Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Tuesday 20 October 2020

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/3C**

Further Mathematics

Advanced

Paper 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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P 6 6 5 0 7 A 0 1 2 8

Pearson

1. A particle P of mass 0.5 kg is moving with velocity $(4\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ when it receives an impulse $\mathbf{J}$ N s. Immediately after receiving the impulse, P is moving with velocity $(-\mathbf{i} + 6\mathbf{j})\,\mathrm{m\,s^{-1}}$.

(a) Find the magnitude of $\mathbf{J}$.

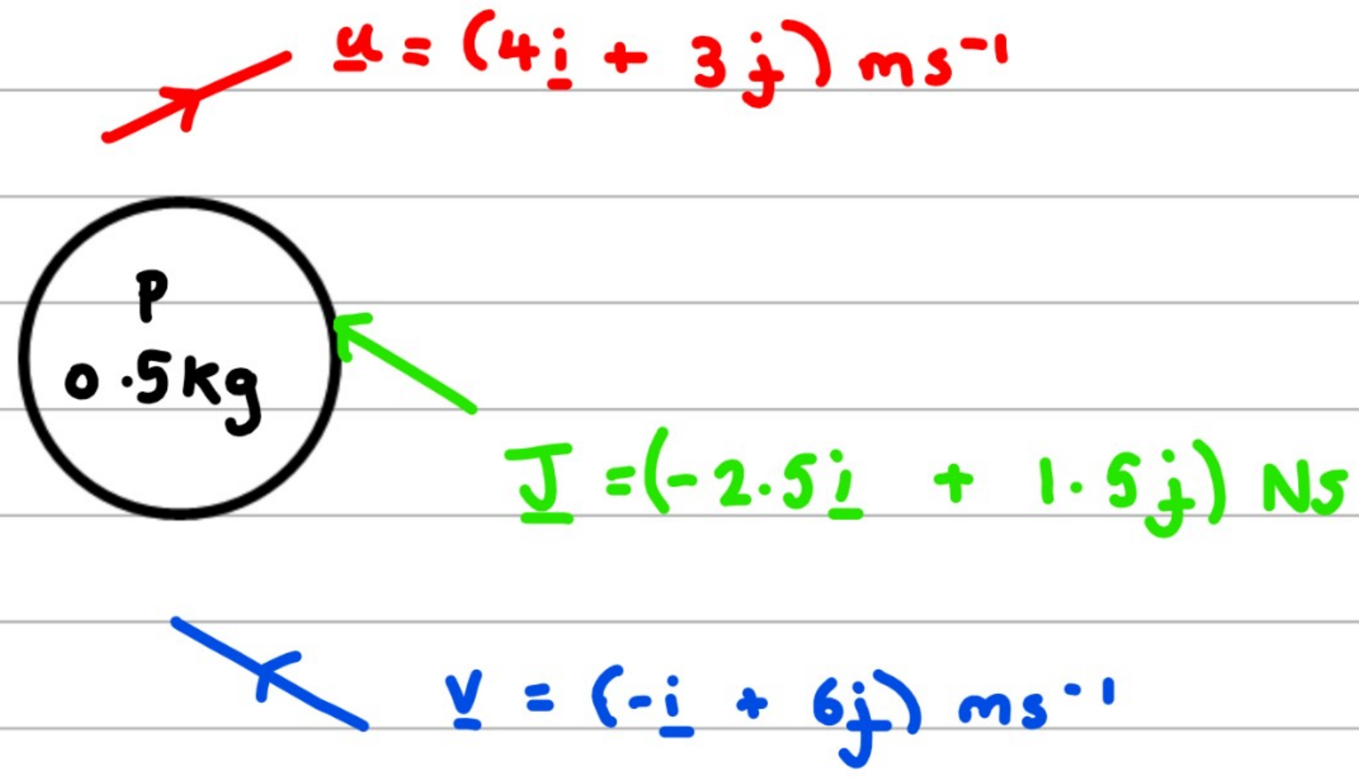
(4)

The angle between the direction of the impulse and the direction of motion of P immediately before receiving the impulse is $\alpha°$

(b) Find the value of α

(3)

(a)



IMPULSE = CHANGE IN MOMENTUM

= MASS × CHANGE IN VELOCITY

$\underline{J} = 0.5\left[(-\underline{i} + 6\underline{j}) - (4\underline{i} + 3\underline{j})\right]$

$= 0.5\left[-5\underline{i} + 3\underline{j}\right]$

$= (-2.5\underline{i} + 1.5\underline{j})$ Ns

$|\underline{J}| = \sqrt{(-2.5)^2 + (1.5)^2}$

$= \frac{1}{2}\sqrt{34}$ Ns

(b)

$\cos\alpha = \dfrac{\underline{u} \cdot \underline{J}}{|\underline{u}| \times |\underline{J}|}$

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Question 1 continued

$\underline{u} \cdot \underline{J} = (4\underline{i} + 3\underline{j}) \cdot (-2.5\underline{i} + 1.5\underline{j})$

$= (4)(-2.5) + (3)(1.5)$

$= -5.5$

$|\underline{u}| = \sqrt{4^2 + 3^2}$

$= 5$

$|\underline{J}| = \frac{1}{2}\sqrt{34}$

$\therefore \cos\alpha = \dfrac{-5.5}{5 \times \frac{1}{2}\sqrt{34}}$

$\therefore \alpha = 112^\circ$ (3sf)

(Total for Question 1 is 7 marks)

P 6 6 5 0 7 A 0 3 2 8

2. A truck of mass 1200 kg is moving along a straight horizontal road.

At the instant when the speed of the truck is v m s^{-1}, the resistance to the motion of the truck is modelled as a force of magnitude $(900 + 9v)$ N.

The engine of the truck is working at a constant rate of 25 kW.

(a) Find the deceleration of the truck at the instant when $v = 25$

(4)

Later on, the truck is moving up a straight road that is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{20}$

At the instant when the speed of the truck is v m s^{-1}, the resistance to the motion of the truck from non-gravitational forces is modelled as a force of magnitude $(900 + 9v)$ N.

When the engine of the truck is working at a constant rate of 25 kW the truck is moving up the road at a constant speed of V m s^{-1}.

(b) Find the value of V.

(5)

(a)

$P = Fv : 25000 = F \times 25 \quad \therefore \quad F = 1000$

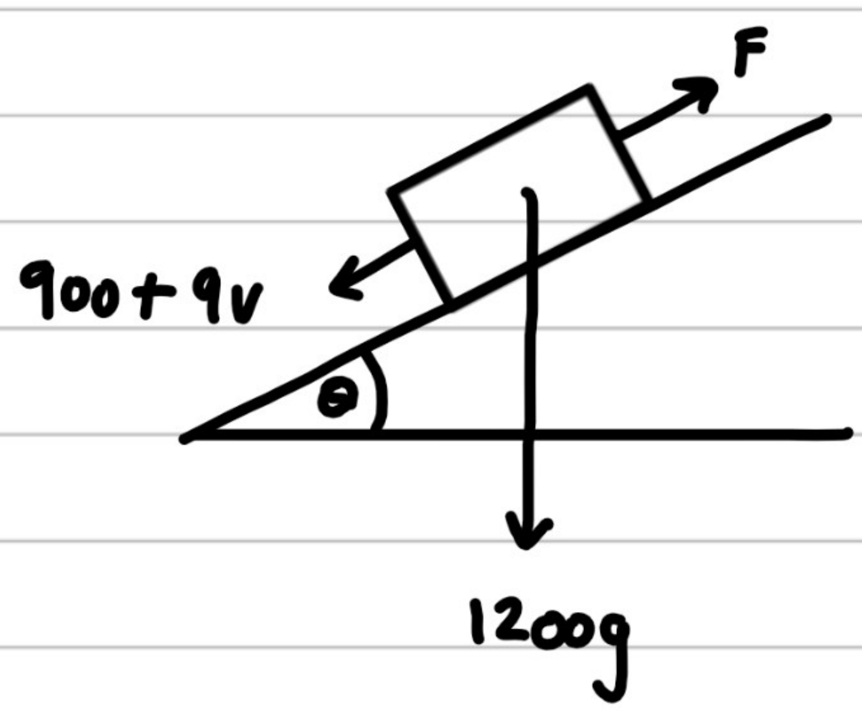
ΣFORCES $= ma : \; F - (900 + 9v) = 1200a$

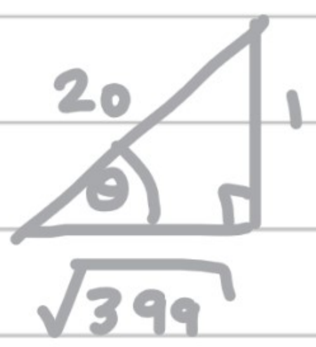
$1000 - (900 + 9 \times 25) = 1200a$

$-125 = 1200a \quad \therefore a = -\frac{5}{48}$

$\therefore$ DECELERATION IS $\frac{5}{48}$ ms^{-2}

(b)





$\sin\theta = 1/20$
$\cos\theta = \sqrt{399}/20$
$\tan\theta = 1/\sqrt{399}$

CONSTANT SPEED IMPLIES $a = 0$

Question 2 continued

$P = Fv$: $25000 = FV$ $\therefore F = \dfrac{25000}{V}$

$\sum$ FORCES $= ma$: $F - 1200g \sin\theta - (900 + 9v) = 1200 \times 0$

$$\frac{25000}{V} - 1200g\left(\frac{1}{20}\right) - 900 - 9V = 0$$

$$\frac{25000}{V} - 1488 - 9V = 0$$

$$9V^2 + 1488V - 25000 = 0$$

$$V = 15.4, -180.7 \quad (1dp)$$

$\therefore V = 15.4$ (1dp)

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Turn over

3. Two particles, A and B, have masses $3m$ and $4m$ respectively. The particles are moving in the same direction along the same straight line on a smooth horizontal surface when they collide directly. Immediately before the collision the speed of A is $2u$ and the speed of B is u.

The coefficient of restitution between A and B is e.

(a) Show that the direction of motion of each of the particles is unchanged by the collision.

(8)

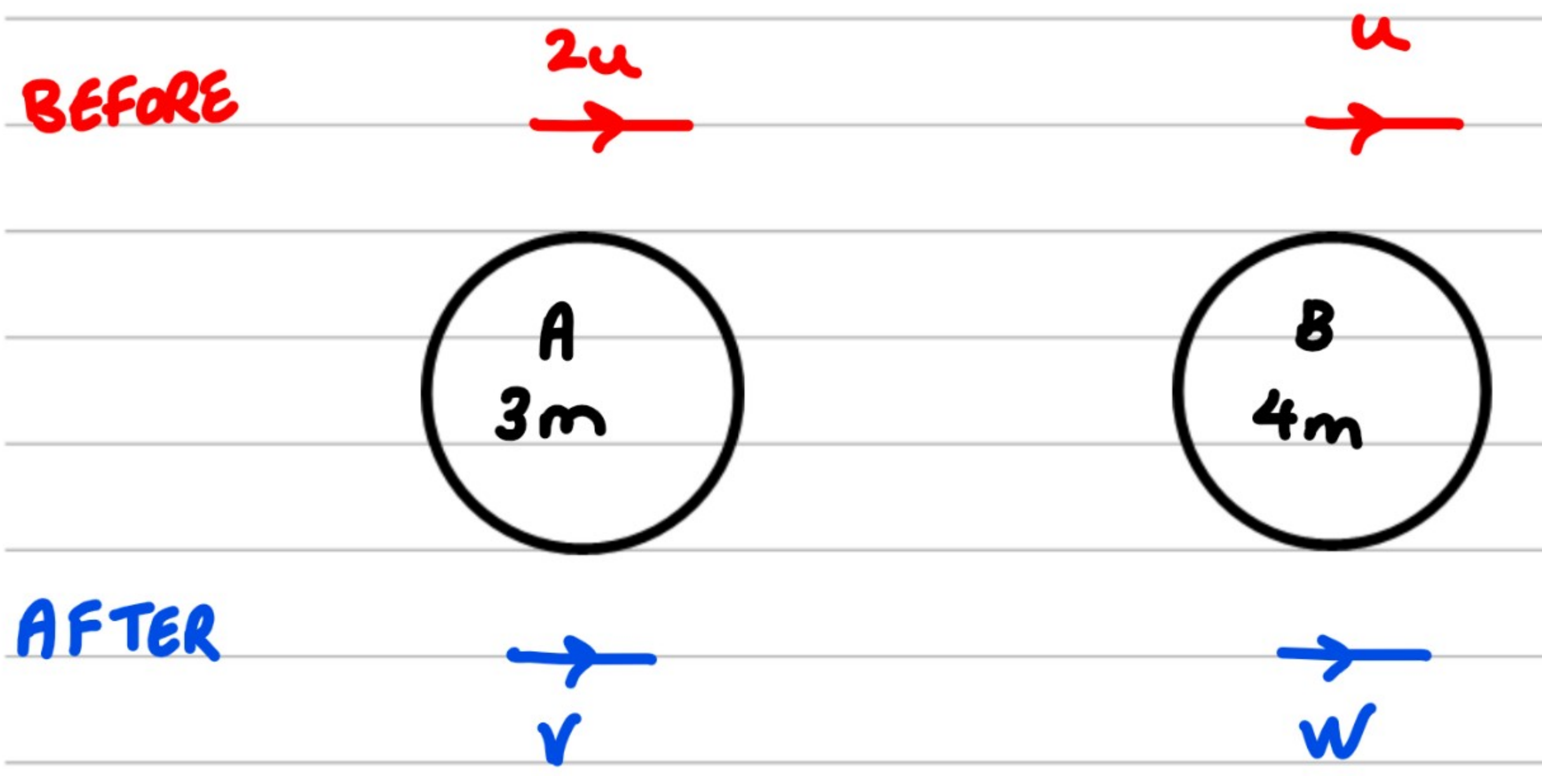
After the collision with A, particle B collides directly with a third particle, C, of mass $2m$, which is at rest on the surface.

The coefficient of restitution between B and C is also e.

(b) Show that there will be a second collision between A and B.

(6)

(a)



CLM : $(3m)(2u) + (4m)(u) = (3m)(v) + (4m)(w)$

$10u = 3v + 4w$ ①

NLR : $e = \dfrac{\text{SPEED OF SEPARATION}}{\text{SPEED OF APPROACH}} = \dfrac{w - v}{2u - u}$

$w - v = eu$ ② $\times 3$

$3w - 3v = 3eu$ ③

① + ③ : $7w = 10u + 3eu$

$7w = u(10 + 3e)$

$\therefore w = \frac{u}{7}(10 + 3e)$ ④

Question 3 continued

FROM ②: $v = w - eu$

$$v = \frac{u}{7}(10 + 3e) - eu$$

$$v = \frac{10u}{7} + \frac{3eu}{7} - eu$$

$$v = \frac{10u}{7} - \frac{4eu}{7}$$

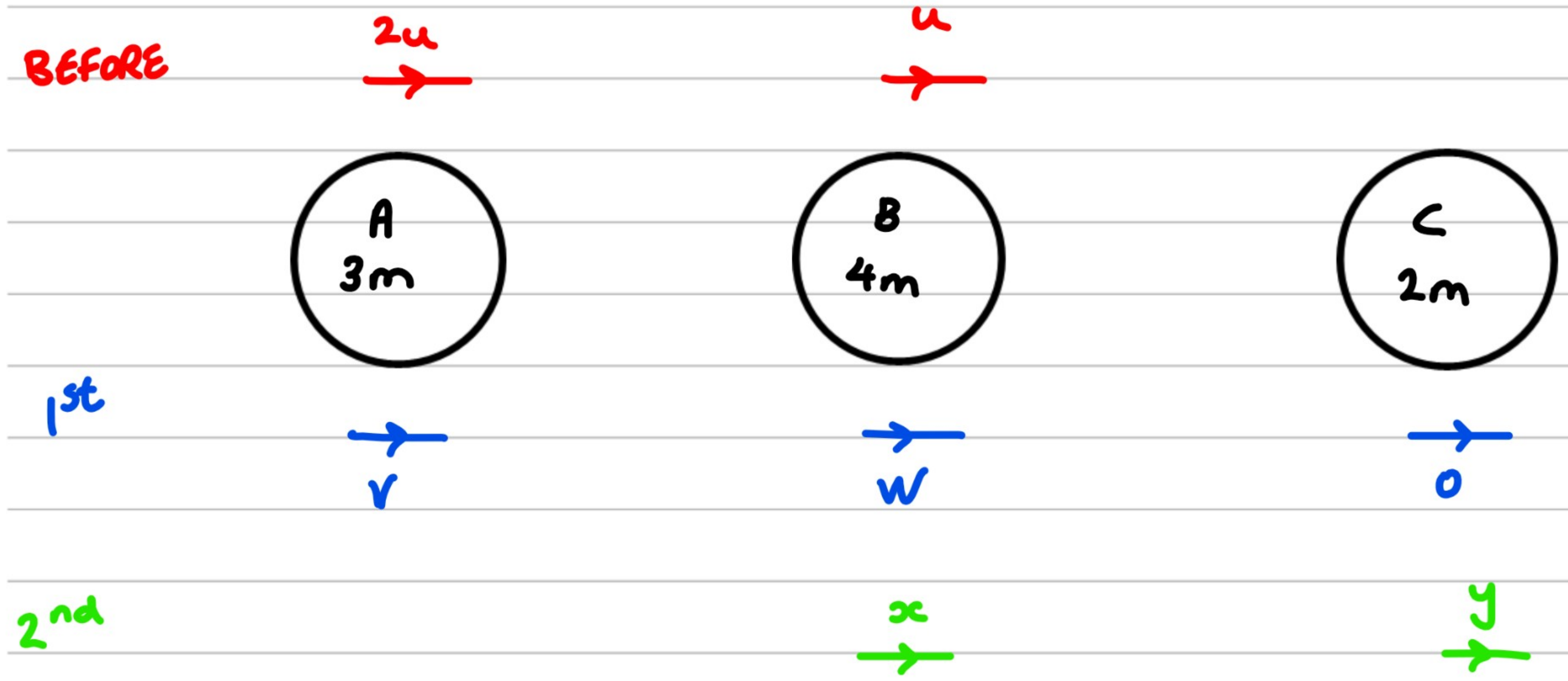
$\therefore v = \frac{u}{7}(10 - 4e)$ ⑤

AS $0 \leq e \leq 1$, $10 + 3e > 0$ and $10 - 4e > 0$

$\therefore v, w > 0$

$\therefore$ BOTH PARTICLES STILL TRAVELLING IN SAME DIRECTION

(b)



WANT TO SHOW $v > x$ IF THERE IS A COLLISION BETWEEN A AND B

CLM: $(4m)(w) + (2m)(0) = (4m)(x) + (2m)(y)$

$$4w = 4x + 2y$$

$$2w = 2x + y \quad ⑥$$

NLR: $e = \dfrac{y - x}{w}$

P 6 6 5 0 7 A 0 9 2 8

Turn over

Question 3 continued

$y - x = ew$ ⑦

⑥ − ⑦ : $2w - ew = 3x$

$w(2-e) = 3x$

$\therefore x = \frac{w}{3}(2-e)$ ⑧

④ INTO ⑧ : $x = \frac{u}{21}(10+3e)(2-e)$ ⑩

i.e. $\frac{u}{7}(10-4e) > \frac{u}{21}(10+3e)(2-e)$

$\frac{1}{7}(10-4e) > \frac{1}{21}(20-4e-3e^2)$

$3(10-4e) > 20-4e-3e^2$

$30-12e > 20-4e-3e^2$

$3e^2 - 8e + 10 > 0$

$e^2 - \frac{8}{3}e + \frac{10}{3} > 0$

$(e-\frac{4}{3})^2 - \frac{16}{9} + \frac{10}{3} > 0$

$(e-\frac{4}{3})^2 + \frac{14}{9} > 0$

SO ⑪ WILL BE TRUE FOR ALL VALUES OF e

$\therefore$ 2nd COLLISION BETWEEN A AND B

4. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

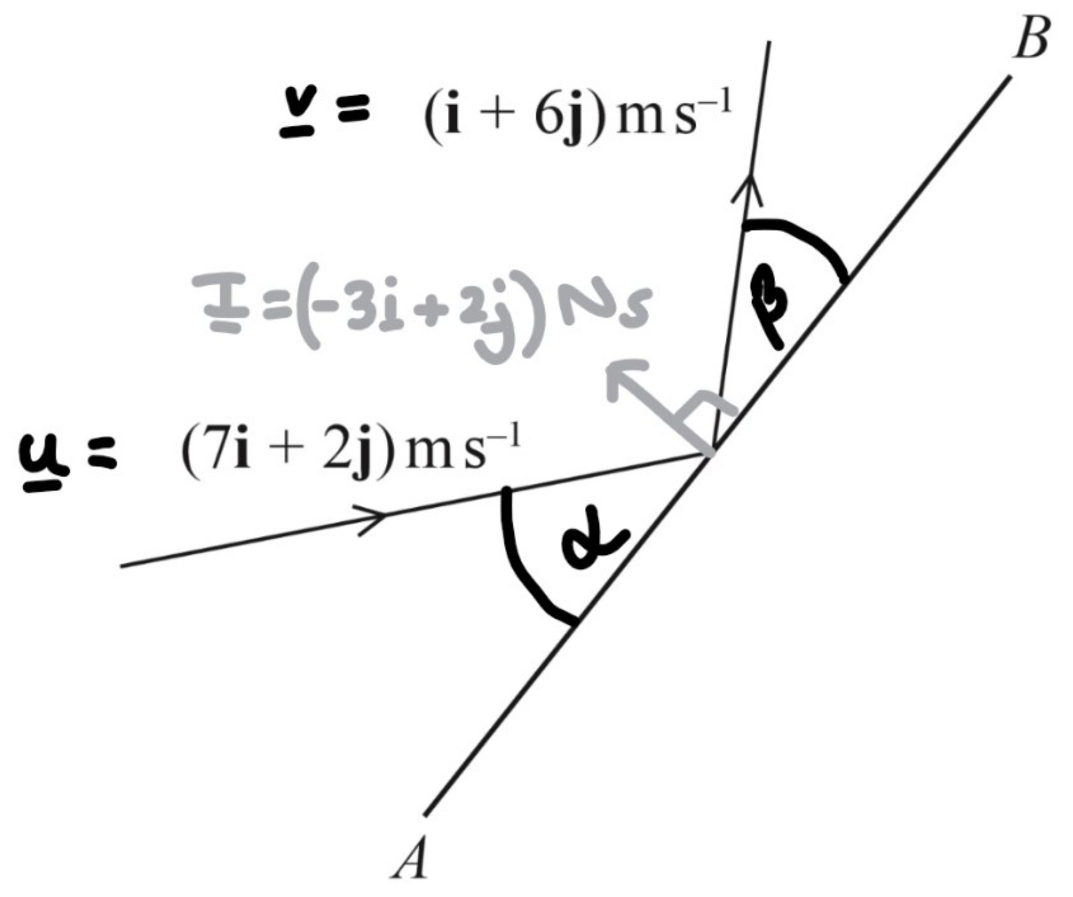


Figure 1

Figure 1 represents the plan view of part of a smooth horizontal floor, where *AB* represents a fixed smooth vertical wall.

A small ball of mass 0.5 kg is moving on the floor when it strikes the wall.

Immediately before the impact the velocity of the ball is $(7\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$.

Immediately after the impact the velocity of the ball is $(\mathbf{i} + 6\mathbf{j})\,\text{m s}^{-1}$.

The coefficient of restitution between the ball and the wall is e.

(a) Show that *AB* is parallel to $(2\mathbf{i} + 3\mathbf{j})$.

(4)

(b) Find the value of e.

(5)

(a) $\underline{I} = m(\underline{v} - \underline{u})$

$= 0.5[(\underline{i} + 6\underline{j}) - (7\underline{i} + 2\underline{j})]$

$= 0.5(-6\underline{i} + 4\underline{j})$

$= (-3\underline{i} + 2\underline{j})$ Ns

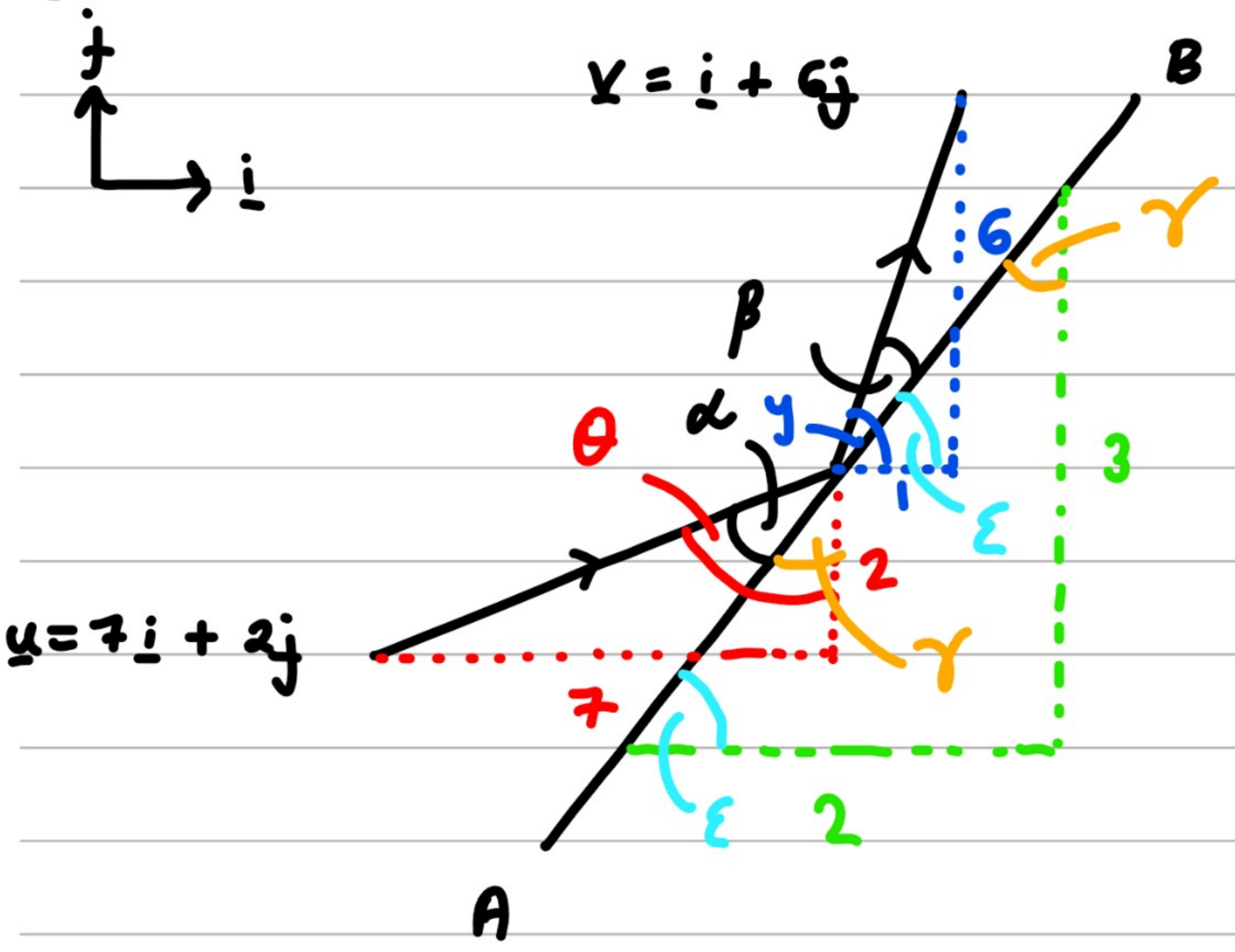
IF AB IS PERPENDICULAR TO IMPULSE, THEN $(2\underline{i} + 3\underline{j}) \cdot \underline{I} = 0$:

$(2\underline{i} + 3\underline{j}) \cdot (-3\underline{i} + 2\underline{j}) = (2)(-3) + (3)(2) = 0$

$\therefore$ AB IS PARALLEL TO $(2\underline{i} + 3\underline{j})$

Question 4 continued

(b)



$\alpha = \theta - \gamma = \tan^{-1}\left(\frac{7}{2}\right) - \tan^{-1}\left(\frac{2}{3}\right) = 40.36\ldots$

$\beta = y - \varepsilon = \tan^{-1}\left(\frac{6}{1}\right) - \tan^{-1}\left(\frac{3}{2}\right) = 24.22\ldots$

i.e. $\tan 24.22\ldots = e \tan 40.36\ldots$

$e = \dfrac{\tan 24.22\ldots}{\tan 40.36\ldots}$

$\therefore e = \dfrac{9}{17}$

P 6 6 5 0 7 A 0 1 3 2 8

5. A smooth uniform sphere P has mass 0.3 kg. Another smooth uniform sphere Q, with the same radius as P, has mass 0.2 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of P is $(4\mathbf{i} + 2\mathbf{j})\,\text{m s}^{-1}$ and the velocity of Q is $(-3\mathbf{i} + \mathbf{j})\,\text{m s}^{-1}$.

At the instant of collision, the line joining the centres of the spheres is parallel to $\mathbf{i}$.

The kinetic energy of Q immediately after the collision is half the kinetic energy of Q immediately before the collision.

(a) Find

(i) the velocity of P immediately after the collision,

(ii) the velocity of Q immediately after the collision,

(iii) the coefficient of restitution between P and Q,

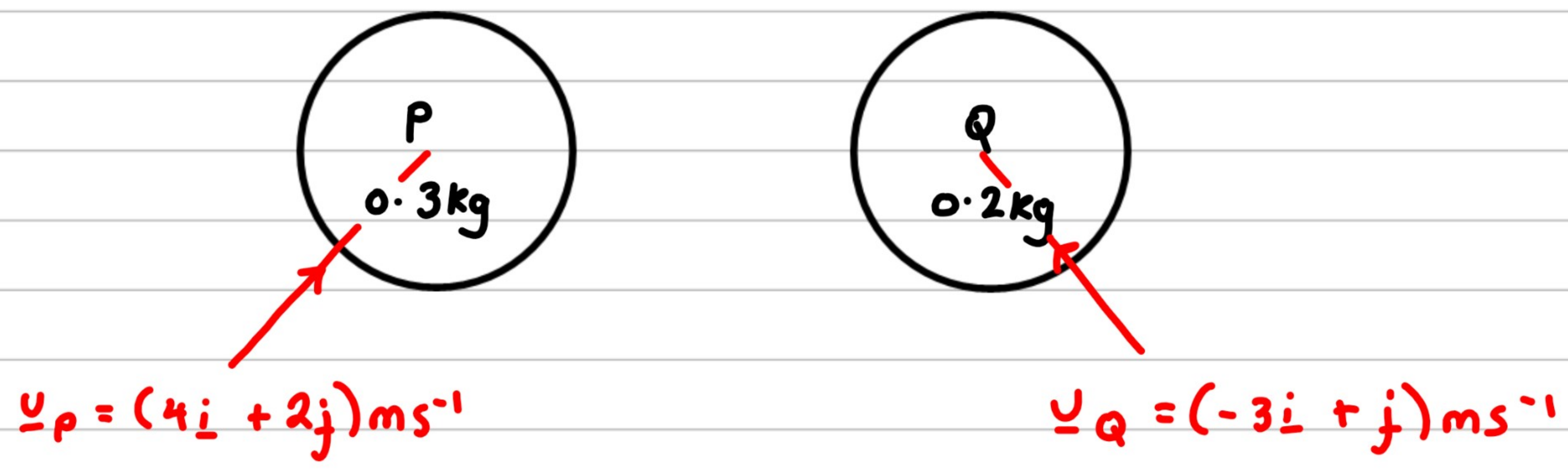
carefully justifying your answers.

(11)

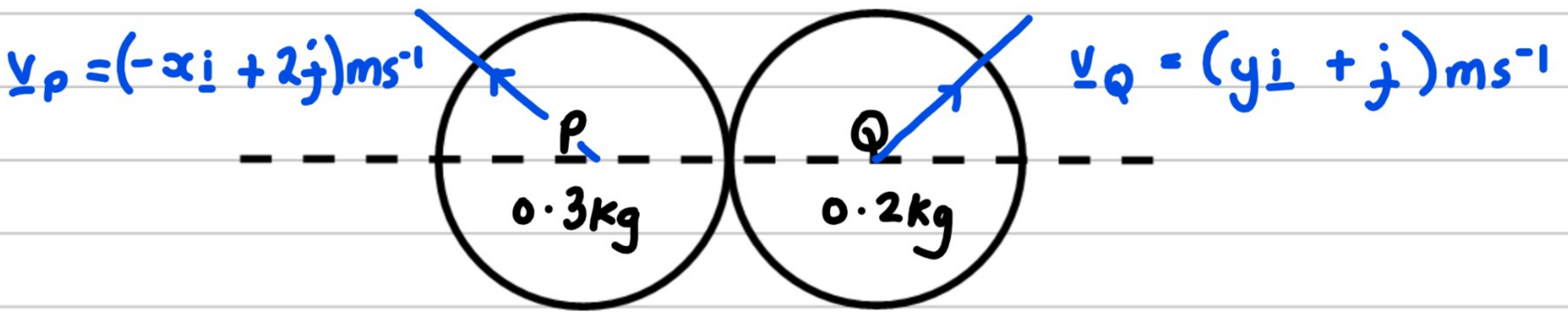
(b) Find the size of the angle through which the direction of motion of P is deflected by the collision.

(3)

BEFORE



AFTER



$|\underline{u}_Q| = \sqrt{(-3)^2 + (1)^2} = \sqrt{10}$

$|\underline{v}_Q| = \sqrt{(y)^2 + (1)^2} = \sqrt{y^2 + 1}$

i.e. $\frac{1}{2}\left(\frac{1}{2} \times 0.2 \times \sqrt{10}^{\,2}\right) = \frac{1}{2} \times 0.2 \times \sqrt{y^2+1}^{\,2}$

Question 5 continued

$5 = y^2 + 1$

$y^2 = 4 \quad \therefore y = \pm 2$

CLM : $(0.3)(4) + (0.2)(-3) = (0.3)(-x) + (0.2)(y)$

$2y - 3x = 6$ ①

NLR : $e = \frac{\text{SPEED OF SEPARATION}}{\text{SPEED OF APPROACH}} = \frac{x+y}{4+3}$

$x + y = 7e$ ②

$y = -2$: $2(-2) - 3x = 6$

$3x = -10$

$x = -\frac{10}{3}$

THIS IMPLIES P AND Q HAVE PASSED THROUGH EACH OTHER, WHICH IS IMPOSSIBLE!

$y = 2$: $2(2) - 3x = 6$

$3x = -2$

$x = -\frac{2}{3}$

$\therefore \underline{v}_P = \left(\frac{2}{3}\underline{i} + 2\underline{j}\right) \text{ms}^{-1}$

$\therefore \underline{v}_Q = (2\underline{i} + \underline{j}) \text{ms}^{-1}$

$x = -\frac{2}{3}, y = 2$: $-\frac{2}{3} + 2 = 7e$

$\frac{4}{3} = 7e \quad \therefore e = \frac{4}{21}$

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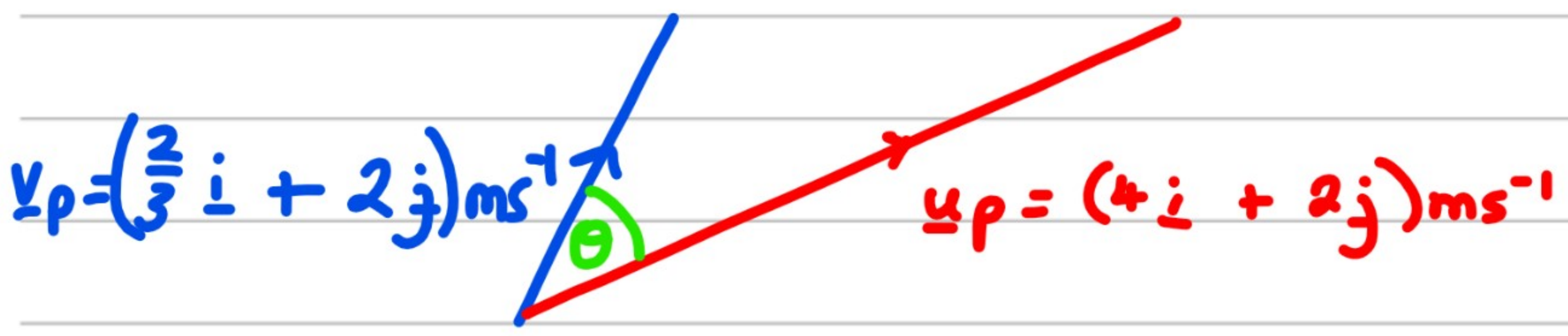
(b)

Question 5 continued



$$\cos\theta = \frac{\underline{u}_p \cdot \underline{v}_p}{|\underline{u}_p| \times |\underline{v}_p|}$$

$$= \frac{(4\underline{i} + 2\underline{j}) \cdot (\frac{2}{3}\underline{i} + 2\underline{j})}{\sqrt{4^2 + 2^2}\sqrt{(2/3)^2 + 2^2}}$$

$$= \frac{8/3 + 4}{\sqrt{20}\sqrt{40/9}}$$

$\therefore \theta = 45°$

6. A light elastic string with natural length l and modulus of elasticity kmg has one end attached to a fixed point A on a rough inclined plane. The other end of the string is attached to a package of mass m.

The plane is inclined at an angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$

The package is initially held at A. The package is then projected with speed $\sqrt{6gl}$ up a line of greatest slope of the plane and first comes to rest at the point B, where $AB = 3l$.

The coefficient of friction between the package and the plane is $\frac{1}{4}$

By modelling the package as a particle,

(a) show that $k = \frac{15}{26}$ **(6)**

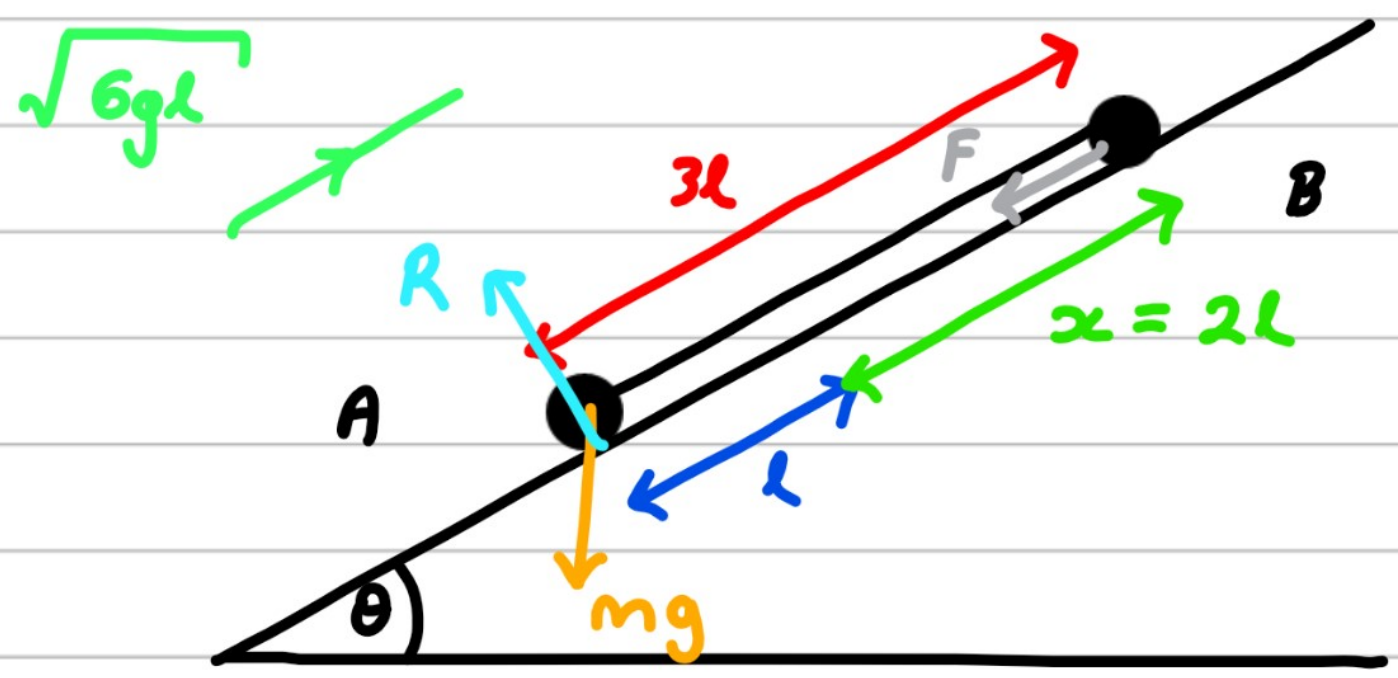
(b) find the acceleration of the package at the instant it starts to move back down the plane from the point B. **(5)**

(a)

$\sin\theta = 5/13$

$\cos\theta = 12/13$

$\tan\theta = 5/12$



$R = mg\cos\theta = \frac{12mg}{13}$

$F = \mu R = \frac{1}{4} \times \frac{12mg}{13} = \frac{3mg}{13}$

initial KE = final GPE + final EPE + WD against friction

$\frac{1}{2}mu^2 = mgh + \frac{\lambda x^2}{2l} + Fs$

Question 6 continued

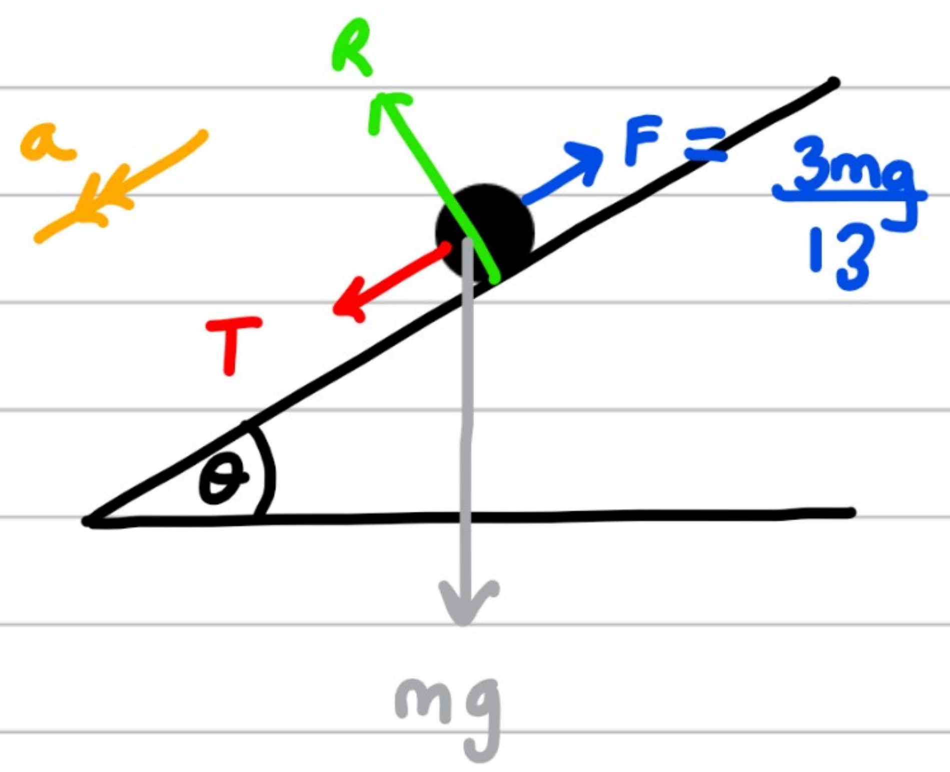
$$\frac{1}{2} \times m \times \sqrt{6gl}^2 = mg \times 3l\sin\theta + \frac{kmg \times (2l)^2}{2 \times l} + \frac{3mg}{13} \times 3l$$

$$\frac{1}{2} \times 6 = 3 \times \frac{5}{13} + 2k + \frac{9}{13}$$

$$3 = \frac{15}{13} + 2k + \frac{9}{13}$$

$$2k = \frac{15}{13} \quad \therefore k = \frac{15}{26}$$

(b) AT B:



$$\Sigma \text{FORCES} = ma : T + mg\sin\theta - F = ma$$

$$\frac{\lambda x}{l} + mg \times \frac{5}{13} - \frac{3mg}{13} = ma$$

$$\frac{\frac{15}{26}mg \times 2l}{l} + \frac{5mg}{13} - \frac{3mg}{13} = ma$$

$$\left(\frac{15}{13} + \frac{5}{13} - \frac{3}{13}\right)g = a \quad \therefore a = \frac{17g}{13}$$

7.

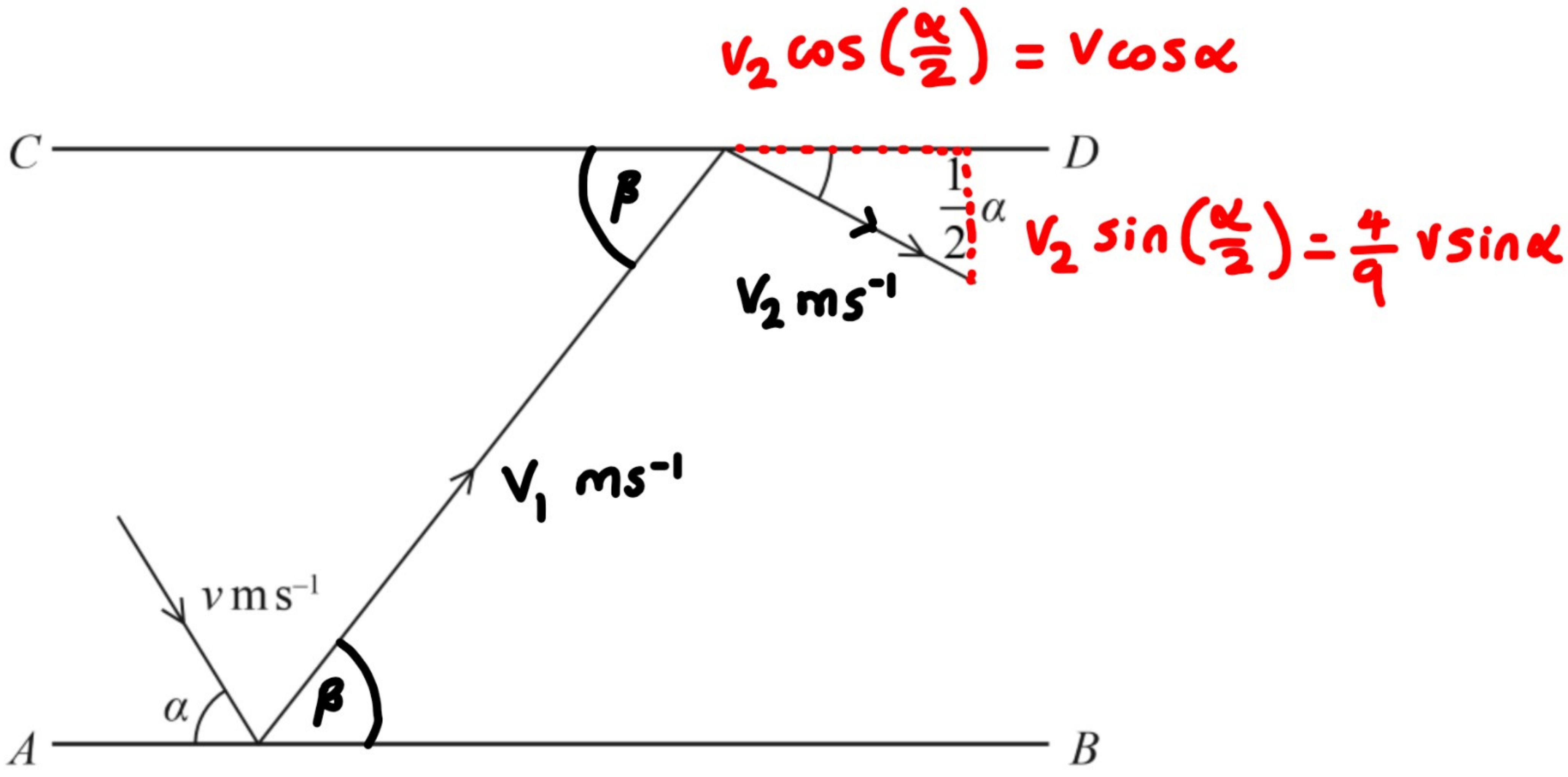


Figure 2

Figure 2 represents the plan view of part of a horizontal floor, where AB and CD represent fixed vertical walls, with AB parallel to CD.

A small ball is projected along the floor towards wall AB. Immediately before hitting wall AB, the ball is moving with speed $v\,\mathrm{m\,s}^{-1}$ at an angle α to AB, where $0 < \alpha < \frac{\pi}{2}$

The ball hits wall AB and then hits wall CD.

After the impact with wall CD, the ball is moving at angle $\frac{1}{2}\alpha$ to CD.

The coefficient of restitution between the ball and wall AB is $\frac{2}{3}$

The coefficient of restitution between the ball and wall CD is also $\frac{2}{3}$

The floor and the walls are modelled as being smooth. The ball is modelled as a particle.

(a) Show that $\tan\left(\frac{1}{2}\alpha\right) = \frac{1}{3}$

(7)

(b) Find the percentage of the initial kinetic energy of the ball that is lost as a result of the two impacts.

(4)

(a) PARALLEL TO AB : $V\cos\alpha = V_1 \cos\beta$ ①

PERPENDICULAR TO AB: $\frac{2}{3} V \sin\alpha = V_1 \sin\beta$ ②

PARALLEL TO CD : $V_1 \cos\beta = V_2 \cos\left(\frac{\alpha}{2}\right)$ ③

PERPENDICULAR TO CD: $\frac{2}{3} V_1 \sin\beta = V_2 \sin\left(\frac{\alpha}{2}\right)$ ④

Question 7 continued

② INTO ④ : $V_2 \sin\left(\frac{\alpha}{2}\right) = \frac{2}{3}\left(\frac{2}{3} V \sin\alpha\right)$

$\therefore V_2 \sin\left(\frac{\alpha}{2}\right) = \frac{4}{9} V \sin\alpha$ ⑤

① INTO ③ : $V_2 \cos\left(\frac{\alpha}{2}\right) = V\cos\alpha$ ⑥

⑤ ÷ ⑥ : $\dfrac{V_2 \sin\left(\frac{\alpha}{2}\right)}{V_2 \cos\left(\frac{\alpha}{2}\right)} = \dfrac{\frac{4}{9} V \sin\alpha}{V\cos\alpha}$

$\tan\left(\frac{\alpha}{2}\right) = \frac{4}{9}\tan\alpha$ ⑦

but $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

i.e. $\tan\alpha = \dfrac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)}$ ⑧

⑧ INTO ⑦ : $\tan\left(\frac{\alpha}{2}\right) = \dfrac{4}{9} \times \dfrac{2\tan\left(\frac{\alpha}{2}\right)}{1-\tan^2\left(\frac{\alpha}{2}\right)}$

LET $t = \tan\left(\frac{\alpha}{2}\right)$: $t = \dfrac{4(2t)}{9(1-t^2)}$

$9 = \dfrac{8}{1-t^2}$

$1-t^2 = \frac{8}{9}$

$t^2 = \frac{1}{9}$ $\quad \therefore t = \frac{1}{3}$

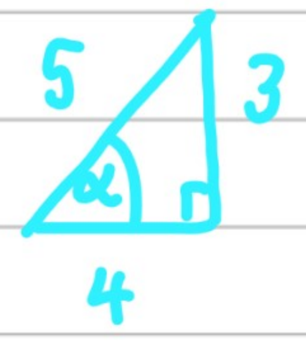
$\therefore \tan\left(\frac{\alpha}{2}\right) = \frac{1}{3}$ ⑨

(b) $V_2 = \sqrt{(V\cos\alpha)^2 + \left(\frac{4}{9}V\sin\alpha\right)^2}$

$V_2^{\,2} = V^2\cos^2\alpha + \frac{16}{81}V^2\sin^2\alpha$ ⑩

Question 7 continued

9 INTO 8 : $\tan\alpha = \dfrac{2(1/3)}{1-(1/3)^2} = \dfrac{3}{4}$



$\sin\alpha = 3/5$

$\cos\alpha = 4/5$

$\tan\alpha = 3/4$

FROM 10 : $v_2{}^2 = v^2(4/5)^2 + \frac{16}{81}v^2(3/5)^2$

$\therefore v_2{}^2 = \dfrac{32v^2}{45}$ 11

KE BEFORE $= \frac{1}{2}mv^2$ 12

KE AFTER $= \frac{1}{2}mv_2{}^2$ 13

$= \frac{1}{2}m\,\frac{32v^2}{45}$ 14

KE LOST $= \dfrac{13mv^2}{90}$

% KE LOST $= \dfrac{13mv^2/90}{mv^2/2} \times 100$

$= 28.9\%$ (3sf)