Broder P. Myers

20/20

yay!

Midterm for PHYS135B Module 2, Spring 2021

Dr. Jordan Hanson - Whittier College Dept. of Physics and Astronomy

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* I like to write everything because it helps me work through problems better.

1 Memory Bank

1. $V = (4/3)\pi r^3$ ... The volume of a sphere.
2. $m = \rho V$ ... The relationship between mass m, density ρ, and volume V.
3. $\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$ ... Coulomb Force
4. $k = 9 \times 10^9$ N C^{-2} m^2 ... Remember $k = 1/(4\pi\epsilon_0)$.
5. $q_e = 1.6 \times 10^{-19}$ C ... Charge of an electron/proton
6. Atomic mass: the number of grams per mole of a substance
7. $N_A = 6.03 \times 10^{23}$ ... Avagadro's number
8. $\vec{F} = q\vec{E}$ ... Electric field and charge
9. $\vec{E}(z) = \frac{\sigma}{\epsilon_0}\hat{z}$ ... Electric field of two oppositely charge planes each with charge density σ
10. $\epsilon_0 \approx 8.85 \times 10^{-12}$ F/m
11. $U = q\Delta V$ ... Potential energy and voltage
12. 1 eV: an electron-Volt is the amount of energy one electron gains through 1 V.
13. $V(r) = k\frac{q}{r}$ ... Voltage of a point charge
14. $\vec{E} = -\frac{\Delta V}{\Delta x}$ ... E-field is the slope or change in voltage with respect to distance
15. $V(x) = -Ex + V_0$ ... Voltage is linear between two charge planes
16. $Q = C\Delta V$ ... Definition of capacitance
17. $C = \frac{\epsilon_0 A}{d}$ ... Capacitance of a parallel plate capacitor
18. $C_{tot}^{-1} = C_1^{-1} + C_2^{-2}$ ... Adding two capacitors *in series.*
19. $C_{tot} = C_1 + C_2$ ... Adding two capacitors *in parallel.*
20. $i(t) = \Delta Q/\Delta t$ ... Definition of current.
21. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
22. $R_{tot}^{-1} = R_1^{-1} + R_2^{-1}$ ... Adding two capacitors *in parallel.*
23. $R_{tot} = R_1 + R_2$ ... Adding two capacitors *in series.*
24. $\Delta V = IR_{\text{tot}}$ ... Ohm's Law
25. $P = IV$ ... Relationship between power, current, and voltage.
26. $V_{\text{C}}(t) = \epsilon_1 (1 - \exp(-t/\tau))$ ... voltage across the capacitor in an RC series circuit. The time constant is $\tau = RC$.
27. $i(t) = \frac{\epsilon_1}{R}\exp(-t/\tau)$ ... Current in an RC series circuit.
28. $i_{\text{in}} = i_{\text{out}}$ ... Kirchhoff's junction rule.
29. $\epsilon_1 + \epsilon_2 + \epsilon_3 + ... = 0$ ... Kirchhoff's loop rule.

2 Electric Charge and Electric Fields

1. **Scaling problem**: (a) Some point charge produces an E-field $E_C = 2.00 \times 10^{-3}$ V/m at a distance of 1 mm. What is the value of E_C at 5 mm produced by the same charge? (b) A 1 μC charge produces an E-field $E_C = 8.00 \times 10^{-3}$ V/m at some distance. What is the value of E_C at the same distance if the charge is 3 μC?

a) We don't know "q" so we can solve for charge first, then use $E_c = Kq/r^2$ to solve for E_c at $r = 5$mm.

→ $E_c = Kq/r^2$

$q = (E_c)(r^2)/K$

$q = (2.00 \times 10^{-3}\,V/m)(0.001m)^2/(8.99 \times 10^9\,N\cdot m^2/C^2)$

$q = (2.0 \times 10^{-9}\,V)/(8.99 \times 10^9\,N\cdot m^2/C^2)$

$q = 2.22 \times 10^{-19}\,C$

* This is the "same charge" the question said to utilize.

you can just scale by $\frac{1}{5^2}$ and fast-forward to

→ $E_c = Kq/r^2$

$E_c = (8.99 \times 10^9\,N\cdot m^2/C^2)(2.22 \times 10^{-19}\,C)/(0.005m)^2$

$E_c = (2.0 \times 10^{-9})/(0.005)^2$

$E_c = 8.0 \times 10^{-5}\,V/m$ ✓

b) Similar to part A, we can rearrange the equation $E_c = Kq/r^2$ to find the distance used. Then, we can calculate the E_c at the derived distance and given charge.

$\rightarrow E_c = Kq/r^2$

$r = \sqrt{Kq/E_c}$

$r = \sqrt{(9.0\times10^9\ N\cdot m^2/C^2)(10^{-6}\ C)/(8.00\times10^{-3}\ V/m)}$

$r = \sqrt{(9000)/(8.00\times10^{-3})}$

$r = 1061\ m$

$\rightarrow$ Find E_c when $r = 1061\ m$ and $q = 3\ \mu C$.

$E_c = Kq/r^2$

$E_c = (9.0\times10^9\ N\cdot m^2/C^2)(3\times10^{-6}\ C)/(1061\ m)^2$

$E_c = 27000/(1061\ m)^2$

$E_c = 2.4\times10^{-2}\ V/m$ ✓

Can just scale by 3 to fast-forward

2. The classic Millikan oil drop experiment was the first to measure the electron charge. Oil drops were suspended against the gravitational force by an electric field. (See Fig. 1.) Suppose the drops have a *mass* of 4×10^{-16} kg, and the E-field is oriented downward, and has a value of 6131.25 N/C. With this exact value, the drops remain suspended in air. (a) How many electrons are on the drops? (b) Suppose a cosmic ray comes along and removes an electron from a droplet. What will the acceleration of the droplet be?

a) We want to isolate the value of "q" because we know the charge for a single electron is 1.6×10^{-19} C.

$\rightarrow qE = ma$

$g = \text{gravity} = 9.8\ m/s^2$

$q = mg/E$

$q = (4\times10^{-16}\,Kg)(9.8\,m/s^2)/6131.25\,N/C$

$q = 6.39\times10^{-19} \approx 6.4\times10^{-19}\,C$

→ # electrons $= 6.4\times10^{-19}\,C / 1.6\times10^{-19}\,C$

$= 3.99$ electrons

≈ 4 electrons.

well done

b) If we remove one electron ($1.6\times10^{-19}C$), then we must take that out of "q"

→ $(6.4\times10^{-19}C) - (1.6\times10^{-19}C) = 4.8\times10^{-19}\,C$

for a now 3 electron system.

- Newton's Second law tells us that $mg = ma$, which allows use to rearrange $F = qE$ to solve for acceleration

→ $ma = qE$

$a = qE/m$, we need to take into consideration the force due to gravity,

$a = \cancel{m}g - qE/m$

$= (4.0\times10^{-16}Kg)(9.8\,m/s^2) - (4.8\times10^{-19}C)(6131.25\,C)/(4\times10^{-16}\,Kg)$

$= 9.77\times10^{-16} \quad / \quad 4.0\times10^{-16}\,Kg$

$\approx 2.4\,m/s^2$.

3 Potential Energy and Voltage, Capacitors

1. A *mass spectrometer* is a device used to accelerate ions to determine atomic masses of chemicals. Suppose two conducting plates with potential difference $\Delta V = 4$ kV are used to accelerate both hydrogen ions and helium ions. Hydrogens have charge $+1q_e$, and helium ions have charge $+2q_e$. (a) What is the total kinetic energy (in electron-volts) gained by the hydrogens and heliums? (b) If the plate separation is $\Delta x = 5$ cm, what is the electric field value? *Hint: think of the E-field as the slope of voltage.*

a) Kinetic energy is the energy of motion and we can use the equation $KE = qV$ and remembering that the charge of an electron or proton equal to 1.6×10^{-19} C.

→ $KE = qV$

$KE_{hydrogen} = (1.6 \times 10^{-19}\,C)(4000\ V)$

$= 6.4 \times 10^{-16}$ J

Volts

$KE_{helium} = (2 \cdot 1.6 \times 10^{-19}\,C)(4000\,J)$

$= 1.28 \times 10^{-15}$ J * Obviously larger KE value.

b) At a distance of 0.05m, we can calculate the Efield value using:

→ $\vec{E} = \Delta V / \Delta X$

$= 4000\ V/0.05m$

$= 8.0 \times 10^{4}\ V/m$

2. Suppose a parallel plate capacitor has an internal E-field of 1 kV/m, and a plate separation of 2 mm. Draw the voltage as a function of distance between the negative and the positive plates. Make sure to label the axes with proper units, and mark the x-value of each plate. What is the y-intercept of this function?

a) We can solve for the voltage using what is given and infer about the slope using the

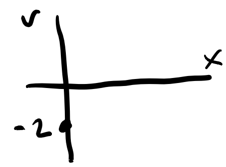
E-field.

→ Solve for voltage using what is given

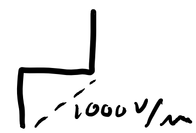
- V = Efield • Distance

$= (1000 V)(0.002 m)$

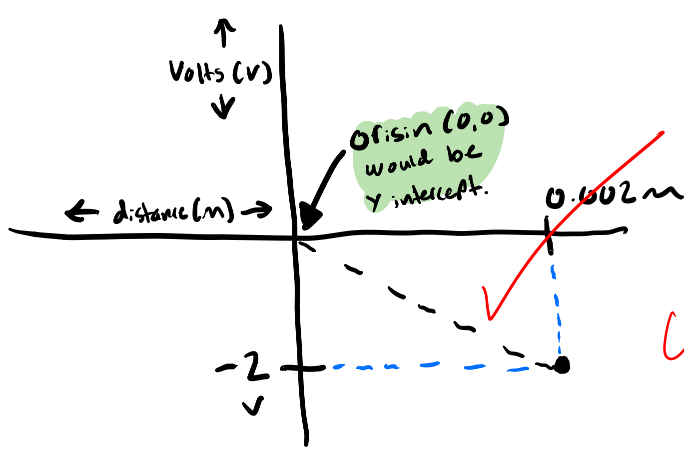
$V = -2$



→ The given Efield can inform us about slope, which states 1 kV/m, which means there are 1000 V per meter aka slope!



→ Since voltage is -2 and the slope is 1000 V/m, we can see that the slope for this function needs to be -1000 V/m from the origin.



Well done

3. Suppose the plates in the previous problem have an area of 1 cm^2. (a) What is the capacitance of the system? (b) How much energy (in Joules) is stored in this capacitor if the voltage is 5 V?

a) Since we are given area (A), we can calculate capacitance using the following equation:

→ $C = \epsilon_0 A / d$

- $\epsilon_0 \approx 8.85 \times 10^{-12}$ F/m
- $A = 0.0001\ m^2 = 1.0 \times 10^{-4}\ m^2$
- $d = 0.002\ m$

$C = (8.85 \times 10^{-12}\ F/m)(1.0 \times 10^{-4}\ m^2) / 0.002\ m$

$C = 4.43 \times 10^{-13}$ Farads.

b) We can calculate the energy of a capacitor when given voltage and capacitance using:

→ $E_{cap} = CV^2/2$

$= (4.43 \times 10^{-13}\ F)(5V)^2/2$

$= 5.54 \times 10^{-12}\ J$

4. Suppose we need a system that can store more energy for the same voltage (in other words, more capacitance). Should we connect an identical capacitor to the first *in series* or *in parallel*?

→ Capacitores in parallel store more energy than those in series. Total capacitance can be thought of as the sum of individual capacitors. Capacitors in parallel are directly connected to the voltage source. I like to think of the series capacitor as a waterfall of energy; the further down the waterfall (series capacitor) the less energy available. You would need to increase the voltage so that the energy further down a series capacitor would be similar to those in a parallel capacitor. thus, at same voltage, a parallel capacitor could store more energy.

4 Current, Resistance, and DC Circuits

1. When dealing with AA batteries, we can either connect them "end to end" (in series), or in parallel (see Fig. 2). Suppose that the internal resistances of the batteries $r_1 = r_2 = 2\Omega$, and that the emfs of the two batteries are both $\epsilon_1 = \epsilon_2 = 1.5$ V. Finally, let $R = 50\Omega$. Suppose R represents a small device that will work at 1.5 V or 3 V (a child's toy, an old CD player, a computer mouse). (a) Using Kirchhoff's rules, find the current through R for the serial case (3 V) (Fig. 2, left), and the parallel case (Fig. 2, right). (b) What is the power consumption in each case? (c) Check your calculations of current using the PhET DC circuit construction modeling kit.

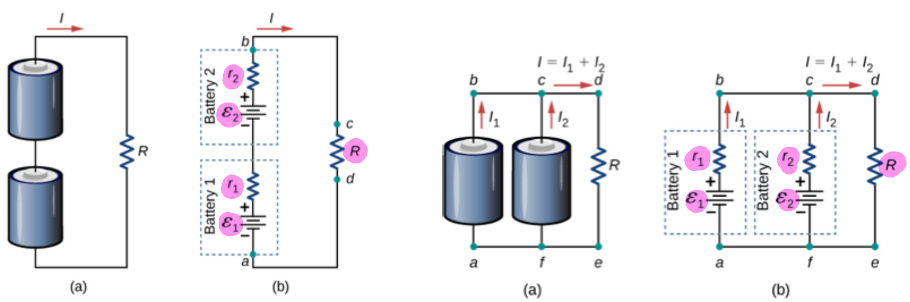


Figure 2: Two ways of connecting batteries. (Left) In series. (Right) In parallel.

a)

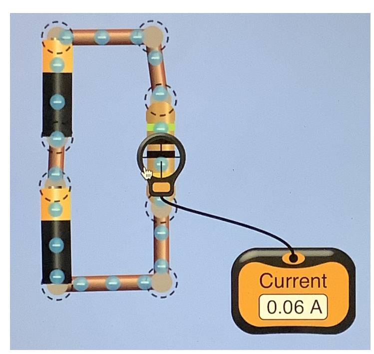
Let's start with the series

- $r = 2\,\Omega \quad \times 2 = 4\,\Omega$ total
- $\varepsilon = 1.5\,V \quad \times 2 = 3V$ total
- $R = 50\,\Omega$

→ $I = \text{emf} / R_{load} + r$

$I = 3V / 50\Omega + 4\Omega$

$= 0.055\,A \approx 0.06\,A.$

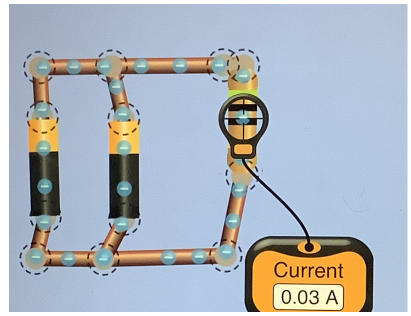


Now, let's find current for parallel

→ $I = \varepsilon / R_{tot}$

$= 1.5 / 50$

$= 0.03A$ ✓



b)

For series circuit

- Total resistance $= 50\Omega$
- Voltage $= 3V$

$P = IV$

$= (0.06A)(3V)$

$= 0.18\ W$ ✓

For parallel circuit

- Total Resistance $= 50\Omega$
- Current $= 0.03A$

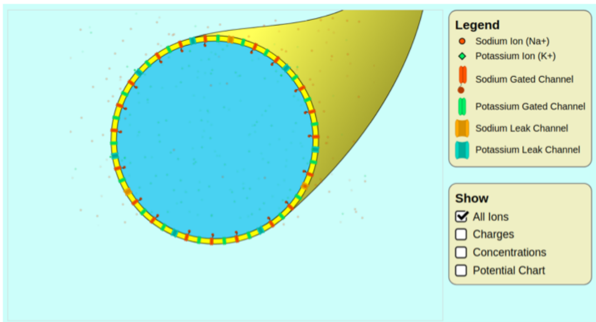
→ $P = I^2R$

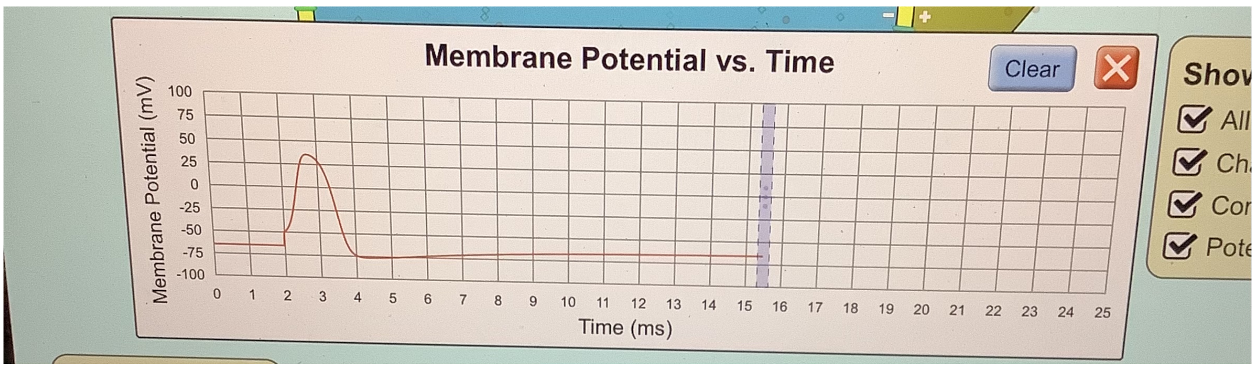
$= (0.03)^2(50)$

$= 0.045\ W$ ✓

c) Listed as the photos next to part A. I built the circuits using the program and took a screenshot. ✓

2. Recall the PhET activity in which we covered nerve stimulation as chemical-driven capacitors. Think of the voltage as a signal versus time that flows down the nerve. If you stimulate the nerve in this calculation, (a) what is the pulse width, in milliseconds? (b) What is the peak-to-peak voltage (greatest voltage minus least) in millivolts?





* Graph produced from simulation.

a) The depolarization begins at 2 ms, while hyperpolarization occurs at 4 ms.

→ t = Final − Initial

= 4 − 2

= 2 ms.

b) The highest positive potential was around +35 mV, while the lowest was around −75 mV.

→ Peak-to Peak V = Voltage between highest (+35 mV) and lowest peak $(-75mV = |75mV|)$

$\rightarrow |75mV| + 35mV$

$= 110mV$

* $+35mV$ and $-75mV$ estimated values from the graph.