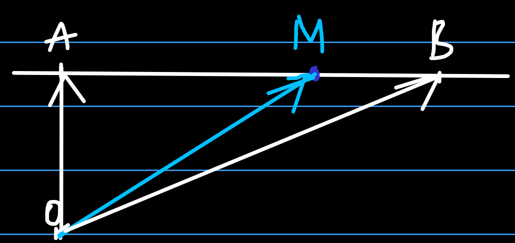
Seminar W3 – 816

ℓ line

$A, B \in \ell,\ A \neq B$



Vector equation

$$\vec{r_M} = \lambda \vec{r_A} + (1-\lambda)\vec{r_B}$$

$$\lambda \in \mathbb{R}$$

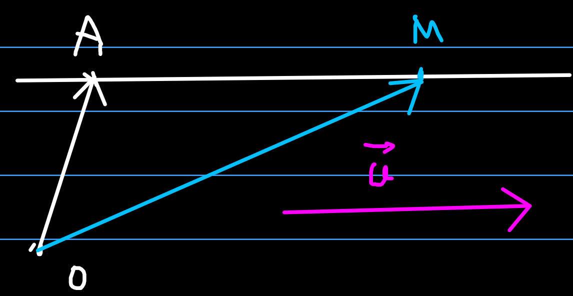
Parametric equation:

$$\begin{cases} x = \lambda x_A + (1-\lambda) x_B \\ y = \lambda y_A + (1-\lambda) y_B \\ z = \lambda z_A + (1-\lambda) z_B \end{cases}$$

Canonical equation:

$$(\lambda =) \frac{x - x_B}{x_A - x_B} = \frac{y-y_B}{y_A - y_B} = \frac{z - z_B}{z_A - z_B}$$

$A \in \ell,\ \vec{u} \parallel \ell$



$$\vec{r_M} = \vec{r_A} + t \cdot \vec{u}$$

$$t \in \mathbb{R}$$

$$\downarrow$$

$$x \cdot \vec{i} + y\vec{j} + z\vec{k} = x_A \cdot \vec{i} + y_A\vec{j} + z_A\vec{k} +$$
$$+ t \cdot x_{\vec{u}} \cdot \vec{i} + t \cdot y_{\vec{u}} \cdot \vec{j} + t \cdot z_{\vec{u}} \cdot \vec{k}\,!$$

$$\begin{cases} x = x_A + t \cdot x_{\vec{u}} \\ y = y_A + t \cdot y_{\vec{u}} \\ z = z_A + t \cdot z_{\vec{u}} \end{cases}$$

$$(t =) \frac{x - x_A}{x_{\vec{u}}} = \frac{y - y_A}{y_{\vec{u}}} = \frac{z - z_A}{z_{\vec{u}}}$$

! The transition parametric $\longrightarrow$ canonical needs to be done with care in the cases where the denominators might be zero.

e.g. if $x_{\vec{u}} = 0$ and $y_{\vec{u}} \neq 0$, $z_{\vec{u}} \neq 0$

$$\begin{cases} x = x_A \\ \dfrac{y - y_A}{y_{\vec{u}}} = \dfrac{z - z_A}{z_{\vec{u}}} \end{cases}$$

if $x_{\vec{u}} = 0$ and $y_{\vec{u}} = 0$, $z_{\vec{u}} \neq 0$

$$\begin{cases} x = x_A \\ y = y_A \end{cases}$$

Implicit form:

3D

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 \\ A_2 x + B_2 y + C_2 z + D_2 = 0 \end{cases}$$

2D

$$Ax + By + C = 0$$

Explicit form: $\longrightarrow$ in the 2D case:

$$y = mx + n$$

slope

3.2. Write the equation of the line which passes through $A(1,-2,6)$ and is parallel to:

(a) the x-axis

(b) (d_1) $\frac{x-1}{2} = \frac{y+5}{-3} = \frac{z-1}{4}$

(c) $\vec{u}(1,0,2)$

(a) $d \parallel Ox \Rightarrow d \parallel \vec{u}(1,0,0)$

parametric

$\Rightarrow \begin{cases} x = 1 + t \\ y = -2 + 0\cdot t \\ z = 6 + 0\cdot t \end{cases}$

canonical

$\begin{cases} y = -2 \\ z = 6 \end{cases}$

(b) $d: \begin{cases} x = 1 + 2\cdot t \\ y = -2 + (-3)\cdot t \\ z = 6 + 4t \end{cases}$

(c) $d: \begin{cases} \frac{x-1}{1} = \frac{z-6}{2} \\ y = -2 \end{cases}$

3.18. The vertices of the triangle ABC are the intersection points of the lines:

$d_1: 4x+3y-5=0$

$d_2: x-3y+10=0$

$d_3: x-2=0$

(a) Find the coordinates of A, B, C

(b) Find the equations of the medians of the triangle

(c) Find the equations of the heights of the triangle.

(a) $\{A\} = d_2 \cap d_3$ $\{B\} = d_3 \cap d_1$ $\{C\} = d_1 \cap d_2$

$$A: \begin{cases} x-2=0 \\ x-3y+10=0 \end{cases} \Rightarrow \begin{cases} x=2 \\ 12-3y=0 \end{cases} \Rightarrow \begin{cases} 3y=12 \\ x=2 \end{cases} \Rightarrow$$

$\Rightarrow A(2,4)$

$$B: \begin{cases} x-2=0 \\ 4x+3y-5=0 \end{cases} \Leftrightarrow \begin{cases} x=2 \\ 8+3y-5=0 \end{cases} \Leftrightarrow \begin{cases} x=2 \\ y=-1 \end{cases}$$

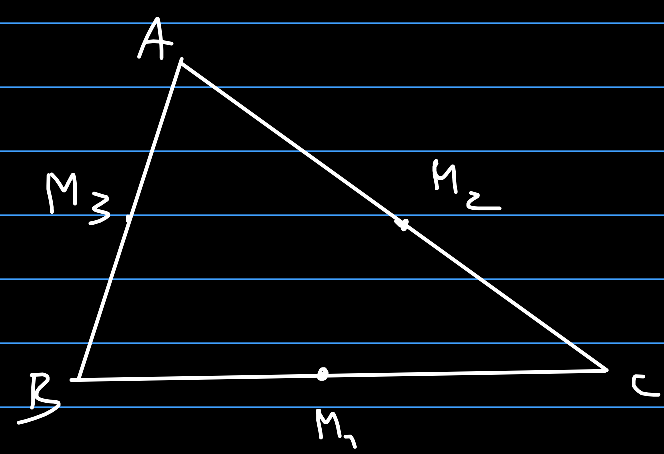
$\Rightarrow B(2,-1)$

$$c : \begin{cases} 4x+3y-5=0 \\ x-3y+10=0 \end{cases} \Rightarrow \begin{cases} 5x+5=0 \\ x-3y+10=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x=-1 \\ -1-3y+10=0 \end{cases} \Rightarrow \begin{cases} x=-1 \\ 3y=9 \end{cases} \Rightarrow \begin{cases} x=-1 \\ y=3 \end{cases}$$

$$\Rightarrow C(-1,3)$$

(b)



$$M_1\left(\frac{2+(-1)}{2}, \frac{-1+3}{2}\right) \Rightarrow M_1\left(\frac{1}{2}, 1\right)$$

$$M_2\left(\frac{2+(-1)}{2}, \frac{4+3}{2}\right) \Rightarrow M_2\left(\frac{1}{2}, \frac{7}{2}\right)$$

$$M_3\left(\frac{2+2}{2}, \frac{4+(-1)}{2}\right) \Rightarrow M_3\left(2, \frac{3}{2}\right)$$

$$m_{AM_1} = \frac{y_A - y_{M_1}}{x_A - x_{M_1}} = \frac{4-1}{2-\frac{1}{2}} = 2$$

$\Rightarrow AM_1: \ y - y_{M_1} = m \cdot (x - x_{M_1})$

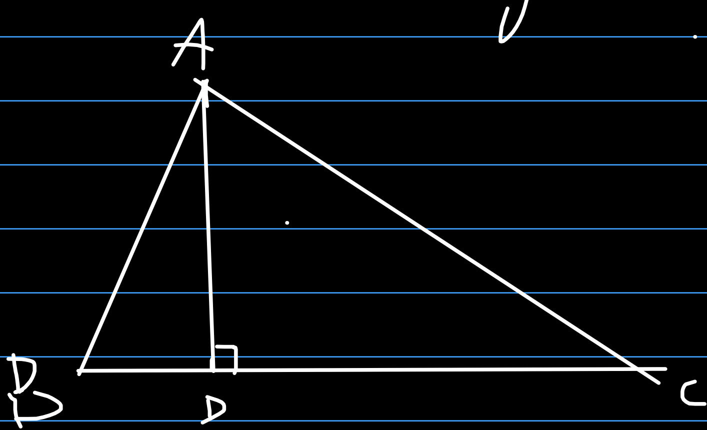
$$y - 1 = 2 \cdot \left(x - \frac{1}{2}\right)$$

$$BM_2: \ \frac{y - y_B}{y_{M_2} - y_B} = \frac{x - x_B}{x_{M_2} - x_B}$$

$$\Rightarrow \ \frac{y + 1}{\frac{7}{2} + 1} = \frac{x - 2}{\frac{1}{2} - 2}$$

CM_3 : Same thing

(c)



$AD \perp BC \Rightarrow m_{AD} \cdot m_{BC} = -1$

$$m_{BC} = \frac{y_B - y_C}{x_B - x_C} = \frac{2 + 1}{-1 - 3} = -\frac{3}{4}$$

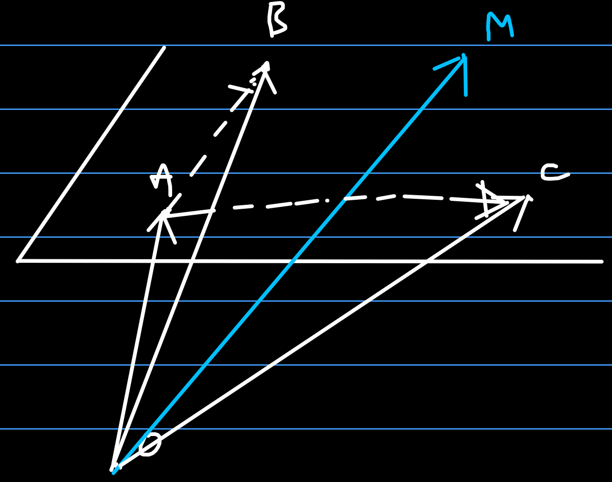
$$\Rightarrow m_{A_1} = \frac{4}{3} \Rightarrow AD: \ y - y_A = m_{AD} \cdot (x - x_A)$$

$$\Rightarrow \ AD: \ y - 4 = \frac{4}{3}(x - 2)$$

Planes

Π plane

$A, B, C \in \Pi$ noncollinear



Vector equation

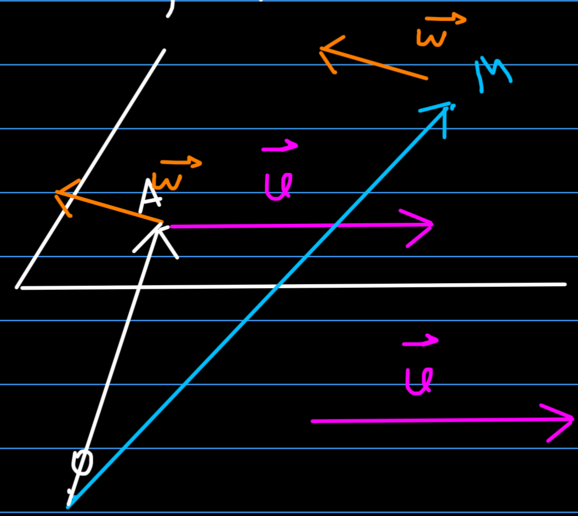
$$\vec{r}_M = (1-\lambda-\mu)\vec{r}_A + \lambda \vec{r}_B + \mu \vec{r}_C$$

$$\lambda, \mu \in \mathbb{R}$$

Parametric equation

$$\begin{cases} x = (1-\lambda-\mu)x_A + \lambda x_B + \mu x_C \\ y = (1-\lambda-\mu)y_A + \lambda y_B + \mu y_C \\ z = (1-\lambda-\mu)z_A + \lambda z_B + \mu z_C \end{cases}$$

$A \in \Pi$, $\vec{u}, \vec{w} \parallel \Pi$



$$\vec{r}_M = \vec{r}_A + \alpha \vec{u} + \beta \cdot \vec{w}$$

$$\alpha, \beta \in \mathbb{R}$$

$$\begin{cases} x = x_A + \alpha x_{\vec{u}} + \beta x_{\vec{w}} \\ y = y_A + \alpha y_{\vec{u}} + \beta y_{\vec{w}} \\ z = z_A + \alpha z_{\vec{u}} + \beta z_{\vec{w}} \end{cases}$$

Canonical equation

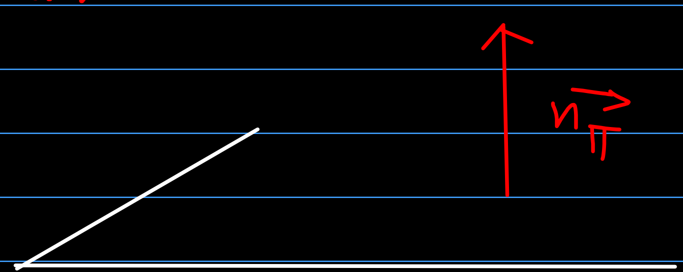
$$\begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_B-x_A & y_B-y_A & z_B-z_A \\ x_C-x_A & y_C-y_A & z_C-z_A \end{vmatrix} = \begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_{\vec{u}} & y_{\vec{u}} & z_{\vec{u}} \\ x_{\vec{w}} & y_{\vec{w}} & z_{\vec{w}} \end{vmatrix} = 0$$

$= 0 \iff$

$$\begin{vmatrix} x & y & z & 1 \\ x_A & y_A & z_A & 1 \\ x_B & y_B & z_B & 1 \\ x_C & y_C & z_C & 1 \end{vmatrix} = 0$$

Implicit equation: $A \cdot x + B \cdot y + C \cdot z + D = 0$

$\longrightarrow$ normal vector $\vec{n_\pi}(A, B, C)$



3.1. Write the equation of the plane which passes through $M_0(-1, 2, 0)$ and is parallel to the vectors $\vec{u_1}(1,2,3)$, $\vec{u_2}(0,-1,6)$.

$$\begin{vmatrix} x+1 & y-2 & z \\ 1 & 2 & 3 \\ 0 & -1 & 6 \end{vmatrix} = 0$$

$$\Rightarrow \quad 12(x+1) - z + 3(x+1) - 6(y-2) = 0$$

$$\Rightarrow \quad 15x - 6y - z + 27 = 0$$

$$\Rightarrow \quad z = 15x - 6y + 27$$

$$\Rightarrow \begin{cases} x = \lambda \\ y = \mu \\ z = 15\lambda - 6\mu + 27 \end{cases}$$

3.3. Write the equation of the plane which contains the line

$$(d_1): \frac{x-3}{2} = \frac{y+4}{1} = \frac{z-2}{-3}$$

and is parallel to the line

$$(d_2): \frac{x+5}{2} = \frac{y-2}{2} = \frac{z-1}{2}$$

$$A(3,-4,2) \in d_1 \subset \Pi \Rightarrow A \in \Pi$$

$$d_1 \subset \Pi \Rightarrow \vec{d_1} \parallel \Pi$$

$$d_2 \parallel \Pi \Rightarrow \vec{d_2} \parallel \Pi$$

$$\Rightarrow \begin{vmatrix} x-3 & y+4 & z-2 \\ 2 & 1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = 0$$