ALGEBRA-BASED PHYSICS-1: MECHANICS (PHYS135A-01): WEEK 6

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WEEK 5 REVIEW

1. Friction
   - Normal force and friction
   - Static, kinetic
2. Drag
   - Terminal velocity
3. Restoring Forces
   - Hooke's Law
   - Young's modulus
   - Shear modulus
   - Bulk modulus

WEEK 5 REVIEW PROBLEM

A car rests on four shock absorbers, and each is like a spring with a spring constant $k = 1000N/cm$. The car weighs 10000 N. By what distance is each spring compressed?

- A: 2.5 cm
- B: 10 cm
- C: 1 meter
- D: 0 cm

A team of workers is pulling a 500 kg load up a ramp with a 30 degree incline, at constant speed, and the coefficient of friction between the load and ramp is 0.6. What is the force with which the workers pull?

- A: 5000 N
- B: 2500 N
- C: 2750 N
- D: 3750 N

WEEK 6 SUMMARY

1. Angular kinematics
    - Angular displacement
    - Angular velocity
    - centripetal acceleration
2. Newton's Law of Gravity and circular orbits
3. Kepler's Laws

ANGULAR KINEMATICS

There is a correspondence between angular and linear kinetmatics, if we deal with accelerations that are constant or zero.

Linear:

$$x(t) = x_0 + v_\mathrm{i} t + \frac{1}{2} a t^2 \tag{1}$$
$$v(t) = v_\mathrm{i} t + a t \tag{2}$$
$$v^2 = v_\mathrm{i}^2 + 2a(x - x_0) \tag{3}$$

Angular:

$$\theta(t) = \theta_0 + \omega_\mathrm{i} t + \frac{1}{2} \alpha t^2 \tag{4}$$
$$\omega(t) = \omega_\mathrm{i} t + \alpha t \tag{5}$$
$$\omega^2 = \omega_\mathrm{i}^2 + 2\alpha \Delta(\theta - \theta_0) \tag{6}$$

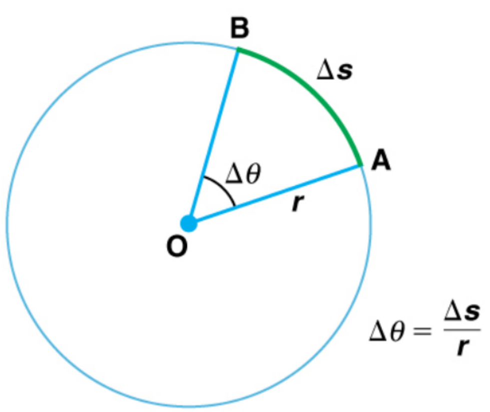

Figure 1: The definitions of arc length, Δs, radius, r, and angular displacement $\Delta\theta$.

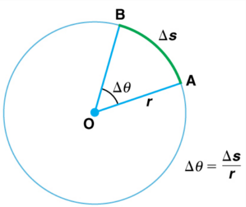

Figure 2: Examining the change in these quantities: $\Delta\theta/\Delta t = \omega$, $\Delta\omega/\Delta t = \alpha$.

Relationship between linear and rotational:

$$v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega \tag{7}$$

$$a = \frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha \tag{8}$$

Notice that the units of angular velocity are s^{-1}, and those of angular acceleration are s^{-2}.

Astromers have now discovered several thousand planets orbiting in star systems other than ours. Suppose we observe a star system face-on, and see a planet orbiting in a circular orbit with constant angular velocity. If it goes halfway around the star in 3 months, what is the angular velocity of the planet?

- A: $\frac{\pi}{3}$ months^{-1}
- B: $\frac{\pi}{6}$ months^{-1}
- C: $\frac{2\pi}{3}$ months^{-1}
- D: 2π months^{-1}

If we define a coordinate system such that at time $t = 0$ months, the planet is along the x-axis, in how many months will the planet cross the negative y-axis?

- A: 3 months
- B: 3.5 months
- C: 4.0 months
- D: 4.5 months

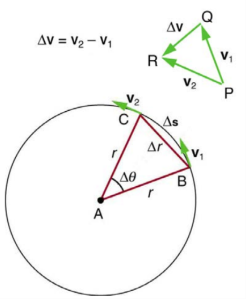

Figure 3: The velocity triangle and the position triangle are *similar*, because they are isosceles with the same angle ($\Delta\theta$).

Similar triangles have equal *ratios of sides*:

$$\frac{\Delta v}{v} = \frac{\Delta s}{r} \tag{9}$$

$$\Delta v = \frac{v}{r}\Delta s \tag{10}$$

$$\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t} \tag{11}$$

$$\Delta t \to 0 \tag{12}$$

$$a_{\mathrm{C}} = \frac{v^2}{r} = r\omega^2 \tag{13}$$

$$\vec{a}_{\mathrm{C}} = -\frac{v^2}{r}\hat{r} \tag{14}$$

With centripetal acceleration comes **centripetal force**, which is the net force for uniform circular motion:

$$\vec{F}_{\mathrm{C}} = -\frac{mv^2}{r}\hat{r} = -mr\omega^2\hat{r} \tag{15}$$

In Eq. 15, the minus sign indicates that the force points towards the center of the circle.

Example problem with centripetal acceleration and force.

- A: 5000 N
- B: 2500 N
- C: 2750 N
- D: 3750 N

CONCLUSION

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    - Angular displacement
    - Angular velocity
    - centripetal acceleration
2. Newton's Law of Gravity and circular orbits
3. Kepler's Laws

ANSWERS

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- 2.5 cm
- 2750 N
- $\frac{\pi}{3}$ months^{-1}
- 4.5 months
- ...