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Midterm 3

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1 Memory Bank

1. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
2. $P = IV$ ... Relationship between power, current, and voltage.
3. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
4. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.
5. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ... Ampère's Law.
6. $\epsilon = -N d\phi/dt$ ... Faraday's Law.
7. $\phi = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux.
8. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
9. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L. Self-inductance: $\Delta V = -L(dI/dt)$.
10. Units of inductance: V s A^{-1}, which is called a Henry, or H.
11. $B = \mu_0 n I$ ... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

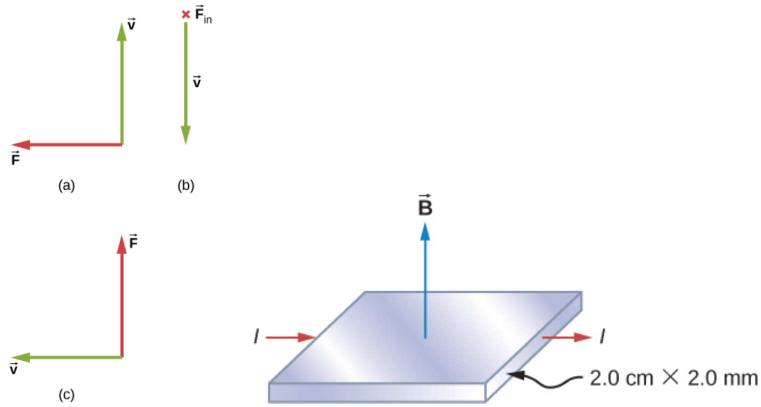

Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.

   - a: into the page
   - b: to the left
   - c: out of the page

2. Consider Fig. 1 (right). **The Hall Effect**. An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that

$$\Delta V = \frac{B\Delta x I}{nq_e A} \tag{1}$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28}$ m^{-3}, $A = 1$ mm^2, and q_e is the charge of an electron.

a) Force of B feild on charge:
$F_B = qvB\sin(\theta)$
As θ equals $90°$
$F_B = qE$
Force of E feild on charges:
$F_E = qE$
$qvB = qE$
$V = E/B$

B.) $I = nq_eAv$
$V = nq_eAv$
$\Delta V = B(1/nq_eA)\Delta x$
$= BI(\Delta x)/nq_eA$

$= \dfrac{1.33\,T(10\,A)(0.02\,m)}{(2\times10^{28}\,m^{-3})(1.6\times10^{-19})(10^{-6}\,m^2)}$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^{4} A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

$\tau = i \cdot B \cdot A$
$A = \pi r^2 = \pi(0.65\times10^{-15})^2 = 1.327\times10^{-30}$
$\tau = (1.05\times10^{4}\,A)(2.50)(1.327\times10^{-30}) = 3.48\times10^{-26}$ N.m.

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

a) $B = \mu_0 n I$

$B = (4\pi \times 10^{-7} N/A)(500\ t/m)(0.3 A)$

$B = 1.88 \times 10^{-4}\ T$

B.) $B' = \mu n I$

$= 5000 \times 1.88 \times 10^{-4}\ T$

$= 0.94\ T$

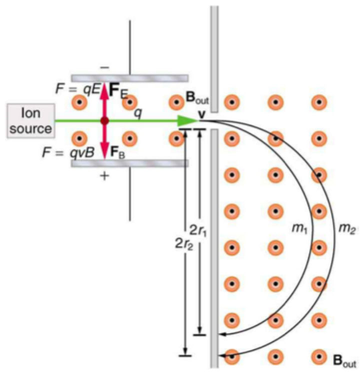

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \tag{2}$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r?

a) netforce 0= n the particle iso

$F_{electrical} + F_{magnetic} = 0$

$F_{electrical} = q\vec{E}$

$F_{magnetic} = q(\vec{v} \times \vec{B})$

$F_{total} = F_{electrical} + F_{magnetic} = 0$

$q[\vec{E} + \vec{v} \times \vec{B}] = 0$

$\hat{v} \times \hat{B}$ is downwards opposite to $\vec{E}$

$F_{total} = q[E - vB] = 0$

$E = vB \Rightarrow v = \frac{E}{B}$ for $F_{net} = 0$

B.) Centripetal force $= mv^2/r$

$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$

$v = \frac{E}{B} \Rightarrow r = \frac{m\frac{E}{B}}{qB} = \frac{mE}{qB^2}$

mass of oxygen nucleus $m = 16 \times m_{Proton} = 16 \times 1.67 \times 10^{-27}$ kg

$q = |e| = 1.602 \times 10^{-19}$ C

$E = 10$ V/m

$B = 0.01$ T

$r = \frac{m\frac{E}{B}}{qB} = \frac{mE}{qB^2} = \frac{16(1.67 \times 10^{-27})(10)}{1.602 \times 10^{-19}(0.01)^2} = 1.67 \times 10^{-2}$ m $= 1.67$ cm

radius

4 Chapter 13: Electromagnetic Induction

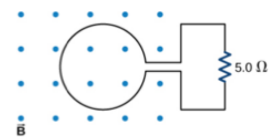

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0}(\sin(2\pi f t)) \tag{3}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1?

a.) Induced voltage $e = \frac{d\Phi}{dt} = \frac{d(BA)}{dt}$
$e = A\frac{dB}{dt}$
$e = \pi r^2 \times \frac{B_0}{T_0} \sin(2\pi f t)$

b.) Induced emf at $t=0$
At $t=0$, $\sin(2\pi f t) = \sin 0 = 0$
induced emf will be zero at $t=0$

c.) At $t_1 = 0.16ms$: $f = 10^3$ Hz, $B_0 = 0.1T$, $r = 0.1m$
$e = \pi(0.1)^2\left(\frac{0.1}{1\times10^{-3}}\right)\sin(2\pi\times10^3\times0.16\times10^{-3})$
$= \pi(0.1)^2\left(\frac{0.1}{1\times10^{-3}}\right)\sin(1.0053) = 0.055\,V$

d.) $I = \frac{e}{R} = \frac{0.055V}{5} = 0.011$ Amp

5 Chapter 14: Inductance

1. What is (a) the rate at which the current though a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

L (inductance) = 0.50 H
$\varepsilon = -L\frac{dI}{dt}$
$\frac{dI}{dt} = \frac{-\varepsilon}{L} = \frac{-0.150V}{0.50\,H} = -0.3\,A/s$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$\varepsilon = L\frac{dI}{dt}$ $\quad dt = \frac{L}{\varepsilon}dI$

$dt = \frac{2.00mH}{500V}(0.100A) = \frac{2.0mH \cdot \frac{10^{-3}H}{1mH}}{500\,V}(0.100A) = 4.00\times10^{-7}s$