Seminar W1 - 917

02 December 2020 12:14

$CC = \max(M, S + \frac{M}{2})$

S = exercises solved in class (2p, 3p)

≤ 12 (1p, 1.5p)

$FG = \max(\frac{4}{10} CC + \frac{6}{10} E, E)$

minimal conditions for passing:

$CC \geq 4.5$, $E \geq 4.5$

Analytic Geometry

↳ "we identify points with vectors of numbers"

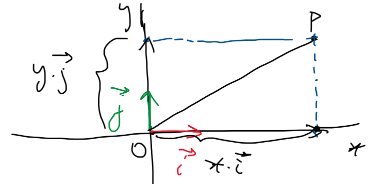
We work in the Euclidean plane (or space).

In practice: $\mathbb{R}^2$, $\mathbb{R}^3$

We have fixed a Cartesian reference system:

$(O; b)$

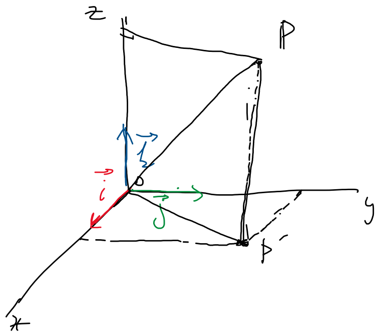
O – point; b ↳ basis of the vector space



In our case: $b = (\vec{i}, \vec{j})$

$[P]_{\mathcal{R}} = [\underbrace{\vec{OP}}_{\vec{r}_P}]_b = \begin{pmatrix} x \\ y \end{pmatrix} \Leftrightarrow \vec{r}_P = x\vec{i} + y\vec{j}$

the position vector of P with regards to the reference system $\mathcal{R}$



1.1. Consider a tetrahedron ABCD. Find the following sums of vectors:

(a) $\vec{AB} + \vec{BC} + \vec{CD}$

(b) $\vec{AD} + \vec{CB} + \vec{DC}$

(c) $\vec{AB} + \vec{BC} + \vec{DA} + \vec{CD}$

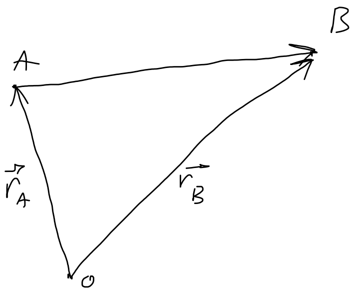
(a) $\vec{AB} + \vec{BC} + \vec{CD} = (\vec{AB} + \vec{BC}) + \vec{CD} =$
$= \vec{AC} + \vec{CD} = \vec{AD}$

(b) $\vec{AB} + \vec{CB} + \vec{DC} = (\vec{AD} + \vec{DC}) + \vec{CB} =$
$= \vec{AC} + \vec{CB} = \vec{AB}$

(c) $\vec{AB} + \vec{BC} + \vec{DA} + \vec{CD} = (\vec{AB} + \vec{BC}) + \vec{DA} + \vec{CD} =$
$= \vec{AC} + \vec{DA} + \vec{CD} = \vec{AC} + (\underbrace{\vec{DA} + \vec{CD}}_{\vec{CA}}) =$
$= \vec{AC} + \vec{CA} = \vec{0}$

$\vec{AC} + \vec{BD} - \vec{BA} + \vec{AD} = \vec{r}_C - \vec{r}_A +$
$+ \vec{r}_D - \vec{r}_B - \vec{r}_A + \vec{r}_B + \vec{r}_D - \vec{r}_A =$
$= -3\vec{r}_A + \vec{r}_C + 2\vec{r}_D$

$\vec{AB} = \vec{r}_B - \vec{r}_A$



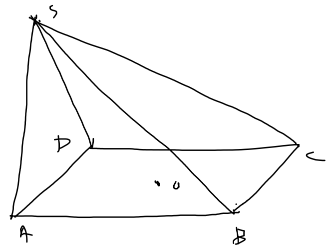
M midpoint of [AB]

$\vec{r}_M = \frac{\vec{r}_A + \vec{r}_B}{2}$

$M \in [AB]$, $\dfrac{AM}{MB} = k$

$$\vec{r}_M = \frac{1}{k+1}\cdot \vec{r}_A + \frac{k}{k+1}\vec{r}_B$$

1.3. Consider a pyramid with the vertex at S and the basis a parallelogram ABCD, whose diagonals are concurrent at O.
Show the equality $\vec{SA}+\vec{SB}+\vec{SC}+\vec{SD}=4\vec{SO}$



$\vec{SA} = \vec{SO} + \vec{OA}$
$\vec{SB} = \vec{SO} + \vec{OB}$
$\vec{SC} = \vec{SO} + \vec{OC}$
$\vec{SD} = \vec{SO} + \vec{OD}$

$\Rightarrow \vec{SA}+\vec{SB}+\vec{SC}+\vec{SD} = 4\vec{SO} + \vec{OA}+\vec{OB}+\vec{OC}+\vec{OD} = 4\vec{SO} + \underbrace{(\vec{OA}+\vec{OC})}_{=\vec{0}} + \underbrace{(\vec{OB}+\vec{OD})}_{=\vec{0}}$

because O is the midpoint of both [AC] and [BD]

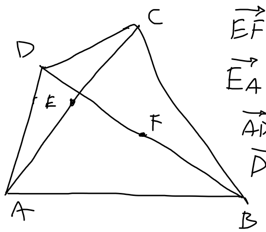
$\Rightarrow \vec{SA}+\vec{SB}+\vec{SC}+\vec{SD} = 4\vec{SO}$

1.4. Let E and F be the midpoints of the diagonals of a quadrilateral ABCD. Show that:

$$\vec{EF} = \frac{1}{2}(\vec{AB}+\vec{CD}) = \frac{1}{2}(\vec{AD}+\vec{CB})$$

($E \in [AC]$, $F \in [BD]$)

QUADRILATERAL



$\vec{EF} = \vec{EA} + \vec{AD} + \vec{DF}$
$\vec{EA} = \frac{1}{2}\vec{CA}$
$\vec{AD} = \vec{AB} + \vec{BC} + \vec{CD}$
$\vec{DF} = \frac{1}{2}\vec{DB}$

$CA = \vec{CD} + DA$