Seminar 9

1. Sketch the level sets $L_c = \{(x,y) \in \mathbb{R}^2 \mid f(x,y) = c\}$ for the function $f(x,y) = \sqrt{x^2+y^2}$ and the values $c = 0, 1, 4$. Compare the graph of this function with the paraboloid $z = x^2 + y^2$.

2. Study the limits of the following functions when $(x,y) \to (0,0)$:

   (a) $\dfrac{x+y}{x^2+y^2}$ (b) $\dfrac{x^2-y^2}{x^2+y^2}$. (c) $\dfrac{x^3+y^3}{x^2+y^2}$. (d) $\dfrac{\sin x - \sin y}{x-y}$.

3. Study the continuity and the partial differentiability of the function $f : \mathbb{R}^2 \to \mathbb{R}$,

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

4. Compute the first order partial derivatives (where they exist) for the following functions:

   (a) $f(x,y) = \sqrt{x^2+y^2}$.
   (b) $f(x,y) = \ln \sqrt{x^2+y^2}$.
   (c) $f(x,y) = \cos x \cos y - \sin x \sin y$.
   (d) $f(x,y,z) = x^2yz + ye^z$.

5. Find the gradient of the function f at the point a for the following:

   (a) $f(x,y) = e^{-x}\sin(x+2y)$, $a = (0, \frac{\pi}{4})$.
   (b) $f(x,y) = \arctan(\frac{y}{x})$, $a = (1,1)$.
   (c) $f(x,y,z) = e^{xyz}$, $a = (0,0,0)$.
   (d) $f(x,y,z) = \sqrt{x^2+y^2+z^2}$, $a = (1,1,1)$

6. Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function and let $g : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$g(x,y) = f(x^2+y^2),\ \forall\,(x,y) \in \mathbb{R}^2.$$

   Prove that

$$x\frac{\partial g}{\partial y}(x,y) = y\frac{\partial g}{\partial x}(x,y).$$