Algebra-Based Physics: Electricity, Magnetism, and Modern Physics (PHYS135B): Unit 0

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November 3, 2023

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Summary

Unit 0 Summary

Reading: Chapters 3.1 - 3.3, 18.1 - 18.5, 19.1 - 19.3

1. Estimation/Approximation
2. Coordinates and Vectors
3. Review of concepts from Newtonian mechanics
    - Kinematics and Newton's Laws
    - Work-energy theorem, energy conservation
    - Momentum, conservation of momentum
4. Electrostatics I: charges and fields
5. Electrostatics II: potential, and potential energy

Estimation/Approximation

Estimation/Approximation

In science and engineering, estimation is to obtain a quantity in the absence of precision, informed by rational constraints.

1. Define relevant unit scales: (mg, g, or kg), (m/s or km/hr)
2. Obtain complex quantities from simple ones
   - Obtain *areas* and *volumes* from *lengths*
   - Obtain *rates* from *numerators* and *denominators*
3. Taking advantage of scaling problems
   - Knowing *relationship* between variables
   - Using that *relationship* to obtain new information
4. Constrain the unknown with upper and lower limits

Unit scale: A generation is about one-third of a lifetime. Determine how many generations have passed since the year 0 AD[1].

- A: 10
- B: 20
- C: 60
- D: 100

[1] What is the appropriate scale here?

Unit scale: (a) What fraction of Earth's diameter[2] is the greatest ocean depth (11 km below sea level)? (b) The greatest mountain height (8.8 km above sea level)?

- A: 8.6×10^{-2}, 6.9×10^{-2}
- B: 8.6×10^{-3}, 6.9×10^{-3}
- C: 8.6×10^{-4}, 6.9×10^{-4}
- D: 8.6×10^{-5}, 6.9×10^{-3}

[2] The diameter of the Earth is 12,800 km.

Estimation/Approximation

Complex quantities: Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

- A: 1 per second (1 Hz)
- B: 30 per second (30 Hz)
- C: 60 per second (60 Hz)
- D: 100 per second (100 Hz)

Estimation/Approximation

Complex quantities: If a Whittier College athlete ran the 5k race at a track meet in 35 minutes, what was her average speed?

- A: 0.3 meters per second
- B: 3 meters per second
- C: 30 meters per second
- D: 300 meters per second

Complex quantities: Suppose you won the lottery and received \$1 billion USD. Because your life is dope, you stack that paper over the Whittier College soccer field. Each stack contains 100 bills, and each bill is worth \$100. If you evenly cover the field, how high is the money level?

- A: 0.5 inch
- B: 1 inch
- C: 2 inches
- D: 1 foot

Estimation/Approximation

Scaling problem: Supposed you have an ideal gas in a cylinder of fixed volume. If the pressure begins as 100 kPa, and you *double* the temperature of the gas, what is the new pressure?

- A: 100 kPa
- B: 50 kPa
- C: 10 kPa
- D: 200 kPa

Scaling problem: Supposed you have an ideal gas in a cylinder of fixed volume. If the pressure begins as 100 kPa, and you *halve* the temperature of the gas, what is the new pressure?

- A: 100 kPa
- B: 50 kPa
- C: 10 kPa
- D: 200 kPa

Estimation/Approximation

Upper/lower limits: How many undergraduate students are there at Whittier College[3]?

- A: 5,000
- B: 1,000
- C: 1,250
- D: 500

[3] What is the absolute lower limit, and what is the upper limit?

Estimation/Approximation

Upper/lower limits: What is the average yearly college tuition in the United States (before subtracting grants and scholarships)?

- A: \$5,000
- B: \$10,000
- C: \$25,000
- D: \$40,000

What information affects the upper and lower limits here?

Coordinates and Vectors

Physics requires mathematical objects to build equations that capture the behavior of nature. Two examples of such objects are scalar and vector quantities. Each type of object obeys similar but different rules.

1. Scalar quantities
    - mass: $m_1 + (m_2 + m_3) = (m_1 + m_2) + m_3$
    - speed: $v_1(v_2 + v_3) = v_1 v_2 + v_1 v_3$
    - charge: $q_1 \left(\frac{1}{q_1}\right) = 1$, $q_1(0) = 0$
2. Vector quantities
    - velocity: $\vec{v}_1 + (\vec{v}_2 + \vec{v}_3) = (\vec{v}_1 + \vec{v}_2) + \vec{v}_3$
    - tension: $\vec{t}_1 \cdot (\vec{t}_2 + \vec{t}_3) = \vec{t}_1 \cdot \vec{t}_2 + \vec{t}_1 \cdot \vec{t}_3$

Examples: break into components, adding two vectors.

A vector may be expressed as *a list of scalars*: $\vec{v} = (4, 2)$ (a vector with two *components*), $\vec{u} = (3, 4, 5)$ (three *components*). Now, we know how to add and subtract scalars. How do we add and subtract vectors?

What is
$(1, 3, 8)+$
$(0, 2, 1)$?
Answer: $(1, 5, 9)$

In other words, when adding vectors, we add them component by component. **Work several examples.**

How do we subtract vectors? In the same fashion:

What is
$(1, 3, 8)-$
$(0, 2, 1)$?
Answer: $(1, 1, 7)$

In other words, when subtracting vectors, we subtract them component by component. **Work several examples.**

How do we multiply vectors? In the same fashion, *for one kind of multiplication*:

What is
$(1, 3, 8) \cdot (0, 2, 1)$?
Answer: $1 \cdot 0 + 3 \cdot 2 + 8 \cdot 1 = 14$

This kind of multiplication is known as the dot-product. There is also the *cross-product*, which we will save for later. **Work several examples.**

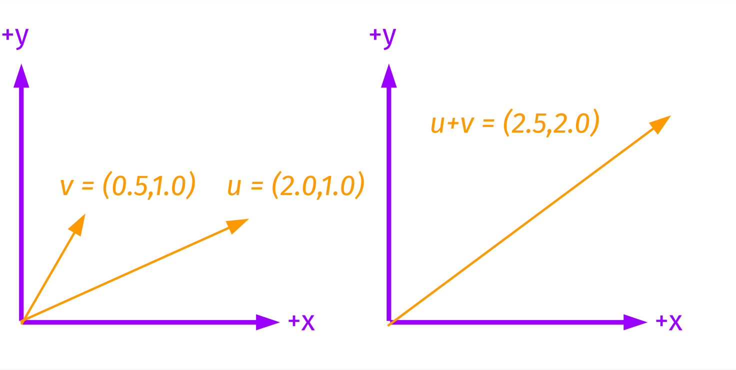
Coordinates and Vectors (Chapters 3.1 - 3.3)

The components of a vector may describe quantities in a coordinate system, such as *Cartesian coordinates* - after René Descartes. Vectors in the 3D Cartesian coordinate system (x,y,z) may be written in the following notation:

$$\boxed{\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}}$$

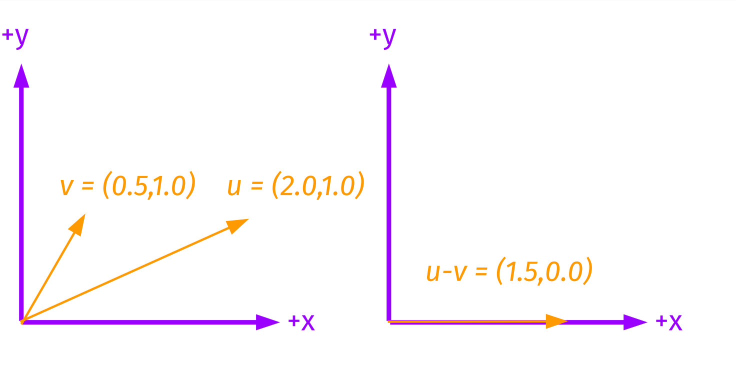
- a: The amount in the +x-direction, $\hat{i}$: a vector of length 1, in the +x-direction
- b: The amount in the +y-direction, $\hat{j}$: a vector of length 1, in the +y-direction
- c: The amount in the +z-direction, $\hat{k}$: a vector of length 1, in the +z-direction

Coordinates and Vectors (Chapters 3.1 - 3.3)



(a) (b)

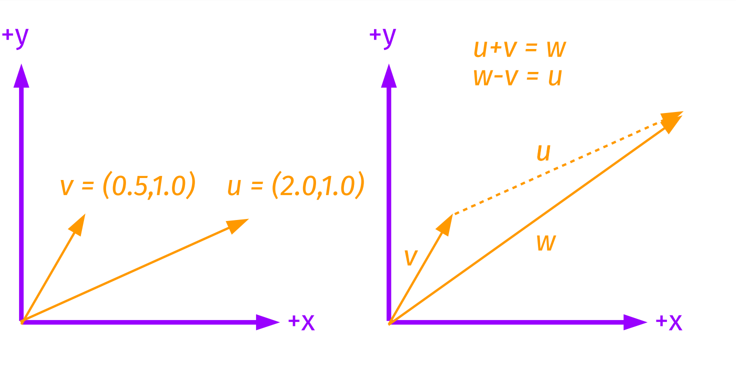
Figure 1: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} + \vec{v}$? Adding components: $\vec{u} + \vec{v} = 2.5\hat{i} + 2.0\hat{j}$.



(a) (b)

Figure 2: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} - \vec{v}$? Subtracting components: $\vec{u} - \vec{v} = 1.5\hat{i} + 0.0\hat{j}$.

Coordinates and Vectors (Chapters 3.1 - 3.3)



(a) (b)

Figure 3: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) To compute $\vec{w} - \vec{v}$, arrange the vectors to get a sense of the result, $\vec{u}$.

Suppose $\vec{x}_i = -3\hat{i} + 2\hat{j}$ km, and $\vec{x}_f = -3\hat{i} - 2\hat{j}$ km. What is the *displacement*?

- A: $4\hat{i}$ km
- B: $-4\hat{i}$ km
- C: $4\hat{j}$ km
- D: $-4\hat{j}$ km

Suppose $\vec{x}_i = 3\hat{i} - 2\hat{j}$ km, and $\vec{x}_f = 3\hat{i} - 2\hat{j}$ km. What is the *displacement*?

- A: 0 km
- B: $0\hat{i} + 0\hat{j}$ km
- C: $1\hat{i}$ km
- D: $1\hat{j}$ km

We define the *position* of an object as a vector locating it in a given coordinate system. The scalar *distance* is the norm of the position vector, that is, the distance to to the origin.

Now we can introduce the concept of displacement: a vector describing a movement of an object.

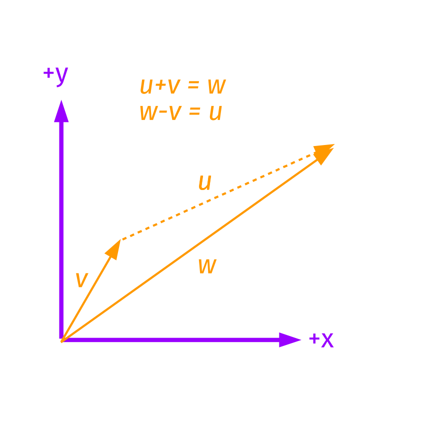


Figure 4: Suppose an object moves from position $\vec{v}$ to $\vec{w}$. In this case, the displacement is $\vec{u}$. **Thus, the final position is the initial position, plus the displacement.**

It follows that the *displacement* is zero if the initial and final positions are the same, but the *distance travelled* is not.

Suppose a jet fighter travelling at 800 km per hour banks such that it flies in a circle of radius 0.5 km. How long does it take to complete the circle? What is the distance traveled, and what is the displacement?

- A: 2π km, 28 seconds, 2π km
- B: π km, 14 seconds, π km
- C: π km, 28 seconds, π km
- D: π km, 14 seconds, 0 km

Average velocity is the ratio of the displacement to the elapsed time.

$$\boxed{\vec{v}_{\mathrm{avg}} = \frac{\Delta \vec{x}}{\Delta t}} \tag{1}$$

The *average speed* is the norm of the average velocity:

$$\boxed{v_{\mathrm{avg}} = \frac{|\Delta \vec{x}|}{\Delta t}} \tag{2}$$

If the motion is in one dimension, then the average speed is

$$v_{\mathrm{avg}} = \frac{x_{\mathrm{f}} - x_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} \tag{3}$$

$\vec{p} = 4\hat{i} + 2\hat{j}$. $\vec{q} = -4\hat{i} + 2\hat{j}$.
Compute $\vec{p} \cdot \vec{q}$.

- A: 12
- B: -12
- C: 4
- D: 8

$\vec{p} = -1\hat{i} + 6\hat{j}$. $\vec{q} = 3\hat{i} + 0.5\hat{j}$.
Compute $\vec{p} \cdot \vec{q}$.

- A: -1
- B: 1
- C: 0
- D: 3

Why was the last answer zero? Look at it graphically:

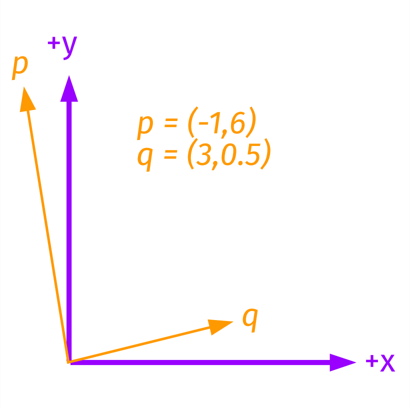


Figure 5: Two vectors $\vec{p}$ and $\vec{q}$ are *orthogonal* if $\vec{p} \cdot \vec{q} = 0$.

The *length* or *norm* of a vector $\vec{v} = a\hat{i} + b\hat{j}$ is $|\vec{v}| = \sqrt{a^2 + b^2}$.

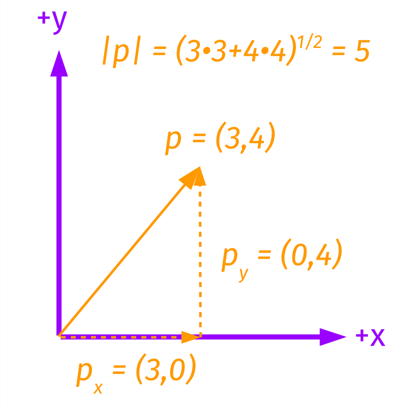


Figure 6: Computing the norm of a vector $\vec{p}$.

Notice that $\sqrt{\vec{p}\cdot\vec{p}} = |\vec{p}|$.
Let θ_p be the angle between $\vec{p}$ and the x-axis.
$p_x = \vec{p}\cdot\hat{i} = |\vec{p}|\cos(\theta_p)$.
$p_y = \vec{p}\cdot\hat{j} = |\vec{p}|\sin(\theta_p)$.

Theorem: The dot product of two vectors $\vec{p}$ and $\vec{q}$ is $|u||v|\cos(\theta)$, if θ is the angle between them.

Proof: $\vec{p}\cdot\vec{q} = p_x q_x + p_y q_y = |p||q|\cos\theta_p\cos\theta_q + |p||q|\sin\theta_q\sin\theta_q$
$= |p||q|(\cos\theta_p\cos\theta_q + \sin\theta_p\sin\theta_q) = |p||q|\cos(\theta_p - \theta_q)$
$= |p||q|\cos\theta$.

$$\boxed{\vec{p}\cdot\vec{q} = |p||q|\cos\theta}$$

An object moves at 2 m/s at $\theta = 60°$ with respect to the x-axis. What is the velocity of the object?

- A: $(1\hat{i} + 1\hat{j})$ m/s
- B: $(\sqrt{3}\hat{i} + 1\hat{j})$ m/s
- C: $(\sqrt{3}\hat{i} + \sqrt{3}\hat{j})$ m/s
- D: $(1\hat{i} + \sqrt{3}\hat{j})$ m/s

An object moves at 2 m/s at $\theta = 120°$ with respect to the x-axis. What is the velocity of the object?

- A: $(-1\hat{i} + \sqrt{3}\hat{j})$ m/s
- B: $(1\hat{i} - \sqrt{3}\hat{j})$ m/s
- C: $(-1\hat{i} + \sqrt{3}\hat{j})$ m/s
- D: $(-1\hat{i} - \sqrt{3}\hat{j})$ m/s

Is it possible to multiply vectors and scalars? Of course: $a_1\vec{p} = a_1 p_x \hat{i} + a_1 p_y \hat{j}$.

Also, multiplication properties still hold. For example: $(a_1 + a_2)\vec{p} = a_1\vec{p} + a_2\vec{p}$.

A spacecraft moves at 400 m/s, at an angle of 30 degrees with respect to the x-axis. If it fires two thrusters that boost the x-component and y-component of the velocity by 25% and 50%, respectively, what is the final velocity?

- A: $(433\hat{i} + 300\hat{j})$ m/s
- B: $(300\hat{i} + 433\hat{j})$ m/s
- C: 400 m/s
- D: $(400\hat{i} + 433\hat{j})$ m/s

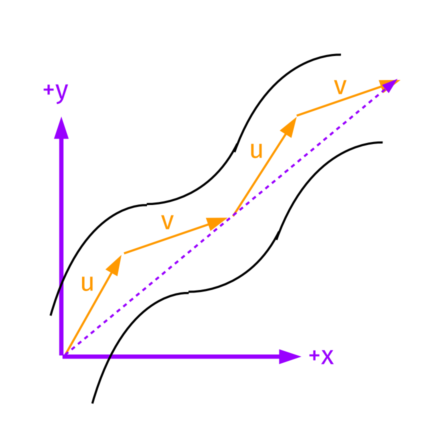


Figure 7: A Formula-1 driver keeps his car on the track by following a path approximated by the position vectors *u*, *v*, *u*, and *v*. The dashed arrow represents the total displacement.

If $\vec{u} = (20\hat{i} + 30\hat{j})$ m, and $\vec{v} = (30\hat{i} + 20\hat{j})$ m, what is the total displacement? If the elapsed time is 10 seconds, what is the magnitude of the average velocity?

- A: $(50\hat{i} + 50\hat{j})$ m, 14 m/s
- B: $(80\hat{i} + 100\hat{j})$ m, 10 m/s
- C: $(100\hat{i} + 100\hat{j})$ m, 14 m/s
- D: $(50\hat{i} + 150\hat{j})$ m, 10 m/s

PhET simulation about vector addition:
`https://phet.colorado.edu/en/simulation/vector-addition`

Kinematics and Newton’s Laws

Kinematics - A description of the motion of particles and systems
Dynamics - An explanation of the motion of particles and systems

What causes an object to move? **Forces**. Forces exist as a result of the **interactions** of objects or systems.

Evolution - A description of the change of biological species
Natural Selection - An explanation of change in biological species

What causes species to evolve? **Natural selection**. Natural selection exists because of selection pressures, numerous offspring, and variation among offspring.

Newton's First Law: A man slides a palette crate across a concrete floor of his shop. He exerts a force of 60.0 N, and the box has a constant velocity of 0.5 m/s. What force cancels his pushing force, and what is the value in Newtons?

- A: wind, 60.0 N
- B: friction: 60.0 N
- C: friction: -60.0 N
- D: weight: -60.0 N

Newton's Second Law: The crate has a mass of 50 kg, and encounters an area where there is no longer friction. If the pushing force is still 60 N, what is the acceleration?

- A: 1.0 m/s^2
- B: 0.8 m/s
- C: 1.2 m/s
- D: 1.2 $m/^2$

Kinematics: If the acceleration is 1.2 m/s^2, and the crate begins with a velocity of 1 m/s, what is the velocity after 5 seconds?

- A: 4 m/s
- B: 5 m/s
- C: 6 m/s
- D: 7 m/s

Newton's Second Law: Suppose there is no pushing force, but the crate moves at 5 m/s through an area with a frictional force that has a magnitude of 5 N. If the crate still weighs 50 kg, what is the acceleration?

- A: 0.2 m/s^2
- B: -0.1 m/s^2
- C: 1 m/s^2
- D: -2 m/s^2

Newton's Third Law: If a person hangs from a horizontal rope (with the ends tied to two walls), and the person has a weight $\vec{w} = -600N$, what is the total upward component of the tension in the rope?

- A: -600 N
- B: 60 N
- C: 600 N
- D: -60 N

Newton's Third Law: If a heavy truck and a light car collide, which exerts the larger force on the other?

- A: The heavy truck exerts a larger force on the car.
- B: The light car exerts a larger force on the heavy truck.
- C: They exert the same force on each other.
- D: Cannot determine.

Work-Energy Theorem and Conservation of Energy

Group board exercise: A firework of mass 1 kg is launched straight upwards. The gunpowder releases 500 J of energy. What is the velocity of the shell as it leaves the launcher? How high does it fly straight upwards?

Three useful concepts: 1) Work equation 2) Work-energy theorem 3) gravitational potential energy.

Work-energy theorem: How high in the air would a 0.1 kg rock go if it was launched straight upward by a spring with $k = 1000$ N/m, if the spring was compressed 0.1 m?

- A: 0.5 m
- B: 5 m
- C: 50 m
- D: 500 m

Note: the potential energy of a spring with spring constant k and displacement x is $U = \frac{1}{2}kx^2$.

Work-energy theorem: How high would it go if the spring was compressed 0.2 m?

- A: 100 m
- B: 200 m
- C: 500 m
- D: 50 m

Note: think of this exercise as a scaling problem.

Momentum

Momentum

A ball with mass 0.1 kg moves at 1 m/s. It strikes a stationary ball with twice the mass and stops. The heavier ball moves with a velocity of

- A: 0.1 m/s
- B: 1 m/s
- C: 5 m/s
- D: 0.5 m/s

A ball with mass 0.1 kg moves at 1 m/s. It strikes a stationary ball with the same mass and they stick together. What is the final velocity of the object?

- A: 0.1 m/s
- B: 1 m/s
- C: 5 m/s
- D: 0.5 m/s

Momentum

If the mass of an object that is rotating around an origin with angular velocity ω decreases by a factor of 2, the new angular velocity will be:

- A: $-\omega$
- B: -3ω
- C: 2ω
- D: ω

Electrostatics I: Chapters 18.1 - 18.5

Electrostatics I - Beginnings

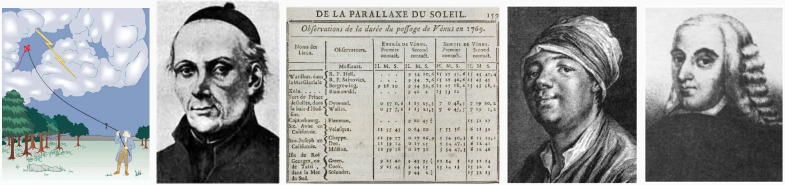


Figure 8: (a) Benjamin Franklin with kite and key, (b) Father José Antonio Alzate y Ramírez, (c) Table of astronomical data for 1769 Venus transit, (d) Jean-Baptiste Chappé d'Auteroche, (e) Joaquín Velázquez de León.

Who first understood the aurora borealis?
https://youtu.be/czMh3BnHFHQ?si=9rd7rpaaUd2Ef_XT

Unit 0 concept trailer:
https://youtu.be/SglzRDD1oNI

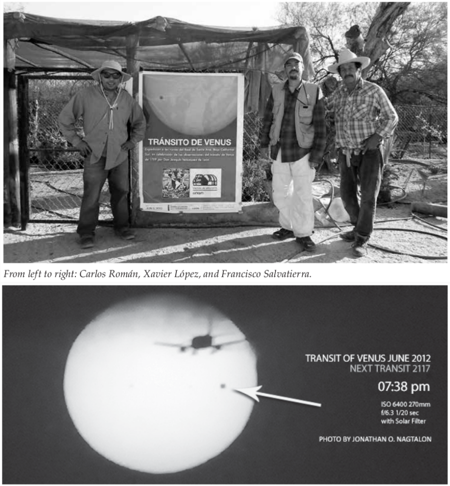


Figure 9: (Top) Carlos Román, Xavier López, and Fransisco Salvatierra. (Bottom) The 2012 Venus transit, from Baja California.

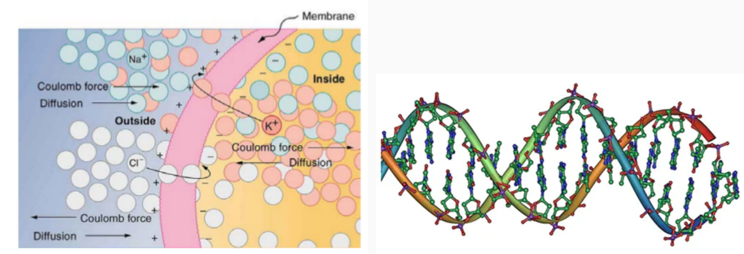


Figure 10: (Left) Nerve signals are caused by electric charge. (Right) The DNA molecule is held together by electrical forces.

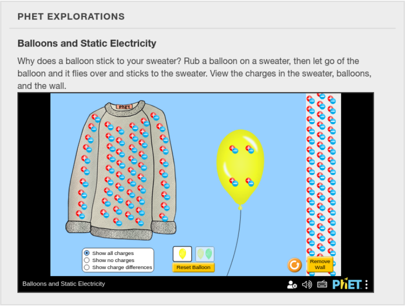


Figure 11: Convervation of charge: https://phet.colorado.edu/en/simulations/balloons-and-static-electricity.

Observations: charge is conserved, there are positive and negative charges, opposite charges attract, like charges repel. *Can you deduce these without seeing the charges?*

- **Conductors** - These can have net charge, and charge moves around freely. Consider *the wall* in the balloon example.
- **Insulators** - These can have net charge, but charge cannot move freely. Consider *the balloon* in the previous example.
- **Unit of charge** - 1 unit of charge is $q_{\mathrm{e}} = 1.60 \times 10^{-19}$ C
- **The Coulomb** - 1 C is $0.625 \times 10^{19} q_{\mathrm{e}}$
- **The Ampère (amp)** - 1 Coulomb is 1 amp of current for 1 second.

Activity: PhET Charges and Fields

At your tables, go to the following URL:

`https://phet.colorado.edu/en/simulation/charges-and-fields`

Click on the java app to get it running. Notice the following:

1. This is a 2D coordinate space, and you can activate the grid lines at right, by clicking *grid*.
2. Clicking *values* gives you the measurement scale.
3. Click *electric field*, or make sure it is activated.
4. Verify the length scale with the **ruler tool**, shaped like a tape measure. It can be dragged from the box at right.

Activity: PhET Charges and Fields

`https://phet.colorado.edu/en/simulation/charges-and-fields`

Click and drag a positive charge into the 2D coordinate system. This is analagous to charging an insulator.

1. Drag the yellow tool at the bottom into the space, and use it to measure the *electric field E*. Notice the units are in V/m and m. The electric field units V/m are equivalent to Newtons per Coulomb (N/C).
2. Copy to excel the field strength (E) versus distance (r). Use 25 cm distance increments, and record 15 data points in two columns.
3. In a third column, compute r^2.

Activity: PhET Charges and Fields

`https://phet.colorado.edu/en/simulation/charges-and-fields`

Click and drag a positive charge into the 2D coordinate system. This is analagous to charging an insulator.

1. Plot E vs. r^2. Do you observe a linear trend? What are some sources of error that contribute to the uncertainty in the slope?
2. Repeat this same exercise, but instead of measuring field strength versus *distance*, measure it in one location, versus *charge*. Take 15 data points in two columns and plot E versus q in Excel. What is the slope of the line? Notice the units of charge are nC.

Example data: See Moodle for sample data drawn from this PhET.

Coulomb's Law describes the force between charges.

Coulomb's Law

The electric force, or **Coulomb force**, between two electrically charged systems with charges q_1 and q_2 separated by a distance r is

$$\vec{F}_{\mathrm{C}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \tag{4}$$

In Eq. 4, $\hat{r} = \vec{r}/|\vec{r}|$, and $\epsilon_0 = 8.85418782 \times 10^{-12} N^{-1} m^{-2} C^2$, called the *perimittivity of free space.*

Coulomb Field

The electric field corresponding to Eq. 4, experienced by a charge q and generated by a charge Q is

$$\vec{E}_{\mathrm{C}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \tag{5}$$

In Eq. 5, r remains the separation between q and Q.

Thus we have: $\vec{F}_{\mathrm{C}} = q\vec{E}_{\mathrm{C}}$.

Suppose a charge $+q$ experiences the Coulomb field of another charge of $-q$, separated by a distance r. Which of the following is true?

- A: The charge $+q$ accelerates the $-q$ charge only.
- B: The charge $-q$ accelerates the $+q$ charge only.
- C: No charges move; the force on one is equal to the force on the other.
- D: Both charges move, and the force on one is equal to the force on the other.

Recall Newton's Third Law:

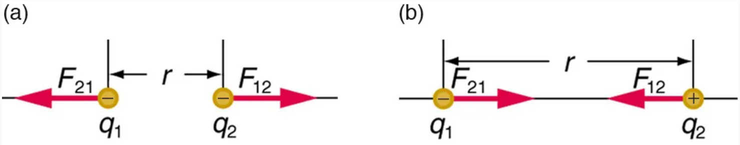


Figure 12: Newton's Third Law still applies to the electrostatic force.

What is the angle of the E-field at point (1,1) in Fig. 13 at right?

- A: 0 deg
- B: 45 deg
- C: 90 deg
- D: 135 deg

What is the fastest way to solve this problem?

- A: Guess
- B: Guess harder
- C: Lots of algebra
- D: Symmetry

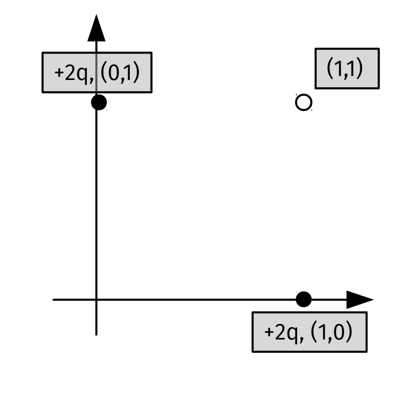


Figure 13: Two charges create a field for a hypothetical *test charge*.

Which of the following is true of the E-field at point (1,1) in Fig. 14 at right?

- A: The angle with respect to the x-axis is 45 degrees
- B: The angle with respect to the x-axis is greater than 45 degrees
- C: The angle with respect to the x-axis is less than 45 degrees
- D: The angle with respect to the x-axis is 90 degrees

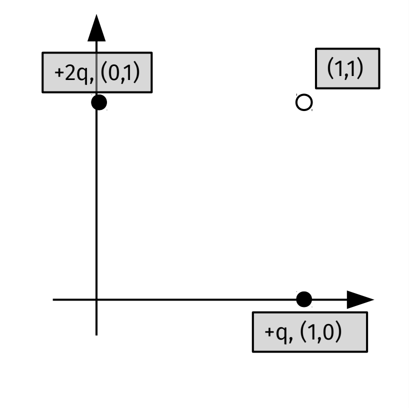


Figure 14: Two charges create a field for a hypothetical *test charge*.

The forces of N fixed charges on a test charge q create a net force, where the individual forces simply add like vectors. This is known as the **superposition principle**.

Superposition Principle

$$\vec{F}_{\mathrm{C,Net}} = \frac{1}{4\pi\epsilon_0} q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i = q\vec{E}_{\mathrm{C,Net}} \tag{6}$$

$$\vec{E}_{\mathrm{C,Net}} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i \tag{7}$$

Short PhET demonstration: create a circle of charges, and determine the field at the center.

The following problem is an example of solving for a field analytically, and *testing various limits*. Upon taking limits results are often simple and intuitive.

Two charges $+q$ are on the fixed in an insulator on the x-axis. Solve for the E-field at $P = (0, 0, z)$.

(Professor demonstrate on board).

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left(z^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}} \hat{k} \quad (8)$$

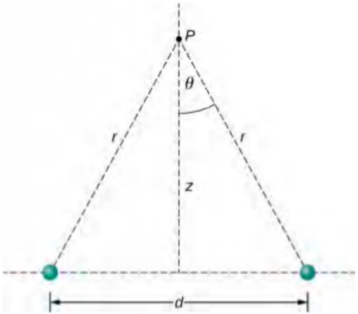


Figure 5.20 Finding the field of two identical source charges at the point P. Due to the symmetry, the net field at P is entirely vertical. (Notice that this is *not* true away from the midline between the charges.)

Figure 15: Solve for the E-field as a function of z, d, and q.

Show that the general solution is

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{2qz}{\left(z^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}}\hat{k} \quad (9)$$

Take the following two limits: 1) $z \gg d$ and 2) $z = 0$. What are the results?

Keep these results in mind, because we are about to start drawing **vector fields,** in order to visualize the algebra.

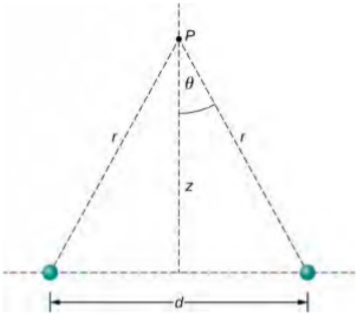


Figure 5.20 Finding the field of two identical source charges at the point P. Due to the symmetry, the net field at P is entirely vertical. (Notice that this is *not* true away from the midline between the charges.)

Figure 16: Solve for the E-field as a function of z, d, and q.

Activity: PhET Charges and Fields

PhET Simulation of E-fields from Charges:

`https://phet.colorado.edu/en/simulation/charges-and-fields`

1. Create the situation in the prior problem, in Fig. 16.
2. Use the yellow sensor object to determine the local direction of the E-field at various points along the z-axis.
   - Do the results match the limit $z \gg d$?
   - Do the results match the limit $z = 0$, halfway between the charges?
   - Where is the field maximal?
3. Make sure you can see above and below the charges, and repeat steps 1 and 2 for negative z-values. What do you find?

Electrostatics II: Chapters 19.1 - 19.3

Recall that the *change in potential energy* is force:

$$F = -\frac{\Delta U}{\Delta x} \tag{10}$$

- The units of U: Joules = Newtons per meter
- The units of x: meters
- The ratio: Newtons

Instructor examples: (a) force of gravity near surface of Earth, (b) force of spring.

The negative change in potential energy gives us the force. Often in electrostatics, we just need the electric field. The negative change in the *potential* gives us the field, because we just factor out the test charge:

$$F = -\frac{\Delta U}{\Delta x} = -\frac{q\Delta V}{\Delta x} \tag{11}$$

$$E = F/q = -\frac{\Delta V}{\Delta x} \tag{12}$$

In Eq. 12, we refer to ΔV as **voltage**. Voltage is the potential energy per unit charge.

Activity: PhET Simulation of Charges and Fields

`https://phet.colorado.edu/en/simulation/charges-and-fields`

1. Place a positive charge, and measure the voltage with the blue tool at right.
2. Voltage should be a *number*, not a *vector.*
3. Measure the voltage in 25 cm increments for 15 data points, and copy the data to Excel. One column should be the distance r in meters, and the other column should be the voltage V in **Volts.**
4. Plot the data in Excel, and fit a *power-law* trendline to the data. What do you notice?
5. Plot V vs. r^{-1}. What do you notice?

Same PhET simulation:

1. Measure the voltage from the same charge and tool position (fixed r), but vary the *amount of charge.*
2. Take 15 measurements, adding a positive red charge each time.
3. Plot the voltage in **Volts** vs. the charge in nano-Coulombs (nC). What do you notice?

Electrostatics II - Potential of a Point Charge

Potential of a Point Charge

The potential a distance r from a charge q is given by

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}} \tag{13}$$

If a potential of 4 Volts is felt a distance of 0.1 m from a charge q, what is the potential at 0.4 m?

- A: 5 Volts
- B: 4 Volts
- C: 3 Volts
- D: 2 Volts

Electrostatics II - Potential of a Point Charge

Potential of a Point Charge

The potential a distance r from a charge q is given by

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}} \tag{14}$$

If a potential of 3 Volts is felt a distance of 3 m from a charge q, what is the potential at 1 m?

- A: 1 Volts
- B: 3 Volts
- C: 6 Volts
- D: 9 Volts

Activity: PhET Simulation of Charges and Fields

Create the system shown in Fig. 17 in the PhET simulator *Charges and Fields*. The lines of charge should be far enough apart to allow several measurement locations, but close enough for the superposition principle to matter.

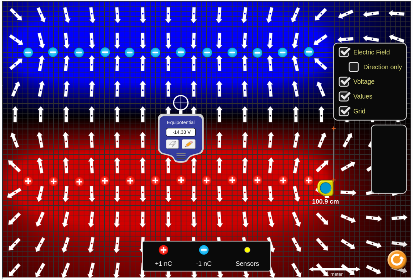


Figure 17: Parallel rows of positive and negative charges. The voltage button illustrates positive voltage in red, and negative voltage in blue.

Activity: PhET Simulation of Charges and Fields

`https://phet.colorado.edu/en/simulation/charges-and-fields`

1. Measure the voltage in 10 cm increments for 10-15 data points, and copy the data to Excel. One column should be the distance r in meters, and the other column should be the voltage V in Volts.
2. Plot the data and fit a *linear* trendline to the data. Is the model a good fit?
3. **Key question:** if the voltage depends linearly on the displacement, how does the E-field depend on displacement?
4. The system we have created is known as a *capacitor.*

Suppose the potential across the parallel-plate capacitor in Fig. 18 is 12 Volts. If a $q = +1$ nC charge is released at the positive side, and pops out from a hole on the negative side, what kinetic energy will q have?

- A: -12 nJ
- B: 0 nJ
- C: 12 nJ
- D: 12 J

Remember: ΔV is the potential energy *per unit charge*, so $\Delta U = q\Delta V$, and $E = -\Delta V/\Delta x$.

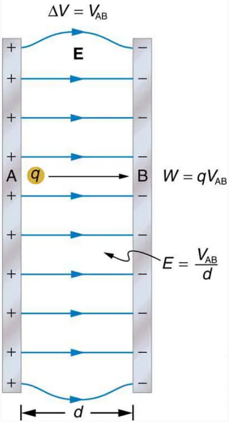


Figure 18: A capacitor with $\Delta V = 12$ V.

If the plates of charge in Fig. 19 are separated by 1 mm, what is the value of the electric field?

- A: -12 V/mm
- B: 0 V/mm
- C: 12 V/m
- D: 12 V/mm

Remember: ΔV is the potential energy *per unit charge*, so $\Delta U = q\Delta V$, and $E = -\Delta V/\Delta x$.

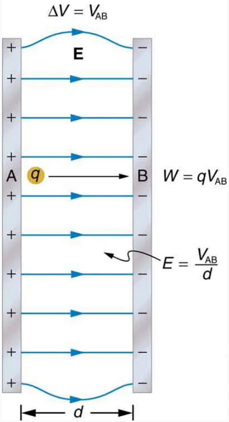


Figure 19: A capacitor with $\Delta V = 12$ V.

There is a difference between a *capacitor* and a *battery* that we will understand soon. For now, assume Fig. 20 represents a 12 V battery, and the total charge is 3600 C. If we connect a 36 Watt light to the voltage, how long before all the charge is gone?

- A: 1 minute
- B: 20 minutes
- C: 1 hour
- D: 12 hours

Remember that 1 Watt is 1 Joule per second.

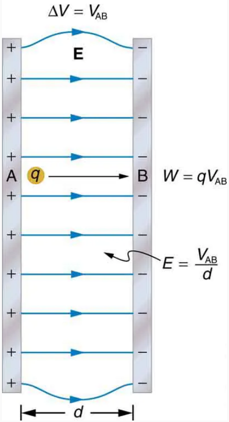


Figure 20: A capacitor with $\Delta V = 12$ V.

Applications of Electrostatics in Biology I

Applications of Electrostatics in Biology I

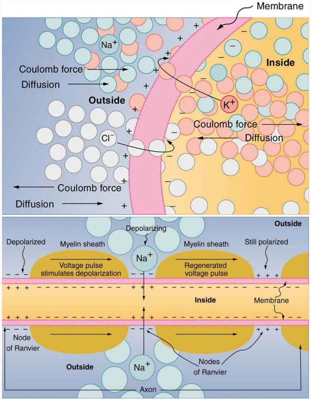


Figure 21: (Top) Nerve cell membrane (Bottom) Axon membrane with myelin sheath, and nodes.

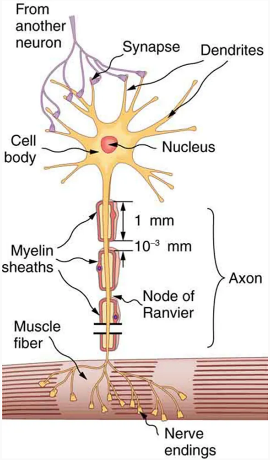


Figure 22: The general structure of a neuron.

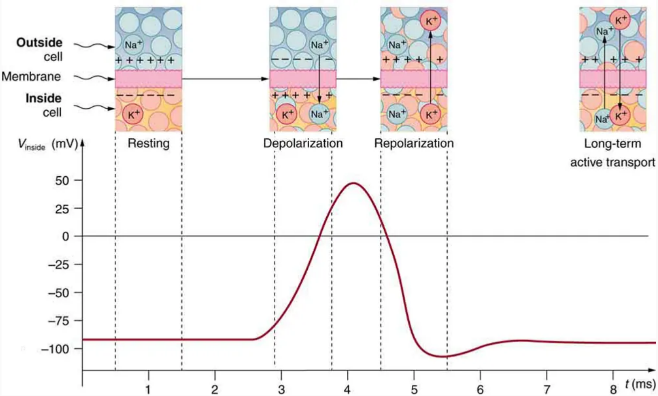


Figure 23: There are four stages to nerve signal propagation: resting, depolarization, repolarization, and long-term active transport.

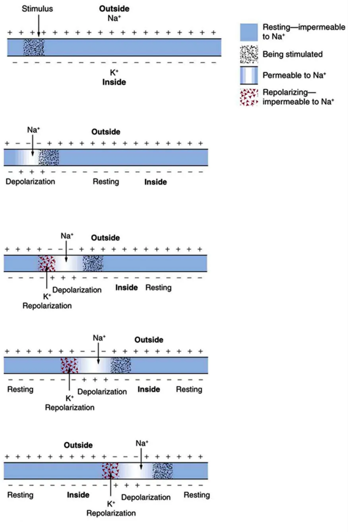


Figure 24

Signal propagation

- Stimulus causes Na-ion permeability
- Na-ions depolarize membrane
- Depolarization starts depolarization in adjacent area

Return to equilibrium

- Repolarization begins at original location
- Each area that is depolarized repolarizes, but in the order stimulated

PhET simulation of the cell membrane:
`https://phet.colorado.edu/en/simulations/neuron`

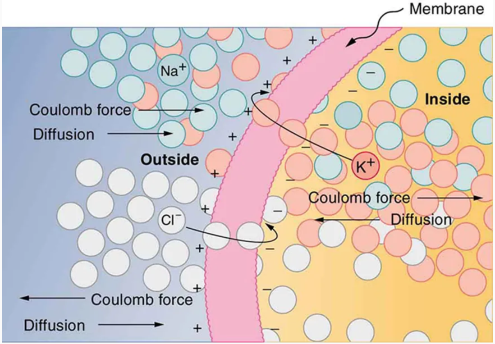


Figure 25: We can model this process and study the electrical and physiological implications.

Conclusion

Unit 0 Summary

Reading: Chapters 3.1 - 3.3, 18.1 - 18.5, 19.1 - 19.3

1. Estimation/Approximation
2. Coordinates and Vectors
3. Review of concepts from Newtonian mechanics
    - Kinematics and Newton's Laws
    - Work-energy theorem, energy conservation
    - Momentum, conservation of momentum
4. Electrostatics I: charges and fields
5. Electrostatics II: potential, and potential energy