

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

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## Pearson Edexcel Level 3 GCE

**Monday 15 May 2023**

Afternoon (Time: 1 hour 40 minutes)

Paper  
reference

**8FM0/01**

## Further Mathematics

### Advanced Subsidiary

### PAPER 1: Core Pure Mathematics



#### You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

#### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

#### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

#### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

**Turn over** ►

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**Pearson**

1.

$$\begin{pmatrix} x & 9 \\ y & z \end{pmatrix} - 3 \begin{pmatrix} z & y \\ z & y \end{pmatrix} = k \mathbf{I}$$

where  $x, y, z$  and  $k$  are constants.

Determine the value of  $x$ , the value of  $y$  and the value of  $z$ .

(4)

$$\begin{bmatrix} x & 9 \\ y & z \end{bmatrix} - 3 \begin{bmatrix} z & y \\ z & y \end{bmatrix} = k \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x - 3z & 9 - 3y \\ y - 3z & z - 3y \end{bmatrix} = \begin{bmatrix} k & 0 \\ 0 & k \end{bmatrix}$$

$$\therefore x - 3z = k$$

$$\therefore 9 - 3y = 0$$

$$\therefore y - 3z = 0$$

$$\therefore z - 3y = k$$

$$\therefore y = 3$$

$$\therefore z = 1$$

$$\therefore k = -8$$

$$\therefore x = -5$$



## **Question 1 continued**

**(Total for Question 1 is 4 marks)**



2.  $f(z) = z^3 + az^2 + bz + 175$  where  $a$  and  $b$  are real constants

Given that  $-3 + 4i$  is a root of the equation  $f(z) = 0$

(a) determine the value of  $a$  and the value of  $b$ .

(4)

(b) Show all the roots of the equation  $f(z) = 0$  on a single Argand diagram.

(2)

(c) Write down the roots of the equation  $f(z + 2) = 0$

(1)

**(a)** If  $z_1 = -3 + 4i$ , THEN  $z_2 = -3 - 4i$

$$\text{QUADRATIC FACTOR is } [z - (-3 + 4i)][z - (-3 - 4i)]$$

$$\equiv [(z + 3) - 4i][(z + 3) + 4i]$$

$$\equiv (z + 3)^2 + 16$$

$$\equiv z^2 + 6z + 25$$

$$\therefore [z^2 + 6z + 25][z + \alpha] \equiv z^3 + az^2 + bz + 175$$

$$\therefore 25\alpha = 175$$

$$\therefore \alpha = 7$$

$$\therefore [z^2 + 6z + 25][z + 7] \equiv z^3 + az^2 + bz + 175$$

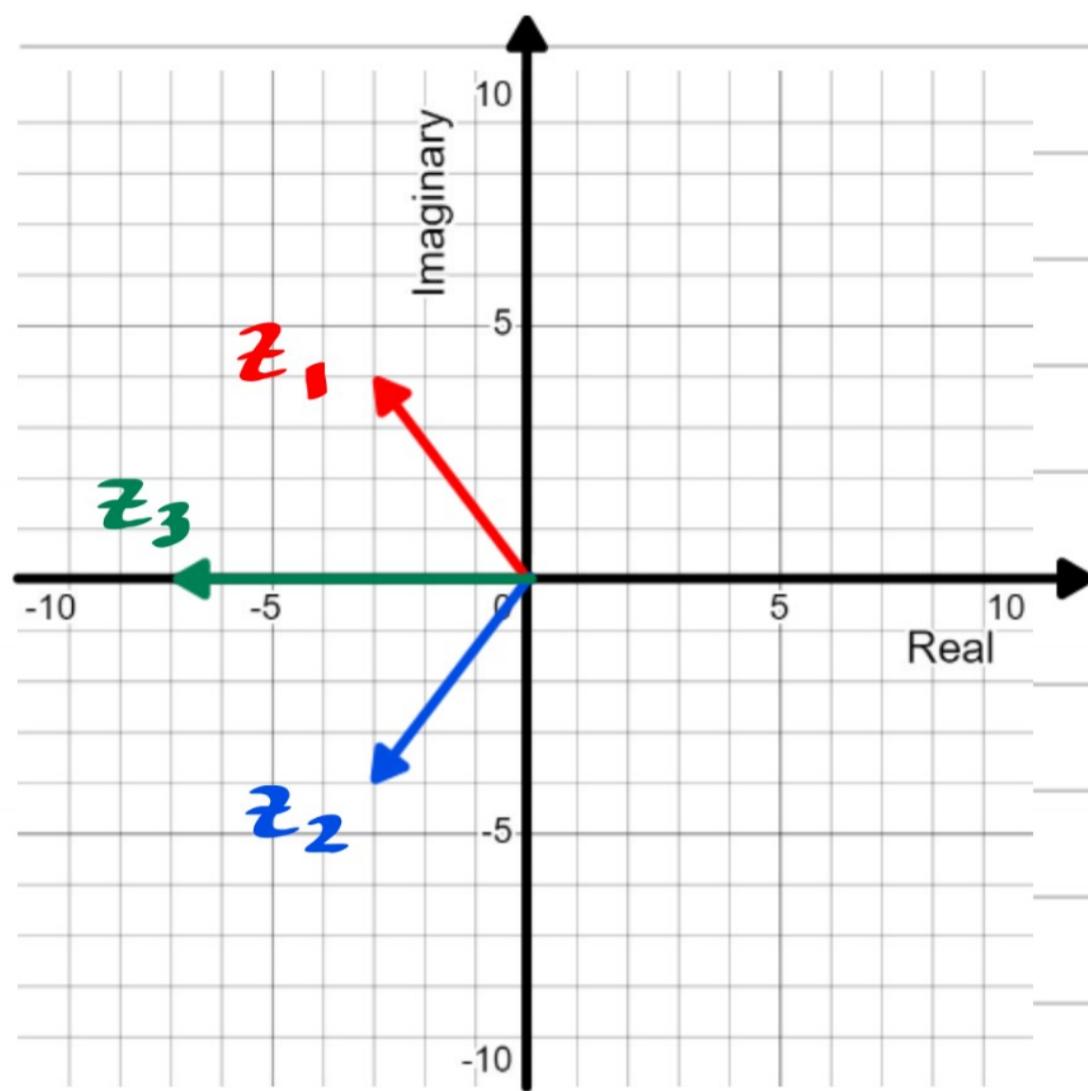
$$\therefore a = 13, b = 67$$

$$\therefore f(z) = z^3 + 13z^2 + 67z + 175$$



Question 2 continued

(b)

 $f(z) = 0$  HAS ROOTS  $z_1 = -3 + 4i$ ,  $z_2 = -3 - 4i$ ,  $z_3 = -7$  $f(z) \rightarrow f(z + 2)$  IS A TRANSLATION BY  $\begin{pmatrix} -2 \\ 0 \end{pmatrix}$  $\therefore f(z + 2) = 0$  HAS ROOTS  $z = -5 \pm 4i, -9$ 

## Question 2 continued

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## **Question 2 continued**

**(Total for Question 2 is 7 marks)**



P 7 2 8 0 6 A 0 7 3 6

3.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

(a) Describe fully the single geometric transformation  $A$  represented by the matrix  $\mathbf{A}$ .

(2)

$$\mathbf{B} = \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

The transformation  $B$  is represented by the matrix  $\mathbf{B}$ .

The transformation  $A$  followed by the transformation  $B$  is the transformation  $C$ , which is represented by the matrix  $\mathbf{C}$ .

To determine matrix  $\mathbf{C}$ , a student attempts the following matrix multiplication.

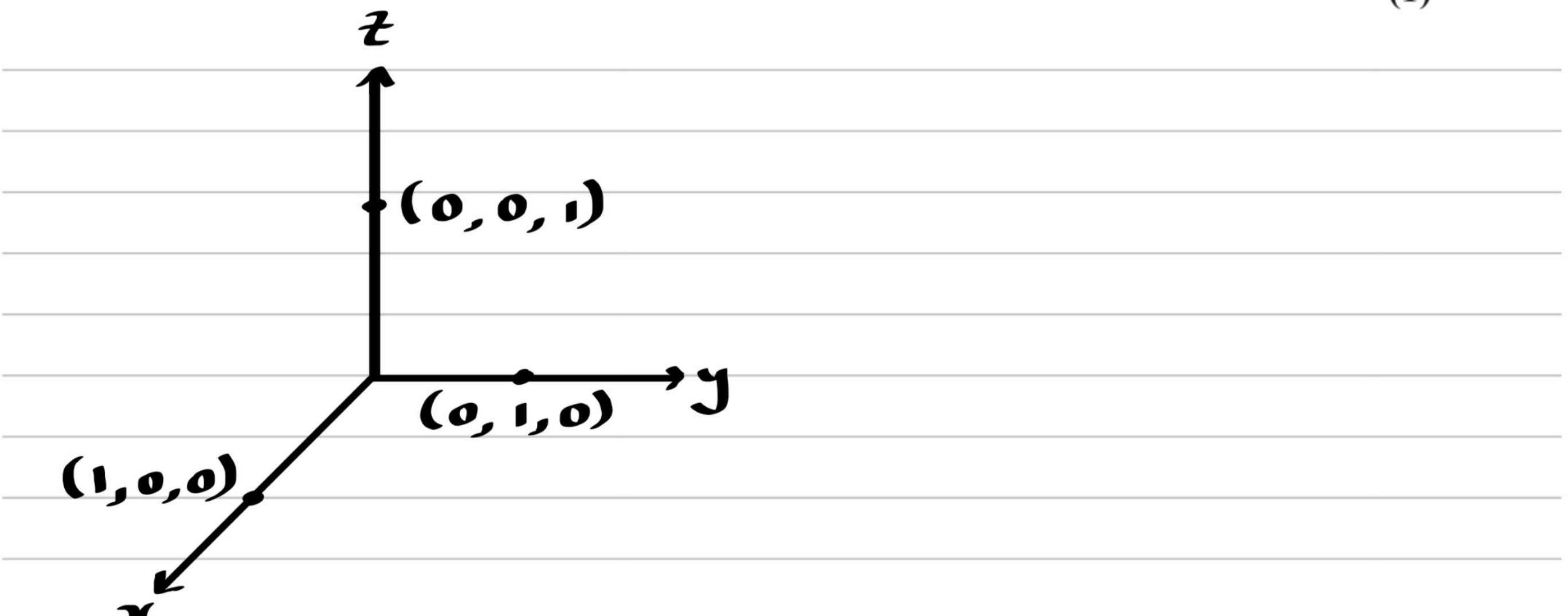
$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix} \begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}$$

(b) State the error made by the student.

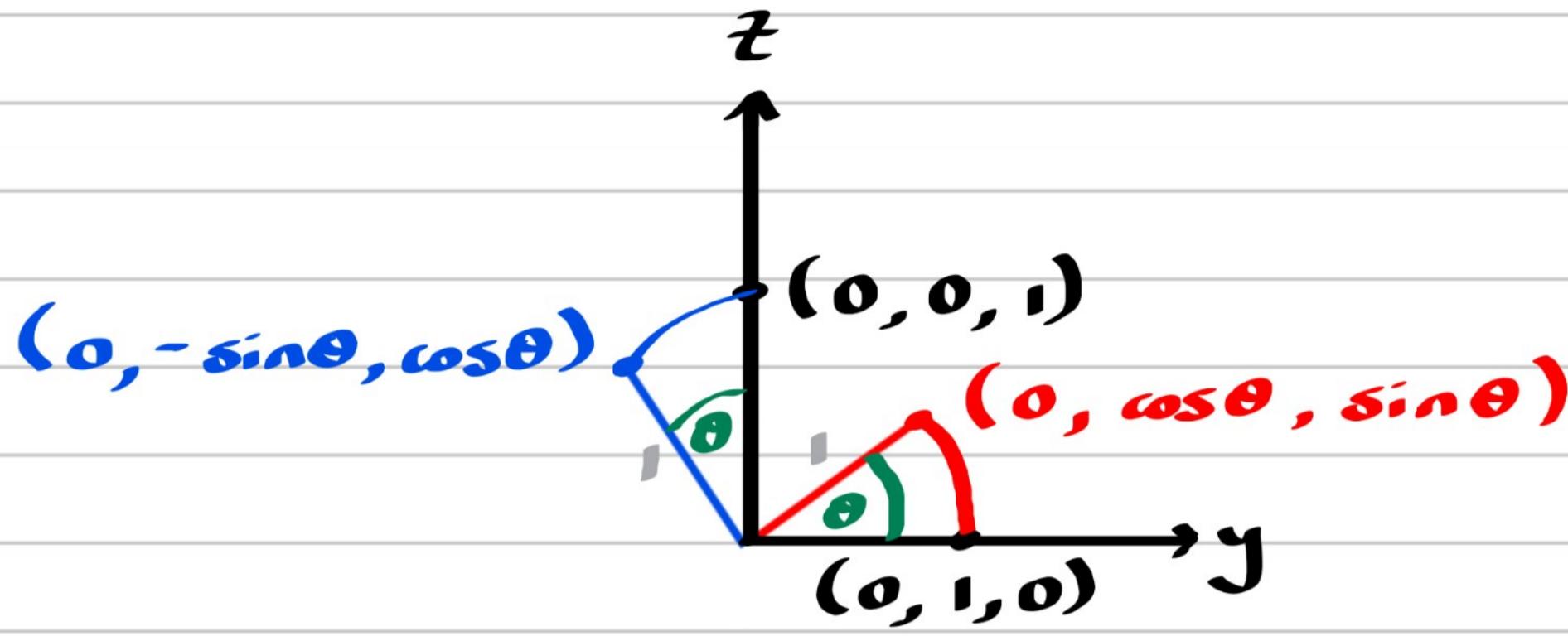
(1)

(c) Determine the correct matrix  $\mathbf{C}$ .

(1)



## Question 3 continued



$\therefore$  ROTATION ABOUT  $x$ -AXIS BY  $\theta$  IS

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$\therefore$  TRANSFORMATION A IS A ROTATION BY  $30^\circ$  IN  $x$ -DIR

THE TRANSFORMATIONS ARE BEING MULTIPLIED IN THE  
WRONG ORDER

$$\begin{bmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & \frac{3\sqrt{3}}{2} & -\frac{3}{2} \\ \sqrt{3} & \frac{5\sqrt{3}}{2} & \frac{15}{2} \\ 1 & \sqrt{3} & -1 \end{bmatrix}$$

(Total for Question 3 is 4 marks)

4. (i) (a) Show that

$$\frac{2+3i}{5+i} = k(1+i)$$

where  $k$  is a constant to be determined.

(Solutions relying on calculator technology are not acceptable.)

(3)

Given that

- $n$  is a positive integer
- $\left(\frac{2+3i}{5+i}\right)^n$  is a real number

(b) use the answer to part (a) to write down the smallest possible value of  $n$ .

(1)

(ii) The complex number  $z = a + bi$  where  $a$  and  $b$  are real constants.

Given that

- $|z^{10}| = 59049$
- $\arg(z^{10}) = -\frac{5\pi}{3}$

determine the value of  $a$  and the value of  $b$ .

(4)

**(a)** 
$$\frac{2+3i}{5+i} = \frac{2+3i}{5+i} \times \frac{5-i}{5-i}$$

$$= \frac{10 - 2i + 15i + 3}{25 + 1}$$

$$= \frac{13 + 13i}{26}$$

$$= \frac{1+i}{2}$$

$$\therefore k = \frac{1}{2}$$

**(b)** 
$$\left[ \frac{2+3i}{5+i} \right]^4 = \frac{1}{4}$$



## Question 4 continued

$$\therefore n = 4$$

$$\begin{aligned} \text{IF } z = a + bi, \text{ THEN } z^{10} &= (a + bi)^{10} \\ &= [r(\cos\theta + i\sin\theta)]^{10} \\ &= r^{10} (\cos 10\theta + i\sin 10\theta) \end{aligned}$$

$$\text{As } \arg(z^{10}) = -\frac{5\pi}{3} \therefore 10\theta = -\frac{5\pi}{3} \therefore \theta = -\frac{\pi}{6}$$

$$\text{As } |z^{10}| = 59049 \therefore r = |z| = 59049^{\frac{1}{10}} = 3$$

$$\begin{aligned} \therefore z &= 3 \left[ \cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right) \right] \\ &= \frac{3\sqrt{3}}{2} - \frac{3i}{2} \end{aligned}$$

$$\therefore a = \frac{3\sqrt{3}}{2}, b = -\frac{3}{2}$$



## Question 4 continued

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## **Question 4 continued**

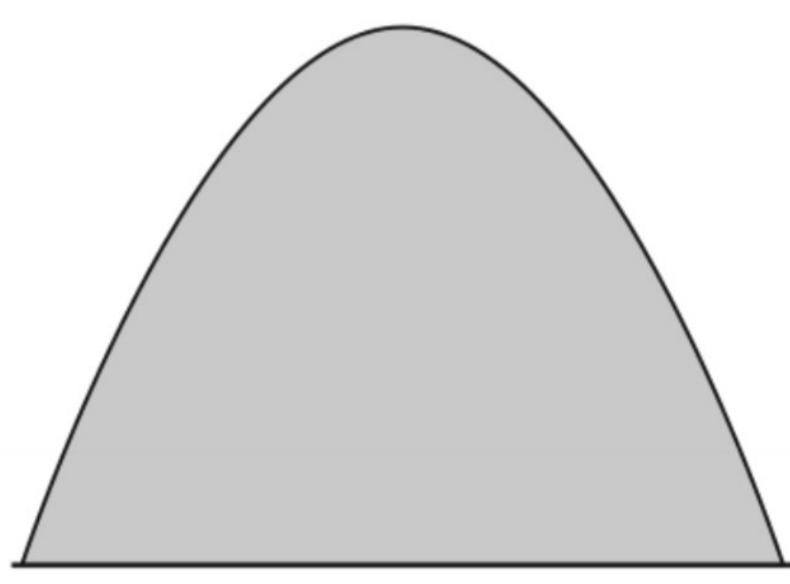
**(Total for Question 4 is 8 marks)**



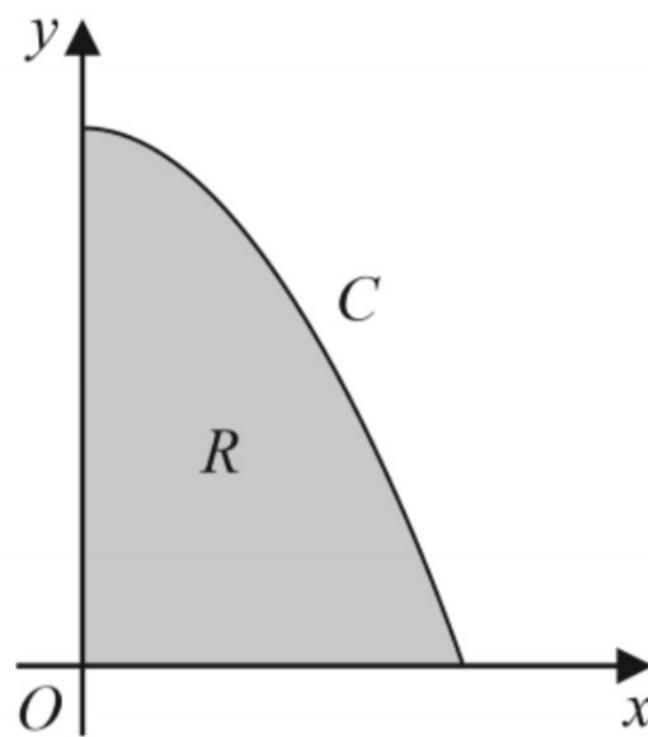
5.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.



**Figure 1**



**Figure 2**

A large pile of concrete waste is created on a building site.

Figure 1 shows a central vertical cross-section of the concrete waste.

The curve  $C$ , shown in Figure 2, has equation

$$y + x^2 = 2 \quad 0 \leq x \leq \sqrt{2}$$

The region  $R$ , shown shaded in Figure 2, is bounded by the  $y$ -axis, the  $x$ -axis and the curve  $C$ .

The volume of concrete waste is modelled by the volume of revolution formed when  $R$  is rotated through  $360^\circ$  about the  $y$ -axis. The units are metres.

The density of the concrete waste is  $900 \text{ kg m}^{-3}$

- (a) Use the model to estimate the mass of the concrete waste. Give your answer to 2 significant figures.

(6)

- (b) Give a limitation of the model.

(1)

The mass of the concrete waste is approximately  $5500 \text{ kg}$ .

- (c) Use this information and your answer to part (a) to evaluate the model, giving a reason for your answer.

(1)

(a)

$$x = 0, y = 2$$

$$x = \sqrt{2}, y = 0$$

$$y + x^2 = 2 \quad \therefore x^2 = 2 - y$$

$$\text{volume} = \pi \int_0^2 x^2 dy$$

Question 5 continued

$$= \pi \int_0^2 (2 - y) dy$$

$$= \pi \left[ 2y - \frac{y^2}{2} \right]_0^2$$

$$= 2\pi \text{ m}^3$$

$$\therefore \text{MASS} = 2\pi \times 900$$

$$= 1800 \pi$$

$$= 5700 \text{ kg (2sf)}$$

THE PILE OF CONCRETE WASTE IS UNLIKELY TO BE  
PERFECTLY SYMMETRICAL

$$\% \text{ ERROR} = \frac{1800 \pi - 5500}{5500} \times 100$$

$$= 2.81\ldots\%$$

$\therefore$  Good model to use



## Question 5 continued

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## **Question 5 continued**

**(Total for Question 5 is 8 marks)**



6. The line  $l_1$  has equation  $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$  where  $\lambda$  is a scalar parameter.

The line  $l_2$  is parallel to  $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

- (a) Show that  $l_1$  and  $l_2$  are perpendicular.

(2)

The plane  $\Pi$  contains the line  $l_1$  and is perpendicular to  $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$

- (b) Determine a Cartesian equation of  $\Pi$

(2)

- (c) Verify that the point  $A(3, 1, 1)$  lies on  $\Pi$

(1)

Given that

- the point of intersection of  $\Pi$  and  $l_2$  has coordinates  $(2, 3, 2)$
- the point  $B(p, q, r)$  lies on  $l_2$
- the distance  $AB$  is  $2\sqrt{5}$
- $p, q$  and  $r$  are positive integers

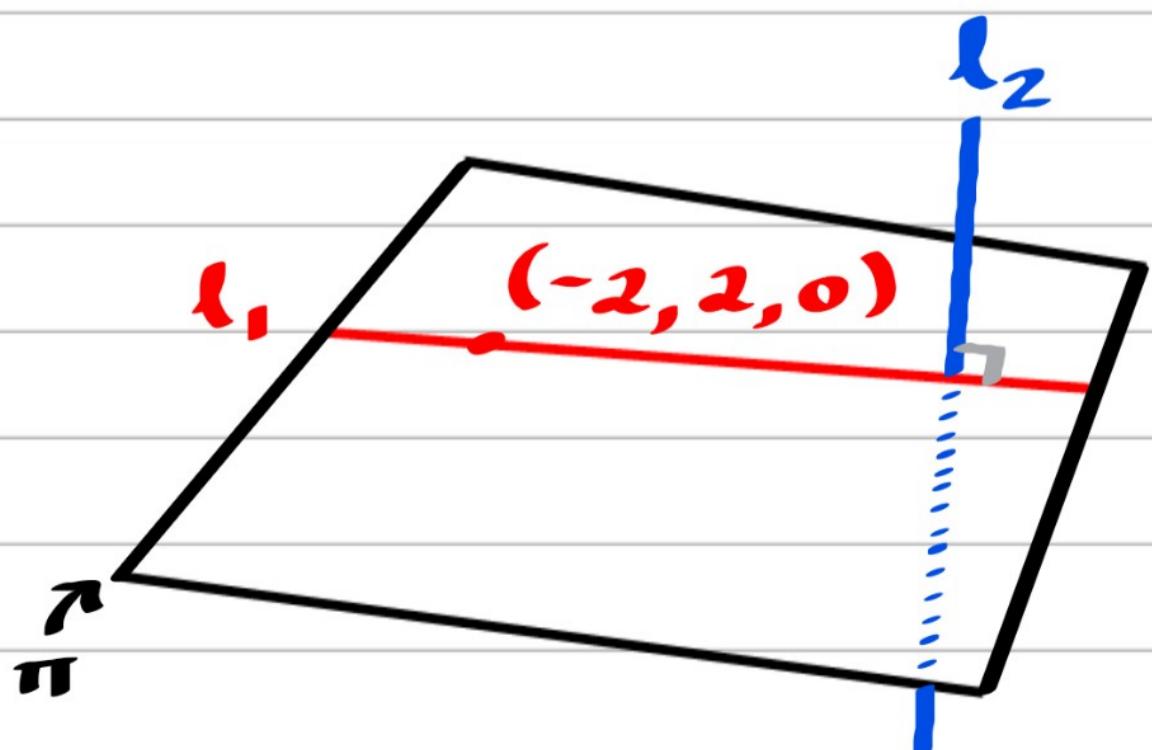
- (d) determine the coordinates of  $B$ .

(6)

$$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} = 1 \times 3 + 2 \times 0 + -3 \times 1 = 0$$

∴ LINES ARE PERPENDICULAR

(b)



Question 6 continued

$$\pi: x + 2y - 3z = 2$$

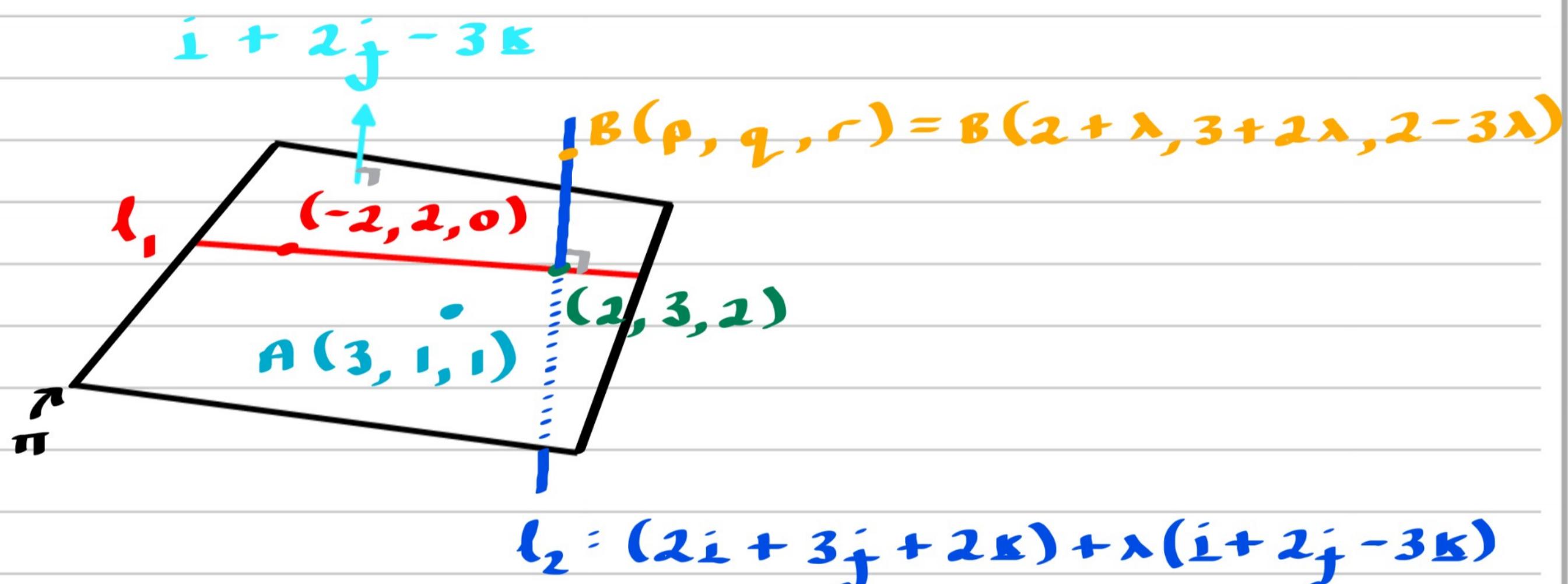
As  $(-2, 2, 0)$  lies on  $\pi \therefore d = (-2) + 2(2) - 3(0) = 2$

$$\therefore \pi: x + 2y - 3z = 2$$

$$A(3, 1, 1) \therefore \text{LHS} = 3 + 2 \times 1 - 3 \times 1 = 2 = \text{RHS}$$

$\therefore A(3, 1, 1)$  lies on plane

(d)



$$\overrightarrow{AB} = \begin{bmatrix} 2+\lambda-3 \\ 3+2\lambda-1 \\ 2-3\lambda-1 \end{bmatrix} = \begin{bmatrix} -1+\lambda \\ 2+2\lambda \\ 1-3\lambda \end{bmatrix}$$

$$|\overrightarrow{AB}| = \sqrt{(-1+\lambda)^2 + (2+2\lambda)^2 + (1-3\lambda)^2} = 5\sqrt{2}$$

$$\therefore (-1+\lambda)^2 + (2+2\lambda)^2 + (1-3\lambda)^2 = 20$$

$$\therefore \lambda^2 - 2\lambda + 1 + 4\lambda^2 + 8\lambda + 4 + 9\lambda^2 - 6\lambda + 1 = 20$$

$$\therefore 14\lambda^2 = 14$$

$$\therefore \lambda = \pm 1, \text{ but as } p, q, r > 0 \therefore \lambda = -1$$

$$\therefore B(1, 1, 5)$$



## Question 6 continued

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## **Question 6 continued**

**(Total for Question 6 is 11 marks)**



7. (i) Shade, on an Argand diagram, the set of points for which

$$|z - 3| \leq |z + 6i|$$

(3)

- (ii) Determine the exact complex number  $w$  which satisfies both

$$\arg(w - 2) = \frac{\pi}{3} \quad \text{and} \quad \arg(w + 1) = \frac{\pi}{6}$$

(6)

(a)  $|z - (3 + 0i)| \leq |z - (0 - 6i)|$

**WE REQUIRE PERPENDICULAR BISECTOR**

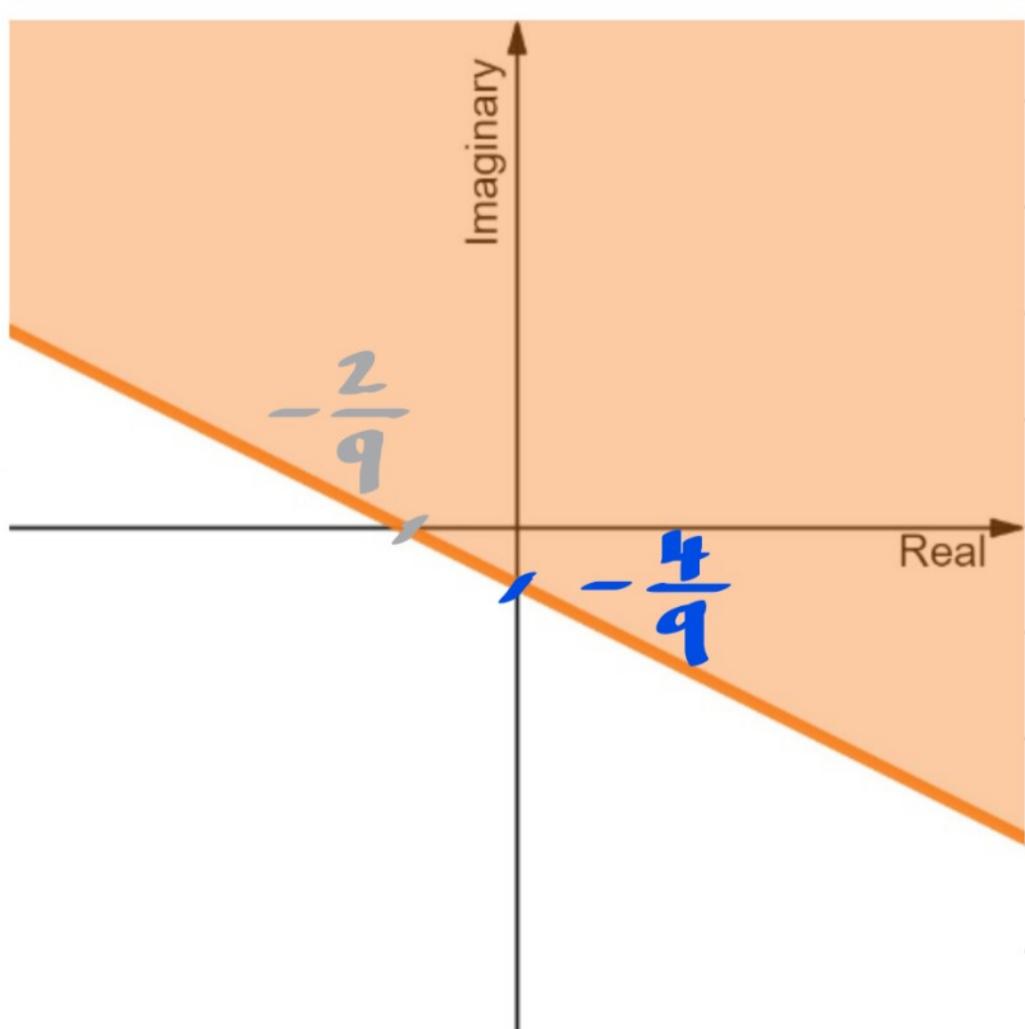
$\therefore \text{GRADIENT} = \frac{0 - -6}{3 - 0} = 2$

$\therefore \text{PERPENDICULAR GRADIENT} = -\frac{1}{2}$

$\therefore \text{MIDPOINT, } M \left[ \frac{3+0}{2}, \frac{0+-6}{2} \right] = M \left[ \frac{3}{2}, -3 \right]$

i.e.  $y - -3 = -\frac{1}{2} \left[ x - \frac{3}{2} \right]$

$\therefore \text{PERPENDICULAR BISECTOR HAS EQUATION } y = -\frac{1}{2}x - \frac{9}{4}$



Question 7 continued

$$\arg(w - 2) = \frac{\pi}{3} \Rightarrow y = \sqrt{3} [x - 2] \quad \textcircled{1}$$

$$\arg(w + 1) = \frac{\pi}{6} \Rightarrow y = \frac{\sqrt{3}}{3} [x + 1] \quad \textcircled{2}$$

SOLVE  $\textcircled{1}$  AND  $\textcircled{2}$  SIMULTANEOUSLY:

$$\therefore \sqrt{3} [x - 2] = \frac{\sqrt{3}}{3} [x + 1]$$

$$\therefore 3[x - 2] = x + 1$$

$$\therefore 3x - 6 = x + 1$$

$$\therefore x = \frac{7}{2}$$

$$\therefore y = \frac{\sqrt{3}}{3} \left[ \frac{7}{2} + 1 \right] = \frac{3\sqrt{3}}{2}$$

$$\therefore w = \frac{7}{2} + \frac{3\sqrt{3}}{2} i$$



## Question 7 continued

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## **Question 7 continued**

**(Total for Question 7 is 9 marks)**



8. (a) Use the standard results for  $\sum_{r=1}^n r^2$  and  $\sum_{r=1}^n r$  to show that, for all positive integers  $n$ ,

$$\sum_{r=1}^n (2r-1)^2 = \frac{n}{3}(an^2 - 1)$$

where  $a$  is a constant to be determined.

(5)

- (b) Hence determine the sum of the squares of all positive odd three-digit integers.

$$(a) \sum_{r=1}^n [2r-1]^2 = \sum_{r=1}^n [4r^2 - 4r + 1] \quad (3)$$

$$= \sum_{r=1}^n 4r^2 - \sum_{r=1}^n 4r + \sum_{r=1}^n 1$$

$$= 4 \sum_{r=1}^n r^2 - 4 \sum_{r=1}^n r + \sum_{r=1}^n 1$$

$$= 4 \left[ \frac{1}{6} n(n+1)(2n+1) \right]$$

$$- 4 \left[ \frac{1}{2} n(n+1) \right] + n$$

$$= n \left[ \frac{2}{3} (n+1)(2n+1) - 2(n+1) + 1 \right]$$

$$= \frac{n}{3} \left[ 2(n+1)(2n+1) - 6(n+1) + 3 \right]$$

$$= \frac{n}{3} \left[ 4n^2 + 6n + 2 - 6n - 6 + 3 \right]$$

$$= \frac{n}{3} \left[ 4n^2 - 1 \right]$$

$$\therefore a = 4$$



**SUM OF SQUARES OF POSITIVE ODD THREE-DIGIT NUMBERS**

$$= \left[ \underline{101} + 103 + 105 + 107 + \dots + 995 + 997 + \underline{999} \right]^2$$

$$= \sum_{r=1}^{500} [x_{r-1}]^2$$

$$= \sum_{r=1}^{500} [x_{r-1}]^2 - \sum_{r=1}^{50} [x_{r-1}]^2$$

$$= \frac{500}{3} \left[ 4(500)^2 - 1 \right] - \frac{50}{3} \left[ 4(50)^2 - 1 \right]$$

$$= 166\ 499\ 850$$

(Total for Question 8 is 8 marks)



9. (i)  $\mathbf{P} = \begin{pmatrix} k & -2 & 7 \\ -3 & -5 & 2 \\ k & k & 4 \end{pmatrix}$  where  $k$  is a constant

Show that  $\mathbf{P}$  is non-singular for all real values of  $k$ .

(4)

(ii)  $\mathbf{Q} = \begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}$

The matrix  $\mathbf{Q}$  represents a linear transformation  $T$

Under  $T$ , the point  $A(a, 2)$  and the point  $B(4, -a)$ , where  $a$  is a constant, are transformed to the points  $A'$  and  $B'$  respectively.

Given that the distance  $A'B'$  is  $\sqrt{58}$ , determine the possible values of  $a$ .

(5)

$$(i) \det(\mathbf{P}) = \begin{vmatrix} k & -2 & 7 \\ -3 & -5 & 2 \\ k & k & 4 \end{vmatrix}$$

$$= k \begin{vmatrix} -5 & 2 \\ k & 4 \end{vmatrix} - 2 \begin{vmatrix} -3 & 2 \\ k & 4 \end{vmatrix} + 7 \begin{vmatrix} -3 & -5 \\ k & k \end{vmatrix}$$

$$= k[-20 - 2k] + 2[-12 - 2k] + 7[-3k + 5k]$$

$$= -2k^2 - 10k - 24$$

$$\text{DISCRIMINANT} = (-10)^2 - 4(-2)(-24)$$

$$= -92 < 0$$

$$\therefore -2k^2 - 10k - 24 \neq 0 \text{ for } k \in \mathbb{R}$$

$\therefore \mathbf{P}$  IS NON-SINGULAR

(b)  $\begin{bmatrix} x_A \\ y_A \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} a \\ 2 \end{bmatrix}$

$$\therefore A' [2a - 2, -3a]$$



## Question 9 continued

$$\begin{bmatrix} x_a \\ y_a \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ -a \end{bmatrix}$$

$$\therefore \mathbf{b}' = [8 + a, -12]$$

$$\therefore \overrightarrow{\mathbf{A}'\mathbf{b}'} = \begin{bmatrix} 8 + a \\ -12 \end{bmatrix} - \begin{bmatrix} 2a - 2 \\ -3a \end{bmatrix}$$

$$= \begin{bmatrix} 10 - a \\ -12 + 3a \end{bmatrix}$$

$$\therefore \sqrt{(10 - a)^2 + (-12 + 3a)^2} = \sqrt{58}$$

$$\therefore (10 - a)^2 + (-12 + 3a)^2 = 58$$

$$\therefore a^2 - 20a + 100 + 9a^2 - 72a + 144 = 58$$

$$\therefore 10a^2 - 92a + 186 = 0$$

$$\therefore a = 3, a = \frac{31}{5}$$



## Question 9 continued

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## **Question 9 continued**

**(Total for Question 9 is 9 marks)**



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10.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

(i) The quartic equation

$$z^4 + 5z^2 - 30 = 0$$

has roots  $p, q, r$  and  $s$ .

Without solving the equation, determine the quartic equation whose roots are

$$(3p - 1), (3q - 1), (3r - 1) \text{ and } (3s - 1)$$

Give your answer in the form  $w^4 + aw^3 + bw^2 + cw + d = 0$ , where  $a, b, c$  and  $d$  are integers to be found.

(5)

(ii) The roots of the cubic equation

$$4x^3 + nx + 81 = 0 \quad \text{where } n \text{ is a real constant}$$

are  $\alpha, 2\alpha$  and  $\alpha - \beta$

Determine

(a) the values of the roots of the equation,

(5)

(b) the value of  $n$ .

(2)

(i)  $w = 3z - 1 \Rightarrow z = \frac{w+1}{3}$

$\therefore$  NEW EQUATION IS:  $\left[ \frac{w+1}{3} \right]^4 + 5 \left[ \frac{w+1}{3} \right]^2 - 30 = 0$

$\therefore [w+1]^4 + 45[w+1]^2 - 2430 = 0$

$\therefore w^4 + 4w^3 + 6w^2 + 4w + 1 + 45[w^2 + 2w + 1]$

$- 2430 = 0$

$\therefore w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$



Question 10 continued

$$\therefore a = 4, b = 51, c = 94, d = -2384$$

$$(ii)(a) 4x^3 + 0x^2 + 1x + 81 = 0$$

$$x^3 + 0x^2 + \frac{1}{4}x + \frac{81}{4} = 0$$

$$\text{sum of Roots: } \alpha + 2\alpha + (\alpha - \beta) = -0$$

$$\therefore \beta = 4\alpha$$

$$\text{Product of Roots: } (\alpha)(2\alpha)(\alpha - \beta) = -\frac{81}{4}$$

$$\therefore (\alpha)(2\alpha)(\alpha - 4\alpha) = -\frac{81}{4}$$

$$\therefore -6\alpha^3 = -\frac{81}{4}$$

$$\therefore \alpha = \frac{3}{2}$$

$$\text{i.e. } 2\alpha = 2 \times \frac{3}{2} = 3$$

$$\beta = 4\alpha = 4 \times \frac{3}{2} = 6$$

$$\alpha - \beta = \frac{3}{2} - 6 = -\frac{9}{2}$$

$$\therefore \text{Roots are: } \frac{3}{2}, 3, -\frac{9}{2}$$



Question 10 continued

(b)

$$\therefore \left[ x - \frac{3}{2} \right] [x - 3] \left[ x + \frac{9}{2} \right] = 0$$
$$\therefore [2x - 3][x - 3][2x + 9] = 0$$
$$\therefore [2x^2 - 9x + 9][2x + 9] = 0$$
$$\therefore 4x^3 + 18x^2 - 18x^2 - 81x + 18x + 81 = 0$$
$$\therefore 4x^3 - 63x + 81 = 0$$
$$\therefore n = -63$$



## **Question 10 continued**

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## **Question 10 continued**

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**(Total for Question 10 is 12 marks)**

**TOTAL FOR PAPER IS 80 MARKS**

