Study Guide for Midterm 1

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1 Estimation and Unit Conversion

1. Which of the following is most likely the speed of a runner on a track?
   - A: 0.5 m/s
   - B: **5 m/s**
   - C: 50 m/s
   - D: 500 m/s
2. Convert the speed you chose in to kilometers per hour.
$$5\left(\frac{m}{s}\right)\left(\frac{1km}{1000m}\right)\left(\frac{3600s}{1hr}\right) = 5\times 3.6km/hr = 18km/hr \tag{1}$$
3. Water flows through a pipe at a rate of 1000 cm^3/s. What is this rate in m^3/hour?
$$10^3\left(\frac{cm^3}{s}\right)\left(\frac{1m}{100cm}\right)^3\left(\frac{3600s}{1hr}\right) = 3.6\times 10^{-6+3+3}\left(\frac{m^3}{hr}\right) = 3.6\left(\frac{m^3}{hr}\right) \tag{2}$$
4. One *knot* is about 0.51 m/s. A submarine travels at 20 knots, and another submarine travels at 25 knots. What is the difference in speed, in meters per second?

   The difference in speed, *in knots*, is 5 knots. So $5kn \times (0.51m/s/kn) = 2.55m/s$.

2 Displacement, Velocity, and Constant Acceleration Vectors

1. An object has an initial position of 3 m, and a final position of -4 m, after 3.5 seconds elapses. What is the average velocity?
$$v_{ave} = \frac{x_f - x_i}{t_f - t_i} = \frac{-4-3}{3.5-0}\ (m/s) = -7/3.5\ (m/s) = -2.0\ (m/s) \tag{3}$$
2. Suppose the position of an object is described by the following equation: $x(t) = 3.0t + 5.0$ m. Which of the following is true of the velocity and acceleration?
   - A: Velocity is positive, acceleration is negative.
   - B: Velocity is negative, acceleration is positive.
   - C: **Velocity is positive, acceleration is zero.** (Remember, if velocity is a *linear* function, acceleration is zero).
   - D: Velocity is negative, acceleration is zero.
3. If $x(t) = 3.0t + 5.0$ m, what is the displacement between $t = 1.0$ sec and $t = 5.0$ sec? What is the acceleration?
   - A: 8 m, 0 m/s^2
   - B: 12 m, 2 m/s^2
   - C: **12 m, 0 m/s**^{2} (It has to be one of the answers with no acceleration, and plug-in to find $x_f - x_i = x(5) - x(1) = 12m$.
   - D: 8 m, 2 m/s^2

4. A basketball is shot horizontally from the top of a 100 m-tall building. The initial vertical velocity is 0 m/s, and the initial horizontal velocity is 3 m/s. How far away from the edge of the building does the ball land? (You can assume that $g = -10$ m/s^2 for this problem).

   To find the horizontal displacement, we'd need to know the horizontal velocity (given, 3 m/s) and the *time*.

   $$\Delta x = v_x \Delta t \tag{4}$$

   We don't know Δt yet. Let's assume $t_i = 0$, so that $\Delta t = t_f - t_i = t_f$. How do we get t_f?

   $$y(t) = \frac{1}{2}at^2 + v_{i,y}t + y_i \tag{5}$$

   This equation is true, since we are applying it to the *vertical direction only.* Since the object is falling, we have an acceleration of $\vec{a} = -g\hat{j}\ m/s^2$. Also, $v_{i,y} = 0$ m/s because the ball is shot *horizontally*, meaning it has no vertical velocity initially. If the final y-position is at the ground, then

   $$y_f - y_i = -\frac{1}{2}gt_f^2 \tag{6}$$

   $$0 - h = -\frac{1}{2}gt_f^2 \tag{7}$$

   $$t_f = \sqrt{2h/g} \tag{8}$$

   Now we have the t_f, so we can plug it in to Eq. 4:

   $$\Delta x = v_x\sqrt{2h/g} = 3\sqrt{20}\ m \tag{9}$$

5. What is the final velocity of the ball?

   To find the final velocity, we need both components of the velocity, v_x and v_y. But v_x is *constant* the entire time, because there is no horizontal acceleration. To find $v_{y,f}$ at the time of landing, we can use

   $$v_{y,f} = v_{i,y} + at = -gt_f = -g\sqrt{2h/g} \tag{10}$$

   Remember, $v_{i,y} = 0$ because the ball was shot horizontally, and $a = -g$ (acceleration is down). Now we have $v_{x,f}$ and $v_{y,f}$, which are components of the velocity vector. If we know the components of the velocity vector, we can use Pythagorean theorem to solve for the magnitude:

   $$|v| = \sqrt{v_{x,f}^2 + v_{y,f}^2} = \sqrt{v_{x,f}^2 + g^2(2h/g)} = \sqrt{v_{x,f}^2 + 2gh} \approx 45\ m/s \tag{11}$$

3 Vectors

1. Let $\vec{x}_f = (3.0, -3.0)$ m, and $\vec{x}_i = (3.0, 3.0)$ m. What is $\Delta\vec{x} = \vec{x}_f - \vec{x}_i$?

   Remember to subtract x's from x's and y's from y's:

   $$\Delta\vec{x} = (3.0 - 3.0, -3.0 - 3.0)\ m = (0.0, -6.0)\ m \tag{12}$$

2. A jet fighter (Maverick) has an initial speed of 100 m/s, at a 60 degree angle with respect to horizontal. Another fighter (Jester) has an initial speed of 100 m/s, but at a 45 degree angle with respect to horizontal. What is the velocity of Maverick, minus the velocity of Jester? *Hint: it's not 0 m/s. Build the velocity vector for each fighter first.*

   First, the velocity vector of Maverick: the hypoteneuse of the triangle is 100 m/s, and the angle is 60 degrees. Thus, the x-component is $100\cos(60°)$ m/s, and the y-component is $100\sin(60°)$ m/s, so

   $$\vec{v}_M = (100/2, \sqrt{3}(100)/2)\ m/s = (50, 50\sqrt{3})\ m/s \tag{13}$$

   The same logic applies to the velocity vector of Jester, except it's 45 degrees instead of 60.

   $$\vec{v}_J = (100/\sqrt{2}, 100/\sqrt{2})\ m/s \tag{14}$$

3. If Maverick accelerates to a velocity of $v = (100, 100)$ m/s, what is his speed?

   Use Pythagorean theorem: $\sqrt{100^2 + 100^2} = \sqrt{2 \times 10^2} = 100\sqrt{2}\ m/s.$

4. Multiply them via the dot-product. Evaluate the dot product $\vec{x}_1 \cdot \vec{x}_2$, if $\vec{x}_1 = (0, 1)$ m, and $\vec{x}_2 = (2, 5)$ m.

   $(0, 1) \cdot (2, 5)\ m^2 = 0 \times 2 + 1 \times 5\ m^2 = 5\ m^2$