Seminar W1 - 914

02 December 2020 12:14

$CC = \max\left(M, S + \frac{M}{2}\right)$

$FG = \max\left(\frac{4}{10}CC + \frac{6}{10}E, E\right)$

Minimal condition:

$CC \geq 4.5$, $E \geq 4.5$

S = ex. solved in class (2/5 pts)

75%

We are working in the Euclidean plane/space: $\mathbb{R}^2 / \mathbb{R}^3$

Analytic Geometry

↳ the study of geometry through a Cartesian line

Descartes → "working with coordinates"

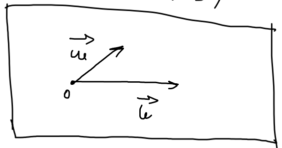
Cartesian reference system:

$(O; b)$

O → a point, the origin of the plane/space

$b = (e_1; e_2)$ basis of $\mathbb{R}^2$

$b = (e_1; e_2; e_3)$ basis of $\mathbb{R}^3$

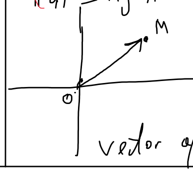


$(\vec{u}, \vec{w})$ basis of $\mathbb{R}^2$ ($\downarrow$ $\mathbb{E}^2$) $\Leftrightarrow$ $\vec{u}, \vec{w}$ lin. indep $\Leftrightarrow$

$\Leftrightarrow \nexists \alpha \in \mathbb{R}: \vec{w} = \alpha \cdot \vec{u} \Leftrightarrow$

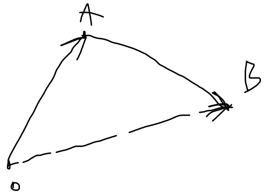
$\Leftrightarrow$ $\vec{u}$ and $\vec{w}$ are not parallel

Fix the reference system $(O; \vec{i}, \vec{j})$ and assume that $\vec{i} \perp \vec{j}$ (the reference system is orthogonal) and $\|\vec{i}\| = \|\vec{j}\| = 1$ (orthonormal)

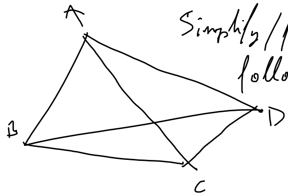


$\vec{OM} = \vec{r_M}$ = "the position vector of the point M"

~~At~~ $\forall A, B \in \mathbb{R}^2$: $\vec{AB} = \vec{r_B} - \vec{r_A}$



1.1: Consider a tetrahedron ABCD. Simplify/find the following sums of vectors:



(a) $\vec{AB} + \vec{BC} + \vec{CD}$

(b) $\vec{AD} + \vec{CB} + \vec{DC}$

(c) $\vec{AB} + \vec{BC} + \vec{DA} + \vec{CD}$

(a) $\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{AC} + \vec{CD} = \vec{AD}$

$\Rightarrow \vec{AB} + \vec{BC} + \vec{CD} = \vec{AD}$

(b) $\vec{AB} + \vec{CB} + \vec{DC} = \vec{r_D} - \vec{r_A} + \vec{r_B} - \vec{r_C} +$

$+ \vec{r_C} - \vec{r_D} = \vec{r_B} - \vec{r_A} = \vec{AB}$

(c) $\vec{AB} + \vec{BC} = \vec{AC}$

$\vec{DA} = -\vec{AD}$

$\vec{AB} + \vec{BC} + \vec{DA} + \vec{CD} = \vec{AC} - \vec{AD} + \vec{CD} =$

$= \vec{AC} - (\vec{AB} + \vec{BC} + \vec{CD}) + \vec{CD} = \underbrace{\vec{AC} + \vec{BA}}_{\vec{BC}} +$

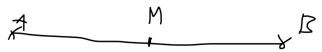
$+ \vec{CB} + \underbrace{\vec{DC} + \vec{CD}}_{= \vec{0}} = \vec{BC} + \vec{CB} = \vec{0}$

If we fix a reference system $\mathcal{R}(O; e_1, e_2)$ then the coordinates of a point ~~$P \in \mathbb{R}^2$~~

$P \in \mathbb{R}^2$

$[P]_{\mathcal{R}} = [\vec{OP}]_b = [\vec{r_P}]_b =$

$= \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow \vec{r_P} = x\vec{e_1} + y\vec{e_2}$



A, B points, M midpoint of [AB]

Let O be a point.

$$\vec{OM} = \frac{\vec{OA}+\vec{OB}}{2}$$

$\Rightarrow$ If we fix a reference system

then $\vec{r_M} = \frac{\vec{r_A}+\vec{r_B}}{2}$

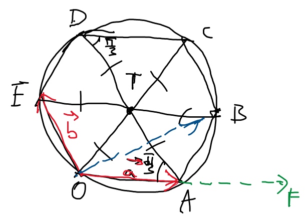
$\frac{AM}{MB} = k \in \mathbb{R}$ $\vec{r_M} = \frac{k\cdot\vec{r_B}+1\cdot\vec{r_A}}{k+1}$

1.2. OABCDE regular hexagon

$\vec{OA} = \vec{a}$, $\vec{OE} = \vec{b}$

Express the vectors $\vec{OB}, \vec{OC}, \vec{OD}$ in terms of $\vec{a}$ and $\vec{b}$.

Show that $\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD}+\vec{OE} = 3\vec{OC}$



Say T is the intersection point of the diagonals. The sides are the same $\Rightarrow$ the angles around T are the same $\Rightarrow$ they measure $\frac{2\pi}{6} = \frac{\pi}{3}$

$\Rightarrow$ all the triangles are equilateral and congruent $\Rightarrow$ $AB \overset{\parallel}{=} DE$

$OA \overset{\parallel}{=} DC$

$BC \overset{\parallel}{=} OE$

$\widehat{CDT} \equiv \widehat{TAO} \Rightarrow CD \parallel OA$
$\triangle CDT \equiv \triangle TAO \Rightarrow CD = OA$ $\Big\} \Rightarrow CD \overset{\parallel}{=} OA$

$\triangle BAF \equiv \triangle ETOA$

$BE = OF$, $BE \parallel OF$ $\Big\} \Rightarrow BFOE$ parallelogram

$\Rightarrow OE \overset{\parallel}{=} FB \Rightarrow \vec{FB} = \vec{b}$

$\vec{OB} = \vec{OA} + \vec{AF} + \vec{FB} = 2\vec{a} + \vec{b}$

$\vec{OC} = \vec{OF} + \vec{FC}$

$DC \overset{\parallel}{=} AF \Rightarrow DCFA$ parallelogram

$\Rightarrow \vec{FC} = \vec{AD} = 2\vec{b}$

$\vec{OC} = 2\vec{a} + 2\vec{b}$

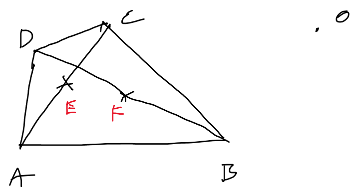
$\vec{OD} = \vec{OC} - \vec{DC} = (2\vec{a}+2\vec{b}) - \vec{a} = \vec{a} + 2\vec{b}$

$\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD}+\vec{OE} = \vec{a} + (2\vec{a}+\vec{b}) + (2\vec{a}+2\vec{b}) + (\vec{a}+2\vec{b}) + \vec{b} =$

$= 6\vec{a} + 6\vec{b} = 3\cdot(2\vec{a}+2\vec{b}) = 3\cdot\vec{OC}$

1.4. Let E and F be the midpoints of the diagonals of a quadrilateral ABCD. Show that:

$$\vec{EF} = \frac{1}{2}\left(\vec{AB}+\vec{CD}\right) = \frac{1}{2}\cdot\left(\vec{AD}+\vec{CB}\right)$$



Let O be an arbitrary point.

$\vec{EF} = \vec{EO} + \vec{OF} = \frac{\vec{AO}+\vec{CO}}{2} + \frac{\vec{OD}+\vec{OB}}{2}$

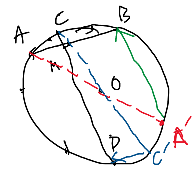
$= \frac{\vec{AO}+\vec{OD}}{2} + \frac{\vec{CO}+\vec{OB}}{2} = \frac{\vec{AD}+\vec{CB}}{2}$

$\vec{EF} = \frac{\vec{AO}+\vec{OB}}{2} + \frac{\vec{CO}+\vec{OD}}{2} = \frac{1}{2}(\vec{AB}+\vec{CD})$

1.7. Consider two perpendicular chords AB and CD of a given circle and $\{M\} = AB \cap CD$. Show that

$$\vec{OA}+\vec{OB}+\vec{OC}+\vec{OD} = 2\vec{OM}$$

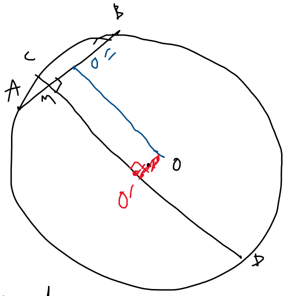
(where O is the center of the circle)



$\vec{OA}+\vec{OB} = \vec{A'O}+\vec{OB} =$

$= \vec{A'B}$

$\vec{OC}+\vec{OD} = \vec{C'D}$



We draw $OO' \perp DC$, $O' \in (DC)$

$OO'' \perp AB$, $O'' \in [AB]$

~~$\Rightarrow OA'MO''$ rectangle~~

$\left.\begin{array}{l} OO' \perp AD \\ BC \perp AD \end{array}\right\} \Rightarrow OO' \parallel BC$

$\left.\begin{array}{l} OO'' \perp BC \\ AD \perp BC \end{array}\right\} \Rightarrow OO'' \parallel AD$

$\Rightarrow OA'MO''$ rectangle