Katelyn Williams
20594548

Midterm 3

Dr. Jordan Hanson - Whittier College Dept. of Physics and Astronomy

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1 Memory Bank

1. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
2. $P = IV$ ... Relationship between power, current, and voltage.
3. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
4. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.
5. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ... Ampère's Law.
6. $\epsilon = -Nd\phi/dt$ ... Faraday's Law.
7. $\phi = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux.
8. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
9. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L. Self-inductance: $\Delta V = -L(dI/dt)$.
10. Units of inductance: V s A^{-1}, which is called a Henry, or H.
11. $B = \mu_0 nI$ ... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

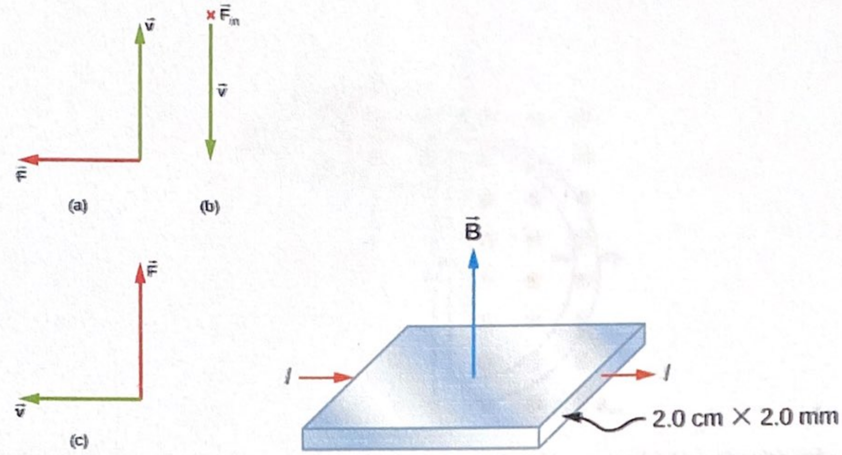

Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the ~~current~~ B-field given that F is the Lorentz force.

- a: into the page
- b: left
- c: out of the page

2. Consider Fig. 1 (right). **The Hall Effect.** An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that

$$\Delta V = \frac{B\Delta x I}{n q_e A} \qquad \Delta V = \frac{B\Delta x I}{n q_e A} \tag{1}$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28}$ m^{-3}, $A = 1$ mm^2, and q_e is the charge of an electron.

a) $v = \frac{E}{B}$

$E = \frac{V}{l}$ $V = Blv, \mathcal{E} = Blv$

$I = nq_eAv_d$

$v_d = \frac{I}{nq_eA}$

$F_B = q\vec{v}\cdot\vec{B}$

$q\vec{E} = qvB$

$E = vB$

$\boxed{v = \frac{E}{B}}$

b) $\Delta V = \frac{B\Delta x I}{nq_e}$

$E = \frac{\Delta V}{\Delta x}$, $v = \frac{E}{B}$

$E = vB$, $v = \frac{I}{nqA}$

$\Delta V - E\Delta x = vB\Delta x$

$\Delta V = \left(\frac{I}{nqA}\right)B\Delta x$

$\Delta V = (B\Delta x I)/(nqA)$

$\Delta V = \frac{.266}{3200} = \boxed{8.31 \times 10^{-5}\ V}$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^4 A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

$1.05 \times 10^4 (\pi)(.65 \times 10^{-15})^2 (2.5)$ $F = IL \times B$

$\boxed{= 3.48 \times 10^{-26}\ N \cdot m}$

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

a) $B = \mu_0 n I$

$n = 500$

$\mu_0 = 5000$

$I = .3A$

$B = (4\pi \times 10^{-7})(500)(.3)$

$\boxed{= 1.88 \times 10^{-4}\ T}$

b) $5000(1.88 \times 10^{-4})$

$\boxed{= 0.942\ T}$

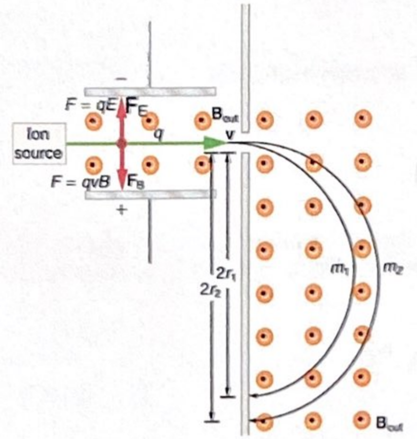

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \tag{2}$$

a) $V = \frac{E}{B}$, $F_{NET} = 0$, $F_B = qVB$

$F_{NET} = F_E + F_B$, $F_E = qE$ $\quad 0 = qE - qVB$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r?

$\frac{mv^2}{r}$, $qVB = \frac{mv^2}{r}$

$r = \frac{mE}{qB^2}$, $r = \frac{mv^2}{qVB}$

$x = \frac{E}{B}$

$r = \frac{m(\frac{E}{B})^2}{q(\frac{E}{B})B}$

$\boxed{r = \frac{mE}{qB^2}}$

$m = 16(1.67 \times 10^{-27}) = 2.67 \times 10^{-26}$ kg

$E = 10$ V/m

$q = 1.6 \times 10^{-19}$ $\quad B = .01$ T

$r = \frac{2.67 \times 10^{-26}(10)}{1.6 \times 10^{-19}(.01)^2}$

$\boxed{r = 1.67 \times 10^{-2} \text{ m}}$

4 Chapter 13: Electromagnetic Induction

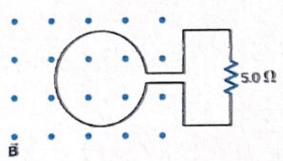

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0}(\sin(2\pi f t)) \tag{3}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1?

a) $\mathcal{E} = -N\frac{d\phi}{dt}$, $E = BLx$

$\mathcal{E} = -N\frac{d\phi}{dt} = -NBA = -NB(\pi r^2)$

$\mathcal{E} = -N\frac{\Delta B}{\Delta t}\pi r^2$

$\boxed{\mathcal{E} = -N\left(\frac{B_0}{T_0}\sin(2\pi f t)\right)(\pi r^2)}$

b) $\sin 2\pi(10^3)(0)$

$\sin 0 = 0$

$\boxed{0 = \mathcal{E}}$

c) $e = \pi(0.1)^2\left(\frac{0.1}{10^{-3}}\right)\sin 2\pi(10^3)(0.16 \times 10^{-3})$

$\boxed{= -0.055 \text{ V}}$

d) $I = \frac{e}{R}$

$\frac{.055}{5} = \boxed{0.011 \text{ A}}$

[1]Molecules that do not have this velocity will hit the sides of this portion of the instrument.

5 Chapter 14: Inductance

1. What is (a) the rate at which the current though a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

$$\frac{.150\text{V}}{.50\text{ H}} = \boxed{.3\text{ A/s}}$$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$I = .100$ A

$L = 2.0$ mH

$\mathcal{E} = 500$V

$$\frac{500\text{ V}}{2.0 \times 10^{3}\text{ H}} = 2.5 \times 10^{5}\text{ A/s}$$

$$\frac{.100\text{ A}}{2.5 \times 10^{5}\text{ A/s}} = \boxed{4.0 \times 10^{-7}\text{ s}}$$