Babeş-Bolyai University

Faculty of Mathematics and Computer Sciences

June, 06, 2020

GEOMETRY FOR FIRST YEAR STUDENTS IN COMPUTER SCIENCE

FINAL EXAM

Name				Group number			
Signature				Registration number $\overline{\alpha\beta\gamma\delta} =$			
Norm	**of. (1p)**	**Q.1 (1p)**	**Q.2 (1.5p)**	**Q.3 (1.5p)**	**Q.4 (1.5p+1.5p)**	**Q.5 (2p)**	Total
Score							

(1) Determine whether the given statements are TRUE or FALSE and circle the correct alternative.

(a) The angle between the tangent line of the parametrized differentiable curve $r : \mathbb{R} \longrightarrow \mathbb{R}^3$, $r(t) = ((\beta+1)\cos t, (\gamma+1)\sin t, (\delta+1)t)$ and the z-axis is constant (**True/False**).

(b) The conic of equation $\frac{x^2}{(\alpha+1)^2} - \frac{y^2}{(\beta+1)^2} = 1$ is a bounded set (**True/False**).

(c) The quadric of equation $\frac{x^2}{(\alpha+1)^2} + \frac{y^2}{(\beta+1)^2} - \frac{z^2}{(\gamma+1)^2} = -1$ admits two families of rectilinear generatrices (**True/False**).

(d) The revolution surface obtained by rotating the line

$$(l) \begin{cases} x = \alpha \\ y = \beta z \end{cases}$$

about the z-axis is a hyperbolouid of one sheet. (**True/False**).

(2) Find the equations of the tangent planes to the ellipsoid

$$(\mathcal{E})\ \frac{x^2}{(\alpha+1)^2} + \frac{y^2}{(\beta+1)^2} + \frac{z^2}{(\gamma+1)^2} = 1$$

which are perpendicular to the line

$$(L)\ \frac{x}{\alpha+1} = \frac{y}{\beta+1} = \frac{z}{\gamma+1}.$$

(3) Consider the curve

$$(C) \begin{cases} \frac{x^2}{(\alpha+1)^2} + \frac{y^2}{(\beta+1)^2} + \frac{z^2}{(\gamma+1)^2} = 1 \\ (\alpha+1)x + (\beta+1)y + (\delta+1)z = 0. \end{cases}$$

Find the equation of:

(a) the cylindrical surface with director curve C, whose generatrices are perpendicular to the plane of the curve.

(b) **(bonus 1p)** the conical surface with director curve C and vertex at $V(\alpha+1, \beta+1, \delta+1)$.

(4) Consider the quadrics $\mathcal{E}$ and $\mathcal{Q}$, where:

$$\mathcal{E} : \frac{x^2}{4} + y^2 + z^2 = 1$$

If your δ is **odd**, then

$$\mathcal{Q} : x^2 + \frac{y^2}{b} + \frac{z^2}{c} = 1$$

If your δ is **even**, then

$$\mathcal{Q} : -2x + \frac{y^2}{b} + \frac{z^2}{c} = 0$$

According to your γ, you will have:

$$(b, c) = \begin{cases} (4, -1), \text{ if } \gamma \in \{0, 2, 6\} \\ (-4, 1), \text{ if } \gamma \in \{1, 3, 9\} \\ (1, -4), \text{ if } \gamma \in \{4, 7\} \\ (-1, 4), \text{ if } \gamma \in \{5, 8\} \end{cases}$$

(a) Find all the rectilinear generatrices of $\mathcal{Q}$ that are parallel to the plane π, where:

$$\pi : \sqrt{|bc|}x + \sqrt{|c|}y + \sqrt{|b|}z + 1 = 0$$

(b) Find the locus of points on $\mathcal{Q}$ whose tangent plane is perpendicular to the tangent plane of $\mathcal{E}$ in the point $(r, s, 0)$, where:

$$(r, s) = \begin{cases} (2, 0), \text{ if } \gamma \in \{1, 4, 6\} \\ (0, 1), \text{ if } \gamma \in \{0, 3, 8\} \\ (-2, 0), \text{ if } \gamma \in \{2, 7\} \\ (0, -1), \text{ if } \gamma \in \{5, 9\} \end{cases}$$

Which conic is it?

Hint: A locus is a set of all points whose location satisfies or is determined by one or more specified conditions.

(5) (a) Let f be an affine transformation in the Euclidean plane that maps the points $A(1, 2)$ and $B(2, 3)$ to the points $f(A) = A'$ and $f(B) = B'$. For each of the following 4 types of transformations, decide if it is a valid candidate for f. If it is, then give an example of such an f. If it is not, then explain why (prove that there is no transformation f of that type that maps A and B to A' and B', respectively).

- translation;
- scaling;

- reflection;
- shear.

According to your δ, you will have:

$$\begin{cases} A'(-1,-1),\ B'(-2,-3) \text{ if } \delta \in \{2,4\} \\ A'(-2,5),\ B'(-3,8) \text{ if } \delta \in \{1,5\} \\ A'(2,4),\ B'(3,5) \text{ if } \delta \in \{3,7,9\} \\ A'(2,3),\ B'(1,2) \text{ if } \delta \in \{0,6,8\} \end{cases}$$

(b) **(bonus 1p)** We consider the line d through the origin $O(0,0)$ that has slope $\delta + 1$. Find the values of $\theta \in [0, 2\pi)$ for which we have

$$R_\theta \circ r_d = r_d \circ R_\theta$$

Hint: R_θ is the rotation around the origin by the angle θ (in the counter-clockwise direction); r_d is the reflection with respect to the line d.