Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 1 hour 40 minutes

Paper reference **8FM0/01**

Further Mathematics

Advanced Subsidiary
PAPER 1: Core Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

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1. $\mathbf{P} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\qquad$ $\mathbf{Q} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

(a) (i) Describe fully the single geometrical transformation P represented by the matrix $\mathbf{P}$.

(ii) Describe fully the single geometrical transformation Q represented by the matrix $\mathbf{Q}$.

(4)

The transformation P followed by the transformation Q is the transformation R, which is represented by the matrix $\mathbf{R}$.

(b) Determine $\mathbf{R}$.

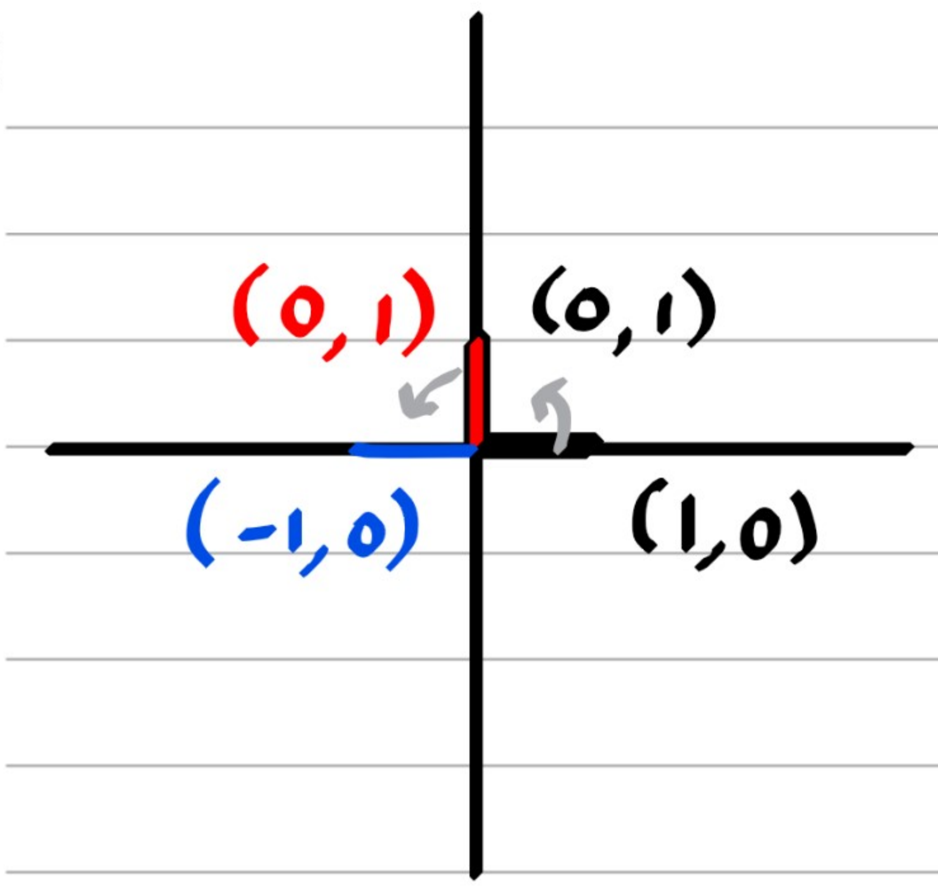
(1)

(c) (i) Evaluate the determinant of $\mathbf{R}$.

(ii) Explain how the value obtained in (c)(i) relates to the transformation R.

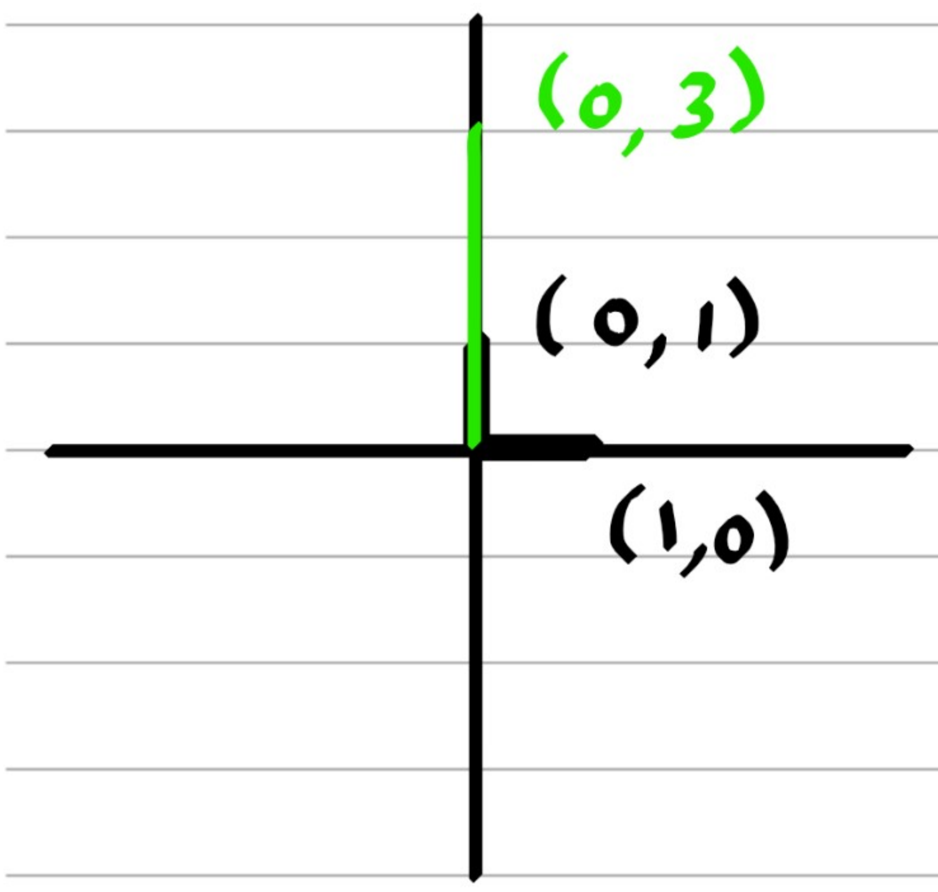
(2)

(a)(i)



ROTATION 90° ACW ABOUT ORIGIN

(ii)



STRETCH SF 3 PARALLEL TO y-AXIS

(b) $R = QP = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 3 & 0 \end{pmatrix}$

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Question 1 continued

(c)(i) $\det(R) = \begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} = 0 \times 0 - -1 \times 3 = 3$

(ii) AREA SCALE FACTOR OF TRANSFORMATION

(Total for Question 1 is 7 marks)

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2. The cubic equation

$$9x^3 - 5x^2 + 4x + 7 = 0$$

has roots α, β and γ.

Without solving the equation, find the cubic equation whose roots are $(3\alpha - 2)$, $(3\beta - 2)$ and $(3\gamma - 2)$, giving your answer in the form $aw^3 + bw^2 + cw + d = 0$, where a, b, c and d are integers to be determined.

(5)

LET $w = 3x - 2 \quad \therefore x = \frac{w+2}{3}$

i.e. $9\left(\frac{w+2}{3}\right)^3 - 5\left(\frac{w+2}{3}\right)^2 + 4\left(\frac{w+2}{3}\right) + 7 = 0$

$\frac{(w+2)^3}{3} - \frac{5(w+2)^2}{9} + \frac{4(w+2)}{3} + 7 = 0$

$\frac{w^3 + 6w^2 + 12w + 8}{3} - \frac{5(w^2 + 4w + 4)}{9} + \frac{4(w+2)}{3} + 7 = 0$

$\frac{w^3}{3} + 2w^2 + 4w + \frac{8}{3} - \frac{5w^2}{9} - \frac{20w}{9} - \frac{20}{9} + \frac{4w}{3} + \frac{8}{3} + 7 = 0$

$\frac{w^3}{3} + \frac{13w^2}{9} + \frac{28w}{9} + \frac{91}{9} = 0$

$\therefore 3w^3 + 13w^2 + 28w + 91 = 0$

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3. (a) Use the standard results for summations to show that for all positive integers n

$$\sum_{r=1}^{n}(5r-2)^2 = \frac{1}{6}n(an^2+bn+c)$$

where a, b and c are integers to be determined.

(5)

(b) Hence determine the value of k for which

$$\sum_{r=1}^{k}(5r-2)^2 = 94k^2$$

(4)

(a) $(5r-2)^2 \equiv 25r^2 - 20r + 4$

$\therefore \sum_{r=1}^{n}(25r^2 - 20r + 4) \equiv 25\sum_{r=1}^{n} r^2 - 20\sum_{r=1}^{n} r + 4\sum_{r=1}^{n} 1$

$= 25 \times \frac{1}{6}n(n+1)(2n+1) - 20 \times \frac{1}{2}n(n+1) + 4n$

$= \frac{25}{6}n(n+1)(2n+1) - 10n(n+1) + 4n$

$= \frac{1}{6}n\left[25(n+1)(2n+1) - 60(n+1) + 24\right]$

$= \frac{1}{6}n\left[25(2n^2+3n+1) - 60n - 60 + 24\right]$

$= \frac{1}{6}n\left[50n^2 + 75n + 25 - 60n - 60 + 24\right]$

$= \frac{1}{6}n(50n^2 + 15n - 11)$

(b) $\sum_{r=1}^{k}(5r-2)^2 = 94k^2$

$\frac{1}{6}k(50k^2 + 15k - 11) = 94k^2$

$50k^3 + 15k^2 - 11k = 564k^2$

$50k^3 - 549k^2 - 11k = 0$

$k(50k^2 - 549k - 11) = 0$

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Question 3 continued

$K(K-11)(50K+1)=0$

$K=0,\ K=11,\ K=-\frac{1}{50}$

$\therefore K=11$ IS ONLY SOLUTION

(Total for Question 3 is 9 marks)

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4.
$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 4 \\ k & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix} \qquad \mathbf{N} = \begin{pmatrix} k-7 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$$

where k is a constant.

(a) Determine, in simplest form in terms of k, the matrix $\mathbf{MN}$.

(2)

(b) Given that $k = 5$

(i) write down $\mathbf{MN}$

(ii) hence write down $\mathbf{M}^{-1}$

(2)

(c) Solve the simultaneous equations

$$\begin{aligned} 2x + y + 4z &= 2 \\ 5x + 2y - 2z &= 3 \\ 4x + y - 2z &= -1 \end{aligned}$$

(2)

(d) Interpret the answer to part (c) geometrically.

(1)

(a)
$$MN = \begin{pmatrix} 2 & 1 & 4 \\ K & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix}\begin{pmatrix} K-7 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 2(K-7)+1(2)+4(-3) & 2(6)+1(-20)+4(2) & 2(-10)+1(24)+4(-1) \\ K(K-7)+2(2)-2(-3) & K(6)+2(-20)-2(2) & K(-10)+2(24)-2(-1) \\ 4(K-7)+1(2)-2(-3) & 4(6)+1(-20)+2(2) & 4(-10)+1(24)-2(-1) \end{pmatrix}$$

$$\therefore MN = \begin{pmatrix} 2K-24 & 0 & 0 \\ K^2-7K+10 & 6K-44 & -10K+50 \\ 4K-20 & 0 & -14 \end{pmatrix}$$

(b) $K = 5$:

(i)
$$MN = \begin{pmatrix} -14 & 0 & 0 \\ 0 & -14 & 0 \\ 0 & 0 & -14 \end{pmatrix}$$

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Question 4 continued

(ii) $M = \begin{pmatrix} 2 & 1 & 4 \\ 5 & 2 & -2 \\ 4 & 2 & -2 \end{pmatrix}$

$\therefore \; M^{-1} = -\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}$

(c)
$$\begin{aligned} 2x + y + 4z &= 2 \\ 5x + 2y - 2z &= 3 \\ 4x + y - 2z &= -1 \end{aligned}$$

$$\begin{pmatrix} 2 & 1 & 4 \\ 5 & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 1 & 4 \\ 5 & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix}^{-1}\begin{pmatrix} 2 & 1 & 4 \\ 5 & 2 & -2 \\ 4 & 2 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & 1 & 4 \\ 5 & 2 & -2 \\ 4 & 1 & -2 \end{pmatrix}^{-1}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{14}\begin{pmatrix} -2 & 6 & -10 \\ 2 & -20 & 24 \\ -3 & 2 & -1 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -12/7 \\ 40/7 \\ -1/14 \end{pmatrix}$$

$$\therefore \left(-\frac{12}{7}, \frac{40}{7}, -\frac{1}{14}\right)$$

(d) THE COORDINATE OF THE ONLY POINT AT WHICH THE PLANES REPRESENTED BY THE EQUATIONS IN PART (c)

5.

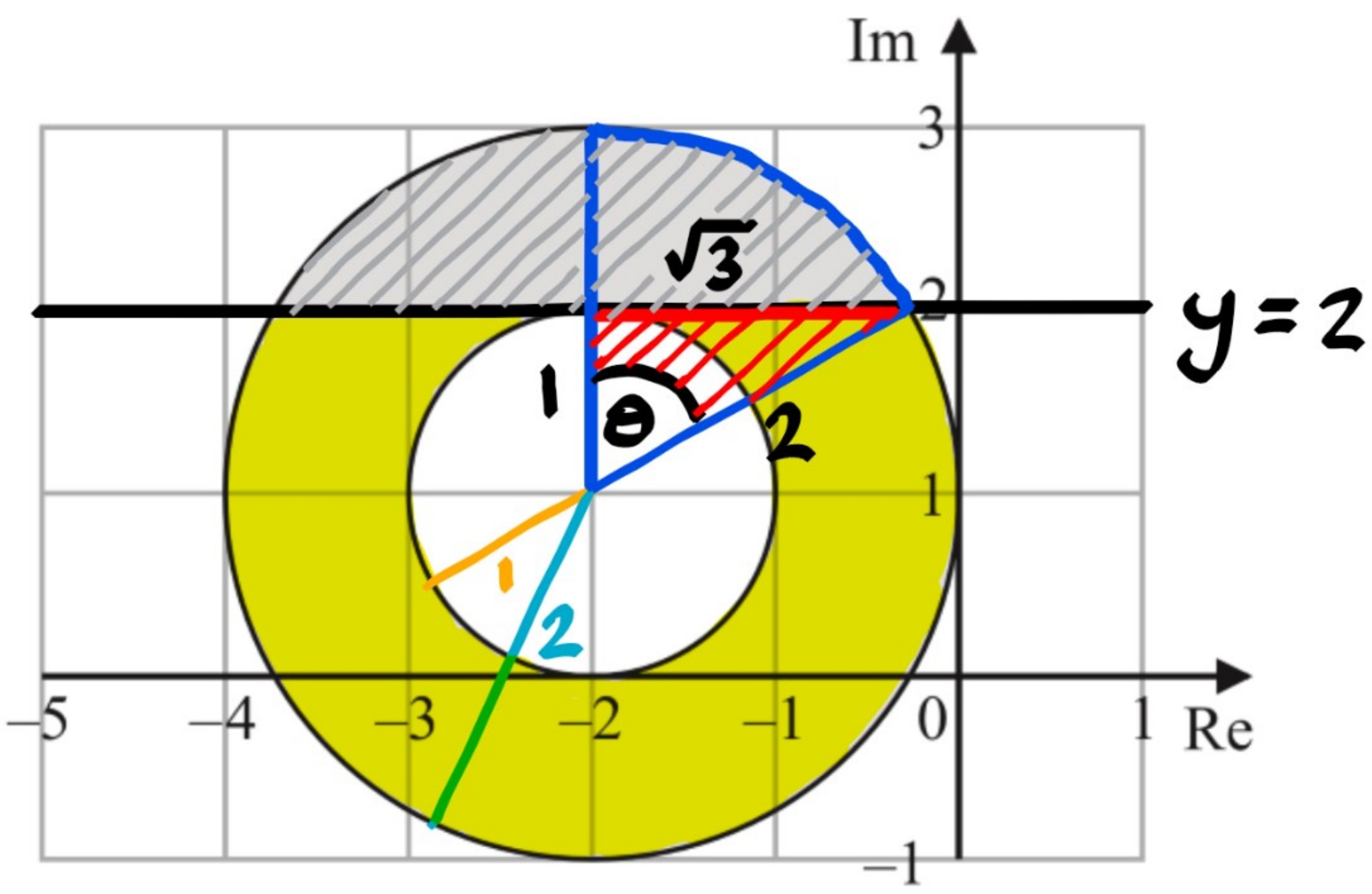

Figure 1

Figure 1 shows an Argand diagram.

The set P, of points that lie within the shaded region including its boundaries, is defined by

$$P = \{z \in \mathbb{C} : a \leqslant |z + b + c\mathrm{i}| \leqslant d\}$$

where a, b, c and d are integers.

(a) Write down the values of a, b, c and d.

(3)

The set Q is defined by

$$Q = \{z \in \mathbb{C} : a \leqslant |z + b + c\mathrm{i}| \leqslant d\} \cap \{z \in \mathbb{C} : |z - \mathrm{i}| \leqslant |z - 3\mathrm{i}|\}$$

(b) Determine the exact area of the region defined by Q, giving your answer in simplest form.

(7)

(a) $\{z \in \mathbb{C} : a \leqslant |z - (-b - c\mathrm{i})| \leqslant d\}$

BY INSPECTION: $a = 1$, $b = 2$, $c = -1$, $d = 2$

(b) $|z - \mathrm{i}| = |z - 3\mathrm{i}|$

$|z - (0 + \mathrm{i})| = |z - (0 + 3\mathrm{i})|$

PERPENDICULAR BISECTOR IS: $y = 2$

$\theta = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

AREA OF TRIANGLE $= \frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$

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Question 5 continued

AREA OF SECTOR $= \frac{1}{2} \times 2^2 \times \frac{\pi}{3} = \frac{2\pi}{3}$

AREA BETWEEN CIRCLES $= \pi \times 2^2 - \pi \times 1^2 = 3\pi$

SHADED AREA $= 2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) = \frac{4\pi}{3} - \sqrt{3}$

$\therefore$ REQUIRED AREA $= 3\pi - \left(\frac{4\pi}{3} - \sqrt{3}\right)$

$= \frac{5\pi}{3} + \sqrt{3}$ UNITS^2

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6. A mining company has identified a mineral layer below ground.

The mining company wishes to drill down to reach the mineral layer and models the situation as follows.

With respect to a fixed origin O,

- the ground is modelled as a horizontal plane with equation $z = 0$
- the mineral layer is modelled as part of the plane containing the points $A(10, 5, -50)$, $B(15, 30, -45)$ and $C(-5, 20, -60)$, where the units are in metres

(a) Determine an equation for the plane containing A, B and C, giving your answer in the form $\mathbf{r}.\mathbf{n} = d$
(5)

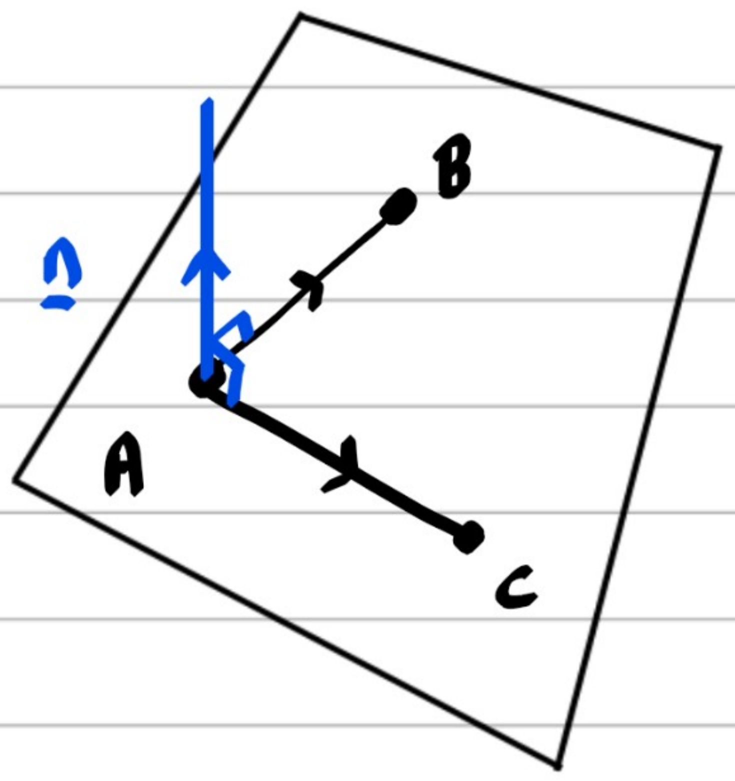
(b) Determine, according to the model, the acute angle between the ground and the plane containing the mineral layer. Give your answer to the nearest degree.
(3)

The mining company plans to drill vertically downwards from the point (5, 12, 0) on the ground to reach the mineral layer.

(c) Using the model, determine, in metres to 1 decimal place, the distance the mining company will need to drill in order to reach the mineral layer.
(2)

(d) State a limitation of the assumption that the mineral layer can be modelled as a plane.
(1)

(a)



$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (15\underline{i} + 30\underline{j} - 45\underline{k}) - (10\underline{i} + 5\underline{j} - 50\underline{k})$

$\therefore \overrightarrow{AB} = 5\underline{i} + 25\underline{j} + 5\underline{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (-5\underline{i} + 20\underline{j} - 60\underline{k}) - (10\underline{i} + 5\underline{j} - 50\underline{k})$

$\therefore \overrightarrow{AC} = -15\underline{i} + 15\underline{j} - 10\underline{k}$

LET NORMAL VECTOR $\underline{n} = a\underline{i} + b\underline{j} + c\underline{k}$

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Question 6 continued

$\vec{AB} \cdot \underline{n} = 0 : (5\underline{i} + 25\underline{j} + 5\underline{k}) \cdot (a\underline{i} + b\underline{j} + c\underline{k}) = 0$

$5a + 25b + 5c = 0$

$\therefore a + 5b + c = 0$ ①

$\vec{AC} \cdot \underline{n} = 0 : (-15\underline{i} + 15\underline{j} - 10\underline{k}) \cdot (a\underline{i} + b\underline{j} + c\underline{k}) = 0$

$-15a + 15b - 10c = 0$

$\therefore -3a + 3b - 2c = 0$ ②

LET $c = 1$: $a + 5b + 1 = 0 \quad \therefore a + 5b = -1$ ③

$-3a + 3b - 2(1) = 0 \quad \therefore -3a + 3b = 2$ ④

SOLVE ③ AND ④ SIMULTANEOUSLY:

$\therefore a = -\frac{13}{18}, \quad b = -\frac{1}{18}, \quad c = 1$

$\therefore \underline{n} = -\frac{13}{18}\underline{i} - \frac{1}{18}\underline{j} + \underline{k}$

$d = -\underline{a} \cdot \underline{n} : d = -(10\underline{i} + 5\underline{j} - 50\underline{k}) \cdot (-\frac{13}{18}\underline{i} - \frac{1}{18}\underline{j} + \underline{k})$

$\therefore d = -(-\frac{115}{2})$

$\therefore d = \frac{115}{2}$

i.e. $\underline{r} \cdot (-\frac{13}{18}\underline{i} - \frac{1}{18}\underline{j} + \underline{k}) + \frac{115}{2} = 0$

$\therefore \Pi_1 : \underline{r} \cdot (-13\underline{i} - \underline{j} + 18\underline{k}) = -1035$ ⑤

(b) $\Pi_2 : \underline{r} \cdot (0\underline{i} + 0\underline{j} + \underline{k}) = 0$ ⑥ (EQN OF PLANE $z = 0$)

REQUIRE ACUTE ANGLE BETWEEN ⑤ AND ⑥

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Question 6 continued

$$\cos\alpha = \left|\frac{\underline{n}_1 \cdot \underline{n}_2}{|\underline{n}_1| \times |\underline{n}_2|}\right|$$

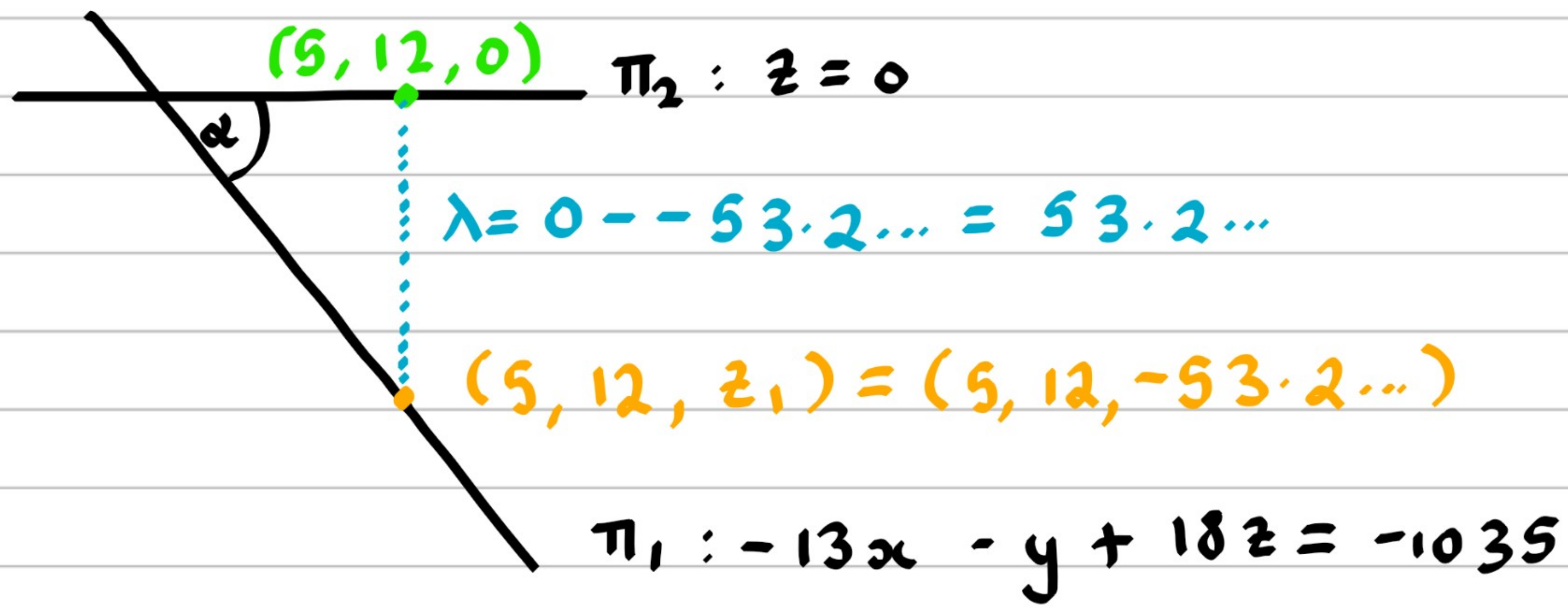
$$\cos\alpha = \left|\frac{(-13\underline{i} - \underline{j} + 18\underline{k}) \cdot (0\underline{i} + 0\underline{j} + \underline{k})}{\sqrt{(-13)^2 + (-1)^2 + (18)^2}\ \sqrt{(0)^2 + (0)^2 + (1)^2}}\right| =$$

$$= 0.809\ldots$$

$$\alpha = 35.9\ldots$$

$\therefore \alpha = 36°$ (NEAREST DEGREE)

(c)



$(5, 12, z_1)$: $-13(5) - (12) + 18(z_1) = -1035$

$$-77 + 18z_1 = -1035$$

$$18z_1 = -958$$

$$\therefore z_1 = -53.2\ldots$$

$\therefore$ DISTANCE, $\lambda = 53.2$ m (1dp)

(d) THE MINERAL LAYER WILL NOT BE PERFECTLY FLAT / SMOOTH AND WILL NOT FORM A PLANE

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7. $$f(z) = z^4 - 6z^3 + pz^2 + qz + r$$

where p, q and r are real constants.

The roots of the equation $f(z) = 0$ are α, β, γ and δ where $\alpha = 3$ and $\beta = 2 + i$

Given that γ is a complex root of $f(z) = 0$

(a) (i) write down the root γ,

(ii) explain why δ must be real.

(2)

(b) Determine the value of δ.

(2)

(c) Hence determine the values of p, q and r.

(3)

(d) Write down the roots of the equation $f(-2z) = 0$

(2)

(a)(i) $\gamma = 2 - i$

(ii) ROOTS OF POLYNOMIALS WITH REAL COEFFICIENTS OCCUR IN CONJUGATE PAIRS

β AND γ FORM A CONJUGATE PAIR

AS α IS REAL, IT FOLLOWS THAT δ MUST BE REAL

(b) SUM OF ROOTS $= \alpha + \beta + \gamma + \delta = -(-6) = 6$

$$(3) + (2 + i) + (2 - i) + \delta = 6$$

$$7 + \delta = 6$$

$\therefore \delta = -1$

(c) $f(z) = z^4 - 6z^3 + pz^2 + qz + r = 0$:

$(z - \alpha)(z - \beta)(z - \gamma)(z - \delta) = 0$

$\alpha = 3$, $\beta = 2 + i$, $\gamma = 2 - i$, $\delta = -1$

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Question 7 continued

i.e. $(z-3)(z-(2+i))(z-(2-i))(z-(-1))=0$

$(z-3)(z+1)[(z-2)-(i)][(z-2)+(i)]=0$

$(z^2-2z-3)(z^2-4z+4--1)=0$

$(z^2-2z-3)(z^2-4z+5)=0$

$z^4-4z^3+5z^2-2z^3+8z^2-10z-3z^2+12z-15=0$

$z^4-6z^3+10z^2+2z-15=0$

COMPARE WITH $z^4-6z^3+pz^2+qz+r=0$:

$\therefore p=10,\ q=2,\ r=-15$

(d) ROOTS OF $f(z)=0$: $z=3,-1,2+i,2-i$

$\therefore$ ROOTS OF $f(-2z)=0$: $z=-\frac{3}{2},\frac{1}{2},-1\pm\frac{1}{2}i$

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WANT TO SHOW FOR END RESULT:
$\frac{1}{2}(K+1)(K+1+1)^2(K+1+2)$

8. (a) Prove by induction that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+1)(2r+1) = \frac{1}{2}n(n+1)^2(n+2)$$

(6)

(b) Hence, show that, for all positive integers n,

$$\sum_{r=n}^{2n} r(r+1)(2r+1) = \frac{1}{2}n(n+1)(an+b)(cn+d)$$

where a, b, c and d are integers to be determined.

(3)

(a) $n=1$: LHS $= \sum_{r=1}^{1} r(r+1)(2r+1) = 1(1+1)(2\times1+1) = 6$

RHS $= \frac{1}{2}\times1\times(1+1)^2\times(1+2) = 6$

$\therefore$ TRUE FOR $n=1$

ASSUME TRUE FOR $n=K$:

$\sum_{r=1}^{K} r(r+1)(2r+1) = \frac{1}{2}K(K+1)^2(K+2)$

TRY FOR $n=K+1$:

$\sum_{r=1}^{K+1} r(r+1)(2r+1) = \sum_{r=1}^{K} r(r+1)(2r+1) + (K+1)(K+1+1)$
$\times(2(K+1)+1)$

$= \frac{1}{2}K(K+1)^2(K+2) + (K+1)(K+1+1)(2(K+1)+1)$

$= \frac{1}{2}K(K+1)^2(K+2) + (K+1)(K+2)(2K+3)$

$= \frac{1}{2}(K+1)(K+2)[K(K+1) + 2(2K+3)]$

$= \frac{1}{2}(K+1)(K+2)(K^2+K+4K+6)$

$= \frac{1}{2}(K+1)(K+2)(K^2+5K+6)$

$= \frac{1}{2}(K+1)(K+2)(K+2)(K+3)$

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Question 8 continued

$= \frac{1}{2}(k+1)(k+2)^2(k+3)$

$= \frac{1}{2}(k+1)(k+1+1)^2(k+1+2)$

IF STATEMENT IS TRUE FOR $n=k$, THEN IT HAS BEEN SHOWN TRUE FOR $n=k+1$. AS TRUE FOR $n=1$, THEN TRUE FOR $n \in \mathbb{Z}^+$

(b) $f(r) = r(r+1)(2r+1)$

$$\sum_{r=n}^{2n} f(r) = \sum_{r=1}^{2n} f(r) - \sum_{r=1}^{n-1} f(r)$$

$= \frac{1}{2}(2n)(2n+1)^2(2n+2) - \frac{1}{2}(n-1)(n-1+1)^2(n-1+2)$

$= \frac{1}{2}(2n)(2n+1)^2(2n+2) - \frac{1}{2}(n-1)(n)^2(n+1)$

$= \frac{1}{2}(2n)(2n+1)^2(n+1) - \frac{1}{2}(n-1)(n)^2(n+1)$

$= \frac{1}{2}n(n+1)[4(2n+1)^2 - n(n-1)]$

$= \frac{1}{2}n(n+1)[4(4n^2+4n+1) - n^2 + n]$

$= \frac{1}{2}n(n+1)(16n^2+16n+4-n^2+n)$

$= \frac{1}{2}n(n+1)(15n^2+17n+4)$

$= \frac{1}{2}n(n+1)(3n+1)(5n+4)$

9.

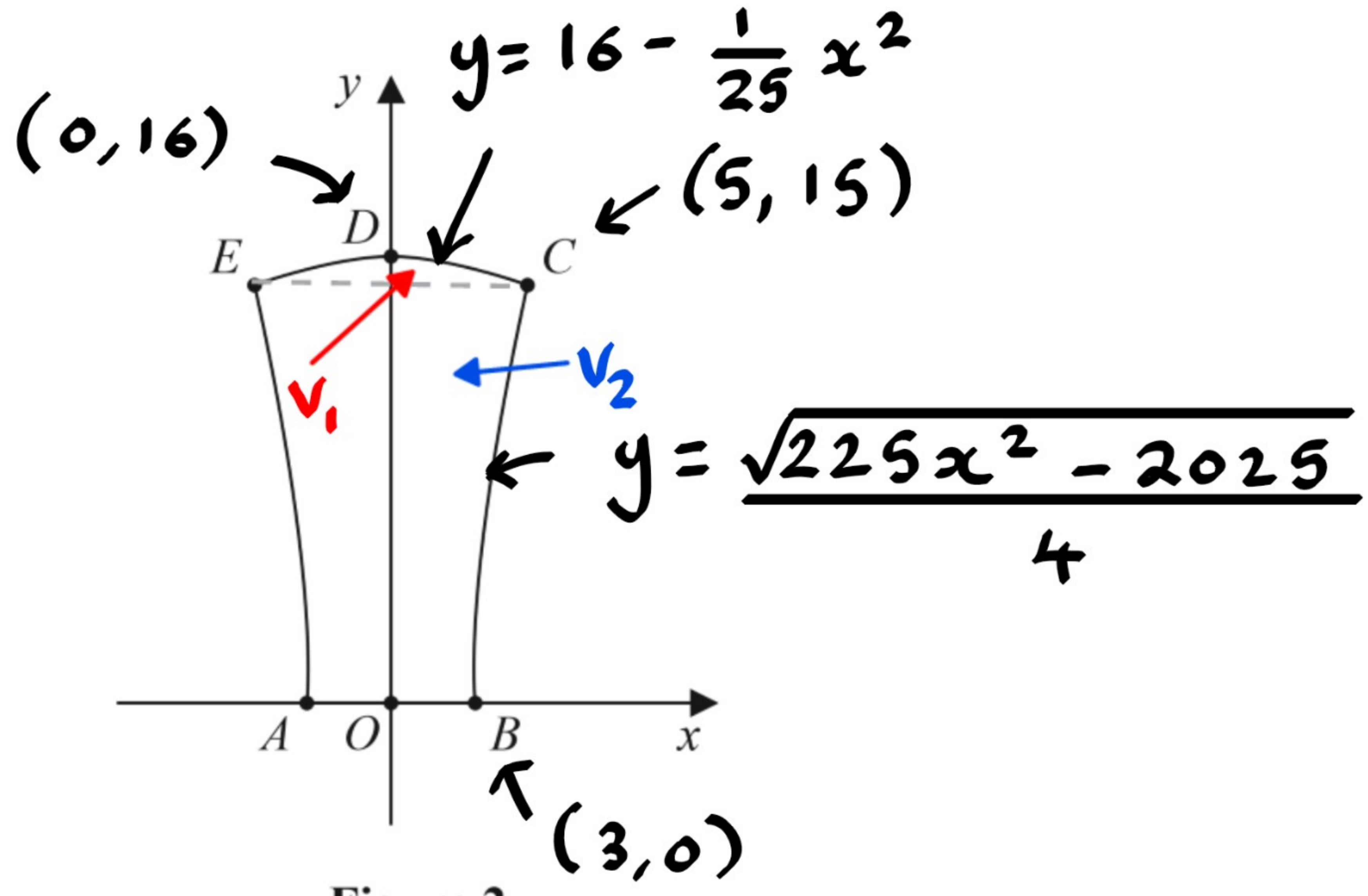

Figure 2

Figure 2 shows the vertical cross-section, *AOBCDE*, through the centre of a wax candle.

In a model, the candle is formed by rotating the region bounded by the y-axis, the line OB, the curve BC, and the curve CD through 360° about the y-axis.

The point B has coordinates (3, 0) and the point C has coordinates (5, 15).

The units are in centimetres.

The curve BC is represented by the equation

$$y = \frac{\sqrt{225x^2 - 2025}}{a} \qquad 3 \leqslant x < 5$$

where a is a constant.

(a) Determine the value of a according to this model.

(2)

The curve CD is represented by the equation

$$y = 16 - 0.04x^2 \qquad 0 \leqslant x < 5$$

(b) Using algebraic integration, determine, according to the model, the exact volume of wax that would be required to make the candle.

(9)

(c) State a limitation of the model.

(1)

When the candle was manufactured, 700 cm³ of wax were required.

(d) Use this information and your answer to part (b) to evaluate the model, explaining your reasoning.

(1)

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Question 9 continued

(a) $(5, 15)$: $15 = \dfrac{\sqrt{225(5)^2 - 2025}}{a}$

$15 = \dfrac{60}{a}$ $\therefore a = 4$

$\therefore y = \dfrac{\sqrt{225x^2 - 2025}}{4}$

(b) VOL OF REVOLUTION : $\pi \int_{\alpha}^{\beta} x^2 \, dy$

BC : $y = \dfrac{\sqrt{225x^2 - 2025}}{4}$

$4y = \sqrt{225x^2 - 2025}$

$16y^2 = 225x^2 - 2025$

$\therefore x^2 = \dfrac{16y^2 + 2025}{225}$

CD : $y = 16 - \dfrac{1}{25}x^2$

$\therefore x^2 = 25(16 - y)$

$V_1 = \pi \int_{15}^{16} 25(16 - y) \, dy$

$= 25\pi \int_{15}^{16} (16 - y) \, dy$

$= 25\pi \left[16y - \frac{1}{2}y^2\right]_{15}^{16}$

$= 25\pi \left[\{16(16) - \frac{1}{2}(16)^2\} - \{16(15) - \frac{1}{2}(15)^2\}\right]$

$= \dfrac{25\pi}{2}$ cm^3

P 6 6 7 9 0 A 0 2 9 3 2

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Question 9 continued

$$V_2 = \pi \int_0^{15} \frac{16y^2 + 2025}{225} \, dy$$

$$= \frac{\pi}{225} \int_0^{15} (16y^2 + 2025) \, dy$$

$$= \frac{\pi}{225} \left[\frac{16}{3} y^3 + 2025y \right]_0^{15}$$

$$= \frac{\pi}{225} \left\{ \frac{16}{3}(15)^3 + 2025(15) \right\}$$

$$= 215\pi \text{ cm}^3$$

$\therefore$ TOTAL VOLUME $= \frac{25\pi}{2} + 215\pi$

$$= \frac{455\pi}{2} \text{ cm}^3$$

$$= 714.7\ldots \text{ cm}^3$$

(c) THE SIDES OF THE CANDLE WILL NOT BE PERFECTLY SMOOTH

(d) FROM (b), $V \approx 715 \text{cm}^3$

$\therefore$ GOOD ESTIMATE

$\because$ 700cm^3 IS ONLY 15cm^3 LESS THAN 715cm^3

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