Algebra-Based Physics-2: Electricity, Magnetism, and Modern Physics (PHYS135B-01): Unit 5

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Summary

Unit 5 Summary

1. Electromagnetic waves - Chapters 24.1 - 24.4
    - Maxwell's Equations
    - Electromagnetic wave production
    - Electromagnetic spectrum and energy
2. Geometric optics - Chapters 25.1 - 25.3, 25.6
    - Ray-tracing and reflection
    - Refraction
    - Lens optics

Unit 5 Summary

1. Wave optics - Chapters 27.1 - 27.3
    - Wave interference
    - Wave diffraction
    - Double slit experiments
2. Nuclear physics in medicine - 32.1 - 32.4
    - Diagnostics and medical imaging
    - Biological effects of ionizing radiation
    - Therapeutic uses of ionizing radiation
    - Food irradiation

Electromagnetic waves: Maxwell's Equations

Maxwell's Equations are a set of four main ideas describing *the entirety* of electromagnetism.

- **Electric fields**, or the force per unit test charge, originate on positive charges and terminate on negative charges. The force is related to the permittivity of free space, ϵ_0. Electric fields give rise to Coulomb's law, or Gauss's law for electricity.
- **Magnetic fields**, or the force per unit test current per unit length, are continuous, having no beginning or end. No magnetic monopoles are known to exist. The strength of the magnetic force is related to the permeability of free space, μ_0. Magnetic fields give rise to Gauss's law for magnetism.

Maxwell's Equations are a set of four main ideas describing *the entirety* of electromagnetism.

- A *changing* **magnetic field** induces an electromotive force (emf) and, hence, an **electric field**. The direction of the emf opposes the change. This is Faraday's law of induction, and includes Lenz's law.
- **Magnetic fields** are generated by *moving charges* (current) or by *changing* **electric fields**. This is Ampère's law, enhanced with the idea that changing **electric fields** without current or charge induce **magnetic fields**.

Ampère's Law is enhanced with the idea that changing **electric fields** without current or charge induce **magnetic fields**.

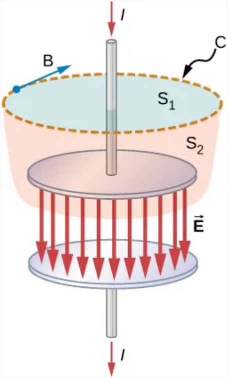

Figure 1: When a current creates a B-field, which surface bounding the B-field line is relevant?

Electromagnetic waves: Maxwell's Equations

The addition to Ampère's Law is called *the displacement current*:

$$I_{\mathrm{d}} = \epsilon_0 \frac{\Delta \Phi_{\mathrm{E}}}{\Delta t} \tag{1}$$

The electric flux is $\Phi_{\mathrm{E}} = \vec{E} \cdot \vec{A}$. Assume we are dealing with surface S_2, meaning $I = 0$. Ampère's Law gives

$$B 2\pi r = \mu_0 \left(I + I_{\mathrm{d}}\right) \tag{2}$$

$$B 2\pi r = \mu_0 \left(0 + I_{\mathrm{d}}\right) \tag{3}$$

$$B 2\pi r = \mu_0 \left(\epsilon_0 \frac{\Delta \Phi_E}{\Delta t}\right) = \mu_0 \epsilon_0 A \left(\frac{\Delta E}{\Delta t}\right) \tag{4}$$

For a parallel-plate capacitor,

$$E = V/d = Q/(Cd) = Qd/(\epsilon_0 A d) = Q/(\epsilon_0 A) \tag{5}$$

Electromagnetic waves: Maxwell's Equations

Insert the magnitude of the E-field into Ampère's Law to find:

$$B2\pi r = \mu_0\epsilon_0 A\left(\frac{\Delta E}{\Delta t}\right) \tag{6}$$

$$B2\pi r = \mu_0\epsilon_0 A\left(\frac{\Delta E}{\epsilon_0 A\Delta t}\right) \tag{7}$$

$$B = \frac{\mu_0}{2\pi r}\frac{\Delta Q}{\Delta t} \tag{8}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \tag{9}$$

A *changing* E-field is responsible for the B-field of a capacitor.

What is the B-field generated 1 cm laterally from a capacitor in an RC circuit that charges from 0 to 10 nJ in 1 μs?

- A: 20 μT
- B: 20 mT
- C: 20 nT
- D: 20 pT

What is the energy stored in the capacitor, if the capacitance is $C = 10$ pF?

- A: 5 μJ
- B: 5 mJ
- C: 5 nJ
- D: 5 pJ

Recall that $U = \frac{1}{2}\frac{Q^2}{C}$.

But *where* is the energy stored in a capacitor? **The E-field.** Consider that we proved the stored energy is

$$U_{\mathrm{C}} = \frac{1}{2}CV^2 \tag{10}$$

The voltage only exists because of the arrangement of charges and the field, and we know that $V = Ed$. Also, the volume is Ad. Thus,

$$U_{\mathrm{C}} = \frac{1}{2}CE^2d^2 \tag{11}$$

$$U_{\mathrm{C}} = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)E^2d^2 \tag{12}$$

$$\frac{U_{\mathrm{C}}}{Ad} = \frac{1}{2}\epsilon_0 E^2 \tag{13}$$

$$\boxed{\epsilon_{\mathrm{C}} = \frac{1}{2}\epsilon_0 E^2} \tag{14}$$

Ampère's Law is enhanced with the idea that changing **electric fields** without current or charge induce **magnetic fields**.

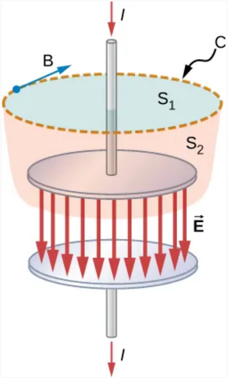

Figure 2: When a current creates a B-field, which surface bounding the B-field line is relevant?

Electromagnetic waves: Maxwell's Equations

What if there was a solenoid inductor ($N = 1$) next to the capacitor, waiting to catch the B-field and become charged (via Faraday's Law)? The solenoid will produce some current I to create the *opposite* B-field:

$$L = \frac{\mu_0 N^2 A}{d} \tag{15}$$

$$U_{\mathrm{L}} = \frac{1}{2} L I^2 \tag{16}$$

$$U_{\mathrm{L}} = \frac{1}{2} \frac{\mu_0 A}{d} I^2 \tag{17}$$

$$B = \mu_0 \frac{N}{d} I = \mu_0 \frac{I}{d} \tag{18}$$

$$I^2 = \frac{d^2 B^2}{\mu_0^2} \tag{19}$$

What if there was a solenoid inductor ($N = 1$) next to the capacitor, waiting to catch the B-field and become charged (via Faraday's Law)? The solenoid will produce some current I to create the *opposite* B-field:

$$U_{\mathrm{L}} = \frac{1}{2}\frac{\mu_0 A}{d}\frac{d^2 B^2}{\mu_0^2} = \frac{1}{2}\frac{B^2 A d}{\mu_0} \tag{20}$$

$$\frac{U_{\mathrm{L}}}{Ad} = \frac{B^2}{2\mu_0} \tag{21}$$

$$\boxed{\epsilon_{\mathrm{L}} = \frac{1}{2\mu_0}B^2} \tag{22}$$

Suppose the inductor catches *all* the energy from the capacitor, so that $\epsilon_{\mathrm{C}} = \epsilon_{\mathrm{L}}$?

Electromagnetic waves: Maxwell's Equations

If that is true, then

$$\epsilon_{\mathrm{C}} = \epsilon_{\mathrm{L}} \tag{23}$$

$$\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2\mu_0}B^2 \tag{24}$$

$$\frac{E}{B} = \frac{1}{\sqrt{\epsilon_0\mu_0}} \tag{25}$$

Show that the units of E/B are m s^{-1}. *Hint: recall $F = qE$, and $F = qvB$.* Knowing that the ratio on the left hand side is a velocity:

$$\boxed{v = \frac{1}{\sqrt{\epsilon_0\mu_0}}} \tag{26}$$

Equation 26 represents the **speed of light.** Now imagine the inductor charging a second capacitor, and that capacitor charging some second inductor ... the energy starts to propagate.

We should be able to observe this effect in the lab.

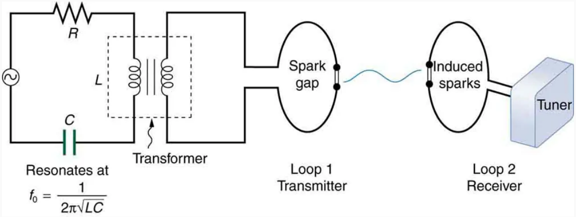

Figure 3: Heinrich Hertz demonstrated the *spark gap* RLC circuit.

- The RLC circuit on the left side is set to resonate.
- The transformer changes the signal to a high voltage that makes a spark in loop 1.
- The RLC circuit in the tuner is set to the same resonance frequency.
- Sparks are induced *even though the circuits are not connected with conductors.*

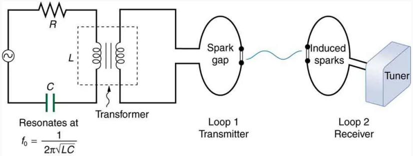

Figure 4: The transmitter and receiver are connected to RLC circuits with the same resonance frequency.

If the transmitter and receiver resonance frequency are the same:

$$f_{\mathrm{L}} = f_{\mathrm{R}} \tag{27}$$

$$\frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2\pi\sqrt{L_2C_2}} \tag{28}$$

$$L_1C_1 = L_2C_2 \tag{29}$$

If the transmitter and receiver resonance frequency are the same:

$$L_{\mathrm{TX}} C_{\mathrm{TX}} = L_{\mathrm{RX}} C_{\mathrm{RX}} \tag{30}$$

If the transmitter (TX) inductance is 1 mH, and the TX capacitance is 0.1 mF, and the receiver (RX) capacitance is 10 mF, what is the RX inductance?

- A: 1 mH
- B: 0.1 mH
- C: 0.01 mH
- D: 0.001 mH

Hint: treat this as a scaling problem.

If the transmitter (TX) inductance is 1 mH, and the TX capacitance is 0.1 mF, and the receiver (RX) capacitance is 0.2 mF, what is the RX inductance?

- A: 5 mH
- B: 0.5 mH
- C: 0.05 mH
- D: 0.005 mH

Hint: treat this as a scaling problem.

Electromagnetic waves: Maxwell's Equations

That electromagnetic fields can *propagate* was strong evidence that they are wavelike. All waves that obey the "wave equation" share a relationship between the speed, v, frequency f, and the *wavelength* λ:

$$v = f\lambda \tag{31}$$

The wavelength is the displacement between wave peaks, and $1/f = T$ is the period in time between peaks. If the speed is $v = 1/\sqrt{\epsilon_0\mu_0}$, and the resonance frequency corresponds to a capacitance of 0.2 μF and inductance of 0.5 μH, what is the wavelength?

- A: 3750 m
- B: 375 m
- C: 37.5 m
- D: 3.75 m

If the speed of light is 3×10^8 m/s, what is this same speed in m/ns?

- A: 30 m/ns
- B: 3 m/ns
- C: 0.3 m/ns
- D: 0.03 m/ns

What is the frequency of electromagnetic radiation with a wavelength comparable to the length of a person (≈ 1 m)?

- A: 0.3 GHz
- B: 3 GHz
- C: 300 MHz
- D: 3000 MHz

Note: is there any reason to expect limitations on the wavelengths and frequencies of electromagnetic waves?

Electromagnetic waves: Electromagnetic wave production

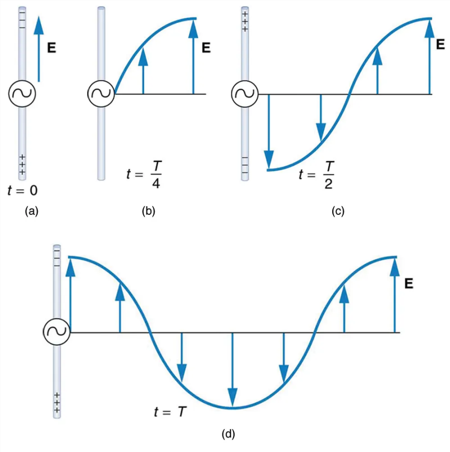

Figure 5: An AC voltage source corresponds to electrons oscilatting, which leads to an oscilatting field.

Electromagnetic waves: Electromagnetic wave production

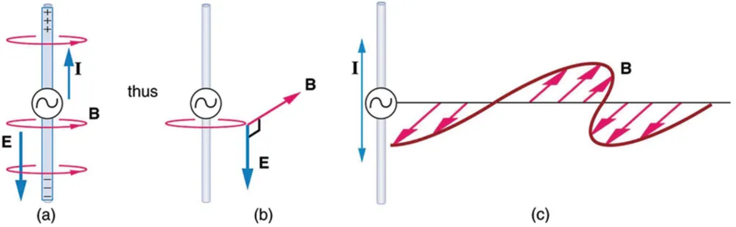

Figure 6: The oscillating E-field generates an orthogonal B-field.

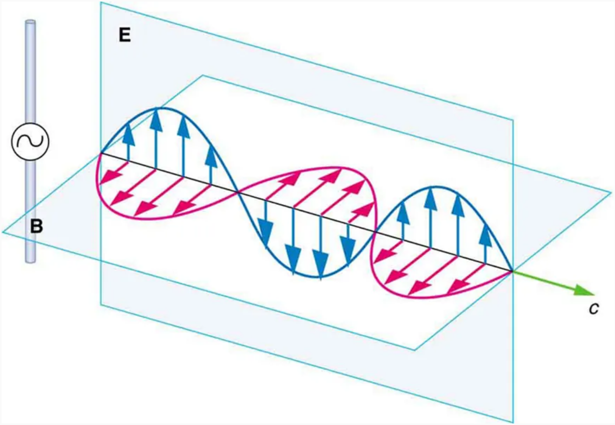

Figure 7: The oscillating B-field generates an orthogonal E-field, continuing the process.

The wave **moves energy** in the direction of the green arrow.

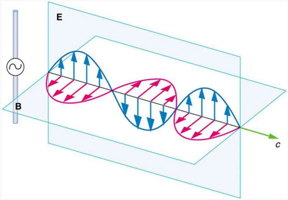

Figure 8: The oscillating B-field generates an orthogonal E-field, continuing the process.

The flux of energy per unit area in this case is (after some length mathematics)

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{32}$$

The wave **moves energy** in the direction of $\vec{S}$. The direction is orthogonal to the E and B, with a magnitude EB/μ_0. The peak B-value is $B = E/c$. Both B and E are sinusoids with the same f and ϕ. Thus, $S \propto \sin^2(2\pi f t + \phi)$, and the average of this is 1/2. This makes the average **intensity**

$$\bar{S} = \frac{1}{2c\mu_0} E^2 \tag{33}$$

The units of intensity are W m^{-2}. This formula is useful:

- Calculate the RX power at a radio given the field strength at the radio station, and the distance to the radio station.
- Calculate the brightness of a star observed at Earth, given the field strength of the light at the star.

Suppose a microwave in the kitchen generates 1 kW of power, projected onto a 10cm x 10cm area at a distance of 1 m. What is the intensity (power per unit area)?

- C: 1 kW
- D: 100 kW
- A: 1 kW m^{-2}
- B: 100 kW m^{-2}

Suppose a microwave in the kitchen generates 1 kW of power, projected onto a 10cm x 10cm area at a distance of 1 m. How long does it take the energy to travel the 1 meter?

- C: 0.333 ns
- D: 3.33 ns
- A: 33.3 ns
- B: 333 ns

Suppose a microwave in the kitchen generates 1 kW of power, projected onto a 10cm x 10cm area at a distance of 1 m. What is the peak E-field at the source?

- C: 870 V/m
- D: 8700 V
- A: 8700 V/m
- B: 870 V

Electromagnetic waves: Electromagnetic spectrum and energy

Electromagnetic waves: Electromagnetic spectrum and energy

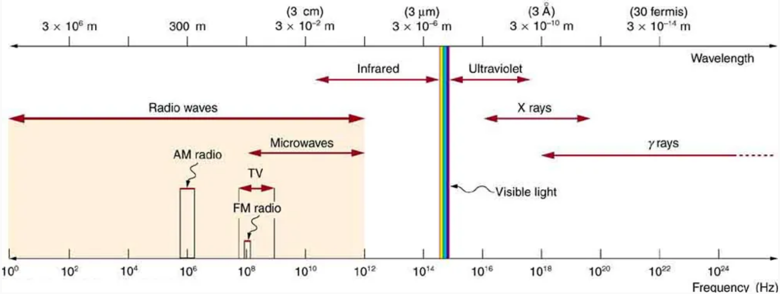

Figure 9: The electromagnetic spectrum maps signal types to wavelength (top) and frequency (bottom).

- Visible spectrum: more than 10^{14} Hz, 400-700 nm wavelengths
- Radio waves: $[10^{-1} - 10^{4}]$ MHz

Amplitude modulation (AM) is a technology that allows audio transmission over the EM spectrum.

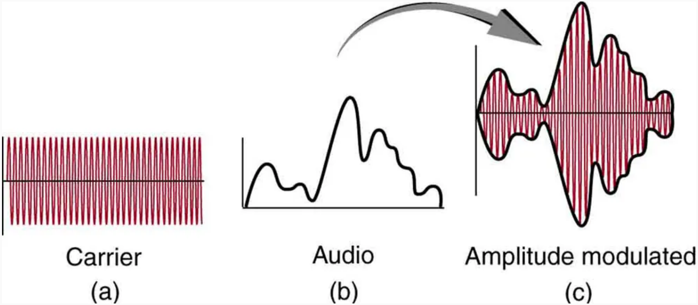

Figure 10: (a) The carrier wave. (b) The audio spectrum. (c) The *modulated* carrier wave.

The carrier is a pure sinusoid at a single frequency, f_c, with amplitude A:

$$c(t) = A\sin(2\pi f_\mathrm{c} t) \tag{34}$$

Let the modulating audio signal (at a given frequency) be

$$m(t) = Am\cos(2\pi f_\mathrm{m} t + \phi) \tag{35}$$

Audio waves are not electromagnetic, and audio frequencies are orders of magnitude smaller than radio frequencies. If there were a way to *mix* (multiply) these signals as voltages, then we get

$$y(t) = \left[1 + \frac{m(t)}{A}\right] c(t) \tag{36}$$

Do you remember the following trigonometric identity?

$$\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right) \tag{37}$$

Group exercise: Substitute Eqs. 34 and 35 into 36, and use the trigonometric identity in Eq. 37 to simplify the result.

1. Look for three waves: the carrier, and two additional ones at two different frequencies.
2. Draw a picture of the spectrum, the amplitude versus frequency of the signal.

Electromagnetic waves: Electromagnetic spectrum and energy

The AM mixing yields three waves:

- The original carrier
- A wave with $f_\mathrm{c} + f_\mathrm{m}$
- A wave with $f_\mathrm{c} - f_\mathrm{m}$

To re-capture the audio, we must *demodulate*, or reverse the process.

How do we create $m(t)$, and how do we modulate and demodulate it?

- Parallel LC circuits that act as resonators
- Diodes, devices that allow current to flow only one way

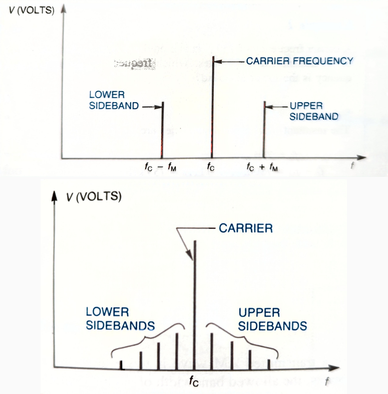

Figure 11: (Top) An example of a single audio frequency mixed into an AM signal. (Bottom) An audio spectrum mixed into an AM signal.

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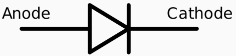


Figure 12: Circuit diagram for the diode.

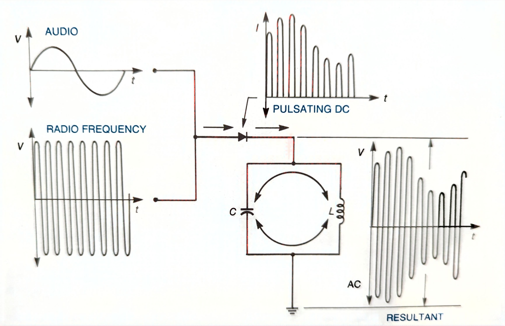


Figure 13: (Upper left) The audio signal is converted to a voltage via a microphone. (Lower left) The radio carrier signal oscillates at a higher frequency. (Middle) The LC resonator and diode mix the two signals. (Right) The final amplitude is modulated by the audio.

Suppose an audio signal exists primarily at 10 kHz, and we need an AM carrier at 1.4 MHz. If this AM carrier is mixed with the audio signal, what frequencies will exist in the final signal?

- A: 1390 and 1400 kHz
- B: 1400 kHz only
- C: 1390, 1400, and 1410 kHz
- D: 1390 and 1410 kHz

Suppose an audio signal exists primarily at 10 kHz, and we need an AM carrier at 1.4 MHz. If this AM carrier is mixed with the audio signal, what is the *bandwidth* required? That is, how much of the EM spectrum is occupied by the final signal?

- A: 10 kHz
- B: 20 kHz
- C: 1400 kHz
- D: 1420 kHz

Suppose an audio signal exists primarily at 10 kHz, and we need an AM carrier at 1.4 MHz. If, in our mixer, $L = 10\ \mu$H is required, what value must we choose for C?

- A: 1.3 mF
- B: 2.6 μF
- C: 1.3 μF
- D: 1.3 nF

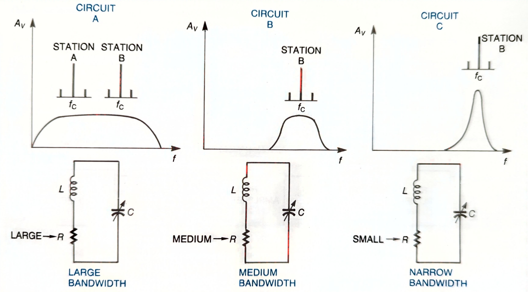


Figure 4-19 Relation of circuit bandwidth and receiver selectivity.

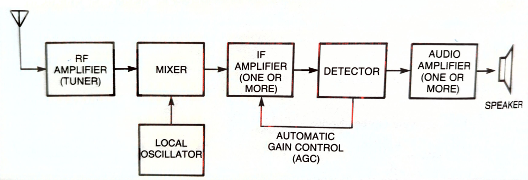


Figure 14: Circuit properties for the AM receiver.

We can show that the *bandwidth* of the RLC response is

$$\frac{\Delta f}{f_0} = \omega_0 \tau = 2\pi f_0 \tau \tag{38}$$

Where $\tau = RC$ and $f_0 = 1/(2\pi\sqrt{LC})$. This simplifies to

$$\frac{\Delta f}{f_0} = R\sqrt{\frac{C}{L}} \tag{39}$$

Thus, bandwidth is proportional to R, as shown in Fig. 14.

Suppose an audio signal exists primarily at 5 kHz, and we need an AM carrier at 1100 kHz. This implies our sidebands will be at 1095 kHz and 1105 kHz, making our bandwidth 10 kHz centered around 1100 kHz. If our resistance R is such that we are capturing the carrier, but not the sidebands, we should:

- A: Decrease R
- B: Increase R
- C: Leave R unchanged
- D: Set R to 0 Ohms

The local oscillator (LO) is a tunable oscillator set to be 455 kHz above the AM channel. For example:

- AM channel: 1200 kHz
- LO: 1655 kHz
- Mixer: 1655 kHz + 1200 kHz, 1655 kHz, and 1655 - 1200 kHz
- 455 kHz is the *intermediate frequency* (IF)
- IF amplifier: responds only to 1655 - 1200 = 455 kHz

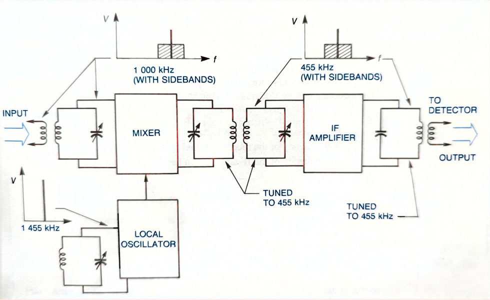
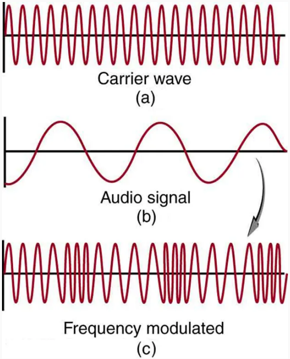


Figure 15: The superheterodyne radio scheme. Channel input is amplified, mixed with a local oscillator, and moved to the IF. The IF is filtered and amplified, then demodulated in the detector.

Suppose an audio signal exists primarily at 10 kHz, and is mixed with a carrier at 1000 kHz. What should our LO be if our IF is 455 kHz?

- A: 1000 kHz
- B: 455 kHz
- C: 545 kHz
- D: 1455 kHz

Frequency modulation (FM) radio transmission converts audio signals into frequency deviations in the carrier.



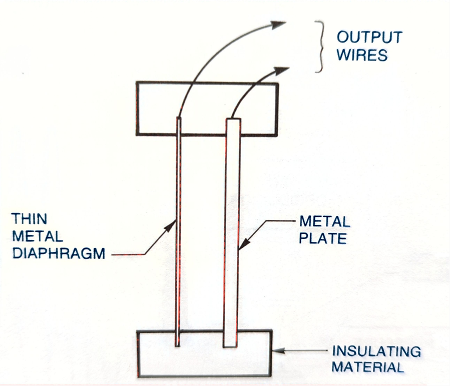


Figure 16: (Left) Frequency modulation. (Right) Capacitance microphone.

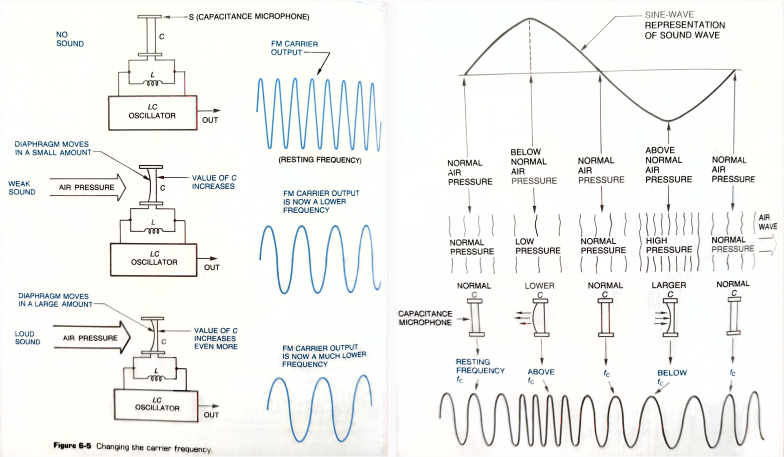


Figure 17: (Left) Lowering C raises f_0, and raising C lowers f_0. (Right) Changes in pressure correspond to changes in C.

In summary,

1. The *C* in the LC oscillator can be made to depend on audio amplitude
2. The audio amplitude corresponds to the frequency deviation
3. The rate at which the frequency changes depends on audio frequency.
4. **For those interested,** a great final project is to assemble a DIY AM transistor radio

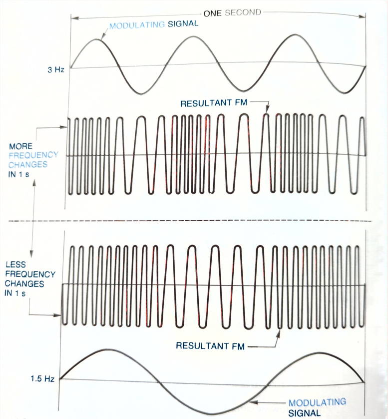


Figure 18: Audio (modulation) frequency determines *how often* the frequency deviates, not the frequency deviation itself.

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Geometric optics: Ray-tracing and Reflection

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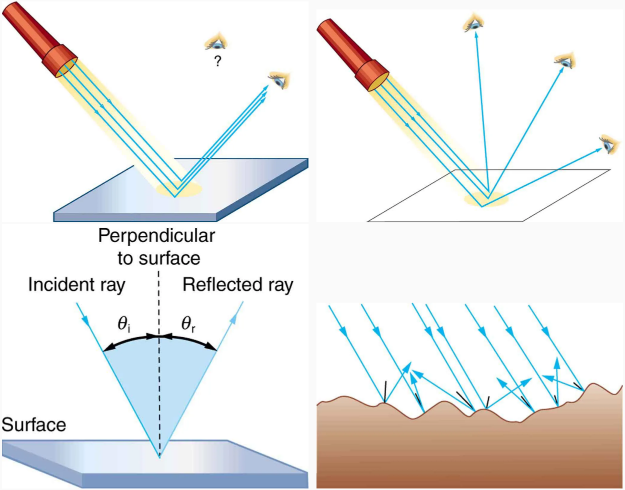


Figure 19: (Top left) Specular reflection (Top right) Diffuse reflection (Bottom left) Smooth surface (Bottom right) Rough surface.

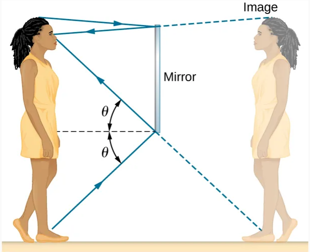


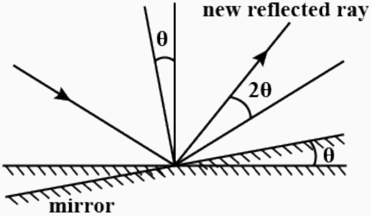
Figure 20: (Left) Your image in a mirror is due to specular reflection. (Right) The image of the moon on the ocean is due to diffuse reflection.

Specular reflection rule: the incident angle and reflected angle are equal.

$$\theta_{\mathrm{i}} = \theta_{\mathrm{r}} \tag{40}$$

Both angles are usually measured with respect to the direction orthogonal to the reflecting surface.

Group exercise: Light shows staged with lasers use moving mirrors to swing beams and create colorful effects. Show that a light ray reflected from a mirror changes direction by 2θ when the mirror is rotated by an angle θ.



Think about rotating a mirror 90 degrees. What happens to the reflected laser light?

The speed of light can be measured independently of electromagnetism, with an *interferometer.*

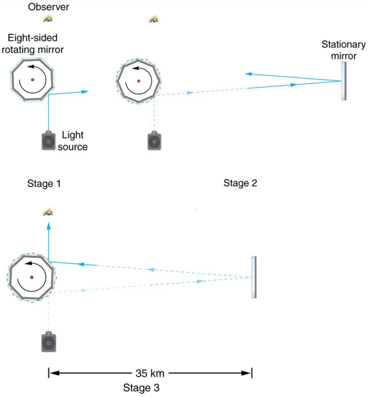


Figure 21: A schematic of the 1887 experiment by Albert Michelson.

The Speed of Light

The speed of light in a vacuum is

$$c = 2.99792458 \times 10^{8} \;\; m \; s^{-1} \tag{41}$$

When light travels through a material with *index of refraction*, n, the speed is

$$v = \frac{c}{n} \tag{42}$$

In the Michelson interferometer, light had to travel 35 km down to a mirror, and 35 km back from the mirror. Assuming the speed of light in a vacuum (3×10^8 m/s), how long does it take for the light to make the round trip?

- A: 116 μs
- B: 116 ns
- C: 233 μs
- D: 233 ns

In airborne radar, aircraft must transmit a radio pulse and record the reflection time to determine the position of other aircraft. If an aircraft records a reflection time of 700 μs, how far away is the object?

- A: 210 km
- B: 210 m
- C: 105 m
- D: 105 km

The index of refraction is a property of materials and substances that depends on the atomic or molecular structure of electrons.

Air	Radio	1.000368
Air	Optical	1.000293
Water, fresh	Optical	1.333
Ice, fresh	Optical	1.31
Ice, fresh	Radio	1.78

Table 1: Indices of refraction for several substances in different parts of the electromagnetic spectrum.

Water, fresh	Radio	1.31
Ice, fresh	Radio	1.78

Table 2: Indices of refraction for several substances.

Suppose we are measuring the ice shelf thickness in Greenland with **radio waves** to determine changes in ice volume due to climate change. If in 2016 we observed a radio echo time of 1150 ns, and in 2020 we observed a time of 1125 ns, by how much has the ice thickness been reduced?

- A: 5.25 cm
- B: 52.5 cm
- C: 525 cm
- D: 0.525 cm

Hint: Assume the radio signal begins from the ice surface, travels down, and reflects from the bottom.

Reflection Coefficient at Normal Incidence

Suppose an electromagnetic wave in a medium with index of refraction n_1 approaches the surface of a medium with index of refraction n_2 at an angle $\theta_i = 0$ degrees. The wave will be reflected at $\theta_r = 0$ degrees. The **reflection coefficient** is the fraction of power or intensity reflected, and it is equal to

$$R = \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2 \tag{43}$$

The **transmission coefficient** is $T = 1 - R$, to conserve power and energy.

Water, fresh	Radio	1.31
Ice, fresh	Radio	1.78

Table 3: Indices of refraction for several substances.

Suppose our aircraft radar is reflecting radar echos from a frozen lake surface. Assuming air has $n_1 = 1.0$, what fraction of the power reflects from the lake?

- A: 100 percent
- B: 7.9 percent
- C: 15.8 percent
- D: 0 percent

Water, fresh	Radio	1.31
Ice, fresh	Radio	1.78

Table 4: Indices of refraction for several substances.

If we cross from a region of ice to fresh water, what is the new result?

- A: 100 percent
- B: 3.6 percent
- C: 1.8 percent
- D: 0 percent

Geometric optics: Ray-tracing and Reflection

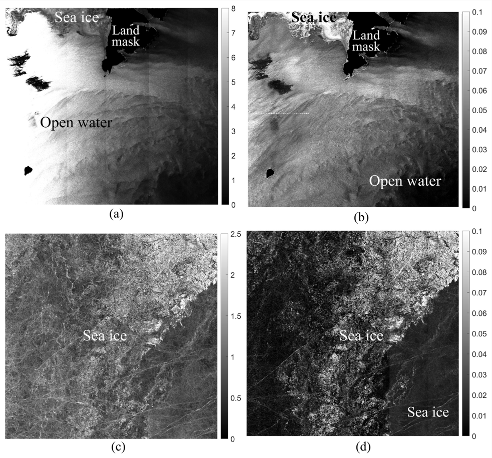


Figure 22: A result from Wang and Li, "Arctic sea ice cover data from spaceborne synthetic aperture radar by deep learning." Earth System Science Data 13(6) 2723-2742, 2021.

PhET: Geometric optics and Reflection Coefficients

PhET: Geometric optics and Reflection Coefficients

Navigate to `https://phet.colorado.edu/en/simulations/bending-light`

1. Click on the More Tools tab, and drag the laser tool to the top so that the light experiences normal incidence.
2. Activate the laser tool by clicking the red button. The reflected light appears above the laser tool.
3. Use the green intensity tool at left to measure the reflected intensity.
4. Measure the reflection coefficient between air and water by controlling the indices with the sliders at right.
5. Copy your result into 100 spreadsheet cells in Google Sheets ($2 \to 4 \to 8 \to \ldots$).

PhET: Geometric optics and Reflection Coefficients

Navigate to `https://phet.colorado.edu/en/simulations/bending-light`

1. Suppose your data exist in A1:A100. In cell B1, type the following:
   `=NORMINV(RAND(),A1,0.01)`
2. The previous step will create a *gaussian random error* with the mean of A1 and standard deviation of 0.01.
3. In cell C1, enter a number that is a few tenths of a percent below the average reflection coefficient (e.g. 3.9). Then, in cell C2, write:
   `=C1+0.01`
4. Click and drag C2 until the C-cells encompass the range of data we find in column B (e.g [3.9:4.1]).

Navigate to `https://phet.colorado.edu/en/simulations/bending-light`

1. Assume the column C cells we created exist in C1:C21. In cell D1, type the following:
   `=FREQUENCY(B1:B100,C1:C21)`
2. Now hit CNTL+Shift+Enter. The formula will be encapsulated by `ArrayFormula`, and hit enter until the `FREQUENCY` function is evaluated.
3. The results are *frequencies* in the sense that, next to each frequency bin, the function calculates *how frequently* data fall into the given bin.
4. Create an x-y scatter graph of frequency bins (column C) versus frequencies (column D).

PhET: Geometric optics and Reflection Coefficients

Navigate to `https://phet.colorado.edu/en/simulations/bending-light`

1. This type of chart is called a *histogram.*
2. Does your histogram peak at the expected reflection coefficient?
3. What does the width of your histogram tell you?

Geometric optics: Refraction

PhET: Geometric optics and Refraction

Navigate to
`https://phet.colorado.edu/en/simulations/bending-light`

1. Perhaps you noticed in the previous activity that transmission angle depends on both n_2 and θ_i. We will now sort out that relationship.
2. Click on the More Tools tab, and click on the yellow protractor at left.
3. Place the center of the protractor at the spot where the laser hits the surface.
4. Select $n_1 = 1.0$, and $n_2 = 1.5$. Take 15-20 data points of θ_i and θ_t, where θ_i is the incident angle with respect to normal, and θ_t is the transmission angle with respect to normal.
5. Graph θ_t vs. θ_i. Do you observe a linear effect?
6. Now graph $\sin\theta_t$ vs. $\sin\theta_i$. Do you observe a linearized trend?
7. Infer a general rule based on your data.

Snell's Law

Let two media have two indices of refraction, n_1 and n_2, and let θ_i and θ_t represent the incident and transmission angles, measured with respect to normal. Snell's Law relates these quantities:

$$n_1 \sin\theta_i = n_2 \sin\theta_t \tag{44}$$

Suppose you see your friend swimming underwater, and you are standing in the water near them. Are they:

- A: Farther away than they appear
- B: Exactly where they appear
- C: Closer than they appear
- D: Deeper than they appear

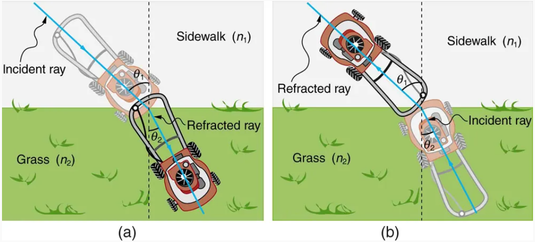


Figure 23: (a) The mower crosses from sidewalk to grass. (b) The mower crosses from grass to sidewalk.

The mower is powered by the two back wheels, and the speed they generate depends on the torque of the motor and axel against the surface. This speed is higher on the sidewalk than the grass. If the mower enters the grass at an angle, the right side goes slower before the left side. The right side goes faster first as the mower enters the sidewalk.

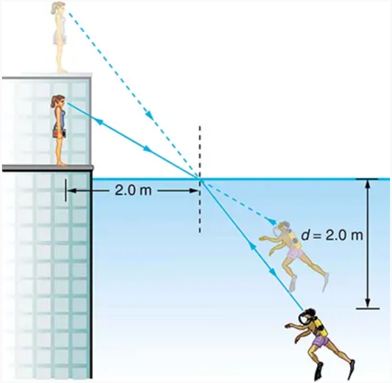


Figure 24: A scuba diver is 2.0 m beneath the water surface ($n = 1.3$).

A scuba diver looks at his instructor. What angle does the ray from the instructor's face make with respect to normal at the point where the ray enters? The angle between the ray in the water and normal is 25.0 degrees.

- A: 15.2 degrees
- B: 25.0 degrees
- C: 30.1 degrees
- D: 34.3 degrees

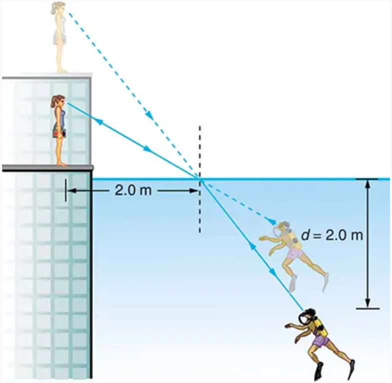


Figure 25: A scuba diver is 2.0 m beneath the water surface ($n = 1.3$).

Is your answer reasonable?

A scuba diver looks at his instructor. How tall is the instructor?

- A: 1.23 meters
- B: 4.56 meters
- C: 7.89 meters
- D: 2.93 meters

Hint: draw your own diagram.

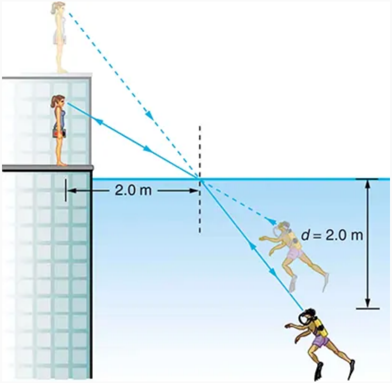
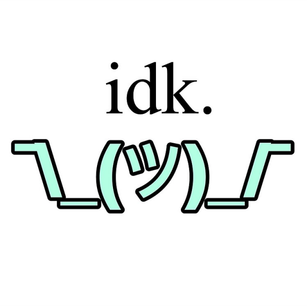


Figure 26: A scuba diver is 2.0 m beneath the water surface ($n = 1.3$).

The answer was 2.93 meters.

No. It really is, according to the Instructor's Solutions Manual.



For reference: there is a lens simulator at `https://phet.colorado.edu/en/simulations/geometric-optics`.

Geometric optics: Lens optics

Geometric optics: Lens optics

Lens optics provides an understanding of image formation by lenses given the laws of refraction and lens properties.

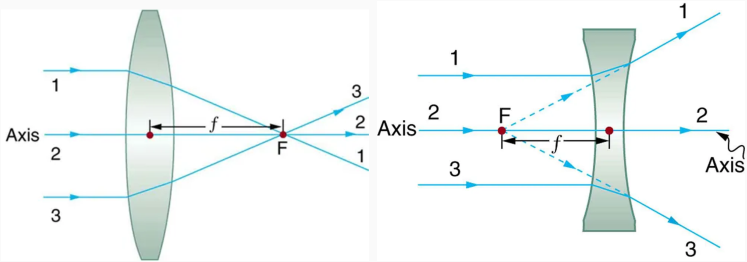


Figure 27: (Left) A converging lens. (Right) A diverging lens.

The *focal length*, f is the distance from the lens center where rays converge (converging lenses), or from where rays appear to diverge (diverging lenses).

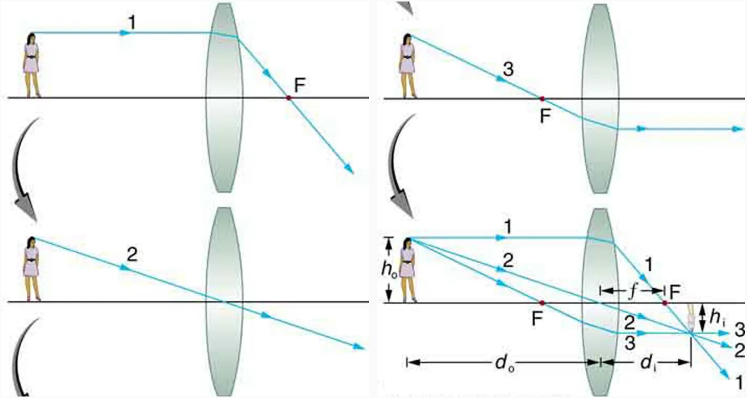


Figure 28: Rays 1, 2, and 3, pass parallel through the lens, through the center of the lens, and through the lens focal length, respectively. A **real image** is formed on the other side.

Thin Lens Equations

Let d_o be the distance from the source to the lens center. Let d_i be the distance from the lens center to the real image. Let f be the focal length. Let h_o be the object height. let h_i be the real image height. The thin lens approximation gives

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \tag{45}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} \tag{46}$$

The ratio m is called the **magnification.**

Suppose we have a lens with $f = 3$ cm. Where will the image be if $d_o = 10$ m?

- A: 0 cm
- B: 3 cm
- C: 10 cm
- D: 10 m

For the same lens, what is the magnification?

- A: -3×10^{0}
- B: 3×10^{-1}
- C: 3×10^{-2}
- D: -3×10^{-3}

Why was the magnification so small in the prior example?

- A: The object was close to the lens
- B: The object was far from the lens
- C: The object was at the focal length
- D: The object was within the focal length

Suppose $d_o = 10$ cm, and $f = 3$ cm. What are d_i and m?

- A: $30/7$ cm, $-3/7$
- B: 30 cm, $-1/2$
- C: $7/30$, $-3/7$ cm
- D: $30/7$, $-3/7$

What happens if $d_o = f$?

- A: $d_i = f$
- B: $d_i = 0$
- C: $d_i \to \infty$
- D: $d_0 = 1/f$

An object within the focal length creates a *virtual image*.

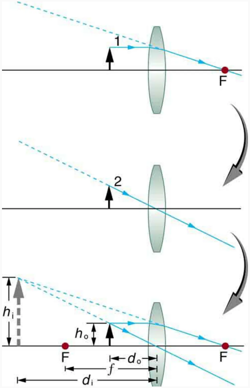


Figure 29: Rays 1 and 2 pass parallel through the lens to the focal point, and through the lens center, respectively. A **virtual image** forms on the same side.

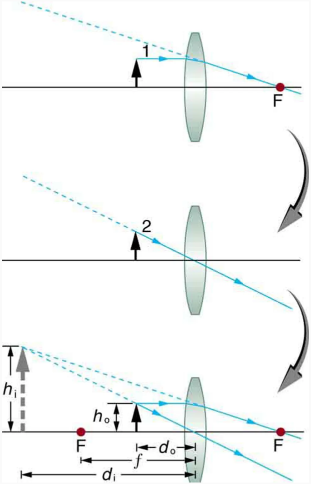


Figure 30: A **virtual image** forms if $d_o < f$.

Group exercise. (a) Show that if $d_0 < f$, that $d_i < 0$. (b) Show that the magnification is positive, and given by

$$m = \frac{f}{f - d_0} \qquad (47)$$

Group exercise. Design a magnifying glass (by solving for f) that gives $m = 10$ for an object 2.7 cm from the lens center.

Convex lenses create *virtual images* when the object when $|d_0| > |f|$.

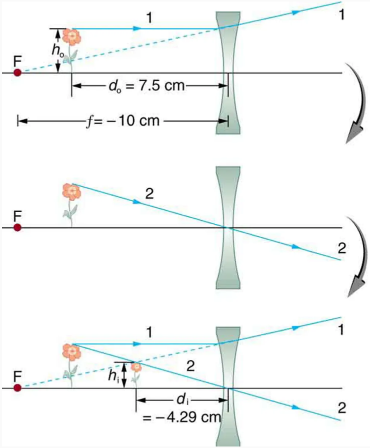


Figure 31: A convex lens diverges rays, and $f < 0$.

Geometric optics: Lens optics

Suppose an object such as a book page is held 7.50 cm from a concave lens of focal length –10.0 cm. Such a lens could be used in eyeglasses to correct pronounced nearsightedness. What magnification is produced?

- A: -0.57
- B: 4.2
- C: -4.2
- D: 0.57

Interesting final project idea: Measure the energy in sunlight by using a lens to concentrate sunlight to heat water by a fixed temperature. For those interested in projects involving vision and medicine, see Chap. 26.

Unit 5 Summary

1. Electromagnetic waves - Chapters 24.1 - 24.4
    - Maxwell's Equations
    - Electromagnetic wave production
    - Electromagnetic spectrum and energy
2. Geometric optics - Chapters 25.1 - 25.3, 25.6
    - Ray-tracing and reflection
    - Refraction
    - Lens optics

Unit 5 Summary

1. Wave optics - Chapters 27.1 - 27.3
    - Wave interference
    - Wave diffraction
    - Double slit experiments
2. Nuclear physics in medicine - 32.1 - 32.4
    - Diagnostics and medical imaging
    - Biological effects of ionizing radiation
    - Therapeutic uses of ionizing radiation
    - Food irradiation

Wave optics: Wave interference and diffraction

We observe that light acts as a *ray* and a *wave*.

- Lasers are used for targeting in optical observatories, precisely because they travel in straight lines.
- When we pass lasers through *diffraction gratings*, the light exhibits a *diffraction pattern*. Rays don't diffract.
- $\lambda_n = \lambda/n$ in a medium with n.

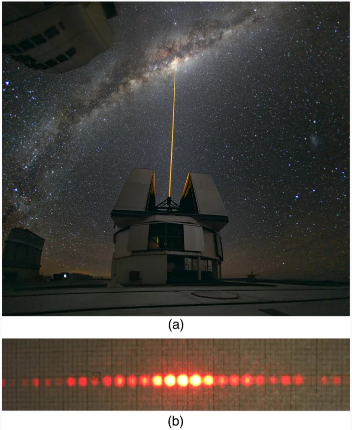


Figure 32: The Paranal Observatory of the ESO, in the Atacama Desert, Chile.

Wave optics: Wave interference and diffraction

We observe that light acts as a *ray* and a *wave*.

- The 3D picture of an electromagnetic wave is that it is a wave in *space* and *time*.
- The electromagnetic wave also has lateral extent.
- *Huygen's principle* states that every point along the wavefront is a point-source.

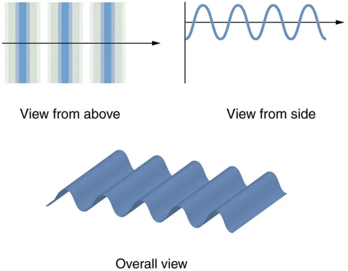


Figure 33: The three-dimensional picture of a transverse wave.

We observe that light acts as a *ray* and a *wave*.

- The 3D picture of an electromagnetic wave is that it is a wave in *space* and *time*.
- The electromagnetic wave also has lateral extent.
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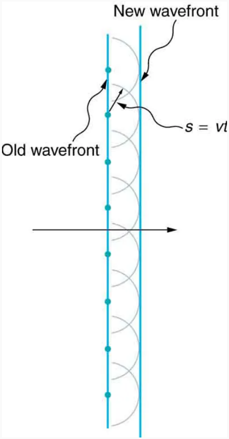


Figure 34: The three-dimensional picture of a transverse wave.

We observe that light acts as a *ray* and a *wave.*

- *Huygen's principle* states that every point along the wavefront is a point-source.
- *Huygen's principle* aligns with observations of specular reflection.

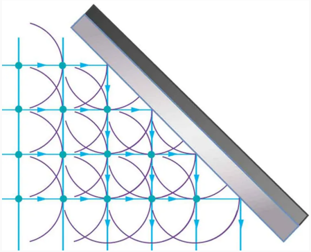


Figure 35: The transverse wave reflecting from a flat mirror.

We observe that light acts as a *ray* and a *wave.*

- *Huygen's principle* states that every point along the wavefront is a point-source.
- *Huygen's principle* aligns with observations of refraction between media with different indices of refraction.
- Notice that the wavelength changes, but the frequency does not.

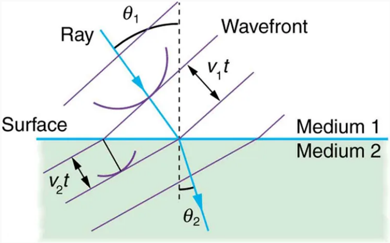


Figure 36: The transverse wave refracting into a medium with a new index of refraction.

We observe that light acts as a *ray* and a *wave*.

- *Huygen's principle* states that every point along the wavefront is a point-source.
- *Huygen's principle* aligns with observations of diffraction through openings that allow light (and sound) limited propagation.

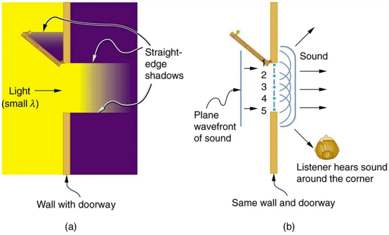


Figure 37: Light waves cause straight shadows, while sound waves diffract. This is why you cannot see in the shadows, but you can hear sounds in another room.

Wave optics: Wave interference and diffraction

We observe that light acts as a *ray* and a *wave.*

- *Huygen's principle* states that every point along the wavefront is a point-source.
- *Huygen's principle* aligns with observations of diffraction through openings that allow light (and sound) limited propagation.
- The key to diffraction is the comparison between wavelength, and length scale of the opening.

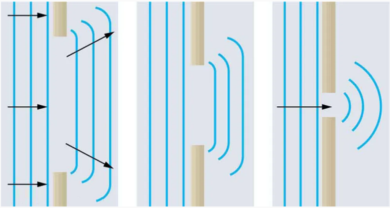


Figure 38: Waves diffract more strongly through openings that are small compared to the wavelength.

Wave optics: Double slit experiments

Wave optics: Double slit experiments

How do we prove that light is a wave? Demonstrate the effects of *constructive* and *deconstructive* interference.

- *Young's double-slit experiment.* In 1801, Thomas Young demonstrated that light exhibits constructive and deconstructive interference.
- The view of Isaac Newton and others was that light was a particle, and that there were other explanations of color and diffraction.

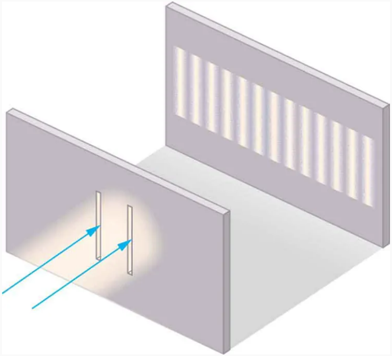


Figure 39: If light is a particle, there should only be two slot images on the wall.

Wave optics: Double slit experiments

How do we prove that light is a wave? Demonstrate the effects of *constructive* and *deconstructive* interference.

- *Young's double-slit experiment.* In 1801, Thomas Young demonstrated that light exhibits constructive and deconstructive interference.
- Constructive and deconstructive interference occur when waves are in and out of phase, respectively.

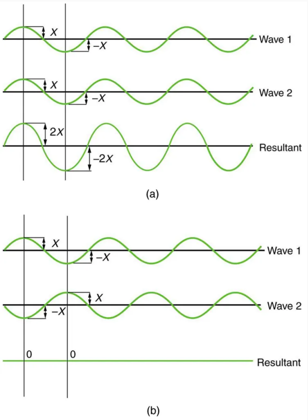


Figure 40: (a) Waves in phase add in amplitude. (b) Waves out of phase subtract in amplitude.

- The double-slit experiment creates two in-phase point sources, following Huygen's Principle.
- The two sources are separated by d about the origin, and a screen is a distance $r \gg d$ from the origin.

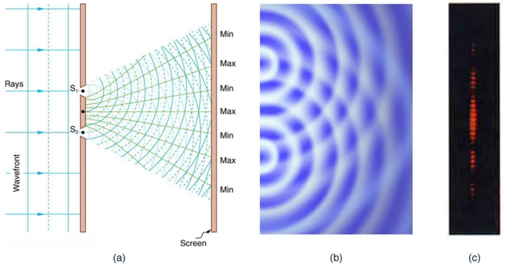


Figure 41: (a) A plane wave arrives at two slits. (b) A view of the wave magnitudes after the slits. (c) The diffraction pattern on the screen.

PhET: Double slit experiments

Wave optics: Double slit experiments

Navigate to `https://phet.colorado.edu/en/simulations/wave-interference`.

1. **Waves tab learning goals:**
   - Learn measurement controls
   - Measure period, frequency, wavelength accurately
   - Measure wave speed, and compare to $v = f\lambda$
   - Perform measurements for water, sound, and light
2. **Interference tab learning goals:**
   - Let θ_m be the angular location of constructive interference fringes. Compare $d \sin \theta_m$ with $m\lambda$, where m is an integer.
   - Repeat this procedure with sound and light waves.

Wave optics: Double slit experiments

Navigate to `https://phet.colorado.edu/en/simulations/wave-interference`.

1. **Slits tab learning goals:**
    - Repeat the procedure from the interference tab to show that the interference relationship is identical, provided certain criteria are met.
    - Repeat this procedure with sound and light waves.
2. **Diffraction tab learning goals:**
    - Explore the diffraction patterns of different combinations of boundary conditions and wavelengths.
    - Explore the idea that the image we see is not necessarily the same as the shape of the source.

Wave optics: Double slit experiments

1. **Waves tab learning goals:**
   - Learn measurement controls.
     1.1 Activate the water droplets with the green button.
     1.2 The color of the pool indicates wave amplitude.
     1.3 The frequency knob at right controls the drops per second.
     1.4 The amplitude knob controls the droplet size, and therefore wave amplitude.
   - Measure period, frequency, wavelength accurately
     1.1 Use the tape measure tool (upper right) to measure wavelength.
     1.2 Use the stopwatch and tape measure to measure wave speed ($\Delta x = v\Delta t$) as the waves proceed across the pool.
   - Measure wave speed, and compare to $v = f\lambda$
     1.1 Compare $v = \Delta x/\Delta t$ to $v = f\lambda$. Do your results agree?
   - Perform measurements for water, sound, and light.

Wave optics: Double slit experiments

1. **Interference tab learning goals:**
   - Let θ_m be the angular location of constructive interference fringes. Compare $d \sin \theta_m$ with $m\lambda$, where m is an integer.
     1.1 The source separation knob is on the right side.
     1.2 Allow the wave interference pattern to stabilize before measuring θ_m, the angle between horizontal and the constructive interference fringe.
     1.3 Use the tape measure to measure the length of the hypoteneuse corresponding to the constructive interference fringe. Also measure the distance to the edge of the pool. Calculate θ_m using the appropriate trigonometric function.
     1.4 Adjust the controls to observe multiple fringes. Compare $d \sin \theta$ to $m\lambda$, where m is an integer. Plot $d \sin \theta_m$ vs. $m\lambda$ while varying λ (by changing the frequency).
   - Repeat this procedure with sound and light waves.

1. Slits tab learning goals:
   - **Repeat the procedure from the interference tab.** Show that the interference relationship is identical, provided that the slit width is small compared to λ, and that the slit separation, d is comparable to λ.
   - Focus this procedure on light waves.
   - Create a graph of $d \sin \theta_1$ versus $m\lambda$, with $m = 1$, for 10-15 different optical wavelengths.
   - Organize your data as follows: λ in Column A, error or imprecision in λ in Column B, $d \sin \theta_1$ in Column C, error or imprecision in $d \sin \theta_1$ in Column D.
   - How do we calculate the propagated error in $d \sin \theta_1$? Let σ_f represent the error in this quantity. Let σ_θ represent the error in θ, and σ_d represent the error in d. The total error is

$$\sigma_f = \left(\sigma_d^2 \sin^2 \theta + \sigma_\theta^2 d^2 \cos^2 \theta\right)^{1/2} \tag{48}$$

Wave optics: Double slit experiments

1. Diffraction tab learning goals:
    - Explore the diffraction patterns of different combinations of boundary conditions and wavelengths.
        1.1 Start with the circle boundary. Adjust the diameter to the largest setting (0.4 mm).
        1.2 Shrink the diameter to the smallest setting (0.04 mm) while leaving the wavelength constant. Explain to your lab partner why the diffraction pattern morphs. Compare this to the 1D case by using a 1 slit system in the Slits Tab, while adjusting the slit width.
    - Explore the idea that the image we see is not necessarily the same as the shape of the source.
        1.1 Now try different boundary conditions. In general, which direction (x or y) should be the *longer* direction in the diffraction pattern, given your boundary conditions?
        1.2 Explain to your lab partner how the pattern is a result of *Huygen's Principle*.

Wave optics: Double slit experiments

Huygen's Principle, combined with the double-slit arrangement, shows the wave-like nature of light.

- Mathematically, the *far-field* approximation gives:

$$d \sin\theta = m\lambda \qquad (49)$$

- Deconstructive interference occurs when:

$$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad (50)$$

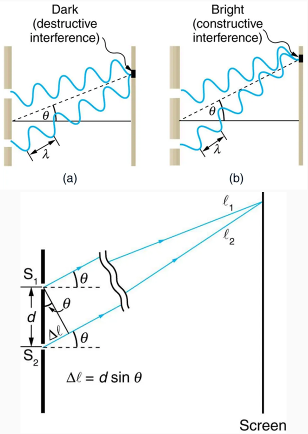


Figure 42: For a path length difference of $(m + 1/2)\lambda$, we observe a dark fringe.

Huygen's Principle, combined with the double-slit arrangement, shows the wave-like nature of light.

- Mathematically, the *far-field* approximation gives:

$$d \sin \theta = m\lambda \tag{51}$$

- Deconstructive interference occurs when:

$$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda \tag{52}$$

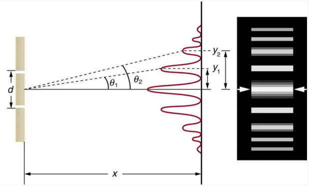


Figure 43: The double-slit interference pattern depends on θ and λ.

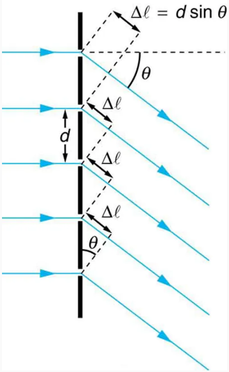


Figure 44: A repeated structure that affects light transmission is called a *diffraction grating*.

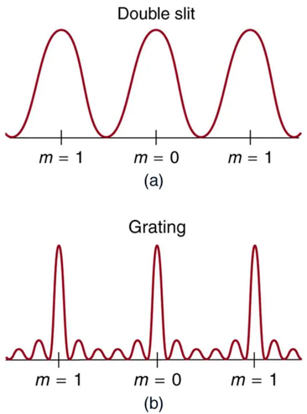


Figure 45: The diffraction pattern still retains peaks that correspond to integers *m*.

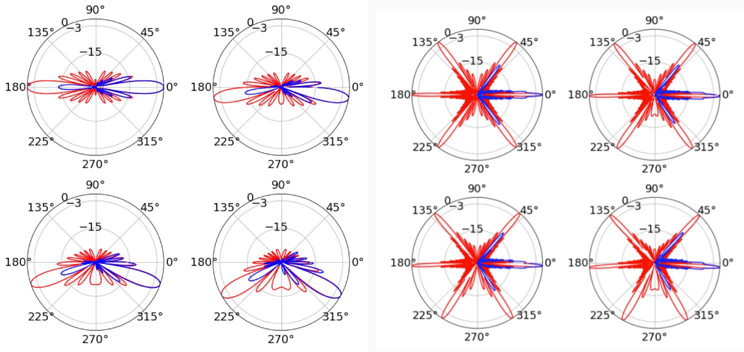


Figure 46: The radiation pattern of a radar phased array design. (Left) Four diffraction patterns for 2.5 GHz radio waves. (Right) Four diffraction patterns for 5.0 GHz radio waves.

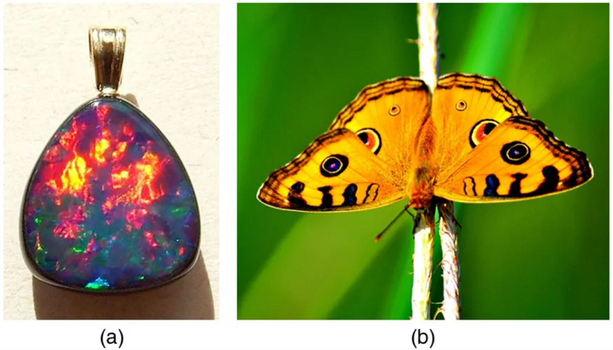


Figure 47: The optical light reflected from gemstones and butterfly wings bears the signature of a diffraction grating that responds to the wavelength.

Unit 5 Summary

1. Electromagnetic waves - Chapters 24.1 - 24.4
    - Maxwell's Equations
    - Electromagnetic wave production
    - Electromagnetic spectrum and energy
2. Geometric optics - Chapters 25.1 - 25.3, 25.6
    - Ray-tracing and reflection
    - Refraction
    - Lens optics

Unit 5 Summary

1. Wave optics - Chapters 27.1 - 27.3
    - Wave interference
    - Wave diffraction
    - Double slit experiments
2. Nuclear physics in medicine - 32.1 - 32.4
    - Diagnostics and medical imaging
    - Biological effects of ionizing radiation
    - Therapeutic uses of ionizing radiation
    - Food irradiation

Nuclear physics in medicine: Introduction to Nuclear Isotopes and Radioactive Decay

One example of *modern physics* deals with radioactivity. Radioactivity refers to the emission and detection of particles or radiation from **radioactive isotopes.** THese are unstable nuclei that release energy in the form of radiation.

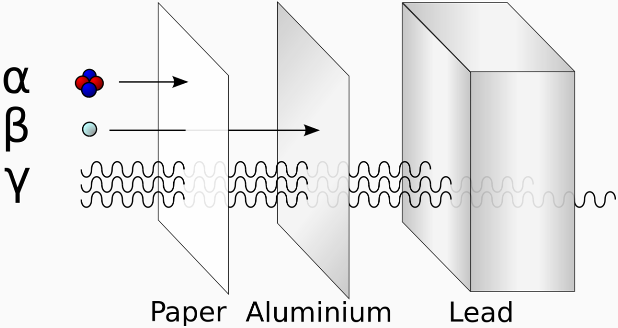


Figure 48: There are three basic types of radioactive decay: alpha particles, beta particles, and gamma particles.

Type	Rest Mass	Charge
α	3.727 GeV/c^2	+2
β	0.511 MeV/c^2	± 1
γ	0	0

Table 5: Radiation from nuclear decay is comprised of three particle types.

Let c be the speed of light, and let E be the energy of a particle at rest. Equation for the *rest mass* of a sub-atomic or radioactive particle:

$$E = mc^2 \tag{53}$$

The probability that **radioactivite particles** penetrate biological tissue or matter depends on *charge* and *energy*. One form of energy is the *rest mass*, given by $E = mc^2$.

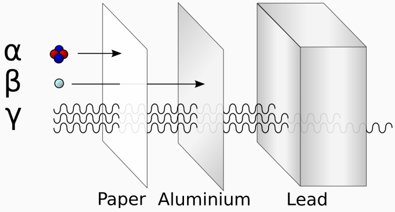


Figure 49: There are three basic types of radioactive decay.

Type	Cross-section (b)	Energy (MeV)
α	$\approx 10^6$	≈ 4
β	≈ 1	≈ 10
γ	≈ 0.2	≈ 1

Table 6: The **barn** is a unit of area, 10^{-24} cm^2.

Let $I(z)$ be the intensity of a beam of particles traveling a distance z through matter. Let n be the number density of atoms or scatterers in the matter. Let I_0 be the original intensity. The total cross-section σ_{tot} of the interactions relates these:

$$\boxed{I(z) = I_0 e^{-\sigma_{\text{tot}} nz}} \tag{54}$$

The probability that radioactivite particles penetrate biological tissue or matter depends on the *cross-section.*

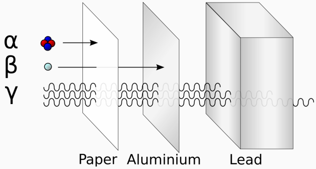


Figure 50: There are three basic types of radioactive decay.

$$I(z) = I_0 e^{-\sigma_{\mathrm{tot}} n z} \tag{55}$$

Suppose a beam of radioactive decay products with total cross-section σ_{tot} is incident on a block of metal with number density n. Show that half the intensity is gone at a distance $z_{1/2}$ into the metal, where

$$z_{1/2} = \frac{\ln 2}{\sigma n} \tag{56}$$

Materials with higher density tend to block more radioactive decay products.

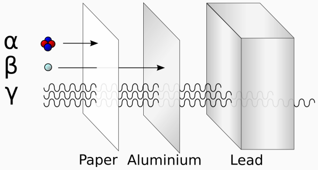


Figure 51: There are three basic types of radioactive decay.

$$z_{1/2} = \frac{\ln 2}{\sigma n} \tag{57}$$

For lead, $n = 3.3 \times 10^{22}$ cm^{-3}, assuming 1 electron ready to scatter a γ-ray per nucleus. If $\sigma = 1$ barn, 10^{-24} cm^2, how far in **lead** will the γ-rays travel before half of them are gone?

- A: 0.021 cm
- B: 0.21 cm
- C: 2.1 cm
- D: 21 cm

Materials with higher density tend to block more radioactive decay products.

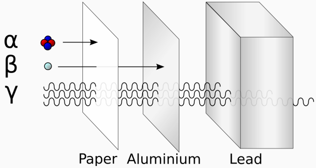


Figure 52: The cross-section is related to how far a stream of particles will penetrate into a substance.

Knowing that $z_{1/2}$ is inversely proportional to the cross-section, what would the result have been if the cross section was 1 Mb instead of 1 b?

- A: 2.1×10^{-5} m
- B: 2.1×10^{-3} cm
- C: 2.1×10^{-5} cm
- D: 2.1×10^{-4} cm

Materials with higher density tend to block more radioactive decay products.

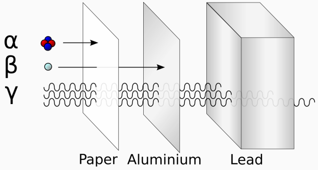


Figure 53: Particles with smaller cross-sections penetrate much deeper into materials.

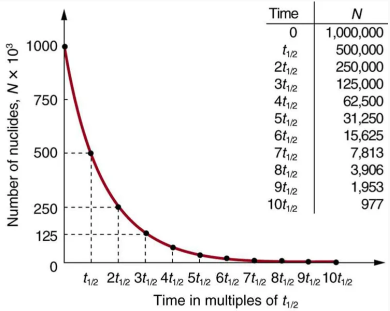
The half-life of a radioactive substance determines *how long* it takes for the number of radioisotopes to reduce by half.

$$N(t) = N_0 e^{-\lambda t} \tag{58}$$

Show that the **half-life** is

$$t_{1/2} = \frac{\ln 2}{\lambda} \tag{59}$$

The half-life of Carbon-14 is 5730 years, and Carbon 12 is the non-radioactive isotope.



Time	N
0	1,000,000
$t_{1/2}$	500,000
$2t_{1/2}$	250,000
$3t_{1/2}$	125,000
$4t_{1/2}$	62,500
$5t_{1/2}$	31,250
$6t_{1/2}$	15,625
$7t_{1/2}$	7,813
$8t_{1/2}$	3,906
$9t_{1/2}$	1,953
$10t_{1/2}$	977

Figure 54: Radioactive substances emit radiation, but have a finite amount of particles that can be radiated.

Group exercise: Note that if we know the half-life value, $t_{1/2}$, we can get λ from $\ln 2/\lambda$. Calculate the age of the Shroud of Turin given that the amount of Carbon-14 found in it is 92% of that in living tissue.

1. Determine λ.
2. Note that $N/N_0 = 0.92$.
3. Solve for t.

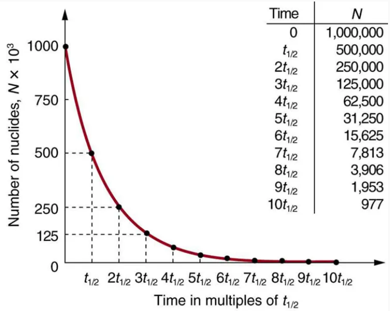


Figure 55: Given a finite amount of radioactive isotopes in a substance, and that decays are equally likely to occur in any given Δt, we can demonstrate exponential decay.

The decay rate can be derived from the $N(t)$ function:

$$R = \frac{\Delta N}{\Delta t} \tag{60}$$

$$R = -\lambda N_0 e^{-\lambda t} \tag{61}$$

$$R = -\left(\frac{\ln 2}{t_{1/2}}\right) N(t) \tag{62}$$

Intuitively, the *decays per second* are proportional to the number of remaining radioactive isotopes, N, and inversely proportional to the half-life. The SI unit of radioactive decay is the **becquerel**, or 1 decay/second.

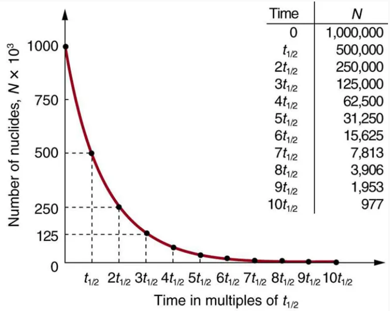


Figure 56: It takes about 10 half-lives to decay 10^6 isotopes.

Nuclear physics in medicine: Diagnostics and medical imaging

Nuclear physics, encompassing radioactive isotopes and radiation, is used in medicine.

Consider the *radioimmunoassay* (RIA) technique.

1. Irradiate a sample of antigen with a radioactive isotope so that the nuclei in the substance become radioactive.
2. Mix with a sample from a patient containing unknown number of antigens.
3. Introduce antibodies that bind to the antigen.
4. Separate bound and unbound antigens.
5. Measure decay rate of unbound antigens.
6. Work out the answer.

Nuclear physics, encompassing radioactive isotopes and radiation, is used in medicine.

Consider an analogy, to measure the number of fish in a pond:

1. Catch, tag, and release n fish, in a pond with unknown number of fish N.
2. Return to the pond, and catch m fish.
3. Count the *untagged fish*, m_u. $(m_u + m_t = m)$.
4. Assert that the uncaught fraction is a constant, $f = m_u/m = (N - n)/N$
5. Work out the answer:

$$N = \frac{n}{1 - f} \tag{63}$$

Nuclear physics, encompassing radioactive isotopes and radiation, is used in medicine.

$$N = \frac{n}{1-f} \tag{64}$$

If we tag $n = 67$ fish, and our untagged fraction is $f = 0.33$, how many fish are in the pond?

- A: 10
- B: 67
- C: 100
- D: 133

Nuclear physics in medicine: Diagnostics and medical imaging

How do we measure **radioactively tagged** molecules? There are several useful isotopes (see text). One is Technetium-99, a metastable isotope with a 6 hour half-life that emits a 0.142 MeV gamma ray.

- The γ rays penetrate Pb, but are eventually stopped.
- A Pb collimator accepts γ rays from the direction of the source.
- The γ rays cause an optical flash in the scintillators.
- The optical flash is amplified by the photomultipliers.

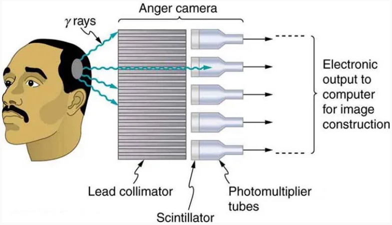


Figure 57: An gamma camera, or Anger camera. The γ rays form a high contrast 2D image.

How do we measure **radioactively tagged** molecules? One useful isotope is Technetium-99, a metastable isotope that emits γ rays.

- The SPET system provides 3D imaging using radioactive isotopes.
- The *spatial resolution* is about 1 cm, but the contrast is high.
- Technetium is convenient, but it is not an element found in the body naturally.

Spatial resolution: the extent to which objects can be distinguished in an image.

Figure 58: A single-photon emission computed tomography (SPET) system forms a 3D image of tissue tagged with radioactive isotopes.

How do we improve spatial resolution? If we can use a β^+ emitting isotope, then the corresponding e^+ will annihilate with an ambient e^-.

- The reaction is $e^- + e^+ \to \gamma + \gamma$.
- To *conserve momentum*, the γ rays are emitted back to back. Knowing this boosts resolution.
- The radioactive isotopes have to be β^+ emitters: C, N, O, and F are common.
- Natural elements bind into substances in the body.

The **spatial resolution** is improved to ≈ 0.5 cm.

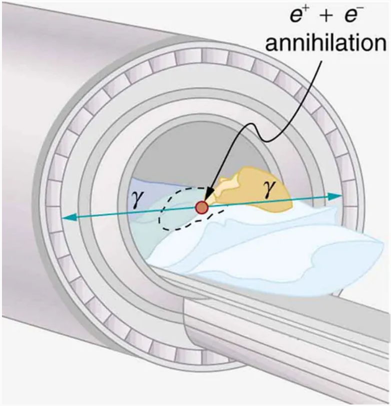


Figure 59: A positron emission tomography (PET) system.

Two γ rays, each with energy 1 MeV, have momenta vectors $\vec{p}_1 = 0\hat{i} + 0\hat{j} + p_{1,z}\hat{k}$ and $\vec{p}_2 = 0\hat{i} + 0\hat{j} + p_{2,z}\hat{k}$. If $p_{1,z} = -p_{2,z}$, and $p_{1,z} = 1$ MeV, what is the total momentum?

- A: $1\hat{z}$ MeV
- B: $-1\hat{z}$ MeV
- C: $0\hat{z}$ MeV
- D: $2\hat{z}$ MeV

Photons have momentum.

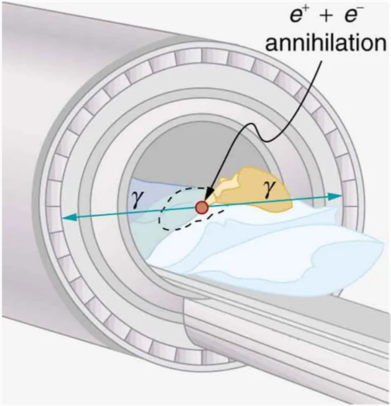


Figure 60: A positron emission tomography (PET) system.

Photons have momentum

Let h be **Planck's constant**. The momentum of a photon with wavelength λ is

$$p = \frac{h}{\lambda} \tag{65}$$

Planck's constant: 6.626×10^{-34} J s, or 4.136×10^{-15} eV s.

Why did we specify γ ray momentum in units of energy, MeV? We set $c = 1$ when dealing with speeds comparable to c. From Eq. 65:

$$[p] = \frac{[J][s]}{[m]} \tag{66}$$

$$c = 1, \quad [m] = [s] \tag{67}$$

$$[p] = [J] \tag{68}$$

What is the momentum of a photon in the beam of a standard HeNe red laser pointer? The wavelength is 633 nm.

- A: 1.5×10^{-7} eV s m^{-1}
- B: 2.5×10^{-8} eV s m^{-1}
- C: 6.5×10^{-9} eV s m^{-1}
- D: 6.5×10^{-9} eV

How many such photons would combine to have a total momentum of 10^{-2} kg m s^{-1}? Recall that 1 eV is 1.6×10^{-19} J.

- A: about 10^{24}
- B: about 10^{25}
- C: about 10^{26}
- D: about 10^{23}

In comparison to the wavelength of the photons in a HeNe laser, a typical γ ray wavelength is 10^5 times smaller. What is, therefore, the momentum of a typical γ ray?

- A: 1.5×10^{-2} eV s m^{-1}
- B: 2.5×10^{-3} eV s m^{-1}
- C: 6.5×10^{-4} eV s m^{-1}
- D: 6.5×10^{-4} eV

How many such photons would combine to have a total momentum of 10^{-2} kg m s^{-1}? Recall that 1 eV is 1.6×10^{-19} J.

- A: about 10^{19}
- B: about 10^{20}
- C: about 10^{21}
- D: about 10^{18}

In a typical laser flux of 10^{15} photons/second, it would take about 1 day for the γ rays to deliver 0.01 kg m s^{-1}. **Does radiation have an effect on health?**

Nuclear physics in medicine: Biological effects of ionizing radiation

Ionizing radiation affects molecules within cells, particularly DNA molecules.

Ionizing radiation is any radiation from (for example) radioactive isotopes that deposits enough energy in matter to free bound electrons from atoms.

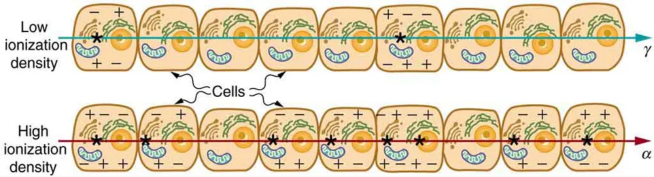


Figure 61: Different classes of ionizing radiation affect cell tissue differently.

Nuclear medicine: Biological effects of ionizing radiation

Ionizing radiation affects DNA molecules. The energy required to break a hydrogen bond is 21 kJ/mole (in H_2O). This is < 1 eV per atom.

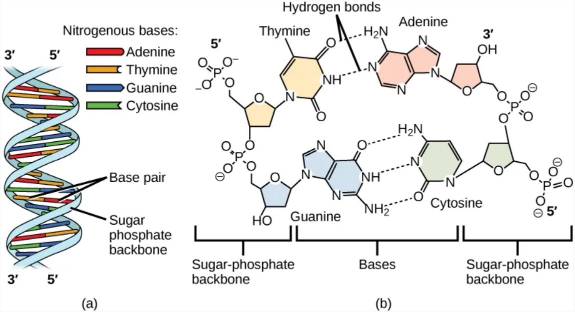


Figure 62: DNA is a molecule with double-helix structure, a sugar-phosphate backbone, and nucleic acid base-pairs bound with hydrogen bonds.

Nuclear medicine: Biological effects of ionizing radiation

Ionizing radiation affects DNA molecules. The energy required to break a hydrogen bond is 21 kJ/mole (in H_2O). This is < 1 eV per atom.

1. DNA is *self-healing*, containing code that checks for errors and code for repair.
2. A typical damage rate is ≈ 1 million errors per cell per day. The response rate depends on cell type and age.
3. Three outcomes after excess damage:
    - Senescence: a state of irreversible dormancy
    - Pre-programmed cell death
    - Unregulated cell division leading to tumors and cancers
4. Cancer is unregulated cell division, which can be both caused and stopped by radiation.

Unit	Definition	Purpose
1 rad	0.01 J/kg	Quantifies deposited ionization energy
1 Gray (Gy)	100 rad	Quantifies deposited ionization energy
RBE[1]	Varies by radiation	Quantifies ionization concentration
rem[2]	rad × RBE	Quantifies health effects of radiation
1 Sv (Sievert)	Gy × RBE, 100 rem	Quantifies health effects of radiation

Table 7: Common units involved in assessing ionizing radiation in the human body.

[1] Relative biological effectiveness
[2] roentgen equivalent man

Nuclear medicine: Biological effects of ionizing radiation

Type and energy of radiation	RBE[1]
X-rays	1
γ rays	1
β rays greater than 32 keV	1
βrays less than 32 keV	1.7
Neutrons, thermal to slow (<20 keV)	2–5
Neutrons, fast (1–10 MeV)	10 (body), 32 (eyes)
Protons (1–10 MeV)	10 (body), 32 (eyes)
α rays from radioactive decay	10–20
Heavy ions from accelerators	10–20

Figure 63: The RBE factors for different classes of radiation. Note that the most common radiation classes have RBEs of 1, so rem = rad, and 1 Sy = 1 Gy. The RBE for α radiation is significantly higher due to shorter, more concentrated range in tissue.

Quantity	SI unit name	Definition	Former unit	Conversion
Activity	Becquerel (bq)	decay/s	Curie (Ci)	$1\ \mathrm{Bq} = 2.7 \times 10^{-11}\ \mathrm{Ci}$
Absorbed dose	Gray (Gy)	1 J/kg	rad	$\mathrm{Gy} = 100\ \mathrm{rad}$
Dose Equivalent	Sievert (Sv)	1 J/kg × RBE	rem	$\mathrm{Sv} = 100\ \mathrm{rem}$

Figure 64: The distinction between **activity**, and absorbed dose and dose equivalent.

Dose in Sv [2]	Effect
0–0.10	No observable effect.
0.1 – 1	Slight to moderate decrease in white blood cell counts.
0.5	Temporary sterility; 0.35 for women, 0.50 for men.
1 – 2	Significant reduction in blood cell counts, brief nausea and vomiting. Rarely fatal.
2 – 5	Nausea, vomiting, hair loss, severe blood damage, hemorrhage, fatalities.
4.5	LD50/32. Lethal to 50% of the population within 32 days after exposure if not treated.
5 – 20	Worst effects due to malfunction of small intestine and blood systems. Limited survival.
>20	Fatal within hours due to collapse of central nervous system.

Figure 65: As the dose equivalent increases (measured in Sv), the health effects become qualitatively more dangerous.

What is the dose (in rad) if 100 mJ of total energy is deposited in the body of a 50 kg adult male?

- A: 0.2 rad
- B: 2.0 rad
- C: 20.0 rad
- D: 200.0 rad

If the above dose was instead concentrated in 1.0 kg of brain tissue, what would be the dose in rad?

- A: 0.1 rad
- B: 1.0 rad
- C: 10 rad
- D: 100 rad

If the RBE is 1 because we are dealing with x-rays, is the **equivalent dose** fatal? Answer: rem = rad, because RBE = 1, then convert to Sv and use Tab, 65.

What is the equivalent dose (in Sv) if 500 mJ of total energy is deposited by α radiation (RBE of 20) in the 1 kg lungs of an adult male?

- A: 0.1 Sv
- B: 1.0 Sv
- C: 10.0 Sv
- D: 50.0 Sv

What can be said about the probability of survival?

- A: $>$50 percent.
- B: $<$50 percent.
- C: It is unknown.
- D: It is 100 percent.

The person is at risk [for cancer] for at least 30 years after the latency period ... the overall risk of a radiation-induced cancer death per year per rem of exposure is about 10 in a million ... $10/10^6$ *rem*$^{-1}$ *yr*$^{-1}$.

If a person receives a dose of 1 rem, his risk each year of dying from radiation-induced cancer is 10 in a million and that risk continues for about 30 years. The lifetime risk is thus 300 in a million, or 0.03 percent. Since about 20 percent of all worldwide deaths are from cancer, the increase due to a 1 rem exposure is impossible to detect demographically. But 100 rem (1 Sv), which was the dose received by the average Hiroshima and Nagasaki survivor, causes a 3 percent risk, which can be observed in the presence of a 20 percent normal or natural incidence rate.

Suppose a woman receives an x-ray, and there is an incidental exposure of 25 rem to her breast tissue. What is her increased lifetime risk of breast cancer due to the exposure?

- A: 75 percent
- B: 7.5 percent
- C: 0.75 percent
- D: 25 percent

If we receive 3.5 mrem (milli-rem) per plane flight from cosmic rays, how many flights can we take before we reach 1 Sv?

- A: 17
- B: 33
- C: 63
- D: 286

Nuclear medicine: Biological effects of ionizing radiation

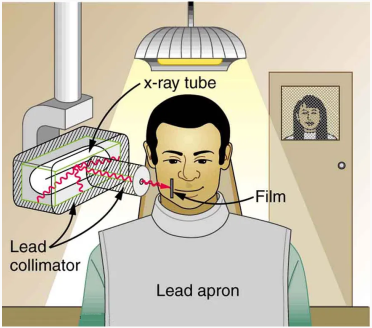


Figure 66: Suppose this dental x-ray source has an **activity** of 1 Curie, or 37×10^9 Bq. Recall that 1 Bq is 1 particle per second in the context of radiation.

Group exercise: Assume the energy of each x-ray is 70 keV, and the exposure lasts 3.5 seconds. Determine the effective dose in mrem to the patient, if the affected tissue has a mass of 1.5 kg.

Nuclear physics in medicine: Therapeutic uses of ionizing radiation

Nuclear medicine: Therapeutic uses of ionizing radiation

Ionizing radiation may be used to treat cancer.

- https://xkcd.com/881/
- https://xkcd.com/931/
- https://xkcd.com/933/
- https://xkcd.com/1217/

Figure 67: Que descanse en paz, suegro.

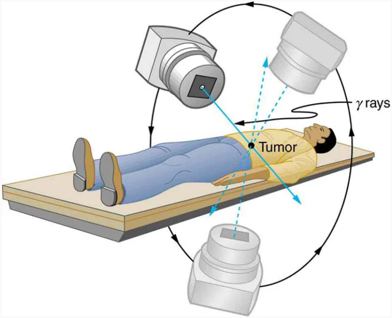


Figure 68: A γ ray cannon is rotated around the patient.

Treatments maximize the **therapeutic ratio**, the ratio of exposure to tumor versus exposure to healthy tissue.

Ionizing radiation may be used to treat cancer.

Type of Cancer	Typical dose (Sv)
Lung	10–20
Hodgkin's disease	40–45
Skin	40–50
Ovarian	50–75
Breast	50–80+
Brain	80+
Neck	80+
Bone	80+
Soft tissue	80+
Thyroid	80+

Figure 69: The equivalent doses of radiation for different cancer tissues.

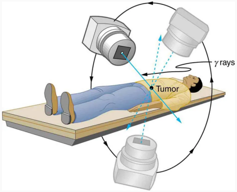


Figure 70: A γ ray cannon is rotated around the patient.

Treatments maximize the **therapeutic ratio**.

Nuclear medicine: Therapeutic uses of ionizing radiation

Recall that the intensity of ionizing radiation through matter goes as

$$I(z) = I_0 e^{-\sigma_{\mathrm{tot}} n z} \tag{69}$$

and that half of the intensity is stopped by the matter after a distance

$$z_{1/2} = \frac{\ln 2}{\sigma n} \tag{70}$$

Group exercise: The treatment in Fig. 71 involves ^{60}Co γ rays with energies of 1.173 and 1.332 MeV. Suppose that $I_0 = 10^{12}$ Bq cm^{-1}, and that $(\sigma n) = 0.1$ cm^{-1} in tissue, **implying that $5I_0$ (in Bq) are deposited in the first 10 cm of tissue**. Using the higher energy, compute the effective dose (RBE = 1) for a 90 second exposure to the 1 kg of tissue in the first 10 cm. *Answer:* $\approx$ *60 Sv.*

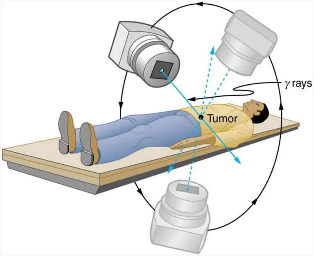


Figure 71: What will be the equivalent dose?

Instructor: demonstrate how we find $5I_0$ for the first 10 cm.

Nuclear physics in medicine: Food irradiation

Nuclear physics in medicine: Food irradiation

Ionizing radiation may be used to improve food quality.

- Food irradiation is not widely accepted.
- Pros: replaces pasteurization, preservatives, and insecticides
- Cons: potentially toxic residues, environmental hazards
- Could increase crop production by 25%, and reduce food spoilage by 25%.
- Currently, in USA we use it on spices, some fruits. Approved in 40 countries.

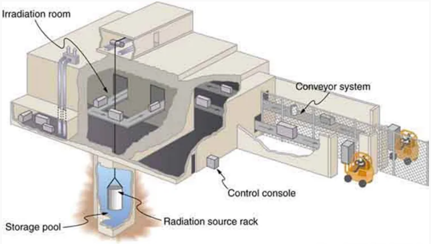


Figure 72: A γ ray food irradiation plant.

Types of radiation: ^{60}Co or ^{137}Cs γ rays (MeV), x-rays (100 keV), electrons (10 MeV).

Typical doses: *Low-dose*: 1000 Gy, kills many microorganisms on fresh produce. *High-dose*: 10,000 Gy, kills bacteria like salmonella, more required for fungi.

Group exercise: Suppose the water container for the γ rays is a cylindar with a radius of 5 m and height of 10 m. Suppose that the γ ray cross-section in water at 1 MeV is ≈ 1 barn. Use Eq. 69, with $\sigma = 1$ barn $= 10^{-28}$ m^2 and $n = 0.33 \times 10^{29}$ m^{-3} to compute the fraction of γ rays that survive after 5 m and 10 m of water.

Answers: for 5 m, the probability is 6×10^{-8}*. For 10 m, it is* 3×10^{-15}*. (Assume a food mass of 1 kg, and* 10^4 *Sv). When not in use, the* γ *source causes 180* γ *rays to escape into the rock after 10 m of water. Then the* γ *rays are blocked by rock.*

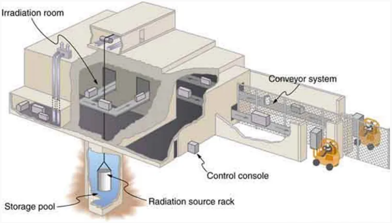


Figure 73: A γ ray food irradiation plant.

Conclusion

Unit 5 Summary

1. Electromagnetic waves - Chapters 24.1 - 24.4
    - Maxwell's Equations
    - Electromagnetic wave production
    - Electromagnetic spectrum and energy
2. Geometric optics - Chapters 25.1 - 25.3, 25.6
    - Ray-tracing and reflection
    - Refraction
    - Lens optics

Unit 5 Summary

1. Wave optics - Chapters 27.1 - 27.3
   - Wave interference
   - Wave diffraction
   - Double slit experiments
2. Nuclear physics in medicine - 32.1 - 32.4
   - Diagnostics and medical imaging
   - Biological effects of ionizing radiation
   - Therapeutic uses of ionizing radiation
   - Food irradiation