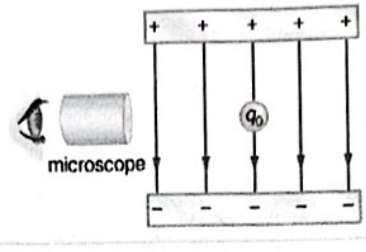



Figure 1: The classic Millikan oil drop experiment was a measurement of the charge of an electron.

## 2 Electric Charge and Electric Fields

distance is inversely proportional to E-field? a)

1. **Scaling problem:** (a) Some point charge produces an E-field $E_C = 2.00 \times 10^{-3}$ V/m at a distance of 1 mm. What is the value of $E_C$ at 5 mm produced by the same charge? (b) A 1 $\mu$C charge produces an E-field $E_C = 8.00 \times 10^{-3}$ V/m at some distance. What is the value of $E_C$ at the same distance if the charge is 3 $\mu$C?

b) charge is proportional to electric field

a) $\vec{F}$ proportional to $E_C$

$\therefore E_C = \frac{kq}{r^2}$

$q = \frac{(E_C)(r^2)}{k}$

$q = \frac{(2\times10^{-3} V/m)(1\times10^{-3} m)^2}{9\times10^9 N/C^2 \cdot m^2} = 2.2\times10^{-19}$ C

$E_C = \frac{(9\times10^9 \frac{N}{C^2 m^2})(2.2\times10^{-19} C)}{.005 m^2}$

$= 8\times10^{-5}$ N/C

$= \boxed{8\times10^{-5} V/m}$

b) $1\mu C \rightarrow E_C = 8.00\times10^{-3}$ V/m

$3\mu C \rightarrow E_C = ?$

$\frac{8.00\times10^{-3} V/m}{1\mu C} = \frac{X}{3\mu C}$

$\boxed{X = 0.024 V/m}$

2. The classic Millikan oil drop experiment was the first ~~to measure the~~ electron charge. Oil drops were suspended against the gravitational force by an electric field. (See Fig. 1.) Suppose the drops have a *mass* of $4 \times 10^{-16}$ kg, and the E-field is oriented downward, and has a value of 6131.25 N/C. With this exact value, the drops remain suspended in air. (a) How many electrons are on the drops? (b) Suppose a cosmic ray comes along and removes an electron from a droplet. What will the acceleration of the droplet be?

$F_E = mg$

$4 - 1 = 3e^-$

$qE$

a) $\vec{E} \cdot q = mg$

$q = (\# n)(e) = n \cdot 1.6\times10^{-19}$ C

# of $e^-$ missing

$\vec{E}(n \cdot e) = mg$

$n = \frac{mg}{E \cdot e}$ $\frac{(4\times10^{-16} kg)(9.81 m/s^2)}{(6{,}131.25 N/C)(1.6\times10^{-19} C)}$ $\frac{kg \cdot m / s^2}{N}$ $= \boxed{n = 4}$

b) $F_e = (1.6\times10^{-19} C)(3e^-) \cdot 6{,}131.25 \frac{N}{C}$

$F_e = 2.943\times10^{-15}$ N

$\frac{F_e}{m} = a = 7.3575$ m/s²

factor in acceleration due to gravity $\rightarrow$ $9.81 - 7.3575$

$= \boxed{2.453 m/s^2}$

## 3 Potential Energy and Voltage, Capacitors

1. A *mass spectrometer* is a device used to accelerate ions to determine atomic masses of chemicals. Suppose two conducting plates with potential difference $\Delta V = 4$ kV are used to accelerate both hydrogen ions and helium ions. Hydrogens have charge $+1q_e$, and helium ions have charge $+2q_e$. (a) What is the total kinetic energy (in electron-volts) gained by the hydrogens and heliums? (b) If the plate separation is $\Delta x = 5$ cm, what is the electric field value? *Hint: think of the E-field as the slope of voltage.*

a) $KE = q \cdot \Delta V$ energy gained

$H^+$ $KE = (+1q_e)(4{,}000 V) = \boxed{4{,}000 eV}$ $\boxed{4 keV}$

He $KE = (+2q_e)(4{,}000 V) = \boxed{8{,}000 eV}$ $\boxed{8 keV}$

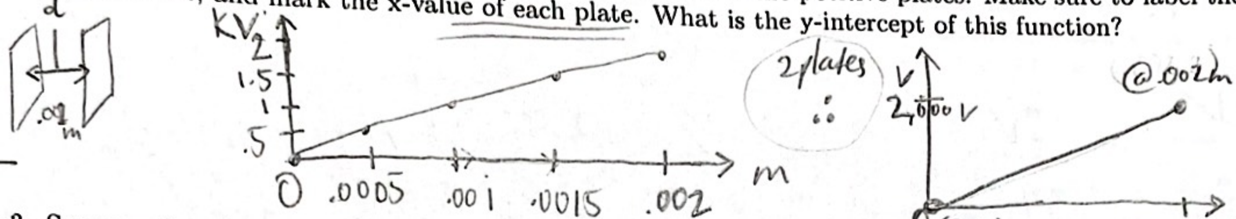
b) $\vec{E} = \frac{-V}{\Delta X}$

$= \frac{-4 kV}{.05 m} = \frac{-4{,}000 V}{.05 m}$

$= \boxed{-80{,}000 V/m}$

$V = \vec{E} \cdot \Delta x \qquad V = 1{,}000\ V/m \cdot \Delta x \quad = 1{,}000 \cdot .002 \quad = 2V$

2. Suppose a parallel plate capacitor has an internal E-field of 1 kV/m, and a plate separation of 2 mm. Draw the voltage as a function of distance between the negative and the positive plates. Make sure to label the axes with proper units, and mark the x-value of each plate. What is the y-intercept of this function?




$\boxed{\text{Y-intercept} = 0}$

| $\Delta x$ | V |
|---|---|
| .0005 | 0.5 |
| .001 | 1 |
| .0015 | 1.5 |
| .002 | 2 |

3. Suppose the plates in the previous problem have an area of 1 cm$^2$. (a) What is the capacitance of the system? (b) How much energy (in Joules) is stored in this capacitor if the voltage is 5 V?

a) $C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ F/m)(0.0001\ m^2)}{(2 \times 10^{-3}\ m)} = \boxed{4.425 \times 10^{-13}\ F}$

$F = C/V$ $\quad J = C \cdot V$ $\quad F = \frac{C}{V}$ $\quad C \cdot V = J$

b) $E = \frac{1}{2} C V^2 = \frac{1}{2}(4.425 \times 10^{-13}\ F)(5V)^2 = \boxed{5.53 \times 10^{-12}\ J}$

Much energy stored in a capacitor $= E = \frac{1}{2} C V^2$

4. Suppose we need a system that can store more energy for the same voltage (in other words, more capacitance). (a) Should we connect an identical capacitor to the first *in series* or *in parallel*? (b) What is the total energy stored in three capacitors connected in parallel, if each capacitor is identical to the one in the prior problem?

a) in parallel, it would give more capacitance ~~since it is not~~ (not inversed.)

$3(\frac{1}{2} C V^2) = 3 \cdot (5.53 \times 10^{-12} J)$

b) each capacitor is identical ∴ $3(5.53 \times 10^{-12} J) = \boxed{E = 1.659 \times 10^{-11} J}$

$C_{tot} = 3(C)$

# 4 Current, Resistance, and DC Circuits

1. When dealing with AA batteries, we can either connect them "end to end" (in series), or in parallel (see Fig. 2). Suppose that the internal resistances of the batteries $r_1 = r_2 = 2\Omega$, and that the emfs of the two batteries are both $\epsilon_1 = \epsilon_2 = 1.5$ V. Finally, let $R = 50\Omega$. Suppose $R$ represents a small device that will work at 1.5 V or 3 V (a child's toy, an old CD player, a computer mouse). (a) Using Kirchhoff's rules, find the current through $R$ for the serial case (3 V) (Fig. 2, left), and the parallel case (Fig. 2, right). (b) What is the power consumption in each case? (c) Check your calculations of current using the PhET DC circuit construction modeling kit.

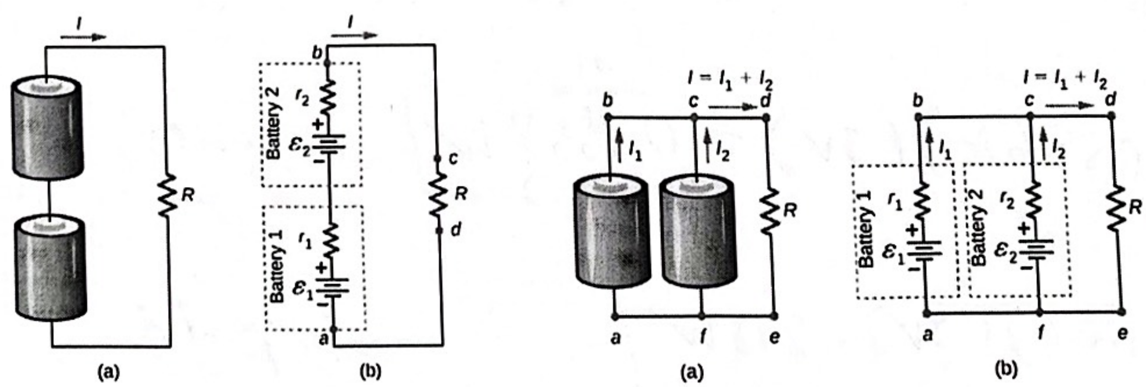



Figure 2: Two ways of connecting batteries. (Left) In series. (Right) In parallel.

4) a) $V = IR$  $R_{tot} = 50 + 2 + 2 = 54$

$\rightarrow$ $I = \frac{3V}{50+2+2} = \frac{3V}{54\Omega} = 0.0556\, V/\Omega = \boxed{0.0556\,A}$ in series

$\rightarrow$ $R_{tot}^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1}$  $I = \frac{V}{R} = \frac{3V}{0.98\Omega} = \boxed{3.06A \text{ in parallel}}$

$= 2^{-1} + 2^{-1} + 50^{-1}$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{50}$

$= \frac{25}{50} + \frac{25}{50} + \frac{1}{50}$

$= \frac{51}{50}$

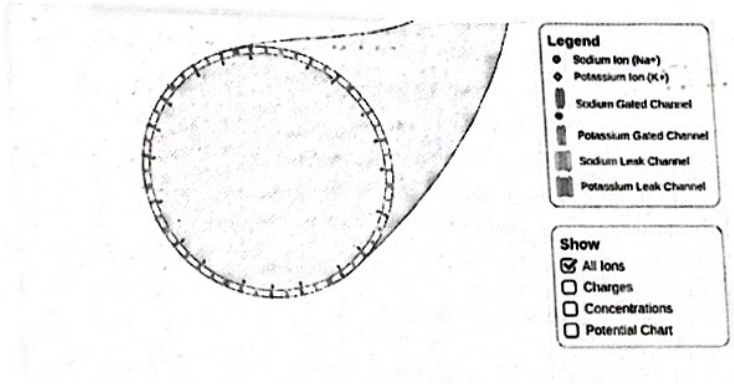
$R_{tot}^{-1} = \frac{50}{51} = .98\,\Omega$

b) $P = IV$

$P = (0.0556A)(3V) = \boxed{0.1668\,W}$ in series

$P = (3.06A)(3V) = \boxed{9.18\,W}$ in parallel

... activity in which we covered nerve stimulation as chemical-driven capacitors. Think of the voltage as a signal versus time that flows down the nerve. If you stimulate the nerve in this calculation, (a) what is the pulse width, in milliseconds? (b) What is the peak-to-peak voltage (greatest voltage minus least) in millivolts? **Bonus:** (c) Estimate the time required for a nerve signal to travel from your toe to your spinal chord.




1/m

Figure 3: Recall the molecular model of the nerve membrane, and the voltage generated across it by chemical valves.

a) pulse width in milliseconds = $\boxed{2\text{ms}}$ (4-2)

b) 30 - (-75) = $\boxed{105\text{mV}}$

c) time = $\frac{\text{distance}}{\text{speed}}$ $\qquad \frac{1\text{ m}}{\frac{1\text{m}}{2\text{ms}}}$

time in ms = $\frac{1\text{m}}{40\text{m/s}}$

average velocity of nerve signal = 40-50m/s

$= 1\cancel{m} \cdot \frac{1\text{s}}{40\cancel{m}}$

$= .025$ sec

$\boxed{= 2.5\text{ms}}$