WRITTEN SOLUTIONS

Write your name here

Surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number	Candidate Number

Further Mathematics
Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)

Thursday 17 May 2018 – Afternoon

Paper Reference
8FM0-25

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions
- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information
- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice
- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

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Pearson

Whenever a numerical value of g is required, take $g = 9.8\,\mathrm{m\,s^{-2}}$ and give your answer to either 2 significant figures or 3 significant figures.

Answer ALL questions. Write your answers in the spaces provided.

1. A small ball of mass 0.3 kg is released from rest from a point 3.6 m above horizontal ground. The ball falls freely under gravity, hits the ground and rebounds vertically upwards.

In the first impact with the ground, the ball receives an impulse of magnitude 4.2 N s. The ball is modelled as a particle.

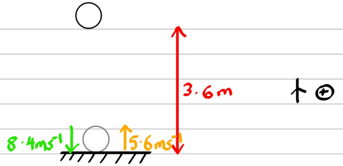
(a) Find the speed of the ball immediately after it first hits the ground.

(5)

(b) Find the kinetic energy lost by the ball as a result of the impact with the ground.

(3)

(a)



speed just before impact: $v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times 9.8 \times 3.6 = 70.56$

$v = \sqrt{70.56} = -8.4\ \mathrm{ms^{-1}}$

impulse = change in momentum

$4.2 = 0.3(w - -8.4)$

$14 = w + 8.4 \quad \therefore w = 5.6\ \mathrm{ms^{-1}}$

(b) KE lost $= \frac{1}{2}mv^2 - \frac{1}{2}mw^2$

$= \frac{1}{2} \times 0.3 \times 8.4^2 - \frac{1}{2} \times 0.3 \times 5.6^2$

$= 5.88\ \mathrm{J}$

2.

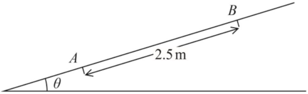


Figure 1

Figure 1 shows a ramp inclined at an angle θ to the horizontal, where $\sin\theta = \dfrac{2}{7}$

A parcel of mass 4 kg is projected, with speed $5\,\text{m}\,\text{s}^{-1}$, from a point A on the ramp.
The parcel moves up a line of greatest slope of the ramp and first comes to instantaneous rest at the point B, where $AB = 2.5$ m.
The parcel is modelled as a particle.

The total resistance to the motion of the parcel from non-gravitational forces is modelled as a constant force of magnitude R newtons.

(a) Use the work-energy principle to show that $R = 8.8$

(4)

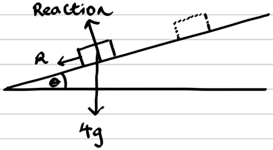
After coming to instantaneous rest at B, the parcel slides back down the ramp. The total resistance to the motion of the particle is modelled as a constant force of magnitude 8.8 N.

(b) Find the speed of the parcel at the instant it returns to A.

(3)

(c) Suggest two improvements that could be made to the model.

(2)



work-energy principle:

KE lost = PE gained + work done

$\frac{1}{2} \times 4 \times 5^2 = 4g \times 2.5\sin\theta + 2.5R$

$\frac{1}{2} \times 4 \times 5^2 = 4 \times 9.8 \times 2.5 \times \frac{2}{7} + 2.5R$

$50 = 28 + 2.5R$

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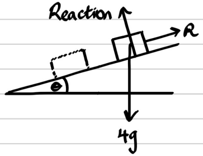
Question 2 continued

$22 = 2.5R \quad \therefore R = 8.8$

(b)



$\underline{\Sigma F = ma:}\ 4g\sin\theta - 8.8 = 4a$

$4g \times \frac{2}{7} - 8.8 = 4a$

$4a = 2.4 \quad \therefore a = 0.6\ ms^{-2}$

$\underline{v^2 = u^2 + 2as:}\ v^2 = 0^2 + 2 \times 0.6 \times 2.5$

$v^2 = 3$

$\therefore v_A = \sqrt{3}\ ms^{-1}$

(c) we have assumed constant resistance; have resistance proportional to speed

do not model parcel as a particle

3. A van of mass 750 kg is moving along a straight horizontal road. At the instant when the van is moving at v m s^{-1}, the resistance to the motion of the van is modelled as a force of magnitude λv N, where λ is a constant.

The engine of the van is working at a constant rate of 18 kW.
At the instant when $v = 15$, the acceleration of the van is 0.6 m s^{-2}

(a) Show that $\lambda = 50$

(4)

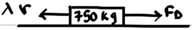
The van now moves up a straight road inclined at an angle to the horizontal, where $\sin\alpha = \frac{1}{15}$

At the instant when the van is moving at v m s^{-1}, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude $50v$ N.
When the engine of the van is working at a constant rate of 12 kW, the van is moving at a constant speed V m s^{-1} $\rightarrow a = 0$

(b) Find the value of V.

(5)

(a)



$\underline{P = F_D v:}$ $18000 = F_D \times 15$ $\therefore F_D = 1200$ N

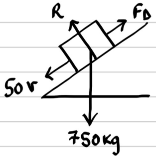
$\underline{\Sigma F = ma:}$ $F_D - \lambda v = 750a$

$1200 - 15\lambda = 750 \times 0.6$

$1200 - 15\lambda = 450$

$15\lambda = 750$ $\therefore \lambda = 50$

(b)



$\underline{P = F_D v:}$

Question 3 continued

$12000 = F_D V$

$F_D = 12000/V$

$\underline{\Sigma F = ma:}$ $F_D - 50v - 750g\sin\alpha = 750 \times 0$

$12000/v - 50v - 750g \times 1/15 = 0$

$12000/v - 50v - 490 = 0 \qquad (\times v)$

$12000 - 50v^2 - 490v = 0 \qquad (\div -10)$

$5v^2 + 49v - 1200 = 0$

$$v = \frac{-(49) \pm \sqrt{(49)^2 - 4(5)(-1200)}}{2(5)}$$

$v = 11.3\ldots, -21.1\ldots$

but $v > 0$

$\therefore v = 11.3 \text{ ms}^{-1}$

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4. A particle P of mass $3m$ is moving in a straight line on a smooth horizontal floor. A particle Q of mass $5m$ is moving in the opposite direction to P along the same straight line.

The particles collide directly.

Immediately before the collision, the speed of P is $2u$ and the speed of Q is u.
The coefficient of restitution between P and Q is e.

(a) Show that the speed of Q immediately after the collision is $\frac{u}{8}(9e+1)$

(6)

$\nearrow v > 0$

(b) Find the range of values of e for which the direction of motion of P is not changed as a result of the collision.

(2)

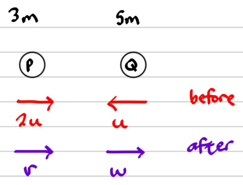
When P and Q collide they are at a distance d from a smooth fixed vertical wall, which is perpendicular to their direction of motion. After the collision with P, particle Q collides directly with the wall and rebounds so that there is a second collision between P and Q. This second collision takes place at a distance x from the wall.

Given that $e = \frac{1}{18}$ and the coefficient of restitution between Q and the wall is $\frac{1}{3}$

(c) find x in terms of d.

(6)

(a)



conservation of linear momentum:

$$\underbrace{(3m)(2u) - (5m)(u)}_{\text{before}} = \underbrace{(3m)(v) + (5m)(w)}_{\text{after}}$$

$\therefore u = 3v + 5w$ ①

newton's law of restitution:

$$e = \frac{\text{speed of seperation}}{\text{speed of approach}} = \frac{w - v}{2u + u} = \frac{w + v}{3u}$$

Question 4 continued

$\therefore w - v = 3eu$ ②

Solve ① and ② simultaneously:

From ②: $3w - 3v = 9eu$ ③

① + ③: $8w = u + 9ue$

$\therefore \quad 8w = u(9e+1)$

$\therefore \quad w = \frac{u}{8}(9e+1)$ ④

(b) From ②: $v = w - 3eu$

$v = \frac{u}{8}(9e+1) - 3eu$

$v = \frac{9}{8}eu + \frac{1}{8}u - 3eu$

$v = \frac{u}{8} - \frac{15eu}{8}$

$\therefore v = \frac{u}{8}(1-15e)$ ⑤

$\underline{v>0}$: $\frac{u}{8}(1-15e) > 0$

$15e < 1 \quad \therefore e < \frac{1}{15}$

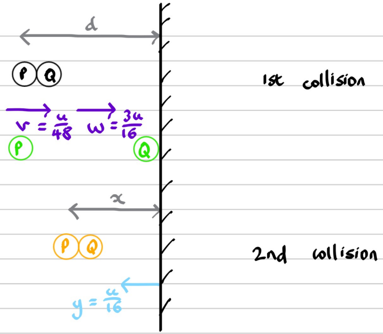
but $e \geq 0$

$\therefore 0 \leq e < \frac{1}{15}$

(c) $\underline{e = \frac{1}{18}}$: $v = \frac{u}{8}\left(1 - 15 \times \frac{1}{18}\right) = \frac{u}{48}$

$w = \frac{u}{8}\left(9 \times \frac{1}{18} + 1\right) = \frac{3u}{16}$

Question 4 continued



$$e = \frac{\text{speed of seperation}}{\text{speed of approach}} = \frac{y}{w} = \frac{1}{3}$$

$$y = \frac{1}{3} w = \frac{1}{3} \times \frac{3u}{16} = \frac{u}{16}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

time for Q to get to 2nd collision

$$= \frac{d}{3u/16} + \frac{x}{u/16} = \frac{16d}{3u} + \frac{16x}{u} = \frac{16d + 48x}{3u}$$

time for P to get to 2nd collision

$$= \frac{d-x}{u/48} = \frac{48(d-x)}{u}$$

both at same place at same time

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Question 4 continued

$$\frac{16d + 48x}{3u} = \frac{48(d-x)}{u}$$

$$\frac{16d + 48x}{3u} = \frac{144d - 144x}{3u}$$

$$16d + 48x = 144d - 144x$$

$$192x = 128d$$

$$\therefore x = \frac{2d}{3}$$