

Midterm 2

Prof. Jordan C. Hanson

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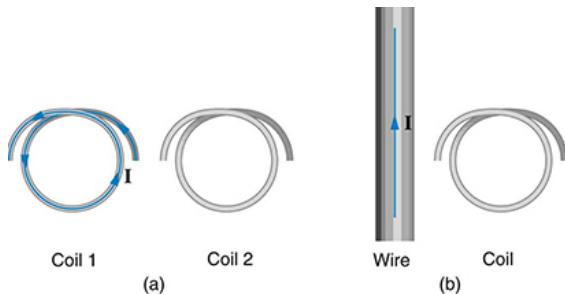


Figure 1: (a) A current I flows in the left coil, in the same plane as the right coil. (b) A wire with current I flows in the same plane as the coil.

1. Consider Fig. 2 (a) What is the direction of the current induced in coil 1 when the switch is closed? If it is left closed for a long time? When it is opened? (b) Repeat exercise (a) for coil 2. (c) Repeat exercise (a) for coil 3.

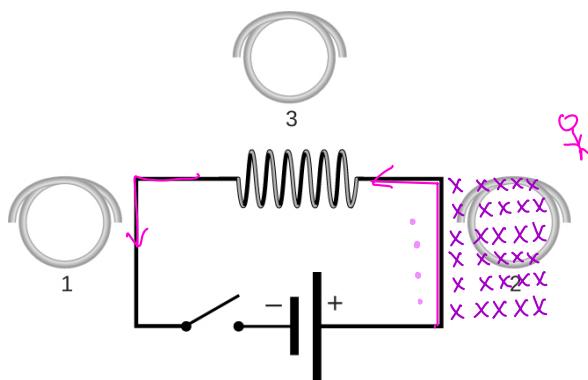


Figure 2: A battery provides emf to a simple circuit with a solenoid, surrounded by three wire loops in the plane of the circuit.

2. Verify that the units of $\Delta\phi/\Delta t$ are Volts.

3. (a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) What current is induced in the ring if its resistance is 0.0100Ω ? (c) Calculate the average power dissipated in the ring.

1 Unit 4: Magnetism II

1. Consider Fig. 1a (a) What is the direction of the current induced in coil 2 if the current in coil 1 increases? If it decreases? Consider Fig. 1b. (b) What is the direction of the current induced in the coil if the current in the wire increases? If it decreases?

- ①
- A) if it increases then Coil 2 would go counter clockwise
 if it decreases then Coil 2 would go clockwise
- B) If wire increases then Coil would go counter clockwise
 if wire decreases then Coil would go clockwise

② Coil 1

a) First Closed : Counter clockwise

Closed for : None
 long time
 Just after open: Clockwise

Coil 2

First Closed : Counter clockwise

Closed for : None
 long time
 Just after open: Clockwise

Coil 3

First Closed : None

Closed for : None
 long time
 Just after open: None

$$\textcircled{3} \quad \frac{\Delta \phi}{\Delta t} = \frac{wb}{s} \rightarrow Iwb = \frac{N \cdot m}{A}$$

$$= \frac{Nm}{As} = \frac{V}{C} = \text{volt}$$

$$F \cdot a = w$$

$$N \cdot m = \textcircled{13}$$

$$C = I \cdot A$$

$$\textcircled{C} F A S$$

(4)

$$\text{Bi=0} \quad \text{Bl=2 A} \rightarrow AB = 2 - 0 = 2$$

$$EMF = -\frac{N \frac{\Delta \phi}{\Delta t}}{\Delta t} = -N \frac{(BA \cos \theta)}{\Delta t} = \frac{-1^2 (\pi \cdot \frac{2 \cdot 2}{2} \times 10^{-2})^2}{0.25} \cdot \cos(0)$$

a) $\mathcal{E}_{EMF_i} = 3.04 \times 10^{-3} \text{ V}$

b) $I = \frac{V}{R} = \frac{3.04 \times 10^{-3}}{0.01} = 3.04 \times 10^{-1} \text{ A}$

c) $P = IV = 3.04 \times 10^{-1} (3.04 \times 10^{-3}) = 9.24 \times 10^{-4} \text{ W}$

5. Prove that when B , l , and v are not mutually perpendicular, motional emf is given by $\text{emf} = Blv \sin \theta$. If v is perpendicular to B , then θ is the angle between l and B . If l is perpendicular to B , then θ is the angle between v and B . Hint: it is helpful to draw a diagram to define the angle θ .

$$V = l v B \sin \theta$$

$$\text{EMF} = -N \frac{\Delta \Phi}{\Delta t}$$

6. (a) A bicycle generator rotates at 1875 rad/s, producing an 18.0 V peak emf. It has a 1.00 by 3.00 cm rectangular coil in a 0.640 T field. How many turns are in the coil? (b) What is the period of the AC output of the generator?

- is aligned with the normal direction of the solenoid ($N = 6$ turns). The system is now acting as a **transformer**. Suppose the emf in the solenoid is given by $V(t) = V_0 \sin(\omega t)$. (a) Draw a graph of $V(t)$, with the x-axis and y-axis units labeled. (b) Add the induced emf in coil 3 to your graph, assuming the magnetic flux in coil 3 is rendered identical to that in the solenoid with an iron core. (c) If $V_0 = 120$ V, and $\omega = (240\pi)$ Hz, when does $V(t) = 0$?

9. Camera flashes charge a capacitor to high voltage by switching the current through an inductor on and off rapidly. In what time must the 0.100 A current through a 2.00 mH inductor be switched on or off to induce a 500 V emf?

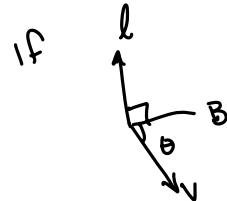
10. A large research solenoid has an inductance of 25.0 H. (a) What induced emf opposes shutting it off when 100 A of current through it is switched off in 80.0 ms? (b) How much energy is stored in the inductor at full current? (c) At what rate in watts must energy be dissipated to switch the current off in 80.0 ms? (d) Considering exercise (c), is it surprising that shutting it down this quickly is difficult? (e) Design a solenoid that has an inductance of 25.0 H by choosing the number of turns N , the cross-sectional area A , and the number of turns l .

7. An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

8. Consider Fig. 2. Suppose coil 3 ($N = 1$ turn) is rotated 90 degrees such that its normal direction

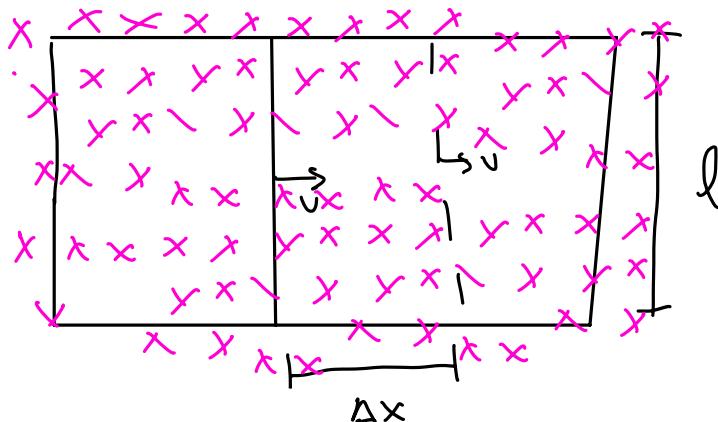
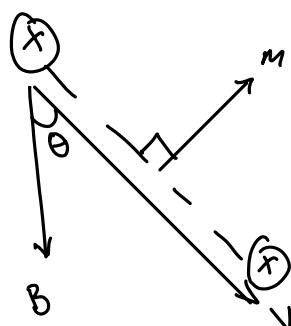
- Your RL circuit has a characteristic time constant

$$\text{motional EMF} = Blv \sin(\theta)$$



Solution

$$\text{EMF} = \frac{\Delta \Phi}{\Delta t}$$



$$\text{so } \Delta \Phi = \text{Area of loop} \cdot B \cos(\theta + 90^\circ)$$

$$\cos(\theta + 90^\circ) = \cos \theta \cdot \cos 90^\circ - \sin \theta \sin 90^\circ$$

$$\cos(\theta + 90^\circ) = -\sin \theta$$

$$\Rightarrow \Delta \Phi = Bl \Delta x (-\sin \theta)$$

$$\Delta \Phi = -Bl \Delta x \sin \theta$$

$$\text{EMF} = -N \frac{\Delta \Phi}{\Delta t} = \frac{-N l - Bl \Delta x \sin \theta}{\Delta t}$$

$$V = \frac{\Delta x}{\Delta t}$$

$$\text{EMF} = -N (Bl V \sin \theta)$$

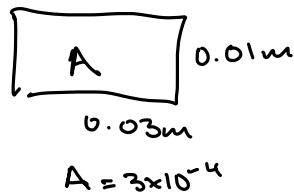
$$\text{EMF} = Bl V \sin \theta$$

(b)
(a)

1875 rod/sec, 18.0V peak EMF, 0.640T field

$$V_p = N \cdot A \cdot B \cdot W$$

$$N = \frac{V_p}{A \cdot B \cdot W} = \frac{18}{3 \times 10^{-4} (0.64) (1875)}$$



$$A = 3 \times 10^{-4}$$

$$N = 50 \text{ turns in coil}$$

$$(b) T = \frac{1}{f} = \frac{2\pi}{W} = \frac{2\pi}{1875 \text{ rod/sec}} = 0.00335 \text{ sec}$$

(7)

$$\frac{V_S}{V_P} = \frac{N_S}{N_P} \rightarrow \frac{V_P}{V_S} = \frac{N_P}{N_S}$$

(a) $\frac{240}{120} = \frac{N_P}{N_S} = 2$

(b) $\frac{I_S}{I_P} = \frac{N_S}{N_P} \rightarrow \frac{I_P}{I_S} = \frac{N_P}{N_S}$

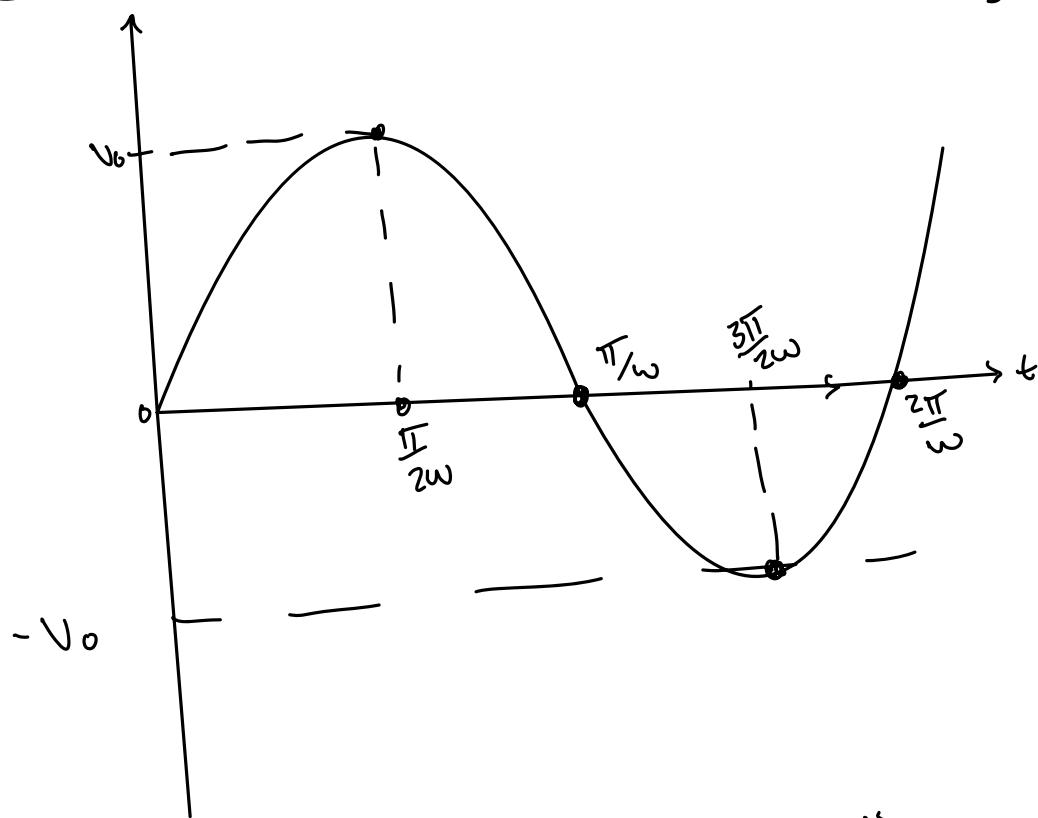
$$\frac{I_P}{I_S} = \frac{1}{2}$$

(c) Switch the coils in the transformer or plug the out pot of one as the input of another

(8)

(a)

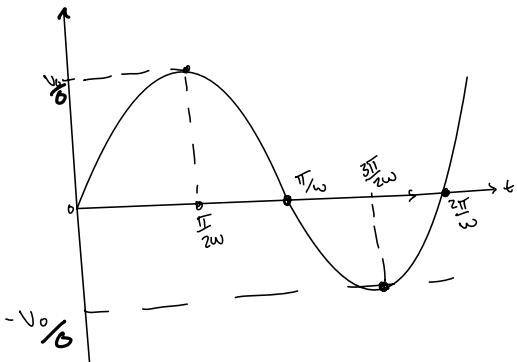
$$V(t) = V_0 \sin(\omega t)$$



(b) $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$$\frac{V_0 \sin(\omega t)}{V_P} = \frac{b}{1}$$

$$V_P = \frac{V_0}{b} \sin(\omega t)$$



$$\textcircled{C} \quad V_p = \frac{V_0}{6} \sin(\omega t)$$

$$\text{if } V_0 = 120 \text{ then } \omega = 240\pi$$

$$V(t) = \frac{120}{6} \sin(240\pi t) = 6$$

$$V(t) = 20 \sin(240\pi t) = 6$$

$$\begin{aligned} \sin(240\pi t) &= 0 \\ 240\pi t &= 0 \text{ or } \pi \text{ or } 2\pi \dots \\ 240t &= 0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \dots \end{aligned}$$

so..

$$\begin{aligned} V(t) &= 0 \text{ when} \\ t &= 0 \text{ or } t = \frac{1}{240} \text{ or } t = \frac{2}{240} \dots \end{aligned}$$

$$t = \frac{k}{240}, k \in \mathbb{N}$$

\textcircled{q}

$$\text{EMF} = L \frac{\Delta I}{\Delta t}$$

$$\Delta t = L \frac{\Delta I}{\text{EMF}}$$

$$= \frac{2 \times 10^{-3} (0.1)}{500}$$

$$= 4 \times 10^{-7} \text{ s}$$

$$\boxed{= 0.4 \mu\text{s}}$$

$$\textcircled{10} \text{ (a)} \quad \text{EMF} = L \frac{\Delta I}{\Delta t}$$

$$\text{EMF} = 25 \left(\frac{100}{80 \times 10^{-3}} \right)$$

$$= 3.13 \times 10^4 \text{ V}$$

$$\text{(b)} \quad E_{\text{ind.}} = \frac{1}{2} L I^2$$

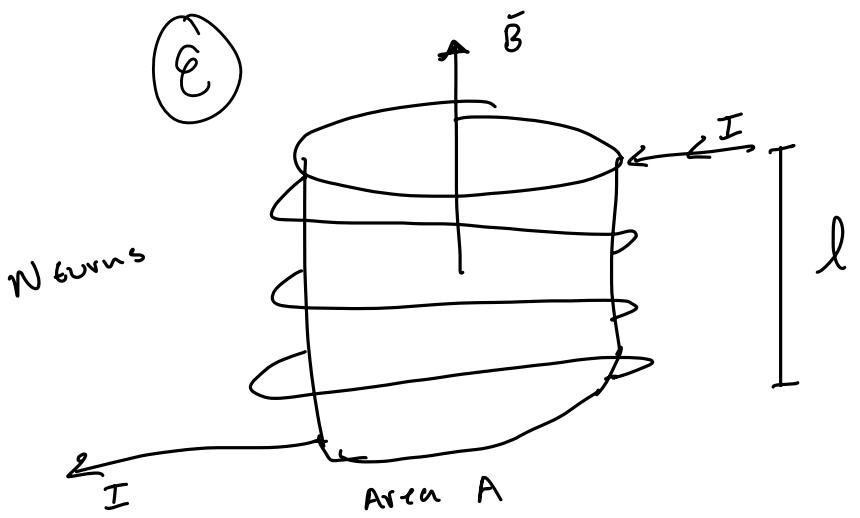
$$E = \frac{1}{2} (25) (100)^2$$

$$= 1.25 \times 10^5 \text{ J}$$

$$\textcircled{C} \quad P = \frac{1}{2} I^2 = \text{Watts}$$

$$= \frac{1.25 \times 10^5}{80 \times 10^{-3}} = 1.56 \times 10^6 \text{ W}$$

\textcircled{d} NO, it is a large value



$$\text{inductance} = 2S +$$

$$\text{turns} = N$$

$$\text{area} = A$$

$$\text{length} = l$$

⑦

$$R \cdot \nabla = L$$

$$5 \times 10^6 \left(20 \times 10^{-9} \right) = L$$

$$0.100 \text{ H} = L$$

$$\textcircled{b} \quad R = \frac{L}{\nabla} = \frac{0.1}{1 \times 10^{-9}} = 1 \times 10^8 \Omega$$

$$\textcircled{c} \quad I(t) = I_0 \cdot \left(1 - e^{-\frac{t}{T}} \right)$$

$$I(3 \text{ ms}) = I_0 \cdot \left(1 - e^{-\frac{3}{T}} \right)$$

$$I = I_0 \cdot 0.95$$

95% of I_0

$$\textcircled{d} \quad f = 10 \times 10^3 \text{ Hz}$$

$$L = 0.1 \text{ H}$$

$$X_L = 2\pi f L$$

$$X_L = 2\pi \times 10^4 \cdot 0.1$$

$$X_L = 6283.19 \Omega$$

of 20.0 ns, and a resistance of 5.00 M Ω . (a) What is the inductance of the circuit? (b) What resistance would give you a 1.00 ns time constant, perhaps needed for quick response in an oscilloscope? (c) Suppose you design for a 1.00 ns time constant. What percentage of the final current I_0 flows 3.0 ns after the circuit is closed? (d) What is the reactance of the inductor at 10 kHz?

14. (a) What are the resonance frequency, f_0 , and resonance width $\Delta f/f_0$ of an RLC circuit with $R = 0.1 \text{ k}\Omega$, $C = 1 \mu\text{F}$, and $L = 10 \text{ mH}$? (b) What is the total impedance of this circuit at $f = 0.1f_0$ and $f = 10f_0$?

15. For an RLC circuit with those values of R , L , and C from the previous problem, an AC source voltage characterized by $V_{\text{rms}} = 120 \text{ V}$, (a) what is I_{rms} at $10f_0$ and $0.1f_0$? (b) What is P_{rms} at these frequencies? Hint: calculate the phase angle between resistance and reactance.

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

12. An inductor designed to filter high frequency noise from power supplied to a personal computer is placed in series with the computer. What minimum inductance should it have to produce a 2.00 k Ω reactance for 15.0 kHz noise? (b) What is its reactance at 60.0 Hz?

16. Suppose we are using an LC resonator and diode combination to create an AM radio signal. If the resonance frequency of our resonator is 1.4 MHz, and our audio signal is at 10 kHz, (a) what three frequencies should be present in our AM radio spectrum? (b) Suppose our receiver was receiving only the carrier frequency, but neither modulation peak. Should we gradually increase or gradually decrease the resistance of the receiver circuit?

13. Consider Fig. 3. Assume the bottom wire is grounded at 0V. (a) Considering the capacitor reactance, describe why the circuit acts as a low-pass filter. That is, AC signals at v_{in} with high frequencies produce very small amplitudes at v_{out} . (b) Using Kirchhoff's loop rule on the left side of the circuit, write an equation that describes changes in potential. (c) Repeat exercise (b) for the right loop. (d) Solve parts (b) and (c) for v_{in} and v_{out} , respectively, and calculate $v_{\text{out}}/v_{\text{in}}$ in terms of R and C . (e) What is the value of $v_{\text{out}}/v_{\text{in}}$ for $f = 100(2\pi RC)^{-1}$? For $f = 0.1(2\pi RC)^{-1}$?

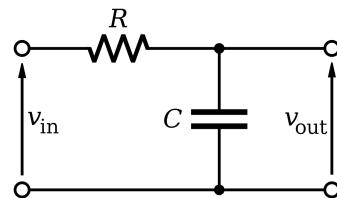


Figure 3: This RC circuit can act as a low-pass filter.

(12)

a) $L = ?$

$$X_L = 2\pi f L$$

$$X_L = 2 \times 10^3$$

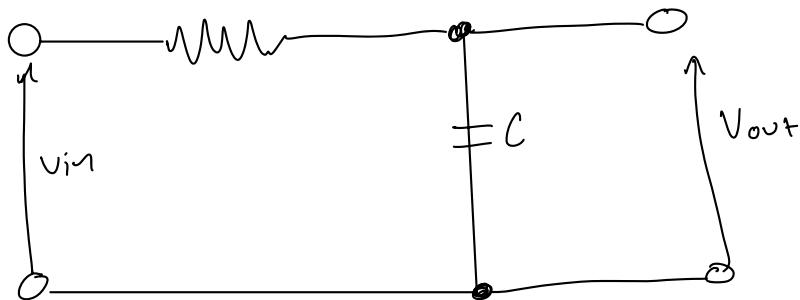
$$L = \frac{X_L}{2\pi f} = \frac{2 \times 10^3}{2\pi (15 \times 10^3)}$$

$$= \frac{2}{2\pi (15)} = 2.12 \times 10^{-2}$$

$$f = 15 \times 10^3$$

b) $X_L = 2\pi f L$
 $= 2\pi (60)(2.12 \times 10^{-2})$
 $X_L = 8.00 \Omega$

(13)



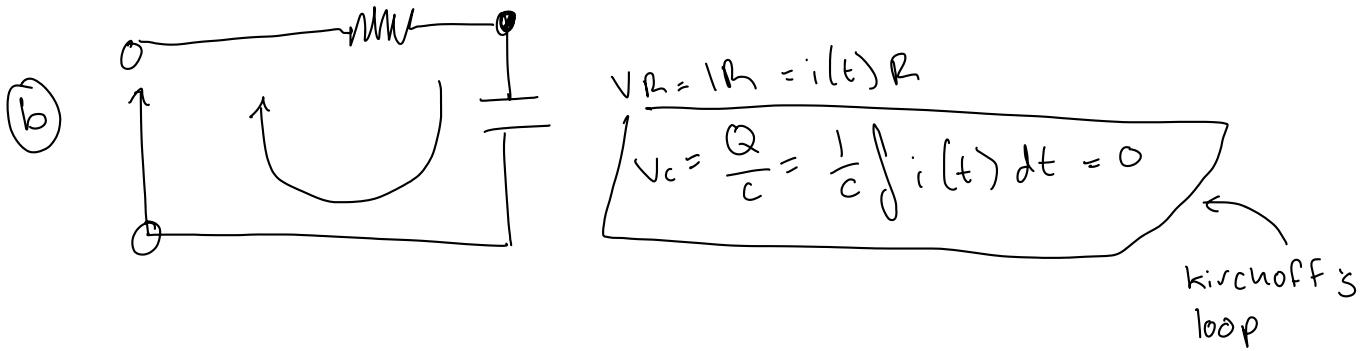
a) $V_{out} = \frac{X_C}{Z} V_{in} = \frac{X_C}{\sqrt{R^2 + (X_L - X_C)^2}} \cdot V_{in}$

Grounded $X_C = \frac{1}{2\pi f C}$ if $f \uparrow \rightarrow X_C \downarrow \rightarrow V_{out} \downarrow$

High freq Signals will pass through the capacitor

Low freq signals are blocked by the capacitor

Output



(c) Right hand

$$V_c = \frac{Q}{C} = \frac{1}{C} \int i(t) dt$$

so kirchoff's loop

$$V_{out} - \frac{1}{C} \int i(t) dt = 0$$

(d)

$$\frac{V_{out}}{V_{in}} = \frac{\frac{X_C}{\sqrt{R^2 + X_C^2}} \cdot V_{in}}{V_{in}}$$

$$\frac{V_{out}}{V_{in}} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \quad \text{but} \quad X_C = \frac{1}{2\pi f C}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{2\pi f C}}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}$$

(e)

$$\frac{V_{out}}{V_{in}} = ? \quad \text{when} \quad f = \frac{100}{2\pi R C}$$

$$\frac{V_{out}}{V_{in}} = \frac{\left(\frac{100}{2\pi R C}\right)}{\sqrt{R^2 + \left(\frac{100}{2\pi R C}\right)^2}} = \frac{\frac{R}{100}}{\sqrt{R^2 + \frac{R^2}{10000}}}$$

$$\frac{V_{out}}{V_{in}} = \frac{\frac{R}{100}}{\sqrt{R^2}} \approx \frac{1}{100}$$

$$\frac{V_{\text{out}}}{V_{\text{in}}} = ? \quad \text{when} \quad \frac{0.1}{2\pi R C}$$

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{\left(\frac{0.1}{2\pi R C} \right)}{\sqrt{R^2 + \left(\frac{0.1}{2\pi R C} \right)^2}} = \frac{\frac{0.1}{100}}{\sqrt{R^2 + \frac{0.1^2}{10000 R^2 C^2}}}$$

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{10 \cdot 0.1}{\sqrt{R^2 + (10 \cdot 0.1)^2}} = \frac{10 \cdot 0.1}{\sqrt{101}} \approx 1$$

(14)

$$\textcircled{a} \quad f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{10 \times 10^{-3} \text{H} \cdot 1 \times 10^{-6} \text{F}}} = 1.6 \times 10^{-3} \text{ kHz}$$

$$L = 10 \times 10^{-3} \text{ H}$$

$$C = 1 \times 10^{-6} \text{ F}$$

$$R = 0.1 \times 10^3 \Omega$$

$$\omega = f_0 \cdot \frac{L}{R} = \frac{f_0}{\Delta f}$$

$$\frac{\Delta f}{f_0} = \frac{R}{f_0 \cdot L} = \frac{0.1 \times 10^3}{0.16 \times 10^{-3} \cdot 10^{-3}} = 62,893.08$$

\textcircled{b}

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_C = \frac{1}{2\pi f C} = 159,154 \cdot \frac{1}{f}$$

$$X_L = \frac{1}{2\pi f L} = 15.92 \cdot \frac{1}{f}$$

$$f = 0.1 f_0 = 1.6 \times 10^{-3} \cdot 1 \times 10^{-4} = 1.6 \times 10^{-4} \rightarrow X_C = 9.947 \times 10^8$$

$$f = 10 f_0 = 1.6 \times 10^{-3} \cdot 1 \times 10^1 = 1.6 \times 10^{-2} \rightarrow X_L = 9.947 \times 10^6$$

$$Z = \sqrt{(0.1 \times 10^3)^2 + (995 - 9.947 \times 10^6)^2} = 9.946 \times 10^6 \quad @ \quad 10f_0$$

$$Z = \sqrt{(0.1 \times 10^3)^2 + (99,500 - 9.947 \times 10^8)^2} = 9.946 \times 10^6 \quad @ \quad 0.1f_0$$

(15)

$$\textcircled{a} \quad I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}$$

$$V_{\text{rms}} = 120V$$

$$Z @ 10f_0 = 9.946 \times 10^6$$

$$Z @ 0.1f_0 = 9.946 \times 10^8$$

$$\frac{120V}{9.946 \times 10^6} = 1.2065 \times 10^{-5}$$

$$\frac{120V}{9.946 \times 10^8} = 1.2065 \times 10^{-7}$$

(b)

$$P_{\text{rms}} = V_{\text{rms}} \times I_{\text{rms}} \times \cos(\phi)$$

$$\phi = \tan^{-1} \left(\frac{x_L - x_C}{R} \right)$$

(1b)

(a)

$$f_C = 1.4 \text{ MHz}$$

$$f_A = 10 \text{ kHz} = 0.01 \text{ MHz}$$

$$\text{Carrier freq } f_C = 1.4 \text{ MHz}$$

$$\text{Upper sideband } f_C + f_A = 1.4 \text{ MHz} + 0.01 \text{ MHz} = 1.41 \text{ MHz}$$

$$\text{Lower sideband } f_C - f_A = 1.4 \text{ MHz} - 0.01 \text{ MHz} = 1.39 \text{ MHz}$$

(b) gradually decreased. It will help broaden the band width of the receiver allowing it to capture the modulation peaks $\frac{1}{3}$ the carrier freq

2 Unit 5: Waves, Optics, Medical Physics

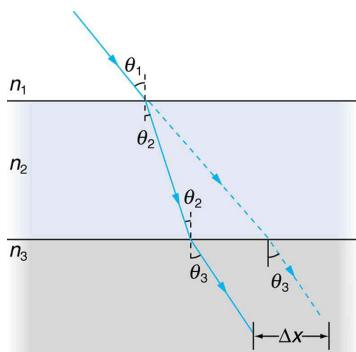


Figure 4: A light ray is in a medium with n_1 , then enters a medium with n_2 , then a medium with n_3 .

3. (a) A microwave generates 1 kW of power, projected onto a 10 cm-squared area at a distance of 1 m. What is the intensity (power per unit area)? (b) How long does it take the energy to travel the 1 meter? (c) What is the peak E-field at the source? **Bonus.** (d) If the intensity drops as the distance squared, what is the intensity 2 meters from the microwave?
1. (a) What is the B-field generated 1 cm laterally from a capacitor in an RC circuit that charges from 0 to 200 nC in 2 μ s? (b) What current is responsible for this B-field, since the capacitor is not passing DC current like a wire?
2. (a) An aircraft receives a pulse reflected from glacial ice below it 10 μ s after transmission. What is the displacement between aircraft and ice? (b) Suppose the main frequency of the radar is $f = 100$ MHz. What is the smallest (approximate) change in glacier height the radar operator can observe? (c) What fraction of the radar power is reflected from the ice, assuming normal incidence? **Bonus.** (d) Suppose the transmitted power at 1 meter is 100 W, and decreases as the inverse of the displacement squared. What is the received power at the aircraft, accounting for distance and reflection?
4. Consider Fig. 4. (a) Show that $\theta_3 = \theta_1$, if $n_1 = n_3$. This proof is the reason why the center of our lenses (like those in our glasses) do not change ray directions. If our glasses changed the apparent location of objects, they would be useless. (b) A doctor examines a child's ear with a 15.0 cm focal length magnifying glass held 13.5 cm from the ear. (c) Where is the image? (d) What is its magnification? (e) How big is the image of a 1.0 cm diameter ear hole?
5. (a) What is the ratio of the speed of light in solid ice versus snow? You may assume fresh snow has the same properties as water for this exercise. (b) If a radio wave transitions from snow to ice with

$$\textcircled{1} \quad B = \frac{\mu_0 I}{2\pi r} = \frac{12.57 \times 10^{-7} \frac{\text{H}}{\text{m}} \left(\frac{100 \text{nC}}{\text{ms}} \right)}{2\pi (0.8 \text{ cm})}$$

$$B = 21.0 \times 10^{-5} \text{ mT}$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{200 \text{nC}}{2 \text{ ms}} = \frac{100 \text{nC}}{\text{ms}}$$

$$\textcircled{2} \quad \text{(a)} \quad v = \frac{c}{n} = \frac{2.997 \times 10^8 \text{ m/s}}{1.0 \times 10^{-6} \text{ s}} \approx 3000 \text{ m/s}$$

$$\frac{3000 \text{ m}}{2} = 1500 \text{ m one way}$$

$$\text{(b)} \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{100 \times 10^6 \text{ Hz}} = 3 \text{ m}$$

$$\text{resolution} = \frac{3 \text{ m}}{2} = 1.5 \text{ m}$$

$$\text{(c)} \quad R_i = \left| \frac{(E-1)^2}{E+1} \right| \approx \left| \frac{3.2-1}{3.2+1} \right|^2 = 0.275$$

↑ smaller & observable Δ

$$\text{(d)} \quad P = P_0 \times \frac{1}{A^2} = 100 \text{ W} \cdot \frac{1}{1500^2} = 0.0000444 \text{ W} \cdot 0.27$$

$$= 12.2 \text{ NW}$$

$$\textcircled{3} \quad \text{(a)} \quad \text{Intensity} = \frac{\text{Power}}{\text{Area}} = \frac{1 \text{ kW}}{4\pi (1)^2}$$

$$= \frac{10^3}{4\pi \text{ m}^2} = \boxed{79.58 \frac{\text{W}}{\text{m}^2}}$$

$$\text{(b)} \quad t = 1 \text{ min} = 60 \text{ seconds}$$

$$\text{(c)} \quad E = IAt = 79.58 \frac{\text{W}}{\text{m}^2} \cdot 10^{-2} \text{ m}^2 \cdot 60 \text{ s} = 217.75 \text{ J}$$

$$A = 100 \times (10^{-2} \text{ m})^2 = 10^{-2} \text{ m}^2$$

$$\text{(d)} \quad \text{Intensity} = \frac{1 \text{ kW}}{4\pi \cdot (2)^2} = \frac{10^3 \text{ W}}{21\pi \cdot 4 \text{ m}^2} = 19.89 \frac{\text{W}}{\text{m}^2}$$

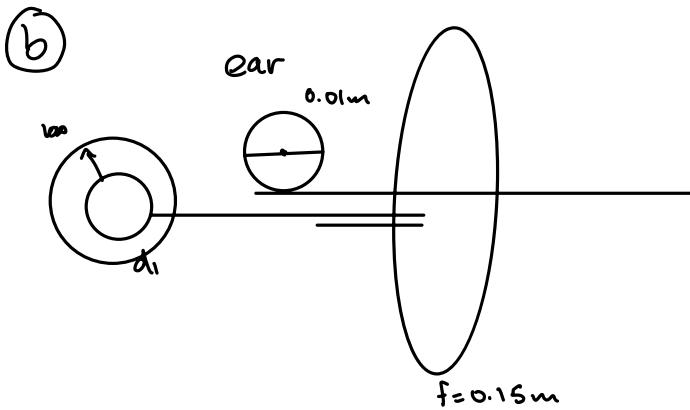
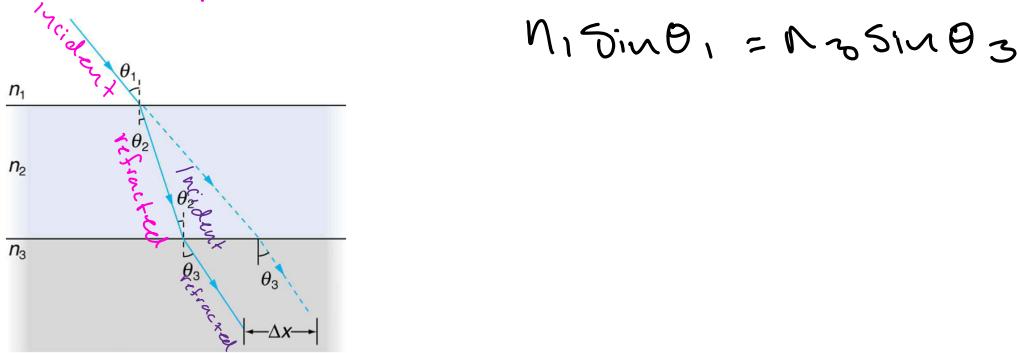
So the intensity dropped

$\textcircled{4} \quad \text{(a)} \quad \text{incident refracted}$

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Incident refracted

$$n_2 \sin \theta_2 = n_3 \sin \theta_3$$



$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$d_i = \frac{1}{\frac{1}{f} - \frac{1}{d}}$$

$$d_i = \frac{1}{\frac{1}{0.15} - \frac{1}{0.135}}$$

$$d_i = -1.35 \text{ m}$$

on same side

(c) $-\frac{d_i}{d_o} = \text{magnification}$

$$\frac{-(-1.35)}{0.135} = m$$

$$10 = m$$

(d) It'd be 10x bigger

~~an initial angle of 30 degrees from normal, what is the transmitted angle?~~

9. **Bonus:** Some nuclear isotopes emit *free neutrons*, neutrons that then propagate outside the nucleus. These neutrons are useful for controlling nuclear fission for power production, and have a half-life of 611 ± 1 seconds in free space. If emitted in free space, what fraction of the neutrons remain after 1 hour?

8. (a) Combine thin lens equations to show that the magnification for a thin lens is determined by its focal length and the object distance and is given by $m = f/(f-d_0)$. (b) Using this equation, determine what conditions lead to $m \rightarrow \infty$. (c) What happens to the image location?¹

10. (a) What is the radioactive dose in rads if 250 mJ of total energy is deposited in the body of a 60 kg adult? (b) What instead would be the dose if the radiation was concentrated in just 2.0 kg of tissue? (c) Assuming the RBE is 1 (x-rays or γ rays), what is the dose from part (b) in Sv? (d) Does this dose represent a serious health risk?

7. **Bonus:** Suppose x rays have a cross section of 200 barns at 10 keV in a lead vest protecting a medical patient. (a) If the vest is 1 cm thick, what fraction of the x-rays pass through it? (b) At what depth in the vest are half of the x-rays gone?

8. **Bonus:** In the previous problem, at what distance would half of the photons be gone if they were γ rays instead of x-rays, with a cross section of 1 barn? Hint: scaling problem.

¹This is why we cannot use lenses to create infinitely large images for free.

(6)

(a)

$$\frac{-d_i}{d_o} = m \quad \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$\frac{1}{d_o} - \frac{1}{m d_o} = \frac{1}{f}$$

$$\frac{1}{d_o} - \frac{1}{f} = \frac{1}{m d_o}$$

$$\frac{f}{fd_o} - \frac{d_o}{fd_o} = \frac{1}{m d_o} \rightarrow \frac{f-d_o}{fd_o} = \frac{1}{m d_o} \rightarrow \frac{fd_o}{f-d_o} = m d_o \rightarrow \frac{fd_o}{(f-d_o) d_o} = m$$

$$\boxed{\frac{f}{f-d_o} = m}$$

(b) For $M \rightarrow \infty$ the focal length would have to be significantly larger than the object distance. However, the d_o would still need to be large to make the denominator smaller.

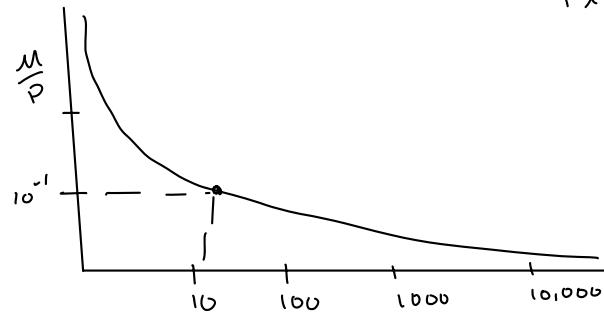
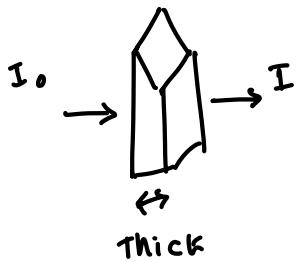
(c) If magnification approaches infinity it means that the image distance will approach 0 making the image very far away.

(7)

(a)

$$I = I_0 \cdot e^{-\mu x}$$

linear attenuation
thickness of material



$$\rho_{lead} = 11.34 \text{ g/cm}^3$$

$$\frac{\mu}{P} = \frac{\text{cm}^2}{\text{g}}$$

$$\mu = \frac{\mu}{P} \times P = 10^{-1} \times 10.34 \text{ cm}^2/\text{g} \cdot \frac{\text{g}}{\text{cm}^3} \cdot \frac{\text{cm}^3}{\text{cm}^{-1}}$$

$$\boxed{\mu = 1.034 \text{ cm}^{-1}}$$

$$\text{Fraction} = e^{-\mu x} = e^{-1.034 \times 10} = 0.355 = 35.5\%$$

(b)

$$e^{-\mu x} = \frac{1}{2}$$

$$e^{-1.034 x} = 0.5$$

$$\rightarrow -1.034 x = \ln(0.5) \rightarrow x = \frac{-\ln(0.5)}{1.034} = \boxed{0.67 \text{ cm}}$$

7

a

$$\mu_x = 1.034 \text{ cm}^{-1}$$

$$n_y = \frac{\sigma_x}{\rho} \cdot \frac{\sigma_x}{\sigma_x}$$

↓

$$\mu_x \cdot \frac{\sigma_x}{\sigma_x} = 1.034 \cdot \frac{1}{200}$$

scaling

$$n_y = \frac{1.034}{200}$$

b

Distance which half are gone

$$e^{-\mu_y x} = \frac{1}{2}$$

$$x = \frac{\ln(1/2)}{\frac{1.034}{200}} = \boxed{134.07 \text{ cm}}$$

9

$$N = N_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{\text{half time}}}$$

$$N = N_0 \times \left(\frac{1}{2}\right)^{\frac{611}{1\text{hr}}}$$

$$N = N_0 \times \left(\frac{1}{2}\right)^{\frac{611}{3600}}$$

$$N = 0.488 \times N_0$$

After 1 hr it remains 48 %

10

②

$$200 \text{ mJ} = 0.25 \text{ J}$$

$$\text{mass} = 60 \text{ kg}$$

$$0.25 \text{ J}$$

$$\text{Dose in rads} = \frac{0.25 \text{ J}}{60 \text{ kg}} \times 100$$

$$= 0.417 \text{ rads}$$

(b)

$$\frac{0.25 \text{ J}}{2 \text{ kg}} = 0.125 \times 100 = 12.5 \text{ rads}$$

(c) to SV

$$= 12.5 \text{ rads} \times \text{RBE} = 12.5 \text{ rads} \times 1 = 12.5 \text{ Sv}$$

(d) 12.5 Sv is high in a health risk