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2nd Semester, 2020-2021

Geometry (Computer Science)

Bonus Exercises : Week 5

Exercise 1 (2p). Consider the lines ℓ_1 and ℓ_2, along with the planes π_1 and π_2, having the following equations:

$$\ell_1 : \begin{cases} x = 2 + 3t \\ y = -3 + t \\ z = 5 - 2t \end{cases} \qquad \ell_2 : \frac{x-1}{6} = \frac{y-4}{-2} = \frac{z-12}{1}$$

$$\pi_1 : 2x + y - z + 1 = 0 \qquad \pi_2 : 4x - y + z - 3 = 0$$

Find all the possible angles between them: $m(\widehat{\ell_1, \ell_2})$, $m(\widehat{\ell_1, \pi_1})$, $m(\widehat{\ell_1, \pi_2})$, $m(\widehat{\ell_2, \pi_1})$, $m(\widehat{\ell_2, \pi_2})$, $m(\widehat{\pi_1, \pi_2})$. Use a calculator to write the final answers correct to four decimal places.

Exercise 2 (2p). Consider the planes:

$$\pi_1 : 3x - 7y + z + 3 = 0 \qquad \pi_2 : 5x + y + 4z - 1 = 0$$

Find the locus of points M in space so that the distance to π_1 is twice the distance to π_2. Find the common line of the planes π_1 and π_2 and explain why it makes sense that it is contained in the locus that you found.

Exercise 3 (2p). Consider the cube $ABCDA'B'C'D'$ with vertices $A(-2,0,0)$, $B(3,0,0)$, $C(3,5,0)$, $D(-2,5,0)$, $A'(-2,0,5)$, $B'(3,0,5)$, $C'(3,5,5)$, $D(-2,5,5)$.

Find the equations of the planes $\pi_1 = (AB'C')$ and $\pi_2 = (BCA')$ and find the plane π that is part of their pencil and contains the point $P = (9, 9, 1)$.

Exercise 4 (2p). In a tetrahedron the **heights** are the segments between the vertices and their orthogonal projections onto the opposite faces. Consider the tetrahedron $ABCD$, where $A(1, 2, 5)$, $B(2, 1, 0)$, $C(20, 1, -2)$ and $D(4, 4, 5)$. Find the orthogonal projections of the vertices on the opposite faces and show that the heights are concurrent.

Exercise 5 (2p). Consider the points $A(3, 8, 4)$, $B(1, 9, 5)$ and $C(7, 7, 1)$. Find the equation of the plane π so that the orthogonal projection of the point A on π is B. Find $\alpha \in \mathbb{R}$ and the equation of the line ℓ that is parallel to the vector $(\alpha, 1, 3)$, so that the orthogonal projection of the point A on ℓ is B.

Exercise 6 (2p). Consider the planes:

$$\pi_1 : x + 2y + 3z - 2 = 0 \qquad \pi_2 : 3x - y + 5z + 4 = 0$$

Find π_3, the orthogonal reflection of the plane π_1 with respect to the plane π_2, and find π_4, the orthogonal reflection of the plane π_2 with respect to the plane π_1. Show that π_3 and π_4 are planes and find the angle between them (use a calculator to write the final answer correct to four decimal places).

Exercise 7 (2p). Consider the plane

$$\pi_1 : x + 2y + 2z + 7 = 0$$

Find the plane π_2 that is perpendicular to π_1 and contains the points $P(2, 1, 0)$ and $Q(2, -4, 2)$. Find all the planes π_3 that contain the common line of π_1 and π_2. Find the bisector planes of the dihedral angle $(\widehat{\pi_1, \pi_2})$.