

# Midterm 1

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## 1 Unit 0: Electrostatics I and II

- ✓ 1. A 50 gram copper wire has a net charge of 2.00  $\mu\text{C}$ . What fraction of the copper's electrons has been removed? (Each atom has 29 protons, and the atomic mass is 63.5.)

Ans.

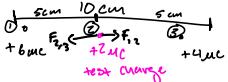
$$2.00 \mu\text{C} \times \frac{1\text{C}}{1.6 \times 10^{-19}\text{C}} \times \frac{1.9}{1.6 \times 10^{-19}\text{C}} = 1.25 \times 10^{25} \text{ excess Protons}$$

*removed electrons*

$$\frac{\# \text{ of } e^- \text{ removed}}{\text{total } \# \text{ of } e^-} = \frac{50\text{g} \times \frac{1\text{mol}}{63.5\text{g}} \times \frac{6.02 \times 10^{23} \text{ atoms}}{1\text{mol}} \times \frac{29\text{p}}{1\text{atom}}}{1.25 \times 10^{25} \text{ p}} = 1.37 \times 10^{25} \text{ p}$$

- ✓ 2. A test charge of +2  $\mu\text{C}$  is placed halfway between a charge of +6  $\mu\text{C}$  and another of +4  $\mu\text{C}$  separated by 10 cm. (a) What is the magnitude of the net force on the test charge? (b) What is the direction of this force (away from or toward the +6  $\mu\text{C}$  charge)?

Ans.



(A)  $F_{1,2}$  repulsive both

$$F_{1,2} = \frac{k|q_1 q_2|}{r^2}$$

$$= \frac{8.99 \times 10^9 |6 \times 10^{-6} \times 2 \times 10^{-6}|}{(0.05)^2}$$

$$= 43.162 \text{ N}$$

(B)  $F_{2,3}$

$$F_{2,3} = \frac{k|q_2 q_3|}{r^2}$$

$$= \frac{8.99 \times 10^9 |2 \times 10^{-6} \times 4 \times 10^{-6}|}{(0.05)^2}$$

$$= F_{2,3} = 28.768 \text{ N}$$

(C) Net force

$$\Sigma F = F_{1,2} - F_{2,3}$$

$$= 43.162 - 28.768$$

$$= 14.4 \text{ N}$$

*Force is away from +6 μC*

- ✓ 3. What is the force on the charge located at  $x = 8.00 \text{ cm}$  in Fig. 1(a) given that  $q = 1.00 \mu\text{C}$ ?

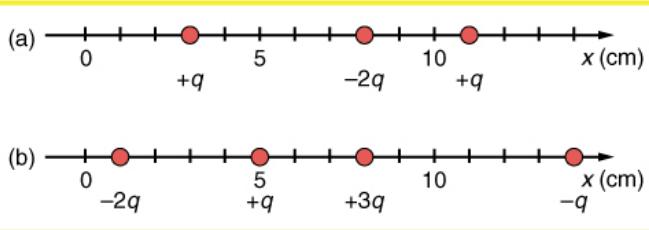


Figure 1: Linear arrangement of charges.

4. Find the total electric field at  $x = 11.00 \text{ cm}$  in Fig. 1(b).

- ✓ 5. Determine the direction of the force on  $q$  in Fig. 2, given that  $q_a = q_b = +7.50 \mu\text{C}$  and  $q_c = q_d = -7.50 \mu\text{C}$ . (b) Calculate the force on the charge  $q$ , given that the square is 10.0 cm on a side and  $q = 2.00 \mu\text{C}$ .

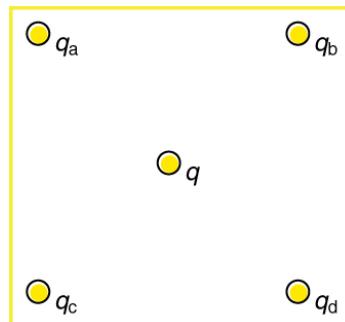


Figure 2: 2D arrangement of charges.

- ✓ 6. (a) An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons? (b) Show that units of V/m and N/C for electric field strength are indeed equivalent.

a)

$$\Delta V = \frac{\Delta PE}{e}$$

$$40,000 \text{ V} = \frac{\Delta PE}{1.6 \times 10^{-19} \text{ C}}$$

$$\Delta PE = 6.4 \times 10^{-18} \text{ J}$$

$$kE = \frac{1}{2} \frac{mv^2}{q}$$

$$= 1.2 \times 10^9 \frac{\text{N}}{\text{C}}$$

b) How  $\frac{V}{m} = \frac{N}{C}$ ?

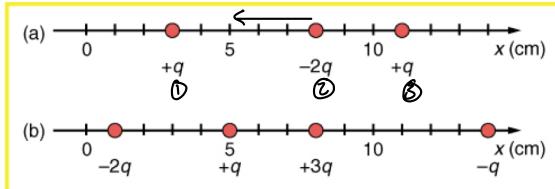
$$V = \frac{E}{q}$$

$$V = \frac{F}{q}$$

$$\frac{V}{m} = \frac{F}{q \cdot m} = \frac{N}{C \cdot m} = \frac{N \cdot m}{C \cdot m} = \frac{J}{C \cdot m} = \frac{W}{C \cdot m}$$

7. The electric field strength between two parallel conducting plates separated by 4.00 cm is 7.50  $\times$

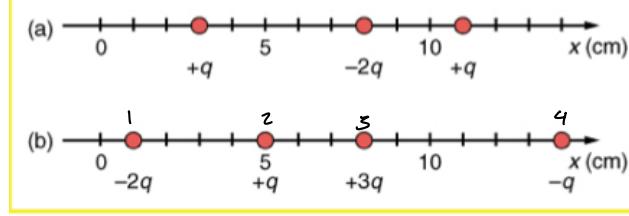
3. What is the force on the charge located at  $x = 8.00$  cm in Fig. 1(a) given that  $q = 1.00 \mu\text{C}$ ?



$$\begin{aligned} \Sigma F_3 &= F_{23} - F_{13} \\ &= k \frac{|q \cdot 2q|}{r_{23}^2} - k \frac{|q \cdot 2q|}{r_{13}^2} = \\ &= \frac{8.99 \times 10^9 |1 \times 10^{-6} \times 2 \times 10^{-6}|}{0.03^2} - \frac{8.99 \times 10^9 |1 \times 10^{-6} \times 2 \times 10^{-6}|}{0.05^2} \\ &= F_3 = \pm 12.48 \text{ N} \end{aligned}$$

4. Find the total electric field at  $x = 11.00$  cm in Fig. 1(b).

$$\begin{matrix} E_1 \\ E_2 \\ E_3 \\ E_4 \end{matrix} \rightarrow$$

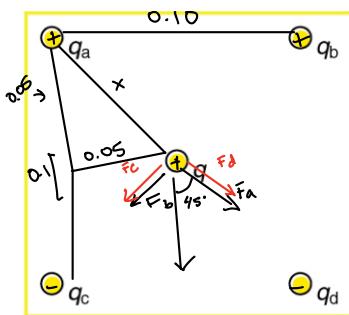


$$E_2 + E_3 + E_4 - E_1 = E_T$$

$$k \left[ \frac{E_2}{0.6^2} + \frac{E_3}{0.08^2} + \frac{E_4}{0.08^2} - \frac{E_1}{0.1^2} \right] = E_T$$

NO  $q$  value. This is the simplest form

5. Determine the direction of the force on  $q$  in Fig. 2, given that  $q_a = q_b = +7.50 \mu\text{C}$  and  $q_c = q_d = -7.50 \mu\text{C}$ . (b) Calculate the force on the charge  $q$ , given that the square is 10.0 cm on a side and  $q = 2.00 \mu\text{C}$ .

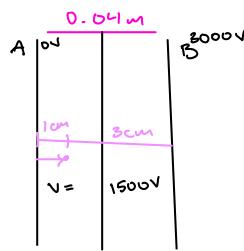


$$\begin{aligned} x &= \sqrt{0.05^2 + 0.05^2} = 0.0707 \text{ m} \\ k \frac{q_1 q_2}{r^2} &= \frac{8.99 \times 10^9 (7.5 \times 10^{-6} \cdot 2 \times 10^{-6})}{0.0707^2} \\ F &= 26.97 \text{ N} \\ F_{ay} &= 26.47 \cos(45^\circ) \\ F_{ay} &= 19.07 \text{ N} \\ \boxed{F_{total} = 26.3 \text{ N}} \end{aligned}$$

Figure 2: 2D arrangement of charges.

7. The electric field strength between two parallel conducting plates separated by 4.00 cm is  $7.50 \times 10^4 \text{ V m}^{-1}$ . (a) What is  $\Delta V$  between the plates?

(b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)? (c) The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength?



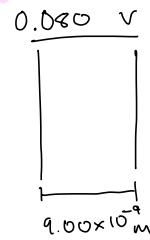
$$A) V_{AB} = Ed$$

$$V_{AB} = 7.5 \times 10^4 [0.04] \text{ V}$$

$$V_{AB} = 3000 \text{ V}$$

$$B) \frac{1}{4}(3000 \text{ V}) = 750 \text{ V}$$

- C) parallel capacitor



$$V_{AB} = Ed$$

$$E = \frac{V}{d}$$

$$E = \frac{0.08}{9 \times 10^{-9}} \text{ N/C}$$

$$E = 8.89 \times 10^6 \text{ N/C}$$

8. A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

$$q = 2(1.6 \times 10^{-19} \text{ C}) = 3.2 \times 10^{-19} \text{ C}$$

$$E = 32.0 \text{ keV} \times \frac{1000 \text{ eV}}{1 \text{ keV}} \times \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} = 5.12 \times 10^{-15} \text{ J} \quad \Delta PE$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

$$\Delta V = \frac{\Delta PE}{q} = \frac{5.12 \times 10^{-15}}{3.2 \times 10^{-19}} = 1.6 \times 10^4 \text{ V}$$

$$V_{AB} = Ed$$

$$E = \frac{V_{AB}}{d} = \frac{1.6 \times 10^4}{0.02} = 8.0 \times 10^5 \text{ N/C}$$

9. In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

$$E = 5 \text{ MeV} \times \frac{10^6 \text{ eV}}{1 \text{ MeV}} \times \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} = 8 \times 10^{-13} \text{ J}$$

$$q = 3.2 \times 10^{-19}$$

$$= 3.2 \times 10^{-19} \text{ C}$$

$$\Delta V = \frac{\Delta PE}{q} = \frac{8 \times 10^{-13}}{3.2 \times 10^{-19}} = 2.5 \times 10^6 \text{ V}$$

$$79 \times 1.6 \times 10^{-19} = 1.26 \times 10^{-17} \text{ C}$$

charge of gold nucleus

$$r = kq = \frac{9 \times 10^9 [1.26 \times 10^{-17}]}{2.5 \times 10^6}$$

$$r = 4.55 \times 10^{-14} \text{ m}$$

✓

- $10^4 \text{ V m}^{-1}$ . (a) What is  $\Delta V$  between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)? (c) The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength?<sup>1</sup>

✓

- (a) What is the energy stored in the  $10.0 \mu\text{F}$  capacitor of a heart defibrillator charged to  $9.00 \times 10^3 \text{ V}$ ? (b) Find the amount of stored charge. (c) In open heart surgery, a much smaller amount of energy will defibrillate the heart. What voltage is applied to the  $8.00 \mu\text{F}$  capacitor of a heart defibrillator that stores 40.0 J of energy? (d) Find the amount of stored charge.

✓

8. A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

✓

- To build up the charge and energy required in part (a) of the previous problem, an AED designer decides to split the charge among four capacitors in parallel. Determine the required capacitance of each individual capacitor, and the charge stored on each, if the voltage remains  $9.00 \times 10^3 \text{ V}$ . Why would the designer choose not to connect the capacitors in series?

✓

9. In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

✓

4. If a 1.0 mm diameter copper wire can have a resistance of no more than  $2.0 \Omega$ , (at 20 degrees C), how long can it be?

## 2 Unit 1: Capacitors, Current, and DC circuits

1. What capacitance is needed to store  $3.00 \mu\text{C}$  of charge at a voltage of 120 V?

✓

5. An LED is connected in series with a  $1 \text{ k}\Omega$  resistor. A 3.0V battery is connected to the resistor, the LED follows the resistor, and the LED is then

<sup>1</sup>The value is surprisingly large, but correct.

6. What capacitance is needed to store  $3.00 \mu\text{C}$  of charge at a voltage of 120 V?

$$C = ?$$

$$q = 3 \times 10^{-6} \text{ C}$$

$$V = 120 \text{ V}$$

$$C = \frac{q}{V}$$

$$C = \frac{3.0 \times 10^{-6} \text{ C}}{120}$$

$$C = 2.5 \times 10^{-8} \text{ F}$$

7. (a) What is the energy stored in the  $10.0 \mu\text{F}$  capacitor of a heart defibrillator charged to  $9.00 \times 10^3$  V? (b) Find the amount of stored charge. (c) In open heart surgery, a much smaller amount of energy will defibrillate the heart. What voltage is applied to the  $8.00 \mu\text{F}$  capacitor of a heart defibrillator that stores 40.0 J of energy? (d) Find the amount of stored charge.

(c)

$$V = ?$$

$$C = 8 \times 10^{-6} \text{ F}$$

$$E = 40 \text{ J}$$

$$E_{\text{cap}} = \frac{CV^2}{2}$$

$$\sqrt{\frac{2E_{\text{cap}}}{C}} = V$$

$$V = \sqrt{\frac{2(40)}{8 \times 10^{-6}}} \text{ V}$$

$$V = 3.16 \times 10^3 \text{ V}$$

$$q = ?$$

$$C = \frac{q}{V}$$

$$q = C \cdot V$$

$$q = 8 \times 10^{-6} \times (3.16 \times 10^3)$$

$$q = 0.0253 \text{ C}$$

$$(a) E = \frac{CV^2}{2}$$

$$= 10 \times 10^{-6} \text{ F}$$

$$= 9 \times 10^3 \text{ V}$$

$$E = \frac{10 \times 10^{-6} (9 \times 10^3)^2}{2}$$

$$E = 405 \text{ J}$$

$$(b) q = ?$$

$$C = \frac{q}{V} \rightarrow q = C \cdot V$$

$$q = 10 \times 10^{-6} (9 \times 10^3) = 9.000 \text{ C}$$

(c)

- To build up the charge and energy required in part (a) of the previous problem, an AED designer decides to split the charge among four capacitors in parallel. Determine the required capacitance of each individual capacitor, and the charge stored on each, if the voltage remains  $9.00 \times 10^3$  V. Why would the designer choose not to connect the capacitors in series?

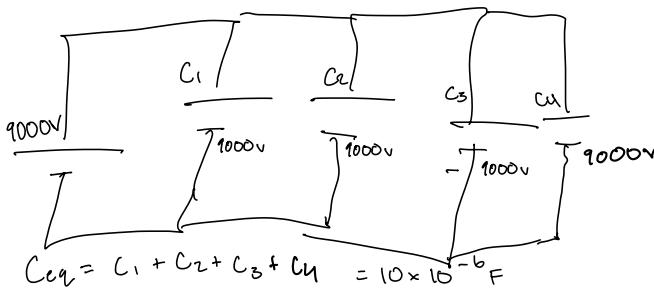
$$405 \text{ J}$$

$$E = \frac{1}{2} CV^2$$

$$405 \text{ J} = \frac{1}{2} \cdot C_1 \cdot 9000^2$$

$$C_{\text{eq}} = 10 \times 10^{-6} \text{ F}$$

$$\frac{4C}{4} = \frac{10 \times 10^{-6}}{4} \text{ F}$$



The voltage needs to be the same across every capacitor. Because KVL for every closed loop circuit the sum of voltages was to equal 0 so every capacitor has to have the same voltage as the battery.

5



- 4 If a 1.0 mm diameter copper wire can have a resistance of no more than 2.0  $\Omega$ , (at 20 degrees C), how long can it be?

$$\text{Copper wire radius} = 0.5 \text{ mm}$$

$$\text{wire length} = ?$$

$$\text{Resistivity} = 1.7 \times 10^{-8} \Omega \text{ m}$$

$$\text{Resistance} = 2.0 \Omega$$

$$\text{Area} = 0.5^2 \cdot \pi = 0.785 \text{ mm}^2$$

$$2 \Omega = \frac{1724 R_{\text{mm}} \cdot L}{0.785 \text{ mm}^2}$$

$$\frac{2 \Omega}{2196 \frac{\Omega}{\text{mm}}} = 2196 \frac{\Omega}{\text{mm}} \cdot L$$

$$L = 9.107 \times 10^{-4} \text{ mm}$$

5. An LED is connected in series with a  $1\text{ k}\Omega$  resistor. A  $3.0\text{V}$  battery is connected to the resistor, the LED follows the resistor, and the LED is then

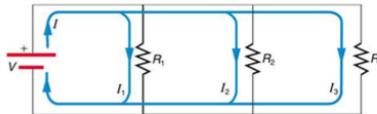


Figure 3: A DC circuit with three resistors.

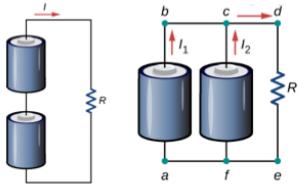
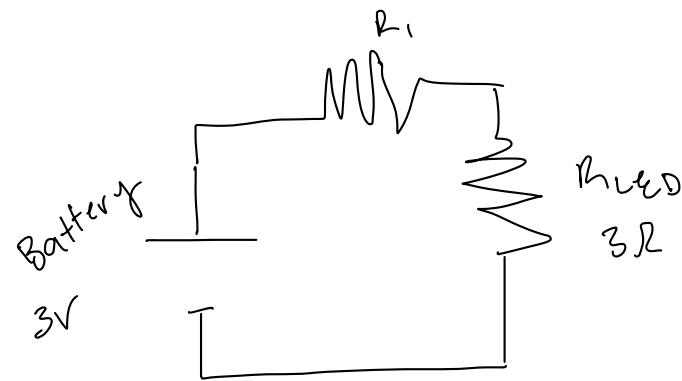


Figure 4: (Left) A DC circuit with two batteries in series, and a resistance  $R = 0.5\text{ k}\Omega$ . (Bottom) A DC circuit with two batteries in parallel, and a resistance  $R = 0.5\text{ k}\Omega$ .

connected to ground. The negative terminal of the battery is also connected to ground. (a) What current flows from the battery, if the LED resistance is  $3\text{ }\Omega$ ? (b) How much power is consumed by the LED? (c) How many Coulombs of charge flow through the LED in 10 minutes?



$$\text{a)} \quad I = \frac{V}{R} = \frac{3\text{V}}{3\Omega} = 1\text{A}$$

$$\text{b)} \quad P = I \cdot V = 1\text{A} \cdot 3\text{V} = \boxed{3\text{W}}$$

$$\begin{aligned} \text{c)} \quad Q &= I \times T \\ &= 1\text{A} \times 600\text{ sec} \\ Q &= 600\text{C} \end{aligned}$$

$$T = 10\text{ min} = 600\text{ sec}$$

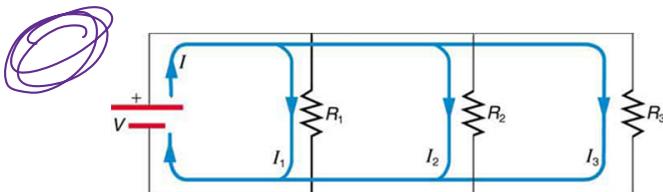


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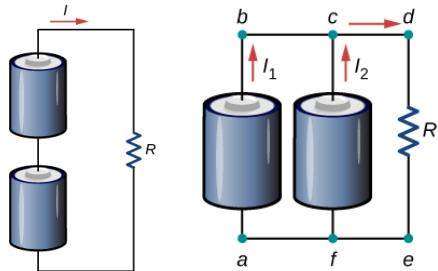


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- 2 Consider Fig. 4. (a) Assuming no internal resistance, calculate the current and power through the resistance  $R$  if each battery has 1.5 V in the series circuit, and 3 V in the parallel circuit. (b) Now repeat part (a) for each circuit, assuming all batteries have an internal resistance of  $5 \Omega$ .

- 3 A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of  $0.02 \Omega$  in series with a 1.53-V carbon-zinc dry cell having a  $0.10 \Omega$  internal resistance. The load resistance is  $10.00 \Omega$ . (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?

- 4 A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

### 3 Unit 2: DC circuits with resistors in series and parallel, RC circuits

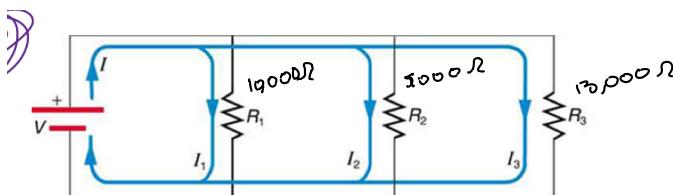
- 1 In Fig. 3, let  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 5 \text{ k}\Omega$ , and  $R_{\text{tot}} = 2 \text{ k}\Omega$ . (a) What is the resistance of  $R_3$ ? (b) If the battery has  $\Delta V = 12 \text{ V}$ , what current flows from the battery? (c) What are the individual currents,  $I_1$ ,  $I_2$ , and  $I_3$ ?

- 5 An ECG monitor must have an RC time constant less than  $1.00 \times 10^2 \mu\text{s}$  to be able to measure

### 3 Unit 2: DC circuits with resistors in series and parallel, RC circuits

1. In Fig. 3, let  $R_1 = 10 \text{ k}\Omega$ ,  $R_2 = 5 \text{ k}\Omega$ , and  $R_{\text{tot}} = 2 \text{ k}\Omega$ . (a) What is the resistance of  $R_3$ ? (b) If the battery has  $\Delta V = 12 \text{ V}$ , what current flows from the battery? (c) What are the individual currents,  $I_1$ ,  $I_2$ , and  $I_3$ ?

$$R_{\text{tot}} = 2 \text{ k}\Omega = 2000 \Omega$$



a)  $R_3 = ?$

$$R_{\text{tot}} = R_1 + R_2 + R_3$$

$$2000 = 10,000 + 5000 + R_3$$

$$R_3 = -13,000 \Omega$$

b)  $\Delta V = 12 \text{ V}$

$$I = \frac{V}{R} = \frac{12 \text{ V}}{2000 \Omega} = 0.006 \text{ Amps}$$

c) individual currents  $I_1$ ,  $I_2$ ,  $I_3$

$$I_1 = \frac{V}{R_1} = \frac{12.0 \text{ V}}{10,000 \Omega} = 0.0012 \text{ Amps}$$

$$I_2 = \frac{V}{R_2} = \frac{12.0 \text{ V}}{5,000 \Omega} = 0.0024 \text{ Amps}$$

$$I_3 = \frac{V}{R_3} = \frac{12.0 \text{ V}}{13,000 \Omega} = -0.000923 \text{ Amps}$$

2. Consider Fig. 4. (a) Assuming no internal resistance, calculate the current and power through the resistance  $R$  if each battery has 1.5 V in the series circuit, and 3 V in the parallel circuit. (b) Now repeat part (a) for each circuit, assuming all batteries have an internal resistance of 5  $\Omega$ .

a)  $R = 500 \Omega$

$$I = \frac{V}{R} = \frac{3 \text{ V}}{500 \Omega} = 0.006 \text{ Amps}$$

$$P = I^2 \cdot R = (0.006)^2 \cdot 500 \Omega$$

$$\text{Power dissipated} = 0.018 \text{ W}$$

$$\text{Power source} = I \cdot E = 0.006 \text{ A} \cdot 3 \text{ V} = 0.018 \text{ W}$$

Parallel

$$I = 3 \text{ V} \left( \frac{1}{500 \Omega} \right)$$

$$I = 0.006 \text{ A}$$

$$P = I^2 \cdot R = (0.006)^2 \cdot 500 \Omega = 0.018 \text{ W}$$

b) Series  $R_{\text{tot}} = 500 \Omega + 5 \Omega + 5 \Omega$

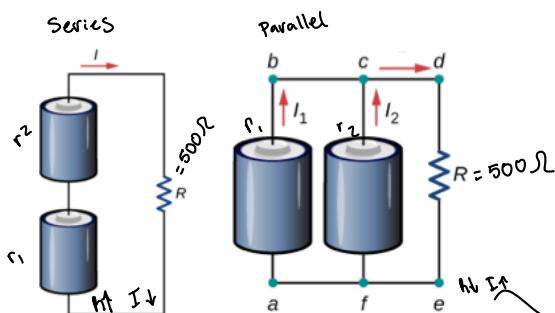
$$I = \frac{3 \text{ V}}{510 \Omega} = 0.00588 \text{ A}$$

$$P = I^2 \cdot R = 0.00588^2 \times 500 \Omega = 1.728 \text{ W}$$

Parallel

$$I = 3 \text{ V} \cdot \left( \frac{1}{500 \Omega} + \frac{1}{5 \Omega} + \frac{1}{5 \Omega} \right) = 3.618 \text{ A}$$

$$P = I^2 \cdot R = (3.618)^2 \cdot 500 \Omega = 6544.96 \text{ W}$$



$$R = 500 \Omega$$

$$I = ?$$

$$3 \text{ V} = I \cdot 500 \Omega$$

$$= 500 \Omega$$

$$I = ?$$

$$R_{\text{int}} = R_{\text{b1}} = 5 \Omega$$

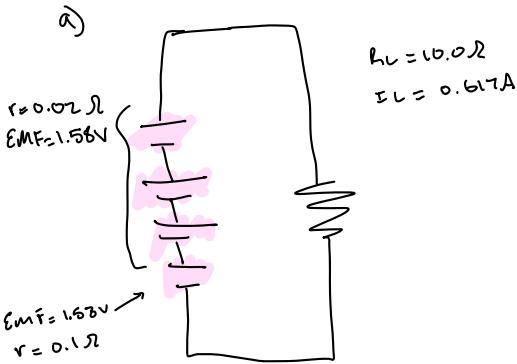
internal resistance

$$V_1 = V_2 = 3 \text{ V} \quad \text{Battery voltage}$$

$i_2 = i_1$   
current through each battery

$i_3 = \text{current through } R$   
 $R = 500 \Omega$

3. A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of  $0.02 \Omega$  in series with a 1.53-V carbon-zinc dry cell having a  $0.10 \Omega$  internal resistance. The load resistance is  $10.00 \Omega$ . (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?



b)  $I_T = \frac{V_T}{R_T}$  ← total voltage  
     total current    total resistance

$$I_T = \frac{(3)(1.58) + 1.53}{3(0.02) + 0.1 + 10}$$

$$= \frac{4.74 + 1.53}{0.16 + 10}$$

$$I_T = 0.617 \text{ Amperes}$$

D)  $P_L = I_L^2 R_L$   
 $0.5 \text{ W} = I_L^2 (10)$

$I_L = 0.224$  ← total current

$$I_T = \frac{V_T}{R_T}$$

$$0.224 = \frac{3(1.58) + 1.53}{3(0.02) + 0.1 + 10}$$

$$10.00 + r = \frac{6.27}{0.224}$$

$r = 19 \Omega$

C)  $P_L = I_L^2 R_L$   
 $P_L = (0.617)^2 (10)$   
 $P_L = 3.808 \text{ W}$

- 4) A heart pacemaker fires 72 times a minute, each time a  $25.0\text{-nF}$  capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

$$\frac{1 \text{ min}}{72 \text{ beats}} = \frac{0.039 \text{ min}}{1 \text{ beat}} \cdot \frac{60 \text{ s}}{1 \text{ min}} = \frac{0.833 \text{ s}}{1 \text{ beat}} = \hat{\tau}$$

$$\hat{\tau} = R \cdot C$$

$$0.833 = R \cdot [25 \times 10^{-9}]$$

$R = 3.33 \times 10^7 \Omega$

- 5) An ECG monitor must have an RC time constant less than  $1.00 \times 10^2 \mu\text{s}$  to be able to measure variations in voltage over small time intervals. If the resistance of the circuit (due mostly to that of the patient's chest) is  $1.00 \text{ k}\Omega$ , what is the maximum capacitance of the circuit?

If  $\hat{\tau} < \text{value found}$  then

the capacitance needs to be less than  $1 \times 10^{-7} \text{ F}$

$$\hat{\tau} = 1 \times 10^{-4} \text{ s}$$

$$r = 1 \times 10^3 \Omega$$

$$C = ?$$

$$\hat{\tau} = R \cdot C \quad \frac{\hat{\tau}}{R} = C$$

$$\frac{1 \times 10^{-4}}{1 \times 10^3} = C$$

$C = 1 \times 10^{-7} \text{ F}$

- b) Since it is such a small value it shouldn't be difficult to limit capacitance

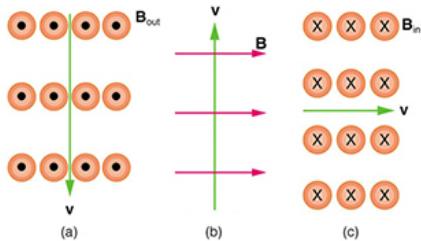


Figure 5: Three cases involving the particle velocity  $\vec{v}$ , and  $\vec{B}$  field.

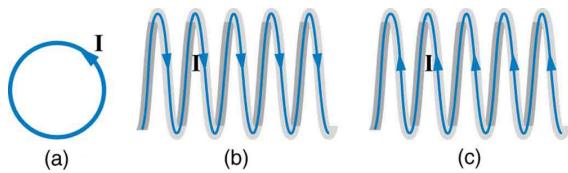


Figure 6: Three currents that create B-fields.

variations in voltage over small time intervals. If the resistance of the circuit (due mostly to that of the patient's chest) is  $1.00 \text{ k}\Omega$ , what is the maximum capacitance of the circuit?

## 4 Unit 3: Magnetism I

- Consider Fig. 5. Fill in Tab. 1 of directions below for the Lorentz force, assuming a **negatively charged** particle. Let  $\hat{i}$  represent right,  $\hat{j}$  represent up, and  $\hat{k}$  represent out of the page.
- An electron moving at  $4.00 \times 10^3 \text{ m s}^{-1}$  in a  $1.25\text{-T}$  magnetic field experiences a magnetic force of  $1.40 \times 10^{-16} \text{ N}$ . What angle does the velocity of the electron make with the magnetic field? There are two possible answers.
- (a) An oxygen-16 ion with a mass of  $2.66 \times 10^{-26} \text{ kg}$  travels at  $5.00 \times 10^6 \text{ m/s}$  perpendicular to a  $1.20\text{-T}$  magnetic field, which makes it move in a circular arc with a  $0.231\text{-m}$  radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer. (d) A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18. Assuming the ions have the same charge, what would the radius of the circular arc be for the oxygen-18? *Hint: this is a scaling problem.*
- What force is exerted on the water in an MHD drive utilizing a  $25.0\text{-cm}$ -diameter tube, if  $100\text{-A}$  current is passed across the tube that is perpendicular to a  $2.00\text{-T}$  magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)
- For this exercise, we are designing an electric motor. Calculate the B-field strength needed on a 200-turn square loop  $20.0 \text{ cm}$  on a side to create a maximum torque of  $300 \text{ N m}$  if the loop has  $25.0 \text{ A}$  of current.

Case	v direction	B direction	F direction
(a)			
(b)			
(c)			
(d)			

Table 1: Table of directions for to Fig. 5.

1. Consider Fig. 5. Fill in Tab. 1 of directions below for the Lorentz force, assuming a **negatively charged** particle. Let  $\hat{i}$  represent right,  $\hat{j}$  represent up, and  $\hat{k}$  represent out of the page.

$\hat{i}$  = right  
 $\hat{j}$  = up  
 $\hat{k}$  = out of the page



Case	v direction	B direction	F direction
(a)	$-\hat{j}$	$\hat{k}$	$\hat{i}$
(b)	$\hat{j}$	$\hat{i}$	$\hat{k}$
(c)	$\hat{i}$	$-\hat{k}$	$-\hat{j}$
(d)			

Table 1: Table of directions for to Fig. 5.

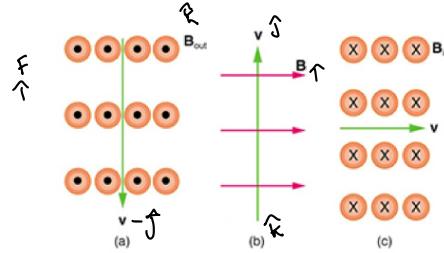
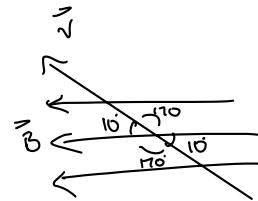


Figure 5: Three cases involving the particle velocity  $\vec{v}$  and  $\vec{B}$  field

2. An electron moving at  $4.00 \times 10^3 \text{ m s}^{-1}$  in a  $1.25\text{-T}$  magnetic field experiences a magnetic force of  $1.40 \times 10^{-16} \text{ N}$ . What angle does the velocity make with the magnetic field? There are two possible answers.

$$\begin{aligned} F &= q v B \sin \theta \\ \frac{F}{q v B} &= \sin \theta \\ \frac{1.40 \times 10^{-16}}{(1.6 \times 10^{-19})(4 \times 10^3)(1.25)} &= \sin \theta \end{aligned}$$

$$\begin{aligned} \sin^{-1}(0.176) &= \theta \quad ① \\ 10.1^\circ &= \theta \end{aligned}$$



$$\begin{aligned} ② \downarrow \\ 170^\circ &= \theta \end{aligned}$$

3. (a) An oxygen-16 ion with a mass of  $2.66 \times 10^{-26}$  kg travels at  $5.00 \times 10^6$  m/s perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer. (d) A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18. Assuming the ions have the same charge, what would the radius of the circular arc be for the oxygen-18? Hint: this is a scaling problem.

$$a) r = \frac{mv}{qB}$$

$$r = \frac{mv}{qB}$$

$$q = \frac{2.66 \times 10^{-26} (5 \times 10^6)}{0.231 (1.2)}$$

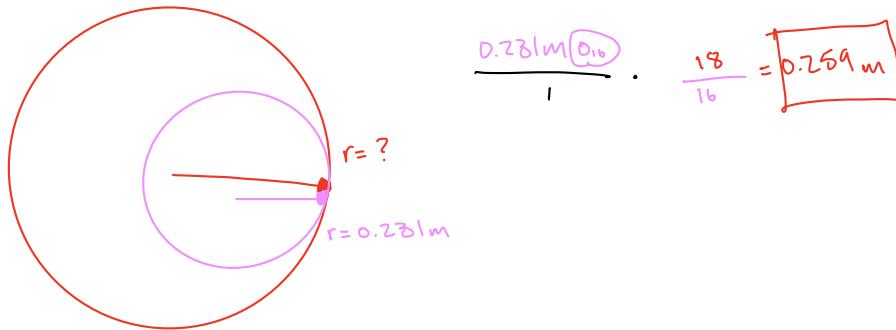
$$q = 4.79 \times 10^{-19} C$$
charge

b)  $\frac{4.79 \times 10^{-19} C}{1.6 \times 10^{-19} C} = \boxed{3}$

c) The value should be an integer because electrons can only come whole, fractional amounts of electrons is not possible

d)  $r = \frac{mv}{qB}$

*oxygen 18*      *oxygen 16*



4. What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

$$F = I l B \sin\theta$$

$$l = 25 \text{ cm} = 0.25 \text{ m}$$

$$F = (100)(0.25)(2) \sin(90)$$

$$F = 50 \text{ N}$$

5. For this exercise, we are designing an electric motor. Calculate the B-field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of 300 N m if the loop has 25.0 A of current.

$$\text{Torque} = (N)(I)(A)B \sin\theta$$

↖ B-field  
 ↑ # turns      ↑ Area = 0.04 m<sup>2</sup>

$$B = \frac{300}{(200)(25)(0.04)} \sin(90)$$

$$B = \frac{300}{200} = 1.5$$

Case	B direction
(a)	
(b)	
(c)	

Table 2: Table of directions for to Fig. 6.

6. Consider Fig. 6. Fill in the B-field directions in Tab. 2 using Ampère's Law, and the appropriate right-hand rule. Let  $\hat{i}$  represent right,  $\hat{j}$  represent up, and  $\hat{k}$  represent out of the page.
7. Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.
  
  
  
  
  
  
8. The B-field in the tokamak reactor in Fig. 7 is given by  $B = \mu_0 NI/(2\pi r)$ , where  $N$  is the total number of loops,  $I$  is the current, and  $r$  is the radius at which we evaluate the B-field. (a) Design your own reactor by specifying practical values of  $N$ ,  $I$ , and  $r$  that achieve a 1.0 T B-field at a radius of 5.0 m. (b) With what frequency will a proton circle the toroidal B-field (1.0 T)? *These ideas will help activate fusion reactors.*

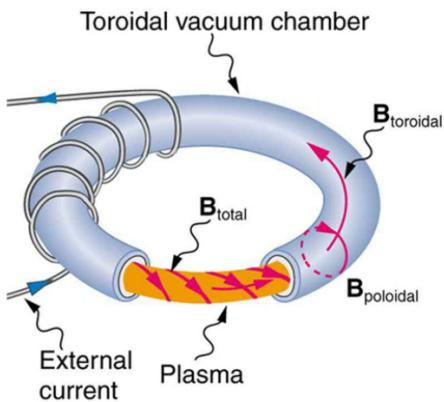


Figure 7: A generalized diagram of a tokamak.

6. Consider Fig. 6. Fill in the B-field directions in Tab. 2 using Ampère's Law, and the appropriate right-hand rule. Let  $\hat{i}$  represent right,  $\hat{j}$  represent up, and  $\hat{k}$  represent out of the page.

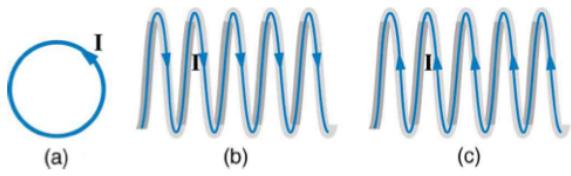


Figure 6: Three currents that create B-fields.

Case	B direction
(a)	$\hat{k}$
(b)	$\hat{j}$
(c)	$-\hat{j}$

Table 2: Table of directions for to Fig. 6.

7. Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.

$$\text{Power} = \text{Voltage} \times \text{Current}$$

$$P = IV$$

$$I = \frac{P}{V} = \frac{450 \times 10^6}{300,000} = 1500 \text{ Amperes}$$

Diagram: A horizontal line with an arrow labeled  $I$  above it, representing a current flowing to the right. A vertical line extends downwards from the center of the horizontal line, ending at a point labeled "20m ? B".

$$B = \frac{\mu_0 I}{2\pi r}$$

$$B = \frac{4\pi \times 10^{-7} (1500)}{2\pi (20)}$$

$$B = 1.50 \times 10^{-5} \text{ T}$$

8. The B-field in the tokamak reactor in Fig. 7 is given by  $B = \mu_0 NI/(2\pi r)$ , where  $N$  is the total number of loops,  $I$  is the current, and  $r$  is the radius at which we evaluate the B-field. (a) Design your own reactor by specifying practical values of  $N$ ,  $I$ , and  $r$  that achieve a 1.0 T B-field at a radius of 5.0 m. (b) With what frequency will a proton circle the toroidal B-field (1.0 T)? These ideas will help activate fusion reactors.

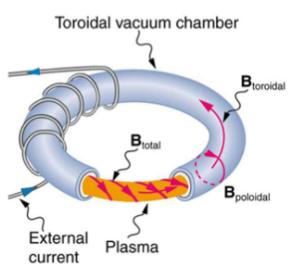


Figure 7: A generalized diagram of a tokamak.

$$\textcircled{a} \quad B = \frac{\mu_0 N I}{2\pi r}$$

$$I = \frac{4\pi \times 10^{-7} \times r^2}{2\pi (5)}$$

$$31.4159 = \frac{1.2566 \times 10^{-6} (x^2)}{31.416}$$

$$\sqrt{249.9262 \cdot 7} = \sqrt{x^2}$$

$$4999.92 = x$$

$$N = \# \text{ of loops} = 5000 \text{ loops}$$

$$I = \text{current} = 5000 \text{ Amperes}$$