Kirchhoff's Rules

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Abstract

Kirchhoff's Rules will help you to understand more complex DC circuits by applying charge and energy conservation.

1 A Complex Circuit

Consider Fig. 1. There are two power sources, a battery providing 15 V and a battery providing 10 V. There are three currents, and they are labeled I_1, I_2, and I_3, and given directions. Positve current flows in the direction of the arrows, but our results for the currents can be negative.

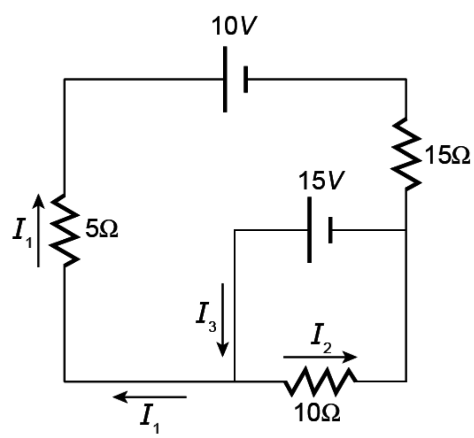

Figure 1: A circuit with three currents and two batteries.

2 Kirchhoff's Rule for Current Junctions

Consider Fig. 1, at the point at the bottom where I_3 splits into I_1 and I_2. The first of Kirchhoff's Rules is the following:

Since charge is conserved, the total current flowing into a junction must equal the total current flowing out of the junction.

This implies that

$$\boxed{I_3 = I_1 + I_2} \tag{1}$$

Can you identify the other junction? **Write the formula relating the three currents at the junction near the 15Ω resistor:**

Does your answer agree with Eq. 1? Why does this make sense?

3 Kirchhoff's Rule for Voltage Loops

Draw an x right behind the 10 V battery, on the right hand side (the negative side). Starting from x, trace a *loop* all the way around the outer edge of the circuit. Any time you encounter a voltage, write it in a list. For example, as you cross the battery, add "10 V" to your list. As you cross the 5Ω resistor, you are traveling against the current so the voltage would be $-I_1R$, where $R = 5\Omega$. As you return to the x, examine your list of voltage changes. If you were to sum the list, what *should* the result be? *Hint: since electric fields are conservative, we know that voltage differences only depend on differences in positiion.*

Use two loops of your choice (one can be the outer edge we just did), develop two equations in addition to Eq. 1 that capture the idea of *energy conservation.* Combine these two equations with Eq. 1 to solve for I_1, I_2, and I_3.