

1. $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$$f(x, y, z) = (4x - 4y - 3z, 4y - 3z, -2x + 3z)$$

$$\text{Im} f = \{ f(u) \mid u \in \mathbb{R}^3 \} =$$

$$= \{ (4x - 4y - 3z, 4y - 3z, -2x + 3z) \mid x, y, z \in \mathbb{R} \} =$$

$$= \{ (4x, 0, -2x) + (-4y, 4y, 0) + (-3z, -3z, 3z) \mid x, y, z \in \mathbb{R} \}$$

$$= \{ x \cdot (4, 0, -2) + y \cdot (-4, 4, 0) + z \cdot (-3, -3, 3) \mid x, y, z \in \mathbb{R} \}$$

$$= \langle (4, 0, -2), (-4, 4, 0), (-3, -3, 3) \rangle$$

$$\text{rank} \begin{pmatrix} 4 & 0 & -2 \\ -4 & 4 & 0 \\ -3 & -3 & 3 \end{pmatrix} = ?$$

$$\begin{vmatrix} 4 & 0 & -2 \\ -4 & 4 & 0 \\ -3 & -3 & 3 \end{vmatrix} = 48 - 24 - 24 + 0 = 0$$

$$\begin{vmatrix} 4 & 0 \\ -4 & 4 \end{vmatrix} = 16 \neq 0 \Rightarrow \begin{pmatrix} 4 & 0 & -2 \\ -4 & 4 & 0 \end{pmatrix} \text{ form a basis of } \text{Im} f$$

$$\Rightarrow \dim \text{Im} f = 2$$