

# Floyd-Warshall alg.:



$$D_0 = \begin{pmatrix} 0 & \infty & 55 & 2 & 22 \\ 2 & 0 & 1 & \infty & \infty \\ \infty & \infty & 0 & 2 & 2 \\ \infty & \infty & \infty & 0 & 4 \\ 4 & \infty & 1 & \infty & 0 \end{pmatrix}$$

$$P_0 = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \\ 5 & 0 & 5 & 0 & 0 \end{pmatrix}$$

$k=1$

$$D_1 = \begin{pmatrix} 0 & \infty & 55 & 2 & 22 \\ 2 & 0 & 1 & \boxed{4} & \boxed{29} \\ \infty & \infty & 0 & 2 & 2 \\ \infty & \infty & \infty & 0 & 4 \\ 4 & \infty & 1 & \boxed{6} & 0 \end{pmatrix}$$

$$P_1 = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 2 & 0 & 2 & \boxed{1} & \boxed{1} \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \\ 5 & 0 & 5 & \boxed{1} & 0 \end{pmatrix}$$

$k=2$

$$D_2 = \begin{pmatrix} 0 & \infty & 55 & 2 & 22 \\ 2 & 0 & 1 & 4 & 29 \\ \infty & \infty & 0 & 2 & 2 \\ \infty & \infty & \infty & 0 & 4 \\ 4 & \infty & 1 & 6 & 0 \end{pmatrix}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 \\ 2 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \\ 5 & 0 & 5 & 1 & 0 \end{pmatrix}$$

$$k=3$$

$$D_3 = \begin{pmatrix} 0 & \infty & 55 & 2 & 2 & 2 \\ 2 & 0 & 1 & \boxed{3} & \boxed{3} & \\ \infty & \infty & 0 & 2 & 2 & \\ \infty & \infty & \infty & 0 & 4 & \\ 4 & \infty & 1 & \boxed{3} & 0 & \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & \\ 2 & 0 & 2 & \boxed{3} & \boxed{3} & \\ 0 & 0 & 0 & 3 & 3 & \\ 0 & 0 & 0 & 0 & 4 & \\ 5 & 0 & 5 & \boxed{3} & 0 & \end{pmatrix}$$

$$k=4$$

$$D_4 = \begin{pmatrix} 0 & \infty & 55 & 2 & \boxed{6} & \\ 2 & 0 & 1 & 3 & 3 & \\ \infty & \infty & 0 & 2 & 2 & \\ \infty & \infty & \infty & 0 & 4 & \\ 4 & \infty & 1 & 3 & 0 & \end{pmatrix}$$

$$P_4 = \begin{pmatrix} 0 & 0 & 1 & 1 & \boxed{4} & \\ 2 & 0 & 2 & 3 & 3 & \\ 0 & 0 & 0 & 3 & 3 & \\ 0 & 0 & 0 & 0 & 4 & \\ 5 & 0 & 5 & 3 & 0 & \end{pmatrix}$$

$$k=5$$

$$D_5 = \begin{pmatrix} 0 & \infty & \boxed{7} & 2 & 6 & \\ 2 & 0 & 1 & 3 & 3 & \\ \boxed{6} & \infty & 0 & 2 & 2 & \\ \boxed{8} & \infty & \boxed{5} & 0 & 4 & \\ 4 & \infty & 1 & 3 & 0 & \end{pmatrix}$$

$$P_5 = \begin{pmatrix} 0 & 0 & \boxed{5} & 1 & 4 & \\ 2 & 0 & 2 & 3 & 3 & \\ \boxed{5} & 0 & 0 & 3 & 3 & \\ \boxed{5} & 0 & \boxed{5} & 0 & 4 & \\ 5 & 0 & 5 & 3 & 0 & \end{pmatrix}$$

$D_4(4, 2) = \infty \Rightarrow$  there is no walk from 4 to 2

The minimum cost walk from 1 to 3 has the cost  $D_4(1, 3) = 7$  and it is obtained from  $P_4$  by using line 1

$$\Rightarrow t=3, P_4(1, 3) = \underline{5}, P_4(1, 5) = \underline{4}, P_4(1, 4) = \underline{1} = \underline{1} \Rightarrow$$

$\Rightarrow$  The min. cost walk from 1 to 3 is:  $1 \rightarrow 4 \rightarrow 5 \rightarrow 3$