Final for Calculus-Based Physics: Electricity and Magnetism

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1 Equations and constants

1. Volume of a sphere: $V_s = \frac{4}{3}\pi r^3$.
2. Density, mass and volume: $m = \rho V$.
3. Charge density, charge and volume: $Q = \rho V$.
4. Coulomb force: $\vec{F}_C = k\frac{q_1 q_2}{r^2}\hat{r}$.
5. Centripetal force: $\vec{F} = \frac{mv^2}{r}$
6. Definition of electric field: $\vec{F}_C = q\vec{E}$.
7. Voltage and electric field, one dimension, uniform field: $|E| = -\frac{\Delta V}{\Delta x}$.
8. Charge and capacitance: $Q = CV$.
9. Parallel plate capacitor: $C = \frac{\epsilon_0 A}{d}$.
10. Ohm's Law: $V = IR$.
11. Electrical power: $P = IV = I^2 R = V^2/R$.
12. Magnetic dipole moment: $\vec{\mu} = I\vec{A}$, where $\vec{A}$ is the area vector. $\mu = NIA$ if there are N loops.
13. Torque on a magnetic dipole: $\tau = \vec{\mu} \times \vec{B}$. The magnitude is $\tau = \mu B \sin(\theta)$.
14. Hall voltage: $emf = Blv$.
15. Definition of magnetic flux: $\phi_m = \vec{B} \cdot \vec{A}$. The units are T m^2, which is called a Weber, or Wb.
16. Faraday's Law: $emf = -N\frac{\Delta\phi}{\Delta t}$.
17. Faraday's Law using **Inductance**, M: $emf = -M\frac{\Delta I}{\Delta t}$.
18. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L.
19. Magnetic permeability: $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}
20. Units of inductance: V s A^{-1}, which is called a Henry, or H.
21. Coulomb constant: $k = 8.9876 \times 10^9$ N m^2 C^{-2}.
22. Fundamental charge: $q_e = 1.602 \times 10^{-19}$ C.
23. Speed of light: $\approx 3 \times 10^8$ m/s.
24. Permittivity of free space: $\epsilon_0 = 8.85 \times 10^{-12}$ N^{-1} C^2 m^{-2}.

2 Exercises

1. **Chapter 18: Electrostatics**

   (a) Two charges 3 μC and 12 μC are fixed 1 m apart, with the second one to the right. Find the magnitude and direction of the net force on a -2 nC charge when placed at the following locations: (a) halfway between the two (b) half a meter to the left of the left charge.

   a) 0.6 mN to the right. b) 0.312 mN to the right.

   (b) A 1 nC charge is fixed at the origin, and a -1 nC charge orbits it at a distance of 1 nm. (a) What is the Coulomb force between them? (b) At what speed is the negative charge required to move if the centripetal force balances the Coulomb force, and it has a mass of 10^{-12} kg? How does this speed compare to the speed of light?

   (a) The Coulomb force is

$$F_c = k\frac{q_1 q_2}{r^2} = 9\times 10^9 \frac{-1^2\times 10^{-18}}{(10^{-9})^2} = -9\times 10^{9-18+18} = -9\times 10^9 \quad N \tag{1}$$

   (b) Set the magnitude of the centripetal force equal to the magnitude Coulomb force:

$$F_C = \frac{mv^2}{r} \tag{2}$$

$$\sqrt{\frac{rF_C}{m}} = v = \sqrt{\frac{10^{-9}\times 9\times 10^9}{10^{-12}}} = 3\times 10^6 \quad m/s \tag{3}$$

$$\tag{4}$$

   This is one percent of the speed of light.

2. **Chapter 19: Voltage and Capacitance**

   (a) The voltage across a capacitor is 100 mV, and the distance between the two charged surfaces is 1 mm. What is the electric field in the capacitor?

   When the electric field is uniform, the field is given by the voltage divided by the distance: $E = \Delta V/\Delta x = 0.1/10^{-3}$ V/m, or 100 V/m.

   (b) A parallel plate capacitor has an area of 25×10^{-6} m^2, and a plate separation of $d = 10^{-4}$ m. (a) What is the capacitance? (b) What is the total capacitance if two such capacitors are connected *in series*? (c) *In parallel*?

   (a) The capacitance is $C = 8.85\times 10^{-12}\times 25\times 10^{-6}\times 10^4 = 8.85\times 25\times 10^{-14} = 2.21$ pF. (b) The rule for identical capacitors in series is that the capacitance drops by a factor of 2, so $C \approx 1.1$ pF. (c) Parallel capacitors add, so 4.42 pF.

3. **Chapters 20-21: Current, Resistance, and DC Circuits**

   (a) Two resistors are connected in series. One has 1000Ω resistance, and the other has 500Ω resistance. If the resistors are connected in series to a 1.5 V battery, (a) what current will flow? (b) Draw a graph of I vs. V in this system, as if V could vary but the total resistance were fixed. Label the axes of the graph and indicate the slope.

   The total resistance is 1500Ω, because the resistors are in series. Using Ohm's law, we have 1.5V/1500Ω = 1.0 mA. The graph is a linear graph, and if we place Volts on the y-axis, and current on the x-axis, the slope will be 1500. However, if we put current on the y-axis and volts on the x-axis, the slope will be 1/1500.

   (b) How much energy in kiloWatt hours does a 10Ω light consume if it draws 1.0 Amp of current for 6 hours?

   The relevant equation is $P = IV = I^2R$. We have the current and the resistance, so this is a $1.0^2\times 10.0 =$ 10.0 W light bulb. Thus, a 10W light running for 6 hours consumes 60 W hours of energy, or 0.06 kW hours. Remember that energy $U = P\Delta t$, so a "Watt hour" or "kW hour" is an energy, not a power.

   (c) Solve for the currents in Fig. 1.

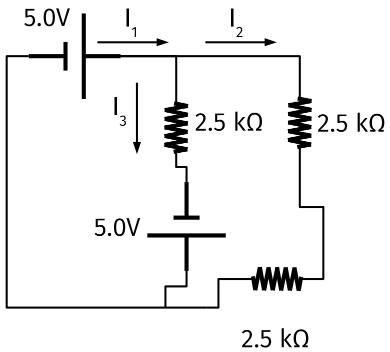

Figure 1: A circuit with two battery voltages and three resistors.

- $I_1 = 5$ mA
- $I_2 = 1$ mA
- $I_3 = 4$ mA

4. **Chapter 22: Magnetism**

   (a) A Hall effect flow probe is placed on an artery, applying a 0.1 T magnetic field across it. What is the blood velocity, given the vessels inside diameter is 4.00 mm and the Hall voltage is 0.8μV?

   The Hall effect follows $emf = Blv$, or $v = emf/(Bl) = 0.8 \times 10^{-6}/(0.1 \times 4 \times 10^{-3}) = 2$ mm/s.

   (b) A circular loop of wire of area 10^{-1} m^2 carries a current of 1.0 A, and it is situated in the xy plane. (a) An external 1.0 T B-field is applied in the positive z-direction. What is the magnitude of the torque? (b) A magnetic field is applied in the x-direction. What is the magnitude of the torque?

   (a) In the first case, the magnetic moment of the loop is in the +z direction, and so is the field, so the torque is zero. (b) $|\vec{\tau}_m| = |\vec{\mu} \times \vec{B}| = \mu B \sin(\theta)$. The angle is 90 degrees, so the torque is $\mu B = IAB = 1.0 \times 10^{-1} \times 1.0 = 10^{-1}$ N m.

5. **Chapter 23: Electromagnetic Induction and Inductance**

   (a) Suppose we encounter two inductors and decide to measure the inductance of each. One is smaller, producing an emf of 5 mV if the current across it is changed at a rate of 50 A/s. The larger produces an emf of 10 mV for a rate of 50 A/s. What is the inductance of each?

   (a) The smaller inductance is $L = emf/(\Delta I/\Delta t) = 5 \times 10^{-3}/50 = 0.1$ mH. Using scaling, we conclude that the second inductor is 0.2 mH.

   (b) Suppose the two inductors were interleved, such that a current through one inductor would generate a solenoidal magnetic field that happened to pass through the other solenoid. If a changing current is passed through the smaller inductor, which of the following is true of the larger?

   - A: Nothing would happen: the emf is only induced in the first inductor.
   - B: An emf would be induced in the second inductor, equal to the emf in the first inductor.
   - C: An emf would be induced in the second inductor, but smaller than the emf in the first inductor.
   - D: An emf would be induced in the second inductor, but larger than the emf in the first inductor.

   The answer is D, because the larger inductor sees the same flux as the smaller, but with more coils. Thus, the induced emf is larger in the larger inductor.

6. Camera flashes charge a capacitor to high voltage by switching the current through an inductor on and off rapidly. In what time must the 0.100 A current through a 2.00 mH inductor be switched on or off to induce a 500 V emf?

   Use $|emf| = L\frac{\Delta I}{\Delta t}$. Solve for $\Delta t = L\frac{\Delta I}{emf} = 2 \times 10^{-3}\frac{0.1}{5\times10^2} = \frac{2}{5} \times 10^{-6} = 0.4\mu$s.