Midterm for PHYS135B Module 2, Spring 2021

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1 Memory Bank

1. $V = (4/3)\pi r^3$ ... The volume of a sphere.
2. $m = \rho V$ ... The relationship between mass m, density ρ, and volume V.
3. $\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$ ... Coulomb Force
4. $k = 9 \times 10^9$ N C^{-2} m^2 ... Remember $k = 1/(4\pi\epsilon_0)$.
5. $q_e = 1.6 \times 10^{-19}$ C ... Charge of an electron/proton
6. Atomic mass: the number of grams per mole of a substance
7. $N_A = 6.03 \times 10^{23}$ ... Avagadro's number
8. $\vec{F} = q\vec{E}$ ... Electric field and charge
9. $\vec{E}(z) = \frac{\sigma}{\epsilon_0}\hat{z}$ ... Electric field of two oppositely charge planes each with charge density σ
10. $\epsilon_0 \approx 8.85 \times 10^{-12}$ F/m
11. $U = q\Delta V$ ... Potential energy and voltage
12. 1 eV: an electron-Volt is the amount of energy one electron gains through 1 V.
13. $V(r) = k\frac{q}{r}$ ... Voltage of a point charge
14. $\vec{E} = -\frac{\Delta V}{\Delta x}$ ... E-field is the slope or change in voltage with respect to distance
15. $V(x) = -Ex + V_0$ ... Voltage is linear between two charge planes
16. $Q = C\Delta V$ ... Definition of capacitance
17. $C = \frac{\epsilon_0 A}{d}$ ... Capacitance of a parallel plate capacitor
18. $C_{tot}^{-1} = C_1^{-1} + C_2^{-2}$ ... Adding two capacitors *in series*.
19. $C_{tot} = C_1 + C_2$ ... Adding two capacitors *in parallel*.
20. $i(t) = \Delta Q/\Delta t$ ... Definition of current.
21. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
22. $R_{tot}^{-1} = R_1^{-1} + R_2^{-1}$ ... Adding two capacitors *in parallel*.
23. $R_{tot} = R_1 + R_2$ ... Adding two capacitors *in series*.
24. $\Delta V = IR_{\text{tot}}$ ... Ohm's Law
25. $P = IV$ ... Relationship between power, current, and voltage.
26. $V_C(t) = \epsilon_1(1 - \exp(-t/\tau))$ ... voltage across the capacitor in an RC series circuit. The time constant is $\tau = RC$.
27. $i(t) = \frac{\epsilon_1}{R}\exp(-t/\tau)$ ... Current in an RC series circuit.
28. $i_{\text{in}} = i_{\text{out}}$ ... Kirchhoff's junction rule.
29. $\epsilon_1 + \epsilon_2 + \epsilon_3 + ... = 0$ ... Kirchhoff's loop rule.

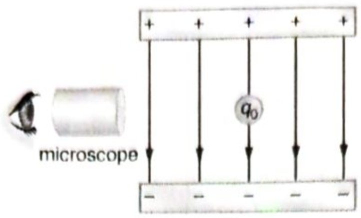


Figure 1: The classic Millikan oil drop experiment was a measurement of the charge of an electron.

2 Electric Charge and Electric Fields

1. **Scaling problem**: (a) Some point charge produces an E-field $E_C = 2.00 \times 10^{-3}$ V/m at a distance of 1 mm. What is the value of E_C at 5 mm produced by the same charge? (b) A 1 μC charge produces an E-field $E_C = 8.00 \times 10^{-3}$ V/m at some distance. What is the value of E_C at the same distance if the charge is 3 μC?

a) $\vec{F} = k\frac{q_1 q_2}{r^2} = E_c$

$E_c = k\frac{q}{r^2}$

$\frac{E_c r^2}{k} = q$

$\frac{2.00\times10^{-3}\text{V/m}(0.001\text{m})^2}{9\times10^9\text{ N C}^{-2}\text{m}^2} = 2.22\times10^{-19}\text{C} = q$

$E_c = \frac{(9\times10^9\text{ N C}^{-2}\text{m}^2)(2.22\times10^{-19}\text{C})}{(0.005\text{m})^2}$

$\boxed{= 7.992\times10^{-5}\text{ V/m}}$

b) $E_c = (8.00\times10^{-3}\text{ V/m})(3)$

$\boxed{= 0.024\text{ V/m}}$

2. The classic Millikan oil drop experiment was the first to measure the electron charge. Oil drops were suspended against the gravitational force by an electric field. (See Fig. 1.) Suppose the drops have a *mass* of 4×10^{-16} kg, and the E-field is oriented downward, and has a value of 6131.25 N/C. With this exact value, the drops remain suspended in air. (a) How many electrons are on the drops? (b) Suppose a cosmic ray comes along and removes an electron from a droplet. What will the acceleration of the droplet be?

a) $W = (4\times10^{-16}\text{kg})(9.81\text{ m/s}^2)$

$W = 3.924\times10^{-15}\text{ N}$

$3.924\times10^{-15}\cancel{\text{N}} \times \frac{1\cancel{\text{C}}}{6131.25\cancel{\text{N}}} \times \frac{1e^-}{1.6\times10^{-19}\cancel{\text{C}}} = \boxed{4e^-}$

b) $3e^- \times \frac{1.6\times10^{-19}\text{C}}{1e^-} \times \frac{6131.25\text{ N}}{1\text{C}}$

$= 2.943\times10^{-15}\text{ N}$

$F = ma$

$2.943\times10^{-15}\text{N} = (4\times10^{-16}\text{kg})a$

$\boxed{a = 7.3575\text{ m/s}^2}$

3 Potential Energy and Voltage, Capacitors

1. A *mass spectrometer* is a device used to accelerate ions to determine atomic masses of chemicals. Suppose two conducting plates with potential difference $\Delta V = 4$ kV are used to accelerate both hydrogen ions and helium ions. Hydrogens have charge $+1q_e$, and helium ions have charge $+2q_e$. (a) What is the total kinetic energy (in electron-volts) gained by the hydrogens and heliums? (b) If the plate separation is $\Delta x = 5$ cm, what is the electric field value? *Hint: think of the E-field as the slope of voltage.*

a) $PE = -KE$

$q\Delta V = -q\Delta V$

$H = -(1)(4000\text{V})$ $\quad$ $He = -(2)(4000\text{V})$

$= -4000\text{eV}$ $\quad$ $= -8000\text{eV}$

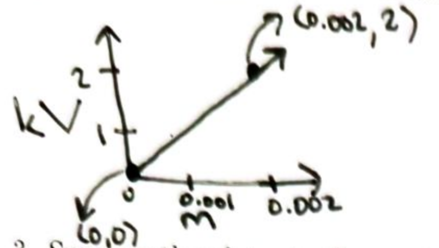
$\boxed{-12{,}000\text{eV} = KE}$ $\boxed{12{,}000\text{eV is gained}}$

b) $\vec{E} = -\frac{\Delta V}{\Delta x}$

$= -\frac{4\text{ kV}}{5\text{ cm}} = -\frac{4000\text{ V}}{0.05\text{m}}$

$= \boxed{-80{,}000\text{ V/m}}$

2. Suppose a parallel plate capacitor has an internal E-field of 1 kV/m, and a plate separation of 2 mm. Draw the voltage as a function of distance between the negative and the positive plates. Make sure to label the axes with proper units, and mark the x-value of each plate. What is the y-intercept of this function?



y-int = 0

3. Suppose the plates in the previous problem have an area of 1 cm^2. (a) What is the capacitance of the system? (b) How much energy (in Joules) is stored in this capacitor if the voltage is 5 V?

$V = J/C \Rightarrow J = CV$

$F = C/V$

a) $C = \frac{\varepsilon_0 A}{d}$

$= \frac{(8.85\times10^{-12}\,F/m)(1\times10^{-4}\,m^2)}{0.002\,m}$

$\boxed{= 4.425\times10^{-13}\,F}$

b) $U = \frac{1}{2}CV^2$

$= \frac{1}{2}(4.425\times10^{-13}\,C/V)(5V)^2$

$= 5.5313\times10^{-12}\,C\cdot V$

$\boxed{= 5.5313\times10^{-12}\,J}$

4. Suppose we need a system that can store more energy for the same voltage (in other words, more capacitance). (a) Should we connect an identical capacitor to the first *in series* or *in parallel*? (b) What is the total energy stored in three capacitors connected in parallel, if each capacitor is identical to the one in the prior problem?

a) We should connect an identical capacitor to the first in parallel because this will result in increasing capacitance.

b) $U = 5.5313\times10^{-7}\,J$

$C_{tot} = C_1 + C_2 + C_3$

$= (5.5313\times10^{-12}J) + (5.5313\times10^{-12}J) + (5.5313\times10^{-12}J)$

$\boxed{= 1.6594\times10^{-11}\,J}$

4 Current, Resistance, and DC Circuits

1. When dealing with AA batteries, we can either connect them "end to end" (in series), or in parallel (see Fig. 2). Suppose that the internal resistances of the batteries $r_1 = r_2 = 2\Omega$, and that the emfs of the two batteries are both $\epsilon_1 = \epsilon_2 = 1.5$ V. Finally, let $R = 50\Omega$. Suppose R represents a small device that will work at 1.5 V or 3 V (a child's toy, an old CD player, a computer mouse). (a) Using Kirchhoff's rules, find the current through R for the serial case (3 V) (Fig. 2, left), and the parallel case (Fig. 2, right). (b) What is the power consumption in each case? (c) Check your calculations of current using the PhET DC circuit construction modeling kit.

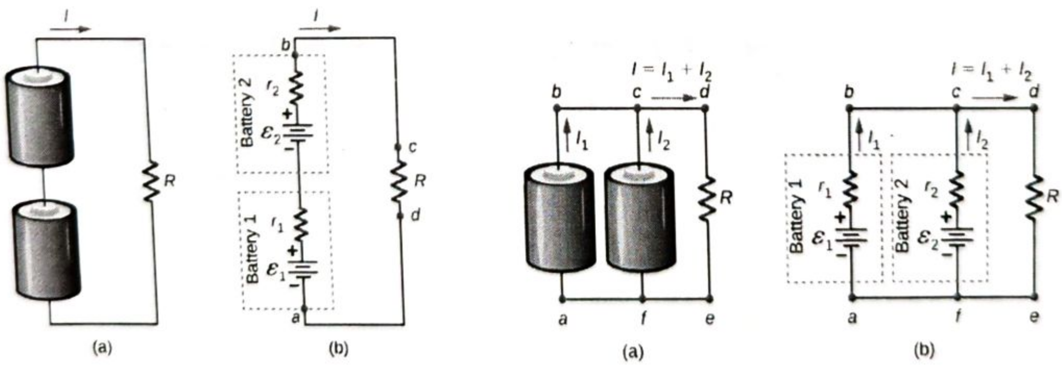


Figure 2: Two ways of connecting batteries. (Left) In series. (Right) In parallel.

Parallel

a) $R_{tot}^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1}$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{50}$

$= 1.02\,\Omega$

$I = \frac{\Delta V}{R_{tot}} = \frac{3V}{1.02\,\Omega} = \boxed{2.941\,A}$

Series

$R_{tot} = R_1 + R_2 + R_3$

$= 2 + 2 + 50$

$= 54\,\Omega$

$I = \frac{3V}{54\,\Omega} = \boxed{0.0556\,A}$

b) Parallel

$P = IV$

$P = (2.941A)(3V)$

$\boxed{= 8.824\,W}$

Series

$P = IV$

$P = (0.0556A)(3V)$

$\boxed{P = 0.1667\,W}$

2. Recall the PhET activity in which we covered nerve stimulation as chemical-driven capacitors. Think of the voltage as a signal versus time that flows down the nerve. If you stimulate the nerve in this calculation, (a) what is the pulse width, in milliseconds? (b) What is the peak-to-peak voltage (greatest voltage minus least) in millivolts? **Bonus:** (c) Estimate the time required for a nerve signal to travel from your toe to your spinal chord.

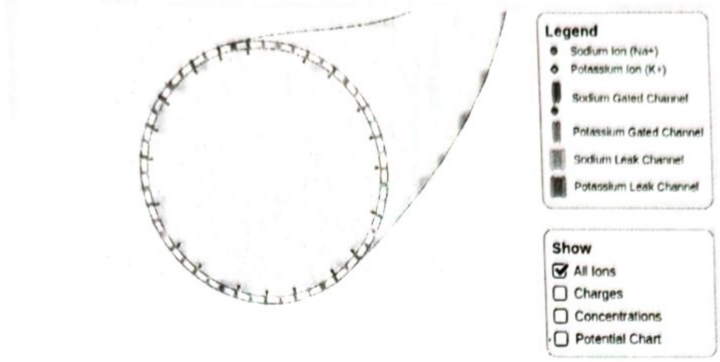


Figure 3: Recall the molecular model of the nerve membrane, and the voltage generated across it by chemical valves.

a) 4ms - 2ms

$\boxed{= 2 \text{ ms}}$

b) 35mV - (-75mV)

= 35mV + 75mV

$\boxed{= 110 \text{ mV}}$

c) Average distance from toe to spinal cord = 1 meter
Average velocity for any given nerve = 50 m/s

$$\frac{1 \text{ m}}{50 \text{ m/s}} \approx 0.02 \text{ s}$$

Approximate time for a nerve signal to travel from your toe to your spinal cord is about 0.02 s, or 20 ms.