

ALGEBRA-BASED PHYSICS-1: MECHANICS (PHYS135A-01): WEEK 9

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WEEK 8 REVIEW

1. Definition of **momentum**
2. *Conservation of momentum*
 - The proof and the assumptions
 - Examples
3. *Classification of collisions*
 - Elastic
 - Inelastic
 - $1 \rightarrow 1, 1 \rightarrow n, n \rightarrow 1, n \rightarrow n$
 - Lab activity
4. Momentum in multiple dimensions
5. Center of mass
 - Derivation of $\vec{F}_{\text{Net}} = \frac{d\vec{p}_{\text{CM}}}{dt}$
 - Center of mass motion

WEEK 8 REVIEW PROBLEMS

WEEK 8 REVIEW PROBLEM

Imagine a space shuttle launch. Rocket propellant is ignited and blasted out of the solid rocket boosters (SRBs). Which of the following is true of the shuttle-propellant system?

- A: The exhaust pushes on the against the ground, and by Newton's 3rd Law, the ground pushes on the rocket, and the rocket accelerates upward.
- B: The exhaust pushes on the air, and by Newton's 3rd Law, the air pushes on the rocket and it accelerates upward.
- C: The momentum of the rocket increases because the ejected mass results in a force on the rocket via Newton's 3rd Law
- D: The rocket pushes out the exhaust, and by Newton's 3rd Law, the exhaust pushes on the rocket

WEEK 8 REVIEW PROBLEM

An Apollo class spacecraft has made it far enough to the Moon that we can neglect Earth's gravity. It is comprised of two pieces of equal mass, and the whole system is moving at speed v . If one piece is launched backward such that it remains stationary, what is the new speed of the craft?

- A: v
- B: $2v$
- C: $4v$
- D: 0

WEEK 9 SUMMARY

We are concluding with chapter 10, regarding angular momentum. Please read chapter 10.

1. Review of **angular motion**
 - Angular velocity and acceleration
 - *Torque*
2. **The moments of inertia**
3. Newton's 2nd Law, rotational case
4. Energy of Rotating Objects
 - **Conservation of energy, revisited**
 - Rotational kinetic energy
 - Examples of work
5. Begin review of all chapters

REVIEW OF ANGULAR MOTION

REVIEW OF ANGULAR MOTION

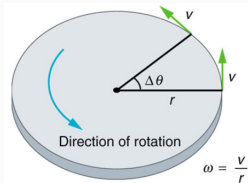


Figure 1: From the definition of a radian we may derive angular velocity and acceleration.

Table 10.1 Rotational and Translational Quantities

Rotational	Translational	Relationship
θ	x	$\theta = \frac{x}{r}$
ω	v	$\omega = \frac{v}{r}$
α	a	$\alpha = \frac{a_t}{r}$

Figure 2: Review of angular kinematic quantities.

REVIEW OF ANGULAR MOTION

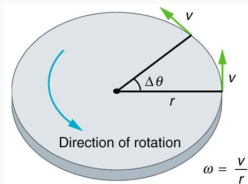


Figure 3: From the definition of a radian we may derive angular velocity and acceleration.

Table 10.2 Rotational Kinematic Equations

Rotational	Translational	
$\theta = \bar{\omega}t$	$x = \bar{v}t$	
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	(constant α , a)
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	(constant α , a)
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$	(constant α , a)

Figure 4: Review of angular kinematic quantities.

A fishing rod is comprised of a pole and a reel for the fishing line. The reel of a particular model has a radius of 5 cm, and a man casts the line, letting out 5 meters in 1 second. What is the angular velocity of the reel?

- A: 5 radians per second
- B: 10 radians per second
- C: 50 radians per second
- D: 100 radians per second

REVIEW OF ANGULAR MOTION

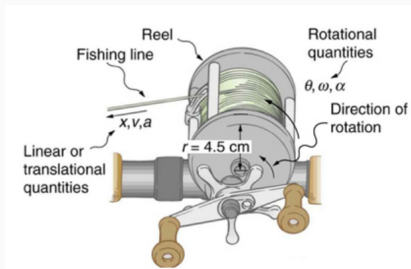


Figure 5: From the definition of a radian we may derive angular velocity and acceleration.

If the man catches a fish on the line, the line will have tension. *What about the reel?*
There is tension on the reel, but not “at the center.”

The concept of torque is that a force leads to *rotational* rather than *linear* motion. **Would it be harder to reel in the fish with a reel of larger or smaller radius?**

REVIEW OF ANGULAR MOTION

Let *torque* be defined as the **cross-product** of a force, F and a moment-arm, r :

$$\vec{\tau} = \vec{r} \times \vec{F} \quad (1)$$

Cross-product: a new way of multiplying vectors: $\hat{i} \times \hat{j} = \hat{k}$, $\hat{k} \times \hat{i} = \hat{j}$, $\hat{j} \times \hat{k} = \hat{i}$. See the pattern? (There's the **right-hand rule**). Although torque is a vector, presently we are concerned with the magnitude of torque:

$$\tau = rF \sin \theta \quad (2)$$

The angle θ is between the force and the moment arm. **The units are: N m.** What other quantity has these units?

If a 1 kg fish is hanging from a fishing pole that is 1.5 meters long and held horizontally, what is the torque?

- A: 15 N m
- B: 10 N m
- C: 1.5 N m
- D: 10 N

If a 1 kg fish is hanging from a fishing pole that is 1.5 meters long and held vertically, what is the torque?

- A: 15 N m
- B: 1.5 N m
- C: 0 N m
- D: 10 N

Previous two questions:

$$\tau = rF \sin \theta \quad (3)$$

The angle θ is between the force and the moment arm, so if $\theta = 0$ then there is no torque, and if $\theta = 90^\circ$ then torque maximizes at $\tau = rF$.

MOMENTS OF INERTIA

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Starting with Newton's 2nd law, we may reveal something called the **moment of inertia**, through the concept of torque:

$$F = ma \quad (4)$$

$$a = r\alpha \quad (5)$$

$$F = mr\alpha \quad (6)$$

$$rF = mr^2\alpha \quad (7)$$

$$\tau = (mr^2)\alpha \quad (8)$$

$$I = mr^2 \quad (9)$$

$$\tau = I\alpha \quad (10)$$

Newton's 2nd Law in rotational form is that torque is equal to the *moment of inertia*, I , times the angular acceleration α .

Newton's 2nd Law in rotational form is that torque is equal to the *moment of inertia*, I , times the angular acceleration α . The moment of inertia describes spatial distribution of mass.

Although the formula $I = mr^2$ only applies to a single piece of matter a distance r along the moment-arm, we may derive the moment of inertia for *any rigid body* through the techniques of calculus.

MOMENTS OF INERTIA

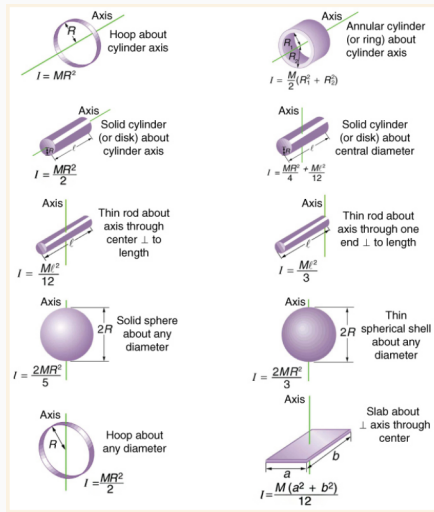


Figure 6: Moments of inertia about different axes of rotation.

Basketball players are taught how to shoot a jump shot with a *backspin*. The moment of inertia of a basketball is like that of a thin spherical shell: $I = \frac{2}{3}MR^2$, where M is the mass of the basketball and R is the radius. If a basketball has a 10 cm radius, and a mass of 0.6 kg, what is the moment of inertia?

- A: $4 \times 10^{-2} \text{ kg m}^2$
- B: $4 \times 10^{-3} \text{ kg m}^2$
- C: $4 \times 10^{-1} \text{ kg m}^2$
- D: $4 \times 10^{-3} \text{ kg}$

Basketball players are taught how to shoot a jump shot with a *backspin*. The moment of inertia of a basketball is $4 \times 10^{-3} \text{ kg m}^2$. What torque is required to make the ball go from 0 radians/sec to 10π radians/sec in 0.1 second? (Angle between force and moment-arm: 90 deg).

- A: $4 \times 10^{-2} \text{ kg m}^2$
- B: $\frac{2\pi}{500} \text{ N m}$
- C: $\frac{2\pi}{5} \text{ N}$
- D: $\frac{2\pi}{5} \text{ N m}$

Basketball players are taught how to shoot a jump shot with a *backspin*. If the radius of the ball were twice as large, by what factor would the torque need to increase to produce the same angular acceleration?

- A: 2
- B: 4
- C: $2 \sin \theta$
- D: No change

CONSERVATION OF ENERGY, REVISITED

In the exact same fashion as the linear case, we can show that work done on a rotating system increases the *rotational kinetic energy*:

$$W = \frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_0^2 \quad (11)$$

In Eq. 11, the rotational kinetic energy is $KE_{\text{rot}} = \frac{1}{2}I\omega^2$. The moment of inertia takes the place of mass, and the angular velocity takes the place of linear velocity.

The presence of rotational kinetic energy in a system requires us to revisit the conservation of energy. One way to state this principal is to assume all potential energy is converted to kinetic energy:

$$PE_i = KE_{\text{trans}} + KE_{\text{rot}} \quad (12)$$

Return to the basketball with *backspin* scenario. Suppose the basketball is shot with some known speed, angle, and backspin. The ball swooshes through the hoop. If the same player shoots the same shot from the same place, but increases the backspin, which of the following is true?

- A: The shot will fall short
- B: The shot will go long and hit the backboard
- C: The shot will still go in
- D: The shot will still go in, but take more time

CONSERVATION OF ENERGY

Suppose two basketballs with different radii roll down a driveway. The moment of inertia for this shape is $I = \frac{2}{3}MR^2$ if M is the mass and R is the radius. Which of the following is true?

- A: The basketball with the larger radius will reach the bottom of the driveway first
- B: The basketball with the smaller radius will reach the bottom of the driveway first
- C: The basketballs will reach the bottom of the driveway at the same time
- D: The basketball with the larger mass

AN EXAMPLE OF WORK

A **grindstone** is used to sharpen knives and cutlery. A grindstone is a disk spinning about the natural axis, with $I = \frac{1}{2}MR^2$. Work done on the grindstone to sap its energy would be $W = \tau\theta$, where τ is the torque and θ is angular displacement.

Newton's 2nd Law gives $\tau = I\alpha$, so $W = \frac{1}{2}\alpha\theta MR^2$.

Suppose we press our pocketknife to a 20 kg grindstone with radius 10 cm, that is spinning at 10 radians per second (no net external force). If we press really hard the grindstone comes to a stop after 10 seconds. How much energy went into sharpening the knife?

Solve in groups at the boards.

CONSERVATION OF ANGULAR MOMENTUM

CONCLUSION

ANSWERS

ANSWERS

- C and D
- $2v$
- 100 radians per second
- 15 N m
- 0 N m
- $4 \times 10^{-3} \text{ kg m}^2$
- $\frac{2\pi}{5} \text{ N m}$
- 2
- The shot will fall short
- The basketball with the smaller radius will reach the bottom of the driveway first
- ZZZ