Midterm 3

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1 Memory Bank

1. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
2. $P = IV$ ... Relationship between power, current, and voltage.
3. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
4. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.
5. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ... Ampère's Law.
6. $\epsilon = -N d\phi/dt$ ... Faraday's Law.
7. $\phi = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux.
8. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
9. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L. Self-inductance: $\Delta V = -L(dI/dt)$.
10. Units of inductance: V s A^{-1}, which is called a Henry, or H.
11. $B = \mu_0 n I$ ... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

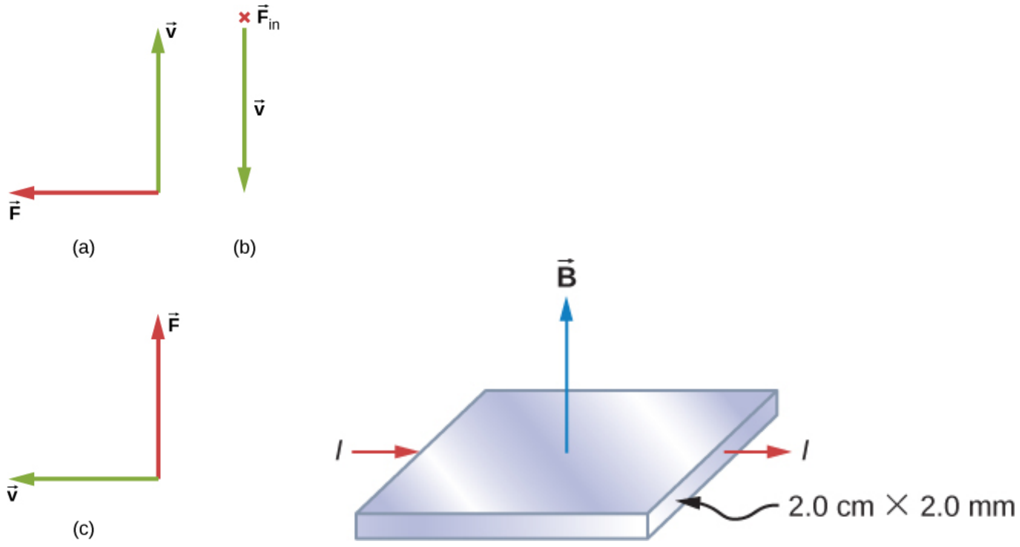

Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.
   - a: into the page
   - b: left
   - c: out of the page

2. Consider Fig. 1 (right). **The Hall Effect.** An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that

$$\Delta V = \frac{B\Delta x I}{nq_e A} \tag{1}$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28}$ m^{-3}, $A = 1$ mm^2, and q_e is the charge of an electron.

(a) $F_B = qvB\sin\theta$

$F_B = qvB$

$F_E = qE$

$qvB = qE$

$vB = E$

$v = \frac{E}{B}$

(b) $E = \frac{\Delta V}{\Delta x}$

$\Delta V = \Delta x E$

$\Delta V = B\Delta x v$

$\Delta V = \frac{B\Delta x I}{nq_e A}$

$\Delta V = \frac{(1.33)(0.02)(10)}{(2\times10^{28})(1.60\times10^{-19})(1\times10^{-6})}$

$\boxed{\Delta V = 8.3125 \times 10^{-5}\ \text{V}}$

$I = nq_e A v$

$v = \frac{I}{nq_e A}$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^4 A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

$\tau = NIAB\sin\theta$

$A = \pi r^2$

$A = \pi(0.65\times10^{-15})^2 = 1.327\times10^{-30}$

$\tau = (1)(1.05\times10^4)(1.327\times10^{-30})(2.50)\sin 90°$

$\boxed{\tau = 3.484\times10^{-26}\ \text{N}\cdot\text{m}}$

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

(a) $\oint B\cdot dl = \mu_0 I_{enc}$

$Bl = \mu_0 I_{enc}$

$B = \frac{\mu_0 I_{enc}}{l}$

$B = \mu_0 n I_{enc}$

$B = (4\pi\times10^{-7})(500)(0.3) = \boxed{1.884\times10^{-4}\ \text{T}}$

(b) $\mu = 5000\mu_0$

$B = 5000(4\pi\times10^{-7})(500)(0.3)$

$\boxed{B = 0.942\ \text{T}}$

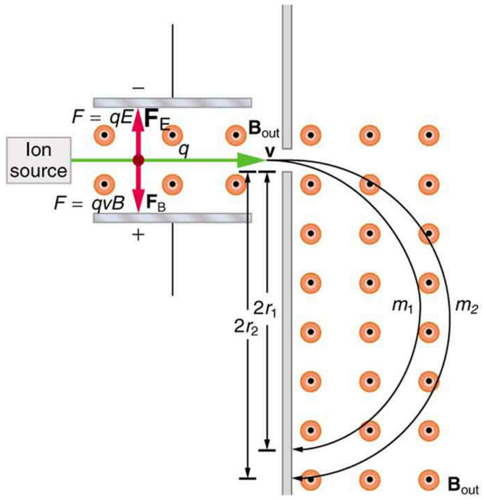

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \tag{2}$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r?

(a) $F_e = qE$

$F_m = q(v \times B)$

$F_{net} = q(E + vB) = 0$

$E = vB$

$v = \frac{E}{B}$

(b) $qvB = \frac{mv^2}{r}$

$r = \frac{mv}{qB}$

$r = \frac{m}{qB} \cdot \frac{E}{B}$

$r = \frac{mE}{qB^2}$

$r = \frac{mE}{qB^2}$

$r = \frac{(16)(1.67 \times 10^{-27})(10)}{(1.602 \times 10^{-19})(0.01)^2}$

$r = 0.0167$ m

4 Chapter 13: Electromagnetic Induction

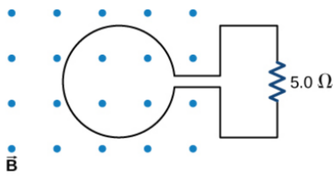

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0}(\sin(2\pi f t)) \tag{3}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1?

(a) $e = A \frac{dB}{dt}$

$e = \pi r^2 \times \frac{B_0}{T_0} \sin(2\pi f t)$

(b) $e = \pi r^2 \times \frac{B_0}{T_0} \sin(2\pi f t)$

$e = \pi(0.1)^2 \times \frac{0.1}{1 \times 10^{-3}} \sin(2\pi(10^3)(0))$

$e = 0$ V

(c) $e = \pi r^2 \times \frac{B_0}{T_0} \sin(2\pi f t)$

$e = \pi(0.1)^2 \times \frac{0.1}{1 \times 10^{-3}} \sin(2\pi(10^3)(0.16 \times 10^{-3}))$

$e = 0.055$ V

(d) $I = \frac{e}{R}$

$I = \frac{0.055}{5}$

$I = 0.011$ A

[1]Molecules that do not have this velocity will hit the sides of this portion of the instrument.

5 Chapter 14: Inductance

1. What is (a) the rate at which the current though a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

$\text{emf} = -M \frac{dI}{dt}$

$\varepsilon = -M \frac{dI}{dt}$

$\frac{dI}{dt} = -\frac{\varepsilon}{M}$

$\frac{dI}{dt} = -\frac{0.150}{0.50}$

$\frac{dI}{dt} = -0.3 \text{ A/s}$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$\text{emf} = -M \frac{dI}{dt}$

$\varepsilon = -M \frac{dI}{dt}$

$dt = -M \frac{dI}{\varepsilon}$

$dt = -(2.00 \times 10^{-3}) \frac{(0.100)}{500}$

$dt = 4.00 \times 10^{-7} \text{ s}$