$n = 2089$

$n-1 = 2088 \Rightarrow 2088 = 2^3 \cdot 261 \Rightarrow s=3,\ t=261$

Iteration $k=1$ for $a=2$

$2^{2^0} = 2 \pmod{2089}$

$2^{2^1} = 4 \pmod{2089}$

$2^{2^2} = 2^{2^1} \cdot 2^{2^1} = 4 \cdot 4 = 16 \pmod{2089}$

$2^{2^3} = 16 \cdot 16 = 256 \pmod{2089}$

$2^{2^4} = 256 \cdot 256 = 65.536 = 777 \pmod{2089}$

$2^{2^5} = 777 \cdot 777 = 603.729 = 8 \pmod{2089}$

$2^{2^6} = 8 \cdot 8 = 64 \pmod{2089}$

$2^{2^7} = 64 \cdot 64 = 4096 = 2007 \pmod{2089}$

$2^{2^8} = 2007 \cdot 2007 = 4.028.049 = 457 \pmod{2089}$

$2^{2^9} = 457 \cdot 457 = 208849 = 2038 \pmod{2089}$

$261 = 2^0 + 2^2 + 2^8 \Rightarrow 2^{261} = 2^{2^0} + 2^{2^2} + 2^{2^8} = 2 \cdot 16 \cdot 457 = 14624 = 1$

$\Rightarrow$ the first number (2^{261}) in the sequence $2^t, 2^{2t}, 2^{2^2 t} \ldots$ is 1

$\Rightarrow n = 2089$ is possible to be prime, $\rightarrow$ try ~~on~~ another base

Iteration $k=2$ for $a=3$

$3^{2^0} = 3 \pmod{2089}$

$3^{2^1} = 3 \cdot 3 = 9 \pmod{2089}$

$3^{2^2} = 9 \cdot 9 = 81 \pmod{2089}$

$3^{2^3} = 81 \cdot 81 = \cancel{6581}\ 6561 = 294$ (-||-)

$3^{2^4} = 294 \cdot 294 = 86436 \equiv 787 \pmod{2089}$

$3^{2^5} = 787 \cdot 787 = 619369 \equiv 1025 \pmod{2089}$

$3^{2^6} = 1025 \cdot 1025 = 1050625 \equiv 1947 \pmod{2089}$

$3^{2^7} = 1947 \cdot 1947 = 3790809 \equiv 1363 \pmod{2089}$

$3^{2^8} = 1363 \cdot 1363 = \cancel{331209}\ 1857769 \equiv 648 \pmod{2089}$

$3^{261} = 3^{2^0+2^2+2^8} = 3 \cdot 81 \cdot 648 = 157464 \equiv 789 \pmod{2089}$

$3^{2 \cdot 261} = (3^{261})^2 = (789)^2 = 622521 \equiv 2088 \equiv -1$ (-11-)

$3^{2^2 \cdot 261} = (3^{2 \cdot 261})^2 = (-1)^2 = 1 \pmod{2089}$

$\Rightarrow$ $n = 2089$ is possible to be prime, as the sequence is 789, -1, 1 $\Rightarrow$ another base

Iteration $k=3$, $a=5$

$5^{2^0} = 5$ (-11-)

$5^{2^1} = 25$ (-11-)

$5^{2^2} = 25 \cdot 25 = 625$ (-11-)

$5^{2^3} = 625 \cdot 625 = 390625 \equiv 2071$ (-11-)

$5^{2^4} = (2071)^2 = 4289041 \equiv 324$ (-11-)

$5^{2^5} = 324 \cdot 324 = 104976 \equiv 526$ (-11-)

$5^{2^6} = (526)^2 = 276676 \equiv 928$ (-11-)

$5^{2^7} = 928 \cdot 928 = 861184 \equiv 516$ (-11-)

$5^{2^8} = 516 \cdot 516 = 266256 \equiv 953$ (-11-)

$5^{261} = 5^{2^0+2^2+2^8} = 5 \cdot 625 \cdot 953 = 2978125 \equiv 1300$ (-11-)

$5^{2 \cdot 261} = (5^{261})^2 = (1300)^2 = 1690000 \equiv 2088 \equiv -1$ (-11-)

$5^{2^2 \cdot 261} = (5^{2\cdot 261})^2 = (-1)^2 = 1$ (-11-)

=) the sequence is 1300, -1, 1 =) $n = 2089$ is probable to be prime

=) the probability of having error is $p < \frac{1}{4^3} = \frac{1}{64}$

$n_2 = 1353$

$n_2 - 1 = 1353 - 1 = 1352 = 2^3 \cdot 169 \to s = 3, t = 169$

Iteration $k = 1, a = 2$

$2^{2^0} = 2 \pmod{1353}$

$2^{2^1} = 4 \pmod{1353}$

$2^{2^2} = 4 \cdot 4 = 16 \pmod{1353}$

$2^{2^3} = 16 \cdot 16 = 256 \pmod{1353}$

$2^{2^4} = (256)^2 = 65536 = \cancel{7741(-11-)}\ 592 \pmod{1353}$

$2^{2^5} = \cancel{(7741)^2 = 603199 = 8}\ (592)^2 = 350464 = 37$ (-11-)

$2^{2^6} = (37)^2 = 1369 = 16$ (-11-)

$2^{2^7} = (16)^2 = 256$ (-11-)

$2^{2^8} = (256)^2 = 65536 = 592$ (-11-)

$2^{2^9} = (592)^2 = 350464 = 37$ (-11-)

$169 = 2^7 + 2^5 + 2^3 + 2^0 \Rightarrow 2^{169} = 2^{2^0 + 2^3 + 2^5 + 2^7} = 2 \cdot 256 \cdot 37 \cdot 256 = 4849664$
$= 512 \pmod{1353}$

$2^{2\cdot 169} = (2^{169})^2 = (512)^2 = \cancel{1024(-11-)}\ 262144 = 1015$ (-11-)

$2^{2^2 \cdot 169} = \cancel{(1024)^2 = 1048576 = 1(-11-)}\ (1015)^2 = 1030225 = 592$

$2^{2^3 \cdot 169} = \cancel{(1)^2 = 1(-11-)}\ (592)^2 = 350464 = 37$ (-11-)

$2^{24 \cdot 169} = (37)^2 = 1369 = 16(\text{-''-})$

the sequence is 512, 1015, 592, 37, 16 $\Rightarrow n_2 = 1353$ is composite