

Algebra-Based Physics: Electricity, Magnetism, and Modern Physics (PHYS135B): Unit 3

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Summary

Unit 3 Summary

1. Magnetostatics I: Chapters 22.1 - 22.4

- 1.1 Magnets, ferromagnetic and electromagnetic
- 1.2 Magnetic fields and field lines, force on moving charge
- 1.3 Magnetic application: [mass spectrometry](#)

2. Magnetostatics II: Chapters 22.7 - 22.9

- 2.1 Forces and torques on conductors with current
- 2.2 Ampère's Law: magnetic fields are created by currents
- 2.3 Magnetic application: [fusion reactors](#)

Magnets and magnetic fields

Magnets and magnetic fields

Review of the Origin of Electric and Magnetic Fields

On the origin of magnetic fields and forces they exert on charge:

<https://www.youtube.com/watch?v=s94suB5uLWw>

On the origin of electric fields and forces they exert on charge:

https://youtu.be/mdulzEfQXDE?si=euGvVjKPT33_E-fI

Key points:

- Some elements are magnetic or can be magnetized
- Current creates magnetic fields
- Current exerts force on moving charge and current

Magnets and magnetic fields

What is a cross-product and how does it work?

Computing the cross product [\[edit \]](#)

Coordinate notation [\[edit \]](#)

The [standard basis](#) vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} satisfy the following equalities in a right hand coordinate system:

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}$$

$$\mathbf{j} \times \mathbf{k} = \mathbf{i}$$

$$\mathbf{k} \times \mathbf{i} = \mathbf{j}$$

which imply, by the [anticommutativity](#) of the cross product, that

$$\mathbf{j} \times \mathbf{i} = -\mathbf{k}$$

$$\mathbf{k} \times \mathbf{j} = -\mathbf{i}$$

$$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$$

The definition of the cross product also implies that

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0} \text{ (the [zero vector](#))}.$$

Figure 1: The cross-product is a way of multiplying unit vectors.

Professor: several examples on board.

Magnets and magnetic fields

Let $\vec{v} = 2\hat{i}$ and $\vec{w} = -2\hat{j}$. What is $\vec{v} \times \vec{w}$?

- A: $-4\hat{k}$
- B: $4\hat{k}$
- C: $-2\hat{i}$
- D: $2\hat{j}$

Magnets and magnetic fields

Let $\vec{v} = 3\hat{j}$ and $w = 5\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $15\hat{i}$
- B: $5\hat{j}$
- C: $3\hat{i}$
- D: $15\hat{k}$

Magnets and magnetic fields

Let $\vec{v} = 3\hat{i} \times 3\hat{j}$ and $w = 2\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $-6\hat{j} + 6\hat{k}$
- B: $-6\hat{j} + 6\hat{i}$
- C: $6\hat{j} + 6\hat{i}$
- D: $6\hat{k} + 6\hat{i}$

Group exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} \quad (1)$$

$$\vec{w} = 4\hat{j} - 4\hat{i} \quad (2)$$

$$\vec{v} \times \vec{w} = ?? \quad (3)$$

Magnets and magnetic fields

The Lorentz Force

Let a particle with charge q and velocity \vec{v} move through a *magnetic field* \vec{B} . The **Lorentz force** on the charged particle is

$$\vec{F} = q\vec{v} \times \vec{B} \quad (4)$$

If θ is the angle between \vec{v} and \vec{B} , the magnitude of \vec{F} is

$$F = qvB \sin \theta \quad (5)$$

As a helpful memory tool, we have the right-hand rule to remember the direction of the cross-product. **The units of the magnetic field are the Telsa**, after Nikola Tesla. We also have the Gauss which is 10^{-4} Tesla.

Magnets and magnetic fields

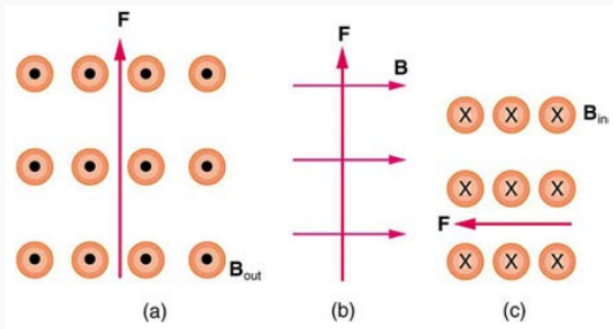


Figure 2: Three different magnetic field and charge scenarios. The vector \vec{F} is the direction of the Lorentz force, and the magnetic field is uniform. A dot indicates that the magnetic field is coming out of the page, and an x indicates that the field is going into the page.

Magnets and magnetic fields

In which of the diagrams is a positively charged particle moving to the left?

- A: A
- B: B
- C: C
- D: Double WAT

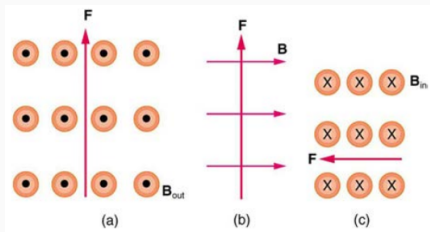


Figure 3: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

In which of the diagrams is a positively charged particle moving upwards?

- A: A
- B: B
- C: C
- D: Double WAT

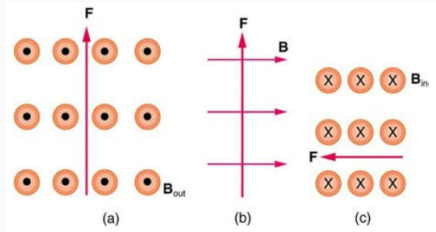


Figure 4: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

In which of the diagrams is a negatively charged particle moving into the page?

- A: A
- B: B
- C: C
- D: Double WAT

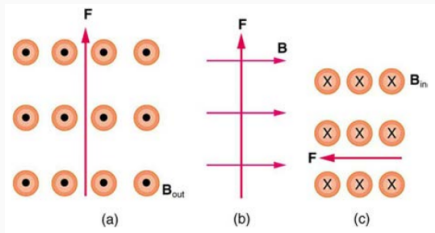


Figure 5: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

In which of the diagrams is a negatively charged particle moving to the right?

- A: A
- B: B
- C: C
- D: Double WAT

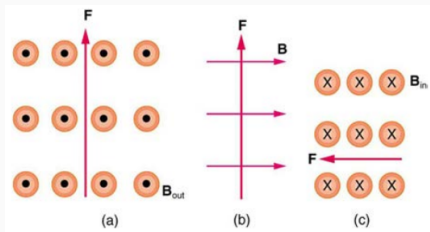


Figure 6: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

A theorem for the magnitude of the cross-product: Let \vec{a} and \vec{b} be vectors and θ be the angle between them. The magnitude of the cross-product is

$$|\vec{a} \times \vec{b}| = ab \sin \theta \quad (6)$$

Thus, the magnitude of the Lorentz force is

$$F_L = qvB \sin \theta \quad (7)$$

The angle θ is between the velocity and the magnetic field.

Magnets and magnetic fields

Suppose a positively charged particle q moves with an initial velocity $\vec{v} = v\hat{i}$, and there is a uniform magnetic field $\vec{B} = -B\hat{k}$. At this moment, what is the direction of the force on q ?

- A: \hat{i}
- B: \hat{j}
- C: \hat{k}
- D: $-\hat{k}$

Magnets and magnetic fields

The force on the positively charged particle q in the \hat{k} -direction eventually causes the velocity to be $\vec{v} = v\hat{j}$. The uniform magnetic field is still $\vec{B} = -B\hat{k}$. At this moment, what is the direction of the force on q ?

- A: \hat{i}
- B: \hat{j}
- C: \hat{k}
- D: $-\hat{i}$

Magnets and magnetic fields

The charge experiences uniform circular motion. At each moment, the acceleration is *perpendicular* to the direction.

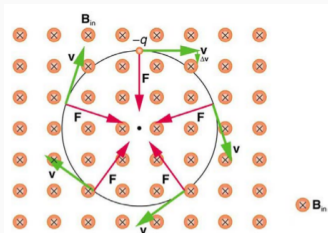


Figure 7: The situation is depicted here with $-q$ (changes direction).

Recall centripetal acceleration, in two equations:

$$F_C = \frac{mv^2}{r} = mr\omega^2 \quad (8)$$

Magnets and magnetic fields

Recall centripetal acceleration, in two equations:

$$F_C = \frac{mv^2}{r} = mr\omega^2 \quad (9)$$

Assume the Lorentz force remains perpendicular to velocity, and set the centripetal force equal to Lorentz force magnitude:

$$\frac{mv^2}{r} = qvB \quad (10)$$

$$qB = \frac{mv}{r} \quad (11)$$

$$r = \frac{mv}{qB} \quad (12)$$

Thus, the radius of curvature is connected to ratio of mass, charge, velocity, and field strength. *What is a scientific application of this?*

Magnetic Applications I: Mass Spectrometry

Magnetic Applications I: Mass Spectrometry

Mass spectrometry. A simplified picture of mass spectrometry involves measuring the radius of curvature of ions in a B-field.

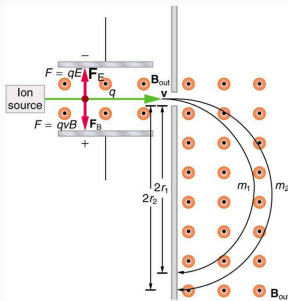


Figure 8: (a) Within the ion source, molecules are turned into an ionized gas, and accelerated through a capacitor. (b) The capacitor creates a uniform E-field, and a coil of current surrounds the capacitor to create a uniform B-field. The E and B-fields are perpendicular. (c) Ions leave the *velocity selector* into an area with just the B-field.

Magnetic Applications I: Mass Spectrometry

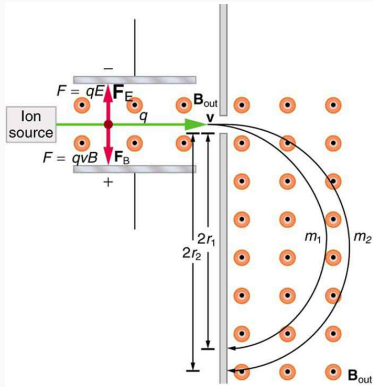


Figure 9: A simplified picture of a mass spectrometer.

1. Show that if $v = E/B$, the velocity in the *velocity selector* is constant.
2. If $E = 100 \text{ V/m}$, and $B = 50 \text{ gauss}$ (5 mT), what velocity is required from the ion source?
3. If q is the equivalent of two electrons ($2e$), and m is 100 amu^a , what is the radius observed after the velocity selector?

^a $1 \text{ amu} = 1.66 \times 10^{-27} \text{ kg}$, and $1e = 1.67 \times 10^{-19} \text{ C}$.

Magnetic Applications I: Mass Spectrometry

Within $\approx 1\%$, what result did you obtain?

- A: 0.02 m
- B: 0.2 m
- C: 2.0 m
- D: 20 m

Magnetic Applications I: Mass Spectrometry

If all other variables remained the same, what would the radius of curvature be if m decreased to 50 amu?

- A: 10 m
- B: 1.0 m
- C: 0.1 m
- D: 0.01 m

(It is not necessary to repeat the calculation. Treat this as a scaling problem.)

Magnetic Applications I: Mass Spectrometry

Notes on the Lorentz force:

1. Magnetic fields do no work.

- Work is defined as

$$W = \vec{F} \cdot \vec{x} \quad (13)$$

- Insert the Lorentz force for \vec{F} :

$$W = q\vec{v} \times \vec{B} \cdot \vec{x} \quad (14)$$

- $\vec{v} \times \vec{B}$ is perpendicular to \vec{x} , which is parallel to \vec{v} .
 - $W = 0$, because the dot-product of perpendicular vectors is zero.
2. The ratio of E to B-fields is a *velocity*, $v = E/B$? What happens for a moving observer? We will postpone this discussion until we cover *inductors*.

22 MAGNETISM

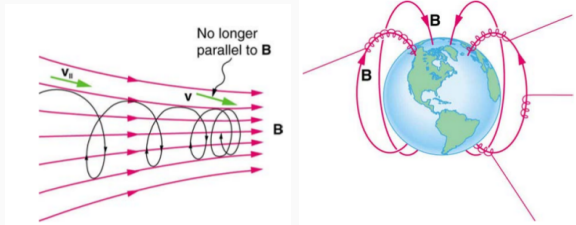


Figure 10: The aurora borealis, or northern lights.

Magnets and magnetic fields

A cool talk on the aurora borealis (time permitting):

<https://youtu.be/czMh3BnHFHQ>



One un-explained piece: what does it mean for the electrons and protons to *high-five* the neutral oxygen and nitrogen atoms?

Force and Torque on a Current Carrying Conductor

Force and Torque on a Current Carrying Conductor

Introduction to magnetic forces on current-carrying conductors:

<https://youtu.be/5fqwJyt4Lus>



Force and Torque on a Current Carrying Conductor

The Lorentz force also effects currents in conductors.

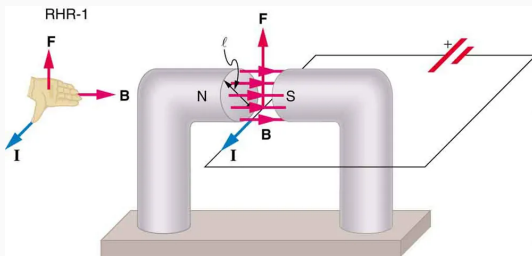


Figure 11: A B-field exerts a force on a wire carrying current.

$$F_q = qv_d B \sin \theta \quad (15)$$

$$F_{\text{tot}} = Nqv_d B \sin \theta \quad (16)$$

$$N = nV \quad (17)$$

$$N = nAL \quad (18)$$

$$F_{\text{tot}} = (nqAv_d) LB \sin \theta \quad (19)$$

$$F_{\text{tot}} = ILB \sin \theta \quad (20)$$

$$F_{\text{tot}} = I\vec{L} \times \vec{B} \quad (21)$$

Force and Torque on a Current Carrying Conductor

What is number density? Number density converts a volume to the number of objects in the volume:

$$N = nV \quad (22)$$

Suppose the Milky Way galaxy is a disc of diameter 100,000 light-years and height of 1,000 light years. We think there are about 10^{11} stars in the Milky Way. What is the number density of stars in the galaxy?

- A: 10 stars per light year
- B: 1 star per light year
- C: 0.1 stars per light year
- D: 0.01 stars per light year



Figure 12: An artist's conception of the diameter of the Milky Way.

Force and Torque on a Current Carrying Conductor

Why does $nqAv_d$ equal

current? Consider the geometry of charges flowing through a conductor:

$$I = \frac{\Delta Q}{\Delta t} \quad (23)$$

$$\Delta Q = Nq = nVq \quad (24)$$

$$\Delta Q = nALq \quad (25)$$

$$L = v_d \Delta t \quad (26)$$

$$I = \frac{nALqv_d}{L} \quad (27)$$

$$\boxed{I = nALqv_d} \quad (28)$$

The n must be the number density of *free charges*.

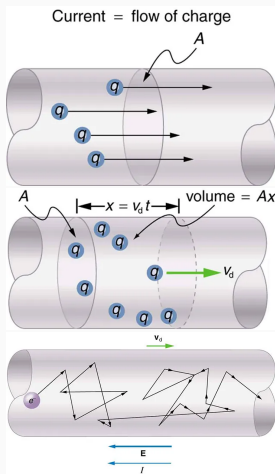


Figure 13: Simple picture of current.

Force and Torque on a Current Carrying Conductor

Lorentz Force on Current Carrying Conductor

The force on a current I of vector length \vec{L} within a magnetic field \vec{B} is

$$\vec{F} = I\vec{L} \times \vec{B} \quad (29)$$

If θ is the angle between \vec{L} and \vec{B} , the magnitude of \vec{F} is

$$F = ILB \sin \theta \quad (30)$$

When using this concept, remember:

- The units of Tesla are $\text{N m}^{-1} \text{A}^{-1}$
- The vector \vec{L} has the length of the conductor
- The vector \vec{L} has the direction of positive current

Force and Torque on a Current Carrying Conductor

A 1 m wire carries a 0.5 A current in the x-direction. A B-field of 0.01 T is in the y-direction. What is the magnitude of the force on the wire?

- A: 5×10^{-2} N
- B: 5×10^{-3} N
- C: 5×10^{-1} N
- D: 5×10^{-4} N

Force and Torque on a Current Carrying Conductor

A 1 m wire carries a 0.5 A current in the x-direction (\hat{i}). A B-field of 0.01 T is in the y-direction (\hat{j}). In what direction is the force?

- A: \hat{k}
- B: $-\hat{k}$
- C: $-\hat{j}$
- D: \hat{j}

Force and Torque on a Current Carrying Conductor

A loop of wire is in the x-y plane, centered at the origin. It carries a current in the counter-clockwise direction. A uniform B-field in the **z-direction** surrounds the wire. The loop of current will

- A: Rotate
- B: Move to the left
- C: Move to the right
- D: Remain stationary

Force and Torque on a Current Carrying Conductor

A loop of wire is in the x-y plane, centered at the origin. It carries a current in the counter-clockwise direction. A uniform B-field in the **y-direction** surrounds the wire. The loop of current will

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Force and Torque on a Current Carrying Conductor

A magnetohydrodynamic (MHD) pump system is shown in Fig. 14. A current is passed through a liquid, and the liquid passes through a B-field. Since the angle θ between \vec{L} and \vec{B} is 90 degrees, $F = ILB$. What B-field is required to produce a pumping force of 10^{-3} N, if the diameter of the pipe is 1 cm, and the achievable current is 0.1 A?

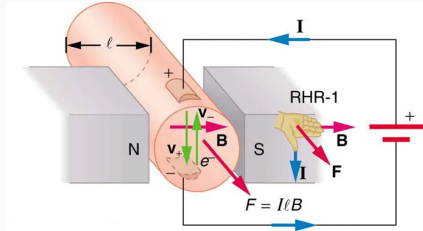


Figure 14: Simple picture of an MHD pump.

- A: 0.01 T
- B: 0.1 T
- C: 1.0 T
- D: 10.0 T

Force and Torque on a Current Carrying Conductor

Returning to the **rotating loop of current**, notice it implies a *torque*:

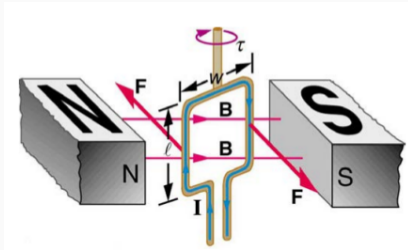


Figure 15: In a loop of current in a uniform magnetic field, we find forces going the opposite directions.

Torque can be defined like:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

(31)

Force and Torque on a Current Carrying Conductor

Returning to the **rotating loop of current**, notice it implies a *torque*:

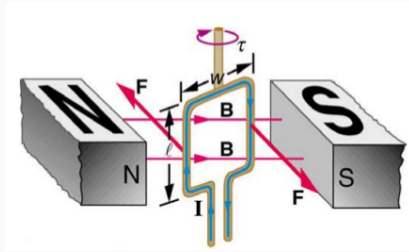


Figure 16: A loop of current in a B-field.

The magnitude follows the usual pattern for cross-products. In Fig. 16, $r = w/2$, and θ is the angle between F and r .

$$\tau = rF \sin \theta \quad (32)$$

Force and Torque on a Current Carrying Conductor

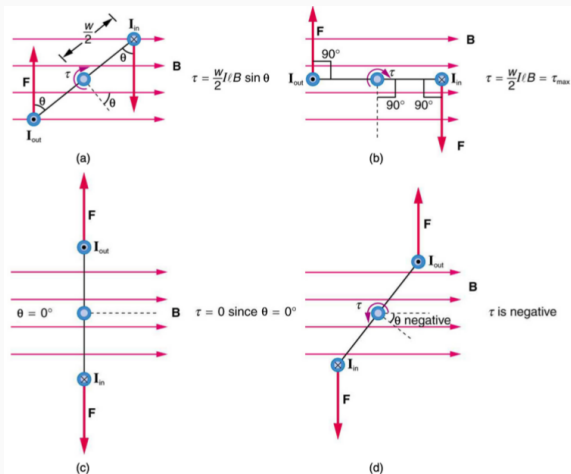


Figure 17: (a) Torque is positive (in). (b) Torque is maximally positive. (c) Torque is zero. (d) Torque is negative.

Force and Torque on a Current Carrying Conductor

The **loop of current**, experiences a *torque* that is proportional to loop area, number of turns (coils), current, B-field magnitude, and the sine of the angle between the loop and the force.

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (33)$$

$$\tau_{L,R} = rF \sin \theta \quad (34)$$

$$\tau = \tau_L + \tau_R \quad (35)$$

$$\tau = \frac{W}{2} F \sin \theta + \frac{W}{2} F \sin \theta = WF \sin \theta \quad (36)$$

$$F = lWB \quad (37)$$

$$\tau = lW^2B = lAB \sin \theta \quad (38)$$

$$\tau = NIAB \sin \theta \quad (39)$$

Force and Torque on a Current Carrying Conductor

Recall that $\tau = NIAB \sin \theta$. What is the maximum torque experienced by a coil of $N = 200$ turns of wire carrying 2.0 A with area $A = 0.2 \text{ m}^2$ in a 2.0 T B-field?

- A: 0 N m
- B: 16 N m
- C: 160 N
- D: 160 N m

What is the *minimum* torque?

Force and Torque on a Current Carrying Conductor

How do we avoid the problem of *negative torque*? If τ is negative when θ is between 180 and 359 degrees, one idea is to periodically disconnect the circuit with *brushes*.

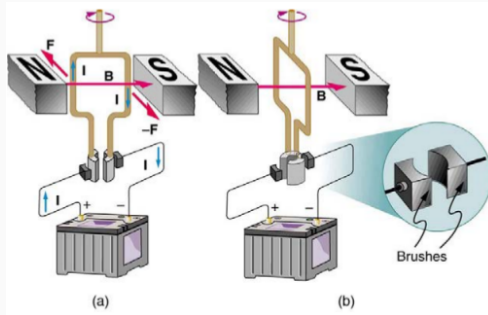


Figure 18: Torque can be used to drive a motor.

Force and Torque on a Current Carrying Conductor

Recall that $\tau = NIAB \sin \theta$. Suppose that $\tau = 0$ when $180 < \theta < 359$ (degrees), because $I = 0$. Suppose further that when $\theta = 90$ (degrees), the torque is 160 N m. What is the torque when $\theta = 30$ (degrees)?

- A: 160 N m
- B: 80 N m
- C: $160/\sqrt{2}$ N m
- D: 80 N

An example of DIY physics experiment to illustrate this concept:
<https://youtu.be/WI0pGk0MMhg?si=D-YjbydOmKPzPJlG>

Ampère's Law: Magnetic Fields Created by Current

Ampère's Law: Magnetic Fields Created by Current

One version of Ampère's Law relates the *magnetic field* created by a *current* in a long straight wire:

Ampère's Law

Let the permeability of free space be $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$. The magnetic field \vec{B} a distance r from a long, straight wire carrying a current I is given by

$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi} \quad (40)$$

How do we sort out the direction, $\hat{\phi}$?

Ampère's Law: Magnetic Fields Created by Current

One version of Ampère's Law relates the *magnetic field* created by a *current* in a long straight wire:

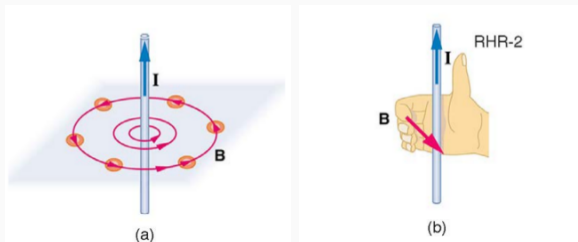


Figure 19: (a) The B-field encircles the current ($\hat{\phi}$). (b) Put thumb in direction of current, curled fingers reveal B-field direction.

Ampère's Law: Magnetic Fields Created by Current

Two wires are 1 cm apart, and the length of each is 100 cm. Each carries a current of 1 A in the same direction. What is the B-field of the left wire at the right wire? What is the B-field of the right wire at the left wire? Assume the currents are in the plane of the page and moving upwards.

- A: 2 Gauss out of the page, 2 Gauss out of the page
- B: 1 Gauss into the page, 1 Gauss out of the page
- C: 2 Gauss into the page, 2 Gauss out of the page
- D: 2 Gauss out of the page, 2 Gauss into the page

Ampère's Law: Magnetic Fields Created by Current

Two wires are 1 cm apart, and the length of each is 100 cm. Each carries a current of 1 A in the same direction. What is the force of the left wire on the right wire? What is the force of the right wire on the left wire? Assume the currents are in the plane of the page and moving upwards.

- A: 2×10^{-4} N (left), 2×10^{-4} N (right)
- B: 2×10^{-4} N (right), 2×10^{-4} N (right)
- C: 2×10^{-4} N (left), 2×10^{-4} N (left)
- D: 1×10^{-4} N (left), 1×10^{-4} N (right)

Ampère's Law: Magnetic Fields Created by Current

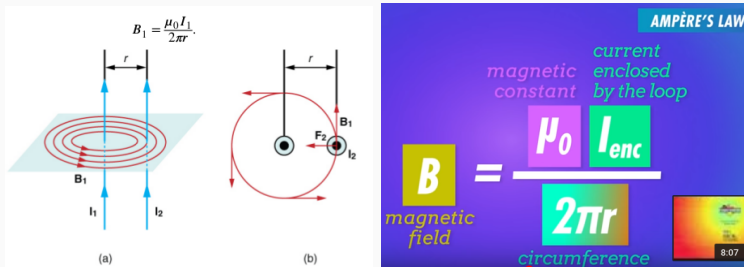


Figure 20: (a) The force of the left wire on the right wire attracts the right wire to the left wire. (b) Ampère's Law summarized in our PBS Crash Course content.

One awesome final project idea: DIY demonstration of Ampère's Law.

Ampère's Law: Magnetic Fields Created by Current

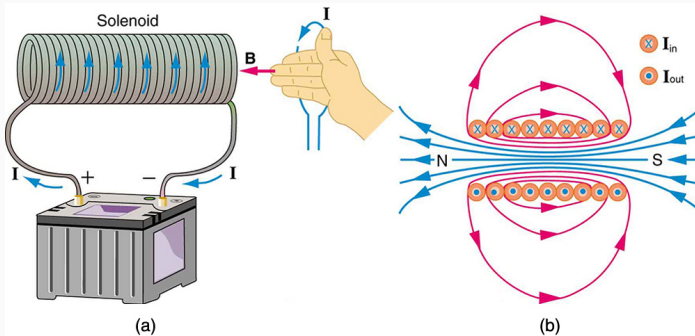


Figure 21: A solenoid is an application of Ampère's Law.

Ampère's Law: Magnetic Fields Created by Current

Ampère's Law may be used to calculate the B-field in Fig. 21.

Magnetic Field of a Solenoid

Let the permeability of free space be $\mu_0 = 4\pi \times 10^{-7} \text{ T m A}^{-1}$. Let N parallel loops of wire with current I be arranged concentrically along a distance $\vec{L} = L\hat{k}$, and let $n = N/L$. This arrangement is called a *solenoid*, and the magnetic field \vec{B} everywhere inside it is

$$\vec{B} = \mu_0 n I \hat{k} \quad (41)$$

PhET Activity: Solenoids

PhET Activity: Solenoids

PhET simulation illustrating solenoids ([run the CheerpJ browser compatible version](#)):

<https://phet.colorado.edu/en/simulations/generator>

- Click the Electromagnet tab along the top line.
- Click Show Field Meter in the grey box on the right side.
- Use the DC battery as the current source, and assume a constant resistance (current is proportional to voltage). Place the Field Meter inside the solenoid, and leave the number of loops constant.
- Create a graph of B-field strength versus voltage, in steps of 1V, from -10 V to 10 V . Is the result linear? How is this explained by Ampère's Law and Eq. 41?

PhET Activity: Solenoids

PhET simulation illustrating solenoids ([run the CheerpJ browser compatible version](https://phet.colorado.edu/en/simulations/generator)):

<https://phet.colorado.edu/en/simulations/generator>

- Click the Generator tab along the top line.
- Activate the water spigot at left to spin the magnetic field.
- Is the fact that the bulb lights up explained by Ampère's Law? In what ways is this experimental setup qualitatively different or similar to the Electromagnet experiment?

We will now perform a laboratory activity based on the simulations.

Magnetic Applications II: Nuclear Fusion

Magnetic Applications II: Nuclear Fusion

Tokamaks provide magnetic containment of charged plasma.

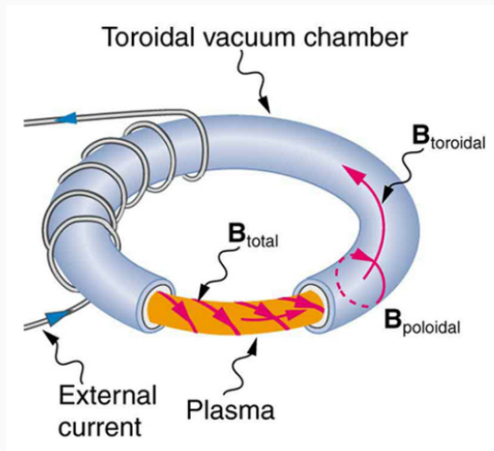


Figure 22: The tokamak contains high-energy plasma.

Magnetic Applications II: Nuclear Fusion

1. First, the external current creates a *toroidal* magnetic field.
2. Use the right hand rule for positive current to show that the toroidal field should be counter-clockwise
3. Suppose a positively charged particle is injected into the toroidal field with velocity \vec{v} . Suppose at least one component of the velocity is tangent to the counter-clockwise direction.

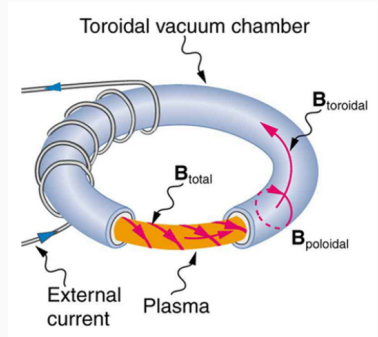


Figure 23: The tokamak contains high-energy plasma.

Magnetic Applications II: Nuclear Fusion

1. How will the positively charged particle begin to move?
2. Use the right hand rule for currents to predict the direction of the *poloidal* B-field created by the ions.
3. Note how the *poloidal* and *toroidal* fields add together to form the spiral field.
4. Can a positively charged particle to escape? Think about the Lorentz force and the cross product of $\vec{v} \times \vec{B}$.

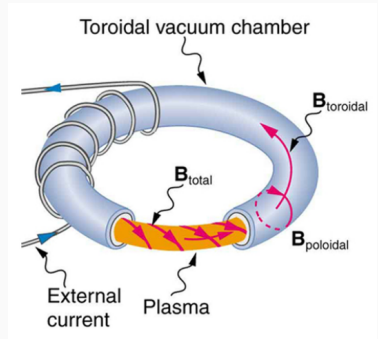


Figure 24: The tokamak contains high-energy plasma.

Conclusion

Unit 3 Summary

1. Magnetostatics I: Chapters 22.1 - 22.4

- 1.1 Magnets, ferromagnetic and electromagnetic
- 1.2 Magnetic fields and field lines, force on moving charge
- 1.3 Magnetic application: [mass spectrometry](#)

2. Magnetostatics II: Chapters 22.7 - 22.9

- 2.1 Forces and torques on conductors with current
- 2.2 Ampère's Law: magnetic fields are created by currents
- 2.3 Magnetic application: [fusion reactors](#)