Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$\frac{177}{45} = 3.933...$ so lower bound is 4	M1 A1 (2)
(b)	Container 1: **16 23 4** Container 2: **18 9** 5 13 Container 3: 20 17 6 Container 4: 35 Container 5: 11	**M1** A1 A1 (3)
(c)	16 23 18 9 4 20 35 5 17 13 6 11 23 18 16 9 20 35 5 17 13 6 11 **4** 23 18 16 20 35 9 17 13 6 11 **5 4** 23 18 20 35 16 17 13 9 11 **6 5 4** 23 20 35 18 17 16 13 11 **9 6 5 4** 23 35 20 18 17 16 13 **11 9 6 5 4** 35 23 20 18 17 16 **13 11 9 6 5 4** 35 23 20 18 17 **16 13 11 9 6 5 4**	M1 A1 A1ft A1cso (4)
(d)	Container 1: **35** 9 Container 2: **23 20** Container 3: **18 17** 6 4 Container 4: **16** 13 11 5	**M1** A1 A1 (3)
		12 marks

Notes for Question
a1M1: Attempt to find the lower bound $(177 \pm 35)/45$. A value of 3.9 or better (for example, 3.93, 3.933, 3.9333…, $\frac{59}{15}$, etc.) seen with no corresponding calculation can imply this mark. Allow this mark for a clear intention of adding **all twelve** values and dividing by 45, for example, $\frac{16+23+18+9+4+20+35+5+17+13+6+11}{45}$ scores M1 **a1A1:** CAO (correct answer only) - correct calculation seen followed by 4 **or** the correct value of 3.9 or better stated (see note above for mark **a1M1**) followed by 4. However, an answer of 4 with no working scores no marks in this part **b1M1:** First five items placed correctly and at least eight values placed in containers. The first five values are those in bold. Condone cumulative totals for M1 only (e.g. for Crate 1: 16 39 43) **b1A1:** First eight items placed correctly (the underlined **and** bold values). This mark cannot be awarded if you see **any** repeated values or more than twelve items placed in crates (even if the first eight items have been placed correctly) **b2A1:** CSO (correct solution only). No additional or repeated values **c1M1:** Bubble sort. Consistent direction, end number (4) in place and the list beginning with the correct first four numbers (23 18 16 9). Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 4 appears at the end of the list is the end of their first pass **c1A1:** The first, second **and** third passes correct – so end three numbers in place

Question Number	Scheme	Marks

c2A1ft: Fourth and fifth passes correct following through from the candidate's third pass – so end five numbers in place

c3A1: CSO (correct solution only – so previous three marks must have been awarded in this part). Must show a 7th pass showing no swaps/changes. Condone if the sort continues until an 11th pass has been completed (but there should be no changes in the 8th to 11th passes) **or** if the pass stops betweeen an 8th and 11th pass (provided there are no changes in the values in any pass after the 6th pass)

d1M1: Must be using correct sorted list in descending order (so no follow through or misreads from an incorrect list from (c)). First six items placed correctly and at least eight values placed in containers. Note that the first six items are the bold values. Condone cumulative totals for M1 only. First-fit increasing scores no marks in this part

d1A1: First nine items placed correctly (the underlined and bold values). No additional or repeated values

d2A1: CSO. So no additional or repeated values

Sorting into ascending order in (c)

- If the candidate sorts the list into ascending order and reverses the list they can score full marks
- If the list is not reversed in (c) then mark as a misread (so mark according to the main scheme and then subtract the final two A marks earned). If the candidate says that the list needs reversing but doesn't actually show the reversed list then remove the final A mark earned

Misreads for part (c) only – if there is a 'misread' of a single number (this could take the form of an extra number, a number missing, or a number changed, for example, 42 rather than 23) before starting the sort in (c) then mark as a misread. If they 'misread' more than one number then M0. If they miscopy one of their own numbers during the sort then this is an accuracy error and loses the corresponding A mark(s) according to the scheme. No misreads permitted in (a), (b) and (d) (so mark to the main scheme)

For reference the sort in ascending order is:

16	23	18	9	4	20	35	5	17	13	6	11
16	18	9	4	20	23	5	17	13	6	11	**35**
16	9	4	18	20	5	17	13	6	11	**23**	**35**
9	4	16	18	5	17	13	6	11	**20**	**23**	**35**
4	9	16	5	17	13	6	11	**18**	**20**	**23**	**35**
4	9	5	16	13	6	11	**17**	**18**	**20**	**23**	**35**
4	5	9	13	6	11	**16**	**17**	**18**	**20**	**23**	**35**
4	5	9	6	11	**13**	**16**	**17**	**18**	**20**	**23**	**35**
4	5	6	9	**11**	**13**	**16**	**17**	**18**	**20**	**23**	**35**
4	5	6	**9**	**11**	**13**	**16**	**17**	**18**	**20**	**23**	**35**

Q2.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>(a)</td><td>$3 < \frac{228}{n} \leq 4$</td><td>M1</td></tr>
<tr><td></td><td>Critical value of 57 and 76 (or 57 and 75)</td><td>A1</td></tr>
<tr><td></td><td>$57 \leq n < 76$ (or $57 \leq n \leq 75$)</td><td>A1 (3)</td></tr>
<tr><td>(b)</td><td>e.g. middle right
<table>
<tr><td>14</td><td>20</td><td>23</td><td>17</td><td>15</td><td>22</td><td>19</td><td>25</td><td>13</td><td>28</td><td>32</td></tr>
<tr><td>23</td><td>25</td><td>28</td><td>32</td><td>22</td><td>14</td><td>20</td><td>17</td><td>15</td><td>19</td><td>13</td></tr>
<tr><td>32</td><td>28</td><td>23</td><td>25</td><td>22</td><td>20</td><td>17</td><td>19</td><td>15</td><td>14</td><td>13</td></tr>
<tr><td>32</td><td>28</td><td>25</td><td>23</td><td>22</td><td>20</td><td>19</td><td>17</td><td>15</td><td>14</td><td>13</td></tr>
<tr><td>32</td><td>28</td><td>25</td><td>23</td><td>22</td><td>20</td><td>19</td><td>17</td><td>15</td><td>14</td><td>13</td></tr>
</table>
e.g. middle left
<table>
<tr><td>14</td><td>20</td><td>23</td><td>17</td><td>15</td><td>22</td><td>19</td><td>25</td><td>13</td><td>28</td><td>32</td></tr>
<tr><td>23</td><td>25</td><td>28</td><td>32</td><td>22</td><td>14</td><td>20</td><td>17</td><td>15</td><td>19</td><td>13</td></tr>
<tr><td>28</td><td>32</td><td>25</td><td>23</td><td>22</td><td>20</td><td>19</td><td>17</td><td>14</td><td>15</td><td>13</td></tr>
<tr><td>32</td><td>28</td><td>25</td><td>23</td><td>22</td><td>20</td><td>19</td><td>17</td><td>15</td><td>14</td><td>13</td></tr>
<tr><td>32</td><td>28</td><td>25</td><td>23</td><td>22</td><td>20</td><td>19</td><td>17</td><td>15</td><td>14</td><td>13</td></tr>
</table></td><td>M1
A1
A1ft
A1 (4)</td></tr>
<tr><td>(c)</td><td>From first-fit Bin 1 could not fit the 17 so $n < 74$ (or $n \leq 73$) but could fit the 15 so n is either 72 (as the largest total is 72 in Bin 1 from first-fit) or 73</td><td>B1</td></tr>
<tr><td></td><td>From first-fit decreasing the 13 could not fit in Bin 1</td><td>B1</td></tr>
<tr><td></td><td>So $n = 72$</td><td>ddB1 (3)</td></tr>
<tr><td></td><td></td><td>10 marks</td></tr>
</table>

Question Number	Scheme	Marks
	Notes for Question	

a1M1: An equation or inequality linking the expression $\frac{228}{n}$ with either 3 or 4

a1A1: Correct critical values of 57 and 76 (or 57 and 75)
a2A1: $57 \leq n < 76$ or $57 \leq n \leq 75$

b1M1: Quick sort – pivots, p, selected and first pass gives >p, p, <p. If only choosing 1 pivot per iteration M1 only. If sorting into ascending order then mark as a misread
b1A1: First pass correct and next pivots chosen correctly/consistently for second pass
b2A1ft: Second and third passes correct (ft from their first pass and choice of pivots)
b3A1: cso (including a fourth pass with 19 used as a pivot if middle right or 14 if middle left)

c1B1: Correct deduction from first-fit that n is at least 72 **or** at most 73 (oe e.g. less than 74). For example, may see (14 + 20 + 23 + 15 =) 72 stated therefore $n \geq 72$ **or** 14 + 20 + 23 + 17 = 74 followed by $n < 74$ (so realising that the 17 did not fit in Bin 1). As a minimum accept the statement that $n < 74$ **or** $n \leq 73$ **or** $n \geq 72$
c2B1: Correct deduction from first-fit decreasing that the 13 was not placed in Bin 1. For example, may see 32 + 28 + 13 = 73 so therefore $n < 73$ or $n \leq 72$. As a minimum accept the statement that $n < 73$ **or** $n \leq 72$ **or** simply stating that '13 did not fit in Bin 1' (give bod here if not clear which Bin 1 they are considering)
c3ddB1: cao (dependent on both previous B marks) – must state that the largest total in any bin is 72 (**or** they need to say or show there exists a bin with 72) **and** that the 13 did not fit in Bin 1 in first-fit decreasing so n cannot be 73 and therefore $n = 72$ (not just '13 does not fit in Bin 1' – must be clear that they are talking about first-fit decreasing)

No marks in (c) if $n = 72$ stated with no working or if all the candidate does is to sum the numbers in each bin

Note that the first B mark in (c) can be implied if the candidate considers the first-fit decreasing packing first or argues with first-fit decreasing before considering first-fit, e.g., 'The 13 does not fit in Bin 1 in the first-fit decreasing packing therefore n is at most 72 and the total of Bin 1 in first-fit is 72' would imply the first two B marks in this part. Stating that therefore n must be 72 would then score all three marks

Q3.

Question Number	Scheme	Marks
(a)	Bin 1: **2.6** **0.8** **1.2** 0.3 Bin 2: **2.1** 0.9 1.7 Bin 3: 2.3 1.8 Bin 4: 2.7	**M1** A1 A1 **(3)**
(b)(i)	First pass: 2.6 2.1 1.2 0.9 1.7 2.3 0.8 1.8 2.7 0.3 Second pass: 2.6 2.1 1.2 1.7 2.3 0.9 1.8 2.7 0.8 0.3	B1 B1
(b)(ii)	<table><tr><th></th><th>Comparisons</th><th>Swaps</th></tr><tr><td>First pass</td><td>9</td><td>7</td></tr><tr><td>Second pass</td><td>8</td><td>4</td></tr></table>	B1 B1 **(4)**
(c)	e.g. middle right	
	2.6 2.1 1.7 2.3 1.2 1.8 2.7 0.9 0.8 0.3 — 1.8 2.6 2.1 2.3 2.7 **1.8** 1.7 1.2 0.9 0.8 0.3 — 2.3, 0.9 2.6 2.7 **2.3** 2.1 **1.8** 1.7 1.2 **0.9** 0.8 0.3 — 2.7, (2.1), 1.2, 0.3 **2.7** 2.6 **2.3** 2.1 **1.8** 1.7 **1.2** **0.9** 0.8 **0.3** — Sort complete	M1 A1 A1 **(3)**
(d)	Bin 1: **2.7** **2.3** Bin 2: **2.6** **2.1** 0.3 Bin 3: 1.8 1.7 1.2 Bin 4: 0.9 0.8	**M1** A1 A1 **(3)**
		13 marks

Notes for Question

a1M1: First four items placed correctly and at least seven values placed in bins. Condone cumulative totals for M1 only (the values in bold)
a1A1: First seven items placed correctly (the underlined and bold values) – any repeated/additional values then A0
a2A1: CSO (correct solution only – so no additional/repeated values)

bi1B1: CAO (first pass) – some candidates may show each comparison/swap within the first pass so take the first pass to be the list when the 0.3 is in the correct position
bi2B1: CAO (second pass) – some candidates may show each comparison/swap within the second pass so take the second pass to be the list when the 0.8 and the 0.3 are in the correct positions

ISW if completing more than two passes

bii1B1: Two correct values in the given table
bii2B1: Fully correct table completed

Mark table on page 2 of the AB (and ignore any other answers given elsewhere) but if table blank then check answer space carefully and mark this attempt instead

c1M1: Quick sort – pivot, p, chosen (must be choosing middle left or middle right – choosing first/last item as a pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then M1 only. If sorting into ascending order then M0**
c1A1: First two passes correct (second pass pivot consistent with choice of pivot in first pass) – but need not be choosing pivots for the third pass
c2A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including a 'sort complete' - this could be shown by the final list being re-written or 'sorted' statement (e.g. 'done', 'complete', etc.) or each item being used as a pivot (which would therefore mean that the final list would have been written twice)

middle left:

2.6	2.1	1.7	2.3	1.2	1.8	2.7	0.9	0.8	0.3	1.2
2.6	2.1	1.7	2.3	1.8	2.7	**1.2**	0.9	0.8	0.3	1.7, 0.8
2.6	2.1	2.3	1.8	2.7	**1.7**	**1.2**	0.9	**0.8**	0.3	2.3, (0.9), (0.3)
2.6	2.7	**2.3**	2.1	1.8	**1.7**	**1.2**	0.9	**0.8**	0.3	2.6, 2.1
2.7	**2.6**	**2.3**	**2.1**	1.8	**1.7**	**1.2**	0.9	**0.8**	0.3	Sort Complete

Two Special Cases for (c): Case I: Those that perform a quick sort on the original list can score M1 only. Case II: Those that perform a quick sort on 2.6 2.1 1.7 2.3 1.2 1.8 2.7 (so not including the last three numbers in the list) can score M1A1 only

No misreads in (d) – mark according to scheme in all cases

d1M1: First four items placed correctly and at least seven values placed in bins – condone cumulative totals for M1 only (the bold values)
d1A1: First seven items placed correctly (the underlined and bold values) – any repeated/additional values is A0
d2A1: CSO (so no additional/repeated values)

Q4.

Question Number	Scheme	Marks
(a)	$\frac{131}{40} = 3.275$ so lower bound is 4	M1 A1 **(2)**
(b)	Container 1: 17 9 8 4 Container 2: 15 20 5 Container 3: 13 12 Container 4: 28	M1 A1 A1 **(3)**
(c)	middle right 17 9 15 8 20 **13** 28 4 12 5 — Pivots: 13 9 8 **4** 12 5 13 17 15 **20** 28 — Pivots: 4, 20 4 9 8 **12** 5 13 17 **15** 20 28 — Pivots: 12, 15, (28) 4 9 **8** 5 12 13 15 17 20 28 — Pivots: 8, (17) 4 5 8 9 12 13 15 17 20 28 — Sort complete	M1 A1 A1 **(3)**
	middle left 17 9 15 8 **20** 13 28 4 12 5 — Pivots: 20 17 9 15 **8** 13 4 12 5 20 28 — Pivots: 8, (28) **4** 5 8 17 9 **15** 13 12 20 28 — Pivots: 4, 15 4 5 8 9 **13** 12 15 17 20 28 — Pivots: (5), 13, (17) 4 5 8 **9** 12 13 15 17 20 28 — Pivots: 9 4 5 8 9 12 13 15 17 20 28 — (Sort Complete)	
(d)	$\left\lceil \frac{1+10}{2} \right\rceil = 6$ 13 – reject 13 – 28 $\left\lceil \frac{1+5}{2} \right\rceil = 3$ 8 – reject 4 – 8 $\left\lceil \frac{4+5}{2} \right\rceil = 5$ 12 – reject 12 $\lceil 4 \rceil = 4$ 9 – found	M1 A1 A1 **(3)**
		11 marks

Notes for Question

a1M1: Attempt to find the lower bound $(131 \pm 28)/40$. A value of 3.275 seen with no corresponding calculation can imply this mark. Allow this mark for a clear intention of adding **all** ten values and dividing by 40, for example,

$\frac{17+9+15+8+20+13+28+4+12+5}{40}$ scores M1

a1A1: cao - correct calculation seen followed by 4 **or** the correct value of 3.275 followed by 4. However, an answer of 4 with no working scores no marks in (a)

b1M1: First four values placed correctly and at least eight values placed in containers. The first four values are those that are boxed. Condone cumulative totals for M1 only (e.g. for Container 1: 17 26 34 38)
b1A1: First eight values placed correctly (the boxed **and** underlined values). This mark cannot be awarded if you see **any** repeated values or more than ten values placed in containers (even if the first eight items have been placed correctly)
b2A1: cso – no additional or repeated values (dependent on both previous marks)

c1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values less than the pivot), pivot, (values greater than the pivot). If sorting into descending order then M1 only (even if the list is reversed at the end of the sort). This mark can be scored if one number only is either missing or incorrect or an additional number is added to the list
c1A1: First two passes correct and next pivots chosen correctly for third pass (but third pass does not need to be attempted or correct)
c2A1: cso (correct solution only – all previous marks in this part must have been awarded) – if middle right then either a fifth pass **or** a 'sort complete' statement (e.g. 'sorted', 'complete', etc. but not just underlining the fourth pass) is required after the fourth pass. If middle left then a fifth pass (with the value of 9) is required (but no 'sort complete' statement is required)

d1M1: Choosing middle right pivot (choosing middle left '12' is M0) **and** an attempt at discarding/retaining half the list (condone if retaining the wrong half of the list or if retaining 1 – 6)
d1A1: First and second passes correct i.e. selecting the 6th item in the first pass **and** using 1st to 5th items in the second pass (must not be using the 6th item for the second pass) and then correctly selecting the 3rd item (the 8) in the second pass and rejecting the 1st to 3rd items
d2A1: cao - search completed correctly (so rejecting the 12 in the third pass) together with 'found'. Condone candidates who say that after the 12 in the third pass has been rejected the only value left is the 9 so it has been found. It must be clear that the 9 has been 'found' and not just stated as the final value

In (d) candidates must be using a correct ordered list (4 5 8 9 12 13 15 17 20 28) – if it is clear that the candidates are not using this list then M0. With regards to using the original (unsorted) list the 6th value is 13 too so what the candidates do next will most likely indicate if the correct list is being used (e.g. if the next pivot is 15 then M0)

Candidates who have sorted the list into descending order can earn full marks in (d) – scheme above applies in the same way for descending – must be choosing middle right (the 12) not middle left (the 13) for M1

28 20 17 15 13 12 9 8 5 4

$\left\lceil \frac{1+10}{2} \right\rceil = 6$ 12 – reject 28 – 12

$\left\lceil \frac{7+10}{2} \right\rceil = 9$ 5 – reject 5 – 4

$\left\lceil \frac{7+8}{2} \right\rceil = 8$ 8 – reject 8

$\lceil 7 \rceil = 7$ 9 – found

Q5.

Question Number	Scheme	Marks
(a)	$\frac{132}{42} = 3.14...$ so lower bound is 4	M1 A1 (2)
(b)	Group 1: **8** **17** **9** 7 Group 2: **14** **18** 10 Group 3: **12** 22 Group 4: 15	M1 A1 (2)
(c)	e.g. middle right 8 17 9 14 18 12 22 10 15 7 17 14 18 22 15 **12** 8 9 10 7 22 **18** 17 14 15 **12** **10** 8 9 7 22 **18** 17 15 **14** **12** **10** **9** 8 7 22 **18** 17 **15** **14** **12** **10** **9** 8 **7** e.g. middle left 8 17 9 14 18 12 22 10 15 7 22 **18** 8 17 9 14 12 10 15 7 22 **18** 17 15 **14** 8 9 12 10 7 22 **18** **17** 15 **14** **12** 8 9 10 7 22 **18** **17** 15 **14** **12** 10 **9** 8 7 22 **18** **17** 15 **14** **12** 10 **9** **8** 7	M1 A1 A1ft A1 (4)
(d)	Group 1: **22** **18** Group 2: **17** **15** 10 Group 3: **14** **12** 9 7 Group 4: 8	M1 A1 (2)
(e)	B(E)C + G(I)H = (11.2 + 14.5) + (8.3 + 17.2) = 51.2* B(F)G + C(EJ)H = (10.3 + 15.2) + (14.5 + 7. 5 + 16.2) = 63.7 B(EJ)H + C(EF)G = (11.2 + 7.5 + 16.2) + (14.5 + 4. 3 + 15.2) = 68.9 Repeat arcs: BE, CE, GI, HI	M1 A1 A1 A1 (4)
(f)	Route e.g. ABEBFECEJIFGIGHIHJDCA Length = 227. 2 + 51.2 = 278.4 (m)	B1 B1ft (2)
(g)	Finishing vertex: C Reduction in lengths: 51.2 – (10.3 + 15.2) = 25.7 (m)	B1 B1 (2)
		18 marks

Notes for Question

PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE FOR ASCENDING IN PART (c)

a1M1: Attempt to find the lower bound $(132 \pm 22)/42$ (a value of 3.14 (or better) seen with no working can imply this mark)
a1A1: CSO - correct calculation seen **or** 3.14 followed by 4 – accept 3.1 if correct calculation seen. An answer of 4 with no working scores M0A0

b1M1: First six items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the values in bold)
b1A1: CSO (so no additional/repeated values)

c1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less that the pivot). **If only choosing one pivot per iteration then M1 only**

c1A1: First pass correct **and** next pivots chosen correctly for the second pass (but the second pass does not need to be correct) – so they must be choosing (if middle right) a pivot values of 18 and 10 for the second pass or (if middle left) a pivot value of 14

c2A1ft: Second and third passes correct (follow through from their first pass and choice of pivots). They do not need to be choosing a pivot for the fourth pass for this mark
c3A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including if middle right a fourth pass with the 15 and 7 **used** as pivots **or** if middle left a fifth pass with the 8 used as a pivot

Sorting list into ascending order in (c)

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in **(c)**
- If the list is not reversed in **(c)** but stated in ascending or descending order in **(d)** then remove the last two A marks earned in **(c)**. If the candidate says that the list needs reversing in **(c)** but does not actually show the reversed list in **(c)** then remove the last A mark earned
- **Note that if sorting into ascending order then a 'sort complete' statement is required – this could be shown by the final list being re-written or 'sorted' statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice) BEFORE list is reversed**

Middle right ascending
(requires sort complete statement– see above)

8 17 9 14 18 12 22 10 15 7
8 9 10 7 **12** 17 14 18 22 15
8 9 7 **10** **12** 17 14 15 **18** 22
8 7 **9** **10** **12** **14** 17 15 **18** 22
7 8 **9** **10** **12** **14** **15** 17 **18** 22

Middle left ascending
(requires sort complete statement – see above)

8 17 9 14 18 12 22 10 15 7
8 17 9 14 12 10 15 7 **18** 22
8 9 12 10 7 **14** 17 15 **18** 22
8 9 10 7 **12** **14** 15 **17** **18** 22
8 7 **9** 10 **12** **14** 15 **17** **18** 22
7 **8** **9** 10 **12** **14** 15 **17** **18** 22

d1M1: First six items placed correctly and at least eight items placed in bins – condone cumulative totals for M1 only (the values in bold)
d1A1: CSO (so no additional/repeated values)

e1M1: Correct three pairings of the correct four odd nodes (B, C, G and H)
e1A1: Any one row correct including pairings **and** totals
e2A1: All three rows correct including pairings **and** totals
e3A1: CAO correct arcs clearly stated: BE, CE, GI and HI – must be these arcs and not e.g. BEC, GIH, or BC via E, etc.

f1B1: Any correct route (checks: 21 vertices, starting and ending at A, BE, CE, GI and HI appearing twice, A(2), B(2), C(2), D(1), E(3), F(2), G(2), H(2), I(3), J(2))
f2B1ft: For 227.2 + their smallest repeat out of a choice of at least **two** totals seen in **(e)** – this mark is dependent on M1 in (e)

g1B1: CAO (C)
g2B1: CAO (25.7) – note that the correct answer can come from incorrect working e.g. 11.2 + 14.5 = 25.7 is B0 (just adding BE and EC together) so this answer need to be checked carefully – correct method is 51.2 – (10.3 + 15.2) (subtracting BF and FG from 51.2) but give bod on a correct answer of 25.7 with no working

Q6.

Question Number	Scheme	Marks
	$\left[\frac{1+10}{2}\right]=6$ Diameter – reject 1 – 6	M1
	$\left[\frac{7+10}{2}\right]=9$ Segment – reject 9 – 10	A1
	$\left[\frac{7+8}{2}\right]=8$ Sector – reject 8	A1
	[7] = 7 Radius – reject Parallelogram is not in list	A1 **(4)**
		4 marks

Notes for Question

1M1: Choosing middle right pivot (choosing middle left 'Circumference' is M0) + an attempt at discarding/retaining half the list (condone if retaining the wrong half of the list or if only rejecting 1 – 5)

1A1: First pass correct i.e. 6th item **and** using 7 – 10 in the second pass (must not be using the 6th item in the second pass) – need not choose the 9th item or reject 9 – 10 for this mark

2A1: Second and third passes correct i.e. 9th (Segment) and 8th (Sector) items (no sticky pivots) – need not be rejecting the 8th item for this mark

3A1: CAO search complete (so rejecting 8th and 7th items) + 'not found' – must consider Radius by name or the '7th item' after rejecting sector (or stating that the seventh item is not Parallelogram). Condone if Radius is rejected on the same line that Sector is rejected (but must be after Sector). Condone those candidates that correctly state that Radius is not Parallelogram or who do not explicitly reject the Radius
e.g. '[7] = 7 Radius therefore Parallelogram is not in the list.'

Allow use of abbreviations provided clear and unambiguous. Also accept the new list of words being re-written after each pass (with or without the corresponding calculations).

For reference:

Arc Centre Chord Circle Circumference Diameter Radius Sector Segment Tangent

Q7.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>(a)</td><td>$\frac{1150}{300} = 3.83...$ so lower bound is 4</td><td>M1 A1 (2)</td></tr>
<tr><td>(b)</td><td>
<table>
<tr><td>175</td><td>135</td><td>210</td><td>105</td><td>100</td><td>150</td><td>60</td><td>20</td><td>70</td><td>125</td></tr>
<tr><td>175</td><td>210</td><td>135</td><td>105</td><td>150</td><td>100</td><td>60</td><td>70</td><td>125</td><td>20</td></tr>
<tr><td>210</td><td>175</td><td>135</td><td>150</td><td>105</td><td>100</td><td>70</td><td>125</td><td>60</td><td>20</td></tr>
<tr><td>210</td><td>175</td><td>150</td><td>135</td><td>105</td><td>100</td><td>125</td><td>70</td><td>60</td><td>20</td></tr>
<tr><td>210</td><td>175</td><td>150</td><td>135</td><td>105</td><td>125</td><td>100</td><td>70</td><td>60</td><td>20</td></tr>
<tr><td>210</td><td>175</td><td>150</td><td>135</td><td>125</td><td>105</td><td>100</td><td>70</td><td>60</td><td>20</td></tr>
<tr><td>210</td><td>175</td><td>150</td><td>135</td><td>125</td><td>105</td><td>100</td><td>70</td><td>60</td><td>20</td></tr>
</table>
</td><td>M1

A1

A1ft

A1cso (4)</td></tr>
<tr><td>(c)</td><td>Truck 1: 210 <u>70</u> 20
Truck 2: 175 <u>125</u>
Truck 3: 150 135
Truck 4: <u>105</u> <u>100</u> 60</td><td>M1 <u>A1</u> A1
(3)</td></tr>
<tr><td></td><td></td><td>9 marks</td></tr>
</table>

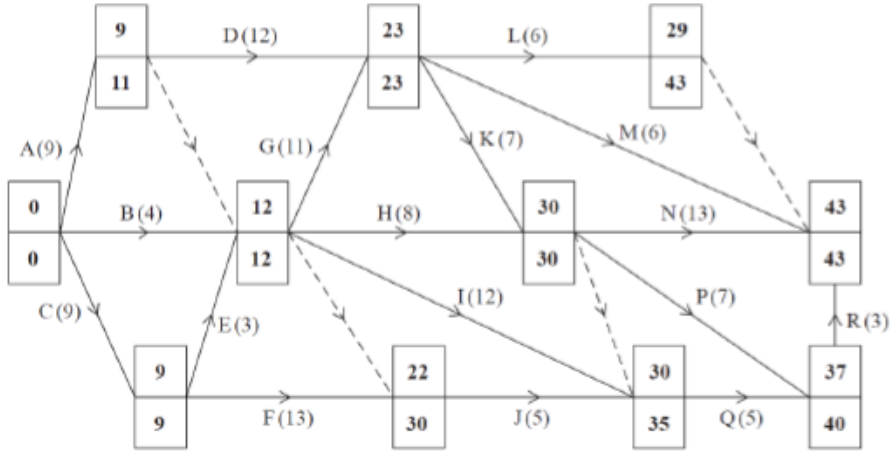
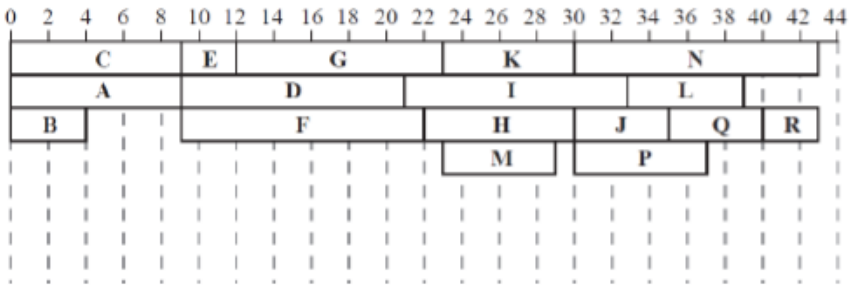
Notes for Question

a1M1: Attempt to find the lower bound $(1150 \pm 210)/300$ (a value of 3.83 (or better) seen with no working can imply this mark)
a1A1: Correct calculation seen **or** 3.83 (or better) followed by a lower bound of 4. An answer of 4 with no working scores M0A0. **Only** seeing 3.8 followed by 4 scores M1A0
b1M1: Bubble sort. Consistent direction, end number (20) in place and the list beginning with the correct first five numbers (175 210 135 105 150). Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 20 appears at the end of the list is the end of their first pass
b1A1: The first, second **and** third passes correct – so end three numbers in place
b2A1ft: Fourth and fifth passes correct following through from the candidate's third pass – so end five numbers in place
b3A1: cso (correct solution only – so previous three marks must have been awarded in this part). Must show a 6th pass showing no swaps/changes (give bod if the passes are not labelled but do not award this mark if it is clear that after the 5th pass the list is simply being written out again (rather than a genuine 6th pass taking place)). Condone if the sort continues until a 9th pass has been completed (but there should be no changes in the 9th pass)

SC in (b) if list is sorted into ascending order (regardless of reversing at the end of the sort) award
M1 for 135 175 105 100 150 60 20 70 125 210 and then
A1 for 135 105 100 150 60 20 70 125 175 210 **and** 105 100 135 60 20 70 125 135 175 210 so 2 marks max.

c1M1: Their first four items placed correctly and at least eight values placed in trucks (if correct this will be the bold items but must check **their** packing if any of **their** first four values are incorrect – note that the maximum weight is 300). Condone cumulative totals for M1 only. First-fit increasing scores no marks in this part. If no sort seen in (b) then mark (c) assuming the correct ordered list in descending order
c1A1: First eight items placed correctly (the underlined and bold values). No additional or repeated values. No follow through or misreads for the A marks in this part. Must be using the correct ten values (so **any** wrong values regardless of where they appear in the trucks is A0)
c2A1: cso. No additional or repeated values

Q8.

Question number	Scheme	Marks
(a)		B2, 1, 0 (2)
(b)		M1 A1 M1 A1 (4)
(c)	Critical activities: C, E, G, K and N	B1 (1)
(d)	Float on J = 35 – 22 – 5 = 8	B1ft (1)
(e)	Lower bound is $\frac{133}{43} = 3.0930... = 4$	B1 (1)
(f)	e.g. 	M1 A1 A1 A1 (4)
		(13 marks)

Activity	Immediately preceded by	Activity	Immediately preceded by	Activity	Immediately preceded by
A	-	G	**A, B, E**	M	D, G
B	-	H	**A, B, E**	N	**H, K**
C	-	I	**A, B, E**	P	**H, K**
D	A	J	**A, B, E, F**	Q	**H, I, J, K**
E	C	K	D, G	R	**P, Q**
F	C	L	D, G		

Notes for Question
a1B1: Any four (of the eight blank) rows correct **a2B1:** All eight rows correct **b1M1:** All top boxes complete, values generally increasing in the direction of the arrows (so generally going from 'left to right' across the network), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of

the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above
b1A1: CAO – **all** values correct in the top boxes
b2M1: All bottom boxes complete (but condone a blank box for the late event time at the end event node for the M mark only). Values generally decreasing in the opposite direction of the arrows (so generally going from 'right to left' across the network), condone one 'rogue' (as described above in **b1M1**)
b2A1: CAO – **all** values correct in the bottom boxes

c1B1: CAO (C, E, G, K and N only)

d1B1ft: correct calculation seen for their J (provided total float is non-negative). Correct answer or the correct answer following through the event times for J with no working seen scores B0 – must see all three numbers in their calculation

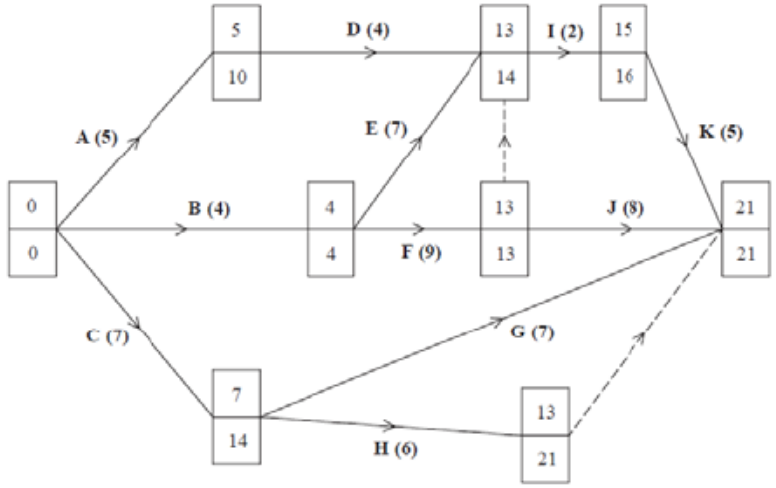
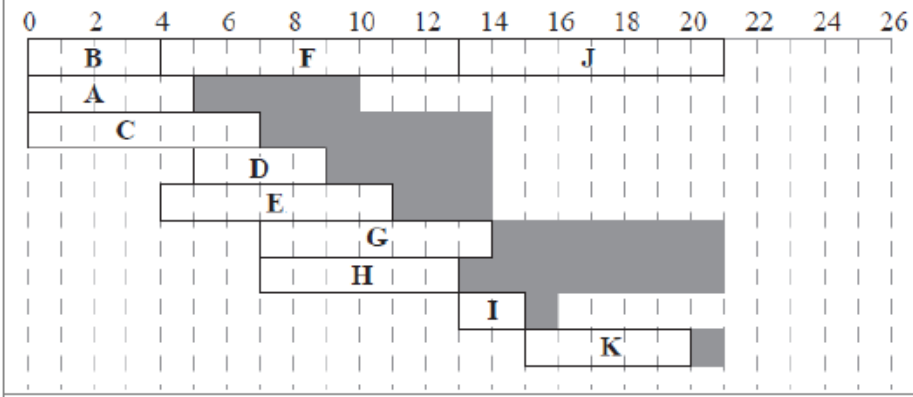
e1B1: CSO – 4 together with **either** a **correct** calculation seen **or** an awrt 3.1. An answer of 4 with no working scores B0. If working seen then it must be correct

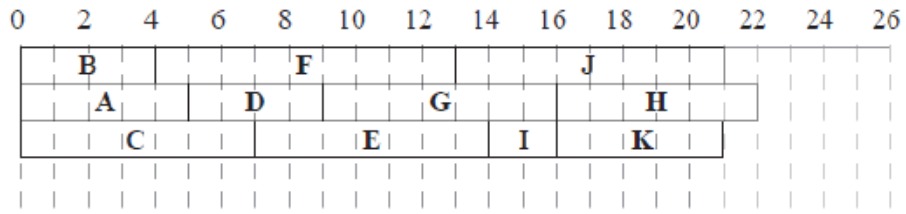
f1M1: Not a cascade (Gantt) chart. 5 'workers' used at most and at least 8 new (11 in total) activities placed
f1A1: 4 workers. All 14 new (17 in total) activities present (just once). Condone at most **three** errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
f2A1: 4 workers. All 14 new (17 in total) activities present (just once). Condone **one** error **either** precedence **or** time interval **or** activity length; An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
f3A1: 4 workers. All 14 new (17 in total) activities present (just once). No errors.

The table below is helpful in checking an activities duration, time interval and IPA

Activity	Duration	Time interval	IPA
D	12	9 – 23	A
E	3	9 – 12	C
F	13	9 – 30	C
G	11	12 – 23	A, B, E
H	8	12 – 30	A, B, E
I	12	12 – 35	A, B, E
J	5	22 – 35	A, B, E, F
K	7	23 – 30	D, G
L	6	23 – 43	D, G
M	6	23 – 43	D, G
N	13	30 – 43	H, K
P	7	30 – 40	H, K
Q	5	30 – 40	H, I, J, K
R	3	37 – 43	P, Q

Q9.

Question	Scheme	Marks
(a)	(i) The dummy from event 5 to event 6 is needed to show that J depends on F but I depends on D, E and F	B1
	(ii) The dummy from event 7 to event 9 is because activities G and H must be able to be described uniquely in terms of the events at each end	B1
		(2)
(b)		M1 A1 M1 A1
		(4)
(c)	21 (hours)	B1
		(1)
(d)	$\frac{64}{21} \approx 3.048$ so at least 4 workers required	M1 A1
		(2)
(e)		M1 A1 M1 A1
		(4)

(f)	e.g.  	M1 A1 A1
		(3)
		(16 marks)

Notes:

(a)

In (a) any use of the terms 'activity' and 'event' must be correct

B1: cao dependency - all relevant activities must be referred to - activities I, J, F and either D or E must be mentioned.

B1: cao uniqueness – please note that, for example, 'so that activities can be defined uniquely' is not sufficient to earn this mark. There must be some mention of describing activities in terms of the event at each end. However, give bod on statements that imply that an activity begins and ends at the same event

(b)

M1: **All** top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue

A1: cao (top boxes)

M1: **All** bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue

A1: cao (bottom boxes)

(c)

B1: cao (21)

(d)

M1: Attempt to find lower bound: (a value in the interval [55 – 73] / their finish time) **or** (sum of the activities / their finish time) **or** (as a minimum) an awrt 3.05 or 3.04 (truncated)

A1: cso – either a **correct** calculation seen **or** awrt 3.05 (or 3.04) **then** 4. An answer of 4 with no working scores M0A0

(e)

M1: At least 8 activities added including 5 floats. Scheduling diagram scores M0

A1: Critical activites dealt with correctly and 4 non-critical activities dealt with correctly

M1: All 11 activities including all 8 floats (on the correct non-critical activities)

A1: cao (all activities correct and present only once)

(f)

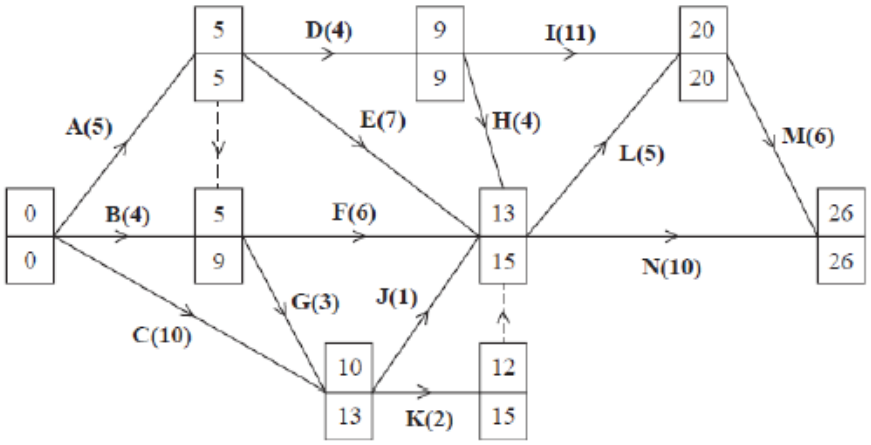
M1: Not a cascade chart. 3 workers used and at least 9 activities placed. The completion time must be no greater than one hour more than the minimum completion time stated in (c) or seen in (b)

A1: 3 workers, All 11 activities present (just once). Condone one error either precedence or activity length. The completion time must be one hour greater than the minimum completion time stated in (c) or seen in (b)

A1: 3 workers. All 11 activities present (just once). No errors. The completion time must be 22

Activity	Duration	IPA
A	5	-
B	4	-
C	7	-
D	4	A
E	7	B
F	9	B
G	7	C
H	6	C
I	2	D, E, F
J	8	F
K	5	I

Q10.

Question Number	Scheme	Marks
(a)(i)	The dummy from event 2 to event 3 is required because activity F (or G) relies on activity A and B but activity D (or E) relies on activity A only	B1
(ii)	The dummy from event 6 to event 7 is required as otherwise activities J and K (which both begin at event 4) would end at the same event	B1 (2)
(b)		M1 A1 M1 A1 (4)
(c)	Minimum completion time: 26 (hours) Critical activities: A, D, I and M	B1 B1 (2)
(d)	The early event time at event 7 is (the larger of) 12 or $9+x$ The late event time at event 7 would then be either 15 or $9+x$	M1 A1 A1 (3)
(e)	$x=10$	B1 (1)
		12 marks

Notes for Question

In (a) any use of the terms 'activity' and 'event' must be correct

ai1B1: CAO dependency - all relevant activities must be referred to - activities A and B and one of D or E and one of F or G (so four activities) must be mentioned
aii1B1: CAO uniqueness – please note that, for example, 'so that activities can be defined uniquely' is not sufficient to earn this mark. There must be some mention of describing activities in terms of the event at each end. However, give bod on statements that imply that an activity begins and ends at the same event (for this mark candidates do not need to explicitly mention activities J and K)

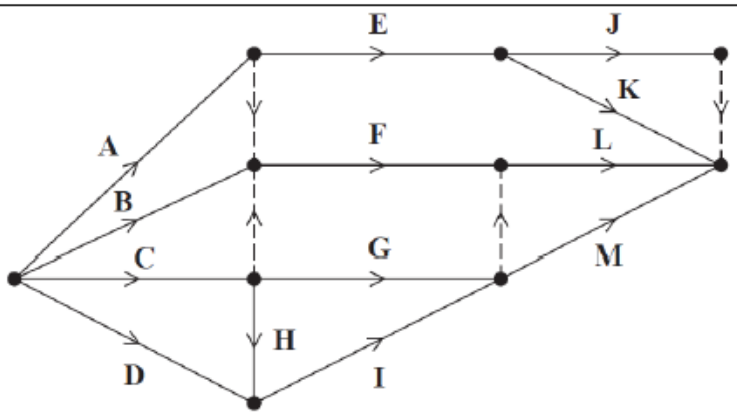
b1M1: All top boxes complete, values in the top boxes generally increasing in the direction of the arrows ('left to right'), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and then the values do increase in the direction of the arrows then this is considered to be only one rogue value)
b1A1: CAO for the top boxes
b2M1: All bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue. Condone missing 0 and/or their 26 (at the end event) for the M only
b2A1: CAO for the bottom boxes

c1B1: CAO (26)
c2B1: CAO (A, D, I and M only)

di1M1: One of 12 or $9+x$ as the early event time for event 7
di1A1: Both correct answers 12, $9+x$ (A0 if 'linked' in some way e.g. $12 > 9+x$ but bod for the M mark)
dii2A1: Both correct answers of 15, $9+x$ for the late event time for event 7

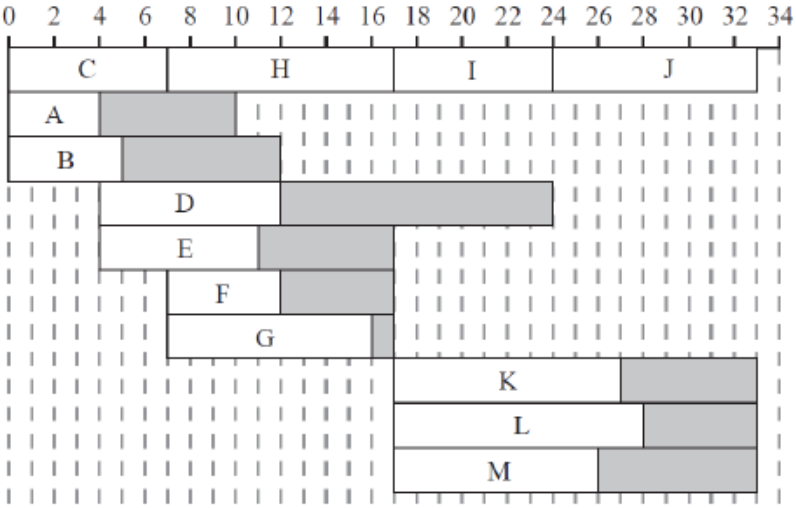
e1B1: CAO (10)

Q11.

Question Number	Scheme	Marks
(a)		M1 A1 A1 A1 A1 (5)
(b)	Critical path: A – E – K	B1 (1)
(c)	First critical path: C – H – I – M Second critical path: C – H – I – L	B1 B1 (2)
		8 marks

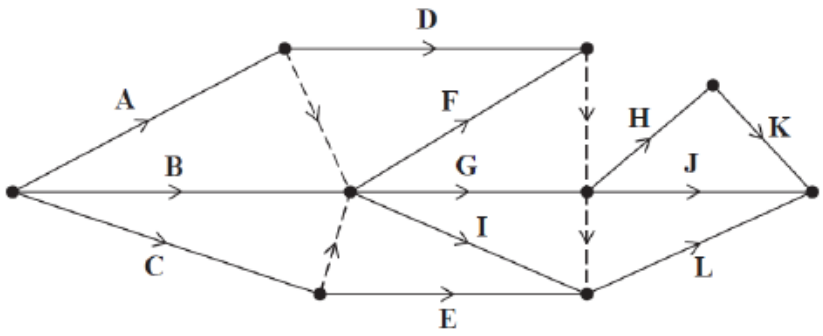
Notes for Question
Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'F dealt with correctly' requires the correct precedences for this activity, i.e. A, B and C labelled correctly and leading into the same node and F starting from that node but do not consider the end event for F. **Activity on node is M0** If an arc is not labelled, for example, if the arc for activity G is not labelled (but the arc is present) then this will lose the second A mark and the final (CSO) A mark – they can still earn the third A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be) **Ignore incorrect or lack of arrows on the activities for the first four marks only** **a1M1:** At least eight activities (labelled on arc), one start and at least two dummies placed **a1A1:** Activities A, B, C, first two dummies (+ correct arrows on these two dummies) and D dealt with correctly – the first two dummies are those that meet at the end of activity B **a2A1:** Activities E, F, G, H and I dealt with correctly **a3A1:** Activities J, K, L and M and 3rd dummy (+ correct arrow on this dummy) dealt with correctly – the 3rd dummy is the one that begins at the end of activity G **a4A1:** CSO – Final dummy + arrow, all arrows correctly placed for each activity with one finish and no additional dummies. Note that this is not a unique solution e.g. J and K could be interchanged, or the dummy could come immediately after E, etc. so please check these carefully. **Please check all arcs carefully for arrows – if there are no arrows on dummies then M1 only.** **Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned** **b1B1:** CAO (A, E and K only) **c1B1:** One correct path (with at most three paths stated) **c2B1:** Both correct with no others

Q12.

Question Number	Scheme	Marks
(a)	$x = 8, y = 12, z = 17$	B3, 2, 1, 0 **(3)**
(b)		M1 A1 A1 A1 **(4)**
(c)	Lower bound is 4 workers e.g. activities J, K, L and M together with $24 < \text{time} < 26$	M1 A1 **(2)**
		9 marks

Notes for Question
a1B1: x value correct **a2B1**: y value correct **a3B1**: z value correct **b1M1**: At least ten activities labelled including at least five floats. A scheduling diagram scores M0 **b1A1**: The critical activities dealt with correctly and appearing just once (C, H, I and J) and three non-critical activities dealt with correctly **b2A1**: Any six non-critical activities correct (this mark is not dependent on the previous A mark) **b3A1**: CSO – completely correct Gantt chart (exactly thirteen activities appearing just once) **c1M1**: Either a statement with the correct number of workers (4) and the correct activities (J, K, L and M) with any numerical time stated **or** the correct number of workers (4) and a time in the interval $24 \leq x \leq 26$ – mark the numerical value only not their use of day/time **c1A1**: A completely correct statement with details of both time and activities. Candidates must give a time within the correct interval of $24 < t < 26$. Please note the strict inequalities for the time interval (e.g. implying a time of 24 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 25 – 26 is A0). Note that 'on day 25' or 'on day 26' are correct but 'on day 24' is not correct. A completely correct statement with an additional incorrect statement scores A0 (so no isw)

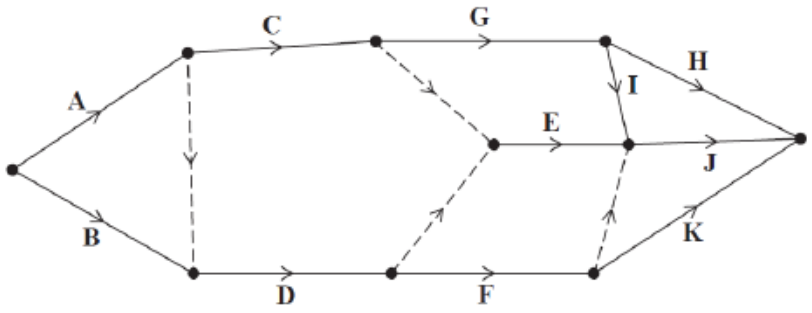
Q13.

Question Number	Scheme	Marks
(a)		M1 A1 A1 A1 A1 (5)
(b)	Activities E, I, J and L cannot be critical	M1 A1 (2)
		7 marks

Notes for Question
Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finishes at the correct event, e.g. 'F dealt with correctly' requires the correct precedences for this activity, i.e. A, B and C labelled correctly and leading into the same node and F starting from that node but do not consider the end event for F. The table below is very useful in checking this point. **Activity on node is M0** If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be) ' **Ignore incorrect or lack of arrows on the activities for the first four marks only** **a1M1**: Seven activities (labelled on arc), one start and at least two dummies placed **a1A1**: Activities A, B, C, 1st two dummies (including correct arrows on these two dummies), D and E dealt with correctly. The first two dummies are those at the end of activities A and C **a2A1**: A dummy linking D, F and G into an event node and a dummy linking D, F, G, I and E into an event node and F, G and I dealt with correctly. The dummies must have arrows **a3A1**: Activities H, J, K and L dealt with correctly **a4A1**: CSO – all arrows correctly placed for each activity with one finish and at most four dummies. Note that this is not a unique solution due to for example the 3rd and 4th dummies so please check these carefully. **Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only. Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned** **b1M1**: At least two correct activities with no more than 5 activities stated **b1A1**: All four activities (E, I, J and L) and no others

A	B	C	D	E	F	G	H	I	J	K	L
-	-	-	A	C	A, B, C	A, B, C	D, F, G	A, B, C	D, F, G	H	D, E, F, G, I

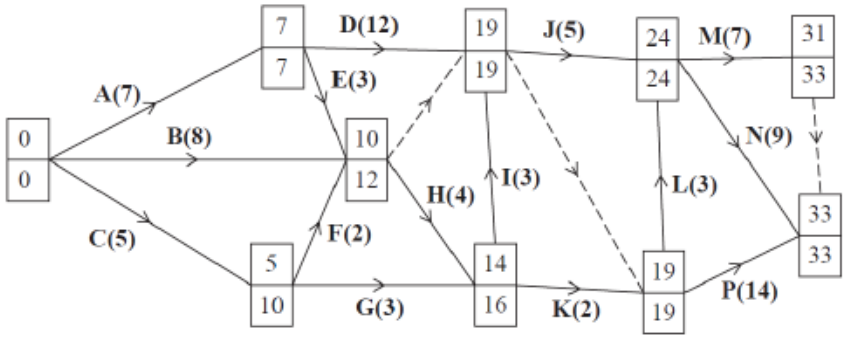
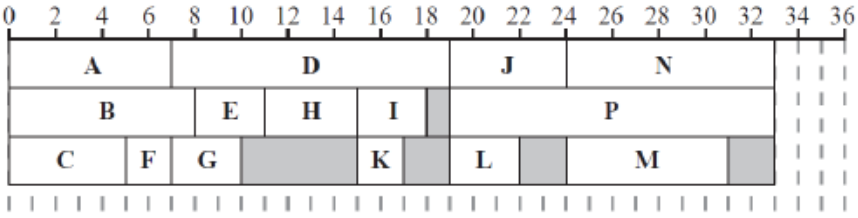
Q14.

Question Number	Scheme	Marks
(a)		M1 A1 A1 A1 A1 (5)
(b)	D and F are guaranteed to be critical	B1 (1)
(c)	Critical path: A – C – G – I – J	B1 (1)
		7 marks

Notes for Question
In (a) condone lack of, or incorrect, numbered events throughout – also 'dealt with correctly' means that the activity starts from the correct event but may not finish at the correct event (use the table below to check this). **Activity on node is M0** If an arc is not labelled, for example, if the arc for activity D is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be) **Ignore incorrect or lack of arrows on the activities for the first four marks only**
a1M1: Eight activities (labelled on arc), one start and at least two dummies placed **a1A1:** Activities A, B, C, 1st dummy (+ correct arrow on this dummy – this is the dummy at the end of A) and D dealt with correctly **a2A1:** 2nd and 3rd dummies (+ correct arrows on these dummies – these dummies are the ones at the end of C and D) and E, F and G dealt with correctly **a3A1:** 4th dummy (+ correct arrow on this dummy – this is the dummy at the end of F) and activities H, I , J and K dealt with correctly **a4A1:** CSO – all arrows present and correctly placed with one finish **Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.** **Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned** **b1B1:** CAO (D and F) and no other activities stated as critical (however, ignore K if stated too) **c1B1:** CAO (A – C – G – I – J)

Activity	A	B	C	D	E	F	G	H	I	J	K
IPA	-	-	A	A, B	C, D	D	C	G	G	E, F, I	F

Q15.

Question Number	Scheme	Marks
(a)		M1 A1 M1 A1 (4)
(b)	Lower bound is $\frac{87}{33} = 2.6363... = 3$	M1 A1 (2)
(c)	e.g. 	M1 A1 A1 A1 (4)
(d)	G is not a critical activity (it has a total float of 16 – 5 – 3 = 8 days) and so there is no benefit from reducing the duration of G by one day Activities D and P are both critical activities	M1
	However, D appears in both critical paths therefore reducing P would not reduce the minimum completion time (as there is still a critical path A – D – J – N of length 33) and so activity D should be shortened by one day	A1 (2)
		12 marks

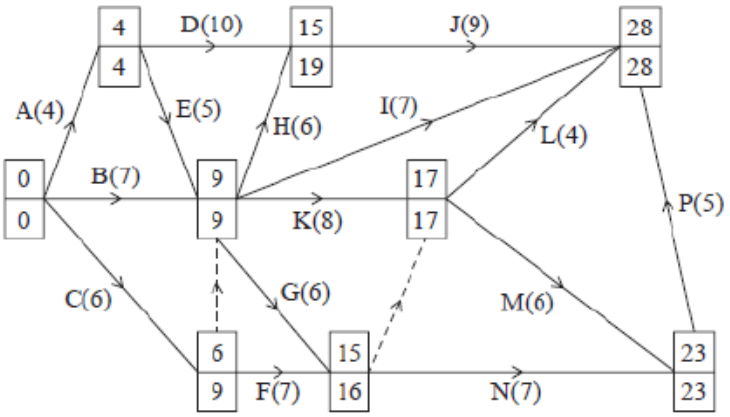
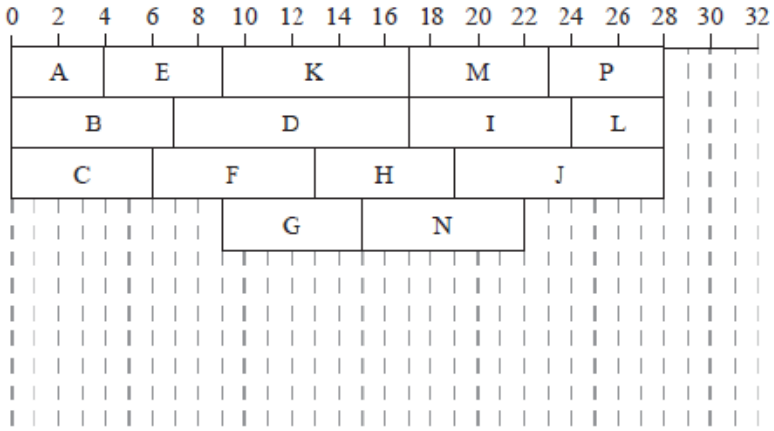
Notes for Question
a1M1: **All** top boxes complete, values generally increasing in the direction of the arrows ('left to right'), condone one rogue
a1A1: CAO (top boxes)
a2M1: **All** bottom boxes complete, values generally decreasing in the opposite direction of the arrows ('right to left'), condone one rogue. Condone missing 0 and/or their 33 (at the end event) for the M mark only
a2A1: CAO (bottom boxes)
b1M1: Attempt to find lower bound: (a value in the interval [73 – 101] / their finish time) **or** (sum of the activities / their finish time) **or** (as a minimum) an awrt 2.6
b1A1: CSO – requires both a **correct** calculation **or** awrt 2.6 seen and 3. An answer of 3 with no working scores no marks
c1M1: Not a cascade chart. 4 workers used at most, at least 10 activities placed
c1A1: 3 workers. All 15 activities present (just once). Condone **two** errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
c2A1: 3 workers. All 15 activities present (just once). Condone **one** error. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA

c3A1: CAO

Activity	Duration	Time interval	IPA
A	7	0 – 7	-
B	8	0 – 12	-
C	5	0 – 10	-
D	12	7 – 19	A
E	3	7 – 12	A
F	2	5 – 12	C
G	3	5 – 16	C
H	4	10 – 16	B, E, F
I	3	14 – 19	G, H
J	5	19 – 24	B, D, E, F, I
K	2	14 – 19	G, H
L	3	19 – 24	B, D, E, F, I, K
M	7	24 – 33	J, L
N	9	24 – 33	J, L
P	14	19 – 33	B, D, E, F, I, K

d1M1: D and P stated as being critical **or** activity G is not critical
d1A1: Correct answer of D with fully correct reason (G not critical, D and P are both critical but D appears in both/all critical paths or P appears in only one critical path)

Q16.

Question Number	Scheme	Marks
(a) and (b)		B1 B1 B1 **(3)** M1 A1 M1 A1 **(4)**
(c)	Critical activities: A, E, K, M, P	B1 **(1)**
(d)	Lower bound $= \frac{97}{28} = 3.46\ldots$ so 4 workers	M1 A1 **(2)**
(e)	e.g. 	M1 A1 A1 **(3)**
		13 marks

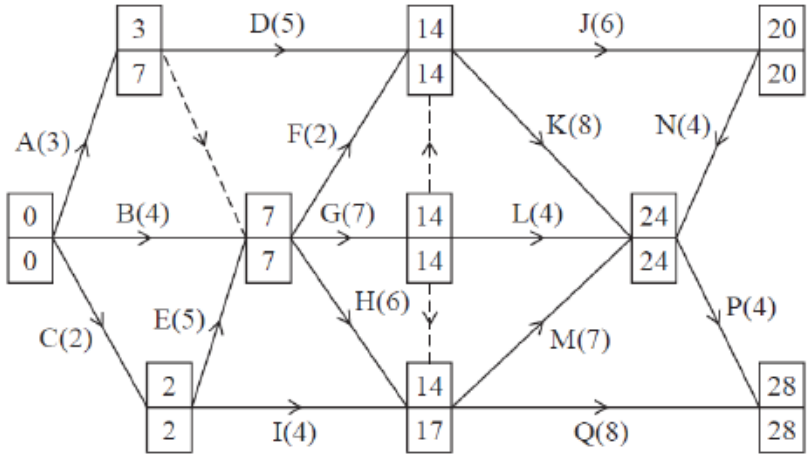
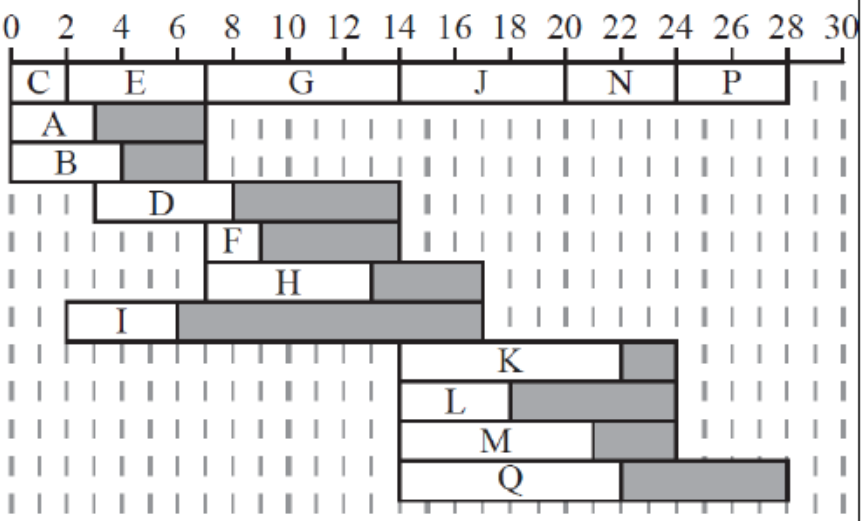
Notes for Question
a1B1: Any two of the five arcs (G, H, I or the two dummies) drawn correctly (from correct vertex to correct vertex) – activities labelled with the correct letter (but condone no or wrong arrows) and the dummies must be shown as dashed lines (or labelled as 'dummy') with no weight (but condone no or wrong arrows) **a2B1**: Four of the five arcs (G, H, I and the two dummies) drawn correctly – activities must be labelled with the correct letter (but condone no or wrong arrow) and the dummies must be shown as dashed lines (or labelled 'dummy') with no weight (but condone no or wrong arrows) **a3B1: CSO** - all three activities (G, H and I) and the two dummies drawn correctly with no extras. Activities must be labelled with the correct letter and weights. The activities **and** dummies (as dashed lines with zero weight) must have the correct arrows (**do check carefully that all arrows are present**) **In (a) condone for full marks activities which are shown as dashed lines provided they are labelled with the correct letter. Condone lack of (or incorrect) weights on the activity arcs for the first two marks only.** **b1M1**: All top boxes complete (condone lack of 0 for the M mark only), values generally increasing in the direction of the arrows ('left to right'), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above – **this mark is dependent on the first mark having being awarded in (a)** **b1A1**: CAO – all values correct in the top boxes **b2M1**: All bottom boxes complete (condone lack of 28 and/or 0 for the M mark only), values generally decreasing in the opposite direction of the arrows ('right to left'), condone one 'rogue' – **this mark is dependent on the first mark having being awarded in (a)** **b2A1**: CAO – all values correct in the bottom boxes **For full marks in (b) all three activities (G, H and I) and the two dummies must have been added correctly in (a) – condone lack of arrows only. If all values in the bottom and top boxes are correct but any arc or dummy is missing or incorrect then award M1A1M1A0 – if all values are not correct (and some arcs are missing) then mark to the scheme above** **c1B1**: CAO (A, E, K, M, P only)

d1M1: Attempt to find lower bound: (a value in the interval [87 – 107] / their finish time) **or** showing the summing of **all** 15 activities divided by their finish time **or** (as a minimum) an awrt 3.5
d1A1: CSO – either a **correct** calculation seen **or** awrt 3.5 **then** 4 (with no incorrect working seen). An answer of 4 with no working scores M0A0
e1M1: Not a cascade (Gantt) chart. 5 'workers' used at most and at least 11 activities placed
e1A1: 4 workers. All 15 activities present (just once). Condone at most two errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
e2A1: 4 workers. All 15 activities present (just once). No errors

Activity	Duration	Time	IPA
A	4	0 – 4	-
B	7	0 – 9	-
C	6	0 – 9	-
D	10	4 – 19	A
E	5	4 – 9	A
F	7	6 – 16	C
G	6	9 – 16	B, C, E
H	6	9 – 19	B, C, E

Activity	Duration	Time	IPA
I	7	9 – 28	B, C, E
J	9	15 – 28	D, H
K	8	9 – 17	B, C, E
L	4	17 – 28	F, G, K
M	6	17 – 23	F, G, K
N	7	15 – 23	F, G
P	5	23 – 28	M, N

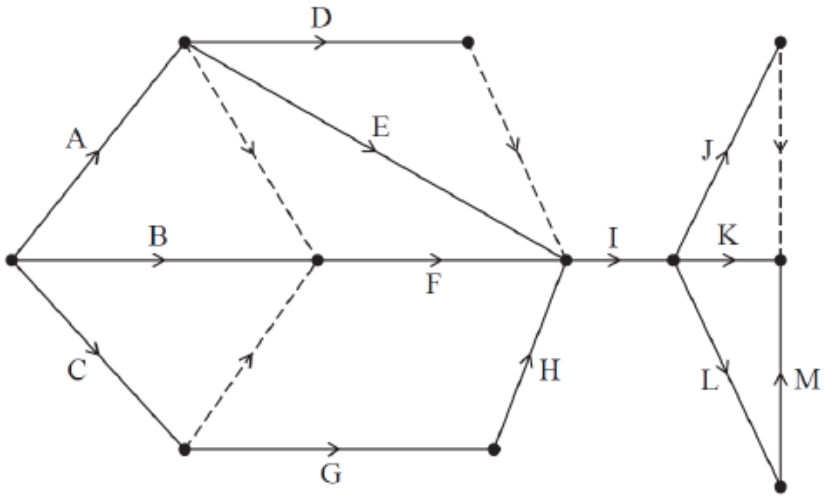
Q17.

Question Number	Scheme	Marks
(a)		M1 A1 M1 A1 (4)
(b)	e.g. 	M1 A1 A1 A1 (4)

(c)	Lower bound is 4 workers e.g. activities K, M, N and Q together with $20 < \text{time} < 21$	M1 A1 (2)
		10 marks

Question Number	Scheme	Marks
	Notes for Question	
	a1M1: All top boxes complete (condone lack of 0 for the M mark only), values generally increasing in the direction of the arrows (so generally going from 'left to right' across the network), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above **a1A1:** CAO – **all** values correct in the top boxes **a2M1:** All bottom boxes complete (but condone a blank box for the late event time at the end of activities P/Q and/or at the start node for this method mark only). Values generally decreasing in the opposite direction of the arrows (so generally going from 'right to left' across the network), condone one 'rogue' (as described above in **a1M1**) **a2A1:** CAO – **all** values correct in the bottom boxes **Note that in (b) it is acceptable for the critical activities (C, E, G, J, N and P) to appear on separate lines or for several activities to appear on the same line as long as their durations and total floats are clear and do not overlap. The floats on the non-critical activities do not need to be shaded but they must be clearly distinguishable from the duration of the activity** **b1M1:** At least ten different activities labelled including at least six floats. A scheduling diagram (so a diagram in which no floats are evident) scores M0 **b1A1:** The critical activities dealt with correctly and appearing just once (C, E, G, J, N and P) and three non-critical activities dealt with correctly (both duration and total float correct) **b2A1:** Any six non-critical activities correct (this mark is not dependent on the previous A mark) **b3A1:** CSO – completely correct Gantt chart (exactly sixteen activities appearing just once) **c1M1:** Either a statement with the correct number of workers (4) and stating the correct activities (K, M, N and Q) with any numerical time stated **or** the correct number of workers (4) and a time in the interval $20 \leq x \leq 21$ – mark the numerical value only not their use of the words 'day/time' (or equivalent) **c1A1:** A completely correct statement with details of both time **and** activities. Candidates must give a time within the correct interval of $20 < t < 21$, e.g. 20.5 and state the correct activities (K, M, N and Q). Please note the strict inequalities for the time interval (e.g. implying a time of 20 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 20 – 21 is A0). A completely correct statement with an additional incorrect statement scores A0 (so do not ignore subsequent working)	

Q18.

Question Number	Scheme	Marks
(a)	e.g.  	M1 A1 A1 A1 A1 (5)

(b)	Activity I is guaranteed to be critical…	M1	
	…because all paths (from source to sink) contain activity I	A1	(2)
(c)	Minimum project completion time is 12 (hours)	B1	
	Critical path is C G H I L M	B1 (2)	
		9 marks	

Notes for Question

In **(a)** condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event. For example, 'I dealt with correctly' requires the correct precedences for this activity, i.e. D, E, F and H labelled correctly **and** that they lead directly into the same node (possibly with the aid of a dummy activity). Activity I then starts from that node but ignore the end event for I. The table below is useful in checking this. **Activity on node is M0**

Activity	A	B	C	D	E	F	G	H	I	J	K	L	M
IPA	-	-	-	A	A	A, B, C	C	G	D, E, F, H	I	I	I	L

If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (correct solution only) A mark – they can still earn the second A mark on the bod (benefit of doubt). If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line which has not been labelled is an activity rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only

a1M1: Eight activities (labelled on arc), one start and at least two dummies placed
a1A1: Activities A, B, C, 1st two dummies, D, E and G dealt with correctly. The first two dummies are those at the end of activities A and C. Both dummies must have arrows pointing in the correct direction
a2A1: Activities F, H and 3rd dummy dealt with correctly. The 3rd dummy is the one at the end of D. The dummy must have an arrow pointing in the correct direction
a3A1: Activities I, J, K, L and M dealt with correctly
a4A1: CSO – all arrows correctly placed for each activity with one finish and a 4th dummy (with a correct arrow) at the end of activity J. **Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only. Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark, if earned, in (a)**

Note additional valid solutions:

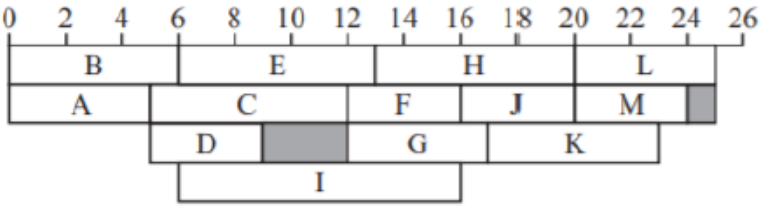
- Activities D and E interchanged **or** J and K interchanged (or both)
- The dummy at the end of D could appear before activity D, similarly for the dummy at the end of J
- A combination of both these points, e.g. D and E interchanged and the corresponding dummy appearing before the activities
- The arrow on the 'final' dummy (the one at the end of J) could be reversed

b1M1: CAO (Activity I **only**) – if more than one activity stated as being critical then M0

b1A1: Correct reasoning – mention that every/all paths/routes (in the network) contain activity I e.g. 'there is no route that doesn't contain I' scores M1A1. Must mention 'path', 'route' etc. so e.g. 'everything passes through I' would score M1A0 (but 'every route passes through I' is fine for both marks)

c1B1: Correct answer only (12)
c2B1: Correct answer only (C G H I L M)

Q19.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>(a)</td><td>
<table>
<tr><th>Activity</th><th>IPA</th></tr>
<tr><td>A</td><td>-</td></tr>
<tr><td>B</td><td>-</td></tr>
<tr><td>C</td><td>A</td></tr>
<tr><td>D</td><td>A</td></tr>
<tr><td>E</td><td>A, B</td></tr>
</table>
<table>
<tr><th>Activity</th><th>IPA</th></tr>
<tr><td>F</td><td>C</td></tr>
<tr><td>G</td><td>C</td></tr>
<tr><td>H</td><td>C, D, E</td></tr>
<tr><td>I</td><td>A, B</td></tr>
<tr><td>J</td><td>A, B</td></tr>
</table>
<table>
<tr><th>Activity</th><th>IPA</th></tr>
<tr><td>K</td><td>F</td></tr>
<tr><td>L</td><td>F, G, H, I</td></tr>
<tr><td>M</td><td>F, G, H, I, J</td></tr>
</table>
</td><td>B2, 1, 0 (2)</td></tr>
<tr><td>(b)</td><td>$v = 7$, $w = 4$, $x = 6$, $y = 16$, $z = 19$</td><td>B3, 2, 1, 0 (3)</td></tr>
<tr><td>(c)</td><td>$\frac{74}{25} = 2.96$ so 3 workers</td><td>B1 (1)</td></tr>
<tr><td>(d)</td><td>e.g.
 
</td><td>M1
A1
A1 (3)</td></tr>
<tr><td>(e)</td><td>New minimum project completion time: 27 (days)</td><td>B1</td></tr>
<tr><td></td><td>Critical path: ACFK</td><td>B1 (2)</td></tr>
<tr><td></td><td></td><td>11 marks</td></tr>
</table>

Notes for Question

a1B1: Any 6 rows correct (not including A and B)
a2B1: cao (condone blank rows for A and B)

b1B1: Any 2 correct values
b2B1: Any 3 correct values
b3B1: All 5 values correct

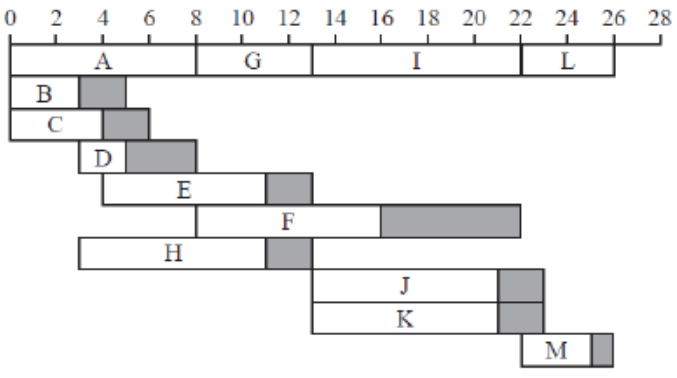
c1B1: cao (3 from correct working) – as a minimum for correct working accept either 2.96 or $\frac{5+6+7+4+7+4+5+7+10+4+6+5+4}{25}$ or $\frac{74}{25}$
d1M1: Not a cascade chart. 4 workers used at most, at least 9 activities placed
d1A1: 4 workers. All 13 activities present (just once). Condone **two** errors **either** precedence **or** time interval **or** activity length. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
d2A1: 4 workers. All 13 activities present (just once). No errors

e1B1: cao (27 only)
e2B1: cao (ACFK or KFCA only)

Question Number	Scheme	Marks

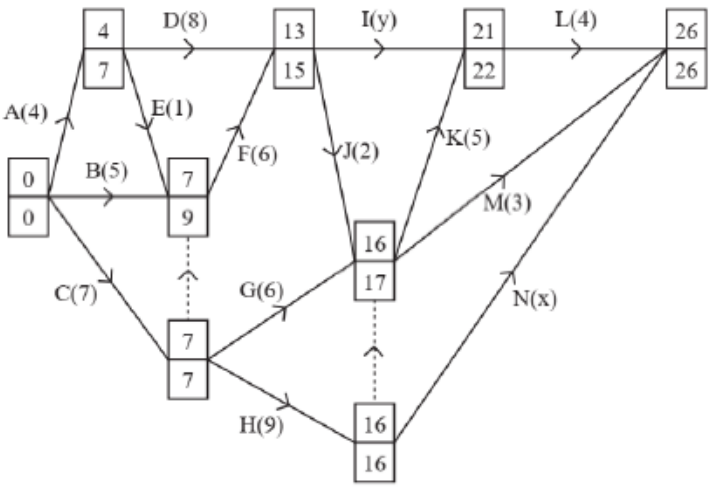
Activity	Duration	Time	IPA
A	5	0 – 6	-
B	6	0 – 6	-
C	7	5 – 13	A
D	4	5 – 13	A
E	7	6 – 13	A, B
F	4	12 – 19	C
G	5	12 – 20	C
H	7	13 – 20	C, D, E
I	10	6 – 20	A, B
J	4	6 – 21	A, B
K	6	16 – 25	F
L	5	20 – 25	F, G, H, I
M	4	20 – 25	F, G, H, I, J

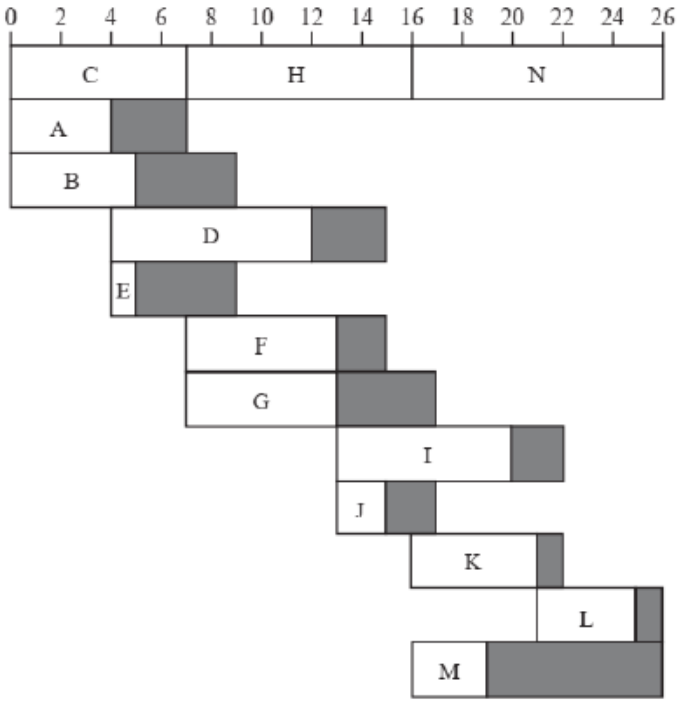
Q20.

Question Number	Scheme	Marks
(a)	Float on F is twice float on D $\Rightarrow 22-8-y=2(8-3-x)$ (oe)	B1
	BFDM is 10 less than critical path	B1
	$\Rightarrow 3+x+y+3=26-10$ (oe)	
	$x+y=10$ $-2x+y=4$ and solve simultaneously to find both x and y	M1
	$x=2, y=8$	A1 (4)
(b)		M1 A1 A1 A1 (4)
(c)	Lower bound is 4 workers e.g. activities F, I, J and K together with 15 < time < 16	M1 A1 (2)
		10 marks

Notes for Question
a1B1: cao (any equivalent form) – allow $14-y=2(5-x)$ or $14-y=10-2x$ **a2B1:** cao (any equivalent form) – allow $x+y+6=16$ **a1M1:** Setting up two equations both including x and y (dependent on one correct equation) and an attempt to solve for both x and y leading to a value for x and a value for y – this mark can be implied if the candidate has two equations, both in x and y (with at least one correct), followed by values for x and y **a1A1:** cao ($x=2$, $y=8$) – if both correct values are stated without any working/justification please send to review
b1M1: At least nine activities labelled including at least five floats. A scheduling diagram scores M0 **b1A1:** The critical activities dealt with correctly and appearing just once (A, G, I and L) and three non-critical activities dealt with correctly **b2A1:** Any six non-critical activities correct (this mark is not dependent on the previous A mark) **b3A1:** Completely correct Gantt chart (exactly thirteen activities appearing just once)
c1M1: Either a statement with the correct number of workers (4) and the correct activities (F, I, J and K) with **any** numerical time (or time interval) stated **or** the correct number of workers (4) and a time in the interval $15 \leqslant t \leqslant 16$ – mark the numerical value only not their use of day/time. In either case they must state the correct number of workers . M0 for 'F, J, K and the critical activity' (they must explicitly state activity I in this case) unless a time in the interval stated above is given too **c1A1:** A completely correct statement with details of both time and activities. Candidates must give a time within the correct interval of $15 < t < 16$. Please note the strict inequalities for the time interval (e.g. implying a time of 15 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 15 – 16 is A0). Note that 'on day 16' is correct but 'on day 15' is not correct. A completely correct statement with an additional incorrect statement scores A0 (so no isw)

Q21.

(a)	$x = 10$ $y = 7$ from $9 - 0 - 5 = 2(22 - 13 - y)$ (oe)	B1 B1	**(2)**
(b)		M1 M1 A1	**(3)**

(c)	 Lower bound is 4 workers e.g. activities H, D, F and G together with $11 < \text{time} < 12$	M1 A1 A1 A1 depM1 A1 **(6)**
		11 marks

Notes for Question

a1B1: cao for x (ignore working for this mark)
a2B1: cao with sufficient working as Answer Given – as a minimum accept $4 = 2(9 - y)$ (oe) but just $4 = 18 - 2y$ is B0

b1M1: All top boxes complete, values generally increasing in the direction of the arrows (so generally going from 'left to right' across the network), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above
b2M1: All bottom boxes complete. Values generally decreasing in the opposite direction of the arrows (so generally going from 'right to left' across the network), condone one 'rogue' (as described above in **b1M1**)
b1A1: cao – **all** values correct

c1M1: At least ten different activities labelled including at least seven floats. A scheduling diagram (so a diagram in which no floats are evident) scores M0
c1A1: The critical activities dealt with correctly and appearing just once (C, H and N) and three non-critical activities dealt with correctly (both duration and total float correct)
c2A1: Any six non-critical activities correct (this mark is not dependent on the previous A mark)
c3A1: cso – completely correct Gantt chart (exactly fourteen activities appearing just once)
c2dM1: Dependent on first M mark in this part. Either a statement with the correct number of workers (4) and stating the correct activities (H, D, F and G) with any numerical time stated **or** the correct number of workers (4) and a time in the interval $11 \leq t \leq 12$ – mark the numerical value only not their use of the words 'day/time' (or equivalent)
c4A1: A completely correct statement with details of both time **and** activities. Candidates must give a time within the correct interval of $11 < t < 12$, e.g. 11.5 (or 'on/during day 12') and state the correct activities (H, D, F and G).
Please note the strict inequalities for the time interval (e.g. implying a time of 11 is incorrect). Answers given as an interval of time are acceptable provided the time interval stated is correct for all its possible values (e.g. time 11 – 12 or 'between 11 and 12' is A0). A completely correct statement with an additional incorrect statement scores A0 (so do not ignore subsequent working)

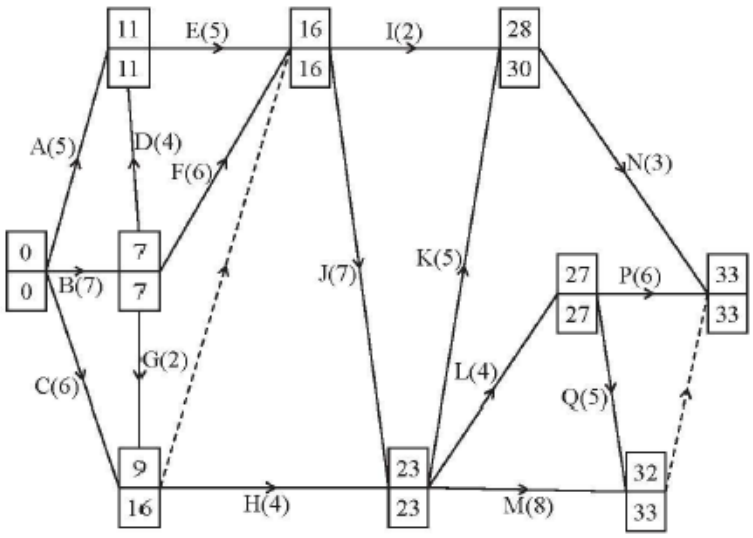
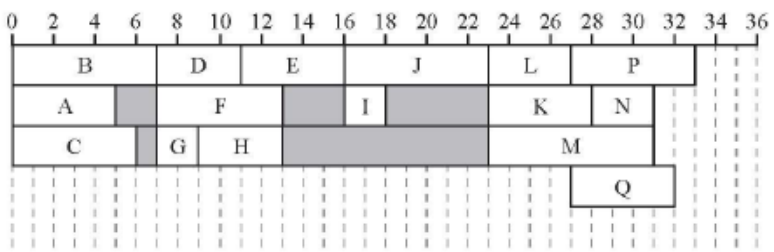
For (c) the following may be useful in checking their cascade chart provided the float is shown after the corresponding activity:

Activity	Duration + Float
A	0 to 4 F: 4 to 7
B	0 to 5 F: 5 to 9
C	0 to 7 Critical
D	4 to 12 F: 12 to 15
E	4 to 5 F: 5 to 9

Activity	Duration + Float
F	7 to 13 F: 13 to 15
G	7 to 13 F: 13 to 17
H	7 to 16 Critical
I	13 to 20 F: 20 to 22
J	13 to 15 F: 15 to 17

Activity	Duration + Float
K	16 to 21 F: 21 to 22
L	21 to 25 F: 25 to 26
M	16 to 19 F: 19 to 26
N	16 to 26 Critical

Q22.

Question Number	Scheme	Marks
(a)		M1 A1 M1 A1 (4)
(b)	Activity H can be delayed by $23 - 9 - 4 = 10$ (days)	B1ft (1)
(c)	Lower bound $= \frac{5+7+6+\ldots+6+5}{33} = \frac{79}{33} = 2.393\ldots = 3$ workers	M1 A1 (2)
(d)	e.g. 	M1 A1 A1 (3)
		10 marks

Notes for Question

a1M1: All top boxes complete, values generally increasing in the direction of the arrows (so generally going from 'left to right' across the network), condone one 'rogue' value (if values do not increase in the direction of the arrows then if one value is ignored and the remaining values do increase in the direction of the arrows then this is considered to be a single rogue value). Note that all values in the top boxes could be incorrect but it can still score the M mark if the values are **increasing** in the way stated above
a1A1: CAO – **all** values correct in the top boxes
a2M1: All bottom boxes complete (but condone a blank box for the late event time at the end event node and/or no zero value for the late event time at the start event node for the **M** mark only). Values generally decreasing in the opposite direction of the arrows (so generally going from 'right to left' across the network), condone one 'rogue' (as described above in **a1M1**)
a2A1: CAO – **all** values correct in the bottom boxes

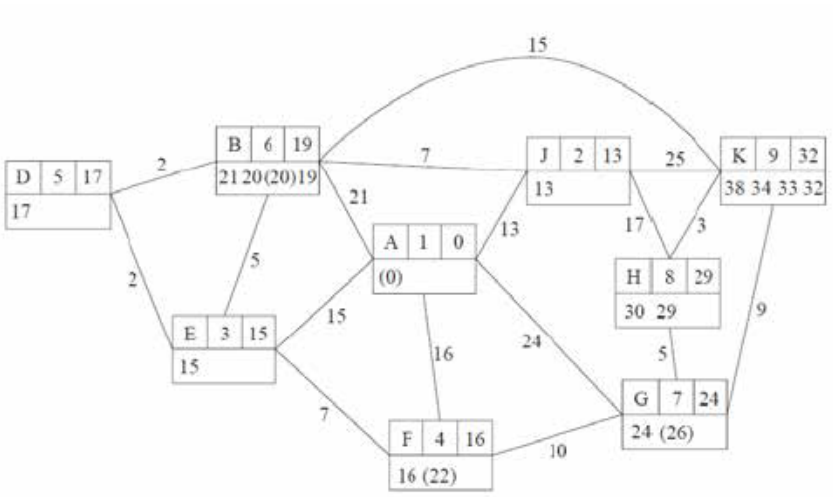
b1B1ft: Correct calculation seen for their H (provided total float is non-negative). Correct answer or the correct answer following through the event times for H with no working seen scores **B0** – must see all three numbers in their calculation (e.g. 23 – 9 – 4 = 10, or 9 + 4 = 13, 23 – 13 = 10, etc.)

c1M1: Attempt to find lower bound: (a value in the interval [71 – 87] / their finish time) **or** (sum of all the activities / their finish time) **or** (as a minimum) an awrt 2.4
c1A1: CSO – requires the correct answer of 3 **and** either a **correct** calculation **or** awrt 2.4. An answer of 3 with no working scores no marks in this part

d1M1: Not a cascade chart. 4 workers used at most, at least 10 activities placed
d1A1: 4 workers. All 16 activities present just once. Condone at most **two** errors. An activity can give rise to at most three errors; one on duration, one on time interval and only one on IPA
d2A1: CAO – no errors, all 16 activities present just once.

Activity	Duration	Time interval	IPA
A	5	0 – 11	-
B	7	0 – 7	-
C	6	0 – 16	-
D	4	7 – 11	B
E	5	11 – 16	A, D
F	6	7 – 16	B
G	2	7 – 16	B
H	4	9 – 23	C, G
I	2	16 – 30	C, E, F, G
J	7	16 – 23	C, E, F, G
K	5	23 – 30	H, J
L	4	23 – 27	H, J
M	8	23 – 33	H, J
N	3	28 – 33	I, K
P	6	27 – 33	L
Q	5	27 – 33	L

Q23.

Question	Scheme	Marks
(a)		M1 A1 (JEFD) A1 (BG) A1ft (HK)
	Quickest route: A – G – H – K	A1
	Shortest time: 32 (mins)	A1ft
		(6)
(b)	Route from B to K via A: B – D – E – A – G – H – K Length: 51 (mins)	B1 B1ft
		(2)
(c)	A(ED)B + F(G)H = 19 + 15 = 34 AF + B(K)H = 16 + 18 = 34 A(G)H + B(DE)F = 29 + 11 = 40 Arcs AF, BK, KH or AE, ED, DB, FG, GH will be traversed twice Route length = 196 + 34 = 230 (mins)	M1 A1ft A1ft A1ft A1A1 A1
		(7)

Notes:	
(a)	
M1:	A larger value replaced by a smaller value at least once in the working values at either B or H or K
A1:	All values in J, E, F and D correct and the working values in the correct order. Penalise order of labelling only once per question. Condone an additional working value at F of 22
A1:	All values in B and G correct and the working values in the correct order. Penalise order of labelling only once per question (B and G must be labelled in that order and B must be labelled after J, E, F, D). Condone an additional working value of 20 at B and an additional working value of 26 at G
A1ft:	All values in H and K correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question (H and K must be labelled in that order and H labelled after all other nodes (excluding K))
A1:	CAO (AGHK)
A1ft:	Follow through on their final value at K – if their answer is not 32 follow through their final value at K (condone lack of units)

(b) **B1:** CAO (BDEAGHK) **B1ft:** 51 or their final value at B + their final value at K (condone lack of units)
(c) **M1:** Three distinct pairings of the correct four odd nodes **A1ft:** One row correct including pairing **and** total (the ft on the first three A marks in (c) is for using their final values at B, F and H from (a) for the lengths of AB, AF and AH only) **A1ft:** Two rows correct including pairing **and** totals **A1ft:** All three rows correct including pairing **and** totals **A1:** CAO one combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working) **A1:** CAO both combination of arcs that need traversing twice (arcs must be explicitly stated and not implied by working) **A1:** CAO (230)

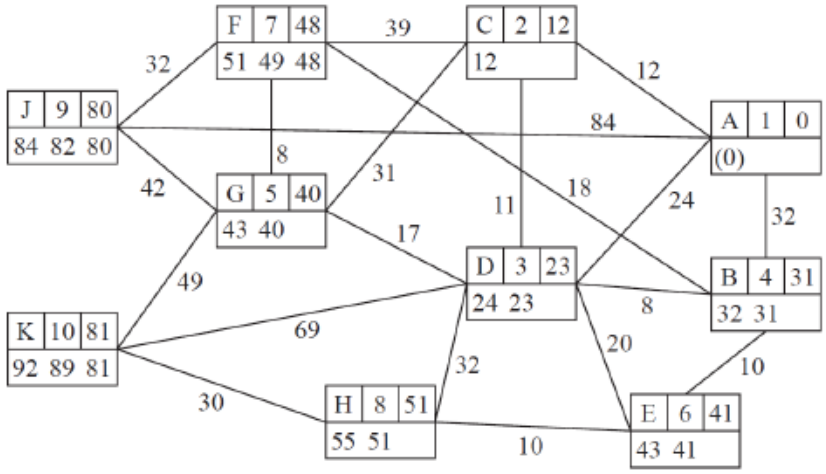
Q24.

Question Number	Scheme	Marks
(a)	(Maximise) $x+y+z$	B1
	Subject to:	
	$14x+8y+12z \leqslant 976$	B1
	$5z \geqslant 2x$	B1
	$\frac{1}{2}(x+y+z) \leqslant x (\Rightarrow -x+y+z \leqslant 0)$	M1
	$\frac{1}{5}(x+y+z) = y (\Rightarrow x-4y+z=0)$	B1
	Substituting $z=4y-x$ into objective and constraints	M1
	Maximise $(P=)5y$ Subject to: $x+28y \leqslant 488$ $7x-20y \leqslant 0$ $2x-5y \geqslant 0$ $x-4y \leqslant 0$ $x \geqslant 0 \quad y \geqslant 0$	A1 A1 **(8)**
(b)	Substituting $y=16$ into constraints gives $x \leqslant 40, x \leqslant \frac{320}{7}, x \geqslant 40, x \leqslant 64$	M1
	$\Rightarrow x=40$ and therefore the maximum number of leadership prizes is 24	A1 **(2)**
		10 marks

Notes for Question
a1B1: CAO $(x+y+z)$ – can be implied by seeing $5y$ as the objective – do not penalise lack of 'maximise' here. Allow equal to any letter but not equal to a numerical value **a2B1:** CAO $(14x+8y+12z \leqslant 976$ or an unsmplified inequality equivalent to $x+28y \leqslant 488)$ **a3B1:** CAO $(5z \geqslant 2x$ (or equivalent) or an unsimplified inequality equivalent to $7x-20y \leqslant 0)$ **a1M1:** Correct method - must see $\frac{1}{2}(x+y+z) \bullet x$ where $\bullet$ is any inequality or =. The bracket must be present or implied by later working – this mark is implied by the stating of a correct/incorrect inequality that would come from $\frac{1}{2}(x+y+z) \bullet x$ (possibly with $z=4y-x$ substituted) **a4B1:** CAO $\frac{1}{5}(x+y+z)=y$ (allow any equivalent unsimplified/simplified form for this mark) – simply stating that e.g. $x+z=4y$ scores this mark **a2M1:** Eliminating z using the correct equation $z=4y-x$ from all three non-trivial constraints and objective **a1A1:** At least two simplified constraints correct + correct objective (in x/y only) – condone lack of 'maximise' for this mark - accept any equivalent forms for the constraints provided that coefficients are integers and only a single term in x and/or y **a2A1:** CAO – must include 'maximise' or 'max' (but not 'maximum') and all 6 constraints – accept any equivalent forms for the constraints provided that coefficients are integers and only a single term in x and/or y **Condone using *a* for *x*, *s* for *y* and *l* for *z* – if any other letter used then send to review**

Must be using inequalities in x and y only in (b) – if inequalities are still in terms of z then z must be eliminated using the correct equation (e.g. $z = 64 - x$). Attempting part (b) with equations (rather than inequalities) scores no marks (even if the correct answer of 24 is seen). **b1M1:** Substituting $y=16$ into at least two correct constraints from the following four: $x+28y \leqslant 488$, $7x-20y \leqslant 0$, $2x-5y \geqslant 0$ and $x-4y \leqslant 0$ **b1A1:** Correctly obtaining $x \leqslant 40$ from $x+28y \leqslant 488$ **and** $x \leqslant \frac{320}{7}$ from $7x-20y \leqslant 0$ **and** $x \geqslant 40$ from $2x-5y \geqslant 0$ (condone lack of $x \leqslant 64$ from $x-4y \leqslant 0$) then stating/implying that $x=40$ and therefore $z=24$ (or equivalent in context e.g. 24 leadership prizes)

Q25.

Question Number	Scheme	Marks
(a)	Start at A e.g. we would be able to find the shortest distance from A to every other vertex e.g. A appears in both required routes	B1 B1dep (2)
(b)		M1 A1 (ACDBG) A1 (EFH) A1ft (JK)
	Shortest path to J: A C D G F J Length: 80 (miles) Shortest path to K: A C D B E H K Length: 81 (miles)	A1 A1 A1ft (7)
(c)	Route from F to H via A: F G D C A C D B E H	B1 (1)
		10 marks
Notes for Question		

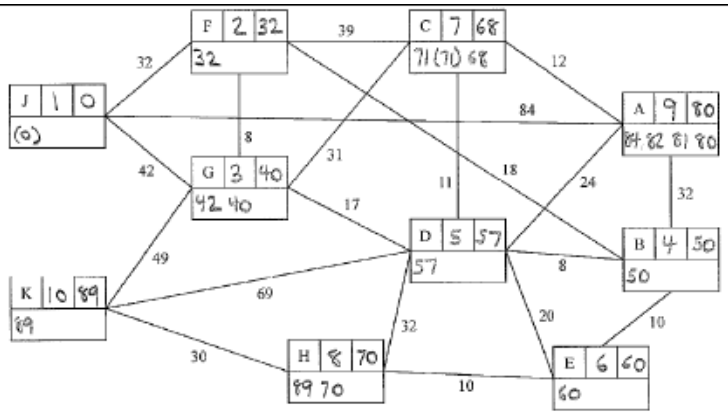
a1B1: CAO (A) – if more than (vertex) A stated then B0
a2B1dep: Correct reason for starting at A (dependent on first B mark) – either need to expicitly mention that A appears on both routes **or** if starting at A then the shortest route to all other vertices (or just to vertices J and K) can be found

In (b) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 51 49 48 in that order (so 51 48 49 is incorrect)
It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

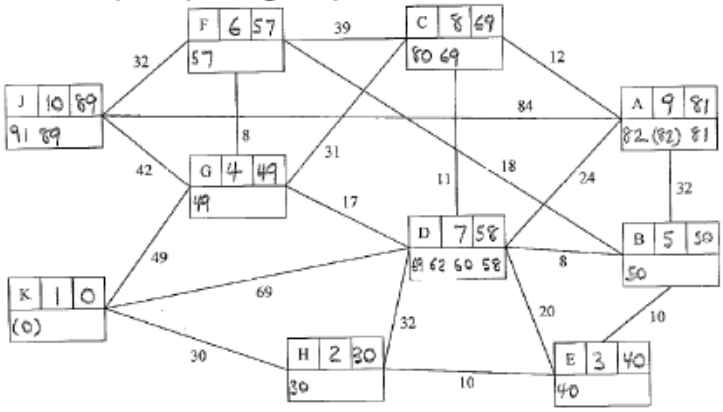
b1M1: Any larger working value replaced by any smaller working value at any two nodes except A and C (for example, **if** correct at B, 32 is replaced by 31 which is a larger value being replaced by a smaller value at one node – as this is a method mark the values do not need to be correct)
b1A1: All values at A, C, D, B and G correct and the working values in the correct order (including order of labelling so nodes must be labelled in the order A, then C, then D, then B, then G). Condone lack of a zero as a working value at A
b2A1: All values at E, F and H correct and the working values in the correct order. Penalise order of labelling only once per question (so E, F and H must be labelled in that order and E must be labelled after A, C, D, B and G)
b3A1ft: All values in J and K correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through J check that the working values at J follow from the candidate's final values for the nodes that are directly attached to J (which are A, F and G). For example, **if** correct then the order of labelling of nodes A, F and G is 1, 7 and 5 respectively so the working values at J should come from A, G and F in that order. The first working value at J should be 0 (the Final value at A) + 84 (the weight of the arc AJ), the second working value at J should be 40 (the Final value at G) + 42 (the weight of the arc GJ) and the final working value at J should be 48 (the Final value at F) + 32 (the weight of the arc FJ). Repeat the process for K (which will have working values from D, G and H with the order of these nodes determined by the candidate's order of labelling at D, G and H)
b4A1: One correct route (either ACDGFJ or ACDBEHK) – allow if reversed (e.g. JFGDCA) and allow if stated in terms of arcs (e.g. AC, CD, DG, GF, FJ)
b5A1: Both routes correct (as for **b4A1** – routes can be reversed and accept in terms of arcs)
b6A1ft: Both lengths correct following through **their** Final values at J and K. Condone correct answers or correct answers following through their diagram even if not explicitly clear which value refers to which path

c1B1: Correct answer only (FGDCACDBEH or FG, GD, DC, CA, AC, CD, DB, BE, EH) – if stated in terms of arcs then arcs AC and CD must appear twice in their route

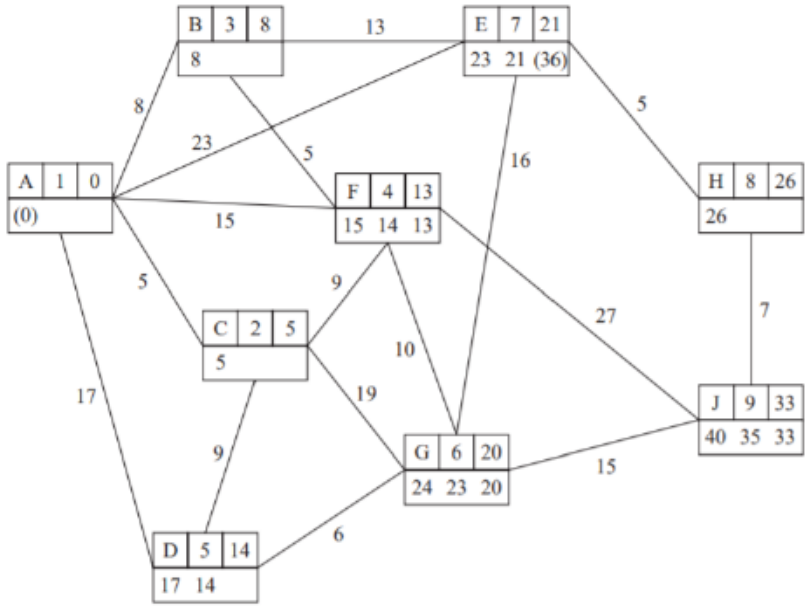
If Dijkstra is completed twice (from J **and** K) then full marks can be awarded. If the candidate uses **just** J or K as the starting vertex in **(b)** then this is not a misread. The candidate can score if starting at J: M1 (as above), A1 (for correct values at J, F, G, B, D), A0, A0, A1 (for route JFGDCA), A1 (for length 80), A0 so 4 out of 7 max.



If starting at K: M1 (as above), A1 (for correct values at K, H, E, G, B), A0, A0, A1 (for route KHEBDCA), A1 (for length 81), A0 so 4 out of 7 max.



Q26.

Question Number	Scheme	Marks
(a)	A path is a (i) finite sequence of edges, such that (ii) the end vertex of one edge in the sequence is the start vertex of the next, and in which (iii) no vertex appears more than once	B2, 1, 0 **(2)**
(b)	 Shortest path: A B E H J Length: 33 (km)	M1 A1 (ACBFD) A1 (GE) A1ft (HJ) A1 A1ft **(6)**
(c)	Shortest path from J to A via G: J G D C A Length: 20 + 15 = 35 (km)	B1 B1ft **(2)**
		10 marks

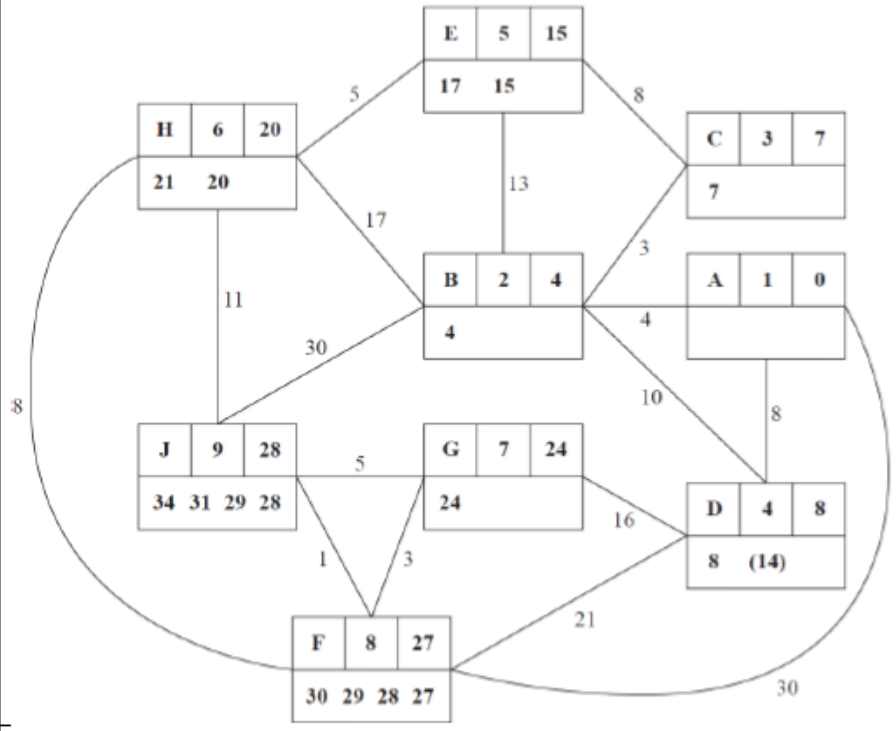
Question Number	Scheme	Marks
	Notes for Question	

a1B1: One of the three points made clearly ('finite, edges', 'end vertex of one edge is the start vertex of the next', 'no vertex appears more than once')
a2B1: All three points made clearly. Candidates who state that a path is a walk in which no vertex appears more than once can score B1B0 only

In (b) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 15 14 13 in that order (so 15 13 14 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling

b1M1: Working values - a larger value replaced by a smaller value for at least two of the five activities D, E, F, G, J
b1A1: All values at A, C, B, F and D correct and the working values in the correct order
b2A1: All values at G and E correct and the working values in the correct order
b3A1ft: All values in H and J correct on the follow through and the working values in the correct order
b4A1: cao (A B E H J only)
b5A1ft: Follow through on their final value at J **only** (condone lack of units)
c1B1: cao (J G D C A only)
c2B1ft: 35 or follow through their final value at G + 15

Q27.

Question number	Scheme	Marks
(a)		M1 A1 (BCDE) A1 (HG) A1ft (FJ)
	Shortest path: A – D – G – F – J	A1
	Length: 28 (miles)	A1ft (6)
(b)	Shortest path: J – F – G – D – A – B – C – E – H	B1
	Length: 28 + 20 = 48 (miles)	B1ft (2)
(c)	A(BC)E + FG = 15 + 3 = 18* A(DG)F + E(HF)G = 27 + 16 = 43 A(D)G + E(H)F = 24 + 13 = 37	M1 A1 A1
	Repeat arcs: AB, BC, CE, FG	A1
	Length: 193 + 18 = 211 (miles)	A1ft (5)
(d)	EF (13) is the shortest link between two odd nodes excluding G Repeat EF (13) since this is the shortest path excluding G	M1
	We finish at A	A1
	Length of route = 193 + 13 = 206 (miles)	A1 (3)
		(16 marks)

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 21 20 in that order (so 20 21 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**

a1M1: A larger value replaced by a smaller value at least twice in the working values at either E, F, H or J
a1A1: All values at B, C, D and E correct and the working values in the correct order
a2A1: All values at H and G correct and the working values in the correct order
a3A1ft: All values in F and J correct on the follow through and the working values in the correct order. To follow through F check that the working values at F follow from the candidate's final values for the nodes that are directly attached to F (which are A, D, H, G (and J)). For example, **if** correct then the order of labelling of nodes A, D, H and G are 1, 4, 6 and 7 respectively so the working values at F should come from A, D, H and G in that order. The first working value at F should be 30 (from A), the second working value at F should be their 29 (the Final value at D) + 21 (the weight of the arc DF), the third working value at F should be their 20 (the Final value at H) + 8 (the weight of the arc HF) and the fourth working value at F should be their 24 (the Final value at G) + 3 (the weight of the arc FG). Repeat the process for J (which will have working values from B, H, G and F with the order of these nodes determined by the candidate's order of labelling at B, H, G and F)
a4A1: CAO (ADGFJ or AD, DG, GF, FJ but **not** JFGDA or equivalent from J to A)
a5A1ft: Follow through their final value at J **only** – if their answer is 28 but this is not the Final Value at J then A0

b1B1: CAO for the route (JFGDABCEH or JF, FG, GD, DA, AB, BC, CE, EH)
b2B1ft: 48 or follow through their final value at J + their final value at H

c1M1: Three distinct pairings of the nodes A, E, F and G
c1A1: Any two rows correct including pairings **and** totals
c2A1: All three rows correct including pairings **and** totals
c3A1: CAO - correct arcs clearly stated and not just in their working as AB, BC, CE and FG (allow BA, CB, etc.) – must be these arcs. Do not accept ABCE or AE via B and C
c4A1ft: Correct answer of 211 **or** follow through 193 + their least total from a choice of three

d1M1: Identifies the need to repeat one path of the three (AE, AF, EF) which does not include G (maybe implicit) or listing of only these possible repeats – **this mark is dependent on either scoring the M mark in (c) or stating all three possible paths**. If stating more than these three paths (AE, AF, EF) then it must be clear from later working that they are only considering these three. **As a minimum stating just one of these three paths (or any combination of these three paths with no others) can score this mark (so, for example, just stating AE and AF scores this mark) provided that they do not further imply that a path including G should be repeated (as this would indicate that mentioning one (or more) of these paths is for the purpose of not repeating it)**
d1A1: Identifies EF as the least **and** A as the finishing point. They have to explicitly state that **EF** is the **least** path that **does not include G**
d2A1: CAO (206)

Q28.

Question Number	Scheme	Marks
(a)	$y \leqslant 2x$ and $3x+y \leqslant 30$	B1
	Correct method for finding the third boundary of the feasible region	M1
	$5y \geqslant 6x+10$	A1 (3)
(b)	(6, 12)	B1
	Solving simultaneous equations to find the other two vertices	M1
	$\left(\frac{5}{2},5\right)$ and $\left(\frac{20}{3},10\right)$	A1 (3)
(c)	$\left(\frac{5}{2},5\right) \rightarrow P=20$ $\left(\frac{20}{3},10\right) \rightarrow P=\frac{130}{3}$ $(6,12) \rightarrow P=48$	M1 A1
	Optimal vertex is (6,12) with $P=48$	A1 (3)
(d)	$12+12k \square 2\left(\frac{20}{3}\right)+10k$ where $\square$ is any inequality or equals	M1
	$k \geqslant \frac{2}{3}$	A1 (2)
		11 marks

Notes for Question

a1B1: Both correct inequalities: $y \leqslant 2x$ and $3x+y \leqslant 30$ (or equivalent if they have been rearranged)

a1M1: Correct equation for the third boundary of the feasible region, e.g. $\frac{y-20}{14-20}=\frac{x-15}{10-15}$ would score **M1** (so does not need to be simplified). Condone any inequality sign used (provided that if the inequality sign was replaced with an 'equals' then the equation ($5y = 6x + 10$) would be correct). ISW once the correct equation has been seen (so condone the correct unsimplified equation if not simplified correctly)

a1A1: CAO for third inequality – must be three terms only but need not be further simplified (e.g. $12x-10y+20 \leqslant 0$, $y \geqslant 1.2x+2$ etc. scores **A1**)

b1B1: (6, 12) **or** $x = 6, y = 12$

b1M1: Correct method for solving their simultaneous equations (so $y = 2x$ with their $5y = 6x + 10$ **or** $3x + y = 30$ with their $5y = 6x + 10$) to find both the x and y-coordinates of at least one of the other two vertices (can be implied by either vertex stated correctly – condone non-exact values (to at least 3 sf) for this mark if no working seen)

b1A1: CAO (2.5, 5) **and** $\left(\frac{20}{3},10\right)$ (must be exact, so accept $(6\frac{2}{3},10)$, $(6.\dot{6},10)$, etc. but not (6.6666…, 10))

c1M1: Testing their three **vertices** (not just points in the FR) in the correct objective function ($P = 2x + 3y$). Condone one slip only when applying the objective function to one of their three vertices (so they must apply the correct objective to at least two of their three points but condone a single slip in the third)

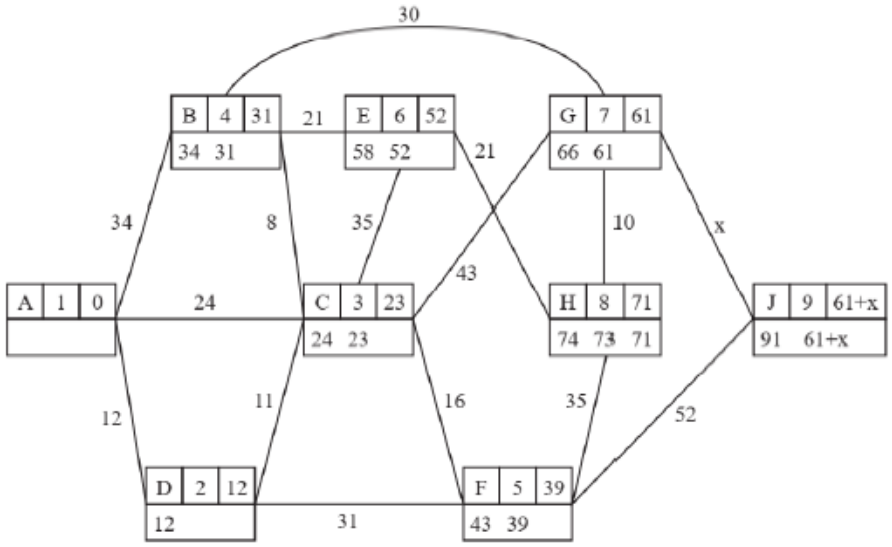
c1A1: At least two correct (therefore exact) points tested correctly (so at least two correct values of P explicitly stated – allow awrt 43.3 when testing (20/3, 10))

c2A1: CSO all three correct (therefore exact) points tested correctly (so all three correct values of P explicitly seen – allow awrt 43.3 when testing (20/3, 10)) with a clear indication of which is the optimal vertex (this can be achieved by either making it clear that (6, 12) is the optimal point **or** that $P = 48$ is the maximum (as one implies the other))

d1M1: (6, 12) **and their** $\left(\frac{20}{3},10\right)$ (does not need to be exact) correctly substituted into $P = 2x + ky$ and compared (by comparing we mean forming an equation or any inequality). **Or** comparing $-\frac{2}{k}$ with -3 (so using the gradient of the objective line) **or** comparing $\frac{2}{k}$ with 3 **or their** reciprocals (e.g. $\frac{k}{2}$ compared with $\frac{1}{3}$ etc.) but comparing $-\frac{2}{k}$ with 3 is **M0** (by comparing we mean forming an equation or any inequality with the **correct** pairs of values)

d1A1: CAO either $k \geqslant \frac{2}{3}$ or $k > \frac{2}{3}$ **only** (or exact equivalents) – if any other answers given with $k \geqslant \frac{2}{3}$ e.g. $k \geqslant -1$ then **A0** (unless clearly rejected). Correct answer with no working can score both marks in this part. Please ensure that if working is shown that the correct answer of $k \geqslant \frac{2}{3}$ comes from correct working e.g. $-\frac{2}{k} \geqslant -3$

Q29.

(a)	 Fastest time: 71 (minutes) Quickest route: ADCBGH	M1 A1 (DCB) A1 (FE) A1ft (GH) A1ft A1 **(6)**
(b)	AD + EH = 12 + 21 = 33*	M1 A1
	A(DCB)E + D(CBG)H = 52 + 59 = 111	A1
	A(DCBG)H + D(CB)E = 71 + 40 = 111	A1
	$(383 + x) + 33 \ldots 440$ Þ $x \ldots\ldots$	depM1
	$24 \leqslant x < 30$	A1 **(6)**
(c)	e.g. ABCEBGCDADFJGHEHFCA	B1 **(1)**
(d)	If a direct road DH opens then only A and E are odd therefore the shortest inspection route is (383 + x) + 25 (DH) + 52 (AE)	M1
	$460 + x = 488$ therefore $x = 28$	A1 **(2)**
		15 marks

	Notes for Question	
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at H the working values must be 74 73 71 in that order (so 74 71 73 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**		

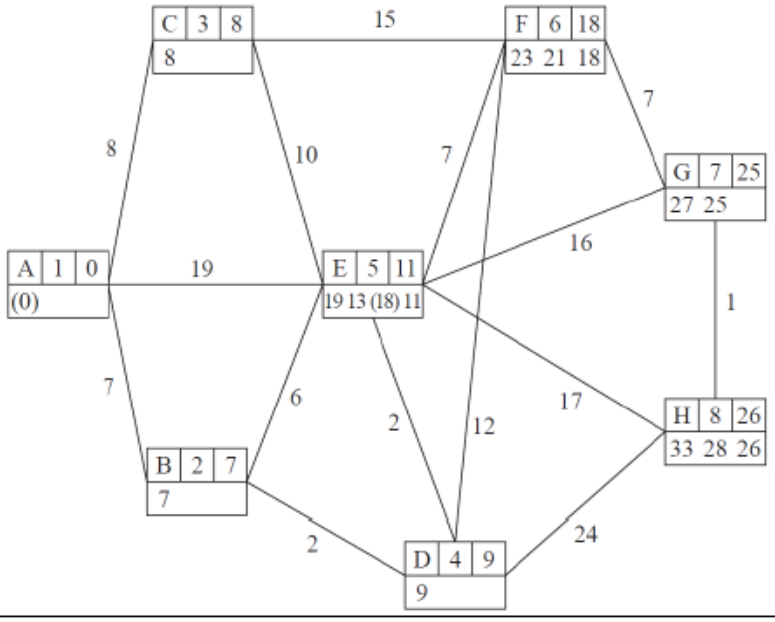
a1M1: A larger value replaced by a smaller value at least twice in the working values at either B, C, E, F, G, H
a1A1: All values at D, C and B correct and the working values in the correct order
a2A1: All values at F and E correct and the working values in the correct order
a3A1ft: All values in G and H correct on the follow through and the working values in the correct order. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are C and B). For example, if correct then the order of labelling of nodes C and B are 3 and 4 respectively so the working values at G should come from C and B in that order. The first working value at G should be their 23 (the Final value at C) + 43 (the weight of the arc CG), the second working value at G should be their 31 (the Final value at B) + 30 (the weight of the arc BG). Repeat the process for H (which will have working values from F, E and G with the order of these nodes determined by the candidate's order of labelling at F, E and G)
a4A1ft: Follow through their final value at H **only** – if answer is 71 but this is not the Final Value at H then A0
a5A1: CAO (ADCBGH)

b1M1: Three distinct pairings of the nodes A, D, E and H
b1A1: Any one row correct including pairing **and** total
b2A1: Any two rows correct including pairings **and** totals
b3A1: All three rows correct including pairings **and** totals
b2dM1: $(383 + x)$ + (their least pairing total) with any inequality sign or equal to 440 – dependent on first M mark in (b). Give bod if not all totals are shown (so if they only give two totals then they should be using the least of these two) but they must have shown all three distinct pairings of the four odd nodes
b4A1: cao $(24 \text{,,} \ x < 30)$ – condone $24 \text{,,} \ x \text{,,} \ 29$

c1B1: cao (check: starting and finishing at A, 19 nodes, AD and EH repeated in route, with A(3), B(2), C(3), D(2), E(2), F(2), G(2), H(2), J(1)) – can be given in terms of arcs

d1M1: $(383 + x) + 25$ + their 52 (where 'their 52' must be the length of their shortest path from A to E in either (a) **or** (b) **or** they state/imply the shortest path from A to E is 52) - a correct value of 28 with no working can imply this mark only
d1A1: cao (28) from correct working **and** correct reasoning that A and E are the only odd nodes **or** that we only need to pair A and E (as a minimum accept mention of A and E only but ignore any mention of the new direct road from D to H)

Q30.

Question Number	Scheme	Marks
(a)		M1 A1 (ABCD) A1 (EF) A1ft (GH) dM1 A1 (6)

	A	B	C	D	E	F	G	H
A	-	**7**	**8**	**9**	**11**	**18**	**25**	**26**
B	**7**	-	14	2	4	11	18	19
C	**8**	14	-	12	10	15	22	23
D	**9**	2	12	-	2	9	16	17
E	**11**	4	10	2	-	7	14	15
F	**18**	11	15	9	7	-	7	8
G	**25**	18	22	16	14	7	-	1
H	**26**	19	23	17	15	8	1	-

(b)	NNA: A – B – D – E – F – G – H – C – A	B1
	7 + 2 + 2 + 7 + 7 + 1 + 23 + 8 = 57 (km)	B1 (2)
(c)(i)	Prim (starting at C): CE, DE, BD, EF, FG, GH	M1 A1
	RMST weight = 10 + 2 + 2 + 7 + 7 + 1 = 29	
(c)(ii)	29 + 7(AB) + 8(AC) = 44 (km)	M1 A1 (4)
(d)	44 ⩽ optimal distance ⩽ 57	B2, 1, 0 (2)
		14 marks

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 23 21 18 in that order (so 23 18 21 is incorrect). It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling
a1M1: Any larger working value replaced by any smaller working value at at **least two nodes** except A, B, C and D **a1A1:** All values at A, B, C and D correct and the working values in the correct order. Condone lack of a zero as a working value at A **a2A1:** All values at E and F correct and the working values in the correct order. Penalise order of labelling only once per question (so E and F must be labelled in that order and E must be labelled after D) **a3A1ft:** All values at G and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working values at G follow from the candidate's final values for the nodes that are directly attached to G (which are E, F and H). For example, **if** correct then the order of labelling of nodes E, F and H are 5, 6 and 8 respectively so the working values at G should come from E and F in that order. The first working value at G should be 18 (the Final value at F) + 7 (the weight of the arc FG) and the second working value at G should be 11 (the Final value at E) + 16 (the weight of the arc EG). Repeat the process for H (which will have working values from D, E and G with the order of these nodes determined by the candidate's order of labelling at D, E and G) **a2M1:** Correct entries in the table following through their final values – dependent on the previous M mark (need only fill in either the A row or A column) **a4A1:** cao **b1B1:** Correct nearest neighbour route starting and finishing at A (A – B – D – E – F – G – H – C – A) **b2B1:** cao (57) on length of route **ci1M1:** First three arcs (CE, DE, BD) **or** all 7 nodes {C, E, D, B, F, G, H} correctly chosen in order. If any explicit rejections seen then M1only in (c)(i). Order of nodes may be seen at the top of the matrix/table {4, 1, 3, 2, 5, 6, 7}. Award M0 for a correct tree with no working. Award M1 only for the first three arcs (oe) selected correctly if starting at a different node than C. If correct values circled in the table but no indication of order of selection then M0 **ci1A1:** cso – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen). If AB and AC added explicitly in (c)(i) then A0 but can score both marks in (c)(ii) **cii2M1:** Weight of RMST + 7 + 8 (two smallest arcs incident to A) with $19 \leqslant \text{RMST} \leqslant 39$ (if clearly not six arcs in RMST then M0). Give bod if 15 is added to the total of six values circled in the table provided those six values sum to a value between 19 and 39 inclusive **cii2A1:** cao (44) – if correct RMST stated in (c)(i) followed by 44 (with no additional working) then award M1A1 in (c)(ii). This mark is dependent on Prim's algorithm being used to find the RMST (allow this mark if rejections seen in (c)(i) when applying Prim). So in (c) M1A0M1A1 is possible e.g. if only stating the node (instead of the arc) selection in order when applying Prim. If the correct six values are circled in the table and added to 15 to give 44 but the order of arc/node selection is not stated (so no indication that Prim has been applied) then A0 (as the qu. says, 'Hence…') **d1B1:** **Any** indication of an interval from their answer to **(c)(ii)** to their answer to **(b)** with one value correct (e.g. $44 \sim 57$ scores B1 but $57 \sim 44$ or $57 \leqslant$ optimal distance $\leqslant 44$ scores B0). **If correct route seen in (b) but the upper bound not stated in (b) allow recovery in part (d) it stated here (but still withold the second mark in (b)).** **d2B1:** cao ($44 \leqslant$ optimal distance $\leqslant 57$) including correct inequalities (allow $44 <$ optimal distance $\leqslant 57$) – allow interval notation e.g., [44, 57] or (44, 57]

Q31.

Question number	Scheme	Marks
(a)	Bin 1: **1.8 1.4 1.6** Bin 2: **2.6** 0.9 0.8 0.6 Bin 3: 2.8 1.2 Bin 4: 3.1 Bin 5: 2.4	**M1** A1 A1 **(3)**
(b)(i)	1.8 2.6 1.6 2.8 1.4 3.1 0.9 1.2 2.4 0.8 0.6	B1
(ii)	Comparisons: 10 Swaps: 6	B1 B1 **(3)**
(c)	e.g. using middle right 2.6 1.8 2.8 1.6 3.1 [1.4] 1.2 2.4 0.9 0.8 0.6 pivot 1.4 2.6 1.8 2.8 [1.6] 3.1 2.4 1.4 1.2 0.9 [0.8] 0.6 pivots 1.6 0.8 2.6 1.8 [2.8] 3.1 2.4 1.6 1.4 1.2 [0.9] 0.8 0.6 pivots 2.8 0.9 (0.6) 3.1 2.8 2.6 [1.8] 2.4 1.6 1.4 1.2 0.9 0.8 0.6 pivot(s) (3.1) 1.8 (1.2) 3.1 2.8 2.6 [2.4] 1.8 1.6 1.4 1.2 0.9 0.8 0.6 pivot 2.4 3.1 2.8 2.6 2.4 1.8 1.6 1.4 1.2 0.9 0.8 0.6 (sort complete) e.g. using middle left 2.6 1.8 2.8 1.6 3.1 [1.4] 1.2 2.4 0.9 0.8 0.6 pivot 1.4 2.6 1.8 [2.8] 1.6 3.1 2.4 1.4 1.2 [0.9] 0.8 0.6 pivots 2.8 0.9 3.1 2.8 2.6 [1.8] 1.6 2.4 1.4 1.2 0.9 [0.8] 0.6 pivots (3.1) 1.8 (1.2) 0.8 3.1 2.8 [2.6] 2.4 1.8 1.6 1.4 1.2 0.9 0.8 0.6 pivot(s) 2.6 (1.6) (0.6) 3.1 2.8 2.6 2.4 1.8 1.6 1.4 1.2 0.9 0.8 0.6 (sort complete)	M1 A1 A1ft A1
(d)	Bin 1: **3.1 1.8** Bin 2: **2.8** 1.6 0.6 Bin 3: **2.6 2.4** Bin 4: 1.4 1.2 0.9 0.8	**M1** A1 A1 **(3)**
		(13 marks)

Notes for Question

PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE IN PART (c) AND THE GUIDANCE FOR THE M MARK IN (d)

a1M1: The **correct** first four items placed correctly (the bold values) and at least eight values placed in bins (allow repeated values). Condone cumulative totals for M1 only
a1A1: First eight values placed correctly (the bold **and** underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if **any** repeated values or incorrect values are seen (even if the first eight values have been placed correctly)
a2A1: CSO – no additional or repeated values (dependent on both previous marks)

bi1B1: CAO – isw after one complete pass. Please check these carefully as some candidates show all the swaps and comparisons in the first pass and some show more than one complete pass. As a guide consider the placement of the 0.8 (when this is the second value from the right-hand side this will indicate the completion of the first pass)
bii1B1: Comparisons correct (10)
bii2B1: Swaps correct (6)
If the comparisons and swaps are not labelled then assume that the first number seen is the comparisons and the second number is the swaps (so seeing after the 1st pass of bubble sort the numbers 10, 6 then award both the second and third B marks in this part). If all they state is 6 then 10 then give **SC B1 B0** for the final two marks in this part

c1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1 only**
c1A1: First pass correct and choosing consistent pivots for the second pass for this mark
c2A1ft: Second and third pass correct following through from their first pass and choice of pivots for the second pass these pivots for the second pass must be consistent (either both middle left or both middle right)
c3A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including a fifth pass shown (not just saying 'sort complete' after the fourth pass) for 'middle right' or a fourth pass shown for 'middle left'

SC for (c): If using the original list or an incorrect list from the start of (c), or after the first pass, with only one error (an error is either one missing number, one extra number, two numbers transposed or one incorrect number) then they can score at most M1A0A1ftA0. If the candidate sorts into ascending order they can score M1 as per the main scheme (but with the values either side of the pivot reversed), A1 for a fully correct sort then A0 A0 even if the list is reversed at the end (so 2 marks max.).

d1M1: Their five largest items placed correctly and at least eight values placed in bins (if correct this will be the bold items but must check **their** packing if any of **their** five largest values are incorrect – note that the maximum weight of a bin is 5). Condone cumulative totals for M1 only. First-fit increasing scores no marks in this part. If no sort seen in (c) then mark (d) assuming the correct ordered list is being used
d1A1: First eight values placed correctly (the bold **and** underlined values) with all eleven correct values only placed in bins. This mark cannot be awarded if **any** repeated values or incorrect values are seen (even if the first eight values have been placed correctly)
d2A1: CSO – no additional or repeated values (dependent on both previous marks)

Q32.

Question Number	Scheme	Marks
(a)	<table><tr><th>a</th><th>b</th><th>Qn</th></tr><tr><td>1</td><td>0.967168(21)</td><td>N</td></tr><tr><td>0.967168(21)</td><td>0.972789(34)</td><td>N</td></tr><tr><td>0.972789(34)</td><td>0.971833(85)</td><td>N</td></tr><tr><td>0.971833(85)</td><td>0.971996(47)</td><td>N</td></tr><tr><td>0.971996(47)</td><td>0.971968(80)</td><td>N</td></tr><tr><td>0.971968(80)</td><td>0.971973(50)</td><td>Y</td></tr></table> Final output is 0.97197	M1 A1 A1 A1 (4)
(b)	Consideration of $12-5a$ being negative (oe)	M1
	$a > 2.4$	A1 (2)
		6 marks

Notes for Question

Candidates may write each value for a, b and N/Y (or No/Yes) in a new row which is fine. Assume that each row begins and ends when a value is changed. For example, the values in row 2 in the table above consists of the a value of 0.967168 and the b value of 0.972789. In (a) for the first three marks the values for a and b must be given to <u>at least</u> 6 decimal places (but accepted either rounded or truncated values)

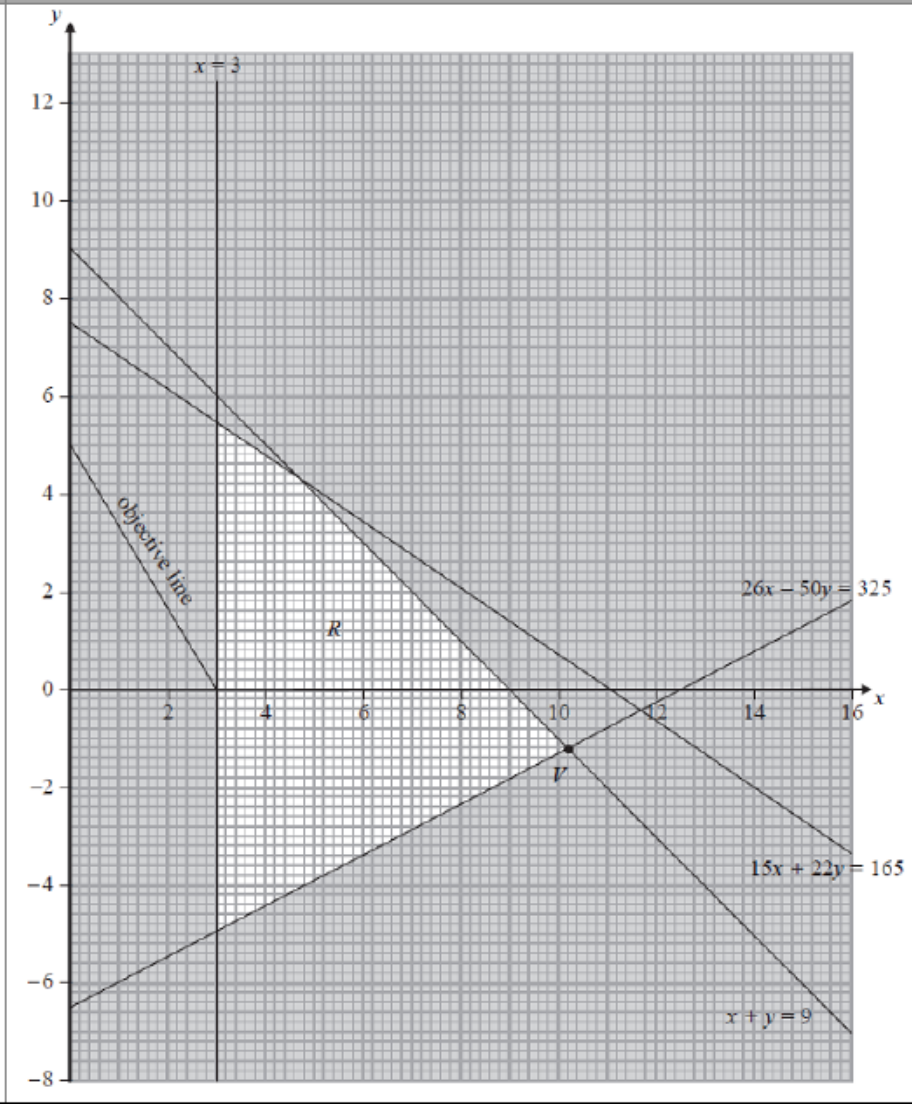
a1M1: At least **three** rows of cells for columns a and b completed with a correct first row (so 1 for a and 0.967168 for b)
a1A1: CAO for the first three rows (for just the a and b columns) – to at least 6 dp
a2A1: CAO for the fourth and fifth rows (for just the a and b columns) – to at least 6 dp
a3A1: CSO - Output **must** be correct and written in the 'Final Output' box at the bottom of the page. This value must be given as 0.97197 (do not accept any other value). Candidates **must** have completed the third column of the table correctly to score this mark

b1M1: Consideration that the fourth root cannot be negative – allow just stating the critical value of 2.4 for this mark or stating or implying one of $f(a) \leq 0$ or $f(a) = 0$ or $f(a) < 0$ where

$$f(a) = 12-5a \text{ or } f(a) = \frac{12-5a}{8} \text{ or } f(a) = (12-5a)^{\frac{1}{4}} \text{ or } f(a) = \left(\frac{12-5a}{8}\right)^{\frac{1}{4}}$$

Allow any letter for a (e.g. x)
b1A1: CAO (must be a strict inequality and must be using a)

Q33.

Question	Scheme	Marks
(a)(b)		B1 B1 B1 B1 (R) **(4)** B1 B1 **(2)**

(c)	$V\left(\frac{775}{76}, -\frac{91}{76}\right)$ $P = \frac{1801}{38}$	M1 A1 A1
		(3)
(d)	$x = 3, y = -4$ minimum value is 3	B1 B1
		(2)
		(11 marks)

Notes:

(a)

In (a), lines must be long enough to define the correct feasible region **and** pass through one small square of the points stated:

$x+y=9$ passes through (5, 4) and (9,0) but in most cases check (0, 9) and (9,0)

$26x-50y=325$ passes through (5, -3.9) and (10, -1.3) but in most cases check (0, -6.5) and (12.5, 0)

$15x+22y=165$ passes through $\left(3,\frac{60}{11}\right)$ and $\left(4,\frac{105}{22}\right)$ but in most cases check (0, 7.5) and (11, 0)

B1: Any two lines correctly drawn

B1: Any three lines correctly drawn

B1: All four lines correctly drawn

B1: Region, R, correctly labelled – not just implied by shading – dependent on scoring the first three marks in (a)

(b)

B1: Drawing the correct objective line on the graph, use line drawing tool to check if necessary. Line must not pass outside of a small square if extended from axis to axis

B1: V labelled clearly on their graph. **This mark is dependent on both the correct feasible region (but maybe not labelled) and the correct objective line**

(c)

M1: Candidates **must** have drawn either the correct objective line **or** its reciprocal. If they have drawn the correct objective line they must be solving $x+y=9$ and $26x-50y=325$. If they have drawn the reciprocal objective line they must be solving $x=3$ and $15x+22y=165$. Must get to either $x=...$ or $y=...$ (condone one error in the solving of the simultaneous equations).

The correct exact answer $\left(\frac{775}{76},-\frac{91}{76}\right)$, or for the reciprocal $\left(3,\frac{60}{11}\right)$, can imply this mark

A1: cao $\left(\frac{775}{76},-\frac{91}{76}\right)$ or $\left(10\frac{15}{76},-1\frac{15}{76}\right)$ (coordinates must be exact) – **if correct answer stated with no working seen then award M1A0 only** (however, they can still earn the next A mark for the corresponding value of P at V). **This mark is dependent on the correct feasible region (but maybe not labelled)**

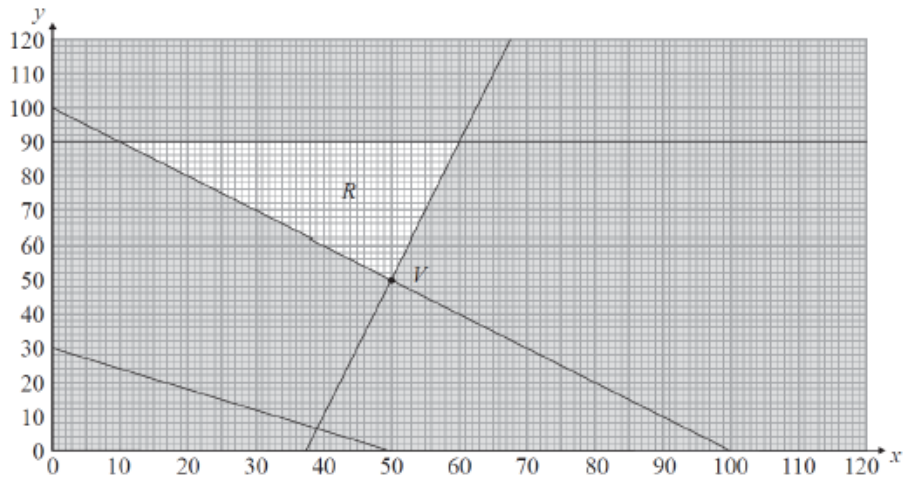
A1: cao $\frac{1801}{38}$ or $47\frac{15}{38}$ (must be exact). **This mark is dependent on the correct feasible region (but maybe not labelled)**

(d)

B1: cao $x=3, y=-4$ or $(3,-4)$

B1: cao of 3

Q34.

Question Number	Scheme	Marks
(a)	$5y \leq 3z$	M1 A1 (2)
(b)	The total number of shirts must be at least 250	B1
	At most 20% of all the shirts should be small	M1 A1 (3)
(c)	(Minimise) $6x+10y+15z$	B1 (1)
(d)(i)	$z=150 \Rightarrow x+y \geq 100$	M1
	$y \leq 90$	
	$4x-y \leq 150$	A1
(d)(ii)		B1 B1 B1 B1 (6)
(e)	Correct objective line	B1
	V correctly labelled	B1 (2)
(f)	50 small and 50 medium shirts	B1
	Cost = £3050	B1 (2)
(g)	$x=50, y=75 \Rightarrow z \geq 125$ therefore minimum number of large shirts is 125	M1
	This leads to a cost of £2925 which is less than the cost in (f)	A1 (2)
		18 marks

Notes for Question

a1M1: Correct method: $5y \square 3z$ where $\square$ is any inequality or equals. An exact equivalent answer (with or without integer coefficients) can score M1 **or** M1 for $3y \leq 5z$ only

a1A1: CAO (or equivalent e.g. $k(5y \leq 3z)$ where k is any positive integer only)

b1B1: CAO oe e.g. the minimum number of shirts is 250 is fine for this mark (note that they must imply that the total number (and not one particular brand of shirt) is **at least** 250)

b1M1: Three of 'at most', '20%', 'all' and 'small' (allow equivalents e.g. fifth or 0.2 for 20%) allow those who imply 'all' provided that it is clear that they aren't talking about one particular brand only

b1A1: CAO (o.e. e.g. the number of small shirts is less than or equal to a fifth of the total number of shirts, the number of small shirts is at most 20% of all the shirts sold) – give bod of these that clearly imply 'all' provided that they aren't talking about only one particular brand. Do not allow statements which contain use of 0.2 or $\frac{1}{5}$ for this mark, e.g. the number of small shirts is at most 0.2 of all the shirts is A0

c1B1: Expression correct (or $600x+1000y+1500z$)
di1M1: Eliminating z from all **their** inequalities by using the substitution $z = 150$ – accept unsimplified (e.g. $x+y+150 \geq 250, x \leq 0.2(x+y+150)$ and **their** $5y \leq 3(150)$)
di1A1: CAO e.g. $x+y \geq 100, 4x \leq y+150, 5y \leq 450$ (oe) - all constraints must be correct with integer coefficients but allow positive multiplies – ignore $x \geq 0, y \geq 0$ but any other additional constraints is A0 – allow recovery in this part if $y \leq 90$ (oe) seen in **(d)** even if their $5y \leq 3z$ is incorrect in **(a)**

In **(d)**, lines must be long enough to define the correct feasible region and would pass through one small square of the points stated:

$x+y=100$ must pass within one small square of its intersection with the axes – (0, 100) and (100, 0)
$y=90$ must pass within one small square of its intersection with the y-axis and (60, 90)
$4x-y=150$ must pass within one small square of (37.5, 0) and (60, 90)

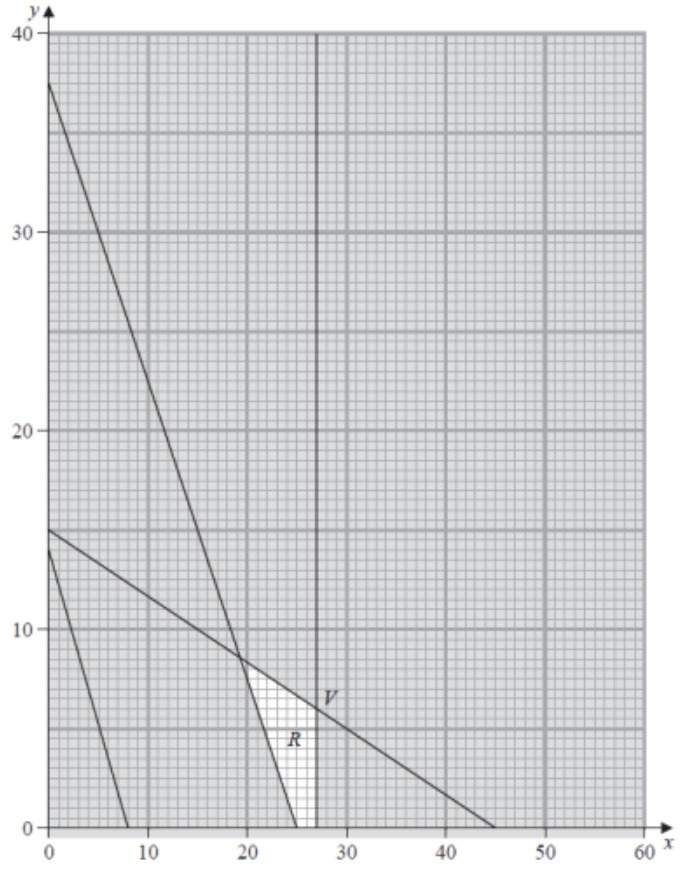
dii1B1: Any one line correctly drawn
dii2B1: Any two lines correctly drawn
dii3B1: All three lines correctly drawn
dii4B1: Region, R, correctly labelled – not just implied by shading – dependent on scoring the three previous B marks in this part

e1B1: Drawing the correct objective line on the graph with gradient of –0.6. Line must be correct to within one small square if extended from axis to axis. If line is shorter than (0, 6) to (10, 0) then B0
e2B1: V correctly labelled – note that this mark is dependent on the correct feasible region in **(d)** (so must have scored at least B1B1B1B0) and the previous B mark in **(e)**

f1B1: CAO – must be in context (50 small and 50 medium and not for $x = y = 50$) note that this mark is dependent on the correct feasible region in **(d)** (so must have scored at least B1B1B1B0 in **(d)**) and the first B mark (for a correct objective line) in **(e)**
f2B1: CAO (3050) – units not required - note that this mark is dependent on the correct feasible region in **(d)** (so must have scored at least B1B1B1B0 in **(d)**) and the first B mark (for a correct objective line) in **(e)** – condone if stated/seen in **(g)**

g1M1: Substitute to obtain the correct value for z of 125 (accept $z \geq 125$ or $>$) – if no method allow 125 seen but M0 if 125 found but a different value of z stated and subsequently used
g1A1: Correct cost (2925) and dependent on the final B mark in **(f)**

Q35.

Question Number	Scheme	Marks
(a)	Minimise $(P=)9x+12y+16z$	B1
	$x+y+z \geqslant 40$	B1
	$z \geqslant 2y$	B1
	$\frac{3}{5}(x+y+z) \geqslant x$ simplifies to $2x \leqslant 3y+3z$	M1 A1
	$x+1.5y+2.5z \leqslant 75$ simplifies to $2x+3y+5z \leqslant 150$	M1 A1
	$(x \geqslant 0, y \geqslant 0, z \geqslant 0)$	**(7)**
(b)	$9x+12y+16(45-x-y)$	M1
	which leads to $-7x-4y+720$	A1
	+720 is a constant so the total value is minimised when $-7x-4y$ is **and** $-7x-4y=-(7x+4y)$, as $-(7x+4y)$ is negative (for all positive values of x and y) and so minimising an expression which is negative is equivalent to maximising the corresponding positive expression $7x+4y$	A1 **(3)**
(c)		B1 B1 B1 B1 **(4)**
(d)	Correct objective line	B1
	V correctly labelled	B1 **(2)**
(e)	27 small containers, 6 medium containers and 12 large containers	B1
	Total cost is (£)507	B1 **(2)**
		18 marks

Notes for Question

a1B1: cao – expression correct and 'minimise' or 'min' but not 'minimum'
a2B1: cao ($x+y+z \geqslant 40$ oe but must be four terms only with integer coefficients e.g. $x+y+z-40 \geqslant 0$)
a3B1: cao ($z \geqslant 2y$ oe $(\text{e.g. } 4y-2z \leqslant 0)$ but must be two terms only with integer coefficients)
a1M1: correct method - must see $\frac{3}{5}(x+y+z) \bullet x$ where $\bullet$ is any inequality or =. The bracket must be present or implied by later working. Allow 0.6 but not 60% (unless implied by later working)
a1A1: cao – simplified (one term only in x, y and z) – answer must have integer coefficients (e.g. $4x-6y-6z \leqslant 0$) - the correct simplified inequality with either no working **or** working with % sign implies M1A1
a2M1: correct method – must see $x+1.5y+2.5z \bullet 75$ (oe) where $\bullet$ is any inequality or =
a2A1: cao – simplified (one term only in x, y and z) – answer must have integer coefficients (e.g. $4x+6y \leqslant 300-10z$) – the correct simplified inequality with no working implies M1A1
b1M1: substituting $z = 45-x-y$ into $9x+12y+16z$
b1A1: cao of $-7x-4y+720$ **and** any attempt at explaining why the minimum total cost is achieved when $7x + 4y$ is maximised
b2A1: stating that 720 is a constant (and so doesn't impact on the optimal values of x, y and z) **and** a correct deduction that minimising a negative expression is equivalent to maximising the corresponding positive expression (so just stating that $-7x-4y$ is minimised when $7x+4y$ is maximised is A0)

The lines in **(c)** must define the correct FR **and if extended** pass within a small square of the points stated:

$x+3y=45$ with points (0, 15) and (45, 0)
$3x+2y=75$ with points (0, 37.5) and (25, 0)
$x=27$ with points (27, 0) and (27, 40) – a common wrong response is to draw either $y = 27$ or $x = 28$ – these are both B0
c1B1: Any one line drawn correctly
c2B1: Any two lines drawn correctly
c3B1: Any three lines drawn correctly
c4B1: Correct *R* labelled – not just implied by shading – dependent on scoring the first three marks in this part (condone if no shading below the x-axis)

d1B1: A correct objective line drawn on the graph with a gradient of –1.75 – line must be at least the length of (2, 0) to (0, 3.5) and within one small square - for reference common intersections points with each axes are given below

x	y
4	7
5	8.75
8	14
10	17.5
12	21
15	26.25
16	28
20	35

x	y
2.85...	5
5.71…	10
8.57…	15
11.4…	20
14.3…	25
17.1…	30
20	35
22.9…	40

d2B1: *V* labelled clearly on their graph. This mark is dependent on
- the first three marks in (c)
- not labelling or implying that any other region is the FR
- the first B mark in (d)

By clearly labelled the vertex should either be labelled '*V*' or circled or clearly distinguishable from any other vertex (but B0 if not clear e.g. another vertex circled too) (note that *V*(27, 6))

e1B1: cao (must be in context – so not in terms of x, y and z) – dependent on first three marks in (c) and the first mark in (d) (27 small, 6 medium and 12 large)
e2B1: cao (507) - dependent on first three marks in (c) and the first mark in (d) – units not required. Condone incorrect units e.g. $

Q36.

Question Number	Scheme	Marks
	Minimise $(C=)2x+3y$	B1
	$x+y\geq 85$	B1
	$y\geq 2x$	M1
	$y\leq \frac{4}{5}(x+y)$	M1
	$y\geq 2x$ and $y\leq 4x$	A1 (5)
		5 marks
	Notes for Question	

1B1: Expression correct $(2x+3y)$ together with 'minimise' or 'min' (but not 'minimum') – if 'simplified' e.g. $x+1.5y$ then must see $2x+3y$ at some point

2B1: CAO – any equivalent form provided integer coefficients and only one term in x and one term in y e.g. $x\geq 85-y$

1M1: $y \square 2x$ where $\square$ is any inequality or equals. Accept $2y\geq x$ for this mark

2M1: $y \square \frac{4}{5}(x+y)$ where $\square$ is any inequality or equals – if no bracket then correct rhs must be implied by later working. $y \square 4x$ where $\square$ is any inequality or equals implies this mark. Use of % symbol only is M0 unless correctly replaced by a fraction or decimal later

1A1: Both $y\geq 2x$ **and** $y\leq 4x$ CAO – must be a single terms in x and y but allow any equivalent form provided integer coefficients e.g. $2x-y\leq 0$, $2y-8x\leq 0$ etc.

Q37.

Question Number	Scheme	Marks
	$y\geq 3x$	B1
	$z-x\geq 50$	B1
	$y\leq 120$	B1
	Sub. $x+y+z=180$	M1
	$2x+y\leq 130$	A1
	Maximise $(P=)x+y$	B1 (6)
		6 marks
	Notes for Question	

1B1: cao $(y\geq 3x)$ oe (two terms only with integer coefficients)

2B1: cao $(z-x\geq 50)$ – may be implied by later working oe (three terms only with integer coefficients)

3B1: $y\leq 120$ oe

1M1: Eliminating z by substituting $x+y+z=180$ into an inequality that involves z and x only

1A1: $2x+y\leq 130$ oe (three terms only with integer coefficients)

4B1: correct objective with 'maximise' or 'max' but not 'maximum' – either the expression $x+y$ or any other letter for P except x, y or z

Q38.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>(a)</td><td>
<table>
<tr><th>N</th><th>A</th><th>B</th><th>C</th><th>D</th></tr>
<tr><td>4217</td><td>421.7</td><td>421</td><td>4210</td><td>7</td></tr>
<tr><td>421</td><td>42.1</td><td>42</td><td>420</td><td>1</td></tr>
<tr><td>42</td><td>4.2</td><td>4</td><td>40</td><td>2</td></tr>
<tr><td>4</td><td>0.4</td><td>0</td><td>0</td><td>4</td></tr>
<tr><td>0</td><td></td><td></td><td></td><td></td></tr>
</table>
Output values: 7, 1, 2, 4
</td><td>M1
A1
A1
A1
(4)</td></tr>
<tr><td>(b)</td><td>The first value is the units digit of N, the second value is the tens digit, the third is the hundreds digit, and so on</td><td>B2, 1, 0
(2)</td></tr>
<tr><td></td><td></td><td>6 marks</td></tr>
</table>

Notes for Question

a1M1: At least three rows of cells completed with a correct first row – condone repeated values in all columns or a single value in each row
a1A1: CAO – the values in the second and third row correct
a2A1: CAO – fourth row correct and a zero only in the fifth *N* row
a3A1: Correct outputs (7, 1, 2, 4) – dependent on the first **four** rows being correct – the output must either be stated on the given answer line or 'output 7, 1, 2, 4' must be clearly written somewhere near the table (do not bod column D being circled, etc.). Condone the output being stated as 7124

b1B1: Indication that the outputs are the digits of *N*
b2B1: Indication that the digits are in the reverse order

Examples of **B1 B1**:

- The output is *N* (or the original input or 4217) in reverse order
- The output is *N* backwards
- The output is (the digits of) *N* written right to left
- The first value is the unit digit of *N*, the second value is the tens digit, and so on

Examples for **B1 B0**:

- The output is each number of *N*
- The output is *N*
- Output values are the values that make up the original input

Examples for **B0 B0**:

- The output comes from/is derived from the original input
- The algorithm removes the last digit step by step
- The output values are the last digits of *N*
- The output is the last digit of *N*
- The output is *N* + contradictory statement (e.g. 'output is *N* or a number that is smaller than *N*')

Example for **B0 B1** (not common):

- The output is in reverse/right to left/backwards (so no mention of *N* or original input or 4217)

Q39.

(a)(i)	Let the point of intersection of $-x+5y=10$ and the unknown line be $A(a_1,a_2)$ Let the point of intersection of $4x+8y=65$ and the unknown line be $B(b_1,b_2)$	
	$a_1+3a_2=10$ **or** $b_1+3b_2=24$ $-a_1+5a_2=10$ $4b_1+8b_2=65$	M1 A1
	$A\left(\frac{5}{2},\frac{5}{2}\right)$ **or** $B\left(\frac{3}{4},\frac{31}{4}\right)$	A1
	$a_1+3a_2=10$ **and** $b_1+3b_2=24$ $-a_1+5a_2=10$ $4b_1+8b_2=65$	depM1
	$A\left(\frac{5}{2},\frac{5}{2}\right)$ **and** $B\left(\frac{3}{4},\frac{31}{4}\right)$	A1
(ii)	$y-\frac{5}{2}=\left(\frac{\frac{31}{4}-\frac{5}{2}}{\frac{3}{4}-\frac{5}{2}}\right)x-\frac{5}{2}\ (y=-3x+10)$	ddM1
	$3x+y\ldots10$	A1
	$-x+5y\ldots10, 4x+8y\ldots65$	B1 **(8)**
(b)	$k=\frac{15/4}{35/4}=\frac{3}{7}$	M1 A1 **(2)**
		10 marks

Notes for Question

ai1M1: Form simultaneous equations to find one of the points of intersection of the unknown line with one of the given lines – allow sign slips only

ai1A1: One correct pair of simultaneous equations (allow any choice of letters for their coordinates)

ai2A1: One correct point (need not be stated as coordinates so $x=\ldots$, $y=\ldots$ is fine)

ai2dM1: Forming both pairs of simultaneous equations (dependent on previous M mark) – allow sign slips only

ai3A1: Both correct points (need not be stated explicitly as coordinates)

aii3ddM1: Find the correct equation of the third line for **their** A and B (dependent on both previous M marks). Allow unsimplified but must be the correct equation for the line passing through their two points. Condone any inequality sign instead of equals

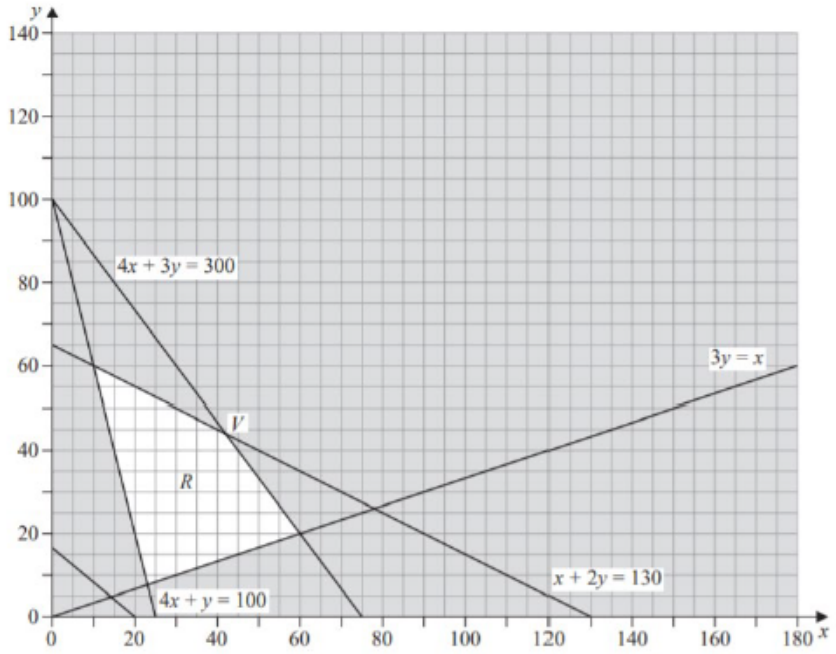
aii4A1: cao (for the third line) – must be three terms but accept any equivalent form e.g. $6x+2y-20\ldots0$(

aii1B1: cao (for the other two given lines) – must be three terms only but accept any equivalent forms

b1M1: Attempt to find the gradient of the line through O and $\left(\frac{35}{4},\frac{15}{4}\right)$ (condone reciprocal) – ignore use of inequalities or k for this mark e.g. seeing $\frac{15/4}{35/4}$ **or** $\frac{15}{35}$ **or** $\frac{35}{15}$ etc. scores M1

b1A1: cao – need not be simplified e.g. $k=\frac{15}{35}$ scores both marks. Allow $y\ldots\frac{3}{7}x$, or just $\frac{3}{7}$ but not $k\ldots\frac{3}{7}$ only. If more than one value of k implied then A0

Q40.

Question Number	Scheme	Marks
(a)		B1 B1 B1 B1 **(4)**
(b)(i)	A correct objective line	B1
	Correct optimal vertex labelled as V	B1 **(2)**
(b)(ii)	Solve correct set of simultaneous equations consistent with either their V or objective line	M1
	$(42, 44)$	A1
	77.6	A1 **(3)**
(c)	$(10, 60) > V \Rightarrow 10k + 60 > 42k + 44$ **or** $(60, 20) > V \Rightarrow 60k + 20 > 42k + 44$ $-\frac{1}{2} < -k$ **or** $-\frac{4}{3} > -k$	M1
	$k < \frac{1}{2}$ **or** $k > \frac{4}{3}$	A1
	$(10, 60) > V \Rightarrow 10k + 60 > 42k + 44$ **and** $(60, 20) > V \Rightarrow 60k + 20 > 42k + 44$ $-\frac{1}{2} < -k$ **and** $-\frac{4}{3} > -k$	M1dep
	$k < \frac{1}{2}$ **and** $k > \frac{4}{3}$ only	A1 **(4)**
		13 marks

Question Number	Scheme	Marks
	Notes for Question	

The lines in **(a)** must define the correct FR and pass within half a square of the points stated:

$4x + 3y = 300$ with points (0, 100) and (75, 0)

$4x + y = 100$ with points (0, 100) and (25, 0)

$x + 2y = 130$ with points (0, 65) and (130, 0)

$3y = x$ with points (0, 0) and (60, 20)

a1B1: Any two lines correctly drawn

a2B1: Any three lines correctly drawn

a3B1: All four lines correctly drawn

a4B1: Correct *R* labelled – dependent on all three previous B marks

bi1B1: A correct objective line drawn on the graph with a gradient of –0.8 – intersections points with each axes given below

x	*y*
10	8
20	16
30	24
40	32
50	40
60	48
70	56
80	64
90	72
100	80

x	*y*
12.5	10
25	20
37.5	30
50	40
62.5	50
75	60
87.5	70
100	80
112.5	90
125	100

bi2B1: cao *V* labelled – dependent on first three B marks in **(a)** and the first B mark in **(b)**

bii1M1: Solving correct pair of simultaneous equations for their *V* (or if not labelled then the vertex consistent with their objective line) – this mark can be implied by (42, 44) but in all cases they must have drawn four lines with at least two correct **and** an attempt at an objective line

bii1A1: cao ((42, 44) only) – dependent on first three B marks in **(a)** and the first B mark in **(b)**

bii2A1: cao (77.6 only) – dependent on first three B marks in **(a)** and the first B mark in **(b)**

SC in (b) if no objective line drawn then can score in (b) B0B0M1A1A0 for both (42, 44) and 77.6 only provided that the first three B marks earned in (a)

Marks in part (c) are not dependent on the marks in (a) and/or (b)

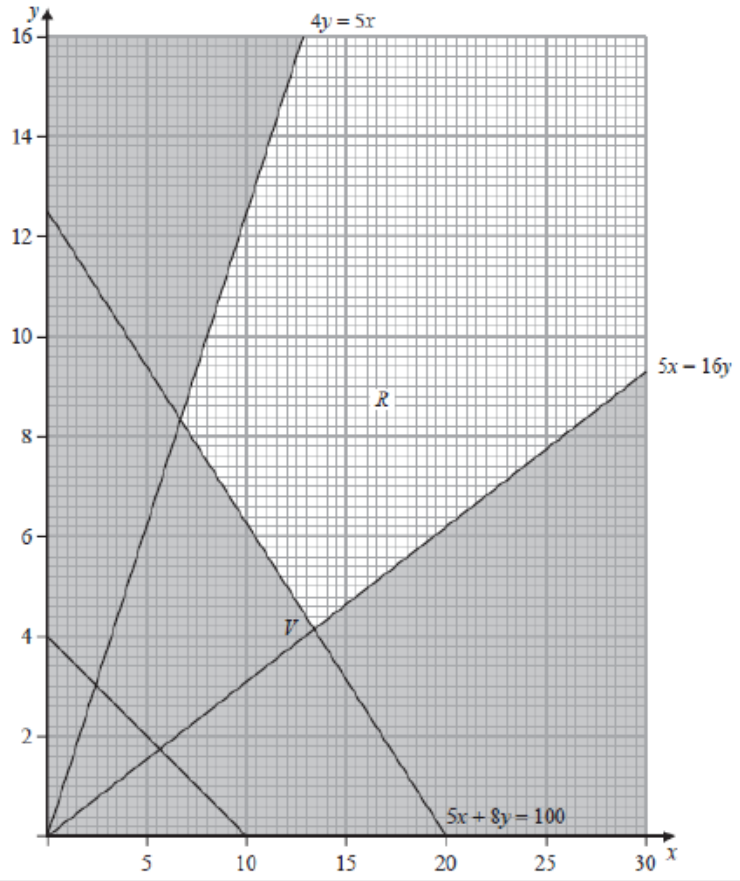
c1M1: Point testing method: $10k + 60 \square kx_1 + y_1$ **or** $60k + 20 \square kx_1 + y_1$ **or** $42k + 44 \square kx_1 + y_1$ where $\square$ is any inequality sign or the equals sign and (x_1, y_1) is their numerical *V* or (42, 44). Objective line method: $-\frac{1}{2} \square -k$ or $-\frac{4}{3} \square -k$ or $\frac{1}{3} \square -k$ where $\square$ is any inequality or equals. Or one correct answer stated

c1A1: One correct answer $(k < \frac{1}{2}, k \leq \frac{1}{2}, k > \frac{4}{3}, k \geq \frac{4}{3})$ - if no method or working (as shown above) then **A0**

c2M1dep: Point testing: $10k + 60 \square kx_1 + y_1$ **and** $60k + 20 \square kx_1 + y_1$ where $\square$ is any inequality sign or the equals sign and (x_1, y_1) is (42, 44) or their *V* (but not ((10, 60) or (60, 20)) (so *V* must now be the intersection of the two lines $4x + 3y = 300$ **and** $x + 2y = 130$). Objective line: $-\frac{1}{2} \square -k$ **and** $-\frac{4}{3} \square -k$ where $\square$ is any inequality or equals. Or both correct answers stated with no working

c2A1: Both correct answers **only** $(k < \frac{1}{2}$ or $k \leq \frac{1}{2}$ **and** $k > \frac{4}{3}$ or $k \geq \frac{4}{3})$ with working as shown above

Q41.

Question Number	Scheme	Marks
(a)	Minimise $400x + 550y + 750z$	B1
	Subject to $x + y + z \geq 20$	B1
	$x \leq 2(y + z)$	M1 A1
	$z \leq \frac{1}{4}(x + y + z) \Rightarrow 3z \leq x + y$	M1 A1 (6)
(b)(i)(ii)	Ratio of 5:3 $\Rightarrow 3y = 5z$	B1
	$x + y + z \geq 20 \Rightarrow x + y + \frac{3}{5}y \geq 20$ and $400x + 550y + 750\left(\frac{3y}{5}\right)$	M1
	$5x + 8y \geq 100$ and $400x + 1000y$	A1 (3)
(c)		B1 B1 B1 B1 (4)

(d)	Drawing an objective line accept reciprocal gradient	M1
	Correct objective line	A1
	V correctly labelled	A1 (3)
(e)(i)	Cost is £9800	B1
(ii)	3 days for senior leaders (12 training days for new teachers, 5 days for middle leaders)	B1 (2)
		18 marks

Question Number	Scheme	Marks
	Notes for Question	
	a1B1: Expression correct <u>together</u> with 'minimise' or 'min' but not 'minimum' – isw if coefficients are subsequently simpified but $400x+550y+750z$ must be seen at some point for this mark to be awarded. The 'min' must appear beside the correct expression **a2B1**: CAO $(x+y+z\geq 20)$ **a1M1**: $x \square 2(y+z)$ where $\square$ is any inequality or equals. Accept $2x \leq y+z$ for this mark **a1A1**: CAO $x \leq 2(y+z)$ or equivalent but only one term in each variable and integer coefficients **a2M1**: $z \square \frac{1}{4}(x+y+z)$ where $\square$ is any inequality or equals – allow 0.25 but do not allow $z \square 25\%(x+y+z)$ unless recovered to a fraction or decimal later **a2A1**: CAO $3z \leq x+y$ or equivalent but only one term in each variable and integer coefficients **bi1B1**: Ratio expressed correctly as an equation $3y = 5z$ (oe) (possibly implied by subsequent working) **bii1M1**: Their linear equation in y and z only (must be of the form $ay = bz$ oe) substituted into their constraint and their objective to eliminate z or one correct answer simplified **b1A1**: CAO $5x+8y\geq 100$ and $400x + 1000y$ (allow ISW if correct objective simplified) In **(c)** $5x+8y=100$ must pass if extended within one small square of its intersection with the axes – (0, 12.5) and (20, 0) but must be long enough to define the correct feasible region $4y=5x$ must pass within one small square of its intersection with the origin and (10, 12.5) $5x=16y$ must pass within one small square of its intersection with the origin and (20, 6.25) **c1B1**: One line correct **c2B1**: Two lines correct **c3B1**: All three lines correct **c4B1**: Region, R, correctly labelled – not just implied by shading – dependent on scoring the first three marks in this part **d1M1**: Drawing their objective line (based on their answer to (b)) or its reciprocal - line must be correct to within one small square if extended from axis to axis. If their line is shorter than (0, 1) to (2.5, 0) then M0 **d1A1**: Drawing a **correct** objective line subject to the conditions above regarding length and if extended from axis to axis **d2A1**: V labelled clearly on their graph (so dependent on the first three marks in (c) and the previous A mark in this part) **eiB1**: CAO – condone lack of units – dependent on the first three marks in (c) and the first two marks in (d) **eiiB1**: CAO (3) – dependent on the first three marks in (c) and the first two marks in (d)	

Q42.

(a)(i)	$z = 14 - 2x - y$ substituted into both $x + 2y + z \geq 15$ and $3x - 4y + 2z \leq 1$	M1	
	$-x + y \geq 1$	A1	
	$x + 6y \geq 27$	A1	
(a)(ii)	The maximum possible value of P is 1	A1	**(4)**
(b)(i)	$-x + y = 1 \Rightarrow x + 6(1 + x) \geq 27$	M1	
	$7x \geq 21 \Rightarrow$ maximum possible value of x is 3	A1	
(b)(ii)	$x = 3, y = 4$ and $z = 4$	A1	**(3)**
		7 marks	

Notes for Question

Mark parts (a) and (b) together (so ignore labelling of parts in this question)

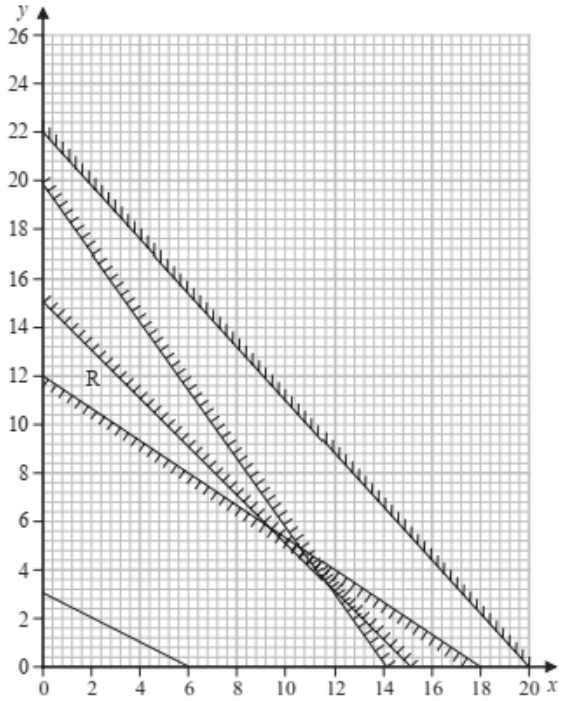
ai1M1: substituting $z = 14 - 2x - y$ into **both** correct inequality constraints – allow sign errors in re-arranging to make z the subject. This mark can also be awarded for **one** correct simplified inequality
ai1A1: cao $(-x + y \geq 1)$ – or equivalent (e.g. $x - y + 1 \leq 0$) but must be three terms only – ISW if candidates incorrectly re-arrange after a correct three term inequality seen
ai2A1: cao $(x + 6y \geq 27)$ – or equivalent but must be three terms only – ISW if candidates incorrectly re-arrange after a correct three term inequality seen

aii3A1: cao - this mark can be awarded after correctly finding x and y – note that this mark is not dependent on the previous A mark. Just stating $P = 1$ is fine (so do not need to mention 'maximum')

bi1M1: substitute their $-x + y = 1$ into their $x + 6y \geq 27$ or their $x + 6y = 27$
bi1A1: correct value of x (if using equations then they do not need to justify that this is the maximum value)
bii2A1: cao for x, y **and** z – accept if seen as a coordinate

Q43.

Question number	Scheme	Marks
(a)	Minimise $(P =)\ 300x + 400y + 400z$	B1
	Subject to:	
	$275x + 200y + 100z \geq 5500\ (\Rightarrow 11x + 8y + 4z \geq 220)$	B1
	$5x + 2y + 3z \geq 70$	B1
	$\frac{x}{15} + \frac{y}{20} + \frac{z}{30} \geq 1 \Rightarrow 4x + 3y + 2z \geq 60$	M1 A1
	$x + y + z \leq 18$ $(x, y, z \geq 0)$	B1 **(6)**
(b)	e.g Martin makes apple cakes and chocolate cakes in the ratio of 2:1 e.g. for every one chocolate cake that Martin makes he has to make 2 apple cakes (assuming that he makes chocolate/apple cakes) e.g. Martin should make twice as many apple cakes as chocolate cakes e.g. The number of apple cakes that Martin makes should be double the number of chocolate cakes he makes	B1 **(1)**

(c)		B1 B1 B1 B1 **(4)**
(d)	Drawing an objective line accept reciprocal gradient	M1
	Correct objective line	A1
	Martin should make 9 carrot cakes, 6 apple cakes and 3 chocolate cakes	A1
	Minimum amount of sugar is 6300 grams (or 6.3 kg)	A1 **(4)**
(e)	Martin has 1525 grams of flour remaining	B1
	Martin also has 4 eggs remaining	B1 **(2)**
		(17 marks)

Notes on Question

a1B1: CAO – expression correct ($300x + 400y + 400z$ or $0.3x + 0.4y + 0.4z$ **only**) and 'minimise' or 'min' but not 'minimum'. ISW if either of these two expressions are seen and then 'simplified'

a2B1: CAO ($275x + 200y + 100z \,„\, 5500$) oe but must be four terms only with integer coefficients e.g. $11x + 8y \,„\, 220 - 4z$

a3B1: CAO ($5x + 2y + 3z \,„\, 70$) oe but must be four terms only with integer coefficents

a1M1: Correct method $\frac{x}{15} + \frac{y}{20} + \frac{z}{30} \bullet 1$ where $\bullet$ is any inequality symbol or =

a1A1: CAO ($4x + 3y + 2z \,„\, 60$) oe must be four terms only with integer coefficients

a4B1: CAO ($x + y + z \ldots 18$) oe must be four terms only with integer coefficients

b1B1: CAO but give bod if intention is correct. Some correct examples include:

- (Martin makes) apple (cakes) and chocolate (cakes) in the ratio of 2:1
- For every one chocolate (cake that Martin makes he has to) make 2 apple (cakes)
- (Martin should make) twice as many apple (cakes) as chocolate (cakes)
- The number of apple cakes (that Martin makes should be) double the number of chocolate cakes (he makes)

Please check these carefully for those candidates that imply incorrectly that Martin should make two chocolate cakes for every one apple cake. Furthermore, do not condone an answer that implies an inequality (e.g. use of words such as, 'at least', 'at most', etc.)

The lines in (c) must define the correct FR **and if extended** would pass within a small square of their point of intersection with the axes

c1B1: Any two lines correctly drawn
c2B1: Any three lines correctly drawn
c3B1: All four lines correctly drawn
c4B1: Correct *R* labelled – not just implied by shading – dependent on scoring the first three marks in this part **and all four lines being drawn from axis to axis (within one small square)**

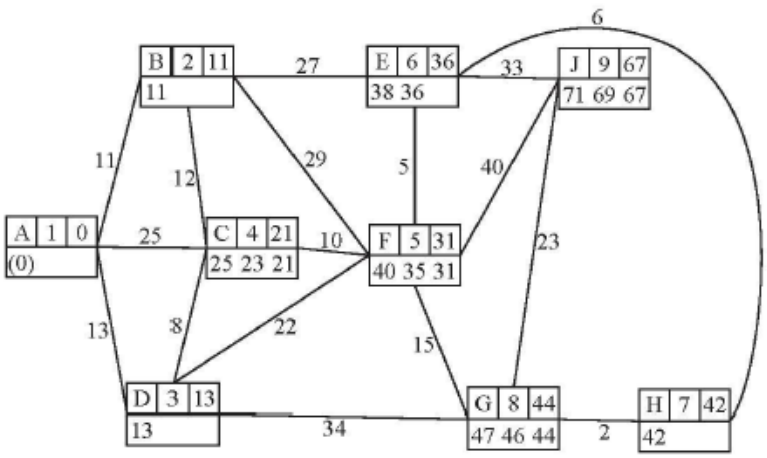
d1M1: Drawing the correct objective line (gradient –0.5) or its reciprocal (gradient –2) on the graph. Line must be correct to within one small square if extended from axis to axis. If line is shorter than (0, 1) to (2, 0) (or for the reciprocal (0, 2) to (1, 0)) then M0
d1A1: Correct objective line – same condition that the line must be correct to within one small square if extended from axis to axis and be no shorter than the line from (0, 1) to (2, 0)

The final 4 marks are all dependent on the first three B marks in (c), the first two marks in part (d) and they must not have implied an incorrect *R* in (c) (but give bod if region not labelled in (c) or if the lines did not go axis to axis in (c) as this was penalised with the final mark in (c))

d2A1: CAO (in context) – as a minimuim accept 9 carrot, 6 apple and 3 chocolate
d3A1: CAO (6300 or 6.3) – no units required but if stated then must be correct – so 6300 kg is A0

e1B1: CAO (1525 (grams) or 1.525 (kg) of flour) – no units required but if stated then must be correct – so 1525 kg is A0
e2B1: CAO 4 (eggs)

Q44.

Question Number	Scheme	Marks
(a)(i)	 Shortest path from A to J: ADCFEHGJ	M1 A1 (ABDC) A1 (FE) A1ft (HGJ) A1
(a)(ii)	Length of shortest path from A to J: 67 (miles)	A1ft **(6)**
(b)	AC + EJ = A(D)C + E(HG)J = 21 + 31 = 52	M1 A1
	AE + CJ = A(DCF)E + C(FEHG)J = 36 + 46 = 82	
	AJ + CE = A(DCFEHG)J + C(F)E = 67 + 15 = 82	A1
	Route length is 315 + 52 = 367 (miles)	A1ft **(4)**
(c)	Pass through G a total of 3 times	B1 **(1)**
(d)	Difference in inspection routes is 67 – 52 = 15 (miles)	B1ft **(1)**
(e)	Arcs CF and EF do not need to be repeated	B1 **(1)**
		13 marks
	Notes for Question	
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 40 35 31 in that order (so 40 31 35 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**		

a1M1: A larger value replaced by a smaller value at least twice in the working values at either C, E, F, G or J
a1A1: All values at A, B, D and C correct and the working values in the correct order
a2A1: All values at F and E correct and the working values in the correct order
a3A1ft: All values in H, G and J correct on the follow through and the working values in the correct order.
To follow through G say check that the working value(s) at G follow from the candidate's final values for the nodes that are directly attached to G (which are D, F, H and J). For example, **if** correct then the order of labelling of nodes D, F and H are 3, 5 and 7 respectively so the working values at G should come from D, F and H in that order. The first working value at G should be their 23 (the Final value at D) + 34 (the weight of the arc DG), the second working value at G should be their 31 (the Final value at F) + 15 (the weight of the arc FG) and the third working value at G should be their 42 (the Final value at H) + 2 (the weight of arc GH). Repeat this exact process for H and J for the follow through for this mark
a4A1: Correct shortest path from A to J (ADCFEHGJ) **only** – not from J to A
a5A1: Follow through their final value at J **only** (condone lack of units) – if their answer is 67 but this is not their Final Value at J then **A0**

Condone for the final two marks in (a) the 'shortest path' and 'length of shortest path' written on the wrong lines

b1M1: Three distinct pairings of nodes A, C, E and J
b1A1: Any one row correct including pairing **and** total
b2A1: All three rows correct including pairings **and** totals
b3A1ft: Correct route length (367) from the correct pairing or follow through 315 + their least total from a choice of three

c1B1: CAO (3)

d1B1ft: Correct answer of 15 either from correct or no incorrect working **or** follow through (their Final value at J from (a) – their least repeat from (b)) – **this mark is dependent on having scored both M marks in (a) and (b)**

e1B1: CAO (CF, EF only)

Q45.

(a)	Kruskal: AB(6), BP(10), CW(11), CP(12), HM(14), AH(15), reject CH(17), reject AC(18), reject AP(20), reject MW(21), LY(21), AS(26), LS(28) (not BS, LM, HL, SY, AL)	M1 A1 A1 **(3)**
(b)	Prim: AB, BP, CP, CW, AH, HM, AS, LS, LY	M1 A1 A1 **(3)**
(c)	143 (miles)	B1 **(1)**
(d)	286 (miles)	B1ft **(1)**
(e)	NNA starting at W: W – C – P – B – A – H – M – L – Y – S – W 11 + 12 + 10 + 6 + 15 + 14 + 40 + 21 + 48 + 55 = 232	M1 A1 **(2)**
(f)	The best upper bound is the one starting at Y as 212 is less than both 232 and 286	B1 **(1)**
(g)	(143 – 11) + 11 + 21 = 164 (miles)	M1 A1 **(2)**
(h)	WCPBAHMLYSACW	B1 **(1)**
		14 marks

Notes for Question
a1M1: First four arcs (AB, BP, CW, CP) correctly chosen and at least one rejection seen at some point **a1A1:** All arcs in tree selected correctly and in the correct order (AB, BP, CW, CP, HM, AH, LY, AS, LS) – no other arcs in MST **a2A1:** cso including all rejections correct and at the correct time – **note that LY can be accepted before MW is rejected**. We do not need to see the explicit rejection of arcs BS to AL but if these are explicitly rejected then they must be in the correct order. Note that a list of all the arcs in the correct order followed by a list of the arcs in the MST can score full marks **b1M1:** First three arcs correctly chosen in order (AB, BP, CP,…) **or** first four nodes {A, B, P, C, …} correctly chosen in order. If any **explicit** rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, 2, 4, - , - , -, 3, -, -, -} so do check carefully for this. Starting at any other node can score M1 only for first three arcs chosen correctly **b1A1:** First six arcs correctly chosen in order (AB, BP, CP, CW, AH, HM,…) **or** all ten nodes {A, B, P, C, W, H, M, S, L, Y} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 2, 4, 6, 9, 7, 3, 8, 5, 10} (**no** missing numbers) **b2A1:** cso – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen) **c1B1:** cao (143) – this mark can be awarded if seen in (b) (although if answered in (c) too then mark according to the answer given in (c)) **d1B1ft:** Follow through double their answer from (c) **e1M1:** Nearest neighbour route starting at W – must have at least W – C – P – B – A – H – … allow if stated in terms of arcs **e1A1:** CAO on length (232) **and** route (must return to W and can be stated in terms of arcs) **f1B1:** An indication that 212 is the minimum (of 212 and the answers to (d) and (e)) – this mark is dependent on the correct values in (d) and (e) so accept an answer of the form 'the one starting at Y (or the route with weight 212) as it is the least' – we do not need to see explicit mention of the values in (d) and (e) provided they are correct in (d) and (e) **g1M1:** (weight of their MST from (c) **or** (b) **or** 132 **only**) – 11 + 11(WC) + 21(MW) (oe so may not see the –11 + 11). A correct answer of 164 can imply this (and the next) mark **g1A1:** 164 **h1B1:** cao – either the route must be written out in full (in terms of nodes or arcs) **or** they must make it absolutely clear that the route begins exactly as in (e) (which must therefore be correct) but after S, towns A and C are visited before (returning to) W. Just stating that A, C, W are visited twice (or similar) is B0

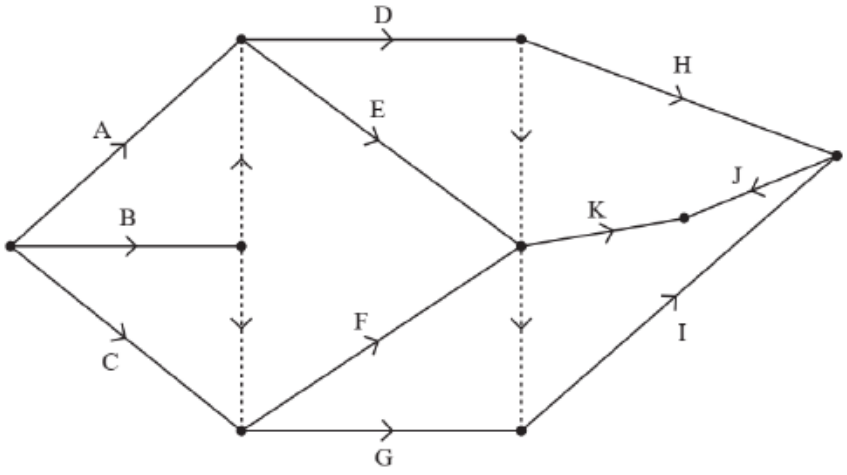
Q46.

Question Number	Scheme	Marks
(a)	Prim: AH, FH, EH, FG, DG, CG, BC	M1 A1 A1 **(3)**
(b)	Initial upper bound 2(201) = 402 (km)	B1ft **(1)**
(c)	A – H – F – G – D – B – C – E – A 27 + 28 + 31 + 29 + 32 + 26 + 38 + 37 = 248	M1 A1 **(2)**
(d)	Nearest neighbour starting at E has a length of $212 + x$ As $x \leqslant 35 \Rightarrow$ the NN route starting at E is at most 247 (km) and therefore the NN starting at E gives the better upper bound as it is less than the one starting at A (which was 248 (km))	M1 A1 **(2)**
(e)	Lower bound is given by $(201-27)+27+x=235$	M1
	$x = 34$	A1
	$235 \leqslant$ optimal length $\leqslant 246$	M1 A1 **(4)**
		12 marks

Notes for Question
a1M1: Prim's – first three arcs correctly chosen in order (AH, FH, EH, …) **or** first four nodes {A, H, F, E, …} correctly chosen in order. If any explicit rejections seen at any point then **M1** (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, -, - , 4 , 3, -, 2} so check there too. Starting at any other node apart from A can score M1 only for first three arcs chosen correctly **a1A1:** First five arcs correctly chosen in order (AH, FH, EH, FG, DG, …) **or** all eight nodes {A, H, F, E, G, D, C, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 7, 6, 4, 3, 5, 2} (**no** missing numbers) **a2A1:** CSO – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen) **b1B1ft:** Follow through double the stated length of their MST **c1M1:** NN starting at A – must have at least A – H – F – G – D –… allow if stated in terms of arcs **c2A1:** CAO on length (248) **and** route (must return to A but can be stated in terms of arcs) **d1M1:** Calculating the correct length of the NN route starting at E ($212 + x$) **and** attempting to use the range of values for x to determining the better upper bound (implied by 247 seen or [244, 247]). **d1A1:** Correct best upper bound stated (the one starting at E) together with a correct comparison of **248** (possibly implicit - if this value is not explicitly stated in (d) then 248 must have been seen in (c)) with 247 (or an indication of 'at most' 247). **For those who obtained an answer of 211 in (c) and say that $212 + x$ is always bigger (without using the given interval for x to find the UB) then no marks in this part** **e1M1:** Correct method for calculating x (which is the weight of MST from (a)/(b) – 27 + two smallest arcs incident to A (the 27 and x) equal to 235). If using the doubled value from (b) then **M0**. If not using the weight of the MST from (a) then they must be using either 174 **or** 26 + 30 + 29 + 31 + 28 + 30 **or** explicitly using the correct six arcs only (BC, CG, GD, GF, FH, HE) so not just circled in one of the tables. The correct value of x (with either no working or no incorrect working) clearly stated can imply this mark **e1A1:** CAO for x (34) – **as a minimum** must have seen the calculation $201 + x = 235$ to award this mark **e2M1:** Any indication of an interval from 235 to either 246, 247 or 248 (this mark is **not** dependent on the previous M mark)

e2A1: CAO (condone $235 <$ optimal length $\leqslant 246$ and allow equivalent interval notation e.g. (235, 246] or [235, 246]) – this mark is dependent on all previous marks in (e) (so must have found that x equals 34). The correct interval (with no others) with no supporting working scores **M0A0M1A0** The minimum requirement for full marks is: $201 + x = 235 \Rightarrow x = 34 \therefore [235, 246]$ For those who simply state $x = 34$ (only) followed by the correct interval they score **M1A0M1A0**

Q47.

(a)	e.g.  	M1 A1 A1 A1 A1	**(5)**
(b)(i)	24 (hours)	B1	
(b)(ii)	C, F, I and J	B1	
(b)(iii)	Total float for G is 3 (hours) Total float for K is 1 (hour)	B1 B1	**(4)**
		9 marks	

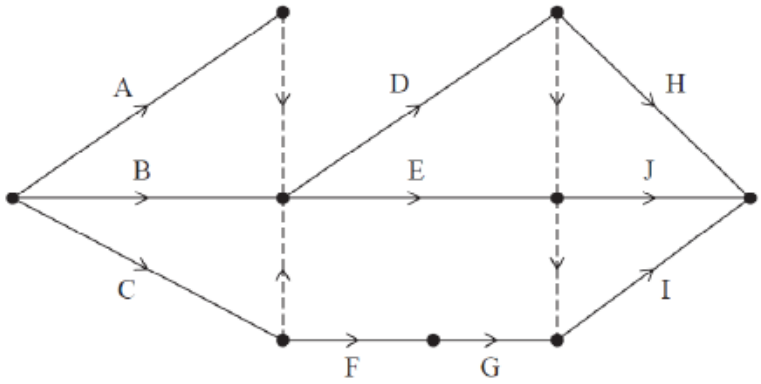
Notes for Question
Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. 'D dealt with correctly' requires the correct precedences for this activity, i.e. A and B labelled correctly and leading into the same node and D starting from that node but do not consider the end event for D. **Activity on node is M0**
If an arc is not labelled, for example, if the arc for activity C is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)
Ignore incorrect or lack of arrows on the activities for the first four marks only
a1M1: At least eight activities (labelled on arc), one start and at least two dummies placed **a1A1:** Activities A, B, C **and** two of activities D, E, F or G dealt with correctly (so at least one dummy (+ correct arrow) required) **a2A1:** Activities D, E, F and G dealt with correctly – so first two dummies (+ correct arrows) are required for this mark **a3A1:** Activities H and I dealt with correctly (so must have the final two dummies + correct arrows) **a4A1:** cso – activities J and K dealt with corrctly. All arrows correctly placed for each activity with one finish and at most four dummies. Note that some candidates are drawing the graph non-planar which is fine **Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only. Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned** **bi1B1:** cao (24) **bii1B1:** cao (C, F, I and J with no others) **biii1B1:** cao (total float for G as 3)

biii2B1: cao (total float for K as 1)

Useful for checking (a):

Activity	A	B	C	D	E	F	G	H	I	J	K
IPA	-	-	-	A, B	A, B	B, C	B, C	D	D, E, F, G	H, I	D, E, F

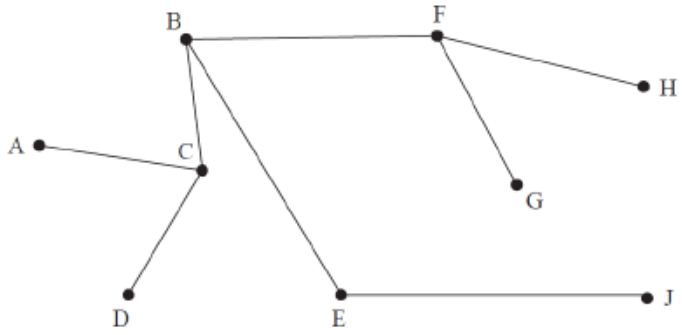
Q48.

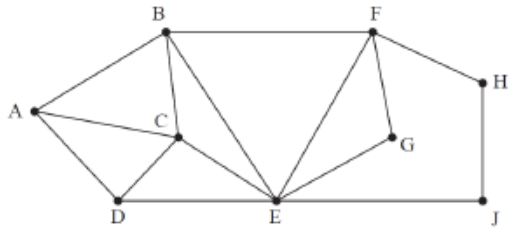
Question Number	Scheme	Marks
	e.g.	M1 A1
	 **Note that this solution is not unique e.g. A and B could be interchanged**	A1 A1 A1 **(5)**
		5 marks

Notes for Question
Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event, e.g. 'D dealt with correctly' requires the correct immediate precedences for this activity, i.e. A, B and C labelled correctly and leading into the same node and D starting from that node but do not consider the end event for D. **Activity on node is M0**
If one arc is not labelled, for example if the arc for activity F is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. **If two or more arcs are not labelled then mark strictly according to the scheme below and therefore no bod as mentioned above.** Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)
Ignore lack of arrows on the activities for the first four marks only
a1M1: Seven activities (labelled on arc), one start and at least two dummies placed **a1A1:** Activities A, B, C, 1st two dummies (including correct arrows on these two dummies) and F dealt with correctly. The first two dummies are those at the end of activities A (or possibly B if A and B are interchanged in their network) and C **a2A1:** D, E, G and H dealt with correctly – this mark can be scored on the bod if the arrows are missing off the first two dummies provided the dummies are in the correct place **a3A1:** I and J dealt with correctly – so this requires the third and fourth dummies (including correct arrows) **a4A1:** cso – all arrows correctly placed for each activity with one finish and at most four dummies (so must have scored the first four marks) **Please check all arcs carefully for arrows – if no dummy has an arrow then M1 only** **Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned** **For reference in checking immediately preceding activities (for 'dealt with correctly')**

A	B	C	D	E	F	G	H	I	J
-	-	-	A, B, C	A, B, C	C	F	D	D, E, G	D, E

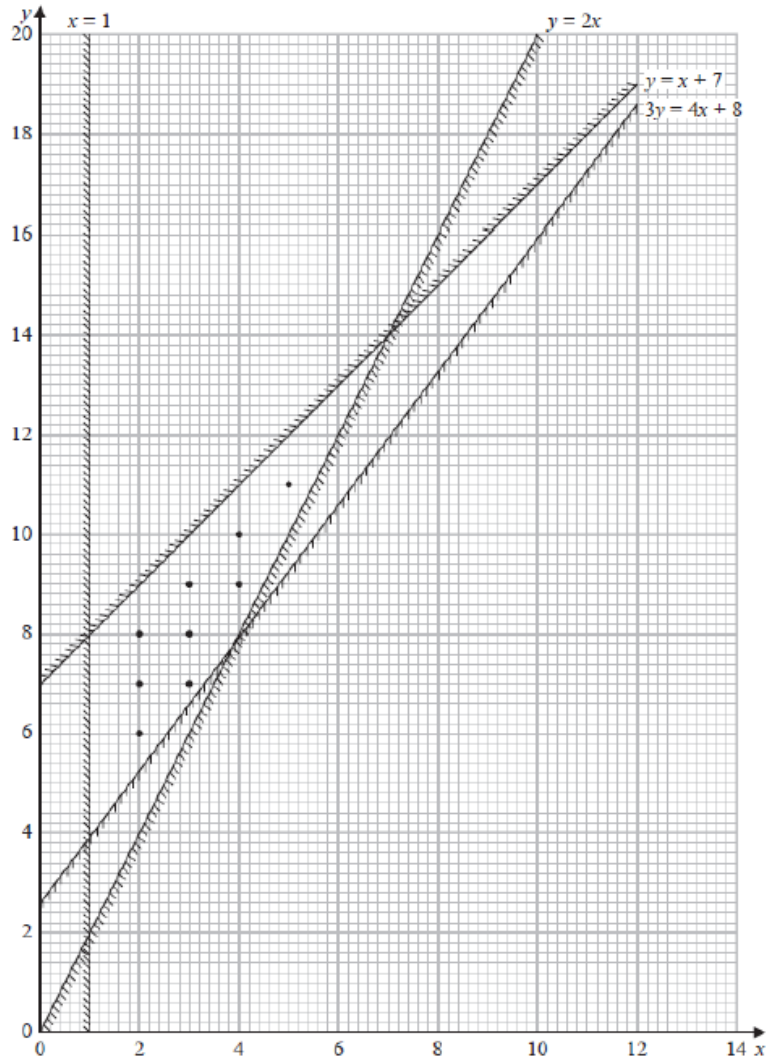
Q49.

Question Number	Scheme	Marks
(a)	e.g. A – B – F – H – J	B1 **(1)**
(b)	A – B – C – D – E – G – F – H – J is not an example of a tour on T as although it contains every vertex it does not return to A	B1 **(1)**
(c)	Kruskal: AC(9), BE(11), BF(12), not EF(14), FG(15), FH(17), not EG(18), EJ(20), {not HJ (21) / BC (21)}, not CE(23), not AB(24), CD(25) (not DE, AD)	M1 A1 A1 **(3)**
(d)		B1 **(1)**
(e)	130 (km)	B1 **(1)**
		7 marks

Notes for Question
a1B1: Any correct example of a path (so no vertex appearing more than once) from A to J **b1B1:** No with a correct reason i.e. must mention that a tour must begin and start at the same vertex, e.g. the route given does not finish at A scores B1 **c1M1:** Kruskal's algorithm - first four arcs (AC, BE, BF, FG) correctly chosen **and** at least one rejection seen at some point (the rejection need not be correct or at the correct time) **c1A1:** All arcs in tree selected correctly in the correct order (AC, BE, BF, FG, FH, EJ, BC, CD) with no additional arcs included in MST **c2A1:** cso - including all rejections correct and at the correct time (do not need to see DE and/or AD rejected but if they are rejected then must be after CD has been selected). Note that BC can be included before HJ rejected **d1B1:** cao **e1B1:** cao (130) – no units required/ignore units even if incorrect
For reference: 

Q50.

Question Number	Scheme	Marks
(a)	If CD is not in the tree then AD must be as these are the only two arcs incident to D or the weight of CD is greater than the weight of AD	M1
	$2y+x>3y-7 \Rightarrow y<x+7$ *	A1 (2)
(b)	$4x+1<2y+1 \Rightarrow y>2x$ *	B1
	$4x+1<8x-3 \Rightarrow x>1$	B1
	$4x+1<3y-7 \Rightarrow 3y>4x+8$	B1 (3)
(c)		

		B1 B1 B1 B1 **(4)**

(d)	$(2,6),(2,7),(2,8)$ $(3,7),(3,8),(3,9)$ $(4,9),(4,10)$ $(5,11)$	B1ft B1 **(2)**
(e)	Arcs in the tree are given as AB, AD, BC, CE, EF and GH – the one remaining arc in the MST is EH (because the weight of arcs EG and FG > weight of arc EH)	M1
	Total weight of the MST is therefore $14x+5y-4$	A1
	$14x+5y-4=73$ and testing integer value points inside FR	M1dep

	$x=3$ and $y=7$	A1 **(4)**
		15 marks

Notes for Question
a1M1: Explaining that if CD is not in the tree than AD must be e.g. 'the MST must contain D so if CD is not in the tree then AD is'. Must explicitly mention arc AD for this mark, so as a minimum accept, 'AD must be in the MST' **a1A1:** Correct reasoning and derivation of the given result ($2y+x>3y-7 \Rightarrow y<x+7$) – as the answer is given we must see at least $2y+x>3y-7$ or $3y-7<2y+x$ before the required answer **SC** (Special Case) **in (a)**: $2y+x>3y-7 \Rightarrow y<x+7$ without any explanation given (or if explanation is incorrect) can score M1A0 **b1B1:** CAO – **must see at least** $4x+1<2y+1$ before the given answer of $y>2x$ and not just arc AB < arc AC or $4x<2y$ **b2B1:** CAO ($x>1$) – but allow equivalents, e.g., $x-1>0$, $1<x$, $4x>4$, etc. but must be two terms only **b3B1:** CAO ($3y>4x+8$) – but allow **exact** equivalents e.g. $y>\frac{4}{3}x+\frac{8}{3}$, $4x-3y<-8$, $4x+8-3y<0$, or equivalent but must be three terms only In (c), the lines can be drawn as either dashed or non-dashed lines (or a combination of the two). The lines must be long enough to define the correct feasible region and pass through one small square of the points stated below: $y=2x$ must pass within one small square of (0, 0) and (7, 14) $y=x+7$ must pass within one small square of (0, 7) and (7, 14) $x=1$ must pass within one small square of (1, 0) and (1, 10) $3y=4x+8$ must pass within one small square of $(1,4)$ and $(7,12)$ **c1B1:** Any one line correctly drawn (ignore any shading) **c2B1:** Any two lines correctly drawn (ignore any shading) **c3B1:** Any three lines correctly drawn (ignore any shading) **c4B1:** All four lines correctly drawn and shading which implies the correct region (but region need not be labelled)

d1B1ft: At least 4 pairs of integer coordinates correctly stated for points inside their region. This mark is dependent on scoring at least the first two marks in (c) (so must have drawn at least two lines correctly) and the candidate must have drawn exactly four lines. The region must not be infinite but need not ncessarily be bounded by all four lines. If the candidate's region does not contain 4 integer coordinates then B0. Note that integer points on the lines that define the boundary of the region are not counted as being inside the region (regardless of if the candidate has strict inequalities or not) **d2B1:** All 9 coordinates correct (and no others) – **dependent on all four lines correctly drawn in (c)** **e1M1:** States that the remaining arc (in the MST) is one of either EH, EG or FG (and no others). Only one of these three arcs need to be stated for this mark. Allow this mark for either the expression $(4x+1)+(3y-7)+(2y-2)+(3x)+(x+y)+(6x-2y+3)+X$ or the equation

$(4x+1)+(3y-7)+(2y-2)+(3x)+(x+y)+(6x-2y+3)+X=73$ where $X=y+1$ or $2y+4$ or $5x+1$ (or equivalent equations/expressions). Their expression/equation need not be simplified but for reference (if correct) they are $14x+4y-5+X$ and $14x+4y-5+X=73$ (with the expression for X as before). Note that stating $14x+4y-5$ (or equivalent) and then separately stating one of the expressions for X would imply this mark

e2A1: A correct expression for the weight of the MST either simplified $(14x+5y-4)$ or not $((4x+1)+(3y-7)+(2y-2)+(3x)+(x+y)+(6x-2y+3)+(y+1))$. This mark can be implied if a correct equation is seen, e.g. $14x + 5y = 77$ (or equivalent and again need not be simplified). **If more than one equation or expression seen then they must clearly select the correct expression/equation for this mark** (so stating more than one expression for the weight of the MST is A0)

e1M1dep: This mark is dependent on the first M mark in this part and the first B mark in (d). Setting their linear expression (in x and y) for the weight of the MST equal to 73 and then substituting into this equation at least one integer pair of values of x and y from (d). This mark can also be awarded for substituting at least one integer pair of values of x and y into their linear expression. The correct answers can imply this mark

e1A1: Correct answers only ($x = 3$ **and** $y = 7$) from correct working – do not accept any other answers stated as well but accept as a coordinate (3, 7) – **must have drawn the correct four lines in (c) but need not have stated all nine correct coordinates in (d)**. As a minimum for full marks in (e) the candidate must have stated a correct expression (e.g. $14x + 5y - 4$ or equivalent) or equation (e.g. $14x + 5y = 77$ or equivalent) before then stating the correct answer

The correct answer with no method or working scores no marks in (e)

Q51.

Question Number	Scheme	Marks
(a)	e.g. in the practical problem each vertex must be visited at least once. In the classical problem each vertex must be visited exactly once	B2, 1, 0 (2)
(b)	NNA starting at A: A – B – D – F – C – G – E – A 25 + 24 + 35 + 27 + 29 + 31 + 35 = 206 (km)	B1 B1 (2)
(c)	The better upper bound is the one starting at D as it is smaller	B1dep (1)
(d)(i)	Prim (starting at A): AB, BD, BE, EF, CF	M1 A1
	RMST weight = 25 + 24 + 27 + 28 + 27 = 131	
(d)(ii)	131 + 29 (CG) + 31 (EG) = 191 (km)	M1 A1 (4)
(e)	The better lower bound is the one found by deleting G as this is the larger of the two	B1dep (1)
(f)	191 ≤ optimal distance ≤ 203	B1ft B1 dep (2)
		12 marks

Notes for Question

a1B1: Understands the difference is connected to the number of times each vertex may be visited – condone 'point' (oe) for vertex (must refer to both problems in their answer but not necessarily by name)
a2B1: Correctly identifies which is classical (each node visited 'exactly once' or 'once') and which is practical (each node visited 'at least once' but B0 for 'more than once' oe – it must be clear that for the practical case that a node may be visited more than once but not necessarily more than once). Must use correct language (e.g. vertex or node) but condone singular/plural confusion e.g. vertex for vertices, or poor spelling (in this part a mark of B0B1 is not possible)

b1B1: Correct nearest neighbour route starting at A (must return to A) – possibly stated in terms of arcs e.g. AB, BD, DF, CF, CG, EG, EA
b2B1: CAO (206) on length of route

c1B1dep: CAO dependent on the correct UB in (b) – allow 'yes it is' (as question asks, 'state whether this (an upper bound of 203) is a better upper bound than the answer to (b)') **and** with some indication that this value is smaller than the one in (b) e.g. '203 < 206 so yes it is' scores B1

di1M1: Must be using Prim's algorithm not NNA. First three arcs (or all 6 nodes / or numbers across the top of the matrix) selected correctly. First three arcs are AB, BD, BE, first six nodes are A, B, D, E, F, C and so numbers across the matrix would be 1, 2, 6, 3, 4, 5. Award M1 only for a correct tree with either no working or if starting at a different node than A
di1A1: CAO (order of arc selection clear) – in terms of arcs only for this mark - AB, BD, BE, EF, CF – condone those that state AB, BD, BE, EF, CF, CG, EG **or** AB, BD, BE, EF, CF, EG, CG (these candidates are most likely adding on the two smallest arcs incident to G for the next part of the question)

dii2M1: Adding two least weighted arcs (CG(29) + EG(31)) to the length of their answer from d(i) (where 100 ≤ d(i) ≤ 160) - condone if parts (d)(i) and (d)(ii) are combined together as a single part (d)
dii2A1: CAO (191)

e1B1dep: CAO dependent on the correct LB in (d)(ii) – allow 'no it isn't' (as question asks, 'state whether this (a lower bound of 188) is a better lower bound than the answer to (d)(ii)') and with some indication that this value is smaller than the one in (d)(ii) e.g. '188 < 191 so no it isn't' scores B1

If the candidate's answer to (b) is less than 188 then no marks can be awarded in (f)

f1B1ft: Their numbers correctly used, accept any inequalities or any indication of an interval from

their largest of the two values (188 or d(ii)) to their smallest of the two values (203 or (b))

e.g. condone for B1 only 203 – 191 = 12

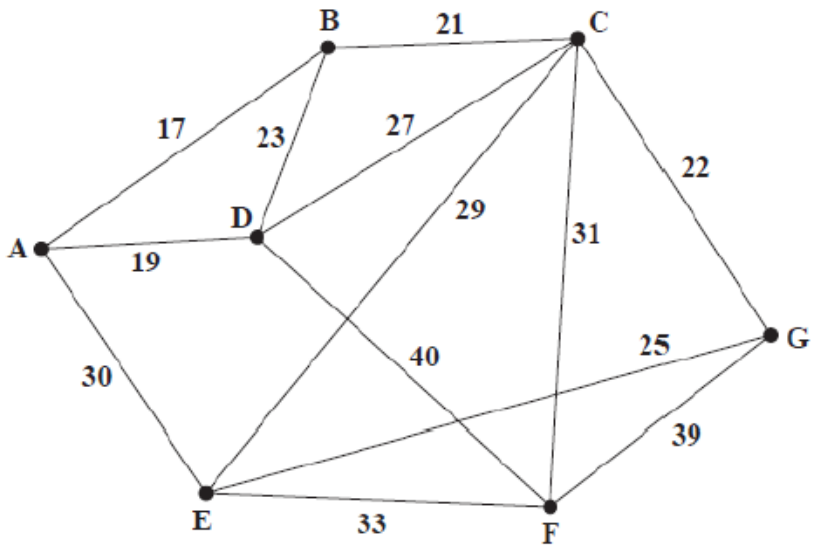
f2B1dep: This mark is dependent on the previous B mark - CAO including correct inequalities (accept either 191 ≤ optimal distance ≤ 203 or 191 < optimal distance ≤ 203) or equivalent notation e.g. [191, 203] or (191, 203]

Q52.

Question Number	Scheme	Marks
(a)	Prim: AE, EG, CE; DG, CF; DH, BF	M1 A1 A1 (3)
(b)	Weight of MST = 197	B1 (1)
(c)	Initial upper bound = 2(197) = 394	B1ft (1)
(d)	A – E – G – D – H – B – F – C – A 23 + 24 + 26 + 33 + 38 + 34 + 32 + 38 = 248	M1 A1
	A – E – G – D – H – F – C – B – A	A1
	23 + 24 + 26 + 33 + 38 + 32 + 35 + 36 = 247	A1 (4)
(e)	247	B1ft (1)
(f)	Weight of RMST is 174	B1ft
	Lower bound = 174 + 23 + 35 = 232	M1 A1 (3)
(g)	232 □ optimal value □ 247	M1 A1 (2)
		15 marks

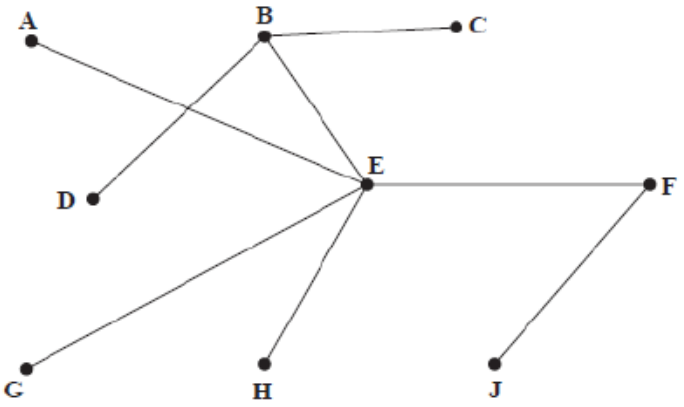
	Notes for Question	
a1M1: Prim's – first three arcs correctly chosen in order (AE, EG, CE, …) **or** first four nodes {A, E, G, C, …} correctly chosen in order. If any explicit rejections seen at some point then M1 (max) only. Order of nodes may be seen at the top of a matrix/table {1, -, 4, - , 2 , -, 3, -}. Starting at any other node can score M1 only for first three arcs chosen correctly **a1A1:** First five arcs correctly chosen in order (AE, EG, CE, DG, CF, …) **or** all eight nodes {A, E, G, C, D, F, H, B} correctly chosen in order. Order of nodes may be seen at the top of a matrix so for the first two marks accept {1, 8, 4, 5, 2, 6, 3, 7} (**no** missing numbers) **a2A1:** cso – all **arcs** correctly **stated** and chosen in the correct order (with no additional arcs). They must be considering arcs for this final mark (do not accept a list of nodes or numbers across the top of the matrix unless the correct list of arcs (in the correct order) is also seen) **b1B1:** cao (197 – ignore units) should come from 23 + 24 + 25 + 26 + 32 + 33 + 34 **c1B1ft:** Follow through double their answer to **(b)** **Mark (d) and (e) together** **d1M1:** Nearest neighbour starting at A with first five nodes correct (A – E – G – D – H –) **d1A1:** One correct route (must return to A) **d2A1:** One correct value **or** both correct routes **d3A1:** Both correct values (do not isw if values doubled) **and** both correct routes (must both return to A) **SC in (d) correct Hamiltonian paths and corresponding weights (AEGDHBFC (210) and AEGDHFCB (211)) scores M1A1A0A0** **e1B1ft:** Follow through their least weight route from **(d)** – must have or imply two Hamiltonian cycles in **(d)** or **(e)** **f1B1ft:** Either 174 **or** 24 + 25 + 26 + 32 + 33 + 34 **or** 197 – 23 **or** the weight of their MST from **(b)** – 23 **f1M1:** Weight of RMST + 23 + 35 (two smallest arcs incident to A) with 151£ RMST £ 197 (if clearly not six arcs in RMST then M0) **f1A1:** cao (232) – if correct answer with no working then awarded B0M1A1 – as a minimum for full marks accept 174 + 23 + 35 = 232 but 174 + 58 = 232 scores B1M1A0 **g1M1:** Any indication of an interval from their answer to **(f)** to their answer to **(e)** with one value correct **g1A1:** cao (either 232£ optimal value £ 247 or 232< optimal value £ 247)	

Q53.

Question	Scheme	Marks
(a)	e.g. accept (i) Every pair of nodes connected by a path (ii) Connected graph with no cycles (iii) All nodes connected	B1 B1 B1
		(3)
(b)	$n - 1$	B1
		(1)
(c)		M1 A1
		(2)
(d)	Kruskal: AB, AD, BC, CG, reject BD, EG, reject CD, reject CE, reject AE, CF	M1 A1 A1
		(3)
(e)	135 (km)	B1
		(1)
		(10 marks)

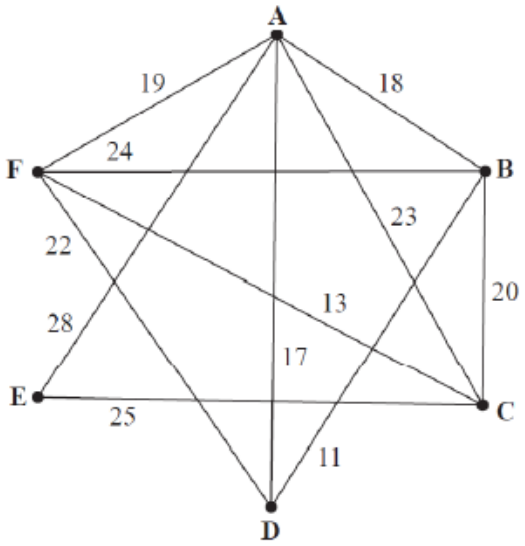
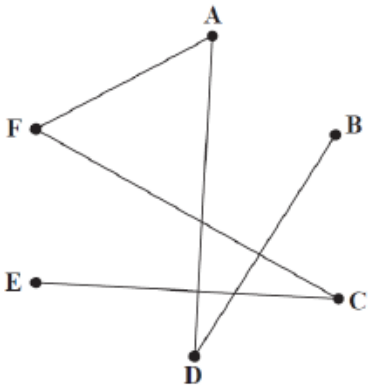
Notes:
(a) **In (a), all technical language used must be correct – for example, do not accept 'point' for node, etc** **(i)B1: every pair** and **path** (or clear definition of path) – no bod - not describing complete graph **(ii)B1: connected** and **no cycles** (not 'loops', 'circles', etc. unless 'cycle' seen as well) **(iii)B1: all nodes** connected (accept definition of minimum spanning tree)
(b) **B1:** cao
(c) **M1:** Either all five arcs correct (ignore weights) or at least three arcs correct (including weights) **A1:** cso (arcs **and** weights) – no additional arcs
(d) **M1:** Kruskal's – first three arcs (AB, AD, BC,… or weights 17, 19, 21, …) chosen correctly **and at least one rejection seen at some point**. **For M1 only**: follow through from their diagram from (c) **A1:** All six arcs (AB, AD, BC, CG, EG, CF or weights 17, 19, 21, 22, 25, 31) chosen correctly and no additional arcs (no follow through from an incorrect network in (c)) **A1:** cso All selections and rejections correct (in correct order and at the correct time) – do not accept weights or a contradiction between arcs and their weights (e.g. AB (16))
B1: cao (ignore lack of units)

Q54.

Question Number	Scheme	Marks
(a)(i)	A tree is a connected graph with no cycles	B1
(ii)	A minimum spanning tree is a tree that contains all vertices The total length of its arcs is as small as possible	B1 B1 (3)
(b)	Kruskal: FJ(11), EG(13), EF(15), EH(17), not GH (18), BC (19), not HJ (20), BD (22), not FH(23), AE (25), BE (29) (not AD, DE, DG, AB, BH)	M1 A1 A1 (3)
(c)		B1
	(Weight of the tree is) 151	B1 (2)
		8 marks

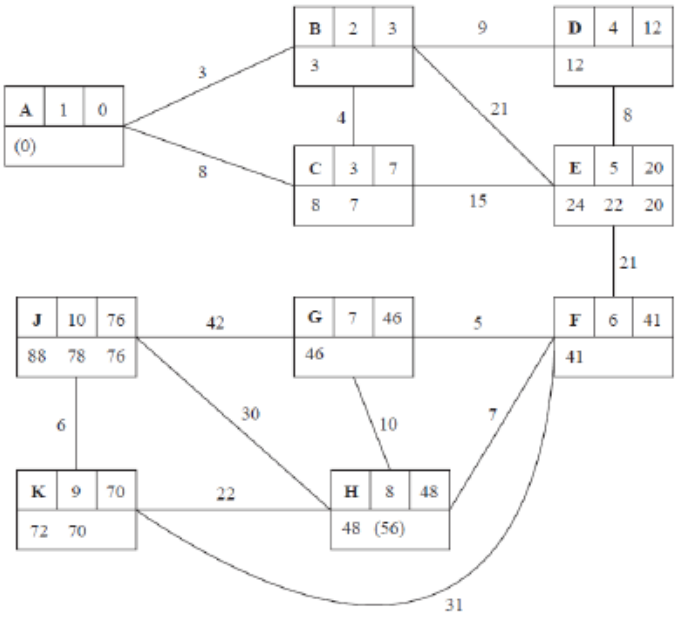
Notes for Question
ai1B1: **Connected + no cycle(s)** (must contain these two points – do not allow 'circle', 'loop' etc. for cycle(s)) – if not using the word 'connected' then allow 'a graph that connects the vertices/nodes' (condone issues with plural or singular e.g. cycle for cycles) **aii2B1**: Contains **all vertices/nodes** (must be clear that all vertices (or nodes) are in a MST) **aii3B1**: Total **length** of arcs is **minimised** (must contain the three points regarding weight/length, arcs/edges and minimised/smallest (oe)) **b1M1**: Kruskal's algorithm - first four arcs (FJ, EG, EF, EH) correctly chosen **and** at least one rejection seen at some point **b1A1**: All arcs in tree selected correctly in the correct order (FJ, EG, EF, EH, BC, BD, AE, BE) with no additional arcs included in MST **b2A1**: CSO including all rejections correct and at the correct time (do not need to see AD, DE, DG, AB, BH rejected but if they are they must be rejected correctly (i.e. in this order) but note that AD, DE have the same weight as do DG and AB so they could appear in either order) Note that stating all the arcs in order (e.g. FJ, EG, EF, EH, GH, BC, HJ, BD, FH, AE, BE, AD, DE, DG, AB, BH) and then stating only those in the tree in the correct order is fine for all three marks in this part **c1B1**: CAO (tree) **c2B1**: CAO (151)

Q55.

Question Number	Scheme	Marks
(a)		M1 A1 (2)
(b)	Kruskal: BD(11), CF(13), AD(17), reject AB(18), AF(19), reject BC(20), reject DF(22), reject AC(23), reject BF(24), CE(25) (not AE)	M1 A1 A1 (3)
(c)		B1
	Weight of MST = 85 (metres)	B1 (2)
		7 marks

Notes for Question
a1M1: At least 8 correct arcs with corresponding correct values **or** all 11 correct arcs **a1A1:** CSO (11 arcs only + correct values) – give bod **b1M1:** Kruskal's: first three arcs (BD, CF, AD) correctly chosen and at least one rejection seen at some point **b1A1:** All arcs in tree selected correctly and in the correct order (BD, CF, AD, AF, CE) – no other arcs in MST **b2A1:** CSO including all rejections correct and at the correct time – AE need not be considered but if AE is considered then it must be rejected after CE has been added to the MST **c1B1:** CAO (tree) **c2B1:** CAO (85)

Q56.

Question Number	Scheme	Marks
(a)	 Route: ABDEFHKJ Length: 76 (km)	M1 A1 (ABCDE) A1 (FGH) A1ft (KJ) A1 A1ft (6)
(b)	Prim: AB, BC; BD, DE	M1; A1 (2)
(c)	Kruskal: FG, JK, FH, not GH, HK, (not HJ), (not FK), (not GJ)	M1; A1 (2)
(d)	Total length: 85 (km)	B1 (1)
		11 marks

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at E the working values must be 24 22 20 in that order (so 24 20 22 is incorrect) **It is also important that the order of labelling is checked carefully – some candidates start with a label of 0 at A (rather than 1) – which is fine. Also the order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling** **a1M1:** A larger value replaced by a smaller value in at least two of the working value boxes at either C or E or J or K **a1A1:** All values in A, B, C, D and E correct and the working values in the correct order at C and E (including order of labelling). Condone lack of 0 in A's working value **a2A1:** All values in F, G and H correct and the working values in the correct order. Penalise order of labelling only once per question (F, G and H must be labelled in that order and F must be labelled after A, B, C, D and E). Note that an additional working value of 56 at H after the 48 is not an error so 48 56 is fine, however, any other number or 56 48 in this order is incorrect and scores A0 in this part **a3A1ft:** All values in K and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through K check that the working value at K follows from the candidate's final values from their feeds into K (which will come from nodes F, H and possibly even J (in the order in which the candidate has labelled them)) and that the final value, and order of

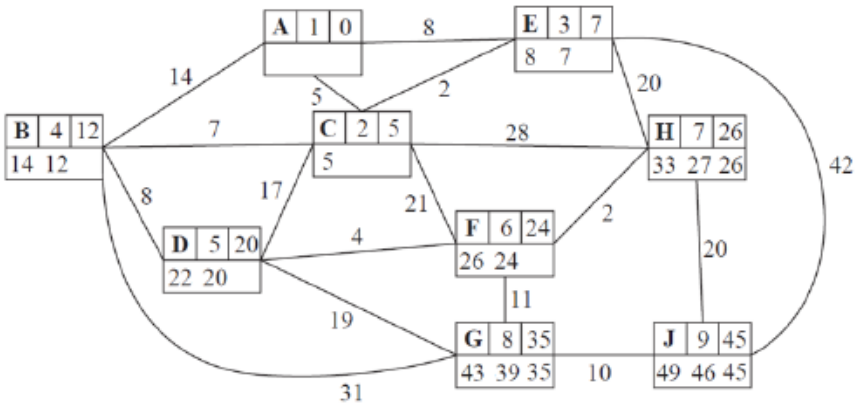
labelling, follows through correctly. Repeat this process for J (which will possibly have working values from G, H and K with the order of these values determined by the candidate's order of labelling at G, H and K)
a4A1: CAO - correct route (ABDEFHKJ) not from J to A
a5A1ft: Follow through on their final value at J only (so if 76 given as the answer and the final value at J is not 76 then A0)

b1M1: First two arcs (AB, BC) chosen correctly in order, or first three nodes (ABC) chosen correctly in order. If any rejections seen at any point, or just a list of **all** the arcs in order, or only a list of weights then M0 (condone for M1only those who find the MST for the entire network)
b1A1: CSO (must be considering arcs so must be AB, BC, BD, DE or BA, BC, etc.) – do not isw if candidates continue and find the MST for the entire network

c1M1: First two arcs (FG, JK) chosen correctly in order **and at least one rejection seen at some point** – no marks in this part if candidates apply Kruskal to the entire network or if only a list of weights given
c1A1: CSO – all selections and rejections correct in the correct order and at the correct time. Note that stating all the arcs in order (e.g. GF, JK, FH, GH, KH, JH, FK, GJ) and then stating only those in the tree in the corret order is fine for both marks in this part

d1B1: CAO (85)

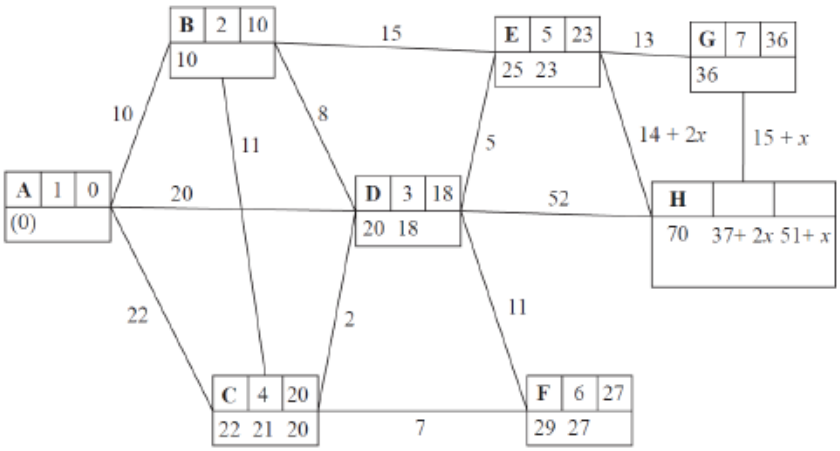
Q57.

Question Number	Scheme	Marks
(a)	 Shortest time: 45 (minutes) Quickest route: A C B D F G J	M1 A1 (CEBD) A1 (FH) A1ft (GJ) A1ft A1 (6)
(b)	A(CB)D + HJ = 20 + 20 = 40 A(CBDF)H + D(FG)J = 26 + 25 = 51 A(CBDFG)J + D(F)H = 45 + 6 = 51 Repeated arcs: AC, BC, BD, HJ	M1 A1 A1 A1 A1 (5)
(c)(i) (ii)	Vertex C: 4 times Vertex D: 3 times	B1 B1 (2)
(d)	H to H requires the consideration of the shortest path from A to J only (as these are the only two odd nodes) 314 > 309 or 45 > 40 so quicker to start at H and finish at D	B1 B1 (2)
		15 marks

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at J the working values must be 49 46 45 in that order (so 49 45 46 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling** **a1M1**: A larger value replaced by a smaller value in at least two of the working value boxes at any node except A or C **a1A1**: All values at C, E, B and D correct and the working values in the correct order (including order of labelling) **a2A1**: All values at F and H correct and the working values in the correct order. Penalise order of labelling only once per question (F and H must be labelled in that order and F must be labelled after C, E, B and D) **a3A1ft:** All values in G and J correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through G check that the working value at G follows from the candidate's final values from their feeds into G (which will mostly likely come from nodes B, D and F (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for J (which will possibly have working values from E, H and G with the order of these values determined by the candidate's order of labelling at E, H and G)

Question Number	Scheme	Marks
a4A1ft: Follow through on their final value at J **only** (condone lack of units) - so if 45 given as the answer and the final value at J is not 45 then A0 **a5A1**: CAO - correct route (from either A to J or J to A) – ACBDFGJ or JGFDBCA **b1M1**: Correct three distinct pairings of the correct four odd nodes A, D, H and J **b1A1**: **Any** row correct including pairing **and** total **b2A1**: **Any** two rows correct including pairings **and** totals **b3A1**: **All** three rows correct including pairings **and** totals **b4A1**: CAO correct edges clearly stated and not just in their working as AC, BC, BD, HJ. Must be these arcs and not AD, ACBD or AD via B and C **ciB1**: CAO (Vertex C: 4) **ciiB1**: CAO (Vertex D: 3) **d1B1**: Correct reasoning (that to travel from H to H) only the shortest path between A and J needs traversing twice – as a minimum must mention either 'A to J only' or refer to A and J being the only odd nodes (e.g. odd nodes: A and J is fine but not 'A and J are odd nodes') **d2B1**: Either 'it will be slower' or 'it will be quicker from H to D' (if saying 'quicker' then it must be clear that they are talking about H to D) + correct numerical argument (not just stating the values 314 and 309 (or 45 and 40) and saying 'slower' - there must be some comparison of these two values)	

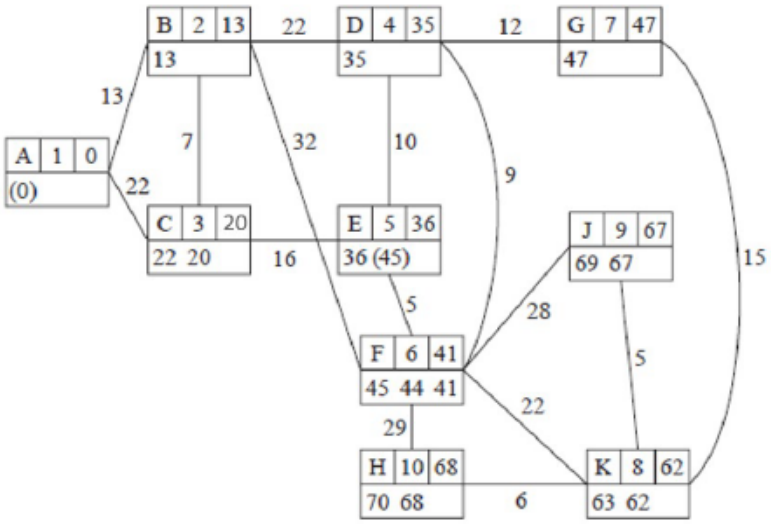
Q58.

Question Number	Scheme	Marks
(a)		M1 A1 A1 A1ft
	Shortest path from A to H via DH: ABDH length: 70	A1
	Shortest path from A to H via EH: ABDEH length: $37 + 2x$	A1
	Shortest path from A to H via GH: ABDEGH length: $51 + x$	A1 (7)
(b)	A to H are the only two odd nodes in the network so repeat arcs in path ABDEH	M1
	$3x + 205 + 37 + 2x = 307$	M1
	$x = 13$	A1
	Time taken is 63 (minutes)	A1 (4)
		11 marks

Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at C the working values must be 22 21 20 in that order (22 20 21 is incorrect) It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling
a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at any node except A, B, G or H (or once with at least two working values seen at H) **a1A1:** All values in A, B, D and C correct and the working values in the correct order at D and C (including order of labelling) **a2A1:** All values E and F correct and the working values in the correct order. Penalise order of labelling only once per question **a3A1ft:** All values in G and H correct on the follow through and the working values in the correct order (the order at H must be correct but give bod). Penalise order of labelling only once per question. Ignore permanent label and final value at H only. Allow unsimplified expressions in x for the working values at H **a4A1:** ABDH and 70 **a5A1:** ABDEH and $37 + 2x$ **a6A1:** ABDEGH and $51 + x$ If A0A0A0 for the final three marks in **(a)** then award A1A0A0 for all 3 routes stated correctly or all 3 correct values stated explicitly (so not just left in the working values at H)

b1M1: Indication of repeating arcs in a path from A to H. As a minimum: stating A and H as the odd nodes for the network (not just stating A and H) **or** stating a route from A to H with 5 nodes only **or** stating the need to repeat a path/route from A to H - this mark is for making their method clear **b2M1:** $3x + 205 +$ (one of their paths involving x) $= 307$ – this mark is for making their working clear **b1A1:** CAO ($x = 13$) – this mark is dependent on the second M mark only **b2A1:** CAO (63) – this mark is dependent on the second M mark only **SC** If M0M0 then **both** correct answers of $x = 13$ and 63 score M0M0A0A1 only (so treating the final mark as a B mark)

Q59.

Question Number	Scheme	Marks
(a)	 Shortest path: A B D G K H Length: 68 (miles)	M1 A1 (ABCDE) A1 (FGK) A1ft (JH) A1 A1ft **(6)**
(b)	Route from F to K via A: F E C B A B D G K Length: 41 + 62 = 103 (miles)	B1 B1ft **(2)**
(c)	AJ + CE = 67 + 16 = 83 AC + EJ = 20 + 32 = 52 AE + CJ = 36 + 48 = 84 Repeated arcs: AB, BC, EF, FK, JK Length: 253 + 52 = 305 (miles)	M1 A1 A1 A1 A1 A1ft **(6)**
(d)	Vertex F: 4 times	B1 **(1)**
(e)	(Start at D and therefore) finish at D	B1 **(1)**
(f)	Difference = 305 – (253 + 16) = 36 (miles)	B1 **(1)**
		17 marks

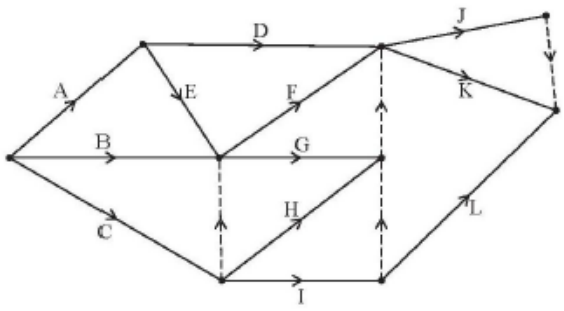
Notes for Question
In (a) it is important that all values at each node are checked very carefully – the order of the working values must be correct for the corresponding A mark to be awarded e.g. at F the working values must be 45 44 41 in that order (so 45 41 44 is incorrect) **It is also important that the order of labelling is checked carefully. The order of labelling must be a strictly increasing sequence – so 1, 2, 3, 3, 4, … will be penalised once (see notes below) but 1, 2, 3, 5, 6, … is fine. Errors in the final values and working values are penalised before errors in the order of labelling**
a1M1: A larger value replaced by a smaller value in at least two of the working value boxes at either C, F, K, J, or H **a1A1**: All values at A, B, C, D and E correct and the working values in the correct order (including order of labelling) – if a working value of 45 appears at E then it must appear after the 36 so therefore 45 36 at E (in this order) is A0 **a2A1**: All values at F, G and K correct and the working values in the correct order (F, G and K must be labelled in that order and F must be labelled after A, B, C, D and E) **a3A1ft**: All values in J and H correct on the follow through and the working values in the correct order. Penalise order of labelling only once per question. To follow through J check that the working value at J follows from the candidate's final values from their feeds into J (which will mostly likely come from nodes F and K (in the order in which the candidate has labelled them)) and that the final value, and order of labelling, follows through correctly. Repeat this process for H (which will possibly have working values from F and K with the order of these values determined by the candidate's order of labelling at F and K) **a4A1**: CAO (ABDGKH or AB, BD, DG, GK, KH) **a5A1ft**: Follow through on their final value at H **only** (condone lack of units) so if answer given as 68 but final value at H is not 68 then A0 **b1B1**: CAO (FECBABDGK or FE, EC, CB, BA, AB, BD, DG, GK) **b2B1ft**: Follow through their final value at F + their final value at K **or** 103 **c1M1**: Correct three distinct pairings of the correct four odd nodes of A, C, E and J **c1A1**: **Any** row correct including pairing **and** total **c2A1**: **Any** two rows correct including pairings **and** totals **c3A1**: **All** three rows correct including pairings **and** totals **c4A1**: CAO correct edges clearly stated and not just in their working as AB, BC, EF, FK and JK – must be these arcs **c5A1ft**: Follow through their value of their smallest pairing total + 253 **d1B1**: CAO (4 only) **e1B1**: CAO (D) **f1B1**: CAO (36)

Q60.

Question Number	Scheme	Marks
(a)	Pair the odd nodes: C, D **or** repeated arcs are CF, FG, DG	B1
	Time = 82 + 7 = 89	B1
	e.g. route GDGJHEADCABEFBCFCGFG	B1 (3)
(b)	BC + DG = B(F)C + DG = 6 + 3 = 9*	M1 A1
	BD + CG = B(FG)D + C(F)G = 11 + 4 = 15	A1
	BG + CD = B(F)G + C(FG)D = 8 + 7 = 15	A1
	Repeat arcs: BF, CF, DG	A1 (5)
(c)	Route starting from G is quicker	B1
	e.g. difference = (82 + 9) – 89 = 2 or 9 – 7 = 2	B1 (2)
		10 marks

Notes for Question
a1B1: cao (correctly stating the two odd nodes **or** correct repeated arcs stated) – so must either state that C and D are odd **or** state the arcs CF, FG, DG only, but B0 if only stating C and D or CD (without mention of 'odd') **a2B1:** cao (89) **a3B1:** Correct route: checks – starts and finishes at G, 20 nodes, CF, FG and DG repeated, A(2), B(2), C(3), D(2), E(2), F(3), G(4), H(1), J(1) **b1M1:** Correct three distinct pairings of the correct four odd nodes B, C, D and G **b1A1:** Any one row correct including pairing **and** total **b2A1:** Any two rows correct including pairings **and** totals **b3A1:** All three rows correct including pairings **and** totals **b4A1:** cao - correct arcs clearly stated and not just in their working as BF, CF and DG – must be these arcs. Do not accept BC, BFC or BC via F **c1B1:** cao (oe e.g. B to G is slower) – dependent on the correct repeats arcs (possibly implied) in (a) and (b) **or** clearly implied in (c) (e.g. correct values compared in this part) – must be clear that it is the route starting at G which is quicker **c2B1:** cao (difference of 2 **or** comparing 89 and 91 **or** comparing 7 with 9)

Q61.

Question Number	Scheme	Marks
(a)	e.g. 	M1 A1 A1 A1 A1 (5)
(b)	Duration of activity K is 33 – 10 – 7 – 8 = 8 **or** the path AEFJ has a duration of < 33	M1
	Therefore the duration of J is less than 8 hours or 0 < dur(J) < 8	A1 (2)
		7 marks

Notes for Question

Condone lack of, or incorrect, numbered events throughout. 'Dealt with correctly' means that the activity starts from the correct event but need not necessarily finish at the correct event and **appears only once in the network**, e.g. 'G dealt with correctly' requires the correct precedences for this activity, i.e. B, C and E labelled correctly and leading into the same node and G starting from that node but do not consider the end event for G **so use the table below for checking as there a number of acceptable answers. Activity on node is M0**

If an arc is not labelled, for example, if the arc for activity E is not labelled (but the arc is present) then this will lose the first A mark and the final (CSO) A mark – they can still earn the second A mark on the bod. If two or more arcs are not labelled then mark according to the scheme. Assume that a solid line is an activity which has not been labelled rather than a dummy (even if in the correct place for where a dummy should be)

Ignore incorrect or lack of arrows on the activities for the first four marks only (but assume that they are in the 'correct' direction for checking purposes)

a1M1: At least seven activities (labelled on arc), one start and at least two dummies placed
a1A1: Activities A, B, C, D and E dealt with correctly
a2A1: Activities F, G, H and I dealt with correctly (so a dummy is required at the end of C + correct arrow)
a3A1: Activities J, K and L dealt with correctly (so **at least** two further dummies required + correct arrows)
a4A1: CSO – all arrows correctly placed for each activity with one finish and at most four dummies.
Please check all arcs carefully for arrows – if there are no arrows on any dummies then M1 only.
Note that additional (but unnecessary) 'correct' dummies that still maintain precedence for the network should only be penalised with the final A mark if earned

Extremely useful for checking (a)

Activity	A	B	C	D	E	F	G	H	I	J	K	L
IPA	-	-	-	A	A	B, C, E	B, C, E	C	C	D, F, G, H, I	D, F, G, H, I	I

b1M1: Either the correct method for calculating the duration of activity K seen (e.g. 33 – 10 – 7 – 8 or an answer of 8 can imply this mark) **or** stating the path AEFJ will have a duration < 33 or $\leqslant 33$

b1A1: Correct indication that the duration of K being 8 implies that J's duration is < 8 (allow $\leqslant 8$ but **A0** for $\leqslant 7$). As a minimum for both marks candidates must either say that the duration of K is 8 (e.g. K = 8) **or** that the path AEFJ has a duration < 33 or $\leqslant 33$, **together** with the duration of J being either < 8 or $\leqslant 8$. A lower limit is not required but if stated then it must be either > 0 or $\geqslant 0$ **only**

Q62.

Question	Scheme	Marks
(a)	Bin 1: 12.1 9.3 10.9 Bin 2: 15.7 6.4 7.9 Bin 3: 17.4 8.1 Bin 4: 20.1 Bin 5: 14.0	M1 A1 A1
		(3)
(b)	(i) 12.1 15.7 10.9 17.4 9.3 20.1 7.9 8.1 14.0 6.4 15.7 12.1 17.4 10.9 20.1 9.3 8.1 14.0 7.9 6.4	M1 A1
	(ii) Comparisons = 9 + 8 = 17 Swaps = 7 + 5 = 12	B1 B1
		(4)
(c)	e.g. middle right 12. 1 9.3 15.7 10.9 17.4 6.4 20.1 7.9 8.1 14.0 Pivot 6.4 12.1 9.3 15.7 10.9 17.4 20.1 7.9 8.1 14.0 6.4 Pivot 17.4 20.1 17.4 12.1 9.3 15.7 10.9 7.9 8.1 14.0 6.4 Pivot (20.1) 10.9 20.1 17.4 12.1 15.7 14.0 10.9 9.3 7.9 8.1 6.4 Pivots 15.7 7.9 20.1 17.4 15.7 12.1 14.0 10.9 9.3 8.1 7.9 6.4 Pivots 14.0 8.1 20.1 17.4 15.7 14.0 12.1 10.9 9.3 8.1 7.9 6.4 Sort complete	M1 (quick) A1 (1st/2nd passes/pivot for 3rd) A1ft (3rd/4th passes/pivot for 5th) A1(cso + 'sort complete')
		(4)

(d)	Bin 1: 20.1 12.1 Bin 2: 17.4 14.0 Bin 3: 15.7 10.9 6.4 Bin 4: 9.3 8.1 7.9	M1 A1 A1
		(3)
(e)	e.g. $\frac{121.9}{33} \approx 3.694$ so yes 4 bins is optimal	B1ft
		(1)
		(15 marks)

Notes:

(a)

M1: First four numbers placed correctly (therefore Bin 1 correct and 15.7 in Bin 2) and at least seven numbers put in bins – condone cumulative totals here only

A1: First eight numbers placed correctly (therefore Bins 1 and 2 correct and 17.4 in Bin 3 and 20.1 in Bin 4)

A1: cso All correct

(b)

(i)M1: Bubble sort – first pass correct

(i)A1: cao both passes correct (ignore additional passes)

(ii)B1: cao on total number of comparisons

(ii)B1: cao on total number of swaps

SC in b(ii): If B0B0, award B1B0 if correct numbers referred to but not summed

(c)

M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – **choosing first/last item as pivot is M0**) and first pass gives >p, p, <p. So after the first pass the list should read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration M1 only**

A1: First and second passes correct **and** next pivot(s) chosen correctly for third pass (but third pass does not need to be correct)

A1ft: Third and fourth passes correct (follow through from their second pass and choice of pivots) – **and** next pivot(s) chosen correctly for the fifth pass

A1: cso (correct solution only – all previous marks in this part **must** have been awarded) including 'sort complete' – this could be shown by the final list being re-written or 'sorted' statement or each item being used (**not** just stated) as a pivot

(d)

M1: **Must be using 'sorted' list** in decreasing order (independent of (c)). First four numbers placed correctly and at least seven numbers put in bins – condone cumulative totals here only. First-fit increasing is M0

A1: First eight numbers placed correctly

A1: cso – all correct

SC for (d): if the 'sorted' list they use in (d) has one 'error' from (c) (e.g. a missing number, an extra number or one number incorrectly placed) then M1 only can be awarded in (d) (for the first four numbers). If there is more than one 'error' then M0. Allow full marks in (d) if a correct list is used in (d) even if the list is incorrect at the end of (c).

(e)

B1ft: $\frac{121.9}{33}$ **or** awrt 3.7 (**or** 3.6 with correct calculation seen) **and** 4 together with a correct conclusion based on their answer to (d) (a correct calcuation etc. with an answer of 4 with no conclusion (as a minimum accept 'yes') scores B0)

middle left

12. 1 9.3 15.7 10.9 [17.4] 6.4 20.1 7.9 8.1 14.0	Pivot 17.4
20.1 17.4 12.1 9.3 15.7 [10.9] 6.4 7.9 8.1 14.0	Pivot (20.1) 10.9
20.1 17.4 12.1 [15.7] 14.0 10.9 9.3 [6.4] 7.9 8.1	Pivots 15.7 6.4
20.1 17.4 15.7 [12.1] 14.0 10.9 9.3 [7.9] 8.1 6.4	Pivots 12.1 7.9
20.1 17.4 15.7 14.0 12.1 10.9 [9.3] 8.1 7.9 6.4	Pivot (14.0) 9.3
20.1 17.4 15.7 14.0 12.1 10.9 9.3 8.1 7.9 6.4	(sort complete (8.1))

Q63.

Question Number	Scheme	Marks
(a)	Bin 1: **35** **17** 7 Bin 2: **10** 28 15 Bin 3: 23 20 Bin 4: 41 Bin 5: 29	M1 A1 A1 **(3)**
(b)	e.g. middle right	Pivot(s)
	35 17 10 7 28 23 41 15 20 29 — 23 35 28 41 29 **23** 17 10 7 15 20 — 41, 7 **41** 35 28 29 **23** 17 10 15 20 7 — 28, 15 **41** 35 29 **28** **23** 17 20 **15** 10 7 — 29, 20, (10) **41** 35 **29** **28** **23** **20** 17 **15** **10** 7 — Sort complete	M1 A1 A1ft A1 **(4)**
(c)	Bin 1: **41** 17 Bin 2: **35** **23** Bin 3: **29** **28** Bin 4: 20 15 10 7	M1 A1 A1 **(3)**
(d)	$8 < x < 12, \quad y > x, \quad (y > 8)$	B3, 2, 1, 0 **(3)**
		13 marks

Notes for Question

PLEASE NOTE NO MISREADS IN THIS QUESTION – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASES IN PART (b) and the guidance for the M mark in (c)

a1M1: First four items placed correctly (the values in bold) and at least seven values placed in bins. Condone cumulative totals for M1 only – if one of the bold values appears in more than one bin then M0
a1A1: First eight items placed correctly (the underlined and bold values) – if one of the underlined values appears in two different bins then this is A0
a2A1: CSO (correct solution only – so no additional/repeated values)

b1M1: Quick sort, pivot, p, chosen (must be choosing middle left or right – choosing first/last item as the pivot is M0). After the first pass the list must read (values greater than the pivot), pivot, (values less than the pivot). **If only choosing one pivot per iteration then M1 only**
b1A1: First two passes correct
b2A1ft: Third pass correct (follow through from their second pass and choice of pivots for the third pass (these pivots for the third pass must be either middle left or middle right)
b3A1: CSO (correct solution only – all previous marks in this part **must** have been awarded) including a 'sort complete' - this could be shown by the final list being re-written or 'sorted' statement or each item being used as a pivot (which would therefore mean that the final list would have been written twice)

middle left

35	17	10	7	28	23	41	15	20	29	28
35	41	29	**28**	17	10	7	23	15	20	41, 7
41	35	29	**28**	17	10	23	15	20	7	35, 23
41	**35**	29	**28**	**23**	17	10	15	20	7	(29), 10
41	**35**	29	**28**	**23**	17	15	20	**10**	7	15
41	**35**	29	**28**	**23**	17	20	**15**	**10**	7	17
41	**35**	29	**28**	**23**	20	**17**	**15**	**10**	7	Sort complete

Question Number	Scheme	Marks

SC for (b): If using an incorrect list from the start of (b) with only one error (an error is either one missing number, one extra number, two numbers transposed or one incorrect number) then the most they can score is M1A0A1ftA0

Sorting list into ascending order in (b)

- If the candidate sorts the list into ascending order and reverses the list in this part then this can score full marks in (b)
- If the list is not reversed in (b) then remove the last two A marks earned in (b). If the list is reversed at the start of (c) but not in (b) then still remove the last two A marks earned in (b). If the list is in ascending order in (b) award no marks for first-fit increasing in (c). If the candidate says that the list needs reversing in (b) but does not actually show the reversed list in (b) then remove the last A mark earned

c1M1: **Must be using a list** that is in strictly descending order. If it is clear that their list is not in descending order then M0. First five items placed correctly (the bold values) and at least eight values placed in bins – condone cumulative totals for M1 only – if one of the bold values appears in more than one bin then M0. If it is clear that their list is not correct (41 35 29 28 23 20 17 15 10 7) then M1 only and for this M mark allow them to be using their final list from (b) which can contain one error and in this case an error is either one missing number or one extra number or one incorrect number (e.g. 24 for 23) but their list must be in descending order.
c1A1: First seven items placed correctly (the underlined and bold values) – if one of the underlined values appears in two different bins then this is A0
c2A1: CSO (so no additional/repeated values)

d1B1: $x > 8$
d2B1: $x < 12$

If B0 B0 then award B1 B0 for $8 \leq x \leq 12$ (oe)

d3B1: $x < y$ but not $y > 8$ only

For full marks in (d) no additional incorrect constraints, for example, if B1B1B1 initially awarded but an additional incorrect constraint seen (e.g. $y > 15$) then award B1B1B0 but do not penalise any additional incorrect constraints unless all three other correct constraints seen

Q64.

Question Number	Scheme	Marks
(a)(i)	In the first pass of a bubble sort we compare the first value with the second and swap if the first is larger than the second. We then compare the value that is second with the third value and swap if the second is larger than the third. Continue like this until the end of the list.	M1 A1
(a)(ii)	Bubble sort stops when we either have a list of length 1 to sort or we have a pass in which no swaps were made.	B1 B1 (4)
(b)	Maximum number of passes is 3 as only the three largest values are in the correct position.	B1 B1dep (2)
(c)	0.9 1.2 1.5 0.5 1.4 1.1 0.7 1.7 2.2 3.2 0.9 1.2 0.5 1.4 1.1 0.7 **1.5** 1.7 2.2 3.2 0.9 0.5 1.2 1.1 0.7 **1.4** **1.5** 1.7 2.2 3.2 0.5 0.9 1.1 0.7 **1.2** **1.4** **1.5** 1.7 2.2 3.2 0.5 0.9 0.7 **1.1** **1.2** **1.4** **1.5** 1.7 2.2 3.2 0.5 0.7 **0.9** **1.1** **1.2** **1.4** **1.5** 1.7 2.2 3.2 0.5 **0.7** **0.9** **1.1** **1.2** **1.4** **1.5** 1.7 2.2 3.2	M1 A1 A1ft A1cso (4)
(d)	Bin 1: 3.2 0.7 Bin 2: 2.2 1.7 Bin 3: 1.5 1.4 1.1 Bin 4: 1.2 0.9 0.5	M1 A1 A1(3)
		13 marks

Notes for Question
ai1M1: Compare first value with second value (allow 'compare first and second') and swap if first is larger (oe) - allow 'wrong order' for M1 only – must be clear that the first value is being compared with the second value **ai1A1:** Compare second with third (not just 'next two') and so on until the end of the list (oe e.g. 'all', 'last two', 'last') – must be clear that **all** the list of numbers has been considered **aii1B1:** CAO – one of 'until only one item left' (oe e.g. 'stops after $n-1$ passes' but not just a statement along the lines of 'until all the required passes have been done') or 'until no swaps' (oe e.g. 'one pass gives the same result as the next pass') – no marks though if they only say 'when the list is in order' (oe) **aii2B1:** CAO – both reasons stated correctly **b1B1:** CAO (3) **b2B1dep:** Correct reasoning – dependent on previous B mark – must mention that the three largest numbers are in the correct position (and not just that 1.7, 2.2 and 3.2 are in the correct position) **c1M1:** Bubble sort. Consistent direction throughout sort, and first pass correct – do check these carefully as some candidates show the result of each comparison and swap in their first pass. No marks for quick sort or descending order **c1A1:** Second and third passes correct – so end six numbers in place after third pass **c2A1ft:** Their fourth and fifth passes correct following through from the candidate's third pass – so end eight numbers in place after fifth pass **c3A1:** CSO (correct solution only) – with sixth pass showing no swaps (not just a statement that the list is in order after a fifth pass) **d1M1:** First four items placed correctly (the boxed values). Condone cumulative totals for M1 only **d1A1:** First eight items placed correctly (the boxed and underlined values) – any additional/repeated values scores M1 only **d2A1:** CSO

Q65.

Question Number	Scheme	Marks
(a)	The 13 was used as a pivot in the first pass as all numbers greater than 13 are on the left (of the 13) and all numbers less than 13 are on the right (of the 13) (and this is not the case for any other value)	B1 (1)
(b)	17 33 14 25 23 28 21 13 9 6 10 33 17 25 23 28 21 14 13 9 10 **6** 33 25 23 28 21 17 14 13 10 **9** **6** 33 25 28 23 21 17 14 13 **10** **9** **6** 33 28 25 23 21 17 14 **13** **10** **9** **6** 33 28 25 23 21 17 14 13 10 9 6	M1 A1 A1 (3)
(c)	Bin 1: **33** **28** **23** Bin 2: **25** **21** 17 14 6 Bin 3: 13 10 9	M1 A1 (2)
		6 marks

Notes for Question

a1B1: pivot value of 13 correctly stated + correct reasoning (give bod provided there is a clear intention of 'values greater than the pivot, pivot, values less than the pivot' e.g. 'after the first pass all the values greater than 13 are to the left and all values smaller are on the right'. Allow for **B1** a statement such as, 'values on one side are bigger than 13 and on the other are smaller', however just a statement like 'all the values greater than 13 are to the left' is **B0** – must have an indication of values on both sides of the 13)

PLEASE NOTE NO MISREADS IN PARTS (b) AND (c) – MARK ACCORDING TO THE SCHEME AND THE SPECIAL CASE IN (b)

b1M1: Bubble sort. Consistent direction, end number (6) in place and the list beginning with the correct first four numbers (33 17 25 23) after the first pass. Do check these carefully as some candidates show the result of each comparison and swap in their first pass. Consider the placement of the candidate's numbers, rather than what the candidate labels each line of their pass. For example, assume that the first time that the 6 appears at the end of the list is the end of their first pass. Their first pass must be of the form 33 17 25 23 x 6 where x is either 5, 6 or 7 numbers
b1A1: The first, second **and** third passes correct
b2A1: Fourth and fifth passes correct – **must include a fifth pass** (ISW after correctly completing the fifth pass). If after the fourth pass they state that the list is in order and simply re-write the list then **A0** (but give bod if it could be interpreted as a fifth pass)

SC in (b): Ascending sort: First two passes correct scores **M1** only in **(b)**

17 14 25 23 28 21 13 9 6 10 33 followed by 14 17 23 25 21 13 9 6 10 28 33

c1M1: The **correct** first five values placed correctly (so must be the 33 28 25 23 and the 21) and at least eight values placed in bins - condone cumulative totals for **M1** only (the bold values)
c1A1: CSO – no additional or repeated values

Q66.

Question number	Scheme	Marks
(a)	A – B – D – F – C – E – G – A 43 + 45 + 49 + 55 + 50 + 48 + 55 = 345 (m) A – B – D – F – G – E – C – A 43 + 45 + 49 + 55 + 48 + 50 + 52 = 342 (m)	M1 A1 A1 (3)
(b)	RMST weight = 237 (m) **or** arcs in RMST are BD, BE, BG, DF, CE 237 + 43 + 47 = 327 (m)	B1 M1 A1 (3)
(c)	327 „ optimal distance „ 342	B1ft (1)
		(7 marks)

Notes for Question

a1M1: First four nodes correct for a nearest neighbour route starting at A – so must have at least A – B – D – F – … **or** in terms of arcs (AB, BD, DF,…) **or** in terms of weights (43, 45, 49,…)
a1A1: One correct route (in terms of arcs or nodes but not just weights), must return to A and corresponding correct length (units not required)
a2A1: Both routes correct (in terms of arcs or nodes but not just weights) and their corresponding correct lengths (units not required)

b1B1: cao for RMST weight (237) **or** correct arcs (BD(45), BE(46), BG(47), DF(49), CE(50) only) **or** 45 + 46 + 47 + 49 + 50 stated
b1M1: Adding the two correct least weighted arcs (AB(43) and AD(47)) to their attempt at RMST weight where their attempt at the RMST has a weight in the interval 224 „ RMST weight „ 250 (give bod if not clear where their attempt at RMST weight comes from but if working shown then must be from summing the weight of exactly five arcs). Allow unsimplified answers which imply the correct two arcs added to the weight of the RMST (e.g. 45 + 46 + 47 + 49 + 50 + 43 + 47 is equivalent to the two smallest arcs (43, 47) added to five arcs which sum to a value in the given interval).

If a candidate uses one of their NN routes from (a) and removes the arcs incident to A from this route and adds on the 43 and 47 (e.g. you may see 45 + 49 + 55 + 50 + 48 + 43 + 47) then this can still score M1 as they have added the weight of five arcs that form a spanning tree (not minimal but this is their attempt - with a total in the required interval) and they have then added on the two correct least weighted arcs

b1A1: CAO (327) – the correct answer of 327 with no working can score all three marks in this part

c1B1ft: Their numbers correctly used (their answer to (b) and their least value from (a)) with correct inequalities **(allow strict inequality for lower bound)** so an answer of 327 – 342 is B0. Lower bound must be less than upper bound. The LB must be 314 „ LB „ 340 and is dependent on scoring the M mark in (b). The UB is dependent on the M mark in (a) and there must be two different values in (a) stated (and they must have chosen the smaller of the two). Allow equivalent notation e.g. [314, 340] or (314, 340]

In (a) allow use of H for G or a combination of Gs and Hs. The following are all examples that would be acceptable for the correct route A – B – D – F – C – E – G – A in (a)

- A – B – D – F – C – E – H – A
- A – B – D – F – C – E – G/H – A
- A – B – D – F – C – E – H – G – A
- A – B – D – F – C – E – G – H – G – A
- A – B – D – F – C – E – H or G – G – A
- AB, BD, DF, FC, CE, EH/G, G/HA
- AB, BD, DF, FC, CE, EG (or EH), HA (or GA)
- AB, BD, DF, FC, CE, EH, HG, GA
- AB, BD, DF, FC, CE, EH, GA

In general accept a cycle of the form A – B – D – F – C – E – X – A where X is G or H or any combination of Gs and Hs (and similarly A – B – D – F – X – E – C – A for the second cycle or their equivalents in terms of arcs). **If any doubt regarding the route in (a) (or their answer to (b) and therefore its implication for (c)) then please contact your Team Leader and send the item to review.**

Q67.

Question Number	Scheme	Marks
(a)	A(DG)C + D(GH)E = 12 + 9 = 21	M1 A1
	AD + C(GH)E = 5 + 10 = 15*	A1
	A(DGH)E + C(G)D = 14 + 7 = 21	A1
	Repeated arcs: AD, CG, GH, EH	A1
	Length of route: 166 + 15 = 181 (km)	A1ft (6)
(b)	Vertex C: 3 times	B1 (1)
(c)	CD (7) is the shortest path between two odd nodes excluding A	M1
	Repeat CGD (7) since this is the shortest path excluding A The route finishes at E	A1
	Length of route = 166 + 7 = 173 (km)	A1 (3)
		10 marks

	Notes for Question	
a1M1: Three distinct pairings of the correct four odd nodes (A, C, D, E) **a1A1:** One row correct including pairings **and** totals **a2A1:** Two rows correct including pairings **and** totals **a3A1:** All three rows correct including pairings **and** totals **a4A1:** The smallest repeat **arcs** (accept AD, CG, GH, EH only) **a5A1ft:** Correct answer of 181 **or** 166 + their least **b1B1:** cao (3) **c1M1:** Identifies the need to repeat one path of the three (DE, CE, CD) which does not include A (this maybe implicit) **or** listing of only these three possible repeats. This mark is dependent on either scoring the M mark in **(a)** or stating all three possible paths in this part. As a minimum accept the stating of one of these three paths **c1A1:** Identifies C(G)D as the least **and** E as the finishing point. They have to explicitly state that C(G)D is the least path of those that do not include A (this can be done by stating that CD is the least of CD, CE, DE only (so with no others) **or** stating that CD is the least of those that don't include A but not for just 'CD is the least') **c2A1:** cao (173)		

Q68.

Question	Scheme	Marks
(a)	E.g. if use CD as shortcut get 807 or if use CF + AD get 793	M1 A1
		(2)
(b)	A F E D B C A	B1
	82 113 98 130 110 217 = 750	B1
		(2)
(c)	length of RMST = 439	B1
	439 + 82 + 113 = 634	M1 A1
		(3)
(d)	634 < optimal ≤ 750	B1ft
		(1)
		(8 marks)

Notes:

(a)

M1: Their plausible shortcut leading to a value < 810 and a length below 810 stated.

A1: cao – shortcut and length must be consistent.

(Examples shortcuts: CD = 807, CF + AD = 793, CF + BD = 664, AD + EF + FC = 715, DF FC = 785 etc.)

(b)

B1: cao

B1: cao

(c)

B1: cao

M1: Adding two least weighted arcs to their RMST length

A1: cao

(d)

B1: An interval that incorporates their lower bound from (c) and their best upper bound from either (a) or (b)

Q69.

Question Number	Scheme	Marks
(a)	NNA: A – D – E – F – B – C – A 27+25+21+34+58+56 = 221 (km)	M1 A1 A1 (3)
(b)	RMST weight = 118 (km)	B1
	118 + 27 + 38 = 183 (km)	M1 A1 (3)
(c)	183 ⩽ length ⩽ 221	M1 A1 (2)
		8 marks

Notes for Question

a1M1: Nearest neighbour A – D – E – F – B or accept 1 5 6 2 3 4 across the top of the table
a1A1: Route correctly stated, must return to A, accept link back to A
a2A1: Length correctly stated. Do not ISW if candidates then go on to double the route length

b1B1: CAO for RMST weight (either 118 or 34 + 21 + 25 + 38) – maybe implied by later working
b1M1: Adding 27 + 38 (the two least weighted arcs) to their RMST length – this mark maybe implied by the correct value for the lower bound – note that their RMST must contain only four arcs
b1A1: CAO - if 183 seen without working then award all 3 marks in **(b)**

c1M1: Their answers from **(a)** and **(b)** correctly used, accept any inequalities or any indication of an interval from their 183 to their 221 (so 183 – 221 can score this mark). Please note that UB > LB for this mark
c1A1: CAO (no follow through on their values) including correct inequalities or equivalent set notation (but condone 183 < length ⩽ 221)

Q70.

Question Number	Scheme	Marks
(a)	Nearest neighbour: A – B – F – D – E – C – A 35 31 44 39 53 42 = 244 (km)	M1 A1 (2)
(b)	MST with B removed: AC, CD, DF, DE gives a RMST weight of 172 (km)	B1
	172 + 31 + 35 = 238 (km)	M1 A1 (3) **5 marks**
	Notes for Question	

a1M1: Nearest neighbour A – B – F – D – E – C – (condone lack of return to start) **or** correct route length of 244. Accept AB, BF, FD, DE, EC but do not accept weights only. Accept 1 2 6 4 5 3 across the top of the table
a1A1: CAO both route (either in terms of vertices (ABFDECA) or arcs (AB, BF, FD, DE, EC, CA) but not weights) **and** length correct (244) do not ISW if this value is then doubled to 488

b1B1: CAO for RMST weight (either 172 or 42 + 47 + 44 + 39) – maybe implied by later working
b1M1: Adding 31 + 35 (the two least weighted arcs) to their RMST length – this mark maybe implied by the correct value for the lower bound – note that their RMST must contain only four arcs
b1A1: CAO - if 238 seen without working then award B0M1A1

Q71.

Question Number	Scheme	Marks
(a)	e.g. add CD and remove AD, BA and BC gives 516 (km) e.g. add EF and remove EB, BA and AF gives 509 (km)	M1 A1 (2)
(b)	NNA: A – B – E – F – D – C – A	B1
	57 66 69 78 71 76 = 417 (km)	B1 (2)
(c)	Length of RMST = 248	B1
	248 + 66 + 69 = 383 (km)	M1 A1 (3)
		7 marks

Notes for Question

a1M1: Must clearly start with 2(length of given MST) and add and subtract at least one arc (to give a network of weight < 628) – graph must be connected and Eulerian

a1A1: CAO – shortcut(s) and length must be consistent (with length stated < 520). The shortcuts must be clearly stated (that is the arcs added and subtracted) and network must be connected and Eulerian

b1B1: CAO (must return to A) – must be stated in terms of either the nodes or arcs (e.g. AB, BE, EF,…) but not just the weights of the arcs

b2B1: CAO (417)

c1B1: Correct length of RMST (248) – maybe implied by later working

c1M1: Adding the two correct least weighted arcs (66 and 69) to their RMST length ($231 \leqslant$ length $\leqslant 265$)

– give bod but their RMST must only contain 4 arcs – this mark can be implied by the correct value for the lower bound

c1A1: CAO (383) – if correct answer with no working then award B0M1A1

Q72.

Question Number	Scheme	Marks
(a)		B2, 1, 0 (2)
(b)	NNA starting at A: A – D – E – F – C – B – G – A 17 + 15 + 6 + 11 + 26 + 20 + 41 = 136 (km)	M1 A1 (2)
(c)	A – D – E – F – C – F – B – G – B – A	B1 (1)
(d)	RMST weight = 12 + 6 + 11 + 15 + 20 = 64	B1
	64 + 17 (AD) + 21 (AB) = 102 (km)	M1 A1 (3)
(e)	AC + EG = 42 + 32 = 74	M1
	AE + CG = 25 + 46 = 71	A1
	AG + CE = 41 + 17 = 58*	A1
	Repeat arcs: AB, BG, CF, EF	A1
	Length: 291 + 58 = 349 (km)	A1ft (5)
		13 marks

	A	B	C	D	E	F	G
A	-	21	**42**	17	25	31	41
B	21	-	26	27	12	15	20
C	**42**	26	-	**32**	17	11	46
D	17	27	**32**	-	15	**21**	47
E	25	12	17	15	-	6	32
F	31	15	11	**21**	6	-	35
G	41	20	46	47	32	35	-

Notes for Question
Condone lack of, or incorrect units throughout this question **a1B1:** At least two of the six values correct (in either table) – these are the bold values. The two values can be the same (for example, 42 in both cells AC and CA would score this mark) **a2B1:** Fully correct (the six bold values in both tables) **b1M1:** Nearest neighbour route starting at A – must have at least A – D – E – F – C – B – … Allow if stated in terms of arcs (AD, DE, EF, FC, CB,…) rather than nodes (note that arcs AD and DA are equivalent and so therefore AD, ED, EF, FC, CB,… is acceptable) **b1A1:** CAO on length (136) **and** route (must return to A and can be stated in terms of arcs rather than nodes) **c1B1:** CAO (ADEFCFBGBA or in terms of arcs, e.g., AD, DE, EF, FC, CF, FB, BG, GB, BA) **d1B1:** CAO on the weight of the RMST (using arcs BE, EF, CF, DE, BG) – accept either 64 **or** 12 + 6 + 11 + 15 + 20 – could be seen or implied by later working as part of the lower bound calculation **d1M1:** Adding correct two least weighted arcs from A (17(AD) + 21(AB)) to their RMST length where their RMST length x is $58 \leq x \leq 70$ **d1A1:** CAO (102). If this answer is stated with no working then award B0M1A1 **e1M1:** Correct three distinct pairings of the correct four odd nodes (A, C, E and G) **e1A1:** Any two rows correct including pairings **and** totals **e2A1:** All three rows correct including pairings **and** totals

e3A1: CAO correct edges clearly stated and not just in their working as AB, BG, CF and EF – must be these arcs. Do not accept AG, ABG or AG via B (and similarly for CE) **e4A1ft:** Follow through their value of their smallest pairing total + 291