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Candidate surname

Other names

**Pearson Edexcel  
Level 3 GCE**

Centre Number

Candidate Number

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**Monday 3 June 2019**

Morning (Time: 1 hour 30 minutes)

Paper Reference **9FM0/01**

## **Further Mathematics**

### **Advanced**

#### **Paper 1: Core Pure Mathematics 1**

**You must have:**

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations.  
Calculators must not have the facility for algebraic manipulation,  
differentiation and integration, or have retrievable mathematical  
formulae stored in them.**

### **Instructions**

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  - *there may be more space than you need.*
- You should show sufficient working to make your methods clear.  
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

### **Information**

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  - *use this as a guide as to how much time to spend on each question.*

### **Advice**

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

**Turn over** ►

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P 6 1 1 7 7 A 0 1 3 2



**Pearson**

Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(z) = z^4 + az^3 + bz^2 + cz + d$$

where  $a, b, c$  and  $d$  are real constants.

Given that  $-1 + 2i$  and  $3 - i$  are two roots of the equation  $f(z) = 0$

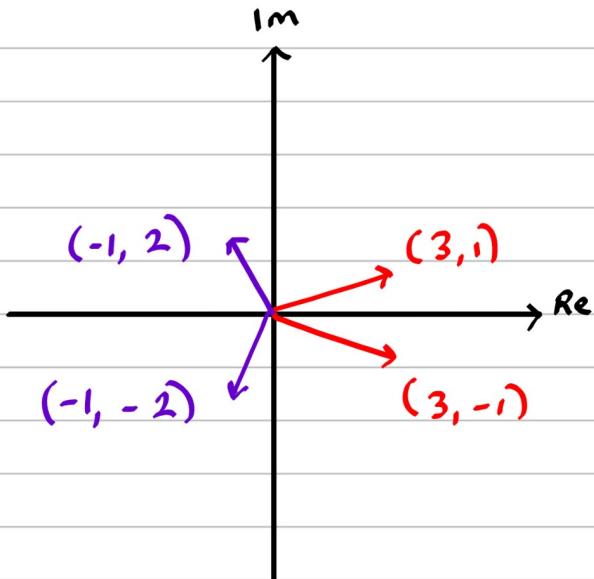
(a) show all the roots of  $f(z) = 0$  on a single Argand diagram,

(4)

(b) find the values of  $a, b, c$  and  $d$ .

(5)

(a)  $z_{1,2} = -1 \pm 2i$  and  $z_{3,4} = 3 \pm i$  are roots of  $f(z) = 0$



(b) we can use complex conjugate roots to find two quadratic factors of the polynomial of degree 4

$$\begin{aligned} z_{1,2} = -1 \pm 2i : \text{sum} &= (-1 + 2i) + (-1 - 2i) = -2 \\ \text{product} &= (-1 + 2i)(-1 - 2i) = 5 \end{aligned}$$

$\therefore$  a quadratic factor of  $f(z) = 0$  is  
 $z^2 + 2z + 5$

$$\begin{aligned} z_{3,4} = 3 \pm i : \text{sum} &= (3 + i) + (3 - i) = 6 \\ \text{product} &= (3 + i)(3 - i) = 10 \end{aligned}$$



## Question 1 continued

$\therefore$  a quadratic factor of  $f(z) = 0$  is  
 $z^2 - 6z + 10$

i.e.  $(z^2 + 2z + 5)(z^2 - 6z + 10) = z^4 + az^3 + bz^2 + cz + d$

now comparing coefficients:

$$z^3: -6 + 2 = a \therefore a = -4$$

$$z^2: 10 + 5 - 12 = b \therefore b = 3$$

$$z: 20 - 30 = c \therefore c = -10$$

$$\text{No: } 50 = d \therefore d = 50$$

$$\therefore f(z) = z^4 - 4z^3 + 3z^2 - 10z + 50$$



P 6 1 1 7 7 A 0 3 3 2

2. Show that

$$\int_0^\infty \frac{8x - 12}{(2x^2 + 3)(x + 1)} dx = \ln k$$

where  $k$  is a rational number to be found.

$$\int_0^\infty \frac{8x - 12}{(2x^2 + 3)(x + 1)} dx \equiv \int_0^\infty \left( \frac{ax + b}{2x^2 + 3} + \frac{c}{x + 1} \right) dx \quad (7)$$

$$\text{i.e. } \frac{8x - 12}{(2x^2 + 3)(x + 1)} \equiv \frac{ax + b}{2x^2 + 3} + \frac{c}{x + 1}$$

$$\text{so: } 8x - 12 \equiv (ax + b)(x + 1) + c(2x^2 + 3)$$

$$x = -1: 8(-1) - 12 = (-a + b)(0) + c(2 + 3) \\ -20 = 5c \therefore c = -4$$

$$x = 0: 8(0) - 12 = b + 3c \\ -12 = b - 12 \therefore b = 0$$

$$x = 1: 8 - 12 = (a + b)(2) + c(2 + 3) \\ -4 = 2a + 5c \\ -4 = 2a - 20 \\ 2a = 16 \therefore a = 8$$

$$\therefore \int_0^\infty \frac{8x - 12}{(2x^2 + 3)(x + 1)} dx \equiv \int_0^\infty \left( \frac{8x}{2x^2 + 3} - \frac{4}{x + 1} \right) dx$$

$$= [2\ln|2x^2 + 3| - 4\ln|x + 1|]_0^\infty$$

$$= [\ln(2x^2 + 3)^2 - \ln(x + 1)^4]_0^\infty$$

$$= \left[ \ln \left( \frac{(2x^2 + 3)^2}{(x + 1)^4} \right) \right]_0^\infty$$

$$= \lim_{t \rightarrow \infty} \left[ \ln \left( \frac{(2x^2 + 3)^2}{(x + 1)^4} \right) \right]_0^t$$

Question 2 continued

$$= \lim_{t \rightarrow \infty} \left[ \ln \left( \frac{(2t^2 + 3)^2}{(t+1)^4} \right) - \ln 9 \right]$$

$$= \lim_{t \rightarrow \infty} \left[ \ln \left( \frac{(2 + 3/t^2)^2}{(1 + 1/t)^4} \right) - \ln 9 \right]$$

$$= \ln \left( \frac{2^2}{1^4} \right) - \ln 9$$

$$= \ln 4 - \ln 9$$

$$= \ln \left( \frac{4}{9} \right)$$

$$\therefore K = \frac{4}{9}$$



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3.

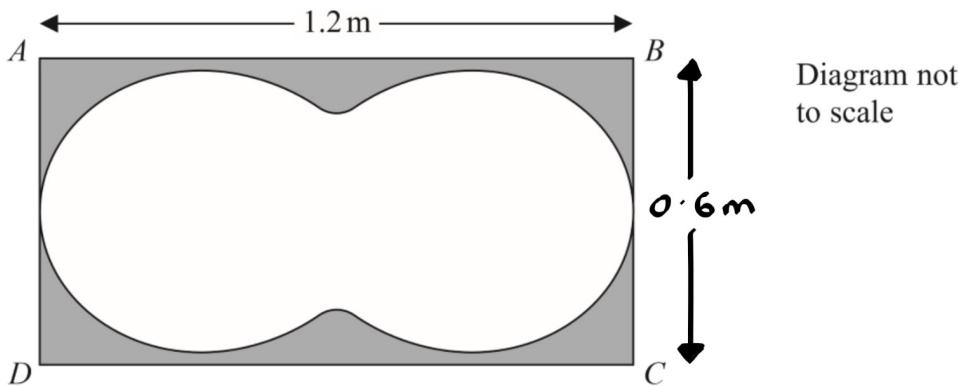


Figure 1

Figure 1 shows the design for a table top in the shape of a rectangle  $ABCD$ . The length of the table,  $AB$ , is 1.2 m. The area inside the closed curve is made of glass and the surrounding area, shown shaded in Figure 1, is made of wood.

The perimeter of the glass is modelled by the curve with polar equation

$$r = 0.4 + a \cos 2\theta \quad 0 \leq \theta < 2\pi$$

where  $a$  is a constant.

- (a) Show that  $a = 0.2$

(2)

Hence, given that  $AD = 60$  cm,

- (b) find the area of the wooden part of the table top, giving your answer in  $\text{m}^2$  to 3 significant figures.

(8)

(a)  $r = 0.4 + a \cos 2\theta ; 0 \leq \theta \leq 2\pi$

maximum value of  $r$  gives half the length of table top

$$\begin{aligned} \text{i.e. } \theta = 0, r = 0.6 : 0.6 &= 0.4 + a \cos 0 \\ 0.6 &= 0.4 + a \therefore a = 0.2 \end{aligned}$$

$$\therefore r = 0.4 + 0.2 \cos 2\theta$$

(b) area of rectangle  $= 1.2 \times 0.6 = 0.72 \text{ m}^2$

$$\text{area of polar curve} = \frac{1}{2} \int_0^{2\pi} r^2 d\theta$$

$$r^2 = (0.4 + 0.2 \cos 2\theta)^2 = 0.16 + 0.16 \cos 2\theta + 0.04 \cos^2 2\theta$$

## Question 3 continued

$$= 0.16 + 0.16 \cos 2\theta + 0.04 \times \frac{1}{2} (\cos 4\theta + 1)$$

$$= 0.18 + 0.16 \cos 2\theta + 0.02 \cos 4\theta$$

$$\text{area of polar curve} = \frac{1}{2} \int_0^{2\pi} (0.18 + 0.16 \cos 2\theta + 0.02 \cos 4\theta) d\theta$$

$$= \frac{1}{2} [0.18\theta + 0.08 \sin 2\theta + 0.005 \sin 4\theta]_0^{2\pi}$$

$$= [0.09\theta + 0.04 \sin 2\theta + 0.0025 \sin 4\theta]_0^{2\pi}$$

$$= \frac{9\pi}{50} \text{ m}^2$$

$$\text{shaded area} = 0.72 - \frac{9\pi}{50} \text{ m}^2$$

$$= 0.15451 \dots \text{m}^2$$

$$= 0.155 \text{ (3sf)} \text{ m}^2$$



4. Prove that, for  $n \in \mathbb{Z}$ ,  $n \geq 0$

$$\sum_{r=0}^n \frac{1}{(r+1)(r+2)(r+3)} = \frac{(n+a)(n+b)}{c(n+2)(n+3)}$$

where  $a$ ,  $b$  and  $c$  are integers to be found.

(5)

$$\frac{1}{(r+1)(r+2)(r+3)} = \frac{A}{r+1} + \frac{B}{r+2} + \frac{C}{r+3}$$

$$\text{i.e. } 1 \equiv A(r+2)(r+3) + B(r+1)(r+3) + C(r+1)(r+2)$$

$$\begin{aligned} r = -1: \quad 1 &\equiv A(-1+2)(-1+3) \\ 1 &\equiv A(1)(2) \\ 1 &= 2A \quad \therefore A = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} r = -2: \quad 1 &\equiv B(-2+1)(-2+3) \\ 1 &\equiv B(-1)(1) \\ 1 &= -B \quad \therefore B = -1 \end{aligned}$$

$$\begin{aligned} r = -3: \quad 1 &\equiv C(-3+1)(-3+2) \\ 1 &\equiv C(-2)(-1) \\ 1 &= 2C \quad \therefore C = \frac{1}{2} \end{aligned}$$

$$\therefore \frac{1}{(r+1)(r+2)(r+3)} = \frac{1}{2(r+1)} - \frac{1}{r+2} + \frac{1}{2(r+3)}$$

$$\text{hence } \sum_{r=0}^n \left( \frac{1}{2(r+1)} - \frac{1}{r+2} + \frac{1}{2(r+3)} \right)$$

$$= r_0 + r_1 + r_2 + r_3 + r_4 + \dots + r_{n-1} + r_n$$

$$= \frac{1}{2 \times 1} - \frac{1}{2} + \frac{1}{2 \times 3} + \frac{1}{2 \times 2} - \frac{1}{3} + \frac{1}{2 \times 4} + \frac{1}{2 \times 3} - \frac{1}{4} + \frac{1}{2 \times 5}$$

$$+ \frac{1}{2 \times 4} - \frac{1}{5} + \frac{1}{2 \times 6} + \frac{1}{2 \times 5} - \frac{1}{6} + \frac{1}{2 \times 7} + \dots + \frac{1}{2(n-1+1)} -$$



## Question 4 continued

$$\frac{1}{n-1+2} + \frac{1}{2(n-1+3)} + \frac{1}{2(n+1)} - \frac{1}{n+2} + \frac{1}{2(n+3)}$$

$$= \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + \cancel{\frac{1}{6}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{8}} + \cancel{\frac{1}{6}} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{10}} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{12}}$$

$$\cancel{\frac{1}{10}} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{14}} + \cdots + \cancel{\frac{1}{2n}} - \cancel{\frac{1}{n+1}} + \cancel{\frac{1}{2(n+2)}} + \cancel{\frac{1}{2(n+1)}} - \cancel{\frac{1}{n+2}} + \cancel{\frac{1}{2(n+3)}}$$

$$= \frac{1}{4} + \frac{1}{2(n+2)} - \frac{1}{n+2} + \frac{1}{2(n+3)} = \frac{1}{4} - \frac{1}{2(n+2)} + \frac{1}{2(n+3)}$$

$$= \frac{1}{4} - \frac{2}{4(n+2)} + \frac{2}{4(n+3)} = \frac{1}{4} \left[ 1 - \frac{2}{n+2} + \frac{2}{n+3} \right]$$

$$= \frac{1}{4} \left[ \frac{(n+2)(n+3)}{(n+2)(n+3)} - \frac{2(n+3)}{(n+2)(n+3)} + \frac{2(n+2)}{(n+2)(n+3)} \right]$$

$$= \frac{1}{4} \left[ \frac{(n+2)(n+3) - 2(n+3) + 2(n+2)}{(n+2)(n+3)} \right]$$

$$= \frac{1}{4} \left[ \frac{n^2 + 5n + 6 - 2n - 6 + 2n + 4}{(n+2)(n+3)} \right] = \frac{n^2 + 5n + 4}{4(n+2)(n+3)}$$

$$= \frac{(n+4)(n+1)}{4(n+2)(n+3)} \quad \therefore a=4, b=1, c=4$$



5. A tank at a chemical plant has a capacity of 250 litres. The tank initially contains 100 litres of pure water.

Salt water enters the tank at a rate of 3 litres every minute. Each litre of salt water entering the tank contains 1 gram of salt.

It is assumed that the salt water mixes instantly with the contents of the tank upon entry.

At the instant when the salt water begins to enter the tank, a valve is opened at the bottom of the tank and the solution in the tank flows out at a rate of 2 litres per minute.

Given that there are  $S$  grams of salt in the tank after  $t$  minutes,

- (a) show that the situation can be modelled by the differential equation

$$\frac{dS}{dt} = 3 - \frac{2S}{100+t} \quad (4)$$

- (b) Hence find the number of grams of salt in the tank after 10 minutes. (5)

When the concentration of salt in the tank reaches 0.9 grams per litre, the valve at the bottom of the tank must be closed.

- (c) Find, to the nearest minute, when the valve would need to be closed. (3)

- (d) Evaluate the model. (1)

(a) The tank initially contains 100 [l]

$3[l]$  of water entering every minute and  $2[l]$  of water leaving every minute  $\therefore$  net flow into tank is  $1[l/min]$

$\therefore$  tank contains  $100 + t$  [l] of water after  $t$  [min]

$$\text{concentration of salt after } t \text{ hours} = \frac{S}{100+t} [\text{g/l}]$$

$$\text{Rate of salt in} = 3 \times 1 = 3 [\text{g/min}]$$

$$\text{Rate of salt out} = 2 \times \frac{S}{100+t} = \frac{2S}{100+t} [\text{g/min}]$$

$$\therefore \frac{dS}{dt} = 3 - \frac{2S}{100+t} [\text{g/min}] \Rightarrow \frac{dS}{dt} + \frac{2}{100+t} S = 3 [\text{g/min}]$$



## Question 5 continued

(b)

INTEGRATING FACTOR:  $\frac{ds}{dt} + Ps = Q$  has solution

$$e^{\int P dt} s = \int e^{\int P dt} Q dt + C$$

$$\text{where IF} = e^{\int P dt}$$

$$P = \frac{2}{100+t} = 2(100+t)^{-1} \quad \text{and} \quad Q = 3$$

$$\text{IF} = e^{\int 2(100+t)^{-1} dt} = e^{2\ln(100+t)} = e^{\ln(100+t)^2} = (100+t)^2$$

$$(100+t)^2 s = \int 3(100+t)^2 dt + C$$

$$(100+t)^2 s = (100+t)^3 + C$$

$$\text{at } t=0, s=0: (100+0)^2 0 = (100+0)^3 + C$$

$$\therefore C = -1000000$$

$$\therefore s(100+t)^2 = (100+t)^3 - 1000000$$

$$\therefore s = \frac{(100+t)^3 - 1000000}{(100+t)^2}$$

$$\text{at } t=10: s = \frac{(100+10)^3 - 1000000}{(100+10)^2}$$

$$s = 27.4 \text{ [g]} \quad (3 \text{ sf})$$

- (c) when salt reaches 0.9 [g/l], the value at the bottom of the tank must be closed, i.e. amount of salt,  $s = 0.9(100+t)$  [g]



Question 5 continued

$$0.9(100+t) = \frac{(100+t)^3 - 1000000}{(100+t)^2}$$

$$0.9(100+t)^3 = (100+t)^3 - 1000000$$

$$0.1(100+t)^3 = 1000000$$

$$(100+t)^3 = 10000000$$

$$100+t = 215.44\dots$$

$$\therefore t = 115[\text{min}] \text{ (3sf)}$$

(d) it is unlikely that the concentration of salt water entering the tank remains exactly the same



6. Prove by induction that for all positive integers  $n$

$$f(n) = 3^{2n+4} - 2^{2n}$$

is divisible by 5

$$n=1: f(1) = 3^{2(1)+4} - 2^{2(1)} = 725 = 145 \times 5 \quad (6)$$

$\therefore f(n)$  is divisible by 5 when  $n=1$

assume true for  $n=k$ , i.e.  $f(k) = 3^{2k+4} - 2^{2k}$  is divisible by 5

try for  $n=k+1$ :

$$\begin{aligned} f(k+1) &= 3^{2(k+1)+4} - 2^{2(k+1)} \\ &= 3^{2k+2+4} - 2^{2k+2} \\ &= 3^{2k+4} \times 3^2 - 2^{2k} \times 2^2 \\ &= 9(3^{2k+4}) - 4(2^{2k}) \end{aligned}$$

$$\text{hence } f(k+1) - f(k) = 9(3^{2k+4}) - 4(2^{2k}) - 3^{2k+4} + 2^{2k}$$

$$= 8(3^{2k+4}) - 3(2^{2k})$$

$$= 5(3^{2k+4}) + 3(3^{2k+4} - 2^{2k})$$

$$= 5(3^{2k+4}) + 3f(k)$$

$$\therefore f(k+1) = 5(3^{2k+4}) + 4f(k)$$

$\therefore$  true for  $n=k+1$

If true for  $n=k$  then it is true for  $n=k+1$  and as it is true for  $n=1$ , the statement is true for all  $n \in \mathbb{Z}^+$



7. The line  $l_1$  has equation

$$\frac{x-1}{2} = \frac{y+1}{-1} = \frac{z-4}{3}$$

The line  $l_2$  has equation

$$\mathbf{r} = \mathbf{i} + 3\mathbf{k} + t(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$$

where  $t$  is a scalar parameter.

(a) Show that  $l_1$  and  $l_2$  lie in the same plane. (3)

(b) Write down a vector equation for the plane containing  $l_1$  and  $l_2$  (1)

(c) Find, to the nearest degree, the acute angle between  $l_1$  and  $l_2$  (3)

(a) If lie on same plane, then either parallel or intersect

$$l_1: \vec{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} \text{ and } l_2: \vec{r} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$l_1$  and  $l_2$  are not parallel, so check if they intersect

$$\begin{aligned} \text{so: } 1 + 2\lambda &= 1 + t & \textcircled{1} \\ -1 - \lambda &= 0 - t & \textcircled{2} \\ 4 + 3\lambda &= 3 + 2t & \textcircled{3} \end{aligned}$$

$$\text{From } \textcircled{1}: t = 2\lambda \quad \textcircled{4}$$

$$\text{From } \textcircled{2}: 1 + \lambda = t \quad \textcircled{5}$$

$$\textcircled{4} \text{ into } \textcircled{5}: 1 + \lambda = 2\lambda$$

$$\therefore \lambda = 1 \quad \textcircled{6}$$

$$\textcircled{6} \text{ into } \textcircled{4}: \therefore t = 2 \quad \textcircled{7}$$

$$\textcircled{6} \text{ and } \textcircled{7} \text{ into } \textcircled{3}: 4 + 3 \times 1 = 3 + 2 \times 2$$



## Question 7 continued

$$\vec{r} = \vec{r}$$

as lines intersect at a single point, the lines lie on the same plane

$$\pi = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$= \frac{(2\hat{i} - \hat{j} + 3\hat{k}) \cdot (\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{(2)^2 + (-1)^2 + (3)^2} \times \sqrt{(1)^2 + (-1)^2 + (2)^2}}$$

$$= \frac{(2)(1) + (-1)(-1) + (3)(2)}{\sqrt{14} \times \sqrt{6}}$$

$$= \frac{3\sqrt{21}}{14}$$

$$\theta = 10.89\dots^\circ$$

$$\therefore \theta = 11^\circ$$



8. A scientist is studying the effect of introducing a population of white-clawed crayfish into a population of signal crayfish.

At time  $t$  years, the number of white-clawed crayfish,  $w$ , and the number of signal crayfish,  $s$ , are modelled by the differential equations

$$\frac{dw}{dt} = \frac{5}{2}(w - s) \quad \textcircled{1}$$

$$\frac{ds}{dt} = \frac{2}{5}w - 90e^{-t} \quad \textcircled{2}$$

- (a) Show that

$$2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w = 450e^{-t} \quad (3)$$

- (b) Find a general solution for the number of white-clawed crayfish at time  $t$  years.

(6)

- (c) Find a general solution for the number of signal crayfish at time  $t$  years.

(2)

The model predicts that, at time  $T$  years, the population of white-clawed crayfish will have died out.

Given that  $w = 65$  and  $s = 85$  when  $t = 0$

- (d) find the value of  $T$ , giving your answer to 3 decimal places.

(6)

- (e) Suggest a limitation of the model.

(1)

(a) Differentiate  $\textcircled{1}$  wrt  $t$ :  $\frac{d^2w}{dt^2} = \frac{5}{2} \left( \frac{dw}{dt} - \frac{ds}{dt} \right) \textcircled{3}$

Sub  $\textcircled{2}$  into  $\textcircled{3}$ :  $\frac{d^2w}{dt^2} = \frac{5}{2} \left( \frac{dw}{dt} - \left[ \frac{2}{5}w - 90e^{-t} \right] \right)$

$$2\frac{d^2w}{dt^2} = 5 \left( \frac{dw}{dt} - \frac{2}{5}w + 90e^{-t} \right)$$

$$2\frac{d^2w}{dt^2} = 5\frac{dw}{dt} - 2w + 450e^{-t}$$

$$\therefore 2\frac{d^2w}{dt^2} - 5\frac{dw}{dt} + 2w = 450e^{-t} \quad \textcircled{4}$$



## Question 8 continued

(b)

$$\text{auxiliary equation: } m^2 - 5m + 2 = 0$$

$$(m-2)(2m-1) = 0$$

$\therefore m = 2, \frac{1}{2}$   $\therefore$  two real, distinct solutions

$$\therefore w_h = Pe^{2t} + Qe^{t/2}$$

for particular integral, try:  $w_{pi} = Ae^{-t}$

$$\frac{dw_{pi}}{dt} = -Ae^{-t}$$

$$\frac{d^2w_{pi}}{dt^2} = Ae^{-t}$$

$$\text{so } ④ \text{ becomes: } 2(Ae^{-t}) - 5(-Ae^{-t}) + 2(Ae^{-t}) = 450e^{-t}$$

$$2A + 5A + 2A = 450$$

$$9A = 450 \quad \therefore A = 50$$

$$\therefore w_{pi} = 50e^{-t}$$

$$\therefore w_{as} = Pe^{2t} + Qe^{t/2} + 50e^{-t} \quad ⑤$$

$$\text{From } ①: s = \omega - \frac{2}{5} \frac{dw}{dt} \quad ⑥$$

$$\text{differentiate } ⑤ \text{ wrt } t: \frac{dw_{as}}{dt} = 2Pe^{2t} + \frac{1}{2}Qe^{t/2} - 50e^{-t} \quad ⑦$$

$$⑤ \text{ and } ⑦ \text{ into } ⑥: s = (Pe^{2t} + Qe^{t/2} + 50e^{-t}) - \frac{2}{5} \left( 2Pe^{2t} + \frac{1}{2}Qe^{t/2} - 50e^{-t} \right)$$



Question 8 continued

$$S_{GS} = \frac{P}{5} e^{2t} + \frac{4Q}{5} e^{-t/2} + 70e^{-t} \quad (8)$$

(d)  $t=0, w=65: 65 = P + Q + 50$

$$\therefore P + Q = 15 \quad (9)$$

$$t=0, S=85: 85 = \frac{P}{5} + \frac{4Q}{5} + 70$$

$$\frac{P}{5} + \frac{4Q}{5} = 15 \quad (\times 5)$$

$$P + 4Q = 75 \quad (10)$$

Solve (9) and (10) simultaneously:

$$3Q = 60 \therefore Q = 20$$

$$P + 20 = 15 \therefore P = -5$$

$$\therefore w_{GS} = -5e^{2t} + 20e^{-t/2} + 50e^{-t} \quad (11)$$

$$w=0, t=T: 0 = -5e^{2T} + 20e^{-T/2} + \frac{50}{e^T} \quad \times -e^T$$

$$e^{3T} - 4e^{3T/2} - 10 = 0 \quad (12)$$

Note (12) is a hidden quadratic!

$$\text{let } \lambda = e^{3T/2} : \lambda^2 - 4\lambda - 10 = 0$$

$$\lambda = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-10)}}{2(1)}$$



**Question 8 continued**

$$\lambda = 2 \pm \sqrt{14}$$

$$\text{but } \lambda > 0 \Rightarrow \lambda = 2 + \sqrt{14}$$

$$e^{3T/2} = 2 + \sqrt{14}$$

$$T = \frac{2}{3} \ln(2 + \sqrt{14})$$

$$T = 1.165 \text{ years} \quad (3 \text{ dp})$$

- (e) When the white clawed crayfish have died out, the model will not be valid.

