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2nd Semester, 2020-2021

Geometry (Computer Science)

Bonus Exercises : Week 7

Exercise 1 (2p). Let $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ and $\overrightarrow{d}$ be vectors in $\mathbb{R}^3$. Show that:

$$((\overrightarrow{a} \times \overrightarrow{b}), (\overrightarrow{b} \times \overrightarrow{c}), (\overrightarrow{c} \times \overrightarrow{d})) = (\overrightarrow{a}, \overrightarrow{b}, \overrightarrow{c}) \cdot (\overrightarrow{b}, \overrightarrow{c}, \overrightarrow{d})$$

Exercise 2 (2p). Find the values of the parameter α for which the lines ℓ_1 and ℓ_2 are coplanar.

$$\ell_1 : \frac{x+1}{4} = \frac{y}{-\alpha} = \frac{z+2}{5}$$

$$\ell_2 : \begin{cases} x + 2y + \alpha z - 3 = 0 \\ 4x - \alpha y + 5z + 1 = 0 \end{cases}$$

Choose a value of α for which they are not coplanar and determine the distance between ℓ_1 and ℓ_2 in that case.

Exercise 3 (2p). Let $ABCD$ be a tetrahedron with $A(1, 0, 1)$, $B(0, -1, 1)$, $C(-1, 1, 1)$ and $D(2, 1, 0)$. We denote by ℓ_1 the common perpendicular of AB and CD and by ℓ_2 the common perpendicular of AD and BC.
Find the distance between ℓ_1 and ℓ_2.

Exercise 4 (2p). Let $VA_0B_0C_0$ be a tetrahedron. Let A_1, B_1 and C_1 be the midpoints of the segments $[B_0C_0]$, $[A_0C_0]$ and $[A_0B_0]$, respectively. We

repeat this process, wherein we choose for every $k \in \mathbb{N}$ the points A_{k+1}, B_{k+1} and C_{k+1} to be the midpoints of the segments $[B_kC_k]$, $[A_kC_k]$ and $[A_kB_k]$, respectively.
Assuming that $\text{Vol}(VA_0B_0C_0) = 1$, find the smallest value of $n \in \mathbb{N}$ for which $\text{Vol}(VA_nB_nC_n) < 0.001$.

Exercise 5 (2p). Let $ABCD$ be a tetrahedron with $A(2,3,1)$, $B(3,2,3)$, $C(0,1,6)$ and $D(2,5,4)$. Find the angle between the common perpendiculars of the lines AB and CD.
<u>Hint:</u> You can do this without actually calculating their equations.

Exercise 6 (2p). Let ℓ_1, ℓ_2, ℓ_3, ℓ_4 be lines given by the equations:

$$\ell_1 : \frac{x+1}{3} = \frac{y}{5} = \frac{z-1}{2} \qquad \ell_2 : \frac{x}{-7} = \frac{y+2}{2} = \frac{z}{4}$$
$$\ell_3 : \frac{x+2}{6} = \frac{y-1}{2} = \frac{z-1}{5} \qquad \ell_4 : \frac{x-1}{4} = \frac{y+6}{2} = \frac{z-4}{4}$$

Let $\ell_{1,2}$ be the common perpendicular of ℓ_1 and ℓ_2 and $\ell_{3,4}$ the common perpendicular of ℓ_3 and ℓ_4. Let ℓ be the common perpendicular of $\ell_{1,2}$ and $\ell_{3,4}$. Find the plane that is perpendicular to ℓ and contains the point $P(1,3,9)$.
<u>Hint:</u> Same hint as for Exercise 5.

Exercise 7 (2p). Consider the lines ℓ_1 and ℓ_2, where:

$$\ell_1 : \frac{x-2}{3} = \frac{y}{3} = \frac{z-1}{2}$$

and $\ell_2 \perp \pi$, where

$$\pi : x + 2y + 3z + 4 = 0$$

If $M(2,6,3)$ is the midpoint of the common perpendicular of ℓ_1 and ℓ_2, find the equation of ℓ_2.