**Truth tables**

**Symbols: V  ˄   Ꞁ   →    ↔   ←** 

**Meta symbols: |= ≡**

**Example 1: Build the truth tables of the propositional formulas:**

*U* (p, q, r) = (Ꞁ p V q) ˄ (r V p)

*V* (p, q, r) = (Ꞁ p ˄ r) V (q ˄ r) V (q ˄ r)





|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | p | q | r | Ꞁ p | Ꞁ p V q | r V p | U |  |  |  |  |  |
| i1 | T | T | T | F | T | T | T |  |  |  |  |  |
| i2 | T | T | F | F | T | T | T |  |  |  |  |  |
| i3 | T | F | T | F | F | T | F |  |  |  |  |  |
| i4 | T | F | F | F | F | T | F |  |  |  |  |  |
| i5 | F | T | T | T | T | T | T |  |  |  |  |  |
| i6 | F | T | F | T | T | F | F |  |  |  |  |  |
| i7 | F | F | T | T | T | T | T |  |  |  |  |  |
| i8 | F | F | F | T | T | F | F |  |  |  |  |  |

U is a contingent formula

**The models of *U*:** i1,i2,i5,i7

i1:{p,q,r}->{T,F}, i1(p)=T, i1(q)=T, i1(r)=T, i1(U)=T

**The anti-models of *U*:** i3,i4,i6,i8

i3:{p,q,r}->{T,F}, i3(p)=T, i3(q)=F, i3(r)=T, i3(U)=F

**Example: conditional rules**

**→  , ↔, Ꞁ**

**p**: It is sunny.

**q**: I will go for a walk.

**X**: **If** it is sunny **then** I will go for a walk. p **→ q**

p is sufficient for q

q is necessary for p

**Y: Only if** it is sunny I will go for a walk. q**→p**

P is necessary for q

Z: I will go for a walk **if and only if** it is sunny. **p↔q**

p is necessary and sufficient for q

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | p | q | X= p **→ q** | Y= q**→p** | Z= **p↔q** |
|  | T | T | T | T | T |
|  | T | F | F | T | F |
|  | F | T | T | F | F |
|  | F | F | T | T | T |

**Remarks:**

Conditional rules are just like game rules, with events that can be true:

“only if” something else is true,

or “if” something else is true.

**Conditional rule: p → q**

* A **sufficient** condition **guarantees** the truth of another condition, but is ***not* necessary** for

that other condition to happen: **p is *sufficient* for q**

* A **necessary** condition ***is* required** for something else to happen, but it **does *not* guarantee**

that the something else happens: **q is *necessary* for p.**

**Sufficient conditions**

* A sufficient condition, if met, *guarantees* another event with no exceptions.
* *But* a sufficient condition is *not necessary* for that event to happen, since there

could be many other conditions that are also sufficient for the resulting event to happen.

* When mapping a conditional rule, the **sufficient condition** is generally put **on the left**.
* **If p, then q** (**p→q)** does not logically imply **If q, then p (q→p)**
* **If p, then q** (**p→q)** does not logically imply **If NOT p, then NOT q (Ꞁ p→ Ꞁ p)**

**Necessary conditions**

* Note how the conditional statement **if p, then q**  (**p→q)** is logically equivalent

to the statement **if NOT q, then NOT p (Ꞁ q→ Ꞁ p).**

This logically equivalent version of a statement is sometimes called its **contrapositive**.

* You’ll notice that the **necessary condition** is always on the **right**.

That’s because the right-hand statement doesn’t lead to another result.

This makes sense because a necessary condition *doesn’t guarantee any event*.

* It’s *necessary* to meet the condition on the right in order for the condition on the left to occur,

but meeting that right-hand necessary condition doesn’t *guarantee* that the left-hand condition occurs.