Training

miercuri, 1 iunie 2022 15:55

$$K_2(x,t)$$

$$\int_{-1}^{1} (\cancel{x}-t)_+ dt - (-1-1)_+ - (-0)_+ =$$

$$= \int_{-1}^{0} (-t)_+ dt + \int_{0}^{1} (-t)_+ dt - 1 =$$

$$= \int_{-1}^{0} -t\, dt + \int_{0}^{1} 0\, dt - 1 =$$

$$= -\frac{t^2}{2}\Big|_{-1}^{0} - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\int_{-1}^{1} (x-t)_+ d\cancel{t} - (\underbrace{-1-t}_{\leq 0})_+ - (\underbrace{1-t}_{\geq 0})_+ =$$

$$\boxed{f = (x-t)_+,\ f(-1) = (-1-t)_+}$$

$$= \int_{-1}^{t} \underbrace{(x-t)_+}_{\leq 0} dx + \int_{t}^{1} \underbrace{(x-t)_+}_{\geq 0} dx - 0 - (1-t)$$

$$-1 \leq t \leq 1 \Leftrightarrow 1 \geq -t \geq -1 \overset{1+1}{\Rightarrow} 2 \geq 1-t \geq 0$$

$$1 \geq -t \geq -1 \overset{1-1}{\Rightarrow} 0 \geq 1-t \geq -2$$

$1 \geq -t \geq -1 \Rightarrow 0 \geq 1-t \geq -2$

$$= 0 + \int_t^1 (x-t)\,dx + t - 1 =$$

$$= t - 1 + \left(\frac{x^2}{2} - tx\right)\Bigg|_{x=t}^{x=1}$$

$$= \cancel{t} - 1 + \frac{1}{2} - \cancel{t} - \frac{t^2}{2} + t^2$$

$$= \frac{1}{2}t^2 - \frac{1}{2} = \frac{1}{2}\underbrace{(t^2-1)}_{\leq 0}$$

$K_2(x,t) \leq 0, \forall x, t \in [-1, 1]$

- constant sign on $[-1, 1]$

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$$\Rightarrow \quad Rf = \frac{1}{2} f''(\xi) \cdot Re_2, \ \xi \in [-1, 1]$$

$$Re_2 = -\frac{4}{3} \qquad Rf = -\frac{2}{3} f''(\xi)$$

$$|Rf| \leq \frac{2}{3} \|f''\|_\infty$$

$$(P_1 f)(x) = \frac{1-x}{2} f(-1) + \frac{x+1}{2} f(1)$$

$$\int_{-1}^{1} (P_1 f)(x)\, dx = f(-1) + f(1)$$

$$Rf = \int_{-1}^{1} f(x)\, dx - (f(-1) + f(1))$$

$$Re_0 = Re_1 = 0 \quad f \in \Pi^n[a,b] \subseteq C^n[a,b]$$

$$Re_1 = \int_{-1}^{1} x\, dx - (-1+1) = 0$$

$f(x) = x^1$ $\qquad f \in C^2[-1, 1]$

$$Re_2 = -\frac{4}{3} \neq 0 \Rightarrow \boxed{d = 1}$$

$$Rf = \int_{-1}^{1} K_2(x,t)\, f''(t)\, dt$$

$$K_n(x,t) = \frac{1}{(n-1)!} L\left[(x-t)_+^{n-1}\right]$$

$$K_2(x,t) = R\left((x-t)_+\right) \qquad f = (x-t)_+$$

$$= \int_{-1}^{1} (x-t)_+ \, dx - (\underbrace{-1-t}_{\leq 0})_+ - (\underbrace{1-t}_{\geq 0})_+$$

$$= \qquad\qquad - \; 0 - (1-t)$$