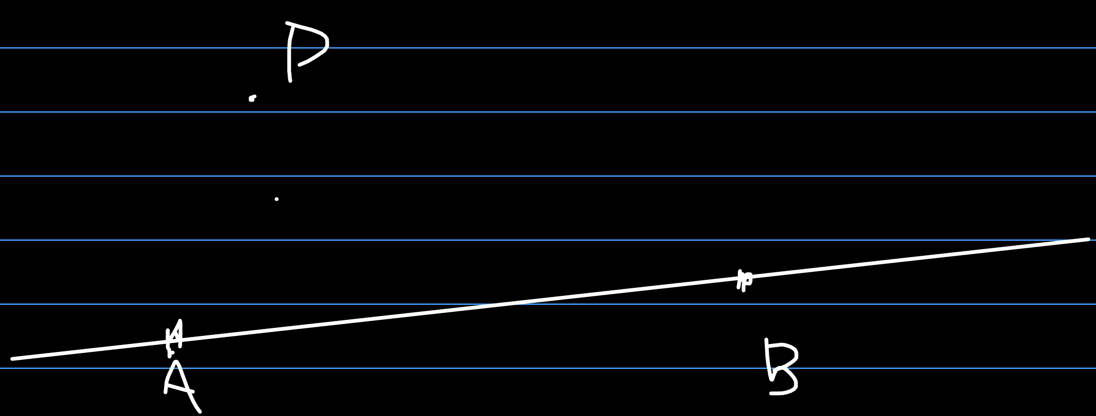
08.04.2021

$$\ell: \begin{cases} x + 2y + 3z - 4 = 0 \\ 2x - y + z + 7 = 0 \end{cases}$$

$P(1, 2, 3)$

$$\mathrm{dist}(P, \ell) = \frac{\|\overrightarrow{PA} \times \overrightarrow{AB}\|}{\|\overrightarrow{AB}\|}$$




$$\begin{cases} x + 2y + 3z - 4 = 0 \\ 2x - y + z + 7 = 0 \end{cases}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & -1 & 1 & -7 \end{pmatrix} \sim$$

$$\overset{L_2 \leftarrow L_2 - 2L_1}{\sim} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & -5 & -5 & -15 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 3 \end{pmatrix} \overset{L_1 \leftarrow L_1 - 2L_2}{\sim}$$

$$\sim \begin{pmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 3 \end{pmatrix}$$

$$\Rightarrow \begin{cases} x + z = -2 \\ y + z = 3 \end{cases} \quad G$$

a) $\begin{cases} x = -2 - z \\ y = 3 - z \end{cases}$

$$\Leftrightarrow \begin{cases} x = -2 - \alpha \\ y = 3 - \alpha \\ z = \alpha \end{cases}$$

$\Rightarrow A(-2, 3, 0) \in l$

$(-1, -1, 1) =: \vec{l}$

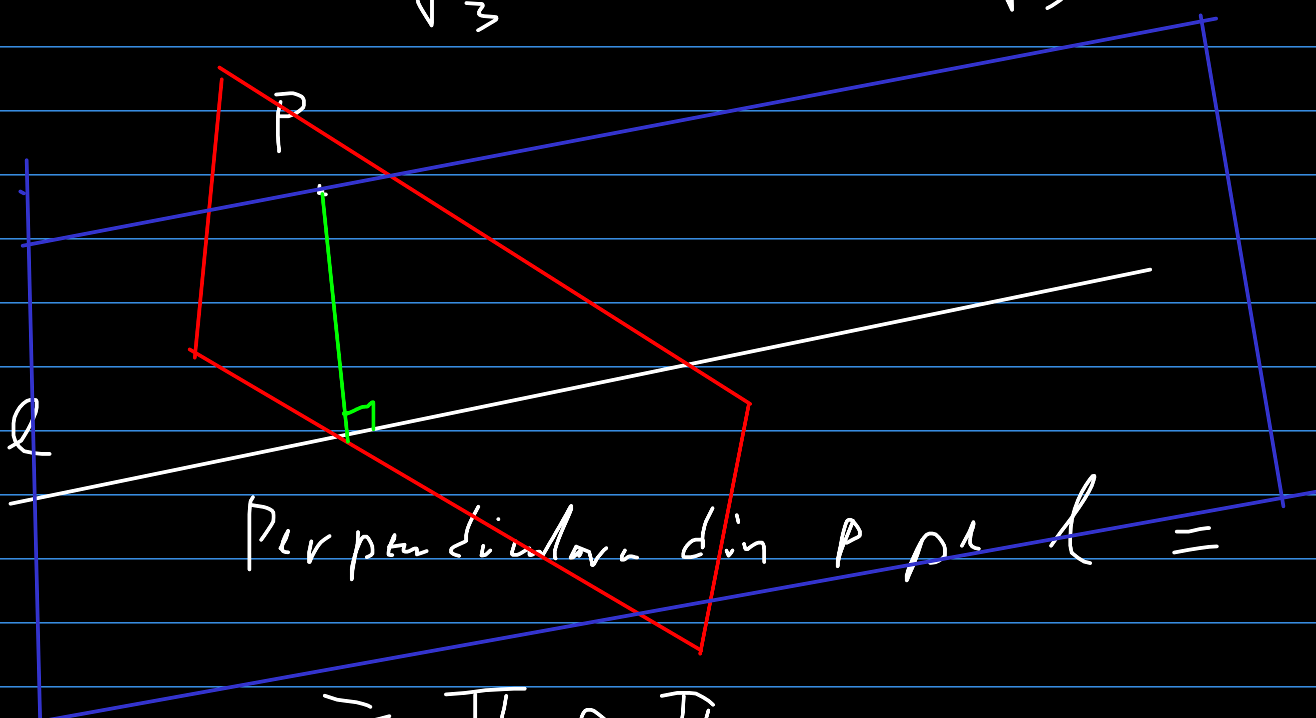
$$\operatorname{dist}(P, l) = \frac{\|\vec{PA} \times \vec{l}\|}{\|\vec{l}\|} =$$

$$= \frac{\|(-3, 1, -3) \times (-1, -1, 1)\|}{\|(-1, -1, 1)\|} =$$

$$= \frac{\left\| \begin{vmatrix} \vec i & \vec j & \vec k \\ -3 & 1 & -3 \\ -1 & -1 & 1 \end{vmatrix} \right\|}{\sqrt{3}} =$$

$$= \frac{\sqrt{\| -2\vec{i} + 6\vec{j} + 4\vec{k} \|}}{\sqrt{3}} =$$

$$= \frac{\sqrt{4+36+16}}{\sqrt{3}} = \frac{\sqrt{56}}{\sqrt{3}}$$




Perpendiculara din $P$ pe $\ell$ =

$\Rightarrow \pi_1 \cap \pi_2$

$$\begin{cases} \Pi_1 : \text{planul perp. pe } \ell \text{ care conține } P \\ \Pi_2 : \text{planul determinat de } P \text{ și } \ell. \end{cases}$$

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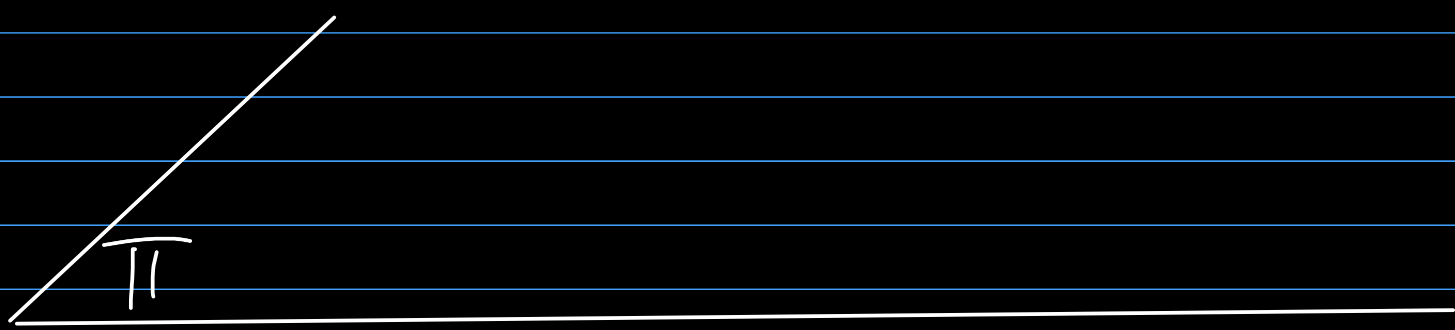
$$\begin{cases} \Pi_1 : \text{planul perp. pe } \ell \text{ care conține } P \\ P' = \Pi_1 \cap \ell \\ \text{perp. este dreapta } PP' \end{cases}$$

$$\ell : \frac{x - x_0}{\alpha} = \frac{y - y_0}{\beta} = \frac{z - z_0}{\gamma}$$

$P(x_1, y_1, z_1)$

$$\Pi_1 : \alpha(x - x_1) + \beta(y - y_1) + \gamma(z - z_1) = 0$$

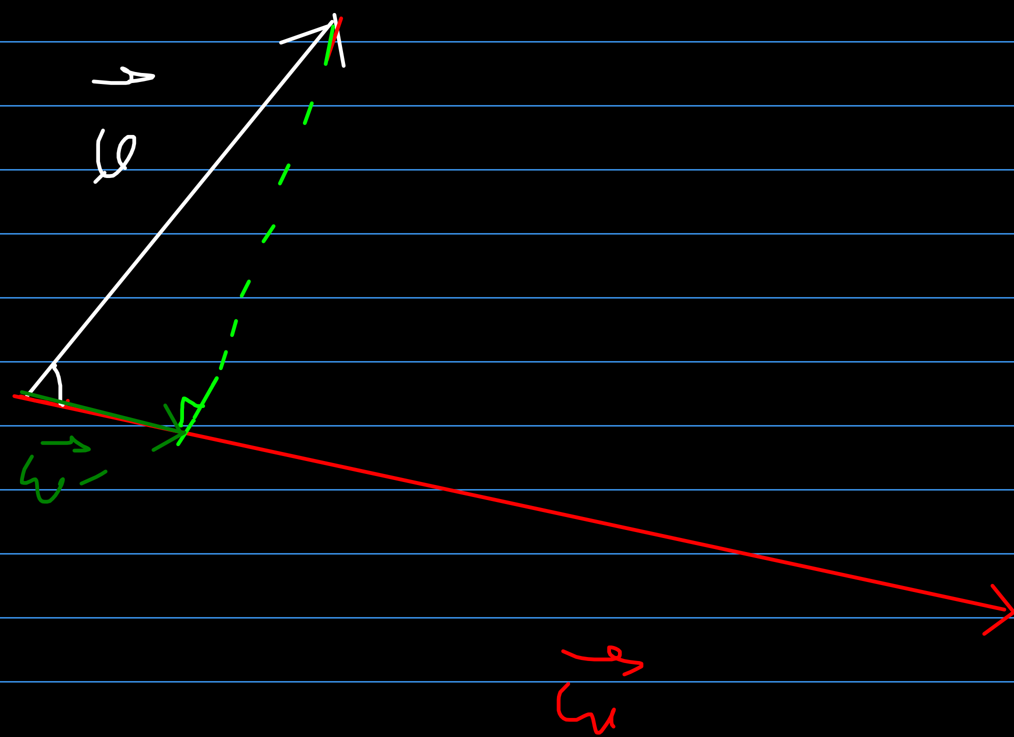



Pentru a găsi $\vec{u} \parallel \Pi$

(altfel spus, $\vec{u} \in \vec{\Pi}$)

→ verificăm $\vec{u} \cdot \vec{n_{\Pi}} = 0$

→ verificăm că $\vec{u}$ combinație liniară a unor vectori din $\vec{\Pi}$




$$\vec{u}\cdot\vec{w} = \|\vec{u}\|\cdot\|\vec{w}\|\cdot\cos(\vec{u},\vec{w}) =$$

$$= \|\vec{u}\|\cdot\cos(\vec{u},\vec{w})\cdot\|\vec{w}\|$$

$$= \|\vec{w}'\|\cdot\|\vec{w}\|$$