$f(1) = 2 \cdot 1 - 5 \cdot 1 + 10 - 11 + 4 > 0$

$x_{n+1} = 2x_n^5 - 5x_n^4 + 10x_n^2 - 10x_n + 4$

$f(x) = 0 \Leftrightarrow \underbrace{2x^5 - 5x^4 + 10x^2 - 10x + 4}_{g(x)} = x$

$\Leftrightarrow \boxed{x = g(x)}$

$x_1 = g(x_0)$

$x_2 = g(x_1)$

....

$x_{n+1} = g(x_n) \leftarrow x_{n+1} = 2x_n^5 - \ldots$

- - - -

$x_n \approx g(x_n)$

$x_0 = 1{,}1 \overset{?}{\Rightarrow} x_{n+1} = g(x_n),\ x_n \to 1$

$g(0.9) \approx 1.02\ldots$

$g(1.1) \approx 1.08\ldots$

g - ~~increasing~~ monotonous on $[0.9, 1.1]$

$\Rightarrow g([0.9, 1.1]) = [1.02\ldots, 1.08\ldots]$

$\Rightarrow$

$\Rightarrow g([0.9, 1.1]) = [1.02\ldots, 1.08\ldots]$
$\subseteq [0.9, 1.1]$

$g'(x) = \ldots$

$g'(x) = 10x^4 - 20x^3 + 20x - 10$
$= 10(x^4 - 2x^3 + 2x - 1)$

$g'(\alpha) = g'(1) = 0$

$g''(x) = 40x^3 - 60x^2 + 20$

$g''(\alpha) = g''(1) = 0$

$g'''(x) = 120x^2 - 120x$

$g'''(1) = 0$

$g^{(4)}(x) = 240x - 120$

$g^{(4)}(1) = 120 \neq 0$

T 3.3 (L12), with $p = 4$

$$\lim_{n\to\infty} \frac{x_{n+1} - \alpha}{(x_n - \alpha)^4} = \frac{1}{24} g^{(4)}(\alpha) \underline{\underline{\alpha = 1}}$$

$= \frac{x_{n+1} - 1}{\ldots} \quad 120 \ldots$

$$= \frac{x_{n+1}-1}{(x_n-1)^4} = \frac{120}{24} = \boxed{5}$$

$$\lim_{n\to\infty} \frac{x_{n+1}-\alpha}{(x_n-\alpha)^4} = 5$$

$f : [0, 2] \to \mathbb{R}$, $f(\underset{x_0}{0}) = 1$, $f(\underset{x_1}{1}) = -1$, $f(\underset{x_2}{2}) = 5$

$n = 3 - 1 = 2$

$$\ell_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j} \qquad (Pf)(x) = \ell_0 f(0) + \ell_1 \cdot f(1) + \ell_2 \cdot f(2)$$

$$\ell_0(x) = \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} = \frac{x-1}{-1} \cdot \frac{x-2}{-2} = \frac{(x-1)(x-2)}{2}$$

$$(Nf)(x) = f(x_0) + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) \cancel{+ f[x_0, x_1, x_2, x_3] \cdot (x-x_0)(x-x_1)(x-x_2)}$$

	f_i	$f[x_i, x_{i+1}]$	$f[x_i, x_{i+1}, x_{i+2}]$
$0 = x_0$	1	$\frac{-1-1}{1-0} = -2$	$\frac{6-(-2)}{2-0} = 4$
$1 = x_1$	-1	$\frac{5-(-1)}{2-1} = 6$	
$2 = x_2$	5		

$$(Nf)(x) = 1 - 2(x-0) + \cancel{8}4(x-0)(x-1)$$
$$= 1 - 2x + \cancel{8}4x^2 - \cancel{8}4x$$
$$\cancel{= 8x^2 - 10x + 1} = \underline{4x^2 - 6x + 1}$$

$$(Nf)(0) = 1$$

$$(Nf)(1) = -1$$

$$(Nf)(2) = 5$$

$$f : [-1, 1] \to \mathbb{R}, \quad f(-1) = -2, \; f'(-1) = 1$$
$$f(1) = 3, \; f'(1) = -2$$

$$(Nf)(x) = f(x_0) + f[x_0, x_0](x - x_0) + f[x_0, x_0, x_1](x - x_0)^2 + f[x_0, x_0, x_1, x_1](x - x_0)^2(x - x_1)$$

$x_0 = -1$	$f_0 = -2$	$f'_0 = 1$	$\frac{5/2 - 1}{1 - (-1)} = \frac{3}{4}$	$-\frac{3}{2}$
$x_0 = -1$	$f_0 = -2$	$\frac{3 - (-2)}{1 - (-1)} = 5/2$	$\frac{-2 - 5/2}{1 - (-1)} = -\frac{9}{4}$	
$x_1 = 1$	$f_1 = 3$	$f'_1 = -2$		
$x_1 = 1$	$f_1 = 3$			

$$Nf = -2 + (x+1) + \frac{3}{4}(x+1)^2 - \frac{3}{2}(x+1)^2(x-1)$$

$$N'f = 1 + \frac{3}{2}(x+1) - 3(x+1)(x-1) - \frac{3}{2}(x+1)^2$$

$$N'f(-1) = 1 + 0 - 3 \cdot 0 \cdot (-1) - 0 = 1 \checkmark$$

$$N'f(1) = 1 + 3 - 0 - 6 = -2 \checkmark$$

$f : [0, 1] \to \mathbb{R}$, $f(\overset{x_0}{0}) = 1$, $f'(\overset{x_0}{0}) = 2$, $f'(\overset{x_1}{1}) = -1$

$n = 3 - 1 = 2 \Rightarrow P(x) = ax^2 + bx + c$

$$\begin{cases} P(0) = 1 \\ P'(0) = 2 \\ P'(1) = -1 \end{cases} \Rightarrow \begin{cases} c = 1 \\ b = 2 \\ 2a + b = -1 \end{cases} \Rightarrow \begin{cases} c = 1 \\ b = 2 \\ a = -\frac{3}{2} \end{cases}$$

$$(Bf)(x) = -\frac{3}{2}x^2 + 2x + 1$$

$$(B'f)(x) = -3x + 2 \checkmark$$

$$(Bf)\left(\frac{1}{2}\right) = -\frac{3}{8} + 1 + 1 = \frac{13}{8}$$

$b_{00} = ax^2 + bx + c$

$b_{01} = dx^2 + ex + f$

$b_{11} = gx^2 + hx + i$

$(Bf)(x) = b_{00}(x)\, f(x_0) + b_{01}(x) \cdot f'(x_0) + b_{11}(x) \cdot f'(x_1)$

$$\begin{cases} b_{00}(0) = 1 \\ b'_{00}(0) = 0 \end{cases} \qquad \begin{cases} b_{01}(0) = 0 \\ b'_{01}(0) = 1 \end{cases} \qquad \begin{cases} b_{11}(0) = 0 \\ b'_{11}(0) = 0 \end{cases}$$

$$\begin{cases} b_{00}(0)=0 \\ b'_{00}(1)=0 \end{cases} \quad \begin{cases} b_{01}(0)=1 \\ b'_{01}(1)=0 \end{cases} \quad \begin{cases} b_{11}(0)=0 \\ b'_{11}(1)=1 \end{cases}$$

$$b_{00}\begin{cases} c=1 \\ b=0 \\ 2a+b=0 \end{cases} \Rightarrow b_{00}(x)=1$$

$$b_{01}\begin{cases} f=0 \\ e=1 \\ 2d+e=0 \end{cases} \Rightarrow b_{01}(x)=-\frac{1}{2}x^2+x$$

$$b_{11}\begin{cases} i=0 \\ h=0 \\ 2g+h=1 \Rightarrow g=\frac{1}{2} \end{cases}, \quad b_{11}(x)=\frac{1}{2}x^2$$

$$(Bf)(x)=1\,f(0)+\left(-\frac{1}{2}x^2+x\right)f'(0)+\frac{1}{2}x^2\cdot f'(1)$$

$$=1\cdot 1-x^2+2x-\frac{1}{2}x^2=-\frac{3}{2}x^2+2x+1$$

$$Rf(x)=f(x)-(Bf)(x)$$

$$=f(x)-f(0)-f'(0)\cdot\left(x-\frac{x^2}{2}\right)-\frac{f'(1)}{2}x^2$$

$$Re_2=x^2-0^2-2\cdot 0\cdot\left(x-\frac{x^2}{2}\right)-\frac{2\cdot 1}{2}\cdot x^2$$

$$=x^2-x^2=0$$

$$Re_3 \overset{f=x^3}{=} x^3-0^3 \ldots \quad x^2 \quad 3\cdot 1^2$$

$$Re_3 \overset{f=x^3}{=} x^3 - 0^3 - 3\cdot 0^2\left(x-\frac{x^2}{2}\right) - \frac{3\cdot 1^2}{2}x^2$$

$$= x^3 - \frac{3}{2}x^2 \neq 0 \Rightarrow d = 2$$

H- $R \in H^3[0,1]$ } $\xRightarrow{\text{Peano}}$
$\mathrm{Ker}\, R = \mathbb{P}_2$ }

$$\Rightarrow Rf(x) = \int_0^1 K_3(x,t)\cdot f'''(t)\,dt$$

$$K_3 = \frac{1}{2} R\left[(x-t)_+^2\right]$$

$$K_3 = \frac{1}{2}\Big((x-t)_+^2 - (\underbrace{0-t}_{\leq 0})_+^2 - 2(\underbrace{-t}_{\leq 0})_+\cdot\left(x-\frac{x^2}{2}\right) - (\underbrace{1-t}_{\geq 0})_+ x^2\Big)$$

$$\frac{\partial}{\partial x}\left((x-t)_+^2\right) = 2\cdot(x-t)_+\cdot\frac{\partial}{\partial x}\left((x-t)_+\right) =$$

$$= 2\cdot(x-t)_+\cdot 1 = 2(x-t)_+$$

$$\frac{\partial}{\partial x}(x-t)_+ = \begin{cases}\frac{\partial}{\partial x}(x-t), & x>t\\ \frac{\partial}{\partial x}0, & x<t\end{cases} = \begin{cases}1, & x>t\\ 0, & x<t\end{cases}$$

$$K_3 = \frac{1}{2}\left((x-t)_+^2 - 0 - 0 - (1-t)x^2\right)$$

$$= \frac{1}{2}(x-t)_+^2 - \frac{1}{2}(1-t)x^2$$

I $t \leq x$:

$$K_3 = \frac{1}{2}(x-t)^2 - \frac{1}{2}(1-t)x^2 =$$

$$= \cancel{\frac{1}{2}x^2} - xt + \frac{t^2}{2} - \cancel{\frac{x^2}{2}} + \frac{tx^2}{2} =$$

$$= \frac{t}{2}x^2 - tx + \frac{t^2}{2}$$

$$\cancel{= tx\left(\frac{x}{2} - 1\right)}$$

$$Rf = \frac{1}{6} f'''(\xi)\left(x^3 - \frac{3}{2}x^2\right)$$

$$g(x) = x^3 - \frac{3}{2}x^2 = \cancel{x^2\left(x - \frac{3}{2}\right)}$$

$$g'(x) = 3x^2 - 3x$$

$$\Rightarrow \left|x^3 - \frac{3}{2}x^2\right| \leq \frac{1}{2}$$

	0		1
$g(x)$	0	$\searrow$	$-\frac{1}{2}$
$g'(x)$	0	- - - - - - - -	0

$$\Rightarrow |Rf| \leq \frac{1}{12} f'''(\xi), \quad \xi \in (0,1)$$