Asist.dr. Tudor Micu

2nd Semester, 2020-2021

Geometry (Computer Science)

Bonus Exercises : Week 4

Exercise 1 (2p). Write the equation of the plane which passes through the point $P(-1,-4,7)$ and is perpendicular to the line given by the parametric equations:

$$\ell : \begin{cases} x = 2 + 3t \\ y = -3 + t \\ z = 5 - 2t \end{cases}$$

Exercise 2 (2p). Let ℓ be the line that passes through the points $A(1,3,0)$ and $B(3,4,3)$. Determine the plane π that contains the line ℓ and passes through the point $C(1,0,1)$.

Exercise 3 (2p). Consider the line ℓ_1, given by the implicit equations:

$$\ell_1 : \begin{cases} 3x + 7y = 9 \\ z = 6 \end{cases}$$

Let ℓ_2 be the line containing the points $A(3,2,4)$ and $B(9,0,-1)$. Find the equation of the plane π which contains ℓ_1 and is parallel to the line ℓ_2.

Exercise 4 (2p). Find the plane π that contains the line:

$$\ell : \frac{x}{2} = \frac{y+5}{3} = \frac{z-1}{5}$$

and is perpendicular to the plane $\rho : 3x + 2y - z + 6 = 0$.

Hint: Two planes are perpendicular if and only if their normal vectors are perpendicular (use the dot product).

Exercise 5 (2p). Write the equation of the line which passes through the point $M(3, -2, 7)$, is parallel to the plane $\pi : 2x - y + z + 1 = 0$ and intersects the line

$$\ell : \frac{x-1}{6} = \frac{y-4}{-2} = \frac{z-12}{1}$$

Exercise 6 (2p). Consider the point $P(1, 4, 1)$, the plane $\pi : x + y - z + 10 = 0$ and the line

$$\ell : \frac{x-1}{-2} = \frac{y+3}{5} = \frac{z-8}{3}$$

that is contained inside it. Find all the vectors $\overrightarrow{v} \in \mathbb{R}^3$ so that ℓ is the projection of a line d, that contains the point P, onto π, parallel to $\overrightarrow{v}$.

(in other words, $p_{\pi, \overrightarrow{v}}(d) = \ell$ and $P \in d$)

Exercise 7 (2p). Let $A(0, 1, 11)$ be a point and ℓ a line with the equation:

$$\ell : \frac{x-5}{2} = \frac{y+3}{1} = \frac{z}{3}$$

Find the equation of the perpendicular line from A to ℓ and its intersection point A' with ℓ.

Hint: You can see this perpendicular as being part of a plane that is perpendicular to ℓ and contains A. Alternatively, you can see A' as the projection of A onto ℓ, parallel with a plane that is perpendicular to ℓ (for this kind of projection, see the relevant section in the lecture notes).

Exercise 8 (3p). Consider a triangle ABC with $A(2, 1)$, $B(18, 31)$, $C(30, 22)$, AA' an angle bisector with $A' \in [BC]$ and I the incenter. Let $D, E \in [BC]$ be so that $ID \parallel AB$ and $IE \parallel AC$. Find the ratio $\dfrac{DE}{BC}$, the area of the triangle IDE and its inradius.