

Figure 1: The classic Millikan oil drop experiment was a measurement of the charge of an electron.

## 2 Electric Charge and Electric Fields

1. **Scaling problem:** (a) Some point charge produces an E-field  $E_C = 2.00 \times 10^{-3} \text{ V/m}$  at a distance of 1 mm. What is the value of  $E_C$  at 5 mm produced by the same charge? (b) A  $1 \mu\text{C}$  charge produces an E-field  $E_C = 8.00 \times 10^{-3} \text{ V/m}$  at some distance. What is the value of  $E_C$  at the same distance if the charge is  $3 \mu\text{C}$ ?

$$E = \left( \frac{1}{4\pi\epsilon_0} \right) \left( \frac{q_1 q_2}{r^2} \right)$$

$$2 \times 10^{-3} \text{ V/m} = \frac{1}{4\pi\epsilon_0} \cdot \left( \frac{q}{(1 \times 10^{-3})^2} \right)^2$$

both on back  $\rightarrow$

2. The classic Millikan oil drop experiment was the first to measure the electron charge. Oil drops were suspended against the gravitational force by an electric field. (See Fig. 1.) Suppose the drops have a mass of  $4 \times 10^{-16} \text{ kg}$  and the E-field is oriented downward, and has a value of  $6131.25 \text{ N/C}$ . With this exact value, the drops remain suspended in air. (a) How many electrons are on the drops? (b) Suppose a cosmic ray comes along and removes an electron from a droplet. What will the acceleration of the droplet be?

$$4 \times 10^{-16} \text{ kg}$$

$$E\text{-field} = 6131.25 \text{ N/C}$$

$$F = mg$$

$$qE = F$$

$$qE = mg$$

$$q = \frac{mg}{E} = \frac{(4 \times 10^{-16} \text{ kg})(10^{-15} \text{ s}^2)}{6131.25}$$

$$q = 6.5 \times 10^{-19}$$

$$q = N \cdot e$$

$$N = \frac{q}{e} = \frac{6.5 \times 10^{-19} \text{ N}}{1.6 \times 10^{-19} \text{ C}} = 4.06$$

b) so  $-1e$

$$q_F = q - e = 6.5 \times 10^{-19} - 1.6 \times 10^{-19} = 4.9 \times 10^{-19}$$

$$F_e = (q_F \cdot E) = 4.9 \times 10^{-19} \cdot 6131.25 = 3 \times 10^{-15} \text{ N}$$

Similarly...

$$m_F = m - m_e = 4 \times 10^{-16} - 9.1 \times 10^{-31} = 4 \times 10^{-16}$$

$$m_e = 9.1 \times 10^{-31}$$

Rest on back  $\rightarrow$

## 3 Potential Energy and Voltage, Capacitors

1. A mass spectrometer is a device used to accelerate ions to determine atomic masses of chemicals. Suppose two conducting plates with potential difference  $\Delta V = 4 \text{ kV}$  are used to accelerate both hydrogen ions and helium ions. Hydrogens have charge  $+1e$ , and helium ions have charge  $+2e$ . (a) What is the total kinetic energy (in electron-volts) gained by the hydrogens and heliums? (b) If the plate separation is  $\Delta x = 5 \text{ cm}$ , what is the electric field value? Hint: think of the E-field as the slope of voltage.

$$\Delta V = 4 \times 10^3$$

$$KE = qV$$

$$U = q\Delta V$$

$$H - 1e$$

$$He - 2e$$

$$a) E = QV = (1.6 \times 10^{-19} \text{ C})(4 \times 10^3 \text{ V}) = 6.4 \times 10^{-16} \text{ J} = 4000 \text{ eV}$$

$$E = QV = 2(1.6 \times 10^{-19} \text{ C})(4 \times 10^3 \text{ V}) = 1.28 \times 10^{-15} \text{ J} = 7990.5 \text{ eV}$$

$$b) \Delta x = 5 \text{ cm}$$

$$E = \frac{\Delta V}{\Delta x} = \frac{4 \times 10^3}{5 \times 10^{-2}} = 8 \times 10^4 \text{ V/m}$$

2. Suppose a parallel plate capacitor has an internal E-field of  $1 \text{ kV/m}$ , and a plate separation of  $2 \text{ mm}$ . Draw the voltage as a function of distance between the negative and the positive plates. Make sure to label the axes with proper units, and mark the x-value of each plate. What is the y-intercept of this function?

$$E = 1000 \text{ V/m}$$

on back  $\rightarrow$

3. Suppose the plates in the previous problem have an area of  $1 \text{ cm}^2$ . (a) What is the capacitance of the system? (b) How much energy (in Joules) is stored in this capacitor if the voltage is  $5 \text{ V}$ ?

on back



$$1) E = \frac{F}{Q}$$

a) So...

is Coulomb force

$$k = \frac{1}{4\pi\epsilon_0}$$

$$E = k \frac{q_1 q_2}{r^2}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{r^2}$$

b)

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \quad q = 1nC$$

$$8 \times 10^{-3} = \frac{1}{4\pi\epsilon_0} \cdot \frac{1 \times 10^{-6}}{r^2} \cdot \frac{1}{1 \times 10^{-6}}$$

$$\frac{1}{1 \times 10^{-6}}$$

$$8 \times 10^3 = \frac{1}{4\pi\epsilon_0}$$

$$a = 3nC$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{3 \times 10^{-6}}{r^2}$$

$$\frac{1}{4\pi\epsilon_0 r^2} \cdot 3 \times 10^{-6}$$

$$E = 8 \times 10^3 \cdot 3 \times 10^{-6}$$

$$E = 2.4 \times 10^{-2} N/C$$

$$2 \times 10^{-3} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1}{(1 \times 10^{-3})^2}$$

$$(2 \times 10^{-3}) = \left( \frac{1}{4\pi\epsilon_0} \cdot q_1 \cdot \frac{1}{(1 \times 10^{-3})^2} \right) 1 \times 10^{-6}$$

$$2 \times 10^{-9} = \frac{q}{4\pi\epsilon_0}$$

$$a + r = 5 \times 10^{-3} m$$

$$E = \left( \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{(5 \times 10^{-3})^2} \right)$$

$$E = 2 \times 10^{-9} \cdot \frac{1}{25 \times 10^{-6}}$$

$$E = 8 \times 10^{-5} \frac{V}{C}$$

2b) Cont..

$$F_g = m_F g = (4 \times 10^{-16}) (10)$$

$$= (4 \times 10^{-15})$$

$$F = m \cdot a$$

$$a = \frac{F}{m}$$

$$= \frac{F_g - F_c}{m} = \frac{4 \times 10^{-15} - 3 \times 10^{-15}}{4 \times 10^{-16}}$$

$$a = 2.5 m/s^2$$

3.2

$$E = 1000 V/m$$

$$V(x) = -Ex + V_0$$

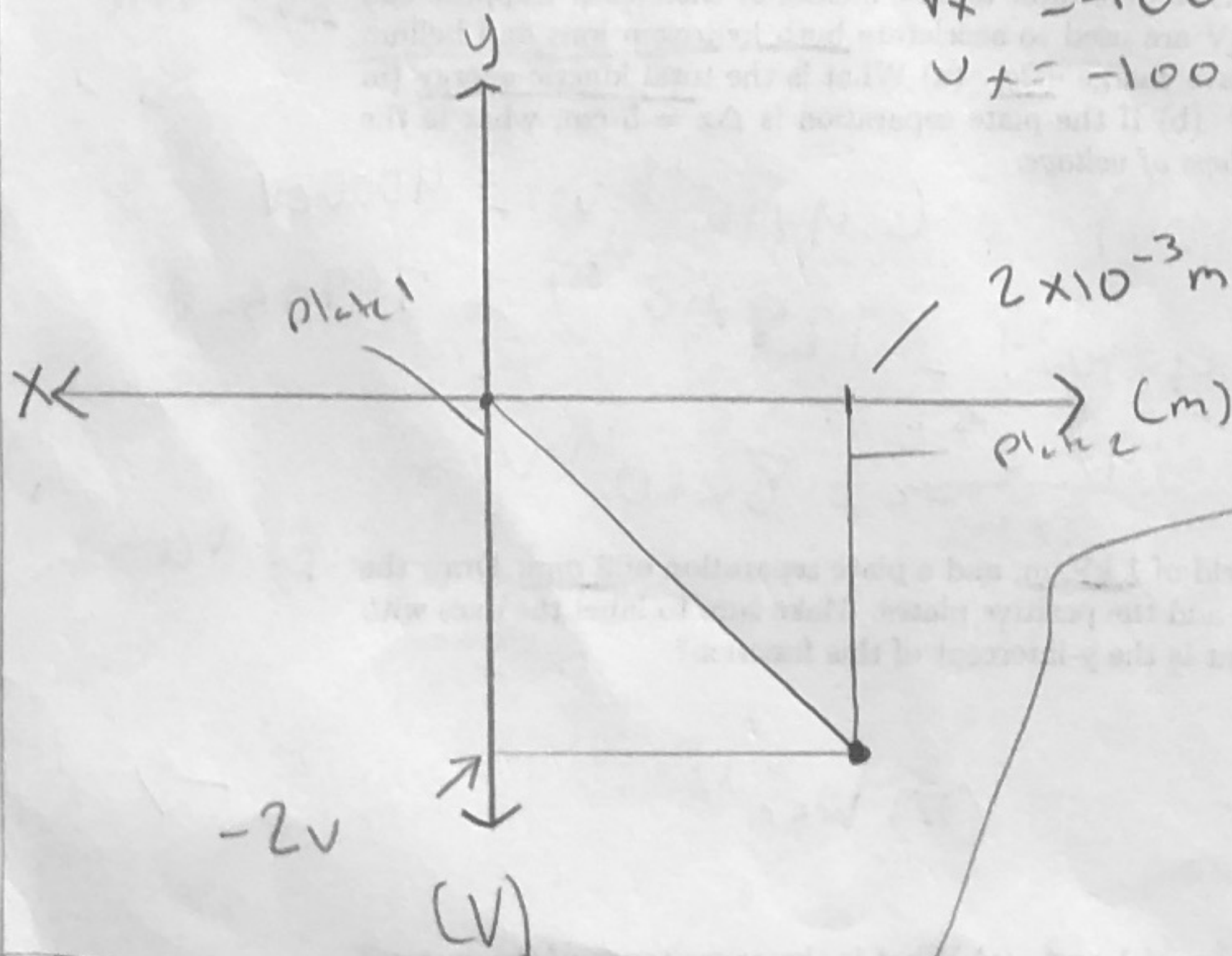
Plate Separation  $2 \times 10^{-3} m$

$$V_x = -1000x + 0 \quad \text{no init } V$$

$$V_x = -1000x$$

$$-1000(2 \times 10^{-3})$$

$$y_{int} = (0, 0)$$



$$3.3 \quad P_1 = A_{rec} = 1 cm^2$$

$$P_2 = A = 1 cm^2$$

$$C = \frac{\epsilon_0 A}{d}$$

$$d = 2 \times 10^{-3} m$$

$$\epsilon_0 = 8.85 \times 10^{-12} F/m$$

$$C = \frac{8.85 \times 10^{-12} \cdot 1 \times 10^{-4} m^2}{2 \times 10^{-3} m}$$

$$C = 4.425 \times 10^{-13} F$$

b) 5V

$$\text{Formula} = E = \frac{1}{2} C V^2$$

$$E = \frac{C V^2}{2} = \frac{4.43 \times 10^{-13} \cdot 5^2}{2}$$

$$= 5.53 \times 10^{-12} J$$



4. Suppose we need a system that can store more energy for the same voltage (in other words, more capacitance). Should we connect an identical capacitor to the first in series or in parallel?

For more capacitance, we should take into account how capacitance is totaled in series and in parallel. Since the inverse of the  $\frac{1}{C}$  is taken in series, we should connect a capacitor in parallel.

#### 4 Current, Resistance, and DC Circuits

1. When dealing with AA batteries, we can either connect them "end to end" (in series), or in parallel (see Fig. 2). Suppose that the internal resistances of the batteries  $r_1 = r_2 = 2\Omega$ , and that the emfs of the two batteries are both  $\mathcal{E}_1 = \mathcal{E}_2 = 1.5\text{ V}$ . Finally, let  $R = 50\Omega$ . Suppose  $R$  represents a small device that will work at 1.5 V or 3 V (a child's toy, an old CD player, a computer mouse). (a) Using Kirchhoff's rules, find the current through  $R$  for the serial case (3 V) (Fig. 2, left), and the parallel case (Fig. 2, right). (b) What is the power consumption in each case? (c) Check your calculations of current using the PhET DC circuit construction modeling kit.

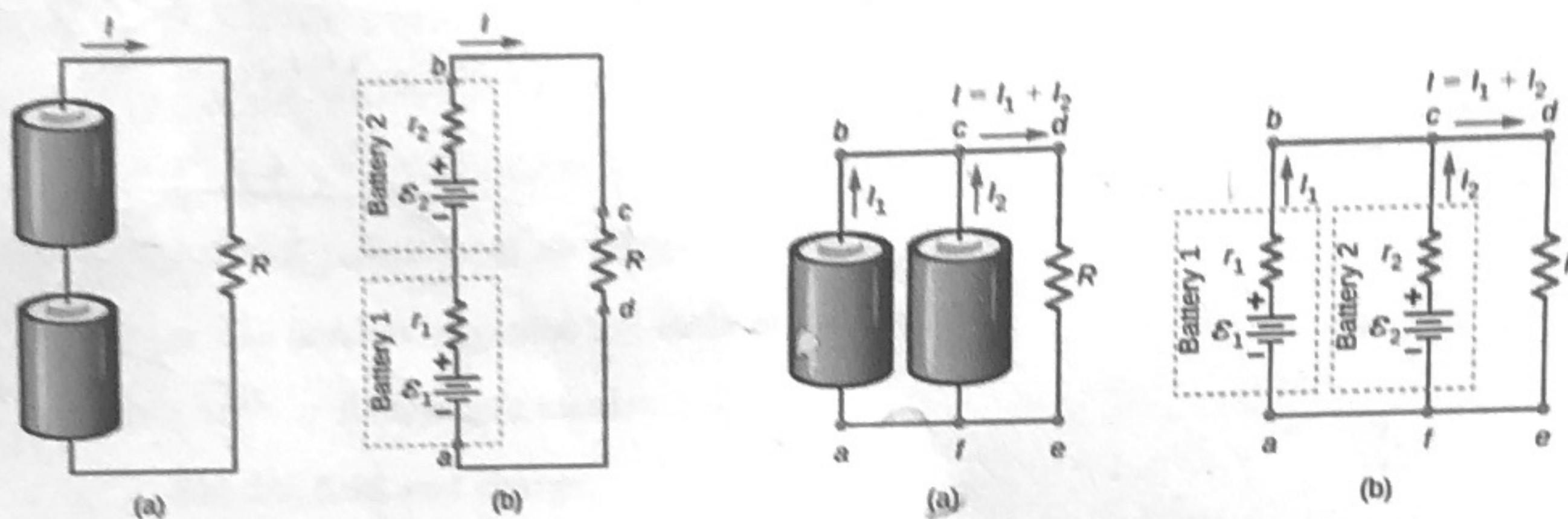


Figure 2: Two ways of connecting batteries. (Left) In series. (Right) In parallel.

2. Recall the PhET activity in which we covered nerve stimulation as chemical-driven capacitors. Think of the voltage as a signal versus time that flows down the nerve. If you stimulate the nerve in this calculation, (a) what is the pulse width, in milliseconds? (b) What is the peak-to-peak voltage (greatest voltage minus least) in millivolts?

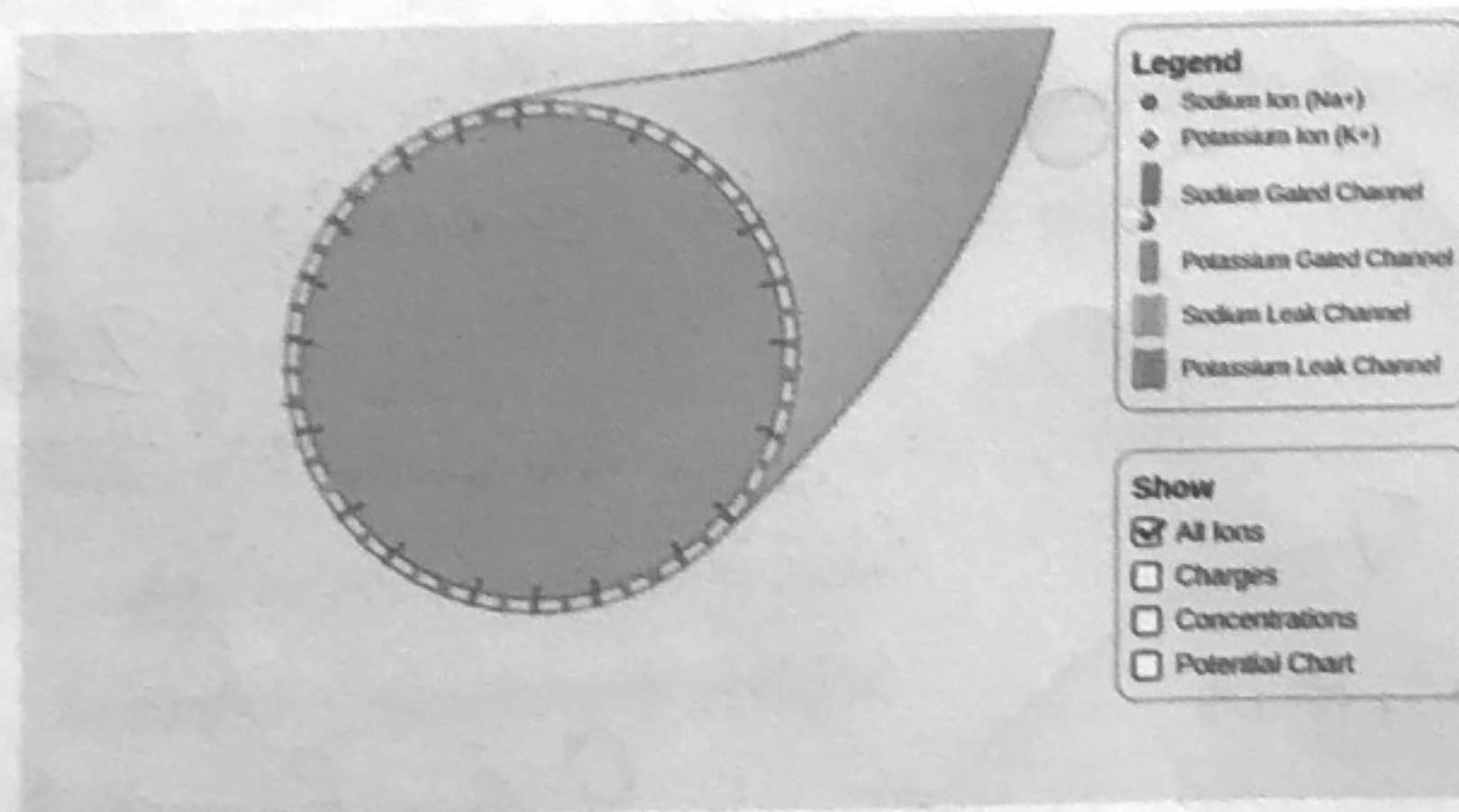
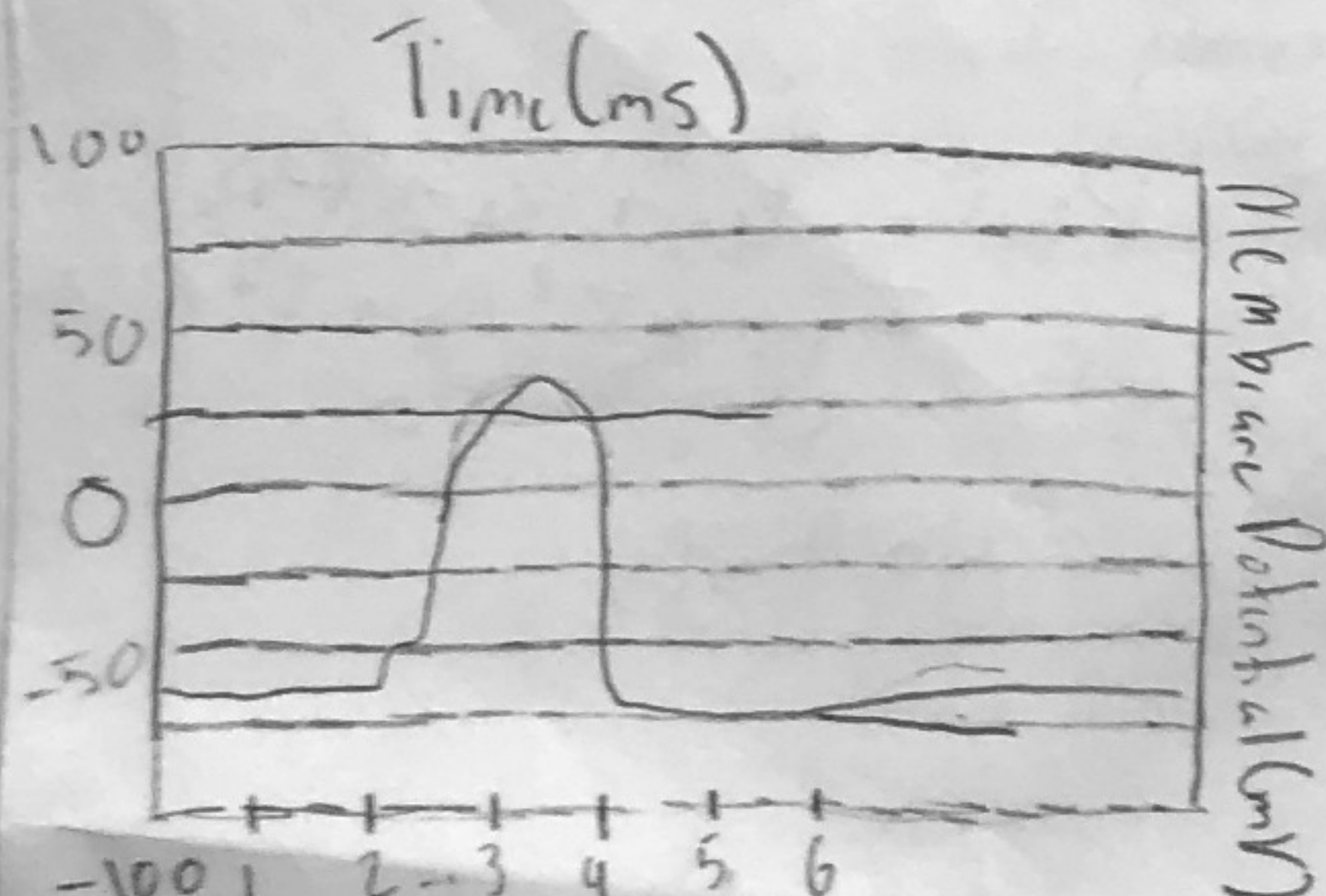


Figure 3: Recall the molecular model of the nerve membrane, and the voltage generated across it by chemical valves.

a) The pulse width when Stimulated is 2 milliseconds

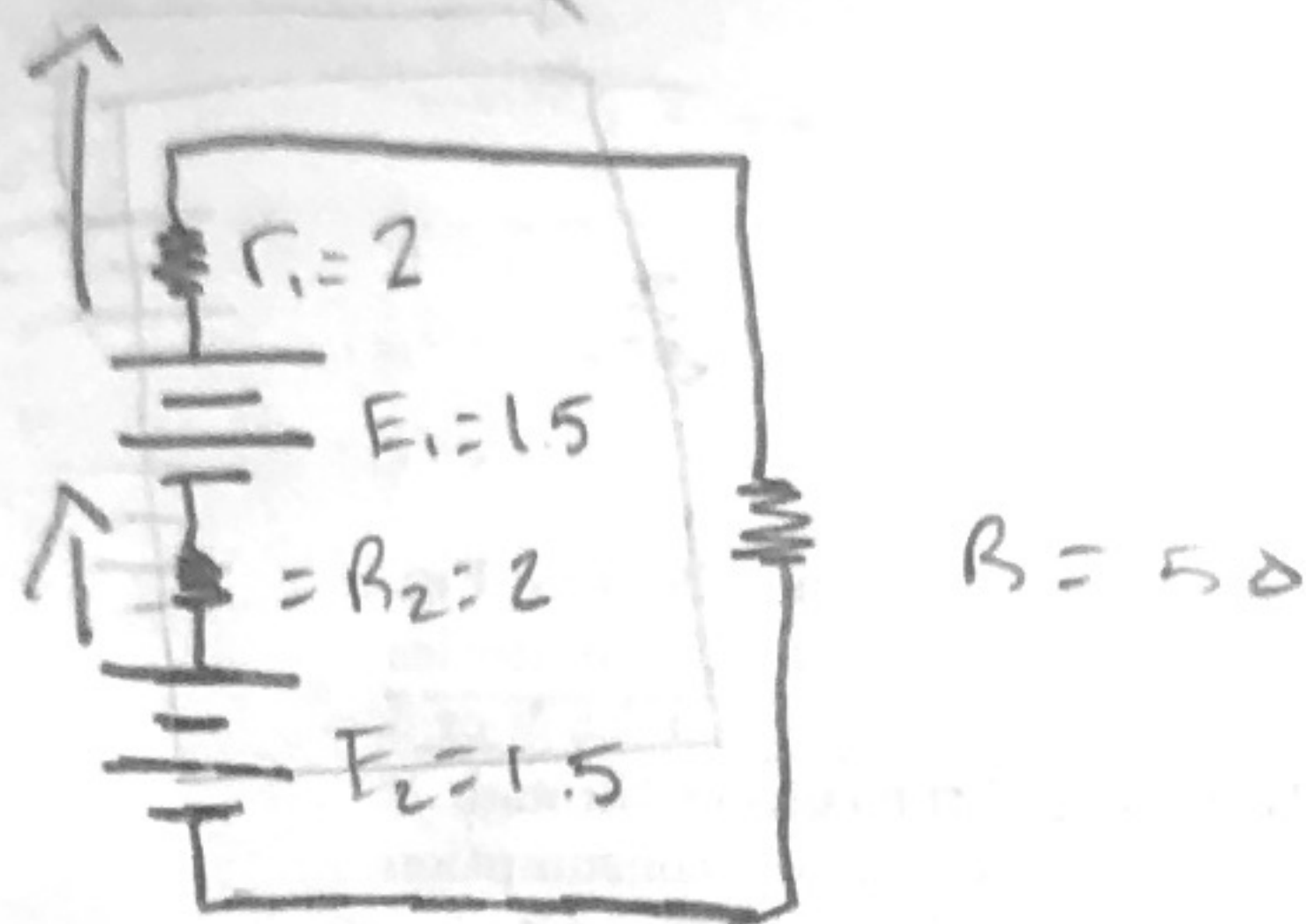
b)  $40 - (-75) = 115$





U.1)

a) Series case



$$V = IR$$

$$1.5V + Ir_1 + Ir_2 + 1.5V + IR = 0$$

$$-3.0V + Ir_1 + Ir_2 + IR$$

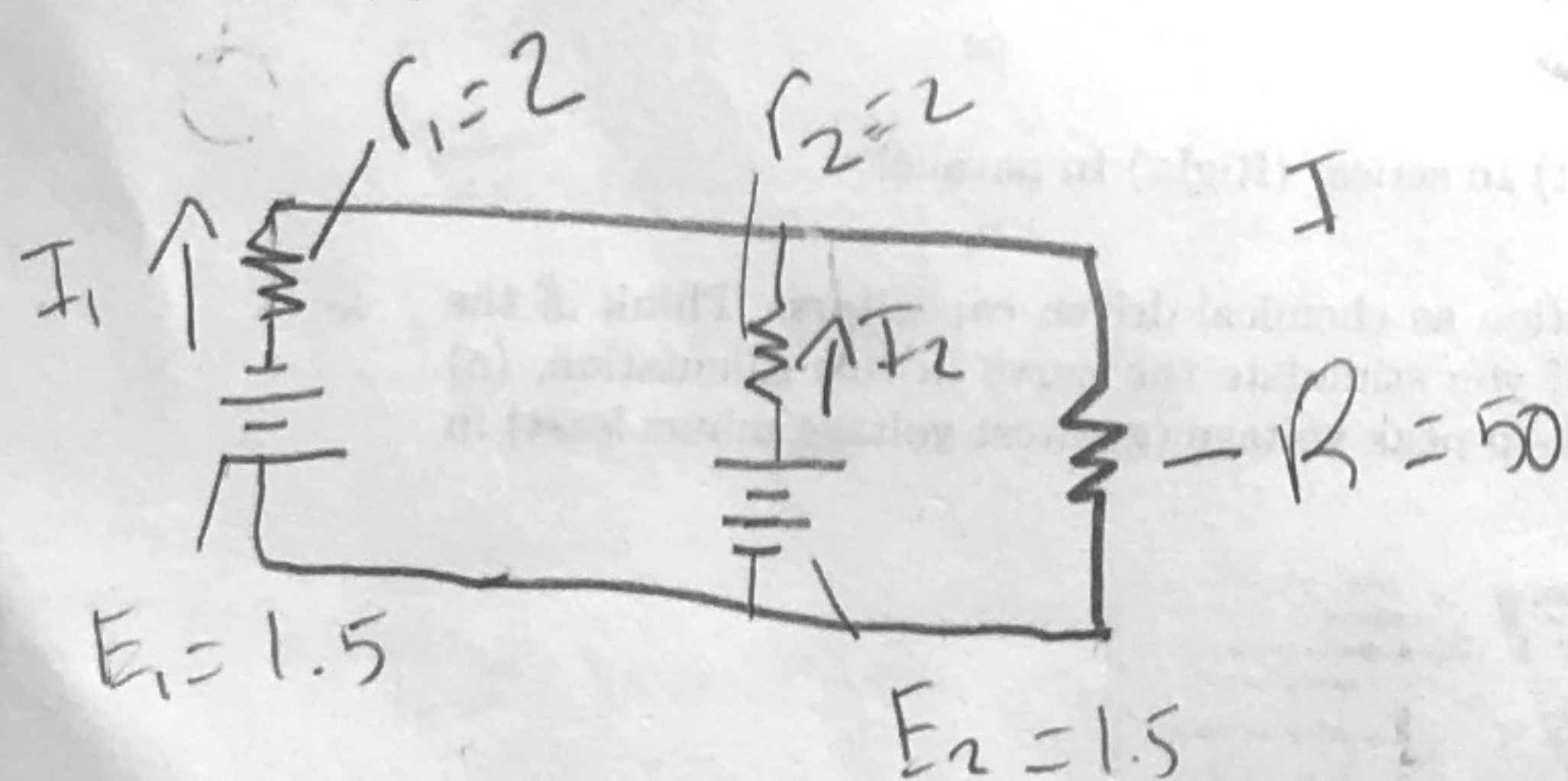
$$-3.0V + I(r_1 + r_2 + R) = 0$$

$$\frac{I(r_1 + r_2 + R)}{(R + r_1 + r_2)} = \frac{3.0}{(R + r_1 + r_2)}$$

$$I = \frac{-3.0}{(50 + 2 + 2)} = -0.0555A$$

$$I = -55.5mA$$

Parallel case



$I$  in Parallel  
and  
add.  
Koopole  
+  
 $V_+ = E_+ + V_1$   
 $E_+ = V_+ - V_0$

$$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$0 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$0 = \frac{V_+ - 1.5}{2} + \frac{V_+ - 1.5}{2} + \frac{V_+}{50}$$

Now  $V = IR$

$$I = \frac{V}{R}$$

$$I_1 = \frac{1.5 - 1.47}{2} = 15mA$$

$$I_2 = \frac{1.5 - 1.47}{2} = 15mA$$

$$0 = \frac{25V_+ - 37.5 + 25V_+ - 37.5 + V_+}{50}$$

$$0 = 75 + 51V_+$$

$$V_+ = 1.47V$$

$$I = 30mA$$

b) Power Consumption for Series

$$P_{tot} = P_{r1} + P_{r2} + P_R$$

$$= I^2 r_1 + I^2 r_2 + I^2 R$$

$$(55.5)^2 \times 2 + (55.5)^2 \times 2$$

$$= 166.33mW$$

$$I = 55.5mA$$

in Parallel

$$P_{tot} = P_{r1} + P_{r2} + P_R$$

$$I^2 R_1 + I^2 R_2 + I^2 R$$

$$(15)^2 \cdot 2 + (15)^2 \cdot 2 + (30)^2 \cdot 50$$

$$+ (55.5)^2 \times 50 = 45.9mW$$