

ALGEBRA-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND MODERN PHYSICS (PHYS135B-01): UNIT 3

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UNIT 2 REVIEW

Reading: Chapters 20 and 21

1. Current, Ohm's Law, resistors and conductors
2. DC circuits I
3. Nerve signals

UNIT 2 REVIEW PROBLEMS

UNIT 2 REVIEW PROBLEMS

Consider a proton with mass m and charge q_p . The proton is located in a region with electric field E , and it has initial velocity v_i at time $t = 0$. What is the acceleration of the proton? What is the proton velocity v_f after a time t ?

- A: $a = \frac{m}{q_p}E$, $v_f = v_i + t$
- B: $a = \frac{m}{q_p}E$, $v_f = v_i + \frac{m}{q_p}Et$
- C: $a = \frac{q_p}{m}E$, $v_f = v_i + \frac{q_p}{m}Et$
- D: $a = \frac{q_p}{m}E$, $v_f = v_i + t$

Consider a 12V battery driving current through a system with a total resistance of $1\text{ k}\Omega$. What is the current? What is the power consumption?

- A: 12 mA, 144 W
- B: 144 mA, 12 mW
- C: 12 mA, 144 mW
- D: 14.4 A, 144 W

SUMMARY

Reading: Chapter 22

This week: 22.1-22.5

1. Magnets and magnetic fields
2. Force on a moving charge in a magnetic field
3. Applications

This weekend: 22.6-22.10

1. The Hall effect
2. Magnetic forces on conductors
3. Torque on a current loop
4. Ampère's Law

MAGNETS AND MAGNETIC FIELDS

Introductory video on the origin of magnetic fields and forces they exert on charge:

<https://www.youtube.com/watch?v=s94suB5uLWw>

What is a cross-product and how does it work?

Computing the cross product [\[edit \]](#)

Coordinate notation [\[edit \]](#)

The [standard basis](#) vectors \mathbf{i} , \mathbf{j} , and \mathbf{k} satisfy the following equalities in a right hand coordinate system:

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}$$

$$\mathbf{j} \times \mathbf{k} = \mathbf{i}$$

$$\mathbf{k} \times \mathbf{i} = \mathbf{j}$$

which imply, by the [anticommutativity](#) of the cross product, that

$$\mathbf{j} \times \mathbf{i} = -\mathbf{k}$$

$$\mathbf{k} \times \mathbf{j} = -\mathbf{i}$$

$$\mathbf{i} \times \mathbf{k} = -\mathbf{j}$$

The definition of the cross product also implies that

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0} \text{ (the zero vector).}$$

Figure 1: The cross-product is a way of multiplying unit vectors.

Professor: several examples on board.

Let $\vec{v} = 2\hat{i}$ and $w = -2\hat{j}$. What is $\vec{v} \times \vec{w}$?

- A: $-4\hat{k}$
- B: $4\hat{k}$
- C: $-2\hat{i}$
- D: $2\hat{j}$

Let $\vec{v} = 3\hat{j}$ and $w = 5\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $15\hat{i}$
- B: $5\hat{j}$
- C: $3\hat{i}$
- D: $15\hat{k}$

Let $\vec{v} = 3\hat{i} \times 3\hat{j}$ and $w = 2\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $-6\hat{j} + 6\hat{k}$
- B: $-6\hat{j} + 6\hat{i}$
- C: $6\hat{j} + 6\hat{i}$
- D: $6\hat{k} + 6\hat{i}$

Group exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} \quad (1)$$

$$\vec{w} = 4\hat{j} - 4\hat{i} \quad (2)$$

$$\vec{v} \times \vec{w} = ?? \quad (3)$$

Group exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} + \hat{k} \quad (4)$$

$$\vec{w} = 4\hat{j} - 4\hat{i} - \hat{k} \quad (5)$$

$$\vec{v} \times \vec{w} = ?? \quad (6)$$

The Lorentz Force

Let a particle with charge q and velocity \vec{v} move through a *magnetic field* \vec{B} . The **Lorentz force** on the charged particle is

$$\vec{F}_L = q\vec{v} \times \vec{B} \quad (7)$$

As a helpful memory tool, we have the right-hand rule to remember the direction of the cross-product. The units of the magnetic field are the Telsa, after Nikola Tesla. We also have the Gauss which is 10^{-4} Tesla.

MAGNETS AND MAGNETIC FIELDS

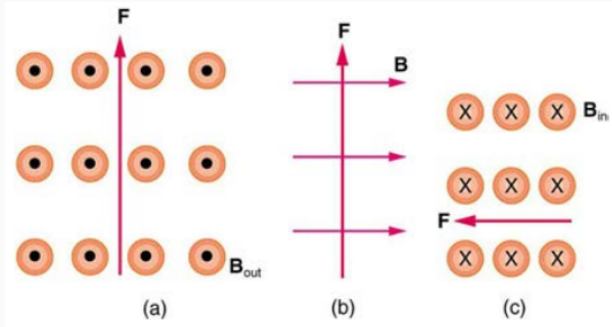


Figure 2: Three different magnetic field and charge scenarios. The vector \vec{F} is the direction of the Lorentz force, and the magnetic field is uniform. A dot indicates that the magnetic field is coming out of the page, and an x indicates that the field is going into the page.

MAGNETS AND MAGNETIC FIELDS

In which of the diagrams is a positively charged particle moving to the left?

- A: A
- B: B
- C: C
- D: Double WAT

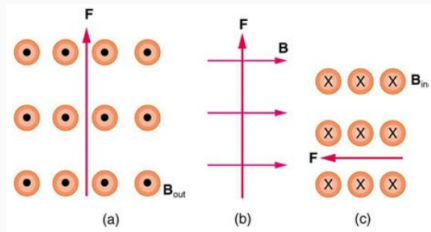


Figure 3: Three different magnetic field and charge scenarios.

MAGNETS AND MAGNETIC FIELDS

In which of the diagrams is a positively charged particle moving upwards?

- A: A
- B: B
- C: C
- D: Double WAT

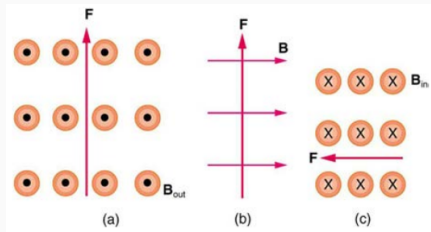


Figure 4: Three different magnetic field and charge scenarios.

MAGNETS AND MAGNETIC FIELDS

In which of the diagrams is a negatively charged particle moving into the page?

- A: A
- B: B
- C: C
- D: Double WAT

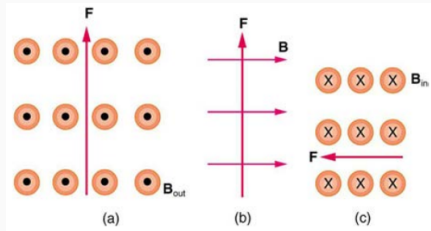


Figure 5: Three different magnetic field and charge scenarios.

MAGNETS AND MAGNETIC FIELDS

In which of the diagrams is a negatively charged particle moving to the right?

- A: A
- B: B
- C: C
- D: Double WAT

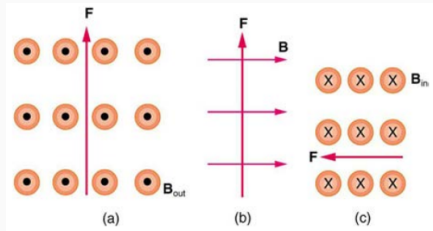


Figure 6: Three different magnetic field and charge scenarios.

A theorem for the magnitude of the cross-product: Let \vec{a} and \vec{b} be vectors and θ be the angle between them. The magnitude of the cross-product is

$$|\vec{a} \times \vec{b}| = ab \sin \theta \quad (8)$$

Thus, the magnitude of the Lorentz force is

$$F_L = qvB \sin \theta \quad (9)$$

The angle θ is between the velocity and the magnetic field.

A cosmic ray proton moving toward the Earth at 3×10^6 m/s experiences a magnetic force of 2×10^{-17} N. What is the strength of the magnetic field if there is a 45 degree angle between it and the proton's velocity? (Remember that q for a proton is 1.6×10^{-19} C).

- A: 0.1 Gauss
- B: 0.6 Gauss
- C: 1 Gauss
- D: 6 Gauss

Other examples:

1. Magnetic fields do no work
2. $v = E/B$
3. q/m circle (potential demonstration)

22 MAGNETISM

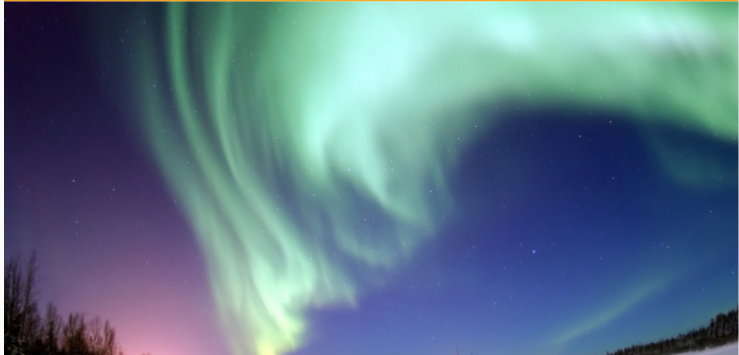
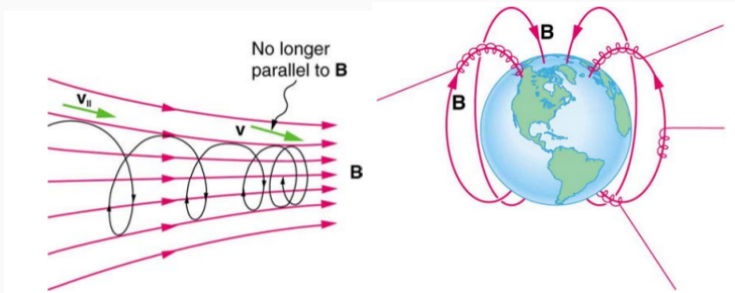


Figure 7: The aurora borealis, or northern lights.

MAGNETS AND MAGNETIC FIELDS

A cool talk on the aurora borealis:
<https://youtu.be/czMh3BnHFHQ>



One un-explained piece: what does it mean for the electrons and protons to *high-five* the neutral oxygen and nitrogen atoms?

FORCE ON A MOVING CHARGES AND CURRENT CARRYING CONDUCTORS

Introduction to magnetic forces on current-carrying conductors:

<https://youtu.be/5fqwJyt4Lus>



Charge to mass ratio, and *cyclotrons*.

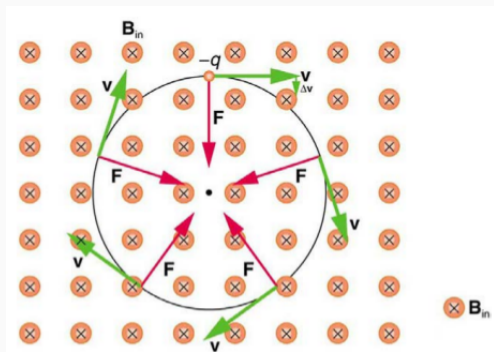


Figure 8: The centripetal force is provided by the Lorentz force.

Group exercise: Suppose we place a gass of unknown particles in the uniform magnetic field of Fig. 8 and get them moving in a circle. The angular frequency is 95.5788 MHz, and the B-field is exactly 1.0 T. (a) Show that the relationship between the angular frequency ω , the B-field strength B , and the q/m ratio is $q/m = \omega/B$. (b) With which particle are we dealing? Is it a proton, a neutron, an electron, or an alpha particle? (*Hint: use the angular frequency and magnetic field to obtain the q/m ratio, and then look up the masses and charges of these particles to make the determination*).

Two unknown particles are moving in helices through a region where there is a magnetic field. Both move clockwise as you observe them. One particle spins around the field line with higher frequency compared to the other. Which of the following is true?

- A: The particles are identical; they just had different initial conditions.
- B: The charge is smaller for the particle with the larger frequency.
- C: The mass is larger for the particle with the larger frequency.
- D: The q/m ratio is larger for the particle with the larger frequency.

Which of the following is true of a charged particle moving in a helical fashion through a magnetic field?

- A: Raising the strength of the B-field increases the period
- B: Raising the strength of the B-field increases the frequency
- C: The particle has a constant velocity parallel to the field
- D: B and C

Two unknown particles are moving in helices through a region where there is a magnetic field. One moves clockwise as you observe it, and the other moves counter-clockwise, and the helices have about the same radius. Which of the following is true?

- A: The particles have identical charge.
- B: The particles have identical charge, and the same mass.
- C: The particles have opposite charge, and the same mass.
- D: The particles have different masses.

Magnetic containment and *tokamaks*.

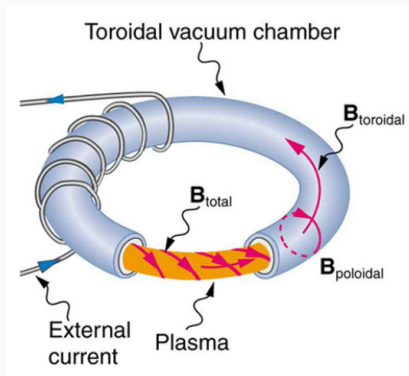


Figure 9: The tokamak contains high-energy plasma, e.g. a charged gas of electrons and protons.

Group exercise: Suppose $\approx 6.7 \times 10^{12}$ protons are circling through the tokamak at a rate of 20 MHz, and the radius of the circle of the tokamak is 20.0 meters. The radius of the *pipe* of the tokamak is 2.0 m. (a) Work out the *current* I going through the tokamak. (b) Using $B = \mu_0 I / (2\pi r_{\text{pipe}})$, compute the poloidal magnetic field of the current.

PHET: ELECTROMAGNETS

Follow the link:

[https://phet.colorado.edu/en/simulation/
magnets-and-electromagnets](https://phet.colorado.edu/en/simulation/magnets-and-electromagnets)

1. Click on the electromagnet tab, and hide the field and compass using the menu in the upper right. Also, display the magnetometer.
2. Place the magnetometer to one side of the *solenoid*. Work out the relationship between the magnetic field strength and voltage. Is it linear, quadratic, or something else?
3. Assuming the circuit has some fixed resistance, is the relationship between current and field strength linear? Why or why not?
4. Now fix the voltage and vary the number of loops. Work out the relationship between magnetic field strength and loop number. Is it linear, quadratic, or something else?
5. Propose an equation for B_{solenoid} based on the prior measurements.

Electromagnets.

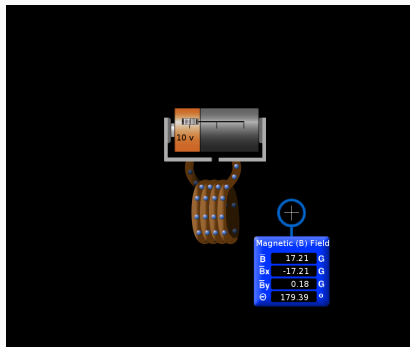


Figure 10: The electromagnet converts charge to magnetic field strength.

The result should be something like:

$$B \propto NI \quad (10)$$

$$B = \mu_0 n I \quad (11)$$

- n : Number of turns per unit length (because we can always change the density and get a different answer).
- I : Current
- μ_0 : Magnetic permeability of free space (solenoid is empty).

MOTORS

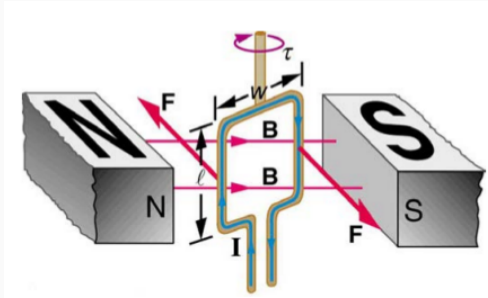


Figure 11: In a loop of current in a uniform magnetic field, we find forces going the opposite directions.

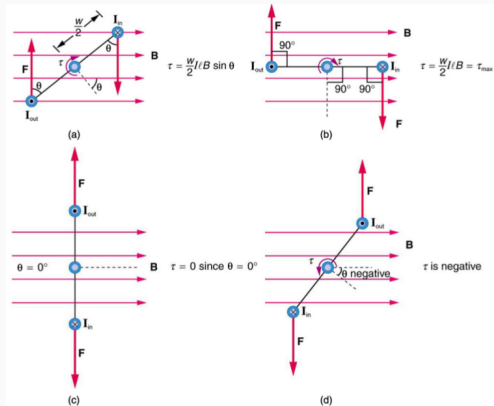


Figure 12: This leads to torque.

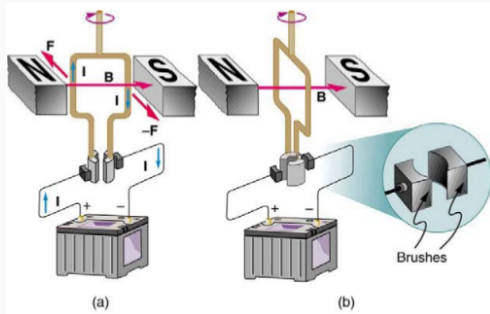


Figure 13: Torque can be used to drive a motor.

Let the number of loops in the coil be N , the current be I , the area of the loops be A , and the magnetic field be B . The angle between the force and loops is θ . The magnitude of the torque τ is

$$\tau = NIAB \sin \theta \quad (12)$$

At what angle between the loops and the B field is the torque maximized?

- A: 0 degrees
- B: 45 degrees
- C: 90 degrees
- D: 135 degrees

Which of the following would boost the torque of a motor?

- A: Increasing the B-field magnitude
- B: Decreasing the number of loops
- C: Increasing the number of loops
- D: A and C

Suppose $I = 10$ amps, $B = 0.01$ T, $N = 200$, and the loops have a common radius of 5 cm. **Group exercise:** what is the maximum torque?

THE HALL EFFECT

THE HALL EFFECT

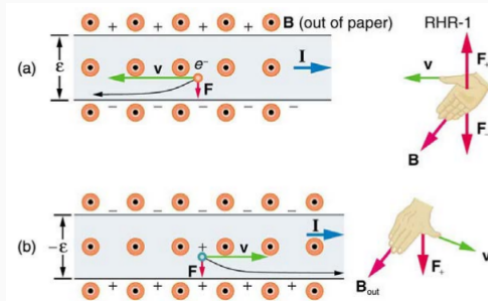


Figure 14: Diagram of the Hall effect. The Hall emf reveals the sign of the moving charges.

The Hall emf is

$$\epsilon = Blv \quad (13)$$

The B is the magnetic field, l is the length across the region where charge is flowing, and v is the velocity of the charges.

Group board exercise: Let n be the charge number density in a conductor. Let q be the charge of an electron. Let A be the cross-sectional area, and I be the current. Recall that the *drift velocity of charges in a wire* is given by the equation

$$v_d = \frac{I}{nqA} \quad (14)$$

The Hall emf is $\epsilon = Blv$. Let $v = v_d$, and substitute the first equation into the second equation, and solve for n . Choose reasonable numbers for the current, diameter of a metal wire, and assume a uniform 0.001 T magnetic field is being created around the wire. Assume a drift velocity of about 1 mm/sec, and solve for n .

AMPÈRE'S LAW

AMPÈRE'S LAW

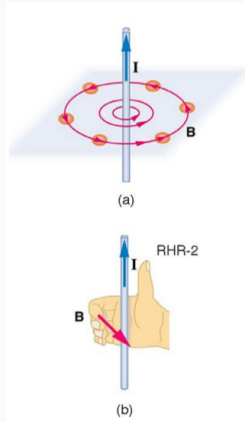


Figure 15: Magnetic fields create currents! To remember the direction, use your right hand.

Ampère's Law states that the current produced by a long straight wire is

$$B = \frac{\mu_0 I}{2\pi r} \quad (15)$$

The current is I , the distance from the wire is r , and $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$.

Group exercise: Suppose we have a wire carrying 1 A, and we are 1 cm away from it. What is the magnetic field? What is the magnetic field if another wire is located 1 cm away from us, but carries -1 A? Should the fields add or subtract?

<https://youtu.be/1JZLKvW00ks>

AMPÈRE'S LAW

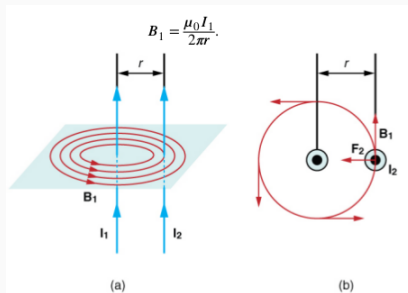


Figure 16: Definition of the amp is derived from this setup.

The B-field at wire 2 due to wire 1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \quad (16)$$

The force on wire 2 is

$$F_2 = I_2 l B_1 \quad (17)$$

Dividing both sides by l and substituting in Eq. 16, we have

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r} \quad (18)$$

Group board exercise: let $I_1 = I_2 = 1.0$ A, and $r = 1$ meter. What is the force per unit length F/l ? (Recall that $\mu_0 = 4\pi \times 10^{-7}$ T m/A by definition).

MAGNETIC FIELDS AND WORK

Notice from the Lorentz force that magnetic fields do not perform work:

$$\vec{F} = q\vec{v} \times \vec{B} \quad (19)$$

$$\vec{v}t = \vec{x} \quad (20)$$

$$\vec{F} = \frac{q}{t}\vec{x} \times \vec{B} \quad (21)$$

$$W = \vec{x} \cdot \vec{F} \quad (22)$$

$$W = \frac{q}{t}\vec{x} \cdot (\vec{x} \times \vec{B}) \quad (23)$$

In the final step, why is the right hand side $\left(\vec{x} \cdot \left(\vec{x} \times \vec{B}\right)\right)$ zero?

- A: Because \vec{x} is parallel to $\vec{x} \times \vec{B}$.
- B: Because \vec{x} is perpendicular to $\vec{x} \times \vec{B}$.
- C: Because $q = 0$ on average.
- D: Because \vec{x} is perpendicular to \vec{B} .

We will continue with the lab activity from last period, in which we began to study how moving magnetic fields *create* emf's (voltages).

1. Connect the voltmeter and wires to the leads on the set of loops of wire, and obtain a ruler and two bar magnets.
2. The goal is to vary the speed of the magnets through the wire loops:
 - Drop the magnets through the wire loops from different heights about the loops.
 - Make measurements of the initial height of the magnet above the loops.
 - Record the maximum voltage observed as the magnet passes through the loops.
 - **Plot the max voltage versus initial height of the magnet, for both polarities of the magnet.**
 - Repeat with two magnets.

CONCLUSION

Reading: Chapter 22

1. Magnets and magnetic fields
2. Force on a moving charge in a magnetic field
3. **The Hall effect**
4. Magnetic forces on conductors
5. Ampère's Law

ANSWERS

• ...

• ...