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2nd Semester, 2020-2021

Geometry (Computer Science)

Bonus Exercises : Week 3

Exercise 1 (2p). Consider the following lines in space:

$$\ell_1 : \begin{cases} x = -\lambda \\ y = -1 - \lambda \qquad \lambda \in \mathbb{R} \\ z = 1 + \lambda \end{cases}$$

$$\ell_2 : \begin{cases} 81x - 16y - 28z + 174 = 0 \\ -7x + 7y - 4z - 3 = 0 \end{cases}$$

Show that they are coplanar (they belong to the same plane) and find the equation of this common plane.

Exercise 2 (2p). Let $ABCD$ be a tetrahedron in space, with $A(1,2,3)$, $B(0,-1,2)$, $C(-5,3,6)$, $D(2,3,0)$. Let G be the centroid of the triangle BCD. Find the equation of the plane π that contains the points A and G and is parallel to the vector $\overrightarrow{v}(1,3,1)$.

Exercise 3 (2p). Let π be a plane given by the parametric equations:

$$\pi : \begin{cases} x = 1 - \lambda + 6\mu \\ y = 3 - \lambda + 5\mu \qquad \lambda, \mu \in \mathbb{R} \\ z = 1 + \lambda + 8\mu \end{cases}$$

Find the equations of all the lines that are parallel to the plane π.

Exercise 4 (2p). Consider the planes:

$$\pi_1 : \ 2x + 3y - z + 1 = 0$$

$$\pi_2 : \ x + 5z - 3 = 0$$

$$\pi_3 : \ -15x + 11y + 3z - 2 = 0$$

Show that they intersect in a point P and write the equation of the line that passes through this point and is parallel to the vector $\overrightarrow{v} = (4, 1, 5)$.

Exercise 5 (2p). Consider the lines:

$$\ell_1 : \frac{x-3}{2} = \frac{y}{5} = \frac{-z+1}{3}$$

$$\ell_2 : \begin{cases} x - 4y - 6z + 3 = 0 \\ 2x + y + 3z - 9 = 0 \end{cases}$$

Show that they are, in fact, identical.

Exercise 6 (2p). Consider the quadrilateral $ABCD$ in the plane, with: $A = (\frac{3}{2}, 3)$, $B = (\frac{3}{2}, 8)$, $C = (4, 7)$, $D = (4, 4)$. Show that it is an isosceles trapezoid. Find the coordinates of the point E, where $\{E\} = BC \cap AD$ and write the equation of the bisector of the angle $\widehat{AEB}$.

Exercise 7 (3p). Let $A(1, -1)$, $B(5, -2)$, $C(9, 6)$ be points in the plane and G the centroid (center of mass) of the triangle ABC. Determine the centroids G_A, G_B, G_C of the triangles GBC, GCA and GAB, find the circumcenter of the triangle $G_AG_BG_C$ and the radius of the circumcircle.