

## Further Mathematics

### Advanced Subsidiary Further Mathematics options 27: Decision Mathematics 1 (Part of options D, F, H and K)

**You must have:**

Mathematical Formulae and Statistical Tables (Green), calculator,  
D1 Answer Book (enclosed)

**Candidates may use any calculator allowed by Pearson regulations.  
Calculators must not have the facility for symbolic algebra manipulation,  
differentiation and integration, or have retrievable mathematical  
formulae stored in them.**

**Instructions**

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided
  - *there may be more space than you need.*
- You should show sufficient working to make your methods clear.  
Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.
- Do not return the question paper with the D1 Answer Book.

**Information**

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

**Advice**

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

**Turn over ►**



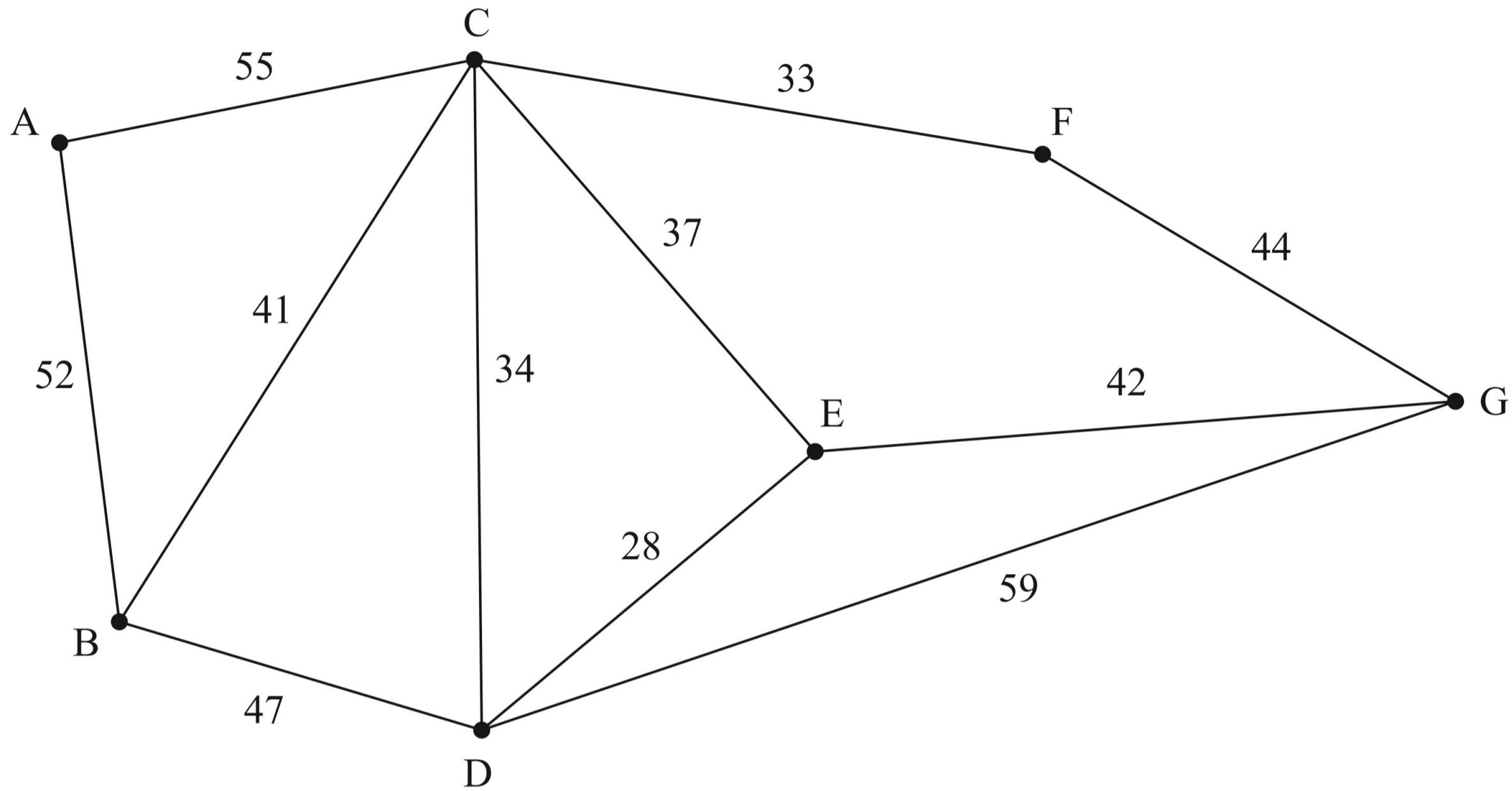
**Answer ALL questions. Write your answers in the answer book provided.**

1.      55    44    34    59    28    37    41    52    33    42    47

The list of eleven numbers shown above is to be sorted into ascending order.

- (a) Carry out a quick sort to produce the sorted list. You should show the result of each pass and identify your pivots clearly.

(4)



**Figure 1**

- (b) Use Kruskal's algorithm to find the minimum spanning tree for the network in Figure 1. You should list the arcs in the order in which you consider them. For each arc, state whether or not you are adding it to your minimum spanning tree.

(3)

- (c) (i) Draw the minimum spanning tree on Diagram 1 in the answer book.

- (ii) State the total weight of the tree.

(2)

**(Total for Question 1 is 9 marks)**

1. 55 44 34 34 59 28 37 41 52 33 42 47

(a) 55 44 34 59 28 37 41 52 33 42 47

34 28 33 37 55 44 59 41 52 42 47

28 34 33 37 41 55 44 59 52 42 47

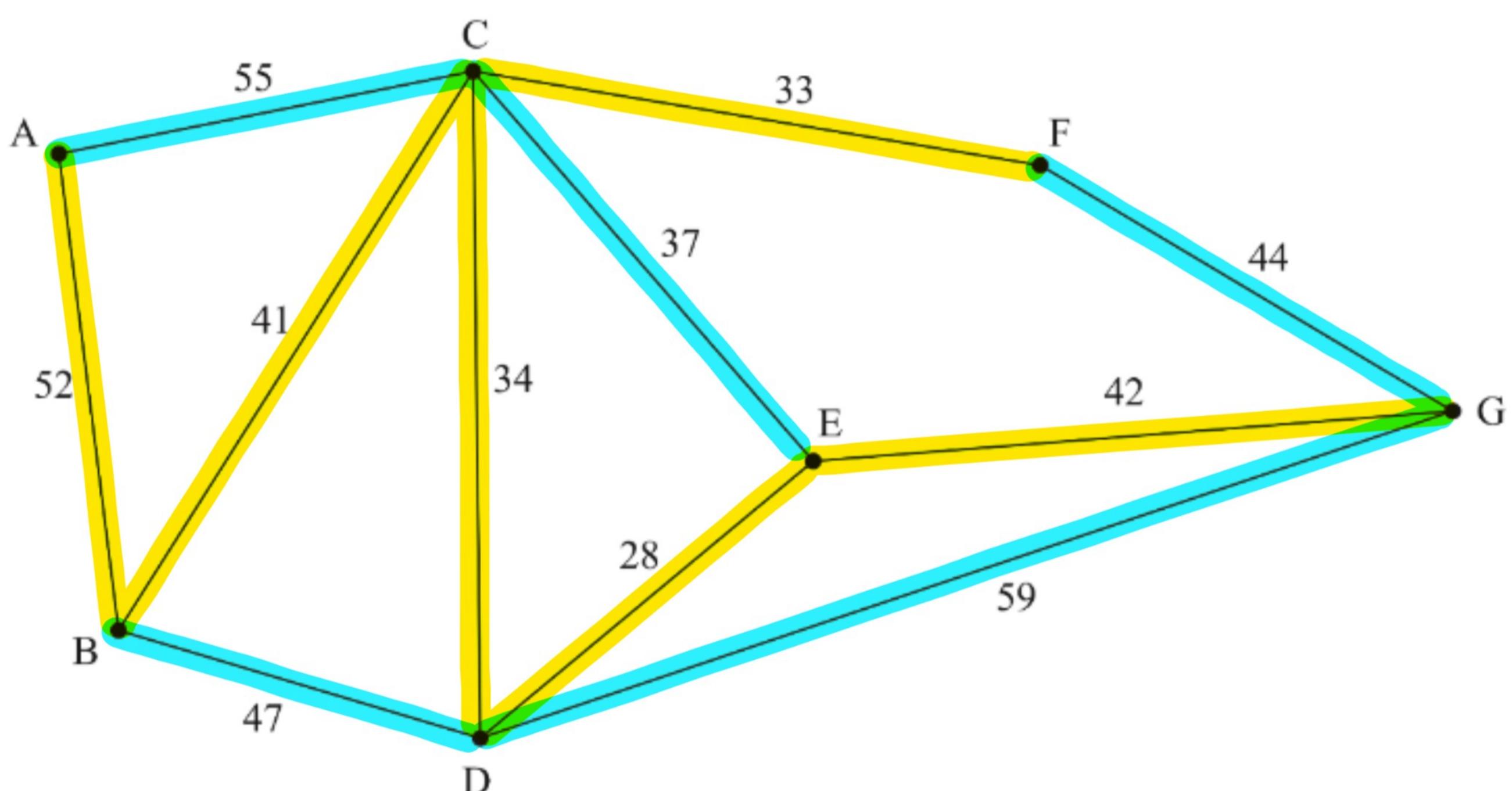
28 33 34 37 41 44 42 47 52 55 59

28 33 34 37 41 42 44 47 52 55 59

28 33 34 37 41 42 44 47 52 55 59

$\therefore$  SORT COMPLETE

(b)



Question 1 continued

DE (28)

CF (33)

CD (34)

CE (37) REJECT

BC (41)

EG (42)

FG (44) REJECT

BD (47) REJECT

AB (52)

AC (55) REJECT

DG (59) REJECT

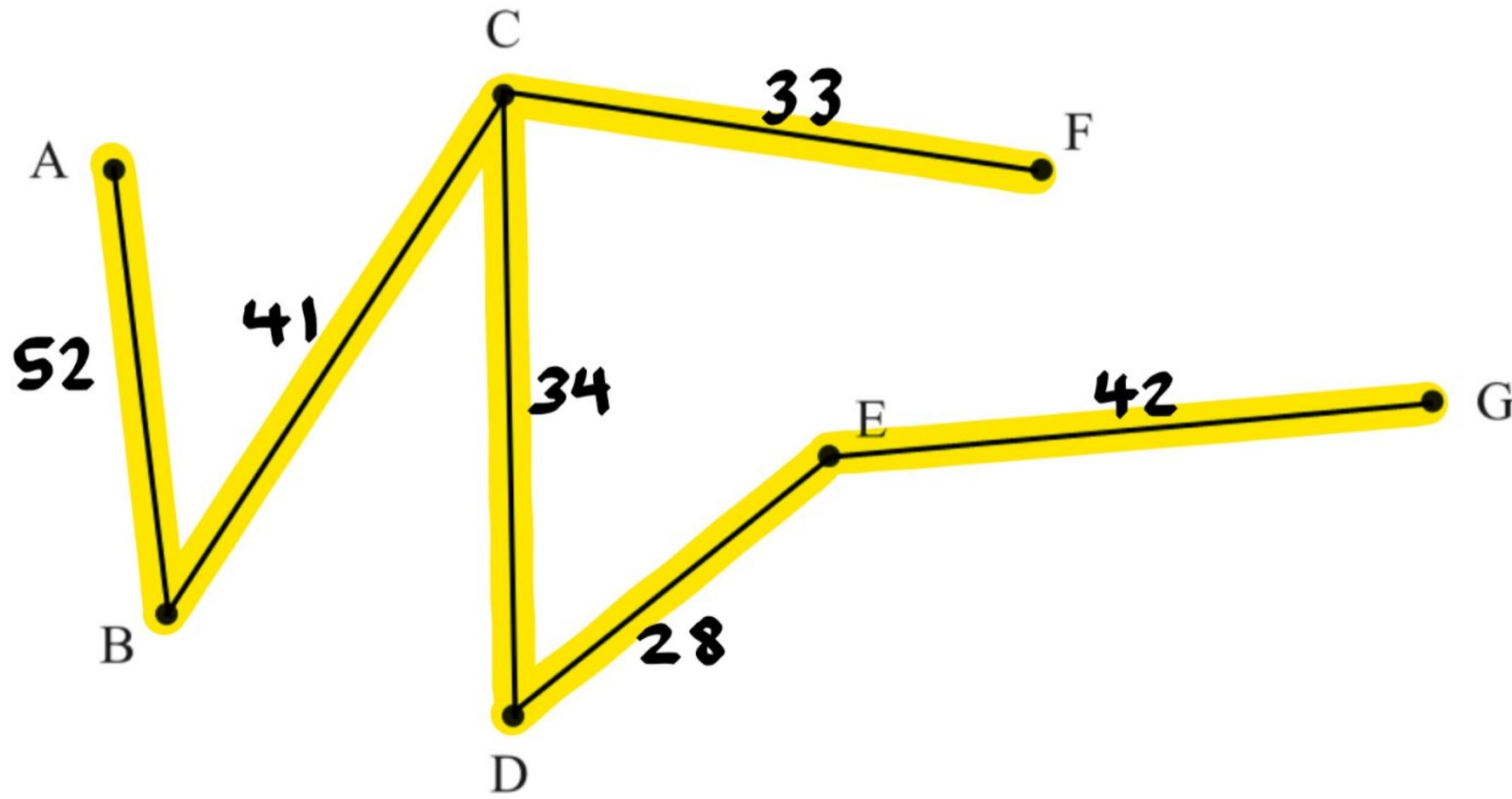


Diagram 1

WEIGHT = 230

(Total for Question 1 is 9 marks)



P 6 8 7 9 2 A 0 3 1 2

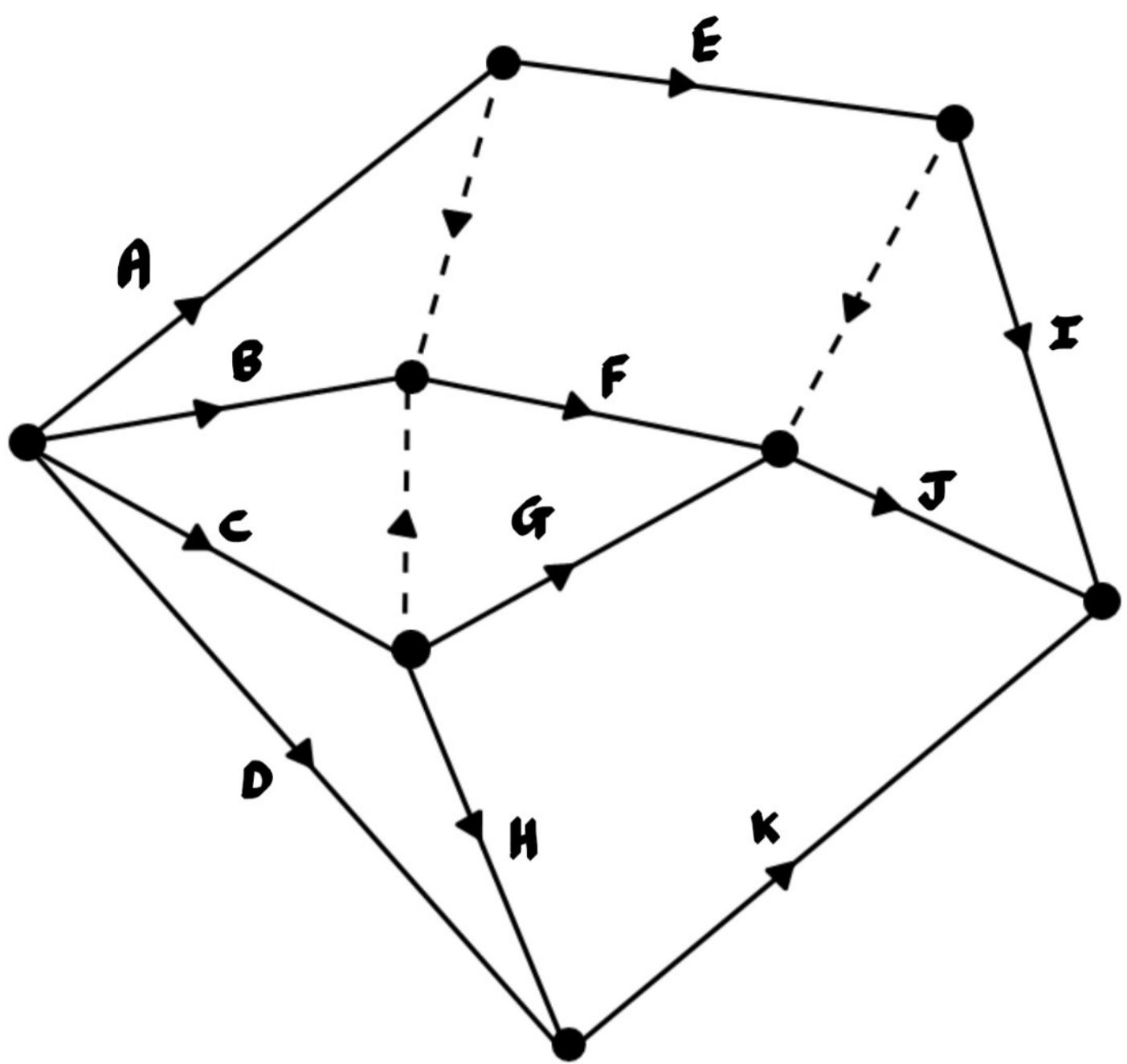
2.

Activity	Immediately preceding activities
A	—
B	—
C	—
D	—
E	A
F	A, B, C
G	C
H	C
I	E
J	E, F, G
K	D, H

- (a) Draw the activity network described in the precedence table above, using activity on arc. Your activity network must contain the minimum number of dummies only. (5)
- (b) Explain why it is necessary to draw a dummy from the end of activity A. (1)
- Every activity shown in the precedence table has the same duration.
- (c) State which activity cannot be critical, justifying your answer. (2)

**(Total for Question 2 is 8 marks)**

2.



- (b) ACTIVITY E DEPENDS ON JUST ACTIVITY A BUT ACTIVITY F DEPENDS ON ACTIVITIES A, B AND C
- (c) ACTIVITY D :: ACTIVITIES C AND H CANNOT BE FINISHED BEFORE ACTIVITY D FINISHES
- ∴ ACTIVITY D MUST HAVE FLOAT AND CANNOT BE CRITICAL



3.

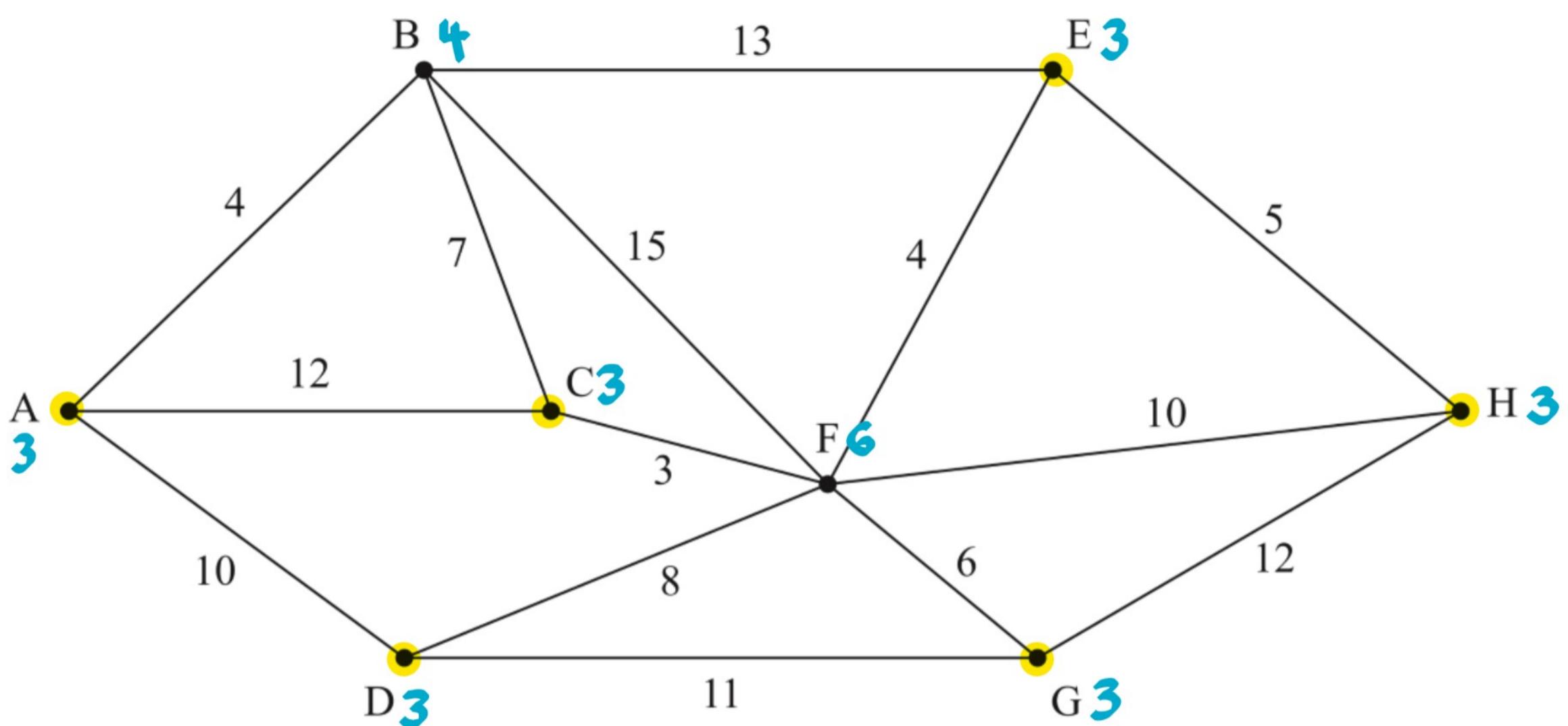


Figure 2

[The total weight of the network is 120]

- (a) Explain what is meant by the term “path”. (2)
- (b) State, with a reason, whether the network in Figure 2 is Eulerian, semi-Eulerian or neither. (1)

Figure 2 represents a network of cycle tracks between eight villages, A, B, C, D, E, F, G and H. The number on each arc represents the length, in km, of the corresponding track. Samira lives in village A, and wishes to visit her friend, Daisy, who lives in village H.

- (c) Use Dijkstra's algorithm to find the shortest path that Samira can take. (5)

An extra cycle track of length 9 km is to be added to the network. It will either go directly between C and D or directly between E and G.

Daisy plans to cycle along every track in the new network, starting and finishing at H.

Given that the addition of either track CD or track EG will not affect the final values obtained in (c),

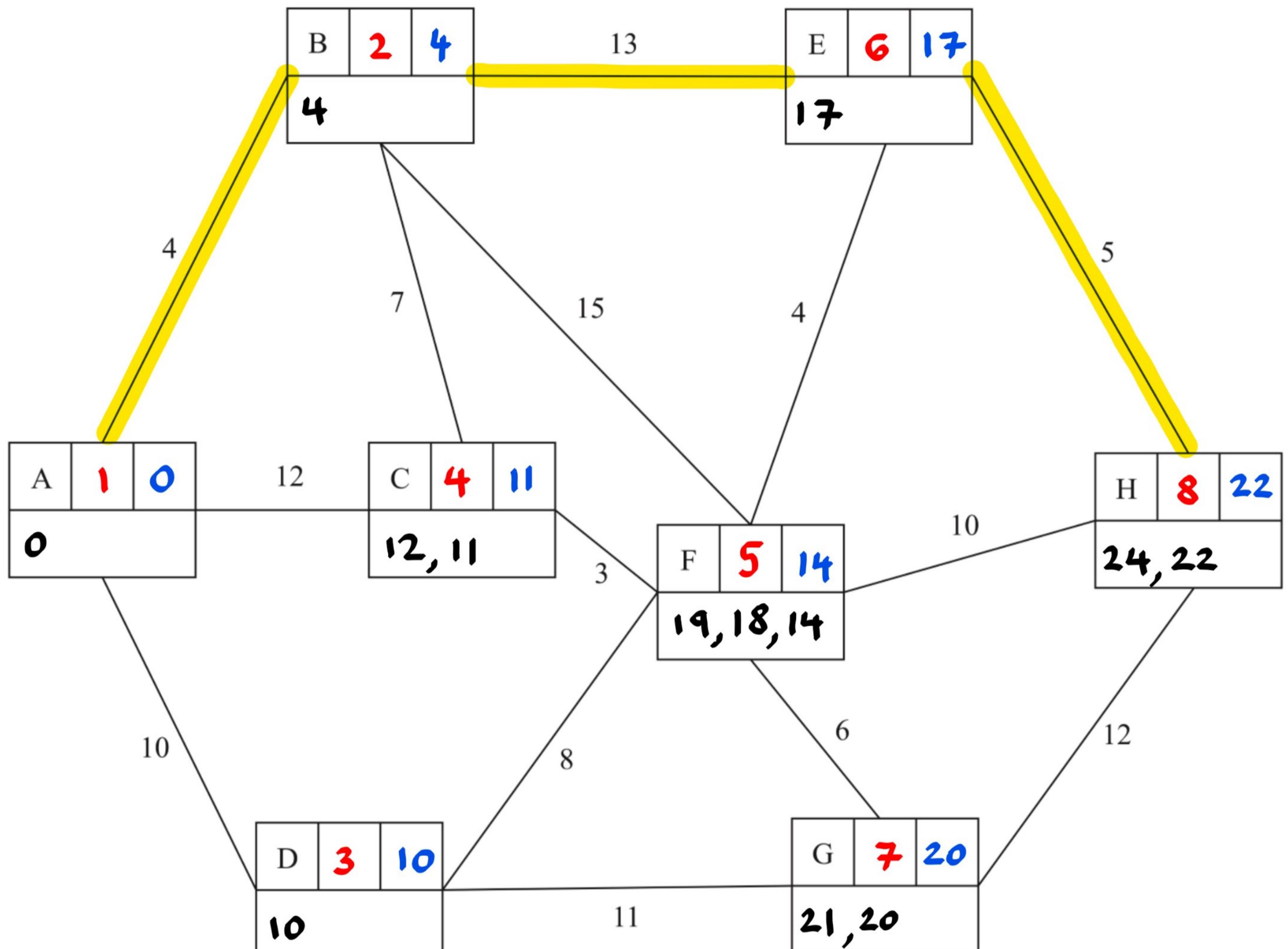
- (d) use a suitable algorithm to find out which of the two possible extra tracks will give Daisy the shortest route, making your method and working clear. You must
- state which tracks Daisy will repeat in her route
  - state the total length of her route
- (6)

(Total for Question 3 is 14 marks)

3.

- (a) PATH = FINITE SEQUENCE OF EDGES SUCH THAT THE END VERTEX OF ONE EDGE IN THE SEQUENCE IS THE START VERTEX OF THE NEXT, AND IN WHICH NO VERTEX APPEARS MORE THAN ONCE
- (b) NEITHER EULARIAN OR SEMI-EULERIAN :: IT HAS SIX ODD VERTICES

(c)



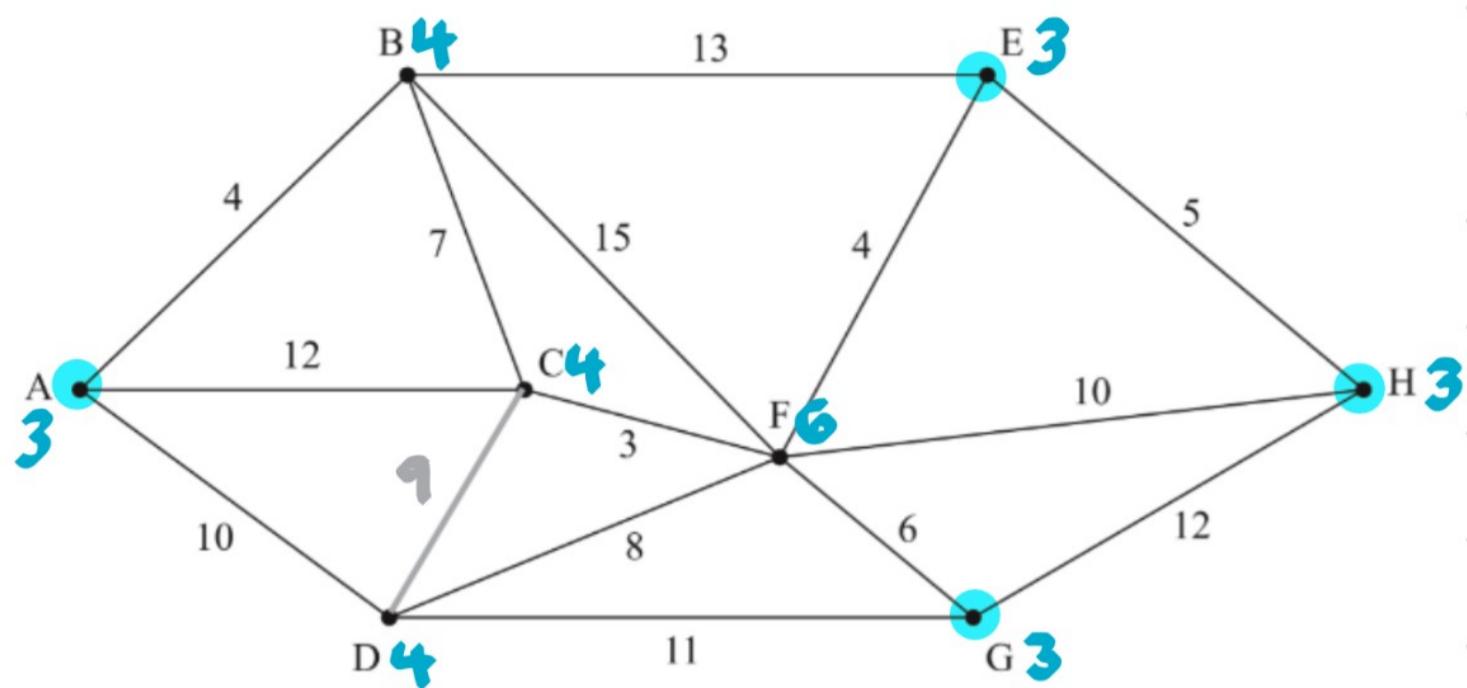
Key:

Vertex	Order of labelling	Final value
Working values		

Shortest path **ABEH**

Question 3 continued

WITH CD INCLUDED:



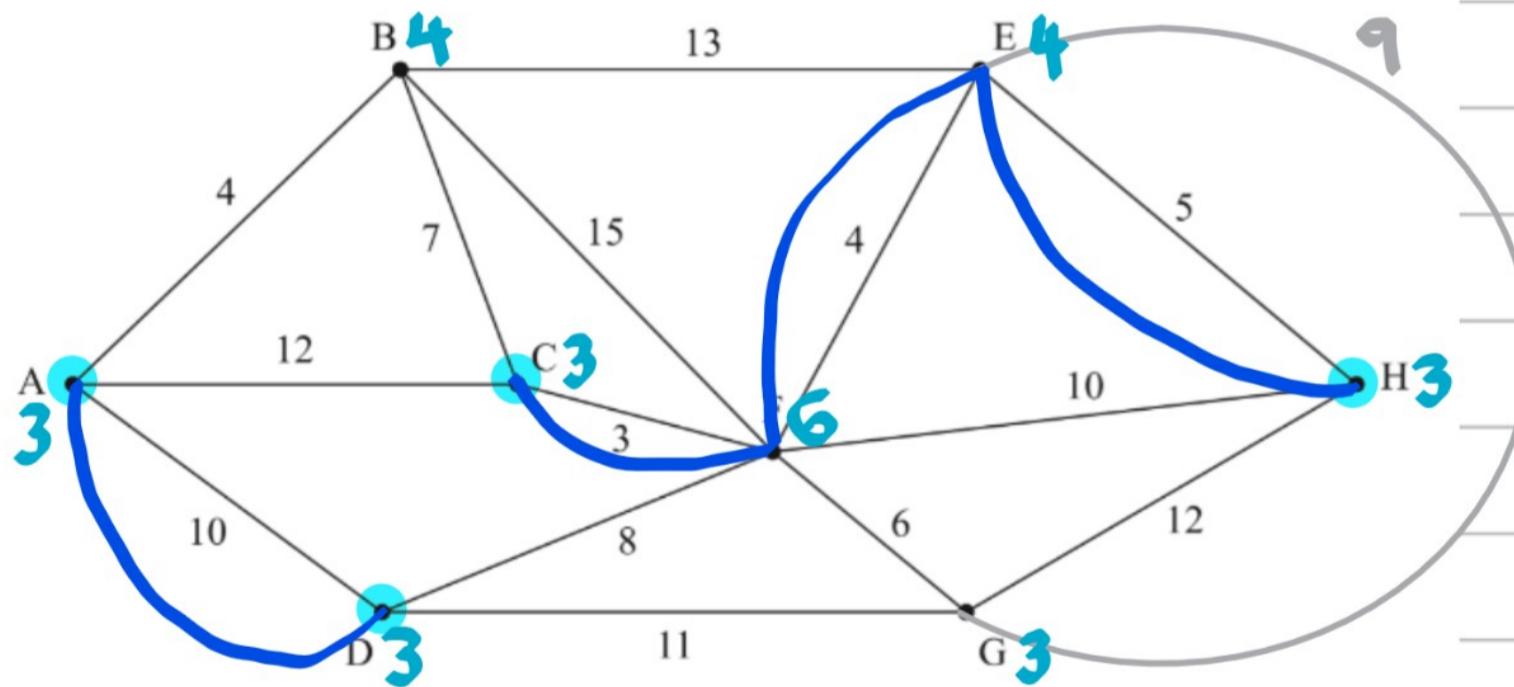
ODD VERTICES: A, E, G, H

$$\text{PAIRINGS: } AE + GH = 17 + 12 = 29$$

$$AG + EH = 20 + 5 = 25$$

$$AH + EG = 22 + 10 = 32$$

WITH FG INCLUDED:



ODD VERTICES: A, C, D, H

$$\text{PAIRINGS: } AC + DH = 11 + 17 = 28$$

$$AD + CH = 10 + 12 = 22 \leftarrow$$

$$AH + CD = 22 + 11 = 33$$

$\therefore$  REPEAT: AD, CF, FE, EH



Question 3 continued

TRACK EG

$$\text{LENGTH} = 120 + 9 + 22 = 151 \text{ Km}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



4.

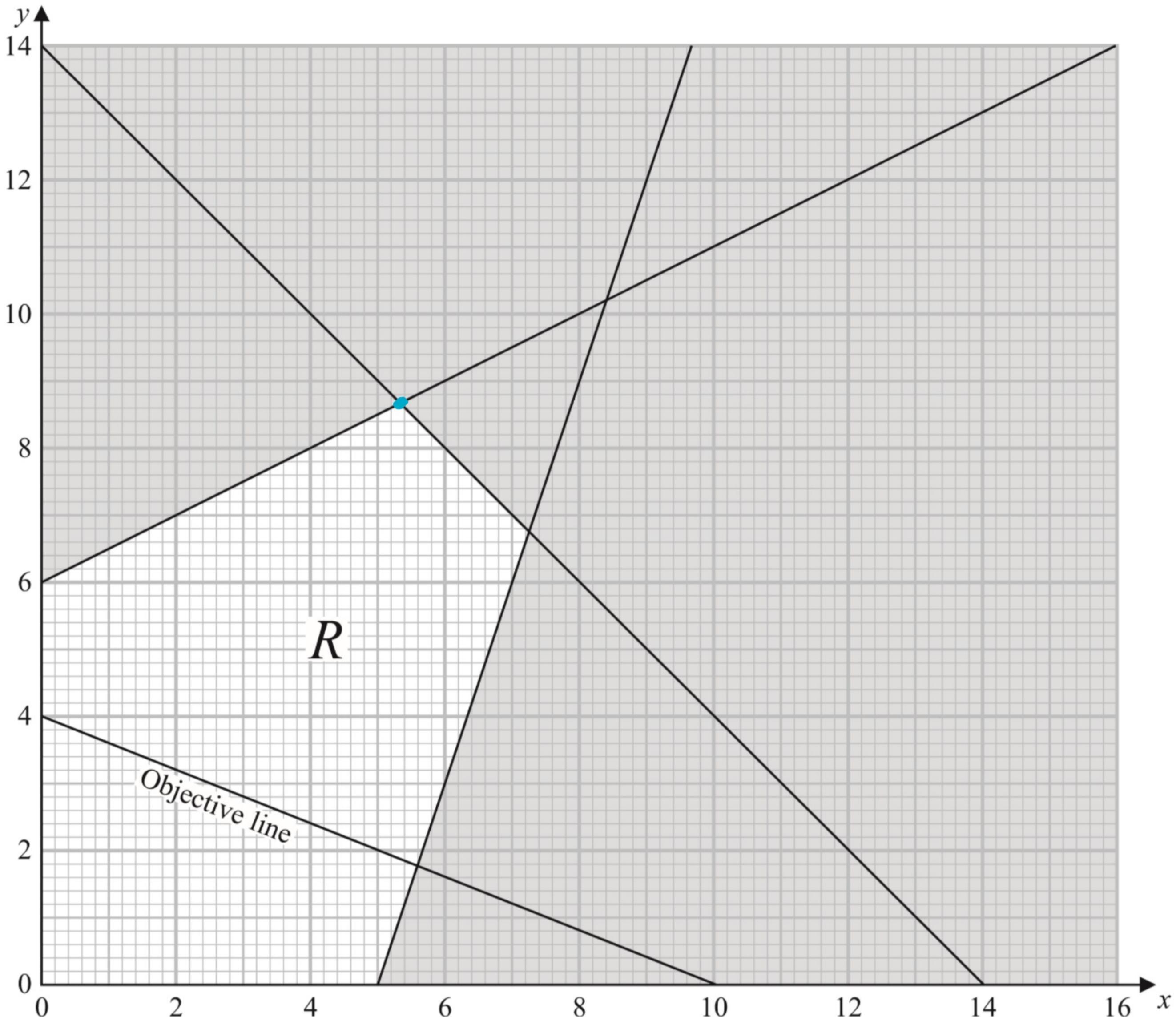
**Figure 3**

Figure 3 shows the constraints of a maximisation linear programming problem in  $x$  and  $y$ , where  $x \geq 0$  and  $y \geq 0$ . The unshaded area, including its boundaries, forms the feasible region,  $R$ . An objective line has been drawn and labelled on the graph.

- (a) List the constraints as simplified inequalities with integer coefficients. (3)

The optimal value of the objective function is 216

- (b) (i) Calculate the exact coordinates of the optimal vertex.  
(ii) Hence derive the objective function. (5)

Given that  $x$  represents the number of small flower pots and  $y$  represents the number of large flower pots supplied to a customer,

(c) deduce the optimal solution to the problem.

(1)

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**(Total for Question 4 is 9 marks)**

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**TOTAL FOR DECISION MATHEMATICS 1 IS 40 MARKS**

**END**

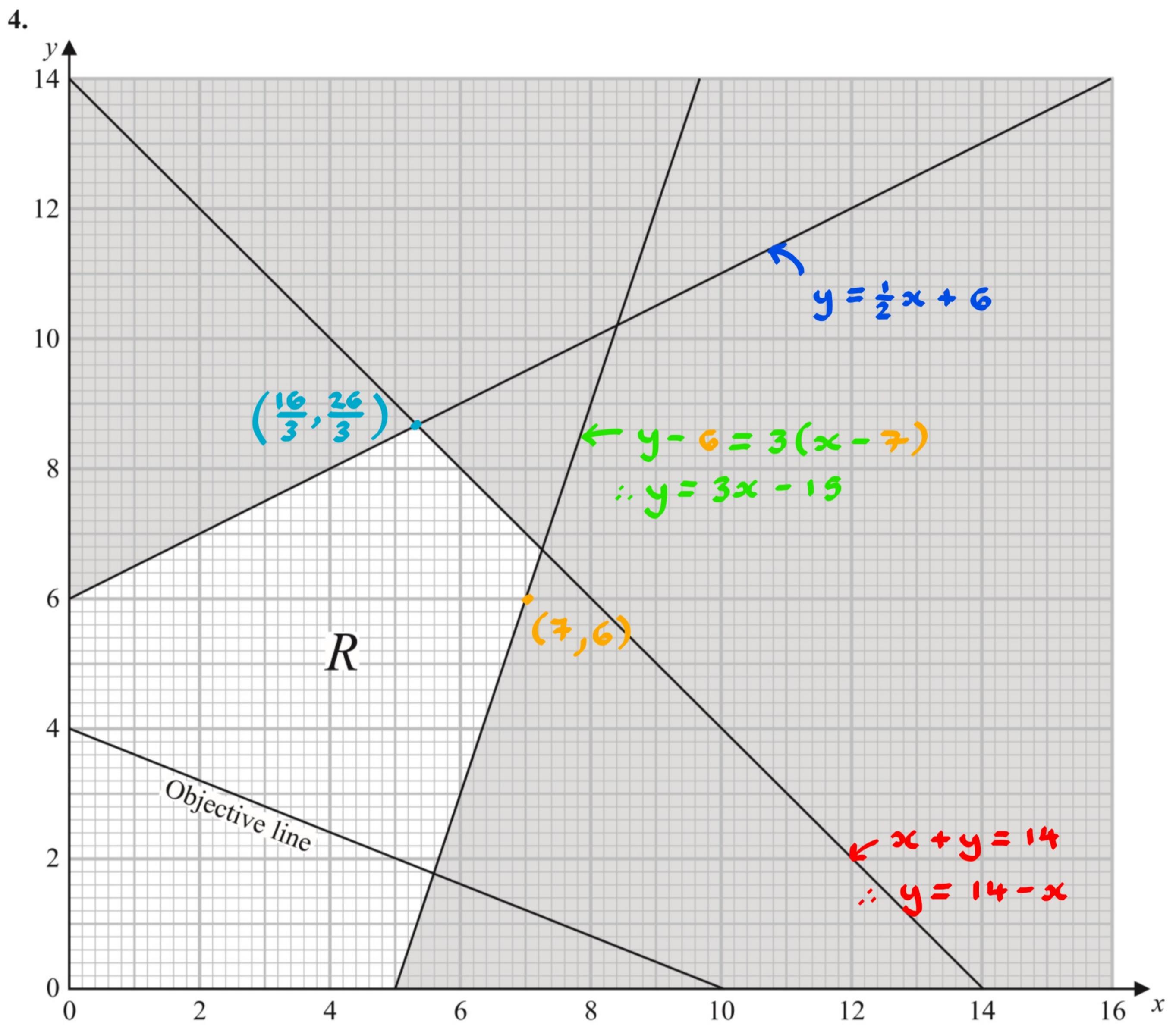


Figure 3

(a)  $y \leq 14 - x$

$y \geq 3x - 15$

$y \leq \frac{1}{2}x + 6 \quad \therefore 2y \leq x + 6$

$x, y \geq 0$

(b)(i)  $\frac{1}{2}x + 6 = 14 - x$

$x + 12 = 28 - 2x$

Question 4 continued

$$3x = 16$$

$$x = \frac{16}{3}$$

$$y = 14 - \frac{16}{3} = \frac{26}{3}$$

∴ OPTIMAL VERTEX IS  $(\frac{16}{3}, \frac{26}{3}) \equiv (5\frac{1}{3}, 8\frac{2}{3})$

(ii)  $P = k(10y + 4x)$

so:  $k(10y + 4x) = 216$

$$k(10 \times \frac{26}{3} + 4 \times \frac{16}{3}) = 216$$

∴  $k = 2$

∴  $P = 8x + 20y$

(c) 6 SMALL FLOWER POTS AND 8 LARGE FLOWER POTS

