

Algebra-Based Physics-2: Electricity, Magnetism, and Modern Physics: Unit 3

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Summary

Unit 3 Summary

1. Magnetostatics I: Chapters 22.1 - 22.4

- 1.1 Magnets, ferromagnetic and electromagnetic
- 1.2 Magnetic fields and field lines, force on moving charge
- 1.3 Magnetic application: [mass spectrometry](#)

2. Magnetostatics II: Chapters 22.7 - 22.9

- 2.1 Forces and torques on conductors with current
- 2.2 Ampère's Law: magnetic fields are created by currents
- 2.3 Magnetic application: [fusion reactors](#)

Magnets and magnetic fields

Magnets and magnetic fields

Review of the Origin of Electric and Magnetic Fields

On the origin of magnetic fields and forces they exert on charge:

<https://www.youtube.com/watch?v=s94suB5uLWw>

On the origin of electric fields and forces they exert on charge:

https://youtu.be/mdulzEfQXDE?si=euGvVjKPT33_E-fI

Key points:

- Some elements are magnetic or can be magnetized
- Current creates magnetic fields
- Current exerts force on moving charge and current

Magnets and magnetic fields

What is a cross-product and how does it work?

Computing the cross product [\[edit \]](#)

Coordinate notation [\[edit \]](#)

The [standard basis](#) vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} satisfy the following equalities in a right hand coordinate system:

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}$$

$$\mathbf{j} \times \mathbf{k} = \mathbf{i}$$

$$\mathbf{k} \times \mathbf{i} = \mathbf{j}$$

which imply, by the [anticommutativity](#) of the cross product, that

$$\mathbf{j} \times \mathbf{i} = -\mathbf{k}$$

$$\mathbf{k} \times \mathbf{j} = -\mathbf{i}$$

$$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$$

The definition of the cross product also implies that

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0} \text{ (the [zero vector](#))}.$$

Figure 1: The cross-product is a way of multiplying unit vectors.

Professor: several examples on board.

Magnets and magnetic fields

Let $\vec{v} = 2\hat{i}$ and $\vec{w} = -2\hat{j}$. What is $\vec{v} \times \vec{w}$?

- A: $-4\hat{k}$
- B: $4\hat{k}$
- C: $-2\hat{i}$
- D: $2\hat{j}$

Magnets and magnetic fields

Let $\vec{v} = 3\hat{j}$ and $w = 5\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $15\hat{i}$
- B: $5\hat{j}$
- C: $3\hat{i}$
- D: $15\hat{k}$

Magnets and magnetic fields

Let $\vec{v} = 3\hat{i} \times 3\hat{j}$ and $w = 2\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $-6\hat{j} + 6\hat{k}$
- B: $-6\hat{j} + 6\hat{i}$
- C: $6\hat{j} + 6\hat{i}$
- D: $6\hat{k} + 6\hat{i}$

Group exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} \quad (1)$$

$$\vec{w} = 4\hat{j} - 4\hat{i} \quad (2)$$

$$\vec{v} \times \vec{w} = ?? \quad (3)$$

Magnets and magnetic fields

The Lorentz Force

Let a particle with charge q and velocity \vec{v} move through a *magnetic field* \vec{B} . The **Lorentz force** on the charged particle is

$$\vec{F}_L = q\vec{v} \times \vec{B} \quad (4)$$

As a helpful memory tool, we have the right-hand rule to remember the direction of the cross-product. The units of the magnetic field are the Telsa, after Nikola Tesla. We also have the Gauss which is 10^{-4} Tesla.

Magnets and magnetic fields

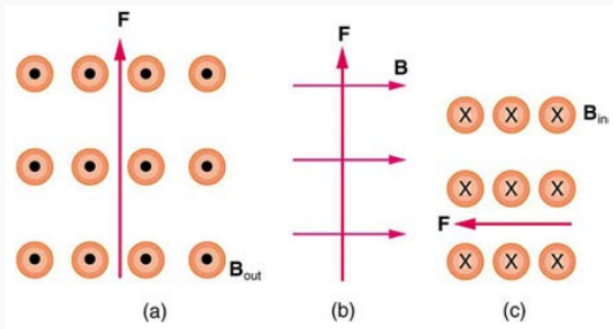


Figure 2: Three different magnetic field and charge scenarios. The vector \vec{F} is the direction of the Lorentz force, and the magnetic field is uniform. A dot indicates that the magnetic field is coming out of the page, and an x indicates that the field is going into the page.

Magnets and magnetic fields

In which of the diagrams is a positively charged particle moving to the left?

- A: A
- B: B
- C: C
- D: Double WAT

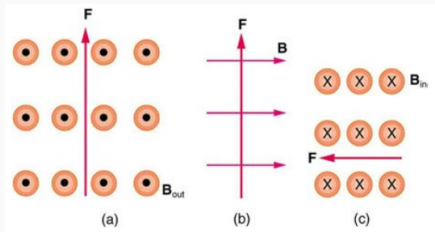


Figure 3: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

In which of the diagrams is a positively charged particle moving upwards?

- A: A
- B: B
- C: C
- D: Double WAT

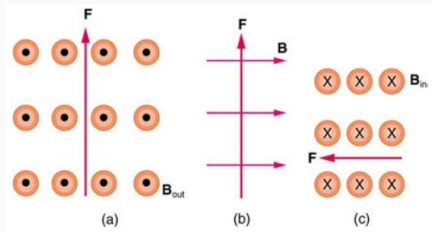


Figure 4: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

In which of the diagrams is a negatively charged particle moving into the page?

- A: A
- B: B
- C: C
- D: Double WAT

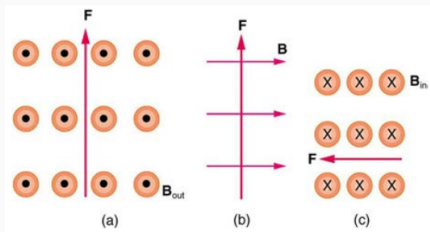


Figure 5: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

In which of the diagrams is a negatively charged particle moving to the right?

- A: A
- B: B
- C: C
- D: Double WAT

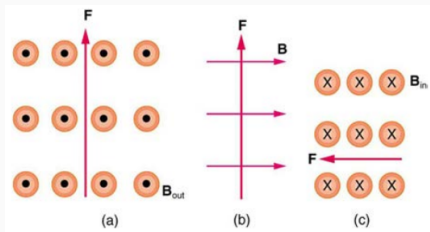


Figure 6: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

A theorem for the magnitude of the cross-product: Let \vec{a} and \vec{b} be vectors and θ be the angle between them. The magnitude of the cross-product is

$$|\vec{a} \times \vec{b}| = ab \sin \theta \quad (5)$$

Thus, the magnitude of the Lorentz force is

$$F_L = qvB \sin \theta \quad (6)$$

The angle θ is between the velocity and the magnetic field.

Magnets and magnetic fields

Suppose a positively charged particle q moves with an initial velocity $\vec{v} = v\hat{i}$, and there is a uniform magnetic field $\vec{B} = -B\hat{k}$. At this moment, what is the direction of the force on q ?

- A: \hat{i}
- B: \hat{j}
- C: \hat{k}
- D: $-\hat{k}$

Magnets and magnetic fields

The force on the positively charged particle q in the \hat{k} -direction eventually causes the velocity to be $\vec{v} = v\hat{j}$. The uniform magnetic field is still $\vec{B} = -B\hat{k}$. At this moment, what is the direction of the force on q ?

- A: \hat{i}
- B: \hat{j}
- C: \hat{k}
- D: $-\hat{i}$

Magnets and magnetic fields

The charge experiences uniform circular motion. At each moment, the acceleration is *perpendicular* to the direction.

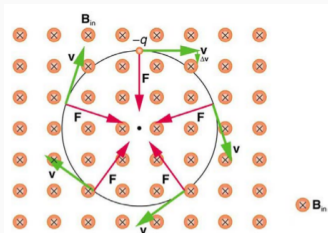


Figure 7: The situation is depicted here with $-q$ (changes direction).

Recall centripetal acceleration, in two equations:

$$F_C = \frac{mv^2}{r} = mr\omega^2 \quad (7)$$

Magnets and magnetic fields

Recall centripetal acceleration, in two equations:

$$F_C = \frac{mv^2}{r} = mr\omega^2 \quad (8)$$

Assume the Lorentz force remains perpendicular to velocity, and set the centripetal force equal to Lorentz force magnitude:

$$\frac{mv^2}{r} = qvB \quad (9)$$

$$qB = \frac{mv}{r} \quad (10)$$

$$r = \frac{mv}{qB} \quad (11)$$

Thus, the radius of curvature is connected to ratio of mass, charge, velocity, and field strength. *What is a scientific application of this?*

Magnetic Applications I: Mass Spectrometry

Magnetic Applications I: Mass Spectrometry

Mass spectrometry. A simplified picture of mass spectrometry involves measuring the radius of curvature of ions in a B-field.

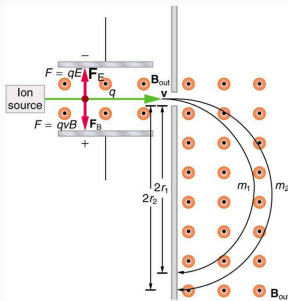


Figure 8: (a) Within the ion source, molecules are turned into an ionized gas, and accelerated through a capacitor. (b) The capacitor creates a uniform E-field, and a coil of current surrounds the capacitor to create a uniform B-field. The E and B-fields are perpendicular. (c) Ions leave the *velocity selector* into an area with just the B-field.

Magnetic Applications I: Mass Spectrometry

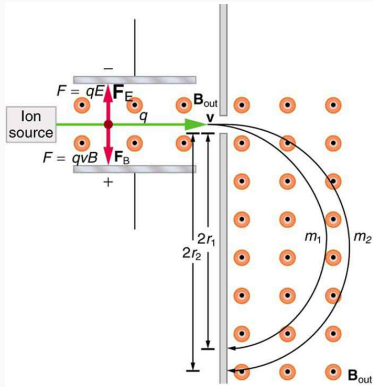


Figure 9: A simplified picture of a mass spectrometer.

1. Show that if $v = E/B$, the velocity in the *velocity selector* is constant.
2. If $E = 100 \text{ V/m}$, and $B = 50 \text{ gauss}$ (5 mT), what velocity is required from the ion source?
3. If q is the equivalent of two electrons ($2e$), and m is 100 amu^a , what is the radius observed after the velocity selector?

^a $1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$, and $1e = 1.67 \times 10^{-19} \text{ C}$.

Magnetic Applications I: Mass Spectrometry

Within $\approx 1\%$, what result did you obtain?

- A: 0.02 m
- B: 0.2 m
- C: 2.0 m
- D: 20 m

Magnetic Applications I: Mass Spectrometry

If all other variables remained the same, what would the radius of curvature be if m decreased to 50 amu?

- A: 10 m
- B: 1.0 m
- C: 0.1 m
- D: 0.01 m

(It is not necessary to repeat the calculation. Treat this as a scaling problem.)

Magnetic Applications I: Mass Spectrometry

Notes on the Lorentz force:

1. Magnetic fields do no work.

- Work is defined as

$$W = \vec{F} \cdot \vec{x} \quad (12)$$

- Insert the Lorentz force for \vec{F} :

$$W = q\vec{v} \times \vec{B} \cdot \vec{x} \quad (13)$$

- $\vec{v} \times \vec{B}$ is perpendicular to \vec{x} , which is parallel to \vec{v} .
 - $W = 0$, because the dot-product of perpendicular vectors is zero.
2. The ratio of E to B-fields is a *velocity*, $v = E/B$? What happens for a moving observer? We will postpone this discussion until we cover *inductors*.

22 MAGNETISM

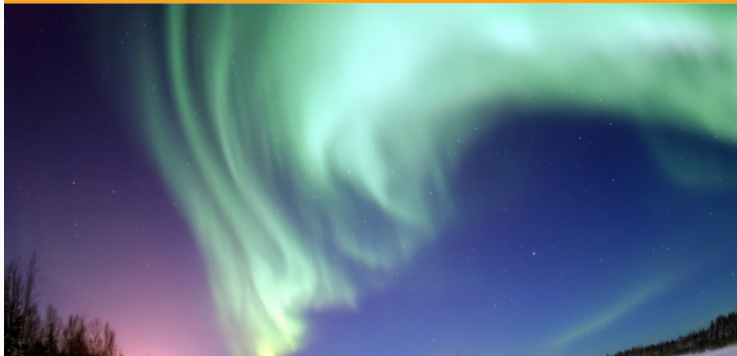
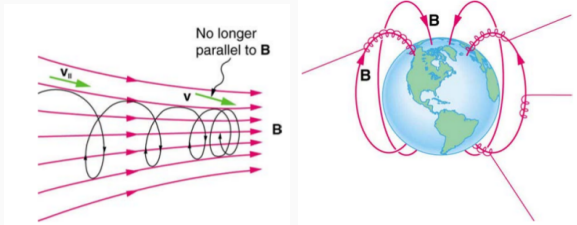


Figure 10: The aurora borealis, or northern lights.

Magnets and magnetic fields

A cool talk on the aurora borealis (time permitting):
<https://youtu.be/czMh3BnHFHQ>



One un-explained piece: what does it mean for the electrons and protons to *high-five* the neutral oxygen and nitrogen atoms?

Force and Torque on a Current Carrying Conductor

Force on a Moving Charges and Current Carrying Conductors

Introduction to magnetic forces on current-carrying conductors:

<https://youtu.be/5fqwJyt4Lus>



Magnetic Applications II: Nuclear Fusion

Force on a Moving Charges and Current Carrying Conductors

Magnetic containment and *tokamaks*.

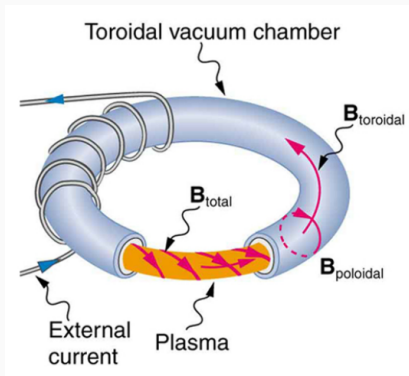


Figure 11: The tokamak contains high-energy plasma, e.g. a charged gas of electrons and protons.

Conclusion

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