Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Monday 13 May 2019

Afternoon (Time: 1 hour 40 minutes) | Paper Reference **8FM0-01**

Further Mathematics

Advanced Subsidiary
Paper 1: Core Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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1.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}$$

(a) Show that the matrix **M** is non-singular.

(2)

The transformation T of the plane is represented by the matrix **M**.

The triangle R is transformed to the triangle S by the transformation T.

Given that the area of S is 63 square units,

(b) find the area of R.

(2)

(c) Show that the line $y = 2x$ is invariant under the transformation T.

(2)

(a) $\det(M) = (4)(-7) - (-5)(2) = -28 - -10 = -18 \neq 0$

$\therefore$ M is non-singular

(b) ASF $= |\det(M)| = |-18| = 18$

area of R $= \dfrac{\text{area of } S}{18} = \dfrac{63}{18} = \dfrac{7}{2}$

(c) $\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix}$

$\begin{pmatrix} 4x & -10x \\ 2x & -14x \end{pmatrix}$

$\begin{pmatrix} -6x \\ -12x \end{pmatrix}$

$-6\begin{pmatrix} x \\ 2x \end{pmatrix}$

$\therefore$ line is invariant

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2. The cubic equation

$$2x^3 + 6x^2 - 3x + 12 = 0$$

has roots α, β and γ. $a=2,\ b=6,\ c=-3,\ d=12$

Without solving the equation, find the cubic equation whose roots are $(\alpha+3)$, $(\beta+3)$ and $(\gamma+3)$, giving your answer in the form $pw^3 + qw^2 + rw + s = 0$, where p, q, r and s are integers to be found.

(5)

$\alpha + \beta + \gamma = -\frac{b}{a} = -\frac{6}{2} = -3$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = -\frac{3}{2}$

$\alpha\beta\gamma = -\frac{d}{a} = -\frac{12}{2} = -6$

sum roots: $(\alpha+3) + (\beta+3) + (\gamma+3)$

$(\alpha + \beta + \gamma) + 9$

$(-3) + 9 = 6$

pair sum: $(\alpha+3)(\beta+3) + (\alpha+3)(\gamma+3) + (\beta+3)(\gamma+3)$

$\alpha\beta + 3\alpha + 3\beta + 9 + \alpha\gamma + 3\alpha + 3\gamma + 9 + \beta\gamma + 3\beta + 3\gamma + 9$

$(\alpha\beta + \alpha\gamma + \beta\gamma) + 6(\alpha + \beta + \gamma) + 27$

$(-\frac{3}{2}) + 6(-3) + 27 = \frac{15}{2}$

product: $(\alpha+3)(\beta+3)(\gamma+3)$

$(\alpha+3)(\beta\gamma + 3\beta + 3\gamma + 9)$

$\alpha\beta\gamma + 3\alpha\beta + 3\alpha\gamma + 9\alpha + 3\beta\gamma + 9\beta + 9\gamma + 27$

$(\alpha\beta\gamma) + 3(\alpha\beta + \alpha\gamma + \beta\gamma) + 9(\alpha + \beta + \gamma) + 27$

$(-6) + 3(-\frac{3}{2}) + 9(-3) + 27 = -\frac{21}{2}$

Question 2 continued

new cubic : $w^3 - 6w^2 + \frac{15}{2}w + \frac{21}{2} = 0$ $(\times 2)$

$2w^3 - 12w^2 + 15w + 21 = 0$

$\therefore p = 2,\ q = -12,\ r = 15,\ s = 21$

(Total for Question 2 is 5 marks)

3. Prove by mathematical induction that, for $n \in \mathbb{N}$

$$\sum_{r=1}^{n} \frac{1}{(2r-1)(2r+1)} = \frac{n}{2n+1}$$

$$\frac{(k+1)}{2(k+1)+1}$$

(6)

$n=1$: LHS $= \sum_{r=1}^{1} \frac{1}{(2\times1-1)(2\times1+1)} = \frac{1}{1\times3} = \frac{1}{3}$

RHS $= \frac{1}{2\times1+1} = \frac{1}{3}$

as LHS = RHS $\therefore$ true for $n=1$

assume true for $n=k$, i.e.

$$\sum_{r=1}^{k} \frac{1}{(2r-1)(2r+1)} = \frac{k}{2k+1}$$

try for $n=k+1$

$$\sum_{r=1}^{k+1} \frac{1}{(2r-1)(2r+1)} = \sum_{r=1}^{k} \frac{1}{(2r-1)(2r+1)} + \frac{1}{(2(k+1)-1)(2(k+1)+1)}$$

$$= \frac{k}{2k+1} + \frac{1}{(2k+1)(2k+3)}$$

$$= \frac{k(2k+3)+1}{(2k+1)(2k+3)}$$

$$= \frac{2k^2+3k+1}{(2k+1)(2k+3)}$$

$$= \frac{\cancel{(2k+1)}(k+1)}{\cancel{(2k+1)}(2k+3)}$$

$$= \frac{k+1}{2k+3}$$

Question 3 continued

$$= \frac{k+1}{2(k+1)+1}$$

$\therefore$ true for $n = k+1$

since true for $n=1$, and true for $n=k$, then true for $n=k+1$ so true for all $n \in \mathbb{Z}^+$

(Total for Question 3 is 6 marks)

4. The line l has equation

$$\frac{x+2}{1} = \frac{y-5}{-1} = \frac{z-4}{-3}$$

The plane Π has equation

$$\mathbf{r}.(\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = -7$$

Determine whether the line l intersects Π at a single point, or lies in Π, or is parallel to Π without intersecting it.

(5)

$$l: \underline{r} = \begin{pmatrix} -2 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 + \lambda \\ 5 - \lambda \\ 4 - 3\lambda \end{pmatrix}$$

meet if $[(-2+\lambda)\underline{i} + (5-\lambda)\underline{j} + (4-3\lambda)\underline{k}] \cdot (\underline{i} - 2\underline{j} + \underline{k}) = -7$

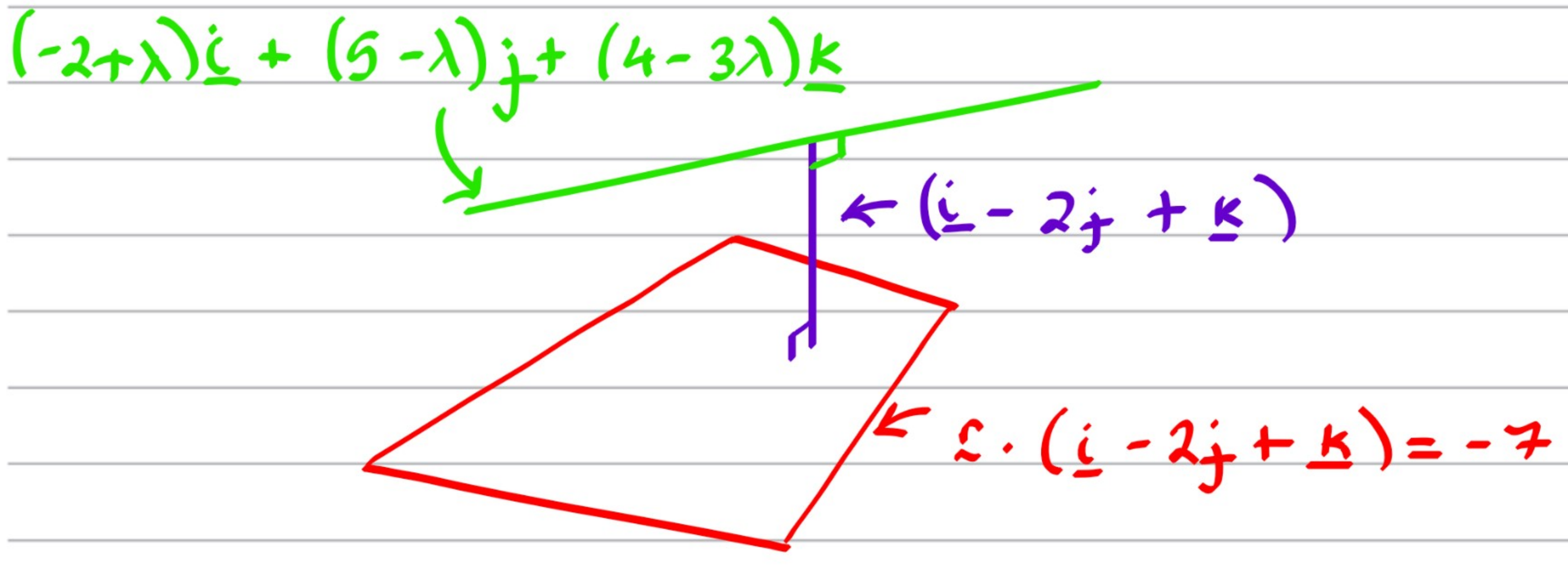
i.e. $1(-2+\lambda) - 2(5-\lambda) + 1(4-3\lambda) = -7$

$-2 + \cancel{\lambda} - 10 + \cancel{2\lambda} + 4 \cancel{- 3\lambda} = -7$

$-8 \neq -7$ $\therefore$ contradiction

$\therefore$ no intersection between line and plane

if a line is parallel to a plane, it will be perpendicular to the plane's normal vector



$(\underline{i} - \underline{j} - 3\underline{k}) \cdot (\underline{i} - 2\underline{j} + \underline{k}) = (1)(1) + (-1)(-2) + (-3)(1) = 0$

$\therefore l$ is parallel to π but not in the plane

5.

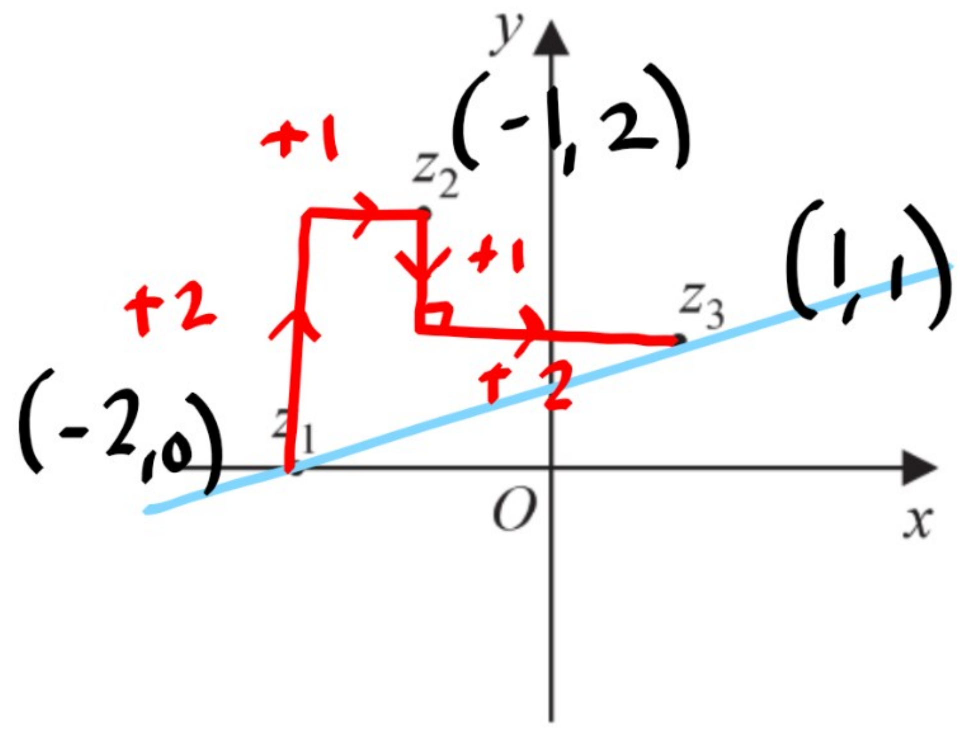


Figure 1

The complex numbers $z_1 = -2$, $z_2 = -1 + 2\text{i}$ and $z_3 = 1 + \text{i}$ are plotted in Figure 1, on an Argand diagram for the complex plane with $z = x + \text{i}y$

(a) Explain why z_1, z_2 and z_3 cannot all be roots of a quartic polynomial equation with real coefficients.

(2)

(b) Show that $\arg\left(\dfrac{z_2 - z_1}{z_3 - z_1}\right) = \dfrac{\pi}{4}$

(3)

(c) Hence show that $\arctan(2) - \arctan\left(\dfrac{1}{3}\right) = \dfrac{\pi}{4}$

(2)

A copy of Figure 1, labelled Diagram 1, is given on page 12.

(d) Shade, on Diagram 1, the set of points of the complex plane that satisfy the inequality

$$|z + 2| \leqslant |z - 1 - \text{i}|$$

(2)

(a) complex roots of a real polynomial occur in conjugate pairs so a polynomial with $z1$, $z2$ and $z3$ as roots need $z2^*$ and $z3^*$ as roots, so 5 roots in total, but a quartic has at most 4 roots, so no quartic can have $z1, z2$ and $z3$ as roots.

(b) $$\frac{z_2 - z_1}{z_3 - z_1} = \frac{-1 + 2i - (-2)}{1 + i - (-2)} = \frac{1 + 2i}{3 + i}$$

$$= \frac{1 + 2i}{3 + i} \cdot \frac{3 - i}{3 - i} = \frac{3 - i + 6i - 2i^2}{9 - i^2}$$

$$= \frac{3 + 5i - 2 \times -1}{9 - -1} = \frac{5 + 5i}{10} = \frac{1}{2} + \frac{1}{2} i$$

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Question 5 continued

hence $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) \equiv \arg\left(\frac{1}{2} + \frac{1}{2}i\right)$

$= \arctan\left(\frac{1/2}{1/2}\right)$

$= \arctan(1)$

$= \pi/4$

(c) $\arg\left(\frac{z_2 - z_1}{z_3 - z_1}\right) \equiv \arg(z_2 - z_1) - \arg(z_3 - z_1)$

$\equiv \arg(1 + 2i) - \arg(3 + i)$

$= \arctan(2/1) - \arctan(1/3)$

$= \arctan(2) - \arctan(1/3)$

$= \pi/4$

(d) $|z + 2| \leq |z - 1 - i|$

$|z - (-2)| \leq |z - (1 + i)|$

$|z - (-2)| = |z - (1 + i)|$ represents a perpendicular bisector joining $(-2, 0)$ and $(1, 1)$

by inspection, perpendicular bisector passes through z_2

Question 5 continued

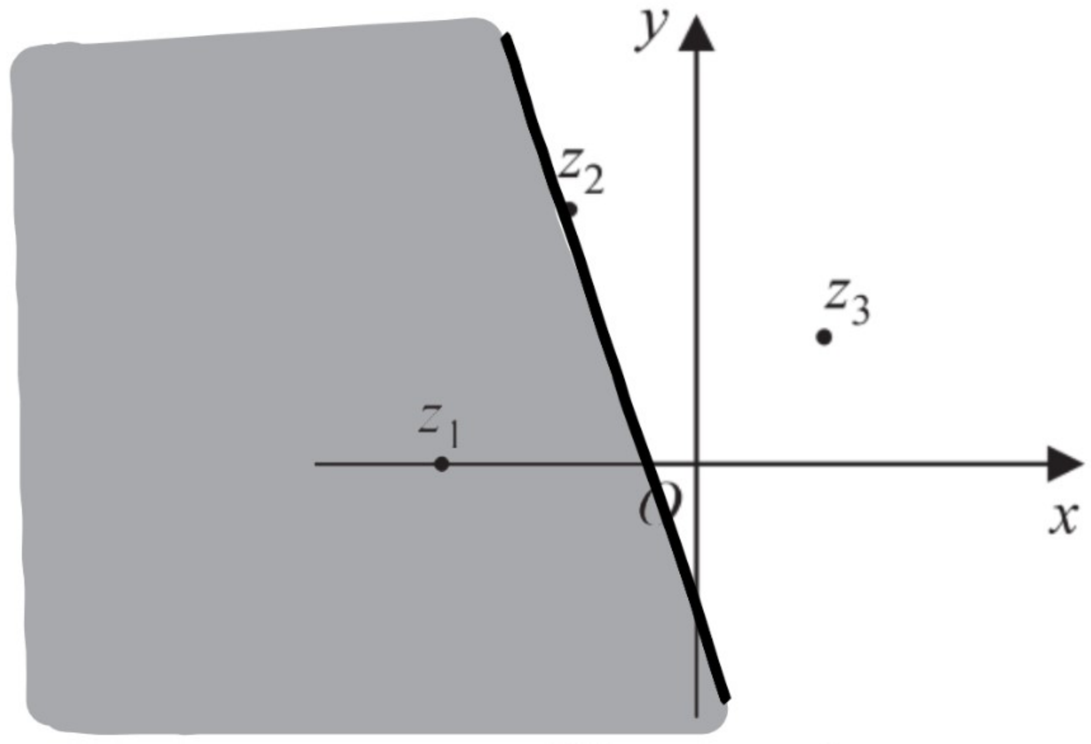

Diagram 1

6. An art display consists of an arrangement of n marbles.

When arranged in ascending order of mass, the mass of the first marble is 10 grams. The mass of each subsequent marble is 3 grams more than the mass of the previous one, so that the rth marble has mass $(7 + 3r)$ grams.

(a) Show that the mean mass, in grams, of the marbles in the display is given by

$$\frac{1}{2}(3n+17)$$

(3)

$\rightarrow n = 85$

Given that there are 85 marbles in the display,

(b) use the standard summation formulae to find the standard deviation of the mass of the marbles in the display, giving your answer, in grams, to one decimal place.

(6)

(a) mean, $\bar{x} = \frac{1}{n}\sum_{r=1}^{n}(7+3r) = \frac{1}{n}\left[7\sum_{r=1}^{n}1 + 3\sum_{r=1}^{n}r\right]$

$= \frac{1}{\cancel{n}}\left[7\cancel{n} + 3\times\frac{\cancel{n}}{2}(n+1)\right] = 7 + \frac{3}{2}(n+1)$

$= 7 + \frac{3}{2}n + \frac{3}{2} = \frac{3n}{2} + \frac{17}{2} = \frac{1}{2}(3n+17)$

(b) $SD = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$

mean, $\bar{x} = \frac{1}{2}\times(3\times 85 + 17) = 136$

$\sum_{r=1}^{n}(7+3r)^2 = \sum_{r=1}^{n}(9r^2 + 42r + 49)$

$\equiv 9\sum_{r=1}^{n}r^2 + 42\sum_{r=1}^{n}r + 49\sum_{r=1}^{n}1$

$= 9\times\frac{1}{6}n(n+1)(2n+1) + 42\times\frac{1}{2}n(n+1) + 49n$

$= \frac{3}{2}n(n+1)(2n+1) + 21n(n+1) + 49n$

hence SD is given by:

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Question 6 continued

$$\sqrt{\frac{\frac{3}{2}\cancel{n}(n+1)(2n+1)+21\cancel{n}(n+1)+49\cancel{n}}{\cancel{n}}-136^2}$$

i.e. $SD = \sqrt{\frac{3}{2}(n+1)(2n+1)+21(n+1)+49-136^2}$

$\underline{n=85:}$ $SD = \sqrt{\frac{3}{2}(85+1)(2\times 85+1)+21(85+1)+49-136^2}$

$= 73.559...$

$\therefore SD = 73.6\text{ g}$ (1dp)

(Total for Question 6 is 9 marks)

P 5 8 3 3 3 A 0 1 5 3 2

7. $$f(z) = z^3 - 8z^2 + pz - 24$$

where p is a real constant. $a=1,\ b=-8,\ c=p,\ d=-24$

Given that the equation $f(z) = 0$ has distinct roots

$$\alpha,\ \beta \text{ and } \left(\alpha + \frac{12}{\alpha} - \beta\right)$$

(a) solve completely the equation $f(z) = 0$

(6)

(b) Hence find the value of p.

(2)

(a) sum of roots: $\alpha + \cancel{\beta} + \left(\alpha + \frac{12}{\alpha} \cancel{-\beta}\right) = \frac{--8}{1} = 8$

$2\alpha + \frac{12}{\alpha} = 8 \qquad (\times\alpha)$

$2\alpha^2 - 8\alpha + 12 = 0 \qquad (\div 2)$

$\alpha^2 - 8\alpha + 6 = 0$

$\therefore \alpha = 2 + \sqrt{2}i,\ \beta = 2 - \sqrt{2}i$

$\therefore z_{1,2} = 2 \pm \sqrt{2}i$

$\therefore z_3 = (2 + \sqrt{2}i) + \frac{12}{2+\sqrt{2}i} - (2 - \sqrt{2}i) = 4$

$\therefore$ roots of $f(z) = 0$ are $2 \pm \sqrt{2}i,\ 4$

(b) $\underline{f(4) = 0}$: $(4)^3 - 8(4)^2 + p(4) - 24 = 0$

$4p - 88 = 0 \quad \therefore p = 22$

$\therefore f(z) = z^3 - 8z^2 + 22z - 24$

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8. A gas company maintains a straight pipeline that passes under a mountain.

The pipeline is modelled as a straight line and one side of the mountain is modelled as a plane.

There are accessways from a control centre to two access points on the pipeline.

Modelling the control centre as the origin O, the two access points on the pipeline have coordinates $P(-300, 400, -150)$ and $Q(300, 300, -50)$, where the units are metres.

(a) Find a vector equation for the line PQ, giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$, where λ is a scalar parameter.

(2)

The equation of the plane modelling the side of the mountain is $2x + 3y - 5z = 300$

The company wants to create a new accessway from this side of the mountain to the pipeline.

The accessway will consist of a tunnel of shortest possible length between the pipeline and the point $M(100, k, 100)$ on this side of the mountain, where k is a constant.

(b) Using the model, find

(i) the coordinates of the point at which this tunnel will meet the pipeline,

(ii) the length of this tunnel.

(7)

It is only practical to construct the new accessway if it will be significantly shorter than both of the existing accessways, OP and OQ.

(c) Determine whether the company should build the new accessway.

(2)

(d) Suggest one limitation of the model.

(1)

(a) $\vec{PQ} = \vec{OQ} - \vec{OP} = (300\underline{i} + 300\underline{j} - 50\underline{k}) - (-300\underline{i} + 400\underline{j} - 150\underline{k})$

$= (600\underline{i} - 100\underline{j} + 100\underline{k})$

$\therefore \underline{r} = \begin{pmatrix} -300 \\ 400 \\ -150 \end{pmatrix} + \lambda \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix}$

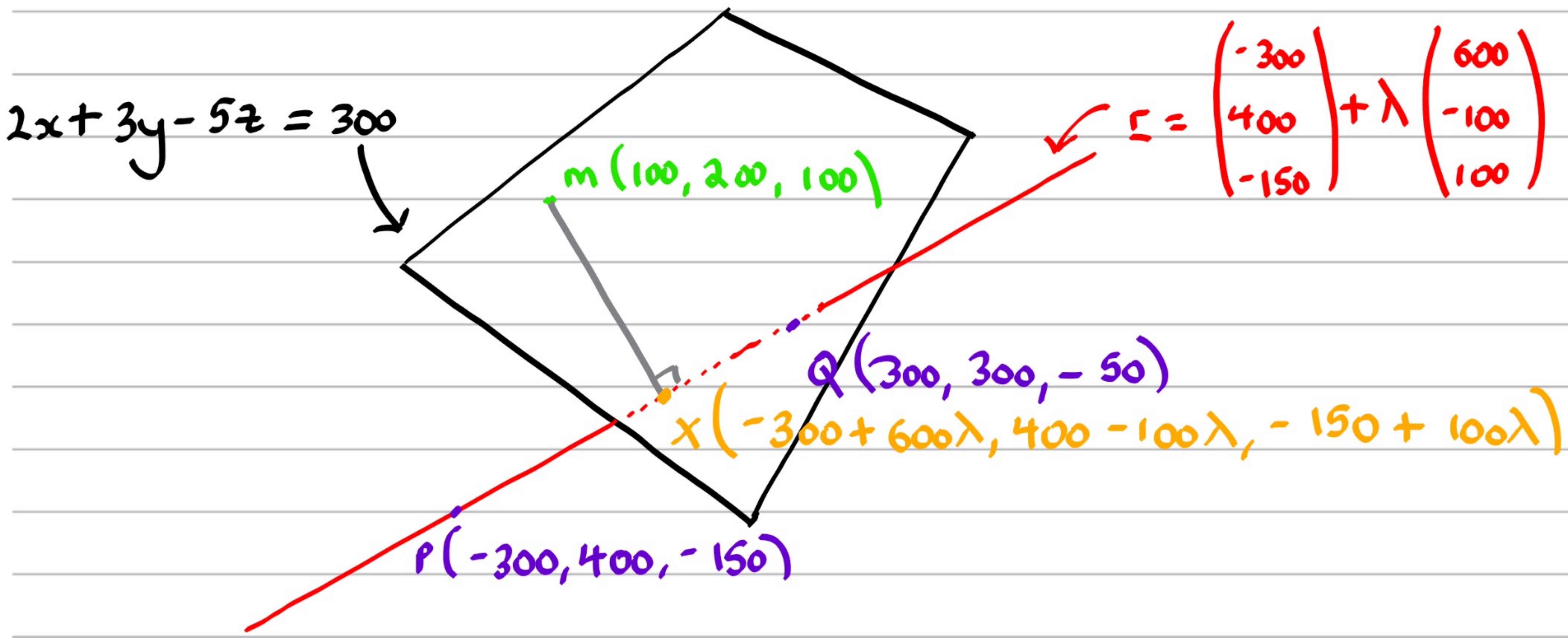
(b)(i) $2x + 3y - 5z = 300$

$\underline{M(100, K, 100)}$: $2(100) + 3(K) - 5(100) = 300$

$200 + 3k - 500 = 300$

$3k = 600 \quad \therefore k = 200$

$\therefore M(100, 200, 100)$



If x is a general point on the line

$$\overrightarrow{MX} = \overrightarrow{0x} - \overrightarrow{0m} = \begin{pmatrix} -300+600\lambda \\ 400-100\lambda \\ -150+100\lambda \end{pmatrix} - \begin{pmatrix} 100 \\ 200 \\ 100 \end{pmatrix} = \begin{pmatrix} -400+600\lambda \\ 200-100\lambda \\ -250+100\lambda \end{pmatrix}$$

for shortest possible length:

$$\begin{pmatrix} -400+600\lambda \\ 200-100\lambda \\ -250+100\lambda \end{pmatrix} \cdot \begin{pmatrix} 600 \\ -100 \\ 100 \end{pmatrix} = 0$$

$600(-400+600\lambda) - 100(200-100\lambda) + 100(-250+100\lambda) = 0$

$-2400 + 3600\lambda - 200 + 100\lambda - 250 + 100\lambda = 0$

$-2850 + 3800\lambda = 0 \quad \therefore \lambda = \frac{3}{4}$

Question 8 continued

$$\overrightarrow{OX} = \begin{pmatrix} -300 + 600 \times 3/4 \\ 400 - 100 \times 3/4 \\ -150 + 100 \times 3/4 \end{pmatrix} = \begin{pmatrix} 150 \\ 325 \\ -75 \end{pmatrix}$$

$\therefore X(150, 325, -75)$

(ii) $$\overrightarrow{MX} = \begin{pmatrix} -400 + 600 \times 3/4 \\ 200 - 100 \times 3/4 \\ -250 + 100 \times 3/4 \end{pmatrix} = \begin{pmatrix} 50 \\ 125 \\ -175 \end{pmatrix}$$

$|\overrightarrow{MX}| = \sqrt{(50)^2 + (125)^2 + (-175)^2} \approx 221\text{ m}$ (3sf)

(c) $|\overrightarrow{OP}| = \sqrt{(-300)^2 + (400)^2 + (-150)^2} \approx 522\text{ m}$ (3sf)

$|\overrightarrow{OQ}| = \sqrt{(300)^2 + (300)^2 + (50)^2} \approx 427\text{ m}$ (3sf)

new tunnel length is significantly shorter than these values so it is likely that the company will decide to build the accessway

(d) the mountainside is not likely to be flat so the plane may not be a good model

9. $$f(x) = 2x^{\frac{1}{3}} + x^{-\frac{2}{3}} \qquad x > 0$$

The finite region bounded by the curve $y = f(x)$, the line $x = \frac{1}{8}$, the x-axis and the line $x = 8$ is rotated through θ radians about the x-axis to form a solid of revolution.

Given that the volume of the solid formed is $\frac{461}{2}$ units cubed, use algebraic integration to find the angle θ through which the region is rotated.

(8)

If rotated by 2π radians about x-axis, then volume is $\pi \int y^2 \, dx$

If rotated by θ radians about x-axis, then volume is $\frac{\theta}{2} \int y^2 \, dx$

$y = 2x^{1/3} + x^{-2/3}$

$y^2 = (2x^{1/3} + x^{-2/3})(2x^{1/3} + x^{-2/3}) \equiv 4x^{2/3} + 4x^{-1/3} + x^{-4/3}$

hence $\frac{\theta}{\cancel{2}} \int_{1/8}^{8} (4x^{2/3} + 4x^{-1/3} + x^{-4/3}) \, dx = \frac{461}{\cancel{2}}$

$\theta \left[\frac{12x^{5/3}}{5} + 6x^{2/3} - 3x^{-1/3} \right]_{1/8}^{8} = 461$

$\theta \left[\left\{ \frac{12(8)^{5/3}}{5} + 6(8)^{2/3} - 3(8)^{-1/3} \right\} - \left\{ \frac{12(1/8)^{5/3}}{5} + 6(1/8)^{2/3} \right. \right.$

$\left. \left. - 3(1/8)^{-1/3} \right\} \right] = 461$

$\theta \left(\frac{993}{10} - \frac{-177}{40} \right) = 461$

$\frac{4149\theta}{40} = 461$

$\therefore \theta = \frac{40}{9}$ radians

10. The population of chimpanzees in a particular country consists of juveniles and adults. Juvenile chimpanzees do not reproduce.

In a study, the numbers of juvenile and adult chimpanzees were estimated at the start of each year. A model for the population satisfies the matrix system

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix} \qquad n = 0, 1, 2, \ldots$$

where a is a constant, and J_n and A_n are the respective numbers of juvenile and adult chimpanzees n years after the start of the study.

(a) Interpret the meaning of the constant a in the context of the model.

(1)

At the start of the study, the total number of chimpanzees in the country was estimated to be 64 000

According to the model, after one year the number of juvenile chimpanzees is 15 360 and the number of adult chimpanzees is 43 008

(b) (i) Find, in terms of a

$$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1}$$

(3)

(ii) Hence, or otherwise, find the value of a.

(3)

(iii) Calculate the change in the number of juvenile chimpanzees in the first year of the study, according to this model.

(2)

Given that the number of juvenile chimpanzees is known to be in decline in the country,

(c) comment on the short-term suitability of this model.

(1)

A study of the population revealed that adult chimpanzees stop reproducing at the age of 40 years.

(d) Refine the matrix system for the model to reflect this information, giving a reason for your answer.

(There is no need to estimate any unknown values for the refined model, but any known values should be made clear.)

(2)

(a) "a" is the proportion of juvenile chimpanzees that survive and remain juvenile chimpanzees the next year

(b)(i) det $= (a)(0.82) - (0.08)(0.15) = 0.82a - 0.012$

Question 10 continued

$$\therefore \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \frac{1}{0.82a - 0.012} \begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$$

(ii) $A_0 + J_0 = 64000$ ①

$$\begin{pmatrix} 15360 \\ 43008 \end{pmatrix} = \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix} \begin{pmatrix} J_0 \\ A_0 \end{pmatrix}$$

i.e. $aJ_0 + 0.15A_0 = 15360$ ②

$0.08J_0 + 0.82A_0 = 43008$ ③

From ①: $A_0 = 64000 - J_0$ ④

④ into ③: $0.08J_0 + 0.82(64000 - J_0) = 43008$

$-0.74J_0 = -9472$

$\therefore J_0 = 12800$, $A_0 = 51200$ (adult population is also decreasing)

sub back into ②: $a(12800) + 0.15(51200) = 15360$

$12800a = 7680$

$\therefore a = 0.6$

(iii) $J_0 = 12800$; $J_1 = 15360$

change $= 15360 - 12800 = 2560$

(c) as the number of juvelines has increased, the model is not initially predicting decline, so is not suitable in the long term

(d) let M_n be the number of mature chimpanzees n years after the start of the study $\therefore$ 3×3 matrix required

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \\ M_{n+1} \end{pmatrix} = \begin{pmatrix} 0.6 & b & \boxed{0} \\ 0.08 & c & \boxed{0} \\ \boxed{0} & d & e \end{pmatrix} \begin{pmatrix} J_n \\ A_n \\ M_n \end{pmatrix}$$

a

no new juvelines arise from mature chimpanzees

mature chimpanzees do not regress to adulthood

juvelines cannot proceed directly to mature chimpanzees

EXAMPLE IN CONTEXT

$$\begin{pmatrix} J_1 \\ A_1 \\ M_1 \end{pmatrix} = \begin{pmatrix} 0.6 & b & \boxed{0} \\ 0.08 & c & \boxed{0} \\ \boxed{0} & d & e \end{pmatrix} \begin{pmatrix} J_0 \\ A_0 \\ M_0 \end{pmatrix}$$

reproducing — don't reproduce

$J_1 = 0.6J_0 + bA_0 + \boxed{0M_0}$

$A_1 = 0.08J_0 + cA_0 + \boxed{0M_0}$ ← mature can't go back to adulthood

$M_1 = dA_0 + eM_0 + \boxed{0J_0}$

juvelines cannot proceed directly to mature chimpanzees

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