Seminar 3

1. Study the convergence and find the sum for each of the following series:

   (a) $\sum_{n\geq 1} \frac{2}{3^n}$.
   (b) $\sum_{n\geq 1} \frac{2n+1}{n!}$.
   (c) $\sum_{n\geq 1} \frac{1}{4n^2-1}$.
   (d) $\sum_{n\geq 1} \frac{1}{n(n+1)(n+2)}$.
   (e) $\sum_{n\geq 1} \frac{n}{2^n}$.
   (f) $\sum_{n\geq 1} \frac{1}{\sqrt[3]{n}}$.

2. ★ Study the convergence and find the sum for each of the following series:

   (a) $\sum_{n=2}^{\infty} \ln\big(1-\frac{1}{n^2}\big)$.
   (b) $\sum_{n\geq 1} \frac{n+1}{3^n}$.
   (c) $\sum_{n\geq 1} \frac{1}{n!+(n+1)!}$.
   (d) $\sum_{n\geq 1} \frac{n}{n^4+n^2+1}$.

3. Study if the following series are convergent or divergent:

   (a) $\sum_{n\geq 1} \big(\frac{n}{n+1}\big)^{n^2}$.
   (b) $\sum_{n\geq 1} \frac{x^n}{n^p}$, $x>0, p\in\mathbb{R}$.
   (c) ★ $\sum_{n\geq 1} \frac{x^n}{\sqrt[n]{n!}}$, $x>0$.
   (d) ★ $\sum_{n\geq 2} \frac{1}{n\ln(n)}$.

4. ★ Start with an equilateral triangle of side 1. For each side, remove the middle third and add there another equilateral triangle. Repeat this process at each iteration (see the figure). How many sides are there at iteration n? What is the limit of the perimeter and the area?

5. $*$ Start with the interval $[0,1]$ and remove the middle third $(1/3, 2/3)$; then remove the middle third of each of the remaining intervals, and so on. What is the total length of the removed intervals? Prove that the set of points that are left – known as Cantor's set – is uncountable.

Homework questions are marked with ★. Bonus questions are marked with $*$.
Solutions should be handed in at the beginning of next week's lecture.