WRITTEN SOLUTIONS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
Level 3 GCE

Centre Number | Candidate Number

Thursday 6 June 2019

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/02**

Further Mathematics

Advanced

Paper 2: Core Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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Pearson

Answer ALL questions. Write your answers in the spaces provided.

1. (a) Prove that

$$\tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad -k < x < k$$

stating the value of the constant k.

(5)

(b) Hence, or otherwise, solve the equation

$$2x = \tanh\left(\ln\sqrt{2-3x}\right)$$

(5)

(a) $\tanh^{-1}(x) = y$

$x = \tanh(y) = \frac{\sinh(y)}{\cosh(y)} = \frac{\cancel{\frac{1}{2}}(e^y - e^{-y})}{\cancel{\frac{1}{2}}(e^y + e^{-y})} = \frac{e^y - e^{-y}}{e^y + e^{-y}} = \frac{e^{2y} - 1}{e^{2y} + 1}$

$x(e^{2y} + 1) = e^{2y} - 1$

$xe^{2y} + x = e^{2y} - 1$

$e^{2y} - xe^{2y} = 1 + x$

$e^{2y}(1 - x) = 1 + x$

$e^{2y} = \frac{1 + x}{1 - x}$

$2y = \ln\left(\frac{1 + x}{1 - x}\right)$

$y = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right)$

$\therefore \tanh^{-1}(x) = \frac{1}{2}\ln\left(\frac{1 + x}{1 - x}\right)$; $-1 < x < 1$ $\therefore K = 1$

(b) $2x = \tanh(\ln\sqrt{2 - 3x})$

P 6 1 1 7 8 A 0 2 2 8

Question 1 continued

$\tanh^{-1}(2x) = \ln\sqrt{2-3x}$

$\frac{1}{2}\ln\left(\frac{1+2x}{1-2x}\right) = \ln(2-3x)^{1/2}$

$\cancel{\frac{1}{2}}\ln\left(\frac{1+2x}{1-2x}\right) = \cancel{\frac{1}{2}}\ln(2-3x)$

$\frac{1+2x}{1-2x} = 2-3x$

$1+2x = (1-2x)(2-3x)$

$1+2x = 6x^2 - 7x + 2$

$6x^2 - 9x + 1 = 0$

$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(6)(1)}}{2(6)} = \frac{9 \pm \sqrt{57}}{12}$

but $-\frac{1}{2} < x < \frac{1}{2}$

$\therefore x = \frac{9-\sqrt{57}}{12}$

(Total for Question 1 is 10 marks)

2. The roots of the equation

$$x^3 - 2x^2 + 4x - 5 = 0$$

are p, q and r.

Without solving the equation, find the value of

(i) $\frac{2}{p} + \frac{2}{q} + \frac{2}{r}$

(ii) $(p - 4)(q - 4)(r - 4)$

(iii) $p^3 + q^3 + r^3$

(8)

$a = 1,\ b = -2,\ c = 4,\ d = -5$

$p + q + r = -\frac{b}{a} = -\frac{-2}{1} = 2$

$pq + pr + qr = \frac{c}{a} = \frac{4}{1} = 4$

$pqr = -\frac{d}{a} = -\frac{-5}{1} = 5$

(i) $\frac{2}{p} + \frac{2}{q} + \frac{2}{r} \equiv 2\left(\frac{pq + pr + qr}{pqr}\right)$

$= 2\left(\frac{4}{5}\right)$

$= \frac{8}{5}$

(ii) $(p-4)(q-4)(r-4) \equiv (pq - 4p - 4q + 16)(r - 4)$

$\equiv pqr - 4pq - 4pr - 4qr + 16p + 16q + 16r - 64$

$\equiv pqr - 4(pq + pr + qr) + 16(p + q + r) - 64$

$= (5) - 4(4) + 16(2) - 64$

$= -43$

Question 2 continued

(iii) $p^3 + q^3 + r^3 \equiv (p+q+r)^3 - 3(p+q+r)(pq+pr+qr) + 3pqr$

$= (2)^3 - 3(2)(4) + 3(5)$

$= -1$

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3.

$$f(x) = \frac{1}{\sqrt{4x^2 + 9}}$$

(a) Using a substitution, that should be stated clearly, show that

$$\int f(x)dx = A\sinh^{-1}(Bx) + c$$

where c is an arbitrary constant and A and B are constants to be found.

(4)

(b) Hence find, in exact form in terms of natural logarithms, the mean value of f(x) over the interval [0, 3].

(2)

(a) $\int \frac{1}{\sqrt{4x^2+9}} dx$

let $x = \frac{1}{2}u \quad \therefore u = 2x$

$\frac{dx}{du} = \frac{1}{2} \quad \therefore dx = \frac{1}{2} du$

so integral becomes:

$\int \frac{1}{\sqrt{u^2+9}} \cdot \frac{1}{2} du \equiv \frac{1}{2} \int \frac{1}{\sqrt{u^2+3^2}} du = \frac{1}{2} \operatorname{arcsinh}\left(\frac{u}{3}\right) + c$

$\therefore \int \frac{1}{\sqrt{4x^2+9}} dx = \frac{1}{2} \operatorname{arcsinh}\left(\frac{2x}{3}\right) + c$

$\therefore A = \frac{1}{2}, \quad B = \frac{2}{3}$

(b) mean value $= \frac{1}{b-a} \int_a^b f(x) dx$

$= \frac{1}{3-0} \int_0^3 \frac{1}{\sqrt{4x^2+9}} dx = \frac{1}{3}\left[\frac{1}{2} \operatorname{arcsinh}\left(\frac{2x}{3}\right)\right]_0^3$

$= \frac{1}{6} \operatorname{arcsinh} 2 = \frac{1}{6} \ln(2 + \sqrt{2^2+1}) = \frac{1}{6} \ln(2 + \sqrt{5})$

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4. The infinite series C and S are defined by

$$C = \cos\theta + \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 9\theta + \frac{1}{8}\cos 13\theta + \ldots$$

$$S = \sin\theta + \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 9\theta + \frac{1}{8}\sin 13\theta + \ldots$$

$i = \sqrt{-1}$

$i^2 = -1$

Given that the series C and S are both convergent,

(a) show that

$$C + iS = \frac{2e^{i\theta}}{2 - e^{4i\theta}}$$

(4)

(b) Hence show that

$$S = \frac{4\sin\theta + 2\sin 3\theta}{5 - 4\cos 4\theta}$$

(4)

(a) $C + iS = (\cos\theta + i\sin\theta) + \frac{1}{2}(\cos 5\theta + i\sin 5\theta) + \frac{1}{4}(\cos 9\theta + i\sin 9\theta)$

$+ \frac{1}{8}(\cos 13\theta + i\sin 13\theta) + \ldots$

$= (\cos\theta + i\sin\theta) + \frac{1}{2}(\cos\theta + i\sin\theta)^5 + \frac{1}{4}(\cos\theta + i\sin\theta)^9$

$+ \frac{1}{8}(\cos\theta + i\sin\theta)^{13} + \ldots$

$C + iS$ is a convergent geometric series to infinity

$S_\infty = \frac{a}{1-r}$; $|r| < 1$

$a = \cos\theta + i\sin\theta$

$r = \frac{1}{2}(\cos\theta + i\sin\theta)^4$

$C + iS = \frac{\cos\theta + i\sin\theta}{1 - \frac{1}{2}(\cos\theta + i\sin\theta)^4} = \frac{e^{i\theta}}{1 - \frac{1}{2}e^{i4\theta}} = \frac{2e^{i\theta}}{2 - e^{i4\theta}}$

Question 4 continued

(b) $c + iS = \dfrac{2e^{i\theta}}{2-e^{i4\theta}} = \dfrac{e^{i\theta}}{1-\frac{1}{2}e^{i4\theta}} = \dfrac{\cos\theta + i\sin\theta}{1-\frac{1}{2}(\cos\theta+i\sin\theta)^4} = \dfrac{2(\cos\theta+i\sin\theta)}{2-(\cos4\theta + i\sin4\theta)}$

$= \dfrac{2\cos\theta + 2i\sin\theta}{2-\cos4\theta - i\sin4\theta} = \dfrac{2\cos\theta + 2i\sin\theta}{(2-\cos4\theta)-i\sin4\theta} \cdot \dfrac{(2-\cos4\theta)+i\sin4\theta}{(2-\cos4\theta)+i\sin4\theta}$

$= \dfrac{4\cos\theta - 2\cos\theta\cos4\theta + 2i\sin4\theta + 4i\sin\theta - 2i\sin\theta\cos4\theta - 2\sin\theta\sin4\theta}{(4-4\cos4\theta+\cos^2 4\theta)+\sin^2 4\theta}$

$= \dfrac{4\cos\theta + 4i\sin\theta - 2\cos\theta\cos4\theta - 2\sin\theta\sin4\theta + 2i\sin4\theta\cos\theta - 2i\sin\theta\cos4\theta}{5-4\cos4\theta}$

now compare imaginary terms:

$S = \dfrac{4\sin\theta + 2\sin4\theta\cos\theta - 2\sin\theta\cos4\theta}{5-4\cos4\theta} = \dfrac{4\sin\theta + 2(\sin4\theta\cos\theta - \sin\theta\cos4\theta)}{5-4\cos4\theta}$

$= \dfrac{4\sin\theta + 2\sin(4\theta-\theta)}{5-4\cos4\theta} = \dfrac{4\sin\theta + 2\sin3\theta}{5-4\cos4\theta}$

(Total for Question 4 is 8 marks)

5. An engineer is investigating the motion of a sprung diving board at a swimming pool.
Let E be the position of the end of the diving board when it is at rest in its equilibrium position and when there is no diver standing on the diving board.
A diver jumps from the diving board.
The vertical displacement, h cm, of the end of the diving board above E is modelled by the differential equation

$$4\frac{\mathrm{d}^2h}{\mathrm{d}t^2} + 4\frac{\mathrm{d}h}{\mathrm{d}t} + 37h = 0$$

where t seconds is the time after the diver jumps.

(a) Find a general solution of the differential equation.
(2)

When $t = 0$, the end of the diving board is 20 cm below E and is moving upwards with a speed of 55 cm s^{-1}.

(b) Find, according to the model, the maximum vertical displacement of the end of the diving board above E.
(8)

(c) Comment on the suitability of the model for large values of t.
(2)

(a) auxiliary equation: $4m^2 + 4m + 37 = 0$

$$m = \frac{-(4) \pm \sqrt{(4)^2 - 4(4)(37)}}{2(4)}$$

$\therefore m = -0.5 \pm 3i \quad \therefore$ complex roots

$\therefore h_{GS} = e^{-0.5t}[A\cos 3t + B\sin 3t]$

(b) $\underline{t=0,\ h=-20:}$ $e^0(A\cos 0 + \cancel{B\sin 0}) = -20$

$\therefore A = -20$

(b) $\frac{dh_{GS}}{dt} = -0.5e^{-0.5t}[A\cos 3t + B\sin 3t] + e^{0.5t}[-3A\sin 3t + 3B\cos 3t]$

$\underline{t=0,\ dh/dt = 55}$ $-0.5e^0(A\cos 0 + B\sin 0) + e^0(-3A\sin 0 + 3B\cos 0) = 55$

$-0.5A + 3B = 55$

Question 5 continued

$-0.5 \times -20 + 3B = 55$

$10 + 3B = 55$

$3B = 45$

$\therefore B = 15$

$\therefore h_{ps} = e^{0.5t}(-20\cos 3t + 15\sin 3t)$

$\therefore dh_{ps}/dt = -0.5e^{-0.5t}(-20\cos 3t + 15\sin 3t) + e^{-0.5t}(60\sin 3t + 45\cos 3t)$

$\underline{dh/dt = 0}$: $-0.5e^{-0.5t}(-20\cos 3t + 15\sin 3t) + e^{-0.5t}(60\sin 3t + 45\cos 3t) = 0$

$e^{-0.5t}(10\cos 3t - 7.5\sin 3t) + e^{-0.5t}(60\sin 3t + 45\cos 3t) = 0$

$e^{-0.5t}(55\cos 3t + 52.5\sin 3t) = 0$

$e^{-0.5t} \neq 0$

$55\cos 3t + 52.5\sin 3t = 0$

$\frac{\sin 3t}{\cos 3t} = -\frac{55}{52.5}$

$\tan 3t = -{}^{22}/_{21}$

$3t = \tan^{-1}(-{}^{22}/_{21}) + \pi$

$3t = 2.3329\ldots$

$\therefore t = 0.7776\ldots$ s

$\underline{t = 0.77\ldots}$: $h = e^{-0.5 \times 0.77\ldots}(-20\cos 2.33\ldots + 15\sin 2.33\ldots)$

Question 5 continued

$h = 16.715\ldots$

$\therefore h_{max} = 16.7$ cm (3sf)

(c) The value of h is very small when t is large and this is likely to be correct $\because$ the displacement of the board should get smaller, suggesting that the model is suitable.

6. In an Argand diagram, the points A, B and C are the vertices of an equilateral triangle with its centre at the origin. The point A represents the complex number $6 + 2i$.

(a) Find the complex numbers represented by the points B and C, giving your answers in the form $x + iy$, where x and y are real and exact.

(6)

The points D, E and F are the midpoints of the sides of triangle ABC.

(b) Find the exact area of triangle DEF.

(3)

(a) If z_1 is one root of the equation $z^n = s$, and $1, \omega, \omega^2, \dots, \omega^{n-1}$ are the nth roots of unity, then the roots of $z^n = s$ are given by $z_1, z_1\omega, z_1\omega^2, \dots, z_1\omega^{n-1}$.

This is the same as rotating z_1 about the origin.

The verticies of triangle correspond to the cube root of $(6+2i)^3$

Hence we can find the roots by multiplying $6+2i$ by the cube roots of unity, which is the same as rotating $6+2i$ about the origin

The cube roots of unity are $1, \omega, \omega^2$ where $\omega = e^{2\pi i/3}$

$$\left.\begin{array}{l} \text{mod}(6+2i) = \sqrt{6^2 + 2^2} = \sqrt{40} \\ \arg(6+2i) = \arctan(2/6) = \arctan(1/3) \end{array}\right\} \sqrt{40}\, e^{i \arctan\frac{1}{3}}$$

so the verticies are at:

$\sqrt{40}\, e^{i\arctan\frac{1}{3}} = 6 + 2i$

$\therefore A(6, 2)$

$\sqrt{40}\, e^{i\arctan\frac{1}{3}} e^{i\frac{2\pi}{3}} = \sqrt{40}\, e^{i(\arctan\frac{1}{3} + \frac{2\pi}{3})}$

* same as rotating $(6+2i)$ by $120° = \frac{2\pi}{3}$ about origin

Question 6 continued

$$\begin{pmatrix} \cos 120 & -\sin 120 \\ \sin 120 & \cos 120 \end{pmatrix}\begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 6\cos 120 - 2\sin 120 \\ 6\sin 120 + 2\cos 120 \end{pmatrix}$$

$$\begin{pmatrix} -3 - \sqrt{3} \\ -1 + 3\sqrt{3} \end{pmatrix}$$

$\therefore B[(-3-\sqrt{3}) + (-1 + 3\sqrt{3})i]$

$\sqrt{40}\, e^{i \arctan\frac{1}{3}} \left(e^{i\frac{2\pi}{3}}\right)^2 = \sqrt{40}\, e^{i\left(\arctan\frac{1}{3} + \frac{4\pi}{3}\right)}$

* same as ~~rotat~~ rotating $6+2i$ by $240° = \frac{4\pi}{3}$ about origin

$$\begin{pmatrix} \cos 240 & -\sin 240 \\ \sin 240 & \cos 240 \end{pmatrix}\begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

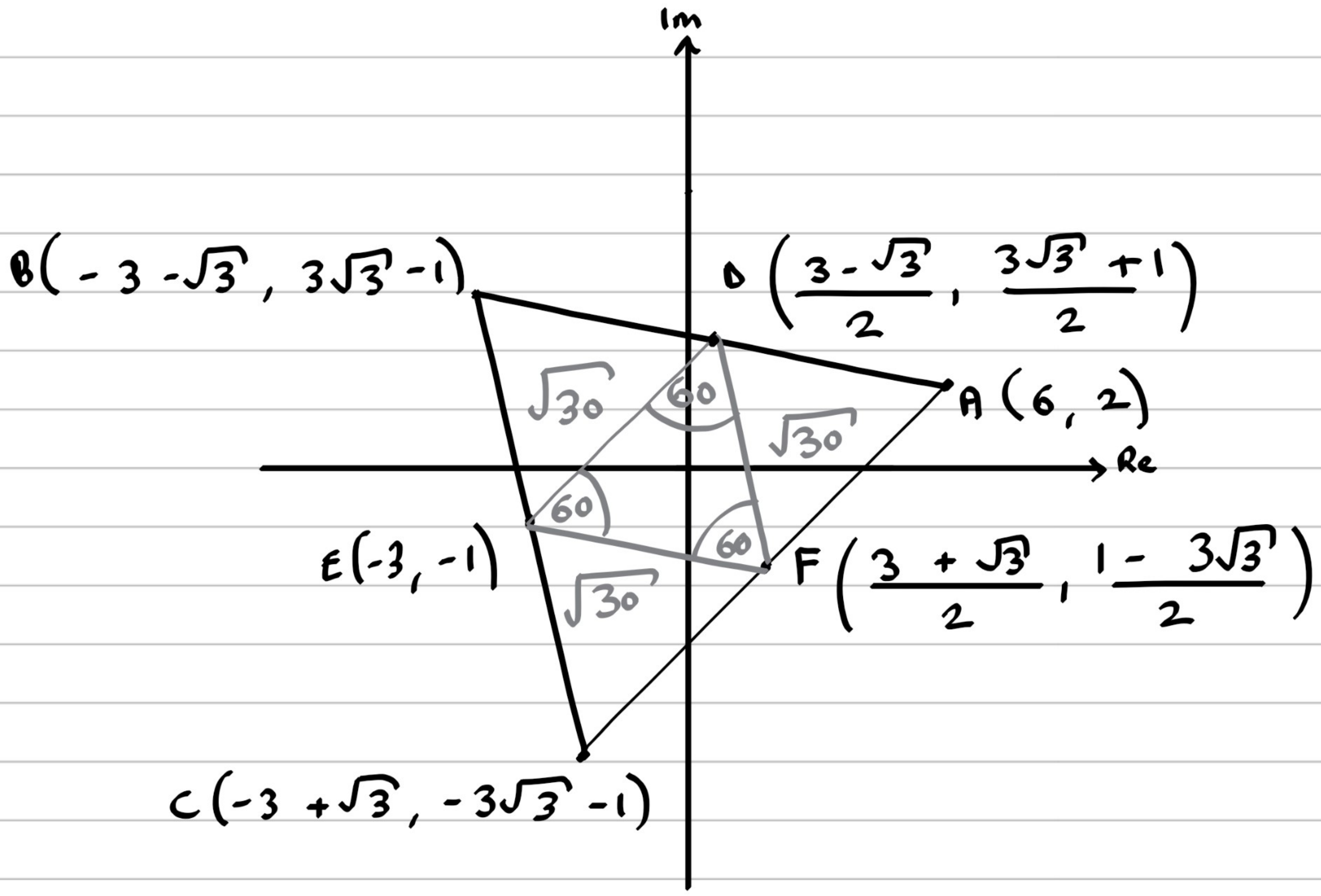
$$\begin{pmatrix} 6\cos 240 - 2\sin 240 \\ 6\sin 240 + 2\cos 240 \end{pmatrix}$$

$$\begin{pmatrix} -3 + \sqrt{3} \\ -1 - 3\sqrt{3} \end{pmatrix}$$

$\therefore C[(-3+\sqrt{3}) + (-1 - 3\sqrt{3})i]$

Question 6 continued

(b)



$$DE = EF = FD = \sqrt{\left(\frac{3+\sqrt{3}}{2} - \frac{3-\sqrt{3}}{2}\right)^2 + \left(\frac{1-3\sqrt{3}}{2} - \frac{3\sqrt{3}+1}{2}\right)^2}$$

$$= \sqrt{(\sqrt{3})^2 + (-3\sqrt{3})^2} = \sqrt{3+27} = \sqrt{30}$$

area of $\triangle DEF = \frac{1}{2} \times \sqrt{30} \times \sqrt{30} \times \sin 60$

$$= \frac{15\sqrt{3}}{2} \text{ units}^2$$

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7.

$$\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & k & 4 \\ 3 & 2 & -1 \end{pmatrix} \qquad \text{where } k \text{ is a constant}$$

(a) Find the values of k for which the matrix $\mathbf{M}$ has an inverse.

(2)

(b) Find, in terms of p, the coordinates of the point where the following planes intersect

$$2x - y + z = p$$
$$3x - 6y + 4z = 1$$
$$3x + 2y - z = 0$$

(5)

(c) (i) Find the value of q for which the set of simultaneous equations

$$2x - y + z = 1$$
$$3x - 5y + 4z = q$$
$$3x + 2y - z = 0$$

can be solved.

(ii) For this value of q, interpret the solution of the set of simultaneous equations geometrically.

(4)

(a) singular matricies do not have an inverse, i.e. det $(\underline{M}) = 0$

i.e. $2\begin{vmatrix} k & 4 \\ 2 & -1 \end{vmatrix} - -1\begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} + 1\begin{vmatrix} 3 & k \\ 3 & 2 \end{vmatrix} = 0$

$2(-k-8) + 1(-3-12) + 1(6-3k) = 0$

$-2k - 16 - 3 - 12 + 6 - 3k = 0$

$-5k - 25 = 0$ NB: $\det(\underline{M}) = -5k - 25$

$\Rightarrow k = -5$

$\therefore \underline{M}$ has an inverse when $k \neq -5$

Question 7 continued

(b) $2x - y + z = p$
$3x - 6y + 4z = 1$
$3x + 2y - z = 0$

$$\begin{pmatrix} 2 & -1 & 1 \\ 3 & -6 & 4 \\ 3 & 2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & -6 & 4 \\ 3 & 2 & -1 \end{pmatrix}^{-1}\begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} -2 & 1 & 2 \\ 15 & -5 & -5 \\ 24 & -7 & -9 \end{pmatrix}\begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix} -2p + 1 + 0 \\ 15p - 5 - 0 \\ 24p - 7 - 0 \end{pmatrix}$$

$$\therefore \left(\frac{-2p+1}{5}, 3p-1, \frac{24p-7}{5} \right)$$

(c)(i) $2x - y + z = 1$ ①
$3x - 5y + 4z = q$ ②
$3x + 2y - z = 0$ ③

assume consistent, i.e. a system of linear equations is consistent if there is at least one set of values that satisfies all the equations simultaneously

try to eliminate z :

① + ③: $5x + y = 1$ ④

③ ×4 : $12x + 8y - 4z = 0$ ⑤

② + ⑤: $15x + 3y = q$ ⑥

Question 7 continued

① × 4 : $8x - 4y + 4z = 4$ ⑦

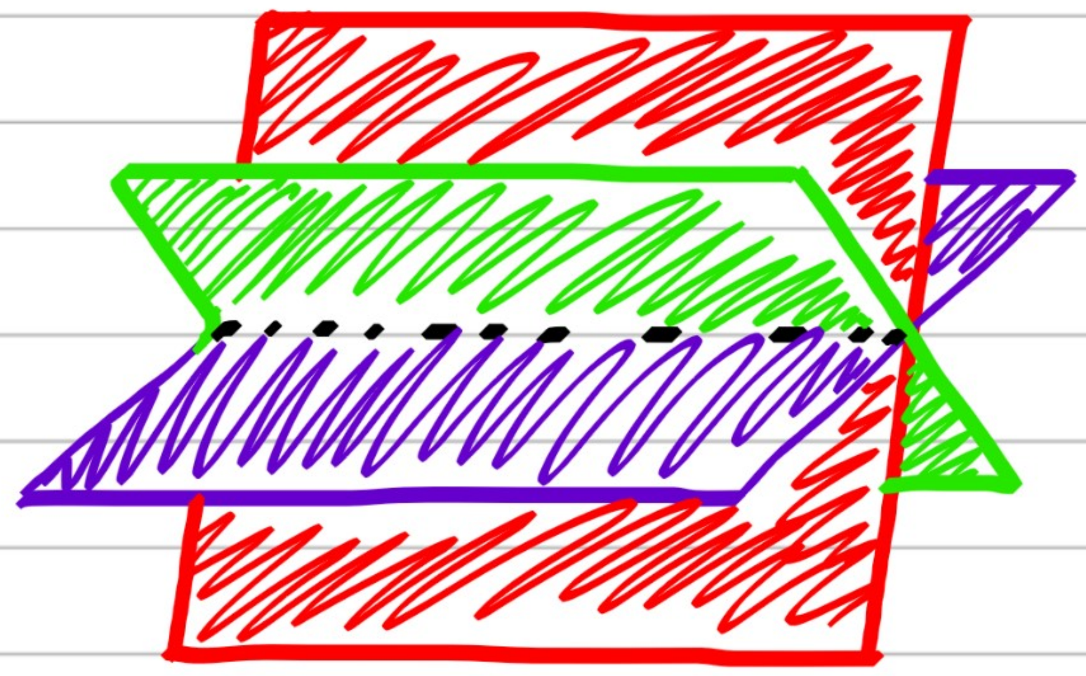
⑦ - ② : $5x + y = 4 - q$ ⑧

set ④ equal to 8 : $1 = 4 - q \therefore q = 3$

compare ⑥ with ④ : ⑥ = 3④

$\therefore q = 3$

(ii) three planes that form a sheaf, i.e. three planes that intersect a line



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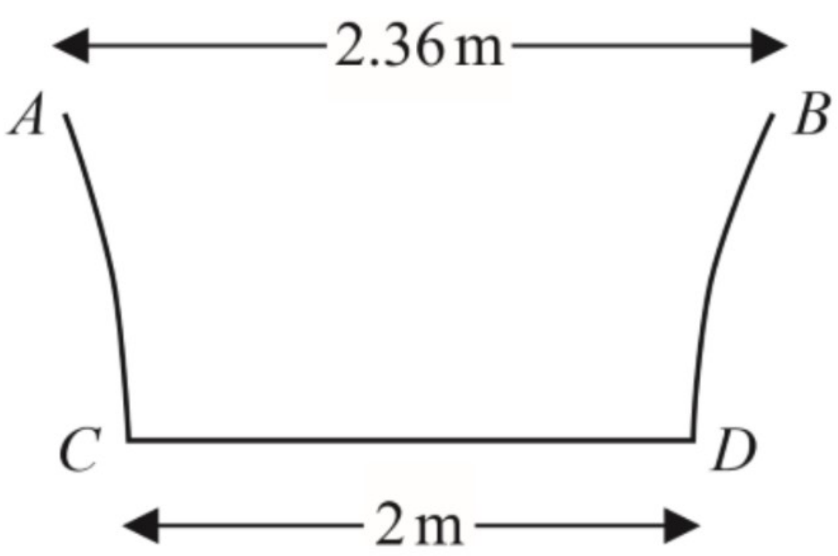

Figure 1

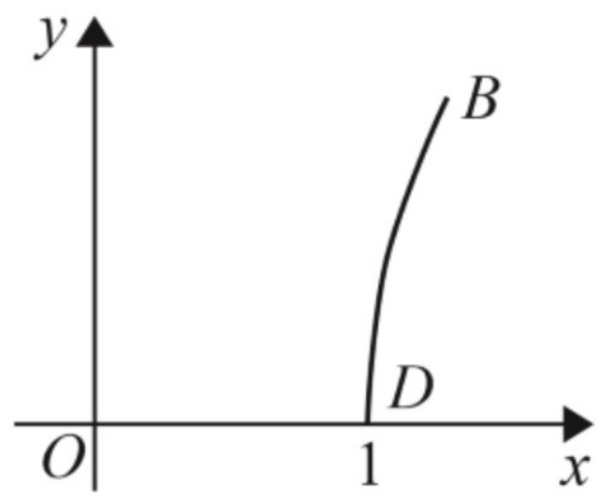

Figure 2

Figure 1 shows the central vertical cross section *ABCD* of a paddling pool that has a circular horizontal cross section. Measurements of the diameters of the top and bottom of the paddling pool have been taken in order to estimate the volume of water that the paddling pool can contain.

Using these measurements, the curve *BD* is modelled by the equation

$$y = \ln(3.6x - k) \qquad 1 \leqslant x \leqslant 1.18$$

as shown in Figure 2.

(a) Find the value of k.

(1)

(b) Find the depth of the paddling pool according to this model.

(2)

The pool is being filled with water from a tap.

(c) Find, in terms of h, the volume of water in the pool when the pool is filled to a depth of h m.

(5)

Given that the pool is being filled at a constant rate of 15 litres every minute,

(d) find, in $\text{cm}\,\text{h}^{-1}$, the rate at which the water level is rising in the pool when the depth of the water is 0.2 m.

(3)

(a) $\underline{x=1,\ y=0:}$ $0 = \ln(3.6 \times 1 - k)$

$e^0 = 3.6 - k \quad \therefore k = 2.6$

$\therefore y = \ln(3.6x - 2.6)$ ①

(b) $\underline{x = 1.18:}$ $y = \ln(3.6 \times 1.18 - 2.6) = 0.4995\ldots$

$\therefore$ depth $= 0.500$ m (3sf)

Question 8 continued

(c) rotation about y-axis: $\pi \int x^2 \, dy$

From ①: $e^y = 3.6x - 2.6$

$\therefore x = \dfrac{e^y + 2.6}{3.6}$ ②

hence $V = \pi \int_0^h \left(\dfrac{e^y + 2.6}{3.6}\right)^2 dy \equiv \dfrac{\pi}{3.6^2} \int_0^h (e^y + 2.6)^2 \, dy$

$\equiv \dfrac{\pi}{3.6^2} \int_0^h \left(e^{2y} + 5.2e^y + 6.76\right) dy$

$= \dfrac{\pi}{3.6^2} \left[0.5e^{2y} + 5.2e^y + 6.76h\right]_0^h$

$= \dfrac{\pi}{3.6^2} \left[\left\{0.5e^{2h} + 5.2e^h + 6.76h\right\} - \left\{0.5 + 5.2\right\}\right]$

$= \dfrac{\pi}{3.6^2} \left(0.5e^{2h} + 5.2e^h + 6.76h - 5.7\right) \; m^3$

$= \pi \left(\dfrac{25e^{2h}}{648} + \dfrac{65e^h}{162} + \dfrac{169h}{324} - \dfrac{95}{216}\right) \; m^3$

(d) $\dfrac{dh}{dt} = ?$ $\quad \dfrac{dV}{dt} = 15 \dfrac{l}{m}$

$\dfrac{dV}{dh} = \dfrac{\pi}{3.6^2} \left(e^{2h} + 5.2e^h + 6.76\right)$

$h = 0.2$: $\dfrac{dV}{dh} = \dfrac{\pi}{3.6^2} \left(e^{2 \times 0.2} + 5.2e^{0.2} + 6.76\right)$

$= 3.539 \ldots \dfrac{l}{min}$

Question 8 continued

$$\frac{dh}{dt} = \frac{dh}{dv}\frac{dv}{dt} = \frac{1}{3.539\ldots} \times \frac{15}{1000} \times 60$$

$$= 0.254\ldots$$

$$= 0.254 \text{ cmh}^{-1} \text{ (3sf)}$$

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