APPENDIX

A. Proof of Theorem 1.

We need to prove that if failing link h changes the forwarding graph of $policy(s,d)$, then h must be in $\mathcal{H}$ (i.e., $H(s,d)$) returned by Alg. 1. For simplicity, we don't consider equal-cost routes, but our proof method can be easily extended.

LEMMA 1. *If failing h changes $Info(s,d)$, then $h \in R(s,d)$ return by Alg. 1 under OSPF.*

PROOF. We prove it by induction hypothesis, which works by the number of routing spreading hops.

Base case. d is connected route on d', s is OSPF neighbor with d', and s to d' is reachable through link $h' = (s,d')$, which has minimum cost. We prove *Base case* with proof by contradiction, and assume that failing h changes $Info(s,d)$, and $h \notin R(s,d)$. When h' is up, fail any link other than h', routing information from device d' can still send to s through h'. Since failing other link only increases the cost of other paths and OSPF is the shortest path protocol, the next hop and cost of optimal route on s is still d' and the value of link h', respectively. Then, if failing h changes the $Info(s,d)$, h have to be equal to h'. $h' \in R(s,d)$ (Line 17), then $h \in R(s,d)$, which is contradictory to assumption.

Step case. a is the OSPF neighbor with d, which send the optimal announcement to s through link $h' = (s,a)$. As an induction hypothesis, $R(a,d)$ contain all h that failing h changes $Info(a,d)$, because of monotonicity of recursion that the path from d to a is without recursive, and the routing hops between s and d is more one hop than between a and d. Then failing h changes $Info(s,d)$, which has two cases. *case(i)* failing h changes $Info(a,d)$. $h \in R(a,d)$ and $R(a,d) \subset R(s,d)$ (Line 21), so $h \in R(s,d)$. *case(ii)* failing h changes the routing propagation through link (a,s) and $Info(a,d)$ is same. The proof of this case is similar to *Base case*.

LEMMA 2. *If failing h changes $Info(s,d)$, then $h \in R(s,d)$ return by Alg. 1 under OSPF and static route.*

PROOF. If the priority of static route on s is higher than OSPF, then $Info(s,d)$ will not change. If the priority of OSPF on s is higher than static route, only OSPF is in effect and we prove it in Lemma 1.

LEMMA 3. *If failing h changes $Path(s,d)$, then $h \in H(s,d)$ return by Alg. 1 under OSPF and static route.*

PROOF. Failing h changes $Info(s,d)$, we prove it in Lemma 2. Failing h changes $Path(s,d)$, but doesn't change $Info(s,d)$, we prove it by induction hypothesis. The proof works by the number of forwarding next hops.

Base case. d is connected route on device d', and the packets from s arrive at d through link $h' = (s,d')$. We prove *Base case* with proof by contradiction, and assume that failing h changes $Path(s,d)$, and $h \notin H(s,d)$. When failing link other than h', packets from s still arrive at d through h', because s still choose d' as the next hop ($Info(s,d)$ isn't changed). Then, we know only failing h' will change $Path(s,d)$. Because failing h changes $Path(s,d)$, h' must be equal to h. $h \in H(s,d)$(Line 6), then $h \in H(s,d)$, which is contradictory to assumption.

Step case. Packets from s arrive at d via the next hop b, while packets from s arrive at b through the link (s,d). As an induction hypothesis, $H(b,d)$ contain all h that failing link h change $Path(b,d)$, because of monotonicity of recursion that the path from b to d is without recursive, and the forwarding hops between s and d is more one hops than between b and d. Then failing h changes $Path(s,d)$, which has two cases. *case(i)* failing h changes $Path(b,d)$. $h \in H(b,d)$ and $H(b,d) \subset H(s,d)$ (Line 12), so $h \in H(s,d)$. *case(ii)* failing h changes the packets forwarding through link (s,b). The proof of this case is similar to *Base case*.

LEMMA 4. *If failing h changes $Info(s,d)$, then $h \in R(s,d)$ return by Alg. 1 under eBGP, iBGP, OSPF, static route.*

PROOF. We prove it by induction hypothesis, which works by the number of routing spreading hops. This proof is similar to the proof of Lemma 1, but the difference is iBGP has dependencies on OSPF and static route. In addition, iBGP have the problem of route racing and route partial order.

Base case. d is connected route on d', s advertises the route to s via protocol $\mathcal{P}$ through link (s,d'), and s is the neighbor of d' under $\mathcal{P}$, and $\mathcal{P} \in \{ospf, ebgp\}$. For OSPF, we prove it in Lemma 1. For eBGP, the proof is similar to the *Base case* of Lemma 1.

Step case. a advertises the route to s via protocol $\mathcal{P}$, and s is the neighbor of a' under $\mathcal{P}$, and $\mathcal{P} \in \{ospf, ibgp, ebgp\}$. Then failing h changes $Info(s,d)$, which has four cases.

case(i) Failing h changes $Path(a, s.ip_a)$, but doesn't change $Info(a,d)$. For $\mathcal{P} = ospf$ or $\mathcal{P} = ebgp$, we prove it in *Base case*. For $\mathcal{P} = ibgp$, $H(a, s.ip_a)$ contain all link h' that failing link h' will change $Path(a, s.ip_a)$, because the packets from a to s is based on OPSF, and we prove it on Lemma 3. Because $h \subset H(a, s.ip_a)$ and $H(a, s.ip_a) \subset H(s,d)$ (Line 18), $h \in H(s,d)$.

case(ii) Failing h changes $Info(a,d)$. As an induction hypothesis, $R(a,d)$ contain all h that failing h changes $Info(a,d)$, because of monotonicity of recursion that the routing hops between s and d is more one hop than between a and d. Because failing h will change $Info(a,d)$, $h \in H(a,d)$ Because $h \subset H(a,d)$ and $H(a,d) \subset H(s,d)$ (Line 19), $h \in H(s,d)$.

case(iii) $Info(a,d)$ and $Path(a, s.ip)$ aren't changed, but s cannot acquire $Info(a,d)$ from a. This case does not existed, because there is no route racing. When there is a path, the route must be arrived.

case(iv) $Info(a,d)$ and $Path(a, s.ip)$ aren't changed, but s accept a more high priority route advertisement $Info'$ of d from c and the next hop of $Info'$ isn't a, i.e., cost of $Info' >$ cost of $Info(a,d)$. This case does not existed, because there is no route partial order. Specifically, $Info'$ is existed when h isn't failed. And $Info(a,d)$ is the best dynamic route when h isn't failed, which is contradictory to cost of $Info' >$ cost of $Info(a,d)$.

LEMMA 5. *If failing h changes $Path(s,d)$, then $h \in H(s,d)$ return by Alg. 1 under eBGP, iBGP, OSPF, static route.*

PROOF. We prove it by induction hypothesis, which works by the number of forwarding next hops. Failing h changes $Info(s, d)$, we prove it in Lemma 4. Failing h changes $Path(s, d)$, but doesn't change $Info(s, d)$, we prove it by induction hypothesis. The proof works by the number of forwarding next hops.

Base case. d is connected route on d', and the packets from s arrive at d based on $\mathcal{P}$ through link (s, d'), and s is the neighbor of d' under $\mathcal{P}$, and $\mathcal{P} \in \{ospf, ebgp\}$. For OSPF, we prove it in Lemma 1. For eBGP, the proof is similar to the *Base case* of Lemma 1.

Step case. Packets from s arrive at d via the next hop b based on protocol $\mathcal{P}$, and $\mathcal{P} \in \{ospf, ibgp, ebgp\}$. Then failing h changes $Path(s, d)$, which has two cases.

case(i) Failing h changes $Path(s, b.ip_s)$. For $\mathcal{P} = ospf$ or $\mathcal{P} = ebgp$, we prove it in *Base case*. For $\mathcal{P} = ibgp$, $H(a, s.ip_a)$ contain all link h' that failing link h' will change $Path(s, b.ip_s)$, because the packets from a to s is based on OPSF, and we prove it on Lemma 3. Because $h \subset H(s, b.ip_s)$ and $H(s, b.ip_s) \subset H(s, d)$ (Line 11), $h \in H(s, d)$.

case(ii) Failing h changes $Path(b, d)$. As an induction hypothesis, $H(b, d)$ contain all h that failing h changes $Path(b, d)$, because of monotonicity of recursion that the forwarding hops between s and d is more one hop than between b and d. Because failing h will change $Path(b, d)$, $h \in H(b, d)$ Because $h \subset H(b, d)$ and $H(b, d) \subset H(s, d)$ (Line 19), $h \in H(s, d)$.