Algebra-Based Physics: Electricity, Magnetism, and Modern Physics (PHYS135B): Unit 4

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November 8, 2023

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Summary

Summary

1. Magnetic induction - Chapters 23.1 - 23.5, 23.7, 23.9
    - Induced EMF and magnetic flux
    - Faraday's Law
    - Motional EMF, generators, and transformers
2. AC circuits - Chapters 23.9 - 23.12
    - Inductors
    - RL circuits
    - RLC circuits

Magnetic induction

First set of observations: a *moving* magnet can induce an emf in a coil of wire. The induced current polarity depends on (a) magnet polarity and (b) direction of magnet velocity. The induced current magnitude

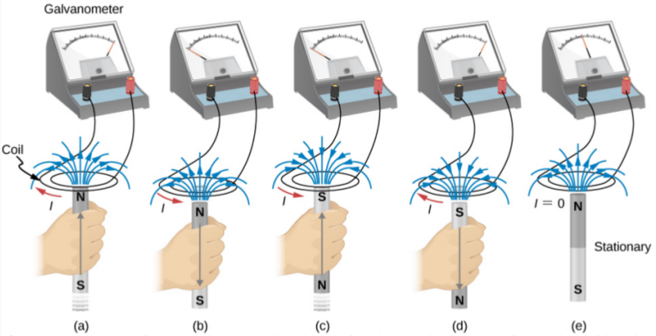

Figure 1: Observations of magnetic induction.

Second set of observations: a *changing* current in a loop can induce an emf in another loop. The induced current polarity depends on (a) inducing current polarity and (b) whether the inducing current is increasing or descreasing.

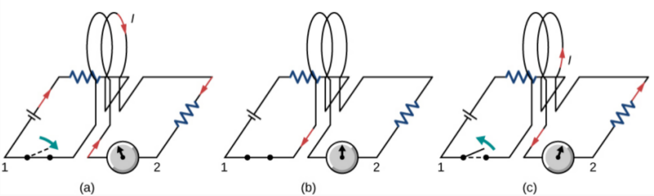

Figure 2: Observations of magnetic induction.

Third observation: a *changing* loop area in a magnetic field induces an emf, and current. The induced current polarity depends on whether the loop area is (a) increasing or (b) descreasing. The current magnitude depends on how quickly the area is changing.

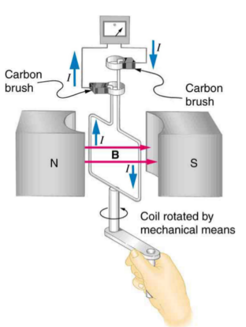

Figure 3: Observations of magnetic induction.

Magnetic induction

Video summary of magnetic induction:
`https://youtu.be/pQp6bmJPU_0`

- Magnet inducing current in loop of wire
- Solenoids inducing current in adjacent solenoids
- Magnetic flux
- Faraday's Law
- Lenz's Law

Magnetic induction

Magnetic flux is the dot-product of the area vector and the magnetic field through loops of wire with area A.

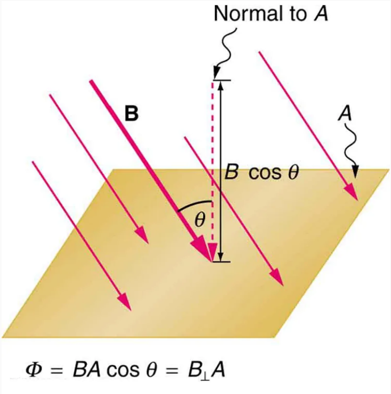

Figure 4: The **area vector** is *normal* to the loop area.

The **area vector** has a magnitude A, the area of the loop. The direction of the area vector is *normal* to the area of the loop.

$$\vec{A} = A\hat{n} \tag{1}$$

The magnetic flux, Φ, is therefore

$$\Phi = \vec{B} \cdot \vec{A} \tag{2}$$

Faraday’s Law

Faraday's Law

Faraday's Law

Let the product of the magnetic field and the vector area be the magnetic flux: $\Phi = \vec{B} \cdot \vec{A}$. The induced emf ϵ in N turns of a conductor will be

$$\epsilon = -\frac{\Delta \Phi}{\Delta t} \tag{3}$$

The induced current from ϵ will create a new B-field that opposes changes in Φ.

The unit of magnetic flux is the Weber, or 1 Wb = 1 T m^2.

Faraday’s Law

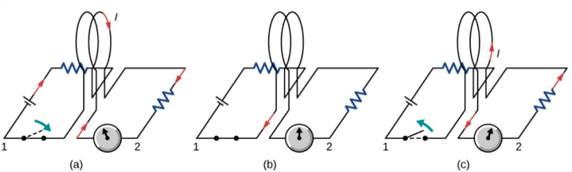

Figure 5: A pickup coil system.

Suppose the switch in Fig. 5 (a) is closed, inducing a current I in the right-hand loop. The B-field directions at the centers of the left and right loops are

- A: Right and left, respectively
- B: Light and right, respectively
- C: Both to the right
- D: Both to the left

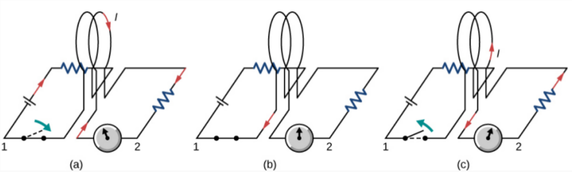

Figure 6: A pickup coil system.

Suppose the switch in Fig. 5 (b) remains closed, and no induced current is observed. This is because

- A: The magnetic flux is zero
- B: The inducing current is zero
- C: The magnetic flux is not changing
- D: The loop area is zero

Faraday's Law

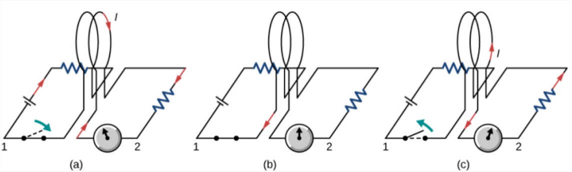

Figure 7: A pickup coil system.

Suppose the switch in Fig. 5 (b) remains closed, and no induced current is observed. This is because

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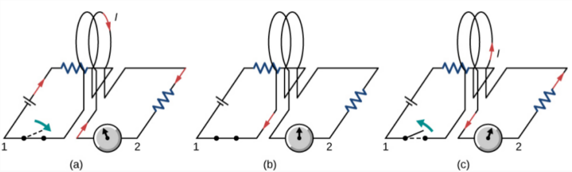

Figure 8: A pickup coil system.

Suppose the switch in Fig. 5 (c) is opened. The induced current in Fig. 5 is in the opposite direction of Fig. 5 (c) because

- A: The magnetic field from the right loop decreased
- B: The magnetic field from the left loop increased
- C: The magnetic field from the left loop is constant
- D: The magnetic field from the left loop decreased

Group board problem:

A magnetic field B passes orthogonally through a circular coil of radius $r = 0.05$ m and $N = 100$ turns. The field magnitude decreases linearly according to

$$B(t) = B_0 - at \tag{4}$$

with $B_0 = 0.015$ T and $a = 0.01$ T s^{-1}. (a) Calculate the magnitude of the emf induced in the coil at the times $t_0 = 0$, and $t_2 = 1.0$ second. (b) Determine the current in the coil if the resistance is 1 Ω.

Sketch this system, and indicate both the direction of the instantaneous B-field, and the direction of current.

Faraday’s Law - PhET Activity

Brief simulation of Faraday’s Law, and Lenz’s Law:

`https://phet.colorado.edu/en/simulations/faradays-law`

1. Learn to control the position and orientation of the bar magnet.
2. Activate the voltmeter in parallel with the light bulb.
3. Use the coil with four loops of wire.
4. Produce the following results:
    - A positve voltage from a moving bar magnet
    - A negative voltage from a moving bar magnet
    - A positive voltage from switching the bar magnet polarity
    - A negative voltage from switching the bar magnet polarity
5. Is your voltage positive or negative when you are increasing Φ? How do you *decrease* Φ?

Motional EMF, Generators, and Transformers

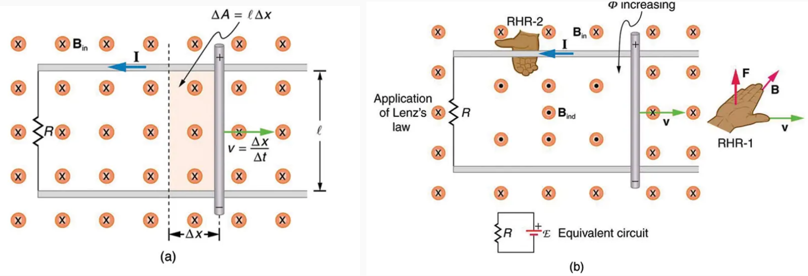

Figure 9: Motional emf in a loop with changing area.

Group board problems:

1. Show that *power* is $P = \vec{F} \cdot \vec{v}$ when acceleration is constant.
2. Show that the emf is $\epsilon = Blv$, where l is the length of the rod.
3. Show that power generated, $P = I^2R = \epsilon/R$, is equal to power injected.

How do we use Faraday's Law to induce power in a generator?

Start with Faraday's Law:

$$\epsilon = -N\frac{\Delta\Phi}{\Delta t} \tag{5}$$

The flux Φ depends on time:

$$\Phi = \vec{B}\cdot\vec{A}(t) = BA\cos(\theta(t)) \tag{6}$$

Let the *angular velocity* be constant: $\theta = \omega t$. Then we have

$$\Phi = BA\cos(\omega t) \tag{7}$$

Thus the emf (with N loops) is (...calculus...)

$$\epsilon = N\omega BA\sin(\omega t) = \epsilon_0\sin(\omega t) \tag{8}$$

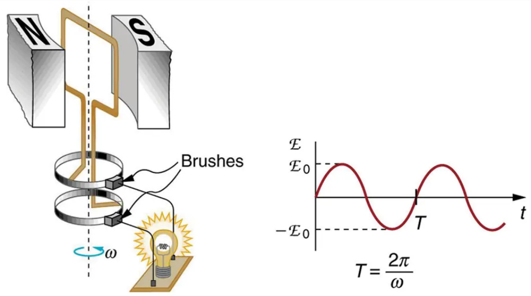

Figure 10: (Left) The AC generator with brushes generates an AC voltage. (Right) This is a diagram of the AC voltage.

- Amplitude: ϵ_0, the maximum value of the AC signal. Units: Volts.
- Period: $T = 2\pi/\omega$, the time to complete one AC cycle. Units: seconds.
- Frequency: $f = 1/T$, the number of cycles per second. Units: Hertz.

Motional EMF, Generators, and Transformers

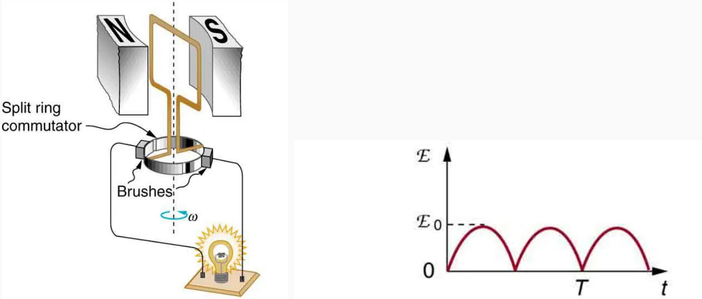

Figure 11: (Left) The AC generator with brushes and *commutator* generates pulsed DC. (Right) This is a diagram of the signal.

- Amplitude: ϵ_0, the maximum value of the AC signal. Units: Volts.
- Period: $T = 2\pi/\omega$, the time to complete one AC cycle. Units: seconds.
- Frequency: $f = 1/T$, the number of cycles per second. Units: Hertz.

Equation 9 is a basic model for the emf from a generator.

$$\epsilon = N\omega BA \sin(\omega t) = \epsilon_0 \sin(\omega t) \tag{9}$$

Which of the following would increase the *amplitude* of the emf?

- A: Turning the shaft more slowly
- B: Turning the shaft more quickly
- C: Decreasing the B-field
- D: Increasing N

Equation 10 is a basic model for the emf from a generator.

$$\epsilon = N\omega BA \sin(\omega t) = \epsilon_0 \sin(\omega t) \tag{10}$$

Which of the following would increase the *frequency* of the emf?

- A: Turning the shaft more slowly
- B: Turning the shaft more quickly
- C: Decreasing the B-field
- D: Increasing N

Equation 11 is a basic model for the emf from a generator.

$$\epsilon = N\omega BA \sin(\omega t) = \epsilon_0 \sin(\omega t) \tag{11}$$

Group exercise: Suppose an AC generator rotates at 200 rpm, in a B-field with 0.1 T, and has 100 loops with radius 5 cm. (a) What is the peak voltage this generator will produce? (b) If the generator powers a system with resistance of 1kΩ, what will be the peak current?

PhET: Motional EMF, Generators, and Transformers

PhET: AC Power generator

Link to the (CheerpJ) simulation:

`https://phet.colorado.edu/en/simulation/generator`

1. Set the water rate such that the meter reads 10 rotations per minute (rpm).
2. Choose the voltage meter under the pickup coil menu.
3. Under loops, choose 1 loop, and under area, leave it at 50%.
4. Choose show field meter in the upper right, and place the tool in the loop center.
5. On the same graph, plot the average B-field and voltage versus time. What is the period and amplitude of your signal? Use the left y-axis for B-field units, and the right y-axis for voltage units.
6. Create the same graph for $N = 3$ loops.
7. Create the same graph for $N = 1$ loop, but for 20 rpm.

Hint: you know the rpm of the magnet, so you know how much time corresponds to one rotation.

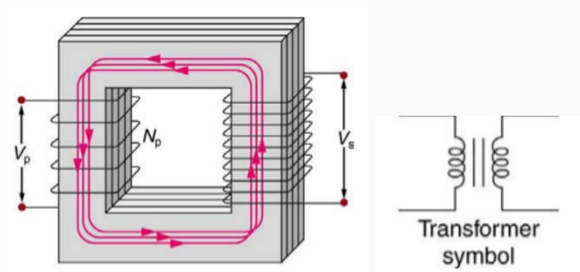

Figure 12: A *transformer* uses Faraday's law to change voltages in AC-generated systems.

The magnetizable core (gray) creates a loop in the B-field that passes through the left and right coils. Use Faraday's law to show that

$$\frac{V_L}{V_R} = \frac{N_L}{N_R} \tag{12}$$

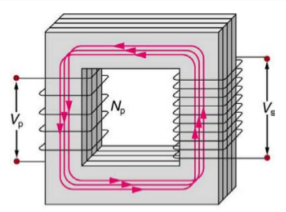

Figure 13: The *transformer* changes AC voltage levels.

Suppose the transformer in Fig. 13 has $N_L = 5$, $N_R = 100$, $V_L = 1$ kV (peak). What is V_R (peak), in kV?

- A: 20 kV
- B: 5 kV
- C: 0.05 kV
- D: 0.05 V

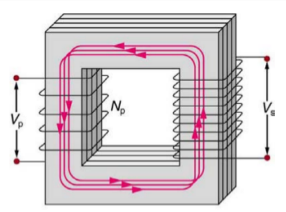

Figure 14: The *transformer* changes AC voltage levels.

Suppose we need the transformer in Fig. 14 to produce $V_R = 120$ V, and $V_L = 12$ kV. Which combination of coils will satisfy the requirement?

- A: $N_L = 3$, $N_R = 10$
- B: $N_L = 10$, $N_R = 1000$
- C: $N_L = 10$, $N_R = 100$
- D: $N_L = 1000$, $N_R = 10$

Motional EMF, Generators, and Transformers

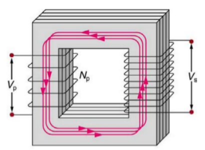

Figure 15: The *transformer* changes AC voltage levels.

If the $V_L \neq V_R$, how are energy and power conserved?

- A: The induced current is larger on the right.
- B: The induced current is smaller on the right.
- C: The induced current is conserved from left to right.
- D: The induced emf is conserved from left to right.

Demonstrate on board how power is conserved by (a) deriving the currents I_L and I_R, then forming the powers P_L and P_R.

Inductors

It turns out you can *reverse* transformers, through symmetry. Between the coils in the transformer in Fig. 15, there is *mutual inductance, M*:

$$\epsilon_R = -M\frac{\Delta I_L}{\Delta t} \tag{13}$$

$$\epsilon_L = -M\frac{\Delta I_R}{\Delta t} \tag{14}$$

The inductance accounts for everything except the current: the geometry, number of turns, and field strength. Inductance is useful for systems with fixed geometry, where we often do not need to know Φ.

Derive an expression that relates M to Φ, given the original version of Faraday's Law.

Consider the case of **self-inductance**, L, which accounts for the magnetic field created inside (for example) a solenoid when current is introduced. New current creates a change in Φ within the solenoid, so Faraday's Law predicts a backwards emf opposing the change:

$$\boxed{\epsilon = -L\frac{\Delta I}{\Delta t}} \tag{15}$$

The inductance unit is the *Henry* (after Joseph Henry), $\mathrm{V\,A^{-1}\,s}$, or $\Omega\,\mathrm{s}$.

Self-inductance applies to increasing and decreasing current:

$$\boxed{\epsilon = -L\frac{\Delta I}{\Delta t}} \tag{16}$$

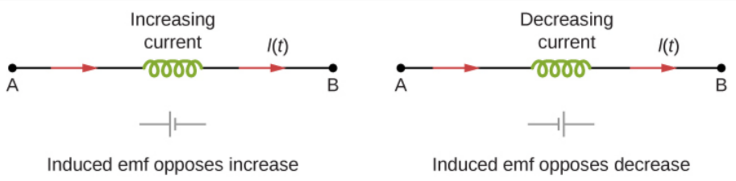

Figure 16: (a) Increasing current yields a negative voltage. (b) Decreasing current yields a positive voltage.

Inductors

Suppose we have an inductor with $L = 0.1$ mH, carrying a current of 100 mA. If we switch off the current in 1 ms, what is the induced emf?

- A: 0.1 mV
- B: 1 mV
- C: 10 mV
- D: 100 mV

(Ahh, units … Got heem! Also, why are the answers *positive*?)

Suppose we have a current that switches from 100 mA to -100 mA in 1 ms. We observe a 100 mV emf across the inductor. What is the inductance?

- A: -0.5 mH
- B: -0.5 H
- C: 0.5 H
- D: 0.5 mH

What is the inductance of a solenoid, given solenoid properties? Recall how the inductance relates to flux, current, and turn number:

$$L = N\frac{\Delta\Phi}{\Delta I} \tag{17}$$

$$L = N\frac{A\Delta B}{\Delta I} \tag{18}$$

$$\Delta B = \mu_0\left(\frac{N}{l}\right)\Delta I \tag{19}$$

$$L = N\frac{A\mu_0(N/l)\Delta I}{\Delta I} \tag{20}$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}} \tag{21}$$

Recall the solenoid used in the Ampère's Law lab activity. Suppose we count $N = 80$ turns, and measure $A = 8 \times 10^{-3}$ m^2, and $l = 0.1$ m. What is the inductance?

- A: 0.6 H
- B: 0.06 H
- C: 6 mH
- D: 0.6 mH

Recall that $\mu_0 = 4\pi \times 10^{-7}$ T A^{-1} m.

What is the inductance of the same solenoid, but with twice the turns?

- A: 2.4 mH
- B: 1.2 mH
- C: 24 mH
- D: 12 mH

This is a scaling problem. How does L depend on N?

Inductors

How much **energy** is stored in an inductor? Suppose potential energy is considered at *a constant voltage*, given some charging circuit that pushes current through an inductor (with some resistance).

$$\Delta U = \Delta q \epsilon \quad \rightarrow \quad \epsilon = \Delta U / \Delta q \tag{22}$$

$$|\epsilon| = L\frac{\Delta I}{\Delta t} \tag{23}$$

$$\frac{\Delta U}{\Delta q} = L\frac{\Delta I}{\Delta t} \tag{24}$$

$$\Delta U = L\Delta I\left(\frac{\Delta q}{\Delta t}\right) \tag{25}$$

$$\Delta U = LI\Delta I \tag{26}$$

$$\boxed{U = \frac{1}{2}LI^2} \tag{27}$$

Inductors

How much energy is stored in an inductor with inductance 50 mH that was charged with a current that reaches 1 A?

- A: 25 J
- B: 2.5 J
- C: 25 mJ
- D: 2.5 mJ

Nom nom nom, more units.

Conclusion

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