Algebra-Based Physics-1: Mechanics (PHYS135A-01): You Have Learned A Lot

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April 24, 2024

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You Have Learned A Lot

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1. Unit 0: **Chapters - 18.1 - 18.5**, **Chapters 19.1 - 19.3**
    - Unit analysis, kinematics, and Newton's Laws
    - Work and energy, momentum
    - Electrostatics, I
    - The Coulomb Force, and Newton's Second Law for electric charges
    - The concept of an electric field
    - Electrostatics, II
    - Potential energy and charge, voltage
    - Potential energy and fields, point charges
    - Electrostatics in biology
2. Unit 1: **Chapters 19.4 - 19.7**, **Chapters 20.1 - 20.5, 20.7**
    - Capacitors and capacitance
    - Equipotential lines, capacitance, and capacitors
    - Capacitors in series and in parallel, energy considerations
    - Current and DC circuits
    - DC current and resistance, Ohm's law
    - Energy and power in DC current
    - AC current and waveforms

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1. Unit 2: **Chapters 21.1 - 21.4, 21.6**
    - DC circuit basics
    - Resistors in series and parallel, electromotive force (EMF)
    - Kirchhoff's rules
    - Voltmeters and ammeters
    - RC circuits
2. Unit 3: **Chapters 22.1 - 22.4**, **Chapters 22.7 - 22.9**
    - Magnetostatics I
    - Magnets, ferromagnetic and electromagnetic
    - Magnetic fields and field lines, force on moving charge
    - Magnetic applications I: mass spectrometry
    - Magnetostatics II
    - Force on current carrying conductor, torque on current loop
    - Ampère's Law: magnetic fields created by current
    - Magnetic applications II: fusion reactors

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1. Unit 4: **Chapters 23.1 - 23.5, 23.7, 23.9**, **Chapters 23.9 - 23.12**
    - Magnetic induction
    - Induced EMF, magnetic flux
    - Faraday's Law
    - Motional EMF, generators, transformers
    - AC circuits
    - Inductors
    - RL circuits
    - RLC circuits
2. Unit 5: **Chapters 24.1 - 24.4**, **Chapters 25.1 - 25.3, 25.6**, **32.1 - 32.4**
    - Electromagnetic waves
    - Maxwell's Equations
    - Electromagnetic wave production
    - Electromagnetic spectrum and energy
    - Geometric optics: ray-tracing, reflection, refraction, lenses
    - Nuclear physics in medicine
    - Diagnostics and medical imaging
    - Biological effects of ionizing radiation

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- $\vec{v}_{\mathrm{ave}} = \Delta\vec{x}/\Delta t$ ... Definition of average velocity involving vectors.
- $x(t) = \frac{1}{2}at^2 + v_{\mathrm{i}}t + x_{\mathrm{i}}$ ... One-dimensional displacement with constant acceleration.
- $v(t) = v_{\mathrm{i}} + at$ ... One-dimensional velocity with constant acceleration.
- $\vec{F}_{\mathrm{net}} = m\vec{a}$ ... Newton's 2nd Law.
- $W = \vec{F} \cdot \vec{x}$ ... Definition of work involving vectors.
- $W = Fx\cos\theta$ ... Definition of work, with θ as angle between force and displacement.
- $KE = \frac{1}{2}mv^2$ ... Kinetic energy.
- $W = \Delta KE = KE_{\mathrm{f}} - KE_{\mathrm{i}}$ ... Work energy theorem.
- $KE_{\mathrm{i}} + PE_{\mathrm{i}} = KE_{\mathrm{f}} + PE_{\mathrm{f}}$ ... Energy conservation.
- $\vec{p} = m\vec{v}$ ... Definition of momentum involving vectors.
- $\vec{p}_{\mathrm{tot,i}} = \vec{p}_{\mathrm{tot,f}}$ ... Conservation of momentum.
- $\tau = I\alpha$ ... Netwon's 2nd Law for rotating objects, with torque, moment of inertia, and angular acceleration.
- $KE_{\mathrm{rot}} = \frac{1}{2}I\omega^2$ ... Rotational kinetic energy.
- $L = I\omega$ ... Angular momentum.

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- $\vec{a} = a_x\hat{i} + a_y\hat{j}$ ... Component notation for 2D vector
- $\vec{a} + \vec{b} = (a_x + b_x)\hat{i} + (a_y + b_y)\hat{j}$ ... Adding vectors
- $\vec{a} - \vec{b} = (a_x - b_x)\hat{i} + (a_y - b_y)\hat{j}$ ... Subtracting vectors
- $|\vec{a}| = \sqrt{a_x^2 + a_y^2}$ .. Magnitude of a 2D vector
- $a_x = |\vec{a}| \cos\theta$ ... x-component of a 2D vector
- $a_y = |\vec{a}| \sin\theta$ ... y-component of a 2D vector
- $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$ ... Dot product of two vectors

- $\vec{E} = \frac{kQ}{r^2}\hat{r}$ ... Coulomb field of a charge Q.
- $k = 8.99 \times 10^9$ N C^{-2} m^2 ... Constant of proportionality for Coulomb field.
- $\vec{F} = q\vec{E}$ ... Force on a charge q in the presence of an $\vec{E}$-field.
- $\vec{F} = m\vec{a}$ ... Newton's 2nd Law.
- $m = \rho V = \frac{4}{3}\pi r^3 \rho$ ... Mass of a sphere with volume V, density ρ, and radius r.
- $PE = qV$ ... Relationship between potential energy, charge, and voltage.
- $V_{AB} = Ed$ ... Relationship between voltage between points A and B, a distance d apart, for a constant E-field E.

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- $Q = CV$ ... Relationship between capacitance, charge, and voltage.
- $C_{tot} = C_1 + C_2 + C_3 + \ldots$ ... Capacitors in parallel.
- $C = A\epsilon_0/d$ ... Capacitance of a parallel-plate capacitor.
- $\epsilon_0 = 8.85 \times 10^{-12}$ F/m
- $C_{tot}^{-1} = C_1^{-1} + C_2^{-1}$ ... Total capacitance of two capacitors in series.
- $I = \Delta Q/\Delta t$ ... Definition of current.
- $\Delta V = IR$ ... Ohm's Law, relating resistance R, current I, and voltage ΔV.
- $U = \frac{1}{2}CV^2$ ... Energy stored in a capacitor.
- $P = U/T$ ... Power consumed in a system, assuming U is energy or work done, and T is time duration.
- $y(x) = mx + b$ ... Linear function with slope m, and y-intercept b
- $m = \Delta y/\Delta x$ ... Formula for slope.
- $V = iR$ ... Ohm's Law, with V for voltage, i for current, and R for resistance.
- $R_{\text{tot}} = R_1 + R_2 + \ldots$ ... Resistors in *series*.
- $R_{tot}^{-1} = R_1^{-1} + R_2^{-1}$ ... Total resistance of two resistors in parallel.
- $P = IV$ ... The power consumed by a device that draws a current I at a voltage V.

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- $(x \pm \sigma_x) + (y \pm \sigma_y) = (x + y) + \sqrt{\sigma_x^2 + \sigma_y^2}$ ... Adding two averages with errors.
- $\hat{i} \times \hat{j} = \hat{k}, \hat{j} \times \hat{k} = \hat{i}, \hat{k} \times \hat{i} = \hat{j}$ ... The direction of the *cross-product* follows this pattern. Reversing the order of any two vectors introduces a minus sign.
- $\hat{i} \times \hat{i} = 0, \hat{j} \times \hat{j} = 0, \hat{k} \times \hat{k} = 0$ ... The *cross-product* is zero when both vectors are parallel.
- $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$ ... The magnitude of the cross-product of two vectors $\vec{a}$ and $\vec{b}$, given the angle θ between them.
- $\vec{F} = i\vec{L} \times \vec{B}$ ... The Lorentz force, by a magnetic field $\vec{B}$ on a *current* i of length and direction $\vec{L}$.
- $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force, one version.
- $\vec{F} = i\vec{L} \times \vec{B}$ ... The Lorentz force, another version.
- $\vec{\tau} = \vec{r} \times \vec{F}$ ... Definition of torque.
- $\vec{\tau} = \vec{\mu} \times \vec{B}$ ... Torque on a current carrying loop in a uniform B-field.
- $\vec{\mu} = Ni\vec{A}$ ... Definition of the magnetic moment.
- $B = (\mu_0 I)/(2\pi r)$ ... The magnetic field B *caused* by the current I a distance r away.

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- $\epsilon = -N\Delta\phi_m/\Delta t$ ... Faraday's Law
- $\phi_m = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux
- $\epsilon = -N\Delta\phi_m/\Delta t$ ... Faraday's Law
- $\phi_m = \vec{B} \cdot \vec{A} = BA\cos(\theta)$ ... Definition of magnetic flux
- $\epsilon(t) = \epsilon_0 \sin(\omega t)$ ... AC voltage generated by generator. ϵ_0 is proportional to ω.
- $P_{max} = V_{max}^2/R$ ... Max power of an AC generator.
- $P_{ave} = \frac{1}{2}P_{max}$ ... Average power of an AC generator.
- $\epsilon(t) = V_{max} \sin(\omega t)$ ... AC voltage signal.
- $\omega = 2\pi f$ ... Angular frequency vs. frequency.
- $\epsilon = -L\Delta I/\Delta t$ ... Faraday's Law, with inductance L
- $\epsilon_2/\epsilon_1 = N_2/N_1$ ... Transformer equation
- $\epsilon(t) = \epsilon_0(2\pi f)\cos(2\pi f t)$ ... Variation of induced voltage as a function of time
- $v(t) = v_0 \sin(2\pi f t - \phi)$ ... Standard AC waveform, with amplitude v_0, in volts, frequency f, in Hertz, time t, in seconds, and phase ϕ, in radians.
- $X_{\mathrm{C}} = 1/(2\pi f C)$ ... The reactance of a capacitor with capacitance C, at frequency f.
- $X_{\mathrm{L}} = 2\pi f L$ ... The reactance of an inductor with inductance L, at frequency f.
- $Z_{\mathrm{tot}} = \sqrt{R^2 + (X_{\mathrm{L}} - X_{\mathrm{C}})^2}$ ... The total impedance of a series circuit with resistance R, and reactances X_{C} and X_{L}.
- $\tau = RC$ ... The time constant of an RC circuit.
- $f_0 = \frac{1}{2\pi\sqrt{LC}}$ ... The resonance frequency of an RLC circuit.

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- $c = f\lambda$ ... The relationship between speed of light, c, the frequency, f, and the wavelength, λ.
- $c = 299,792,458$ m s^{-1}, or $c \approx 0.3$ m ns^{-1}.
- $\bar{S} = E^2/(2c\mu_0)$ ... Average **intensity**, or W m^{-2}, of electromagnetic energy.
- $c/n = f\lambda$ ... Relationship between the speed of light, c, the index of refraction, n, the frequency of an electromagnetic wave, f, and its wavelength, λ.
- $c = 3.0 \times 10^8$ m s^{-1} ... The speed of light in a vacuum, within 1 percent error.
- $\sin(x)\cos(y) = \frac{1}{2}(\sin(x+y) + \sin(x-y))$ ... Trigonometric identity.
- $\Delta x = v\Delta t$ ... The relationship between displacement, constant velocity, and time duration.
- **Optics:** Snell's Law states that, for an interface between two media with *indices of refraction* n_1 and n_2, light leaves one medium with an angle θ_1 and enters the second medium with an angle θ_2 as follows:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \tag{1}$$

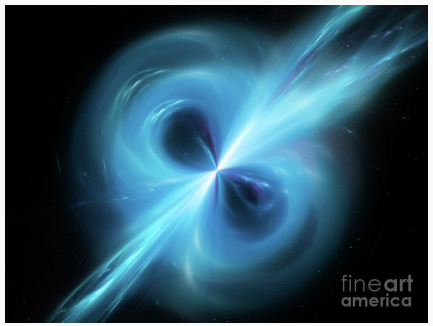
  The two angles are defined with respect to vertical.
- **Optics:** Fresnel's equations give the fraction of power (in light) that is reflected (R) or transmitted (T) by an interface between media with indices of refraction n_1 and n_2:

$$R = \left|\frac{n_1 - n_2}{n_1 + n_2}\right|^2 \tag{2}$$

Think About it Another Way

Think About it Another Way

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial \mathbf{E}}{\partial t}\right)$$



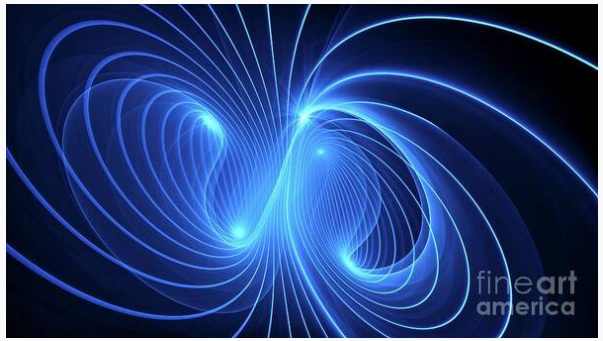


Figure 1: All of the knowledge we have covered this semester can be derived from Maxwell's Equations, and the conservation of charge and energy.

Is the Universe really like that? ... I sure hope so.

Thank You

Think About it Another Way

Figure 2: We are greatful to be able to share this endeavor with you.