MODEL ANSWERS

Please check the examination details below before entering your candidate information

Candidate surname | Other names

**Pearson Edexcel
Level 3 GCE**

Centre Number | Candidate Number

Wednesday 20 May 2020

Afternoon | Paper Reference **8MA0/22**

Mathematics

**Advanced Subsidiary
Paper 22: Mechanics**

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations.
Calculators must not have the facility for symbolic algebra
manipulation, differentiation and integration, or have retrievable
mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name,
  centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions
  are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You should show sufficient working to make your methods clear.
  Answers without working may not gain full credit.
- Unless otherwise indicated, wherever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$
  and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 30. There are 3 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P 6 3 9 3 6 A 0 1 1 2

Pearson

1. At time $t = 0$, a small ball is projected vertically upwards with speed $U\,\mathrm{m\,s^{-1}}$ from a point A that is $16.8\,\mathrm{m}$ above horizontal ground.

   The speed of the ball at the instant immediately before it hits the ground for the first time is $19\,\mathrm{m\,s^{-1}}$

   The ball hits the ground for the first time at time $t = T$ seconds.

   The motion of the ball, from the instant it is projected until the instant just before it hits the ground for the first time, is modelled as that of a particle moving freely under gravity.
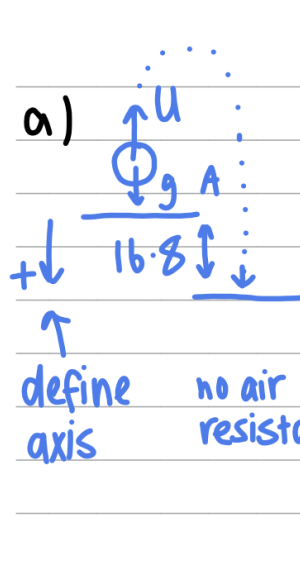
   The acceleration due to gravity is modelled as having magnitude $10\,\mathrm{m\,s^{-2}}$

   Using the model,

   (a) show that $U = 5$

   (2)

   (b) find the value of T,

   (2)

   (c) find the time from the instant the ball is projected until the instant when the ball is $1.2\,\mathrm{m}$ below A.

   (4)

   (d) Sketch a velocity-time graph for the motion of the ball for $0 \leqslant t \leqslant T$, stating the coordinates of the start point and the end point of your graph.

   (2)

   In a refinement of the model of the motion of the ball, the effect of air resistance on the ball is included and this refined model is now used to find the value of U.

   (e) State, with a reason, how this new value of U would compare with the value found in part (a), using the initial unrefined model.

   (1)

   (f) Suggest one further refinement that could be made to the model, apart from including air resistance, that would make the model more realistic.

   (1)

(Handwritten working below:)

— opposite direction to U @ start

a)   ↑U

$S\ 16.8\quad V\text{-}u\quad V\ 19\quad A\ g\quad T$

When ball returns to start point, its velocity is U downwards

$\rightarrow v^2 = u^2 + 2as$

$19^2 = (-u)^2 + 2 \times 10 \times 16.8$

$361 = u^2 + 336$

$u^2 = 25 \quad \therefore \quad U = 5$

define axis   no air resistance

$\downarrow$ 16.8 $\updownarrow$

Question 1 continued

b) $v = u + at \implies 19 = -5 + 10T$

remember the direction of positive velocity

$\hookrightarrow$ down is positive, so when ball moves upwards, its velocity is negative

$$24 = 10T$$

$$T = 2.4$$

c) S 1.2   U -5   V   A g   T t

$S = ut + \frac{1}{2}at^2$

$1.2 = -5t + \frac{1}{2} \times 10 \times t^2$

$1.2 = -5t + 5t^2$

$5t^2 - 5t - 1.2 = 0$

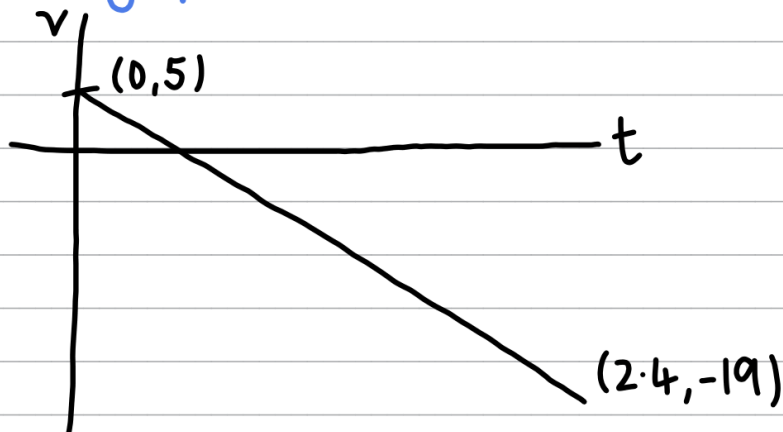$t = \dfrac{5 \pm \sqrt{5^2 - 4 \times 5 \times -1.2}}{10}$

$= 1.2$ or -0.2

t must be >0 so $t = 1.2$s

Question 1 continued

d) v-t graph ⟹ constant acceleration ⟹ constant gradient

(0,5) & (2.4,-19)   (can also give (0,-5) & (2.4,+19))

e) air resistance will slow the ball down, so to have a speed of 19ms⁻¹ after a time T, the ball must start with a faster speed ⟹ larger U calculated.

f) e.g. spin, wind effects, use more accurate value of g.

Question 1 continued
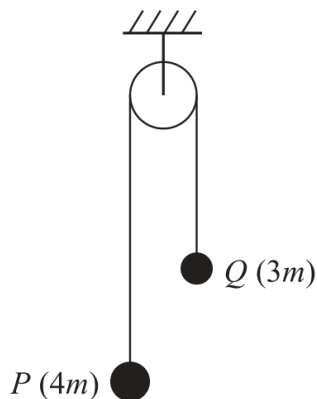
(Total for Question 1 is 12 marks)

2.

Figure 1

One end of a string is attached to a small ball P of mass $4m$.

The other end of the string is attached to another small ball Q of mass $3m$.

The string passes over a fixed pulley.

Ball P is held at rest with the string taut and the hanging parts of the string vertical, as shown in Figure 1.

Ball P is released.

The string is modelled as being light and inextensible, the balls are modelled as particles, the pulley is modelled as being smooth and air resistance is ignored.

(a) Using the model, find, in terms of m and g, the magnitude of the force exerted on the pulley by the string while P is falling and before Q hits the pulley.

(8)

(b) State one limitation of the model, apart from ignoring air resistance, that will affect the accuracy of your answer to part (a).

(1)

a)

T constant throughout (inextensible string)

E.O.M. for P: $4mg - T = 4ma$

P&Q are linked so have same a

E.O.M. for Q: $T - 3mg = 3ma$

T acts in opposite direction

define axis

Make a the subject $a = g - \dfrac{T}{4m} = \dfrac{T}{3m} - g$

$\Rightarrow 2g = \dfrac{T}{4m} + \dfrac{T}{3m}$

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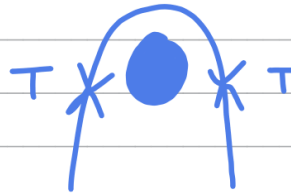
Question 2 continued

$$2g = \frac{(3+4)T}{m(3 \times 4)}$$

$$2g = \frac{7T}{12m}$$

$$\therefore T = \frac{24mg}{7}$$

total force on pulley = 2T

$$= \frac{48mg}{7}$$

b) the weight or extensibility of the rope, or the pulley not being

smooth

P 6 3 9 3 6 A 0 7 1 2

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3. A particle P moves along a straight line such that at time t seconds, $t \geqslant 0$, after leaving the point O on the line, the velocity, $v\,\mathrm{m\,s^{-1}}$, of P is modelled as

$$v = (7 - 2t)(t + 2)$$

(a) Find the value of t at the instant when P stops accelerating.

(4)

(b) Find the distance of P from O at the instant when P changes its direction of motion.

(5)

In this question, solutions relying on calculator technology are not acceptable.

a) 'stops accelerating': $a = \dfrac{dv}{dt} = 0$

$\quad v = (7-2t)(t+2) = 3t - 2t^2 + 14$

$\quad \dfrac{dv}{dt} = 3 - (2 \times 2)t$

$\quad\quad = 3 - 4t$

so $3 - 4t = 0$

$\quad\quad t = \dfrac{3}{4}s$

b) change of direction: $v = 0$

$\quad 3t - 2t^2 + 14 = 0 \Rightarrow (2t - 7)(t + 2) = 0$

$\quad\quad\quad\quad t = \dfrac{7}{2} \text{ or } t = -2$

$t > 0$ so $t = \dfrac{7}{2}$ find t @ which direction changes

$s = \displaystyle\int v\,dt$ integrate to find distance

$\quad = \displaystyle\int_0^{\frac{7}{2}} (3t - 2t^2 + 14)\,dt$

$\quad = \left[\dfrac{3}{2}t^2 - \dfrac{2t^3}{3} + 14t \right]_0^{\frac{7}{2}}$

P 6 3 9 3 6 A 0 1 0 1 2

Question 3 continued

$$= \frac{3}{2}\left(\frac{7}{2}\right)^2 - \frac{2}{3}\left(\frac{7}{2}\right)^3 + 14\left(\frac{7}{2}\right)$$

$$= \frac{931}{24} \, m = 38.79166\ldots m$$

Question 3 continued

(Total for Question 3 is 9 marks)

TOTAL FOR MECHANICS IS 30 MARKS

P 6 3 9 3 6 A 0 1 2 1 2