ALGEBRA-BASED PHYSICS-1: MECHANICS (PHYS135A-01): WEEK 6

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WEEK 5 REVIEW

1. Friction
    - Normal force and friction
    - Static, kinetic
2. Drag
    - Terminal velocity
3. Restoring Forces
    - Hooke's Law
    - Young's modulus
    - Shear modulus
    - Bulk modulus

WEEK 5 REVIEW PROBLEM

A car rests on four shock absorbers, and each is like a spring with a spring constant $k = 1000 N/cm$. The car weighs 10000 N. By what distance is each spring compressed?

- A: 2.5 cm
- B: 10 cm
- C: 1 meter
- D: 0 cm

A team of workers is pulling a 500 kg load up a ramp with a 30 degree incline, at constant speed, and the coefficient of friction between the load and ramp is 0.6. What is the force with which the workers pull?

- A: 5000 N
- B: 2500 N
- C: 2750 N
- D: 3750 N

Week 6 Summary

1. Angular kinematics and dynamics
    - Angular displacement
    - Angular velocity
    - Centripetal acceleration
2. Newton's Law of Gravity and circular orbits
3. Kepler's Laws

ANGULAR KINEMATICS AND DYNAMICS

There is a correspondence between angular and linear kinetmatics, if we deal with accelerations that are constant or zero.

Linear:

$$x(t) = x_0 + v_\mathrm{i} t + \frac{1}{2} a t^2 \quad (1)$$

$$v(t) = v_\mathrm{i} t + a t \quad (2)$$

$$v^2 = v_\mathrm{i}^2 + 2a(x - x_0) \quad (3)$$

Angular:

$$\theta(t) = \theta_0 + \omega_\mathrm{i} t + \frac{1}{2} \alpha t^2 \quad (4)$$

$$\omega(t) = \omega_\mathrm{i} t + \alpha t \quad (5)$$

$$\omega^2 = \omega_\mathrm{i}^2 + 2\alpha \Delta(\theta - \theta_0) \quad (6)$$

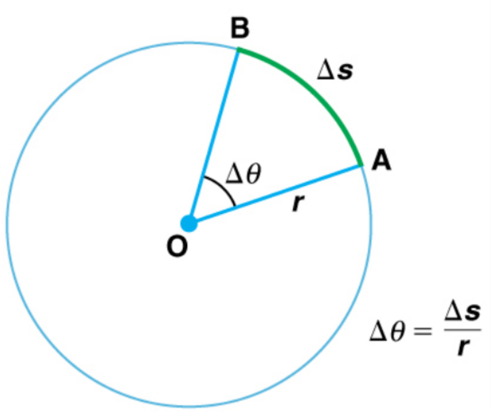


Figure 1: The definitions of arc length, Δs, radius, r, and angular displacement $\Delta\theta$.

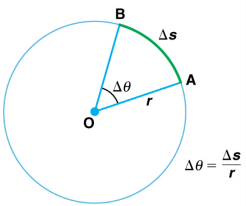


Figure 2: Examining the change in these quantities: $\Delta\theta/\Delta t = \omega$, $\Delta\omega/\Delta t = \alpha$.

Relationship between linear and rotational:

$$v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega \quad (7)$$

$$a = \frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha \quad (8)$$

Notice that the units of angular velocity are s^{-1}, and those of angular acceleration are s^{-2}.

Astromers have now discovered several thousand planets orbiting in star systems other than ours. Suppose we observe a star system face-on, and see a planet orbiting in a circular orbit with constant angular velocity. If it goes halfway around the star in 3 months, what is the angular velocity of the planet?

- A: $\frac{\pi}{3}$ months^{-1}
- B: $\frac{\pi}{6}$ months^{-1}
- C: $\frac{2\pi}{3}$ months^{-1}
- D: 2π months^{-1}

If we define a coordinate system such that at time $t = 0$ months, the planet is along the x-axis, in how many months will the planet cross the negative y-axis?

- A: 3 months
- B: 3.5 months
- C: 4.0 months
- D: 4.5 months

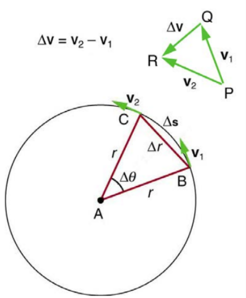


Figure 3: The velocity triangle and the position triangle are *similar*, because they are isosceles with the same angle ($\Delta\theta$).

Similar triangles have equal *ratios of sides*:

$$\frac{\Delta v}{v} = \frac{\Delta s}{r} \tag{9}$$

$$\Delta v = \frac{v}{r}\Delta s \tag{10}$$

$$\frac{\Delta v}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t} \tag{11}$$

$$\Delta t \to 0 \tag{12}$$

$$a_{\mathrm{C}} = \frac{v^2}{r} = r\omega^2 \tag{13}$$

$$\vec{a}_{\mathrm{C}} = -\frac{v^2}{r}\hat{r} \tag{14}$$

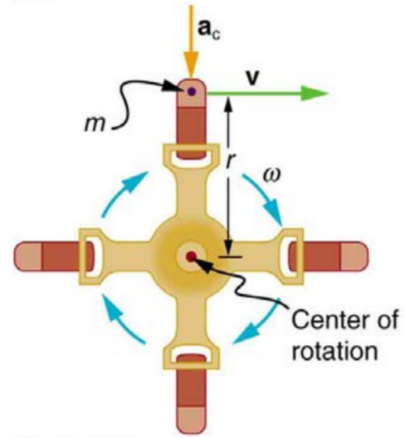
With centripetal acceleration comes centripetal force, which is the net force for uniform circular motion:

$$\vec{F}_{\mathrm{C}} = -\frac{mv^2}{r}\hat{r} = -mr\omega^2\hat{r} \tag{15}$$

In Eq. 15, the minus sign indicates that the force points towards the center of the circle.

Calculate the centripetal acceleration of a point 7.50 cm from the axis of an centrifuge spinning at 7.5×10^4 revolutions per minute. What is this acceleration in g's?

- A: 5×10^3 m/s^2, $\approx 500g$'s
- B: 4.6×10^3 m/s, $\approx 460g$'s
- C: 4.6×10^6 m/s^2, $\approx 460,000g$'s
- D: 3.2×10^5 m/s^2, $\approx 32,000g$'s



Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.

- A: 900 N
- B: 9000 N
- C: 1050 N
- D: 1125 N

What is the coefficient of friction required to keep the car from sliding? (This is an example of *static* fricion).

- A: 0.0
- B: 0.1
- C: 0.125
- D: 0.25

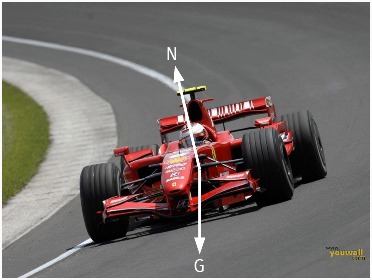


Figure 4: There are two *external* forces on the automobile: normal force and weight.

Suppose the angle between the surface of a banked curve and horizontal is θ.

$$N\cos\theta = mg \tag{16}$$

$$N\sin\theta = m\frac{v^2}{r} \tag{17}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{v^2}{rg} \tag{18}$$

$$\tan\theta = \frac{v^2}{rg} \tag{19}$$

Notice that centripetal force is not an *external* force.

If the banked curve makes an angle of 30 degrees and has a radius of 500 meters, what is the maximum speed the F1 car may have?

- A: 54 m/s
- B: 110 m/s
- C: 10 m/s
- D: 42 m/s

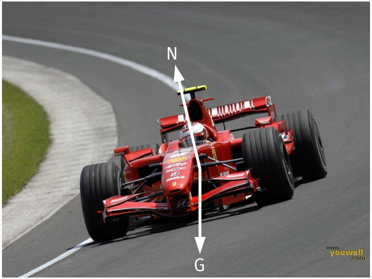


Figure 5: There are two *external* forces on the automobile: normal force and weight.

Group activity: Suppose the banked curve has a coefficient of static friction μ, a radius r, and a bank angle θ. Show that the maximum velocity the F1 car may have without sliding is

$$v = \sqrt{rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)} \qquad (20)$$

Google plotting: Using Google, plot the term in parentheses versus θ for different fixed μ-values, and discuss the differences in the shape of the curves.

NEWTON'S LAW OF GRAVITY AND CIRCULAR ORBITS

Figure 6: Sir Isaac Newton realizing that the force between the ground and the tree could extend to the sun...

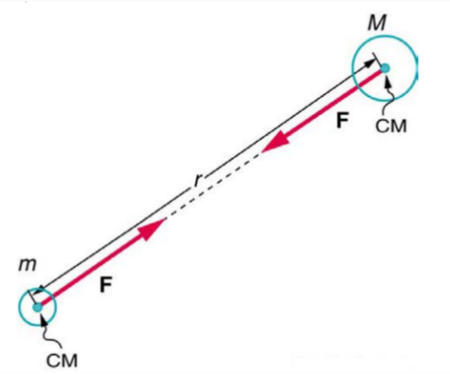


Figure 7: Remember: the force of gravity must obey Newton's Third Law, like any other force.

Newton's Law of Gravitation

Let two systems have masses m_1 and m_2, the distance between them be r, and $G = 6.674 \times 10^{-11}\ \mathrm{N\ m^2\ kg^{-2}}$. The force of gravity between them is $\vec{F} = G\frac{m_1 m_2}{r^2}\hat{r}$

The radius of the Earth is 6.38×10^6 m, the mass of the Earth is 5.98×10^24 kg, and $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}. Using Newton's Second Law and Newton's Law of Gravity, show that the acceleration of systems with mass m at the radius of the Earth is ≈ 9.8 m/s^2.

(Set $F_{\mathrm{Net}} = G\frac{Mm}{r^2}$, where M is the mass of the Earth).

What is the period of the moon's orbit around the Earth? Start by setting gravitational force equal to the centripetal force. (Earth-moon distance is 3.84×10^{8} m, mass of the Earth is 5.98×10^{24} kg, and $G = 6.674 \times 10^{-11}$ N m^2 kg^{-2}).

- A: 25.5 days
- B: 27.4 days
- C: 29.1 days
- D: 31.0 days

Orbital velocity: What is the speed of the International Space Station in orbit around the Earth? (Assume it is orbiting at the radius of the Earth).

- A: 100 m/s
- B: 1 km/s
- C: 10 km/s
- D: 100 km/hr

Kepler's Laws and Newton's Laws of Gravity

The development of the law of gravitation played a "pivotal role in the history of ideas," according to the text. One simple observation and the ensuing analysis led to explanations for:

- The orbit of the moon
- The tides
- The heliocentric universe
- The orbits of planets, including Kepler's Laws
- Comets (not always in orbit)
- The story of Edmond Halley, and measuring 1 AU in meters
- How to get to Mars
- All the things

Kepler's first law:

The orbit of each planet about the Sun is an ellipse with the Sun at one focus.

The approximation of circular orbits holds when one system is much more massive than the other, and there are no perturbations from other orbiting systems.

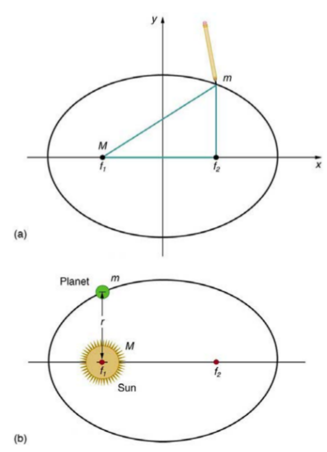


Figure 8: Diagram of Kepler's first law. The ellipse does not have a center and a radius, but two *focii* and a radius that varies.

Kepler's second law:

Each planet moves so that an imaginary line drawn from the Sun to the planet sweeps out equal areas in equal times.

This rule accounts for the fact that the centripetal acceleration increases if the radius (which can vary) decreases. That means that when the orbiting system is closer to more massive system, the orbital velocity must increase.

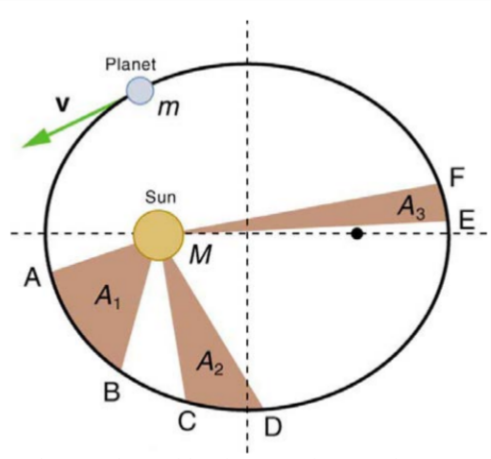


Figure 9: Diagram of Kepler's second law. Each shaded area has equal area, and corresponds to equal time, but not equal orbital distance.

Kepler's third law:

The product of the orbital period squared and the orbital radius cubed of the orbiting system remains constant.

We can derive this fact from Newton's Law of Gravity...

Group activity: Setting the law of gravity equal to the centripetal force, show that

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2} \tag{21}$$

G is the gravity constant, and M is the mass of the system being orbited. *Notice that the right-hand side is a constant.* This implies that ratios of radii cubed and period squared should never change.

Table 6.2 Orbital Data and Kepler's Third Law

Parent	Satellite	Average orbital radius r(km)	Period T(y)	r^3 / T^2 (km^3 / y^2)
Earth	Moon	3.84×10^5	0.07481	1.01×10^{19}
Sun	Mercury	5.79×10^7	0.2409	3.34×10^{24}
	Venus	1.082×10^8	0.6150	3.35×10^{24}
	Earth	1.496×10^8	1.000	3.35×10^{24}
	Mars	2.279×10^8	1.881	3.35×10^{24}
	Jupiter	7.783×10^8	11.86	3.35×10^{24}
	Saturn	1.427×10^9	29.46	3.35×10^{24}
	Neptune	4.497×10^9	164.8	3.35×10^{24}
	Pluto	5.90×10^9	248.3	3.33×10^{24}
Jupiter	Io	4.22×10^5	0.00485 (1.77 d)	3.19×10^{21}
	Europa	6.71×10^5	0.00972 (3.55 d)	3.20×10^{21}
	Ganymede	1.07×10^6	0.0196 (7.16 d)	3.19×10^{21}
	Callisto	1.88×10^6	0.0457 (16.19 d)	3.20×10^{21}

Figure 10: Kepler and his advisor Brache collected data such as this, and determined empirically the third law *before* Sir Isaac Newton published the law of gravity.

CONCLUSION

1. Angular kinematics
    - Angular displacement
    - Angular velocity
    - Centripetal acceleration
2. Newton's Law of Gravity and circular orbits
3. Kepler's Laws

ANSWERS

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- 2.5 cm
- 2750 N
- $\frac{\pi}{3}$ months^{-1}
- 4.5 months
- 4.6×10^6 m/s^2,
  $\approx 460,000g$'s
- 1125 N
- 0.125
- 54 m/s
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- 10 km/s
- ...