Algebra-Based Physics: Electricity, Magnetism, and Modern Physics (PHYS135B): Unit 2

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Summary

Unit 2 Summary

Reading: Chapters 21.1 - 21.4, 21.6

1. Resistors in series and parallel
2. Electromotive force (EMF) and terminal voltage
3. Kirchhoff's rules
4. Voltmeters and ammeters
5. RC circuits

Resistors in Series and Parallel

Series and parallel resistances change how current and power flow in a circuit.

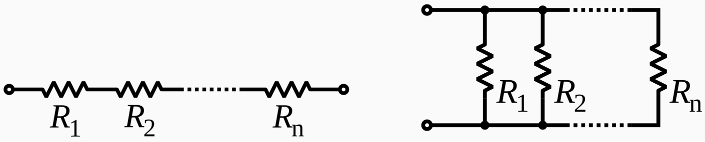


Figure 1: (Left) Resistors in series. (Right) Resistors in parallel.

Resistors in Series and Parallel

Series resistances:

- Share one current
- Represent different voltage changes

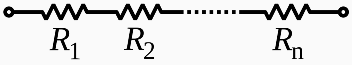


Figure 2: Resistors in series.

$$q\Delta V_{\mathrm{tot}} = q\Delta V_1 + q\Delta V_2 + \ldots + q\Delta V_{\mathrm{n}} \tag{1}$$

$$\Delta V_{\mathrm{tot}} = \Delta V_1 + \Delta V_2 + \ldots + \Delta V_{\mathrm{n}} \tag{2}$$

$$IR_{\mathrm{tot}} = IR_1 + IR_2 + \ldots + IR_{\mathrm{n}} \tag{3}$$

$$R_{\mathrm{tot}} = R_1 + R_2 + \ldots + R_{\mathrm{n}} \tag{4}$$

Resistors in Series and Parallel

Parallel resistances:

- Share one voltage
- Represent different currents

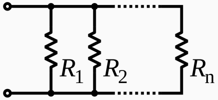


Figure 3: Resistors in parallel.

$$I_{\mathrm{tot}} = I_1 + I_2 + ... + I_{\mathrm{n}} \tag{5}$$

$$\frac{\Delta V}{R_{\mathrm{tot}}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} + ... + \frac{\Delta V}{R_{\mathrm{n}}} \tag{6}$$

$$\frac{1}{R_{\mathrm{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_{\mathrm{n}}} \tag{7}$$

Resistors in Series and Parallel

Series resistances:

$$\boxed{R_{\mathrm{tot}} = R_1 + R_2 + ... + R_{\mathrm{n}}} \tag{8}$$

Parallel resistances:

$$\boxed{\frac{1}{R_{\mathrm{tot}}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_{\mathrm{n}}}} \tag{9}$$

Two 50Ω resistors are connected in *series*. What is the total resistance? Two 50Ω resistors are connected in *parallel*, What is the total resistance?

- A: 25Ω
- B: 50Ω
- C: 100Ω
- D: 150Ω

Resistors in Series and Parallel

A 1 kΩ resistor and a 0.1 kΩ resistor are connected in *parallel*. What is the total resistance?

- A: 1.1 kΩ
- B: 0.091 kΩ
- C: 1 kΩ
- D: 0.1 kΩ

A 1 kΩ resistor and a 0.1 kΩ resistor are connected in *parallel*. Which resistor has the higher current? If the voltage is 12V, what is the larger current?

- A: Smaller, 120 mA
- B: Larger, 12 mA
- C: Smaller, 12 mA
- D: Larger, 120 mA

Hint: draw the circuit if you are not sure.

Parallel resistances make sense in circuits with multiple devices requiring the same voltage:

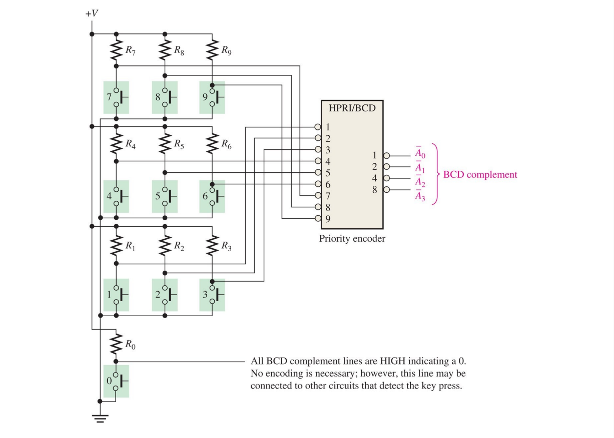


Figure 4: Resistors in parallel arranged in a keypad system.

PhET Activity: Series and Parallel Resistors

PhET Activity: DC Circuits and Ohm's Law

How do we deal with complex circuits?

1. Create a circuit that involves four identical resistors *in series*.
2. Use an ammeter to show that the current is the same in all resistors.
3. Calculate the *effective total resistance* by plotting an $i - V$ curve of the system. Measure i and V by changing the voltage and using the voltmeter and ammeter. Does the slope of the $i - V$ curve match your expectation?
4. Create a circuit that involves four identical resistors *in parallel*.
5. Use the voltmeter to show that the change in voltage is the same across all resistors.
6. Calculate the *effective total resistance* by plotting an $i - V$ curve of the system. Measure i and V by changing the voltage and using the voltmeter and ammeter. Does the slope of the $i - V$ curve match your expectation?

Electromotive Force (EMF) and Terminal Voltage

Electromotive Force (EMF) and Terminal Voltage

The electromotive force (EMF) of a battery is the potential difference that induces current through the circuit. It is *almost* the same as the battery voltage.

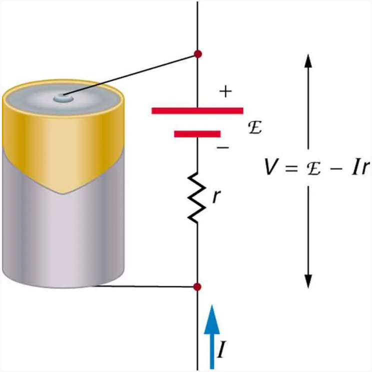


Figure 5: *Internal resistance* is one internal property of a battery.

Electromotive Force (EMF) and Terminal Voltage

The electromotive force (EMF) of a battery is the potential difference that induces current through the circuit.

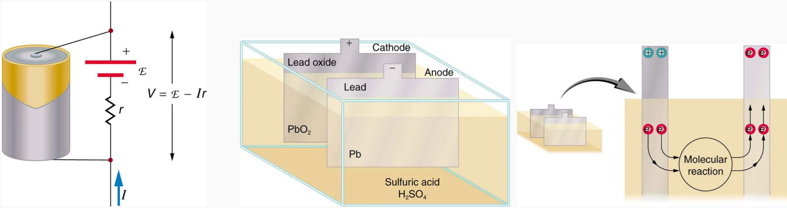


Figure 6: *Internal resistance* arises from battery chemistry, designed to pull electrons from the cathode to the anode before terminal connection.

$$V_{\text{out}} = \mathcal{E} - Ir \tag{10}$$

Electromotive Force (EMF) and Terminal Voltage

The electromotive force (EMF) of a battery is the potential difference that induces current through the circuit.

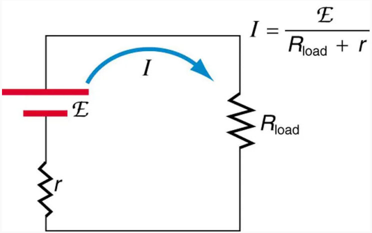


Figure 7: *Internal resistance* is one internal property of a battery.

$$I = \frac{\mathcal{E}}{R_{\mathrm{load}} + r} \tag{11}$$

Reduces to Ohm's law if r is insignificant.

If we treat the internal resistance of a 3.3 V battery as 0 Ω, and we connect $R_{\mathrm{load}} = 100\Omega$ in a simple circuit, what will be the current?

- A: 3.3 A
- B: 3.3 mA
- C: 33 mA
- D: 33 A

Hint: draw the circuit if you are not sure.

Internal resistance tends to rise if the battery is depleted of chemical energy. If we treat the internal resistance of a 3.3 V battery as 10 Ω, and we connect $R_{\mathrm{load}} = 100\Omega$ in a simple circuit, what will be the current?

- A: 3.3 A
- B: 3.3 mA
- C: 30 mA
- D: 30 A

Hint: draw the circuit if you are not sure.

Electromotive Force (EMF) and Terminal Voltage

Suppose we have a device that operates at 9V, and we have a 9V battery with $r = 1\Omega$. If we connect a 9Ω load resistance, what will be the terminal voltage? Will the device operate normally?

- A: 8.1V
- B: 8V
- C: 9.1V
- D: 9V

Will the device operate normally?

In the previous problem, what fraction of the power is disspated in the internal resistor?

Hint: think of dissipated power as $P = I^2R$ and $P = I^2r$ for the load and internal resistance, respectively.

- A: 5%
- B: 10%
- C: 1%
- D: 11%

Electromotive Force (EMF) and Terminal Voltage

Connecting real batteries in series raises the effective terminal voltage, but there are now two internal resistances.

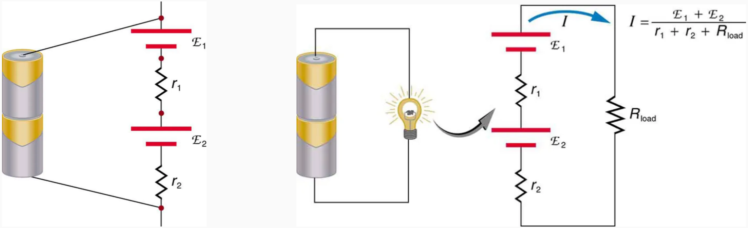


Figure 8: (Left) Two batteries in series. (Right) Two batteries in series connected to a load resistance.

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R_{\text{load}} + r_1 + r_2} \tag{12}$$

Electromotive Force (EMF) and Terminal Voltage

Suppose we have a flashlight powered by four AA batteries *in series*. The EMF of each AA battery is 1.5V. The internal resistance is $r = 0.5\Omega$ for each battery. The load resistance of the flashlight bulb is 30Ω. What is the current flow?

- A: 1.67 A
- B: 0.167 A
- C: 0.2 A
- D: 2.0 A

Remember to draw a picture of the whole circuit if you are stuck.

Electromotive Force (EMF) and Terminal Voltage

Suppose we have a flashlight powered by four AA batteries *in series.* The EMF of each AA battery is 1.5V. The internal resistance is $r = 0.5\Omega$ for each battery. The load resistance of the flashlight bulb is 30Ω. What is the terminal voltage?

- A: 6.0 V
- B: 1.5 V
- C: 3.33 V
- D: 5.67 V

Remember to draw a picture of the whole circuit if you are stuck.

Kirchhoff’s Rules

Kirchhoff's Rules

Kirchhoff's Rules

The following two rules govern complex circuits:

- The sum of all currents entering a junction must equal the sum of all currents leaving the junction: $\sum_n I_{\mathrm{n,in}} = \sum_n I_{\mathrm{n,out}}$.
- The algebraic sum of changes in potential around any closed loop must be zero: $\sum_{n,loop} V_n = 0$.

- The first rule follows from charge conservation, and is called the *junction rule*.
- The second rule follows from energy conservation, and is called the *loop rule*.

Kirchhoff's rules help organize calculations for complex circuits.

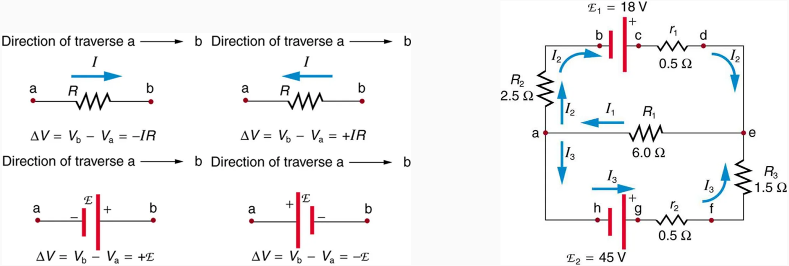


Figure 9: (Left) A change in voltage across a resistor is negative in the direction of current flow. The change is positive if aligned with the polarity of the battery. (Right) The complex circuit has two loops, three load resistors, two batteries, and two internal resistances.

Kirchhoff's rules help organize calculations for complex circuits.

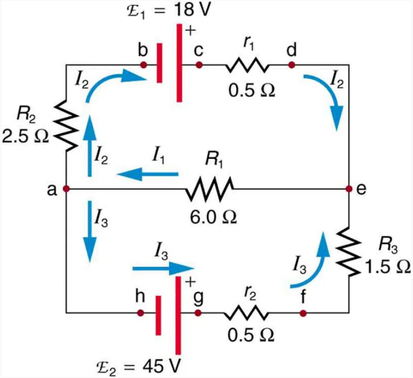
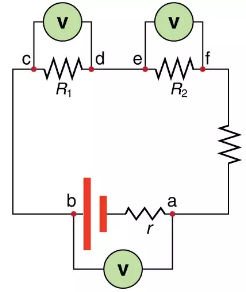


Figure 10: (a) Solve this one in small groups. (b) Model this in DC Circuit Constructor PhET.

Voltmeters and Ammeters

Voltmeters and Ammeters

A galvanometer is a device that deflects a needle based on the strength of a current. Galvanometers can be used to create *voltmeters* and *ammeters*.



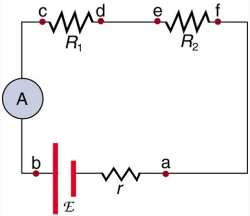


Figure 11: (Left) A voltmeter is connected in parallel. (Right) An ammeter is connected in series. These connections are necessary to make the measurements.

Voltmeters and Ammeters

A galvanometer is a device that deflects a needle based on the strength of a current. This effect is caused by the force of a magnetic field on a current-carrying wire.

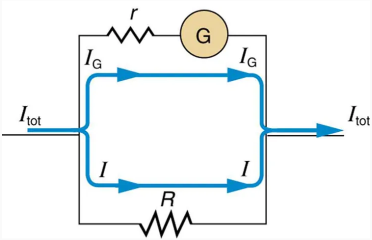
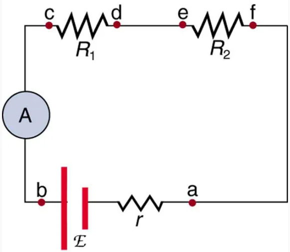


Figure 12: (Left) A galvonometer (G) within a voltmeter is protected by a large resistor, R. (Right) A galvonometer (G) within an ammeter is protected by a large resistor, R. Each G has an internal resistance, r. The value of R also calibrates the measurement in both situations.

Suppose the ammeter below has an internal resistance $r = 10\Omega$, and a "shunt" resistance of $R = 0.1\Omega$, $R_1 = 1\ \text{k}\Omega$, $R_2 = 1\ \text{k}\Omega$, and the battery has $\mathcal{E} = 12\text{V}$ and an internal resistance of $r_b = 1\Omega$.



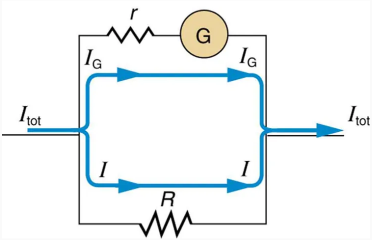


Figure 13: (Left) An ammeter connceted in series with loads represented by R_1 and R_2, and a battery with emf $\mathcal{E}$ and internal resistance r_b. (Right) The internal structure of the ammeter, with galvanometer G, internal resistance r and shunt resistance R.

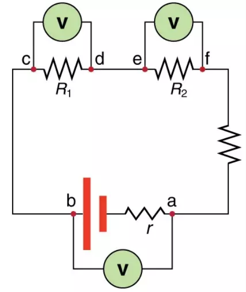
Suppose the ammeter below has an internal resistance $r = 10\Omega$, and a "shunt" resistance of $R = 0.1\Omega$, $R_1 = 0.5$ kΩ, $R_2 = 0.5$ kΩ, and the battery has $\mathcal{E} = 12$V and an internal resitance of $r_b = 1\Omega$.

1. Calculate the total resistance of the ammeter, R_{A}.
2. Calculate the total current flow from the battery, I_{total}.
3. What is the decrease in voltage across the ammeter?
4. What is the current flow to G, the galvanometer, I_{G}?

Solve each of these exercises in small groups. *Does the shunt resistor draw more current than G?* After we have solutions, we can model the current flow in the DC Circuits PhET.

Voltmeters and Ammeters

Suppose the voltmeter below has an internal resistance $r = 10\Omega$, and a "blocking" resistance of $R = 100$ kΩ, $R_1 = 0.5$ kΩ, $R_2 = 0.5$ kΩ, and the battery has $\mathcal{E} = 12$V and an internal resitance of $r_b = 1\Omega$.



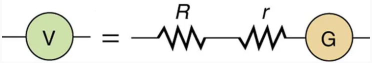


Figure 14: (Left) The voltmeter probes spots in the DC circuit. Assume there is no third resistor after R_2. (Right) The voltmeter has a galvanometer, G, a blocking resistor R, and internal resistance r.

Voltmeters and Ammeters

Suppose the voltmeter below has an internal resistance $r = 10\Omega$, and a "blocking" resistance of $R = 100\ \mathrm{k}\Omega$, $R_1 = 0.5\ \mathrm{k}\Omega$, $R_2 = 0.5\ \mathrm{k}\Omega$, and the battery has $\mathcal{E} = 12\mathrm{V}$ and an internal resitance of $r_b = 1\Omega$.

1. What is the effective resistance of the voltmeter?
2. What is the resistance of the voltmeter combined in parallel with R_1?
3. What is the current flow, I, from the battery?
4. What is the voltage decrease across the R_1 plus voltmeter complex?
5. What is the current flow to G, the galvanometer, I_{G}?

Solve each of these exercises in small groups. *Does the shunt resistor draw more current than G?* After we have solutions, we can model the current flow in the DC Circuits PhET.

RC Circuits

An RC circuit is a simply connected circuit that has resistance and capacitance.

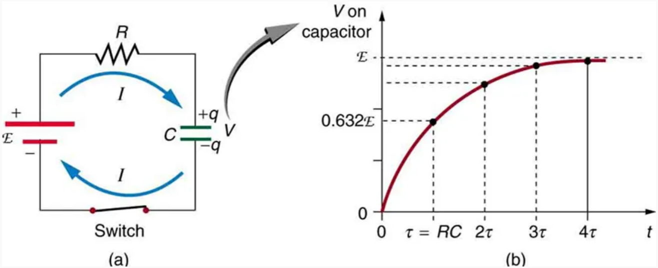


Figure 15: (Left) A battery with emf $\mathcal{E}$ induces current through R and charges C. (Right) The voltage on the capacitor versus time. Note that RC has units of time.

RC Circuits

An RC circuit is a simply connected circuit that has resistance and capacitance.

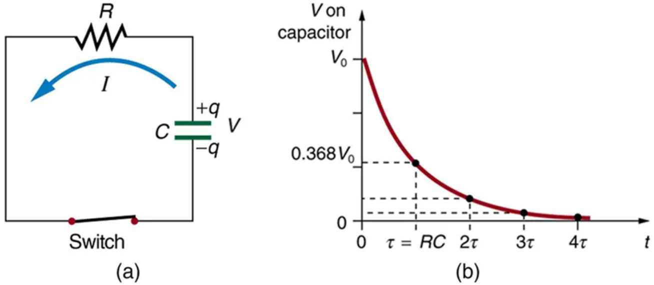


Figure 16: (Left) A charged capacitor induces current through R. (Right) The voltage on the capacitor versus time. Note that RC has units of time.

RC Circuits

RC Circuit Equations

Let R and C be the resitance and capacitance in an RC circuit, let $\tau = RC$ be the time constant, and let $\mathcal{E}$ be the emf when charging. The following two equations may be proven for RC cicuits:

$$V_{\mathrm{C}} = \mathcal{E}\left(1 - e^{-t/\tau}\right) \tag{13}$$

$$V_{\mathrm{C}} = \mathcal{E}e^{-t/\tau} \tag{14}$$

RC Circuits

Suppose an RC circuit is built from a 1 kΩ resistor, a 1 μF capacitor, and a 5 V battery. What is the time constant?

- A: 1 second
- B: 0.1 seconds
- C: 0.01 seconds
- D: 1 ms

Suppose the same RC circuit is connected and charges. What is the voltage on the capacitor after $t = 10\tau$?

- A: 5V
- B: 4.9V
- C: 2.5V
- D: 0V

Suppose the same charged RC circuit is disconnected from the battery and connected to a load. What is the voltage on the capacitor after $t = 10\tau$?

- A: 5V
- B: 4.9V
- C: 2.5V
- D: 0V

After two time constants, what percentage of the final voltage, emf, is on an initially uncharged capacitor C, charged through a resistance R?

- A: 13.2 percent
- B: 26.7 percent
- C: 86.5 percent
- D: 99.5 percent

Think back to the prior unit, when we explored capacitance. Which of the circuits in Fig. 17 takes longer to charge and discharge? Explain why this is true in terms of $\tau = RC$. (Group discussion). **We will now construct and test an RC circuit.**

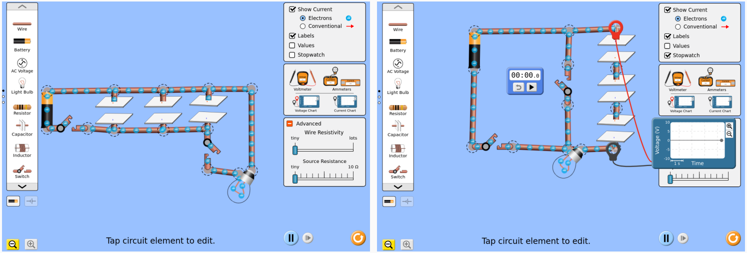


Figure 17: (Left) Three identical capacitors are charged in parallel. (Right) Three identical capacitors are charged in series.

Conclusion

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