WRITTEN SOLUTIONS

Pearson Edexcel Level 3 GCE

Thursday 16 May 2019

Afternoon	Paper Reference **8FM0-27**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
27: Decision Mathematics 1
(Part of options D, F, H and K)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
D1 Answer Book (enclosed)

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams / sketches / graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the Answer Book provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for each question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. (*a*) Draw the graph K_5

(1)

(*b*) (i) In the context of graph theory explain what is meant by 'semi-Eulerian'.

(ii) Draw two semi-Eulerian subgraphs of K_5, each having five vertices but with a different number of edges.

(3)

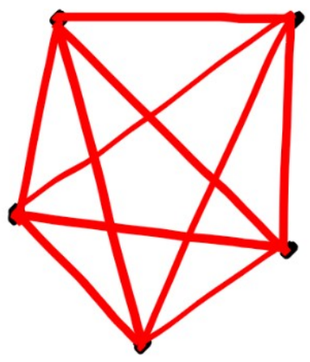
(*c*) Explain why a graph with exactly five vertices with vertex orders 1, 2, 2, 3 and 4 cannot be a tree.

(2)

(Total for Question 1 is 6 marks)

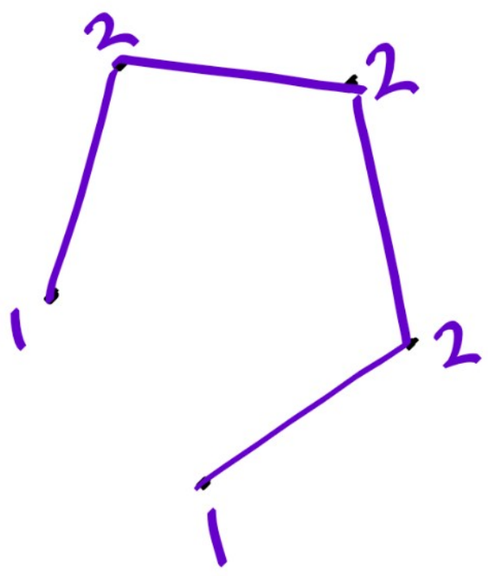
1.

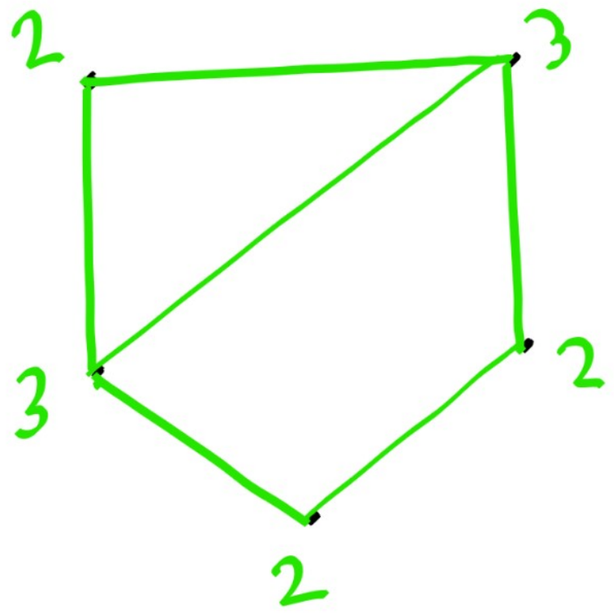
(a)



(b)(i) semi-eulerian = graph contains exactly two nodes of odd order (and any number of nodes of even order)

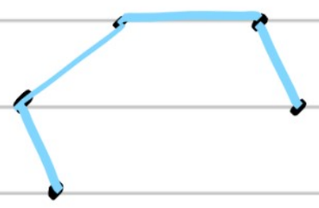
(ii)





(c) vertex order : {1, 2, 2, 3, 4} ∴ 5 nodes

tree = connected graph with no cycles



a tree with 5 nodes has 4 arcs

handshaking lemma = the sum of all the order of verticies of any graph is twice the number of verticies

number of arcs = $\frac{1 + 2 + 2 + 3 + 4}{2} = 5$

5 ≠ 4 ∴ cannot be a tree

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2. The following algorithm produces a numerical approximation for the integral

$$I = \int_{A}^{B} x^4 \, \mathrm{d}x$$

Step 1	Start
Step 2	Input the values of A, B and N
Step 3	Let H = (B – A) / N
Step 4	Let C = H / 2
Step 5	Let D = 0
Step 6	Let D = D + A^4 + B^4
Step 7	Let E = A
Step 8	Let E = E + H
Step 9	If E = B go to Step 12
Step 10	Let D = D + 2 × E^4
Step 11	Go to Step 8
Step 12	Let F = C × D
Step 13	Output F
Step 14	Stop

For the case when A = 1, B = 3 and N = 4,

(*a*) (i) complete the table in the answer book to show the results obtained at each step of the algorithm.

(ii) State the final output.

(4)

(*b*) Calculate, to 3 significant figures, the percentage error between the exact value of *I* and the value obtained from using the approximation to *I* in this case.

(3)

(Total for Question 2 is 7 marks)

2.

You may not need to use all the rows in this table.

It may not be necessary to complete all boxes in each row.

	A	B	N	H	C	D	E	F
2	1	3	4					
3				0.5				
4					0.25			
5						0		
6						82		
7							1	
8							1.5	
10						92.125		
8							2	
10						124.125		
8							2.5	
10						202.25		
8							3	
13								50.5625

Final output: 50.5625

P 6 1 6 3 9 A 0 4 1 2

(b) $\int_1^3 x^4 \, dx = \left[\frac{1}{5} x^5 \right]_1^3$

$= \frac{1}{5} \times 3^5 - \frac{1}{5} \times 1^5$

$= 48.4$

$PE = \frac{50.5625 - 48.4}{48.4} \times 100$

$= 4.4679\ldots$

$= 4.47\%$ (3sf)

(Total for Question 2 is 7 marks)

3.

Activity	Immediately preceding activities
A	-
B	-
C	A
D	A
E	A
F	B, C
G	B, C
H	D
I	D, E, F, G
J	D, E, F, G
K	G

(*a*) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain the minimum number of dummies.

(5)

Every activity shown in the precedence table has the same duration.

(*b*) Explain why activity B cannot be critical.

(1)

(*c*) State which other activities are not critical.

(1)

(Total for Question 3 is 7 marks)

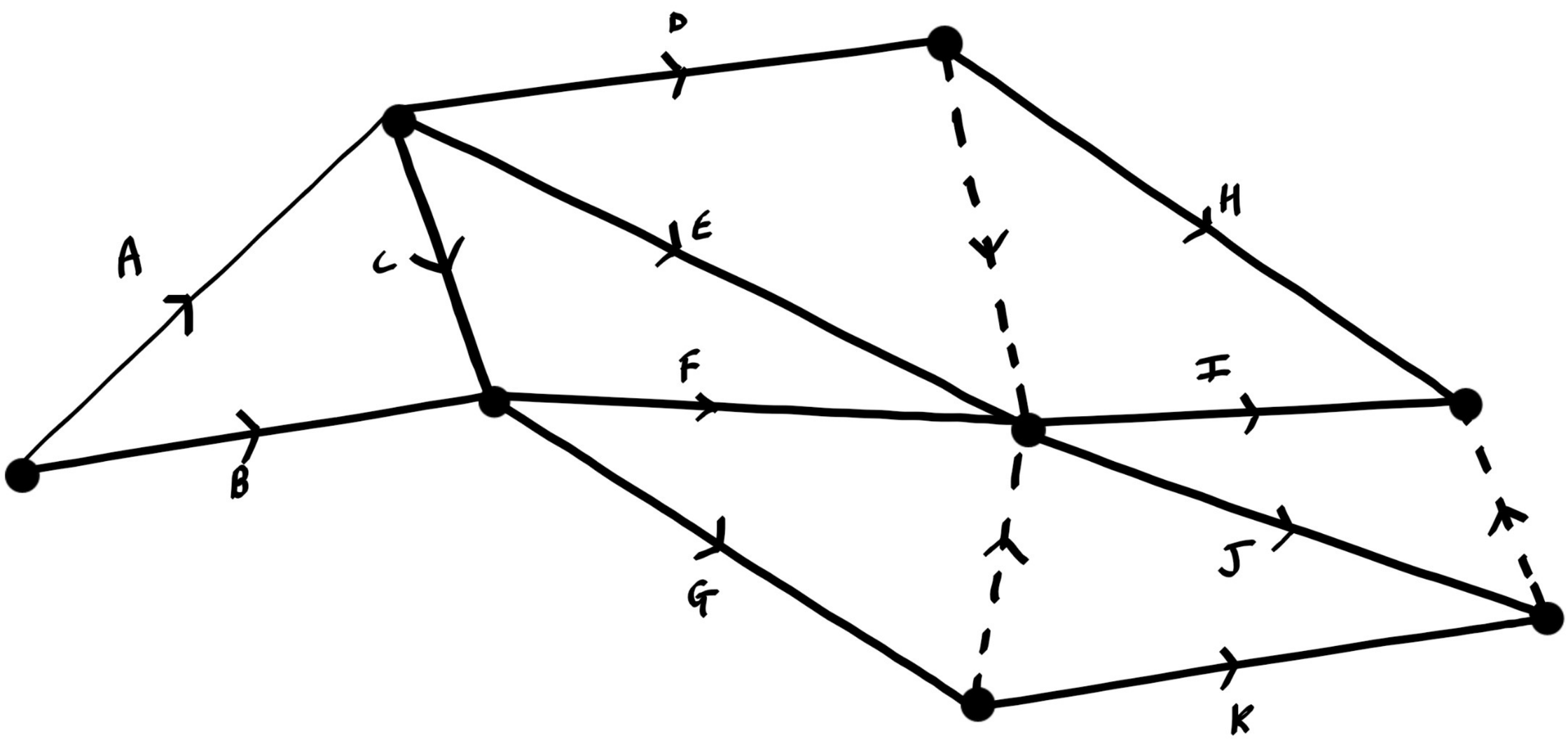
3.

(a)

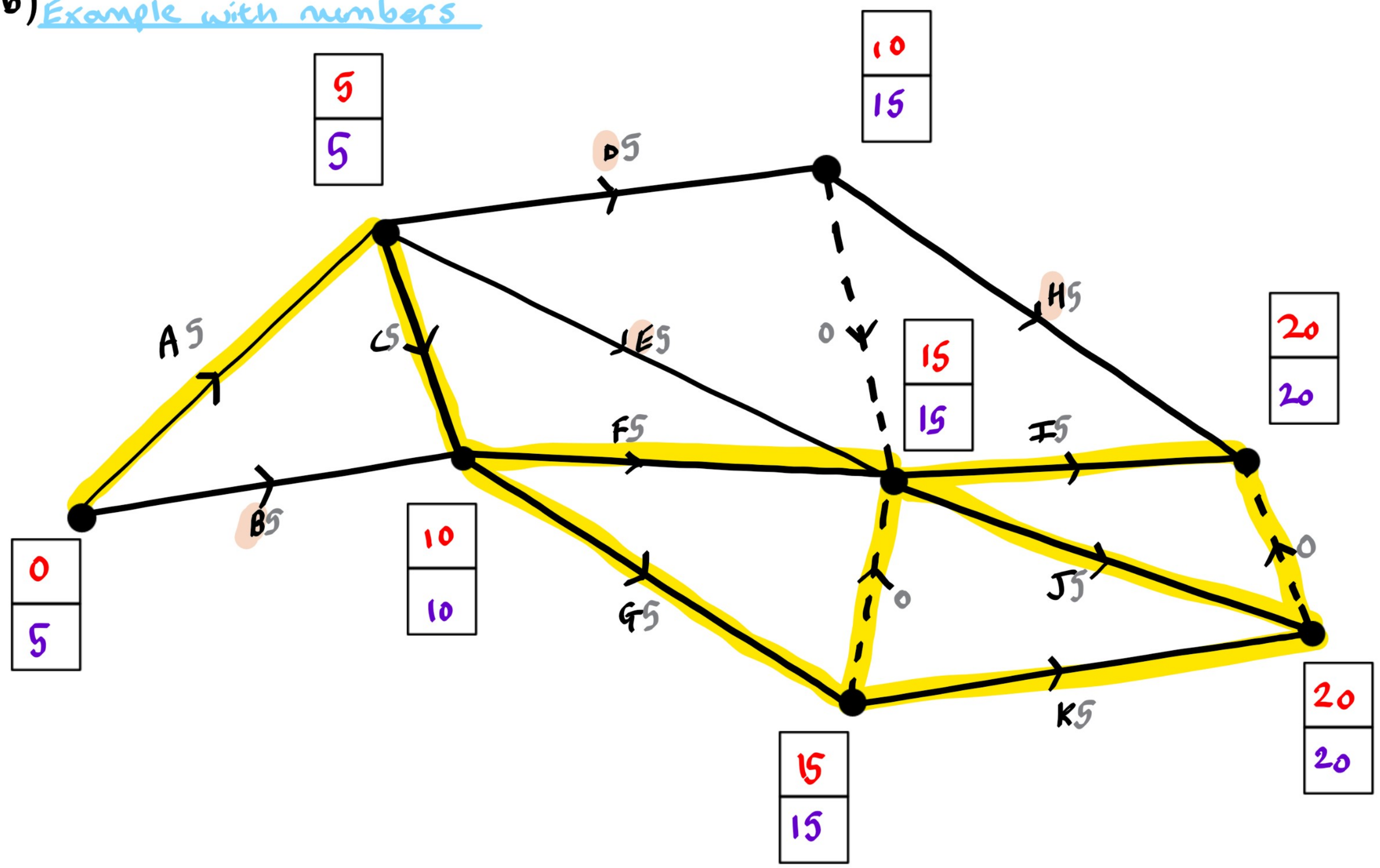
Activity	Immediately preceding activities
A	-
B	-
C	A
D	A
E	A
F	B, C
G	B, C
H	D
I	D, E, F, G
J	D, E, F, G
K	G

Dummy required

* at most one activity between two events
* two or more activities depend on a common activity finishing but also depend on activities that are not common in finishing



(b) Example with numbers



Activity F and G depend on B

Activity A and C need to be completed before activity F and G

Time to complete activity A and C is double that of B

Activity B can be delayed to wait for activity A and C to finish

∴ B is not critical

(c) D, E, H, B are not critical

4.

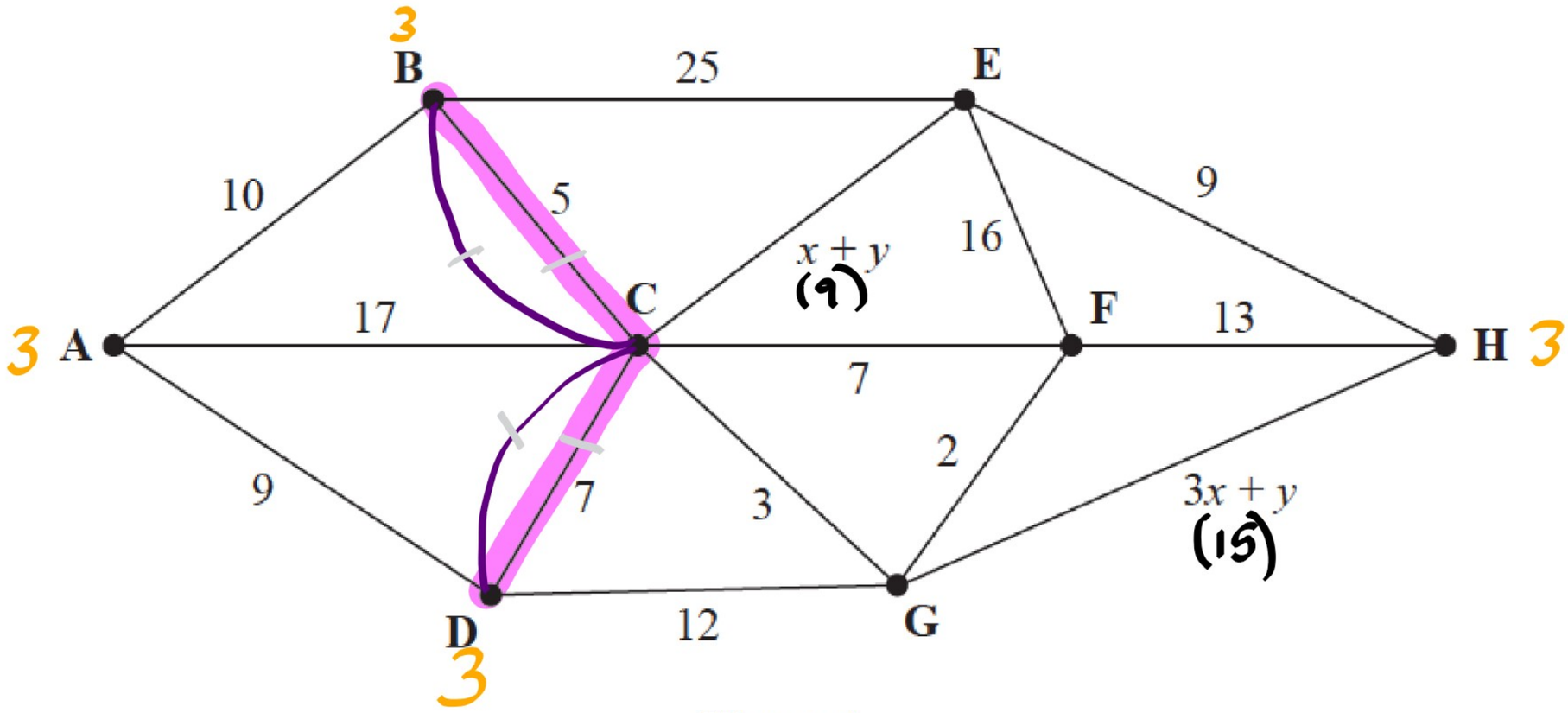


Figure 1

[The total weight of the network is $135 + 4x + 2y$]

The weights on the arcs in Figure 1 represent distances. The weights on the arcs **CE** and **GH** are given in terms of x and y, where x and y are positive constants and $7 < x + y < 20$

There are three paths from **A** to **H** that have the same minimum length.

(*a*) Use Dijkstra's algorithm to find x and y.

(7)

An inspection route starting at **A** and finishing at **H** is found. The route traverses each arc at least once and is of minimum length.

(*b*) State the arcs that are traversed twice.

(1)

(*c*) State the number of times that vertex **C** appears in the inspection route.

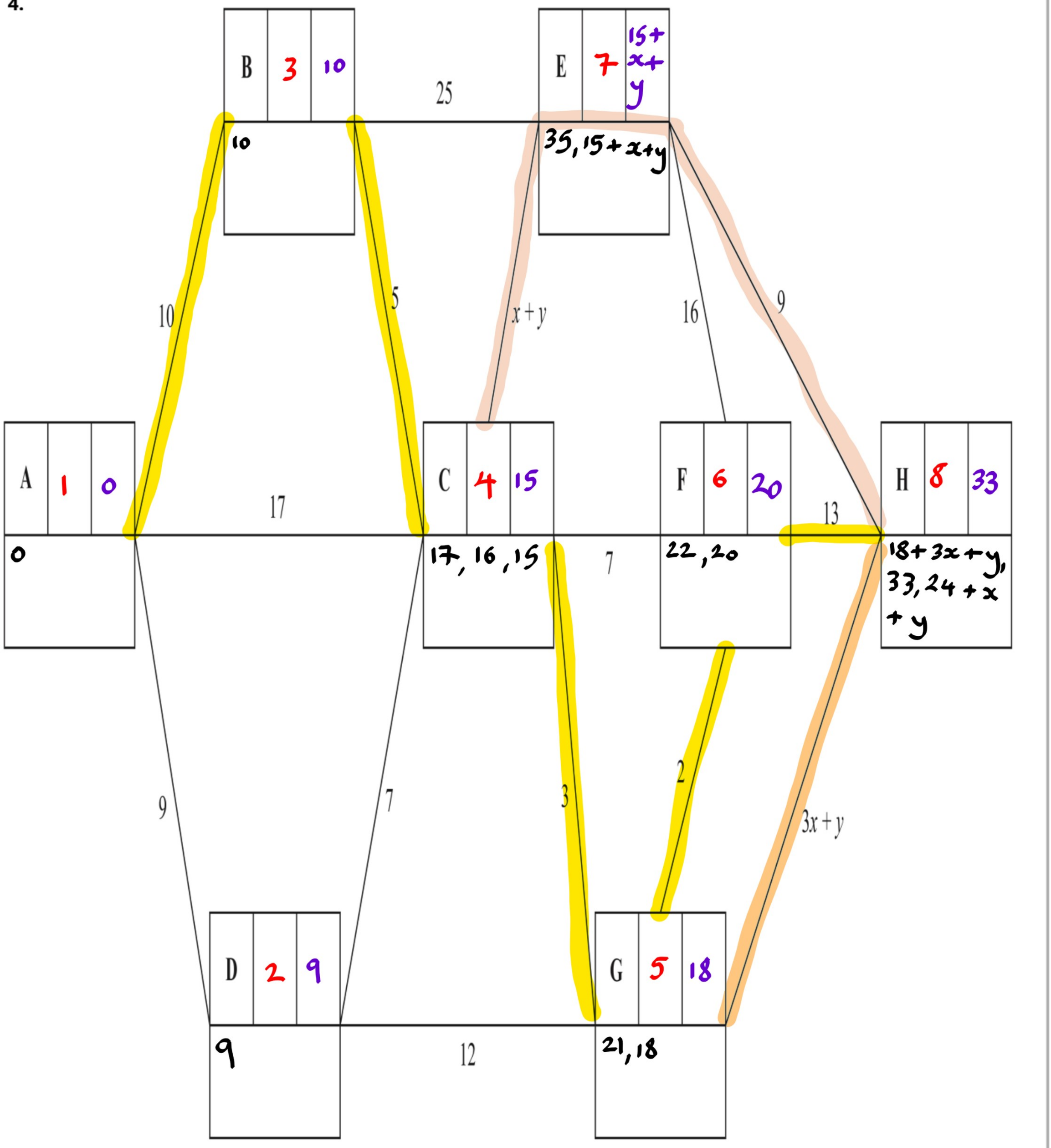
(1)

(*d*) Determine the length of the inspection route.

(1)

(Total for Question 4 is 10 marks)

4.



Key:

Vertex	Order of labelling	Final value
Working values		

Question 4 continued

A → H = 33

$15 + x + y = 33$

$x + y = 9$ ①

$18 + 3x + y = 33$

$3x + y = 15$ ②

solve ① and ② simultaneously:

$2x = 6 \quad \therefore x = 3$

$3 + y = 9 \quad \therefore y = 6$

(b) odd: A, B, D, H

↑ start ↑ finish

∴ repeat BD (BCD) = 12

arcs BC and CD need to be traversed twice

(c) vertex C would appear 4 times

(d) $(135 + 4x + 2y) + 12$ ↑ BD

$135 + 4 \times 3 + 2 \times 6 + 12 = 171$

(Total for Question 4 is 10 marks)

5. Ben is a wedding planner. He needs to order flowers for the weddings that are taking place next month. The three types of flower he needs to order are roses, hydrangeas and peonies.

Based on his experience, Ben forms the following constraints on the number of each type of flower he will need to order.

- At least three-fifths of all the flowers must be roses.
- For every 2 hydrangeas there must be at most 3 peonies.
- The total number of flowers must be exactly 1000

The cost of each rose is £1, the cost of each hydrangea is £5 and the cost of each peony is £4

Ben wants to minimise the cost of the flowers.

Let x represent the number of roses, let y represent the number of hydrangeas and let z represent the number of peonies that he will order.

(*a*) Formulate this as a linear programming problem in x and y only, stating the objective function and listing the constraints as simplified inequalities with integer coefficients.

(7)

Ben decides to order the minimum number of roses that satisfy his constraints.

(*b*) (i) Calculate the number of each type of flower that he will order to minimise the cost of the flowers.

(ii) Calculate the corresponding total cost of this order.

(3)

(Total for Question 5 is 10 marks)

TOTAL FOR DECISION MATHEMATICS 1 IS 40 MARKS

END

5.

(a) minimise $P = x + 5y + 4z$ ① subject to:

$x \geq \frac{3}{5}(x + y + z) \Rightarrow 2x \geq 3y + 3z$ ②

$3y \geq 2z$ ③

$x + y + z = 1000 \Rightarrow z = 1000 - x - y$ ④

④ into ①: $P = x + 5y + 4(1000 - x - y)$
$P = x + 5y + 4000 - 4x - 4y$
$P = y - 3x + 4000$ ⑤

④ into ②: $2x \geq 3y + 3(1000 - x - y)$
$2x \geq 3y + 3000 - 3x - 3y$
$5x \geq 3000$
$x \geq 600$ ⑥

④ into ③: $3y \geq 2(1000 - x - y)$
$3y \geq 2000 - 2x - 2y$
$5y + 2x \geq 2000$ ⑦

minimise $P = y - 3x + 4000$ subject to:

$x \geq 600$ and $5y + 2x \geq 2000$

(b)(i) $x \geq 600$ ∴ 600 roses $(x = 600)$

$5y + 2(600) \geq 2000$
$5y \geq 800$
$y \geq 160$ ∴ 160 hyarageans $(y = 160)$

$z = 1000 - 600 - 160 = 240$ ∴ 240 penies $(z = 240)$

(ii) $P = 600 + 5 \times 160 + 4 \times 240 = 2360$ ∴ cost is £2360