

Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.

• a: $B = F \times v = (-\hat{j} \times \hat{j}) = -\hat{k}$ into the page
 • b: $B = F \times v = (-\hat{k}) \times (-\hat{j}) = -\hat{i}$ to the left
 • c: $B = F \times v = \hat{j} \times (\hat{i}) = \hat{k}$ out of the page

2. Consider Fig. 1 (right). **The Hall Effect.** An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V / \Delta x$. Show that

$$\Delta V = \frac{B \Delta x I}{n q_e A} \quad (1)$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33 \text{ T}$, $\Delta x = 2 \text{ cm}$, $I = 10 \text{ A}$, $n = 2 \times 10^{28} \text{ m}^{-3}$, $A = 1 \text{ mm}^2$, and q_e is the charge of an electron.

a. $F_e = F_B$
 $q_e E = q_e v B \sin \theta$
 $\theta = 90$

$q_e E = q_e v B \rightarrow E = v B$
 $v = \frac{E}{B}$

b. $\Delta V = \frac{B \Delta x I}{n q_e A}$

$\Delta V = \frac{1.33 (2 \times 10^{-2}) \times 10}{(2 \times 10^{28}) \times (1.6 \times 10^{-19}) \times (1 \times 10^{-3})^2}$
 $\Delta V = 8.31 \times 10^{-5} \text{ V}$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop $0.65 \times 10^{-15} \text{ m}$ in radius with a current of $1.05 \times 10^4 \text{ A}$. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

$\tau = N I A B \sin \theta$

$A = \pi r^2$ $A = \pi (0.65 \times 10^{-15} \text{ m})^2 = 1.33 \times 10^{-30} \text{ m}^2$

$\tau = 1 (1.05 \times 10^4 \text{ A}) (1.33 \times 10^{-30} \text{ m}^2) (2.50 \text{ T}) (\sin 90)$

$3.48 \times 10^{-26} \text{ Nm}$

Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

$\mu_0 I = 4\pi \times 10^{-7} (500) (0.3)$
 $1.884 \times 10^{-4} = 1.88 \times 10^{-4} \text{ T}$

b. $5000 \times \mu_0 I$
 $5000 \times 1.88 \times 10^{-4} = 0.94 \text{ T}$

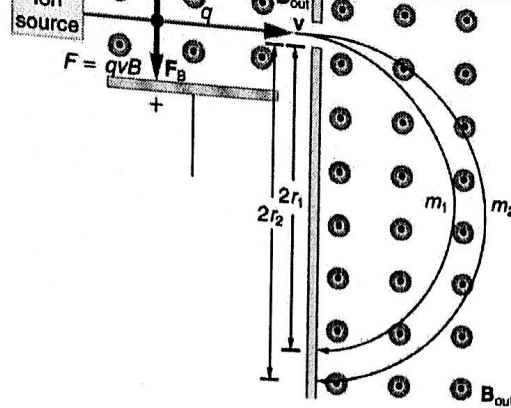


Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer.** Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left¹. (b) Recall that the centripetal force on a particle of mass m is mv^2/r . Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \quad (2)$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-septometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r ?

$F_{total} = F_{electrical} + F_{magnetic} = 0$
 $q[E + v \times B] = 0$
 $F_{total} = q(E - vB) = 0$
 $E = vB \rightarrow v = \frac{E}{B}$ for $F_{net} = 0$
 $b. r = \frac{mE}{qB^2} = \frac{ME}{qB^2} \rightarrow$
 $(16)(1.67 \times 10^{-27})(10) / ((1.602 \times 10^{-19})(0.01)^2) = 1.67 \text{ cm}$

Chapter 13: Electromagnetic Induction

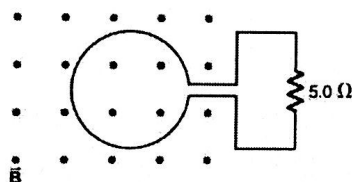


Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0} (\sin(2\pi ft)) \quad (3)$$

The loop has a radius r . (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1 ?

$\frac{dd}{dt} = \frac{d(CBA)}{dt}$
 $e = \pi r^2 \cdot \frac{B_0}{T_0} \sin(2\pi ft)$
 b. induced emf at $t=0$ is zero
 c. $e = \pi (0.1)^2 \cdot \frac{0.1}{1 \text{ ms}} \sin(2\pi \times 10^3 \times 0.16 \times 10^{-3})$
 $e = 3.14 \times \frac{(0.1)^3}{1 \text{ ms}} \times 0.0175 = 0.055 \text{ V}$
 d. $I = \frac{e}{R} = \frac{0.055}{5} = 0.011 \text{ A}$

¹ Molecules that do not have this velocity will hit the sides of this portion of the instrument

Chapter 14: Inductance

What is (a) the rate at which the current through a 0.50-H coil is changing if an emf of 0.150 V is induced in the coil?

$$\frac{\text{emf}}{\text{Inductive}} = - \frac{0.15}{0.50} = \boxed{-0.3}$$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time does a 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$$\mathcal{E} = L \frac{dI}{dt} \quad dt = \frac{L}{\mathcal{E}} dI$$

$$dt = \frac{2.00 \text{ mH}}{500 \text{ V}} (0.100 \text{ A})$$

$$= \boxed{4.00 \times 10^{-7} \text{ s}}$$