ALGEBRA-BASED PHYSICS-1: MECHANICS (PHYS135A-01): WEEK 3

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WEEK 2 REVIEW

1. Displacement, average velocity and acceleration
    - *Mathematics review*: slope of a function
2. The case of constant acceleration
    - An *an equation of motion* for constant acceleration
    - Derivation of common equations of motion
    - Average quantities and exercises
3. **Lab Activity: Measuring acceleration of gravity: g**
4. Exercises with vectors, graphs, and equations of motion

WEEK 2 REVIEW PROBLEMS

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

- A: To the right, positive
- B: To the right, negative
- C: To the left, positive
- D: To the left, negative

An object that is thrown straight up falls back to Earth. When is its velocity zero? Does its velocity change direction? Does the acceleration change sign?

- A: During flight, yes, no
- B: At the peak height, yes, yes
- C: At the peak height, yes, no
- D: During flight, no, no

WEEK 3 SUMMARY

1. Working with vectors: displacement, velocity and acceleration
   - Breaking into components, graphical methods
   - Analytical methods
   - **Lab-activity: measuring different accelerations**
   - **Lab-activity: testing component independence**
2. Combining free-fall and vector components: projectile motion
3. Relative motion and addition of velocities

WORKING WITH VECTORS: DISPLACEMENT, VELOCITY AND ACCELERATION

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In general, the displacement of an object depends on time:

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} \tag{1}$$

- $x(t)$ is the displacement in the x-direction
- $y(t)$ is the displacement in the y-direction
- $z(t)$ is the displacement in the z-direction

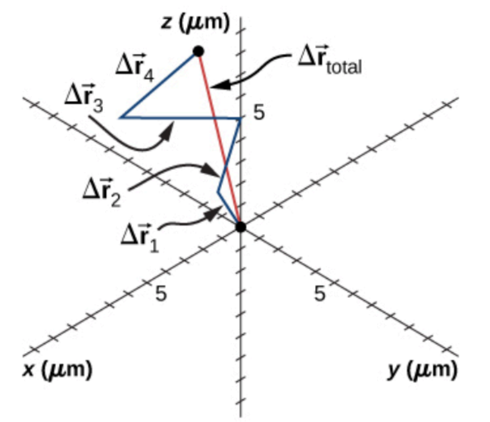

Figure 1: An example of a displacement vector at different moments in time.

WORKING WITH VECTORS: DISPLACEMENT, VELOCITY AND ACCELERATION

The particle in Fig. 1 has four displacement vectors at four moments in time:

- $\vec{r}_1 = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k} \quad (\mu m)$ at t_1
- $\vec{r}_2 = -1.0\hat{i} + 0.0\hat{j} + 3.0\hat{k} \quad (\mu m)$ at t_2
- $\vec{r}_3 = 4.0\hat{i} + -2.0\hat{j} + 1.0\hat{k} \quad (\mu m)$ at t_3
- $\vec{r}_4 = -3.0\hat{i} + 1.0\hat{j} + 2.0\hat{k} \quad (\mu m)$ at t_4

What is the total displacement of the particle from the origin?

We can think of this type of problem as an accounting problem, lining up columns (units: μm):

t_i	$\vec{r}_\mathrm{i}(t_\mathrm{i})$	$x(t_\mathrm{i})$	$y(t_\mathrm{i})$	$y(t_\mathrm{i})$
t_1	$\vec{r}_1(t_1)$	2.0	1.0	3.0
t_2	$\vec{r}_2(t_2)$	-1.0	0.0	3.0
t_3	$\vec{r}_3(t_3)$	4.0	-2.0	1.0
t_4	$\vec{r}_4(t_4)$	-3.0	1.0	2.0
t_total	$\vec{r}_\mathrm{total}(t_\mathrm{total})$	2.0	0.0	9.0

Figure 2: Accounting for the different displacement components, in units of μm.

WORKING WITH VECTORS: DISPLACEMENT, VELOCITY AND ACCELERATION

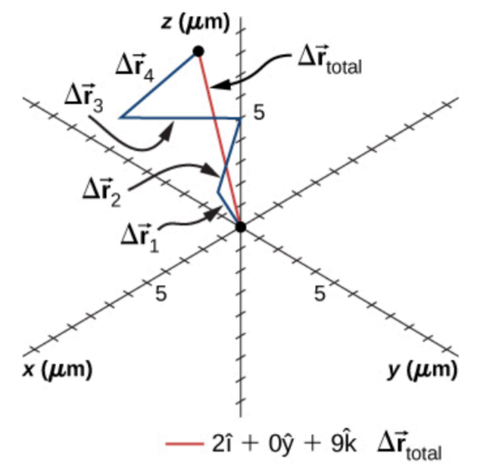

Figure 3: The total displacement of the particle is $\vec{r}_{\rm total} = 2.0\hat{i} + 0.0\hat{k} + 9.0\hat{k} \quad (\mu m)$.

The 18th hole at Pebble Beach Golf Course is a dogleg to the left of length 496.0 meters. The fairway off the tee is taken to be the x direction. A golfer hits his tee shot a distance of 300 meters, corresponding to a displacement of $\vec{r}_1 = 300.0\hat{i} \quad (m)$, and then hits a second shot 189.0 meters with $\vec{r}_2 = 150.0\hat{i} + 80.0\hat{j} \quad m$. What is the final displacement from the tee?

- A: $\vec{r}_{\text{final}} = 150.0\hat{i} + 80.0\hat{j} \quad (m)$
- B: $\vec{r}_{\text{final}} = 450.0\hat{i} + 230.0\hat{j} \quad (m)$
- C: $\vec{r}_{\text{final}} = 230.0\hat{i} + 0.0\hat{j} \quad (m)$
- D: $\vec{r}_{\text{final}} = 450.0\hat{i} + 80.0\hat{j} \quad (m)$

WORKING WITH VECTORS: DISPLACEMENT, VELOCITY AND ACCELERATION

If the first shot takes 5.0 seconds, the second shot takes 4.0 seconds, and the walking time in between the shots is 60.0 seconds, what is the average velocity vector for the ball after the two shots?

- A: $\vec{r}_{\text{final}} = 50.7\hat{i} + 11.6\hat{j} \quad (m/s)$
- B: $\vec{v}_{\text{final}} = 17.0\hat{i} + 80.3\hat{j} \quad (m/s)$
- C: $\vec{v}_{\text{final}} = 6.5\hat{i} + 1.2\hat{j} \quad (m)$
- D: $\vec{v}_{\text{final}} = 6.5\hat{i} + 1.2\hat{j} \quad (m/s)$

The prior problem indicates something you may already have guessed:

$$\vec{v}_{\mathrm{avg}}(t) = v_{\mathrm{x}}(t)\hat{i} + v_{\mathrm{y}}(t)\hat{j} + v_{\mathrm{z}}(t)\hat{k} = \frac{\Delta\vec{r}}{\Delta t} \tag{2}$$

- $v_{\mathrm{x}}(t)$ is the avg. velocity in the x-direction
- $v_{\mathrm{y}}(t)$ is the avg. velocity in the y-direction
- $v_{\mathrm{z}}(t)$ is the avg. velocity in the z-direction

In other words, we divide each displacement component by the time, to get a vector where each component is the average velocity in that direction. $\Delta\vec{r} = \vec{r}_{\mathrm{f}} - \vec{r}_{\mathrm{i}}$. These next problems help us practice breaking vectors into components.

WORKING WITH VECTORS: DISPLACEMENT, VELOCITY AND ACCELERATION

A gamma ray is radiated from a radioactive source, and travels at the speed of light (0.3 m/ns) 60 degrees with respect to the x-axis, in the positive direction. A detection screen is 1.0 to the right of the radioactive source. When does the gamma ray hit the screen?

- A: $20/\sqrt{3}$ ns
- B: $20/(3\sqrt{3})$ ns
- C: $20/3$ ns
- D: 10 ns

A person changes lanes on a highway. Her vehicle is traveling at 100 km/hr. She turns the wheel so that the car's velocity points 20 degrees from the direction down the highway. By what percentage must she increase her speed in order to maintain 100 km/hr *down the highway*?

- A: 1%
- B: 2%
- C: 6%
- D: 10%

A particle would travel with velocity of 4 m/s, 30 degrees above horizontal ($+\hat{i}$ direction), but the medium in which it moves has a velocity of 2 m/s, 225 degrees from the horizontal ($+\hat{i}$ direction). To find the actual velocity of the particle, add the velocity vectors:

- A: $(2.45)\hat{i} - (0.38)\hat{j} \quad (m/s)$
- B: $(1.45)\hat{i} + (0.38)\hat{j} \quad (m/s)$
- C: $(-2.45)\hat{i} + (0.38)\hat{j} \quad (m/s)$
- D: $(2.45)\hat{i} - (1.38)\hat{j} \quad (m/s)$

WORKING WITH VECTORS: DISPLACEMENT, VELOCITY AND ACCELERATION

In the previous example we had to subtract vectors. There are several approaches to doing this. Consider two vectors $\vec{A}$ and $\vec{B}$.

- Break into components: $\vec{A} - \vec{B} = \hat{i}(A_x - B_x) + \hat{j}(A_y - B_y)$
- Add the opposite: $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$ (flip one vector head to tail).
- Rearrange equation: $\vec{C} = \vec{A} - \vec{B} \rightarrow \vec{A} = \vec{C} + \vec{B}$ (usually graphical).

(Note to self: draw example of each).

Suppose a paricle experiences a displacement equivalent to $\vec{R} - \vec{B}$. What is the final position of the particle? ($|R| = 5$ m, $|B| = 4$ m).

- A: $3\hat{i}$ m
- B: $4\hat{i}$ m
- C: $-3\hat{i}$ m
- D: $-5\hat{i}$ m

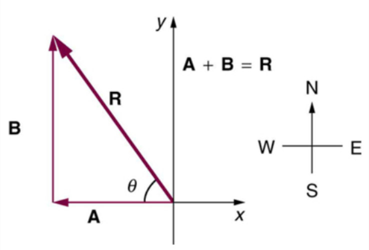

Figure 4: The path of the particle.

Two effects give a particle two velocity vectors: $\vec{v}_{\mathrm{A}}$ and $\vec{v}_{\mathrm{B}}$. The total velocity is observed (see figure). First, solve for v_{A}, v_{B}. (Hint: use the law of sines).

- A: 3.45 m/s, 3.94 m/s
- B: 3.81 m/s, 3.2 m/s
- C: 2.90 m/s, 4.51 m/s
- D: 3.94 m/s, 3.45 m/s

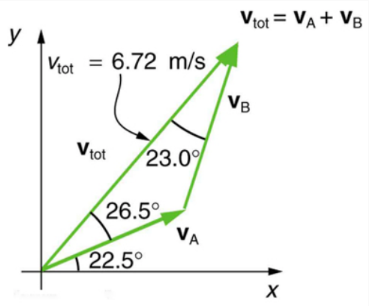

Figure 5: The velocities of the particle.

Solve for $\vec{v}_{\mathrm{A}}$.

- A: (2.19,2.32) m/s
- B: (3.19,3.32) m/s
- C: (3.19,1.32) m/s
- D: (1.32,3.25) m/s

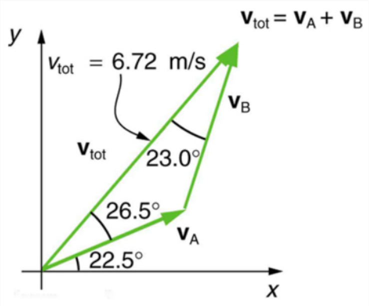

Figure 6: The velocities of the particle.

Solve for $\vec{v}_{\mathrm{B}}$ by using $\vec{v}_{\mathrm{tot}} - \vec{v}_{\mathrm{A}}$.

- A: (1.22,3.75)
- B: (1.51,4.01)
- C: (3.75,1.22)
- D: (4.01,1.32)

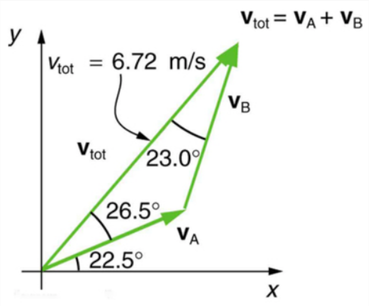

Figure 7: The velocities of the particle.

The following **lab activity** will provide practice for deriving kinematic quantities graphically. Set up a ramp with a cart, and a motion detector on the bottom:

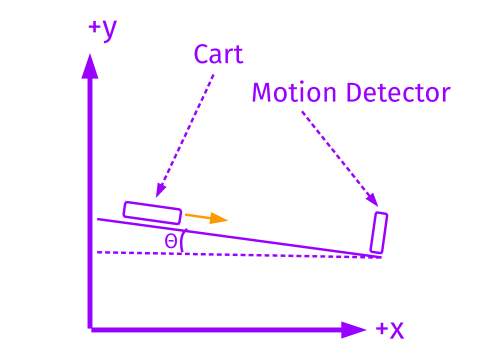

Figure 8: Cart with ramp and motion detector.

1. Once the setup is complete, release the cart and stop it, recording the position with LoggerPro.
2. Grab the cart before it hits the motion detector.
3. Record in your notebooks the position and velocity graphs; describe where the acceleration is constant in the data, and note whether it is positive or negative.
4. Repeat for a different value of θ, the ramp incline angle. Compare acceleration versus θ from the graphs.

WORKING WITH VECTORS: DISPLACEMENT, VELOCITY AND ACCELERATION

In the kinematic description of motion, *we are able to treat the different components of motion separately.* In many cases, motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

Motions in displacement components are independent.

(Exception: non-conservative forces. More on this later.)

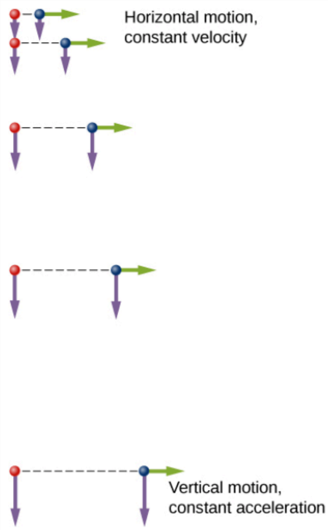

Figure 9: Independence of motion in two dimensions.

Is this true? Figure 9 is testable by experiment.

Procedure:

1. Obtain two marbles, a meter stick, and a stopwatch.
2. Measure the height of the lab bench, Δx.
3. We are going to drop a marble from this height (Δx) and record the time. Show first algebraically that the predicted time for the marble to fall is $t = \sqrt{2\Delta x/g}$.
4. Measure t for several trials. Does it match the expected result $\sqrt{2\Delta x/g}$? What are sources of error?
5. Repeat the measurement, but **roll the marble off of the table at varying speed**. Does the average result for t change?

COMBINING FREE-FALL AND VECTOR COMPONENTS: PROJECTILE MOTION

We now have learned that (a) motions in displacement components are *independent*, and (b) when acceleration is in one direction (vertical) only, the motion is *projectile motion*. Our usual equations of motion for no acceleration (horizontal), and constant acceleration (vertical) apply *independently*:

$$y(t) = y_0 + v_{0,\mathrm{y}} t - \frac{1}{2} g t^2 \tag{3}$$

$$v_{\mathrm{y}}(t) = -gt + v_{0,\mathrm{y}} \tag{4}$$

$$v_{\mathrm{y}}^2 = v_{\mathrm{y},0}^2 - 2g(y - y_0) \tag{5}$$

$$x(t) = x_0 + v_{0,\mathrm{x}} t \tag{6}$$

$$v_{\mathrm{x}}(t) = v_{0,\mathrm{x}} \tag{7}$$

Projectile motion is a good topic to introduce the concept of *boundary conditions*. The *physics* of projectile motion is the same for all situations, but the *individual cases and numbers* might not be the same.

Suppose we are given the initial velocity and angle of a object that undergoes projectile motion. To use Eqs. 3-7, we need $v_{0,\mathrm{x}}$ and $v_{0,\mathrm{y}}$, the initial horizontal and vertical velocity components, respectively.

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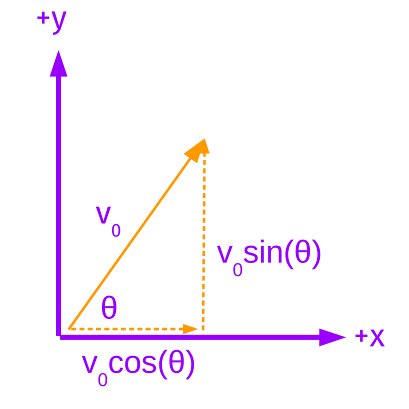

Figure 10: The initial velocity v_0 is broken into components.

During a fireworks display, a shell is shot into the air with an initial speed of 50 m/s, at an angle of 60° above horizontal. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. Calculate the height at which the shell explodes.

- A: 190 m
- B: 100 m
- C: 110 m
- D: 250 m

How much time passes between the launch and the explosion?

- A: 3.9 seconds
- B: 4.3 seconds
- C: 5.1 seconds
- D: 10.0 seconds

What is the horizontal displacement of the shell when it explodes?

- A: 108 meters
- B: 98 meters
- C: 98 degrees
- D: 150 meters

Let's try gaining visual intuition about projectile motion through the following program:

`http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/Projectile/projectile.html`

1. First, set air resistance to zero, at bottom right.
2. Make ten measurements of g by creating some projectile trajectories, and taking the ratio $g = v_{0,\mathrm{y}}^2/(2\Delta y)$. What value do you obtain, on average?
3. Now, set air resistance to $b/m \approx 0.02$, and repeat the ten measurements. What value do you obtain?
4. Explain why this value is smaller, larger, or equal to the first set of measurements.

Projectile motion in two dimensions, with constant acceleration in one dimension, produces *quadratic curves*.

How do we obtain the trajectory, or $y(x)$ for these curves?

Looking at the x-direction:

$$x = v_0 \cos(\theta) t \tag{8}$$

$$t = \frac{x}{v_0 \cos(\theta)} \tag{9}$$

Substituting in Eq. 9 for t into the equation for vertical displacement gives:

$$y(t) - y_0 = -\frac{1}{2} g \frac{x^2}{v_0^2 \cos^2(\theta)} + \tan(\theta) x \tag{10}$$

$$y(t) - y_0 = -\left(\frac{g}{2v_0^2 \cos^2(\theta)}\right) x^2 + \tan(\theta) x \tag{11}$$

$$y(x) = -kx^2 + bx + y_0 \tag{12}$$

In Eq. 12, we are simply saying that $y(x)$ is some quadratic. (It's still true that y and x are both functions of *time*, however, those functions of time are related).

A space explorer is on a moon around another planet, and wants to measure g. She tosses a pebble from an initial height of 2 meter, at an angle of 45 degrees above horizontal, with an initial velocity of 2 m/s. When it lands, the horizontal displacement is 10 meters. What is the gravitational acceleration g?

- A: 0.125 m/s^2
- B: 0.25 m/s^2
- C: 0.5 m/s^2
- D: 1.0 m/s^2

Other useful equations are for the *time-of-flight*, and the *range*, concepts we've already seen in several examples:

$$T_{\mathrm{tof}} = \frac{2v_0 \sin\theta}{g} \tag{13}$$

$$R = \frac{v_0^2 \sin 2\theta}{g} \tag{14}$$

Algebraic challenge: Show that the ratio of the range to the time is just the horizontal velocity, using the trigonometric identity $\sin(2\theta) = 2\sin\theta\cos\theta$.

RELATIVE MOTION AND ADDITION OF VELOCITIES

Thus far, we have been discussing *kinematics* with respect to a *fixed frame of reference*. Usually, we think of this frame of reference as the Earth. If we kick a soccer ball, we know how to use equations to describe the motion. What if we are moving, and the ball is moving with us, when we kick it?

Michael is running at 5 m/s, with a soccer ball rolling with him at the same speed. He shoots the ball, such that he judges the speed to be 10 m/s. What is the speed of the ball for someone who is observing, and standing still?

- A: 5 m/s
- B: 10 m/s
- C: 15 m/s
- D: Cannot determine.

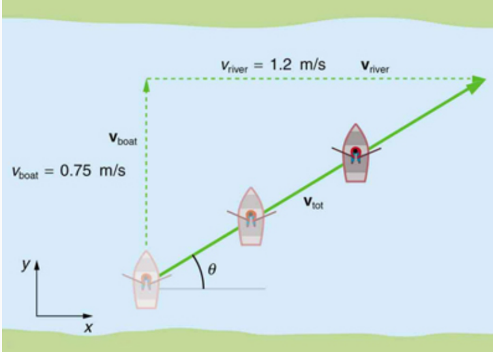

Figure 11: Motion with respect to the ground requires the addition of the velocity of the river and the boat.

The old train-robber problem: a robber is riding a horse alongside a train moving in the $\hat{j}$ direction, at 5 m/s initially. The robber on on the $-\hat{i}$ side of the train, and angles his run at 30 degrees with respect to the $\hat{j}$ direction toward the train. His horse runs at a speed of 6 m/s. What is the horse and robber's velocity, relative to the train?

- A: $3\hat{i}$ m/s
- B: $\sqrt{2}\hat{i} + \sqrt{2}\hat{j}$ m/s
- C: $(3\sqrt{3} - 5)\hat{i} + 3\hat{j}$ m/s
- D: $3\hat{i} + (3\sqrt{3} - 5)\hat{j}$ m/s

The old train-robber problem: The conductor realizes the robber is there! When the robber is 6 m away from the train, the conductor accelerates by 1 m/s^2. What is the train's speed along the track *relative to the robber* when the robber reaches the train? (Check: is this faster or slower than what it was initially?)

- A: $7 - 3\sqrt{3}$ m/s
- B: $5 - 3\sqrt{3}$ m/s
- C: $3\sqrt{3}$ m/s
- D: $\sqrt{3}$ m/s

The old train-robber problem: The conductor sees that the horse no longer has a rider, and assumes the robber made it onto the train. He holds on tight, and pulls the emergency brake. The ensuing deceleration is -3 m/s^2 from an initial train speed of 10 m/s. At this moment the robber is dropping down from the hatch to the car carrying the gold bars. The drop takes 0.5 seconds. What is the horizontal velocity of the bars relative to the robber just before she lands?

- A: -7 m/s
- B: -1.5 m/s
- C: 7 m/s
- D: 10 m/s

The robber tumbles, but reaches her feet and pounces on the gold, and begins to load gold into her pack. The conductor, however, accelerates from 8 m/s at a rate of 1 m/s^2, to more quickly reach a tunnel 0.5 km ahead. How much time does the robber have to load gold before the train reaches the tunnel? What would be her velocity relative to the ground if she dropped vertically out of the train at that time?

- A: 20 s, 10 m/s
- B: 25 s, 15 m/s
- C: 31 s, 17 m/s
- D: 25 s, 33 m/s

The old train-robber problem: Looking ahead from the train, the robber realizes she cannot jump without hitting the tunnel wall. The conductor opens the door with a pistol aimed, and shouts “Hands up!” The robber concludes which of the following?

- A: Crime doesn’t pay
- B: Studying physics helps robbers commit crimes more efficiently
- C: She should have gone into a career in science
- D: Gold is heavy

Relative velocities is also a good model for the difference between "air-speed" and "ground-speed." Suppose a plane can fly at 100 kph in still air, but experiences a tailwind of 20 kph. What is the "ground-speed" of the aircraft?

- A: 80 kph
- B: 120 kph
- C: 100 kph
- D: 20 kph

Suppose the wind changes. The plane is traveling due East ($\hat{i}$ direction), and the wind moves towards Northeast (45 degrees with respect to East), and has a new speed of 40 kph. What is the ground speed of the aircraft?

- A: $100\hat{i} + 40/\sqrt{2}\hat{j}$ kph
- B: $(100 + 40/\sqrt{2})\hat{i} + 40/\sqrt{2}\hat{j}$ kph
- C: $40/\sqrt{2}\hat{i} + 40/\sqrt{2}\hat{j}$ kph
- D: $(100 + 40/\sqrt{2})\hat{i}$ kph

CONCLUSION

1. Working with vectors: displacement, velocity and acceleration
    - Breaking into components, graphical methods
    - Analytical methods
    - **Lab-activity: measuring different accelerations**
    - **Lab-activity: testing component independence**
2. Combining free-fall and vector components: projectile motion
3. Relative motion and addition of velocities

ANSWERS

- To the right, positive
- At the peak height, yes, no
- $\vec{r}_{\text{final}} = 450.0\hat{i} + 80.0\hat{j} \quad (m)$
- $\vec{v}_{\text{final}} = 6.5\hat{i} + 1.2\hat{j} \quad (m/s)$
- 20/3 ns
- 6%
- $(2.45)\hat{i} - (0.38)\hat{j} \quad (m/s)$
- $-3\hat{i}$
- 3.45 m/s, 3.94 m/s
- (3.19,1.32) m/s
- (1.22,3.75)
- 100 m
- 4.3 s
- 108 m
- 0.5 m/s^2
- $3\hat{i} + (3\sqrt{3} - 5)\hat{j}$ m/s
- $7 - 3\sqrt{3}$ m/s
- -1.5 m/s
- 25 s, 33 m/s
- Gold is heavy...jokes
- 120 kph
- $(100 + 40/\sqrt{2})\hat{i} + 40/\sqrt{2}\hat{j}$ kph