Algebra-Based Physics-2: Electricity, Magnetism, and Modern Physics (PHYS135B-01): Unit 1

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Summary

Unit 1 Summary

Reading: Chapters 19.4 - 19.7, 20.1 - 20.5, 20.7

1. Capacitors and capacitance
    - Equipotential lines
    - Capacitance
    - Capacitors
2. Current and DC circuits
    - DC current, resistance, Ohm's law
    - Energy and power in DC current
    - AC current and waveforms

Equipotential Lines and Capacitance

Equipotential Lines

Equipotential lines represent locations of constant potential, and run perpendicular to electric field lines.

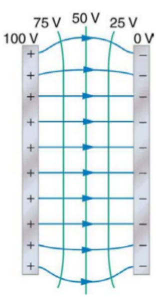

Figure 1: Equipotential lines in a parallel plate capacitor.

PhET *demonstration*:
`https://phet.colorado.edu/en/simulations/charges-and-fields`

Capacitance

What voltage is required to store Q? Let ΔV be the voltage difference, E be the electric field strength, and Δx be the capacitor voltage. We already know that

$$\Delta V = E\Delta x \tag{1}$$

We observe that the voltage is linear, so the E-field is constant and proportional to charge per unit area[1]

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \tag{2}$$

Substitute E for $Q/(\epsilon_0 A)$, and solve for Q:

$$Q = \left(\frac{\epsilon_0 A}{\Delta x}\right)\Delta V \tag{3}$$

The term in parentheses is called *the capacitance, C.*

[1] Professor: we can prove this if time permits.

Capacitance

Let the capacitor have an area A, and plate separation d. The capacitance is

$$C = \frac{\epsilon_0 A}{d} \tag{4}$$

$$Q = C\Delta V \tag{5}$$

Capacitance epresents the ability to store charge. The unit of capacitance is **the farad**, after Michael Faraday (encounter mid-semester). Scaling problems in a moment...

PhET Activity: Capacitor Basics

Go to the activity:

`https://phet.colorado.edu/en/simulation/capacitor-lab-basics`

1. Go to the capacitance tab to find a capacitor and battery.
2. Charge the capacitor under different conditions: $d = 10$ mm, $d = 2$ mm, $A = 100$ mm^2, $A = 400$ mm^2.
3. The **voltmeter** at right is the yellow and black tool. It allows the measurement of voltage between two points. Measure the battery voltage and the capacitor voltage. Why are they equal?
4. What is the charge stored for $d = 2$ mm, and $A = 400$ mm^2?
5. The light bulb tab in the bottom center contains a circuit that operates a light bulb with the energy stored in the capacitor. Measure the voltage across the lightbulb as it is powered with the capacitor. Sketch the voltage as a function of time.

Capacitance

Two capacitors store the same charge. One only needs half the voltage, though. What is true of the capacitor with the lower voltage?

- A: It has half the capacitance
- B: It has the same capacitance but more charges
- C: It has twice the capacitance
- D: It has half the energy

Capacitance

Two capacitors have the same capacitance. Capacitor 1 has half the area A as capacitor 2. Which of the following is true of capacitor 2?

- A: The plates are half the distance (Δx) apart
- B: The plates are twice the distance (Δx) apart
- C: The plates have half the voltage
- D: The plates have twice the voltage

Capacitance

Two identical capacitors power two light bulbs. Capacitor 1 powers the bulb for 20 seconds, and capacitor 2 powers the bulb for 10 seconds. Which of the following is true?

- A: Capacitor 1 was charged at a lower voltage than capacitor 2
- B: Capacitor 1 was charged at the same voltage as capacitor 2
- C: Capacitor 1 was charged at a higher voltage than capacitor 2
- D: Capacitor 2 was not charged

PhET Activity: Capacitor Lab

PhET Activity: Capacitor Lab

Similar to *Capacitor basics* activity:

`https://phet.colorado.edu/en/simulations/circuit-construction-kit-ac`

What is the relationship between total capacitance and individual capacitances?

1. How do capacitors add *in series*?
2. How do capacitors add *in parallel*?

PhET Activity: Capacitor Lab

Instructions:

1. Click the Lab tab. Click and drag three capacitors from the white column at left into the main area.
2. Click on each capacitor and minimize the capacitance to 0.05 F.
3. Orient each capacitor *vertically*, and connect the top sides of the capacitors with wires.
4. Connect the bottom sides of the capacitors with wires.
5. Click and drag a battery from the white column at left into the main area, and do the same for a switch.
6. Connect the battery to the first capacitor, and add a switch between the battery and first capacitor.
7. Leave the switch open.

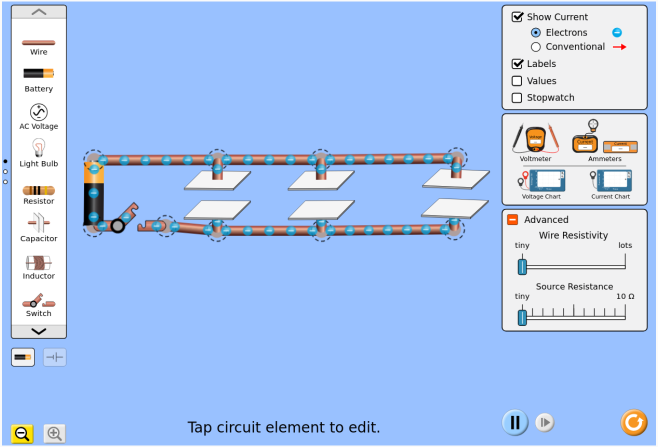

Figure 2: Three capacitors connected in parallel.

Instructions:

1. Click and drag a light bulb from the white column at left, and use wires to connect it to the final capacitor via a switch.
2. Leave both switches open. If your capacitors are charged, close the light bulb switch until the capacitors are fully discharged.
3. With both switches open, close the battery switch to charge the capacitor.
4. Click and drag the voltage charge tool from the white column at right, and place the red and black probes on either side of the light bulb.

PhET Activity: Capacitor Lab

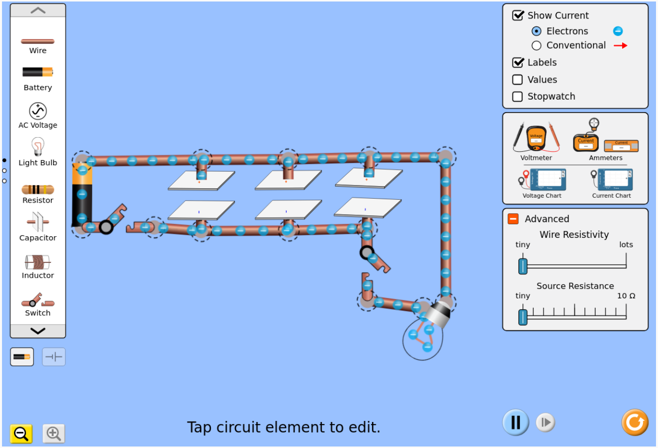

Figure 3: Three capacitors in parallel, with light and switch.

PhET Activity: Capacitor Lab

Instructions:

1. Click stopwatch in the white column on the right side.
2. Use the voltage chart tool to measure *how long* it takes for the bulb to drain the three capacitors, according to the chart and stopwatch.
    2.1 Since your capacitors are charged, leave the battery switch open and close the light bulb switch. Just after closing the switch, click the play button on the stopwatch.
    2.2 Measure the average time required for the bulb voltage to drop to zero by collecting 10 measurements.

PhET Activity: Capacitor Lab

Instructions:

1. Now we will repeat the same experiment, but with the capacitors connected *in series*.
2. Rearrange the circuit so that the bottom of the first capacitor connects to the top of the second, and the bottom of the second connects to the top of the third. Connect the bulb to the bottom capacitor, and add a switch to the other side of the bulb.
3. Connect the top of the top capacitor to the switch, but leave the switch open.
4. Using separate wires, connect the battery to the top and bottom of the circuit, and add a switch to one side of the battery. Leave both switches open.

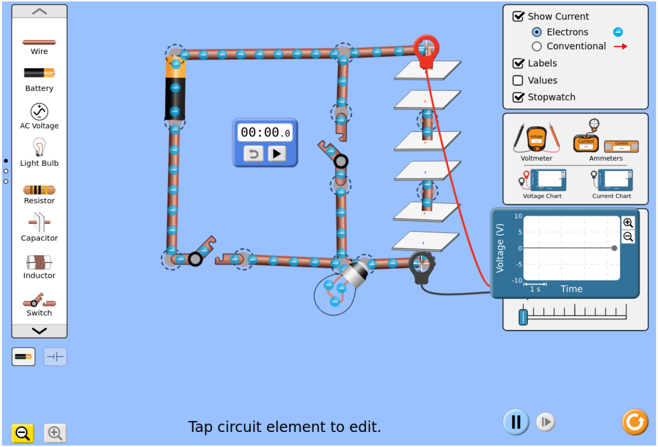

Figure 4: Three capacitors in series, with light and switch.

PhET Activity: Capacitor Lab

1. Use the voltage chart tool to measure *how long* it takes for the bulb to drain the three capacitors, according to the chart and stopwatch.
   1.1 Charge the capacitors by closing the battery switch, and then leave it open.
   1.2 Close the light bulb switch. Measure the average time required for the bulb voltage to drop to zero by collecting 10 measurements.
2. Which capacitor arrangement takes longer to drain?
3. Given that we used the same voltage for both arrangements, which arrangement (parallel or series) stores more charge?

Energy in Capacitors

Energy in Capacitors

Energy stored in a *an electric field*, E:

$$\mathcal{E} = \frac{1}{2}\epsilon_0 E^2 \tag{6}$$

The proof of Eq. 6 will come later in the course. We also know that the E-field is constant because the voltage is linear. In a parallel-plate capacitor, $E = \sigma/\epsilon_0 = Q/(\epsilon_0 A)$. Use $Q = CV$ and Eq. 6 to show that the total energy in a capacitor with capacitance C is

$$U_C = \frac{1}{2}\frac{Q^2}{C} \tag{7}$$

$$U_C = \frac{1}{2}CV^2 \tag{8}$$

Observe proof on board. The variable σ is known as the *surface charge density.*

Energy in Capacitors

Suppose we have a 10 nF capacitor at 5V. How many Joules are stored in it?

- A: 100 nJ
- B: 100 pJ
- C: 125 nJ
- D: 125 pJ

Capacitors, Series and Parallel

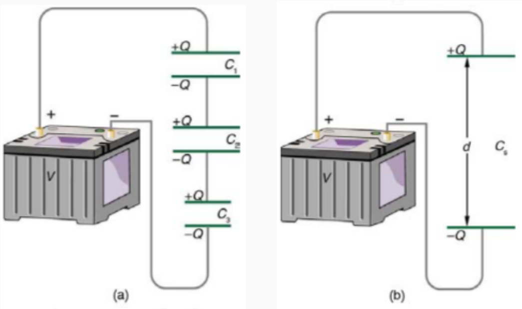

Figure 5: How do we compute the total capacitance of (a) such that we know the charge stored for a given voltage in (b)?

Capacitors, Series and Parallel

Notice that *charge is conserved*, so Q has to be the same on all capacitors But that means the *voltage* on the different capacitors has to obey

$$Q = Q_1 = Q_2 = Q_3 \tag{9}$$

$$C_{\rm tot} V_{\rm tot} = C_1 V_1 = C_2 V_2 = C_3 V_3 \tag{10}$$

To conserve energy, we know that

$$V_{\rm tot} = V_1 + V_2 + V_3 \tag{11}$$

$$\frac{Q}{C_{\rm tot}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \tag{12}$$

$$\boxed{\frac{1}{C_{\rm tot}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} \tag{13}$$

This is the how that capacitance adds for capacitors **in series**.

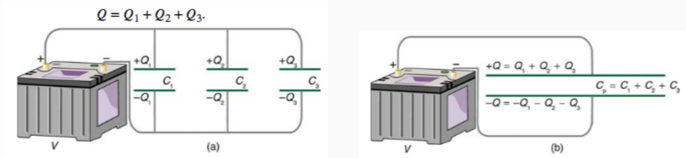

Figure 6: How do we compute the total capacitance of (a) such that we know the charge stored for a given voltage in (b)?

Note that all the capacitors are at the same voltage, because wires are equipotentials. Thus,

$$V_{\mathrm{tot}} = V_1 = V_2 = V_3 = V \tag{14}$$

The total charge stored is

$$Q_{\mathrm{tot}} = Q_1 + Q_2 + Q_3 \tag{15}$$

$$C_{\mathrm{tot}} V = C_1 V + C_1 V + C_2 V \tag{16}$$

$$\boxed{C_{\mathrm{total}} = C_1 + C_2 + C_3} \tag{17}$$

This is the how that capacitance adds for capacitors **in parallel**.

Current

Current

Notions of current:

- $I = \frac{\Delta Q}{\Delta t}$ - The derivative of charge
- The *movement* of electrons
- The *flow* of charge
- Number of Coulombs per second (1 Amp = C/s)

Speed of typical electronic signals: $\approx 10^8$ m/s

Typical speed of actual charges passing through a conductor under voltage: $\approx 10^{-4}$ m/s

Since there is a 12 order of magnitude range, it's probably a good idea to ponder...Observe proof on board.

Ohm’s Law

Energy is transferred to objects that are not perfect conductors in a simple circuit.

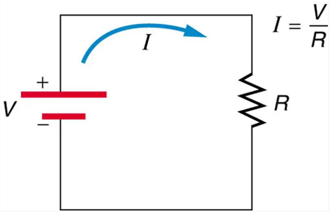

Figure 7: Current transfers energy to *resistors*.

Ohm's law: for a *resistance* R, the voltage V and the current i are related by:

$$V = iR \tag{18}$$

Instructor: Perform a few examples.

Conclusion

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