Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 30 minutes

Paper reference **9FM0/3C**

Further Mathematics

Advanced
PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

Pearson

1. A van of mass 900 kg is moving along a straight horizontal road.

At the instant when the speed of the van is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the van is modelled as a force of magnitude $(500 + 7v)$ N.

When the engine of the van is working at a constant rate of 18 kW, the van is moving along the road at a constant speed $V\,\mathrm{m\,s^{-1}}$

(a) Find the value of V.

(5)

Later on, the van is moving up a straight road that is inclined to the horizontal at an angle θ, where $\sin\theta = \frac{1}{21}$

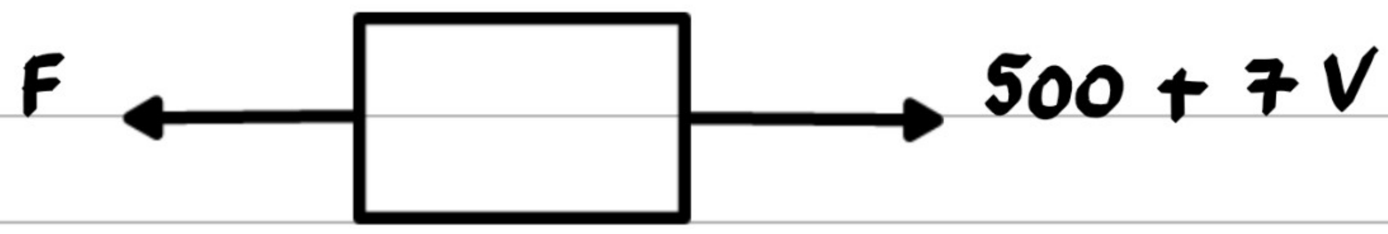
At the instant when the speed of the van is $v\,\mathrm{m\,s^{-1}}$, the resistance to the motion of the van from non-gravitational forces is modelled as a force of magnitude $(500 + 7v)$ N.

The engine of the van is again working at a constant rate of 18 kW.

(b) Find the acceleration of the van at the instant when $v = 15$

(4)

(a)



Σ FORCES = MASS × ACC : $F - (500 + 7V) = 0$ ①

$P = Fv$: $18000 = FV$

$\therefore F = \dfrac{18000}{V}$ ②

② → ① : $\dfrac{18000}{V} - (500 + 7V) = 0$

$18000 - 500V - 7V^2 = 0$

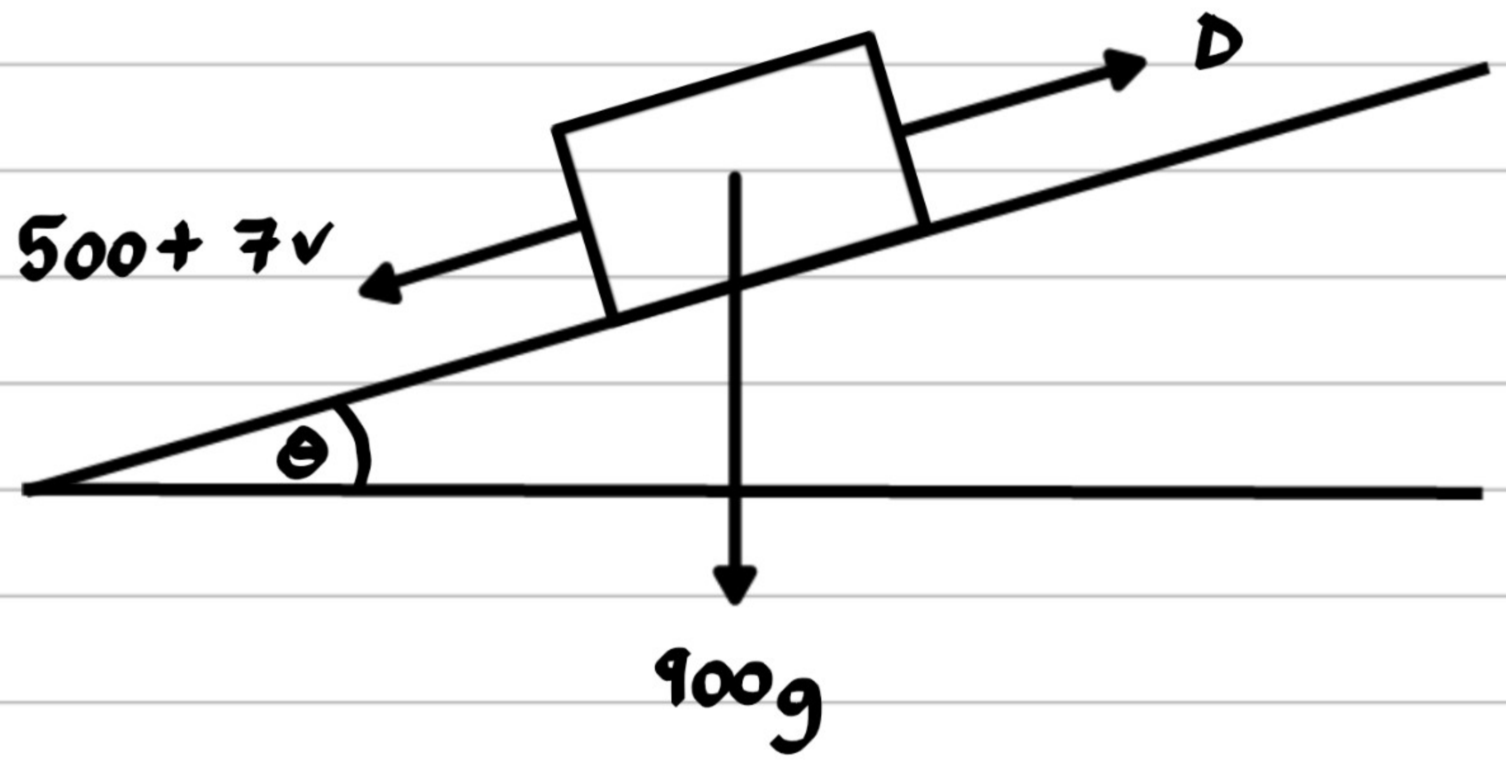
$V = 26.309\ldots, -97.737\ldots$

$\therefore V = 26.3$ (3sf)

(b) $P = Dv$: $18000 = D \times 15$

$\therefore D = 1200$

Question 1 continued



Σ FORCES = MASS × ACC : $D - 900g\sin\theta - (500 + 7v) = 900a$

$1200 - 900g \times \frac{1}{21} - (500 + 7 \times 15) = 900a$

$175 = 900a$

$\therefore a = 0.194 \text{ ms}^{-2}$ (3sf)

2. Two particles, A and B, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.

Particle A has mass $5m$ and particle B has mass $3m$.

The coefficient of restitution between A and B is e, where $e > 0$

Immediately **after** the collision the speed of A is v and the speed of B is $2v$.

Given that A and B are moving in the same direction after the collision,

(a) find the set of possible values of e.

(8)

Given also that the kinetic energy of A immediately after the collision is 16% of the kinetic energy of A immediately before the collision,

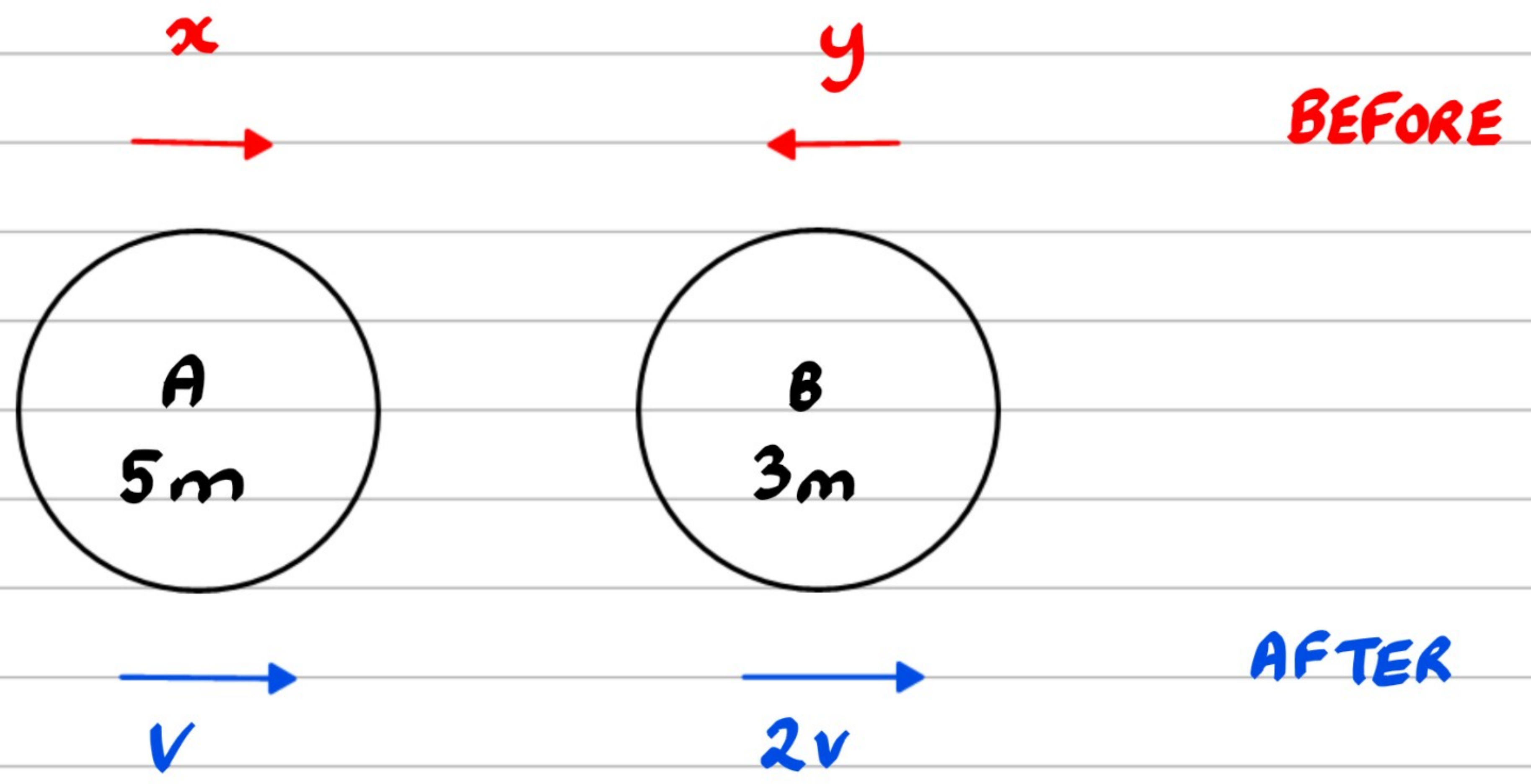
(b) find

(i) the value of e,

(ii) the magnitude of the impulse received by A in the collision, giving your answer in terms of m and v.

(6)

(a)



CLM: $(5m)(x) + (3m)(-y) = (5m)(v) + (3m)(2v)$

$\therefore 5x - 3y = 11v$ ①

NLR: $e = \dfrac{\text{SPEED OF SEPARATION}}{\text{SPEED OF APPROACH}}$

$e = \dfrac{2v - v}{x - -y}$

$e = \dfrac{v}{x+y}$

Question 2 continued

$\therefore v = ex + ey$ ②

① $\times e$: $5ex - 3ey = 11ev$ ③

② $\times 3$: $3ex + 3ey = 3v$ ④

③ + ④ : $8ex = 11ev + 3v$

$\therefore x = \frac{v}{8e}(11e + 3)$ ⑤

⑤ → ② : $v = e \times \frac{v}{8e}(11e + 3) + ey$

$v = \frac{v}{8}(11e + 3) + ey$

$ey = v - \frac{v}{8}(11e + 3)$

$ey = v - \frac{11ve}{8} - \frac{3v}{8}$

$ey = \frac{5v}{8} - \frac{11ve}{8}$

$\therefore y = \frac{v}{8e}(5 - 11e)$ ⑥

$e > 0 \therefore x > 0 \Rightarrow y > 0$

$\therefore \frac{v}{8e}(5 - 11e) > 0$

$\therefore 5 - 11e > 0$

Question 2 continued

$\therefore e < \frac{5}{11}$

$\therefore 0 < e < \frac{5}{11}$

(b)(i) $\frac{1}{2}(5m)(v)^2 = \frac{16}{100} \times \frac{1}{2}(5m)(x)^2$

$v^2 = \frac{16}{100} x^2$

$v^2 = \frac{16}{100} \left\{ \frac{v^2}{64e^2} (11e + 3)^2 \right\}$

$1 = \frac{1}{400e^2} (11e + 3)^2$

$400e^2 = 121e^2 + 66e + 9$

$279e^2 - 66e - 9 = 0$

$(3e - 1)(31e + 3) = 0$

$e = \frac{1}{3}, -\frac{3}{31}$

$\therefore e = \frac{1}{3}$ ONLY

(ii) $|I| = |5m(v - x)|$

$= \left| 5m \left\{ v - \frac{v}{8 \times \frac{1}{3}} (11 \times \frac{1}{3} + 3) \right\} \right|$

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Question 2 continued

$= \left| -\frac{15mv}{2} \right|$

$= \frac{15mv}{2}$

(Total for Question 2 is 14 marks)

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3. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

A smooth uniform sphere P has mass 0.3 kg. Another smooth uniform sphere Q, with the same radius as P, has mass 0.5 kg.

The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision the velocity of P is $(u\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$, where u is a positive constant, and the velocity of Q is $(-4\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$

At the instant when the spheres collide, the line joining their centres is parallel to **i**.

The coefficient of restitution between P and Q is $\dfrac{3}{5}$

As a result of the collision, the direction of motion of P is deflected through an angle of 90° and the direction of motion of Q is deflected through an angle of α°

(a) Find the value of u

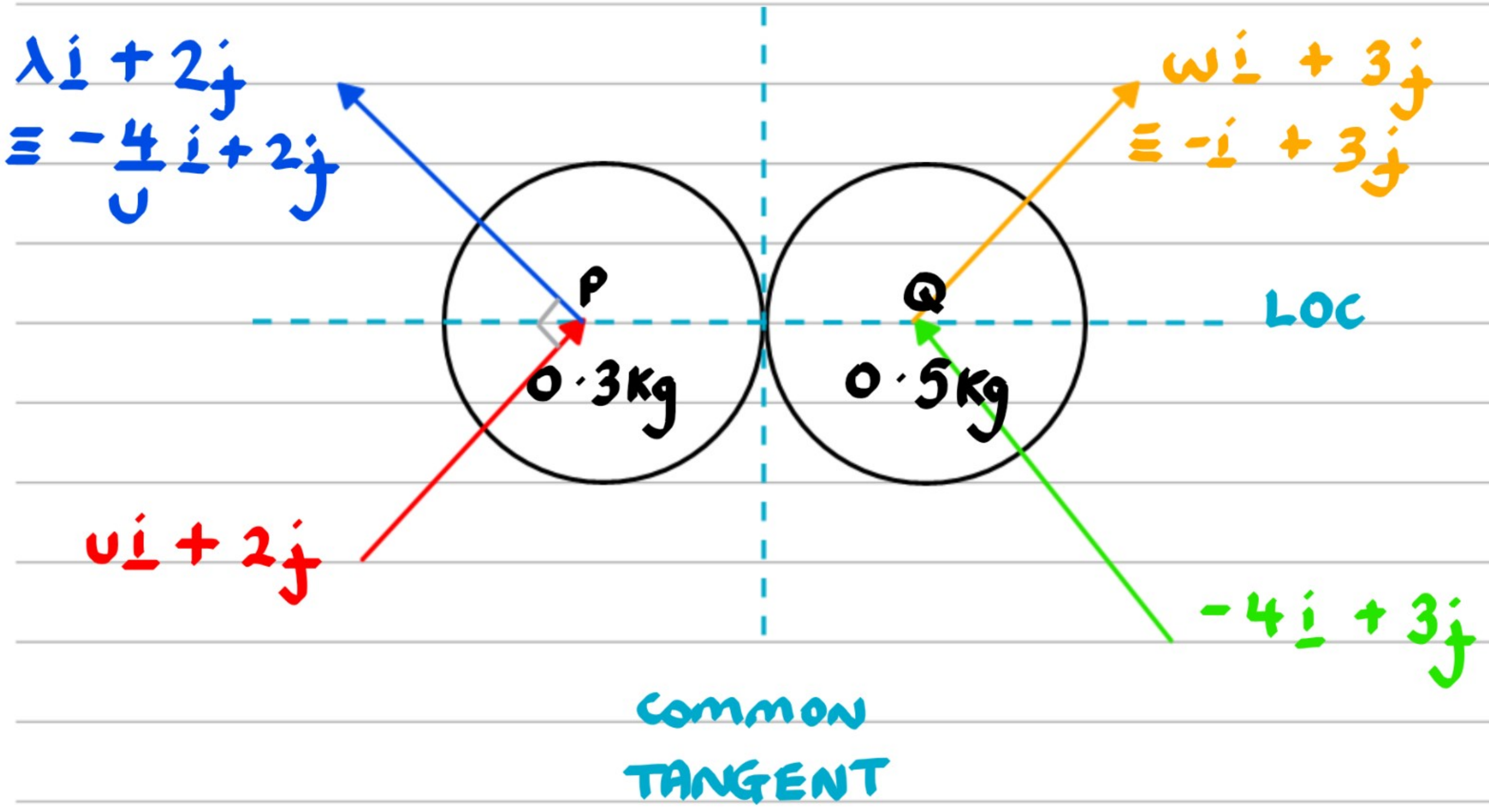
(8)

(b) Find the value of α

(5)

(c) State how you have used the fact that P and Q have equal radii.

(1)



$(u\underline{i} + 2\underline{j}) \cdot (\lambda\underline{i} + 2\underline{j}) = 0$ ∵ DEFLECTED BY 90°

$\Rightarrow \lambda u + 4 = 0 \quad \therefore \lambda = -\dfrac{4}{u}$

CLM (PARALLEL TO LOC):

$(u)(0.3) + (-4)(0.5) = \left(-\dfrac{4}{u}\right)(0.3) + (w)(0.5)$

Question 3 continued

$0.3u - 2 = -\frac{1.2}{u} + 0.5w$

$\therefore 3u + \frac{12}{u} = 20 + 5w$ ①

NLR : $e = \frac{\text{SPEED OF SEPARATION}}{\text{SPEED OF APPROACH}}$

$e = \frac{w - \frac{-4}{u}}{u - -4}$

$\frac{3}{5} = \frac{w + \frac{4}{u}}{u + 4}$

$\frac{3}{5}(u + 4) = w + \frac{4}{u}$

$\therefore \frac{20}{u} + 5w = 3u + 12$ ②

① - ② : $\frac{12}{u} - 12 = 20 - \frac{20}{u}$

$\frac{32}{u} = 32 \quad \therefore u = 1$

i.e. $\frac{20}{(1)} + 5w = 3(1) + 12$

$5w = -5 \quad \therefore w = -1$

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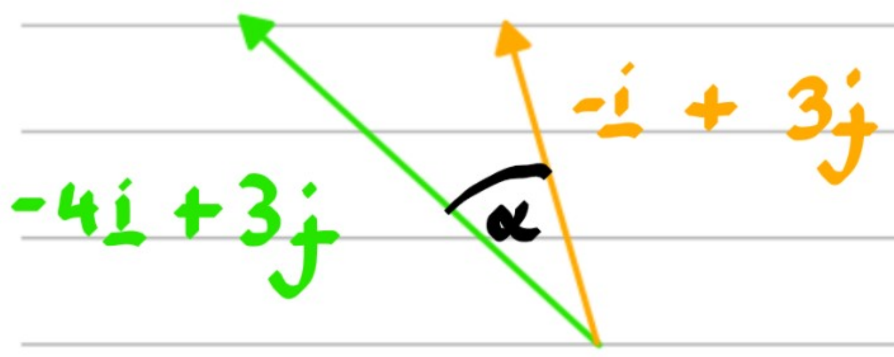
Question 3 continued

(b)



$$\cos\alpha = \frac{(-4\underline{i} + 3\underline{j}) \cdot (-\underline{i} + 3\underline{j})}{\sqrt{(-4)^2 + (3)^2} \times \sqrt{(-1)^2 + (3)^2}}$$

$$= \frac{(-4)(-1) + (3)(3)}{5 \times \sqrt{10}}$$

$$= \frac{13}{5\sqrt{10}}$$

$\therefore \alpha = 34.7^\circ$ (3sf)

(c) THE LOC IS PARALLEL TO THE SPHERES THE SPHERES ARE MOVING ON, SO THE IMPULSE ACTS PARALLEL TO THE SURFACE

4. A particle P has mass 0.5 kg. It is moving in the xy plane with velocity $8\mathbf{i}\,\mathrm{m\,s^{-1}}$ when it receives an impulse $\lambda(-\mathbf{i}+\mathbf{j})\,\mathrm{N\,s}$, where λ is a positive constant.

The angle between the direction of motion of P immediately before receiving the impulse and the direction of motion of P immediately after receiving the impulse is $\theta°$

Immediately after receiving the impulse, P is moving with speed $4\sqrt{10}\,\mathrm{m\,s^{-1}}$

Find (i) the value of λ

(ii) the value of θ

(8)

$-\lambda \underline{i} + \lambda \underline{j} = 0.5(\underline{v} - 8\underline{i})$

$\therefore \underline{v} = (-2\lambda + 8)\underline{i} + 2\lambda \underline{j}$

$|\underline{v}| = 4\sqrt{10} : \sqrt{(-2\lambda+8)^2 + (2\lambda)^2} = 4\sqrt{10}$

$(-2\lambda + 8)^2 + (2\lambda)^2 = 160$

$4\lambda^2 - 32\lambda + 64 + 4\lambda^2 = 160$

$8\lambda^2 - 32\lambda - 96 = 0$

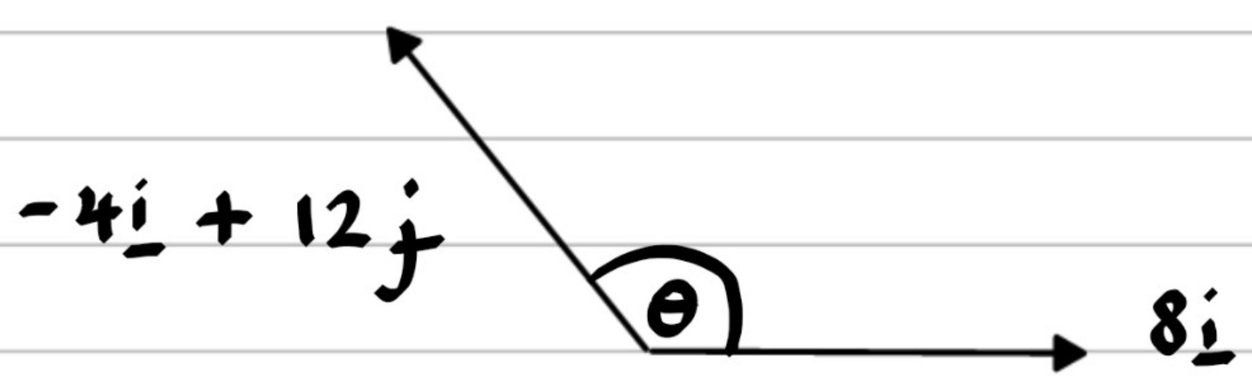
$\lambda^2 - 4\lambda - 12 = 0$

$(\lambda - 6)(\lambda + 2) = 0$

$\lambda = 6, -2$

$\therefore \lambda = 6$ ONLY

$\therefore \underline{v} = -4\underline{i} + 12\underline{j}$



$\cos\theta = \dfrac{(8\underline{i} + 0\underline{j}) \cdot (-4\underline{i} + 12\underline{j})}{\sqrt{8^2 + 0^2} \times \sqrt{(-4)^2 + (12)^2}}$

Question 4 continued

$$= \frac{8 \times -4 + 0 \times 12}{8 \times 4\sqrt{10}}$$

$$= \frac{-32}{32\sqrt{10}}$$

$\therefore \theta = 108^\circ$ (3sf)

(Total for Question 4 is 8 marks)

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5.

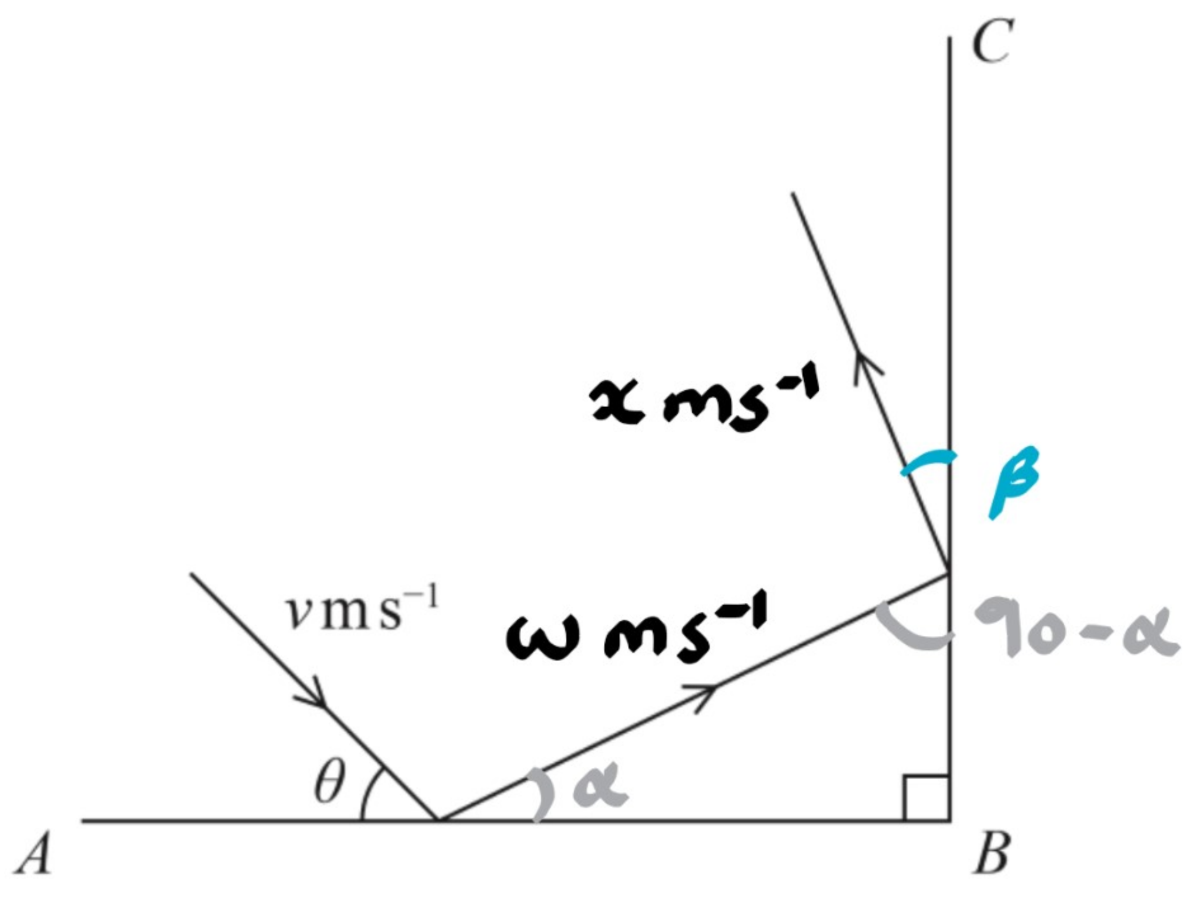

Figure 1

Figure 1 represents the plan view of part of a horizontal floor, where *AB* and *BC* represent fixed vertical walls, with *AB* perpendicular to *BC*.

A small ball is projected along the floor towards the wall *AB*. Immediately before hitting the wall *AB* the ball is moving with speed $v\,\text{m s}^{-1}$ at an angle θ to *AB*.

The ball hits the wall *AB* and then hits the wall *BC*.

The coefficient of restitution between the ball and the wall *AB* is $\frac{1}{3}$

The coefficient of restitution between the ball and the wall *BC* is e.

The floor and the walls are modelled as being smooth.

The ball is modelled as a particle.

The ball loses half of its kinetic energy in the impact with the wall *AB*.

(a) Find the exact value of $\cos\theta$.

(5)

The ball loses half of its remaining kinetic energy in the impact with the wall *BC*.

(b) Find the exact value of e.

(5)

(a) PARALLEL TO AB AFTER 1ST COLLISION:

$w\cos\alpha = v\cos\theta$ ①

PERPENDICULAR TO AB AFTER 1ST COLLISION:

$w\sin\alpha = e\, v\sin\theta$

$w\sin\alpha = \frac{1}{3}\, v\sin\theta$ ②

P 6 6 8 0 0 A 0 1 6 2 8

i.e. $w^2\cos^2\alpha + w^2\sin^2\alpha = v^2\cos^2\theta + \frac{1}{9}v^2\sin^2\theta$

$w^2 = v^2\left(\cos^2\theta + \frac{1}{9}\sin^2\theta\right)$

i.e. $\frac{1}{2} \times \frac{1}{2}mv^2 = \frac{1}{2}mw^2$

$\frac{1}{2}v^2 = v^2\left(\cos^2\theta + \frac{1}{9}\sin^2\theta\right)$

$\frac{1}{2} = \cos^2\theta + \frac{1}{9}(1-\cos^2\theta)$

$\frac{1}{2} = \cos^2\theta + \frac{1}{9} - \frac{1}{9}\cos^2\theta$

$\cos^2\theta = \frac{7}{16}$

$\therefore \cos\theta = \frac{\sqrt{7}}{4}$

(b) PARALLEL TO BC AFTER 2^{ND} COLLISION:

$x\cos\beta = w\cos(90-\alpha)$

$x\cos\beta = w\sin\alpha$

$x\cos\beta = \frac{1}{3}v\sin\theta$ ③

PERPENDICULAR TO BC AFTER 2^{ND} COLLISION:

$x\sin\beta = e\,w\sin(90-\alpha)$

$x\sin\beta = e\,w\cos\alpha$

$x\sin\beta = e\,v\cos\theta$ ④

Question 5 continued

i.e. $x^2\cos^2\beta + x^2\sin^2\beta = \frac{1}{9}v^2\sin^2\theta + e^2v^2\cos^2\theta$

$\therefore\ x^2 = \frac{1}{9}v^2\sin^2\theta + e^2v^2\cos^2\theta$

i.e. $\frac{1}{2}\left(\frac{1}{2}\times\frac{1}{2}mv^2\right) = \frac{1}{2}mx^2$

$\frac{1}{4}v^2 = \frac{1}{9}v^2\sin^2\theta + e^2v^2\cos^2\theta$

$\frac{1}{4} = \frac{1}{9}\sin^2\theta + e^2\cos^2\theta$

$\frac{1}{4} = \frac{1}{9}(1-\cos^2\theta) + e^2\cos^2\theta$

$\frac{1}{4} = \frac{1}{9}\left(1-\frac{7}{16}\right) + e^2\left(\frac{7}{16}\right)$

$e^2 = \frac{3}{7}$

$\therefore e = \sqrt{\frac{3}{7}}$

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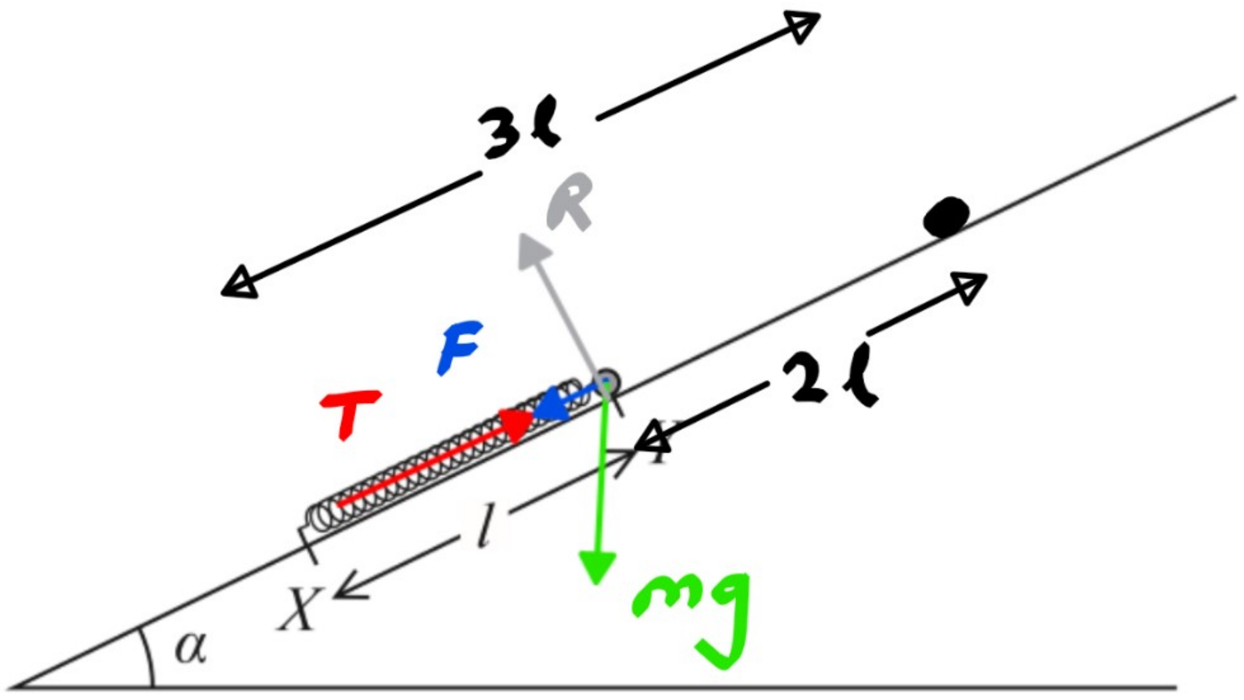
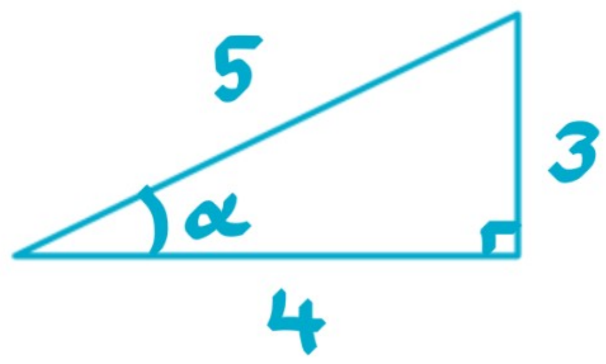
6.





$\sin\alpha = 3/5$

$\cos\alpha = 4/5$

$\tan\alpha = 3/4$

Figure 2

A light elastic spring has natural length $3l$ and modulus of elasticity $3mg$.

One end of the spring is attached to a fixed point X on a rough inclined plane.

The other end of the spring is attached to a package P of mass m.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

COMPRESSION = $2l$

The package is initially held at the point Y on the plane, where $XY = l$. The point Y is higher than X and XY is a line of greatest slope of the plane, as shown in Figure 2.

The package is released from rest at Y and moves up the plane.

The coefficient of friction between P and the plane is $\frac{1}{3}$

By modelling P as a particle,

(a) show that the acceleration of P at the instant when P is released from rest is $\frac{17}{15}g$

(5)

(b) find, in terms of g and l, the speed of P at the instant when the spring first reaches its natural length of $3l$.

(6)

(a) $\perp$ TO PLANE : $R = mg\cos\alpha$

$\therefore R = \frac{4mg}{5}$

$F = \mu R$: $F = \frac{1}{3} \times \frac{4mg}{5}$

$\therefore F = \frac{4mg}{15}$

Question 6 continued

THRUST, $T = \frac{3mg \times 2l}{3l}$

$\therefore T = 2mg$

Σ FORCES = MASS × ACC: $T - F - mg\sin\alpha = ma$

$$2mg - \frac{4mg}{15} - \frac{3mg}{5} = ma$$

$\therefore a = \frac{17g}{15}\ ms^{-2}$

(b) INITIAL EPE $= \frac{3mg \times (2l)^2}{2 \times (3l)} = 2mgl$

FINAL GPE $= 2l\sin\alpha \times mg = 2l \times \frac{3}{5} \times mg = \frac{6mgl}{5}$

FINAL KE $= \frac{1}{2}mv^2$

WD AGAINST FRICTION $= F \times 2l = \frac{4mg}{15} \times 2l = \frac{8mgl}{15}$

WORK-ENERGY PRINCIPLE: $2mgl - \frac{6mgl}{5} - \frac{1}{2}mv^2 = \frac{8mgl}{15}$

$$\frac{1}{2}v^2 = 2gl - \frac{6gl}{5} - \frac{8gl}{15}$$

$$v^2 = \frac{8gl}{15}$$

$\therefore v = \sqrt{\frac{8gl}{15}}\ ms^{-1}$

P 6 6 8 0 0 A 0 2 1 2 8

7. [*In this question,* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane.*]

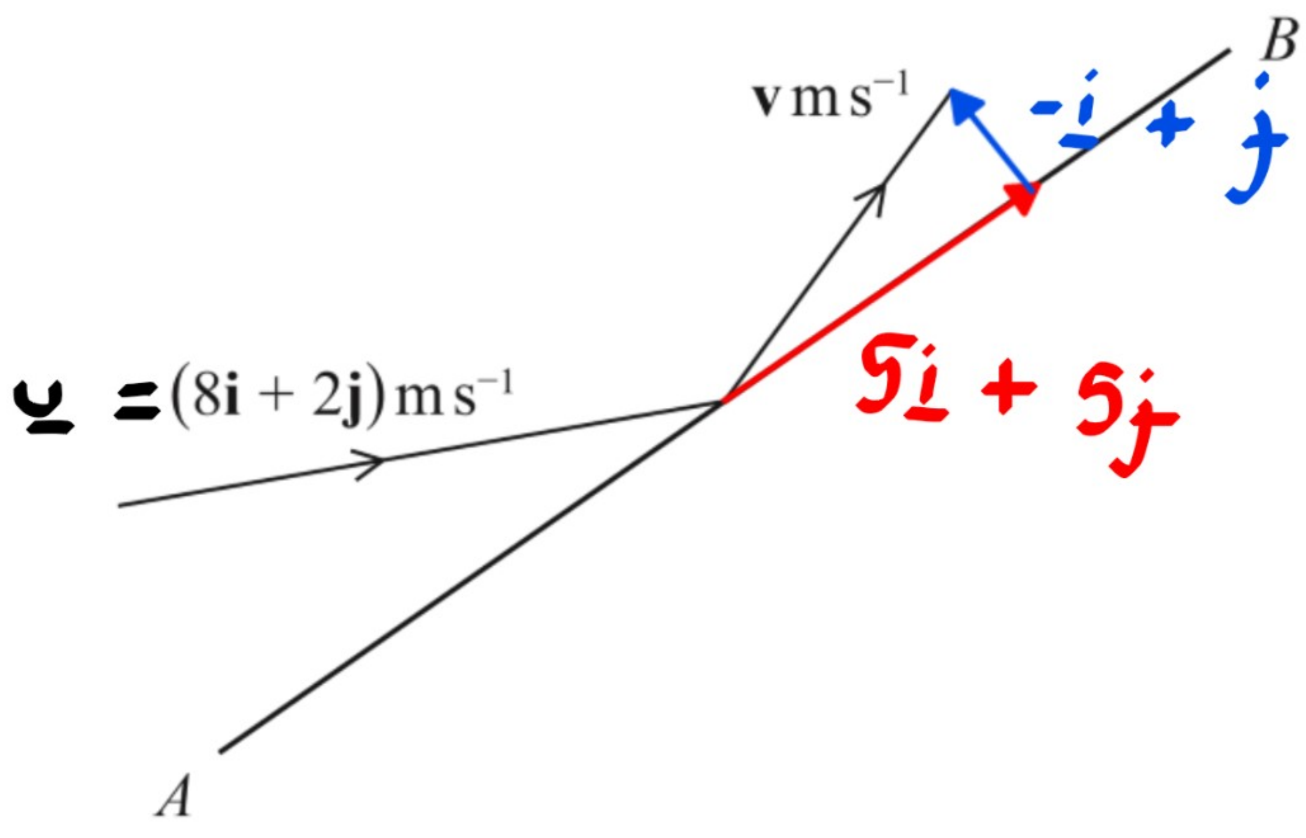


Figure 3

Figure 3 represents the plan view of part of a smooth horizontal floor, where AB is a fixed smooth vertical wall.

The direction of $\overrightarrow{AB}$ is in the direction of the vector $(\mathbf{i} + \mathbf{j})$

A small ball of mass 0.25 kg is moving on the floor when it strikes the wall AB.

Immediately before its impact with the wall AB, the velocity of the ball is $(8\mathbf{i} + 2\mathbf{j})\,\mathrm{m\,s^{-1}}$

Immediately after its impact with the wall AB, the velocity of the ball is $\mathbf{v}\,\mathrm{m\,s^{-1}}$

The coefficient of restitution between the ball and the wall is $\frac{1}{3}$

By modelling the ball as a particle,

(a) show that $\mathbf{v} = 4\mathbf{i} + 6\mathbf{j}$

(6)

(b) Find the magnitude of the impulse received by the ball in the impact.

(3)

(a) UNIT VECTOR PARALLEL TO AB : $\dfrac{\underline{i} + \underline{j}}{\sqrt{2}}$

UNIT VECTOR PERPENDICULAR TO AB : $\dfrac{-\underline{i} + \underline{j}}{\sqrt{2}}$

// TO AB : $\left[\dfrac{(\underline{i} + \underline{j})}{\sqrt{2}} \cdot (8\underline{i} + 2\underline{j})\right] \times \dfrac{\underline{i} + \underline{j}}{\sqrt{2}}$

$= \left\{\dfrac{1 \times 8 + 1 \times 2}{\sqrt{2}}\right\} \times \dfrac{\underline{i} + \underline{j}}{\sqrt{2}}$

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Question 7 continued

$= 5\underline{i} + 5\underline{j}$

$\perp$ To AB : $-\frac{1}{3}\left[\frac{(-\underline{i}+\underline{j})}{\sqrt{2}}\cdot(8\underline{i}+2\underline{j})\right]\times\frac{-\underline{i}+\underline{j}}{\sqrt{2}}$

$= -\frac{1}{3}\left\{\frac{-1\times 8 + 1\times 2}{\sqrt{2}}\right\}\times\frac{-\underline{i}+\underline{j}}{\sqrt{2}}$

$= -\underline{i} + \underline{j}$

$\therefore \underline{v} = (5\underline{i} + 5\underline{j}) + (-\underline{i} + \underline{j})$

$\therefore \underline{v} = 4\underline{i} + 6\underline{j}$

(b) $\underline{I} = 0.25(\underline{v} - \underline{u})$

$= 0.25[(4\underline{i} + 6\underline{j}) - (8\underline{i} + 2\underline{j})]$

$= 0.25(-4\underline{i} + 4\underline{j})$

$= -\underline{i} + \underline{j}$

$\therefore |\underline{I}| = \sqrt{(-1)^2 + (1)^2}$

$= \sqrt{2}$ Ns