Seminar 9

Compute by applying elementary operations the ranks of the matrices:

1. $\begin{pmatrix} 0 & 2 & 3 \\ 2 & 4 & 3 \\ 1 & 1 & 1 \\ 2 & 2 & 4 \end{pmatrix}$. **2.** $\begin{pmatrix} 1 & -1 & 3 & 2 \\ -2 & 0 & 3 & -1 \\ -1 & 2 & 0 & -1 \end{pmatrix}$. **3.** $\begin{pmatrix} \beta & 1 & 3 & 4 \\ 1 & \alpha & 3 & 3 \\ 2 & 3\alpha & 4 & 7 \end{pmatrix}$ $(\alpha, \beta \in \mathbb{R})$.

Compute by applying elementary operations the inverses of the matrices:

4. $\begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix}$. **5.** $\begin{pmatrix} 1 & 4 & 2 \\ 2 & 3 & 1 \\ 3 & 0 & -1 \end{pmatrix}$.

6. Let K be a field, let $B = (e_1, e_2, e_3, e_4)$ be a basis and let $X = (v_1, v_2, v_3)$ be a list in the canonical K-vector space K^4, where

$$v_1 = 3e_1 + 2e_2 - 5e_3 + 4e_4\,,$$

$$v_2 = 3e_1 - e_2 + 3e_3 - 3e_4\,,$$

$$v_3 = 3e_1 + 5e_2 - 13e_3 + 11e_4\,.$$

Write the matrix of the list X in the basis B, determine an echelon form for it and deduce that X is linearly dependent.

For the following exercises, for a list X of vectors in a canonical vector space $\mathbb{R}^n$, use that dim $< X >$ is equal to the rank of an echelon form C of the matrix consisting of the components of the vectors of X, and a basis of $< X >$ is given by the non-zero rows of C.

7. In the real vector space $\mathbb{R}^3$ consider the list $X = (v_1, v_2, v_3, v_4)$, where $v_1 = (1, 0, 4)$, $v_2 = (2, 1, 0)$, $v_3 = (1, 5, -36)$ and $v_4 = (2, 10, -72)$. Determine *dim* $< X >$ and a basis of $< X >$.

8. In the real vector space $\mathbb{R}^4$ consider the list $X = (v_1, v_2, v_3)$, where $v_1 = (1, 0, 4, 3)$, $v_2 = (0, 2, 3, 1)$ and $v_3 = (0, 4, 6, 2)$. Determine *dim* $< X >$ and a basis of $< X >$.

9. Determine the dimension of the subspaces S, T, $S + T$ and $S \cap T$ of the real vector space $\mathbb{R}^3$ and a basis for the first three of them, where

$$S =< (1, 0, 4), (2, 1, 0), (1, 1, -4) >\,,$$

$$T =< (-3, -2, 4), (5, 2, 4), (-2, 0, -8) > \,.$$

10. Determine the dimension of the subspaces S, T, $S + T$ and $S \cap T$ of the real vector space $\mathbb{R}^4$ and a basis for the first three of them, where

$$S =< (1, 2, -1, -2), (3, 1, 1, 1), (-1, 0, 1, -1) >\,,$$

$$T =< (2, 5, -6, -5), (-1, 2, -7, -3) > \,.$$