

9FM0/3C: Further Mechanics 1 (replaced paper) mark scheme – Summer 2019

Question	Scheme	Marks	AOs
2(a)	$F_{\max} = \frac{1}{4}mg \cos \alpha = \frac{1}{5}mg$	B1	1.2
	$mg \sin \alpha = \frac{3}{5}mg > \frac{1}{5}mg \Rightarrow$ slides down	B1	2.2a
		(2)	
	Using work-energy principle to solve the problem	M1	3.4
	$\frac{1}{2}m \times (7^2 - V^2) = \frac{1}{5}mg \times 2 \times \frac{25}{8}$	A1	1.1b
	OR: $mg \times \frac{25}{8} \times \frac{3}{5} - \frac{1}{2}mV^2 = \frac{1}{5}mg \times \frac{25}{8}$	A1	1.1b
	$V = 4.9$ or 4.95	A1	1.1b
		(4)	
(c)	e.g. Include air resistance in the model.	B1	3.5c
		(1)	
(7 marks)			
Notes:			
(a) B1: Correct expression for max friction B1: Correct deduction from comparing weight component with F_{\max}			
(b) M1: Using the work-energy principle with correct no. of terms (either start to finish or descent only) A1: Correct equation, condone 1 error A1: Correct equation A1: 4.9 or 4.95 (m)			
(c) B1: Other refinements e.g. allow for spin of box, dimensions of box, more accurate value of g			

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Question	Scheme	Marks	AOs
3	Use impulse-momentum principle	M1	3.1a
	$\lambda(2\mathbf{i} - \mathbf{j}) = 0.5(\mathbf{v} - (-\mathbf{i} + 2\mathbf{j}))$	A1	1.1b
	$\mathbf{v} = (4\lambda - 1)\mathbf{i} + (2 - 2\lambda)\mathbf{j}$	A1	1.1b
	Use of change in KE to set up quadratic equation in λ only.	M1	2.1
	$12 = \frac{1}{2} \times \frac{1}{2} \{ (4\lambda - 1)^2 + (2 - 2\lambda)^2 - ((-1)^2 + 2^2) \}$	A1ft	1.1b
	Simplifying to $5\lambda^2 - 4\lambda - 12 = 0$ and solving	M1	1.1b
	$\mathbf{I} = 4\mathbf{i} - 2\mathbf{j}$	A1	2.2a
		(7)	
(7 marks)			
Notes: Allow column vectors throughout			
<p>M1: Allow $\mathbf{I} = \dots$ but must be a <i>difference</i> in momenta and dimensionally correct</p> <p>A1: For LHS (This may be awarded later)</p> <p>A1: For RHS</p> <p>M1: All terms present but allow difference reversed</p> <p>A1ft: Follow through their \mathbf{v}</p> <p>M1: Attempt to solve a 3 term quadratic</p> <p>A1: $4\mathbf{i} - 2\mathbf{j}$ only</p>			

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Question	Scheme	Marks	AOs
4(a)	Using CLM	M1	3.1b
	$2mu = 2mv_A + 3mv_B$	A1	1.1b
	Using NIL	M1	3.4
	$eu = -v_A + v_B$	A1	1.1b
	Overall strategy for setting up two equations and solving for v_A or v_B	M1	3.1b
	Speed of A is $\frac{u(3e-2)}{5}$	A1	1.1b
	Speed of B is $\frac{2u(1+e)}{5}$	A1	1.1b
		(7)	
(b)	Direction of motion of A is reversed by the collision, since $\frac{u(3e-2)}{5}$ is positive when $e > \frac{2}{3}$	B1	2.4
		(1)	
(c)	Speed of $A = \frac{u}{10}$ Speed of $B = \frac{11u}{15}$	B1ft	1.1b
	Calculation of KE loss with all terms; condone 'increase'	M1	2.1
	$= \frac{1}{2} \times 2mu^2 - \left\{ \frac{1}{2} \times 2m \left(\frac{u}{10} \right)^2 + \frac{1}{2} \times 3m \left(\frac{11u}{15} \right)^2 \right\}$	A1ft	1.1b
	$= \frac{11mu^2}{60}$	A1	1.1b
		(4)	
(12 marks)			
Notes:			
(a) M1: Correct no.of appropriate terms, condone sign errors A1: Correct equation M1: Need e on the correct side of the equation A1: Correct equation M1: Solving for either A1: Correct speed of A A1: Correct speed of B			
(b) B1: Correct direction with appropriate justification			

(c)

B1ft: Follow through their answers for (a)

M1: All terms but condone a negative loss

A1ft: Follow their speeds

A1: cao

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Question	Scheme	Marks	AOs
5(a)	CLM parallel to line of centres (loc)	M1	3.1a
	$-4mv_P + 5mv_Q = 4mu \cos \alpha - 5mu \cos \alpha$ $-4v_P + 5v_Q = -u \cos \alpha$	A1	1.1b
	Correct use of NIL	M1	3.4
	$v_P + v_Q = 2ue \cos \alpha$	A1	1.1b
	Solve for v_Q	M1	1.1b
	$v_Q = \frac{(8e-1)u \cos \alpha}{9}$	A1	1.1b
	Velocity component of Q perp to loc = $u \sin \alpha$	B1	3.4
	$\tan \theta = \frac{u \sin \alpha}{v_Q}$	M1	3.1a
	$\tan \theta = \frac{u \sin \alpha}{\frac{(8e-1)u \cos \alpha}{9}}$	M1	1.1b
	$\tan \theta = \frac{9 \tan \alpha}{8e-1} *$	A1*	2.1
(b) (i)		(10)	
	Perp to loc $\Rightarrow v_Q = 0 \Rightarrow 8e-1=0 \Rightarrow e = \frac{1}{8}$	B1	2.2a
	$v_P = \frac{1}{4}u \cos \alpha$	B1	1.1b
	$\tan \phi = \frac{u \sin \alpha}{v_P} = \frac{u \sin \alpha}{\frac{1}{4}u \cos \alpha} = 4 \tan \alpha = 4$	M1	3.1a
	$\phi = \tan^{-1} 4 = 76^\circ$ or better (1.3°) to the line of centres oe	A1	1.1b
		(4)	
	Impulse between spheres acts horizontally i.e. parallel to the plane \Rightarrow momentum conserved horizontally	B1	2.4
		(1)	
(15 marks)			
Notes:			
(a)			
M1: Need all four terms			

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A1: Correct unsimplified equation

M1: e must be on the correct side of the equation

A1: Correct unsimplified equation

M1: Solve for v_Q

A1: Correct unsimplified equation

B1: Use the model to find the velocity component perpendicular to loc

M1: Overall strategy to find $\tan \theta$

M1: Sub for v_Q and simplify

A1*: Given answer

(b)(i)

B1: Clear explanation. May use $\theta = 90 \Rightarrow 8e - 1 = 0 \Rightarrow e = \frac{1}{8}$

(b)(ii)

B1: Use $v_Q = 0$ to find v_P

M1: Complete method to solve the problem and find the angle

A1: Answers in degrees (76°) or rads (1.3) or better, are acceptable.

(c)

B1: Clear explanation

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Question	Scheme	Marks	AOs
6(a)	Overall strategy to set up an equation in one unknown using equilibrium condition and resolving vertically: $2T \times \frac{4}{5} = 4mg$	M1	3.1a
	$T = \frac{5mg}{2}$	A1	1.1b
	Use of Hooke's Law	M1	3.1a
	$\frac{5mg}{2} = \frac{5mg}{3} \left(5a - \frac{1}{2}l\right)$ OR $\frac{5mg}{3} \frac{(10a-l)}{l}$	A1	1.1b
	$l = 4a^*$	A1*	1.1b
		(5)	
(b)	Max speed is at equilibrium position	B1	3.1a
	Use of EPE = $\frac{\lambda x^2}{2l}$	M1	3.1a
	Use of conservation of energy principle	M1	3.1a
	$\frac{5mg}{3 \times 8a} \{(6a)^2 - (2a)^2\} = 4mg \times 4a - \frac{1}{2} 4mv^2$	A1	1.1b
		A1	1.1b
	$v = \sqrt{\frac{14ag}{3}}$	A1	1.1b
		(6)	

(11 marks)

Notes:

(a)

M1: Correct no. of terms with T resolved and correct equation in T only

A1: Correct tension

M1: Use of Hooke's Law

A1: Correct unsimplified equation

A1*: Given answer

(b)

B1: Use of max speed at equilm to solve the problem

M1: Use of EPE formula

M1: Use of Conservation of energy to solve the problem

A1: Correct unsimplified equation with one error

A1: Correct unsimplified equation

A1: cao oe

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Question	Scheme	Marks	AOs
7(a)	At A_1 : Horiz component = $14\cos\alpha$	B1	3.4
	At A_1 : Vert component = $\frac{1}{2} \cdot 14\sin\alpha$	B1	3.4
	$\tan\beta = \frac{\text{vert component}}{\text{horiz component}} \left(= \frac{1}{2} \tan\alpha = \frac{3}{8} \right)$	M1	3.1b
	$\beta = 20.6^\circ$ or 0.359 rad (or better)	A1	1.1b
		(4)	
(b)	Since no air resistance, motion symmetrical so vertical component down at A_1 is equal to vertical component up at O ,	B1	2.4
		(1)	
(c)	$(\uparrow): -14\sin\alpha = 14\sin\alpha - gt_1$	M1	3.4
	$t_1 = \frac{2 \times 14\sin\alpha}{g}$	A1	1.1b
	$t_2 = \frac{2 \times 7\sin\alpha}{g}$	A1	1.1b
	Total time = 2.6 or 2.57 (s)	A1	1.1b
		(4)	
(d)	At A_n : Horiz component = $14\cos\alpha$	B1	3.4
	At A_n : Vert component = $\left(\frac{1}{2}\right)^n 14\sin\alpha$	B1	3.4
	$\tan\phi = \frac{3}{2^{n+2}}$ oe	B1	3.1b
		(3)	
(e)	Ball continues to bounce with the size of the angle to the ground decreasing	B1	3.2a
		(1)	
(f)	After hitting the ground at A_1 , the ball moves along the ground at a constant speed of 11.2 m s^{-1} .	B1	2.4
		B1	2.4
		(2)	
(15 marks)			
Notes:			
(a)			
B1: Using NIL as a model to obtain the horiz component at A_1			

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B1: Using NIL as a model to obtain the vert component at A_1

M1: Using the components found above and tan to solve the problem – allow reciprocal for this mark

A1: Accept degrees or radians

(b)

B1: No air resistance means motion is symmetrical

(c)

M1: Using the model and vert motion to find the time from O to A_1

A1: $\sin \alpha$ does not need to be substituted

A1: $\sin \alpha$ does not need to be substituted

A1: Either 2 or 3 sf answers only

(d)

B1: Using NIL as the model to obtain the horiz component at A_n

B1: Using NIL to obtain the vert component at A_n

B1: Solving the problem to produce any equivalent form

(e)

B1: A clear explanation

(f)

B1: Clear description

B1: Constant speed and $11.2 \text{ (m s}^{-1}\text{)}$