Warm-Up: Units 5 and 6, Kepler's Law, Orbits, Work, and Energy

Prof. Jordan C. Hanson

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1 Memory Bank

- $T^2 \propto r^3$ ... Kepler's Third Law
- Given two planets, we can use this like:

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3 \tag{1}$$

- Let $\vec{u} = u_x\hat{i} + u_y\hat{j}$, and $\vec{v} = v_x\hat{i} + v_y\hat{j}$, and θ be the angle between them. The *dot-product* of $\vec{u}$ and $\vec{v}$ is

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y = |\vec{u}||\vec{v}|\cos\theta \tag{2}$$

- Definition of the work, W, done by a net external force $\vec{F}$ through a displacement $\Delta\vec{x}$ is:

$$W = \vec{F} \cdot \Delta\vec{x} = F\Delta x\cos\theta \tag{3}$$

2 Kepler's Laws

1. Suppose we define a unit called an "Astronomical Unit" that is equal to 1.496×10^8 km. This is the distance between the Earth and the Sun. So we can say that the Earth is 1 AU from the Sun. It turns out that Venus is 0.72 AU from the Sun (it's closer). The orbit of the Earth is 1 year. Let $T_1 = 1$ year, $r_1 = 1.0$ AU for the Earth, and $r_2 = 0.72$ AU for Venus. Use Eq. 1. to find the orbital period of Venus, T_2.

2. The orbital period of Jupiter is observed to be 11.8 years. How far in AU is Jupiter from the Sun? (*Hint: it's the same procedure as the prior problem using Earth's numbers, but solving for* T_2).

3 Work and Energy

1. Suppose a person pushes a box with a force $\vec{F} = 40\hat{i} - 30\hat{j}$ N of force, and the box moves $\Delta\vec{x} = 5\hat{i}$ m. (a) What is the work done?[1] (b) Draw a diagram of the crate, force vector, and displacement vector. (c) What is the angle between $\vec{F}$ and $\Delta\vec{x}$? (d) What can the person do to increase the work done?

[1] Note the units are the product of force and displacement, so Newton meters. One Newton meter is called a Joule.