdot \frac{1}{n^2}$.

   (c) $\sum_{n\geq 1} a^{\ln n}$, $a>0$.

   (d) $\sum_{n\geq 1} \frac{a^n n!}{n^n}$ $a>0$.

4. Prove that the alternating harmonic series $\sum_{n\geq 1} \frac{(-1)^{n+1}}{n} = \ln 2$. Show that changing the order of summation in this series can lead to a different sum.

5. ★ [Python programming] Show numerically (plots allowed) that

$$\sum_{n\geq 1} \frac{(-1)^{n+1}}{n} = \ln 2.$$

   Illustrate computationally that changing the order of summation in this series can lead to a different sum.

6. $*$ Rearrange the terms in the alternating harmonic series such that the sum is $s \in \overline{\mathbb{R}}$.

Homework questions are marked with ★. Bonus questions are marked with $*$.
Solutions should be handed in at the beginning of next week's lecture.