Pearson Edexcel Level 3 GCE

Time 1 hour 30 minutes

Paper reference **9FM0/3D**

Further Mathematics

Advanced
PAPER 3D: Decision Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1/1/1/1/

P 7 2 0 9 4 A

Pearson

1. A gardener needs the following lengths of string. All lengths are in metres.

4.3 6.1 5.1 4.7 2.5 5.9 3.4 1.7 2.1 0.4 1.3

She cuts the lengths from balls of string. Each ball contains 10 m of string.

(a) Calculate a lower bound for the number of balls of string the gardener needs. You must make your method clear.

(2)

(b) Use the first-fit bin packing algorithm to determine how the lengths could be cut from the balls of string.

(3)

(Total for Question 1 is 5 marks)

1.

4.3 6.1 5.1 4.7 2.5 5.9 3.4 1.7 2.1 0.4 1.3

(a) $\frac{4.3 + 6.1 + 5.1 + 4.7 + 2.5 + 5.9 + 3.4 + 1.7 + 2.1 + 0.4 + 1.3}{10}$

$= 3.75$

$\therefore$ LB IS 4 BALLS OF STRING

(b) $\cancel{4.3}$ $\cancel{6.1}$ $\cancel{5.1}$ $\cancel{4.7}$ $\cancel{2.5}$ $\cancel{5.9}$ $\cancel{3.4}$ $\cancel{1.7}$ $\cancel{2.1}$ $\cancel{0.4}$ $\cancel{1.3}$

BIN 1	4.3, 5.1, 0.4	10, 5.7, 0.6, 0.2
BIN 2	6.1, 2.5, 1.3	10, 3.9, 1.4, 0.1
BIN 3	4.7, 3.4, 1.7	10, 5.3, 1.9, 0.2
BIN 4	5.9, 2.1,	10, 4.1, 2.0

(Total for Question 1 is 5 marks)

2.

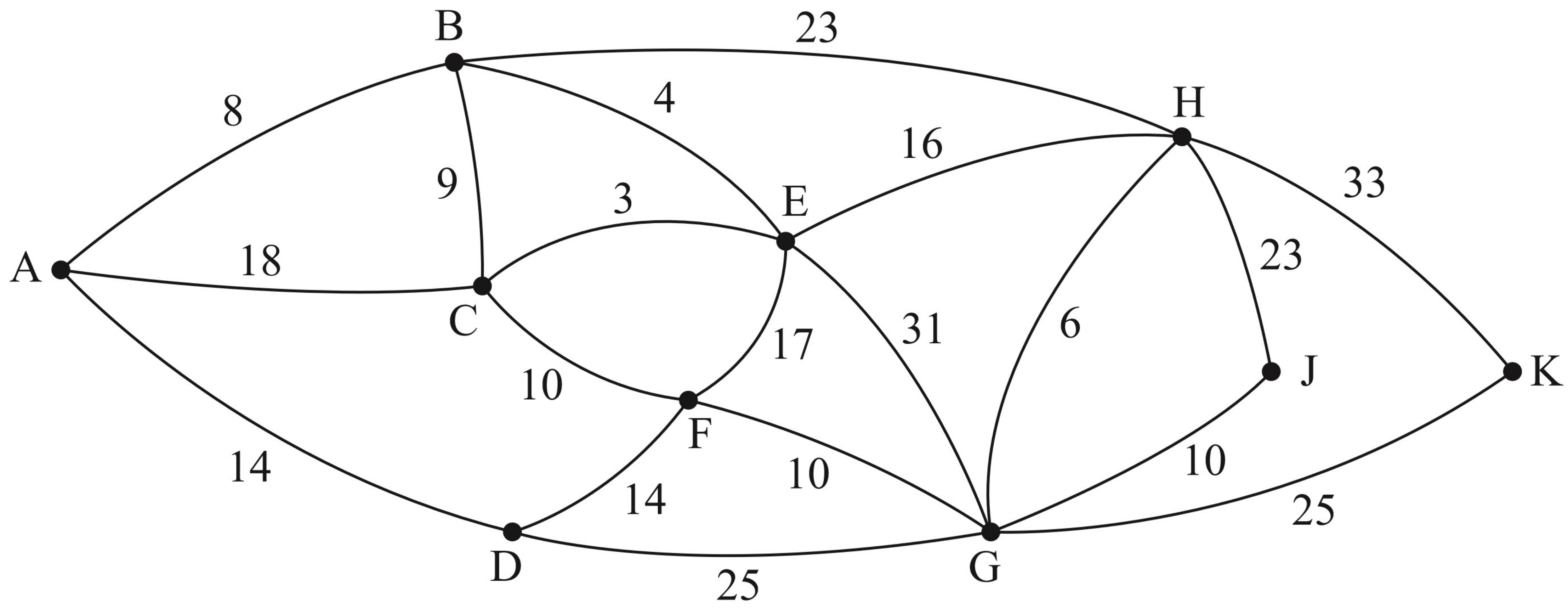


Figure 1

[*The total weight of the network is* 299]

Figure 1 represents a network of cycle tracks between 10 landmarks, A, B, C, D, E, F, G, H, J and K. The number on each edge represents the length, in kilometres, of the corresponding track.

One day, Blanche wishes to cycle from A to K. She wishes to minimise the distance she travels.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to K.

(ii) State the length of the shortest path from A to K.

(6)

The cycle tracks between the landmarks now need to be inspected. Blanche must travel along each track at least once. She wishes to minimise the length of her inspection route. Blanche will start her inspection route at D and finish at E.

(b) (i) State the edges that will need to be traversed twice.

(ii) Find the length of Blanche's route.

(2)

It is now decided to start the inspection route at A and finish at K. Blanche must minimise the length of her route and travel along each track at least once.

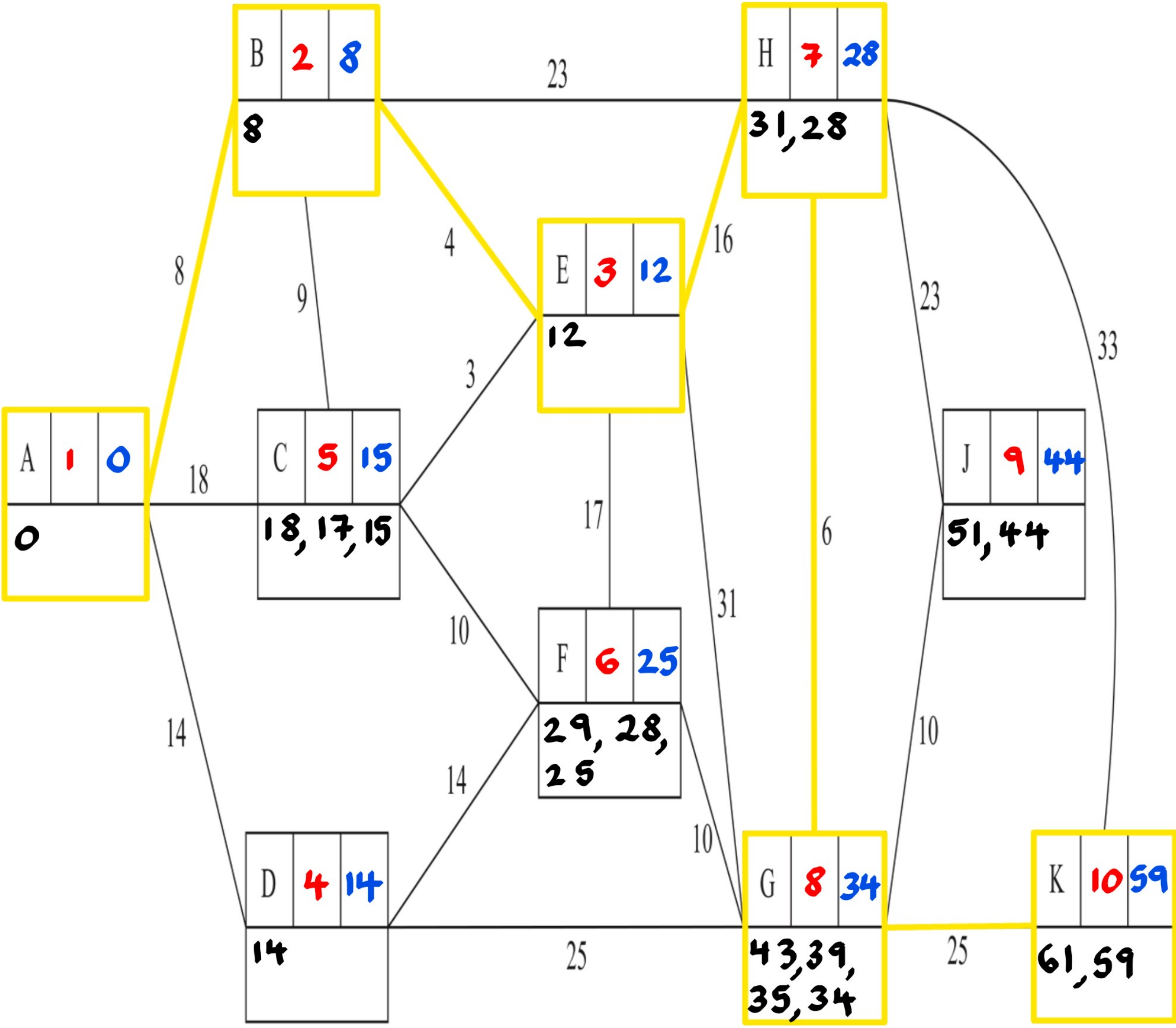
(c) By considering the pairings of all relevant nodes, find the length of Blanche's new route. You must make your method and working clear.

(5)

(Total for Question 2 is 13 marks)

2.

(a)(i)



Key:

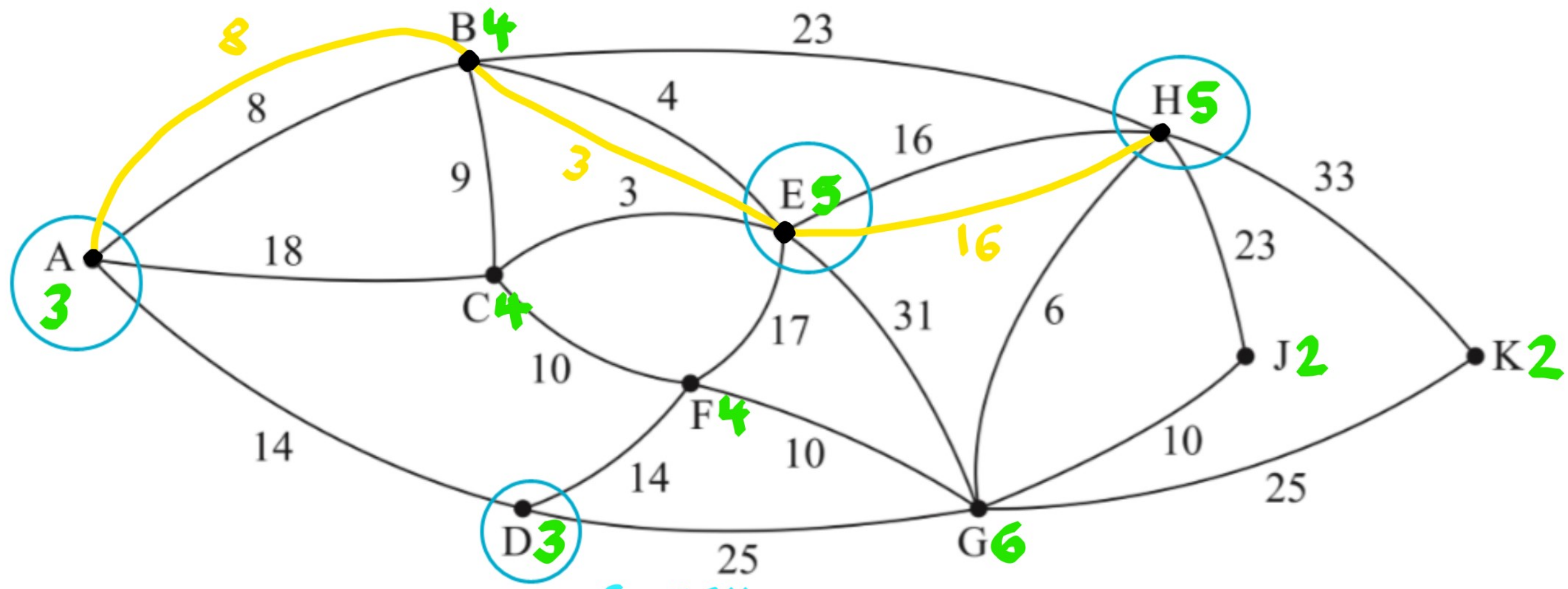
Vertex	Order of labelling	Final value
Working value		

(ii) Shortest path from A to K: ABEHGK

(iii) Length of shortest path from A to K: 59

Question 2 continued

(b)(i)



FINISH

ODD NODES ARE: A, D, E, H

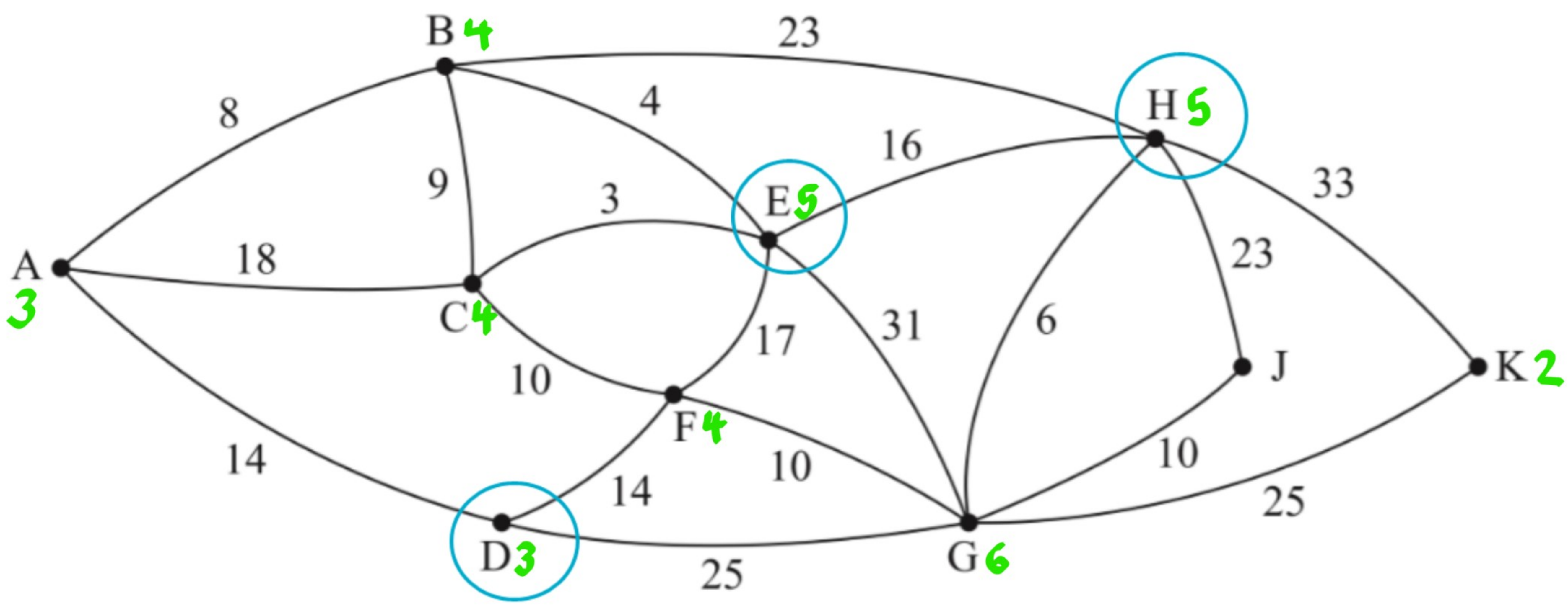
START

EDGES THAT NEED TO BE TRAVERSED TWICE ARE SHORTEST ROUTES FROM A TO H. THESE ARE: AB, BE, EH

(ii) LENGTH = 299 + 8 + 3 + 16

= 327 km

(c)



START AT A AND FINISH AT K

P 7 2 0 9 4 A 0 4 2 0

Question 2 continued

NOTE: MAKE K ODD AND D, E, H EVEN BY REPEATING EDGES

A IS ALREADY ODD SO DO NOT INCLUDE IN PAIRINGS

CONSIDER PAIRINGS OF: D, E, H, K

D(AB)E + H(G)K = 26 + 31 = 57

D(FG)H + E(HG)K = 30 + 47 = 77

D(FG)K + EK = 49 + 16 = 65

∴ LENGTH OF NEW ROUTE = 299 + 57 = 356 km

(Total for Question 2 is 13 marks)

3. The initial distance matrix (Table 1) shows the lengths, in metres, of the corridors connecting six classrooms, A, B, C, D, E and F, in a school. For safety reasons, some of the corridors are one-way only.

	A	**B**	**C**	**D**	**E**	**F**
A	–	12	32	24	29	11
B	12	–	17	8	∞	∞
C	32	17	–	4	12	∞
D	24	∞	4	–	∞	13
E	∞	∞	12	18	–	12
F	11	∞	∞	13	12	–

Table 1

(a) By adding the arcs from vertex A, along with their weights, complete the drawing of this network on Diagram 1 in the answer book.

(2)

Floyd's algorithm is to be used to find the complete network of shortest distances between the six classrooms.

The distance matrix after **two** iterations of Floyd's algorithm is shown in Table 2.

	A	**B**	**C**	**D**	**E**	**F**
A	–	12	29	20	29	11
B	12	–	17	8	41	23
C	29	17	–	4	12	40
D	24	36	4	–	53	13
E	∞	∞	12	18	–	12
F	11	23	40	13	12	–

Table 2

(b) Perform the next two iterations of Floyd's algorithm that follow from Table 2. You should show the distance matrix after each iteration.

(4)

The final distance matrix after completion of Floyd's algorithm is shown in Table 3.

	A	B	C	D	E	F
A	–	12	24	20	23	11
B	12	–	12	8	24	21
C	28	17	–	4	12	17
D	24	21	4	–	16	13
E	23	29	12	16	–	12
F	11	23	17	13	12	–

Table 3

Yinka must visit each classroom. He will start and finish at E and wishes to minimise the total distance travelled.

(c) (i) Use the nearest neighbour algorithm, starting at E, to find two Hamiltonian cycles in the completed network of shortest distances.

(ii) Find the length of each of the two cycles.

(iii) State, with a reason, which of the two cycles provides the better upper bound for the length of Yinka's route.

(4)

(Total for Question 3 is 10 marks)

3.

(a)

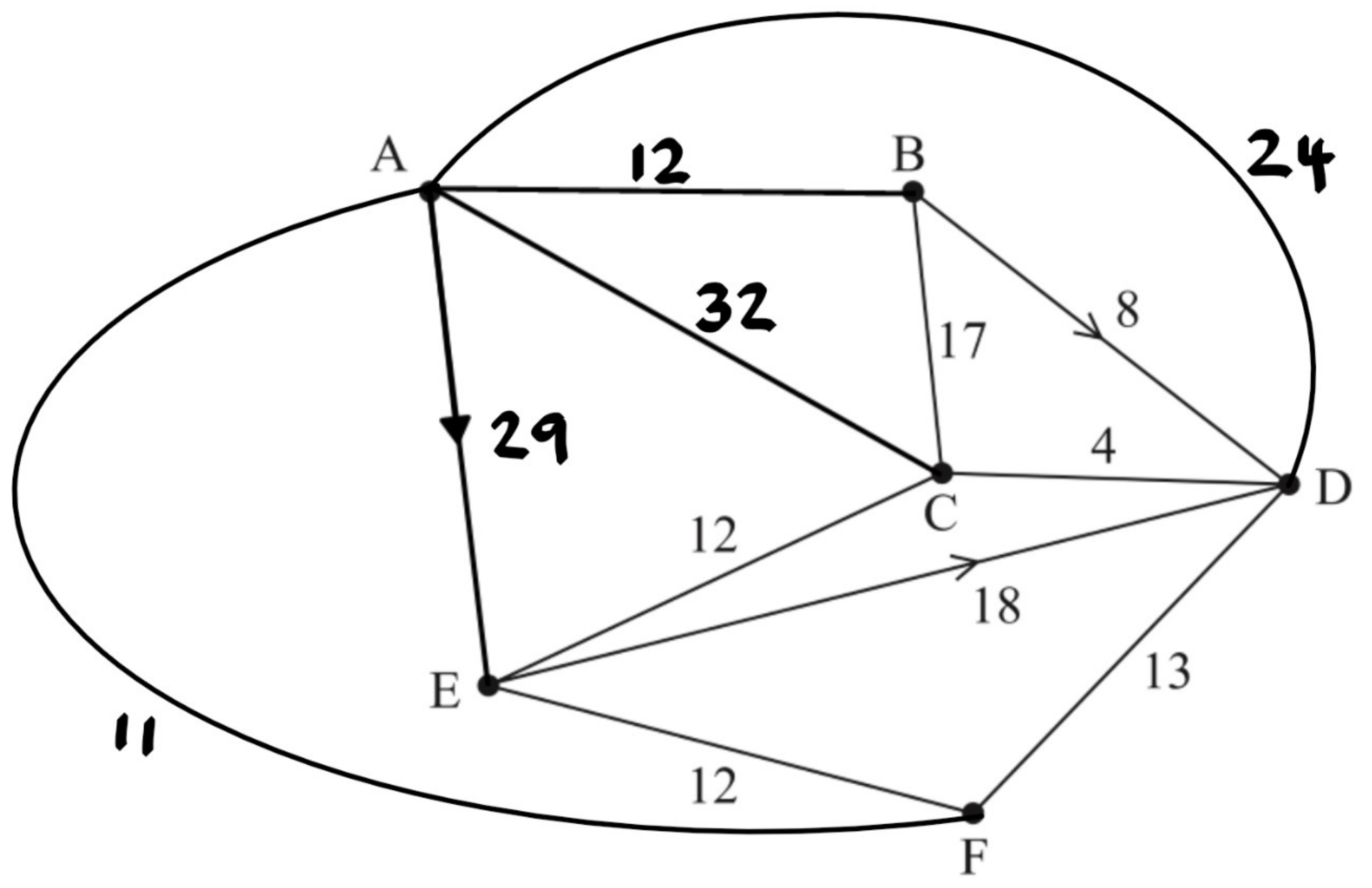


Diagram 1

(b)

There are spare copies of these tables, if required, on Page 9.

3rd iteration

	A	B	C	D	E	F
A	–	12	29	20	29	11
B	12	–	17	8	29	23
C	29	17	–	4	12	40
D	24	21	4	–	16	13
E	41	29	12	16	–	12
F	11	23	40	13	12	–

4th iteration

	A	B	C	D	E	F
A	–	12	24	20	29	11
B	12	–	12	8	24	21
C	28	17	–	4	12	17
D	24	21	4	–	16	13
E	40	29	12	16	–	12
F	11	23	17	13	12	–

P 7 2 0 9 4 A 0 6 2 0

Question 3 continued

(c)

		A	B	C	D	E	F
5	**A**	–	12	24	20	23	11
6	**B**	12	–	12	8	24	21
2	**C**	28	17	–	4	12	17
3	**D**	24	21	4	–	16	13
1	**E**	23	29	12	16	–	12
4	**F**	11	23	17	13	12	–

E → C → D → F → A → B → E

12 + 4 + 13 + 11 + 12 + 24 = 76 m

		A	B	C	D	E	F
3	**A**	–	12	24	20	23	11
4	**B**	12	–	12	8	24	21
6	**C**	28	17	–	4	12	17
5	**D**	24	21	4	–	16	13
1	**E**	23	29	12	16	–	12
2	**F**	11	23	17	13	12	–

E → F → A → B → D → C → E

12 + 11 + 12 + 8 + 4 + 12 = 59 m

∴ THE CYCLE EFABDCE IS THE BETTER UPPER BOUND ∵ THE VALUE IS THE SMALLER OF THE TWO

4. A linear programming problem in x, y and z is to be solved using the big-M method. The initial tableau is shown below.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	–2	2	0	–1	0	1	0	8
a_2	3	0	–4	0	0	–1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

(a) Using the information in the above tableau, formulate the linear programming problem. You should

- list each of the constraints as an inequality
- state the two possible objectives

(4)

(b) Obtain the most efficient pivot for a first iteration of the big-M method. You must give reasons for your answer.

(2)

(Total for Question 4 is 6 marks)

4.

b.v.	x	y	z	s_1	s_2	s_3	a_1	a_2	Value
s_1	2	3	4	1	0	0	0	0	13
a_1	1	–2	2	0	–1	0	1	0	8
a_2	3	0	–4	0	0	–1	0	1	12
P	$2 - 4M$	$-3 + 2M$	$-1 + 2M$	0	M	M	0	0	$-20M$

(a) INEQUALITIES: $2x + 3y + 4z \leq 13 \therefore 2x + 3y + 4z + s_1 = 13$

$x - 2y + 2z \geq 8 \therefore x - 2y + 2z - s_2 + a_1 = 8$

$3x - 4z \geq 12 \therefore 3x - 4z - s_3 + a_2 = 12$

$x, y, z \geq 0$ (NON-NEGATIVITY)

$P - (2-4M)x - (-3+2M)y - (-1+2M)z - Ms_2 - Ms_3 = -20M$

$P - 2x + 3y + z + 4Mx - 2My - 2Mz - Ms_2 - Ms_3 = -20M$

$\therefore$ MAXIMISE $-2x + 3y + z$ OR MINIMISE $2x - 3y - z$

(b) AS M IS BIG, THE ONLY NEGATIVE IN THE OBJECTIVE ROW IS $2 - 4M$

$\therefore$ PIVOT IS FROM x-COLUMN

APPLY THE RATIO TEST: $13 \div 2 = 6.5$

$8 \div 1 = 8$

$12 \div 3 = 4$

$\therefore$ THE 3 IN THE a_2 ROW IS PIVOT

5.

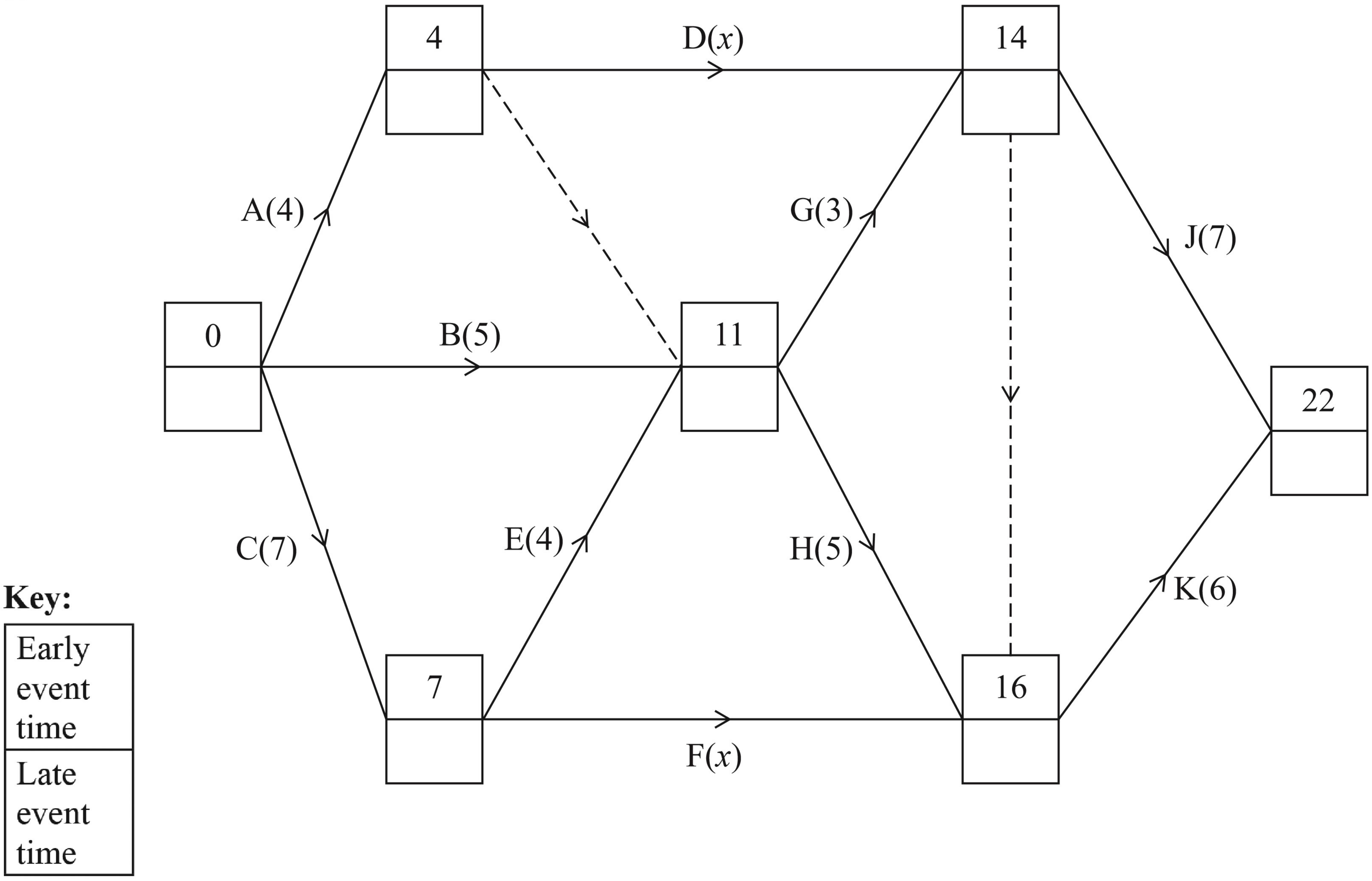


Figure 2

The network in Figure 2 shows the activities that need to be completed for a project. Each activity is represented by an arc and the duration of the activity, in days, is shown in brackets. The early event times are shown in Figure 2.

(a) Complete Table 1 in the answer book to show the immediately preceding activities for each activity.

(2)

It is given that $4 < x \leqslant m$

(b) State the largest possible integer value of m.

(1)

(c) (i) Complete Diagram 1 in the answer book to show the late event times.

(ii) State the activities that must be critical.

(3)

(d) Calculate the total float for activity G.

(1)

The resource histogram in Figure 3 shows the number of workers required when each activity starts at its earliest possible time. The histogram also shows which activities happen at each time.

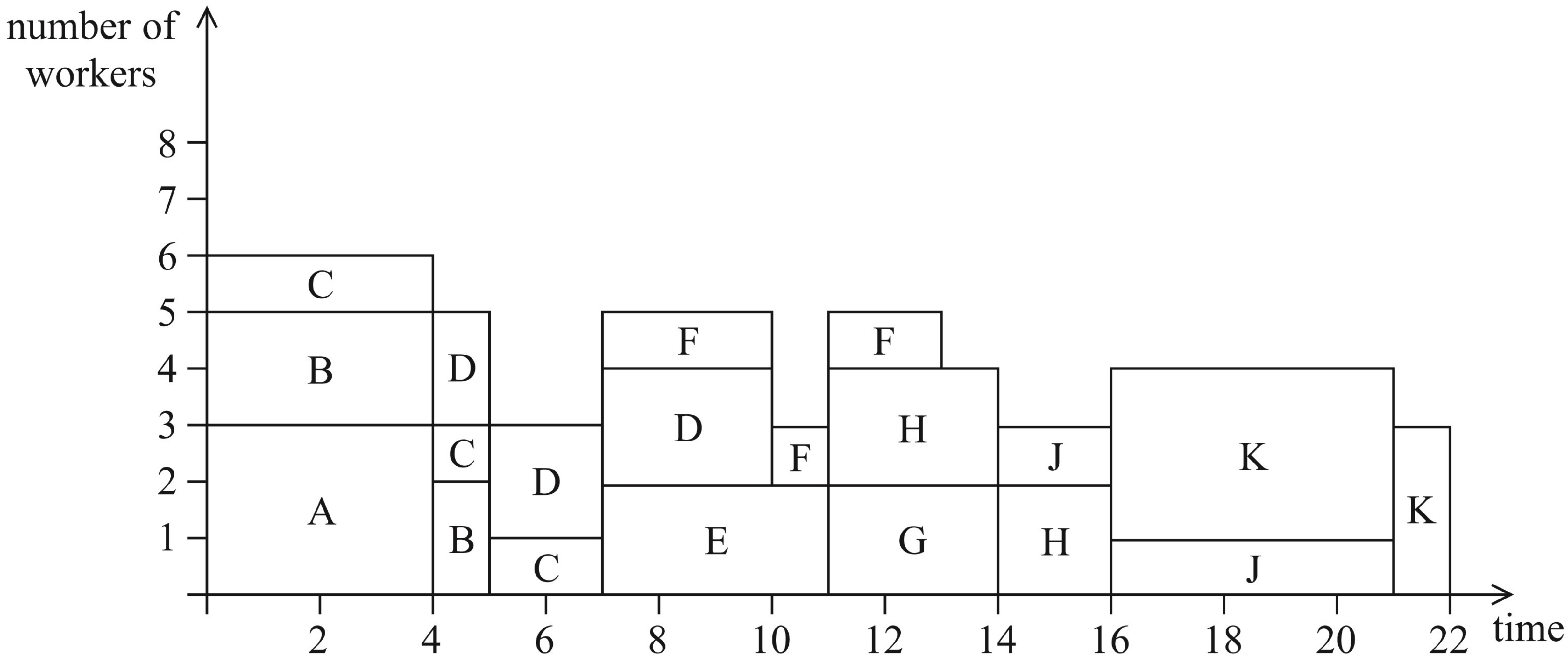


Figure 3

(e) Complete Table 2 in the answer book to show the number of workers required for each activity of the project.

(2)

(f) Draw a Gantt chart on Grid 1 in the answer book to represent the activity network.

(5)

(Total for Question 5 is 14 marks)

5.

Activity	Immediately preceding activity
A	–
B	–
C	–
D	A
E	C

Activity	Immediately preceding activity
F	C
G	A, B, E
H	A, B, E
J	D, G
K	D, G, H, F

Table 1

(b)

$m = 9$

$\therefore 4 < x \leq 9$

(c)(i)

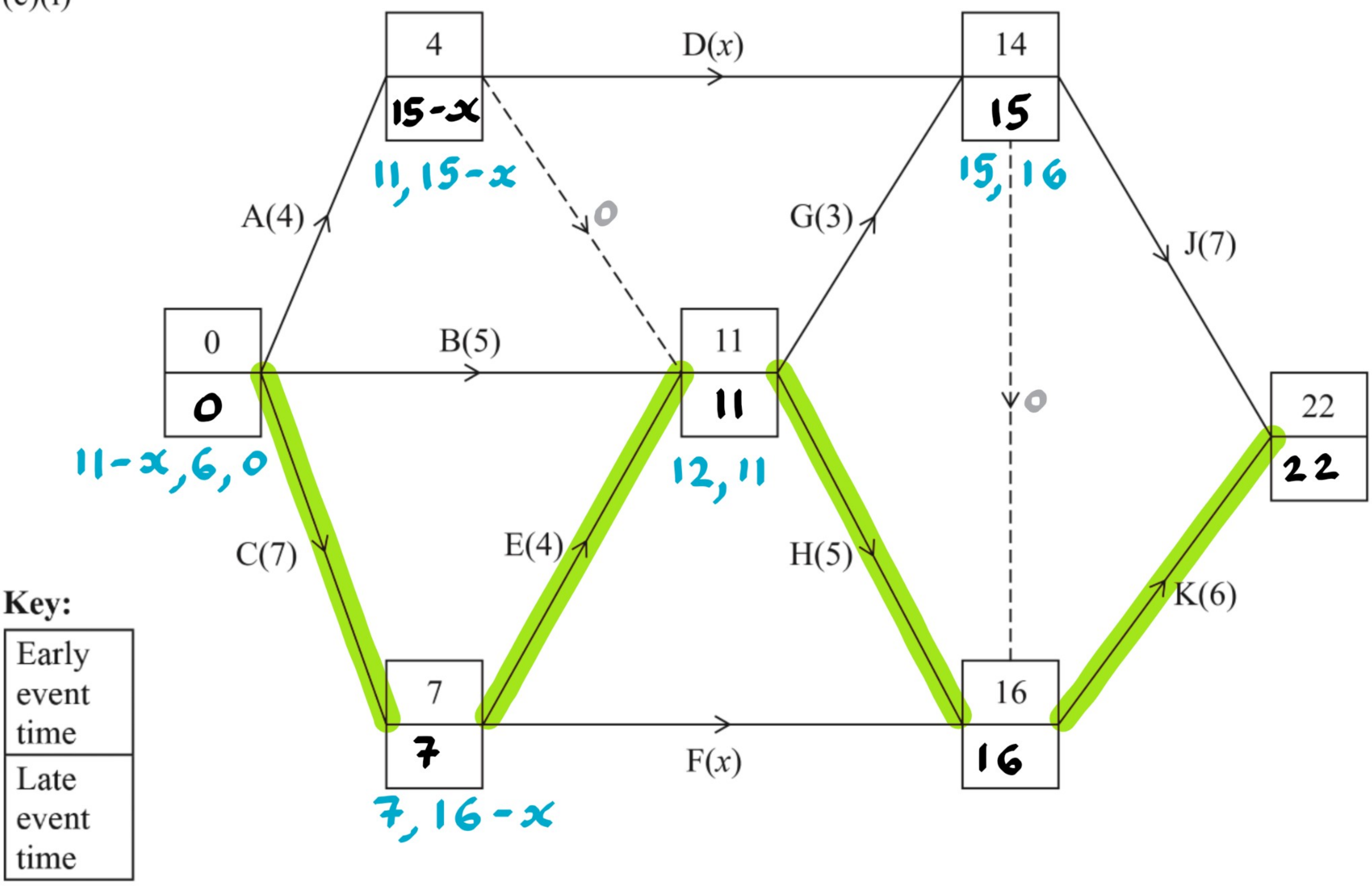


Diagram 1

(c)(ii)

C, E, H, K

(d)

FLOAT OF G = 15 − 3 − 11 = 1

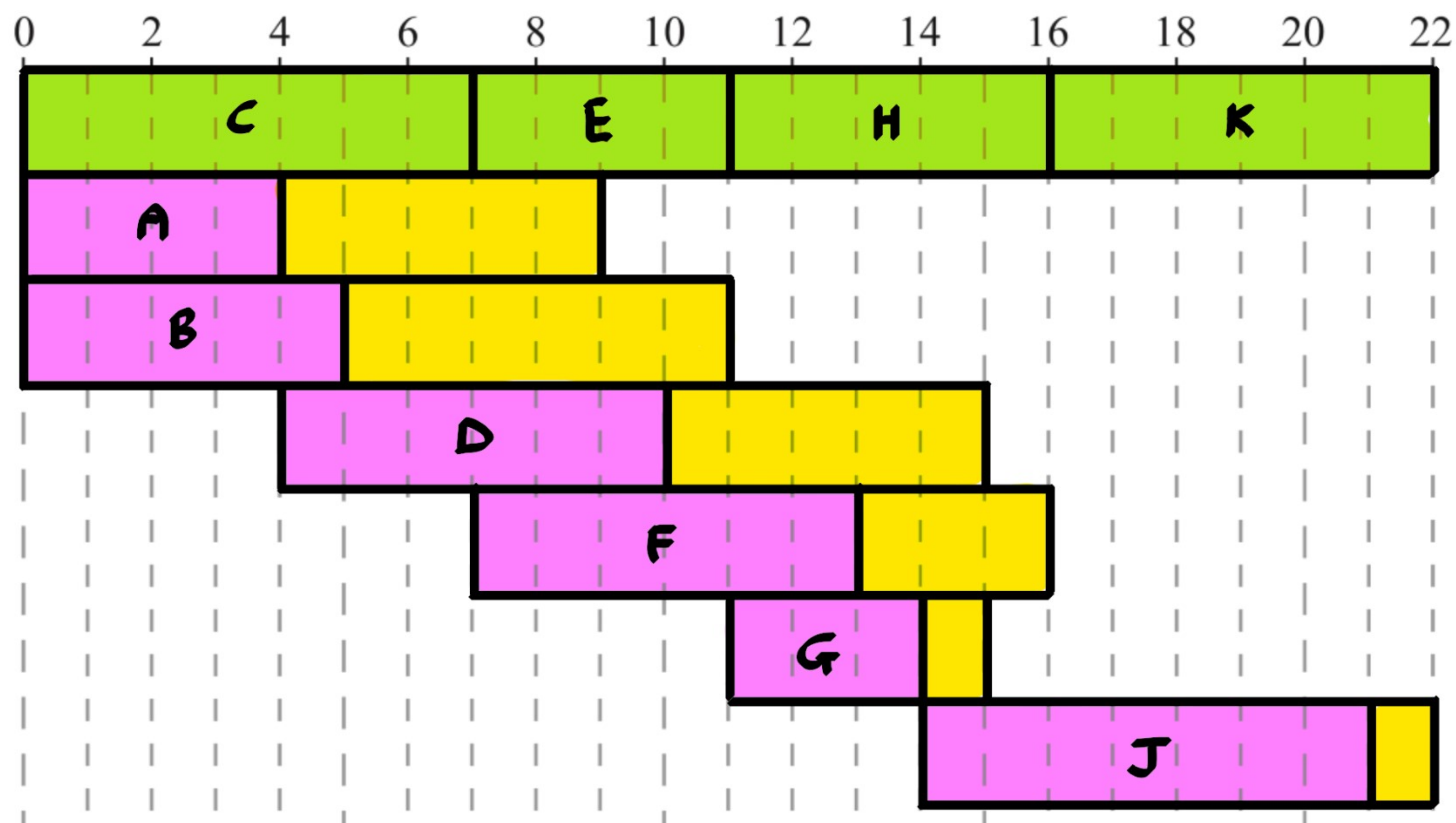
(e)

Activity	Number of workers
A	3
B	2
C	1
D	2
E	2

Activity	Number of workers
F	1
G	2
H	2
J	1
K	3

Table 2

(f)



NOTE : $x = 6$ FROM RESOURCE HISTOGRAM; D STARTS AT 4 AND FINISHES AT 10 SO DURATION IS 6

Grid 1

(Total for Question 5 is 14 marks)

6. The following algorithm determines the number of comparisons made when Prim's algorithm is applied to K_n

Step 1	Start
Step 2	Input the value of n
Step 3	Let $a = 1$
Step 4	Let $b = n - 2$
Step 5	Let $c = b$
Step 6	Let $a = a + 1$
Step 7	Let $b = b - 1$
Step 8	Let $c = c + (a \times b) + (a - 1)$
Step 9	If $b > 0$ go to Step 6
Step 10	Output c
Step 11	Stop

(a) For K_5, complete the table in the answer book to show the results obtained at each step of the algorithm.

(3)

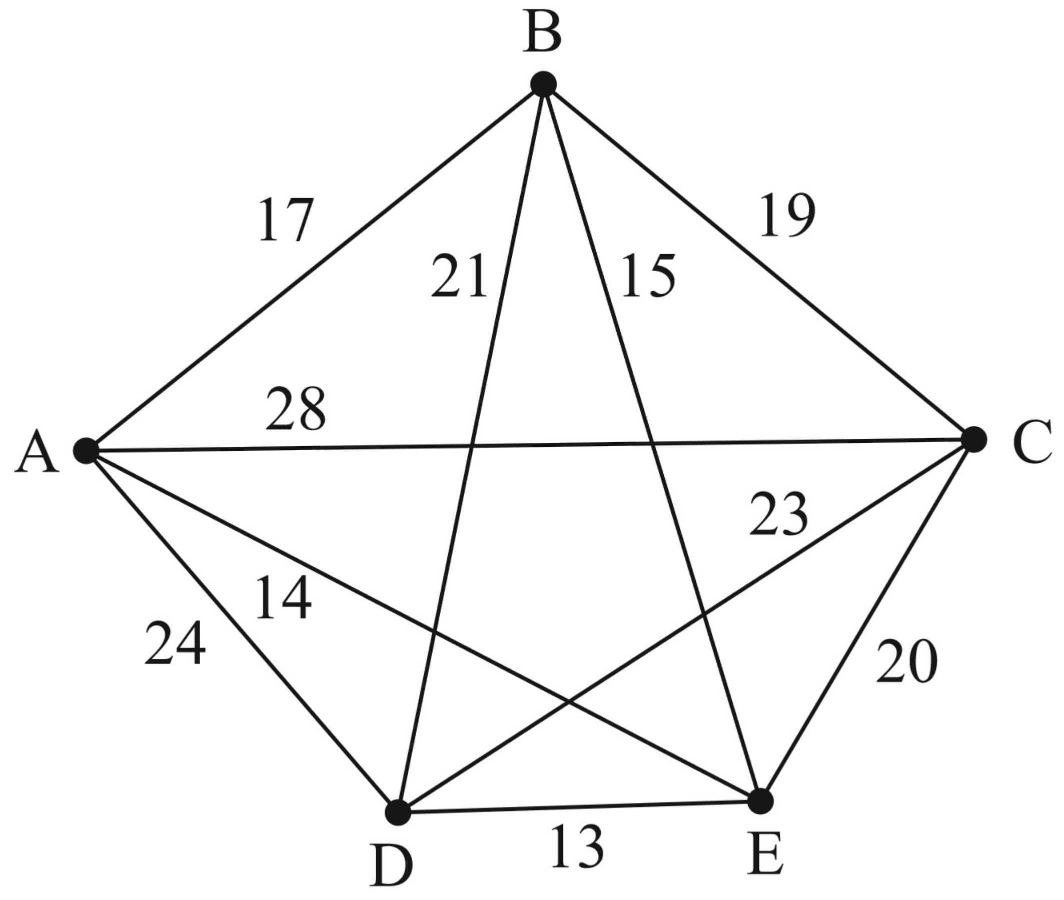


Figure 4

The weights of the ten arcs in Figure 4 are

17 21 24 14 23 13 15 19 28 20

(b) (i) Starting at the left-hand end of the above list, sort the list into ascending order using bubble sort. You need only write down the state of the list at the end of each pass.

(ii) Find the total number of comparisons performed during the sort.

(5)

(c) Find the maximum total number of comparisons required to sort the weights of the 10 arcs of K_5 into ascending order using bubble sort.

(1)

It is given that the maximum total number of comparisons required to sort the weights of the arcs of K_n into ascending order using bubble sort is

$$\lambda n(n-1)(n+1)(n-2)$$

where λ is a constant.

(d) Determine the maximum total number of comparisons required to sort the weights of the arcs of K_{50} into ascending order using bubble sort. You must make your method and working clear.

(3)

(Total for Question 6 is 12 marks)

6. *You may not need to use all the rows in this table.*

It may not be necessary to complete all the boxes in each row.

(a)

STEP	n	a	b	c
2	5			
3		1		
4			3	
5				3
6		2		
7			2	
8				8
6		3		
7			1	
8				13
6		4		
7			0	
8				16

Output: 16

↑

STEP 10

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Question 6 continued

17 21 24 14 23 13 15 19 28 20

(b)(i)

PASS	17	21	24	14	23	13	15	19	28	20	COMPARISONS
1	17	21	14	23	13	15	19	24	20	28	9
2	17	14	21	13	15	19	23	20	24	28	8
3	14	17	13	15	19	21	20	23	24	28	7
4	14	13	15	17	19	20	21	23	24	28	6
5	13	14	15	17	19	20	21	23	24	28	5
6	13	14	15	17	19	20	21	23	24	28	4

(ii) $9 + 8 + 7 + 6 + 5 + 4 = 39$

(c) $\dfrac{10(10-1)}{2} = 45$

(d) $n = 5 : 45 = \lambda \times 5 \times (5-1) \times (5+1) \times (5-2)$

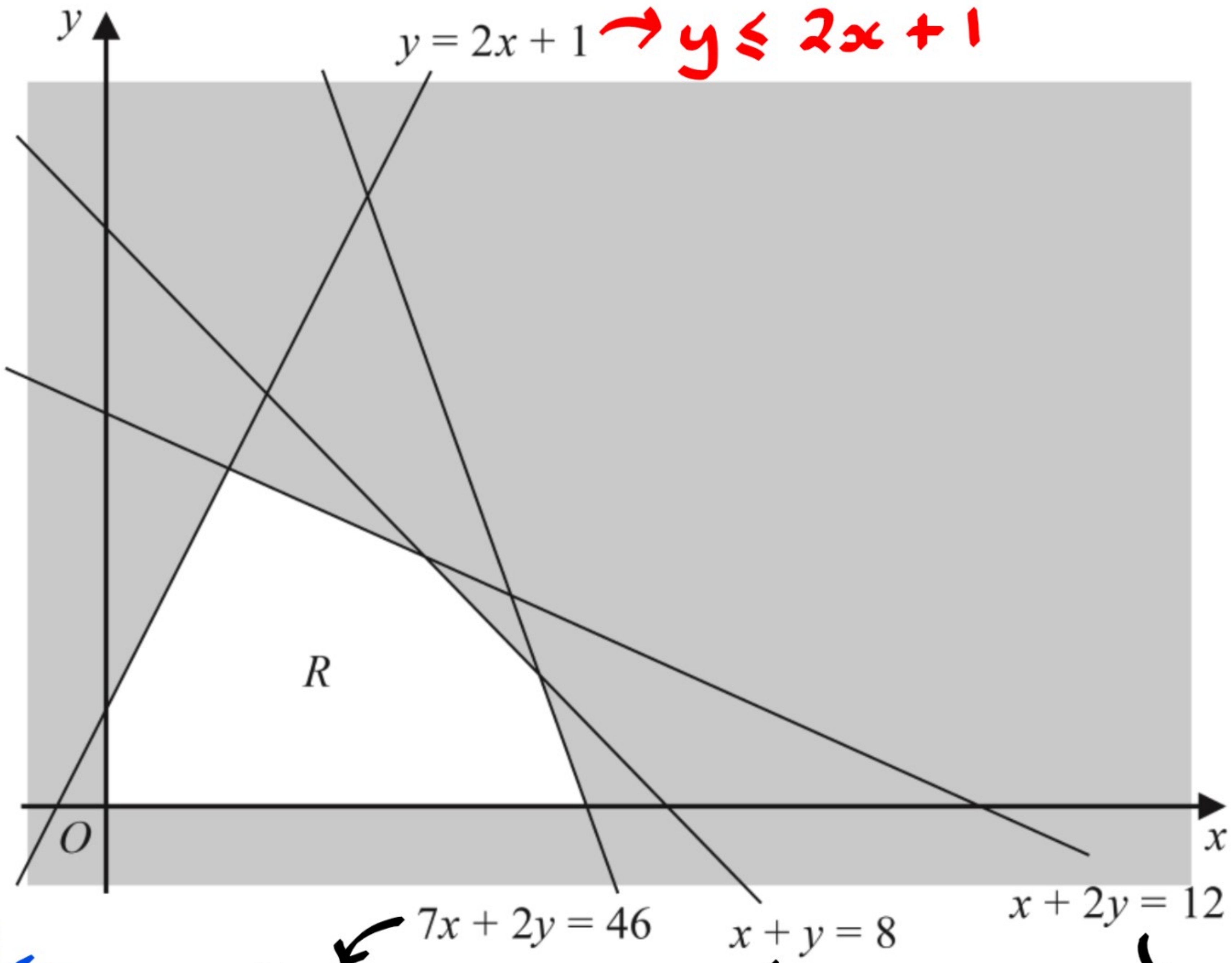
$45 = 360\lambda$

$\therefore \lambda = \dfrac{1}{8}$

$n = 50 : \dfrac{1}{8} \times 50 \times (50-1) \times (50+1) \times (50-2) = 749700$

$\therefore$ 749700 COMPARISONS

7.



$y \leq 2x + 1$

$y \leq -\frac{7}{2}x + 23$

$y = -\frac{7}{2}x + 23$

Figure 5

$y = -x + 8$

$y = -\frac{1}{2}x + 6$

$y \leq -x + 8$

$y \leq -\frac{1}{2}x + 6$

Figure 5 shows the constraints of a linear programming problem in x and y where R is the feasible region.

The objective is to maximise $P = x + ky$, where k is a positive constant.

The optimal vertex of R is to be found using the Simplex algorithm.

(a) Set up an initial tableau for solving this linear programming problem using the Simplex algorithm.

(5)

After two iterations of the Simplex algorithm a possible tableau T is

b.v.	x	y	s_1	s_2	s_3	s_4	Value
s_1	0	0	1	$-\frac{3}{5}$	0	$\frac{1}{5}$	1
x	1	0	0	$\frac{1}{5}$	0	$-\frac{2}{5}$	2
s_3	0	0	0	$-\frac{11}{5}$	1	$\frac{12}{5}$	22
y	0	1	0	$\frac{2}{5}$	0	$\frac{1}{5}$	5
P	0	0	0	$\frac{1}{5}+\frac{2}{5}k$	0	$-\frac{2}{5}+\frac{1}{5}k$	$5k+2$

(b) State the value of each variable after the second iteration.

(1)

It is given that T does not give an optimal solution to the linear programming problem.

After a third iteration of the Simplex algorithm the resulting tableau does give an optimal solution to the problem.

(c) Perform the third iteration of the Simplex algorithm and hence determine the range of possible values for P. You should state the row operations you use and make your method and working clear.

(9)

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS

7.

(a) $y \le 2x + 1 \therefore -2x + y \le 1 \therefore -2x + y + s_1 = 1$

$y \le -\frac{7}{2}x + 23 \therefore \frac{7}{2}x + y \le 23 \therefore 7x + 2y + s_2 = 46$

$y \le -x + 8 \therefore x + y \le 8 \therefore x + y + s_3 = 8$

$y \le -\frac{1}{2}x + 6 \therefore \frac{1}{2}x + y \le 6 \therefore x + 2y + s_4 = 12$

$P = x + ky \therefore P - x - ky = 0$

b.v.	x	y	s_1	s_2	s_3	s_4	Value
s_1	-2	1	1	0	0	0	1
s_2	7	2	0	1	0	0	46
s_3	1	1	0	0	1	0	8
s_4	1	2	0	0	0	1	12
P	-1	$-k$	0	0	0	0	0

(b)

$x = 2, y = 5, P = 5k + 2, s_1 = 1, s_2 = 0, s_3 = 22, s_4 = 0$

Question 7 continued

(c)

b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row	θ
s_1	0	0	1	$-\frac{3}{5}$	0	$\frac{1}{5}$	1	R_1	5
x	1	0	0	$\frac{1}{5}$	0	$-\frac{2}{5}$	2	R_2	-5
s_3	0	0	0	$-\frac{11}{5}$	1	$\frac{12}{5}$	22	R_3	$\frac{55}{6}$
y	0	1	0	$\frac{2}{5}$	0	$\frac{1}{5}$	5	R_4	25
P	0	0	0	$\frac{1}{5}+\frac{2}{5}k$	0	$-\frac{2}{5}+\frac{1}{5}k$	$5k+2$	R_5	

↑

b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row Ops
s_4	0	0	5	-3	0	1	5	$R_6 = 5R_1$
x	1	0	2	-1	0	0	4	$R_7 = R_2 + \frac{2}{5}R_6$
s_3	0	0	-12	5	1	0	10	$R_8 = R_3 - \frac{12}{5}R_6$
y	0	1	-1	1	0	0	4	$R_9 = R_4 - \frac{1}{5}R_6$
P	0	0	$2-k$	$k-1$	0	0	$4k+4$	$R_{10} = R_5 - \left(-\frac{2}{5}+\frac{1}{5}k\right)R_6$

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b.v.	x	y	s_1	s_2	s_3	s_4	Value	Row Ops
P								

Question 7 continued

OPTIMAL VALUE OF P IS $4K + 4$ AT $x = y = 4$

SINCE THIS IS AN OPTIMAL SOLUTION $\therefore 1 \leq \underbrace{K < 2}$

ONE OF THE ENTRIES IN THE PIVOT ROW OF T MUST BE NEGATIVE

HENCE, AS $P = 4K + 4$

$\therefore 8 \leq P < 12$

(Total for Question 7 is 15 marks)

TOTAL FOR PAPER IS 75 MARKS