$-3e^{5t}$ sol $\Rightarrow e^{-5t}$ sol $\Leftrightarrow x'' - 2x' - 15x = 0$

c) $5e^{-3t} - 3e^{5t}$ sol of a LHDE with CC $\overset{ch.eq}{\Leftrightarrow}$ 2

$\Rightarrow e^{-3t}$ and e^{5t} are solutions

d), g), h)

$5te^{-3t}$ - sol $\Rightarrow$ te^{-3t} - sol $\Rightarrow$ -3 - double root

$-3e^{5t}$ - sol $\Rightarrow$ 5 - root

$\Rightarrow$ ch. eq: $(r+3)^2(r-5)=0$

$(r^2+6r+9)(r-5)=0$

$r^3+6r^2+9r-5r^2-30r-45=0$

$r^3+r^2-21r-45=0$

$\Rightarrow$ diff eq: $x'''+x''-21x'-45x=0$

gen. sol: $x = c_1 \cdot e^{-3t} + c_2 \cdot te^{-3t} + c_3 \cdot e^{5t}$

g) $(5-3t+2t^2)e^{-3t} = 5e^{-3t} - 3te^{-3t} + \underbrace{2t^2e^{-3t}}$

this gives us a root of multiplicity 3

$\Rightarrow -3$ - root of multiplicity 3

ch.eq: $(r+3)^3 = 0 \Leftrightarrow \cancel{(r^3} \cancel{r^3+6r^2+9r}$

$r^3 + 9r^2 + 27r + 27 = 0$

$\Rightarrow$ diff eq: $x''' + 9x'' + 27x' + 27x$

gen. sol: $x = c_1 e^{-3t} + c_2 te^{-3t} + c_3 t^2 e^{-3t}$, $c_{1,2,3} \in \mathbb{R}$

h) $\sin 3t$ - sol

$\Rightarrow$ order of multiplicity - 2

~~$\Rightarrow \cos 3t$ - sol~~

~~$\Rightarrow r_1 = 3i, r_2 = -3i$~~

$r = \alpha + i\beta$ - root $\Rightarrow e^{\alpha t}\cos\beta t,\ e^{\alpha t}\sin\beta t$ - sol

since $\sin 3t$ - sol $\Rightarrow \cos 3t$ - sol

$\Rightarrow$ ~~$r = 0 + 3\beta$ - sol~~,

~~$0 + 3i$~~

$\Rightarrow r_1 = 3i,\ r_2 = -3i$

ch eq: $(r+3i)(r-3i) = 0$

$= r^2 + 3ir - 3ir - 9i^2 = 0 \Leftrightarrow$

$\Leftrightarrow r^2 + 9 = 0$

diff eq: $x'' + 9x = 0$

gen sol: $c_1 \cdot \cos 3t + c_2 \cancel{\cos} \sin 3t$, $c_{1,2} \in \mathbb{R}$