Questions

Q1.

16 23 18 9 4 20 35 5 17 13 6 11

The numbers in the list represent the weights, in kilograms, of twelve parcels. The parcels are to be transported in containers that will each hold a maximum weight of 45 kg.

(a) Calculate a lower bound for the number of containers needed. You must make your method clear.

(2)

(b) Use the first-fit bin packing algorithm to allocate the parcels to the containers.

(3)

(c) Carry out a bubble sort, starting at the left-hand end of the list, to produce a list of the weights in descending order.
You should only give the state of the list after each pass.

(4)

(d) Use the first-fit decreasing bin packing algorithm to allocate the parcels to the containers.

(3)

(Total for question = 12 marks)

Q2.

The numbers listed below are to be packed into bins of size n, where n is a positive integer.

14 20 23 17 15 22 19 25 13 28 32

A lower bound for the number of bins required is 4

(a) Determine the range of possible values of n. You must make your method clear.

(3)

(b) Carry out a quick sort to produce a list of the numbers in descending order. You should show the result of each pass and identify your pivots clearly.

(4)

When the first-fit bin packing algorithm is applied to the **original** list of numbers, the following allocation is achieved.

Bin 1:	14	20	23	15
Bin 2:	17	22	19	13
Bin 3:	25	28		
Bin 4:	32			

When the first-fit decreasing bin packing algorithm is applied to the sorted list of numbers, the following allocation is achieved.

Bin 1:	32	28		
Bin 2:	25	23	22	
Bin 3:	20	19	17	15
Bin 4:	14	13		

(c) Determine the value of n. You must explain your reasoning fully.

(3)

(Total for question = 10 marks)

Q3.

2.6 0.8 2.1 1.2 0.9 1.7 2.3 0.3 1.8 2.7

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

(3)

The list is to be sorted into **descending** order.

(b) (i) Starting at the left-hand end of the above list, perform **two** passes through the list using a bubble sort. Write down the lists that result at the end of the first pass and the second pass.

(ii) Write down, in the table in the answer book, the number of comparisons and the number of swaps performed during each of these two passes.

(4)

After a third pass using this bubble sort, the updated list is

2.6 2.1 1.7 2.3 1.2 1.8 2.7 0.9 0.8 0.3

(c) Use a quick sort on this updated list to obtain the fully sorted list. You must make your pivots clear.

(3)

(d) Apply the first-fit decreasing bin packing algorithm to the fully sorted list to pack the numbers into bins of size 5

(3)

(Total for question = 13 marks)

Q4.

17 9 15 8 20 13 28 4 12 5

The numbers in the list shown above are the weights, in kilograms, of ten boxes.
The boxes are to be transported in containers that will each hold a maximum weight of 40 kilograms.

(a) Calculate a lower bound for the number of containers that will be needed to transport the boxes. You must show your working.

(2)

(b) Use the first-fit bin packing algorithm to allocate the boxes to the containers.

(3)

(c) Using the list provided, carry out a quick sort to produce a list of the weights in **ascending** order. You must make your pivots clear.

(3)

(d) Use the binary search algorithm to try to locate the weight of 9 in the sorted list. Clearly indicate how you choose your pivots and which part of the list is being rejected at each stage.

(3)

(Total for question = 11 marks)

Q5.

Pupils from ten schools are visiting a museum on the same day. The museum needs to allocate each school to a tour group. The maximum size of each tour group is 42 pupils. A group may include pupils from more than one school. Pupils from each school must be kept in the same tour group. The numbers of pupils visiting from each school are given below.

8 17 9 14 18 12 22 10 15 7

(a) Calculate a lower bound for the number of tour groups required. You must make your method clear.

(2)

(b) Using the above list, apply the first-fit bin packing algorithm to allocate the pupils visiting from each school to tour groups.

(2)

The above list of numbers is to be sorted into descending order.

(c) Perform a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

(d) Using your sorted list from (c), apply the first-fit decreasing bin packing algorithm to obtain a second allocation of pupils to tour groups.

(2)

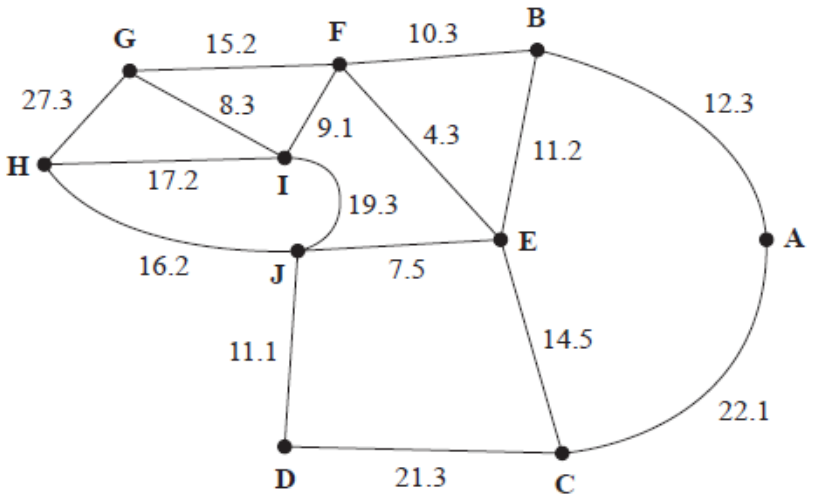


Figure 2

[*The total weight of the network is 227.2*]

Figure 2 represents the corridors in the museum. The number on each arc is the length, in metres, of the corresponding corridor. Sally is a tour guide in the museum and she must travel along each corridor at least once during each tour. Sally wishes to minimise the length of her route. She must start and finish at the museum's entrance at A.

(e) Use an appropriate algorithm to find the corridors that Sally will need to traverse twice. You should make your method and working clear.

(4)

(f) Write down a possible shortest route, giving its length.

(2)

Sally is now allowed to start at H and finish her route at a different vertex. A route of minimum length that includes each corridor at least once needs to be found.

(g) State the finishing vertex of Sally's new route and calculate the difference in length between this new route and the route found in (f).

(2)

(Total for question = 18 marks)

Q6.

Use the binary search algorithm to try to locate the word "Parallelogram" in the following alphabetical list. Clearly indicate how you choose your pivots and which part of the list you are rejecting at each stage.

Arc

Centre

Chord

Circle

Circumference

Diameter

Radius

Sector

Segment

Tangent

(Total for question = 4 marks)

Q7.

175 135 210 105 100 150 60 20 70 125

The numbers in the list above represent the weights, in kilograms, of ten crates. The crates are to be transported in trucks that can each hold a maximum total crate weight of 300 kg.

(a) Calculate a lower bound for the number of trucks that will be needed to transport the crates.

(2)

(b) Using the list provided, carry out a bubble sort to produce a list of the weights in descending order. You need only give the state of the list after each pass.

(4)

(c) Use the first-fit decreasing bin packing algorithm to allocate the crates to the trucks.

(3)

(Total for question = 9 marks)

Q8.

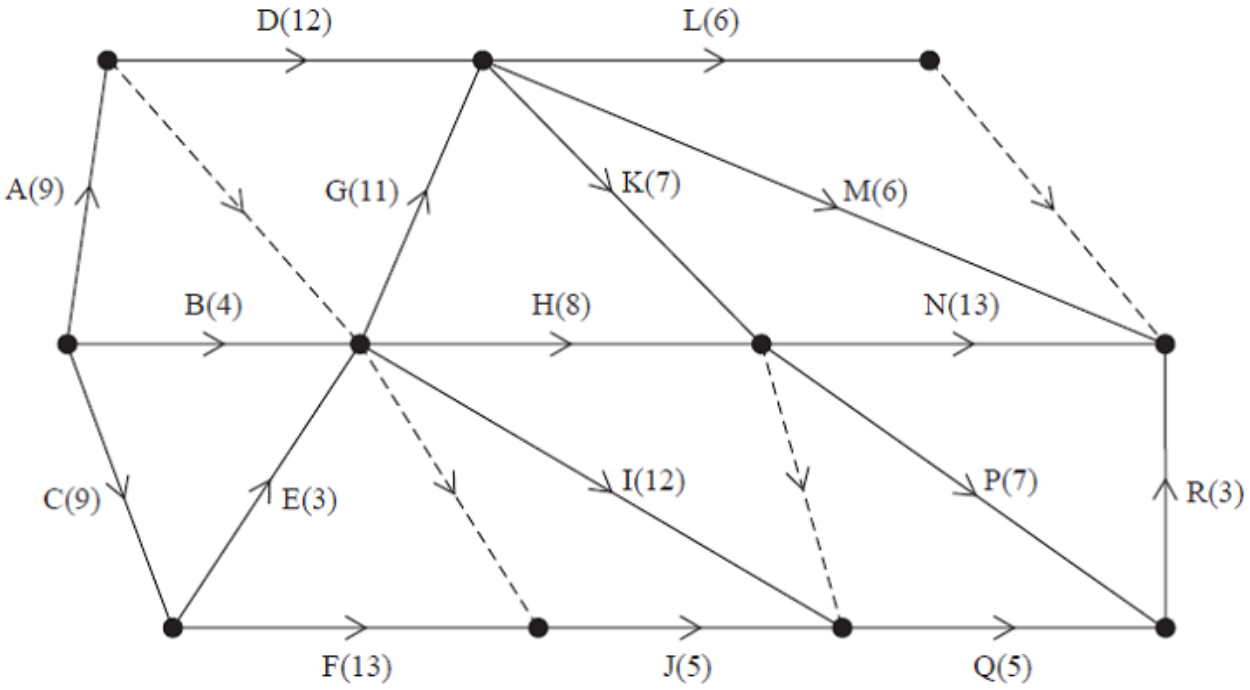


Figure 2

[*The sum of the durations of all the activities is* 133 *days*]

A project is modelled by the activity network shown in Figure 2. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete the precedence table in the answer book.

(2)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(c) State the critical activities.

(1)

(d) Calculate the total float for activity J. You must make the numbers you use in your calculation clear.

(1)

(e) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(1)

Diagram 2 in the answer book shows a partly completed scheduling diagram for this project.

(f) Complete the scheduling diagram, using the minimum number of workers, so that the project is completed in the minimum time.

(4)

(Total for question = 13 marks)

Q9.

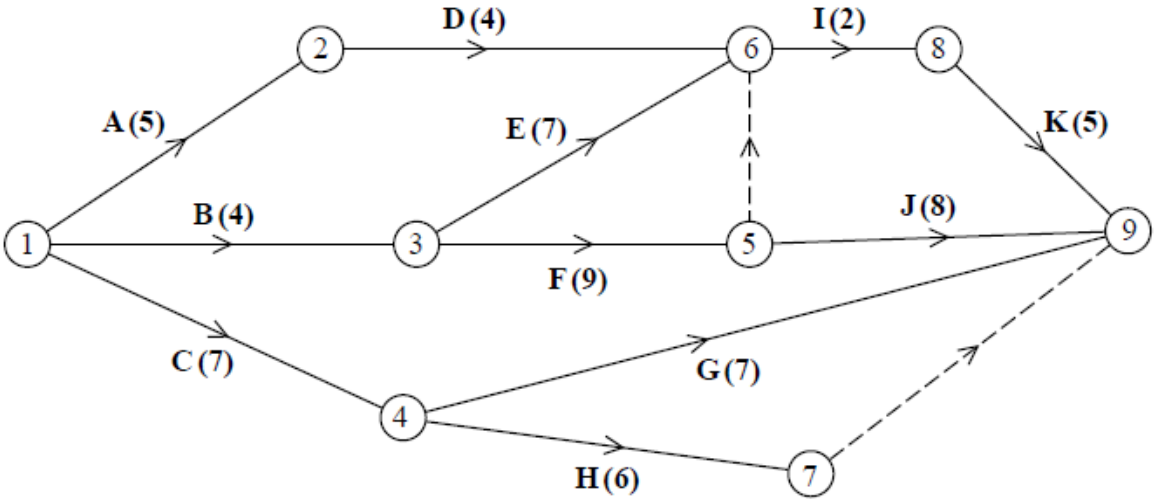


Figure 2

A project is modelled by the activity network shown in Figure 2. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.

(a) Explain the significance of the dummy activity

(i) from event 5 to event 6

(ii) from event 7 to event 9.

(2)

(b) Complete Diagram 3 in the answer book to show the early event times and the late event times.

(4)

(c) State the minimum project completion time.

(1)

(d) Calculate a lower bound for the minimum number of workers required to complete the project in the minimum time. You must show your working.

(2)

(e) On Grid 1 in your answer book, draw a cascade (Gantt) chart for this project.

(4)

(f) On Grid 2 in your answer book, construct a scheduling diagram to show that this project can be completed with three workers in just one more hour than the minimum project completion time.

(3)

(Total for question = 16 marks)

Q10.

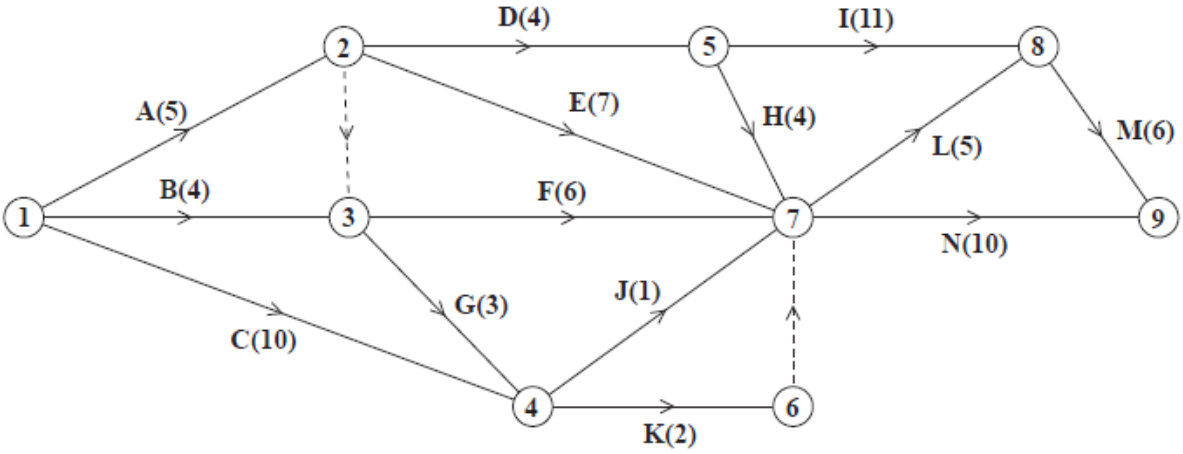


Figure 3

A project is modelled by the activity network shown in Figure 3. The activities are represented by the arcs. The number in brackets on each arc gives the time required, in hours, to complete the corresponding activity. The numbers in circles are the event numbers.

(a) Explain the significance of the dummy activity

(i) from event 2 to event 3

(ii) from event 6 to event 7

(2)

(b) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(c) State the minimum project completion time and list the critical activities.

(2)

The duration of activity H changes to x hours.

(d) Find, in terms of x where necessary,

(i) the possible new early event time for event 7

(ii) the possible new late event time for event 7

(3)

Given that the duration of activity H is such that the minimum project completion time is four hours greater than the time found in (c),

(e) determine the value of x.

(1)

(Total for question = 12 marks)

Q11.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	–
E	A
F	A, B, C
G	C
H	C
I	D, H
J	E
K	E
L	F, G, I
M	G, I

(a) Draw the activity network described in the precedence table, using activity on arc and exactly four dummies.

(5)

Given that there is a unique critical path for the network and that K is a critical activity,

(b) state the critical path for the network.

(1)

Given instead that all the activities shown in the precedence table have the same duration and K is not necessarily critical,

(c) state all the possible critical paths for the network.

(2)

(Total for question = 8 marks)

Q12.

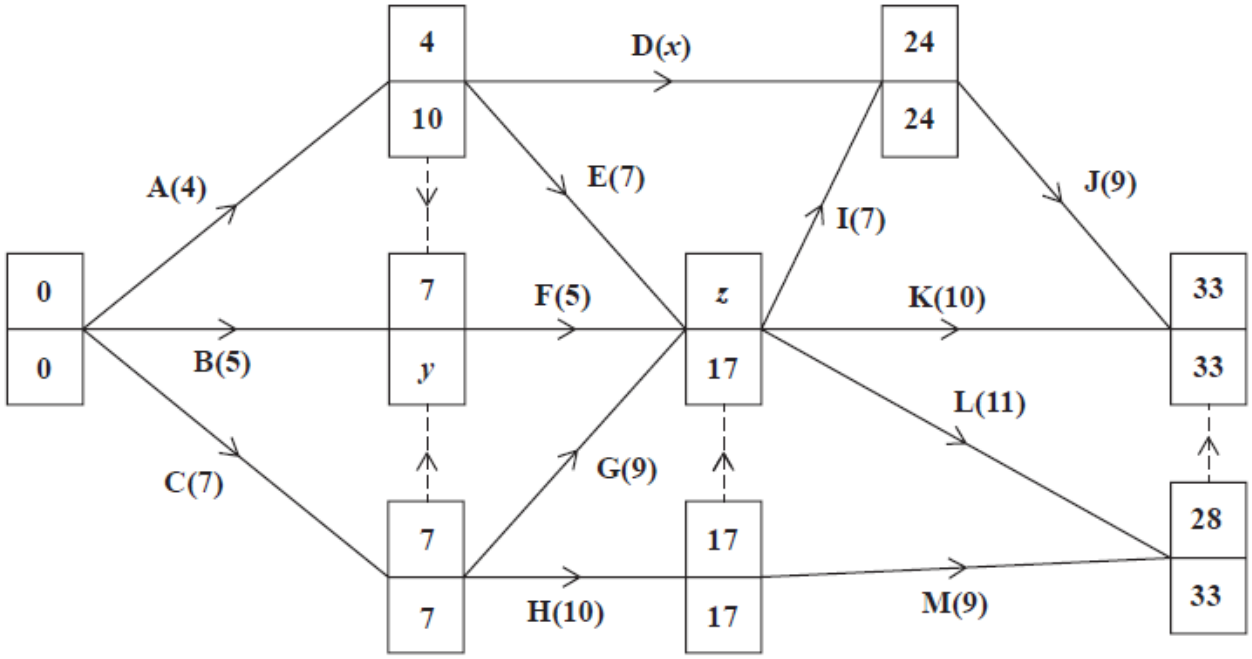


Figure 2

The network in Figure 2 shows the activities that need to be undertaken by a company to complete a project. Each activity is represented by an arc and the duration, in days, is shown in brackets. Each activity requires one worker. The early event times and late event times are shown at each vertex.

The total float on activity D is twice the total float on activity E.

(a) Find the values of x, y and z.

(3)

(b) Draw a cascade chart for this project on Grid 1 in the answer book.

(4)

(c) Use your cascade chart to determine a lower bound for the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities. (You do **not** need to provide a schedule of the activities.)

(2)

(Total for question = 9 marks)

Q13.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	C
F	A, B, C
G	A, B, C
H	D, F, G
I	A, B, C
J	D, F, G
K	H
L	D, E, F, G, I

(a) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain only the minimum number of dummies.

(5)

Given that all critical paths for the network include activity H,

(b) state which activities cannot be critical.

(2)

(Total for question = 7 marks)

Q14.

(a) Draw the activity network described by the precedence table below, using activity on arc. Use dummies only where necessary.

(5)

Activity	Immediately preceding activities
A	–
B	–
C	A
D	A, B
E	C, D
F	D
G	C
H	G
I	G
J	E, F, I
K	F

Given that K is a critical activity,

(b) state which other activities must also be critical.

(1)

Given instead that all activities shown in the precedence table have the same duration and K is not necessarily critical,

(c) state the critical path for the network.

(1)

(Total for question = 7 marks)

Q15.

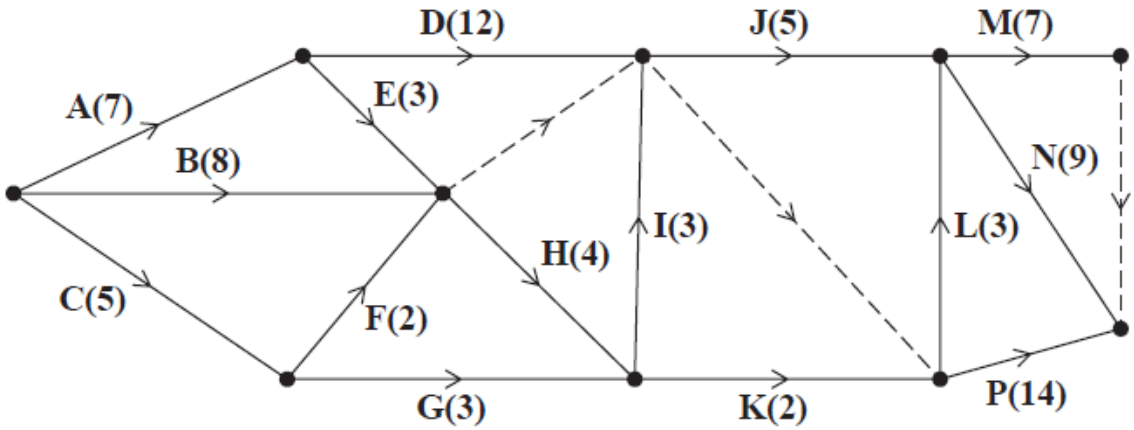


Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the corresponding activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(b) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

(c) Schedule the activities on Grid 1 in the answer book using the minimum number of workers so that the project is completed in the minimum time.

(4)

Additional resources become available, which can shorten the duration of one of activities D, G or P by one day.

(d) Determine which of these three activities should be shortened to allow the project to be completed in the minimum time. You must give reasons for your answer.

(2)

(Total for question = 12 marks)

Q16.

Activity	Duration (days)	Immediately preceding activities
A	4	–
B	7	–
C	6	–
D	10	A
E	5	A
F	7	C
G	6	B, C, E
H	6	B, C, E
I	7	B, C, E
J	9	D, H
K	8	B, C, E
L	4	F, G, K
M	6	F, G, K
N	7	F, G
P	5	M, N

The table above shows the activities required for the completion of a building project. For each activity the table shows the duration, in days, and the immediately preceding activities. Each activity requires one worker. The project is to be completed in the shortest possible time.

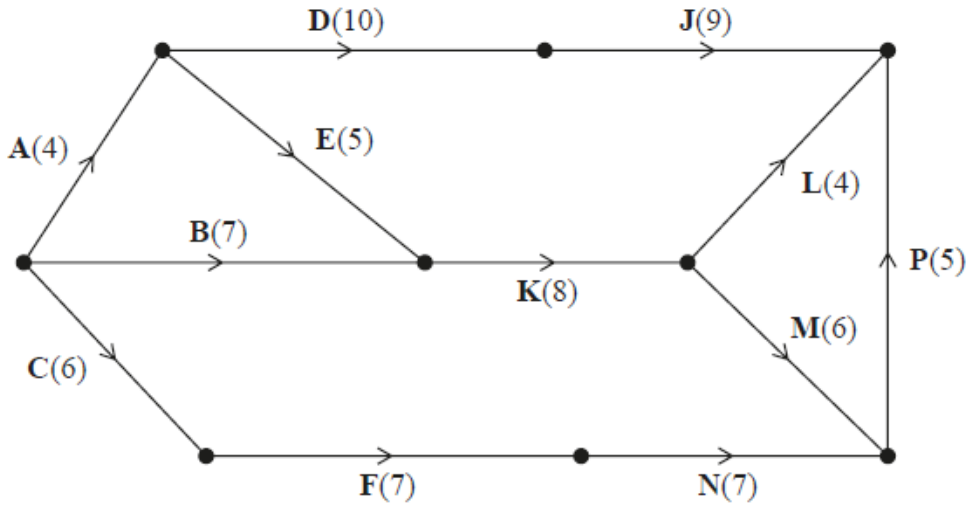


Figure 2

Figure 2 shows a **partially completed** activity network used to model the project. The activities are represented by the arcs and the numbers in brackets on the arcs are the times taken, in days, to complete each activity.

(a) Complete the network in Diagram 1 in the answer book by adding activities G, H and I and the minimum number of dummies.

(3)

(b) Add the early event times and the late event times to Diagram 1 in the answer book.

(4)

(c) State the critical activities.

(1)

(d) Calculate a lower bound for the number of workers needed to complete the project in the shortest possible time. You must show your working.

(2)

(e) Schedule the activities on Grid 1 in the answer book, using the minimum number of workers, so that the project is completed in the minimum time.

(3)

(Total for question = 13 marks)

Q17.

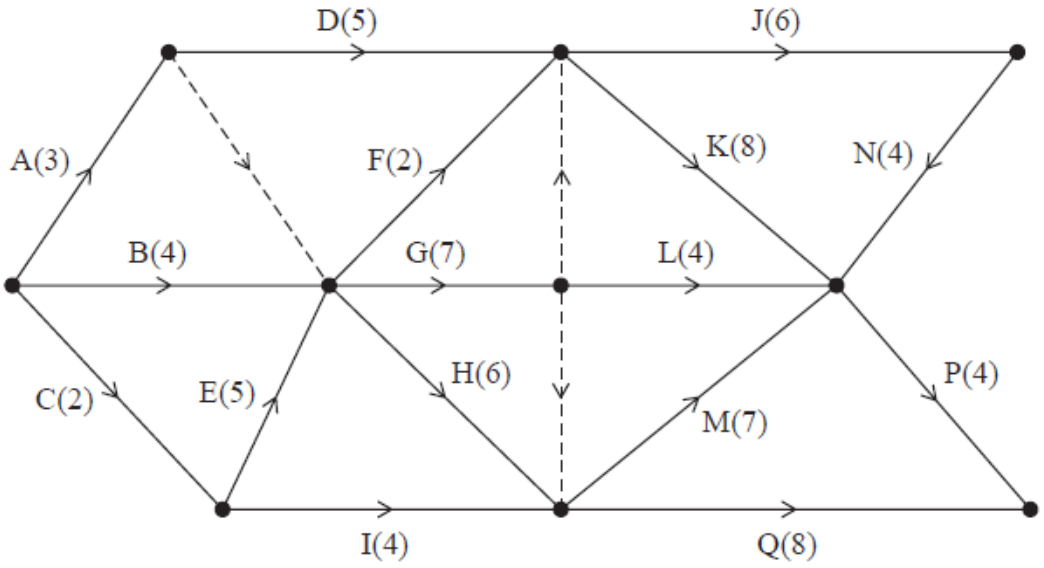


Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the corresponding activity. Each activity requires one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(b) Draw a cascade chart for this project on Grid 1 in the answer book.

(4)

(c) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities. (You do not need to provide a schedule of the activities.)

(2)

(Total for question = 10 marks)

Q18.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	A
F	A, B, C
G	C
H	G
I	D, E, F, H
J	I
K	I
L	I
M	L

(a) Draw the activity network for the project described in the precedence table above, using activity on arc and the minimum number of dummies.

(5)

(b) State which activity is guaranteed to be critical, giving a reason for your answer.

(2)

It is given that each activity in the table takes two hours to complete.

(c) State the minimum completion time and write down the critical path for the project.

(2)

(Total for question = 9 marks)

Q19.

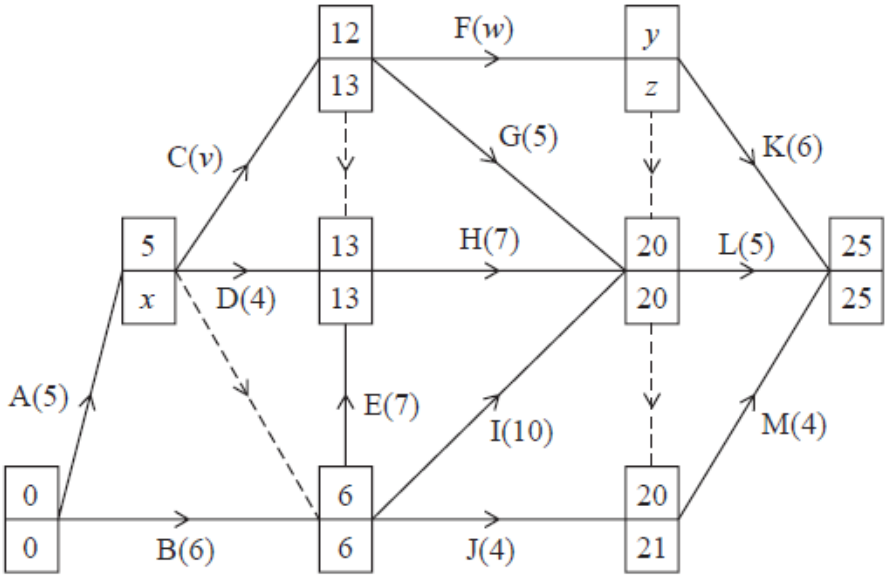


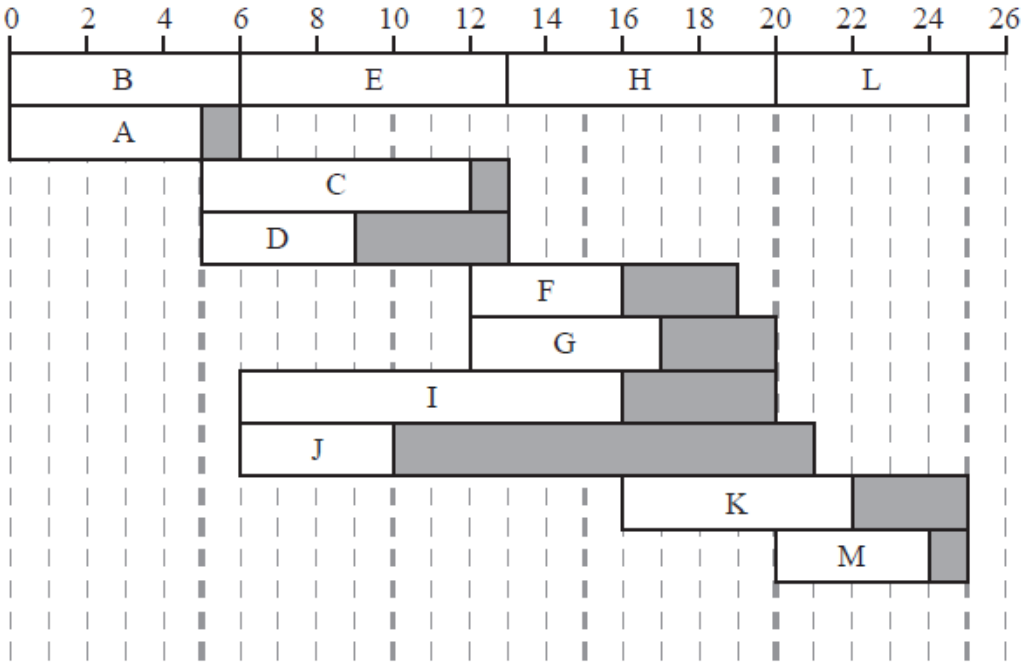
Figure 2

The network in Figure 2 shows the activities that need to be carried out by a company to complete a project. Each activity is represented by an arc, and the duration, in days, is shown in brackets. Each activity requires one worker. The early event times and the late event times are shown at each vertex.

(a) Complete the precedence table in the answer book.

(2)

A cascade chart for this project is shown on Grid 1.



Grid 1

(b) Use Figure 2 and Grid 1 to find the values of v, w, x, y and z.

(3)

The project is to be completed in the minimum time using as few workers as possible.

(c) Calculate a lower bound for the minimum number of workers required. You must show your working.

(1)

(d) On Grid 2 in your answer book, construct a scheduling diagram for this project.

(3)

Before the project begins it is found that activity F will require an additional 5 hours to complete. The durations of all other activities are unchanged. The project is still to be completed in the shortest possible time using as few workers as possible.

(e) State the new minimum project completion time and state the new critical path.

(2)

(Total for question = 11 marks)

Q20.

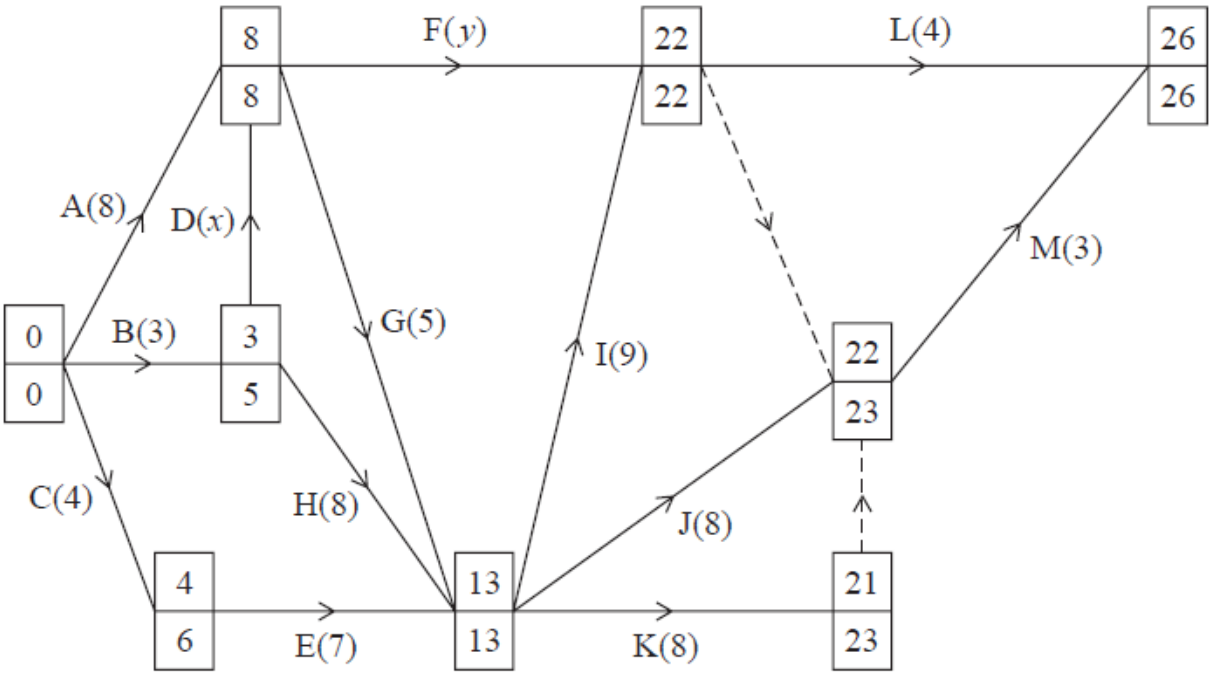


Figure 3

The network in Figure 3 shows the activities that need to be undertaken by a company to complete a project. Each activity is represented by an arc. The duration of the activity, in days, is shown in brackets. Each activity requires exactly one worker. The early event times and the late event times are shown at the vertices.

It is given that the total float on activity F is twice the total float on activity D.

It is also given that the total duration of the activities on the path BDFM is 10 days less than the duration of the critical path.

(a) Determine the value of x and the value of y. You must make your method and working clear.

(4)

(b) Draw a cascade chart for this project on Grid 1 in the answer book.

(4)

(c) Use your cascade chart to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities. (You do not need to provide a schedule of the activities.)

(2)

(Total for question = 10 marks)

Q21.

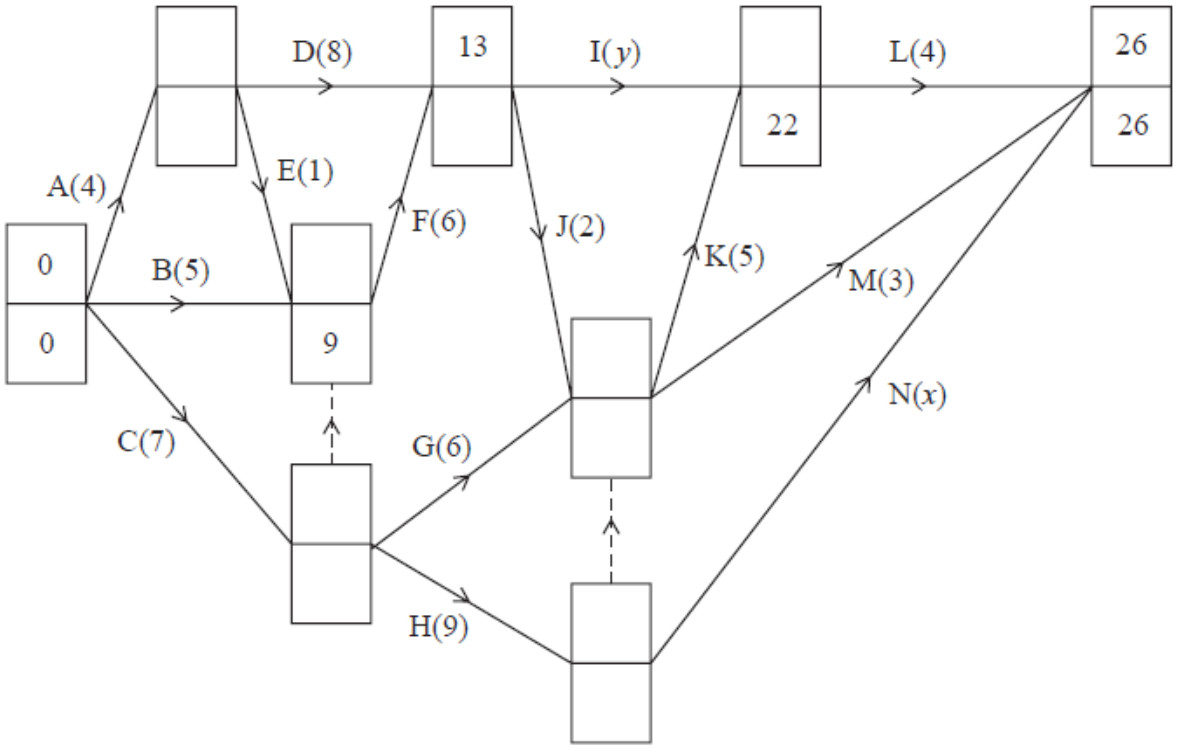


Figure 1

The network in Figure 1 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times and late event times are to be shown at each vertex and some have been completed.

Given that

- CHN is the critical path for the project
- the total float on activity B is twice the duration of the total float on activity I

(a) find the value of x and show that the value of y is 7

(2)

(b) Calculate the missing early event times and late event times and hence complete Diagram 1 in your answer book.

(3)

Each activity requires one worker, and the project must be completed in the shortest possible time.

(c) Draw a cascade chart for this project on Grid 1 in your answer book, and use it to determine the minimum number of workers needed to complete the project in the shortest possible time. You must make specific reference to time and activities.

(6)

(Total for question = 11 marks)

Q22.

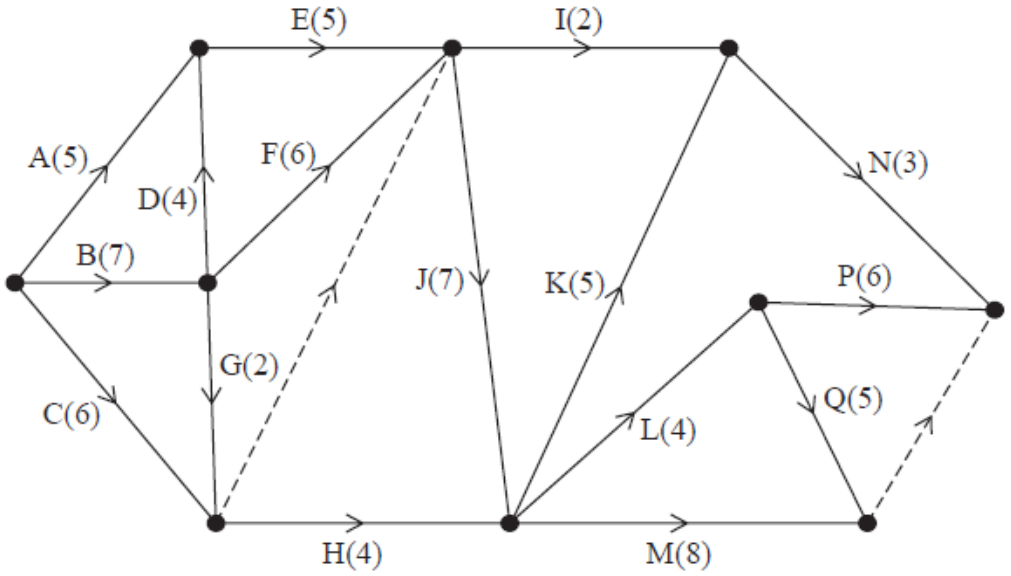


Figure 1

A project is modelled by the activity network shown in Figure 1. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires exactly one worker. The project is to be completed in the shortest possible time.

(a) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(4)

(b) Calculate the maximum number of days by which activity H could be delayed without lengthening the completion time of the project. You must make the numbers used in your calculation clear.

(1)

(c) Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

(2)

(d) Schedule the activities on Grid 1 in the answer book, using the minimum number of workers, so that the project is completed in the minimum time.

(3)

(Total for question = 10 marks)

Q23.

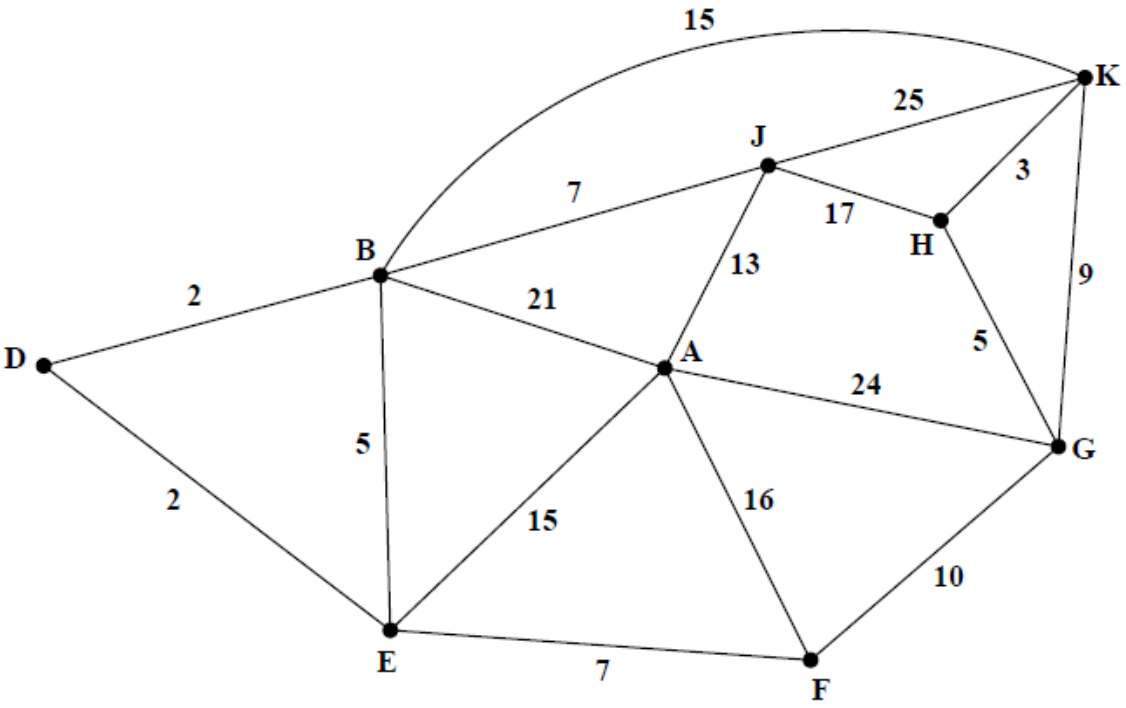


Figure 1

[The total weight of the network is 196]

Figure 1 models a network of roads. The number on each edge gives the time, in minutes, taken to travel along that road. Oliver wishes to travel by road from A to K as quickly as possible.

(a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to K. State the quickest route.

(6)

On a particular day Oliver must travel from B to K via A.

(b) Find a route of minimal time from B to K that includes A, and state its length.

(2)

Oliver needs to travel along each road to check that it is in good repair. He wishes to minimise the total time required to traverse the network.

(c) Use the route inspection algorithm to find the shortest time needed. You must state all combinations of edges that Oliver could repeat, making your method and working clear.

(7)

(Total for question = 15 marks)

Q24.

A headteacher is deciding how to allocate prizes to the students who are leaving at the end of the school year.

There are three categories of prize: academic, sport, and leadership.

- Each academic prize costs £14, each sport prize costs £8, and each leadership prize costs £12. The total amount available to spend on all prizes is £976
- For every 5 academic prizes there must be at least 2 leadership prizes
- At least half the prizes must be academic
- 20% of the prizes must be for sport

The headteacher wishes to maximise the total number of prizes.

Let x, y and z represent the number of academic, sport and leadership prizes respectively.

(a) Formulate this as a linear programming problem in x and y only, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(8)

Given that the headteacher awards 16 sport prizes,

(b) calculate the corresponding number of leadership prizes that the headteacher awards. You must show your working.

(2)

(Total for question = 10 marks)

Q25.

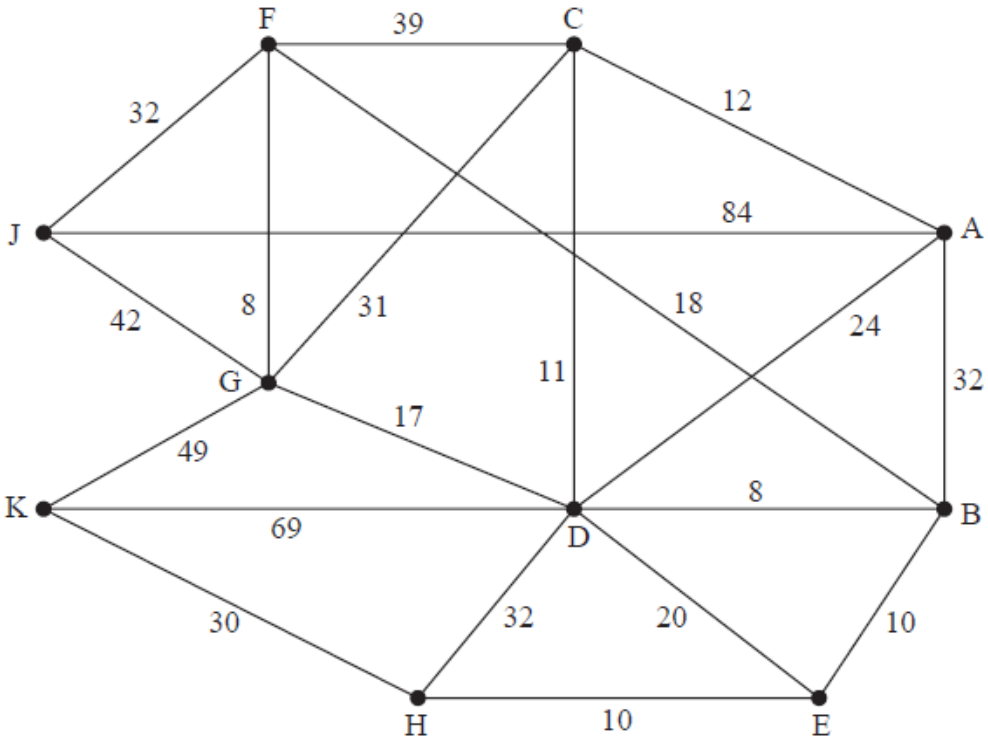


Figure 4

Figure 4 represents a network of roads. The number on each arc represents the length, in miles, of the corresponding road. Tamasi, who lives at A, needs to collect a caravan. Tamasi can collect a caravan from either J or K.

Tamasi decides to use Dijkstra's algorithm once to find the shortest routes between A and J and between A and K.

(a) State, with a reason, which vertex should be chosen as the starting vertex for the algorithm.

(2)

(b) Use Dijkstra's algorithm to find the shortest routes from A to J and from A to K. You should state the routes and their
corresponding lengths.

(7)

Tamasi's brother lives at F. He needs to visit Tamasi at A and then visit their mother who lives at H.

(c) Find a route of minimal length that goes from F to H via A.

(1)

(Total for question = 10 marks)

Q26.

(a) Explain what is meant by the term 'path'.

(2)

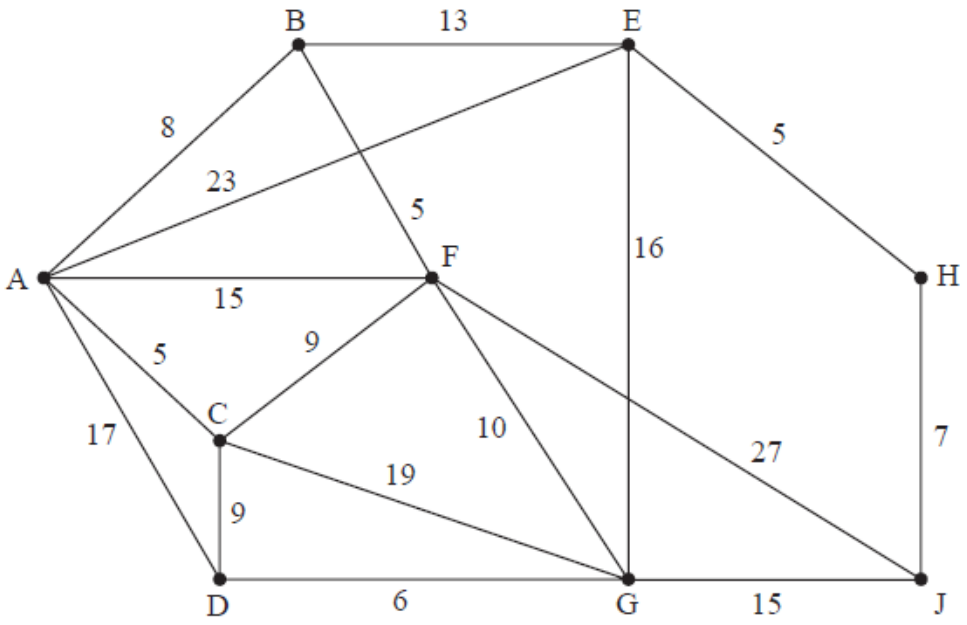


Figure 1

Figure 1 represents a network of roads. The number on each arc represents the length, in km, of the corresponding road. Piatrice wishes to travel from A to J.

(b) Use Dijkstra's algorithm to find the shortest path Piatrice could take from A to J.

State your path and its length.

(6)

Piatrice needs to return from J to A via G.

(c) Find the shortest path Piatrice could take from J to A via G and state its length.

(2)

(Total for question = 10 marks)

Q27.

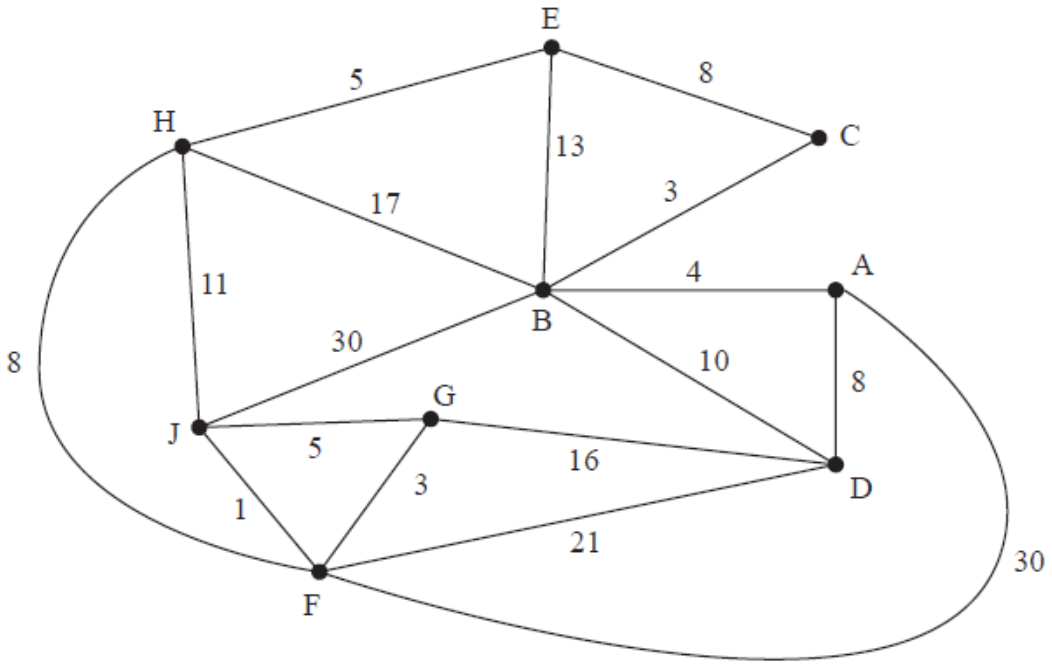


Figure 1

[*The total weight of the network is* 193]

Figure 1 represents a network of roads. The number on each edge represents the length, in miles, of the corresponding road. Jan wishes to travel from A to J. She wishes to minimise the distance she travels.

(a) Use Dijkstra's algorithm to find the shortest path from A to J. Obtain the shortest path and state its length.

(6)

On Monday, Jan needs to travel from her gym at J to her home at H via her office at A.

(b) State the shortest path from J to H via A and its length.

(2)

On Tuesday, Jan needs to check each road. She must travel along each road at least once. Jan must start and finish at A.

(c) Use the route inspection algorithm to find the length of the shortest inspection route. State the roads that should be repeated. You should make your method and working clear.

(5)

On Wednesday, Jan decides to start her inspection route at G but can finish her route at a different node. The inspection route must still traverse each road at least once.

(d) Determine where the route should finish so that the length of the inspection route is minimised. You must give reasons for your answer and state the length of the route.

(3)

(Total for question = 16 marks)

Q28.

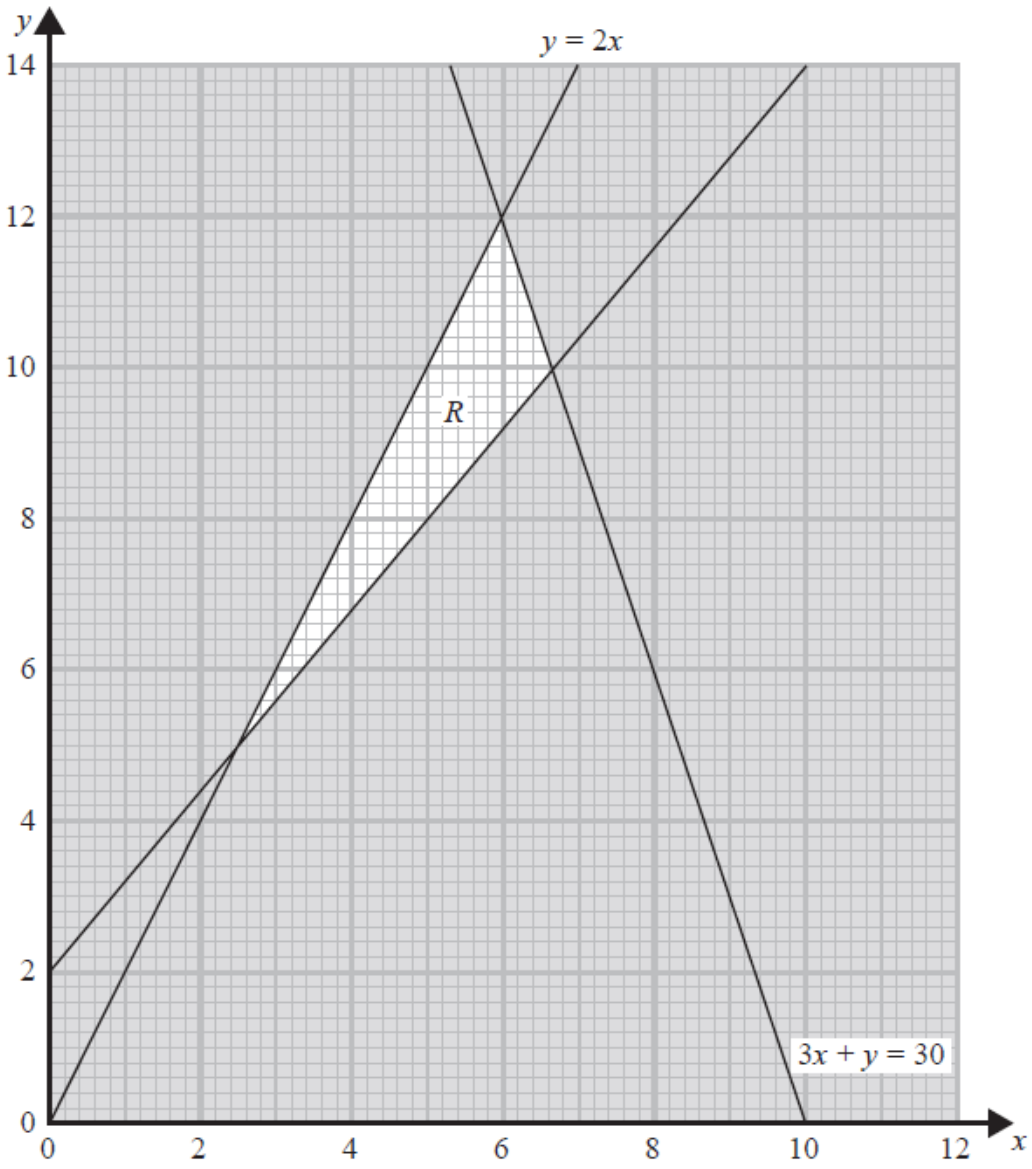


Figure 2

Figure 2 shows the constraints of a linear programming problem in x and y, where R is the feasible region. The equations of two of the lines are shown on the graph.

(a) Determine the inequalities that define the feasible region.

(3)

(b) Find the exact coordinates of the vertices of the feasible region.

(3)

The objective is to maximise P, where $P = 2x + ky$

(c) For the case $k = 3$, use the point testing method to find the optimal vertex of the feasible region and state the corresponding value of P.

(3)

(d) Determine the range of values for k for which the optimal vertex found in (c) is still optimal.

(2)

(Total for question = 11 marks)

Q29.

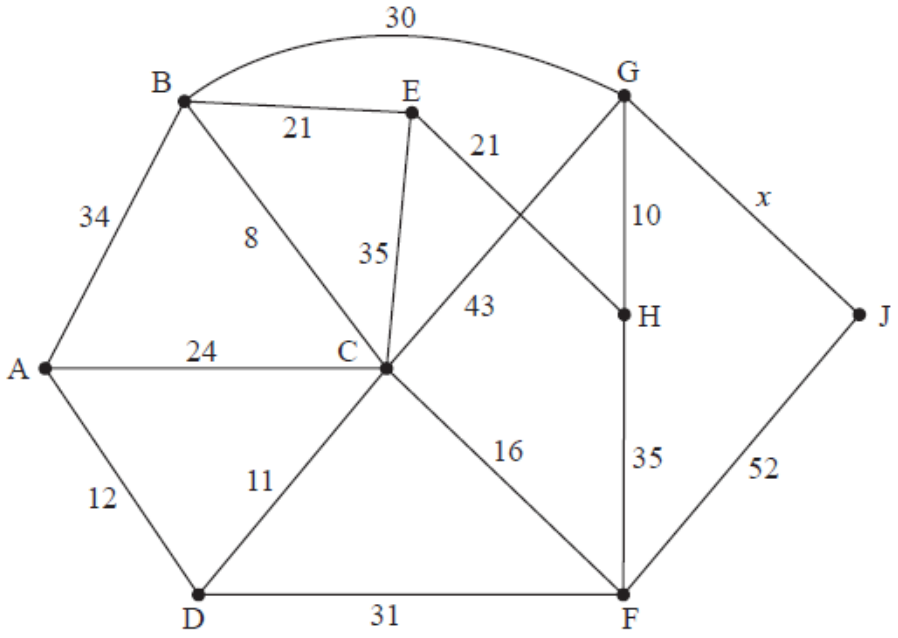


Figure 4

[*The total weight of the network* is $383 + x$]

Figure 4 models a network of roads. The number on each edge gives the time, in minutes, to travel along the corresponding road. The vertices, A, B, C, D, E, F, G, H and J represent nine towns. Ezra wishes to travel from A to H as fast as possible.

The time taken to travel between towns G and J is unknown and is denoted by x minutes.

Dijkstra's algorithm is to be used to find the fastest time to travel from A to H. On Diagram 1 in the answer book the "Order of labelling" and "Final value" at A and J, and the "Working values" at J, have already been completed.

(a) Use Dijkstra's algorithm to find the fastest time to travel from A to H. State the quickest route.

(6)

Ezra needs to travel along each road to check it is in good repair. He wishes to minimise the total time required to traverse the network. Ezra plans to start and finish his inspection route at A. It is given that his route will take at least 440 minutes.

(b) Use the route inspection algorithm and the completed Diagram 1 to find the range of possible values of x.

(6)

(c) Write down a possible route for Ezra.

(1)

A new direct road from D to H is under construction and will take 25 minutes to travel along. Ezra will include this new road in a minimum length inspection route starting and finishing at A. It is given that this inspection route takes exactly 488 minutes.

(d) Determine the value of x. You must give reasons for your answer.

(2)

(Total for question = 15 marks)

Q30.

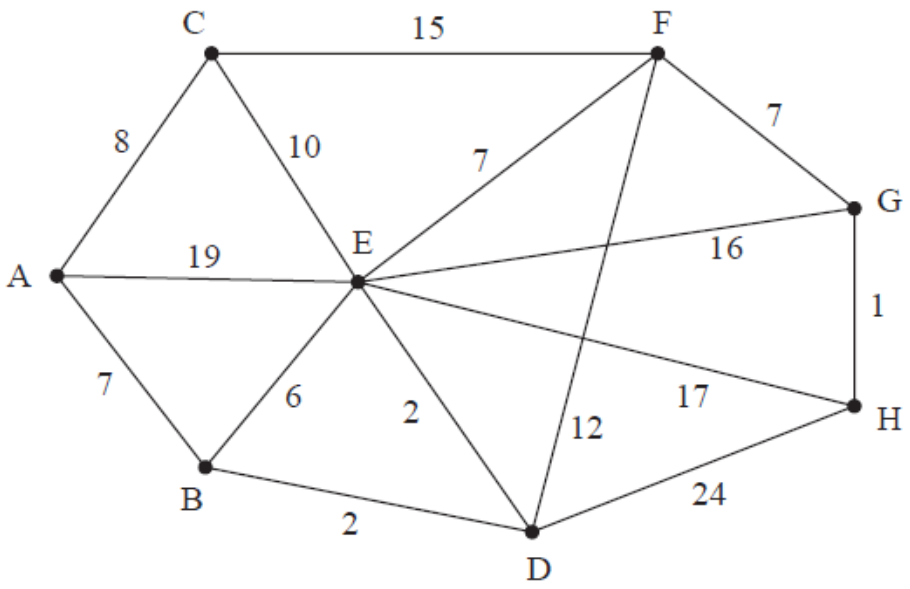


Figure 5

Figure 5 models a network of roads. The number on each edge gives the length, in km, of the corresponding road. The vertices, A, B, C, D, E, F, G and H, represent eight towns. Bronwen

needs to visit each town. She will start and finish at A and wishes to minimise the total distance travelled.

(a) By applying Dijkstra's algorithm, starting at A, complete the table of least distances in the answer book.

(6)

(b) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of Bronwen's route. Write down the route that gives this upper bound.

(2)

A reduced network is formed by deleting A and all arcs that are directly joined to A.

(c) (i) Use Prim's algorithm, starting at C, to construct a minimum spanning tree for the reduced network. You must clearly state the order in which you select the arcs of your tree.

(ii) Hence, calculate a lower bound for the length of Bronwen's route.

(4)

(d) Using only the results from (b) and (c), write down the smallest interval that you can be confident contains the length of Bronwen's optimal route.

(2)

(Total for question = 14 marks)

Q31.

1.8 1.4 2.6 1.6 2.8 0.9 3.1 0.8 1.2 2.4 0.6

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 5

(3)

The list is to be sorted into descending order.

(b) (i) Perform one pass of a bubble sort, starting at the left-hand end of the list. You must write down the list that results at the end of the first pass.

(ii) Write down the number of comparisons and the number of swaps performed during the first pass.

(3)

After a second pass using this bubble sort, the updated list is

2.6 1.8 2.8 1.6 3.1 1.4 1.2 2.4 0.9 0.8 0.6

(c) Use a quick sort on this updated list to obtain the fully sorted list in descending order. You must make your pivots clear.

(4)

(d) Apply the first-fit decreasing bin packing algorithm to the fully sorted list to pack the numbers into bins of size 5

(3)

(Total for question = 13 marks)

Q32.

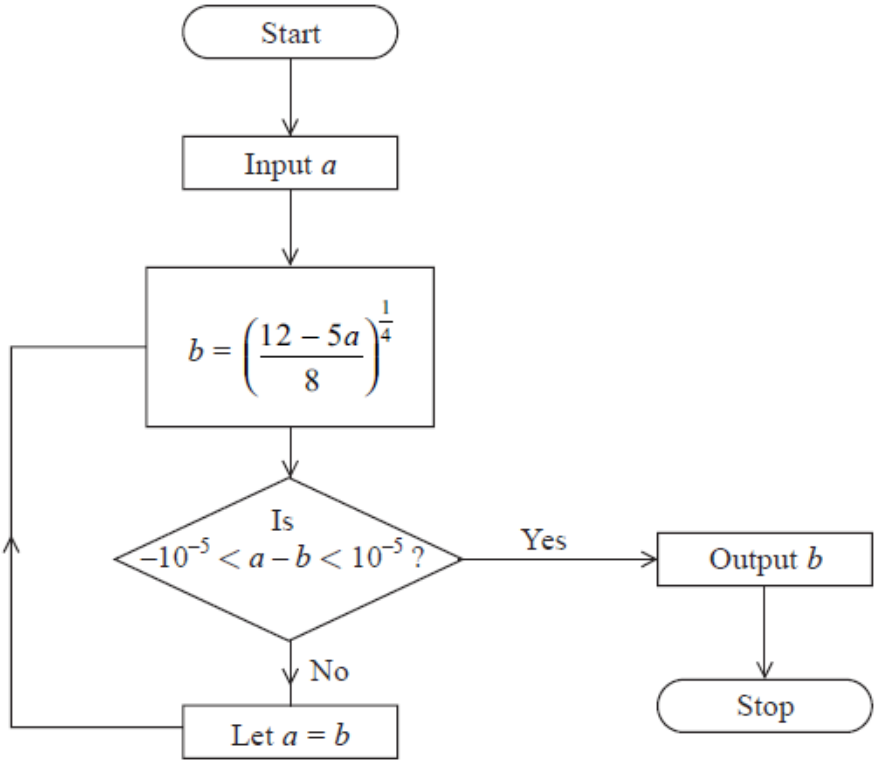


Figure 2

An algorithm for finding the positive real root of the equation $8x^4 + 5x - 12 = 0$ is described by the flow chart shown in Figure 2.

(a) Use the flow chart, with $a = 1$, to complete the table in the answer book, stating values to at least 6 decimal places.

Give the final output correct to 5 decimal places.

(4)

Given that the value of the input a is a non-negative real number,

(b) determine the set of values for a that **cannot** be used to find the positive real root of $8x^4 + 5x - 12 = 0$ using this flow chart.

(2)

(Total for question = 6 marks)

Q33.

A linear programming problem in x and y is described as follows.

Maximise $P = 5x + 3y$

subject to: $x \geqslant 3$

$x + y \leqslant 9$

$15x + 22y \leqslant 165$

$26x - 50y \leqslant 325$

(a) Add lines and shading to Diagram 2 in the answer book to represent these constraints. Hence determine the feasible region and label it R.

(4)

(b) Use the objective line method to find the optimal vertex, V, of the feasible region. You must draw and label your objective line and label vertex V clearly.

(2)

(c) Calculate the exact coordinates of vertex V and hence calculate the corresponding value of P at V.

(3)

The objective is now to **minimise** $5x + 3y$, where x and y are **integers**.

(d) Write down the minimum value of $5x + 3y$ and the corresponding value of x and corresponding value of y.

(2)

(Total for question = 11 marks)

Q34.

A clothing shop sells a particular brand of shirt, which comes in three different sizes, small, medium and large.

Each month the manager of the shop orders x small shirts, y medium shirts and z large shirts.

The manager forms constraints on the number of each size of shirts he will have to order.

One constraint is that for every 3 medium shirts he will order at least 5 large shirts.

(a) Write down an inequality, with integer coefficients, to model this constraint.

(2)

Two further constraints are

$$x + y + z \geq 250 \text{ and } x \leq 0.2(x + y + z)$$

(b) Use these two constraints to write down statements, in context, that describe the number of different sizes of shirt the manager will order.

(3)

The cost of each small shirt is £6, the cost of each medium shirt is £10 and the cost of each large shirt is £15

The manager must minimise the total cost of all the shirts he will order.

(c) Write down the objective function.

(1)

Initially, the manager decides to order exactly 150 large shirts.

(d) (i) Rewrite the constraints, as simplified inequalities with integer coefficients, in terms of x and y only.

(ii) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region R.

(6)

(e) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label V.

(2)

(f) Write down the number of each size of shirt the manager should order. Calculate the total cost of this order.

(2)

Later, the manager decides to order exactly 50 small shirts and exactly 75 medium shirts instead of 150 large shirts.

(g) Find the minimum number of large shirts the manager should order and show that this leads to a lower cost than the cost found in (f).

(2)

(Total for question = 18 marks)

Q35.

A company makes three types of storage container, small, medium and large.

The company owner knows that each week she should make

- at least 40 containers in total
- at least twice as many large containers as medium containers
- at most 60% small containers

Each small container requires 1 hour to make, each medium container requires 1.5 hours to make, and each large container requires 2.5 hours to make. The company has a total of 75 hours per week available to make all the containers.

Each small container costs £9 to make, each medium container costs £12 to make and each large container costs £16 to make.

The company owner wants to minimise her total cost.

- Let x represent the number of small containers made
- Let y represent the number of medium containers made
- Let z represent the number of large containers made

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(7)

The company owner now decides to make exactly 45 containers.

(b) Explain why the minimum total cost is achieved when $7x + 4y$ is maximised.

(3)

The requirement to make exactly 45 containers reduces the constraints of the problem to the following:

$$x + 3y \leq 45$$

$$0 \leq x \leq 27$$

$$3x + 2y \geq 75$$

$$y \geq 0$$

(c) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(d) Use the objective line method to find the optimal vertex, V, of the feasible region. You must make your objective line clear and label V.

(2)

(e) Write down the number of each type of container the company should make. Calculate the corresponding total cost.

(2)

(Total for question = 18 marks)

Q36.

A restaurant sells two sizes of pizza, small and large. The restaurant owner knows that, each evening, she needs to make

- at least 85 pizzas in total
- at least twice as many large pizzas as small pizzas

In addition, at most 80% of the pizzas must be large.

Each small pizza costs £2 to make and each large pizza costs £3 to make.

The restaurant owner wants to minimise her costs.

Let x represent the number of small pizzas made each evening and let y represent the number of large pizzas made each evening.

Formulate the information above as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients. You should **not** attempt to solve the problem.

(Total for question = 5 marks)

Q37.

Chris has been asked to design a badge in the shape of a triangle *XYZ* subject to the following constraints.

- Angle *Y* should be at least three times the size of angle *X*
- Angle *Z* should be at least 50° larger than angle *X*
- Angle *Y* must be at most 120°

Chris has been asked to maximise the sum of the angles *X* and *Y*.

Let *x* be the size of angle *X* in degrees.

Let *y* be the size of angle *Y* in degrees.

Let *z* be the size of angle *Z* in degrees.

Formulate this information as a linear programming problem in *x* and *y* **only**. State the objective and list the constraints as simplified inequalities with integer coefficients.

You are **not** required to solve this problem.

(6)

(Total for question = 6 marks)

Q38.

In this question, the function INT(*X*) is the largest integer less than or equal to *X*.

For example,

$$\text{INT}(5.7) = 5$$

$$\text{INT}(8) = 8$$

$$\text{INT}(-2.3) = -3$$

Consider the following algorithm.

Step 1 Input N

Step 2 Calculate $A = N \div 10$

Step 3 Let $B = \text{INT}(A)$

Step 4 Calculate $C = B \times 10$

Step 5 Calculate $D = N - C$

Step 6 Output D

Step 7 Replace N by B

Step 8 If $N = 0$ then STOP, otherwise go back to Step 2

(a) Complete the table in the answer book, using $N = 4217$, to show the results obtained at each step of the algorithm.

(4)

(b) Explain how the output values of the algorithm relate to the original input N, where N is any positive integer.

(2)

(Total for question = 6 marks)

Q39.

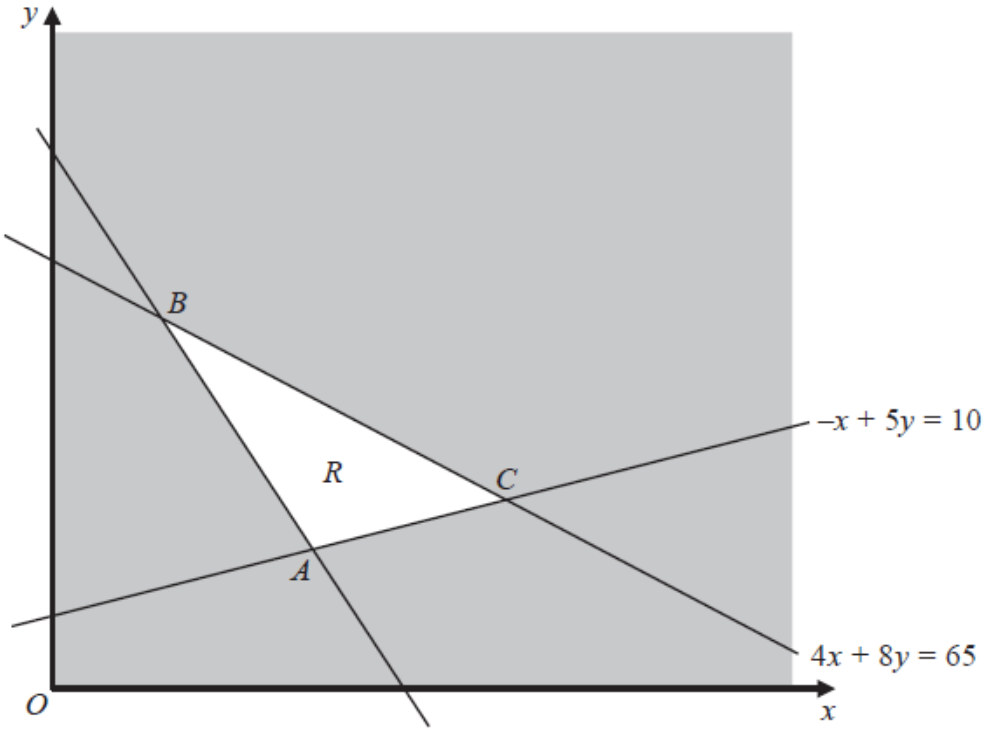


Figure 5

Figure 5 shows the constraints of a linear programming problem in x and y, where R is the feasible region. The equations of two of the lines and the three intersection points, A, B and C, are shown. The coordinates of C are $\left(\frac{35}{4}, \frac{15}{4}\right)$

The objective function is $P = x + 3y$

When the objective is to maximise $x + 3y$, the value of P is 24

When the objective is to minimise $x + 3y$, the value of P is 10

(a) (i) Find the coordinates of A and B.

(ii) Determine the inequalities that define R.

(8)

An additional constraint, $y \geq kx$, where k is a positive constant, is added to the linear programming problem.

(b) Determine the greatest value of k for which this additional constraint does not affect the feasible region.

(2)

(Total for question = 10 marks)

Q40.

A linear programming problem in x and y is described as follows.

Maximise $P = kx + y$, where k is a constant

subject to: $3y \geqslant x$

$x + 2y \leqslant 130$

$4x + y \geqslant 100$

$4x + 3y \leqslant 300$

(a) Add lines and shading to Diagram 1 in the answer book to represent these constraints. Hence determine the feasible region and label it R.

(4)

(b) For the case when $k = 0.8$

(i) use the objective line method to find the optimal vertex, V, of the feasible region. You must draw and label your objective line and label vertex V clearly.

(2)

(ii) calculate the coordinates of V and hence calculate the corresponding value of P at V.

(3)

Given that for a different value of k, V is not the optimal vertex of R,

(c) determine the range of possible values for k. You must make your method and working clear.

(4)

(Total for question = 13 marks)

Q41.

A school is planning to run several training days next year for its new teachers, middle leaders and senior leaders.

Next year, the school will need to run

- at least 20 training days in total,
- at most twice as many training days for new teachers when compared to the total

number of training days required for both middle and senior leaders,
- at most 25% of the training days for senior leaders.

The costs of running a training day for new teachers, middle leaders and senior leaders are £400, £550 and £750 respectively.

The school wants to minimise the total cost of running the training days.

Let x be the number of training days required for new teachers.
Let y be the number of training days required for middle leaders.
Let z be the number of training days required for senior leaders.

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(6)

The school decides that the number of training days for middle leaders and the number of training days for senior leaders should be in the ratio 5 : 3
This reduces two of the constraints to $5x \leq 16y$ and $4y \leq 5x$.

(b) (i) Express the constraint representing the requirement for a total of at least 20 training days as a simplified inequality with integer coefficients in terms of x and y only.

(ii) Express the objective in the form $ax + by$ where a and b are integers.

(3)

(c) Represent the constraints in x and y only on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(d) Use the objective line method to locate the optimal vertex, V, of the feasible region. You must make your objective line clear and label the optimal vertex, V.

(3)

(e) Hence, determine

(i) the total cost of running the training days,

(ii) the number of training days required for senior leaders.

(2)

(Total for question = 18 marks)

Q42.

A linear programming problem in x, y and z is described as follows.

Maximise $P = -x + y$

subject to

$x + 2y + z \leq 15$

$3x - 4y + 2z \geq 1$

$2x + y + z = 14$

$x \geq 0, y \geq 0, z \geq 0$

(a) (i) Eliminate z from the first two inequality constraints, simplifying your answers.

(ii) Hence state the maximum possible value of P

(4)

Given that P takes the maximum possible value found in (a)(ii),

(b) (i) determine the maximum possible value of x

(ii) Hence find a solution to the linear programming problem.

(3)

(Total for question = 7 marks)

Q43.

Martin is making three types of cake for a picnic. The three types of cake are carrot cake, apple cake and chocolate cake. Along with other ingredients,

- each carrot cake contains 275 grams of flour, 300 grams of sugar and 5 eggs
- each apple cake contains 200 grams of flour, 400 grams of sugar and 2 eggs
- each chocolate cake contains 100 grams of flour, 400 grams of sugar and 3 eggs

If Martin makes only one type of cake then he has enough time to prepare 15 carrot cakes or 20 apple cakes or 30 chocolate cakes.

Martin has 5.5 kilograms of flour and 70 eggs available and he has promised the picnic organisers that he will make at least 18 cakes in total.

Martin plans to make a selection of these cakes and wants to minimise the total amount of sugar that he uses.

Let x be the number of carrot cakes made, y the number of apple cakes made and z the number of chocolate cakes made.

(a) Formulate this information as a linear programming problem. State the objective and list the constraints as simplified inequalities with integer coefficients.

(6)

A further constraint is that $y = 2z$

(b) Explain what this constraint means in the context of the question.

The constraint $y = 2z$ reduces the problem to the following

(1)

Minimise $P = 300x + 600y$

subject to $11x + 10y \leqslant 220$

$10x + 7y \leqslant 140$

$x + y \leqslant 15$

$2x + 3y \geqslant 36$

$x \geqslant 0,\ y \geqslant 0$

(c) Represent these constraints on Diagram 1 in the answer book. Hence determine, and label, the feasible region, R.

(4)

(d) Use the objective line method to find the optimal number of each type of cake that Martin should make, and the amount of sugar used.

(4)

(e) Determine how much flour and how many eggs Martin will have left over after making the optimal number of cakes.

(2)

(Total for question = 17 marks)

Q44.

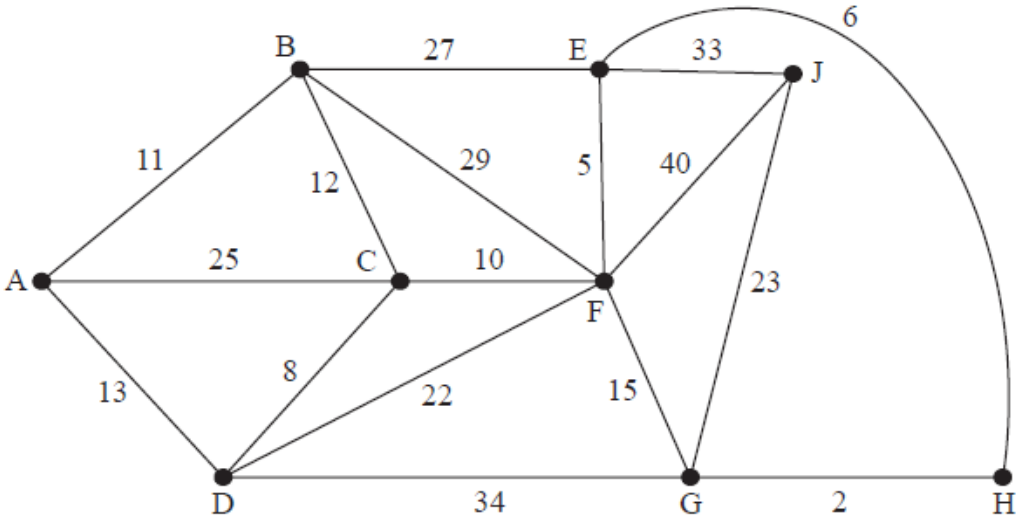


Figure 3

[*The total weight of the network is* 315]

Figure 3 represents a network of roads between nine parks, A, B, C, D, E, F, G, H and J. The number on each edge represents the length, in miles, of the corresponding road.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.

(ii) State the length of the shortest path from A to J.

(6)

The roads between the parks need to be inspected. Robin must travel along each road at least once. Robin wishes to minimise the length of the inspection route. Robin will start the inspection route at C and finish at E.

(b) By considering the pairings of all relevant nodes, find the length of Robin's route.

(4)

(c) State the number of times Robin will pass through G.

(1)

It is now decided to start and finish the inspection route at A. Robin must still minimise the length of the route and travel along each road at least once.

(d) Calculate the difference between the lengths of the two inspection routes.

(1)

(e) State the edges that need to be traversed twice in the route that starts and finishes at A, but do not need to be traversed twice in the route that starts at C and finishes at E.

(1)

(Total for question = 13 marks)

Q45.

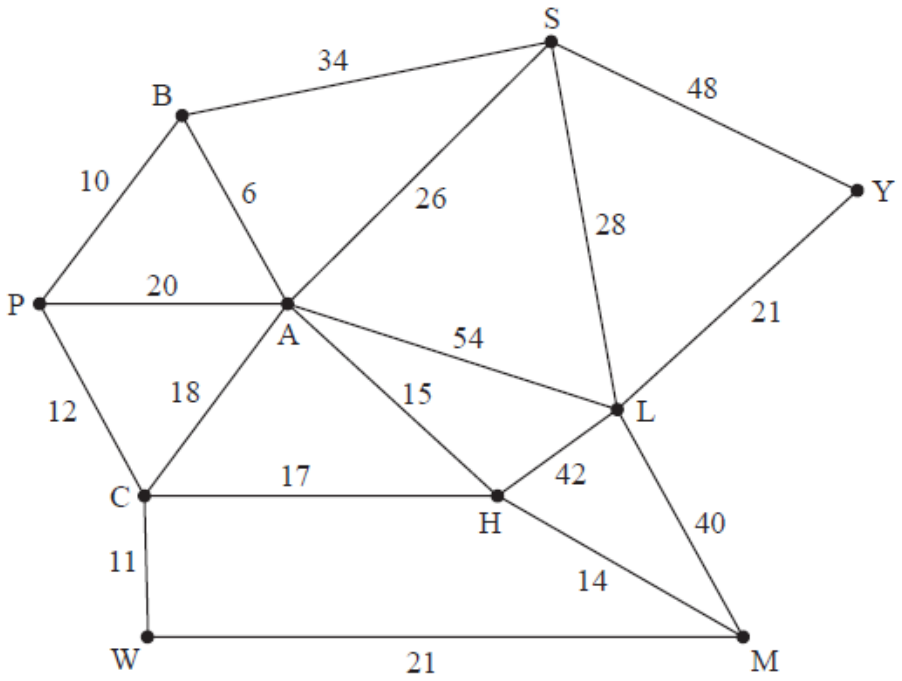


Figure 2

The network in Figure 2 shows the distances, in miles, between ten towns, A, B, C, H, L, M, P, S, W and Y.

(a) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order in which you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

	A	B	C	H	L	M	P	S	W	Y
A	–	6	18	15	54	29	16	26	29	74
B	6	–	22	21	60	35	10	32	33	80
C	18	22	–	17	59	31	12	44	11	80
H	15	21	17	–	42	14	29	41	28	63
L	54	60	59	42	–	40	70	28	61	21
M	29	35	31	14	40	–	43	55	21	61
P	16	10	12	29	70	43	–	42	23	90
S	26	32	44	41	28	55	42	–	55	48
W	29	33	11	28	61	21	23	55	–	82
Y	74	80	80	63	21	61	90	48	82	–

The table shows the shortest distances, in miles, between the ten towns.

(b) Use Prim's algorithm on the table, starting at A, to find the minimum spanning tree for this

network. You must clearly state the order in which you select the arcs of your tree.

(3)

(c) State the weight of the minimum spanning tree found in (b).

(1)

Sharon needs to visit all of the towns, starting and finishing in the same town, and wishes to minimise the total distance she travels.

(d) Use your answer to (c) to calculate an initial upper bound for the length of Sharon's route.

(1)

(e) Use the nearest neighbour algorithm on the table, starting at W, to find an upper bound for the length of Sharon's route. Write down the route which gives this upper bound.

(2)

Using the nearest neighbour algorithm, starting at Y, an upper bound of length 212 miles was found.

(f) State the best upper bound that can be obtained by using this information and your answers from (d) and (e). Give the reason for your answer.

(1)

(g) By deleting W and all of its arcs, find a lower bound for the length of Sharon's route.

(2)

Sharon decides to take the route found in (e).

(h) Interpret this route in terms of the actual towns visited.

(1)

(Total for question = 14 marks)

Q46.

	A	B	C	D	E	F	G	H
A	–	38	37	x	37	42	41	27
B	38	–	26	32	33	38	37	34
C	37	26	–	39	38	39	30	39
D	x	32	39	–	37	36	29	36
E	37	33	38	37	–	32	33	30
F	42	38	39	36	32	–	31	28
G	41	37	30	29	33	31	–	33
H	27	34	39	36	30	28	33	–

The network represented by the table shows the least distances, in km, between eight museums, A, B, C, D, E, F, G and H.

A tourist wants to visit each museum at least once, starting and finishing at A. The tourist wishes to minimise the total distance travelled. The shortest distance between A and D is x km where $32 \leq x \leq 35$

(a) Using Prim's algorithm, starting at A, obtain a minimum spanning tree for the network. You must clearly state the order in which you select the arcs of your tree.

(3)

(b) Use your answer to (a) to determine an initial upper bound for the length of the tourist's route.

(1)

(c) Starting at A, use the nearest neighbour algorithm to find another upper bound for the length of the tourist's route. Write down the route that gives this upper bound.

(2)

The nearest neighbour algorithm starting at E gives a route of

E – H – A – D – G – C – B – F – E

(d) State which of these two nearest neighbour routes gives the better upper bound. Give reasons for your answer.

(2)

Starting by deleting A, and all of its arcs, a lower bound of 235 km for the length of the route is found.

(e) Determine the smallest interval that must contain the optimal length of the tourist's route. You must make your method and working clear.

(4)

(Total for question = 12 marks)

Q47.

The precedence table shows the eleven activities required to complete a project.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A, B
E	A, B
F	B, C
G	B, C
H	D
I	D, E, F, G
J	H, I
K	D, E, F

(a) Draw the activity network for the project, using activity on arc and the minimum number of dummies.

(5)

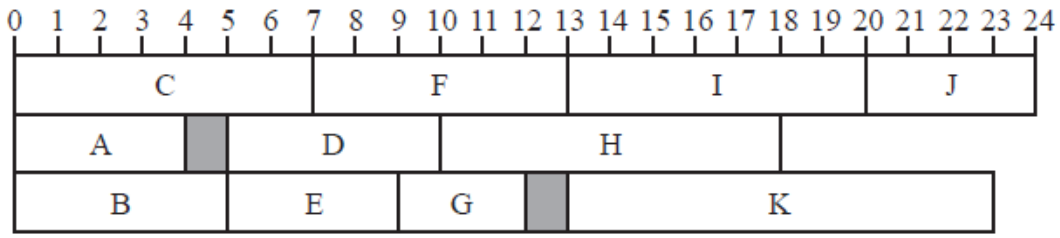


Figure 3

Figure 3 shows a schedule for the project. Each of the activities shown in the precedence table requires one worker. The time taken to complete each activity is in hours and the project is to be completed in the minimum possible time.

(b) (i) State the minimum completion time for the project.

(ii) State the critical activities.

(iii) State the total float on activity G and the total float on activity K.

(4)

(Total for question = 9 marks)

Q48.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A, B, C
E	A, B, C
F	C
G	F
H	D
I	D, E, G
J	D, E

Draw the activity network described in the precedence table above, using activity on arc and exactly 4 dummies.

(Total for question = 5 marks)

Q49.

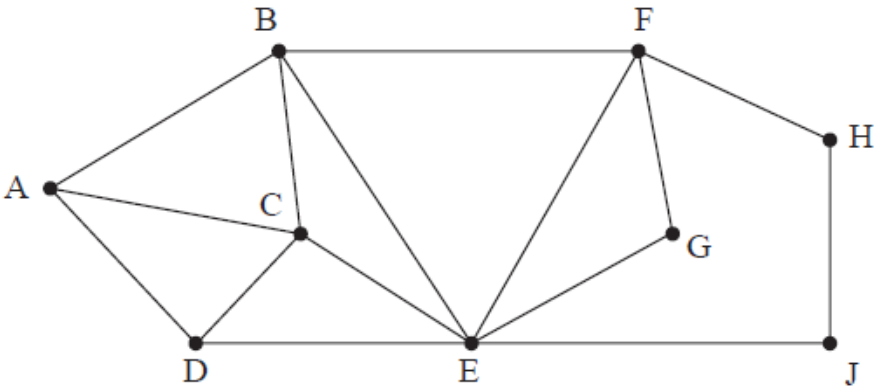


Figure 1

Figure 1 shows a graph, T.

(a) Write down an example of a path from A to J on T.

(1)

(b) State, with a reason, whether A – B – C – D – E – G – F – H – J is an example of a tour on T.

(1)

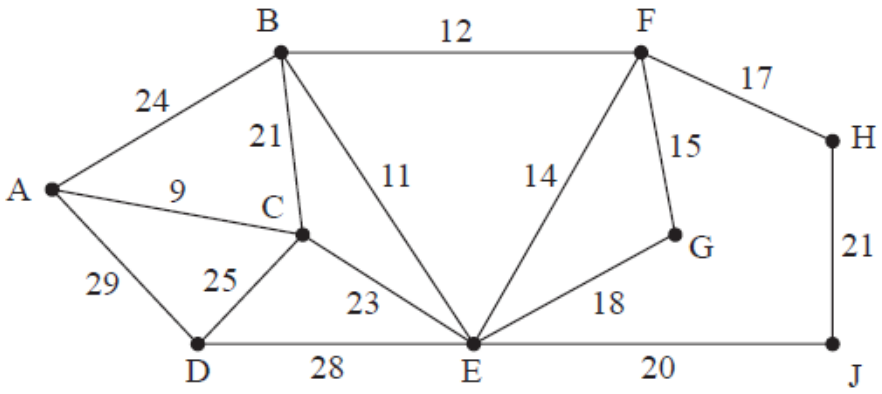


Figure 2

The numbers on the 15 arcs in Figure 2 represent the distances, in km, between nine vertices, A, B, C, D, E, F, G, H and J, in a network.

(c) Use Kruskal's algorithm to find the minimum spanning tree for the network.
You should list the arcs in the order in which you consider them. In each case, state whether or not you are adding the arc to the minimum spanning tree.

(3)

(d) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book.

(1)

(e) State the weight of the minimum spanning tree.

(1)

(Total for question = 7 marks)

Q50.

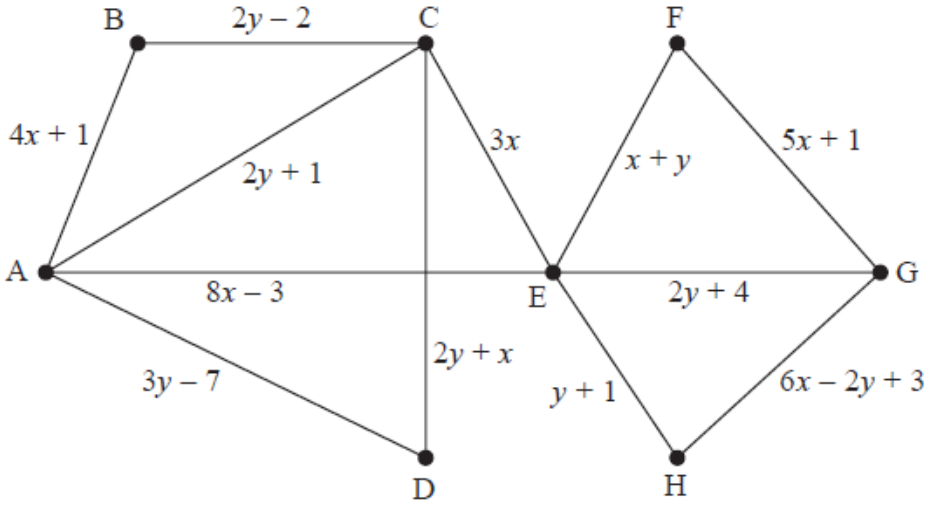


Figure 5

Figure 5 shows a weighted graph that contains 12 arcs and 8 vertices.

It is given that

- no two arcs have the same weight
- x and y are positive integers
- arc CD is not in the minimum spanning tree for the graph

(a) Explain why $y < x + 7$

(2)

It is also given that when Prim's algorithm, starting at A, is applied to the weighted graph, AB is the first arc selected.

(b) Show that $y > 2x$ and write down and simplify two further constraints on the values of x and y.

(3)

(c) Represent these four constraints on Diagram 1 in the answer book.

(4)

(d) Using Diagram 1 **only**, write down the possible pairs of values that x and y can take in the form (x, y).

(2)

The minimum spanning tree for the weighted graph in Figure 5 has total weight 73
Six of the seven arcs in the minimum spanning tree are AB, AD, BC, CE, EF and GH.

(e) Determine the value of x and the value of y. You must make your method and working clear.

(4)

(Total for question = 15 marks)

Q51.

(a) Explain the difference between the classical and the practical travelling salesperson problems.

(2)

The table below shows the distances, in km, between seven museums, A, B, C, D, E, F and G.

	A	B	C	D	E	F	G
A	–	25	31	28	35	30	32
B	25	–	34	24	27	32	39
C	31	34	–	40	35	27	29
D	28	24	40	–	37	35	36
E	35	27	35	37	–	28	31
F	30	32	27	35	28	–	33
G	32	39	29	36	31	33	–

Fran must visit each museum. She will start and finish at A and wishes to minimise the total distance travelled.

(b) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the length of Fran's route. Make your method clear.

(2)

Starting at D, a second upper bound of 203 km was found.

(c) State whether this is a better upper bound than the answer to (b), giving a reason for your answer.

(1)

A reduced network is formed by deleting G and all the arcs that are directly joined to G.

(d) (i) Use Prim's algorithm, starting at A, to construct a minimum spanning tree for the reduced network. You must clearly state the order in which you select the arcs of your tree.

(ii) Hence calculate a lower bound for the length of Fran's route.

(4)

By deleting A, a second lower bound was found to be 188 km.

(e) State whether this is a better lower bound than the answer to (d)(ii), giving a reason for your answer.

(1)

(f) Using only the results from (c) and (e), write down the smallest interval that you can be confident contains the length of Fran's optimal route.

(2)

(Total for question = 12 marks)

Q52.

The table below represents a complete network that shows the least costs of travelling between eight cities, A, B, C, D, E, F, G and H.

	A	B	C	D	E	F	G	H
A	–	36	38	40	23	39	38	35
B	36	–	35	36	35	34	41	38
C	38	35	–	39	25	32	40	40
D	40	36	39	–	37	37	26	33
E	23	35	25	37	–	42	24	43
F	39	34	32	37	42	–	45	38
G	38	41	40	26	24	45	–	40
H	35	38	40	33	43	38	40	–

Srinjoy must visit each city at least once. He will start and finish at A and wishes to minimise his total cost.

(a) Use Prim's algorithm, starting at A, to find a minimum spanning tree for this network. You must list the arcs that form the tree in the order in which you select them.

(3)

(b) State the weight of the minimum spanning tree.

(1)

(c) Use your answer to (b) to help you calculate an initial upper bound for the total cost of Srinjoy's route.

(1)

(d) Show that there are two nearest neighbour routes that start from A. You must make the routes and their corresponding costs clear.

(4)

(e) State the best upper bound that can be obtained by using your answers to (c) and (d).

(1)

(f) Starting by deleting A and all of its arcs, find a lower bound for the total cost of Srinjoy's route. You must make your method and working clear.

(3)

(g) Use your results to write down the smallest interval that must contain the optimal cost of Srinjoy's route.

(2)

(Total for question = 15 marks)

Q53.

Kruskal's algorithm finds a minimum spanning tree for a connected graph with n vertices.

(a) Explain the terms

(i) connected graph,

(ii) tree,

(iii) spanning tree.

(3)

(b) Write down, in terms of n, the number of arcs in the minimum spanning tree.

(1)

The table below shows the lengths, in km, of a network of roads between seven villages, A, B, C,

D, E, F and G.

	A	B	C	D	E	F	G
A	–	17	–	19	30	–	–
B	17	–	21	23	–	–	–
C	–	21	–	27	29	31	22
D	19	23	27	–	–	40	–
E	30	–	29	–	–	33	25
F	–	–	31	40	33	–	39
G	–	–	22	–	25	39	–

(c) Complete the drawing of the network on Diagram 1 in the answer book by adding the necessary arcs from vertex C together with their weights.

(2)

(d) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the arcs in the order that you consider them. In each case, state whether you are adding the arc to your minimum spanning tree.

(3)

(e) State the weight of the minimum spanning tree.

(1)

(Total for question = 10 marks)

Q54.

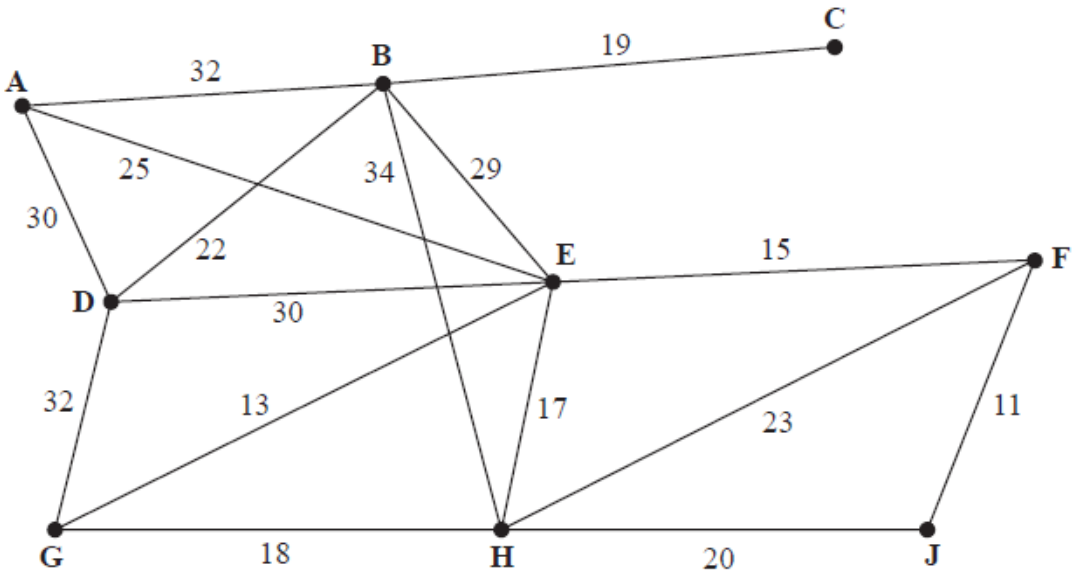


Figure 1

(a) Define the terms

(i) tree,

(ii) minimum spanning tree.

(3)

(b) Use Kruskal's algorithm to find the minimum spanning tree for the network shown in Figure 1. You must clearly show the order in which you consider the edges. For each edge, state whether or not you are including it in the minimum spanning tree.

(3)

(c) Draw the minimum spanning tree using the vertices given in Diagram 1 in the answer book and state the weight of the minimum spanning tree.

(2)

(Total for question = 8 marks)

Q55.

The table below shows the distances, in metres, between six vertices, A, B, C, D, E and F, in a network.

	A	B	C	D	E	F
A	–	18	23	17	28	19
B	18	–	20	11	–	24
C	23	20	–	–	25	13
D	17	11	–	–	–	22
E	28	–	25	–	–	–
F	19	24	13	22	–	–

(a) Draw the weighted network using the vertices given in Diagram 1 in the answer book.

(2)

(b) Use Kruskal's algorithm to find a minimum spanning tree for the network. You should list the edges in the order that you consider them and state whether you are adding them to your minimum spanning tree.

(3)

(c) Draw the minimum spanning tree on Diagram 2 in the answer book and state its total weight.

(2)

(Total for question = 7 marks)

Q56.

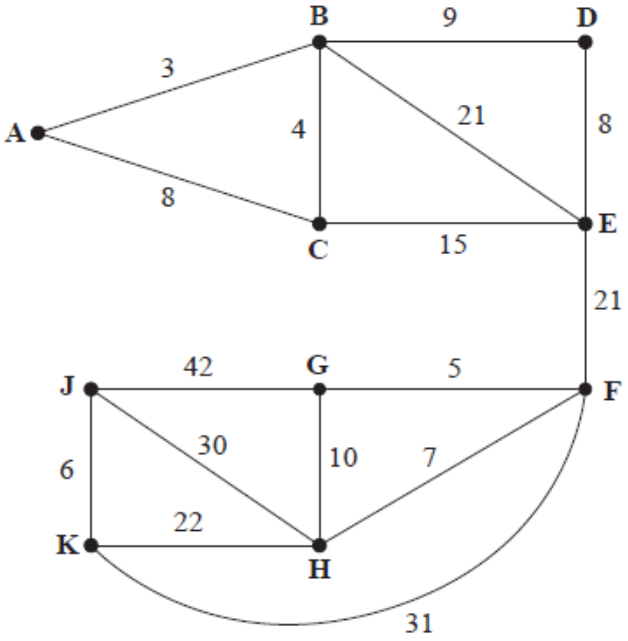


Figure 1

Figure 1 represents a network of roads between ten villages, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in kilometres, of the corresponding road. The local council needs to find the shortest route from A to J.

(a) Use Dijkstra's algorithm to find the shortest route from A to J. State the route and its length.

(6)

During the winter, the council needs to ensure that all ten villages are accessible by road even if there is heavy snow. The council wishes to minimise the total length of road it needs to keep clear.

(b) Use Prim's algorithm, starting at A, to find a minimum connector for the five villages A, B, C, D and E. You must clearly state the order in which you select the edges of your minimum connector.

(2)

(c) Use Kruskal's algorithm to find a minimum connector for the five villages F, G, H, J and K. You must clearly show the order in which you consider the edges. For each edge, state whether or not you are including it in your minimum connector.

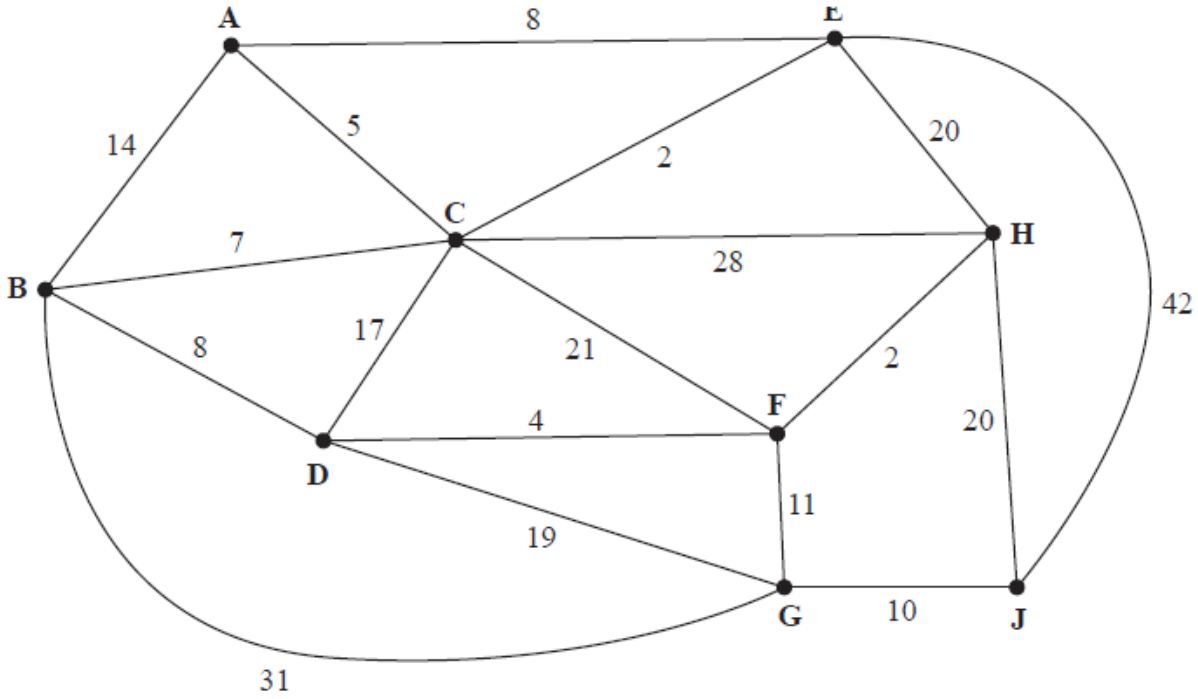
(2)

(d) Calculate the total length of road that the council must keep clear of snow to ensure that all ten villages are accessible.

(1)

(Total for question = 11 marks)

Q57.



[The total weight of the network is 269]

Figure 3

Figure 3 models a network of roads. The number on each edge gives the time taken, in minutes, to travel along the corresponding road.

(a) Use Dijkstra's algorithm to find the shortest time needed to travel from A to J. State the quickest route.

(6)

Alan needs to travel along all the roads to check that they are in good repair. He wishes to complete his route as quickly as possible and will start at his home, H, and finish at his workplace, D.

(b) By considering the pairings of all relevant nodes, find the arcs that will need to be traversed twice in Alan's inspection route from H to D. You must make your method and working clear.

(5)

For Alan's inspection route from H to D

(c) (i) state the number of times vertex C will appear,

(ii) state the number of times vertex D will appear.

(2)

(d) Determine whether it would be quicker for Alan to start and finish his inspection route at H, instead of starting at H and finishing at D. You must explain your reasoning and show all your working.

(2)

(Total for question = 15 marks)

Q58.

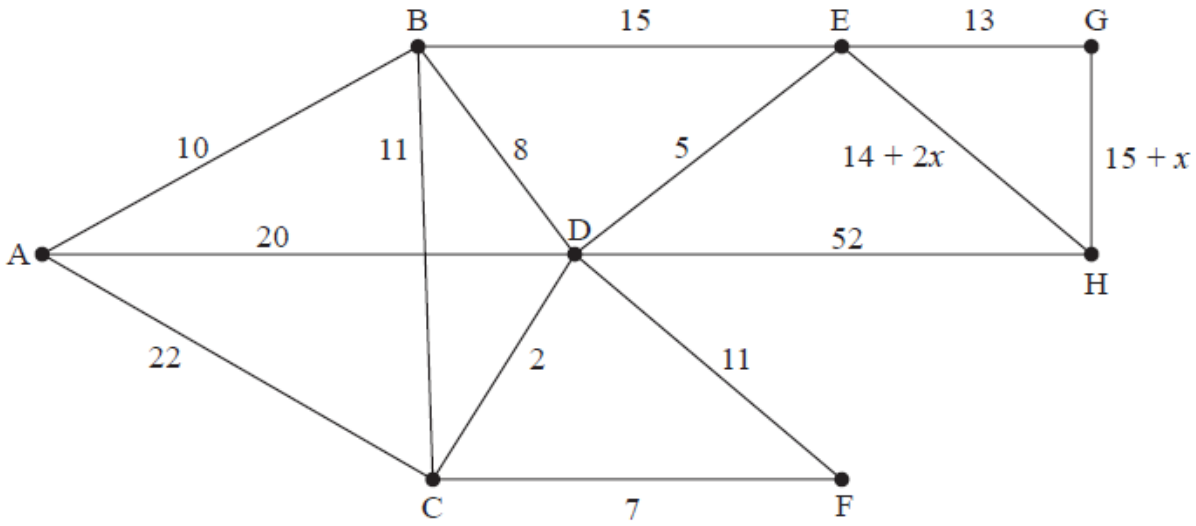


Figure 3

[*The total weight of the network is* $205 + 3x$]

Figure 3 represents a network of roads. The number on each arc represents the time taken, in minutes, to drive along the corresponding road.

Malcolm wishes to minimise the time spent driving from his home at A to his office at H. The delays from roadworks on two of the roads leading in to H vary daily, and so the time taken to drive along these roads is expressed in terms of x, where x is fixed for any given day and $x > 0$

(a) Use Dijkstra's algorithm to find the possible routes that minimise the driving time from A to H. State the length of each route, leaving your answer in terms of x where necessary.

(7)

On Monday, Malcolm needs to check each road. He must travel along each road at least once. He must start and finish at H and minimise the total time taken for his inspection route.

Malcolm finds that his minimum duration inspection route requires him to traverse exactly four roads twice and the total time it takes to complete his inspection route is 307 minutes.

(b) Calculate the minimum time taken for Malcolm to travel from A to H on Monday. You must make your method and working clear.

(4)

(Total for question = 11 marks)

Q59.

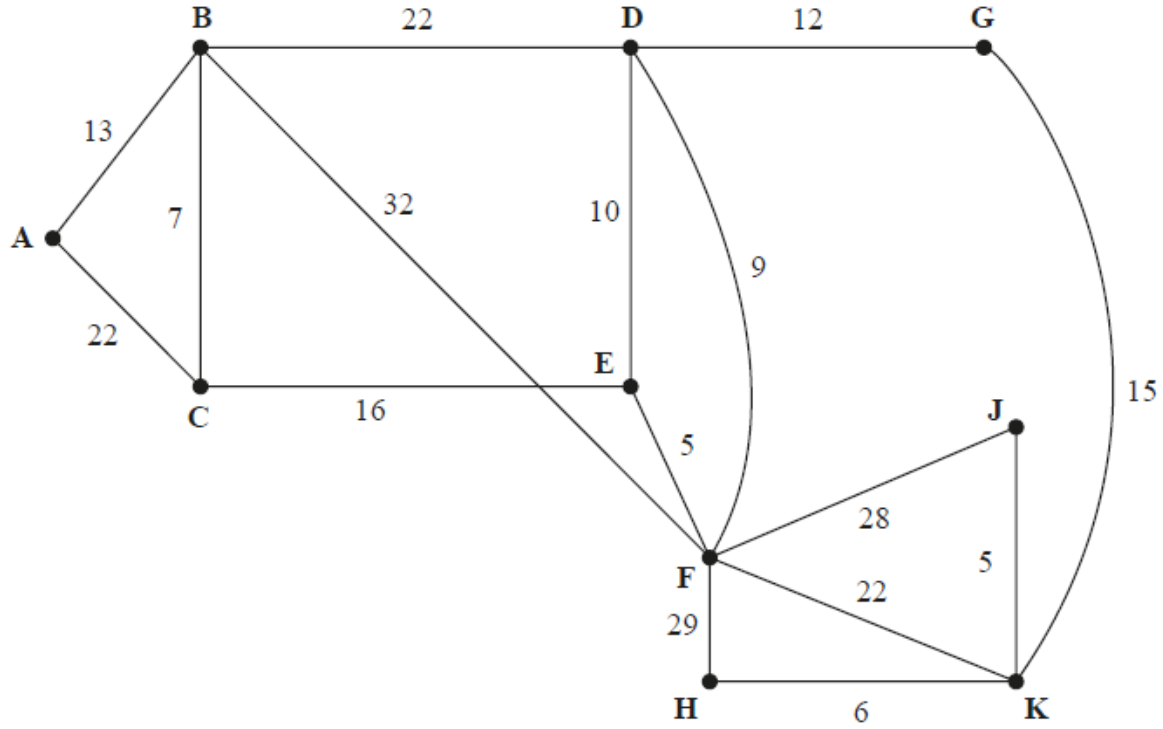


Figure 1

[*The total weight of the network is* 253]

Figure 1 represents a network of roads between 10 cities, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in miles, of the corresponding road.

One day, Mabintou wishes to travel from A to H. She wishes to minimise the distance she travels.

(a) Use Dijkstra's algorithm to find the shortest path from A to H. State your path and its length.

(6)

On another day, Mabintou wishes to travel from F to K via A.

(b) Find a route of minimum length from F to K via A and state its length.

(2)

The roads between the cities need to be inspected. James must travel along each road at least once. He wishes to minimise the length of his inspection route. James will start his inspection route at A and finish at J.

(c) By considering the pairings of all relevant nodes, find the length of James' route. State the arcs that will need to be traversed twice. You must make your method and working clear.

(6)

(d) State the number of times that James will pass through F.

(1)

It is now decided to start the inspection route at D. James must minimise the length of his route. He must travel along each road at least once but may finish at any vertex.

(e) State the vertex where the new inspection route will finish.

(1)

(f) Calculate the difference between the lengths of the two inspection routes.

(1)

(Total for question = 17 marks)

Q60.

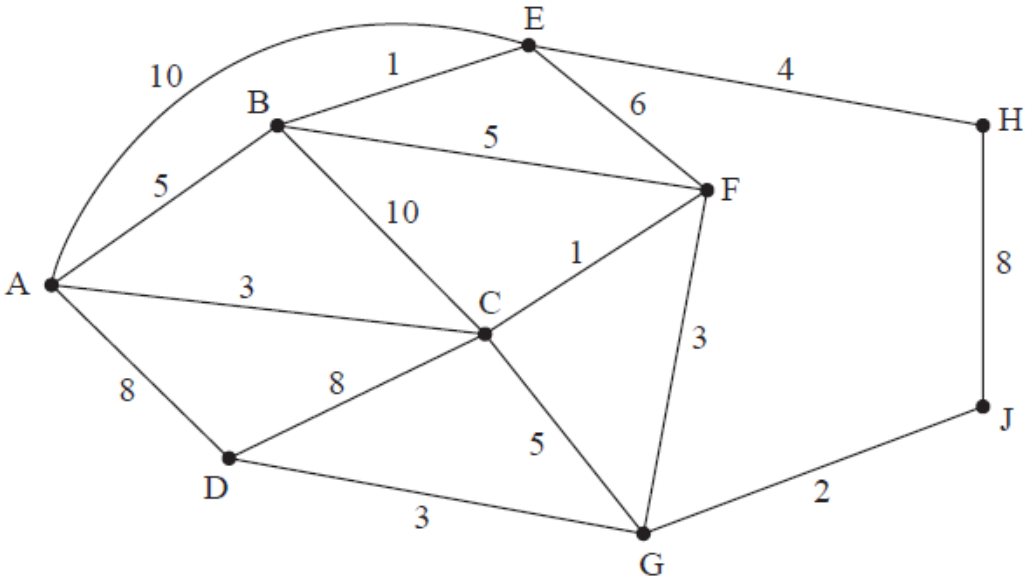


Figure 4

[*The total weight of the network is 82*]

Figure 4 represents a network of 16 roads in a city. The number on each arc represents the time taken, in minutes, to travel along the corresponding road.

Chan needs to check that the roads are in good repair. He must travel along each road at least once. Chan will start and finish at his office at G and must minimise the total time taken for his inspection route.

For this inspection route,

(a) find the time taken and state a possible route. You must make your method and reasoning clear.

(3)

Chan wonders if he can reduce his travel time by starting from his home at B, travelling along each road at least once and finishing at his office at G.

(b) By considering the pairings of all relevant nodes, find any arcs that would need to be traversed twice in the minimum inspection route from B to G. You must make your method clear, showing your working.

(5)

(c) Determine which of the two routes ending at G is quicker, the one starting at G or the one starting at B. You must justify your answer.

(2)

(Total for question = 10 marks)

Q61.

Activity	Immediately preceding activities
A	-
B	-
C	-
D	A
E	A
F	B, C, E
G	B, C, E
H	C
I	C
J	D, F, G, H, I
K	D, F, G, H, I
L	I

(a) Draw the activity network described in the precedence table above, using activity on arc and the minimum number of dummies.

(5)

A project is modelled by the activity network drawn in (a). Each activity requires exactly one worker. The project is to be completed in the shortest possible time. The table below gives the time, in hours, to complete three of the activities.

Activity	Duration (in hours)
A	10
E	7
F	8

The length of the critical path AEFK is 33 hours.

(b) Determine the range of possible values for the duration of activity J. You must make your method and working clear.

(2)

(Total for question = 7 marks)

Q62.

12.1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 33

(3)

The list is to be sorted into **descending** order.

(b) (i) Starting at the left-hand end of the list, perform two passes through the list using a bubble sort. Write down the state of the list that results at the end of each pass.

(ii) Write down the total number of comparisons and the total number of swaps performed during your two passes.

(4)

(c) Use a quick sort on the **original** list to obtain a fully sorted list in **descending** order. You must make your pivots clear.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how the numbers listed can be packed into bins of size 33

(3)

(e) Determine whether your answer to (d) uses the minimum number of bins. You must justify your answer.

(1)

(Total for question = 15 marks)

Q63.

35 17 10 7 28 23 41 15 20 29

(a) Use the first-fit bin packing algorithm to determine how the numbers listed above can be packed into bins of size 60

(3)

(b) The list of numbers is to be sorted into descending order. Use a quick sort to obtain the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)

(c) Use the first-fit decreasing bin packing algorithm on your ordered list to pack the numbers into bins of size 60

(3)

The ten distinct numbers below are to be sorted into descending order.

20 24 17 26 8 15 x y 19 12

A bubble sort, starting at the left-hand end of the list, is to be used to obtain the sorted list.

After the second complete pass the list is

24 26 20 17 15 y 19 12 x 8

(d) Find the constraints on the values of x and y.

(3)

(Total for question = 13 marks)

Q64.

(a) (i) Describe how to carry out the first pass of a bubble sort when it is used to sort a list of n numbers into ascending order.

(ii) Write down the circumstances under which a bubble sort stops.

(4)

A bubble sort, starting at the left-hand end of the list, is used to sort a list of ten numbers into ascending order. After a number of passes the list reads

0.9 1.2 1.5 0.5 1.4 1.1 0.7 1.7 2.2 3.2

(b) Determine the maximum number of passes that could have taken place on this list. You must give a reason for your answer.

(2)

(c) Complete the bubble sort to produce a list of the numbers in ascending order. You only need to give the state of the list after each complete pass.

(4)

(d) Use the first-fit decreasing bin packing algorithm to determine how the ten numbers listed above can be packed into bins of size 4

(3)

(Total for question = 13 marks)

Q65.

A list of eleven numbers is to be sorted into descending order.

After one pass, the quick sort algorithm produces the following list

17 33 14 25 23 28 21 13 9 6 10

(a) State, with a reason, which number was used as a pivot for the first pass.

(1)

(b) Starting at the left-hand end of the above list, obtain the fully sorted list using a bubble sort. You need to write down only the list that results at the end of each pass.

(3)

(c) Apply the first-fit decreasing bin packing algorithm to the fully sorted list to pack the numbers into bins of size 85

(2)

(Total for question = 6 marks)

Q66.

	A	B	C	D	E	F	G
A	–	43	52	47	59	53	55
B	43	–	59	45	46	52	47
C	52	59	–	51	50	55	51
D	47	45	51	–	52	49	55
E	59	46	50	52	–	57	48
F	53	52	55	49	57	–	55
G	55	47	51	55	48	55	–

The table above shows the least distances, in metres, between seven classrooms, A, B, C, D, E, F and G. A teacher needs to visit each classroom, starting and finishing at A, and wishes to minimise the total distance travelled.

(a) Show that there are two nearest neighbour routes that start from A. State these routes and their corresponding lengths.

(3)

(b) Starting by deleting A, and all of its arcs, find a lower bound for the length of the teacher's route.

(3)

(c) Use your results to write down the smallest interval which you can be confident contains the optimal length of the teacher's route.

(1)

(Total for question = 7 marks)

Q67.

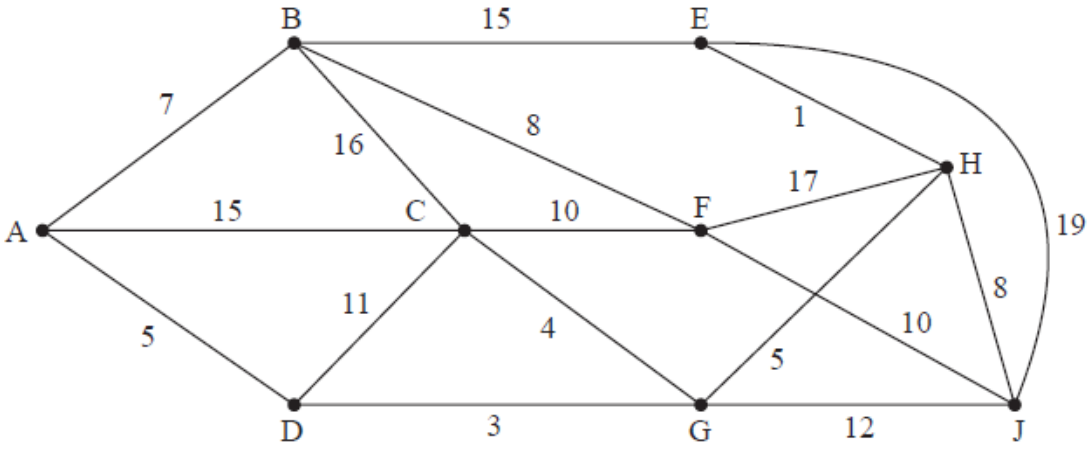


Figure 3

[*The total weight of the network is* 166]

Figure 3 models a network of cycle lanes that must be inspected. The number on each arc represents the length, in km, of the corresponding cycle lane. Lance needs to cycle along each lane at least once and wishes to minimise the length of his inspection route.

He must start and finish at A.

(a) Use an appropriate algorithm to find the length of the route. State the cycle lanes that Lance will need to traverse twice. You should make your method and working clear.

(6)

(b) State the number of times that vertex C appears in Lance's route.

(1)

It is now decided that the inspection route may finish at any vertex. Lance will still start at A and must cycle along each lane at least once.

(c) Determine the finishing point so that the length of the route is minimised. You must give reasons for your answer and state the length of this new minimum route.

(3)

(Total for question = 10 marks)

Q68.

The table shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	122	217	137	109	82
B	122	–	110	130	128	204
C	217	110	–	204	238	135
D	137	130	204	–	98	211
E	109	128	238	98	–	113
F	82	204	135	211	113	–

Liz must visit each town at least once. She will start and finish at A and wishes to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree given in your answer book, use the shortcut method to find an upper bound below 810 km for Liz's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Liz's route.

(2)

(c) Starting by deleting F, and all of its arcs, find a lower bound for the length of Liz's route.

(3)

(d) Use your results to write down the smallest interval which you are confident contains the optimal length of the route.

(1)

(Total for question = 8 marks)

Q69.

	A	B	C	D	E	F
A	–	73	56	27	38	48
B	73	–	58	59	43	34
C	56	58	–	46	38	42
D	27	59	46	–	25	32
E	38	43	38	25	–	21
F	48	34	42	32	21	–

The table above shows the least distances, in km, between six cities, A, B, C, D, E and F. Mohsen needs to visit each city, starting and finishing at A, and wishes to minimise the total distance he will travel.

(a) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the length of Mohsen's route. You must state your route and its length.

(3)

(b) Starting by deleting A and all of its arcs, find a lower bound for the length of Mohsen's route.

(3)

(c) Use your answers from (a) and (b) to write down the smallest interval that you can be confident contains the optimal length of the route.

(2)

(Total for question = 8 marks)

Q70.

The table below shows the distances, in km, between six data collection points, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	35	42	55	48	50
B	35	–	40	49	52	31
C	42	40	–	47	53	49
D	55	49	47	–	39	44
E	48	52	53	39	–	52
F	50	31	49	44	52	–

Ferhana must visit each data collection point. She will start and finish at A and wishes to minimise the total distance she travels.

(a) Starting at A, use the nearest neighbour algorithm to obtain an upper bound for the distance Ferhana must travel. Make your method clear.

(2)

(b) Starting by deleting B, and all of its arcs, find a lower bound for the distance Ferhana must travel. Make your calculation clear.

(3)

(Total for question = 5 marks)

Q71.

The table below shows the least distances, in km, between six towns, A, B, C, D, E and F.

	A	B	C	D	E	F
A	–	57	76	59	72	65
B	57	–	67	80	66	76
C	76	67	–	71	83	80
D	59	80	71	–	77	78
E	72	66	83	77	–	69
F	65	76	80	78	69	–

Mei must visit each town at least once. She will start and finish at A and wishes her route to minimise the total distance she will travel.

(a) Starting with the minimum spanning tree in the answer book, use the shortcut method to find an upper bound below 520 km for Mei's route. You must state the shortcut(s) you use and the length of your upper bound.

(2)

(b) Use the nearest neighbour algorithm, starting at A, to find another upper bound for the length of Mei's route.

(2)

(c) Starting by deleting E, and all of its arcs, find a lower bound for the length of Mei's route.

Make your method clear.

(3)

(Total for question = 7 marks)

Q72.

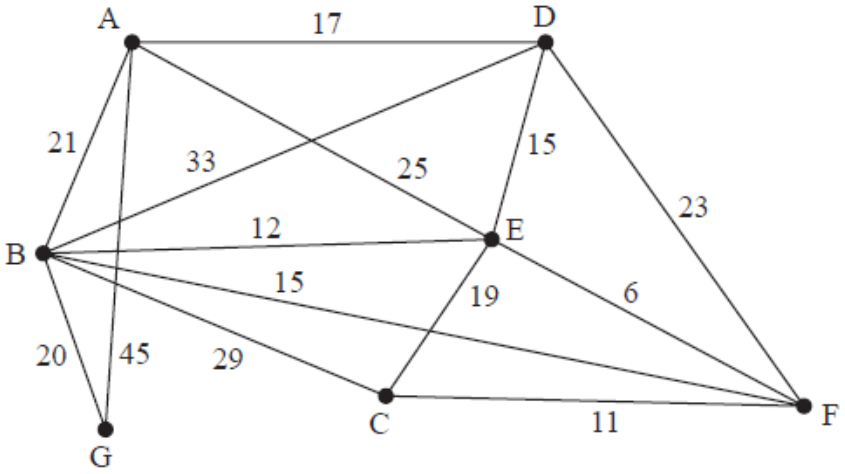


Figure 3

[*The total weight of the network is* 291]

Figure 3 models a network of roads. The number on each edge gives the length, in km, of the corresponding road. The vertices, A, B, C, D, E, F and G, represent seven towns. Derek needs to visit each town. He will start and finish at A and wishes to minimise the total distance travelled.

(a) By inspection, complete the two copies of the table of least distances in the answer book.

(2)

(b) Starting at A, use the nearest neighbour algorithm to find an upper bound for the length of Derek's route. Write down
the route that gives this upper bound.

(2)

(c) Interpret the route found in (b) in terms of the towns actually visited.

(1)

(d) Starting by deleting A and all of its arcs, find a lower bound for the route length.

(3)

Clive needs to travel along the roads to check that they are in good repair. He wishes to

minimise the total distance travelled and must start at A and finish at G.

(e) By considering the pairings of all relevant nodes, find the length of Clive's route. State the edges that need to be traversed twice. You must make your method and working clear.

(5)

(Total for question = 13 marks)