Asist.dr. Tudor Micu

2nd Semester, 2020-2021

Geometry (Computer Science)

Bonus Exercises : Week 6

Exercise 1 (2p). Consider the lines ℓ_1, ℓ_2 and ℓ_3, given by the equations:

$$\ell_1 : \begin{cases} x + y - z - 3 = 0 \\ 3x - 7y + 3z + 3 = 0 \end{cases}$$

$$\ell_2 : \begin{cases} 5x - y - z + 3 = 0 \\ 7x + y - 3z - 11 = 0 \end{cases}$$

$$\ell_3 : \begin{cases} 4x - y + z - 17 = 0 \\ 8x - y - z - 23 = 0 \end{cases}$$

Construct the line ℓ that passes through the point $P(2, 2, 3)$, is perpendicular to ℓ_3 and to every line ℓ' that is perpendicular to ℓ_1 and ℓ_2.

Hint: You could try writing the equation of such a line ℓ' and reason from there, but I would prefer if you tried a more creative approach. I'd prefer it, but you do not absolutely need to cater to my preferences.

Exercise 2 (2p). We consider the tetrahedron $ABCD$, whose vertices have the coordinates $A(2, 3, 6)$, $B(3, 2, 2)$, $C(3, 4, 7)$ and $D(5, 1, 8)$. Find the lateral surface area of the tetrahedron (i.e. the sums of the areas of its faces) and find the volume of the tetrahedron.

Hint: The volume of a tetrahedron $ABCD$ is calculated as:

$$V_{ABCD} = \frac{1}{3}\text{dist}(A,(BCD)) \cdot S_{BCD} = \frac{1}{3}\text{dist}(B,(CDA)) \cdot S_{CDA} =$$
$$= \frac{1}{3}\text{dist}(C,(DAB)) \cdot S_{DAB} = \frac{1}{3}\text{dist}(D,(ABC)) \cdot S_{ABC}$$

Exercise 3 (2p). Consider the lines:

$$\ell_1 : \begin{cases} 2x + y - 2z + 2 = 0 \\ 4x - y - z + 1 = 0 \end{cases}$$

$$\ell_2 : \begin{cases} 3x - 4y + z - 3 = 0 \\ 6x - 2y - z - 6 = 0 \end{cases}$$

Find the equation of the locus of points in space that are equidistant to the two lines and bring it to the simplest form: $f(x,y,z) = 0$, where f is a quadratic function.

Exercise 4 (2p). Consider an orthonormal reference system $(O,[\vec{i},\vec{j},\vec{k}])$. Assume that the system is **inverse**, that is, that

$$\vec{i} \times \vec{j} = -\vec{k}$$

Deduce the formula for the cross product of two vectors $v_1 = (a_1, b_1, c_1)$ and $v_2 = (a_2, b_2, c_2)$ in determinant form.

Assume, now, that the reference system $(O,[\vec{i},\vec{j},\vec{k}])$ is orthogonal and direct, but

$$||\vec{i}|| = ||\vec{j}|| = \frac{1}{2}||\vec{k}|| = 1$$

Deduce the formula of the cross product of two vectors $v_1 = (a_1, b_1, c_1)$ and $v_2 = (a_2, b_2, c_2)$ in determinant form.

Generalize the arguments to the case where the system is orthogonal and direct and we have $||\vec{i}|| = \alpha$, $||\vec{j}|| = \beta$ and $||\vec{k}|| = \gamma$ with $\alpha, \beta, \gamma \in \mathbb{R}_{>0}$

Exercise 5 (**5p**; proceed at your own risk)**.** Consider the plane

$$\pi : x + 2y + 3z = 0$$

and the points $P(1, 9, 4)$ and $A(1, 1, -1) \in \pi$. Find the locus of the orthogonal projections of P onto the lines from the plane π that contain A.

Hint: You might want to choose a convenient reference system, to make everything easier.

Exercise 6 (**3p**)**.** Consider the convex pentagon $ABCDE$, whose vertices have coordinates $A(-3, 8, 3)$, $B(1, 2, 5)$, $C(3, -2, 5)$, $D(2, -3, 2)$, and $E(1, -3, 0)$ and are all situated in the plane $\pi : 2x + y - z + 1 = 0$. Find the area of the pentagon $ABCDE$.

Hint: The **centroid** of a convex polygon is always contained inside it. Every convex polygon is **star-shaped**, that is, if we pick any point inside the polygon, the segments between the point and the vertices are inside the polygon. Triangulation is your friend.