Algebra-Based Physics-2: Electricity, Magnetism, and Modern Physics (PHYS135B-01): Unit 5

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Summary

Unit 5 Summary

1. Electromagnetic waves - **Chapters 24.1 - 24.4**
    - Maxwell's Equations
    - Electromagnetic wave production
    - Electromagnetic spectrum and energy
2. Geometric optics - **Chapters 25.1 - 25.3, 25.6**
    - Ray-tracing
    - Reflection
    - Refraction
    - Lens optics

Unit 5 Summary

1. Wave optics - Chapters 27.1 - 27.3
   - Wave interferance
   - Wave diffraction
   - Double slit experiments
2. Nuclear physics in medicine - 32.1 - 32.4
   - Diagnostics and medical imaging
   - Biological effects of ionizing radiation
   - Therapeutic uses of ionizing radiation
   - Food irradiation

Electromagnetic waves: Maxwell’s Equations

Maxwell's Equations are a set of four main ideas describing *the entirety* of electromagnetism.

- **Electric fields**, or the force per unit test charge, originate on positive charges and terminate on negative charges. The force is related to the permittivity of free space, ϵ_0. Electric fields give rise to Coulomb's law, or Gauss's law for electricity.
- **Magnetic fields**, or the force per unit test current per unit length, are continuous, having no beginning or end. No magnetic monopoles are known to exist. The strength of the magnetic force is related to the permeability of free space, μ_0. Magnetic fields give rise to Gauss's law for magnetism.

Maxwell's Equations are a set of four main ideas describing *the entirety* of electromagnetism.

- A *changing* **magnetic field** induces an electromotive force (emf) and, hence, an **electric field**. The direction of the emf opposes the change. This is Faraday's law of induction, and includes Lenz's law.
- **Magnetic fields** are generated by *moving charges* (current) or by *changing* **electric fields**. This is Ampère's law, enhanced with the idea that changing **electric fields** without current or charge induce **magnetic fields**.

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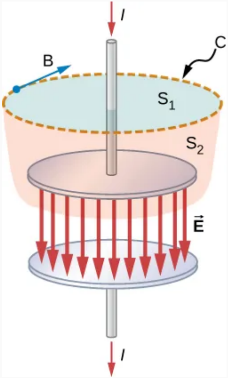

Figure 1: When a current creates a B-field, which surface bounding the B-field line is relevant?

The addition to Ampère's Law is called *the displacement current*:

$$I_{\mathrm{d}} = \epsilon_0 \frac{\Delta\Phi_{\mathrm{E}}}{\Delta t} \tag{1}$$

The electric flux is $\Phi_{\mathrm{E}} = \vec{E} \cdot \vec{A}$. Assume we are dealing with surface S_2, meaning $I = 0$. Ampère's Law gives

$$B 2\pi r = \mu_0 \left(I + I_{\mathrm{d}}\right) \tag{2}$$

$$B 2\pi r = \mu_0 \left(0 + I_{\mathrm{d}}\right) \tag{3}$$

$$B 2\pi r = \mu_0 \left(\epsilon_0 \frac{\Delta\Phi_E}{\Delta t}\right) = \mu_0 \epsilon_0 A \left(\frac{\Delta E}{\Delta t}\right) \tag{4}$$

For a parallel-plate capacitor,

$$E = V/d = Q/(Cd) = Qd/(\epsilon_0 A d) = Q/(\epsilon_0 A) \tag{5}$$

Insert the magnitude of the E-field into Ampère's Law to find:

$$B 2\pi r = \mu_0 \epsilon_0 A \left(\frac{\Delta E}{\Delta t} \right) \tag{6}$$

$$B 2\pi r = \mu_0 \epsilon_0 A \left(\frac{\Delta E}{\epsilon_0 A \Delta t} \right) \tag{7}$$

$$B = \frac{\mu_0}{2\pi r} \frac{\Delta Q}{\Delta t} \tag{8}$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \tag{9}$$

A *changing* E-field is responsible for the B-field of a capacitor.

What is the B-field generated 1 cm laterally from a capacitor in an RC circuit that charges from 0 to 10 nJ in 1 μs?

- A: 20 μT
- B: 20 mT
- C: 20 nT
- D: 20 pT

What is the energy stored in the capacitor, if the capacitance is $C = 10$ pF?

- A: 5 μJ
- B: 5 mJ
- C: 5 nJ
- D: 5 pJ

Recall that $U = \frac{1}{2}\frac{Q^2}{C}$.

But *where* is the energy stored in a capacitor? **The E-field.** Consider that we proved the stored energy is

$$U_{\mathrm{C}} = \frac{1}{2}CV^2 \tag{10}$$

The voltage only exists because of the arrangement of charges and the field, and we know that $V = Ed$. Also, the volume is Ad. Thus,

$$U_{\mathrm{C}} = \frac{1}{2}CE^2d^2 \tag{11}$$

$$U_{\mathrm{C}} = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)E^2d^2 \tag{12}$$

$$\frac{U_{\mathrm{C}}}{Ad} = \frac{1}{2}\epsilon_0 E^2 \tag{13}$$

$$\boxed{\epsilon_{\mathrm{C}} = \frac{1}{2}\epsilon_0 E^2} \tag{14}$$

Ampère's Law is enhanced with the idea that changing **electric fields** without current or charge induce **magnetic fields**.

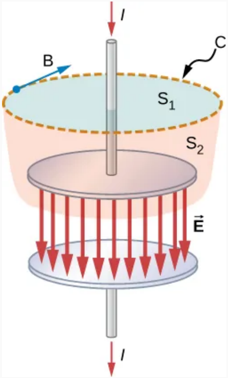

Figure 2: When a current creates a B-field, which surface bounding the B-field line is relevant?

What if there was a solenoid inductor ($N = 1$) next to the capacitor, waiting to catch the B-field and become charged (via Faraday's Law)? The solenoid will produce some current I to create the *opposite* B-field:

$$L = \frac{\mu_0 N^2 A}{d} \tag{15}$$

$$U_{\mathrm{L}} = \frac{1}{2} L I^2 \tag{16}$$

$$U_{\mathrm{L}} = \frac{1}{2} \frac{\mu_0 A}{d} I^2 \tag{17}$$

$$B = \mu_0 \frac{N}{d} I = \mu_0 \frac{I}{d} \tag{18}$$

$$I^2 = \frac{d^2 B^2}{\mu_0^2} \tag{19}$$

What if there was a solenoid inductor ($N = 1$) next to the capacitor, waiting to catch the B-field and become charged (via Faraday's Law)? The solenoid will produce some current I to create the *opposite* B-field:

$$U_{\mathrm{L}} = \frac{1}{2}\frac{\mu_0 A}{d}\frac{d^2 B^2}{\mu_0^2} = \frac{1}{2}\frac{B^2 A d}{\mu_0} \tag{20}$$

$$\frac{U_{\mathrm{L}}}{Ad} = \frac{B^2}{2\mu_0} \tag{21}$$

$$\boxed{\epsilon_{\mathrm{L}} = \frac{1}{2\mu_0}B^2} \tag{22}$$

Suppose the inductor catches *all* the energy from the capacitor, so that $\epsilon_{\mathrm{C}} = \epsilon_{\mathrm{L}}$?

If that is true, then

$$\epsilon_{\mathrm{C}} = \epsilon_{\mathrm{L}} \tag{23}$$

$$\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2\mu_0} B^2 \tag{24}$$

$$\frac{E}{B} = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \tag{25}$$

Show that the units of E/B are m s^{-1}. *Hint: recall* $F = qE$, *and* $F = qvB$. Knowing that the ratio on the left hand side is a velocity:

$$\boxed{v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}} \tag{26}$$

Equation 26 represents the **speed of light.** Now imagine the inductor charging a second capacitor, and that capacitor charging some second inductor ... the energy starts to propagate.

We should be able to observe this effect in the lab.

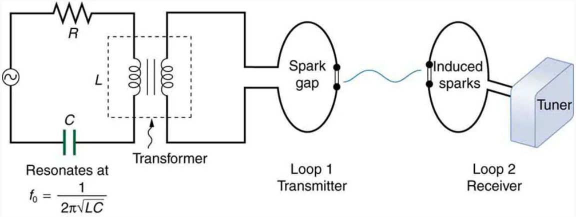

Figure 3: Heinrich Hertz demonstrated the *spark gap* RLC circuit.

- The RLC circuit on the left side is set to resonate.
- The transformer changes the signal to a high voltage that makes a spark in loop 1.
- The RLC circuit in the tuner is set to the same resonance frequency.
- Sparks are induced *even though the circuits are not connected with conductors.*

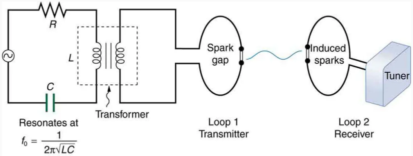

Figure 4: The transmitter and receiver are connected to RLC circuits with the same resonance frequency.

If the transmitter and receiver resonance frequency are the same:

$$f_{\mathrm{L}} = f_{\mathrm{R}} \tag{27}$$

$$\frac{1}{2\pi\sqrt{L_1C_1}} = \frac{1}{2\pi\sqrt{L_2C_2}} \tag{28}$$

$$L_1C_1 = L_2C_2 \tag{29}$$

If the transmitter and receiver resonance frequency are the same:

$$L_{\mathrm{TX}}C_{\mathrm{TX}} = L_{\mathrm{RX}}C_{\mathrm{RX}} \tag{30}$$

If the transmitter (TX) inductance is 1 mH, and the TX capacitance is 0.1 mF, and the receiver (RX) capacitance is 10 mF, what is the RX inductance?

- A: 1 mH
- B: 0.1 mH
- C: 0.01 mH
- D: 0.001 mH

Hint: treat this as a scaling problem.

If the transmitter (TX) inductance is 1 mH, and the TX capacitance is 0.1 mF, and the receiver (RX) capacitance is 0.2 mF, what is the RX inductance?

- A: 5 mH
- B: 0.5 mH
- C: 0.05 mH
- D: 0.005 mH

Hint: treat this as a scaling problem.

Electromagnetic waves: Maxwell's Equations

That electromagnetic fields can *propagate* was strong evidence that they are wavelike. All waves that obey the "wave equation" share a relationship between the speed, v, frequency f, and the *wavelength* λ:

$$v = f\lambda \tag{31}$$

The wavelength is the displacement between wave peaks, and $1/f = T$ is the period in time between peaks. If the speed is $v = 1/\sqrt{\epsilon_0\mu_0}$, and the resonance frequency corresponds to a capacitance of 0.2 μF and inductance of 0.5 μH, what is the wavelength?

- A: 3750 m
- B: 375 m
- C: 37.5 m
- D: 3.75 m

If the speed of light is 3×10^8 m/s, what is this same speed in m/ns?

- A: 30 m/ns
- B: 3 m/ns
- C: 0.3 m/ns
- D: 0.03 m/ns

What is the frequency of electromagnetic radiation with a wavelength comparable to the length of a person (≈ 1 m)?

- A: 0.3 GHz
- B: 3 GHz
- C: 300 MHz
- D: 3000 MHz

Note: is there any reason to expect limitations on the wavelengths and frequencies of electromagnetic waves?

Electromagnetic waves: Electromagnetic wave production

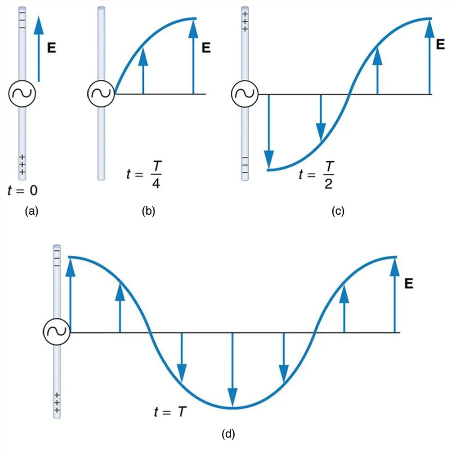

Figure 5: An AC voltage source corresponds to electrons oscilatting, which leads to an oscilatting field.

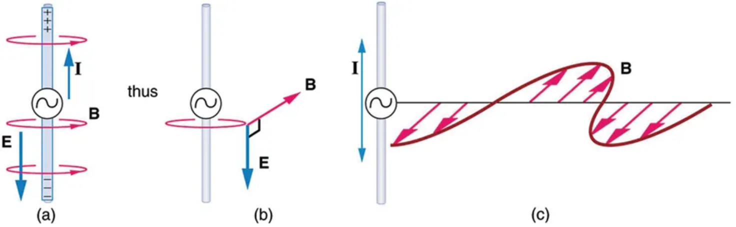

Figure 6: The oscillating E-field generates an orthogonal B-field.

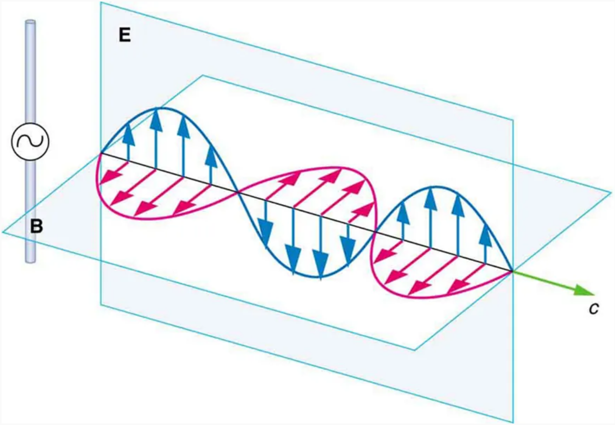

Figure 7: The oscillating B-field generates an orthogonal E-field, continuing the process.

The wave **moves energy** in the direction of the green arrow.

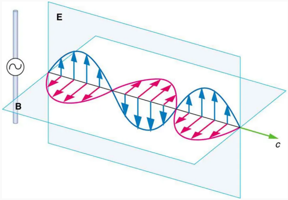

Figure 8: The oscillating B-field generates an orthogonal E-field, continuing the process.

The flux of energy per unit area in this case is (after some length mathematics)

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{32}$$

The wave **moves energy** in the direction of $\vec{S}$. The direction is orthogonal to the E and B, with a magnitude EB/μ_0. The peak B-value is $B = E/c$. Both B and E are sinusoids with the same f and ϕ. Thus, $S \propto \sin^2(2\pi f t + \phi)$, and the average of this is 1/2. This makes the average **intensity**

$$\bar{S} = \frac{1}{2c\mu_0}E^2 \tag{33}$$

The units of intensity are W m^{-2}. This formula is useful:

- Calculate the RX power at a radio given the field strength at the radio station, and the distance to the radio station.
- Calculate the brightness of a star observed at Earth, given the field strength of the light at the star.

Suppose a microwave in the kitchen generates 1 kW of power, projected onto a 10cm x 10cm area at a distance of 1 m. What is the intensity (power per unit area)?

- C: 1 kW
- D: 100 kW
- A: 1 kW m^{-2}
- B: 100 kW m^{-2}

Suppose a microwave in the kitchen generates 1 kW of power, projected onto a 10cm x 10cm area at a distance of 1 m. How long does it take the energy to travel the 1 meter?

- C: 0.333 ns
- D: 3.33 ns
- A: 33.3 ns
- B: 333 ns

Suppose a microwave in the kitchen generates 1 kW of power, projected onto a 10cm x 10cm area at a distance of 1 m. What is the peak E-field at the source?

- C: 870 V/m
- D: 8700 V
- A: 8700 V/m
- B: 870 V

Electromagnetic waves: Electromagnetic spectrum and energy

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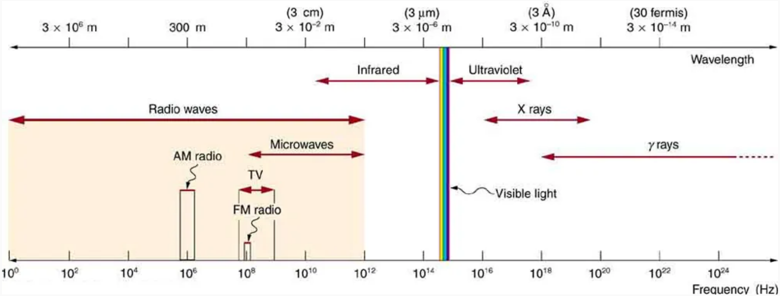

Figure 9: The electromagnetic spectrum maps signal types to wavelength (top) and frequency (bottom).

- Visible spectrum: more than 10^{14} Hz, 400-700 nm wavelengths
- Radio waves: $[10^{-1} - 10^{4}]$ MHz

Amplitude modulation (AM) is a technology that allows audio transmission over the EM spectrum.

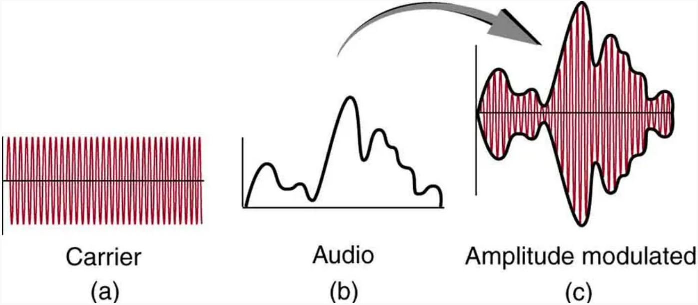

Figure 10: (a) The carrier wave. (b) The audio spectrum. (c) The *modulated* carrier wave.

The carrier is a pure sinusoid at a single frequency, f_{c}, with amplitude A:

$$c(t) = A \sin(2\pi f_{\mathrm{c}} t) \tag{34}$$

Let the modulating audio signal (at a given frequency) be

$$m(t) = Am \cos(2\pi f_{\mathrm{m}} t + \phi) \tag{35}$$

Audio waves are not electromagnetic, and audio frequencies are orders of magnitude smaller than radio frequencies. If there were a way to *mix* (multiply) these signals as voltages, then we get

$$y(t) = \left[1 + \frac{m(t)}{A}\right] c(t) \tag{36}$$

Do you remember the following trigonometric identity?

$$\sin(A)\cos(B) = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right) \tag{37}$$

Group exercise: Substitute Eqs. 34 and 35 into 36, and use the trigonometric identity in Eq. 37 to simplify the result.

1. Look for three waves: the carrier, and two additional ones at two different frequencies.
2. Draw a picture of the spectrum, the amplitude versus frequency of the signal.

Electromagnetic waves: Electromagnetic spectrum and energy

The AM mixing yields three waves:

- The original carrier
- A wave with $f_\mathrm{c} + f_\mathrm{m}$
- A wave with $f_\mathrm{c} - f_\mathrm{m}$

To re-capture the audio, we must *demodulate*, or reverse the process.

How do we create $m(t)$, and how do we modulate and demodulate it?

- Parallel LC circuits that act as resonators
- Diodes, devices that allow current to flow only one way

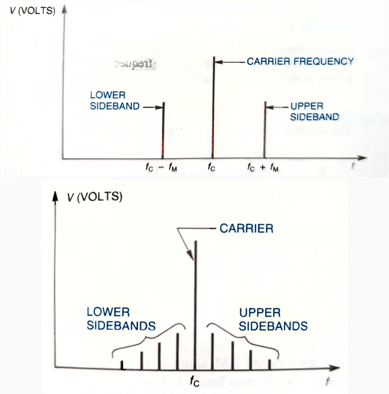

Figure 11: (Top) An example of a single audio frequency mixed into an AM signal. (Bottom) An example of an audio spectrum mixed into an AM signal.

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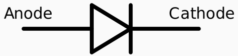

Figure 12: Circuit diagram for the diode.

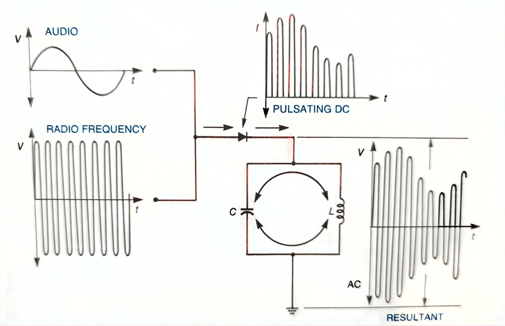

Figure 13: (Upper left) The audio signal is converted to a voltage via a microphone. (Lower left) The radio carrier signal oscillates at a higher frequency. (Middle) The LC resonator and diode mix the two signals. (Right) The final amplitude is modulated by the audio.

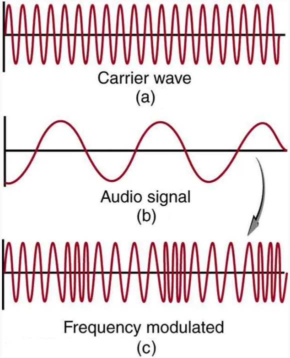

Figure 14

Geometric optics: Ray-tracing

Geometric optics: Reflection

Geometric optics: Refraction

Geometric optics: Lens optics

Wave optics: Wave interferance

Wave optics: Wave diffraction

Wave optics: Double slit experiments

Nuclear physics in medicine: Diagnostics and medical imaging

Nuclear physics in medicine: Biological effects of ionizing radiation

Nuclear physics in medicine: Therapeutic uses of ionizing radiation

Nuclear physics in medicine: Food irradiation

Conclusion

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