Laboratory Activity: Unit 1, Measuring g

Prof. Jordan C. Hanson

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1   The Acceleration of Gravity, First Measurement

The goal of this laboratory activity is to measure g, the acceleration due to gravity. Let g be the acceleration downward, and assume it is constant. Let $v_{i,y}$ be the initial velocity in the y-axis, and assume the y-axis is vertical. Let y_i be the initial vertical position. The position of a vertically accelerating object, in general, is given by

$$y(t) = -\frac{1}{2}gt^2 + v_{i,y}t + y_i \tag{1}$$

In Eq. 1, $-g$ is the acceleration. The vector form of acceleration points down, so we give g a minus sign. We begin to observe the system at time $t = 0$. If a stationary marble is dropped and Eq. 1 is used to predict the position $y(t)$, then $v_{i,y} = 0$. Let the change in height be $h = y(t) - y_i$. Show that

$$h = -\frac{1}{2}gt^2 \tag{2}$$

Use this equation to solve for g. The result should be

$$g = \frac{-2h}{t^2} \tag{3}$$

Use the following procedure to measure g:

1. Use the ruler to measure the vertical displacement of the marble.

2. Use a stopwatch to time the descent of the marble.

3. Insert the measured values of h and t into Eq. 3 to calculate g.

4. Repeat 10 times and compute the average for $g = g_{ave}$.

5. Calculate the *percent error* of g, using the $g = 9.81$ m s^{-2}.

$$\Delta g \ (\%) = \frac{g_{ave} - g}{g} \times 100 \tag{4}$$

2   The Acceleration of Gravity, Second Measurement

Now measure g using a pendulum, constructed in the same fashion as the previous lab activity. Let T be the period of the pendulum, and L be the length. The relationship between T, L, and g is

$$T = 2\pi\sqrt{L/g} \tag{5}$$

Solve Eq. 5 for g, and repeat the above procedure to obtain g_{ave} and the percent error using the pendulum. Compare the results for g_{ave} from each technique.

3   The Acceleration of Gravity, Third Measurement

Using your pendulum data, we will measure g more precisely now. Use the following procedure to extract the g measurement from the data.

1. Estimate the error on your T measurements. Assume, for example, that your precision on measuring T is 5%, fractionally. Thus, if you measure T to be 1.8 seconds, then a 50% error is ±0.9 seconds, and 5% error is ±0.09 seconds.

2. Create a spreadsheet with five columns. Name them "Length (m)," "Period (sec)," "Error (sec)," "Theory (sec)," and "chi squared." In the first column, enter your length data, converting to meters. In the second column, enter your raw, measured period data in seconds. In the third column, enter your estimate of the error on the period measurements, in seconds.

3. In the fourth column, enter the following: `=2*3.14*SQRT(A2/9.81)`. This assumes your first length measurement is in cell A2. Click and drag the formula down to repeat the calculation for each length measurement. Note the formula assumes $g = 9.81$ m s^{-2}.

4. In the fifth column, enter the following: `=(B2-D2)^2/C2^2`. This assumes your first period measurement is in cell B2, the associated error is in C2, and that the theoretical prediction is in D2.

5. In the cell below the "chi-squared" data, enter the following: `=SUM(E2:E12)/N`. Trade "N" for the number of measurements minus one. **Does your sum result in a number close to one?** Why should this occur if the theoretical prediction is a *good fit* to the measured data?

6. The summed quantity is known as the *reduced χ^2 value*. Summing the fractional squared difference between theory and measurement in this way results in a number that follows the χ^2 distribution, a well-known probability distribution in the subject of probability and statistics. **If your chi-squared value is not near 1.0, try tuning your g value within the Theory column. Tune g until χ^2 is minimized.**

7. Create a graph of your data, including errors and theoretical prediction, below.

8. To explore χ^2 in more depth, follow this link: `https://phet.colorado.edu/en/simulations/curve-fitting`.