ALGEBRA-BASED PHYSICS-1: MECHANICS (PHYS135A-01): WEEK 2

Jordan Hanson

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Whittier College Department of Physics and Astronomy

WEEK 1 REVIEW

1. Methods of approximation
    - Estimating the correct order of magnitude
    - Function approximation
    - Unit analysis
2. Coordinates and vectors
    - Scalars and vectors
    - Cartesian (rectangular) coordinates, displacement
    - Vector addition, subtraction, and multiplication
3. Review of geometry and trigonometry techniques
    - Similar triangles
    - Pythagorean theorem
    - Sine, cosine, tangent …

WEEK 1 REVIEW PROBLEMS

Given the displacement vector $\vec{D} = (3\hat{i} - 4\hat{j})$ m, find the displacement vector $\vec{R}$ so that $\vec{D} + \vec{R} = -4D\hat{j}$.

- A: $\vec{R} = (-3\hat{i} - 16\hat{j})$ m
- B: $\vec{R} = (3\hat{i} + 16\hat{j})$ m
- C: $\vec{R} = (-3\hat{i} + 12\hat{j})$ m
- D: $\vec{R} = (-6\hat{i} + 6\hat{j})$ m

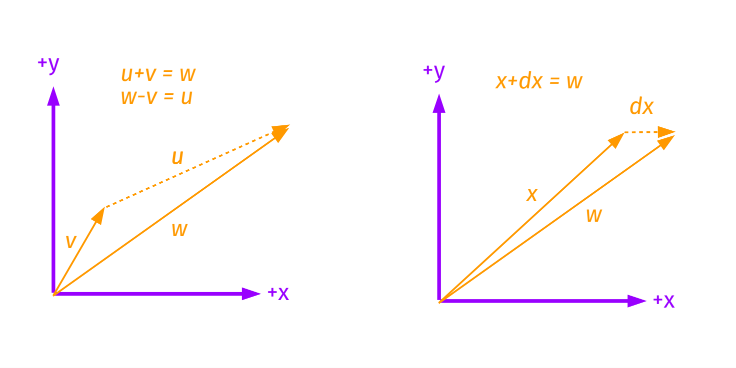
Estimate the surface area of a person.

- A: 0.2 m^2
- B: 2 m^2
- C: 5 m^2
- D: 10 m^2

Week 2 Summary

1. Displacement, average velocity and acceleration
    - *Mathematics review*: slope of a function
2. The case of constant acceleration
    - An *an equation of motion* for constant acceleration
    - Derivation of common equations of motion
    - Average quantities and exercises
3. **Lab Activity: Measuring acceleration of gravity: g**
4. Exercises with vectors, graphs, and equations of motion

DISPLACEMENT, AVERAGE VELOCITY AND ACCELERATION



(a)

(b)

Figure 1: (Left): The displacement vector is $\vec{u}$. (Right) Treat displacement for a small change in time, dt, and call it $d\vec{x}$.

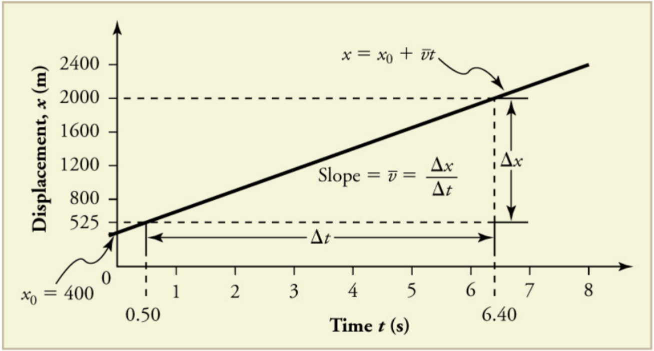


Figure 2: We can think of the velocity of a system as the *slope* of the displacement versus time.

Definition of average velocity vector:

$$\boxed{\bar{v} = \frac{\Delta\vec{x}}{\Delta t}} \tag{1}$$

Simple example: Let the initial and final positions of a particle be 4.0 m and 8.0 m, and the initial and final times be 0.0 sec and 8.0 sec. Then we have

$$\Delta\vec{x} = 8.0 - 4.0 = 4.0 \quad m \tag{2}$$

$$\Delta t = 8.0 - 0.0 = 8.0 \quad sec \tag{3}$$

Then

$$\bar{v} = 4.0/8.0 = 0.5 \quad m/s \tag{4}$$

Definition of average *acceleration* vector:

$$\boxed{\bar{a} = \frac{\Delta\vec{v}}{\Delta t}} \tag{5}$$

Simple example: Let the initial and final velocity of a system be 2.0 m/s and 3.0 m/s, and the initial and final times be 0.0 sec and 1.0 sec. Then we have

$$\Delta\vec{v} = 3.0 - 2.0 = 1.0 \quad m/s \tag{6}$$

$$\Delta t = 1.0 - 0.0 = 1.0 \quad sec \tag{7}$$

Then

$$\bar{a} = 1.0/1.0 = 1.0 \quad m/s^2 \tag{8}$$

These quantities are examples of rates, derived from initial and final states.

The situation at right depicts an object that initially travels upward, but under the acceleration of gravity.

Notice that *constant* acceleration yields linear velocity and quadratic displacement (a property of calculus).

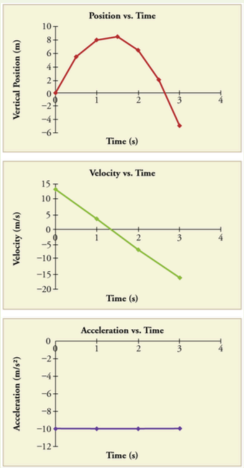


Figure 3: Rate comparisons.

THE CASE OF CONSTANT ACCELERATION

Consider the following three equations for a system experiencing constant acceleration:

$$\vec{y}(t) = (-\frac{1}{2}gt^2 + v_\mathrm{i}t + y_0)\hat{j} \quad (m) \tag{9}$$

$$\vec{v}(t) = (-gt + v_\mathrm{i})\hat{j} \quad (m/s) \tag{10}$$

$$\vec{a}(t) = (-g)\hat{j} \quad (m/s^2) \tag{11}$$

What if we solve for time in Eq. 10, after taking the magnitude of the vector?

$$\frac{v - v_\mathrm{i}}{-g} = t \tag{12}$$

Now substitute Eq. 12 into Eq. 9:

$$
\begin{aligned}
y &= -\frac{1}{2}g\left(\frac{v-v_{\mathrm{i}}}{-g}\right)^2 + v_{\mathrm{i}}\left(\frac{v-v_{\mathrm{i}}}{-g}\right) + y_0 && (13)\\
-2g(y-y_0) &= (v-v_{\mathrm{i}})^2 + 2v_{\mathrm{i}}(v-v_{\mathrm{i}}) && (14)\\
-2g(y-y_0) &= v^2 - v_{\mathrm{i}}^2 && (15)\\
-2g(y-y_0) + v_{\mathrm{i}}^2 &= v^2 && (16)
\end{aligned}
$$

Equation 16 provides a way to obtain the velocity of an accelerating system at some displacement without knowing the time.

A particle moves along the x-axis according to $x(t) = (10t - 2t^2)\hat{i}$ m. Where is the particle at 3.0 seconds? What is the average velocity over the time inverval 0.0-3.0 seconds?

- A: 12 m, 4 m/s
- B: 4 m, 12 m/s
- C: 30 m, 4 m/s
- D: 12 m, 12 m/s

Let $x(t) = (10t - 2t^2)\hat{i}$ m, from prior exercise. What is the displacement between $t = 2$ seconds and $t = 3$ seconds? What is the average velocity in that period?

- A: 0 m, 0 m/s
- B: 10 m, 10 m/s
- C: 0 m, 4 m/s
- D: 4 m, 0 m/s

Notice in the previous two problems: the *instantaneous velocity* is not the *average velocity*. The average velocity between two and three seconds was 0 m/s, but the instantaneous velocity was not zero at either point. However, the *displacement* was 0 m in this time interval. Take care not to confuse *instantaneous* quantities with *average* quantities.

On February 15, 2013, a meteor entered Earth's atmosphere over Chelyabinsk, Russia, and exploded at an altitude of 23.5 km. Eyewitnesses could feel the intense heat from the fireball, and the blast wave from the explosion blew out windows in buildings. The blast wave took approximately 2 minutes 30 seconds to reach ground level. What was the average velocity of the blast wave? Compare this with the speed of sound, which is 343 m/s at sea level.

- A: 35 m/s (10% speed of sound)
- B: 100 m/s (30% speed of sound)
- C: 150 m/s (40% speed of sound)
- D: 350 m/s (100% speed of sound)

$$\vec{x}(t) = \frac{1}{2}at^2 + v_{\mathrm{i}}t + y_0\hat{j} \quad (m) \tag{17}$$

$$\vec{v}(t) = at + v_{\mathrm{i}}\hat{j} \quad (m/s) \tag{18}$$

$$\vec{a} = a\hat{j} \quad (m/s^2) \tag{19}$$

$$v_{\mathrm{f}}^2 - v_{\mathrm{i}}^2 = 2a(x_{\mathrm{f}} - x_{\mathrm{i}}) \tag{20}$$

A particle moves along the x-axis according to $v(t) = (10 - 4t)\hat{i}$ m. What is the average acceleration between $t = 2$ seconds and $t = 3$ seconds?

- A: 4 m/s^2
- B: 2 m/s^2
- C: -2 m/s^2
- D: -4 m/s^2

Same example. What is the average acceleration between $t = 0$ seconds and $t = 3$ seconds?

- A: 4 m/s^2
- B: 2 m/s^2
- C: -2 m/s^2
- D: -4 m/s^2

Notice that the *average acceleration* doesn't depend on time (because it is constant). Similar to the definition of *average velocity*, we have the *average acceleration*:

$$\bar{a} = \boxed{\frac{v_\mathrm{f} - v_\mathrm{i}}{t_\mathrm{f} - t_\mathrm{i}}} \tag{21}$$

So if $a = constant$, $a(t) = \bar{a}$ and the numbers will always arrange themselves to give the same answer...

A cheetah can accelerate from rest to a speed of 35.0 m/s in 7.00 s. What is its average acceleration, if it's headed in the $-\hat{i}$ direction?

- A: -5 m/s^2
- B: 2 m/s^2
- C: 10 m/s^2
- D: 5 m/s^2

Dr. John Paul Stapp was U.S. Air Force officer who studied the effects of extreme deceleration on the human body. On December 10, 1954, Stapp rode a rocket sled, accelerating from rest to a top speed of 282 m/s (1015 km/h) in 5.00 s, and was brought jarringly back to rest in only 1.40 s! Calculate his acceleration and deceleration. Express each in multiples of g, (9.8 m/s^2).

- A: 5.75 g's, 20.6 g's
- B: 56.4 g's, 201 g's
- C: 4.75 g's, 2.6 g's
- D: 5.7 g's, 12.2 g's

While entering a freeway, a car accelerates from rest at a rate of 2.40 m/s^2 for 12.0 s. How far does the car travel in that time? What is the car's final velocity?

- A: 14.4 m, 29 m/s
- B: 346 m, 29 m/s
- C: 173 m, 14.5 m/s
- D: 173 m, 29 m/s

LAB ACTIVITY: MEASURING ACCELERATION OF GRAVITY

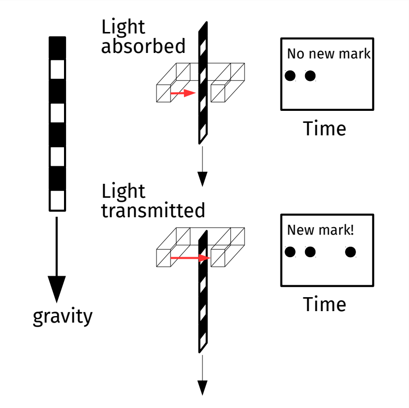


Figure 4: (Left) A *picket* is marked at regular intervals with black strips. (Right) Upon dropping the picket through a *photo-gate*, the strips will block the photo-gate and we will record when this happens on a clock.

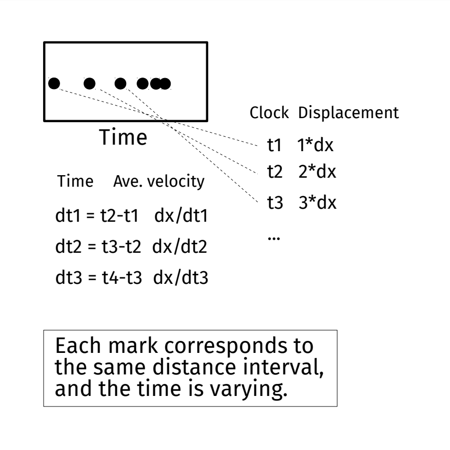


Figure 5: We can measure the velocity versus time of the picket by taking the ratio of *displacements* to *times*.

Acceleration is the change in velocity, so once we have the velocities (dx/dt_{i}), we can take more ratios:

$$t1' = (t2 + t1)/2 \quad dx/dt1 \tag{22}$$
$$t2' = (t3 + t2)/2 \quad dx/dt2 \tag{23}$$
$$t3' = (t4 + t3)/2 \quad dx/dt3 \tag{24}$$
$$\ldots \tag{25}$$

$$t1' \quad \frac{dx/dt2 - dx/dt1}{t2' - t1'} \tag{26}$$
$$t2' \quad \frac{dx/dt3 - dx/dt2}{t3' - t2'} \tag{27}$$
$$t3' \quad \frac{dx/dt4 - dx/dt3}{t4' - t3'} \tag{28}$$
$$\ldots \tag{29}$$

Once we have the acclerations $\frac{dx/dt2 - dx/dt1}{t2' - t1'}$, ..., we can compare them with each other and compute the *average* and *standard deviation.*

$$\bar{a}_{\mathrm{meas}} = N^{-1} \sum_{\mathrm{i}}^{N} a_{\mathrm{meas,i}} \tag{30}$$

$$\sigma^2_{\mathrm{meas}} = N^{-1} \sum_{\mathrm{i}}^{N} (a_{\mathrm{meas,i}} - \bar{a}_{\mathrm{meas}})^2 \tag{31}$$

Quote the result like this: $\bar{a}_{\mathrm{meas}} \pm \sigma_{\mathrm{meas}}$. *The mean plus or minus one standard deviation.* The result of this experiment is $g = \bar{a}_{\mathrm{meas}}$, the acceleration due to gravity near the Earth's surface.

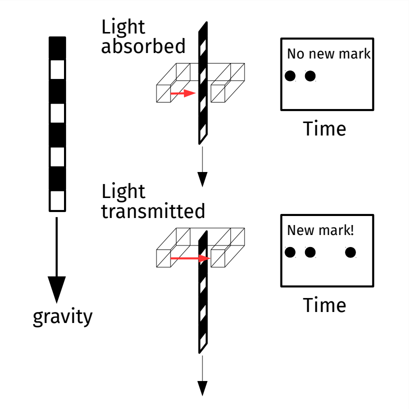


Figure 6: Does adding more mass to the picket change the answers?

EXERCISES WITH VECTORS, GRAPHS, AND EQUATIONS OF MOTION

We have a system of equations describing motion of classical particles undergoing constant acceleration:

$$x = x_0 + \bar{v}t \tag{32}$$

$$\bar{v} = (v + v_0)/2 \tag{33}$$

$$v = v_0 + at \tag{34}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \tag{35}$$

$$v^2 = v_0^2 + 2a(x - x_0) \tag{36}$$

A particle moves in a straight line with an initial velocity of 30 m/s and a constant acceleration of 30 m/s^2. What is the displacement at $t = 5$ seconds? What is the velocity at $t = 5$ seconds?

- A: 900 m, 180 m
- B: 180 m, 525 m/s
- C: 525 m, 180 m/s
- D: 700 m, 200 m/s

A particle is moving at 5 m/s, 60 degrees with respect to the x-axis. At $t = 0$ seconds, it begins to accelerate at 1 m/s^2. What is the speed after 3 seconds?

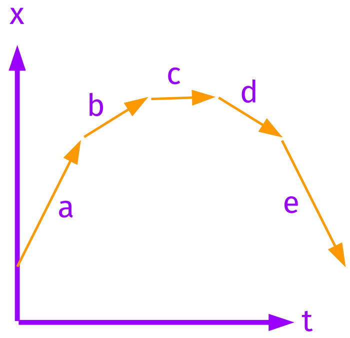
- A: 8 m/s
- B: 4 m/s
- C: -4 m/s
- D: -8 m/s

If the particle is at (0,0) at $t = 0$, where is the particle at $t = 3$ seconds?

- A: (7.5,13) m
- B: (16.9,9.75) m
- C: (9.75,16.9) m
- D: (13,7.5) m

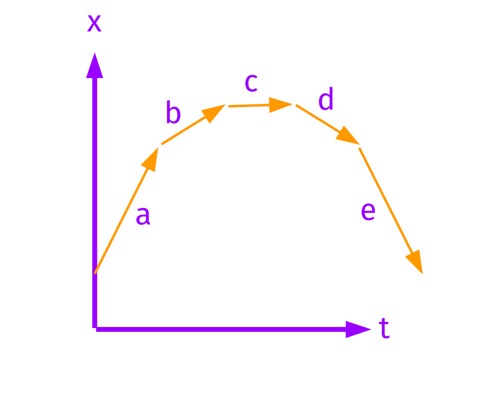
Which segment(s) of the motion described by the plot at right has $v \approx 0$ (m/s)?

- A
- B and D
- C
- E



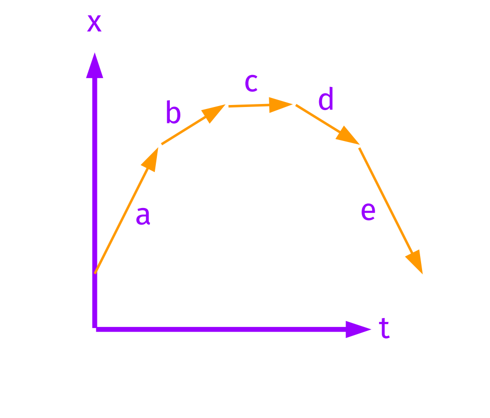
Which segment(s) of the motion described by the plot at right has the largest velocity?

- A
- B
- D
- E



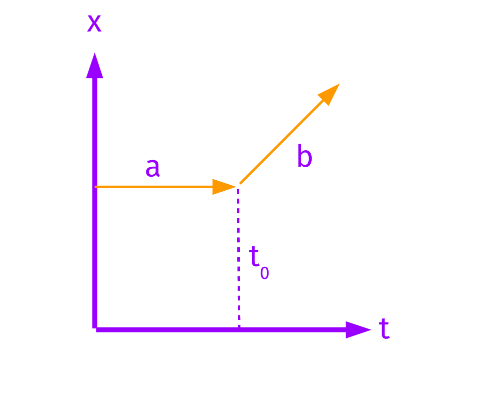
Does the motion described by the plot correspond to negative or positive acceleration?

- Negative
- Positive



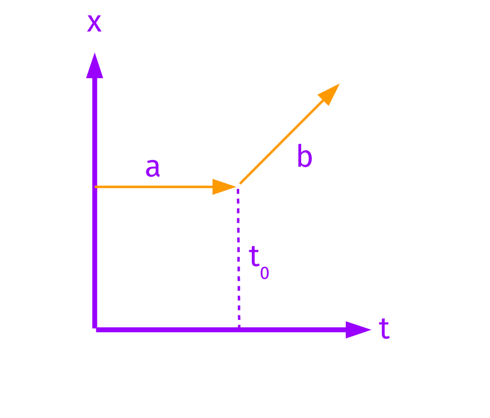
In which region(s) is the acceleration zero m/s^2?

- A only
- B only
- Both A and B
- None



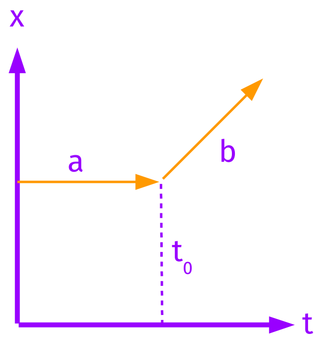
At $t = t_0$, what is the acceleration?

- Negative and large
- Positive and large
- Positive, but small
- Negative, but small



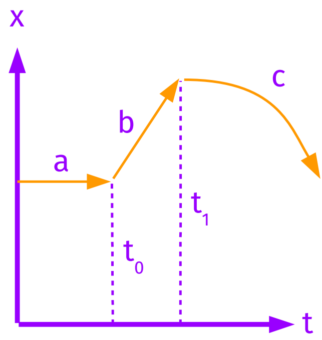
What is the average velocity?

- Less than the slope of region B
- Greater than the slope of region B
- Zero



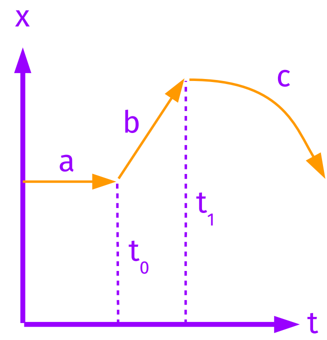
What is the sign of the acceleration at $t = t_1$? What is the sign of the acceleration at $t = t_0$?

- Negative, positive
- Positive, negative
- Positive, positive
- Negative, negative



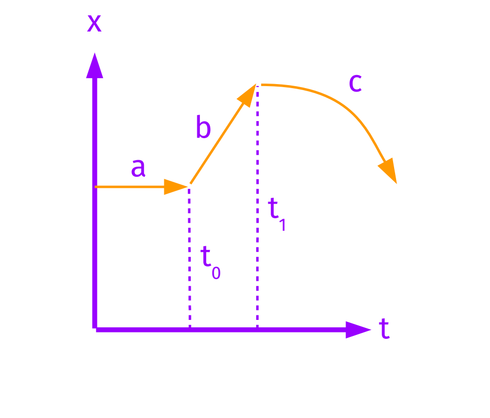
What is most likely the total displacement?

- Positive and large
- Negative and large
- Zero
- Cannot discern from graph



What is most likely the acceleration during segment C?

- Positive and increasing
- Negative and increasing
- Negative and decreasing
- Negative and constant



A particle is moving along the y-axis, with an initial speed of 10 m/s, and an acceleration of -10 m/s^2. What is the displacement when the speed has decreased to 1 m/s?

- A: 3 m
- B: 4 m
- C: 5 m
- D: 6 m

A particle is moving along the y-axis, with an initial speed of 10 m/s, and an acceleration of -10 m/s^2. How much time has elapsed when the particle has a speed of 1 m/s?

- A: 0.9 seconds
- B: 10 seconds
- C: 0.5 seconds
- D: 1.5 seconds

CONCLUSION

1. Displacement, average velocity and acceleration
    - *Mathematics review*: slope of a function
2. The case of constant acceleration
    - An *an equation of motion* for constant acceleration
    - Derivation of common equations of motion
    - Average quantities and exercises
3. **Lab Activity: Measuring acceleration of gravity: g**
4. Exercises with vectors, graphs, and equations of motion

ANSWERS

ANSWERS

- $\vec{R} = (-3\hat{i} - 16\hat{j})$ m
- 2 m^2
- 12 m, 4 m/s
- 0 m, 0 m/s
- 150 m/s (40% speed of sound)
- -4 m/s^2
- -4 m/s^2
- -5 m/s^2
- 5.75 g's, 20.6 g's
- 173 m, 29 m/s
- 525 m, 180 m/s
- 8 m/s
- (9.75,16.9) m
- C
- Negative
- Both A and B
- Positive and large
- Less than the slope of region B
- Negative, positive
- Zero
- Negative and constant
- 5 m