Cara Puravat

Midterm 3

Dr. Jordan Hanson - Whittier College Dept. of Physics and Astronomy

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1 Memory Bank

1. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
2. $P = IV$ ... Relationship between power, current, and voltage.
3. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
4. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.
5. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ... Ampère's Law.
6. $\epsilon = -N d\phi/dt$ ... Faraday's Law.
7. $\phi = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux.
8. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
9. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L. Self-inductance: $\Delta V = -L(dI/dt)$.
10. Units of inductance: V s A^{-1}, which is called a Henry, or H.
11. $B = \mu_0 n I$ ... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

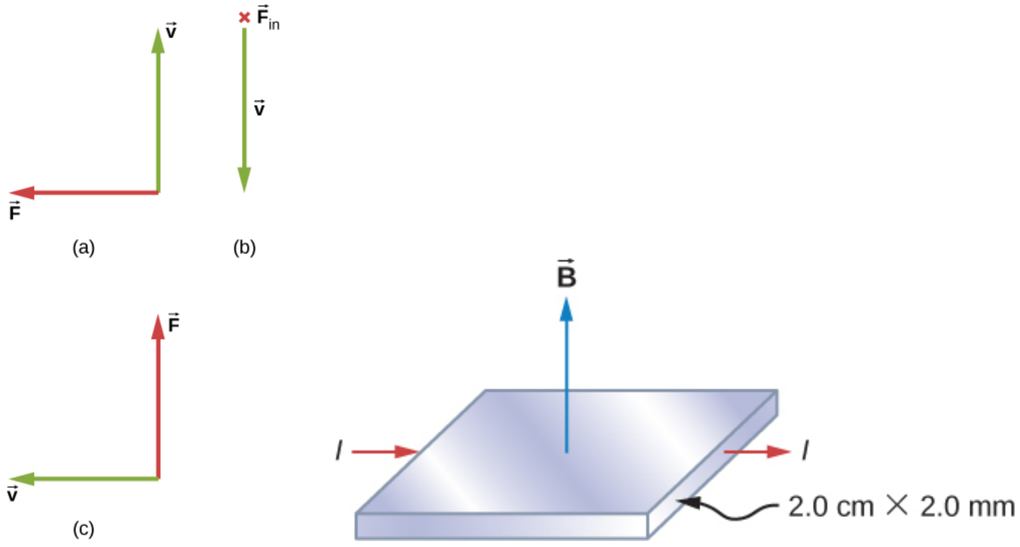


Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.

   - a: into the pg.
   - b: left
   - c: out of the pg.

2. Consider Fig. 1 (right). **The Hall Effect**. An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that

   $E = \Delta V / \Delta x$
   $\Delta V = E \Delta x$
   $\Delta V = v B \Delta x \Rightarrow$

   $$\Delta V = \frac{B \Delta x I}{n q_e A} \tag{1}$$

   $B = 1.33\,T$ $\quad n = 2 \times 10^{28}\,m^{-3}$
   $\Delta x = 2\,cm$ $\quad A = 1\,mm^2$
   $I = 10\,A$

   where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm (0.2 m), $I = 10$ A, $n = 2 \times 10^{28}$ m^{-3}, $A = 1$ mm^2 ($(.001)^2$ 10^{-6}), and q_e is the charge of an electron.

   a. $\vec{F} = q\vec{v} \times \vec{B}$
   $F_B = qv\cancel{B}$
   $F_E = qE$

   $\cancel{q}vB = \cancel{q}E$

   $E = vB$

   b. $\Delta V = \dfrac{(1.33\,T)(0.02m)(10\,A)}{(2 \times 10^{-28} m^{-3})(10^{-6}\,m)(1.6 \times 10^{-19})}$

   $= \dfrac{0.266}{3200}$

   $= 8.31 \times 10^{-5}\,V$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^{4} A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

   $\tau = NIAB\sin\theta$
   $A = \pi r^2$
   $A = \pi(0.65 \times 10^{-15})^2 \Rightarrow 1.33 \times 10^{-30}\,m^2$

   $\tau = (1.05 \times 10^4\,A)(1.33 \times 10^{-30}\,m^2)(2.50\,T)\sin 90°$

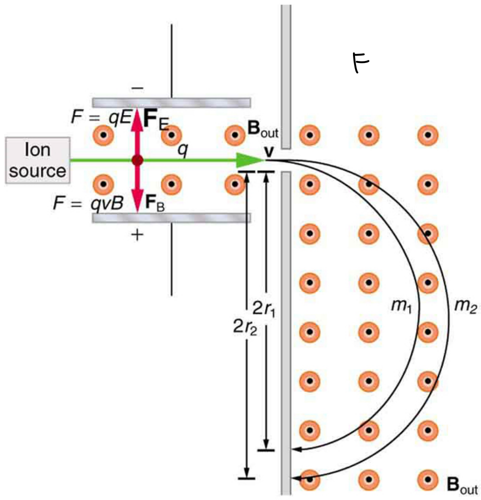
   $= 3.50 \times 10^{-26}\,Nm$

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

   a. $B = \mu_0 n I$
   $\mu_0 = 4\pi \times 10^{-7}$
   $B = (4\pi \times 10^{-7})(500)(0.3A)$
   $B = 1.88 \times 10^{-4}\,T$

   b) $B = (5000)(4\pi \times 10^{-7})(500)(0.3)$
   $B = 0.94T$



F

V

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$F = q\vec{v} \times \vec{B}$

$m = 16(1.67 \times 10^{-27})$

$= 2.67 \times 10^{-26}$ Kg

$$r = \frac{mE}{qB^2} \tag{2}$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r?

$q = 1.6 \times 10^{-19}$

$E = 10$ V/m

$B = 0.01$ T

a. $F_{net} = qE - qvB = 0$

$qvB = qE$

$E = vB \Rightarrow v = \frac{E}{B}$

b. $qvB = \frac{mv^2}{r}$

$r = \frac{mv}{qB}$ $\quad r = \frac{m\,E/B}{qB} \Rightarrow r = \frac{mE}{qB^2}$

c. $r = \frac{(2.67 \times 10^{-26}\text{ Kg})(10\text{ V/m})}{(1.6 \times 10^{-19}\text{ C})(0.01\text{T})^2} = 0.017$ m

4 Chapter 13: Electromagnetic Induction

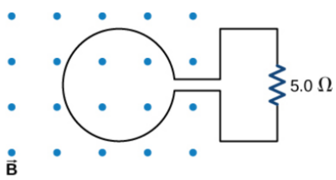


Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0}(\sin(2\pi f t)) \tag{3}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1?

a. $\epsilon = -N\,d\phi/dt$

$\epsilon = (1)\frac{d\phi}{dt} = \frac{d(BA)}{dt}$ $\quad \pi r^2$

$\epsilon = \pi r^2 \times \frac{B_0}{T_0}(\sin(2\pi f t))$

b. $\sin 2\pi f t$

$t = 0$

induced emf $(\epsilon) = 0$

c. $t_1 = .16$ ms $\to$ 160 s

$B_0 = 0.1$ T

$r = 0.1$ m

$f = 10^3$ Hz

$T_0 = 1$ ms $\to$ .001 s

$\epsilon = \pi r^2 \times \frac{B_0}{T_0}(\sin(2\pi f t))$

$= \pi(0.1)^2 \times \frac{0.1\text{ T}}{.001\text{ s}}(\sin(2\pi 10^3 \times 0.16 \times 10^{-3}))$

$= .056$ V

d. $I = \frac{V}{R} = \frac{0.056\text{ V}}{5\,\Omega} = 0.012$ A

[1] Molecules that do not have this velocity will hit the sides of this portion of the instrument.

5 Chapter 14: Inductance

1. What is (a) the rate at which the current though a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

use Faraday's Law or self inductance equation?

$$emf = -M\left(\frac{dI}{dt}\right) \Rightarrow \frac{emf}{-M} = \frac{dI}{dt} \qquad \left|\frac{dI}{dt}\right| = \frac{-.150 \text{ V}}{0.50 \text{ H}} = 0.30 \text{ A/s}$$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$dI = 0.100$ A

$L = 2.00$ mH

$.002$ H

$\Delta V = 500$ V

$$emf = -M\left(\frac{dI}{dt}\right)$$

$$dt = \frac{-M(dI)}{\Delta V}$$

$$|dt| = \frac{(.002 \text{ H})(0.100 \text{ A})}{(500 \text{ V})} \Rightarrow 4.00 \times 10^{-7} \text{ s}$$