Taylor Watanabe
20594796

Midterm 3

Dr. Jordan Hanson - Whittier College Dept. of Physics and Astronomy

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1 Memory Bank

1. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
2. $P = IV$ ... Relationship between power, current, and voltage.
3. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
4. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.
5. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ... Ampère's Law.
6. $\epsilon = -N d\phi/dt$ ... Faraday's Law.
7. $\phi = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux.
8. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
9. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L. Self-inductance: $\Delta V = -L(dI/dt)$.
10. Units of inductance: V s A^{-1}, which is called a Henry, or H.
11. $B = \mu_0 n I$ ... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

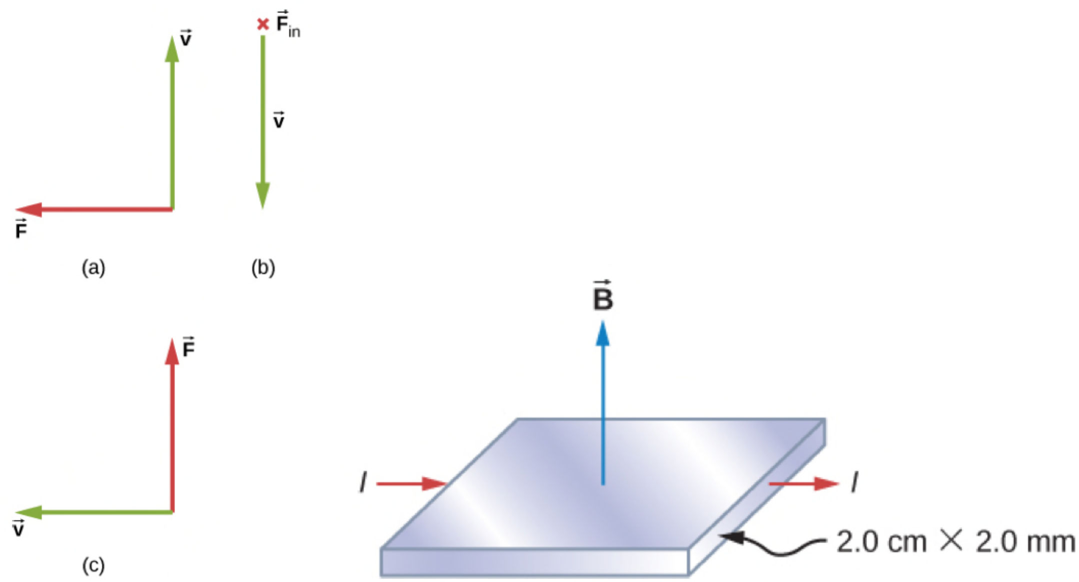

Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.

$F_L = q\vec{v} \times \vec{B}$

- a: B-field is $\boxed{\text{into the page}}$
- b: B-field is $\boxed{\text{left}}$
- c: B-field is $\boxed{\text{out of the page}}$

2. Consider Fig. 1 (right). **The Hall Effect.** An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that

$$\Delta V = \frac{B\Delta x I}{n q_e A} \tag{1}$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28}$ m^{-3}, $A = 1$ mm^2, and q_e is the charge of an electron.

a) Force of B-field : $F_B = qvB\sin\theta$

b/c $\theta = 90° \Rightarrow F_B = qvB$

Force of E : $F_E = qE$

where E field & B-field forces have same magnitude. So,

$$\cancel{q}E = \cancel{q}vB$$

$$\frac{E}{B} = \frac{vB}{B}$$

$$v = E/B$$

b) $E = \Delta V / \Delta x$

Solve for ΔV : $\Delta V = E(\Delta x)$

Plug in $E = v/B$: $\Delta V = (vB)(\Delta x)$

where $v_d = \dfrac{I}{nq_eA}$, plug in for v

So, $\Delta V = \dfrac{B\Delta x I}{nq_eA}$

$$\Delta V = \frac{(1.33T)\left(2cm\left(\frac{10^{-2}m}{1cm}\right)\right)(10A)}{(2\times10^{28}m^{-3})(1.6\times10^{-19}C)\left(1mm^2\left(\frac{10^{-3}m}{1mm}\right)^2\right)}$$

$$\boxed{\Delta V = 8.31 \times 10^{-5} V}$$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^4 A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

$\tau = NIAB\sin\theta$

given $N = 1$ turn ; $\theta = 90°$

$A = \pi r^2$

$A = \pi(0.65 \times 10^{-15} m)^2$

$A = 1.33 \times 10^{30} m^2$

$\tau = (1)(1.05 \times 10^4 A)(1.33 \times 10^{30} m^2)(2.5T)\cancel{\sin(90)}^{=1}$

$\boxed{\tau = 3.5 \times 10^{-26} N \cdot m}$

$A \cdot m^2 \cdot T$

$T = N/Am$

$= \cancel{A}m^{\cancel{2}} \cdot \frac{N}{\cancel{Am}}$

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

a) $B = \mu_0 n I$

$n = N/L$

$= 500 m^{-1}$

$B = (4\pi \times 10^{-7} T \, \cancel{m}/\cancel{A})(500 \cancel{m^{-1}})(0.3\cancel{A})$

$\boxed{B = 1.88 \times 10^{-4} T}$

b) $B = ((5000)\mu_0) n I$

$(4\pi(5) \times 10^{-4} T \cdot m/A)(500 m^{-1})(0.3A)$

$\boxed{B = 0.942 T}$

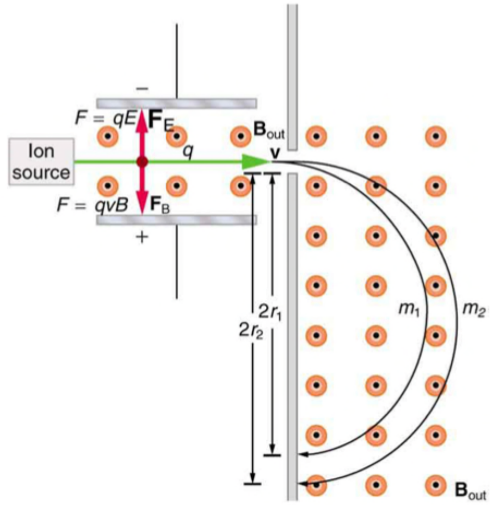

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \tag{2}$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r?

a) "Velocity selector" : path $r \propto$ mass

if $F_{Net} = 0 \Rightarrow$ forces balance + $F_e + F_m = 0$

$F_E = qE$

$F_m = qvB$

$qE + qvB = 0$

$q(E + vB) = 0$

$E = vB$

$v = E/B$ when $F_{Net} = 0$

b) $F_c = mv^2/r$

$F = qvB$

$qvB = mv^2/r$

$r = \frac{mv^{\cancel{2}}}{q\cancel{v}B}$

$r = \frac{mv}{qB}$

but, $v = E/B$

so, $r = \frac{m(E/B)}{qB}$

$r = \frac{mE}{qB^2}$

$r = \frac{16(1.67 \times 10^{-27}\,kg)(10\,V/m)}{(1.6 \times 10^{-19}\,C)(0.01\,T)^2}$

$\boxed{r = 0.017\,m}$

$T = N/Am$

$C = A/s$

$N = kg \cdot m\,s^{-2}$

$V = kg\,m^2 \cdot s^{-3} \cdot A^{-1}$

4 Chapter 13: Electromagnetic Induction

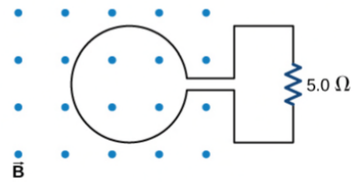

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0}(\sin(2\pi f t)) \tag{3}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1?

a)

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta BA}{\Delta t}$$

$$\Delta B = \frac{\mathcal{E}\cdot\Delta t}{A}$$

$$\frac{\mathcal{E}\cancel{\Delta t}}{A\cancel{\Delta t}} = \frac{B_0}{T_0}(\sin(2\pi f t))$$

$$\mathcal{E} = A\,{}^{B_0}/T_0\,(\sin(2\pi f t))$$

$$\boxed{\mathcal{E} = \pi r^2 \times \frac{B_0}{T_0}(\sin(2\pi f t))}$$

b) @ $t = 0s$

$$= \pi r^2 \times \frac{B_0}{T_0}(\sin(2\pi f(0)))$$

$$= \pi r^2 \times B_0/T_0(\sin(0))$$

$$= 0$$

$\boxed{\text{induced emf} = 0}$

c) $\mathcal{E} = \pi(0.1m)^2 \times \frac{(0.1T)}{(0.001s)} \times \sin(2\pi(10^3 Hz)(0.00016s))$

$$= \pi \times \frac{(0.1)^{\cancel{3}}}{\cancel{(0.001)}} \times \sin(1.01)$$

$$\boxed{\mathcal{E} = 2.65V}$$

d) $I = \mathcal{E}/R$

$$= (2.65V)/(5\Omega)$$

$$\boxed{I = 0.53A}$$

5 Chapter 14: Inductance

1. What is (a) the rate at which the current though a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

a) $\mathcal{E} = -L\frac{\Delta I}{\Delta t}$ $\quad |\mathcal{E}| = L\frac{\Delta I}{\Delta t} \Rightarrow \frac{\Delta I}{\Delta t} = \frac{\mathcal{E}}{L} = \frac{0.15V}{0.5H}$ $\quad \boxed{\left|\frac{\Delta I}{\Delta t}\right| = 0.3\,A/s}$ $\quad V/\frac{V\cdot s}{A}$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$$\mathcal{E} = L\frac{\Delta I}{\Delta t}$$

$$\Delta t = \frac{L}{\mathcal{E}}\Delta I$$

$$\Delta t = \frac{(2mH(10^{-3}H/1mH))}{500V}(0.1A)$$

$$\boxed{\Delta t = 4\times10^{-7}s}$$