

Midterm 1

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1 Unit 0: Electrostatics I and II

1. A 50 gram copper wire has a net charge of $2.00 \mu\text{C}$. What fraction of the copper's electrons has been removed? (Each atom has 29 protons, and the atomic mass is 63.5.)

Divide 50 grams by the atomic weight to obtain the number of moles. Then, multiply by Avogadro's number to obtain the number of atoms. Next, multiply by the atomic number (electrons per atom) to obtain total electrons in 50 grams of copper. Now convert the $2 \mu\text{C}$ into number of electrons via the charge of 1 electron. The electrons in 50 grams of copper, and the electrons in $2 \mu\text{C}$, are both numbers of order 10^{25} . It turns out that 83 percent of the electrons remain on the wire, so 17 percent have been removed.

2. A test charge of $+2 \mu\text{C}$ is placed halfway between a charge of $+6 \mu\text{C}$ and another of $+4 \mu\text{C}$ separated by 10 cm. (a) What is the magnitude of the net force on the test charge? (b) What is the direction of this force (away from or toward the $+6 \mu\text{C}$ charge)?

(a) Draw a free body diagram, and find that the two forces are opposing one another. With equal distances, the stronger force is just the one generated by the larger charge. We can show that $F_{\text{Net}} = kq\Delta q/r^2$, where Δq is the difference between the left and right charges: $2 \mu\text{C}$. The result is 14.4 N . (b) The force points away from the $q = 6 \mu\text{C}$ charge.

3. What is the force on the charge located at $x = 8.00 \text{ cm}$ in Fig. 1(a) given that $q = 1.00 \mu\text{C}$?

Use the same logic as the previous problem, balancing Coulomb forces. The charge at $x = 3 \text{ cm}$ is located 5 cm from the test charge. The

charge at $x = 11 \text{ cm}$ is located 3 cm from the test charge. The 5 cm and 3 cm separations are what enter the denominators of the Coulomb force equations. Note that the attractive force by the charge on the left points left, while the attractive force by the charge on the right points to the right. The result is 12.8 N .

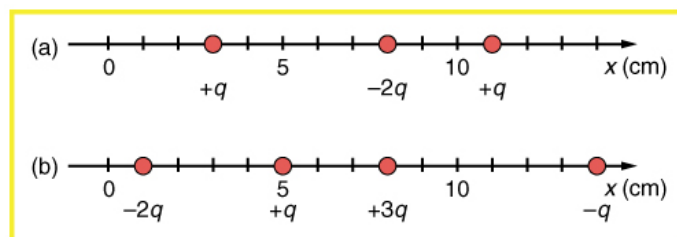


Figure 1: Linear arrangement of charges.

4. Find the total electric field at $x = 11.00 \text{ cm}$ in Fig. 1(b).

Assume $q = 1 \mu\text{C}$. The superposition principle says the electric fields add like vectors, so the positive test charge experiences a negative electric field from the first charge, and positive electric fields from the right three charges. The result is strikingly large: $4.1 \times 10^7 \text{ V/m}$. This is large enough to break down air and cause a spark.

5. Determine the direction of the force on q in Fig. 2, given that $q_a = q_b = +7.50 \mu\text{C}$ and $q_c = q_d = -7.50 \mu\text{C}$. (b) Calculate the force on the charge q , given that the square is 10.0 cm on a side and $q = 2.00 \mu\text{C}$.

(a) Down. The positive charges each create an E-field that cancels the horizontal component of the E-field of the other charge. Similarly, the negative charges each create an E-field that cancels the horizontal component of the E-field of the other negative charge. (b) 76.4 N .

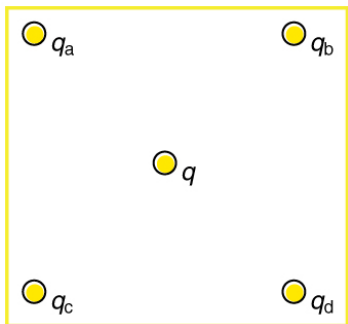


Figure 2: 2D arrangement of charges.

6. (a) An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons? (b) Show that units of V/m and N/C for electric field strength are indeed equivalent.

Non-relativistically, we can assume the energy given to the electron by the voltage is $U = qV$, without accounting for the rest mass of the electron or the “boost factor.” Set $U = KE = (1/2)mv^2$, and solve for v . We find that v is 40% of the speed of light. (b) 1 V m^{-1} is $1 \text{ J C}^{-1} \text{ m}^{-1}$. Now use $1 \text{ J} = 1 \text{ N m}$, and substitute: $1 \text{ N m C}^{-1} \text{ m}^{-1}$, so 1 N C^{-1} .

7. The electric field strength between two parallel conducting plates separated by 4.00 cm is $7.50 \times 10^4 \text{ V m}^{-1}$. (a) What is ΔV between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)? (c) The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength?¹

(a) Note that $V = E\Delta x + V_0$, so $\Delta V = E\Delta x$, if Δx represents displacement between the plates. Multiplying the E-field strength and displacement, accounting for units, gives 300 Volts. (b) Because $\Delta V = E\Delta x$, we can linearly scale the maximum ΔV to obtain the potential difference one-quarter of the way across the gap between plates (1 cm of the 4 cm): $300\text{V}/4 = 75 \text{ V}$. (c) Examine the units. E-fields are a potential divided by a distance, so take 80 mV divided by 9 nm: $80/9 \text{ mV nm}^{-1}$. In V m^{-1} , we have $8.9 \times 10^6 \text{ V m}^{-1}$.

¹The value is surprisingly large, but correct.

8. A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

We have $U = q\Delta V$, and $\Delta V = E\Delta x$. Substituting and solving for E gives $E = U/(q\Delta x)$. U is 32 electron kiloVolts (keV), and q is the equivalent of 2 electrons. With electrons in the numerator and denominator, the electrons cancel. Also, $\Delta x = 2 \text{ cm}$, and we can leave the final units as kV cm^{-1} . The result is 8 kV cm^{-1} .

9. In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

If the alpha has kinetic energy, it will approach a nucleus until all of that kinetic energy is converted to potential energy. The potential energy is $U = q_\alpha V(r)$, where q_α is the charge of the alpha and $V(r)$ is the nuclear potential as a function of distance from the nucleus. We know that $V(r) = kq_N/r$, where k is the usual constant, q_N is the nuclear charge, and r is the distance. Thus, $r = kq_N q_\alpha / U$, and we find $r = 45.5 \times 10^{-15} \text{ m}$. It is encouraging that this is the right length scale, because it is ≈ 16 times the classical electron radius.

2 Unit 1: Capacitors, Current, and DC circuits

1. What capacitance is needed to store 3.00 μC of charge at a voltage of 120 V?

$Q = CV$, and the result is 25 nF.

2. (a) What is the energy stored in the 10.0 μF capacitor of a heart defibrillator charged to $9.00 \times 10^3 \text{ V}$? (b) Find the amount of stored charge. (c) In open heart surgery, a much smaller amount of energy will defibrillate the heart. What voltage is applied to the 8.00 μF capacitor of a heart defibrillator that stores 40.0 J of energy? (d) Find the amount of stored charge.

- (a) Using $U_C = (1/2)CV^2$, we find $U_C = 405 \text{ J}$.
 (b) Using $Q = CV$, we find 0.09 C . (c) Using $U_C = (1/2)CV^2$, and solving for V , we find 3.16 kV (3.16 is the square root of 10). (d) 25.3 mC .

3. To build up the charge and energy required in part (a) of the previous problem, an AED designer decides to split the charge among four capacitors in parallel. Determine the required capacitance of each individual capacitor, and the charge stored on each, if the voltage remains $9.00 \times 10^3 \text{ V}$. Why would the designer choose not to connect the capacitors in series?

The charge is one fourth of the total charge, or 0.0225 C . This is because the parallel capacitors add like $C_1 + C_2 + C_3 + C_4 = C_{\text{tot}}$. If they're all the same, the capacitance of each is $2.5 \mu\text{F}$, so that $C_{\text{tot}} = 10 \mu\text{F}$. Connecting the capacitors in series would weaken, not strengthen, the total capacitance. Each capacitor would be required to have $C > 10 \mu\text{F}$.

4. If a 1.0 mm diameter copper wire can have a resistance of no more than 2.0Ω , (at 20 degrees C), how long can it be?

The relevant concept is the relationship between resistivity and total resistance. The equation is $R = \rho(L/A)$. The value of ρ for copper is $1.68 \times 10^{-8} \Omega \text{ m}$. Using $R = 2.0 \Omega$, and solving for L gives $L = RA/\rho$. The area A is πr^2 , and $r = 0.5 \text{ mm}$. Combining numbers gives $L = 93.5 \text{ m}$.

5. An LED is connected in series with a $1 \text{ k}\Omega$ resistor. A 3.0V battery is connected to the resistor, the LED follows the resistor, and the LED is then connected to ground. The negative terminal of the battery is also connected to ground. (a) What current flows from the battery, if the LED resistance is 3Ω ? (b) How much power is consumed by the LED? (c) How many Coulombs of charge flow through the LED in 10 minutes?

(a) The total resistance is 1003Ω , and the voltage is 3.0 V , so Ohm's law gives $i = 2.99 \text{ mA}$. (b) $P = iV = 2.99 \times 10^{-3} \times 3.0 \text{ W}$, so 8.97 mW . (c) Total charge is current times time, so 2.99 mA times 600 seconds, or about 1.8 C .

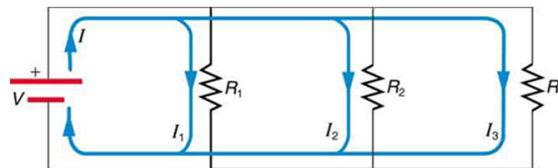


Figure 3: A DC circuit with three resistors.

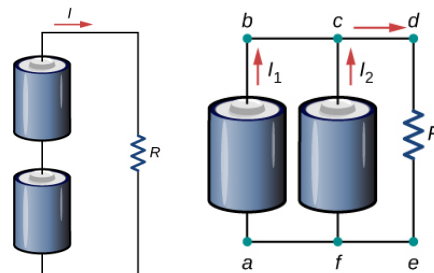


Figure 4: (Left) A DC circuit with two batteries in series, and a resistance $R = 0.5 \text{ k}\Omega$. (Bottom) A DC circuit with two batteries in parallel, and a resistance $R = 0.5 \text{ k}\Omega$.

3 Unit 2: DC circuits with resistors in series and parallel, RC circuits

1. In Fig. 3, let $R_1 = 10 \text{ k}\Omega$, $R_2 = 5 \text{ k}\Omega$, and $R_{\text{tot}} = 2 \text{ k}\Omega$. (a) What is the resistance of R_3 ? (b) If the battery has $\Delta V = 12 \text{ V}$, what current flows from the battery? (c) What are the individual currents, I_1 , I_2 , and I_3 ?

(a) $R_{\text{tot}}^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1}$. Solving for R_3 gives $5 \text{ k}\Omega$. (b) Ohm's law gives 6 mA . (c) $i_1 = 1.2 \text{ mA}$, $i_2 = i_3 = 2.4 \text{ mA}$. As a check, we see that $i_1 + i_2 + i_3 = 6 \text{ mA}$.

2. Consider Fig. 4. (a) Assuming no internal resistance, calculate the current and power through the resistance R if each battery has 1.5 V in the series circuit, and 3 V in the parallel circuit. (b) Now repeat part (a) for each circuit, assuming all batteries have an internal resistance of 5Ω .

(a) For the series batteries, the voltages add and we have $3\text{V}/0.5 \text{ k}\Omega$, giving 6 mA . For the parallel batteries, we have 1.5 V across the resistor, so 3 mA (each battery provides 1.5 mA). (b) For the series circuit, the total resistance is $0.510 \text{ k}\Omega$, and the total emf is 3V . The current is 5.88 mA . For the parallel circuit, we find 2.9985 mA . The reasoning is that if the emf's and internal resistances are equal, then

the current provided by each battery to R is equal. From Kirchhoff's rules, $i = i_1 + i_2 = 2i_1$ ($i_1 = i_2$). Thus, $i_1 = i/2$. Using either the inner or outer loop for Kirchhoff's loop rule gives $\epsilon - (i/2)r - iR = 0$. Solve for i .

3. A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of $0.02\ \Omega$ in series with a 1.53-V carbon-zinc dry cell having a $0.10\ \Omega$ internal resistance. The load resistance is $10.00\ \Omega$. (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only $0.500\ \text{W}$ being supplied to the load?

(a) The circuit is one big series circuit, with three batteries that have the same emf, and a fourth that has a different emf. Similarly, there should be three equal internal resistances, and one that is different than the others. Finally, there should be a load resistor. (b) Kirchhoff's loop rule gives an equation that involves the voltages, current, and all resistances. The final result is $(3\epsilon_1 + \epsilon_2)/(3r_1 + r_2 + R) = 617\ \text{mA}$. (c) $P = i^2 R = (0.617)^2(10) = 3.81\ \text{W}$. (d) First, we need to find the current. If $P = 0.5\ \text{W}$, then use $P = i^2 R$ to find i . The result is $1/\sqrt{20}\ \text{A}$. Then, solve the loop equation for r_2 and plug in i to find $18\ \Omega$. So the internal resistance has jumped above the load resistance, meaning it should be taking the bulk of the power.

4. A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

The information given implies that the pacemaker fires every $5/6$ seconds. This is the RC time of the circuit, because each time it fires we reach 0.632 of full voltage. We find $R = \tau/C = 33\ \text{M}\Omega$.

5. An ECG monitor must have an RC time constant less than $1.00 \times 10^2\ \mu\text{s}$ to be able to measure variations in voltage over small time intervals. If the resistance of the circuit (due mostly to that of the patient's chest) is $1.00\ \text{k}\Omega$, what is the maximum capacitance of the circuit?

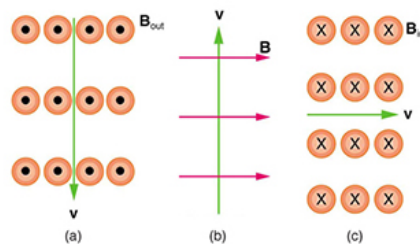


Figure 5: Three cases involving the particle velocity \vec{v} , and \vec{B} field.

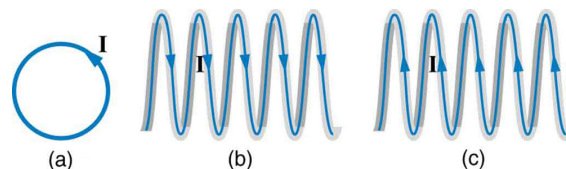


Figure 6: Three currents that create B-fields.

4 Unit 3: Magnetism I

1. Consider Fig. 5. Fill in Tab. 1 of directions below for the Lorentz force, assuming a **negatively charged** particle. Let \hat{i} represent right, \hat{j} represent up, and \hat{k} represent out of the page.
2. An electron moving at $4.00 \times 10^3\ \text{m s}^{-1}$ in a 1.25-T magnetic field experiences a magnetic force of $1.40 \times 10^{-16}\ \text{N}$. What angle does the velocity of the electron make with the magnetic field? There are two possible answers.

Case	v direction	B direction	F direction
(a)			
(b)			
(c)			
(d)			

Table 1: Table of directions for to Fig. 5.

3. (a) An oxygen-16 ion with a mass of 2.66×10^{-26} kg travels at 5.00×10^6 m/s perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer. (d) A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18. Assuming the ions have the same charge, what would the radius of the circular arc be for the oxygen-18? *Hint: this is a scaling problem.*

4. What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

5. For this exercise, we are designing an electric motor. Calculate the B-field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of 300 N m if the loop has 25.0 A of current.

Case	B direction
(a)	
(b)	
(c)	

Table 2: Table of directions for to Fig. 6.

6. Consider Fig. 6. Fill in the B-field directions in Tab. 2 using Ampère's Law, and the appropriate right-hand rule. Let \hat{i} represent right, \hat{j} represent up, and \hat{k} represent out of the page.
7. Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.
8. The B-field in the tokamak reactor in Fig. 7 is given by $B = \mu_0 NI / (2\pi r)$, where N is the total number of loops, I is the current, and r is the radius at which we evaluate the B-field. (a) Design your own reactor by specifying practical values of N , I , and r that achieve a 1.0 T B-field at a radius of 5.0 m. (b) With what frequency will a proton circle the toroidal B-field (1.0 T)? *These ideas will help activate fusion reactors.*

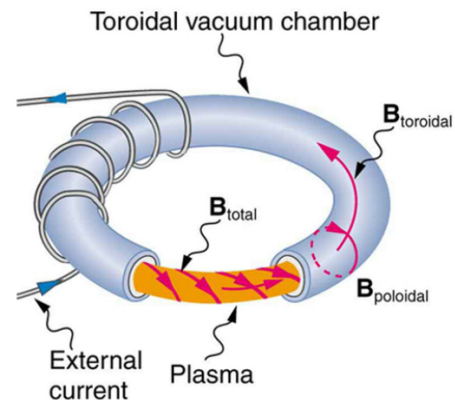


Figure 7: A generalized diagram of a tokamak.