Algebra-Based Physics-1: Mechanics (PHYS135A-01): Unit 2

Jordan Hanson

September 22, 2019

Whittier College Department of Physics and Astronomy

Week 3 Summary

Week 3 Summary

1. Working with vectors: displacement, velocity and acceleration
   - Breaking into components, graphical methods
   - Analytical methods
   - **Lab-activity: testing component independence**
2. Combining free-fall and vector components: projectile motion

Working with vectors: displacement, velocity and acceleration

Working with vectors: displacement, velocity and acceleration

In general, the displacement of an object depends on time:

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} \tag{1}$$

- $x(t)$ is the displacement in the x-direction
- $y(t)$ is the displacement in the y-direction
- $z(t)$ is the displacement in the z-direction

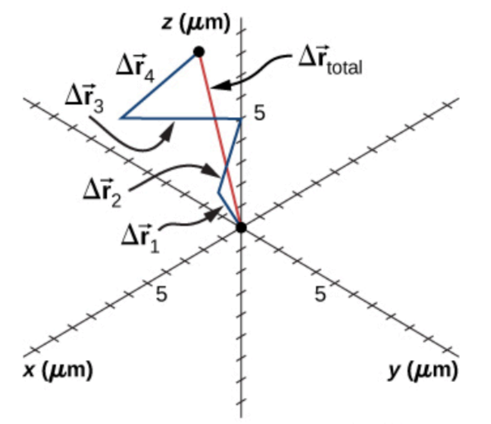

Figure 1: An example of a displacement vector at different moments in time.

Working with vectors: displacement, velocity and acceleration

The particle in Fig. 1 has four displacement vectors at four moments in time:

- $\vec{r}_1 = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k} \quad (\mu m)$ at t_1
- $\vec{r}_2 = -1.0\hat{i} + 0.0\hat{j} + 3.0\hat{k} \quad (\mu m)$ at t_2
- $\vec{r}_3 = 4.0\hat{i} + -2.0\hat{j} + 1.0\hat{k} \quad (\mu m)$ at t_3
- $\vec{r}_4 = -3.0\hat{i} + 1.0\hat{j} + 2.0\hat{k} \quad (\mu m)$ at t_4

What is the total displacement of the particle from the origin?

Working with vectors: displacement, velocity and acceleration

We can think of this type of problem as an accounting problem, lining up columns (units: μm):

t_{i}	$\vec{r}_{\mathrm{i}}(t_{\mathrm{i}})$	$x(t_{\mathrm{i}})$	$y(t_{\mathrm{i}})$	$y(t_{\mathrm{i}})$
t_1	$\vec{r}_1(t_1)$	2.0	1.0	3.0
t_2	$\vec{r}_2(t_2)$	-1.0	0.0	3.0
t_3	$\vec{r}_3(t_3)$	4.0	-2.0	1.0
t_4	$\vec{r}_4(t_4)$	-3.0	1.0	2.0
t_{total}	$\vec{r}_{\mathrm{total}}(t_{\mathrm{total}})$	2.0	0.0	9.0

Figure 2: Accounting for the different displacement components, in units of μm.

Working with vectors: displacement, velocity and acceleration

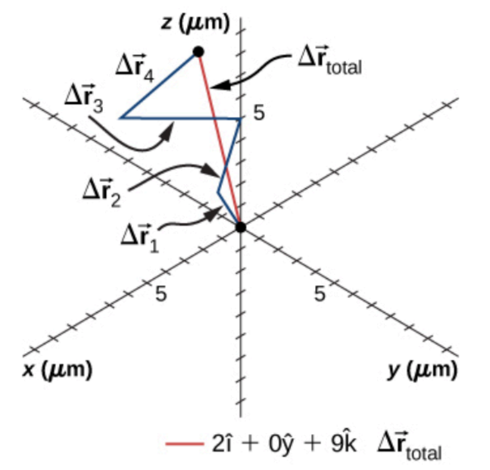

Figure 3: The total displacement of the particle is $\vec{r}_{\text{total}} = 2.0\hat{i} + 0.0\hat{k} + 9.0\hat{k} \quad (\mu m)$.

Working with vectors: displacement, velocity and acceleration

The 18th hole at Pebble Beach Golf Course is a dogleg to the left of length 496.0 meters. The fairway off the tee is taken to be the x direction. A golfer hits his tee shot a distance of 300 meters, corresponding to a displacement of $\vec{r}_1 = 300.0\hat{i} \quad (m)$, and then hits a second shot 189.0 meters with $\vec{r}_2 = 150.0\hat{i} + 80.0\hat{j} \quad m$. What is the final displacement from the tee?

- A: $\vec{r}_{\rm final} = 150.0\hat{i} + 80.0\hat{j} \quad (m)$
- B: $\vec{r}_{\rm final} = 450.0\hat{i} + 230.0\hat{j} \quad (m)$
- C: $\vec{r}_{\rm final} = 230.0\hat{i} + 0.0\hat{j} \quad (m)$
- D: $\vec{r}_{\rm final} = 450.0\hat{i} + 80.0\hat{j} \quad (m)$

Working with vectors: displacement, velocity and acceleration

If the first shot takes 5.0 seconds, the second shot takes 4.0 seconds, and the walking time in between the shots is 60.0 seconds, what is the average velocity vector for the ball after the two shots?

- A: $\vec{r}_{\text{final}} = 50.7\hat{i} + 11.6\hat{j} \quad (m/s)$
- B: $\vec{v}_{\text{final}} = 17.0\hat{i} + 80.3\hat{j} \quad (m/s)$
- C: $\vec{v}_{\text{final}} = 6.5\hat{i} + 1.2\hat{j} \quad (m)$
- D: $\vec{v}_{\text{final}} = 6.5\hat{i} + 1.2\hat{j} \quad (m/s)$

Working with vectors: displacement, velocity and acceleration

The prior problem indicates something you may already have guessed:

$$\vec{v}_{\rm avg}(t) = v_{\rm x}(t)\hat{i} + v_{\rm y}(t)\hat{j} + v_{\rm z}(t)\hat{k} = \frac{\Delta \vec{r}}{\Delta t} \tag{2}$$

- $v_{\rm x}(t)$ is the avg. velocity in the x-direction
- $v_{\rm y}(t)$ is the avg. velocity in the y-direction
- $v_{\rm z}(t)$ is the avg. velocity in the z-direction

In other words, we divide each displacement component by the time, to get a vector where each component is the average velocity in that direction. $\Delta \vec{r} = \vec{r}_{\rm f} - \vec{r}_{\rm i}$. **Professor: work several examples.**

Working with vectors: displacement, velocity and acceleration

A gamma ray is radiated from a radioactive source, and travels at the speed of light (0.3 m/ns) 60 degrees with respect to the x-axis, in the positive direction. A detection screen is 1.0 m to the right of the radioactive source. When does the gamma ray hit the screen?

- A: $20/\sqrt{3}$ ns
- B: $20/(3\sqrt{3})$ ns
- C: $20/3$ ns
- D: 10 ns

A person changes lanes on a highway. Her vehicle is traveling at 100 km/hr. She turns the wheel so that the car's velocity points 20 degrees from the direction down the highway. By what percentage must she increase her speed in order to maintain 100 km/hr *down the highway*?

- A: 1%
- B: 2%
- C: 6%
- D: 10%

Working with vectors: displacement, velocity and acceleration

In the kinematic description of motion, *we are able to treat the different components of motion separately.* In many cases, motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

Motions in displacement components are independent.

(Exception: non-conservative forces. More on this later.)

Projectile motion

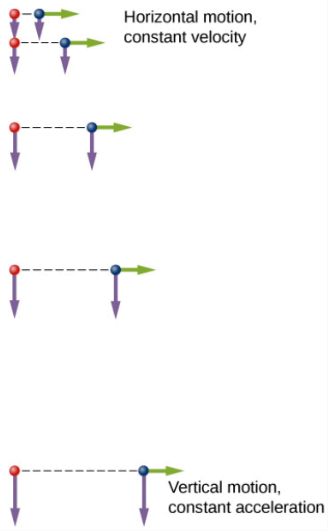

Figure 4: Independence of motion in two dimensions.

Projectile motion

Is this true? Figure 4 is testable by experiment.

Procedure:

1. Obtain two marbles, a meter stick, and a stopwatch.
2. Measure the height of the lab bench, Δx.
3. We are going to drop a marble from this height (Δx) and record the time. Show first algebraically that the predicted time for the marble to fall is $t = \sqrt{2\Delta x/g}$.
4. Measure t for several trials. Does it match the expected result $\sqrt{2\Delta x/g}$? What are sources of error?
5. Repeat the measurement, but **roll the marble off of the table at varying speed**. Does the average result for t change?

Combining free-fall and vector components: projectile motion

Projectile motion

We now have learned that (a) motions in displacement components are *independent*, and (b) when acceleration is in one direction (vertical) only, the motion is *projectile motion*. Our usual equations of motion for no acceleration (horizontal), and constant acceleration (vertical) apply *independently*:

$$y(t) = y_0 + v_{0,\mathrm{y}}t - \frac{1}{2}gt^2 \quad (3)$$

$$v_{\mathrm{y}}(t) = -gt + v_{0,\mathrm{y}} \quad (4)$$

$$v_{\mathrm{y}}^2 = v_{\mathrm{y},0}^2 - 2g(y - y_0) \quad (5)$$

$$x(t) = x_0 + v_{0,\mathrm{x}}t \quad (6)$$

$$v_{\mathrm{x}}(t) = v_{0,\mathrm{x}} \quad (7)$$

Projectile Motion

Projectile motion is a good topic to introduce the concept of *boundary conditions*. The *physics* of projectile motion is the same for all situations, but the *individual cases and numbers* might not be the same.

Suppose we are given the initial velocity and angle of a object that undergoes projectile motion. To use Eqs. 3-7, we need $v_{0,\mathrm{x}}$ and $v_{0,\mathrm{y}}$, the initial horizontal and vertical velocity components, respectively.

Projectile Motion

Suppose we are given the initial velocity and angle of a object that undergoes projectile motion. To use Eqs. 3-7, we need $v_{0,\mathrm{x}}$ and $v_{0,\mathrm{y}}$, the initial horizontal and vertical velocity components, respectively.

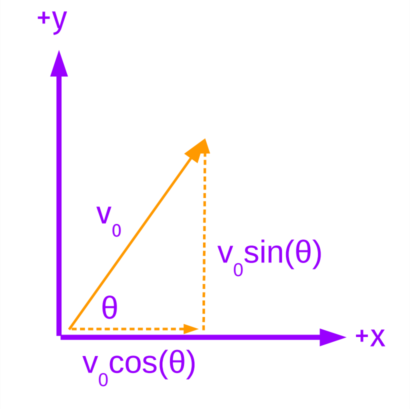

Figure 5: The initial velocity v_0 is broken into components.

Projectile Motion

During a fireworks display, a shell is shot into the air with an initial speed of 50 m/s, at an angle of 60° above horizontal. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. Calculate the height at which the shell explodes.

- A: 190 m
- B: 100 m
- C: 110 m
- D: 250 m

Projectile Motion

How much time passes between the launch and the explosion?

- A: 3.9 seconds
- B: 4.3 seconds
- C: 5.1 seconds
- D: 10.0 seconds

What is the horizontal displacement of the shell when it explodes?

- A: 108 meters
- B: 98 meters
- C: 98 degrees
- D: 150 meters

Projectile Motion

Let's try gaining visual intuition about projectile motion through the following program:

`http://galileoandeinstein.physics.virginia.edu/more_stuff/Applets/Projectile/projectile.html`

1. First, set air resistance to zero, at bottom right.
2. Make ten measurements of g by creating some projectile trajectories, and taking the ratio $g = v_{0,\mathrm{y}}^2/(2\Delta y)$. What value do you obtain, on average?
3. Now, set air resistance to $b/m \approx 0.02$, and repeat the ten measurements. What value do you obtain?
4. Explain why this value is smaller, larger, or equal to the first set of measurements.

Projectile Motion

Projectile motion in two dimensions, with constant acceleration in one dimension, produces *quadratic curves*. How do we obtain the trajectory, or $y(x)$ for these curves? Looking at the x-direction:

$$x = v_0 \cos(\theta) t \tag{8}$$

$$t = \frac{x}{v_0 \cos(\theta)} \tag{9}$$

Projectile Motion

Substituting in Eq. 9 for t into the equation for vertical displacement gives:

$$y(t) - y_0 = -\frac{1}{2}g\frac{x^2}{v_0^2\cos^2(\theta)} + \tan(\theta)x \tag{10}$$

$$y(t) - y_0 = -\left(\frac{g}{2v_0^2\cos^2(\theta)}\right)x^2 + \tan(\theta)x \tag{11}$$

$$y(x) = -kx^2 + bx + y_0 \tag{12}$$

In Eq. 12, we are simply saying that $y(x)$ is some quadratic. (It's still true that y and x are both functions of *time*, however, those functions of time are related).

Projectile Motion

A space explorer is on a moon around another planet, and wants to measure g. She tosses a pebble from an initial height of 2 meter, at an angle of 45 degrees above horizontal, with an initial velocity of 2 m/s. When it lands, the horizontal displacement is 10 meters. What is the gravitational acceleration g?

- A: 0.125 m/s^2
- B: 0.25 m/s^2
- C: 0.5 m/s^2
- D: 1.0 m/s^2

Projectile Motion

Other useful equations are for the *time-of-flight*, and the *range*, concepts we've already seen in several examples:

$$T_{\mathrm{tof}} = \frac{2v_0 \sin\theta}{g} \tag{13}$$

$$R = \frac{v_0^2 \sin 2\theta}{g} \tag{14}$$

Projectile Motion

Algebraic challenge: Show that the ratio of the range to the time is just the horizontal velocity, using the trigonometric identity $\sin(2\theta) = 2\sin\theta\cos\theta$.

Conclusion

Week 3 Summary

1. Working with vectors: displacement, velocity and acceleration
   - Breaking into components, graphical methods
   - Analytical methods
   - **Lab-activity: testing component independence**
2. Combining free-fall and vector components: projectile motion