Midterm 2 *Solutions*

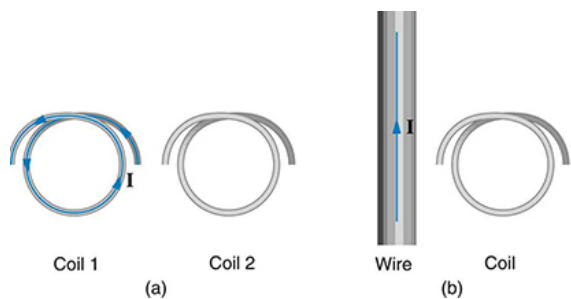## Prof. Jordan C. Hanson

December 11, 2023

Figure 1: (a) A current I flows in the left coil, in the same plane as the right coil. (b) A wire with current I flows in the same plane as the coil.

Figure 2: A battery provides and emf to a simple circuit with a solenoid, surrounded by three wire loops *in the plane of the circuit.*

1 Unit 4: Magnetism II

1. Consider Fig. 1a. (a) What is the direction of the current induced in coil 2 if the current in coil 1 increases? If it decreases? Consider Fig. 1b. (b) What is the direction of the current induced in the coil if the current in the wire increases? If it decreases?

   (a) CCW, CW, to oppose changes. (b) CCW, CW, to oppose changes.

2. Consider Fig. 2. (a) What is the direction of the current induced in coil 1 when the switch is closed? If it is left closed for a long time? When it is opened? (b) Repeat exercise (a) for coil 2. (c) Repeat exercise (a) for coil 3.

   (a) CCW, no current, CW. (b) CCW, no current, CW. (c) No current, no current, no current.

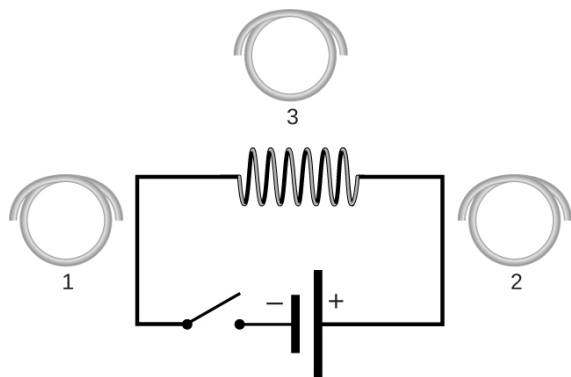3. Verify that the units of $\Delta\phi/\Delta t$ are Volts.

$$V = \frac{Tm^2}{s} \tag{1}$$
$$V = \frac{NA^{-1}m^{-1}m^2}{s} \tag{2}$$
$$V = \frac{Jm^{-1}A^{-1}m^{-1}m^2}{s} \tag{3}$$
$$V = \frac{JA^{-1}}{s} \tag{4}$$
$$V = \frac{CVA^{-1}}{s} \tag{5}$$
$$V = AVA^{-1} \tag{6}$$
$$V = V \quad QED \tag{7}$$

4. (a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) What current is induced in the ring if its resistance is 0.0100 Ω? (c) Calculate the average power dissipated in the ring.

   If the fingers are aligned, then assume angle between ring area and B-field is 90 degrees, so

$\phi = BA\cos\theta = BA$. *We have*

$$\epsilon = \frac{\Delta\phi}{\Delta t} = \frac{BA}{\Delta t} \tag{8}$$

$$\epsilon_{\text{ave}} = (\epsilon_{\text{f}} + \epsilon_{\text{i}})/2 \tag{9}$$

$$\epsilon_{\text{f}} = (2.0T \times 3.8 \times 10^{-4} m^2)/0.250s \tag{10}$$

$$\epsilon_{\text{i}} = 0.0V \tag{11}$$

$$\epsilon_{\text{ave}} = 0.5\epsilon_{\text{f}} = 1.52mV \tag{12}$$

5. Prove that when B, l, and v are not mutually perpendicular, motional emf is given by $emf = Blv\sin\theta$. If v is perpendicular to B, then θ is the angle between l and B. If l is perpendicular to B, then θ is the angle between v and B. *Hint: it is helpful to draw a diagram to define the angle θ.*

Treat two cases: (i) $\vec{l}$ is $\perp$ to B, $\vec{v}$ is not, and (ii) $\vec{v}$ is $\perp$ to B, $\vec{l}$ is not. (i) Imagine a uniform $\vec{B}$, with a square loop of wire with area vector aligned with $\vec{B}$. If the velocity makes an angle θ with $\vec{B}$, the Faraday's Law says:

$$\epsilon = \frac{\Delta\phi}{\Delta t} = B\frac{\Delta A}{\Delta t} \tag{13}$$

$$\epsilon = Bl\frac{\Delta x}{\Delta t} = Blv_\perp \tag{14}$$

$$\epsilon = Blv\sin\theta \tag{15}$$

In the last step, we use the fact that the only velocity component that changes A is $v\sin\theta$, the component perpendicular to $\vec{B}$. For case (ii), make the same argument but instead treat only the component of $\vec{B}$ that is perpendicular to $\vec{l}$: $B\sin\theta$.

6. (a) A bicycle generator rotates at 1875 rad/s, producing an 18.0 V peak emf. It has a 1.00 by 3.00 cm rectangular coil in a 0.640 T field. How many turns are in the coil? (b) What is the period of the AC output of the generator?

(a) $N = 50$, using $\epsilon_{\text{max}} = NBA\omega$. (b) Every 3.35 ms, using $T = 2\pi/\omega$.

7. An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

(a) Use $\epsilon_2/\epsilon_1 = N_2/N_1 = 240/120 = 2$. (b) Output current is $1/2$ of input current. (c) Use the transformer in reverse: output becomes input.

8. Consider Fig. 2. Suppose coil 3 ($N = 1$ turn) is rotated 90 degrees such that its normal direction is aligned with the normal direction of the solenoid ($N = 6$ turns). The system is now acting as a **transformer**. Suppose the emf in the solenoid is given by $V(t) = V_0\sin(\omega t)$. (a) Draw a graph of $V(t)$, with the x-axis and y-axis units labeled. (b) Add the induced emf in coil 3 to your graph, assuming the magnetic flux in coil 3 is rendered identical to that in the solenoid with an iron core. (c) If $V_0 = 120$ V, and $\omega = (240\pi)$ Hz, when does $V(t) = 0$?

*(a) The graph should be a sinusoid, with x-axis labeled with time units, and y-axis labeled with voltage units. (b) If $V(t)$ is a sine wave, then (due to Faradays's Law), coil 3 should have a negative cosine, because the coil responds to **changes** in $V(t)$. Also, the amplitude should be smaller. We do not know the distance between coil 3 and the solenoid, so we cannot say how much smaller. (c) If $\omega = 240\pi$ Hz, then $T = 1/120$ s, so at $t = 0$ $V(0) = 0$, and every $1/120$ seconds.*

9. Camera flashes charge a capacitor to high voltage by switching the current through an inductor on and off rapidly. In what time must the 0.100 A current through a 2.00 mH inductor be switched on or off to induce a 500 V emf?

Solve Faraday's Law with inductance for Δt, which gives 0.4 μs.

10. A large research solenoid has an inductance of 25.0 H. (a) What induced emf opposes shutting it off when 100 A of current through it is switched off in 80.0 ms? (b) How much energy is stored in the inductor at full current? (c) At what rate in watts must energy be dissipated to switch the current off in 80.0 ms? (d) Considering exercise (c), is it surprising that shutting it down this quickly is difficult? (d) Design a solenoid that has an inductance of 25.0 H by choosing the number of turns N, the cross-sectional area A, and the number of turns l.

(a) 31.25 kV. (b) 125 kJ. (c) 1.56 MW.

(d) I happened to pick a 5cm radius solenoid that is 5 m long, with 25.0 H. This implies $N = 112.5k$ turns. That's a lot of turns, like 225 turns per cm. This is possible with small wire, but the wire must be thick enough to carry 100.0 A.

11. Your RL circuit has a characteristic time constant of 20.0 ns, and a resistance of 5.00 MΩ. (a) What is the inductance of the circuit? (b) What resistance would give you a 1.00 ns time constant, perhaps needed for quick response in an oscilloscope? (c) Suppose you design for a 1.00 ns time constant. What percentage of the final current I_0 flows 3.0 ns after the circuit is closed? (d) What is the reactance of the inductor at 10 kHz?

(a) $\tau = L/R$, so $L = R\tau = 5 \times 10^6 \times 20 \times 10^{-9} = 0.1$ H. (b) Scaling: 100 MΩ, because a larger resistance gives a smaller time constant. (c) 95%. (d) $X_L = 2\pi f L = 2\pi(10^4)10^{-1} = 2\pi$ kΩ.

12. An inductor designed to filter high-frequency noise from power supplied to a personal computer is placed in series with the computer. What minimum inductance should it have to produce a 2.00 kΩ reactance for 15.0 kHz noise? (b) What is its reactance at 60.0 Hz?

(a) Using the formula for inductor reactance, we find $1/(15\pi)$ H, or 21 mH. (b) Using $L = 1/(15\pi)$ H, we find with $f = 60$ Hz that $X_L = 8$ Ω.

13. Consider Fig. 3. Assume the bottom wire is grounded at 0V. (a) Considering the capacitor reactance, describe why the circuit acts as a *low-pass filter*. That is, AC signals at v_{in} with *high* frequencies produce very small amplitudes at v_{out}. (b) Using Kirchhoff's loop rule on the left side of the circuit, write an equation that describes changes in potential. (c) Repeat exercise (b) for the right loop. (d) Solve parts (b) and (c) for v_{in} and v_{out}, respectively, and calculate $v_{\text{out}}/v_{\text{in}}$ in terms of R and C. (e) What is the value of $v_{\text{out}}/v_{\text{in}}$ for $f = 100(2\pi RC)^{-1}$? For $f = 0.1(2\pi RC)^{-1}$?

(a) For signals with $\omega \gg 1/\tau = 1/RC$, signals at v_{in} feel no resistance from the capacitor, and flow to ground. Put another way, whatever load is connected to v_{out} will inevitably have a higher resistance or reactance than the capacitor at high
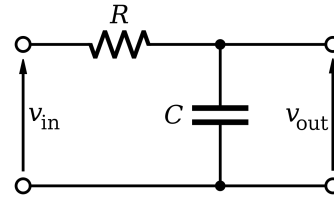

Figure 3: This RC circuit can act as a low-pass filter.

frequencies, and will get less of the signal. (b) $v_{\text{in}} - iR - i(2\pi f C)^{-1} = 0$. (c) $v_{\text{out}} - i(2\pi f C)^{-1} = 0$. (d) $v_{\text{out}}/v_{\text{in}} = 1/(1 + 2\pi f RC)$. (e) 0.0099, and 0.909, so the lower frequency has the better ratio.

14. (a) What are the resonance frequency, f_0, and resonance width, $\Delta f/f_0$, of an RLC circuit with $R = 0.1$ kΩ, $C = 1$ μF, and $L = 10$ mH? (b) What is the total impedance of this circuit at $f = 0.1f_0$ and $f = 10f_0$?

(a) $f_0 = 5/\pi$ kHz, and $\Delta f/f_0 = 1$. (b) For $f = 0.1f_0$, impedance is dominated by the capacitor, and $Z \approx 1$ kΩ. For $f = 10f_0$, the impedance is dominated by the inductor, and $Z \approx 1$ kΩ.

15. For an RLC circuit with that values of R, L, and C from the previous problem, an AC source voltage characterized by $V_{\text{rms}} = 120$ V, (a) what is I_{rms} at $10f_0$ and $0.1f_0$? (b) What is P_{rms} at these frequencies? *Hint: calculate the phase angle between resistance and reactance.*

(a) 120 mA for both frequencies. (b) For both frequencies, $\cos\phi \approx 0.1$, so $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos\phi = 120 \times 10^{-3} 120 \times 10^{-1} = 1.44$ W.

16. Suppose we are using an LC resonator and diode combination to create an AM radio signal. If the resonance frequency of our resonator is 1.4 MHz, and our audio signal is at 10 kHz, (a) what three frequencies should be present in our AM radio spectrum? (b) Suppose our receiver was receiving only the carrier frequency, but neither modulation peak. Should we gradually increase or gradually decrease the resistance of the receiver circuit?

(a) 1390, 1400, and 1410 kHz. (b) Gradually increase the resistance until the modulations are received.
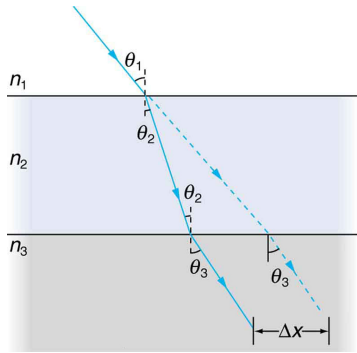
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2 Unit 5: Waves, Optics, Medical Physics

Figure 4: A light ray is in a medium with n_1, then enters a medium with n_2, then a medium with n_3.

1. (a) What is the B-field generated 1 cm laterally from a capacitor in an RC circuit that charges from 0 to 200 nC in 2 μs? (b) What *current* is responsible for this B-field, since the capacitor is not passing DC current like a wire?

   (a) 2 μT. (b) The displacement current.

2. (a) An aircraft receives a pulse reflected from glacial ice below it 10 μs after transmission. What is the displacement between aircraft and ice? (b) Suppose the main frequency of the radar is $f = 100$ MHz. What is the smallest (approximate) change in glacier height the radar operator can observe? (c) What fraction of the radar power is reflected from the ice, assuming normal incidence? **Bonus.** (d) Suppose the transmitted power at 1 meter is 100 W, and decreases as the inverse of the displacement squared. What is the received power at the aircraft, accounting for distance and reflection?

   (a) 1.5 km. (b) 30 cm. (c) 7.9%, from the reflection coefficient. (d) 3.5×10^{-6}, or about -55 dB.

3. (a) A microwave generates 1 kW of power, projected onto a 10 cm-squared area at a distance of 1 m. What is the intensity (power per unit area)? (b) How long does it take the energy to travel the 1 meter? (c) What is the peak E-field at the source? **Bonus.** (d) If the intensity drops as the distance squared, what is the intensity 2 meters from the microwave?[1]

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[1] Keep in mind that microwaves have protective screens

(a) Divide the power by the area to obtain intensity: 0.1 kW cm^{-2}. (b) 10/3 ns. (c) Use $\bar{S} = E^2/(2c\mu_0)$ to find $E = 27.5$ kV/m.

4. Consider Fig. 4. (a) Show that $\theta_3 = \theta_1$, if $n_1 = n_3$. *This proof is the reason why the center of our lenses (like those in our glasses) do not change ray directions. If our glasses changed the apparent location of objects, they would be useless.* (b) A doctor examines a child's ear with a 15.0 cm focal length magnifying glass held 13.5 cm from the ear. (a) Where is the image? (b) What is its magnification? (c) How big is the image of a 1.0 cm diameter ear hole?

   (a) Using Snell's Law at the first interface gives $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Using Snell's Law at the second interface gives $n_2 \sin\theta_2 = n_3 \sin\theta_3$. Transitive equality implies $n_1 \sin\theta_1 = n_3 \sin\theta_3$. If $n_1 = n_3$, then $\sin\theta_1 = \sin\theta_3$, so $\theta_1 = \theta_3$. (b) Using the thin lens equations, we find $d_i = -135$ cm, so 1.35 m virtual image on the far side of the lens. (c) Using the thin lens equations, we find $m = 10$. (d) 10 cm, larger by a factor of 10.

5. (a) What is the ratio of the speed of light in solid ice versus snow? You may assume fresh snow has the same properties as water for this exercise. (b) If a radio wave transitions from snow to ice with an initial angle of 30 degrees from normal, what is the transmitted angle?

   (a) The ratio of speeds is the ratio of indices of refraction: $n_{snow} = 1.31$, and $n_{ice} = 1.78$. Light is $1.78/1.31 = 1.36$ times faster in snow, or 36% faster. (b) Use Snell's Law to solve for the transmitted angle: $\theta_i = \arcsin(n_s/n_i \sin\theta_s)$. The result is $\theta_i = 21.6$ degrees. The final angle should be smaller than 30 degrees because the wave slows down.

6. (a) Combine thin lens equations to show that the magnification for a thin lens is determined by its focal length and the object distance and is given by $m = f/(f - d_o)$. (b) Using this equation, determine what conditions lead to $m \to \infty$. (c) What happens to the image location as this condition is met?[2]

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that attenuate this radiation even more than the distance effect does.

[2] This is why we cannot use lenses to create infinitely large images for free.

Solve the focal length equation for d_i: $d_i = d_o f/(d_o - f)$, then divide d_i by d_o to obtain m: $f/(d_o - f)$. (b) $m \to \infty$ if $d_o \to f$. (c) If the magnification is infinitely large, the image is infinitely far away.

7. Analysis of an interference effect in a clear solid shows that the wavelength of light in the solid is 329 nm. Knowing this light comes from a He-Ne laser and has a wavelength of 633 nm in air, is the substance zircon or diamond? *Maybe this is how they tell if a diamond ring is real ... .*

   Got heem! Using $v = f\lambda$, and $v = c/n$, we can show that $n = \lambda_0/\lambda_n = 633/329 = 1.92$. This matches closely the index for zircon. Should have bought a real one.

8. (a) Suppose we are demonstrating the wave-nature of light with a double-slit experiment. At what angle θ is the first-order maximum for 400-nm wavelength blue light falling on double slits separated by 0.05 mm? (b) If we double the wavelength to 800 nm, will θ increase or decrease? (c) Calculate θ for $\lambda = 800$ nm, $d = 0.05$ mm, and $m = 1$. **Bonus.** (d) How would the diffraction pattern change if a glass block with $n = 1.3$ was placed in front of the slits?

   (a) Using $d\sin\theta = n\lambda$ with $n = 1$, we find $\theta = 0.46$ degrees. (b) Increase. (c) 0.92 degrees, so it doubles. (d) The wavelength decreases in the glass, and if the wavelength decreases then the angular spread decreases.

9. Suppose x-rays have a cross section of 200 barns at 10 keV in a lead vest protecting a medical patient. (a) If the vest is 1 cm thick, what fraction of the x-rays pass through it? (b) At what depth in the vest are half of the x-rays gone?

   *(a) The relevant formula is $I/I_0 = \exp(-n\sigma z)$. The hard part is calculating n, the number density of atoms in Pb. Answer the following questions: what number of atoms of Pb atoms are in 1 cm^3? How many gm of Pb are in 1 cm^3? That's the density: 11.35 gm cm^{-3}. But how do we convert grams to number of atoms? That's the **atomic mass**: 207 gm $mole^{-1}$. Divide these numbers in the right way gives 11.35/207 mole cm^{-3}. Multiply by Avogadro's number, 6.02×10^{23} atoms $mole^{-1}$, to convert moles to atoms. The result is*

3.3×10^{22} atoms per cm^3. 1 barn is 10^{-24} cm^2, and $z = 1$ cm. Plugging in to the formula gives 0.136 percent. (b) $z = -\ln(I/I_0)/n\sigma$ gives 0.1 cm.

10. In the previous problem, at what distance would half of the photons be gone if they were γ rays instead of x-rays, with a cross section of 1 barn? *Hint: scaling problem.*

    20 cm, by scaling the last result up by a factor of 200, since the cross-section is smaller by a factor of 200.

11. **Bonus.** The wavelength of a 10 keV x-ray is about 1.2 angstroms, or 1.2×10^{-10} m. (a) What is the momentum of this x-ray? (b) Suppose this x-ray smacks an electron and gives it 50 percent of the momentum. What is the final speed of the electron, as a fraction of the speed of light?

    (a) $p = h/\lambda$, where h is Planck's constant, or 4.136×10^{-15} eV s. Thus, $p = 3.4 \times 10^{-5}$ eV s m^{-1}. (b) 50 percent of this is 1.7×10^{-5} eV s m^{-1}. To solve for the velocity, we need to divide by electron mass: 0.511 MeV/c^2, and use $c = 3 \times 10^8$ m s^{-1} to find $v_e = 3 \times 10^4$ m s^{-1}. Divide this by c to get the fraction: 10^{-4} or 0.01 percent of the speed of light. This fact justifies treating the x-ray/electron interaction as classical.

12. Some nuclear isotopes emit *free neutrons*, neutrons that then propagate outside the nucleus. These neutrons are useful for controlling nuclear fission for power production, and have a half-life of 611 ± 1 seconds in free space. (a) If emitted in free space, what fraction of the neutrons remain after 1 hour? (b) Suppose we start with 10^6 neutron emitted. What is the decay **rate** after 1222 seconds?

    (a) The relevant formula is $N/N_0 = \exp(-\lambda t)$, but we do not know the decay constant λ. We can convert from half-life to decay constant via $\lambda = \ln(2)/t_{1/2}$, and we find $\lambda = 1.13 \times 10^{-3}$ Hz. Now we can insert $t = 3600$ seconds (1 hour), and λ to find $N/N_0 = 1.7$ percent. (b) The formula for the decay rate that involves the original number of neutrons and λ is $R = -\lambda N_0 \exp(-\lambda t) = 284$ neutrons per second.

13. (a) What is the radioactive dose in rads if 250 mJ of total energy is deposited in the body of a

60 kg adult? (b) What instead would be the dose if the radiation was concentrated in just 2.0 kg of tissue? (c) Assuming the RBE is 1 (x-rays or γ rays), what is the dose from part (b) in Sv? (d) Does this dose represent a serious health risk?

(a) $0.25 J/60 kg * 100 = 0.42$ rads. (b) Scale up by a factor of 30: 12.5 rads. (c) $12.5 * 1/100 = 0.125$ Sv. (d) Probably not, just a decrease in white blood cell count.