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Midterm 3

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1 Memory Bank

1. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
2. $P = IV$ ... Relationship between power, current, and voltage.
3. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
4. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.
5. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ... Ampère's Law.
6. $\epsilon = -N d\phi/dt$ ... Faraday's Law.
7. $\phi = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux.
8. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
9. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L. Self-inductance: $\Delta V = -L(dI/dt)$.
10. Units of inductance: V s A^{-1}, which is called a Henry, or H.
11. $B = \mu_0 n I$ ... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

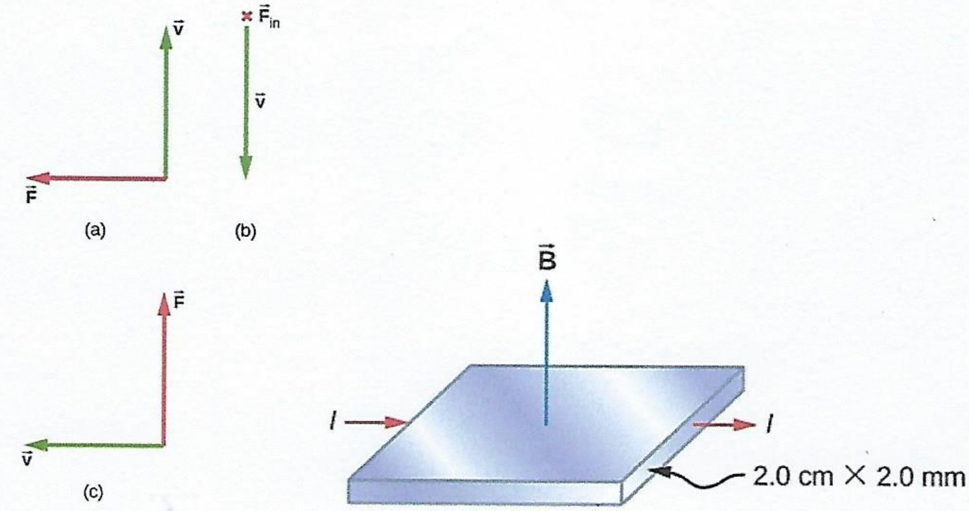


Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.
   - a: Into the page
   - b: Left
   - c: Out of page

2. Consider Fig. 1 (right). **The Hall Effect**. An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that

$$\Delta V = \frac{B \Delta x I}{n q_e A} \tag{1}$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28}$ m^{-3}, $A = 1$ mm^2, and q_e is the charge of an electron.

a) Magnetic field is always 90° to the velocity, force is $F = qvB$ Cross product;
$F = qE \quad F = qvB$
$qE = qvB$
$E = vB$
$\boxed{v = \frac{E}{B}}$

b) $\Delta V = \frac{(1.33T)(0.02m)(10A)}{(2\times10^{28} m^{-3})(1.6\times10^{-19}C)(0.001)^2}$
$\boxed{\Delta V = 8.3\times10^{-5}\ V}$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^4 A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

$A = \pi r^2$
$A = (0.650\times10^{-15} m)^2(\pi)$
$A = 1.327\times10^{-30} m^2$

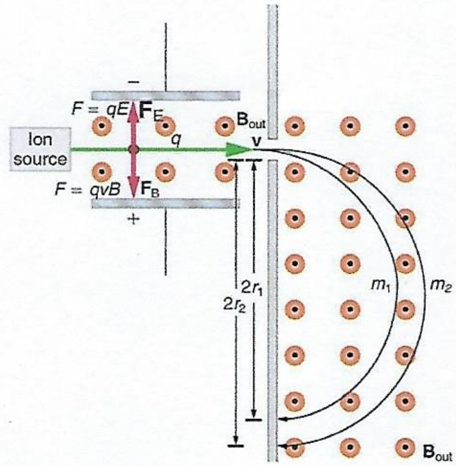
$\tau = (1)(1.05\times10^4 A)(1.327\times10^{-30} m^2)(2.5)(\sin 90°)$
$\boxed{\tau = 3.48\times10^{-26}\ N\cdot m}$

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

a) $B = \mu_0 n I$ where μ_0 is $4\pi\times10^{-7}$ T/amp m
$B = (4\pi\times10^{-7}$ T/amp m$)(500$ turns/m$)(0.3A)$
$\boxed{B = 1.8\times10^{-4}\ T}$

B) $(4\pi\times10^{-7}$ T/amp m$)(5000)$
6.28×10^{-3} T/amp m
$B = (6.28\times10^{-3}$ T/amp m$)(500)(0.3)$
$\boxed{B = 0.942\ T}$



a) If $F_E = F_B$, then $qE = qvB$

$E = vB$

$v = \frac{E}{B}$

$F_{net} = 0$

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \tag{2}$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r?

b) $\frac{mv^2}{r} = qvB$ $\quad r = \frac{mv^2}{qvB}$ $\quad r = \frac{m(\frac{E}{B})}{Bq}$ $\quad \boxed{r = \frac{mE}{B^2 q}}$

$mv^2 = r(qvB)$ $\quad r = \frac{mv}{Bq}$

$r = \frac{mE}{B^2 q}$

$r = \frac{(16)(1.67 \times 10^{-27} kg)(10 V/m)}{(0.01 T)^2 (1.67 \times 10^{-27} C)}$

$\boxed{r = 1.6 \times 10^{-2} m}$

4 Chapter 13: Electromagnetic Induction

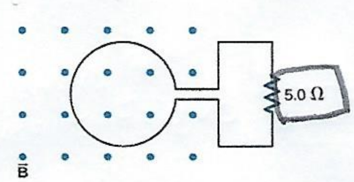


Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0} (\sin(2\pi f t)) \tag{3}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1?

a) $\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0}(\sin(2\pi f t))$

If $N=1$, then area must be πr^2

$\boxed{e = \frac{B_0}{T_0}(\sin(2\pi f t)) \times \pi r^2}$

B) $\sin(2\pi(10^3 Hz)(0))$

$\boxed{= 0}$

c) $\frac{0.1T}{0.001s}(\sin(2\pi(10^3 Hz)(1.6 \times 10^{-4} s))) \times \pi (0.1m)^2$

$\boxed{= 2.65 v}$

D) $I = \frac{E}{R} = \frac{2.65 V}{5 \Omega} = \boxed{0.53 A}$

[1] Molecules that do not have this velocity will hit the sides of this portion of the instrument.

5 Chapter 14: Inductance

$H = V s A^{-1}$ $\quad \frac{\cancel{V}}{\cancel{V} s A^{-1}} = A/s$

1. What is (a) the rate at which the current though a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

$\Delta V = -L(dI/dt)$

$\mathcal{E} = -L\left(\frac{d\phi}{dt}\right) \quad \frac{d\phi}{dt} = \frac{\mathcal{E}}{-L} = \left|-\frac{(0.150V)}{(0.50H)}\right| = \boxed{0.3 A/s}$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$\mathcal{E} = -L(dI/dt)$

$\mathcal{E} = -L\left(\frac{dI}{dt}\right)$

$\left|-\frac{\mathcal{E}}{L}\right| = \frac{dI}{dt}$

$\left|-\frac{500V}{2\times10^{-3}H}\right| = 2.5\times10^{5} A/s$

$\frac{dI}{dt} = \frac{I}{t}$

$t\left(\frac{dI}{dt}\right) = I$

$t = \frac{I}{dI/dt}$

$t = \frac{0.100\cancel{A}}{2.5\times10^{5}\cancel{A}/s}$

$\boxed{t = 4\times10^{-7} s}$