

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

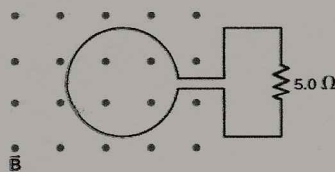
2. Consider Fig. 2. **Mass spectrometer.** Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left¹. (b) Recall that the centripetal force on a particle of mass m is mv^2/r . Set this equal to the magnitude of the Lorentz force to prove that

a) Electrical $= qE$
 $F_{mag} = qvB$ (downwards)
 $(qE + qvB = 0)$
 $r = \frac{mv}{qB}$
 $r = \frac{mE}{qB^2}$ (2)

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-septometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r ?

b) centripetal force $= mv^2/r$
 $qVB = \frac{mv^2}{r}$
 $r = \frac{mv}{qB}$
 $v = E/B$
 $r = \frac{mE}{qB^2}$

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$r = \frac{(1.67 \cdot 10^{-27})(10 \text{ V/m})}{(1.6 \cdot 10^{-19} \text{ C})(.01)^2}$
 $r = .0167 \text{ m}$

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0} (\sin(2\pi ft))$$
 (3)

The loop has a radius r . (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about, $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1 ?

a) Induced voltage $V \cdot s/A = \text{Vs}$
 $\mathcal{E} = -\frac{d\Phi}{dt}$
 $\mathcal{E} = -N \frac{d\Phi}{dt} = -NBA \frac{dB}{dt}$
 $\mathcal{E} = -N \left(\frac{dB}{dt} \right) (\pi r^2)$
 $\mathcal{E} = -N \left(\frac{B_0}{T_0} \sin(2\pi ft) \right) (\pi r^2)$
 $\mathcal{E} = -1 \left(\frac{0.1}{0.001} \right) \sin(2\pi \cdot 10^3 \cdot 0) \pi (0.1)^2$
 $\mathcal{E} = 0 \text{ V}$
 $\mathcal{E} = -1 \left(\frac{0.1}{0.001} \right) \sin(2\pi \cdot 10^3 \cdot 0.16 \cdot 10^{-3}) \pi (0.1)^2$
 $\mathcal{E} = -100 \text{ T/s} \sin(1.0053 \text{ rad}) \cdot 0.0314 \text{ m}^2$
 $\mathcal{E} = -3.14 \text{ V}$
 $I = \frac{\mathcal{E}}{R} = \frac{-3.14 \text{ V}}{5.0 \Omega} = -0.628 \text{ A}$

¹Molecules that do not have this velocity will hit the sides of this portion of the instrument.