A.I. Assignment 5

Fuzzy control of inverted pendulum

Consider an inverted pendulum on a cart. The cart is moving on the edge of a table only in one direction. Stabilize the pendulum in a vertical position by applying a proper force **F** to the cart.

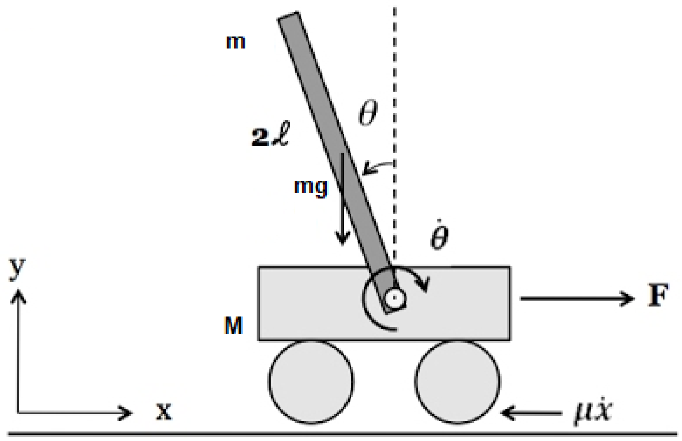

Figure 1. Inverted pendulum on a cart

In the physical model of the system, depicted in Figure 1, the parameters have the following meaning:

2l	length of inverted pendulum
F	traction force
x	position of the cart
θ	angle of the inverted pendulum
m	mass of the pendulum
M	mass of the cart
g	acceleration of gravity

μ	the coefficient of friction for the cart
$\omega = \dot{\theta}$	angular speed

Task

Using RBS, with Mandani's minimum inference engine, write the *solver* function, from file solver.py, in order to control the cart.

In:

- angle θ
- angular speed $\boldsymbol{\omega} = \dot{\theta}$

Out:

- the traction force **F** (or value None if there is a division by zero inside the function *solver*)

Data for the solver:

1. For angle θ we have the sets {**NVB**, **NB**, **N**, **ZO**, **P**, **PB**, **PVB**}. The sets are depicted in Figure 2.

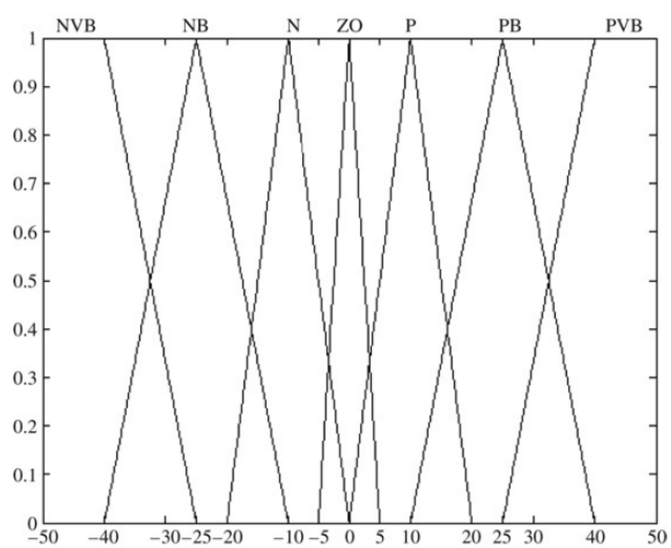

Figure 2. Membership functions for angle θ

2. For angular speed **ω** (= $\dot{\theta}$) we have the sets {**NB**, **N**, **ZO**, **P**, **PB**}, see Figure 3.

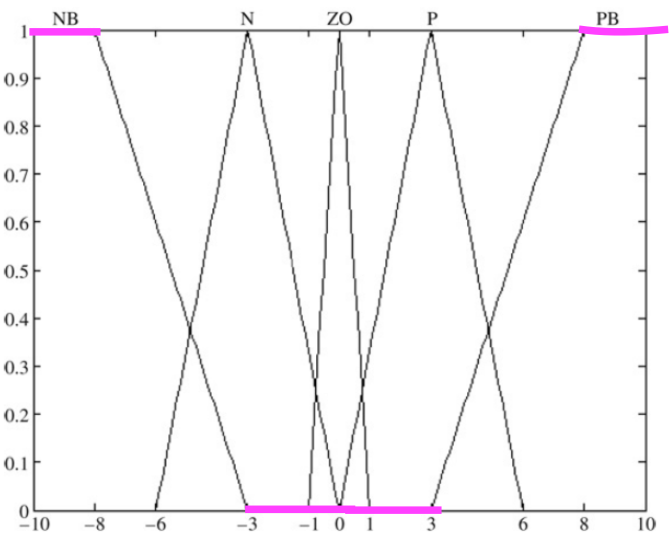

Figure 3. Membership functions for angular speed **ω**

3. For the traction force **F** we have the sets, depicted in Figure 4, {**NVVB**, **NVB**, **NB**, **N**, **ZO**, **P**, **PB**, **PVB**, **PVVB**}

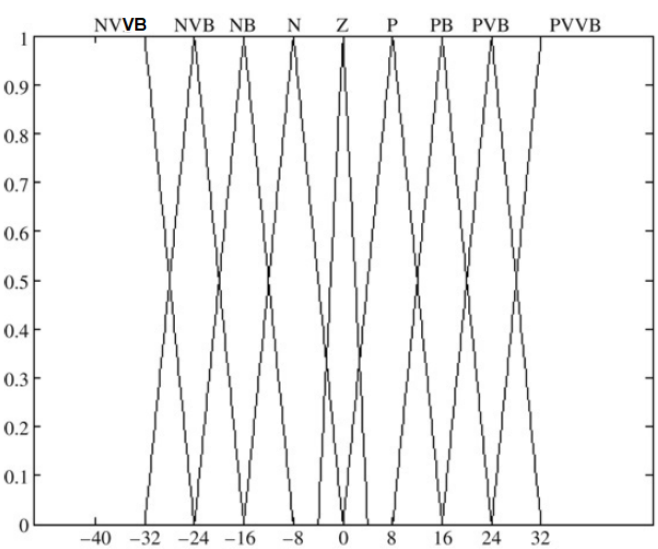

Figure 4. Membership functions for traction force **F**

The abbreviations describe the sets - NVB negative very big, NB negative big, Z or ZO zero, PVB positive very big, PVVB positive very very big.

The rules for the fuzzy system are in Table 1.

θ	ω				
	PB	P	ZO	N	NB
PVB	PVVB	PVVB	PVB	PB	P
PB	PVVB	PVB	PB	P	Z
P	PVB	PB	P	Z	N
ZO	PB	P	Z	N	NB
N	P	Z	N	NB	NVB
NB	Z	N	NB	NVB	NVVB
NVB	N	NB	NVB	NVVB	NVVB

Table 1. Table for the inverted pendulum fuzzy control system rule base

Example:

Let's consider that we have the angle $\theta = 7^o$ and $\omega = -0.5$.

We look for the force **F**.

Steps:

- compute the membership degrees for θ and ω to each set using the data from Figures 2 and 3, and using the formula for triangles from the lecture.

$\mu_{NVB}(\theta = 7^o) = \mu_{NB}(\theta = 7^o) = \mu_{N}(\theta = 7^o) = \mu_{ZO}(\theta = 7^o) = \mu_{PB}(\theta = 7^o) = \mu_{PVB}(\theta = 7^o) = 0;$

$\mu_{P}(\theta = 7^o) = 0.7;$

$\mu_{NB}(\omega =- 0.5) = \mu_{P}(\omega =- 0.5) = \mu_{PB}(\omega =- 0.5) = 0;$

$\mu_N(\omega =- 0.5) = 0.1(6); \mu_{ZO}(\omega = \; - 0.5) = 0.5$

- compute according to Table 1 the membership degree of **F** to each set.

Look in the table and for each cell we take the minimum of the membership values of the index set. So for cell [1,1] we take $min(\mu_{PB}(\omega =- 0.5), \mu_{PVB}(\theta = 7^o)) = min(0, 0) \; = \; 0$

We compute the values for all the cells of the table, thus evaluating each rule.

The membership degree of F to each class will be the maximum value for that class taken from the rules' table.

$$\mu_{ZO}(F) = \; max(cells\ that\ are\ labeled\ Z) = max(0,\ 0,\ 0,\ 0.1(6),\ 0) = 0.1(6)$$

The results for **F** are:

$\mu_{NVVB}(F) = \mu_{NVB}(F) = \mu_{NB}(F) = \mu_N(F) = \mu_{PB}(F) = \mu_{PVB}(F) = \mu_{VVB}(F) = 0;$

$\mu_Z(F) = 0.1(6); \mu_P(F) = 0.5.$

- defuzzify the results for **F** using a weighted average of the membership degrees and the b values of the sets.

$$F \; = \; \frac{0*(-32)+0*(-24)+0*(-16)+0*(-8)+0.16*(0)+0.5*(8)+0*(16)+0*(24)+0*(32)}{0+0+0+0+0.16+0.5+0+0+0} \approx 6.0$$

Bibliography:

Becerikli, Yasar, and B. Koray Celik. "Fuzzy control of inverted pendulum and concept of stability using Java application." *Mathematical and Computer Modelling* 46.1-2 (2007): 24-37.

For this assignment one can get a maximum **100** points.

Due time:

1 week for the final solution.

IF is not done in the first week you will have a penalty of 10 points.

The solution can not be turned in after the 2 weeks.