ALGEBRA-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND MODERN PHYSICS (PHYS135B-01): UNIT 3

Jordan Hanson

March 25, 2019

Whittier College Department of Physics and Astronomy

UNIT 2 REVIEW

Reading: Chapters 20 and 21

1. Current, Ohm's Law, resistors and conductors
2. DC circuits I
3. Nerve signals

UNIT 2 REVIEW PROBLEMS

Consider a proton with mass m and charge q_{p}. The proton is located in a region with electric field E, and it has initial velocity v_{i} at time $t = 0$. What is the acceleration of the proton? What is the proton velocity v_{f} after a time t?

- A: $a = \frac{m}{q_{\mathrm{p}}} E$, $v_{\mathrm{f}} = v_{\mathrm{i}} + t$
- B: $a = \frac{m}{q_{\mathrm{p}}} E$, $v_{\mathrm{f}} = v_{\mathrm{i}} + \frac{m}{q_{\mathrm{p}}} Et$
- C: $a = \frac{q_{\mathrm{p}}}{m} E$, $v_{\mathrm{f}} = v_{\mathrm{i}} + \frac{q_{\mathrm{p}}}{m} Et$
- D: $a = \frac{q_{\mathrm{p}}}{m} E$, $v_{\mathrm{f}} = v_{\mathrm{i}} + t$

Consider a 12V battery driving current through a system with a total resistance of 1 kΩ. What is the current? What is the power consumption?

- A: 12 mA, 144 W
- B: 144 mA, 12 mW
- C: 12 mA, 144 mW
- D: 14.4 A, 144 W

SUMMARY

Reading: Chapter 22

This week: 22.1-22.5

1. Magnets and magnetic fields
2. Force on a moving charge in a magnetic field
3. Applications

This weekend: 22.6-22.10

1. The Hall effect
2. Magnetic forces on conductors
3. Torque on a current loop
4. Ampère's Law

MAGNETS AND MAGNETIC FIELDS

Introductory video on the origin of magnetic fields and forces they exert on charge:

`https://www.youtube.com/watch?v=s94suB5uLWw`

What is a cross-product and how does it work?

Computing the cross product [edit]

Coordinate notation [edit]

The standard basis vectors **i**, **j**, and **k** satisfy the following equalities in a right hand coordinate system:

$$\begin{aligned}\mathbf{i}\times\mathbf{j}&=\mathbf{k}\\ \mathbf{j}\times\mathbf{k}&=\mathbf{i}\\ \mathbf{k}\times\mathbf{i}&=\mathbf{j}\end{aligned}$$

which imply, by the anticommutativity of the cross product, that

$$\begin{aligned}\mathbf{j}\times\mathbf{i}&=-\mathbf{k}\\ \mathbf{k}\times\mathbf{j}&=-\mathbf{i}\\ \mathbf{i}\times\mathbf{k}&=-\mathbf{j}\end{aligned}$$

The definition of the cross product also implies that

$\mathbf{i}\times\mathbf{i}=\mathbf{j}\times\mathbf{j}=\mathbf{k}\times\mathbf{k}=\mathbf{0}$ (the zero vector).

Figure 1: The cross-product is a way of multiplying unit vectors.

Professor: several examples on board.

Let $\vec{v} = 2\hat{i}$ and $w = -2\hat{j}$. What is $\vec{v} \times \vec{w}$?

- A: $-4\hat{k}$
- B: $4\hat{k}$
- C: $-2\hat{i}$
- D: $2\hat{j}$

Let $\vec{v} = 3\hat{j}$ and $w = 5\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $15\hat{i}$
- B: $5\hat{j}$
- C: $3\hat{i}$
- D: $15\hat{k}$

Let $\vec{v} = 3\hat{i} \times 3\hat{j}$ and $w = 2\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $-6\hat{j} + 6\hat{k}$
- B: $-6\hat{j} + 6\hat{i}$
- C: $6\hat{j} + 6\hat{i}$
- D: $6\hat{k} + 6\hat{i}$

Group exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} \tag{1}$$

$$\vec{w} = 4\hat{j} - 4\hat{i} \tag{2}$$

$$\vec{v} \times \vec{w} = ?? \tag{3}$$

Group exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} + \hat{k} \tag{4}$$

$$\vec{w} = 4\hat{j} - 4\hat{i} - \hat{k} \tag{5}$$

$$\vec{v} \times \vec{w} = ?? \tag{6}$$

The Lorentz Force

Let a particle with charge q and velocity $\vec{v}$ move through a *magnetic field* $\vec{B}$. The **Lorentz force** on the charged particle is

$$\vec{F}_{\mathrm{L}} = q\vec{v} \times \vec{B} \tag{7}$$

As a helpful memory tool, we have the right-hand rule to remember the direction of the cross-product. **The units of the magnetic field are the Telsa**, after Nikola Tesla. We also have the Gauss which is 10^{-4} Tesla.

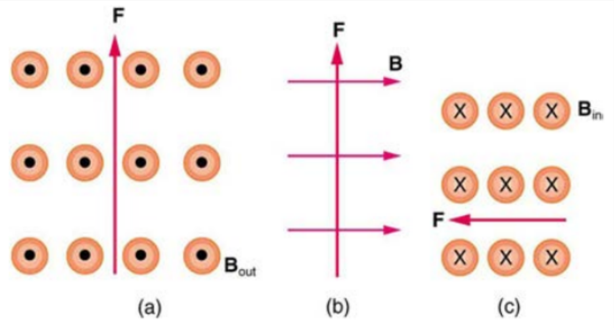


Figure 2: Three different magnetic field and charge scenarios. The vector $\vec{F}$ is the direction of the Lorentz force, and the magnetic field is uniform. A dot indicates that the magnetic field is coming out of the page, and an x indicates that the field is going into the page.

In which of the diagrams is a positively charged particle moving to the left?

- A: A
- B: B
- C: C
- D: Double WAT

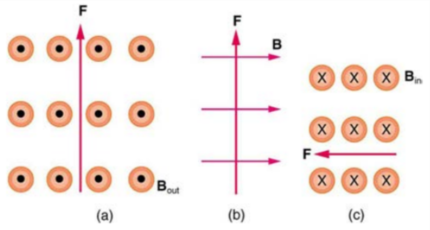


Figure 3: Three different magnetic field and charge scenarios.

In which of the diagrams is a positively charged particle moving upwards?

- A: A
- B: B
- C: C
- D: Double WAT

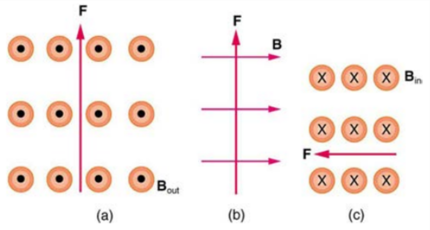


Figure 4: Three different magnetic field and charge scenarios.

In which of the diagrams is a negatively charged particle moving into the page?

- A: A
- B: B
- C: C
- D: Double WAT

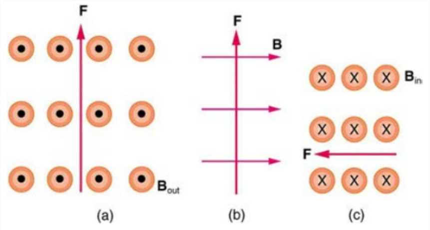


Figure 5: Three different magnetic field and charge scenarios.

In which of the diagrams is a negatively charged particle moving to the right?

- A: A
- B: B
- C: C
- D: Double WAT

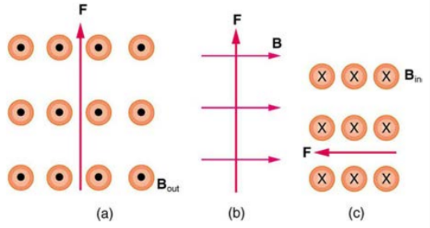


Figure 6: Three different magnetic field and charge scenarios.

A theorem for the magnitude of the cross-product: Let $\vec{a}$ and $\vec{b}$ be vectors and θ be the angle between them. The magnitude of the cross-product is

$$|\vec{a} \times \vec{b}| = ab\sin\theta \tag{8}$$

Thus, the magnitude of the Lorentz force is

$$F_{\mathrm{L}} = qvB\sin\theta \tag{9}$$

The angle θ is between the velocity and the magnetic field.

A cosmic ray proton moving toward the Earth at 3×10^6 m/s experiences a magnetic force of 2×10^{-17} N. What is the strength of the magnetic field if there is a 45 degree angle between it and the proton's velocity? (Remember that q for a proton is 1.6×10^{-19} C).

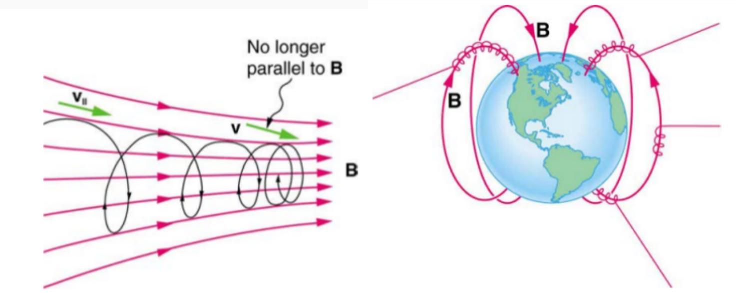
- A: 0.1 Gauss
- B: 0.6 Gauss
- C: 1 Gauss
- D: 6 Gauss

Other examples:

1. Magnetic fields do no work
2. $v = E/B$
3. q/m circle (potential demonstration)

Figure 7: The aurora borealis, or northern lights.

A cool talk on the aurora borealis:
`https://youtu.be/czMh3BnHFHQ`



One un-explained piece: what does it mean for the electrons and protons to *high-five* the neutral oxygen and nitrogen atoms?

THE HALL EFFECT

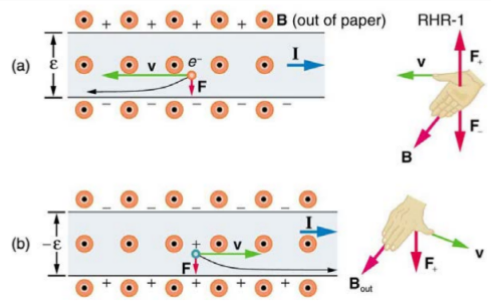


Figure 8: Diagram of the Hall effect. The Hall emf reveals the sign of the moving charges.

The Hall emf is

$$\epsilon = Blv \tag{10}$$

The B is the magnetic field, l is the length across the region where charge is flowing, and v is the velocity of the charges.

Group board exercise: Let n be the charge number density in a conductor. Let q be the charge of an electron. Let A be the cross-sectional area, and I be the current. Recall that the *drift velocity of charges in a wire* is given by the equation

$$v_d = \frac{I}{nqA} \tag{11}$$

The Hall emf is $\epsilon = Blv$. Let $v = v_d$, and substitute the first equation into the second equation, and solve for n. Choose reasonable numbers for the current, diameter of a metal wire, and assume a uniform 0.001 T magnetic field is being created around the wire. Assume a drift velocity of about 1 mm/sec, and solve for n.

MOTORS

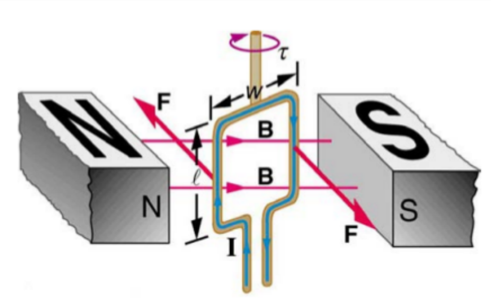


Figure 9: In a loop of current in a uniform magnetic field, we find forces going the opposite directions.

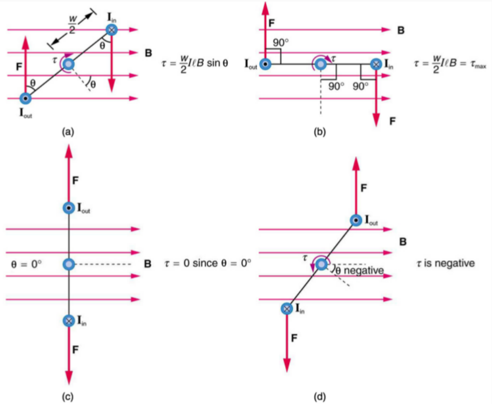


Figure 10: This leads to torque.

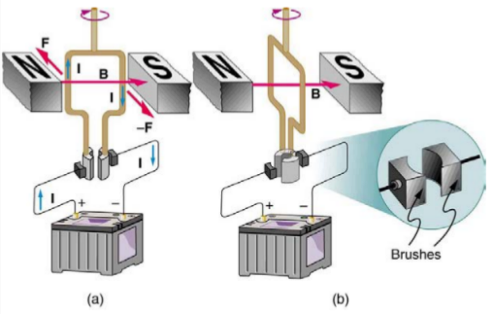


Figure 11: Torque can be used to drive a motor.

Let the number of loops in the coil be N, the current be I, the area of the loops be A, and the magnetic field be B. The angle between the force and loops is θ. The magnitude of the torque τ is

$$\tau = NIAB \sin\theta \tag{12}$$

At what angle between the loops and the B field is the torque maximized?

- A: 0 degrees
- B: 45 degrees
- C: 90 degrees
- D: 135 degrees

Which of the following would boost the torque of a motor?

- A: Increasing the B-field magnitude
- B: Decreasing the number of loops
- C: Increasing the number of loops
- D: A and C

Suppose $I = 10$ amps, $B = 0.01$ T, $N = 200$, and the loops have a common radius of 5 cm. **Group exercise:** what is the maximum torque?

Ampère's Law

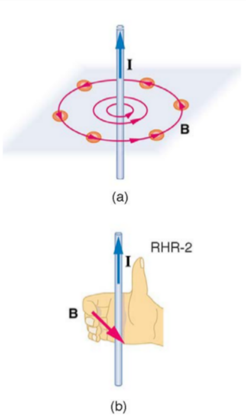


Figure 12: Magnetic fields creat currents! To remember the direction, use your right hand.

Ampère's Law states that the current produced by a long straight wire is

$$B = \frac{\mu_0 I}{2\pi r} \tag{13}$$

The current is I, the distance from the wire is r, and $\mu_0 = 4\pi \times 10^{-7}$ T m/A.

Group exercise: Suppose we have a wire carrying 1 A, and we are 1 cm away from it. What is the magnetic field? What is the magnetic field if another wire is located 1 cm away from us, but carries -1 A? Should the fields add or subtract?

`https://youtu.be/1JZLKvWO0ks`

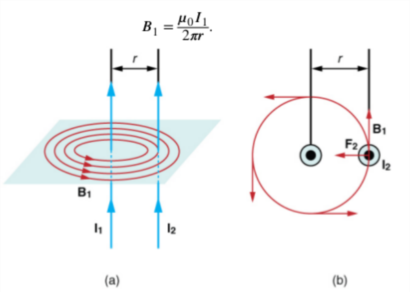


Figure 13: Definition of the amp is derived from this setup.

The B-field at wire 2 due to wire 1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi r} \tag{14}$$

The force on wire 2 is

$$F_2 = I_2 l B_1 \tag{15}$$

Dividing both sides by l and substituting in Eq. 14, we have

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r} \tag{16}$$

Group board exercise: let $I_1 = I_2 = 1.0$ A, and $r = 1$ meter. What is the force per unit length F/l? (Recall that $\mu_0 = 4\pi \times 10^{-7}$ T m/A by definition).

MAGNETIC FIELDS AND WORK

Notice from the Lorentz force that magnetic fields do not perform work:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{17}$$

$$\vec{v}t = \vec{x} \tag{18}$$

$$\vec{F} = \frac{q}{t}\vec{x} \times \vec{B} \tag{19}$$

$$W = \vec{x} \cdot \vec{F} \tag{20}$$

$$W = \frac{q}{t}\vec{x} \cdot \left(\vec{x} \times \vec{B}\right) \tag{21}$$

In the final step, why is the right hand side $\left(\vec{x} \cdot \left(\vec{x} \times \vec{B}\right)\right)$ zero?

- A: Because $\vec{x}$ is parallel to $\vec{x} \times \vec{B}$.
- B: Because $\vec{x}$ is perpendicular to $\vec{x} \times \vec{B}$.
- C: Because $q = 0$ on average.
- D: Because $\vec{x}$ is perpendicular to $\vec{B}$.

We will continue with the lab activity from last period, in which we began to study how moving magnetic fields *create* emf's (voltages).

1. Connect the voltmeter and wires to the leads on the set of loops of wire, and obtain a ruler and two bar magnets.
2. The goal is to vary the speed of the magnets through the wire loops:
    - Drop the magnets through the wire loops from different heights about the loops.
    - Make measurements of the initial height of the magnet above the loops.
    - Record the maximum voltage observed as the magnet passes through the loops.
    - **Plot the max voltage versus initial height of the magnet, for both polarities of the magnet.**
    - Repeat with two magnets.

CONCLUSION

Reading: Chapter 22

1. Magnets and magnetic fields
2. Force on a moving charge in a magnetic field
3. **The Hall effect**
4. Magnetic forces on conductors
5. Ampère's Law

ANSWERS

ANSWERS

- $a = \frac{q_{\mathrm{p}}}{m} E$, $v_{\mathrm{f}} = v_{\mathrm{i}} + \frac{q_{\mathrm{p}}}{m} Et$
- 12 mA, 144 mW
- $-4\hat{k}$
- $15\hat{i}$
- $-6\hat{j} + 6\hat{i}$
- A
- C
- B
- A
- 0.6 Gauss
- 90 degrees
- A and C