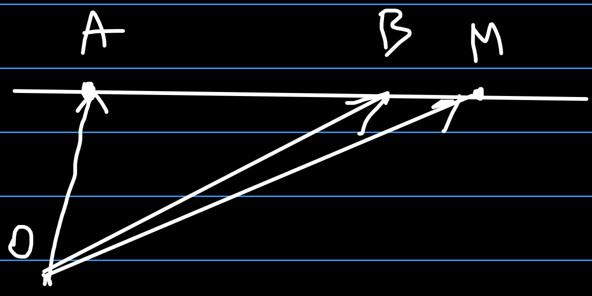
Seminar W3 – 913

ℓ line in 2D/3D

$A, B \in \ell$ distinct

Vector eqn:

$$\vec{r_M} = \lambda \vec{r_A} + (1-\lambda)\vec{r_B}$$



Parametric equation:

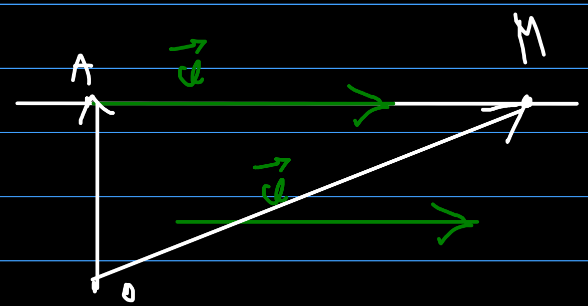
$$\begin{cases} x = \lambda x_A + (1-\lambda) x_B \\ y = \lambda y_A + (1-\lambda) y_B \\ z = \lambda z_A + (1-\lambda) z_B \end{cases}$$

Canonical equation

$$(\Leftarrow)\ \frac{x - x_B}{x_A - x_B} = \frac{y - y_B}{y_A - y_B} = \frac{z - z_B}{z_A - z_B}$$

$A \in \ell$, $\vec{u} \parallel \ell$

$$\vec{r_M} = \vec{r_A} + t \cdot \vec{u}$$



$$\begin{cases} x = x_A + t \cdot x_{\vec{u}} \\ y = y_A + t \cdot y_{\vec{u}} \\ z = z_A + t \cdot z_{\vec{u}} \end{cases}$$

$$(t =)\ \frac{x - x_A}{x_{\vec{u}}} = \frac{y - y_A}{y_{\vec{u}}} = \frac{z - z_A}{z_{\vec{u}}}$$

Watch out for the cases where the denominators are 0

Implicit equation

- in 2D: $Ax + By + c = 0$
- in 3D: $\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 \\ A_2 x + B_2 y + C_2 z + D_2 = 0 \end{cases}$

a plane

another plane

in 2D:

Explicit equation: $y = mx + n$

the slope of the line

$m = \tan\alpha$

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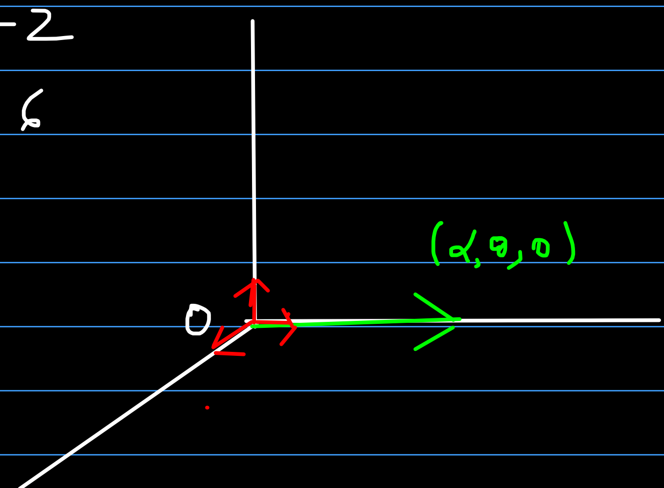
(a) the x-axis

(b) $(d_1): \dfrac{x-1}{2} = \dfrac{y+5}{-3} = \dfrac{z-1}{4}$

(c) $\vec{u}\,(1,0,2)$

(a) $\begin{cases} x = \lambda + 1 \\ y = 0\cdot\lambda + (-2) \\ z = 0\cdot\lambda + 6 \end{cases} \Rightarrow \begin{cases} x = \lambda + 1 \\ y = -2 \\ z = 6 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} y = -2 \\ z = 6 \end{cases}$



(b) $d: \dfrac{x-1}{2} = \dfrac{y+2}{-3} = \dfrac{z-6}{4}$

$d: \begin{cases} x = 2\cdot t + 1 \\ y = -3\cdot t - 2 \\ z = 4t + 6 \end{cases}$

(c)
$$\begin{cases} x = 1 + t\cdot 1 \\ y = -2 + t\cdot 0 \\ z = 6 + t\cdot 2 \end{cases} \Rightarrow \begin{cases} x = 1+t \\ y = -2 \\ z = 6+2t \end{cases} \Rightarrow$$

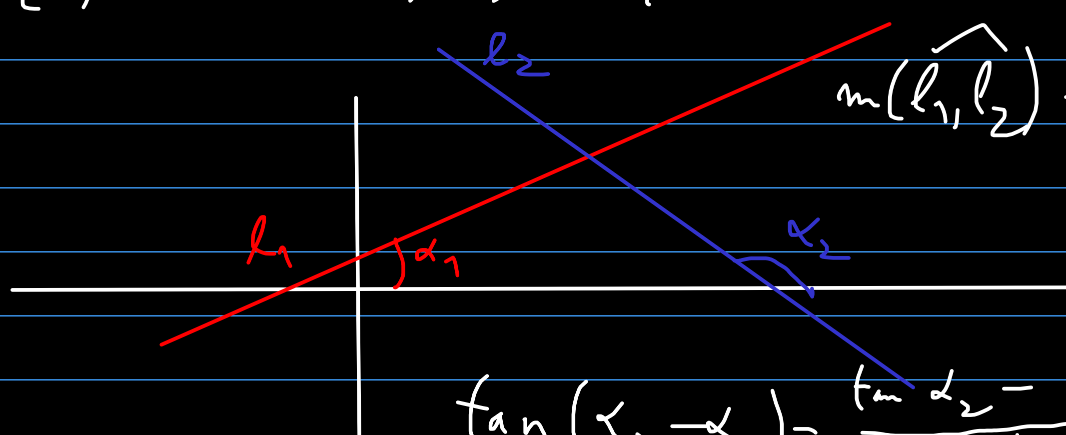
$$\Rightarrow \begin{cases} \dfrac{x-1}{1} = \dfrac{z-6}{2} \\ y = -2 \end{cases}$$

(These parameterizations are actually functions
$$f : \mathbb{R} \to \mathbb{R}^3$$
$$t \mapsto (x(t), y(t), z(t))$$
)

3.17. $d : 2x + 3y + 4 = 0$

Find the equation of a line d_1 through the point $M_0(2,1)$, in the following situations:

(a) $d_1 \parallel d$
(b) $d_1 \perp d$
(c) $m(\widehat{d_1, d}) = \dfrac{\pi}{4}$



$$m(\widehat{l_1, l_2}) = \alpha_2 - \alpha_1$$

$$\tan(\alpha_2 - \alpha_1) = \frac{\tan\alpha_2 - \tan\alpha_1}{1 + \tan\alpha_1 \cdot \tan\alpha_2}$$

$d: 2x+3y+4=0$

$d: y=-\frac{2}{3}x-\frac{4}{3}$

(a) $d \parallel d_1 \Rightarrow m_d = m_{d_1} \Rightarrow m_{d_1} = -\frac{2}{3}$

$d_1: y-y_0 = m(x-x_0)$

$\Rightarrow d_1: y-1 = -\frac{2}{3}(x-2)$

$\Rightarrow d_1: y = -\frac{2}{3}x+\frac{7}{3}$

(b) $d_1: y = mx+n$

$d \perp d_1 \Leftrightarrow m_d \cdot m_{d_1} = -1$

So $m_{d_1} = \frac{3}{2} \Rightarrow d_1: y = \frac{3}{2}x+n$

We plug in the coordinates of M_0:

$1 = \frac{3}{2}\cdot 2+n \Rightarrow n=-2$

$\Rightarrow d_1: y = \frac{3}{2}x-2$

(c) $\frac{\pi}{4} = m(\widehat{d,d_1}) = \alpha_{d_1} - \alpha_d$

$$m_{d_1} = \tan(\alpha_{d_1}) \quad , \quad m_d = \tan(\alpha_d)$$

$$m_d = -\frac{2}{3}$$

$$\tan(\alpha_d - \alpha_{d_1}) = \frac{\tan\alpha_d - \tan\alpha_{d_1}}{1 + \tan\alpha_d \cdot \tan\cdot\alpha_{d_1}}$$

$$\tan\frac{\pi}{4} = 1 = \frac{-\frac{2}{3} - m_{d_1}}{1 - \frac{2}{3} m_{d_1}} \Rightarrow -\frac{2}{3} - m_{d_1} = 1 - \frac{2}{3} m_{d_1} \Rightarrow$$

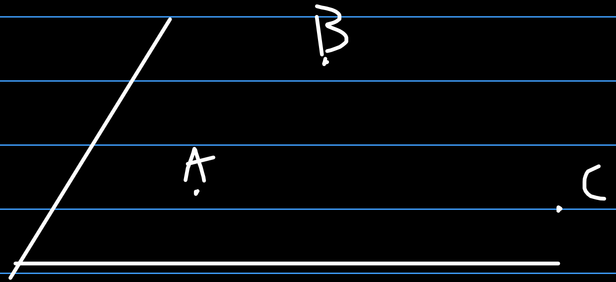
$$\Rightarrow -m_{d_1} + \frac{2}{3} m_{d_1} = 1 + \frac{2}{3} \Rightarrow -\frac{m_{d_1}}{3} = \frac{5}{3} \Rightarrow$$

$$\Rightarrow m_{d_1} = -5 \Rightarrow d_1 : y - 1 = -5(x-2)$$

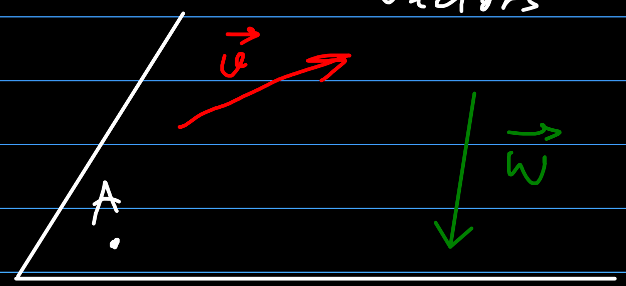
The plane equation

Π plane

A, B, C noncollinear



A point $\vec{u}, \vec{w}$ nonparallel vectors



Vector equation:

$$\vec{r_M} = (1-\lambda-\mu)\vec{r_A} + \lambda\vec{r_B} + \mu\vec{r_C}$$

$$\lambda, \mu \in \mathbb{R}$$

$$\vec{r_M} = \vec{r_A} + \alpha\vec{u} + \beta\cdot\vec{w}$$

$$\alpha, \beta \in \mathbb{R}$$

Parametric equation

$$\begin{cases} x = (1-\lambda-\mu)x_A + \lambda x_B + \mu x_C \\ y = (1-\lambda-\mu)y_A + \lambda y_B + \mu y_C \\ z = (1-\lambda-\mu)z_A + \lambda z_B + \mu\cdot z_C \end{cases}$$

$$\begin{cases} x = x_A + \alpha x_{\vec{u}} + \beta\cdot x_{\vec{w}} \\ y = y_A + \alpha y_{\vec{u}} + \beta\cdot y_{\vec{w}} \\ z = z_A + \alpha z_{\vec{u}} + \beta z_{\vec{w}} \end{cases}$$

Canonical equation:

$$\begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_B-x_A & y_B-y_A & z_B-z_A \\ x_C-x_A & y_C-y_A & z_C-z_A \end{vmatrix} = 0$$

$$\Updownarrow$$

$$\begin{vmatrix} x & y & z & 1 \\ x_A & y_A & z_A & 1 \\ x_B & y_B & z_B & 1 \\ x_C & y_C & z_C & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_{\vec{u}} & y_{\vec{u}} & z_{\vec{u}} \\ x_{\vec{w}} & y_{\vec{w}} & z_{\vec{w}} \end{vmatrix} = 0$$

Implicit form: $Ax+By+Cz+D=0$

$\to$ Advantage: $\vec{n}_{\pi} = (A, B, C)$

3.1. Write the equation of the plane which passes through $M_0(-1, 2, 3)$ and is parallel to the vectors $\vec{v}_1(2, 3, 5)$ and $\vec{v}_2(1, -1, 0)$

$$\Pi: \begin{vmatrix} x+1 & y-2 & z-3 \\ 2 & 3 & 5 \\ 1 & -1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \Pi: \begin{vmatrix} x+y-1 & y-2 & z-3 \\ 5 & 3 & 5 \\ 0 & -1 & 0 \end{vmatrix} = 0$$

$$\Rightarrow \Pi: \begin{vmatrix} x+y-1 & z-3 \\ 5 & 5 \end{vmatrix} = 0$$

$$\Rightarrow \Pi: 5(x+y-1) - 5(z-3) = 0$$

$$\Rightarrow \Pi: 5x + 5y - 5z + 10 = 0$$

$$\Rightarrow \Pi: x + y - z + 2 = 0$$

3.3. Write the equation of the plane which contains the line

$$(d_1)\quad \frac{x-3}{2} = \frac{y+4}{1} = \frac{z-2}{-3}$$

and is parallel to the line

$$(d_2)\quad \frac{x+5}{2} = \frac{y-2}{2} = \frac{z-1}{2}$$

$$A(3,-4,2) \in d_1 \subset \Pi \Rightarrow A \in \Pi$$

$$d_1 \subset \Pi \Rightarrow \vec{d_1} \parallel \Pi$$

We also know that $\vec{d_2} \parallel \Pi$

$$\Rightarrow \begin{vmatrix} x-3 & y+4 & z-2 \\ 2 & 1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2x - 6 + 4z - 8 - 6y - 24 - 2z + 4 +$$
$$+ 6x - 18 - 4y - 16 = 0$$

$$\Rightarrow 8x - 10y + 2z - 66 = 0$$