Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Monday 5 Oct 2020

Morning (Time: 1 hour 30 minutes) | Paper Reference **9FM0/01**

Further Mathematics

Advanced

Paper 1: Core Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. $$f(z) = 3z^3 + pz^2 + 57z + q$$

where p and q are real constants. $a = 3,\ b = p,\ c = 57,\ d = q$

Given that $3 - 2\sqrt{2}\mathrm{i}$ is a root of the equation $f(z) = 0$

(a) show all the roots of $f(z) = 0$ on a single Argand diagram,

(7)

(b) find the value of p and the value of q.

(3)

(a) $\alpha = 3 - 2\sqrt{2}i,\quad \beta = 3 + 2\sqrt{2}i,\quad \gamma = x + iy$

$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{p}{3}$ ①

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \frac{57}{3} = 19$ ②

$\alpha\beta\gamma = -\frac{d}{a} = -\frac{q}{3}$ ③

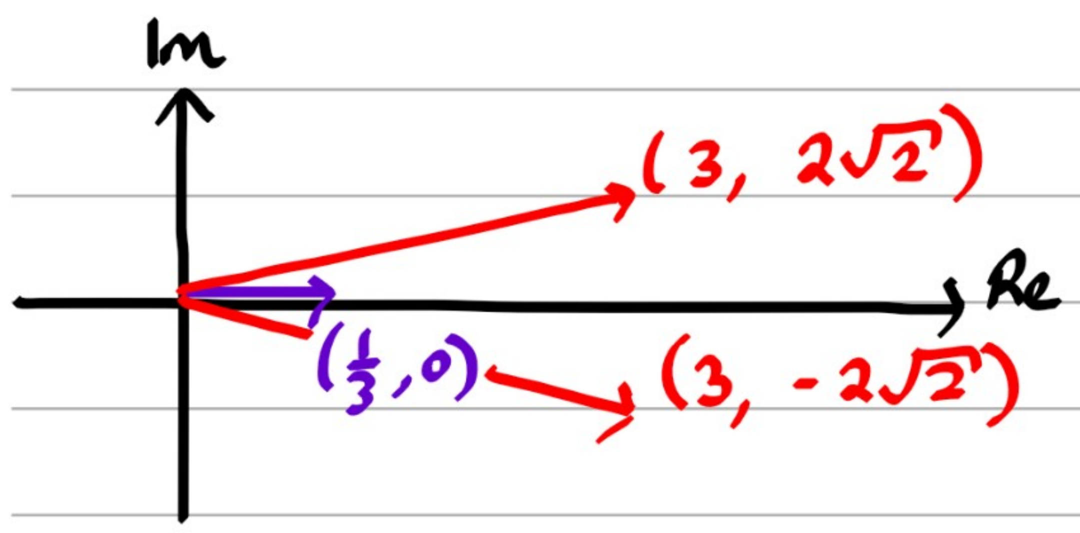
From ② : $(3 - 2\sqrt{2}i)(3 + 2\sqrt{2}i) + (3 + 2\sqrt{2}i)(x + iy) + (x + iy)(3 - 2\sqrt{2}i) = 19$

$17 + 3x + 3yi + \cancel{2\sqrt{2}xi} - \cancel{2\sqrt{2}y} + 3x - \cancel{2\sqrt{2}xi} + 3yi + \cancel{2\sqrt{2}y} = 19$

$17 + 6x + 6yi = 19$

Re : $17 + 6x = 19$

$\left.\begin{array}{l} 6x = 2 \quad \therefore x = \frac{1}{3} \\ \text{Im} : 6y = 0 \quad \therefore y = 0 \end{array}\right\} \therefore \gamma = \frac{1}{3}$



Question 1 continued

(b) From ①: $(3 - 2\sqrt{2}i) + (3 + 2\sqrt{2}i) + (\frac{1}{3}) = -\frac{p}{3}$

$6 + \frac{1}{3} = -\frac{p}{3}$

$18 + 1 = -p$

$19 = -p \therefore p = -19$

From ③: $(3 - 2\sqrt{2}i)(3 + 2\sqrt{2}i)(\frac{1}{3}) = -\frac{q}{3}$

$\frac{17}{3} = -\frac{q}{3}$

$17 = -q \therefore q = -17$

$\therefore f(z) = 3z^3 - 19z^2 + 57z - 17$

2. (a) Explain why $\int_1^{\infty} \frac{1}{x(2x+5)}\,\mathrm{d}x$ is an improper integral.

(1)

(b) Prove that

$$\int_1^{\infty} \frac{1}{x(2x+5)}\,\mathrm{d}x = a\ln b$$

where a and b are rational numbers to be determined.

(6)

(a) The integral being integrated over is bounded

(b) $\frac{1}{x(2x+5)} \equiv \frac{A}{x} + \frac{B}{2x+5}$

$1 \equiv A(2x+5) + B(x)$

$x = 0: 1 = 5A \therefore A = \frac{1}{5}$

$x = -\frac{5}{2}: 1 = -\frac{5}{2}B \therefore B = -\frac{2}{5}$

$\therefore \frac{1}{x(2x+5)} \equiv \frac{1}{5x} - \frac{2}{5(2x+5)}$

ie. $\int_1^{\infty} \frac{1}{x(2x+5)}\,dx \equiv \int_1^{\infty} \frac{1}{5x} - \frac{2}{5(2x+5)}\,dx \equiv \frac{1}{5}\int_0^{\infty} \frac{1}{x} - \frac{2}{2x+5}\,dx$

$= \lim_{t\to\infty} \frac{1}{5}\int_1^{t} \frac{1}{x} - \frac{2}{2x+5}\,dx$

$= \lim_{t\to\infty} \frac{1}{5}\Big[\ln|x| - \ln|2x+5|\Big]_1^t$

$= \lim_{t\to\infty} \frac{1}{5}\Big[\ln\left|\frac{x}{2x+5}\right|\Big]_1^t$

$= \lim_{t\to\infty} \frac{1}{5}\ln\left|\frac{t}{2t+5}\right| - \frac{1}{5}\ln\frac{1}{7}$

Question 2 continued

$$= \lim_{t\to\infty} \frac{1}{5} \ln\left|\frac{t/t}{2t/t + 5/t}\right| - \frac{1}{5}\ln\frac{1}{7}$$

$$= \lim_{t\to\infty} \frac{1}{5} \ln\left(\frac{1}{2+5/t}\right) - \frac{1}{5}\ln\frac{1}{7}$$

$$= \frac{1}{5}\ln\frac{1}{2} - \frac{1}{5}\ln\frac{1}{7}$$

$$= \frac{1}{5}\ln\frac{1/2}{1/7}$$

$$= \frac{1}{5}\ln\frac{7}{2}$$

(Total for Question 2 is 7 marks)

3.

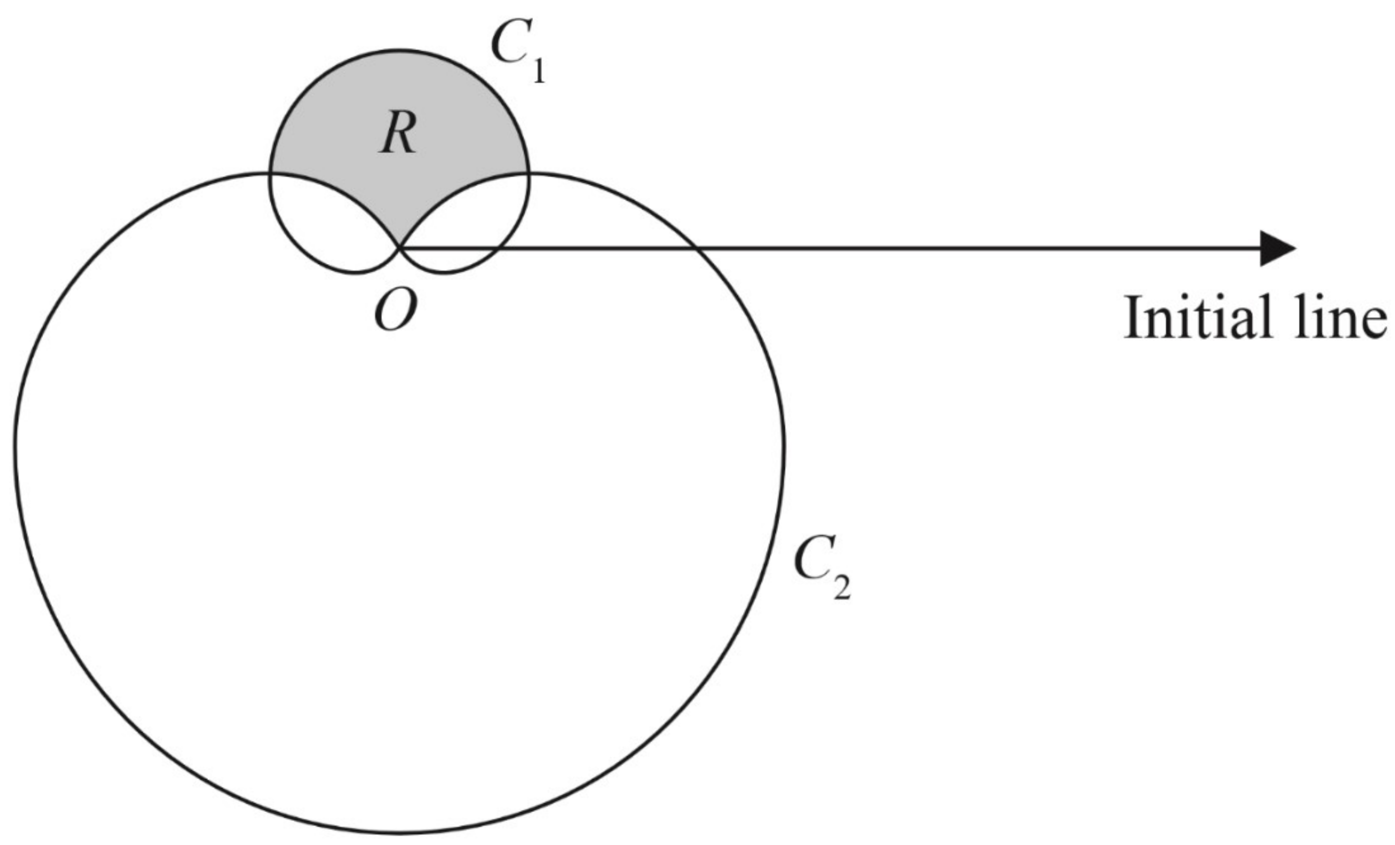


Figure 1

Figure 1 shows a sketch of two curves C_1 and C_2 with polar equations

$$C_1: r = (1 + \sin\theta) \qquad 0 \leqslant \theta < 2\pi$$

$$C_2: r = 3(1 - \sin\theta) \qquad 0 \leqslant \theta < 2\pi$$

The region R lies inside C_1 and outside C_2 and is shown shaded in Figure 1.

Show that the area of R is

$$p\sqrt{3} - q\pi$$

where p and q are integers to be determined.

(9)

$1 + \sin\theta = 3(1 - \sin\theta)$

$1 + \sin\theta = 3 - 3\sin\theta$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2} \quad \therefore \theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$\text{area} = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (1 + \sin\theta)^2 \, d\theta - \frac{1}{2}\int_{\pi/6}^{5\pi/6} [3(1 - \sin\theta)]^2 \, d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left[(1 + \sin\theta)^2 - 9(1 - \sin\theta)^2\right] d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left[1 + 2\sin\theta + \sin^2\theta - 9(1 - 2\sin\theta + \sin^2\theta)\right] d\theta$

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left[1 + 2\sin\theta + \sin^2\theta - 9 + 18\sin\theta - 9\sin^2\theta\right] d\theta$

Question 3 continued

$= \frac{1}{2}\int_{\pi/6}^{5\pi/6}\left[-8 + 20\sin\theta - 8\sin^2\theta\right] d\theta$

$= \int_{\pi/6}^{5\pi/6}\left[-4 + 10\sin\theta - 4\sin^2\theta\right] d\theta$

$\cos 2A \equiv 1 - 2\sin^2 A \quad \therefore \sin^2 A \equiv \frac{1}{2}(1-\cos 2A)$

$= \int_{\pi/6}^{5\pi/6}\left[-4 + 10\sin\theta - 4\times\frac{1}{2}(1-\cos 2\theta)\right] d\theta$

$= \int_{\pi/6}^{5\pi/6}\left[-6 + 10\sin\theta + 2\cos 2\theta\right] d\theta$

$= \left[-6\theta - 10\cos\theta + \sin 2\theta\right]_{\pi/6}^{5\pi/6}$

$= \left\{-6\left(\frac{5\pi}{6}\right) - 10\cos\left(\frac{5\pi}{6}\right) + \sin\left(2\times\frac{5\pi}{6}\right)\right\} - \left\{-6\left(\frac{\pi}{6}\right) - 10\cos\left(\frac{\pi}{6}\right) + \sin\left(2\times\frac{\pi}{6}\right)\right\}$

$= \left\{-5\pi + \frac{9}{2}\sqrt{3}\right\} - \left\{-\pi - \frac{9}{2}\sqrt{3}\right\}$

$= -4\pi + 9\sqrt{3}$

$\therefore p = 9, q = 4$

4. The plane Π_1 has equation

$$\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}) + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Find a Cartesian equation for Π_1 $\quad \vec{r}\cdot\vec{n} = d$ **(4)**

The line l has equation

$$\frac{x-1}{5} = \frac{y-3}{-3} = \frac{z+2}{4}$$

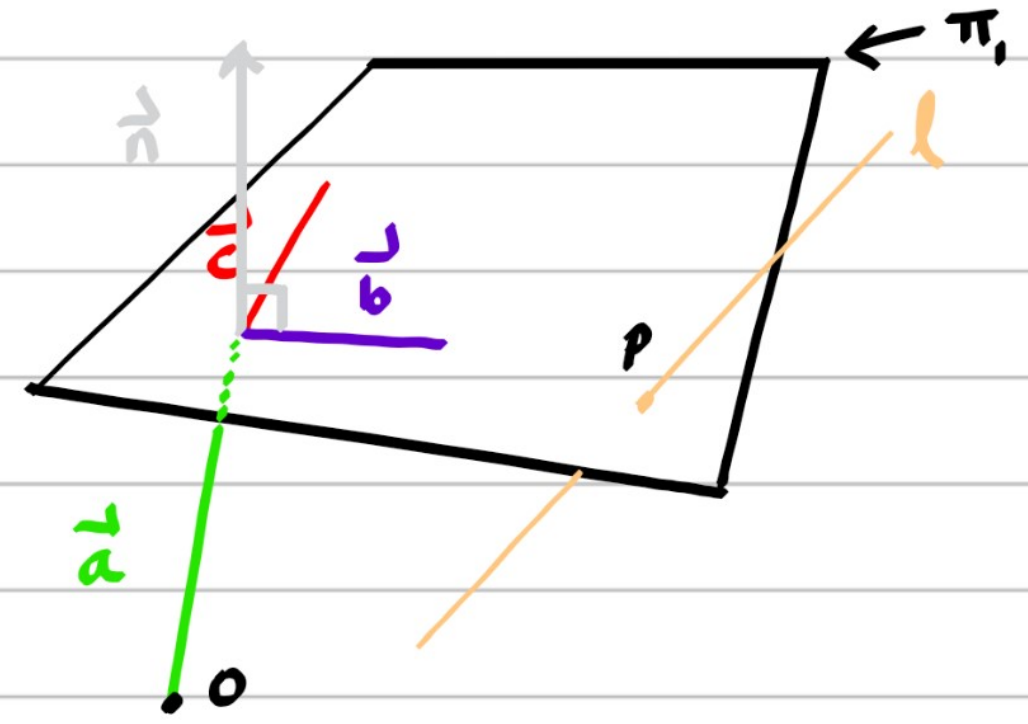
(b) Find the coordinates of the point of intersection of l with Π_1 **(3)**

The plane Π_2 has equation

$$\mathbf{r}.(2\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 5$$

(c) Find, to the nearest degree, the acute angle between Π_1 and Π_2 **(2)**

(a)



$$\vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$$

$$\vec{r} = \begin{pmatrix}2\\4\\-1\end{pmatrix} + \lambda\begin{pmatrix}1\\2\\-3\end{pmatrix} + \mu\begin{pmatrix}-1\\2\\1\end{pmatrix}$$

$$\vec{n} = \begin{pmatrix}1\\2\\-3\end{pmatrix} \times \begin{pmatrix}-1\\2\\1\end{pmatrix} = \begin{pmatrix}8\\2\\4\end{pmatrix}$$

$$d = \vec{a}\cdot\vec{n} = \begin{pmatrix}2\\4\\-1\end{pmatrix}\cdot\begin{pmatrix}8\\2\\4\end{pmatrix} = (2)(8) + (4)(2) + (-1)(4) = 20$$

$$\therefore \begin{pmatrix}x\\y\\z\end{pmatrix}\cdot\begin{pmatrix}8\\2\\4\end{pmatrix} = 20 \quad \text{or} \quad 8x + 2y + 4z = 20 \;\therefore\; 4x + y + 2z = 10$$

Question 4 continued

(b) $\dfrac{x-1}{5} = \dfrac{y-3}{-3} = \dfrac{z+2}{4}$

in vector form : $\vec{r} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + t \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix}$

plane meets line when:

$\begin{pmatrix} 1+5t \\ 3-3t \\ -2+4t \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 2 \\ 4 \end{pmatrix} = 20$

$8(1+5t) + 2(3-3t) + 4(-2+4t) = 20 \qquad (\div 2)$

$4(1+5t) + 1(3-3t) + 2(-2+4t) = 10$

$4 + 20t + 3 - 3t - 4 + 8t = 10$

$25t = 7 \quad \therefore t = 7/25$

i.e. $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} + 7/25 \begin{pmatrix} 5 \\ -3 \\ 4 \end{pmatrix} = \begin{pmatrix} 12/5 \\ 54/25 \\ -22/25 \end{pmatrix}$

$\therefore P\left(\dfrac{12}{5}, \dfrac{54}{25}, \dfrac{-22}{25}\right)$

(c) acute angle is $\cos\theta = \left| \dfrac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \times |\vec{n}_2|} \right|$

$\vec{n}_1 = 8\underline{i} + 2\underline{j} + 4\underline{k}$, $\vec{n}_2 = 2\underline{i} - \underline{j} + 3\underline{k}$

$|\vec{n}_1| = \sqrt{8^2 + 2^2 + 4^2} = 2\sqrt{21}$

Question 4 continued

$|\vec{n}_2| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$

$\vec{n}_1 \cdot \vec{n}_2 = (8\underline{i} + 2\underline{j} + 4\underline{k}) \cdot (2\underline{i} - \underline{j} + 3\underline{k})$

$= (8)(2) + (2)(-1) + (4)(3) = 26$

$\cos\theta = \dfrac{26}{2\sqrt{21}\sqrt{14}}$

$\therefore \theta = 41^\circ$ (nearest degree)

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5. Two compounds, X and Y, are involved in a chemical reaction. The amounts in grams of these compounds, t minutes after the reaction starts, are x and y respectively and are modelled by the differential equations

$$\frac{dx}{dt} = -5x + 10y - 30 \quad ①$$

$$\frac{dy}{dt} = -2x + 3y - 4 \quad ②$$

(a) Show that

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 50$$

(3)

(b) Find, according to the model, a general solution for the amount in grams of compound X present at time t minutes.

(6)

(c) Find, according to the model, a general solution for the amount in grams of compound Y present at time t minutes.

(3)

Given that $x = 2$ and $y = 5$ when $t = 0$

(d) find

(i) the particular solution for x,

(ii) the particular solution for y.

(4)

A scientist thinks that the chemical reaction will have stopped after 8 minutes.

(e) Explain whether this is supported by the model.

(1)

(a) From ①: $\frac{d^2x}{dt^2} = -5\frac{dx}{dt} + 10\frac{dy}{dt}$ ③

② into ③: $\frac{d^2x}{dt^2} = -5\frac{dx}{dt} + 10(-2x + 3y - 4)$

$\frac{d^2x}{dt^2} = -5\frac{dx}{dt} - 20x + 30y - 40$ ④

From ①: $10y = \frac{dx}{dt} + 5x + 30$ ⑤

⑤ into ④: $\frac{d^2x}{dt^2} = -5\frac{dx}{dt} - 20x + 3\left(\frac{dx}{dt} + 5x + 30\right) - 40$

$= -5\frac{dx}{dt} - 20x + 3\frac{dx}{dt} + 15x + 90 - 40$

$= -2\frac{dx}{dt} - 5x + 50$

$\therefore \frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 50$ ⑥

(b) auxiliary equation: $m^2 + 2m + 5 = 0$

$\therefore m = -1 \pm 2i$

$\therefore x_H = e^{-t}[A\cos 2t + B\sin 2t]$

For particular integral, try $x_{PI} = P$

$\frac{dx_{PI}}{dt} = 0$

$\frac{d^2x_{PI}}{dt^2} = 0$

sub into ⑥: $0 + 2(0) + 5x_{PI} = 50$

$5x_{PI} = 50 \quad \therefore x_{PI} = 10$

so $x_{GS} = x_H + x_{PI}$

$\therefore x_{GS} = e^{-t}[A\cos 2t + B\sin 2t] + 10$ ⑦

(c) From ⑦: $\frac{dx}{dt} = e^{-t}(2B\cos 2t - 2A\sin 2t) - e^{-t}(A\cos 2t + B\sin 2t)$ ⑧

Question 5 continued

⑧ into ⑤: $10y = e^{-t}(2B\cos 2t - 2A\sin 2t) - e^{-t}(A\cos 2t + B\sin 2t) +$

$5\left(e^{-t}[A\cos 2t + B\sin 2t] + 10\right) + 30$

$10y = e^{-t}\{(2B\cos 2t - 2A\sin 2t) - (A\cos 2t + B\sin 2t) +$

$(5A\cos 2t + 5B\sin 2t)\} + 50 + 30$

$10y = e^{-t}[(4A + 2B)\cos 2t + (4B - 2A)\sin 2t] + 80$

$\therefore y_{GS} = \frac{1}{10}e^{-t}[(4A + 2B)\cos 2t + (4B - 2A)\sin 2t] + 8$ ⑧

(d) $t = 0, x = 2$: $2 = e^{0}(A\cos 0 + B\sin 0) + 10$

$2 = A + 10 \quad \therefore A = -8$

$t = 0, y = 5$: $5 = \frac{1}{10}e^{0}[(4A + 2B)\cos 0 + (4B - 2A)\sin 0] + 8$

$5 = \frac{1}{10}(4A + 2B) + 8$

$-3 = \frac{1}{10}(4A + 2B)$

$-30 = 4A + 2B$

$-30 = 4 \times -8 + 2B$

$2 = 2B \quad \therefore B = 1$

(i) $x_{PS} = e^{-t}(-8\cos 2t + \sin 2t) + 10$

(ii) $y_{PS} = \frac{1}{10}e^{-t}(-30\cos 2t + 20\sin 2t) + 8$

(e) *When t>8, the amount of compound X and the amount of compound Y remain (approximately) constant at 10 and 8 respectively, which suggests that the chemical reaction has stopped.*

This supports the scientists claim.

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6. (i) Prove by induction that for $n \in \mathbb{Z}^+$

$$\sum_{r=1}^{n}(3r+1)(r+2) = n(n+2)(n+3)$$

(6)

(ii) Prove by induction that for all positive **odd** integers n

$$f(n) = 4^n + 5^n + 6^n$$

is divisible by 15

(6)

(i) AIM IS TO SHOW: $[k+1][(k+1)+2][(k+1)+3]$

$n=1$: LHS $= \sum_{r=1}^{1}(3\times1+1)(1+2) = 4\times3 = 12$

RHS $= 1(1+2)(1+3) = 1\times3\times4 = 12$

$\therefore$ statement is true for $n=1$

assume true for $n=k$, i.e.

$$\sum_{r=1}^{k}(3r+1)(r+2) = k(k+2)(k+3)$$

now try for $n=k+1$:

$$\sum_{r=1}^{k+1}(3r+1)(r+2) = \sum_{r=1}^{k}(3r+1)(r+2) + [3(k+1)+1][(k+1)+2]$$

$$= k(k+2)(k+3) + [3(k+1)+1][(k+1)+2]$$

$$= k(k+2)(k+3) + (3k+4)(k+3)$$

$$= (k+3)[k(k+2) + (3k+4)]$$

$$= (k+3)(k^2+5k+4)$$

$$= (k+3)(k+1)(k+4)$$

$$= [k+1][(k+1)+2][(k+1)+3]$$

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Question 6 continued

If statement is true for n=k, then it has been shown to be true for n=k+1 and as it is true for n=1, the statement is true for all positive integers n.

(ii) $f(n) = 4^n + 5^n + 6^n$

$n=1: f(1) = 4^1 + 5^1 + 6^1 = 15$

$\therefore$ statement is true for $n=1$

assume true for $n = 2k+1$, i.e.

$f(2k+1) = 4^{2k+1} + 5^{2k+1} + 6^{2k+1}$ is divisible by 15

try for $n = 2k+3$:

$f(2k+3) = 4^{2k+3} + 5^{2k+3} + 6^{2k+3}$

$= 4^{2k+1+2} + 5^{2k+1+2} + 6^{2k+1+2}$

$= 4^{2k+1} \cdot 4^2 + 5^{2k+1} \cdot 5^2 + 6^{2k+1} \cdot 6^2$

$= 16(4^{2k+1}) + 25(5^{2k+1}) + 36(6^{2k+1})$

$= 16(4^{2k+1} + 5^{2k+1} + 6^{2k+1}) + 9(5^{2k+1}) + 20(6^{2k+1})$

$= 16f(2k+1) + \underbrace{9(5^{2k+1}) + 20(6^{2k+1})}_{\text{want to show this is divisible by 15}}$

$= 16f(2k+1) + 9(5^{2k} \cdot 5^1) + 20(6^{2k} \cdot 6^1)$

$= 16f(2k+1) + 45(5^{2k}) + 120(6^{2k})$

$= 16f(2k+1) + 15[3 \times 5^{2k} + 8 \times 6^{2k}]$

If true for n=2k+1 then true for n=2k+3, true for n=1 so true for all positive odd integers n

7. A sample of bacteria in a sealed container is being studied.

The number of bacteria, P, in thousands, is modelled by the differential equation

$$(1+t)\frac{dP}{dt} + P = t^{\frac{1}{2}}(1+t)$$

where t is the time in hours after the start of the study.

Initially, there are exactly 5000 bacteria in the container.

(a) Determine, according to the model, the number of bacteria in the container 8 hours after the start of the study.
(6)

(b) Find, according to the model, the rate of change of the number of bacteria in the container 4 hours after the start of the study.
(4)

(c) State a limitation of the model.
(1)

(a) $\frac{dP}{dt} + \frac{1}{1+t}P = \frac{t^{1/2}(1+t)}{1+t} \quad \therefore \frac{dP}{dt} + \frac{1}{1+t}P = t^{1/2}$

INTEGRATING FACTOR : $\frac{dP}{dt} + XP = Y$ HAS SOLUTION

$$e^{\int X dt}P = \int e^{\int X dt} Y dt + C$$

WHERE $IF = e^{\int X dt}$

$X = \frac{1}{1+t}, \; Y = t^{1/2}$

$IF = e^{\int \frac{1}{1+t} dt} = e^{\ln(1+t)} = 1+t$

i.e. $(1+t)P = \int (1+t)t^{1/2} \, dt + c$

$(1+t)P = \int (t^{1/2} + t^{3/2}) dt + c$

$(1+t)P = \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + c$

$t=0, P=5 : (1+0)\times 5 = \frac{2}{3}\times 0^{3/2} + \frac{2}{5}\times 0^{5} + c \quad \therefore c = 5$

Question 7 continued

$\therefore (1+t)P = \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + 5$

$\therefore P = \dfrac{\frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + 5}{(1+t)}$

$t = 8: \quad P = \dfrac{\frac{2}{3} \times 8^{3/2} + \frac{2}{5} \times 8^{5/2} + 5}{1+8} = 10.27\ldots$

$\therefore$ 10277 bacteria

(b) $u = \frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + 5 \qquad v = 1+t$

$u' = t^{1/2} + t^{3/2} \qquad v' = 1$

$\dfrac{dP}{dt} = \dfrac{(1+t)(t^{1/2} + t^{3/2}) - (\frac{2}{3}t^{3/2} + \frac{2}{5}t^{5/2} + 5)}{(1+t)^2}$

$t = 4: \quad \dfrac{dP}{dt} = \dfrac{(1+4)(4^{1/2} + 4^{3/2}) - (\frac{2}{3} \times 4^{3/2} + \frac{2}{5} \times 4^{5/2} + 5)}{(1+4)^2}$

$\therefore \dfrac{dP}{dt} = \dfrac{403}{375}$

$\therefore$ rate is 1075 bacteria per hour

(c) *The number of bacteria increases indefinitely which is not realistic.*