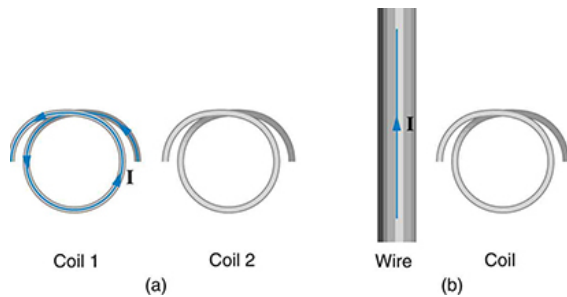# Midterm 2

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Figure 1: (a) A current I flows in the left coil, in the same plane as the right coil. (b) A wire with current I flows in the same plane as the coil.
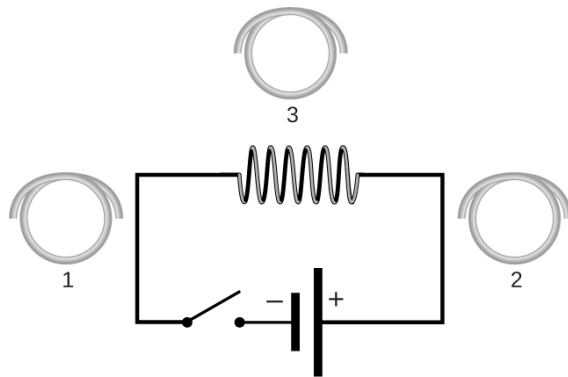

Figure 2: A battery provides and emf to a simple circuit with a solenoid, surrounded by three wire loops *in the plane of the circuit*.

1 Unit 4: Magnetism II

1. Consider Fig. 1a. (a) What is the direction of the current induced in coil 2 if the current in coil 1 increases? If it decreases? Consider Fig. 1b. (b) What is the direction of the current induced in the coil if the current in the wire increases? If it decreases?

Coil 2 is opposite to coil 1.
Coil 1 incr. → increase
Coil 1 decr. → decrease

2. Consider Fig. 2. (a) What is the direction of the current induced in coil 1 when the switch is closed? If it is left closed for a long time? When it is opened? (b) Repeat exercise (a) for coil 2. (c) Repeat exercise (a) for coil 3.

a) no current when closed
   Coil 2 decrease when open

b) no current closed, Coil 1 decreases when open

c) no current closed, coil 2 decreases when open.

3. Verify that the units of $\Delta\phi/\Delta t$ are Volts.

$$\text{emf} = -\frac{\Delta\phi}{\Delta t} \qquad \Delta\phi/\Delta t$$
$$1V = 1Wb/s$$

4. (a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) What current is induced in the ring if its resistance is 0.0100 Ω? (c) Calculate the average power dissipated in the ring.

$$\Delta B = B_f - B_i = 2 - 0 = \boxed{2}$$

a) $$EMF_i = \frac{-N\Delta\phi}{\Delta t} = -N\frac{\Delta BA\cos\theta}{\Delta t} = \frac{-1 \cdot 2(\pi(\frac{2.2}{2}\times10^{-2})^2\cos(0)}{0.25}$$

$$\boxed{EMF = -3.04\times10^{-3}V}$$

b) $$\boxed{NO}$$

5. Prove that when B, l, and v are not mutually perpendicular, motional emf is given by $emf = Blv \sin\theta$. If v is perpendicular to B, then θ is the angle between l and B. If l is perpendicular to B, then θ is the angle between v and B. *Hint: it is helpful to draw a diagram to define the angle θ.*

$$\varnothing = \vec{B} \cdot \vec{A}$$

$$\frac{\Delta\varphi}{\Delta t} = \frac{d(\varnothing)}{dt} = \frac{d(Bl)}{dt}$$

$$emf = -\frac{d(Bl)}{dt} = -B\frac{dl}{dt}$$

$$v = \frac{dl}{dt} = v\sin\theta$$

$$emf = -B(v\sin\theta) = Blv\sin\theta$$

6. (a) A bicycle generator rotates at 1875 rad/s, producing an 18.0 V peak emf. It has a 1.00 by 3.00 cm rectangular coil in a 0.640 T field. How many turns are in the coil? (b) What is the period of the AC output of the generator?

$$V_p = N \cdot A \cdot B \cdot \omega$$

$$N = \frac{V_p}{A \cdot B \cdot \omega} = \frac{18}{3\times10^{-4}(.64)(1875)} = \boxed{50}$$

7. An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

$$\left(\frac{V_s}{V_p} = \frac{N_s}{N_p}\right) \quad \frac{V_p}{V_s} = \frac{N_p}{N_s} = \frac{240}{120} = \boxed{2}$$

$$\frac{I_s}{I_p} = \frac{N_p}{N_s} \quad \frac{I_p}{I_s} = \frac{N_s}{N_p} = \boxed{\frac{1}{2}}$$

8. Consider Fig. 2. Suppose coil 3 ($N = 1$ turn) is rotated 90 degrees such that its normal direction is aligned with the normal direction of the solenoid ($N = 6$ turns). The system is now acting as a **transformer**. Suppose the emf in the solenoid is given by $V(t) = V_0 \sin(\omega t)$. (a) Draw a graph of $V(t)$, with the x-axis and y-axis units labeled. (b) Add the induced emf in coil 3 to your graph, assuming the magnetic flux in coil 3 is rendered identical to that in the solenoid with an iron core. (c) If $V_0 = 120$ V, and $\omega = (240\pi)$ Hz, when does $V(t) = 0$?

$$V(t) = V_0 \sin(\omega t)$$
$$t = 0, \quad V(t) = 0$$
$$t = \frac{1}{240}, \quad V(t) = 120_v$$
$$t = \frac{1}{120}, \quad V(t) = 0$$

$$V(t) = 0 \text{ at } t = \frac{n}{240} s$$

$$\omega t = n\pi$$
$$t = \frac{n\pi}{\omega}$$
$$n = 0, t = 0$$
$$n = 1, t = \pi/\omega$$
$$n = 2, t = \frac{2\pi}{\omega}$$
$$\omega = 240\pi Hz, \quad t = \frac{n}{240} s$$

9. Camera flashes charge a capacitor to high voltage by switching the current through an inductor on and off rapidly. In what time must the 0.100 A current through a 2.00 mH inductor be switched on or off to induce a 500 V emf?

$$EMF = L\frac{DI}{\Delta t}$$
$$\Delta t = \frac{2\times10^{-3}(0.1)}{500} = \boxed{4\times10^{-7}s}$$

10. A large research solenoid has an inductance of 25.0 H. (a) What induced emf opposes shutting it off when 100 A of current through it is switched off in 80.0 ms? (b) How much energy is stored in the inductor at full current? (c) At what rate in watts must energy be dissipated to switch the current off in 80.0 ms? (d) Considering exercise (c), is it surprising that shutting it down this quickly is difficult? (d) Design a solenoid that has an inductance of 25.0 H by choosing the number of turns N, the cross-sectional area A, and the number of turns l.

a)
$$EMF = L\frac{\Delta I}{\Delta t} = 25\left(\frac{100}{80\times10^{-3}}\right) = \boxed{3.13\times10^4 V}$$
b)
$$E_{ind} = \frac{1}{2}LI^2 = \frac{1}{2}(25)(100)^2 = \boxed{1.25\times10^5 J}$$
$$P = \frac{J}{s} = watts = \frac{1.25\times10^5}{80\times10^{-3}} = \boxed{1.56\times10^6 W}$$

11. Your RL circuit has a characteristic time constant

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of 20.0 ns, and a resistance of 5.00 MΩ. (a) What is the inductance of the circuit? (b) What resistance would give you a 1.00 ns time constant, perhaps needed for quick response in an oscilloscope? (c) Suppose you design for a 1.00 ns time constant. What percentage of the final current I_0 flows 3.0 ns after the circuit is closed? (d) What is the reactance of the inductor at 10 kHz?

a)

$T = \frac{L}{R}$

$L = T \times R$

$L = (20 \times 10^{-9})(5 \times 10^6)$

$L = 100 H$

b) $R = \frac{L}{T}$

$R = \frac{100\,H}{1.00 \times 10^{-9}\,s}$

$R = 100 \times 10^9$

$R = 100 G\Omega$

c) $I(t) = I_0(1 - e^{-t/T})$

$\frac{I(3.0)}{I_0} = 1 - e^{-3.0/1.00}$

$\frac{I(3.0)}{I_0} = 1 - e^{-3} = 0.9502$

$X_L = 2\pi f L = 2\pi(10^4)(100) = 2\times 10^6 \pi \Omega$

$= 6.28 \times 10^6 \Omega$

12. An inductor designed to filter high-frequency noise from power supplied to a personal computer is placed in series with the computer. What minimum inductance should it have to produce a 2.00 kΩ reactance for 15.0 kHz noise? (b) What is its reactance at 60.0 Hz?

$X_L = 2\pi f L$

$L = \frac{X_L}{2\pi f} = \frac{2000\,\Omega}{2\pi \times 15 \times 10^3} = \frac{2000}{94.247} = \boxed{21.25\,mH}$

$X_L = 2\pi \times 60 \times 21.25 \times 10^{-3} = 8.00\,\Omega$

13. Consider Fig. 3. Assume the bottom wire is grounded at 0V. (a) Considering the capacitor reactance, describe why the circuit acts as a *low-pass filter*. That is, AC signals at v_in with *high* frequencies produce very small amplitudes at v_out. (b) Using Kirchhoff's loop rule on the left side of the circuit, write an equation that describes changes in potential. (c) Repeat exercise (b) for the right loop. (d) Solve parts (b) and (c) for v_in and v_out, respectively, and calculate v_out/v_in in terms of R and C. (e) What is the value of v_out/v_in for $f = 100(2\pi RC)^{-1}$? For $f = 0.1(2\pi RC)^{-1}$?

$X_C = \frac{1}{2\pi f C}$

$V_\text{in} - iR - V_\text{out} = 0$

$V_\text{out} - iX_C = 0$

$X_C = \frac{1}{2\pi(100 \times 2\pi R_C)} = R$

$X_C = \frac{1}{2\pi(.1 \times 2\pi R_C)} = 10R$

$\frac{V_\text{out}}{V_\text{in}} = \frac{R}{R+R} = \frac{1}{2}$

$\frac{10R}{10R+R} = \frac{10}{11}$

14. (a) What are the resonance frequency, f_0, and resonance width, $\Delta f/f_0$, of an RLC circuit with $R = 0.1$ kΩ, $C = 1\,\mu$F, and $L = 10$ mH? (b) What is the total impedance of this circuit at $f = 0.1 f_0$ and $f = 10 f_0$?

$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10\times10^{-3}(1\times10^{-6})}} = 15.92\,kHz$

$\frac{\Delta f}{15.92} = \frac{0.1 \times 10^3}{10 \times 10^{-3}}$ $\Delta f = 1.592 \times 10^5\,Hz$

15. For an RLC circuit with that values of R, L, and C from the previous problem, an AC source voltage characterized by $V_\text{rms} = 120$ V, (a) what is I_rms at $10 f_0$ and $0.1 f_0$? (b) What is P_rms at these frequencies? *Hint: calculate the phase angle between resistance and reactance.*

$I_\text{rms} = \frac{V_\text{rms}}{Z}$ $Z = \sqrt{(0.1 \times 10^3)^2 + \left(10^4 \pi - \frac{1}{2\pi \times 159.29\mu}\right)^2}$

$Z = 0.1 \times 10^3 \Omega$

$I_\text{rms} = 1.2\,A$

$P_\text{rms} = V_\text{rms} \times I_\text{rms} \times \cos(\phi)$

$= 120 \times 1.2 \times \cos(0)$

$P_\text{rms} = 144\,w$

16. Suppose we are using an LC resonator and diode combination to create an AM radio signal. If the resonance frequency of our resonator is 1.4 MHz, and our audio signal is at 10 kHz, (a) what three frequencies should be present in our AM radio spectrum? (b) Suppose our receiver was receiving only the carrier frequency, but neither modulation peak. Should we gradually increase or gradually decrease the resistance of the receiver circuit?

$f_\text{carrier} = 1.4\,MHz$

$f_\text{upper} = f_c + f_a = 1.4 + 10$
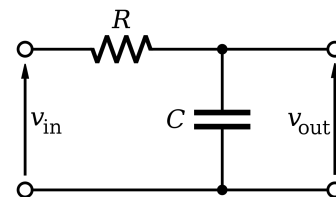
$f_\text{lower} = f_c - f_a = 1.4 - 10\,kHz$

Figure 3: This RC circuit can act as a low-pass filter.
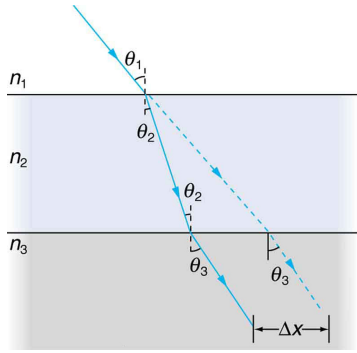
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2 Unit 5: Waves, Optics, Medical Physics

Figure 4: A light ray is in a medium with n_1, then enters a medium with n_2, then a medium with n_3.

1. (a) What is the B-field generated 1 cm laterally from a capacitor in an RC circuit that charges from 0 to 200 nC in 2 μs? (b) What *current* is responsible for this B-field, since the capacitor is not passing DC current like a wire?

$$I_c = \frac{\Delta Q}{\Delta t} = \frac{200\times10^{-9}}{2\times10^{-6}} = 100\,\text{mA}$$

$$B = \frac{(4\pi\times10^{-7})(100)}{2\pi(.01)} = 10^{-5}\text{T}$$

2. (a) An aircraft receives a pulse reflected from glacial ice below it 10 μs after transmission. What is the displacement between aircraft and ice? (b) Suppose the main frequency of the radar is $f = 100$ MHz. What is the smallest (approximate) change in glacier height the radar operator can observe? (c) What fraction of the radar power is reflected from the ice, assuming normal incidence? **Bonus.** (d) Suppose the transmitted power at 1 meter is 100 W, and decreases as the inverse of the displacement squared. What is the received power at the aircraft, accounting for distance and reflection?

$$\text{Distance} = \tfrac{1}{2}\text{speed}\times\text{Time} = \tfrac{1}{2}(3\times10^8)(10\times10^{-6}) = 150\text{m}$$

$$\text{Resolution} = \frac{c}{2f} = \frac{3\times10^8}{2\times100\times10^6} = 1.5\text{m}$$

$$P_{recieved} = \frac{P_{transmitted}}{(distance)^2} = \frac{100}{(150)^2} = 0.0044\text{w}$$

3. (a) A microwave generates 1 kW of power, projected onto a 10 cm-squared area at a distance of 1 m. What is the intensity (power per unit area)? (b) How long does it take the energy to travel the 1 meter? (c) What is the peak E-field at the source? **Bonus.** (d) If the intensity drops as the distance squared, what is the intensity 2 meters from the microwave?

$$I = \frac{P}{A} = \frac{1000}{.001} = 1\times10^6 \text{ W/m}^2$$

$$t = d/c = 1/3\times10^8 = 3.33\times10^{-9}$$

$$E_{peak} = \sqrt{\frac{2I}{c\cdot\varepsilon_0}} = \sqrt{\frac{2\times1\times10^6}{3\times10^8\times8.85\times10^{-12}}} = 274.5\text{ V/m}$$

$$I_2 = I_1 \times \left(\frac{r_1}{r_2}\right)^2$$

$$I_2 = 1\times10^6 \times \left(\tfrac{1}{2}\right)^2 = 2.5\times10^5 \text{ W/m}^2$$

4. Consider Fig. 4. (a) Show that $\theta_3 = \theta_1$, if $n_1 = n_3$. *This proof is the reason why the center of our lenses (like those in our glasses) do not change ray directions. If our glasses changed the apparent location of objects, they would be useless.* (b) A doctor examines a child's ear with a 15.0 cm focal length magnifying glass held 13.5 cm from the ear. (c) Where is the image? (d) What is its magnification? (c) How big is the image of a 1.0 cm diameter ear hole?

$$n_1 \sin\theta_1 = n_3 \sin\theta_3$$

$$n_1 \sin\theta_1 = n_1 \sin\theta_3$$

$$\sin\theta_1 = \sin\theta_3$$

$$\frac{1}{d_i} = \frac{1}{15} - \frac{1}{-13.5} = 7.17\text{cm}$$

$$M = \frac{-d_i}{d_o} = \frac{-7.17}{-13.5} = 0.53$$

5. (a) What is the ratio of the speed of light in solid ice versus snow? You may assume fresh snow has the same properties as water for this exercise. (b) If a radio wave transitions from snow to ice with

an initial angle of 30 degrees from normal, what is the transmitted angle?

$$n = \frac{c}{v}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{n_{snow}}{n_{ice}} = \frac{1.33}{1.33} = 1 \qquad \theta_1 = 30° \qquad n_1 = n_2 = 1.33$$

$$1.33 \times \sin 30° = 1.33 \sin\theta_2$$

$$\sin\theta_2 = \frac{1.33 \times \sin 30°}{1.33}$$

$$\sin\theta_2 = \sin 30°$$

$$\theta_2 = 30°$$

6. (a) Combine thin lens equations to show that the magnification for a thin lens is determined by its focal length and the object distance and is given by $m = f/(f - d_o)$. (b) Using this equation, determine what conditions lead to $m \to \infty$. (c) What happens to the image location?[1]

$$d_i = \frac{f \cdot d_o}{d_o - f} \qquad m = \frac{f}{f - d_o}$$

if $d_i \to f$, then $f - d_o \to 0$ therefore $m \to \infty$

7. **Bonus:** Suppose x-rays have a cross section of 200 barns at 10 keV in a lead vest protecting a medical patient. (a) If the vest is 1 cm thick, what fraction of the x-rays pass through it? (b) At what depth in the vest are half of the x-rays gone?

8. **Bonus:** In the previous problem, at what distance would half of the photons be gone if they were γ rays insted of x-rays, with a cross section of 1 barn? *Hint: scaling problem.*

9. **Bonus:** Some nuclear isotopes emit *free neutrons*, neutrons that then propagate outside the nucleus. These neutrons are useful for controlling nuclear fission for power production, and have a half-life of 611 ± 1 seconds in free space. If emitted in free space, what fraction of the neutrons remain after 1 hour?

10. (a) What is the radioactive dose in rads if 250 mJ of total energy is deposited in the body of a 60 kg adult? (b) What instead would be the dose if the radiation was concentrated in just 2.0 kg of tissue? (c) Assuming the RBE is 1 (x-rays or γ rays), what is the dose from part (b) in Sv? (d) Does this dose represent a serious health risk?

$$Dose = \frac{energy\ deposited}{mass}$$

$$energy = 250\ mJ$$
$$mass = 60\ kg$$

$$Dose = \frac{250 \times 10^{-3}}{60} = 4.17 \times 10^{-3}\ J/kg$$

$$Dose(Sv) = Dose(rads) \times Q$$
$$Dose\ (Sv = 1.25 \times 10^{-3}\ Sv$$

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[1]This is why we cannot use lenses to create infinitely large images for free.