ALGEBRA-BASED PHYSICS-1: MECHANICS (PHYS135A-01): WEEK 1

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5. Text: College Physics (openstax.org)
6. Homework: ExpertTA (theexpertta.com)

Physics - $\phi\upsilon\sigma\iota\kappa\acute{\eta}$ - "phusiké": *knowledge of nature*
from $\phi\acute{\upsilon}\sigma\iota\varsigma$ - "phúsis": *nature*

1. Methods of approximation
   - Estimating the correct order of magnitude
   - Function approximation
   - Unit analysis
2. Coordinates and vectors
   - Scalars and vectors
   - Cartesian (rectangular) coordinates, displacement
   - Vector addition, subtraction, and multiplication
3. Review of geometry and trigonometry techniques
   - Similar triangles
   - Pythagorean theorem
   - Sine, cosine, tangent ...

HOMEWORK PROBLEMS (DUE MONDAY, SEPTEMBER 11)

- Chapter 1, exercise 1
- Chapter 1, exercise 9
- Chapter 1, exercise 10
- Chapter 1, exercise 29
- Chapter 1, exercise 35

METHODS OF APPROXIMATION

In science and engineering, estimation is to obtain a quantity in the absence of precision, informed by rational constraints.

1. Define relevant scales
    - 1 *AU* for the solar system (distance from Sun to Earth)
    - 1 *angstrom* (10^{-10} meters) for cell membranes
2. Obtain complex quantities from simple ones
    - Obtain *areas* and *volumes* from *lengths*
    - Obtain *rates* from *numerators* and *denominators*
3. Constrain the unknown with upper and lower limits
    - The solar system is *less than one light-year across*
    - An insect is *at least one millimeter* long

Estimate the mass of ants in an ant colony. Assume that the colony is a species known to have 10^5 ants (roughly) per colony.

- A: 0.01 kg
- B: 0.1 kg
- C: 1 kg
- D: 10 kg

An adult blue whale is about 30 meters long. What is the mass of a blue whale calf? (1 tonne = 1000 kg).

- A: 100 kg
- B: 0.5 tonnes
- C: 5 tonnes
- D: 20 tonnes

Table 1.3 Approximate Values of Length, Mass, and Time

Lengths in meters		Masses in kilograms (more precise values in parentheses)		Times in seconds (more precise values in parentheses)	
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron $(9.11\times10^{-31}\ \text{kg})$	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom $(1.67\times10^{-27}\ \text{kg})$	10^{-22}	Mean life of an extremely unstable nucleus
10^{-14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cells of living organisms	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day $(8.64\times10^{4}\ \text{s})$
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (y) $(3.16\times10^{7}\ \text{s})$
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of the Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from the Earth to the Sun	10^{23}	Mass of the Moon $(7.35\times10^{22}\ \text{kg})$	10^{17}	Age of the Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of the Earth $(5.97\times10^{24}\ \text{kg})$	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way galaxy	10^{30}	Mass of the Sun $(1.99\times10^{30}\ \text{kg})$		
10^{22}	Distance from the Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from the Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Figure 1: Table 1.3 from the text.

How long does it take an airliner to fly across the Atlantic ocean? Assume the velocity is 500 mph, and the radius of the Earth is 7000 km.

- A: 10 hours
- B: 15 hours
- C: 2 hours
- D: 4 hours

A flock of birds takes one minute to pass overhead, and it is about 100 meters wide, with most birds flying at roughly the same altitude. How many birds are in the flock?

- A: 100 birds
- B: 1,000 birds
- C: 10,000 birds
- D: 100,000 birds

In science and engineering, function approximation or expanding a function is a technique in which a simple function is used obtain the value of a more complicated function near a point where they are approximately equal.

1. Memorizing special cases
    - $\sin(x) \approx x$, when $|x| < 1$
    - $\tan(x) \approx x$, when $|x| < 1$
    - $(1+x)^{1/2} \approx 1 + \frac{1}{2}x$, when $|x| < 1$
    - $\exp(x) \approx 1 + x$, when $|x| < 1$
2. Utilizing the **Taylor Series** (encounter in calculus...)
    - $f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$

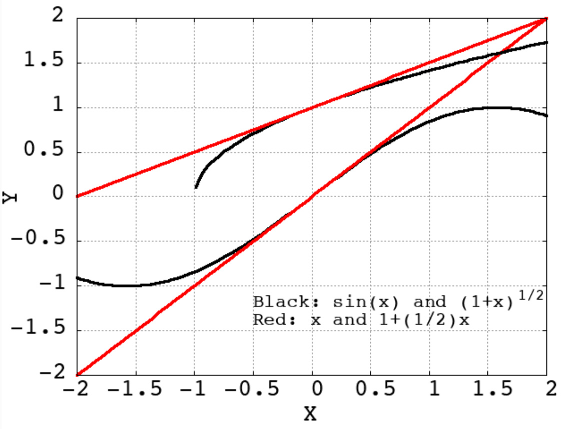


Figure 2: Certain functions may be approximated by simpler ones. In this case, $\sin(x)$ is approximated by x near $x = 0$, and $(1+x)^{1/2}$ is approximated by $1 + \frac{1}{2}x$ near $x = 0$.

The height in meters of a surfer above some average height as he bobs in the waves is described by $h(t) = sin(t)$. What is his height at 1.0 second? What is his height at -1.0 second?

- A: 1 meter, -1 meter
- B: π meters, -π meters
- C: -1 meter, 1 meter
- D: -π meters, π meters

The value of an investment in dollars, v, versus time in years, t, follows the form $v(t) = P\exp(rt)$, where P is the value at $t = 0$, and $r = 1/3$. What is $v(1)$, the value after one year?

- A: $\approx 1/3P$
- B: $\approx 2/3P$
- C: $\approx 3/2P$
- D: $\approx 4/3P$

Physics requires units to relate ideas to the real world, and unit analysis is a powerful tool to eliminate incorrect results and to facilitate estimation.

1. SI units, and kilogram-meter-second unit set
    - mass: kilogram (gram = 10^{-3} kg, milligram = 10^{-6} kg)
    - length: meter (millimeter = 10^{-3} m, kilometer = 10^{3} m)
    - time: second (1 year $\approx \pi \times 10^7$ sec, 1 hour = 3600 sec)
2. Unit analysis
    - If we are calculating a density, the units should work out to be kg/m^3
    - Identifying the fundamental unit in a complex calculation often simplifies it (when done properly, this reveals the beauty of physics)

A millenium is 1000 years. If a glacier retreats at a pace of 10 cm per year, what is this rate in meters per millenium?

- A: 0.1 meter per millenium
- B: 1 meter per millenium
- C: 10 meters per millenium
- D: 100 meters per millenium

Ice has a density of 0.917 grams per centimeter cubed. What is this density in kilograms per meter cubed?

- A: 91.7 kg/m^3
- B: 917 kg/m^3
- C: 9170 kg/m^3
- D: 9.17 kg/m^3

Sometimes, the beauty of physics arrises from choosing the right unit.

http : //joshworth.com/dev/pixelspace/pixelspace_solarsystem.html

The Sun in this ruler is at 0 km, and Jupiter is at about 780,000,000 km (good luck finding it). Clearly, the kilometer is the wrong unit to choose for interplanetary distances. What if we defined a new unit, the astronomical unit, as the distance between the Earth and the Sun?

Planetary orbital radii in AU (geometric means):

Mercury	0.379
Venus	0.722
Earth	1.00
Mars	1.52
Jupiter	5.20
Saturn	9.54
Uranus	19.2
Neptune	30.1

Figure 3: Why such simple numbers? There is a set of simple relationships between the *orbital period* and the *orbital radius* of planets called Kepler's Laws, which led to the discovery of Newton's Law of Gravity.

COORDINATES AND VECTORS

Physics requires mathematical objects to build equations that capture the behavior of nature. Two examples of such objects are scalar and vector quantities. Each type of object obeys similar but different rules.

1. Scalar quantities
    - mass: $m_1 + (m_2 + m_3) = (m_1 + m_2) + m_3$
    - speed: $v_1(v_2 + v_3) = v_1 v_2 + v_1 v_3$
    - charge: $q_1 \left(\frac{1}{q_1}\right) = 1$, $q_1(0) = 0$
2. Vector quantities
    - velocity: $\vec{v}_1 + (\vec{v}_2 + \vec{v}_3) = (\vec{v}_1 + \vec{v}_2) + \vec{v}_3$
    - tension: $\vec{t}_1 \cdot (\vec{t}_2 + \vec{t}_3) = \vec{t}_1 \cdot \vec{t}_2 + \vec{t}_1 \cdot \vec{t}_3$

A vector may be expressed as *a list of scalars*: $\vec{v} = (4, 2)$ (a vector with two *components*), $\vec{u} = (3, 4, 5)$ (three *components*). Now, we know how to add and subtract scalars. How do we add and subtract vectors?

What is
$(1, 3, 8)+$
$(0, 2, 1)$?
Answer: $(1, 5, 9)$

In other words, when adding vectors, we add them component by component.

How do we subtract vectors? In the same fashion:

What is
$(1, 3, 8)-$
$(0, 2, 1)$?
Answer: $(1, 1, 7)$

In other words, when subtracting vectors, we subtract them component by component.

How do we multiply vectors? In the same fashion, *for one kind of multiplication*:

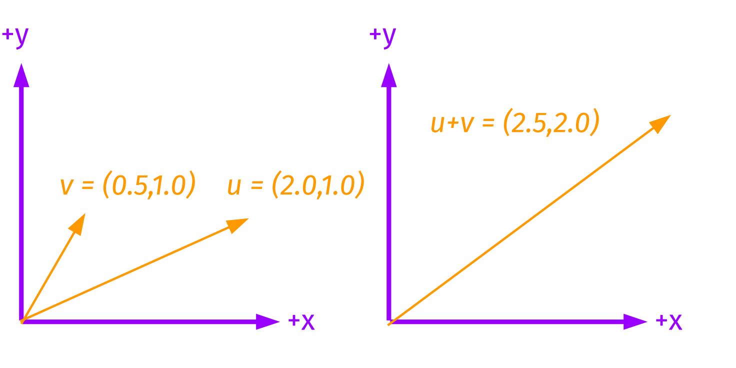
What is
$(1, 3, 8) \cdot (0, 2, 1)$?
Answer: $1 \cdot 0 + 3 \cdot 2 + 8 \cdot 1 = 14$

This kind of multiplication is known as the dot-product. There is also the *cross-product*, which we will save for later.

The components of a vector may describe quantities in a coordinate system, such as *Cartesian coordinates* - after René Descartes. Vectors in the 3D Cartesian coordinate system (x,y,z) may be written in the following notation:

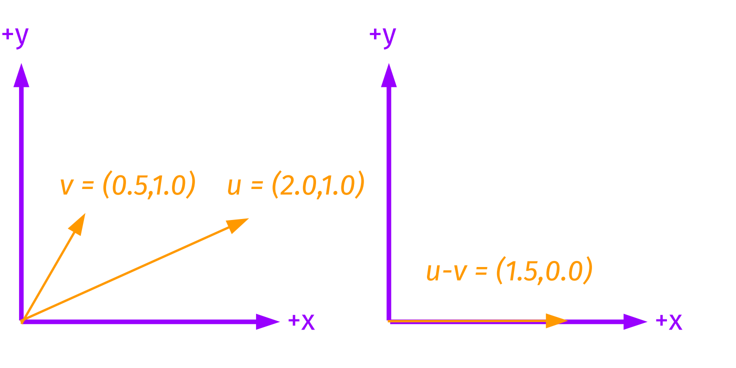
$$\boxed{\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}}$$

- a: The amount in the +x-direction, $\hat{i}$: a vector of length 1, in the +x-direction
- b: The amount in the +y-direction, $\hat{j}$: a vector of length 1, in the +y-direction
- c: The amount in the +z-direction, $\hat{k}$: a vector of length 1, in the +z-direction



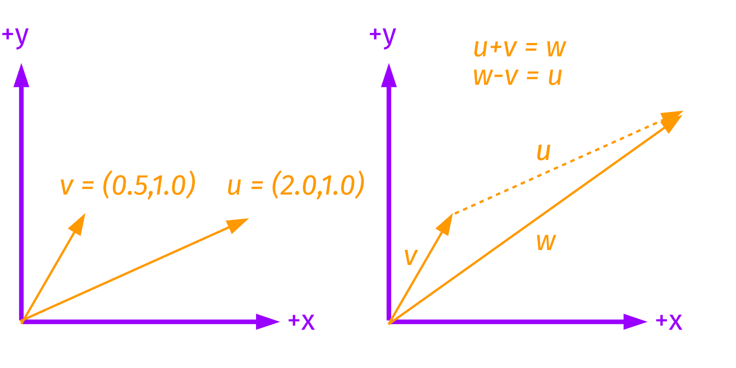
(a) (b)

Figure 4: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} + \vec{v}$? Adding components: $\vec{u} + \vec{v} = 2.5\hat{i} + 2.0\hat{j}$.



(a) (b)

Figure 5: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} - \vec{v}$? Subtracting components: $\vec{u} - \vec{v} = 1.5\hat{i} + 0.0\hat{j}$.



(a) (b)

Figure 6: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) To compute $\vec{w} - \vec{v}$, arrange the vectors to get a sense of the result, $\vec{u}$.

$\vec{p} = 4\hat{i} + 2\hat{j}$. $\vec{q} = -4\hat{i} + 2\hat{j}$.
Compute $\vec{p} \cdot \vec{q}$.

- A: 12
- B: -12
- C: 4
- D: 8

$\vec{p} = -1\hat{i} + 6\hat{j}$. $\vec{q} = 3\hat{i} + 0.5\hat{j}$.
Compute $\vec{p} \cdot \vec{q}$.

- A: -1
- B: 1
- C: 0
- D: 3

Why was the last answer zero? Look at it graphically:

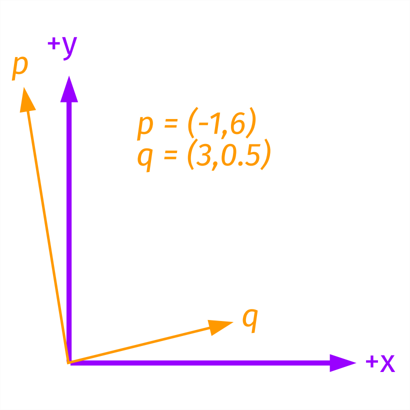


Figure 7: Two vectors $\vec{p}$ and $\vec{q}$ are *orthogonal* if $\vec{p} \cdot \vec{q} = 0$.

What if the vectors are parallel? Look at it graphically:

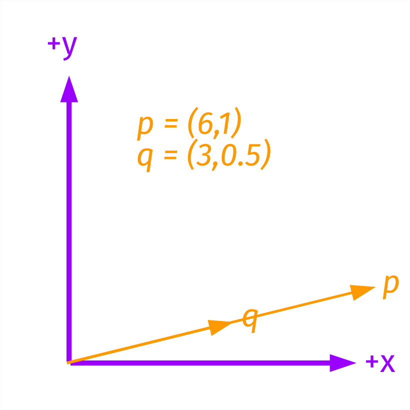


Figure 8: Two vectors $\vec{p}$ and $\vec{q}$ are *parallel* if $\vec{p} \cdot \vec{q}$ is maximal.

The *length* or *norm* of a vector $\vec{v} = a\hat{i} + b\hat{j}$ is $|\vec{v}| = \sqrt{a^2 + b^2}$.

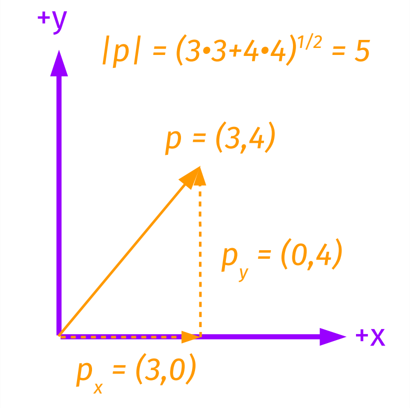


Figure 9: Computing the norm of a vector $\vec{p}$.

Notice that $\sqrt{\vec{p} \cdot \vec{p}} = |\vec{p}|$.
Let θ_p be the angle between $\vec{p}$ and the x-axis.
$p_x = \vec{p} \cdot \hat{i} = |\vec{p}| \cos(\theta_p)$.
$p_y = \vec{p} \cdot \hat{j} = |\vec{p}| \sin(\theta_p)$.

Theorem: The dot product of two vectors $\vec{p}$ and $\vec{q}$ is $|u||v| \cos(\theta)$, if θ is the angle between them.

Proof:
$\vec{p} \cdot \vec{q} = p_x q_x + p_y q_y = |p||q| \cos\theta_p \cos\theta_q + |p||q| \sin\theta_q \sin\theta_q$
$= |p||q|(\cos\theta_p \cos\theta_q + \sin\theta_p \sin\theta_q) = |p||q| \cos(\theta_p - \theta_q)$
$= |p||q| \cos\theta$.
$\boxed{\vec{p} \cdot \vec{q} = |p||q| \cos\theta}$

An object moves at 2 m/s at $\theta = 60°$ with respect to the x-axis. What is the velocity of the object?

- A: $(1\hat{i} + 1\hat{j})$ m/s
- B: $(\sqrt{3}\hat{i} + 1\hat{j})$ m/s
- C: $(\sqrt{3}\hat{i} + \sqrt{3}\hat{j})$ m/s
- D: $(1\hat{i} + \sqrt{3}\hat{j})$ m/s

What is the dot product of this velocity with another velocity: 5 m/s along the x-axis?

- A: 1 $(\text{m/s})^2$
- B: 5 $(\text{m/s})^2$
- C: 10 $(\text{m/s})^2$
- D: 5 (m/s)

Is it possible to multiply vectors and scalars? Of course: $a_1\vec{p} = a_1 p_x \hat{i} + a_1 p_y \hat{j}$.

Also, multiplication properties still hold. For example: $(a_1 + a_2)\vec{p} = a_1\vec{p} + a_2\vec{p}$.

A spacecraft moves at 400 m/s, at an angle of 30 degrees with respect to the x-axis. If it fires two thrusters that boost the x-component and y-component of the velocity by 25% and 50%, respectively, what is the final velocity?

- A: $(433\hat{i} + 300\hat{j})$ m/s
- B: $(300\hat{i} + 433\hat{j})$ m/s
- C: 400 m/s
- D: $(400\hat{i} + 433\hat{j})$ m/s

We define the *position* of an object as a vector locating it in a given coordinate system. The scalar *distance* is the norm of the position vector, that is, the distance to to the origin.

Now we can introduce the concept of dislacement: a vector describing a movement of an object.

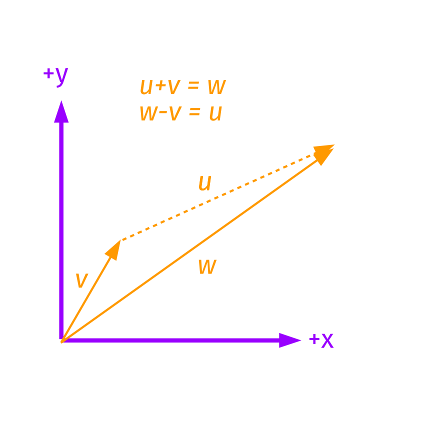


Figure 10: Suppose an object moves from position $\vec{v}$ to $\vec{w}$. In this case, the displacement is $\vec{u}$. **Thus, the final position is the initial position, plus the displacement.**

It follows that the *displacement* is zero if the initial and final positions are the same, but the *distance travelled* is not.

Suppose a jet fighter travelling at 800 km per hour banks such that it flies in a circle of radius 0.5 km. How long does it take to complete the circle? What is the distance traveled, and what is the displacement?

- A: 2π km, 28 seconds, 2π km
- B: π km, 14 seconds, π km
- C: π km, 28 seconds, π km
- D: π km, 14 seconds, 0 km

Download the Java applet (you may need to update Java):
https://phet.colorado.edu/en/simulation/vector-addition

Average velocity is the ratio of the displacement to the elapsed time.

$$\boxed{\vec{v}_{\mathrm{avg}} = \frac{\Delta \vec{x}}{\Delta t}} \tag{1}$$

The *average speed* is the norm of the average velocity:

$$\boxed{v_{\mathrm{avg}} = \frac{|\Delta \vec{x}|}{\Delta t}} \tag{2}$$

If the motion is in one dimension, then the average speed is

$$v_{\mathrm{avg}} = \frac{x_{\mathrm{f}} - x_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} \tag{3}$$

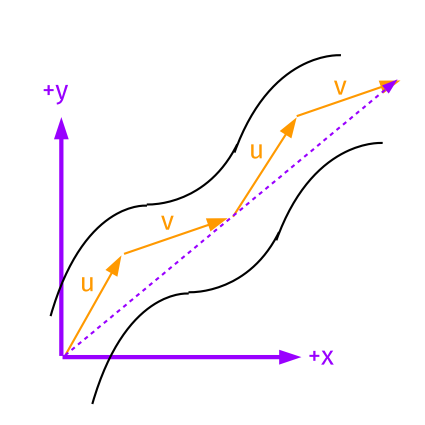


Figure 11: A Formula-1 driver keeps his car on the track by following a path approximated by the position vectors *u*, *v*, *u*, and *v*. The dashed arrow represents the total displacement.

If $\vec{u} = (20\hat{i} + 30\hat{j})$ m, and $\vec{v} = (30\hat{i} + 20\hat{j})$ m, what is the total displacement? If the elapsed time is 10 seconds, what is the average velocity?

- A: $(50\hat{i} + 50\hat{j})$ m, 14 m/s
- B: $(80\hat{i} + 100\hat{j})$ m, 10 m/s
- C: $(100\hat{i} + 100\hat{j})$ m, 14 m/s
- D: $(50\hat{i} + 150\hat{j})$ m, 10 m/s

REVIEW OF GEOMETRY AND TRIGONOMETRY TECHNIQUES

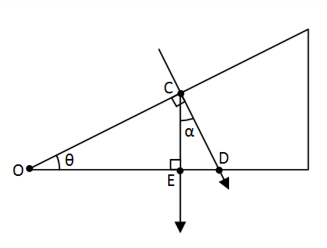


Figure 12: Angles of a triangle add up to π (180°).

$$\angle OCD = \pi/2 = \angle OCE + \angle DCE = \angle OCE + \alpha \tag{4}$$

$$\angle OCE + \angle COE + \pi/2 = \pi = \angle OCE + \theta + \pi/2 \tag{5}$$

$$\angle OCE + \theta = \pi/2 \tag{6}$$

$$\theta = \alpha \tag{7}$$

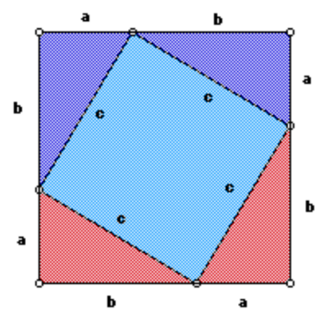


Figure 13: Proof of Pythagorean theorem.

$$A_1 = (a+b)^2 = a^2 + b^2 + 4((1/2)ab) \tag{8}$$

$$A_2 = c^2 + 4((1/2)ab) = A_1 \tag{9}$$

$$a^2 + b^2 = c^2 \tag{10}$$

One soccer teammate passes the ball to another. The player without the ball is 7 meters away from the player with the ball, and they are both running in the same direction. The player without the ball runs ahead by 24 meters before the pass. How far does the ball travel?

- A: 7 meters
- B: 24 meters
- C: 25 meters
- D: 17 meters

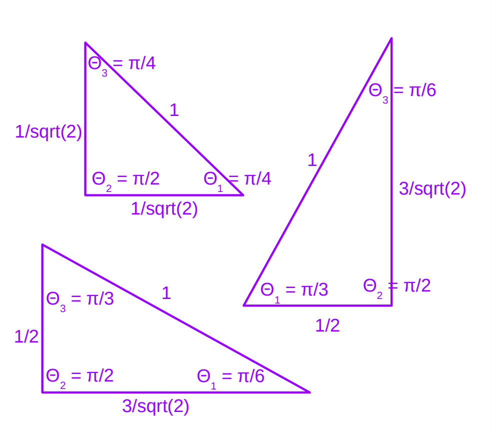


Figure 14: Memorize the properties of these special triangles.

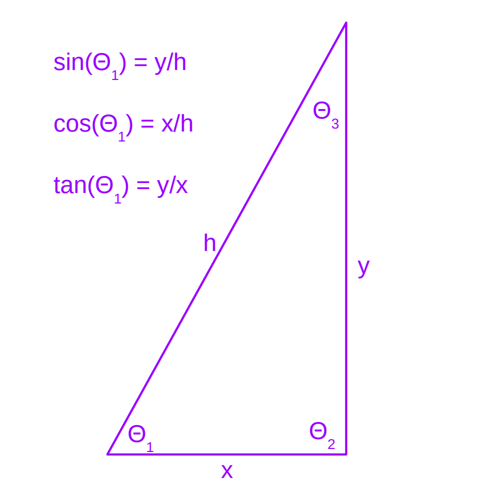


Figure 15: Working definitions of trigonometric functions.

What is $\sin(30^\circ)$?

- A: $1/2$
- B: $\sqrt{3}/2$
- C: 0
- D: 1

What is $\tan(45^\circ)$?

- A: $1/2$
- B: $\sqrt{3}/2$
- C: 0
- D: 1

What is $\sin(30^\circ)^2 + \cos(30^\circ)^2$?

- A: 1/2
- B: $\sqrt{3}/2$
- C: 0
- D: 1

A right-triangle has sides of length 3, 4, and a hypoteneuse of 5. What are the angles inside the triangle?

- A: arctan(5/4), arctan(4/5), $\pi/2$
- B: arctan(1), arctan(4/5), $\pi/2$
- C: arctan(4/3), arctan(3/4), $\pi/2$
- D: arctan(3/4), arctan(3/4), $\pi/2$

CONCLUSION

Chapters 1, 2.

1. Methods of approximation
    - Estimating the correct order of magnitude
    - Function approximation
    - Unit analysis
2. Coordinates and vectors
    - Scalars and vectors
    - Cartesian (rectangular) coordinates, displacement
    - Vector addition, subtraction, and multiplication
3. Review of geometry and trigonometry techniques
    - Parallel lines, similar triangles
    - Pythagorean theorem
    - Sine, cosine, tangent …

ANSWERS

- Mass of ants: 0.1 kg
- Mass of baby whale: 5 tonnes
- Length of flight is 10 hours
- Number of birds is 10,000
- Height of surfer is 1.0 meter, -1.0 meter
- Value of investment is $4/3P$
- The glacier is retreating at 100 meters per millenium
- Ice has a density of 917 kg/m^3
- -12
- 0
- $(1\hat{i} + \sqrt{3}\hat{j})$ m/s
- 10 (m/s)^2
- $(433\hat{i} + 300\hat{j})$ m/s
- π km, 14 seconds, 0 km
- $(100\hat{i} + 100\hat{j})$ m, 14 m/s
- 25 meters
- 1/2
- 1
- arctan(4/3), arctan(3/4), $\pi/2$