## Paper 2 Option **H**

Further Mechanics 1 Mark Scheme (Section A)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1(a)** | Using the model and $v^2 = u^2 + 2as$ to find $v$ | M1 | 3.4 |
| | $v^2 = 2as = 2g \times 2.4 = 4.8g \implies v = \sqrt{(4.8g)}$ | A1 | 1.1b |
| | Using the model and $v^2 = u^2 + 2as$ to find $u$ | M1 | 3.4 |
| | $0^2 = u^2 - 2g \times 0.6 \implies u = \sqrt{(1.2g)}$ | A1 | 1.1b |
| | Using the correct strategy to solve the problem by finding the sep. speed and app. speed and applying NLR | M1 | 3.1b |
| | $e = \sqrt{(1.2g)} / \sqrt{(4.8g)} = 0.5$ * | A1* | 1.1b |
| | | **(6)** | |
| **(b)** | Using the model and $e = $ sep. speed / app. speed, $v = 0.5\sqrt{(1.2g)}$ | M1 | 3.4 |
| | Using the model and $v^2 = u^2 + 2as$ | M1 | 3.4 |
| | $0^2 = 0.25\,(1.2g) - 2gh \implies h = 0.15$ (m) | A1 | 1.1b |
| | | **(3)** | |
| **(c)** | Ball continues to bounce with the height of each bounce being a quarter of the previous one | B1 | 2.2b |
| | | **(1)** | |
| | | **(10 marks)** | |

| Notes: |
|---|

**(a)**
**M1:** For a complete method to find $v$
**A1:** For a correct value (may be numerical)
**M1:** For a complete method to find $u$
**A1:** For a correct value (may be numerical)
**M1:** For finding both $v$ and $u$ and use of Newton's Law of Restitution
**A1*:** For the given answer

**(b)**
**M1:** For use of Newton's Law of Restitution to find rebound speed
**M1:** For a complete method to find $h$
**A1:** For 0.15 (m) oe

**(c)**
**B1:** For a clear description including reference to a quarter

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 2(a) | Energy Loss = KE Loss – PE Gain | M1 | 3.3 |
| | $= \frac{1}{2} \times 0.5 \times 25^2 - 0.5\,g \times 20$ | A1 | 1.1b |
| | $= 58.25 = 58$ (J) or 58.3 (J) | A1 | 1.1b |
| | | **(3)** | |
| (b) | Using work-energy principle, $20\,R = 58.25$ | M1 | 3.3 |
| | $R = 2.9125 = 2.9$ or 2.91 | A1ft | 1.1b |
| | | **(2)** | |
| (c) | Make resistance variable (dependent on speed) | B1 | 3.5c |
| | | **(1)** | |
| | | **(6 marks)** | |

Notes:

**(a)**
**M1:** For a difference in KE and PE
**A1:** For a correct expression
**A1:** For either 58 (2sf) or 58.3(3sf)

**(b)**
**M1:** For use of work-energy principle
**A1ft:** For either 2.9 (2sf) or 2.91 (3sf) follow through on their answer to (a)

**(c)**
**B1:** For variable resistance oe

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | Force = Resistance  (since no acceleration) =  30 | B1 | 3.1b |
| | Power  = Force $\times$ Speed = 30 $\times$ 4 | M1 | 1.1b |
| | = 120 W | A1 **ft** | 1.1b |
| | | **(3)** | |
| **(b)** | Resolving parallel to the slope | M1 | 3.1b |
| | $F - 60g\sin\alpha - 30 = 0$ | A1 | 1.1b |
| | $F = 70$ | A1 | 1.1b |
| | Power = Force $\times$ Speed = 70 $\times$ 3 | M1 | 1.1b |
| | = 210 W | A1 **ft** | 1.1b |
| | | **(5)** | |
| | | **(8 marks)** | |

Notes:

**(a)**
**B1:** For force = 30 seen
**M1:** For use of $P = Fv$
**A1ft:** For 120 (W), follow through on their '30'

**(b)**
**M1:** For resolving parallel to the slope with correct no. of terms and 60g resolved
**A1:** For a correct equation
**A1:** For $F = 70$
**M1:** For use of $P = Fv$
**A1ft:** For 210 (W), follow through on their '70'

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | Use of conservation of momentum | M1 | 3.1a |
| | $3mu - 2mu = 3mv + mw$ | A1 | 1.1b |
| | Use of NLR | M1 | 3.1a |
| | $3ue = -v + w$ | A1 | 1.1b |
| | Using a correct strategy to solve the problem by setting up two equations (need both) in $u$ and $v$ and solving for $v$ | M1 | 3.1b |
| | $v = \frac{u}{4}(1 - 3e)$ | A1 | 1.1b |
| | | **(6)** | |
| **(b)** | $\frac{u}{4}(1 - 3e) < 0$ | M1 | 3.1b |
| | $\frac{1}{3} < e \leq 1$ | A1 | 1.1b |
| | | **(2)** | |
| **(c)** | Solving for $w$ | M1 | 2.1 |
| | $w = \frac{u}{4}(1 + 9e) *$ | A1 * | 1.1b |
| | | **(2)** | |
| **(d)** | Substitute $e = \frac{5}{9}$ | M1 | 1.1b |
| | $v = -\frac{u}{6}, w = \frac{3u}{2}$ | A1 | 1.1b |
| | Use NLR for impact with wall, $x = fw$ | M1 | 1.1b |
| | Further collision if $x > -v$ | M1 | 3.4 |
| | $f\frac{3u}{2} > \frac{u}{6}$ | A1 | 1.1b |
| | $1 \geq f > \frac{1}{9}$ | A1 | 1.1b |
| | | **(6)** | |
| | | **(16 marks)** | |

Notes:

**(a)**
**M1:** For use of CLM, with correct no. of terms, condone sign errors
**A1:** For a correct equation
**M1:** For use of Newton's Law of Restitution, with $e$ on the correct side
**A1:** For a correct equation
**M1:** For setting up *two* equations and solving their equations for $v$
**A1:** For a correct expression for $v$

**(b)**
**M1:** For use of an appropriate inequality
**A1:** For a complete range of values of $e$

**(c)**
**M1:** For solving their equations for $w$
**A1:** For the given answer

| Question 4 notes continued: |
|---|
| **(d)** |
| **M1:**    For substituting $e = \dfrac{5}{9}$ into their $v$ and $w$ |
| **A1:**    For correct expressions for $v$ and $w$ |
| **M1:**    For use of Newton's Law of Restitution, with $e$ on the correct side |
| **M1:**    For use of appropriate inequality |
| **A1:**    For a correct inequality |
| **A1:**    For a correct range |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(a)** |  | M1<br><br>A1<br><br>A1 | 1.1b<br><br>1.1b<br><br>1.1b |
| | Path: ABECDGF | A1 | 1.1b |
| | Length: 55 (metres) | A1ft | 1.1b |
| | | **(5)** | |
| **(b)** | AB + DG = 13 + 11 = 24 ← | M1 | 1.1b |
| | A(BEC)D + B(ECD)G = 34 + 32 = 66 | A1 | 1.1b |
| | A(BECD)G + B(EC)D = 45 + 21 = 66 | A1 | 1.1b |
| | Repeat arcs: AB, DG | A1ft | 2.2a |
| | | **(4)** | |
| **(c)** | Length = 189 + 24 = 213 (metres) | B1ft | 1.1b |
| | | **(1)** | |
| **(d)** | $189 + x + 34 = 213 + 2x$ | M1 | 3.1b |
| | $x = 10$          so BG is 10 m | A1 | 1.1b |
| | | **(2)** | |
| | | **(12 marks)** | |

Notes:

**(a)**
**M1:** For a larger number replaced by a smaller one in the working values boxes at C, D, F or G
**A1:** For all values correct (and in correct order) at A, B, C and D
**A1:** For all values correct (and in correct order) at E, F & G
**A1:** For the correct path
**A1ft:** For 55 or ft their final value at F

**(b)**
**M1:** For 3 correct pairings of the four odd nodes (A,B, D & G)
**A1:** At least two pairings and totals correct
**A2:** All three pairings and totals correct
**A3ft:** Selecting their shortest pairing, and stating that these arcs should be repeated
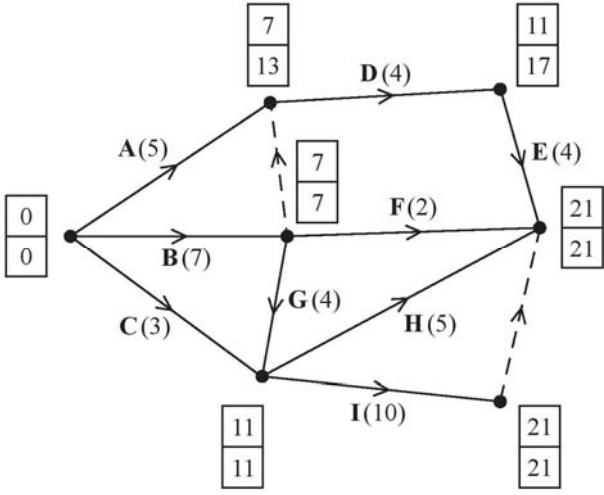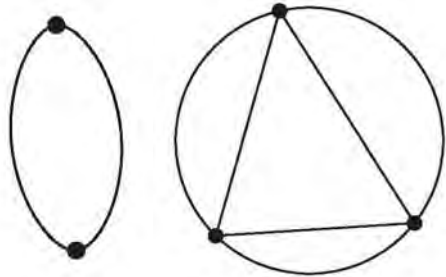
| Question **5** notes continued: |
| :--- |
| **(c)** |
| **B1ft:** For 213 or 189 + their shortest repeat |
| **M1:** For translating the information in the question in to an equation involving $x$, $2x$ and 34 |
| **A1:** For a correct equation leading to BG = 10 (m) |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **6** | Objective line drawn or at least two vertices tested | M1 | 3.1a |
| | For solving y = 4x and 8x + 7y = 560 to find the exact co-ordinate of the optimal point, must reach either x = or y = | M1 | 1.1a |
| | $x = 15\frac{5}{9}$ and $y = 62\frac{2}{9}$ | A1 | 1.1b |
| | Finding at least two points with integer co-ordinates from $(15 \pm 1, \; 63 \pm 2)$ | M1 | 1.1b |
| | Testing at least two points with integer co-ordinates | M1 | 1.1b |
| | x = 15 and y = 63 | A1 | 2.2a |
| | So the teacher should buy 15 pens and 63 pencils | A1ft | 3.2a |

**(7 marks)**

Notes:

**M1:** Selecting an appropriate mathematical process to solve the problem – either drawing an objective line with the correct gradient (or reciprocal gradient), or testing at least two vertices in C

**M1:** Solving simultaneous equations

**A1:** cao

**M1:** Recognition that outcome from this model is non-integer and integer solutions are required – testing two points with integer co-ordinates in at least one of $y \geq 4x$ and $8x + 7y \geq 560$

**M1:** Testing at least two integer solutions in $y \geq 4x$ or $8x + 7y \geq 560$ and C

**A1:** cao – deducing from tests which integer solution is both valid and optimal

**A1ft:** Interpreting solution in the context of the question – gives their integer values for x and y in the context of pens and pencils

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **7(a)(b)** |  | M1<br>A1<br>A1 | 1.1b<br>1.1b<br>1.1b |
| | | **(3)** | |
| | | M1<br>A1 | 2.1<br>1.1b |
| | The number(s) at the end of activity E indicate this project can be completed in 21 days | A1ft | 2.2a |
| | Critical activities: B, G, I | A1 | 1.1b |
| | | **(4)** | |
| | | **(7 marks)** | |

Notes:

| | |
|---|---|
| **M1:** | At least 5 activities and one dummy, one start |
| **A1:** | A,B,C,D,F,G and first dummy correct |
| **A1:** | E,H,I correct, second dummy correct and one finish |

| | |
|---|---|
| **M1:** | All boxes completed, number generally increasing L to R (condone one "rogue") |
| **A1:** | All values cao |
| **A1:** | Deduction that result in diagram indicates that project can be completed in 21 days (all boxes completed, numbers generally increasing in the direction of the arrows for the top boxes and generally decreasing in the opposite direction of the arrow for the bottom boxes) |
| **A1:** | Critical activities correct |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **8(a)** | e.g. a graph cannot contain an odd number of odd nodes <br><br> e.g. number of arcs $=\dfrac{1+3+4+4+5}{2}=8.5 \notin \mathbb{Z}$ | B1 | 2.4 |
| | | **(1)** | |
| **(b)(i)** | $\left(2^{2x}-1\right)+\left(2^{x}\right)+(x+1)+\left(2^{x+1}-3\right)+(11-x)=2(18)$ | M1 | 1.1b |
| | $2^{2x}+3\left(2^{x}\right)-28=0 \Rightarrow x=...$ | M1 | 1.1b |
| | $\left(2^{x}+7\right)\left(2^{x}-4\right)=0 \Rightarrow x=2$ | A1 | 1.1b |
| | | **(3)** | |
| **(b)(ii)** | The order of the nodes are 9, 15, 3, 4, 5 | M1 | 2.1 |
| | Therefore the graph is neither Eulerian nor semi-Eulerian as there are more than two odd nodes | A1 | 2.4 |
| | | A1 | 2.2a |
| | | **(3)** | |
| **(c)** |  | M1 | 2.5 |
| | | A1 | 2.2a |
| | | **(2)** | |
| | | **(9 marks)** | |

| Notes: |
|---|
| **(a)** <br> **B1:** Explanation referring to need for an even number of odd nodes oe |
| **(b)** <br> **M1:** Forming an equation involving the orders of the 5 odd nodes and 2(18) <br> **M1:** Simplifies to a quadratic in $2^{x}$ and attempts to solve <br> **A1:** 2 cao <br> **M1:** Construct an argument involving the order of the 5 nodes <br> **A1:** Explanation considering the number of odd nodes <br> **A1:** Deduction that therefore it is neither Eulerian nor semi-Eulerian |
| **(c)** <br> **M1:** Interprets mathematical language to construct a disconnected graph <br> **A1:** Deduce a correct graph |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **9** | Minimise $(C =) 25x + 35y$ | B1 | 3.3 |
| | Subject to:<br>$(500x + 800y \geqslant 150\,000 \Rightarrow) \; 5x + 8y \geqslant 1500$ | B1 | 3.3 |
| | $\dfrac{7}{20}(x+y) \leqslant x \leqslant \dfrac{13}{20}(x+y)$ | M1<br>M1 | 3.3<br>3.3 |
| | Which simplifies to $7y \leqslant 13x$ **and** $13y \geqslant 7x$ | A1 | 1.1b |
| | $x, y \geqslant 0$ | | |

**(5 marks)**

Notes:

**B1:**  A correct objective function + minimise
**B1:**  Translate information in to a correct inequality
**M1:**  For translating the information given into the LHS inequality
**M1:**  For translating the information given in to the RHS inequality
**A1:**  Simplifying to the correct inequalities