Algebra-Based Physics-2: Electricity, Magnetism, and Modern Physics: Unit 3

Jordan Hanson

November 3, 2023

Whittier College Department of Physics and Astronomy

Summary

Unit 3 Summary

1. **Magnetostatics I: Chapters 22.1 - 22.4**
    1.1 Magnets, ferromagnetic and electromagnetic
    1.2 Magnetic fields and field lines, force on moving charge
    1.3 Magnetic application: mass spectrometry
2. **Magnetostatics II: Chapters 22.7 - 22.9**
    2.1 Forces and torques on conductors with current
    2.2 Ampère's Law: magnetic fields are created by currents
    2.3 Magnetic application: fusion reactors

Magnets and magnetic fields

Magnets and magnetic fields

Review of the Origin of Electric and Magnetic Fields

On the origin of magnetic fields and forces they exert on charge:

`https://www.youtube.com/watch?v=s94suB5uLWw`

On the origin of electric fields and forces they exert on charge:

`https://youtu.be/mdulzEfQXDE?si=euGvVjKPT33_E-fI`

Key points:

- Some elements are magnetic or can be magnetized
- Current creates magnetic fields
- Current exerts force on moving charge and current

Magnets and magnetic fields

What is a cross-product and how does it work?

Computing the cross product [edit]

Coordinate notation [edit]

The standard basis vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ satisfy the following equalities in a right hand coordinate system:

$$\begin{aligned} \mathbf{i} \times \mathbf{j} &= \mathbf{k} \\ \mathbf{j} \times \mathbf{k} &= \mathbf{i} \\ \mathbf{k} \times \mathbf{i} &= \mathbf{j} \end{aligned}$$

which imply, by the anticommutativity of the cross product, that

$$\begin{aligned} \mathbf{j} \times \mathbf{i} &= -\mathbf{k} \\ \mathbf{k} \times \mathbf{j} &= -\mathbf{i} \\ \mathbf{i} \times \mathbf{k} &= -\mathbf{j} \end{aligned}$$

The definition of the cross product also implies that

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0} \text{ (the zero vector).}$$

Figure 1: The cross-product is a way of multiplying unit vectors.

Professor: several examples on board.

Magnets and magnetic fields

Let $\vec{v} = 2\hat{i}$ and $w = -2\hat{j}$. What is $\vec{v} \times \vec{w}$?

- A: $-4\hat{k}$
- B: $4\hat{k}$
- C: $-2\hat{i}$
- D: $2\hat{j}$

Magnets and magnetic fields

Let $\vec{v} = 3\hat{j}$ and $w = 5\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $15\hat{i}$
- B: $5\hat{j}$
- C: $3\hat{i}$
- D: $15\hat{k}$

Magnets and magnetic fields

Let $\vec{v} = 3\hat{i} \times 3\hat{j}$ and $w = 2\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $-6\hat{j} + 6\hat{k}$
- B: $-6\hat{j} + 6\hat{i}$
- C: $6\hat{j} + 6\hat{i}$
- D: $6\hat{k} + 6\hat{i}$

Group exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} \tag{1}$$

$$\vec{w} = 4\hat{j} - 4\hat{i} \tag{2}$$

$$\vec{v} \times \vec{w} = ?? \tag{3}$$

The Lorentz Force

Let a particle with charge q and velocity $\vec{v}$ move through a *magnetic field* $\vec{B}$. The **Lorentz force** on the charged particle is

$$\vec{F}_{\mathrm{L}} = q\vec{v} \times \vec{B} \tag{4}$$

As a helpful memory tool, we have the right-hand rule to remember the direction of the cross-product. **The units of the magnetic field are the Telsa**, after Nikola Tesla. We also have the Gauss which is 10^{-4} Tesla.

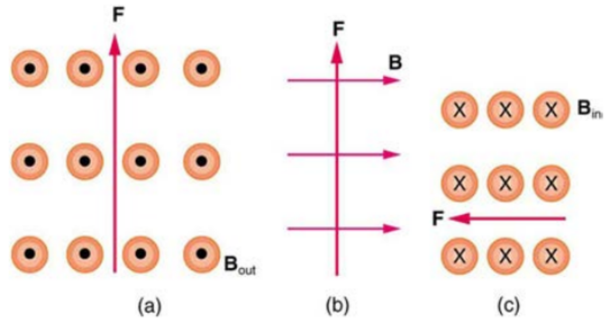


Figure 2: Three different magnetic field and charge scenarios. The vector $\vec{F}$ is the direction of the Lorentz force, and the magnetic field is uniform. A dot indicates that the magnetic field is coming out of the page, and an x indicates that the field is going into the page.

In which of the diagrams is a positively charged particle moving to the left?

- A: A
- B: B
- C: C
- D: Double WAT

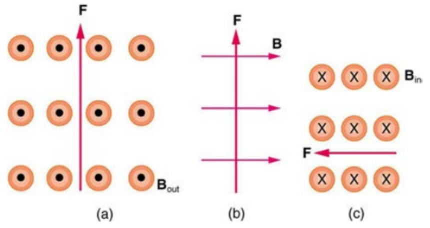


Figure 3: Three different magnetic field and charge scenarios.

In which of the diagrams is a positively charged particle moving upwards?

- A: A
- B: B
- C: C
- D: Double WAT

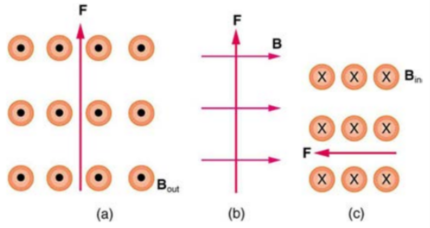


Figure 4: Three different magnetic field and charge scenarios.

Magnets and magnetic fields

In which of the diagrams is a negatively charged particle moving into the page?

- A: A
- B: B
- C: C
- D: Double WAT

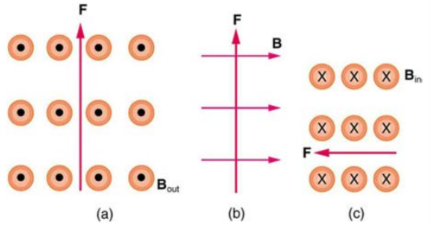


Figure 5: Three different magnetic field and charge scenarios.

In which of the diagrams is a negatively charged particle moving to the right?

- A: A
- B: B
- C: C
- D: Double WAT

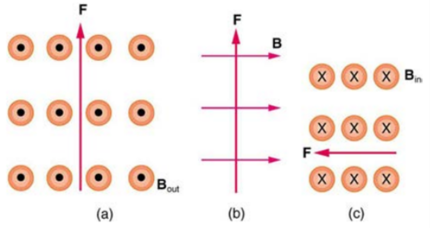


Figure 6: Three different magnetic field and charge scenarios.

A theorem for the magnitude of the cross-product: Let $\vec{a}$ and $\vec{b}$ be vectors and θ be the angle between them. The magnitude of the cross-product is

$$|\vec{a} \times \vec{b}| = ab \sin\theta \tag{5}$$

Thus, the magnitude of the Lorentz force is

$$F_{\mathrm{L}} = qvB \sin\theta \tag{6}$$

The angle θ is between the velocity and the magnetic field.

Magnets and magnetic fields

Suppose a positively charged particle q moves with an initial velocity $\vec{v} = v\hat{i}$, and there is a uniform magnetic field $\vec{B} = -B\hat{k}$. At this moment, what is the direction of the force on q?

- A: $\hat{i}$
- B: $\hat{j}$
- C: $\hat{k}$
- D: $-\hat{k}$

The force on the positively charged particle q in the $\hat{k}$-direction eventually causes the velocity to be $\vec{v} = v\hat{j}$. The uniform magnetic field is still $\vec{B} = -B\hat{k}$. At this moment, what is the direction of the force on q?

- A: $\hat{i}$
- B: $\hat{j}$
- C: $\hat{k}$
- D: $-\hat{i}$

The charge experiences uniform circular motion. At each moment, the acceleration is *perpendicular* to the direction.

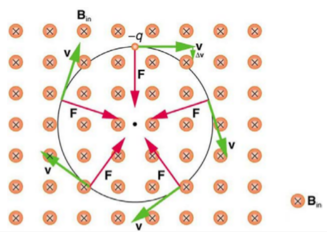


Figure 7: The situation is depicted here with $-q$ (changes direction).

Recall centripetal acceleration, in two equations:

$$F_{\mathrm{C}} = \frac{mv^2}{r} = mr\omega^2 \tag{7}$$

Magnets and magnetic fields

Recall centripetal acceleration, in two equations:

$$F_{\mathrm{C}} = \frac{mv^2}{r} = mr\omega^2 \tag{8}$$

Assume the Lorentz force remains perpendicular to velocity, and set the centripetal force equal to Lorentz force magnetiude:

$$\frac{mv^2}{r} = qvB \tag{9}$$

$$qB = \frac{mv}{r} \tag{10}$$

$$r = \frac{mv}{qB} \tag{11}$$

Thus, the radius of curvature is connected to ratio of mass, charge, velocity, and field strength. *What is a scientific application of this?*

Magnetic Applications I: Mass Spectrometry

Magnetic Applications I: Mass Spectrometry

Mass spectrometry. A simplified picture of mass spectrometry involves measuring the radius of curvature of ions in a B-field.

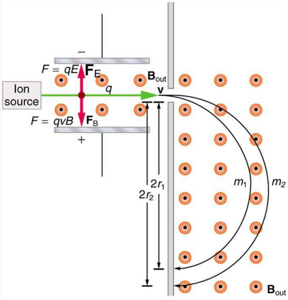


Figure 8: (a) Within the ion source, molecules are turned into an ionized gas, and accelerated through a capacitor. (b) The capacitor creates a uniform E-field, and a coil of current surrounds the capacitor to create a uniform B-field. The E and B-fields are perpendicular. (c) Ions leave the *velocity selector* into an area with just the B-field.

Magnetic Applications I: Mass Spectrometry

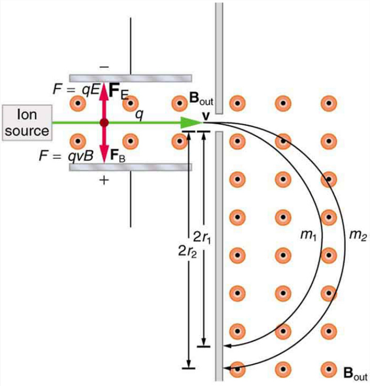


Figure 9: A simplified picture of a mass spectrometer.

1. Show that if $v = E/B$, the velocity in the *velocity selector* is constant.
2. If $E = 100$ V/m, and $B = 50$ gauss (5 mT), what velocity is required from the ion source?
3. If q is the equivalent of two electrons ($2e$), and m is 100 amu[a], what is the radius observed after the velocity selector?

[a] 1 amu = 1.66×10^{-27} kg, and $1e$ = 1.67×10^{-19} C.

Magnetic Applications I: Mass Spectrometry

Within $\approx 1\,\%$, what result did you obtain?

- A: 0.02 m
- B: 0.2 m
- C: 2.0 m
- D: 20 m

If all other variables remained the same, what would the radius of curvature be if *m* decreased to 50 amu?

- A: 10 m
- B: 1.0 m
- C: 0.1 m
- D: 0.01 m

(It is not necessary to repeat the calculation. Treat this as a scaling problem.)

Magnetic Applications I: Mass Spectrometry

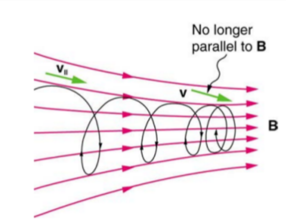
Notes on the Lorentz force:

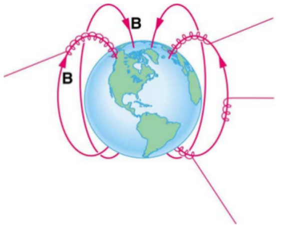
1. Magnetic fields do no work.
   - Work is defined as
   $$W = \vec{F} \cdot \vec{x} \tag{12}$$
   - Insert the Lorentz force for $\vec{F}$:
   $$W = q\vec{v} \times \vec{B} \cdot \vec{x} \tag{13}$$
   - $\vec{v} \times \vec{B}$ is perpendicular to $\vec{x}$, which is parallel to $\vec{v}$.
   - $W = 0$, because the dot-product of perpendicular vectors is zero.
2. The radio of E to B-fields is a *velocity*, $v = E/B$? What happens for a moving observer? We will postpone this discussion until we cover *inductors*.

Figure 10: The aurora borealis, or northern lights.

Magnets and magnetic fields

A cool talk on the aurora borealis (*time permitting*):
`https://youtu.be/czMh3BnHFHQ`





One un-explained piece: what does it mean for the electrons and protons to *high-five* the neutral oxygen and nitrogen atoms?

Force and Torque on a Current Carrying Conductor

Introduction to magnetic forces on current-carrying conductors:

`https://youtu.be/5fqwJyt4Lus`

Force and Torque on a Current Carrying Conductor

The Lorentz force also effects currents in conductors.

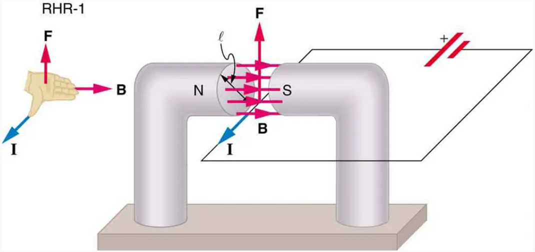


Figure 11: A B-field exerts a force on a wire carrying current.

$$F_q = q v_\mathrm{d} B \sin\theta \quad (14)$$
$$F_\mathrm{tot} = N q v_\mathrm{d} B \sin\theta \quad (15)$$
$$N = nV \quad (16)$$
$$N = nAL \quad (17)$$
$$F_\mathrm{tot} = (nqAv_\mathrm{d})\, LB \sin\theta \quad (18)$$
$$F_\mathrm{tot} = ILB\sin\theta \quad (19)$$
$$F_\mathrm{tot} = I\vec{L}\times\vec{B} \quad (20)$$

What is number density? Number density converts a volume to the number of objects in the volume:

$$\boxed{N = nV} \tag{21}$$

Suppose the Milky Way galaxy is a disc of diameter 100,000 light-years and height of 1,000 light years. We think there are about 10^{11} stars in the Milky Way. What is the number density of stars in the galaxy?

- A: 10 stars per light year
- B: 1 star per light year
- C: 0.1 stars per light year
- D: 0.01 stars per light year

Figure 12: An artist's conception of the diameter of the Milky Way.

Force and Torque on a Current Carrying Conductor

Why does $nqAv_\mathrm{d}$ equal current? Consider the geometry of charges flowing through a conductor:

$$I = \frac{\Delta Q}{\Delta t} \tag{22}$$

$$\Delta Q = Nq = nVq \tag{23}$$

$$\Delta Q = nALq \tag{24}$$

$$L = v_\mathrm{d}\Delta t \tag{25}$$

$$I = \frac{nALqv_\mathrm{d}}{L} \tag{26}$$

$$\boxed{I = nALqv_\mathrm{d}} \tag{27}$$

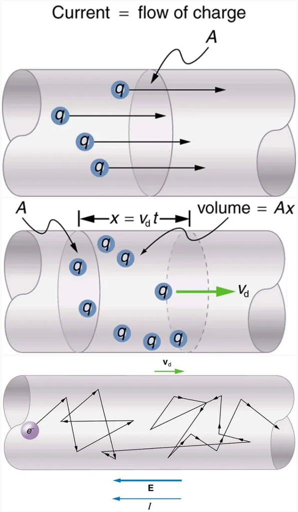


Figure 13: Simple picture of current.

Magnetic Applications II: Nuclear Fusion

Magnetic containment and *tokamaks*.

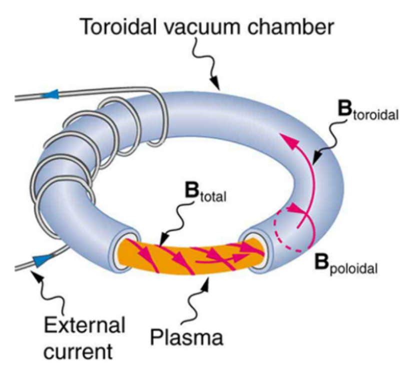


Figure 14: The tokamak contains high-energy plasma, e.g. a charged gas of electrons and protons.

Conclusion

Unit 3 Summary

1. **Magnetostatics I: Chapters 22.1 - 22.4**
   1.1 Magnets, ferromagnetic and electromagnetic
   1.2 Magnetic fields and field lines, force on moving charge
   1.3 Magnetic application: mass spectrometry
2. **Magnetostatics II: Chapters 22.7 - 22.9**
   2.1 Forces and torques on conductors with current
   2.2 Ampère's Law: magnetic fields are created by currents
   2.3 Magnetic application: fusion reactors