Seminar W3 - 914

l line

Vector eqns.:

- 2 points A, B: $\vec{r}_M = \lambda \vec{r}_A + 1-\lambda\ \vec{r}_B$
- 1 point + 1 vector: $A, \vec{v}$; $\vec{r}_M = \vec{r}_A + t \cdot \vec{v}$

$\Downarrow$ **Parametric equation**

A, B:

$$\begin{cases} x = \lambda x_A + (1-\lambda) x_B \\ y = \lambda y_A + (1-\lambda) y_B \\ z = \lambda z_A + (1-\lambda) z_B \end{cases}$$

$A, \vec{u}$:

$$\begin{cases} x = x_A + t\, x_{\vec{u}} \\ y = y_A + t \cdot y_{\vec{u}} \\ z = z_A + t \cdot z_{\vec{u}} \end{cases}$$

$\Downarrow$ **Canonical equation**

$$\frac{x - x_A}{x_B - x_A} = \frac{y - y_A}{y_B - y_A} \left(= \frac{z - z_A}{z_B - z_A} \right)$$

$$(t =) \frac{x - x_A}{x_{\vec{u}}} = \frac{y - y_A}{y_{\vec{u}}} = \frac{z - z_A}{z_{\vec{u}}}$$

! if $x_{\vec{u}} \cdot y_{\vec{u}} \cdot z_{\vec{u}} = 0$ we need to be careful

$\Downarrow$ **implicit equation**

2D: $Ax + By + C = 0$

3D:

$$\begin{cases} A_1 x + B_1 y + C_1 z + D_1 = 0 \\ A_2 x + B_2 y + C_2 z + D_2 = 0 \end{cases}$$

$\cap$ plane

plane

implicit equation

2D: $Ax+By+c=0$

3D: $\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$

plane

plane

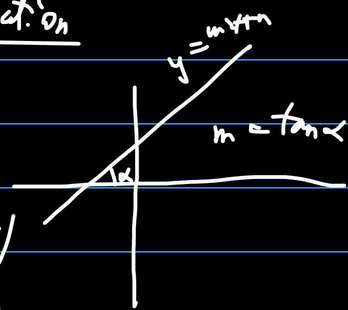
$\Downarrow$ explicit equation

$y = mx+n$

$\downarrow$

the slope, $\tan(\alpha)$

$\alpha = (\widehat{l, Ox})$



3.2. Write the equation of the line which passes through $A(1,-2,6)$ and is parallel to:

(a) The x-axis

(b) (d_1): $\dfrac{x-1}{2} = \dfrac{y+5}{-3} = \dfrac{z-1}{4}$

(c) $\vec{u}(1,0,2)$

(a) $\vec{u}_{Ox} = (1,0,0)$

$\Rightarrow \vec{r_M} = \vec{r_A} + \lambda \cdot \vec{u} \Rightarrow$

$\Rightarrow \begin{cases} x = 1+\lambda \\ y = -2 \\ z = 6 \end{cases}$

(b) $\vec{d_1}(2,-3,4) \Rightarrow \begin{cases} x = 1+2\lambda \\ y = -2-3\lambda \\ z = 6+4\lambda \end{cases}$

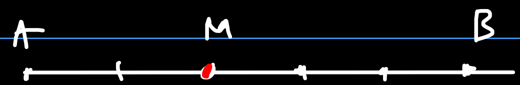
(c) $\begin{cases} x = 1 + \lambda \\ y = -2 \\ z = 6 + 2\lambda \end{cases} \Rightarrow \begin{cases} \dfrac{x-1}{1} = \dfrac{z-6}{2} \\ y = -2 \end{cases}$

3.10. Find the equation of the line passing through the intersection point of the lines

$d_1: 2x - 5y - 1 = 0$

$d_2: x + 4y - 7 = 0$

and through a point $M \in [AB]$, $A(4,-3)$, $B(-1,2)$ which divides $[AB]$ into the ratio $k = \frac{2}{3}$



Let $\{C\} = d_1 \cap d_2 \Rightarrow C: \begin{cases} 2x - 5y - 1 = 0 \\ x + 4y - 7 = 0 \end{cases} \Rightarrow$

$\Rightarrow \begin{cases} x = 7 - 4y \\ 14 - 8y - 5y - 1 = 0 \end{cases} \Rightarrow \begin{cases} x = 7 - 4y \\ 13 - 13y = 0 \end{cases}$

$\Rightarrow C(3, 1)$

$\begin{cases} x_M = x_A \cdot \dfrac{1}{1+k} + x_B \cdot \dfrac{k}{1+k} = 4 \cdot \dfrac{3}{5} + (-1) \cdot \dfrac{2}{5} = 2 \\ y_M = y_A \cdot \dfrac{1}{1+k} + y_B \cdot \dfrac{k}{1+k} = (-3) \cdot \dfrac{3}{5} + 2 \cdot \dfrac{2}{5} = -1 \end{cases}$

$\Rightarrow M(2, -1)$

$\Rightarrow CM: \dfrac{x-3}{2-3} = \dfrac{y-1}{-1-1} \Rightarrow \dfrac{x-3}{-1} = \dfrac{y-1}{-2}$

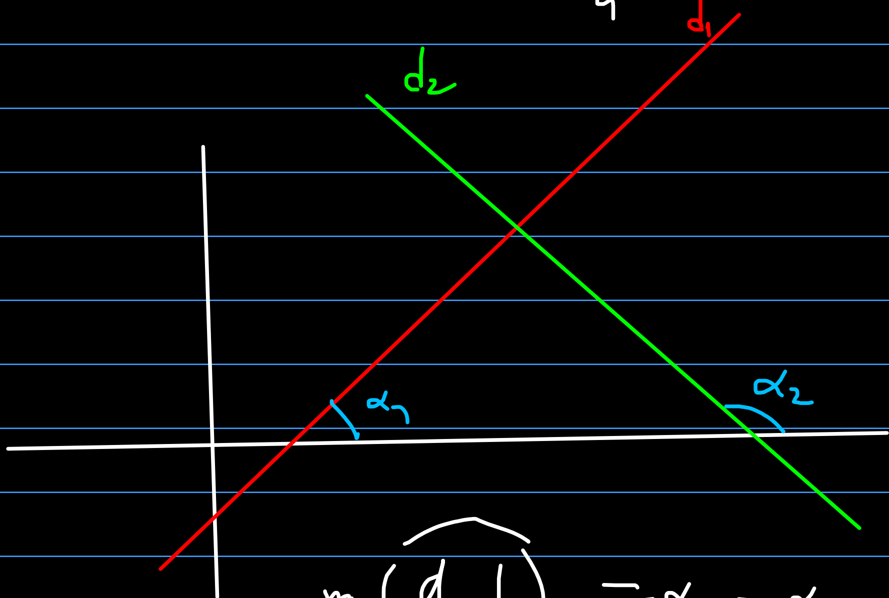
317. $d: 2x+3y+4=0$

Find the equation of a line d_1 through the point $M_0(2,1)$, in the following situations:

(a) $d_1 \parallel d$

(b) $d_1 \perp d$

(c) $m(\widehat{d, d_1}) = \frac{\pi}{4}$



$$m(\widehat{d_1, d_2}) = \alpha_2 - \alpha_1$$

$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \cdot \tan y}$$

$d: 2x+3y+4=0$

$d: \; y = -\frac{2}{3}x - \frac{4}{3} \Rightarrow m = -\frac{2}{3}$

(a) $d \parallel d_1 \Rightarrow m_{d_1} = m_d = -\frac{2}{3}$

$\Rightarrow d_1: \; y = -\frac{2}{3}x + n_1$

Because $M_0 \in d_1 \Rightarrow 1 = -\frac{2}{3} \cdot 2 + n_1$

$\Rightarrow n_1 = \frac{7}{3} \Rightarrow d_1 : y = -\frac{2}{3}x + \frac{7}{3}$

(b) $d \perp d_1 \Leftrightarrow m_d \cdot m_{d_1} = -1$

$\Rightarrow m_{d_1} = \frac{-1}{-\frac{2}{3}} = \frac{3}{2} \Rightarrow d_1 : y - y_0 = m(x - x_0)$

$\Rightarrow d_1 : y - 1 = \frac{3}{2} \cdot (x - 2)$

$\Rightarrow d_1 : y = \frac{3}{2}x - 2$

(c) $m(\widehat{d, d_1}) = \frac{\pi}{4}$, $\alpha_d = m(\widehat{d, Ox})$

$\alpha_{d_1} = m(\widehat{d_1, Ox})$

$\tan \alpha_d = m_d = -\frac{2}{3}$

$\tan \alpha_{d_1} =: t$

$\Rightarrow 1 = \tan\left(\frac{\pi}{4}\right) = \tan\left(\alpha_d - \alpha_{d_1}\right) =$

$= \frac{\tan \alpha_d - \tan \alpha_{d_1}}{1 + \tan \alpha_d \cdot \tan \alpha_{d_1}} = \frac{-\frac{2}{3} - t}{1 - \frac{2}{3}t}$

$\Rightarrow -\frac{2}{3} - t = 1 - \frac{2}{3}t \Rightarrow t = \frac{-\frac{2}{3} - 1}{1 - \frac{2}{3}} \Leftrightarrow$

(c) $f = \dfrac{-\frac{5}{3}}{\frac{1}{3}} = -5 \Rightarrow m_{t} = -5 \Rightarrow$

$\Rightarrow \quad y - 1 = -5(x - 2)$

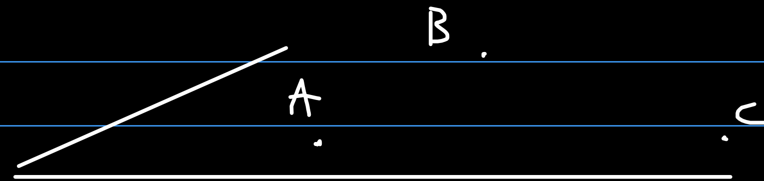
$\Rightarrow \quad y = -5x + 11$

Plane equations: Let π be a plane

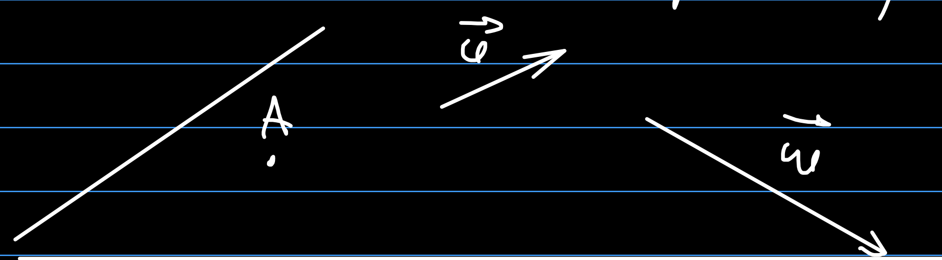
Vector form:

- 3 noncollinear points A, B, C

$$\vec{r}_M = \alpha \vec{r}_A + \beta \vec{r}_B + (1-\alpha-\beta)\vec{r}_C$$



- 1 point + 2 vectors (nonparallel)



$$\vec{r}_M = \vec{r}_A + \lambda \vec{u} + \mu \cdot \vec{w}$$

Parametric form:

- $$\begin{cases} x = \alpha x_A + \beta x_B + (1-\alpha-\beta) x_C \\ y = \alpha y_A + \beta y_B + (1-\alpha-\beta) y_C \\ z = \alpha z_A + \beta z_B + (1-\alpha-\beta) z_C \end{cases}$$

$$\begin{cases} x = x_A + \lambda\, x_{\vec{u}} + \mu \cdot x_{\vec{w}} \\ y = y_A + \lambda\, y_{\vec{u}} + \mu \cdot y_{\vec{w}} \\ z = z_A + \lambda\, z_{\vec{u}} + \mu \cdot z_{\vec{w}} \end{cases}$$

Canonical form

...

$$\begin{vmatrix} x - x_C & y - y_C & z - z_C \\ x_A - x_C & y_A - y_C & z_A - z_C \\ x_B - x_C & y_B - y_C & z_B - z_C \end{vmatrix} = 0$$

$$\begin{vmatrix} x - x_A & y - y_A & z - z_A \\ x_{\vec{u}} & y_{\vec{u}} & z_{\vec{u}} \\ x_{\vec{w}} & y_{\vec{w}} & z_{\vec{w}} \end{vmatrix} = 0$$

The implicit form: $Ax + By + Cz + D = 0$

→ ADVANTAGE: $\vec{n_{\pi}} = (A, B, C)$

3.1. Write the equation of the plane which passes through $M_0(-1, 2, 3)$ and is parallel to the vectors $\vec{u_1}(2, 1, 0)$ and $\vec{u_2}(5, 2, 3)$.

$$\Pi: \begin{vmatrix} x+1 & y-2 & z-3 \\ 2 & 1 & 0 \\ 5 & 2 & 3 \end{vmatrix} = 0$$

$$\Pi: (-1)^{1+1} \cdot (x+1) \cdot \begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} + (-1)^{1+2} \cdot (y-2) \cdot \begin{vmatrix} 2 & 0 \\ 5 & 3 \end{vmatrix} +$$

$$+ (-1)^{1+3} \cdot (z-3) \cdot \begin{vmatrix} 2 & 1 \\ 5 & 2 \end{vmatrix} = 0$$

$$\Rightarrow \Pi: 3(x+1) - 6(y-2) + (-1)(z-3) = 0$$

$$\Rightarrow \Pi: 3x - 6y - z + 18 = 0$$

3.3. Write the equation of the plane which contains the line:

$$(d_1)\ \frac{x-3}{2} = \frac{y+4}{1} = \frac{z-2}{-3}$$

and is parallel to the line

$$(d_2)\ \frac{x+5}{2} = \frac{y-2}{2} = \frac{z-1}{2}$$

$d_1 \subset \Pi \Rightarrow (3,-4,2) \in d_1 \Rightarrow (3,-4,2) \in \Pi$

$\Rightarrow \vec{d_1} \parallel \Pi$

We also know that $d_2 \parallel \Pi$, so Π is the plane given by $A(3,-4,2)$ and the vectors $\vec{d_1}$ and $\vec{d_2}$:

$$\begin{vmatrix} x-3 & y+4 & z-2 \\ 2 & 1 & -3 \\ 2 & 2 & 2 \end{vmatrix} = 0$$