ALGEBRA-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND MODERN PHYSICS (PHYS135B-01): UNIT 1

Jordan Hanson

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Whittier College Department of Physics and Astronomy

UNIT 0 REVIEW

Physics - φυσική - "phusiké": knowledge of nature
from *φύσις - "phúsis": nature*
Reading: Chapters 18 and 19 (for Unit 1)

1. Estimation/Approximation
   - Estimating the correct order of magnitude
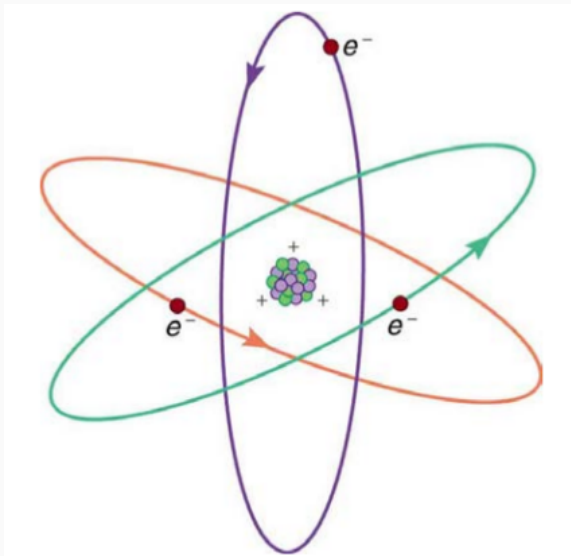   - Building complex quantities
   - Unit analysis

2. Review of concepts from Newtonian mechanics
   - Kinematics and Newton's Laws
   - Work-energy theorem, energy conservation
   - Momentum, conservation of momentum

UNIT 0 REVIEW PROBLEMS

SUMMARY

Reading: Chapters 18 and 19

1. Charge, mass, the Coulomb force, and the gravitational force
2. Force fields
3. Electric potential and capacitance

JITT - READING QUIZ RESULTS

CHARGE, CONDUCTORS AND INSULATORS

Let's begin this topic in a special way: comparison to *gravity*. What do electricity and gravity have in common? The answer lies in a notion we call *charge...*

Charge: the constant of proportionality between the strength of a *field* and the force a field exerts on an *object*.

Gravity

1. Force: $\vec{F} = G\frac{mM}{r^2}\hat{r}$

2. Parameters: r is absolute distance between two objects with masses m and M, and the direction is $\hat{r}$

3. *Charge* of one object: m

4. *Field felt by that object*: $\vec{G} = G\frac{M}{r^2}\hat{r}$

5. $\vec{F} = m\vec{G}$

Electricity

1. Force: $\vec{F} = k\frac{qQ}{r^2}\hat{r}$

2. Parameters: r is absolute distance between two objects with electric charges q and Q, and the direction is $\hat{r}$

3. *Charge* of one object: q

4. *Field felt by that object*: $\vec{E} = G\frac{Q}{r^2}\hat{r}$

5. $\vec{F} = q\vec{E}$

Charge: the constant of proportionality between the strength of a *field* and the force a field exerts on an *object*.

In the field paradigm, objects with charges *emanate* fields, causing other objects with charge to experience force.

Gravity

How many *types of charge*, or how many charges, exist under the force of gravity?

One. We call it mass.

Electricity

How many *types of charge*, or how many charges, exist under the force of electricity?

Two. We call one positive, and one negative.

Charge: the constant of proportionality between the strength of a *field* and the force a field exerts on an *object.*

In the field paradigm, objects with charges *emanate* fields, causing other objects with charge to experience force.

In the field paradigm, gravity has one charge (mass), and electricity has two charges (positive and negative).

There is one fundamental fact that is puzzling. What about Newton's 2nd law? Acceleration is not a field, it is a kinematic function.

$$\vec{F}_{\text{net}} = m\vec{a} \tag{1}$$

Aparently there are *two kinds of mass*: **inertial** and **gravitational**.

Equivalence principle:

There are *two kinds of mass*: **inertial** and **gravitational**, with **equal value** for a given object.

`https://en.wikipedia.org/wiki/Equivalence_principle`

There is no similar principle for charge. If the electric force on a charged object is calculated, that force must still be inserted into **Newton's 2nd Law** to obtain the acceleration, and the inertial mass must be known.

CHARGE, CONDUCTORS AND INSULATORS

Charge has other properties, some similar to gravitational mass:

1. Charge is conserved globally (charge cannot be created nor destroyed). Mass has the same property.

2. Charge is conserved locally (if we pull charge out of the system, charge will flow into the system).

3. Charge is quantized, with an electron (for example) having the fundamental negative unit, and a proton (for example) having the fundamental positive unit.

4. The laws of physics are the same for positive and negative charges.

5. The two kinds of charge emit fields that attract each other; fields emitted by charges of the same type repel such charges.

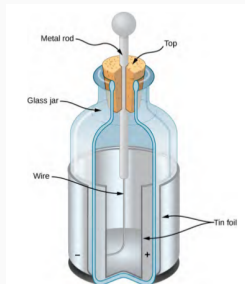Benjamin Franklin and the Leyden Jar. (Good paper topic).

Figure 5.8 A Leyden jar (an early version of what is now called a capacitor) allowed experimenters to store large amounts of electric charge. Benjamin Franklin used such a jar to demonstrate that lightning behaved exactly like the electricity he got from the equipment in his laboratory.

Figure 1: A Leyden jar was an early version of a capacitor. Benjamin Franklin guessed that one type of charge moves and another remains stationary, explaining several behaviors of charged objects.

The rest of the properties of charge are connected to the development of the structure of the atom, and we will return to this topic at the end of the semeter.

Figure 5.8 The nucleus of a carbon atom is composed of six protons and six neutrons. As in hydrogen, the surrounding six electrons do not have definite locations and so can be considered to be a sort of cloud surrounding the nucleus.
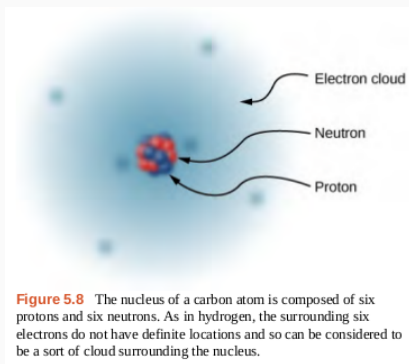
Figure 2: A sketch of our current atomic paradigm.

Suppose an ion is composed of six protons, eight neutrons, and five electrons. What is the net charge?

- A: +1
- B: 0
- C: −1
- D: −2

A rod with a positive charge is held next to a *conductor* (an object were charge can move around freely). Which of the following is true?

- A: The charges in the conductor all remain in place because charge is conserved.
- B: The negative charges in the conductor move toward the positive charges in the rod.
- C: The positive charges remain in place but the negative charges move away from the rod.
- D: The positive charges move toward the rod and the negative charges remain in place.

COULOMB'S LAW AND ELECTRIC FIELDS

Coulomb's Law describes the force between charges.

Coulomb's Law

The electric force, or **Coulomb force**, between two electrically charged systems with charges q_1 and q_2 separated by a distance r is

$$\vec{F}_{\mathrm{C}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2} \hat{r} \tag{2}$$

In Eq. 2, $\hat{r} = \vec{r}/|\vec{r}|$, and $\epsilon_0 = 8.85418782 \times 10^{-12} N^{-1} m^{-2} C^2$, called the *perimittivity of free space.*

Coulomb Field

The electric field corresponding to Eq. 2, experienced by a charge q and generated by a charge Q is

$$\vec{E}_C = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \qquad (3)$$

In Eq. 3, r remains the separation between q and Q.

Thus we have: $\vec{F}_C = q\vec{E}_C$.

The SI Unit of charge is the Coulomb, which is equal to the amount of charge in a "current" of 1 amp for 1 second (more on this later). **The charge of an electron is** 1.6×10^{-19} **Coulombs, or C.**

Have you noticed that the Coulomb force does not depend on kinematic variables like velocity, or have any dissipative effect? Perhaps it is **conservative**. In your own words at your table, discuss the meaning of a conservative force.

If a force F is conservative, what is the relationship between F and the potential energy, U?

- A: F is proportional to the kinetic energy, which is related to U.

- B: F is proportional to the change in $-U$

- C: Work done on a system going against F is path independent, so U is the same if the system's path is closed

- D: B and C

If a force is conservative, it is true that

$$F = -\frac{\Delta U}{\Delta x} \qquad (4)$$

In kinematics, U is the potential energy. In electrostatics, we write it as a *voltage*.

$$E = -\frac{\Delta V}{\Delta x} \qquad (5)$$

Suppose a charge $+q$ experiences the Coulomb field of another charge of $-2q$, separated by a distance r. Which of the following is true?

- A: The charge $+q$ accelerates towards the other charge, and the charge $-2q$ remains stationary, because opposite charges attract.
- B: The charge $-2q$ accelerates towards the other charge, and the charge $+q$ remains stationary, because opposite charges attract.
- C: No charges move; the force on one is equal to the force on the other.
- D: The charges accelerate towards each other.

The answer to the previous problem involves Newton's Third Law.

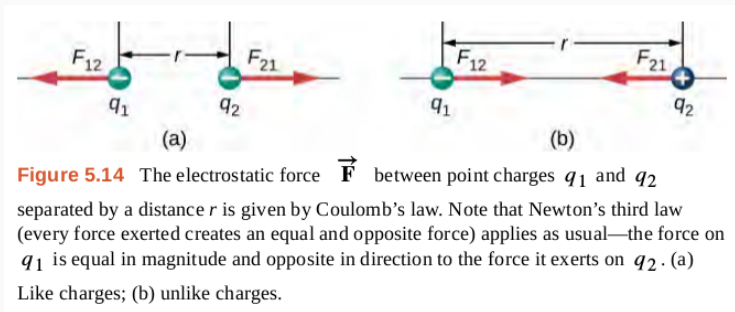**Figure 5.14** The electrostatic force $\vec{F}$ between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2. (a) Like charges; (b) unlike charges.

Figure 3: Newton's Third Law still applies.

Suppose a charge $+q$ experiences the Coulomb fields of two other charges of $-2q$, each located a distance r from $+q$. The charges are all colinear (on the same line). Which of the following is true?

- A: The charge $+q$ accelerates towards one of the other charge, because opposite charges attract.
- B: The charges $-2q$ accelerate towards the charge $+q$.
- C: The charges $-2q$ accelerate towards the charge $+q$, but eventually repel each other back in the other direction.
- D: The charges $-2q$ repel each other from the start.

The Coulomb force equation gives a vector, and so does the corresponding electric field. Like a gravitational field, this effect has a vector at each point in space, so we refer to the Coulomb force and the Coulomb field as *vector fields*.

Vector field: An assignment of a vector to each point in a subset of space.

Which of the following is true of vectors $\vec{v}_i$ in the lower left-hand corner of the figure at right?

- A: They are probably $\vec{v}_i = -\hat{i} - \hat{j}$

- B: They are probably $\vec{v}_i = \hat{i} + \hat{j}$

- C: They are probably $\vec{v}_i = -\hat{i} + \hat{j}$

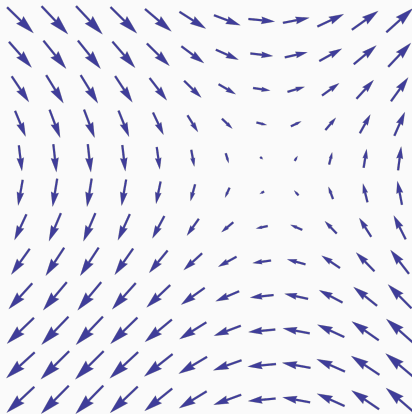- D: They are probably $\vec{v}_i = \hat{i} - \hat{j}$
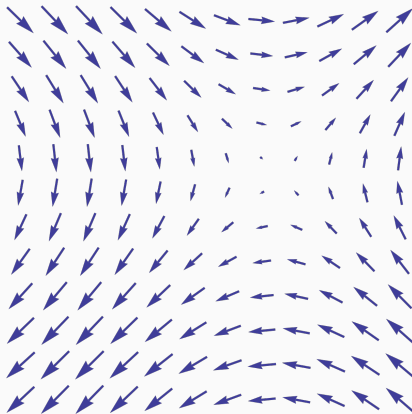
Figure 4: A vector field of vectors $\vec{v}_i$. Let $\hat{j}$ represent up, and $\hat{i}$ represent right.

Which of the following is true of vectors $\vec{v}_i$ in the upper left-hand corner of the figure at right?

- A: They are probably $\vec{v}_i = -\hat{i} - \hat{j}$

- B: They are probably $\vec{v}_i = \hat{i} + \hat{j}$

- C: They are probably $\vec{v}_i = -\hat{i} + \hat{j}$

- D: They are probably $\vec{v}_i = \hat{i} - \hat{j}$

Figure 5: A vector field of vectors $\vec{v}_i$. Let $\hat{j}$ represent up, and $\hat{i}$ represent right.

Group board exercise: What is the angle of the net electric field for the *test charge* at the point (1,1) in Fig. 6?

Group board exercise: What is the magnitude of the net electric field for the *test charge* at the point (1,1) in Fig. 6, if the distances have units of nanometers, and q is the charge of an electron, 1.6×10^{-19} C? (Let $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N m^2 C^{-2}).
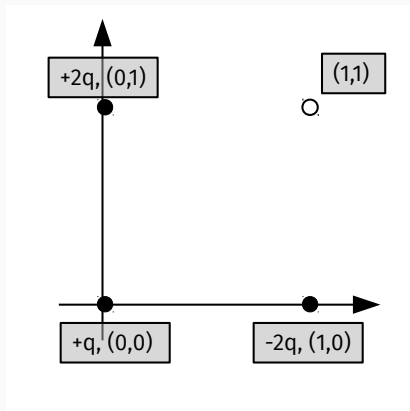

Figure 6: Three charges create a field for a hypothetical *test charge*.

What is the angle of the E-field at point (1,1) in Fig. 7 at right?

- A: 0 deg
- B: 45 deg
- C: 90 deg
- D: 135 deg

What is the fastest way to solve this problem?

- A: Blind luck
- B: Do the algebra
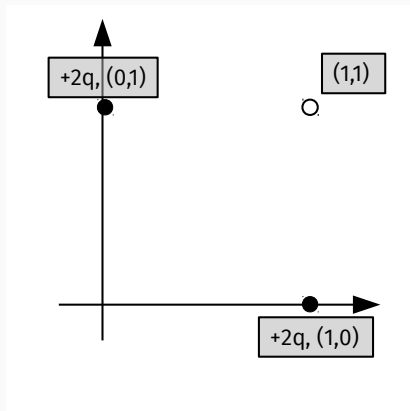- C: Symmetry
- D: Numerical estimation

Figure 7: Two charges create a field for a hypothetical *test charge*.

What is the angle of the E-field at point (1,1) in Fig. 8 at right?

- A: About 0 deg
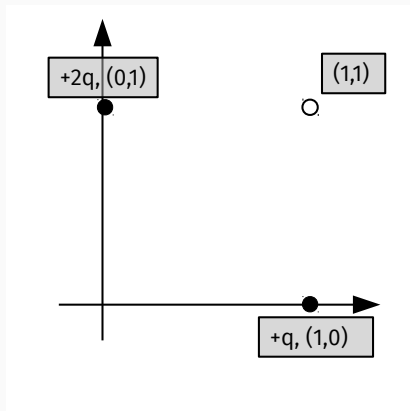- B: About 25 deg
- C: About 45 deg
- D: About 60 deg

Figure 8: Two charges create a field for a hypothetical *test charge*.

The forces of N fixed charges on a test charge Q create a net force, where the individual forces simply add like vectors. This is known as the **superposition principle**.

$$\vec{F}_{\text{C,Net}} = \frac{1}{4\pi\epsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i = Q\vec{E}_{\text{C,Net}} \tag{6}$$

$$\vec{E}_{\text{C,Net}} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i \tag{7}$$

For the expressions of fields built from the superposition principle, let's adopt a notation:

$$\vec{E}_{\text{C,Net}}(P) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i \tag{8}$$

Equation 8 represents the field at a *position $P = P(x, y, z)$*, relative to the positions $\vec{r}_i$ of the source charges.

Table exercise: Calculate $\vec{E}_{\mathrm{C,Net}}(P)$, if $P = (1,1)$.

Table exercise: Calculate $\vec{E}_{\mathrm{C,Net}}(P)$, if $P = (-1,-1)$.

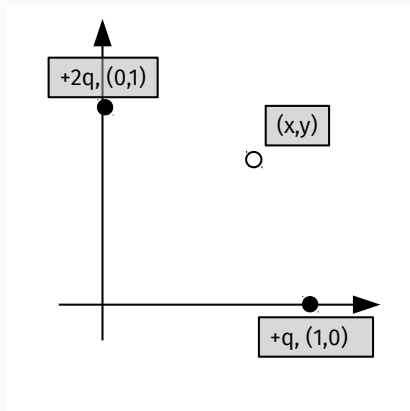**Group discussion:** What does it mean if $P = (1,0)$?

Figure 9: Two charges create a field for a hypothetical *test charge*.

Notice in the prior examples of *fixed* charges, we need an *explanation* for why the fixed charges remain fixed, even though they are obviously subject to Coulomb forces.

Insulator: A material in which there are no free charges available to conduct electricity. Charges may be fixed in position within an insulator.

Conductor: A material in which there are free charges available to conduct electricity. Charges may not be fixed in position within a conductor.

Semi-conductor: A material in which there are free charges available to conduct electricity if certain requirements are met.

The following problem is an example of solving for a field analytically, and *testing various limits*. Upon taking limits results are often simple and intuitive.

Two charges $+q$ are on the fixed in an insulator on the x-axis. Solve for the E-field at $P = (0, 0, z)$.

Show that the general solution is

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left(z^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}} \hat{k} \quad (9)$$
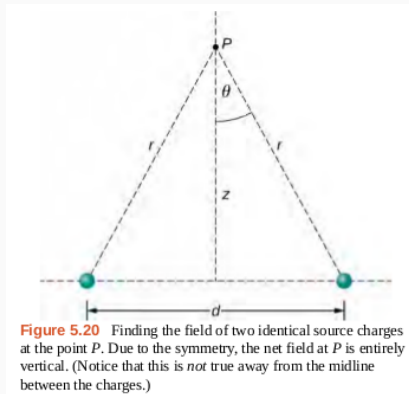

Figure 5.20 Finding the field of two identical source charges at the point P. Due to the symmetry, the net field at P is entirely vertical. (Notice that this is *not* true away from the midline between the charges.)

Figure 10: Solve for the E-field as a function of z, d, and q.

Show that the general solution is

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left(z^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}} \hat{k} \quad (10)$$

Take the following two limits:
1) $z \gg d$ and 2) $z = 0$. What are
the results?

Keep these results in mind,
because we are about to start
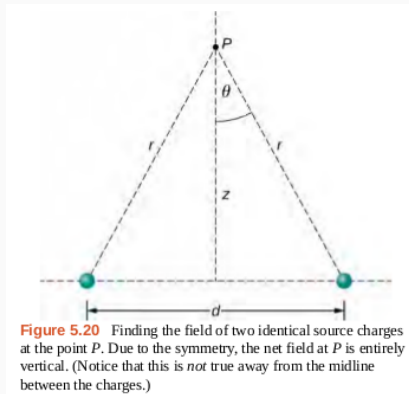drawing **vector fields,** in order to
visualize the algebra.

Figure 5.20 Finding the field of two identical source charges at the point P. Due to the symmetry, the net field at P is entirely vertical. (Notice that this is *not* true away from the midline between the charges.)

Figure 11: Solve for the E-field as a function of z, d, and q.

PhET Simulation of E-fields from Charges:

https://phet.colorado.edu/en/simulation/
charges-and-fields

1. Create the situation in the prior problem, in Fig. 11.
2. Use the yellow sensor object to determine the local direction of the E-field at various points along the z-axis.
   - Do the results match the limit $z \gg d$?
   - Do the results match the limit $z = 0$, halfway between the charges?
   - Where is the field maximal?
3. Make sure you can see above and below the charges, and repeat steps 1 and 2 for negative z-values. What do you find?

PhET Simulation of E-fields from Charges:

Build E-fields with the following properties, by adding single charges. Let the *z-axis be upwards*, and let the *x-axis be to the right.*

1. Build an electric field that has **reflection symmetry** across the z-axis, with at least five charges.

2. Build an electric field that has *radial symmetry* about the origin, with at least six charges.

3. Build an electric field that would be the same if I rotated the picture by 90 degrees (**4-fold symmetry**) with at least four charges, some negative and some positive.

4. Build an electric field that would be the same if I rotated the picture by 45 degrees (**8-fold symmetry**) with at least eight charges, some negative and some positive.

PhET Simulation of E-fields from Charges:

The lesson is that the E-field has the *symmetry properties* of the *charge distribution*.

When we connect the vectors in a vector field, the results are figures like Fig. 12. Fields by convention originate from positive charges and terminate on negative ones.
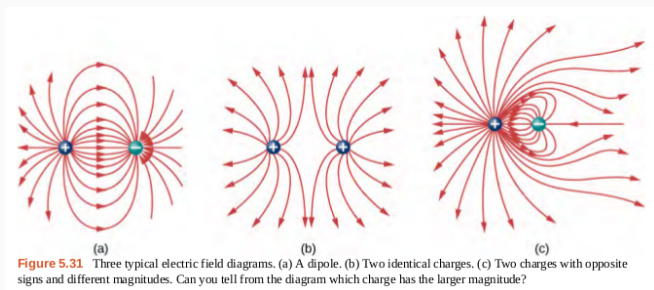

Figure 5.31 Three typical electric field diagrams. (a) A dipole. (b) Two identical charges. (c) Two charges with opposite signs and different magnitudes. Can you tell from the diagram which charge has the larger magnitude?

Figure 12: Field-line diagrams. The density of lines indicates electric field strength.

E-FIELDS OF CHARGE DISTRIBUTIONS

Welcome to calculus! Let $k = 1/(4\pi\epsilon_0)$.

$$\vec{E}(P) = k \sum_{i=1}^{N} \left(\frac{q_i}{r_i^2} \right) \hat{r} \tag{11}$$

$$\vec{E}(P) = k \int_{line} \left(\frac{\lambda dl}{r^2} \right) \hat{r} \tag{12}$$

$$\vec{E}(P) = k \int_{surface} \left(\frac{\sigma dA}{r^2} \right) \hat{r} \tag{13}$$

$$\vec{E}(P) = k \int_{volume} \left(\frac{\rho dV}{r^2} \right) \hat{r} \tag{14}$$

The functions λ, σ, and ρ are just charge densities. They decribe where charge is, and how much there is.
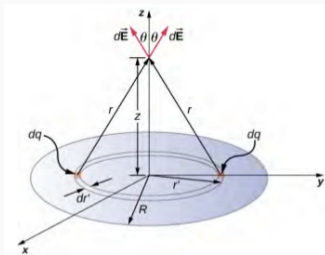
Figure 5.25 A uniformly charged disk. As in the line charge example, the field above the center of this disk can be calculated by taking advantage of the symmetry of the charge distribution.

Figure 13: We are going to work this example together, and other examples will be left to homework.

Observe on board.

Figure 5.25 A uniformly charged disk. As in the line charge example, the field above the center of this disk can be calculated by taking advantage of the symmetry of the charge distribution.

Figure 14: We are going to work this example together, and other examples will be left to homework.

Result:

$$\vec{E} = k\left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2 + z^2}}\right)\hat{k} \tag{15}$$

$$\vec{E} = k \left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2 + z^2}} \right) \hat{k} \qquad (16)$$

Which of the following not true of Eq. 16?

- A: Taking the limit $R \to \infty$ yields a constant field.
- B: Taking the limit $z \to 0$ yields a constant field.
- C: The charge distribution has radial symmetry, so the field cannot have horizontal components.
- D: Taking the value $z = R$ represents a minimum in the field strength.

$$\vec{E} = k \left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2 + z^2}} \right) \hat{k} \qquad (17)$$

What happens to Eq. 17, in the limit that $R \to \infty$?

- A: The field decreases to zero.
- B: The field is constant.
- C: The field grows increasingly positive.
- D: The field grows increasingly negative.

In the limit that $R \to \infty$,

$$\vec{E} = 2\pi\sigma k\hat{k} = \frac{\sigma}{2\epsilon_0}\hat{k} \qquad (18)$$

Equation for the electric field of a uniform infinite disk.

Imagine two infinite disks with equal uniform charge distributions, some distance apart. One has positive charge, the other negative charge. What is the E-field between them?

- A: 0
- B: $\frac{\sigma}{2\epsilon_0}$
- C: $\frac{\sigma}{\epsilon_0}$
- D: $\frac{\sigma}{4\epsilon_0}$

Imagine two infinite disks with equal uniform charge distributions, some distance apart. Both have positive charge. What is the E-field between them?

- A: 0
- B: $\frac{\sigma}{2\epsilon_0}$
- C: $\frac{\sigma}{\epsilon_0}$
- D: $\frac{\sigma}{4\epsilon_0}$

Other interesting charge distributions:

- A line of charge with length L and total charge $Q = \lambda L$, where $P = (0, 0, z)$ above midpoint:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{\lambda L}{z\sqrt{z^2 + \frac{1}{4}L^2}}\hat{k} \tag{19}$$

- Equation 19, but with $L \to \infty$:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z}\hat{k} \tag{20}$$

Other interesting charge distributions:

- A ring of radius R and total charge $Q = 2\pi R \lambda$, where $P = (0, 0, z)$ above midpoint:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2\pi R \lambda z}{\left(z^2 + R^2\right)^{3/2}} \hat{k} \qquad (21)$$

- Equation 21, but with $z \gg R$:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2\pi R \lambda}{z^2} \hat{k} \qquad (22)$$

In Eq. 21, what does the quantity $2\pi R\lambda$ represent?

- A: The total charge density on the ring
- B: The circumference of the ring
- C: The magnitude of the electric field from the ring
- D: The total charge on the ring

Let $Q_{\text{tot}} = 2\pi R\lambda$. That makes Eq. 22

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{Q_{\text{tot}}}{z^2}\hat{k} \qquad (23)$$

This is identical to the electric field of what charge distribution? (Think back to the definition of the electric field).

- A: A plane with charge density Q_{tot}/A, where A is the area
- B: A line with total charge Q_{tot}
- C: A dipole of charge $\pm Q_{\text{tot}}$
- D: A point charge Q_{tot}

ELECTRIC POTENTIAL, POTENTIAL ENERGY

Recall that the *change in potential energy* is force:

$$F = -\frac{\Delta U}{\Delta x} \qquad (24)$$

- The units of *U*: Joules = Newtons per meter
- The units of *x*: meters
- The ratio: Newtons

An object of mass m is a height h above the ground, in Earth's gravity field. What is the *potential* energy of the object?

- A: $U = \frac{1}{2}mv^2$ (v is the velocity)
- B: $U = \frac{1}{2}mv^2 + mgh$ (v is the velocity)
- C: $U = mgh$
- D: U is zero, because the object is at rest.

What is the expression for the force of gravity, if $\Delta U = mgh$, and $\Delta x = h$?

- A: mg
- B: $-mg$
- C: g
- D: $-g$

If the potential energy is a function of displacement, $U = U(\vec{x})$, it may be called a potential energy *surface*.
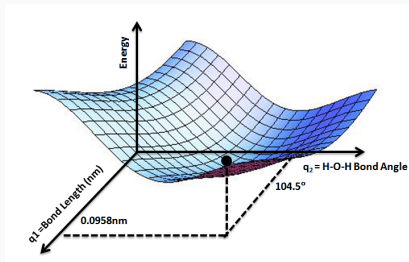
Figure 15: An example of a potential energy surface.

Considering *Newton's Second Law*, however, if $F = ma$ then $ma = -\frac{\Delta U}{\Delta x}$, and
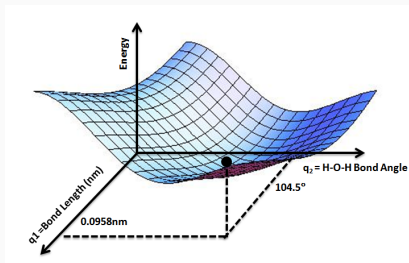
$$a = -\frac{1}{m}\frac{\Delta U}{\Delta x} \tag{25}$$

Figure 16: Imagine dividing the numbers on the z-axis by the mass, and treating this surface as the potential energy *per unit mass*.

Let's just scale the z-axis by the mass of the system:

$$a = -\frac{1}{m}\frac{\Delta U}{\Delta x} \tag{26}$$

$$\Delta V = \frac{\Delta U}{m} \tag{27}$$

$$a = -\frac{\Delta V}{\Delta x} \tag{28}$$

Instead of calling ΔV the potential energy, let's just call it *the potential*. Recall that if the force is a vector field, then acceleration is a vector field as well (the object has a given acceleration vector for all points in the space).

Newtonian mechanics

$$F = ma \tag{29}$$

$$a = -\frac{\Delta V}{\Delta x} \tag{30}$$

Electrostatics

$$F = qE \tag{31}$$

$$E = -\frac{\Delta V}{\Delta x} \tag{32}$$

In Eq. 32, we refer to ΔV as **voltage.**

Good paper topic:

Figure 17: (Lago di Como, Italia) Monument to Alessandro Volta, inventor of the electric battery. Debunked claim that electricity only generated by life.

Voltage is like a potential energy surface → *potential energy per unit charge.*

https://phet.colorado.edu/en/simulation/
charges-and-fields
Using the PhET simulation about charges and fields:

1. Explore the voltage associated with fields generated by charges using the voltage button.
2. Add a single point charge, and use the ruler and voltmeter (potentiometer) to measure voltage versus distance, and plot it.
3. What function describes the relationship between voltage and distance?

https://phet.colorado.edu/en/simulation/
charges-and-fields

Using the PhET simulation about charges and fields:

1. Note that the units of ϵ_0 are N m^2 C^{-2}, and the value is 8.854×10^{-12}

2. We know from prior equations that the units of voltage are J C^{-1}

3. Using your measurements, show that the voltage due to a point charge is

$$V = \pm \frac{1}{4\pi\epsilon_0} \frac{q}{r} \qquad (33)$$

(Where the sign depends on the charge, just like E-fields)

Voltage due to a point charge:

$$V = \pm\frac{1}{4\pi\epsilon_0}\frac{q}{r} \tag{34}$$

Equation 34 follows from the form of the electric field of a point charge, and $E = -\Delta V/\Delta x$ (derivative, calculus).

Voltage is an example of a **scalar field**, whereas the electric field is an example of a **vector field.** Recall the result for the electric field of a large plane of charge, with charge σ per unit area:

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{k} \qquad (35)$$

What should the voltage be, if the field is constant? (Think of the situation of gravity, with electric field standing in for acceleration).

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https://phet.colorado.edu/en/simulation/
charges-and-fields
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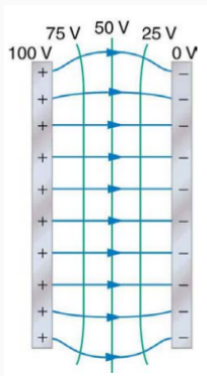
Figure 18: Parallel plates of charge, electric field, and potential. Notice the linear decrease in voltage. Did you see this in the PhET?

Two parallel plates, opposite charge:

$$V = -\frac{\sigma}{\epsilon_0}z + C \qquad (36)$$

With the boundary condition that $V = V_0$ when $z = 0$, we have

$$V(z) - V_0 = -\frac{\sigma}{\epsilon_0}z \qquad (37)$$

Let $\Delta V(z) = V(z) - V_0$, and $\Delta z = z$:

$$\frac{\Delta V}{\Delta z} = -\frac{\sigma}{\epsilon_0} = E \qquad (38)$$

VOLTAGE, POTENTIAL ENERGY, AND WORK

What follows is a series of memorizable equations and mental images comparing voltage to gravitational potential energy. This analogy runs deep because the Coulomb force is conservative.
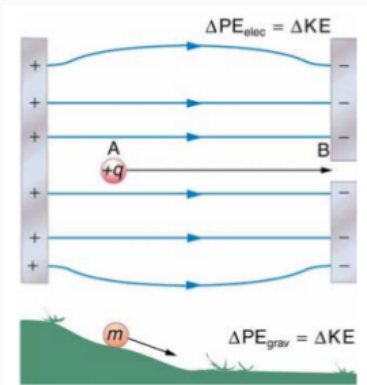
Figure 19: Remembering the connection of voltage to gravity.

We can think of work as the opposite in the change in potential energy: $W = -\Delta PE$. If gravity pulls an object down a hill, that object is losing gravitational potential energy, and the energy is being converted to kinetic energy. Which of the following formulas describes the relationship between work and displacement?

- A: $W = \vec{F} \cdot \vec{x} = Fx \cos\theta$
- B: $W = \frac{\Delta F}{\Delta x}$
- C: $W = mgxF$
- D: I'm confused.

Imagine trying to calculate the work done ($W = -\Delta PE$) on a charge q as it moves through a complex vector field ($F = qE$, where E can come from some complex charge distribution). It's easier to take the test charge q out of it, and think of just the potential surface as a surface, rather than proportional to qE times the path.

$$V = \frac{PE}{q} \tag{39}$$

This makes the unit of one Volt equal to one Joule per Coulomb. (1 V = 1 J/C). The volt reminds us of all those times the *mass drops out* of the problems when we are not concerned with how many Newtons, just where things are going.

For potential energy calculations involving gravity, **we** get to decide the location of "zero energy," and we're only interested in *changes* in potential energy.

$$\Delta V = V_{\mathrm{B}} - V_{\mathrm{A}} = \frac{\Delta PE}{q} \qquad (40)$$

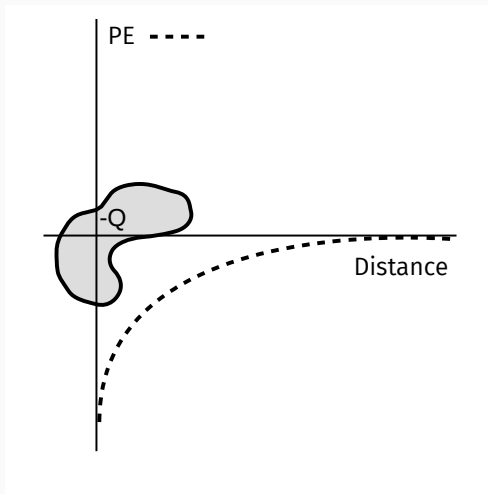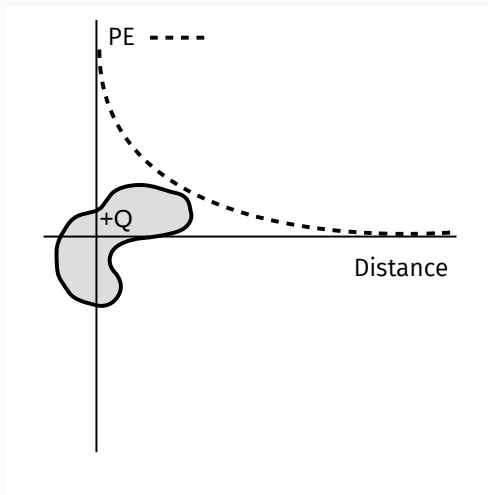Where is the most logical place to put the voltage zero point?

Figure 20: Where should we place the zero point of potential energy here? Which way do positive charges fall? Which way do negative charges fall?

Figure 21: Where should we place the zero point of potential energy here? Which way do positive charges fall? Which way do negative charges fall?

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver?

- A: 60 kJ and 720 kJ, respectively
- B: 600 kJ and 7200 kJ, respectively
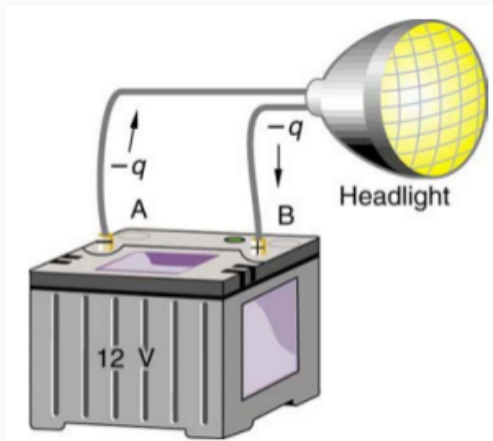- C: 6 kJ and 72 kJ, respectively
- D: 0.6 kJ and 7.2 kJ, respectively

Figure 22: Here is a battery circuit that has a potential difference of 12 V. Where would you put the zero point?

Consider the battery headlight circuit. The car batter has 60,000 C stored, with $\Delta V = 12.0$ V. The headlight requires 30 W of power. We know how much energy is in the battery, so how long can the headlight run?

- A: About 1 hour
- B: About 2 hours
- C: About 7 hours
- D: About 9 hours

Smaller than a motorcycle battery: Suppose an AA battery has 1.5 V, and 7200 C of charge. How much energy is stored inside it?

- A: About 5 kJ
- B: About 10 kJ
- C: About 15 kJ
- D: About 20 kJ

Same headlight: requires 30 W of power. We know how much energy is in the battery, so how long can the headlight run?

- A: About 1 hour
- B: About 30 minutes
- C: About 15 minutes
- D: About 6 minutes

Scaling problems: Consider two batteries. Battery 1 has 5000 C at 12 V. Battery 2 has the same charge but only 3/4 the energy. What is the voltage of battery 2?

- A: 5 V
- B: 7 V
- C: 9 V
- D: 12 V

Scaling problems: Two AA batteries have 1.5 V each. Placed end-to-end, the voltages add together. If each had 7200 C of charge, what's the new energy of the double battery system?

- A: 5 kJ
- B: 10 kJ
- C: 15 kJ
- D: 20 kJ

Scaling problems: A 24V solar-cell battery can run a 10 W system for 24 hours. If we add a second 24V battery with half the charge of the first, for how long can the system run?

- A: 24 hours
- B: 36 hours
- C: 48 hours
- D: 12 hours

The electron-Volt, eV: Since charge times voltage is energy, why not have a unit where it's the charge of one electron through a potential difference of one volt?

$q_e \approx 1.6 \times 10^{-19}$ C, 1 eV = 1.6×10^{-19} J.
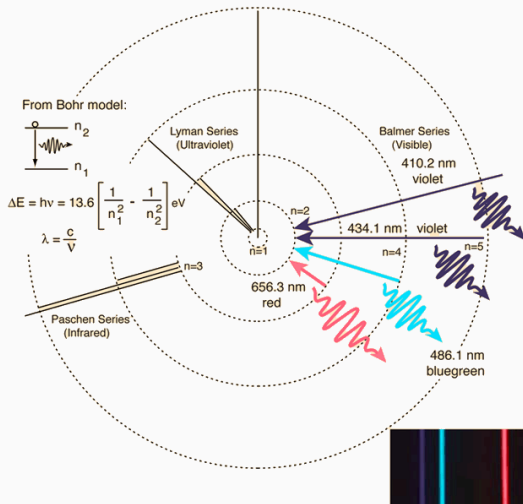
This unit comes in handy!

Figure 23: Energies of order 1 electron-Volt. Our whole visual range…

Different amounts of eV:

- Atomic energy transitions: $\approx$ 1 eV

- X-ray energies: $\approx$ 1 keV

- Nuclear energy transitions: $\approx$ 1 MeV

- *Mass of protons and neutrons*: $\approx$ 1 GeV (wait, what?...)[1]

- *Mass of other famous particles*: $\approx$ 100 GeV

- But what else is out there in nature?

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[1]This is a use of Einsten's equation, $E = mc^2$. We will return to this point later. For another example, $m_e = 511$ keV.
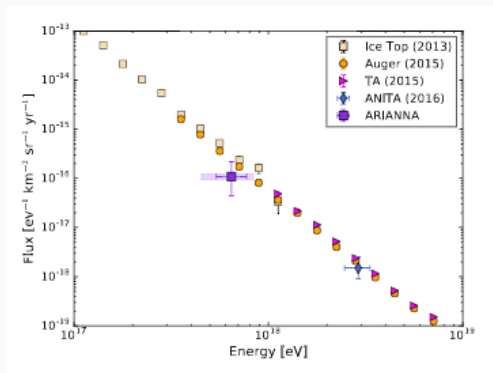
Figure 24: My research on the ARIANNA project has measured cosmic rays with energies of up to 10^{18} eV!

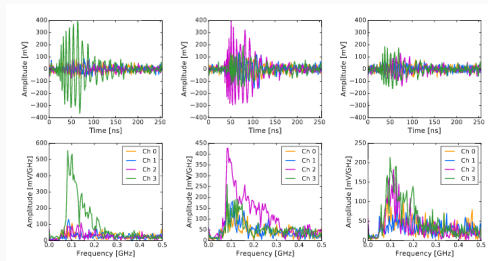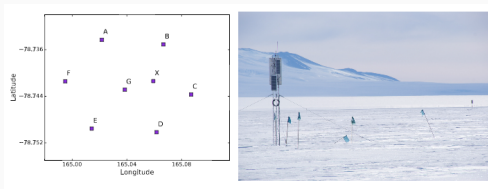Figure 25: These cosmic rays (protons) have so much energy, that they make radio pulses through the atmosphere.

Figure 26: How do we do it? We build autonomous radio detectors in the Antarctic wilderness… This would make a nice paper topic.

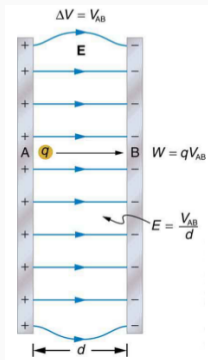One electron-Volt is the energy gained by a charge of 1 electron falling through a potential of 1 Volt.

Figure 27: Fields, voltage, and energy.

Suppose two objects have the same charge, $-e = -1.6 \times 10^{-19}$ C, but one has twice the mass as the other. If each is released through a potential of 1 V, which object gains more energy?

- A: The lighter one
- B: The heavier one
- C: They gain the same energy
- D: Cannot determine

Suppose two objects have the same charge, $-e = -1.6 \times 10^{-19}$ C, but one has twice the mass as the other. If each is released through a potential of 1 V, which object achieves the highest speed?

- A: The lighter one
- B: The heavier one
- C: They gain the same energy
- D: Cannot determine

Consider the plates of Fig. 27:

$$W = q\Delta V \tag{41}$$

$$W = F\Delta x = qE\Delta x \tag{42}$$

$$qE\Delta x = q\Delta V \tag{43}$$

$$E\Delta x = \Delta V \tag{44}$$

$$E = \frac{\Delta V}{\Delta x} \tag{45}$$

This derivation is good for a uniform constant field, for example, between two *parallel plates of charge*. **We proved that this field is constant back when considering charge distributions.**

So what happens if we place an electric field across two parallel plates? A charge +Q on one side and -Q on the other side develops, storing charge. This is called a *capacitor*.
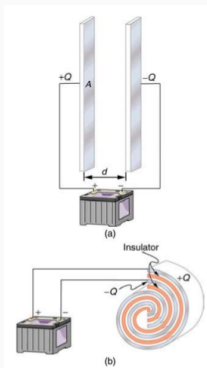

Figure 28: A battery is like a parallel plate *capacitor*.

CAPACITANCE

What voltage is required to store Q? Let ΔV be the voltage difference, E be the electric field strength, and Δx be the voltage between the plates. We already know that

$$\Delta V = E\Delta x \qquad (46)$$

But we also know from Coulomb's Law that

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A} \qquad (47)$$

So we have

$$\Delta V \left(\epsilon_0 \frac{A}{\Delta x} \right) = Q \qquad (48)$$

So now we know how much charge to expect for a given voltage.

$$Q = \Delta V \left(\epsilon_0 \frac{A}{\Delta x} \right) = C \Delta V \qquad (49)$$

We call the quantity in parentheses the *capacitance* of the capacitor, representing the ability to store charge. The unit of capacitance is **the farad**, after Michael Faraday (encounter mid-semester). Scaling problems in a moment...

Good paper topic:

Figure 29: (Lago di Como, Italia) Monument to Alessandro Volta, inventor of the electric battery.

Two batteries store the same charge. One only needs half the voltage, though. What is true of the capacitance of the battery with the lower voltage?

- · A: It has half the capacitance
- · B: It has the same capacitance but more charges.
- · C: It has twice the capacitance
- · D: It has half the energy

Two batteries have the same capacitance. Battery 1 has half the area A as battery 2. Which of the following is true of battery 2?

- A: The plates are half the distance (Δx) apart
- B: The plates are twice the distance (Δx) apart
- C: The plates have half the voltage
- D: The plates have twice the voltage

MORE ON POINT CHARGES AND VOLTAGE

We showed in the PhET simulation that the voltage due to a point charge Q is

$$V(r) = \pm k \frac{Q}{r} \qquad (50)$$

Open the PhET again: `https://phet.colorado.edu/en/simulation/charges-and-fields`

- Re-verify that voltage acts like a scalar field: voltage due different point charges just add like numbers
- Build a battery. Plot the voltage as a function of distance between the plates.
- (The program only handles 32 separate point charges)

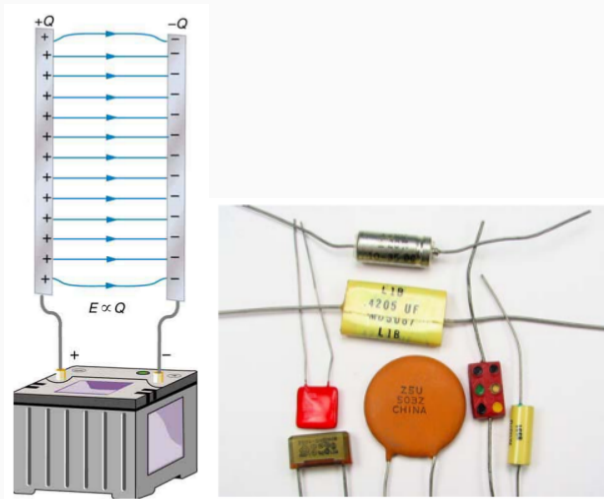# CAPACITANCE AND DIELECTRICS

Figure 30: A battery and a capacitor are not quite the same thing.

Parallel plate capacitor:

$$Q = \Delta V \left(\epsilon_0 \frac{A}{\Delta x} \right) = C \Delta V \tag{51}$$

How can we boost the charge? Can we increase that ϵ_0? (It's a small number: 8.85×10^{-12} N^{-1} C^2 m^{-2}). It turns out that by stuffing some insulating material between the plates *boosts the capacitance...*

Because *dielectric* material reduces the field between the plates for the same charge, the voltage corresponding to that charge is reduced. But that means *higher capacitance.*
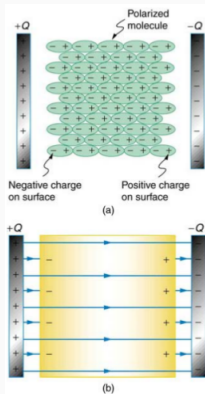

Figure 31: Polarized dielectric.

Table 19.1 Dielectric Constants and Dielectric Strengths for Various Materials at 20°C

Material	Dielectric constant κ	Dielectric strength (V/m)
Vacuum	1.00000	—
Air	1.00059	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Polystyrene	2.56	24×10^6
Pyrex glass	5.6	14×10^6
Silicon oil	2.5	15×10^6
Strontium titanate	233	8×10^6
Teflon	2.1	60×10^6
Water	80	—

Figure 32: The middle column is the number by which ϵ_0 is scaled up when dielectric is introduced. We can't play this game indefinitely, because materials have a maximum electric field (third column) they can handle before they're ripped apart.

CONCLUSION

ANSWERS

- It is zero.
- Entropy has increased, but the internal energy returns to the original value
- 0.5
- The system does work equal to 0.5 liters times 1 atm
- +1
- The negative charges in the conductor move toward the positive charges in the rod.
- B and C
- The charges accelerate towards each other.
- The charges $-2q$ accelerate towards the charge $+q$.
- They are probably $\vec{v}_i = -\hat{i} - \hat{j}$
- They are probably $\vec{v}_i = \hat{i} - \hat{j}$
- 45 deg
- Symmetry
- About 25 deg (26.56 degrees)
- Taking the value $z = R$ represents a minimum in the field strength.
- The field is constant.
- $\frac{\sigma}{\epsilon_0}$ N/C
- 0 N/C

- The total charge on the ring
- A point charge Q_{tot}
- mgh
- $-mg$
- $W = \vec{F} \cdot \vec{x} = Fx \cos \theta$
- 60 kJ and 720 kJ, respectively
- About 7 hours
- About 10 kJ
- About 6 minutes
- 9 V
- 20 kJ
- 36 hours
- They gain the same energy
- The lighter one
- It has twice the capacitance
- The plates are half the distance (Δx) apart