Algebra-Based Physics: Electricity, Magnetism, and Modern Physics (PHYS135B): Unit 4

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Summary

Summary

1. Magnetic induction - Chapters 23.1 - 23.5, 23.7, 23.9
    - Induced EMF and magnetic flux
    - Faraday's Law
    - Motional EMF, generators, and transformers
2. AC circuits - Chapters 23.9 - 23.12
    - Inductors
    - RL circuits
    - RLC circuits

Magnetic induction

First set of observations: a *moving* magnet can induce an emf in a coil of wire. The induced current polarity depends on (a) magnet polarity and (b) direction of magnet velocity. The induced current magnitude

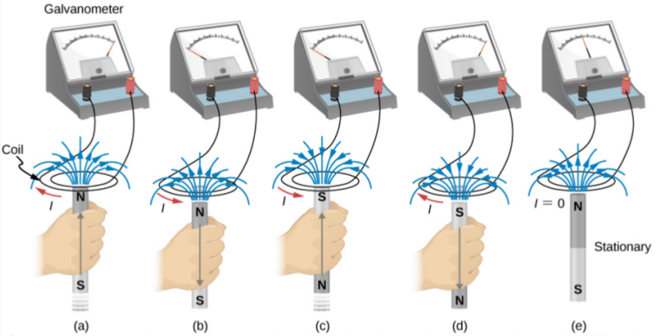

Figure 1: Observations of magnetic induction.

Magnetic induction

Second set of observations: a *changing* current in a loop can induce an emf in another loop. The induced current polarity depends on (a) inducing current polarity and (b) whether the inducing current is increasing or descreasing.

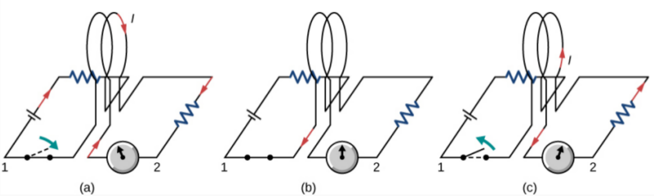

Figure 2: Observations of magnetic induction.

Magnetic induction

Third observation: a *changing* loop area in a magnetic field induces an emf, and current. The induced current polarity depends on whether the loop area is (a) increasing or (b) descreasing. The current magnitude depends on how quickly the area is changing.

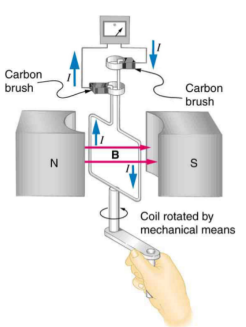

Figure 3: Observations of magnetic induction.

Video summary of magnetic induction:
`https://youtu.be/pQp6bmJPU_0`

- Magnet inducing current in loop of wire
- Solenoids inducing current in adjacent solenoids
- Magnetic flux
- Faraday's Law
- Lenz's Law

Magnetic induction

Magnetic flux is the dot-product of the area vector and the magnetic field through loops of wire with area A.

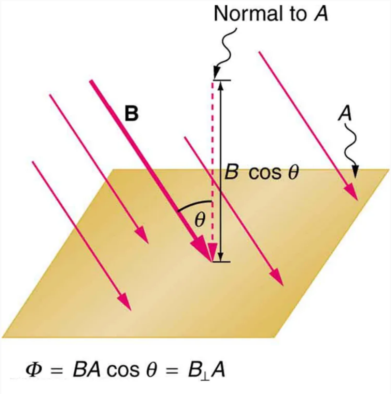

Figure 4: The **area vector** is *normal* to the loop area.

The **area vector** has a magnitude A, the area of the loop. The direction of the area vector is *normal* to the area of the loop.

$$\vec{A} = A\hat{n} \tag{1}$$

The magnetic flux, Φ, is therefore

$$\Phi = \vec{B} \cdot \vec{A} \tag{2}$$

Faraday’s Law

Faraday's Law

Let the product of the magnetic field and the vector area be the magnetic flux: $\Phi = \vec{B} \cdot \vec{A}$. The induced emf ϵ in N turns of a conductor will be

$$\epsilon = -\frac{\Delta\Phi}{\Delta t} \tag{3}$$

The induced current from ϵ will create a new B-field that opposes changes in Φ.

The unit of magnetic flux is the Weber, or 1 Wb = 1 T m^2.

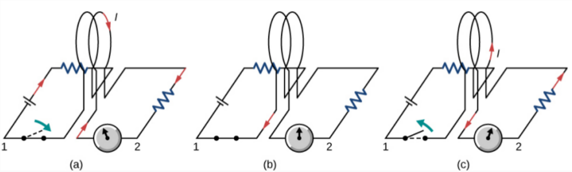

Figure 5: A pickup coil system.

Suppose the switch in Fig. 5 (a) is closed, inducing a current *I* in the right-hand loop. The B-field directions at the centers of the left and right loops are

- A: Right and left, respectively
- B: Light and right, respectively
- C: Both to the right
- D: Both to the left

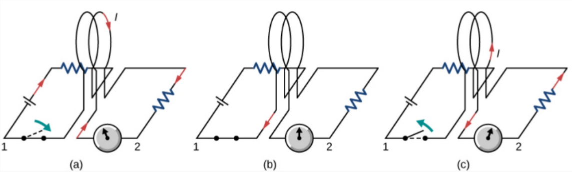

Figure 6: A pickup coil system.

Suppose the switch in Fig. 5 (b) remains closed, and no induced current is observed. This is because

- A: The magnetic flux is zero
- B: The inducing current is zero
- C: The magnetic flux is not changing
- D: The loop area is zero

Faraday's Law

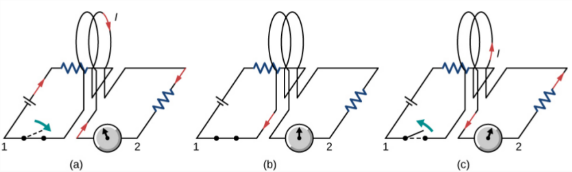

Figure 7: A pickup coil system.

Suppose the switch in Fig. 5 (b) remains closed, and no induced current is observed. This is because

- A: The magnetic flux is zero
- B: The inducing current is zero
- C: The magnetic flux is not changing
- D: The loop area is zero

Faraday's Law

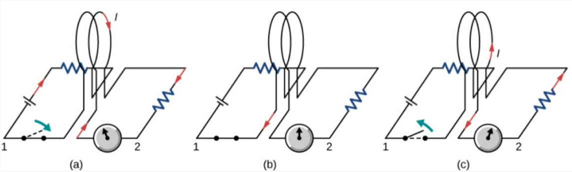

Figure 8: A pickup coil system.

Suppose the switch in Fig. 5 (c) is opened. The induced current in Fig. 5 is in the opposite direction of Fig. 5 (c) because

- A: The magnetic field from the right loop decreased
- B: The magnetic field from the left loop increased
- C: The magnetic field from the left loop is constant
- D: The magnetic field from the left loop decreased

Group board problem:

A magnetic field B passes orthogonally through a circular coil of radius $r = 0.05$ m and $N = 100$ turns. The field magnitude decreases linearly according to

$$B(t) = B_0 - at \tag{4}$$

with $B_0 = 0.015$ T and $a = 0.01$ T s^{-1}. (a) Calculate the magnitude of the emf induced in the coil at the times $t_0 = 0$, and $t_2 = 1.0$ second. (b) Determine the current in the coil if the resistance is 1 Ω.

Sketch this system, and indicate both the direction of the instantaneous B-field, and the direction of current.

Faraday's Law - PhET Activity

Brief simulation of Faraday's Law, and Lenz's Law:

`https://phet.colorado.edu/en/simulations/faradays-law`

1. Learn to control the position and orientation of the bar magnet.
2. Activate the voltmeter in parallel with the light bulb.
3. Use the coil with four loops of wire.
4. Produce the following results:
    - A positve voltage from a moving bar magnet
    - A negative voltage from a moving bar magnet
    - A positive voltage from switching the bar magnet polarity
    - A negative voltage from switching the bar magnet polarity
5. Is your voltage positive or negative when you are increasing Φ? How do you *decrease* Φ?

Motional EMF, Generators, and Transformers

Motional EMF, Generators, and Transformers

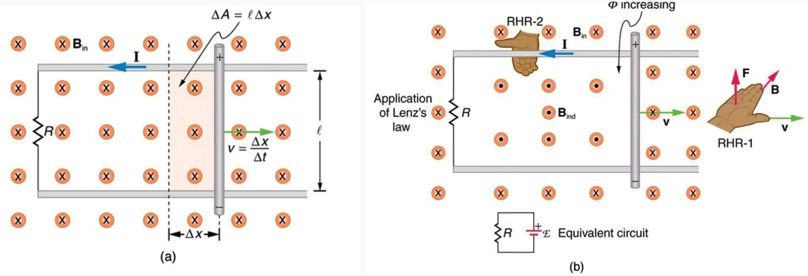

Figure 9: Motional emf in a loop with changing area.

Group board problems:

1. Show that *power* is $P = \vec{F} \cdot \vec{v}$ when acceleration is constant.
2. Show that the emf is $\epsilon = Blv$, where l is the length of the rod.
3. Show that power generated, $P = I^2R = \epsilon/R$, is equal to power injected.

Motional EMF, Generators, and Transformers

How do we use Faraday's Law to induce power in a generator?
Start with Faraday's Law:

$$\epsilon = -N\frac{\Delta\Phi}{\Delta t} \tag{5}$$

The flux Φ depends on time:

$$\Phi = \vec{B}\cdot\vec{A}(t) = BA\cos(\theta(t)) \tag{6}$$

Let the *angular velocity* be constant: $\theta = \omega t$. Then we have

$$\Phi = BA\cos(\omega t) \tag{7}$$

Thus the emf (with N loops) is (...calculus...)

$$\epsilon = N\omega BA\sin(\omega t) = \epsilon_0\sin(\omega t) \tag{8}$$

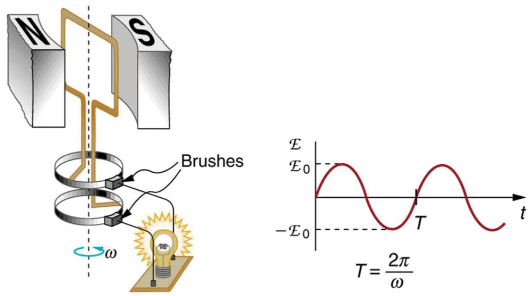

Figure 10: (Left) The AC generator with brushes generates an AC voltage. (Right) This is a diagram of the AC voltage.

- Amplitude: ϵ_0, the maximum value of the AC signal. Units: Volts.
- Period: $T = 2\pi/\omega$, the time to complete one AC cycle. Units: seconds.
- Frequency: $f = 1/T$, the number of cycles per second. Units: Hertz.

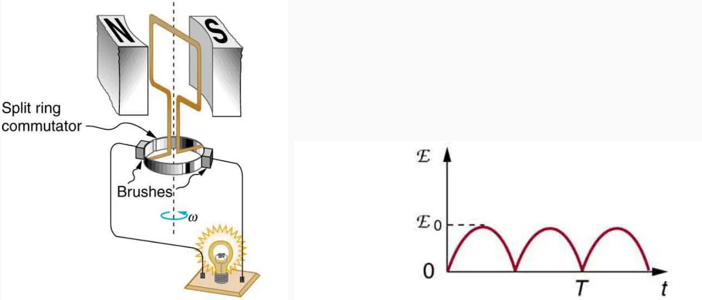

Figure 11: (Left) The AC generator with brushes and *commutator* generates pulsed DC. (Right) This is a diagram of the signal.

- Amplitude: ϵ_0, the maximum value of the AC signal. Units: Volts.
- Period: $T = 2\pi/\omega$, the time to complete one AC cycle. Units: seconds.
- Frequency: $f = 1/T$, the number of cycles per second. Units: Hertz.

Equation 9 is a basic model for the emf from a generator.

$$\epsilon = N\omega BA \sin(\omega t) = \epsilon_0 \sin(\omega t) \tag{9}$$

Which of the following would increase the *amplitude* of the emf?

- A: Turning the shaft more slowly
- B: Turning the shaft more quickly
- C: Decreasing the B-field
- D: Increasing N

Equation 10 is a basic model for the emf from a generator.

$$\epsilon = N\omega BA \sin(\omega t) = \epsilon_0 \sin(\omega t) \tag{10}$$

Which of the following would increase the *frequency* of the emf?

- A: Turning the shaft more slowly
- B: Turning the shaft more quickly
- C: Decreasing the B-field
- D: Increasing N

Equation 11 is a basic model for the emf from a generator.

$$\epsilon = N\omega BA\sin(\omega t) = \epsilon_0 \sin(\omega t) \tag{11}$$

Group exercise: Suppose an AC generator rotates at 200 rpm, in a B-field with 0.1 T, and has 100 loops with radius 5 cm. (a) What is the peak voltage this generator will produce? (b) If the generator powers a system with resistance of 1kΩ, what will be the peak current?

PhET: Motional EMF, Generators, and Transformers

PhET: AC Power generator

Link to the (CheerpJ) simulation:

`https://phet.colorado.edu/en/simulation/generator`

1. Set the water rate such that the meter reads 10 rotations per minute (rpm).
2. Choose the voltage meter under the pickup coil menu.
3. Under loops, choose 1 loop, and under area, leave it at 50%.
4. Choose show field meter in the upper right, and place the tool in the loop center.
5. On the same graph, plot the average B-field and voltage versus time. What is the period and amplitude of your signal? Use the left y-axis for B-field units, and the right y-axis for voltage units.
6. Create the same graph for $N = 3$ loops.
7. Create the same graph for $N = 1$ loop, but for 20 rpm.

Hint: you know the rpm of the magnet, so you know how much time corresponds to one rotation.

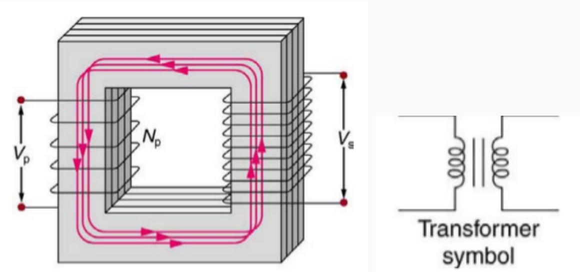

Figure 12: A *transformer* uses Faraday's law to change voltages in AC-generated systems.

The magnetizable core (gray) creates a loop in the B-field that passes through the left and right coils. Use Faraday's law to show that

$$\frac{V_L}{V_R} = \frac{N_L}{N_R} \tag{12}$$

Motional EMF, Generators, and Transformers

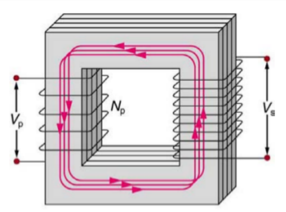

Figure 13: The *transformer* changes AC voltage levels.

Suppose the transformer in Fig. 13 has $N_L = 5$, $N_R = 100$, $V_L = 1$ kV (peak). What is V_R (peak), in kV?

- A: 20 kV
- B: 5 kV
- C: 0.05 kV
- D: 0.05 V

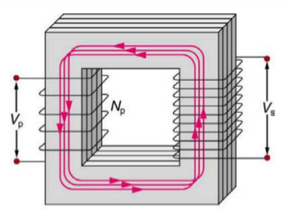

Figure 14: The *transformer* changes AC voltage levels.

Suppose we need the transformer in Fig. 14 to produce $V_R = 120$ V, and $V_L = 12$ kV. Which combination of coils will satisfy the requirement?

- A: $N_L = 3$, $N_R = 10$
- B: $N_L = 10$, $N_R = 1000$
- C: $N_L = 10$, $N_R = 100$
- D: $N_L = 1000$, $N_R = 10$

Motional EMF, Generators, and Transformers

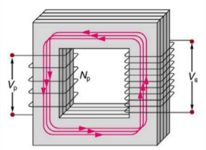

Figure 15: The *transformer* changes AC voltage levels.

If the $V_L \neq V_R$, how are energy and power conserved?

- A: The induced current is larger on the right.
- B: The induced current is smaller on the right.
- C: The induced current is conserved from left to right.
- D: The induced emf is conserved from left to right.

Demonstrate on board how power is conserved by (a) deriving the currents I_L and I_R, then forming the powers P_L and P_R.

Inductors

It turns out you can *reverse* transformers, through symmetry. Between the coils in the transformer in Fig. 15, there is *mutual inductance, M*:

$$\epsilon_R = -M\frac{\Delta I_L}{\Delta t} \tag{13}$$

$$\epsilon_L = -M\frac{\Delta I_R}{\Delta t} \tag{14}$$

The inductance accounts for everything except the current: the geometry, number of turns, and field strength. Inductance is useful for systems with fixed geometry, where we often do not need to know Φ.

Derive an expression that relates M to Φ, given the original version of Faraday's Law.

Consider the case of **self-inductance**, L, which accounts for the magnetic field created inside (for example) a solenoid when current is introduced. New current creates a change in Φ within the solenoid, so Faraday's Law predicts a backwards emf opposing the change:

$$\boxed{\epsilon = -L\frac{\Delta I}{\Delta t}} \tag{15}$$

The inductance unit is the *Henry* (after Joseph Henry), $\mathrm{V\,A^{-1}\,s}$, or $\Omega\,\mathrm{s}$.

Self-inductance applies to increasing and decreasing current:

$$\boxed{\epsilon = -L\frac{\Delta I}{\Delta t}} \tag{16}$$

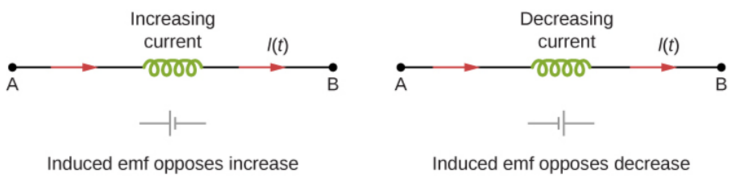

Figure 16: (a) Increasing current yields a negative voltage. (b) Decreasing current yields a positive voltage.

Inductors

Suppose we have an inductor with $L = 0.1$ mH, carrying a current of 100 mA. If we switch off the current in 1 ms, what is the induced emf?

- A: 0.1 mV
- B: 1 mV
- C: 10 mV
- D: 100 mV

(Ahh, units … Got heem! Also, why are the answers *positive*?)

Inductors

Suppose we have a current that switches from 100 mA to -100 mA in 1 ms. We observe a 100 mV emf across the inductor. What is the inductance?

- A: -0.5 mH
- B: -0.5 H
- C: 0.5 H
- D: 0.5 mH

Inductors

What is the inductance of a solenoid, given solenoid properties? Recall how the inductance relates to flux, current, and turn number:

$$L = N\frac{\Delta\Phi}{\Delta I} \tag{17}$$

$$L = N\frac{A\Delta B}{\Delta I} \tag{18}$$

$$\Delta B = \mu_0 \left(\frac{N}{l}\right) \Delta I \tag{19}$$

$$L = N\frac{A\mu_0(N/l)\Delta I}{\Delta I} \tag{20}$$

$$\boxed{L = \frac{\mu_0 N^2 A}{l}} \tag{21}$$

Recall the solenoid used in the Ampère's Law lab activity. Suppose we count $N = 80$ turns, and measure $A = 8 \times 10^{-3}\ \mathrm{m}^2$, and $l = 0.1$ m. What is the inductance?

- A: 0.6 H
- B: 0.06 H
- C: 6 mH
- D: 0.6 mH

Recall that $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\ A^{-1}\ m}$.

What is the inductance of the same solenoid, but with twice the turns?

- A: 2.4 mH
- B: 1.2 mH
- C: 24 mH
- D: 12 mH

This is a scaling problem. How does L depend on N?

Inductors

How much **energy** is stored in an inductor? Suppose potential energy is considered at *a constant voltage*, given some charging circuit that pushes current through an inductor (with some resistance).

$$\Delta U = \Delta q \epsilon \quad \rightarrow \quad \epsilon = \Delta U / \Delta q \tag{22}$$

$$|\epsilon| = \frac{\Delta U}{\Delta q} = L \frac{\Delta I}{\Delta t} \tag{23}$$

$$\Delta U = L \Delta I \left(\frac{\Delta q}{\Delta t} \right) \tag{24}$$

$$\Delta U = L I \Delta I \tag{25}$$

$$\boxed{U = \frac{1}{2} L I^2} \tag{26}$$

If you do not know how to integrate in the last step, consider the analogy of the area of a triangle.

How much energy is stored in an inductor with inductance 50 mH that was charged with a current that reaches 1 A?

- A: 25 J
- B: 2.5 J
- C: 25 mJ
- D: 2.5 mJ

Nom nom nom, more units.

A Massive Clue about Electricity and Magnetism

A Massive Clue

The energy stored in a **capacitor:**

$$\Delta U = \Delta q \epsilon \;\rightarrow\; \epsilon = \Delta U / \Delta q \tag{27}$$

$$q = C\epsilon \tag{28}$$

$$q = C\frac{\Delta U}{\Delta q} \tag{29}$$

$$C^{-1} q \Delta q = \Delta U \tag{30}$$

$$\boxed{U = \frac{1}{2}\frac{q^2}{C}} \tag{31}$$

The energy stored in an **inductor:**

$$\Delta U = \Delta q \epsilon \;\rightarrow\; \epsilon = \Delta U / \Delta q \tag{32}$$

$$|\epsilon| = \frac{\Delta U}{\Delta q} = L\frac{\Delta I}{\Delta t} \tag{33}$$

$$\Delta U = L \Delta I \left(\frac{\Delta q}{\Delta t}\right) \tag{34}$$

$$\Delta U = L I \Delta I \tag{35}$$

$$\boxed{U = \frac{1}{2} L I^2} \tag{36}$$

It seems the energy stored in a capacitor is *in the E-field*, and the energy stored in the inductor is *in the B-field*. Suppose we charge a parallel plate capacitor and solenoid ($N = 1$) charged with the same current, I, that have the same cross-sectional area A, and the same length d, such that they have equal energies?

Assume the proper formulas for capacitance and inductance, and equate energies.

$$C = \frac{\epsilon_0 A}{d} \tag{37}$$

$$L = \frac{\mu_0 N^2 A}{d} \tag{38}$$

$$U_{\mathrm{C}} = U_{\mathrm{L}} \tag{39}$$

$$\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}LI^2 \tag{40}$$

$$\frac{Q^2 d}{\epsilon_0 A} = \frac{\mu_0 N^2 A I^2}{d} \tag{41}$$

$$N = 1 \tag{42}$$

$$\frac{1}{\epsilon_0 \mu_0}\frac{Q^2}{I^2} = \frac{A^2}{d^2} \tag{43}$$

Recall that the currents used to charge the capacitor and inductor are the same.

$$\frac{Q^2}{I^2} = t^2 \tag{44}$$

Note that the volumes of the capacitor and inductor are both $V = Ad$. Let the lateral dimension of the capacitors (parallel to A) be x, and

$$A = \pi x^2 \tag{45}$$

Combining formulas,

$$\frac{1}{\epsilon_0 \mu_0} = \frac{\pi x^2}{t^2} = v^2 \tag{46}$$

The result is a constant **velocity!** Solving for v,

$$\boxed{v = \frac{1}{\sqrt{\epsilon_0 \mu_0}}} \tag{47}$$

Insert known values for ϵ_0 and μ_0. What speed do you obtain?

- A: 3×10^6 m/s
- B: 3×10^7 m/s
- C: 3×10^8 m/s
- D: 3×10^9 m/s

The speed of light appears in electromagnetic energy

RL Circuits

An RL circuit is a circuit with some resistance R and some inductance L.

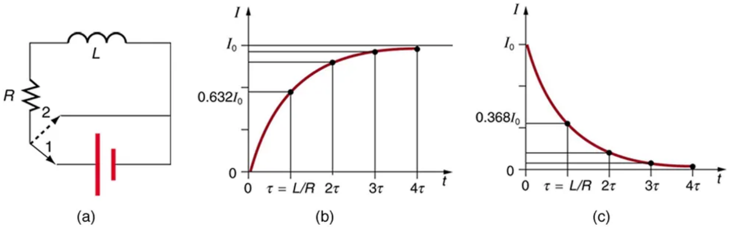

Figure 17: (a) Put switch in lower position to charge inductor, and put switch in upper position to discharge it. (b) The charging current takes a characteristic time to reach the steady state. (d) The discharging current takes a characteristic time to reach zero.

RL Circuits

Applying Kirchhoff's loop rule to Fig. 17 (a), we have

$$\mathcal{E} - iR - L\frac{\Delta I}{\Delta t} = 0 \tag{48}$$

- $\mathcal{E}$ is a positve emf in the loop direction.
- The second term is negative because the resistor *decreases the voltage* by consuming energy. The side with the larger voltage is towards the emf.
- The third term is negative because the inductor *produces an emf* counteracting the positive emf. The side with the larger voltage is towards the emf.
- There is an exact solution to this equation. Let $\tau = L/R$:

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-tR/L}\right) \tag{49}$$

Letting $i_0 = \mathcal{E}/R$, and $\tau = L/R$,

$$i(t) = i_0 \left(1 - e^{-t/\tau}\right) \tag{50}$$

Suppose an RL circuit has an inductance of 1 mH and a resistance of 1 kΩ. What is the time constant?

- A: 1 millisecond
- B: 1 microsecond
- C: 1 nanosecond
- D: 1 second

Letting $i_0 = \mathcal{E}/R$, and $\tau = L/R$,

$$i(t) = i_0 \left(1 - e^{-t/\tau}\right) \tag{51}$$

If $\tau = 1\mu$s, and $R = 1$ kΩ, with $\mathcal{E} = 5$V, what will the current be after 100 μs?

- A: 1 mA
- B: 5 A
- C: 5 mA
- D: 1 A

Letting $i_0 = \mathcal{E}/R$, and $\tau = L/R$,

$$i(t) = i_0 \left(1 - e^{-t/\tau}\right) \tag{52}$$

If $\tau = 1\mu$s, and $R = 1$ kΩ, with $\mathcal{E} = 5$V, what will the current be at 1 μs?

- A: 1.20 mA
- B: 3.16 mA
- C: 4.12 mA
- D: 10.1 mA

RL Circuits

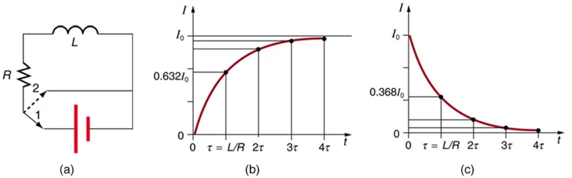

Figure 18: Current discharge is not instantaneous (c).

Inductor discharge in RL circuit:

$$i(t) = i_0 e^{-t/\tau} \tag{53}$$

What is the current as $t \to \infty$? What is the current at $t = 0$?

- A: 0, $R/\mathcal{E}$
- B: 0, $\mathcal{E}/R$
- C: $\mathcal{E}/R$
- D: $R/\mathcal{E}$, 0

PhET: RL Circuits as Signal Filters

RC and RL circuits can act as *high-pass* and *low-pass* signal filters.

- In the RC circuit, with $\tau = RC$, the capacitor *voltage* charges and discharges like

$$V_{charge}(t) = V_0 \left(1 - e^{-t/\tau}\right) \tag{54}$$

$$V_{discharge}(t) = V_0 e^{-t/\tau} \tag{55}$$

- In the RL circuit, with $\tau = L/R$, the inductor *current* charges and discharges like

$$i_{charge}(t) = i_0 \left(1 - e^{-t/\tau}\right) \tag{56}$$

$$i_{discharge}(t) = i_0 e^{-t/\tau} \tag{57}$$

PhET: RL and RC Circuits as Filters

Go to `https://phet.colorado.edu/en/simulations/circuit-construction-kit-ac`.

- In the AC Circuits PhET simulator, create an RC circuit, with $R = 8\Omega$, and $C = 0.05$ F (50 mF), and a AC voltage source set to 120V amplitude.
    1. Click and drag a voltage chart from the right side to measure the AC source voltage.
    2. Click and drag a voltage chart from the right side to measure the capacitor voltage.
    3. Create a spreadsheet with three columns: (1) frequency of the AC voltage source, (2) peak AC voltage source amplitude, and (3) peak capacitor voltage amplitude.

PhET: RL and RC Circuits as Filters

Go to `https://phet.colorado.edu/en/simulations/circuit-construction-kit-ac`.

- In the AC Circuits PhET simulator, create an RL circuit, with $R = 10\Omega$, and $L = 1.0$ H, and a AC voltage source set to 120V amplitude.
  1. Click and drag a voltage chart from the right side to measure the AC source voltage.
  2. Click and drag a voltage chart from the right side to measure the inductor voltage.
  3. Create a spreadsheet with three columns: (1) frequency of the AC voltage source, (2) peak AC voltage source amplitude, and (3) peak capacitor voltage amplitude.

PhET: RL and RC Circuits as Filters

1. Create a graph with a range [0.0,1.0] for the y-axis, for a unitless ratio, and a domain of [0.0,2.0] Hz for the x-axis.
2. Plot the ratio of $V_{\mathrm{C}}/V_{\mathrm{source}}$ versus frequency on your graph.
3. Plot the ratio of $V_{\mathrm{L}}/V_{\mathrm{source}}$ versus frequency on your graph.
4. For a **bonus point**, convert your ratio to decibels (dB, $20\,\mathbf{log}_{10}(\mathit{ratio})$).
5. Which AC circuit is the *low-pass* filter, and which AC circuit is the *high-pass* filter? How is energy conserved if the voltage changes?

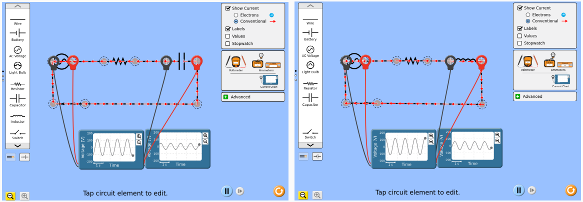

Figure 19: (Left) Example of the RC circuit. (Right) Example of the RL circuit.

Reactance, Inductive and Capacitive

Reactance is resistance that is associated with a change in the signal *phase*. What is the phase?

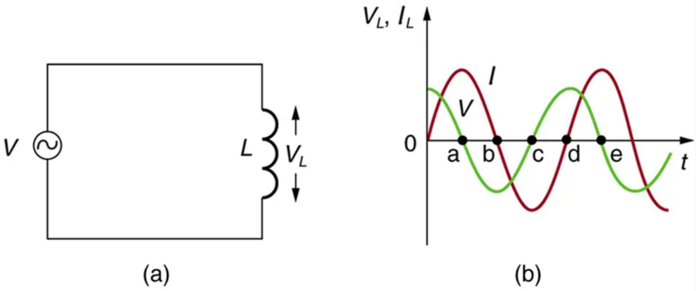

Figure 20: (a) An AC source connected to an inductor. (b) Voltage and current at the inductor are no longer *in-phase*. Voltage leads the current by a 90 degree phase shift.

Phase Shift of a Sinusoid

Let a time-dependent signal have an amplitude A, frequency f, and phase ϕ:

$$s(t) = A\cos(2\pi ft + \phi) \tag{58}$$

Two signals with phases ϕ_1 and ϕ_2 have a relative *phase shift* $\Delta\phi = \phi_2 - \phi_1$, measured in degrees or radians.

Reactance, Inductive and Capacitive

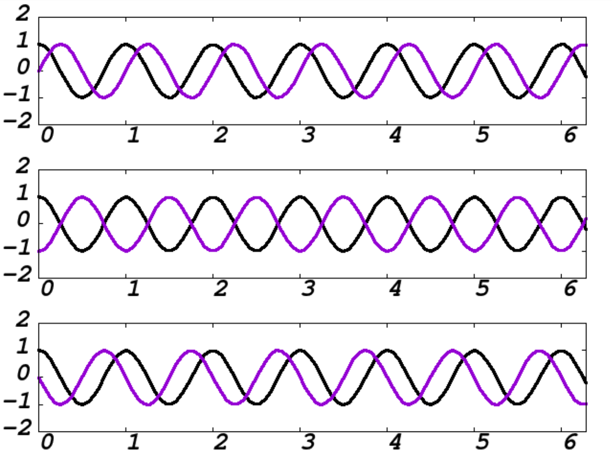

Figure 21: (Top) $\Delta\phi = 90$ deg. (Middle) $\Delta\phi = 180$ deg. (Bottom) $\Delta\phi = 270$ deg.

Reactance and Inductors

Let ϕ_{V} and ϕ_{I} be the phase of the voltage and current. When a sinusoidal voltage is applied to an inductor:

$$\phi_{\mathrm{V}} - \phi_{\mathrm{I}} = \pi/2 \tag{59}$$

The voltage leads the current by a 90 degree phase shift. The reactance from the inductor is X_{L}, and fits into Ohm's law like $V = IX_{\mathrm{L}}$, where V and I are the rms voltage and current. Finally,

$$X_{\mathrm{L}} = 2\pi f L \tag{60}$$

Units: 1 H is 1Ω s, and frequency is 1 Hz, or 1 s^{-1}.

Reactance, Inductive and Capacitive

Suppose the reactance of an inductor is 1 kΩ at a frequency of 1 kHz. What is the inductance?

- A: 2π Henries
- B: 2π Ohms
- C: $1/(2\pi)$ Henries
- D: $1/(2\pi)$ Ohms

What is the reactance at half the frequency, 0.5 kHz?

- A: π Henries
- B: π Ohms
- C: $1/(4\pi)$ Henries
- D: $1/(4\pi)$ Ohms

Hint, treat this like a scaling problem.

Suppose we are dealing with a *series* RL circuit. The resistor has $R = 1$ kΩ, and the frequency is $1/(2\pi)$ MHz. If the inductance is 1 mH, what is the total resistance, in kΩ?

- A: 2 kΩ
- B: 4 kΩ
- C: $1/(2\pi)$ kΩ
- D: $1/(4\pi)$ kΩ

Hint, deal with the units via powers of ten.

Suppose we are dealing with the same resistance and inductance, but they are connected *in parallel.* What is the total resistance?

- A: 0.25 kΩ
- B: 1 kΩ
- C: 0.1 kΩ
- D: 0.5 kΩ

Recall that series resistances (and reactances) sum differently when they are connected in parallel.

What is the rms current in a series RL circuit with $R = 1\ \mathrm{k\Omega}$, and $X_{\mathrm{L}} = 1\ \mathrm{k\Omega}$, if the rms voltage is 120 V?

- A: 60 A
- B: 120 mA
- C: 60 mA
- D: 120 mA

Reactance is resistance that is associated with a change in the signal *phase*. What is the phase?

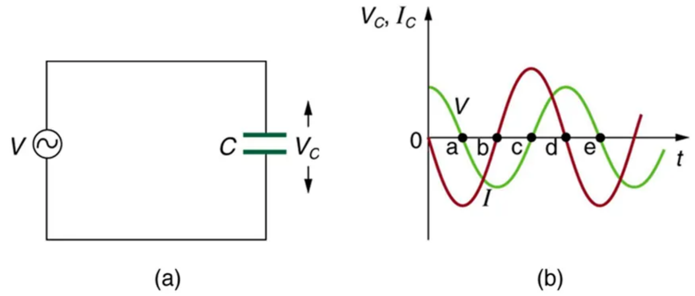

Figure 22: (a) An AC source connected to a capacitor. (b) Voltage and current at the inductor are no longer *in-phase*. Current leads the voltage by a 90 degree phase shift.

Reactance and Capacitors

Let ϕ_{V} and ϕ_{I} be the phase of the voltage and current. When a sinusoidal voltage is applied to a capacitor:

$$\phi_{\mathrm{V}} - \phi_{\mathrm{I}} = -\pi/2 \tag{61}$$

The voltage lags the current by a 90 degree phase shift. The reactance from the capacitor is X_{C}, and fits into Ohm's law like $V = IX_{\mathrm{C}}$, where V and I are the rms voltage and current. Finally,

$$X_{\mathrm{C}} = \frac{1}{2\pi f C} \tag{62}$$

Units: 1 F = C V^{-1}, $f \to 1\ \mathrm{s}^{-1}$, and 1 A V^{-1} is Ω^{-1}, $X_{\mathrm{C}} \to 1/(1/\Omega) = \Omega$.

Reactance, Inductive and Capacitive

Suppose the reactance of a capacitor is 1 kΩ at a frequency of 1 kHz. What is the capacitance?

- A: $2\pi\ \mu$F
- B: 2π F
- C: $1/(2\pi)\ \mu$F
- D: $1/(2\pi$ F

What is the reactance at half the frequency, 0.5 kHz?

- A: π F
- B: π μF
- C: $1/(\pi)$ μF
- D: $1/(\pi)$ F

Hint, treat this like a scaling problem.

Suppose we are dealing with a *series* RC circuit. The resistor has $R = 1$ kΩ, and the frequency is $1/(2\pi)$ kHz. If the capacitance is 1 μF, what is the total resistance, in kΩ?

- A: $1/(4\pi)$ kΩ
- B: 4 kΩ
- C: $1/(2\pi)$ kΩ
- D: 2 kΩ

Hint, deal with the units via powers of ten.

Suppose we are dealing with the same resistance and inductance, but they are connected *in parallel.* What is the total resistance?

- A: 0.25 kΩ
- B: 1 kΩ
- C: 0.1 kΩ
- D: 0.5 kΩ

Recall that series resistances (and reactances) sum differently when they are connected in parallel.

What is the rms current in a series RC circuit with $R = 1\ \mathrm{k}\Omega$, and $X_\mathrm{C} = 1\ \mathrm{k}\Omega$, if the rms voltage is 120 V?

- A: 60 mA
- B: 120 mA
- C: 60 A
- D: 120 mA

Reactance is resistance that is associated with a change in the signal *phase*. What is the phase?

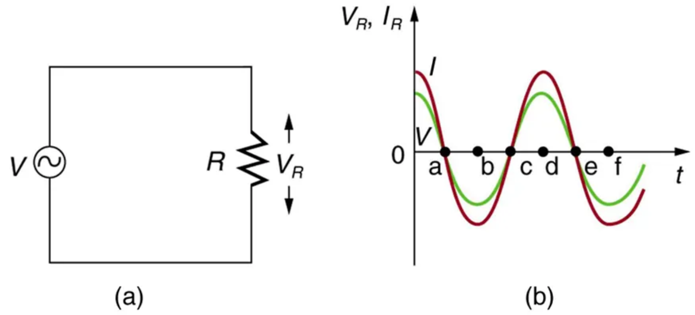

Figure 23: (a) An AC source connected to a resistor. (b) Voltage and current at the inductor are *in-phase*. Current and voltage have a phase shift of 0 degrees.

RLC Circuits

A generalization of Ohm's Law for AC circuits that involve resistance, capacitance, and inductance is

$$V_{\mathrm{rms}} = I_{\mathrm{rms}} Z \tag{63}$$

$$V_0 = I_0 Z \tag{64}$$

The impedance is Z, and subscripts refer to the rms and peak values.

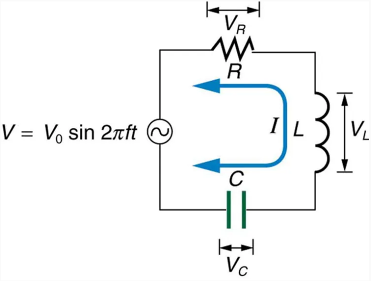

Figure 24: An AC source connected to an RLC circuit.

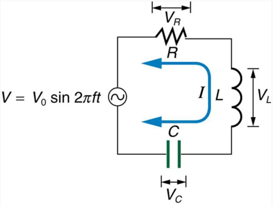

Figure 25: How do we calculate impedance in the RLC circuit, and how do we predict the behavior of current?

We cannot simply sum the impedances of the R, L, and C components. The reason is *phase*:

$$\phi_{\mathrm{V,RL}} - \phi_{\mathrm{I,RL}} = \pi/2 \tag{65}$$

$$\phi_{\mathrm{V,RC}} - \phi_{\mathrm{I,RC}} = -\pi/2 \tag{66}$$

$$\phi_{\mathrm{V,RLC}} - \phi_{\mathrm{I,RLC}} = \pi \tag{67}$$

The phase shift between the inductor and capacitor voltages is 180 degrees.

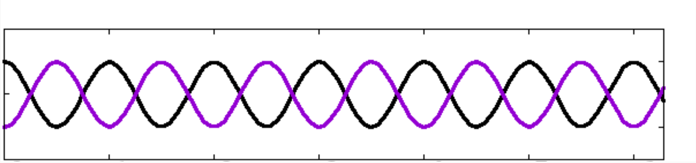

Figure 26: Two sinusoids with a relative phase shift of 180 degrees.

Using complex numbers to represent Z_{R}, Z_{L}, and Z_{C}, we can show that the **total impedance** follows

$$Z = \sqrt{R^2 + (X_{\mathrm{L}} - X_{\mathrm{C}})^2} \tag{68}$$

An RLC circuit is set up at $f = 10$ kHz with $R = 1$ kΩ, $X_{\mathrm{L}} = 0.5$ kΩ and $X_{\mathrm{C}} = 2$ kΩ, what is the total Z?

- A: 1.0 kΩ
- B: 1.8 kΩ
- C: 0.9 kΩ
- D: 2.0 kΩ

What is the total Z if the frequency is *doubled* to 20 kHz?

- A: 1.0 kΩ
- B: 1.8 kΩ
- C: 0.9 kΩ
- D: 2.0 kΩ

What do you notice about this impedance?

- A: It is the minimum total impedance.
- B: It is the maximum total impedance.
- C: $Z = R$
- D: A and C

Consider Ohm's law, and the RLC impedance:

$$I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{\sqrt{R^2 + (X_{\mathrm{L}} - X_{\mathrm{C}})^2}} \tag{69}$$

Setting $X_{\mathrm{L}} = X_{\mathrm{C}}$, we find the original form of Ohm's Law for AC circuits: $I_{\mathrm{rms}} = V_{\mathrm{rms}}/R$. Because $(X_{\mathrm{L}} - X_{\mathrm{C}})^2$ is always positive, current is maximized when the inductor and capacitor reactance are equal. This implies

$$X_{\mathrm{L}} = X_{\mathrm{C}} \tag{70}$$

$$2\pi f L = \frac{1}{2\pi f C} \tag{71}$$

$$\boxed{f = \frac{1}{2\pi\sqrt{LC}}} \tag{72}$$

Given L and C, there is a *resonance frequency* that maximizes current.

The resonance frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{73}$$

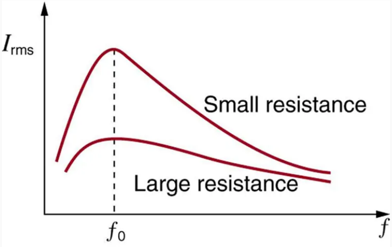

Figure 27: The peak and width of I_{rms} vs. f is controlled by choices for R, L, and C.

What is the resonance frequency for an RLC circuit with $L = 1\ \mu$H and $C = 1\ \mu$F?

- A: $1/(2\pi)$ Hz
- B: 1 kHz
- C: $1/(2\pi)$ GHz
- D: $1/(2\pi)$ MHz

PhET: RLC Circuits and Resonators

PhET: RL and RC Circuits as Filters

Go to `https://phet.colorado.edu/en/simulations/circuit-construction-kit-ac`.

- In the AC Circuits PhET simulator (virtual lab), create a series RLC circuit, with $R = 1\Omega$, $L = 1.0$ H, $C = 0.05$ F, and AC voltage source amplitude of 12 V.
- Measure the AC source voltage with a voltage chart.
- Measure the current with a current chart and ammeter between the inductor and capacitor.
- Create a spreadsheet with two columns: (1) frequency of the AC voltage source, and (2) peak current. Make measurements in frequency steps of 0.1 Hz, from [0.1-1.4] Hz.
- Graph peak current versus frequency, and identify the resonance frequency. Does it match Eq. 74?

Using complex numbers for impedances, we can show the power in the series RLC circuit is

$$P_{\mathrm{ave}} = I_{\mathrm{rms}} V_{\mathrm{rms}} \cos\phi \tag{74}$$

The angle ϕ is the phase shift between voltage and current. When $f = 1/(2\pi\sqrt{LC})$, $\phi = 0$ degrees, so $\cos\phi = 1$. When $\phi \neq 0$ degrees, voltage and current are out of phase, and the usable power is reduced.

Final project proposal: Build a series RLC circuit, and simulate it with the AC circuit PhET. Measure power vs. phase angle.

Conclusion

Summary

1. Magnetic induction - **Chapters 23.1 - 23.5, 23.7, 23.9**
    - Induced EMF and magnetic flux
    - Faraday's Law
    - Motional EMF, generators, and transformers
2. AC circuits - **Chapters 23.9 - 23.12**
    - Inductors
    - RL circuits
    - RLC circuits