Momentum

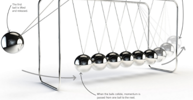
When objects collide, the effect one object has on another depends on a quantity called momentum. The greater the mass of a moving object or the faster the object is moving, the greater its momentum and the greater the effect it can have.

Key facts

- ✓ The greater an object's mass or the faster it is moving, the more momentum it has.
- ✓ Momentum = mass × velocity.
- ✓ The law of conservation of momentum says that in a system not affected by external forces, total momentum is the same before and after a collision.
- ✓ Momentum is a vector, so calculations must take into account the direction the object is moving in.

Conservation of momentum

Newton's cradle is a device that demonstrates a law known as conservation of momentum. According to this law, when a system is not affected by external forces, the total momentum in the system is the same before and after a collision. When one of the metal balls is lifted and allowed to hit the others, its momentum passes from ball to ball, making the last ball rise and repeat the cycle.



Formula for momentum

Both velocity and mass affect an object's momentum, as the equation below shows. Shooting stars are often no bigger than grains of sand, but they have great momentum because of their speed. Large vehicles such as freight trains have enormous momentum due to their great mass and can cause dangerous collisions even when moving slowly. Momentum is a vector, so calculations must take into account the direction in which the object is moving.

$$\text{momentum (kg m/s)} = \text{mass (kg)} \times \text{velocity (m/s)}$$
$$p = m \times v$$

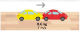
Calculating momentum

Question 1

A rhinoceros has a mass of 1000 kg and is traveling at 15 m/s. How much momentum does it have?

Answer 1

$p = m \times v$
$= 1000 \text{ kg} \times 15 \text{ m/s}$
$= 15\,000 \text{ kg m/s}$



Question 2

A toy car with a mass of 0.2 kg hits another toy car with a mass of 0.4 kg while traveling at 2 m/s. The two cars stick together and continue moving in the same direction. What speed are they going at?

Answer 2

The total momentum is conserved, so use the following equation to work out the answer:

momentum before collision = momentum after collision

momentum before = (0.2 kg × 2 m/s) + (0.4 kg × 0 m/s)
= 0.4 kg m/s
momentum after = 0.4 kg m/s

$$v = \frac{p}{m} = \frac{0.4 \text{ kg m/s}}{0.2 \text{ kg} + 0.4 \text{ kg}} = 0.67 \text{ m/s}$$

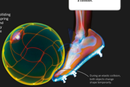
Elastic and inelastic collisions

When objects collide, the total momentum before and after the collision is conserved (see page 100). However, kinetic energy may not be. Whether kinetic energy is conserved or not depends on whether a collision is elastic or inelastic.

Key facts

- ✓ Collisions can be elastic or inelastic.
- ✓ Kinetic energy is conserved in an elastic collision and lost in an inelastic collision.
- ✓ Total momentum is the same before and after a collision.

Elastic collisions
During an elastic collision, the colliding objects change shape but then spring back into their original shapes and separate. The total kinetic energy of the moving objects is the same before and after the collision. Few collisions in the real world are perfectly elastic, as some kinetic energy is usually lost. For example, when a foot kicks a ball, some kinetic energy is transferred to sound.



During an elastic collision, both objects change shape temporarily.

Calculating kinetic energy

Most collisions result in a loss of some kinetic energy. We can find out how much is lost by calculating the kinetic energy before and after the collision.

Question
During a game of billiards, a 0.17 kg white ball traveling at 1.5 m/s hits a stationary red ball with a mass of 0.16 kg. The red ball moves forward at 1.2 m/s and the white ball at 0.37 m/s. How much kinetic energy was lost?

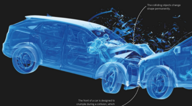
Answer
Use the equation for kinetic energy from page 52 ($E_k = ½ \times m \times v^2$) to work out the total kinetic energy before and after the collision.

Kinetic energy before collision $= ½ \times 0.17 \times 1.5^2$
$= 0.19$ J

Kinetic energy after collision
$= (½ \times 0.17 \times 0.37^2) + (½ \times 0.16 \times 1.2^2)$
$= 0.13$ J

The energy lost $= 0.19$ J $- 0.13$ J
$= 0.06$ J

Inelastic collisions
In an inelastic collision, the colliding objects can change shape permanently and may join together. Kinetic energy is transferred to sound, internal energy, and other energy stores. For example, the car collision shown below is inelastic. Instead of rebounding like a soccer ball off a shoe, the cars lose kinetic energy and come to a halt. The shape of both cars is permanently changed.



The colliding objects change shape permanently.

The front of a car is designed to crumple during a collision, which reduces the forces on passengers.

Explosions

In an explosion, momentum is conserved but kinetic energy is not. An unexploded bomb has zero kinetic energy when it's stationary, but the exploding fragments have a huge amount of kinetic energy. However, momentum stays the same. The total momentum of a stationary bomb is zero, and the total momentum of the fragments is also zero. (Momentum is a vector quantity, and the fragments all travel outward in different directions.)

Changing momentum

Changing the momentum of a moving object—whether stopping a car or striking a tennis ball—requires a force. The greater the change in momentum or the more quickly the momentum changes, the greater the force required. Car crashes are dangerous because the very rapid change in momentum involves huge forces.

> **Key facts**
> - ✓ Changing an object's momentum requires a force.
> - ✓ The more momentum an object has, the larger the force needed to stop it, or the longer the stopping force must be applied.

Force and momentum

When a car comes to a halt, its momentum falls to zero. We can calculate the force needed to change the car's momentum using the equation here. As the examples below demonstrate, a far greater force is needed to stop a car suddenly than to slow it down gradually.

$$\text{force (N)} = \frac{\text{change in momentum (kg m/s)}}{\text{time (s)}}$$

Final velocity

$$F = \frac{mv_f - mv_i}{t}$$

Initial velocity

Stopping gradually

A car with a mass of 1000 kg is traveling at 14 m/s (about 31 mph [50 km/h]). The driver brakes for 10 seconds, bringing the car to a stop. What is the force acting on the car?

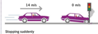
Remember to take the initial momentum away from the final momentum.

$$F = \frac{(1000\ \text{kg} \times 0\ \text{m/s}) - (1000\ \text{kg} \times 14\ \text{m/s})}{10\ \text{s}}$$

$$= \frac{0 - 14\,000\ \text{kg m/s}}{10\ \text{s}}$$

$$= -1400\ \text{N}$$

The force is negative because it acts in the opposite direction to the motion of the car.



Stopping suddenly

A car of the same mass is also traveling at 14 m/s. It hits a traffic light and decelerates to 0 m/s in 0.07 seconds. What is the force acting on this car?

$$F = \frac{(1000\ \text{kg} \times 0\ \text{m/s}) - (1000\ \text{kg} \times 14\ \text{m/s})}{0.07\ \text{s}}$$

$$= \frac{0 - 14\,000\ \text{kg m/s}}{0.07\ \text{s}}$$

$$= -200\,000\ \text{N}$$

The force that stops the car is equivalent to the weight of five elephants.

Stopping distance

In an emergency, a driver may see a hazard and have to stop the car very quickly. The distance the car travels between the driver seeing the hazard and the car coming to a stop is called the stopping distance and is affected by the car's speed, mass, and other factors.

> **Key facts**
> - ✓ Stopping distance is the distance covered between the driver seeing a hazard and the vehicle stopping.
> - ✓ Stopping distance is the sum of thinking distance and braking distance.
> - ✓ Factors affecting thinking distance include tiredness, use of drugs or alcohol, distractions, and the vehicle's speed.
> - ✓ Factors affecting braking distance include the vehicle's speed, mass, condition, and road and weather conditions.

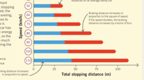
Thinking and braking

Total stopping distance can be divided into two parts: thinking distance and braking distance. Thinking distance is the distance the car travels during the time a driver takes to react and use the brakes after seeing a hazard. Braking distance is the distance the car travels after braking begins.

Stopping distance and speed

The most important factor affecting stopping distance is speed; the faster a car is traveling, the longer it takes to stop safely. This is because a fast car has far more kinetic energy than a slow car, so the brakes must do much more work to bring the car to a stop.

Thinking distance
Braking distance



Reaction time

Most drivers take about 0.7 seconds to react to a hazard, but this time can more than triple if a driver has been drinking alcohol, has taken drugs, or is distracted by a mobile phone. Reaction times also vary from person to person and are affected by how tired we are. You can assess your reaction time with a simple experiment: ask a helper to drop a ruler without warning and see how quickly you can catch it.

Factors affecting braking distance

Massive vehicles have more kinetic energy than smaller vehicles, so they require a longer braking distance.

Fast vehicles have more kinetic energy than slow vehicles, so they require a longer braking distance.

Tire and brake condition affect braking distance. If they are worn or in poor condition, they create less friction, which means less force to reduce kinetic energy.

Wet or icy roads reduce friction, resulting in a smaller stopping force. They may also cause skids.

Supersonic stopping distance
Braking distance increases in proportion to the square of a car's speed. (Doubling the speed causes braking distance to increase by a factor of four.) As a result, the supersonic car *Bloodhound LSR*, which is designed to break the land speed record, has a braking distance of around 7.2 km, even with the assistance of a braking parachute.

Car safety features

During a car crash, the car and everything inside it undergo a rapid change in momentum. The force on the people in the car from the collision is equal to the rate of change of their momentum. In order to minimize this large force, which can cause serious injuries, cars have safety features that slow the change in momentum.

Key facts

- ✓ **Car crashes involve extreme changes in momentum and large, dangerous forces.**
- ✓ **Slowing down the change in momentum reduces the forces in a crash.**
- ✓ **Car safety features include airbags, seatbelts, and crumple zones.**

Crash test
Simulating car crashes allows engineers to measure the forces on different parts of a passenger's body and ensure that safety features are effective. The main safety features are seatbelts, the front and rear crumple zones, and airbags. Airbags and crumple zones increase the time it takes for the person's body to come to a stop, which reduces the change in momentum and hence the forces on the passengers.

Safety cage

Many parts of a modern car are designed to deform safely during a collision, reducing the forces on the passengers. However, some parts must be protected from crumpling, including the cabin containing the occupants and the fuel tank or battery. These areas are enclosed by a rigid steel frame called a safety cage, which can withstand huge forces without significant deformation.

Braking distance and energy

To stop a car, its kinetic energy must be transferred to other energy stores. The faster a car is moving, the more kinetic energy it has to transfer away and the greater its braking distance.

Key facts

- ✓ A braking force does work on a vehicle to change its motion.
- ✓ The work done braking is equal to the change in kinetic energy.
- ✓ The braking distance required to stop a car safely increases in proportion to the square of the car's speed.

Kinetic energy
A moving car has a store of kinetic energy. We can calculate how much by using the equation from page 52: kinetic energy = ½ × m × v^2. When the brakes are used to slow down the car, they exert a force and do work. Because the work done braking is equal to the change in kinetic energy, we can combine the equation for work (see page 55) with the equation for kinetic energy to make a new equation:

$$\text{work (J)} = \text{force (N)} \times \text{distance (m)}$$

$$\text{kinetic energy (J)} = \frac{1}{2} \times \text{mass (kg)} \times \text{speed}^2\ \text{(m/s)}^2$$

$$\text{force (N)} \times \text{distance (m)} = \frac{1}{2} \times \text{mass (kg)} \times \text{speed}^2\ \text{(m/s)}^2$$

$$F \times d = \frac{1}{2} \times m \times v^2$$

The combined equation is useful because it allows us to calculate braking distance (d) if we know a car's speed, mass, and the force of its brakes.

Calculating braking distance

Question
A uniform braking force of 2000 N is applied to the wheels of a 1100 kg car traveling at 13 m/s (about 29 mph/47 km/h). What is its braking distance? What would the braking distance be at twice that speed?

Answer 1
Rearrange the equation to make braking distance the subject.

$$d = \frac{\frac{1}{2} \times m \times v^2}{F}$$

$$= \frac{\frac{1}{2} \times 1100\text{ kg} \times 13\text{ m/s} \times 13\text{ m/s}}{2000\text{ N}}$$

$$= 46\text{ m}$$

Answer 2

$$d = \frac{\frac{1}{2} \times 1100\text{ kg} \times 26\text{ m/s} \times 26\text{ m/s}}{2000\text{ N}}$$

$$= 186\text{ m}$$

At twice the speed, braking distance is four times greater. Braking distance is proportional to the square of the speed.

Speed and safety

The combined equation shows that braking distance increases in proportion to the square of a car's speed. In other words, if you double the speed, the braking distance is four times greater; if you triple the speed, the braking distance is nine times greater. This is one of the reasons that driving at high speeds is dangerous—a faster car not only has a lot more momentum than a slow car, but also needs far more room to stop.

Brake disks
When a car brakes, energy is transferred from its store of kinetic energy to thermal energy stores, making the brakes hot. Formula 1 cars are built to accelerate and decelerate incredibly quickly. Huge amounts of energy are transferred while braking, causing the brake disks to glow red hot.

The disk brakes in the wheels glow red-hot when braking.