Theoretical part

An interpretation i which evaluates the formula U as true is called a model of U.

Distributive laws

$$U \wedge (V \vee Z) = (U \wedge V) \vee (U \wedge Z)$$
$$U \vee (V \wedge Z) = (U \vee V) \wedge (U \vee Z)$$

Idempotery laws

$$V \wedge V \equiv V$$
$$U \wedge U \equiv U$$

Commutative laws

$$U \wedge V \equiv V \wedge U$$
$$U \vee V \equiv V \vee U$$

De Morgan Laws

$$\neg(U \wedge V) = \neg U \vee \neg V$$
$$\neg(U \vee V) = \neg U \wedge \neg V$$

$$U \rightarrow V = \neg U \vee V$$

A Cube is a conjunction of a finite number of literals

A formula is in DNF if it is written as a disjunction of cubes

Reduction rule

$$\neg\neg X \equiv X$$

DNF of a formula provides ~~of~~ all the models of that formula, finding all the interpretations that evaluates the cubes as true.

Solution

Ex 6. Using the appropriate normal form write all the models of the following formula:

$V_7 = (q \vee r \xrightarrow{1} p) \xrightarrow{3} (p \xrightarrow{3} r) \wedge q$

replace 2 using $U \to V = \neg U \vee V$

$V_7 = \neg(q \vee r \xrightarrow{1} p) \vee (p \xrightarrow{3} r) \wedge q$

replace 1,3

$V_7 = \neg[\neg(q \vee r) \vee p) \vee (\neg p \vee r) \wedge q$

apply de Morgan

$V_7 = ((q \vee r) \wedge \neg p) \vee (\neg p \vee r) \wedge q$

apply distributivity

$V_7 = (q \wedge \neg p) \vee (r \wedge \neg p) \vee (\neg p \wedge q) \vee (r \wedge q)$

apply idempocity

$V_7 = (q \wedge \neg p) \vee (r \wedge \neg p) \vee (r \wedge q)$ DNF with 3 cubes

Cube: $q \wedge \neg p$ 2 models

$i_1 : \{p, q, r\} \to \{T, F\}$ $i_1(p) = F$ $i_1(q) = T$ $i_1(r) = F$

$i_2 : \{p, q, r\} \to \{T, F\}$ $i_2(p) = F$ $i_2(q) = T$ $i_2(r) = T$

Cube: $\neg p \wedge r$

$i_3 : \{p, q, r\} \to \{T, F\}$ $i_3(p) = F$ $i_3(q) = F$ $i_3(r) = T$

$i_4 : \{p, q, r\} \to \{T, F\}$ $i_4(p) = F$ $i_4(q) = T$ $i_4(r) = T$

Cube: $q \wedge r$

$i_5 : \{p, q, r\} \to \{T, F\}$ $i_5(p) = F$ $i_5(q) = T$ $i_5(r) = T$

$i_6 : \{p, q, r\} \to \{T, F\}$ $i_6(p) = T$ $i_6(q) = T$ $i_6(r) = T$

We notice that $i_2 = i_4 = i_5$ => The models of V_7 are the interpretations i_1, i_2, i_3, i_6

$i_1(V_7) = i_2(V_7) = i_3(V_7) = i_6(V_7) = T$

In conclusion, the models of the formula V_7 are i_1, i_2, i_3, i_6.