Algebra-Based Physics-2: Electricity, Magnetism, and Modern Physics (PHYS135B-01): Unit 5

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Nuclear physics in medicine: Introduction to Nuclear Isotopes and Radioactive Decay

Introduction to Nuclear Isotopes and Radioactive Decay

One example of *modern physics* deals with radioactivity. Radioactivity refers to the emission and detection of particles or radiation from **radioactive isotopes.** THese are unstable nuclei that release energy in the form of radiation.

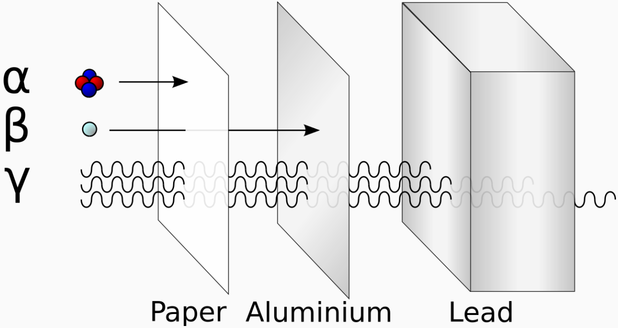


Figure 1: There are three basic types of radioactive decay: alpha particles, beta particles, and gamma particles.

Type	Rest Mass	Charge
α	3.727 GeV/c^2	+2
β	0.5111 MeV/c^2	$\pm$1
γ	0	0

Table 1: Radiation from nuclear decay is comprised of three particle types.

Let c be the speed of light, and let E be the energy of a particle at rest. Equation for the *rest mass* of a sub-atomic or radioactive particle:

$$E = mc^2 \quad (1)$$

The probability that **radioactivite particles** penetrate biological tissue or matter depends on *charge* and *energy*. One form of energy is the *rest mass*, given by $E = mc^2$.

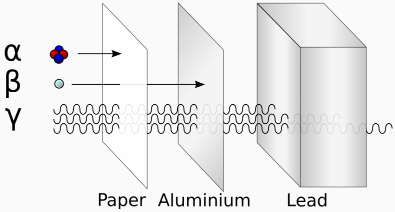


Figure 2: There are three basic types of radioactive decay.

Introduction to Nuclear Isotopes and Radioactive Decay

Type	Cross-section (b)	Energy (MeV)
α	$\approx 10^6$	≈ 4
β	≈ 1	≈ 10
γ	≈ 0.2	≈ 1

Table 2: The **barn** is a unit of area, 10^{-24} cm^2.

Let $I(z)$ be the intensity of a beam of particles traveling a distance z through matter. Let n be the number density of atoms or scatterers in the matter. Let I_0 be the original intensity. The total cross-section σ_{tot} of the interactions relates these:

$$\boxed{I(z) = I_0 e^{-\sigma_{\text{tot}} nz}} \tag{2}$$

The probability that **radioactivite particles** penetrate biological tissue or matter depends on the *cross-section.*

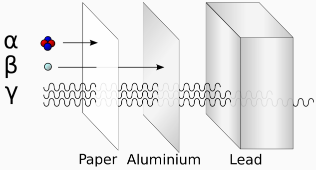


Figure 3: There are three basic types of radioactive decay.

$$I(z) = I_0 e^{-\sigma_{\mathrm{tot}} n z} \tag{3}$$

Suppose a beam of radioactive decay products with total cross-section σ_{tot} is incident on a block of metal with number density n. Show that half the intensity is gone at a distance $z_{1/2}$ into the metal, where

$$z_{1/2} = \frac{\ln 2}{\sigma n} \tag{4}$$

Materials with higher density tend to block more radioactive decay products.

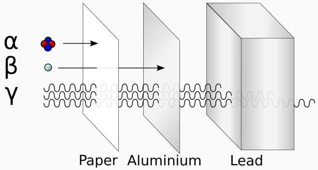


Figure 4: There are three basic types of radioactive decay.

$$z_{1/2} = \frac{\ln 2}{\sigma n} \tag{5}$$

For lead, $n = 3.3 \times 10^{22}$ cm^{-3}, assuming 1 electron ready to scatter a γ-ray per nucleus. If $\sigma = 1$ barn, 10^{-24} cm^2, how far in **lead** will the γ-rays travel before half of them are gone?

- A: 0.021 cm
- B: 0.21 cm
- C: 2.1 cm
- D: 21 cm

Materials with higher density tend to block more radioactive decay products.

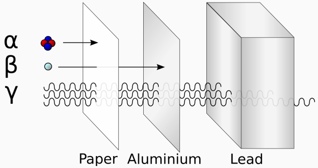


Figure 5: The cross-section is related to how far a stream of particles will penetrate into a substance.

Knowing that $z_{1/2}$ is inversely proportional to the cross-section, what would the result have been if the cross section was 1 Mb instead of 1 b?

- A: 2.1×10^{-5} m
- B: 2.1×10^{-3} cm
- C: 2.1×10^{-5} cm
- D: 2.1×10^{-4} cm

Materials with higher density tend to block more radioactive decay products.

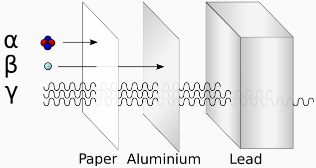


Figure 6: Particles with smaller cross-sections penetrate much deeper into materials.

The half-life of a radioactive substance determines *how long* it takes for the number of radioisotopes to reduce by half.

$$N(t) = N_0 e^{-\lambda t} \tag{6}$$

Show that the **half-life** is

$$t_{1/2} = \frac{\ln 2}{\lambda} \tag{7}$$

The half-life of Carbon-14 is 5730 years, and Carbon 12 is the non-radioactive isotope.

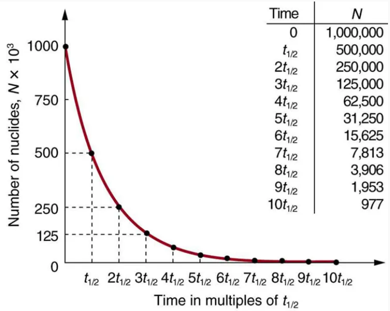


Figure 7: Radioactive substances emit radiation, but have a finite amount of particles that can be radiated.

Group exercise: Note that if we know the half-life value, $t_{1/2}$, we can get λ from $\ln 2/\lambda$. Calculate the age of the Shroud of Turin given that the amount of Carbon-14 found in it is 92% of that in living tissue.

1. Determine λ.
2. Note that $N/N_0 = 0.92$.
3. Solve for t.

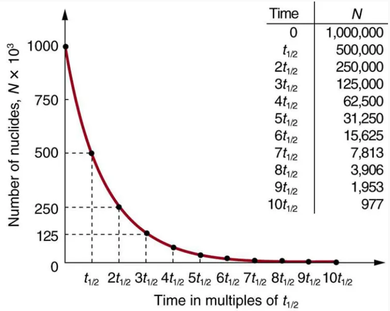


Figure 8: Given a finite amount of radioactive isotopes in a substance, and that decays are equally likely to occur in any given Δt, we can demonstrate exponential decay.

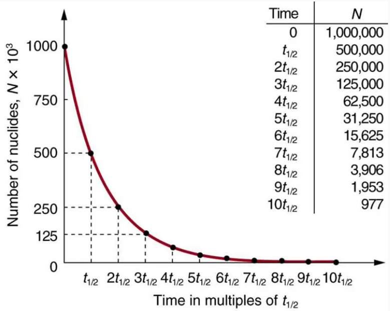
The decay rate can be derived from the $N(t)$ function:

$$R = \frac{\Delta N}{\Delta t} \tag{8}$$

$$R = -\lambda N_0 e^{-\lambda t} \tag{9}$$

$$R = -\left(\frac{\ln 2}{t_{1/2}}\right) N(t) \tag{10}$$

Intuitively, the *decays per second* are proportional to the number of remaining radioactive isotopes, N, and inversely proportional to the half-life. The SI unit of radioactive decay is the **becquerel**, or 1 decay/second.



Time	N
0	1,000,000
$t_{1/2}$	500,000
$2t_{1/2}$	250,000
$3t_{1/2}$	125,000
$4t_{1/2}$	62,500
$5t_{1/2}$	31,250
$6t_{1/2}$	15,625
$7t_{1/2}$	7,813
$8t_{1/2}$	3,906
$9t_{1/2}$	1,953
$10t_{1/2}$	977

Figure 9: It takes about 10 half-lives to decay 10^6 isotopes.

Nuclear physics in medicine: Diagnostics and medical imaging

Nuclear physics in medicine: Diagnostics and medical imaging

Nuclear physics, encompassing radioactive isotopes and radiation, is used in medicine.

Consider the *radioimmunoassay* (RIA) technique.

1. Irradiate a sample of antigen with a radioactive isotope so that the nuclei in the substance become radioactive.
2. Mix with a sample from a patient containing unknown number of antigens.
3. Introduce antibodies that bind to the antigen.
4. Separate bound and unbound antigens.
5. Measure decay rate of unbound antigens.
6. Work out the answer.

Nuclear physics in medicine: Diagnostics and medical imaging

Nuclear physics, encompassing radioactive isotopes and radiation, is used in medicine.

Consider an analogy, to measure the number of fish in a pond:

1. Catch, tag, and release n fish, in a pond with unknown number of fish N.
2. Return to the pond, and catch m fish.
3. Count the *untagged fish*, m_u. ($m_u + m_t = m$).
4. Assert that the uncaught fraction is a constant, $f = m_u/m = (N - n)/N$
5. Work out the answer:

$$N = \frac{n}{1 - f} \tag{11}$$

Nuclear physics in medicine: Diagnostics and medical imaging

Nuclear physics, encompassing radioactive isotopes and radiation, is used in medicine.

$$N = \frac{n}{1 - f} \tag{12}$$

If we tag $n = 67$ fish, and our untagged fraction is $f = 0.33$, how many fish are in the pond?

- A: 10
- B: 67
- C: 100
- D: 133

How do we measure **radioactively tagged** molecules? There are several useful isotopes (see text). One is Technetium-99, a metastable isotope with a 6 hour half-life that emits a 0.142 MeV gamma ray.

- The γ rays penetrate Pb, but are eventually stopped.
- A Pb collimator accepts γ rays from the direction of the source.
- The γ rays cause an optical flash in the scintillators.
- The optical flash is amplified by the photomultipliers.

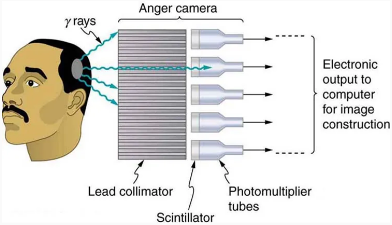


Figure 10: An gamma camera, or Anger camera. The γ rays form a high contrast 2D image.

Nuclear physics in medicine: Diagnostics and medical imaging

How do we measure **radioactively tagged** molecules? One useful isotope is Technetium-99, a metastable isotope that emits γ rays.

- The SPET system provides 3D imaging using radioactive isotopes.
- The *spatial resolution* is about 1 cm, but the contrast is high.
- Technetium is convenient, but it is not an element found in the body naturally.

Spatial resolution: the extent to which objects can be distinguished in an image.

Figure 11: A single-photon emission computed tomography (SPET) system forms a 3D image of tissue tagged with radioactive isotopes.

How do we improve spatial resolution? If we can use a β^+ emitting isotope, then the corresponding e^+ will annihilate with an ambient e^-.

- The reaction is $e^- + e^+ \to \gamma + \gamma$.
- To *conserve momentum*, the γ rays are emitted back to back. Knowing this boosts resolution.
- The radioactive isotopes have to be β^+ emitters: C, N, O, and F are common.
- Natural elements bind into substances in the body.

The **spatial resolution** is improved to ≈ 0.5 cm.

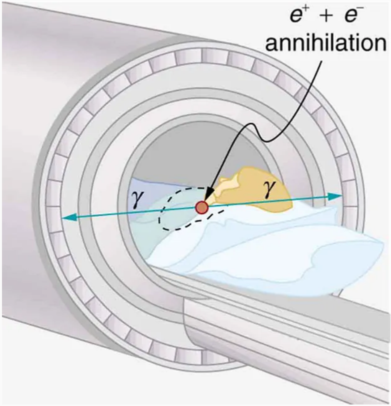


Figure 12: A positron emission tomography (PET) system.

Two γ rays, each with energy 1 MeV, have momenta vectors $\vec{p}_1 = 0\hat{i} + 0\hat{j} + p_{1,z}\hat{k}$ and $\vec{p}_2 = 0\hat{i} + 0\hat{j} + p_{2,z}\hat{k}$. If $p_{1,z} = -p_{2,z}$, and $p_{1,z} = 1$ MeV, what is the total momentum?

- A: $1\hat{z}$ MeV
- B: $-1\hat{z}$ MeV
- C: $0\hat{z}$ MeV
- D: $2\hat{z}$ MeV

Photons have momentum.

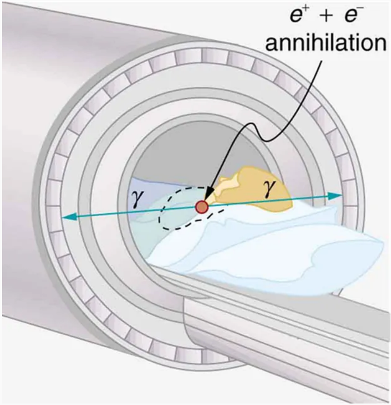


Figure 13: A positron emission tomography (PET) system.

Photons have momentum

Let h be **Planck's constant**. The momentum of a photon with wavelength λ is

$$p = \frac{h}{\lambda} \tag{13}$$

Planck's constant: 6.626×10^{-34} J s, or 4.136×10^{-15} eV s.

Why did we specify γ ray momentum in units of energy, MeV? We set $c = 1$ when dealing with speeds comparable to c. From Eq. 13:

$$[p] = \frac{[J][s]}{[m]} \tag{14}$$

$$c = 1, \quad [m] = [s] \tag{15}$$

$$[p] = [J] \tag{16}$$

What is the momentum of a photon in the beam of a standard HeNe red laser pointer? The wavelength is 633 nm.

- A: 1.5×10^{-7} eV s m^{-1}
- B: 2.5×10^{-8} eV s m^{-1}
- C: 6.5×10^{-9} eV s m^{-1}
- D: 6.5×10^{-9} eV

How many such photons would combine to have a total momentum of 10^{-2} kg m s^{-1}? Recall that 1 eV is 1.6×10^{-19} J.

- A: about 10^{24}
- B: about 10^{25}
- C: about 10^{26}
- D: about 10^{23}

In comparison to the wavelength of the photons in a HeNe laser, a typical γ ray wavelength is 10^5 times smaller. What is, therefore, the momentum of a typical γ ray?

- A: 1.5×10^{-2} eV s m^{-1}
- B: 2.5×10^{-3} eV s m^{-1}
- C: 6.5×10^{-4} eV s m^{-1}
- D: 6.5×10^{-4} eV

How many such photons would combine to have a total momentum of 10^{-2} kg m s^{-1}? Recall that 1 eV is 1.6×10^{-19} J.

- A: about 10^{19}
- B: about 10^{20}
- C: about 10^{21}
- D: about 10^{18}

In a typical laser flux of 10^{15} photons/second, it would take about 1 day for the γ rays to deliver 0.01 kg m s^{-1}. **Does radiation have an effect on health?**

Nuclear physics in medicine: Biological effects of ionizing radiation

Ionizing radiation affects molecules within cells, particularly DNA molecules.

Ionizing radiation is any radiation from (for example) radioactive isotopes that deposits enough energy in matter to free bound electrons from atoms.

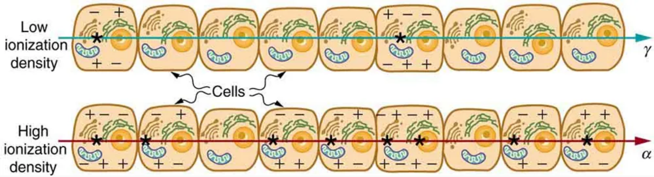


Figure 14: Different classes of ionizing radiation affect cell tissue differently.

Ionizing radiation affects DNA molecules. The energy required to break a hydrogen bond is 21 kJ/mole (in H_2O). This is < 1 eV per atom.

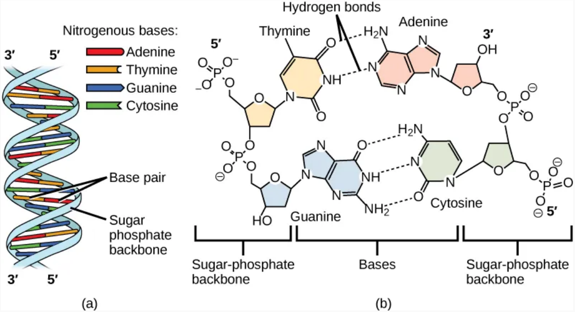


Figure 15: DNA is a molecule with double-helix structure, a sugar-phosphate backbone, and nucleic acid base-pairs bound with hydrogen bonds.

Nuclear medicine: Biological effects of ionizing radiation

Ionizing radiation affects DNA molecules. The energy required to break a hydrogen bond is 21 kJ/mole (in H_2O). This is < 1 eV per atom.

1. DNA is *self-healing*, containing code that checks for errors and code for repair.
2. A typical damage rate is ≈ 1 million errors per cell per day. The response rate depends on cell type and age.
3. Three outcomes after excess damage:
    - Senescence: a state of irreversible dormancy
    - Pre-programmed cell death
    - Unregulated cell division leading to tumors and cancers
4. Cancer is unregulated cell division, which can be both caused and stopped by radiation.

Nuclear medicine: Biological effects of ionizing radiation

Unit	Definition	Purpose
1 rad	0.01 J/kg	Quantifies deposited ionization energy
1 Gray (Gy)	100 rad	Quantifies deposited ionization energy
RBE[1]	Varies by radiation	Quantifies ionization concentration
rem[2]	rad $\times$ RBE	Quantifies health effects of radiation
1 Sv (Sievert)	Gy $\times$ RBE, 100 rem	Quantifies health effects of radiation

Table 3: Common units involved in assessing ionizing radiation in the human body.

[1] Relative biological effectiveness
[2] roentgen equivalent man

Nuclear medicine: Biological effects of ionizing radiation

Type and energy of radiation	RBE[1]
X-rays	1
γ rays	1
β rays greater than 32 keV	1
βrays less than 32 keV	1.7
Neutrons, thermal to slow (<20 keV)	2–5
Neutrons, fast (1–10 MeV)	10 (body), 32 (eyes)
Protons (1–10 MeV)	10 (body), 32 (eyes)
α rays from radioactive decay	10–20
Heavy ions from accelerators	10–20

Figure 16: The RBE factors for different classes of radiation. Note that the most common radiation classes have RBEs of 1, so rem = rad, and 1 Sy = 1 Gy. The RBE for α radiation is significantly higher due to shorter, more concentrated range in tissue.

Quantity	SI unit name	Definition	Former unit	Conversion
Activity	Becquerel (bq)	decay/s	Curie (Ci)	$1 \text{ Bq} = 2.7 \times 10^{-11} \text{ Ci}$
Absorbed dose	Gray (Gy)	1 J/kg	rad	$\text{Gy} = 100 \text{ rad}$
Dose Equivalent	Sievert (Sv)	1 J/kg × RBE	rem	$\text{Sv} = 100 \text{ rem}$

Figure 17: The distinction between activity, and absorbed dose and dose equivalent.

Nuclear medicine: Biological effects of ionizing radiation

Dose in Sv 2	Effect
0–0.10	No observable effect.
0.1 – 1	Slight to moderate decrease in white blood cell counts.
0.5	Temporary sterility; 0.35 for women, 0.50 for men.
1 – 2	Significant reduction in blood cell counts, brief nausea and vomiting. Rarely fatal.
2 – 5	Nausea, vomiting, hair loss, severe blood damage, hemorrhage, fatalities.
4.5	LD50/32. Lethal to 50% of the population within 32 days after exposure if not treated.
5 – 20	Worst effects due to malfunction of small intestine and blood systems. Limited survival.
>20	Fatal within hours due to collapse of central nervous system.

Figure 18: As the dose equivalent increases (measured in Sv), the health effects become qualitatively more dangerous.

What is the dose (in rad) if 100 mJ of total energy is deposited in the body of a 50 kg adult male?

- A: 0.2 rad
- B: 2.0 rad
- C: 20.0 rad
- D: 200.0 rad

If the above dose was instead concentrated in 1.0 kg of brain tissue, what would be the dose in rad?

- A: 0.1 rad
- B: 1.0 rad
- C: 10 rad
- D: 100 rad

If the RBE is 1 because we are dealing with x-rays, is the **equivalent dose** fatal? Answer: rem = rad, because RBE = 1, then convert to Sv and use Tab, 18.

What is the equivalent dose (in Sv) if 500 mJ of total energy is deposited by α radiation (RBE of 20) in the 1 kg lungs of an adult male?

- A: 0.1 Sv
- B: 1.0 Sv
- C: 10.0 Sv
- D: 50.0 Sv

What can be said about the probability of survival?

- A: $>$50 percent.
- B: $<$50 percent.
- C: It is unknown.
- D: It is 100 percent.

The person is at risk [for cancer] for at least 30 years after the latency period ... the overall risk of a radiation-induced cancer death per year per rem of exposure is about 10 in a million ... $10/10^6$ *rem*$^{-1}$ *yr*$^{-1}$.

If a person receives a dose of 1 rem, his risk each year of dying from radiation-induced cancer is 10 in a million and that risk continues for about 30 years. The lifetime risk is thus 300 in a million, or 0.03 percent. Since about 20 percent of all worldwide deaths are from cancer, the increase due to a 1 rem exposure is impossible to detect demographically. But 100 rem (1 Sv), which was the dose received by the average Hiroshima and Nagasaki survivor, causes a 3 percent risk, which can be observed in the presence of a 20 percent normal or natural incidence rate.

Suppose a woman receives an x-ray, and there is an incidental exposure of 25 rem to her breast tissue. What is her increased lifetime risk of breast cancer due to the exposure?

- A: 75 percent
- B: 7.5 percent
- C: 0.75 percent
- D: 25 percent

If we receive 3.5 mrem (milli-rem) per plane flight from cosmic rays, how many flights can we take before we reach 1 Sv?

- A: 17
- B: 33
- C: 63
- D: 286

Nuclear medicine: Biological effects of ionizing radiation

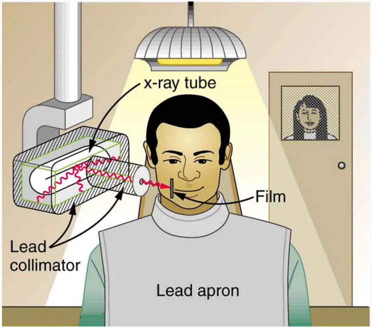


Figure 19: Suppose this dental x-ray source has an **activity** of 1 Curie, or 37×10^9 Bq. Recall that 1 Bq is 1 particle per second in the context of radiation.

Group exercise: Assume the energy of each x-ray is 70 keV, and the exposure lasts 3.5 seconds. Determine the effective dose in mrem to the patient, if the affected tissue has a mass of 1.5 kg.

Nuclear physics in medicine: Therapeutic uses of ionizing radiation

Nuclear physics in medicine: Food irradiation

Conclusion

Unit 5 Summary

1. Electromagnetic waves - Chapters 24.1 - 24.4
   - Maxwell's Equations
   - Electromagnetic wave production
   - Electromagnetic spectrum and energy
2. Geometric optics - Chapters 25.1 - 25.3, 25.6
   - Ray-tracing and reflection
   - Refraction
   - Lens optics

Unit 5 Summary

1. Wave optics - Chapters 27.1 - 27.3
    - Wave interference
    - Wave diffraction
    - Double slit experiments
2. Nuclear physics in medicine - 32.1 - 32.4
    - Diagnostics and medical imaging
    - Biological effects of ionizing radiation
    - Therapeutic uses of ionizing radiation
    - Food irradiation