

Figure 1: (Left) A current I experiences a force F in a B -field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the ~~current~~ ^{B-field} given that F is the Lorentz force.

- a: INTO page
- b: Left
- c: OUT of page

2. Consider Fig. 1 (right). **The Hall Effect.** An E -field exists in the vertical direction and a B -field is perpendicular to the direction of charge velocity. (a) Show that if the E -field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E -field is constant, $E = \Delta V / \Delta x$. Show that

$$\Delta V = \frac{B \Delta x I}{n q_e A} \quad (1)$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28} \text{ m}^{-3}$, $A = 1 \text{ mm}^2$, and q_e is the charge of an electron.

a) $F_E = F_B \Rightarrow qE = qvB \sin \theta$
 $(\theta = 90^\circ)$
 $\Rightarrow qE = qvB \Rightarrow E = vB$
 $V = \frac{E}{B}$

b) $E = \frac{\Delta V}{\Delta x}$
 $\Delta V = (vB) \Delta x$
 $\Rightarrow \Delta V = B (\Delta x) v$
 $v = \frac{I}{n q_e A}$

$\Delta V = B (\Delta x) \cdot \frac{I}{n q_e A}$
 $\Rightarrow \Delta V = \frac{B \Delta x I}{n q_e A}$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^4 A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

$T_{\max} = N I A B = N I \pi r^2 B$
 $\Rightarrow (1) (1.05 \times 10^4 \text{ A}) (\pi) (0.65 \times 10^{-15} \text{ m})^2 (2.50 \text{ T})$
 $= 3.48 \times 10^{-26} \text{ N}\cdot\text{m}$

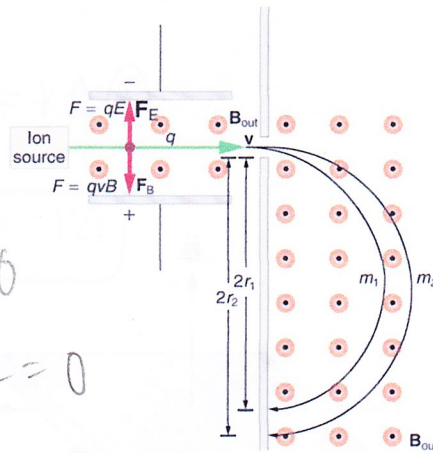
3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B -field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B -field?

a) $B = \mu_0 n I$
 $= (4\pi \times 10^{-7}) (500) (0.3) = 1.88 \times 10^{-4} \text{ T}$

b) $(4\pi \times 10^{-7}) (5000) (0.3)$
 $= 9.42 \times 10^{-1} \text{ T}$
 $\rightarrow (1.88 \times 10^{-4} \text{ T}) (5000) = 0.94 \text{ T}$

2) $F = q\vec{E}$
 $F_{\text{mag}} = q(\vec{v} \times \vec{B})$
 $F_{\text{tot}} = F_{\text{elec}} + F_{\text{mag}} = 0$
 $q[\vec{E} + \vec{v} \times \vec{B}] = 0$
 $|F_{\text{tot}}| = q[E - vB] = 0$
 $E = vB \Rightarrow v = \frac{E}{B}; F_{\text{net}} = 0$



b) Cent. force = $\frac{mv^2}{r}$
 $qvB = \frac{mv^2}{r}$
 $\Rightarrow r = \frac{mv}{qB}$
 $v = \frac{E}{B} \Rightarrow r = \frac{mE}{qB^2} = \frac{mE}{qB^2}$

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer.** Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{\text{net}} = 0$ in the region in the top left¹. (b) Recall that the centripetal force on a particle of mass m is mv^2/r . Set this equal to the magnitude of the Lorentz force to prove that

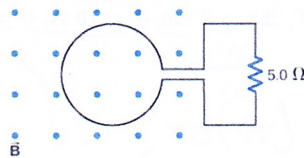
$$r = \frac{mE}{qB^2} \quad (2)$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-septometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r ?

$$r = \frac{mE}{qB^2} = \frac{ME}{qB^2} = \frac{(16)(1.67 \times 10^{-27})(10)}{(1.62 \times 10^{-19})(0.01)^2} = 1.65 \times 10^{-2} = 0.0165 \text{ m} = 1.65 \text{ cm}$$

4 Chapter 13: Electromagnetic Induction

a) $\mathcal{E} = \left| \frac{d\Phi}{dt} \right| = \frac{d(BA)}{dt}$
 $\mathcal{E} = A \frac{dB}{dt}$



b) $t = 0 \text{ sec}$
 $\sin 0 = 0$
 So, induced emf $\mathcal{E} = 0$

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0} (\sin(2\pi ft)) \quad (3)$$

The loop has a radius r . (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1 \text{ T}$, $r = 0.1 \text{ m}$, $f = 10^3 \text{ Hz}$, and $T = 1 \text{ ms}$, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16 \text{ ms}$? (d) What is the current through the resistor at t_1 ?

c) $\mathcal{E} = \pi (0.1)^2 \times \frac{0.1}{1 \times 10^{-3}} \times \sin(2\pi \times 10^3 \times 0.16 \times 10^{-3}) = 0.055 \text{ V}$

d) $I = \frac{\mathcal{E}}{R} = \frac{0.055}{5} = 0.011 \text{ A}$

¹Molecules that do not have this velocity will hit the sides of this portion of the instrument.

5 Chapter 14: Inductance

1. What is (a) the rate at which the current through a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$$\textcircled{1} \quad \mathcal{E} = -L \frac{dI}{dt}$$

$$\frac{dI}{dt} = \left| \frac{-\mathcal{E}}{L} \right| = \frac{0.15 \text{ V}}{0.50 \text{ H}} = \boxed{0.3 \frac{\text{A}}{\text{s}}}$$

$$\textcircled{2} \quad I = 0.100 \text{ A}$$

$$L = 2.00 \text{ mH} = 2.00 \times 10^{-3} \text{ H}$$

$$\mathcal{E} = 500 \text{ V}$$

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$\frac{dI}{dt} = \left| \frac{-\mathcal{E}}{L} \right| = \frac{500 \text{ V}}{(2.00 \times 10^{-3} \text{ H})} = 250 \times 10^{-3} \frac{\text{A}}{\text{s}}$$

$$\frac{dI}{dt} = \frac{I}{t} \Rightarrow t = \frac{I}{dI/dt}$$

$$t = \frac{I}{dI/dt} = \frac{0.100 \text{ A}}{250 \times 10^{-3} \text{ A/s}} = \boxed{4.0 \times 10^{-7} \text{ sec}}$$