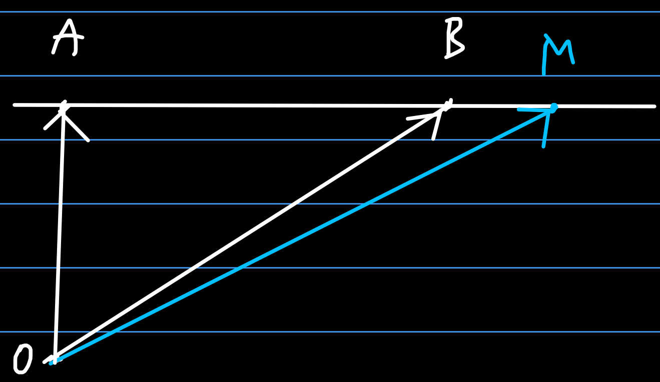
# Seminar W3 - 917

$\ell$ line

$A, B \in \ell, \quad A \neq B$




**Vector equation:**

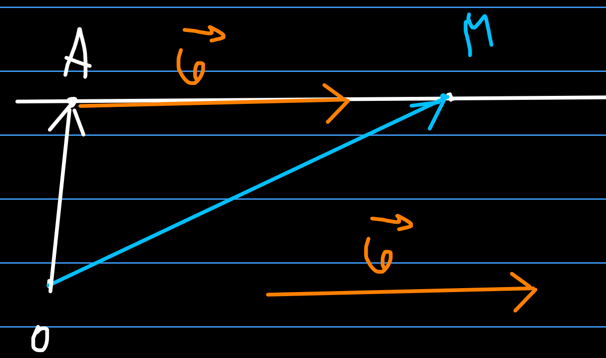
$$\vec{r}_M = \lambda \vec{r}_A + (1-\lambda)\vec{r}_B$$

$$\lambda \in \mathbb{R}$$

**Parametric equation:**

$$\begin{cases} x = \lambda x_A + (1-\lambda) x_B \\ y = \lambda y_A + (1-\lambda) y_B \\ z = \lambda z_A + (1-\lambda) z_B \end{cases}$$

$A \in \ell, \quad \vec{u} \parallel \ell$




$$\vec{r}_M = \vec{r}_A + t \cdot \vec{u}$$

$$t \in \mathbb{R}$$

$$\begin{cases} x = x_A + t \cdot x_{\vec{u}} \\ y = y_A + t \cdot y_{\vec{u}} \\ z = z_A + t \cdot z_{\vec{u}} \end{cases}$$

## Canonical equation:

$$(d=)\ \frac{x-x_B}{x_A-x_B}=\frac{y-y_B}{y_A-y_B}=\frac{z-z_B}{z_A-z_B} \qquad (f=)\ \frac{x-x_A}{x_{\vec{u}}}=\frac{y-y_A}{y_{\vec{u}}}=\frac{z-z_A}{z_{\vec{u}}}$$

! We have to be careful about the cases where the denominators might be 0.

→ if $x_{\vec{u}}=0$, $y_{\vec{u}}\neq 0$, $z_{\vec{u}}\neq 0$:

$$\begin{cases} x=x_A \\ \dfrac{y-y_A}{y_{\vec{u}}}=\dfrac{z-z_A}{z_{\vec{u}}} \end{cases}$$

→ if $x_{\vec{u}}=0$, $y_{\vec{u}}=0$, $z_{\vec{u}}\neq 0$

$$\begin{cases} x=x_A \\ y=y_A \end{cases}$$

## Implicit equation:

3D
$$\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$$

2D
$$Ax+By+C=0$$

## Explicit equation $\xrightarrow{\text{in 2D}}$

$$y=\overset{\text{slope}}{m}x+n$$

3.2. Write the equation of the line which passes through $A(1,-2,6)$ and is parallel to:

(a) the $x$-axis

(b) the line $(d_1)$ : $\frac{x-1}{2} = \frac{y+5}{-3} = \frac{z-1}{4}$

(c) the vector $\vec{u}(1,0,2)$

(a) $d \parallel Ox \Rightarrow \vec{d} = (1,0,0)$

parametric canonical

$$\Rightarrow \begin{cases} x = 1 + t\cdot 1 \\ y = -2 + t\cdot 0 \\ z = 6 + t\cdot 0 \end{cases} \Rightarrow \begin{cases} y = -2 \\ z = 6 \end{cases}$$

(b) $d: \frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-6}{4}$

(c) $d: \begin{cases} y = -2 \\ \frac{x-1}{1} = \frac{z-6}{2} = t \end{cases}$

$$d: \begin{cases} x = 1+t \\ y = -2 \\ z = 6+2t \end{cases}$$

3.18. The vertices of the triangle ABC are the intersection points of the lines

$$d_1 : 4x + 3y - 5 = 0$$

$$d_2 : x - 3y + 10 = 0$$

$$d_3 : x - 2 = 0$$

$$\{A\} = d_2 \cap d_3, \quad \{B\} = d_3 \cap d_1, \quad \{C\} = d_1 \cap d_2$$

(a) Find the coordinates of A, B, C

(b) Find the equations of the medians

(c) Find the equations of the heights

(a) $A : \begin{cases} x - 3y + 10 = 0 \\ x - 2 = 0 \end{cases} \Leftrightarrow \begin{cases} x = 2 \\ 2 - 3y + 10 = 0 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x = 2 \\ y = 4 \end{cases} \Rightarrow A(2, 4)$

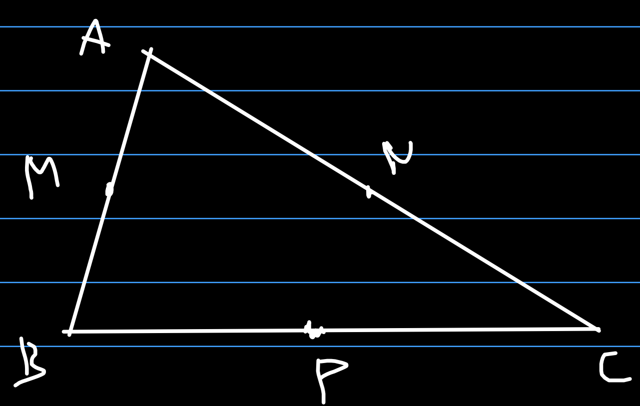
$B : \begin{cases} x - 2 = 0 \\ 4x + 3y - 5 = 0 \end{cases} \Rightarrow \begin{cases} x = 2 \\ 8 + 3y - 5 = 0 \end{cases} \Rightarrow \begin{cases} x = 2 \\ y = -1 \end{cases}$

$\Rightarrow B(2, -1)$

$$C: \begin{cases} 4x+3y-5=0 \\ x-3y+10=0 \end{cases} \Leftrightarrow \begin{cases} 5x+5=0 \\ x-3y+10=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x=-1 \\ -1-3y+10=0 \end{cases} \Leftrightarrow \begin{cases} x=-1 \\ y=3 \end{cases}$$

(b)




$A(2, 4)$

$B(2, -1)$

$C(-1, 3)$

$$M\left(\frac{2+2}{2}, \frac{4-1}{2}\right) \Rightarrow M\left(2, \frac{3}{2}\right)$$

$$N\left(\frac{2-1}{2}, \frac{4+3}{2}\right) \Rightarrow N\left(\frac{1}{2}, \frac{7}{2}\right)$$

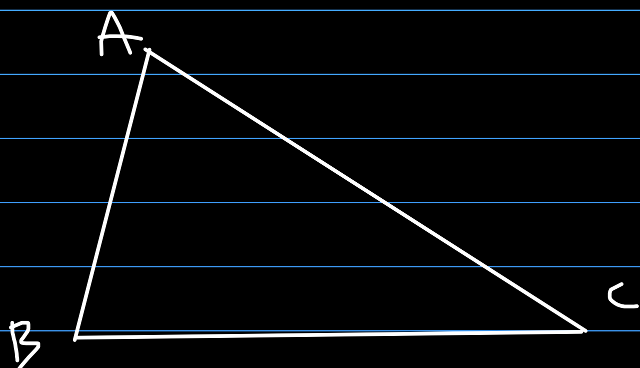
$$P\left(\frac{2-1}{2}, \frac{-1+3}{2}\right) \Rightarrow P\left(\frac{1}{2}, 1\right)$$

$$CM: \frac{y-3}{\frac{3}{2}-3} = \frac{x+1}{2+1}$$

$$BN: \frac{y-\frac{7}{2}}{-1-\frac{7}{2}} = \frac{x-\frac{1}{2}}{2-\frac{1}{2}}$$

$$AP: \frac{y-1}{2-1} = \frac{x-\frac{1}{2}}{4-\frac{1}{2}}$$

(c)




$d_1: \ 4x+3y-5=0 \Rightarrow d_1: y = -\frac{4}{3}x+\frac{5}{3}$

$d_2: \ x-3y+10=0 \Rightarrow d_2: y = \frac{1}{3}x+\frac{10}{3}$

$d_3: \ x-2=0$

$$m_{h_A} = -\frac{1}{m_{d_1}} = \frac{3}{4} \Rightarrow h_A: y-4 = \frac{3}{4}\cdot(x-2)$$

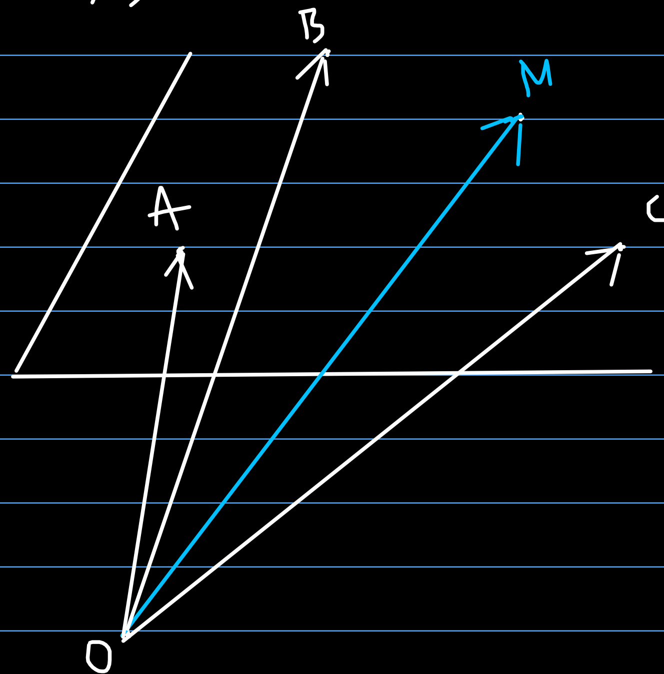
$$m_{h_B} = -\frac{1}{m_{d_2}} = -3 \Rightarrow h_B: y+1 = -3(x-2)$$

$$m_{h_C} = 0 \qquad h_C: y=3$$

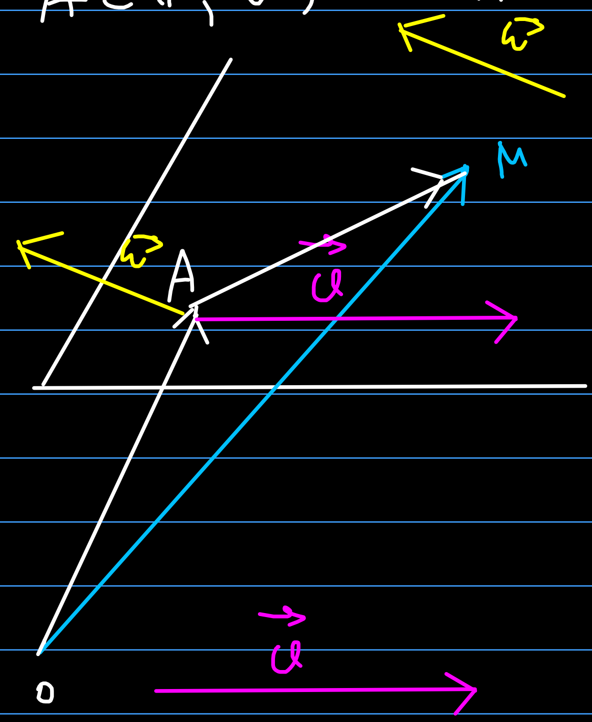
# Planes

$\Pi$ plane

$A, B, C \in \Pi$ noncollinear




$A \in \Pi$, $\vec{u}, \vec{w} \parallel \Pi$




## Vector equation:

$$\vec{r_M} = (1-\lambda-\mu)\vec{r_A} + \lambda\vec{r_B} + \mu\vec{r_C}$$

$$\vec{r_M} = \vec{r_A} + \alpha\vec{u} + \beta\cdot\vec{w}$$

## Parametric equations:

$$\begin{cases} x = (1-\lambda-\mu)x_A + \lambda x_B + \mu x_C \\ y = (1-\lambda-\mu)y_A + \lambda y_B + \mu y_C \\ z = (1-\lambda-\mu)z_A + \lambda z_B + \mu z_C \end{cases}$$

$$\begin{cases} x = x_A + \alpha x_{\vec{u}} + \beta x_{\vec{w}} \\ y = y_A + \alpha y_{\vec{u}} + \beta y_{\vec{w}} \\ z = z_A + \alpha z_{\vec{u}} + \beta z_{\vec{w}} \end{cases}$$

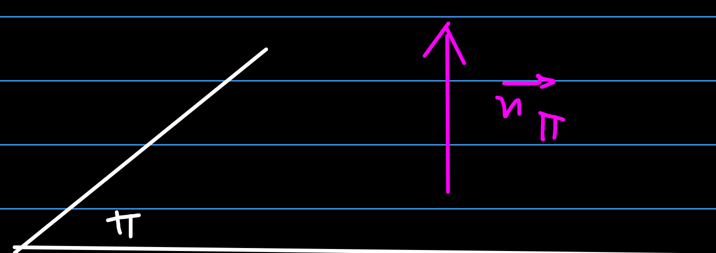
Canonical equations:

$$\begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_B-x_A & y_B-y_A & z_B-z_A \\ x_C-x_A & y_C-y_A & z_C-z_A \end{vmatrix}=0 \qquad \begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_{\vec{u}} & y_{\vec{u}} & z_{\vec{u}} \\ x_{\vec{w}} & y_{\vec{w}} & z_{\vec{w}} \end{vmatrix}=0$$

$$\Leftrightarrow \begin{vmatrix} x & y & z & 1 \\ x_A & y_A & z_A & 1 \\ x_B & y_B & z_B & 1 \\ x_C & y_C & z_C & 1 \end{vmatrix}=0$$

Implicit equation: $A\cdot x+B\cdot y+C\cdot z+D=0$

$\longrightarrow$ normal vector: $\vec{n}_\pi=(A,B,C)$



3.1. Write the equation of the plane which passes through $M_0(1,5,-6)$ and is parallel to the vectors $\vec{u}_1(1,0,7)$ and $\vec{u}_2(-8,2,10)$

$$\begin{vmatrix} x-1 & y-5 & z+6 \\ 1 & 0 & 7 \\ -8 & 2 & 10 \end{vmatrix} = 0 \iff$$

$$\iff 0 + (-56)(y-5) + 2(z+6) - 0 -$$
$$-10(y-5) - 14(x-1) = 0 \iff$$

$$\iff -14x - 66y + 2z + 356 = 0 \iff$$

$$\iff -7x - 33y + z + 178 = 0$$

3.3, Write the equation of the plane which contains the line:

$$(d_1) \quad \frac{x-3}{2} = \frac{y+4}{1} = \frac{z-2}{-3}$$

and is parallel to

$$(d_2) \quad \frac{x+5}{2} = \frac{y-2}{2} = \frac{z-1}{2}$$

$$\left.\begin{array}{l} A(3,-4,2) \in d_1 \subset \Pi \Rightarrow A \in \Pi \\ d_1 \subset \Pi \Rightarrow \vec{d_1} \parallel \Pi \\ d_2 \parallel \Pi \Rightarrow \vec{d_2} \parallel \Pi \end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{vmatrix} x-3 & y+4 & z-2 \\ 2 & 1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = 0 \Rightarrow$$

$$\Rightarrow \begin{vmatrix} x-3 & y-x+7 & z-x+1 \\ 2 & -1 & -5 \\ 2 & 0 & 0 \end{vmatrix} = 0 \Rightarrow$$

$$\Rightarrow \quad 2 \cdot \begin{vmatrix} y-x+7 & z-x+1 \\ -1 & -5 \end{vmatrix} = 0$$

$$\Rightarrow -5y + 5x - 35 + z - x + 1 = 0 \Rightarrow$$

$$\Rightarrow \quad 4x - 5y + z - 34 = 0$$