ALGEBRA-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND MODERN PHYSICS (PHYS135B-01): UNIT 2

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February 6, 2018

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UNIT 1 REVIEW

Reading: Chapters 18 and 19

1. Charge, mass, the Coulomb force, and the gravitational force
2. Force fields
3. Electric potential and capacitance

UNIT 1 REVIEW PROBLEMS

Charged black holes: Suppose two black holes with the same mass are pulled towards each other by gravity. Each, however, has a slight positive charge. If the Coulomb force balances with gravity, what is the charge of the black holes? Each black hole has a mass of 6×10^{30} kg, $G = 7 \times 10^{-11}$ m^3 s^{-2} kg^{-1}, and $\epsilon_0 = 9 \times 10^{-12}$ N^{-1} m^{-2} C^2.

- A: 5×10^{40} C
- B: 5×10^{30} C
- C: 5×10^{20} C
- D: 5×10^{10} C

Is this number surprisingly small, or surprisingly large?

SUMMARY

Reading: Chapters 20 and 21

1. Current, Ohm's Law, resistors and conductors
2. DC circuits I
3. Nerve signals
4. DC circuits II

JITT - READING QUIZ RESULTS

CURRENT

Notions of current:

- $I = \frac{\Delta Q}{\Delta t}$ - The derivative of charge
- The *movement* of electrons
- The *flow* of charge
- Number of Coulombs per second (1 Amp = C/s)

There is an interesting problem with the notion of current as movement of charges.

Speed of typical electronic signals: $\approx 10^8$ m/s

Typical speed of actual charges passing through a conductor under voltage: $\approx 10^{-4}$ m/s

Since there is a 12 order of magnitude range, it's probably a good idea to ponder...

Are the electrons colliding/interacting to form electrical signals? Or just moving all together?

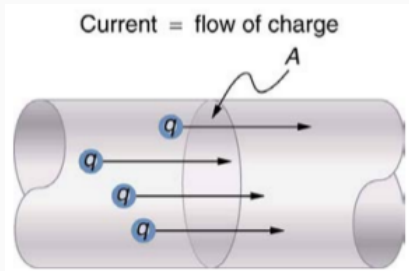**Figure 1:** The *drift velocity* is the average velocity of an electron, and current is derived from this average velocity.

https://youtu.be/8dgyPRA86K0

The answer lies in the fact that we are no longer dealing with contact forces, but long-range interactions like the Coulomb force.
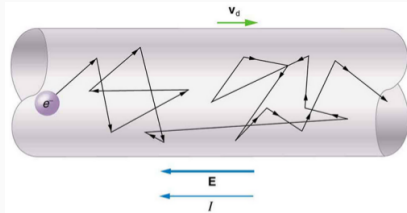

Figure 2: The *drift velocity* is the average velocity of an electron, and current is derived from this average velocity.

Electrical signals are more like a wave on a string:
`https://phet.colorado.edu/en/simulation/legacy/`
`wave-on-a-string`

So we see how electrical signals can move near the speed of light, but we measure the movements of electrons in circuits to be slow. Can we make a calculation to understand the speed of the electrons?
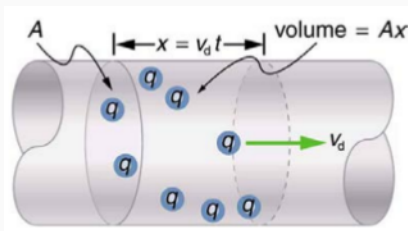

Figure 3: Consider the volume V of conductor with cross-sectional area A and length Δx, having n free electrons per unit volume.

An **amp** is one *Coulomb* per *second*. The definition of current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{qnA\Delta x}{\Delta t} = qnAv_{\mathrm{d}} \qquad (1)$$

Solving for drift velocity:

$$v_{\mathrm{d}} = \frac{I}{qnA} \qquad (2)$$

Suppose our conductor is a wire with radius r and $A = \pi r^2$. Substituting,

$$v_{\mathrm{d}} = \frac{I}{\pi qnr^2} \qquad (3)$$

Remember that $q = 1.6 \times 10^{-19}$ C, and n is the number of free electrons *per atom per unit volume.* How do we get this number?

Number density: Let's examine copper, a common wire material with one free electron per atom. Copper has a density of 8800 kg/m^3, and 0.06354 kg/mol. There are 6.02×10^{23} atoms/mol. How many free electrons per m^3 of copper?

- A: 8.342×10^{26} free electrons per kg
- B: 8.342×10^{27} free electrons per kg
- C: 8.342×10^{28} free electrons per kg
- D: 8.342×10^{29} free electrons per kg

Consider a copper wire with radius $r = 2.053$ mm that is carrying 20.0 A of current. Using $q = 1.6 \times 10^{19}$, and $n = 8.342 \times 10^{28}$ electrons/m^3, and $v_d = I/(\pi q n r^2)$, compute the drift velocity of charge in the wire. *This is a common situation in household wiring.*

- A: 4.53×10^1 m/s
- B: 2.25×10^{-2} m/s
- C: 2.25×10^{-3} m/s
- D: 4.53×10^{-4} m/s

OHM'S LAW

Electrons are now moving through our copper wire ($v_d \propto I$, $v_d \propto A^{-1}$). What happens when the electrons, which have had some PE converted to KE, encounter objects that are not conductors? **They deposit energy and move forward.**
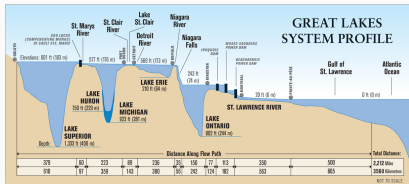

Figure 4: Current is comprised of electrons that deposit energy as they move to lower voltages.

PhET: https://phet.colorado.edu/en/simulation/
circuit-construction-kit-dc

PhET: Create a DC circuit involving a battery, resistor (the brown striped object), a light bulb, and a switch.

1. Place the battery and connect to it a wire, and attach a resistor to that wire.

2. To the other end of the resistor, connect a switch and leave it open.

3. Connect a light bulb to the other end of the switch, and connect a wire from the light bulb to the battery.

4. The properties of each circuit element can be edited by clicking on the element.

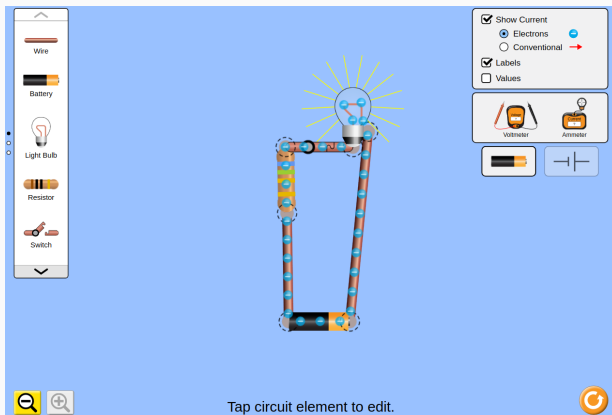**Figure 5:** Your circuit should resemble this.

PhET: Make observations.

1. What happens to the drift velocity of the electrons as you raise and lower the resistance? Why do we call light bulbs and the brown striped objects "resistors?"

2. Since we cannot change the cross-sectional area of the wire independently, we can treat $v_{\mathrm{d}} \propto I$.

3. How does the current change if you increase the voltage?

PhET: The unit of resistance is the Ohm. We use the symbol Ω for Ohms, and $1\Omega = 1\text{V/A}$.

1. There are two devices available: the *voltmeter*, and the *ammeter*. Using these devices, measure the voltage drop across the resistor and the light bulb, and the current flowing through the circuit.

2. How are voltage and current and resistance related? Derive an equation from the data.

Ohm's Law: Let *V* be the voltage change across a resistor with resistance *R*, and let *i* be the current flowing through the resistor. Ohm's law states that

$$V = iR \qquad (4)$$

for materials that fall into the category of *Ohmic*.

PhET: **How do we deal with more complex circuits?** There must be a way to "add" resistors.

1. Create a circuit that involves just a hairy mess of resistors. Connect them *in series* and *in parallel*.
2. Calculate the *effective total resistance* by plotting an $i - V$ curve of the system. Measure i and V by changing the voltage and using the voltmeter and ammeter.
3. What is the effective total resistance of the circuit? How did you obtain this number from the $i - V$ curve?

As you may have discovered, *resistors in series* **add**:

$$R_{\text{tot}} = R_1 + R_2 + ... \tag{5}$$

Resistors in parallel **add their reciprocals**:

$$\frac{1}{R_{\text{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + ... \tag{6}$$

Resitance is not an *intrinsic property* of materials. Imagine a 0.1 m-long wire (which is a conductor) actually having a small resistance. What about that same wire, but 1 kilometer long?

- Electrons lose some fixed energy per unit length in a given material (*Joule heating*)
- Electrons lose more energy if the wire is thinner (*Joule heating*)

Resistivity ρ is defined in terms of resistance R, length L and cross-sectional area A as

$$R = \rho \left(\frac{L}{A} \right) \tag{7}$$

Material	Resistivity ρ ($\Omega \cdot m$)
Conductors	
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Platinum	10.6×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}
Manganin (Cu, Mn, Ni alloy)	44×10^{-8}
Constantan (Cu, Ni alloy)	49×10^{-8}
Mercury	96×10^{-8}
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}

Figure 6: Conductor resistivities are in units of Ω m, and are small but non-zero.

Semiconductors[1]	
Carbon (pure)	3.5×10^{5}
Carbon	$(3.5 - 60) \times 10^{5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1 - 600) \times 10^{-3}$
Silicon (pure)	2300
Silicon	$0.1 - 2300$
Insulators	
Amber	5×10^{14}
Glass	$10^{9} - 10^{14}$
Lucite	$> 10^{13}$
Mica	$10^{11} - 10^{15}$
Quartz (fused)	75×10^{16}
Rubber (hard)	$10^{13} - 10^{16}$
Sulfur	10^{15}
Teflon	$> 10^{13}$
Wood	$10^{8} - 10^{11}$

Figure 7: Semiconductor resistivities are in units of Ω m, and are larger.

Two copper wires need to be attached to carry current to the top floor of a building. One has a cross-sectional area of 2 mm and is 10 meters long, while the other has a cross-sectional area of 4 mm and is 15 meters long. What is the total resistance of the two wires attached in series?

- A: about 1 mΩ
- B: about 20 mΩ
- C: about 200 mΩ
- D: about 2 Ω

Consider the same system. If we attach a battery and use the wire to feed voltage to some circuit drawing 3.0 A of current, what is the voltage drop due to just the wire?

- A: about 60 mV
- B: about 600 mV
- C: about 6 V
- D: Current will not flow at all

What would the resistance be if the wire system was 10 times as long?

- A: about 60 mV
- B: about 600 mV
- C: about 6 V
- D: Current will not flow at all

So you can start to see that resistance matters even for conductors, if the current is traveling for long distances. Often manufacturers quote the Ohms per foot in wire data sheets.

Resistivity depends on temperature in the following way:

$$\rho = \rho_0 \left(1 + \alpha \Delta T\right) \tag{8}$$

For most conductors, α is small, on the order of $10^{-3}\ {}^\circ C^{-1}$.

Material	Coefficient α (1/°C) [2]
Conductors	
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.93×10^{-3}
Lead	3.9×10^{-3}
Manganin (Cu, Mn, Ni alloy)	0.000×10^{-3}
Constantan (Cu, Ni alloy)	0.002×10^{-3}
Mercury	0.89×10^{-3}
Nichrome (Ni, Fe, Cr alloy)	0.4×10^{-3}
Semiconductors	
Carbon (pure)	-0.5×10^{-3}
Germanium (pure)	-50×10^{-3}
Silicon (pure)	-70×10^{-3}

Figure 8: Conductor resistivities depend on temperature.

Continuing with the same example of the long copper wires attached together (10.0 m and 15.0 m), if the temperature increases by 50.0 deg C, what is the new resistance?

- A: 16 mΩ
- B: 20 mΩ
- C: 24 mΩ
- D: 30 mΩ

Power is consumed in resistors, since charges are losing energy and new charges are showing up at a certain rate. Consider the PE converted to work in a resistor:

$$\Delta PE = \Delta q \Delta V \tag{9}$$

$$\Delta W = \Delta q i R \tag{10}$$

$$\frac{\Delta W}{\Delta t} = \frac{\Delta q}{\Delta t} i R = i^2 R \tag{11}$$

$$\frac{\Delta W}{\Delta t} = iV \tag{12}$$

$$P = iV \tag{13}$$

The formula $P = iV$ shows that the wattage required by some device in a circuit will pull current according to the voltage of the battery.

How much current is required by a 50 W light bulb if the voltage supplying it is 120 V?

- A: 420 mA
- B: 120 mA
- C: 50 mA
- D: 50 V

GRAPHICAL ANALYSIS OF SIMPLE CIRCUITS

CONCLUSION

Reading: Chapters 20 and 21

1. Current, Ohm's Law, resistors and conductors
2. DC circuits I
3. Nerve signals
4. DC circuits II

ANSWERS

ANSWERS

- 5×10^{30} C (double check this)
- 8.342×10^{28} free electrons per kg
- 4.53×10^{-4} m/s
- about 20 mΩ
- about 60 mV
- about 600 mV
- 24 mΩ
- 420 mA