Study Guide for Algebra-Based Physics-1: Mechanics (PHYS135A-01)

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1 Definition of Momentum

1. Remember that the units of momentum are **kg m/s**.
2. There are two requirements for momentum to be conserved. First, there must be *no net external force*. Second, the *masses of the particles cannot change*.
3. In vector form, the momentum is $\vec{p} = m\vec{v}$.

2 Conservation of Momentum

1. If momentum is conserved, the total initial momentum is equal to the total final momentum. In vector form, $\vec{P}_{\mathrm{f}} = \vec{P}_{\mathrm{i}}$. If the interaction is a $2 \rightarrow 2$ type interaction, then $\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$, where the prime notation indicates the final state.
2. Another way to state the conservation of momentum is $\frac{d\vec{p}}{dt} = 0$. Since force is the derivative of momentum, this is equivalent to saying that there is no net external force.

3 Classifying Interactions

1. An *elastic interaction* is one in which kinetic energy is conserved as well as momentum.
2. An *inelastic interaction* is one in which kinetic energy is not conserved, but momentum is still conserved.
3. *Elastic interactions* are usually of type $n \rightarrow n$, and *inelastic interactions* are usually of type $n \rightarrow 1$.
4. **Example problem**: A 1 kg particle has $v_1 = -1$ m/s, and it interacts with a 1 kg particle with velocity $v_2 = 1$ m/s. If the collision is elastic, what is the final velocity of each particle?

   Momentum is conserved, so let's write:
   $\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2'$
   $m\vec{v}_1 + m\vec{v}_2 = m\vec{v}_1' + m\vec{v}_2'$
   $-1 + 1 = 0 = \vec{v}_1' + \vec{v}_2'$
   $\vec{v}_1' = -\vec{v}_2'$
   We cannot procede without adding additional facts. However, we know that this is an elastic interaction, meaning that kinetic energy is conserved:
   $\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$
   $v_1^2 + v_2^2 = v_1'^2 + v_2'^2$
   $2 = v_1'^2 + v_2'^2$
   Putting in the conclusion from momentum conservation, that the final velocities are equal and opposite, we find that the magnitude of each is 1 m/s. Thus, $v_1' = 1$ m/s and $v_2' = -1$ m/s.
   This is intuitive, right? They have equal mass and equal speed so they just bounce off of each other and go the other way.