Algebra-Based Physics: Electricity, Magnetism, and Modern Physics (PHYS135B): Unit 0

Jordan Hanson

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Whittier College Department of Physics and Astronomy

Summary

Unit 0 Summary

Reading: Chapters 3.1 - 3.3, 18.1 - 18.5, 19.1 - 19.3

1. Estimation/Approximation
2. Coordinates and Vectors
3. Review of concepts from Newtonian mechanics
    - Kinematics and Newton's Laws
    - Work-energy theorem, energy conservation
    - Momentum, conservation of momentum
4. Electrostatics I: charges and fields
5. Electrostatics II: potential, and potential energy

Estimation/Approximation

Estimation/Approximation

In science and engineering, estimation is to obtain a quantity in the absence of precision, informed by rational constraints.

1. Define relevant unit scales: (mg, g, or kg), (m/s or km/hr)
2. Obtain complex quantities from simple ones
   - Obtain *areas* and *volumes* from *lengths*
   - Obtain *rates* from *numerators* and *denominators*
3. Taking advantage of scaling problems
   - Knowing *relationship* between variables
   - Using that *relationship* to obtain new information
4. Constrain the unknown with upper and lower limits

Unit scale: A generation is about one-third of a lifetime. Determine how many generations have passed since the year 0 AD[1].

- A: 10
- B: 20
- C: 60
- D: 100

[1] What is the appropriate scale here?

Unit scale: (a) What fraction of Earth's diameter[2] is the greatest ocean depth (11 km below sea level)? (b) The greatest mountain height (8.8 km above sea level)?

- A: 8.6×10^{-2}, 6.9×10^{-2}
- B: 8.6×10^{-3}, 6.9×10^{-3}
- C: 8.6×10^{-4}, 6.9×10^{-4}
- D: 8.6×10^{-5}, 6.9×10^{-3}

[2] The diameter of the Earth is 12,800 km.

Complex quantities: Assuming one nerve impulse must end before another can begin, what is the maximum firing rate of a nerve in impulses per second?

- A: 1 per second (1 Hz)
- B: 30 per second (30 Hz)
- C: 60 per second (60 Hz)
- D: 100 per second (100 Hz)

Complex quantities: If a Whittier College athlete ran the 5k race at a track meet in 35 minutes, what was her average speed?

- A: 0.3 meters per second
- B: 3 meters per second
- C: 30 meters per second
- D: 300 meters per second

Estimation/Approximation

Complex quantities: Suppose you won the lottery and received \$1 billion USD. Because your life is dope, you stack that paper over the Whittier College soccer field. Each stack contains 100 bills, and each bill is worth \$100. If you evenly cover the field, how high is the money level?

- A: 0.5 inch
- B: 1 inch
- C: 2 inches
- D: 1 foot

Scaling problem: Supposed you have an ideal gas in a cylinder of fixed volume. If the pressure begins as 100 kPa, and you *double* the temperature of the gas, what is the new pressure?

- A: 100 kPa
- B: 50 kPa
- C: 10 kPa
- D: 200 kPa

Scaling problem: Supposed you have an ideal gas in a cylinder of fixed volume. If the pressure begins as 100 kPa, and you *halve* the temperature of the gas, what is the new pressure?

- A: 100 kPa
- B: 50 kPa
- C: 10 kPa
- D: 200 kPa

Upper/lower limits: How many undergraduate students are there at Whittier College[3]?

- A: 5,000
- B: 1,000
- C: 1,250
- D: 500

[3] What is the absolute lower limit, and what is the upper limit?

Upper/lower limits: What is the average yearly college tuition in the United States (before subtracting grants and scholarships)?

- A: \$5,000
- B: \$10,000
- C: \$25,000
- D: \$40,000

What information affects the upper and lower limits here?

Coordinates and Vectors

Coordinates and Vectors (Chapters 3.1 - 3.3)

Physics requires mathematical objects to build equations that capture the behavior of nature. Two examples of such objects are scalar and vector quantities. Each type of object obeys similar but different rules.

1. Scalar quantities
   - mass: $m_1 + (m_2 + m_3) = (m_1 + m_2) + m_3$
   - speed: $v_1(v_2 + v_3) = v_1 v_2 + v_1 v_3$
   - charge: $q_1 \left(\frac{1}{q_1} \right) = 1$, $q_1(0) = 0$
2. Vector quantities
   - velocity: $\vec{v}_1 + (\vec{v}_2 + \vec{v}_3) = (\vec{v}_1 + \vec{v}_2) + \vec{v}_3$
   - tension: $\vec{t}_1 \cdot (\vec{t}_2 + \vec{t}_3) = \vec{t}_1 \cdot \vec{t}_2 + \vec{t}_1 \cdot \vec{t}_3$

Examples: break into components, adding two vectors.

A vector may be expressed as *a list of scalars*: $\vec{v} = (4, 2)$ (a vector with two *components*), $\vec{u} = (3, 4, 5)$ (three *components*). Now, we know how to add and subtract scalars. How do we add and subtract vectors?

What is
$(1, 3, 8)+$
$(0, 2, 1)$?
Answer: $(1, 5, 9)$

In other words, when adding vectors, we add them component by component. **Work several examples.**

How do we subtract vectors? In the same fashion:

What is
$(1,3,8)-$
$(0,2,1)$?
Answer: $(1,1,7)$

In other words, when subtracting vectors, we subtract them component by component. **Work several examples.**

How do we multiply vectors? In the same fashion, *for one kind of multiplication*:

What is
$(1, 3, 8) \cdot (0, 2, 1)$?
Answer: $1 \cdot 0 + 3 \cdot 2 + 8 \cdot 1 = 14$

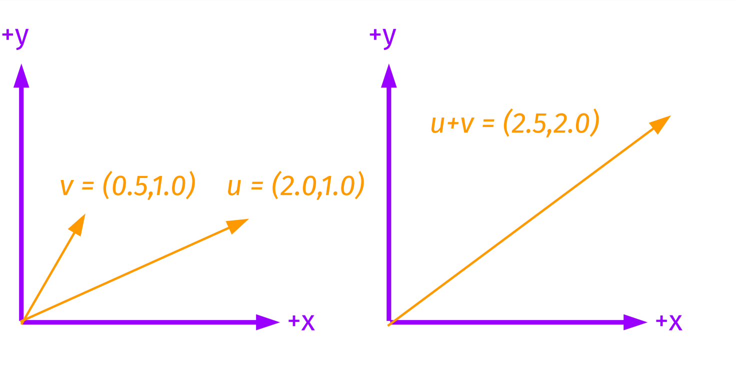
This kind of multiplication is known as the dot-product. There is also the *cross-product*, which we will save for later. **Work several examples.**

Coordinates and Vectors (Chapters 3.1 - 3.3)

The components of a vector may describe quantities in a coordinate system, such as *Cartesian coordinates* - after René Descartes. Vectors in the 3D Cartesian coordinate system (x,y,z) may be written in the following notation:

$$\boxed{\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}}$$

- a: The amount in the +x-direction, $\hat{i}$: a vector of length 1, in the +x-direction
- b: The amount in the +y-direction, $\hat{j}$: a vector of length 1, in the +y-direction
- c: The amount in the +z-direction, $\hat{k}$: a vector of length 1, in the +z-direction



(a) (b)

Figure 1: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} + \vec{v}$? Adding components: $\vec{u} + \vec{v} = 2.5\hat{i} + 2.0\hat{j}$.

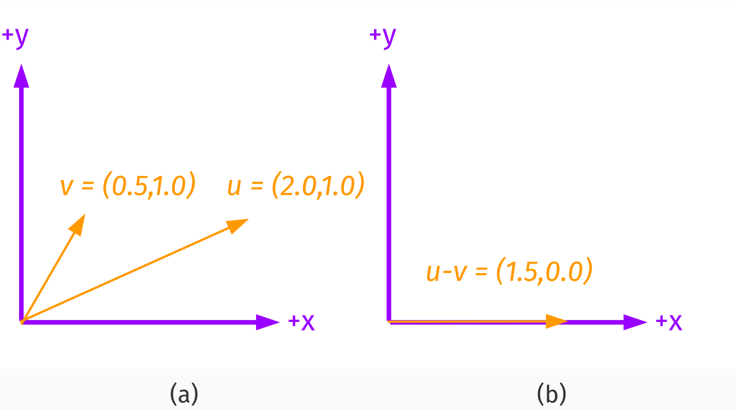
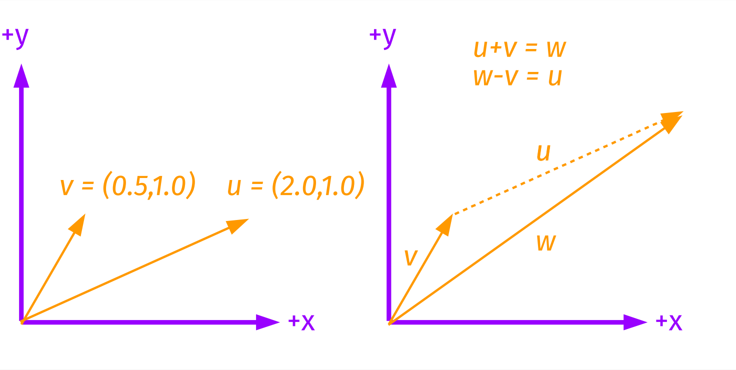


Figure 2: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} - \vec{v}$? Subtracting components: $\vec{u} - \vec{v} = 1.5\hat{i} + 0.0\hat{j}$.

Coordinates and Vectors (Chapters 3.1 - 3.3)



(a) (b)

Figure 3: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) To compute $\vec{w} - \vec{v}$, arrange the vectors to get a sense of the result, $\vec{u}$.

Suppose $\vec{x}_i = -3\hat{i} + 2\hat{j}$ km, and $\vec{x}_f = -3\hat{i} - 2\hat{j}$ km. What is the *displacement*?

- A: $4\hat{i}$ km
- B: $-4\hat{i}$ km
- C: $4\hat{j}$ km
- D: $-4\hat{j}$ km

Suppose $\vec{x}_i = 3\hat{i} - 2\hat{j}$ km, and $\vec{x}_f = 3\hat{i} - 2\hat{j}$ km. What is the *displacement*?

- A: 0 km
- B: $0\hat{i} + 0\hat{j}$ km
- C: $1\hat{i}$ km
- D: $1\hat{j}$ km

We define the *position* of an object as a vector locating it in a given coordinate system. The scalar *distance* is the norm of the position vector, that is, the distance to to the origin.

Now we can introduce the concept of displacement: a vector describing a movement of an object.

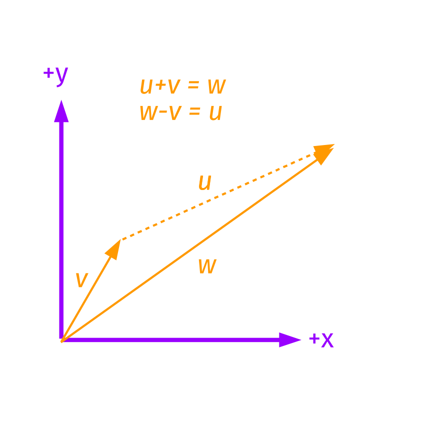


Figure 4: Suppose an object moves from position $\vec{v}$ to $\vec{w}$. In this case, the displacement is $\vec{u}$. **Thus, the final position is the initial position, plus the displacement.**

It follows that the *displacement* is zero if the initial and final positions are the same, but the *distance travelled* is not.

Suppose a jet fighter travelling at 800 km per hour banks such that it flies in a circle of radius 0.5 km. How long does it take to complete the circle? What is the distance traveled, and what is the displacement?

- A: 2π km, 28 seconds, 2π km
- B: π km, 14 seconds, π km
- C: π km, 28 seconds, π km
- D: π km, 14 seconds, 0 km

Average velocity is the ratio of the displacement to the elapsed time.

$$\boxed{\vec{v}_{\mathrm{avg}} = \frac{\Delta \vec{x}}{\Delta t}} \tag{1}$$

The *average speed* is the norm of the average velocity:

$$\boxed{v_{\mathrm{avg}} = \frac{|\Delta \vec{x}|}{\Delta t}} \tag{2}$$

If the motion is in one dimension, then the average speed is

$$v_{\mathrm{avg}} = \frac{x_{\mathrm{f}} - x_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} \tag{3}$$

$\vec{p} = 4\hat{i} + 2\hat{j}$. $\vec{q} = -4\hat{i} + 2\hat{j}$.
Compute $\vec{p} \cdot \vec{q}$.

- A: 12
- B: -12
- C: 4
- D: 8

$\vec{p} = -1\hat{i} + 6\hat{j}$. $\vec{q} = 3\hat{i} + 0.5\hat{j}$.
Compute $\vec{p} \cdot \vec{q}$.

- A: -1
- B: 1
- C: 0
- D: 3

Why was the last answer zero? Look at it graphically:

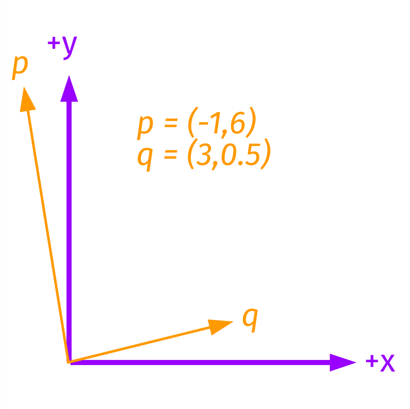


Figure 5: Two vectors $\vec{p}$ and $\vec{q}$ are *orthogonal* if $\vec{p} \cdot \vec{q} = 0$.

The *length* or *norm* of a vector $\vec{v} = a\hat{i} + b\hat{j}$ is $|\vec{v}| = \sqrt{a^2 + b^2}$.

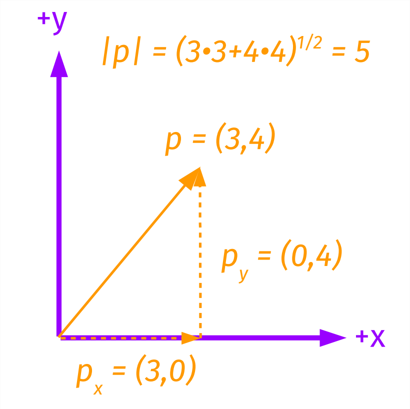


Figure 6: Computing the norm of a vector $\vec{p}$.

Notice that $\sqrt{\vec{p} \cdot \vec{p}} = |\vec{p}|$.
Let θ_p be the angle between $\vec{p}$ and the x-axis.
$p_x = \vec{p} \cdot \hat{i} = |\vec{p}| \cos(\theta_p)$.
$p_y = \vec{p} \cdot \hat{j} = |\vec{p}| \sin(\theta_p)$.

Theorem: The dot product of two vectors $\vec{p}$ and $\vec{q}$ is $|u||v| \cos(\theta)$, if θ is the angle between them.

Proof: $\vec{p} \cdot \vec{q} = p_x q_x + p_y q_y = |p||q| \cos\theta_p \cos\theta_q + |p||q| \sin\theta_q \sin\theta_q$
$= |p||q|(\cos\theta_p \cos\theta_q + \sin\theta_p \sin\theta_q) = |p||q| \cos(\theta_p - \theta_q)$
$= |p||q| \cos\theta$.

$\boxed{\vec{p} \cdot \vec{q} = |p||q| \cos\theta}$

An object moves at 2 m/s at $\theta = 60°$ with respect to the x-axis. What is the velocity of the object?

- A: $(1\hat{i} + 1\hat{j})$ m/s
- B: $(\sqrt{3}\hat{i} + 1\hat{j})$ m/s
- C: $(\sqrt{3}\hat{i} + \sqrt{3}\hat{j})$ m/s
- D: $(1\hat{i} + \sqrt{3}\hat{j})$ m/s

An object moves at 2 m/s at $\theta = 120°$ with respect to the x-axis. What is the velocity of the object?

- A: $(-1\hat{i} + \sqrt{3}\hat{j})$ m/s
- B: $(1\hat{i} - \sqrt{3}\hat{j})$ m/s
- C: $(-1\hat{i} + \sqrt{3}\hat{j})$ m/s
- D: $(-1\hat{i} - \sqrt{3}\hat{j})$ m/s

Is it possible to multiply vectors and scalars? Of course: $a_1\vec{p} = a_1 p_x \hat{i} + a_1 p_y \hat{j}$.

Also, multiplication properties still hold. For example: $(a_1 + a_2)\vec{p} = a_1\vec{p} + a_2\vec{p}$.

A spacecraft moves at 400 m/s, at an angle of 30 degrees with respect to the x-axis. If it fires two thrusters that boost the x-component and y-component of the velocity by 25% and 50%, respectively, what is the final velocity?

- A: $(433\hat{i} + 300\hat{j})$ m/s
- B: $(300\hat{i} + 433\hat{j})$ m/s
- C: 400 m/s
- D: $(400\hat{i} + 433\hat{j})$ m/s

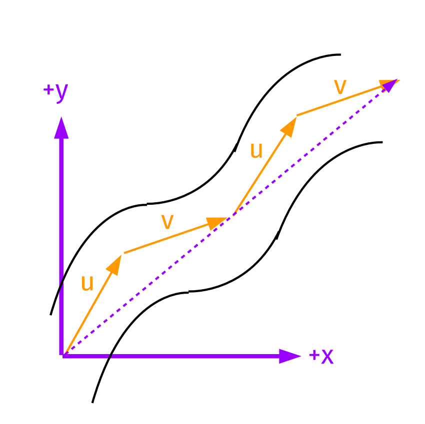


Figure 7: A Formula-1 driver keeps his car on the track by following a path approximated by the position vectors *u*, *v*, *u*, and *v*. The dashed arrow represents the total displacement.

If $\vec{u} = (20\hat{i} + 30\hat{j})$ m, and $\vec{v} = (30\hat{i} + 20\hat{j})$ m, what is the total displacement? If the elapsed time is 10 seconds, what is the magnitude of the average velocity?

- A: $(50\hat{i} + 50\hat{j})$ m, 14 m/s
- B: $(80\hat{i} + 100\hat{j})$ m, 10 m/s
- C: $(100\hat{i} + 100\hat{j})$ m, 14 m/s
- D: $(50\hat{i} + 150\hat{j})$ m, 10 m/s

PhET simulation about vector addition:
`https://phet.colorado.edu/en/simulation/vector-addition`

Kinematics and Newton’s Laws

Kinematics - A description of the motion of particles and systems
Dynamics - An explanation of the motion of particles and systems

What causes an object to move? **Forces**. Forces exist as a result of the **interactions** of objects or systems.

Evolution - A description of the change of biological species
Natural Selection - An explanation of change in biological species

What causes species to evolve? **Natural selection**. Natural selection exists because of election pressures, numerous offspring, and variation among offspring.

Newton's First Law: A man slides a palette crate across a concrete floor of his shop. He exerts a force of 60.0 N, and the box has a constant velocity of 0.5 m/s. What force cancels his pushing force, and what is the value in Newtons?

- A: wind, 60.0 N
- B: friction: 60.0 N
- C: friction: -60.0 N
- D: weight: -60.0 N

Newton's Second Law: The crate has a mass of 50 kg, and encounters an area where there is no longer friction. If the pushing force is still 60 N, what is the acceleration?

- A: $1.0\ m/s^2$
- B: $0.8\ m/s$
- C: $1.2\ m/s$
- D: $1.2\ m/^2$

Kinematics: If the acceleration is 1.2 m/s^2, and the crate begins with a velocity of 1 m/s, what is the velocity after 5 seconds?

- A: 4 m/s
- B: 5 m/s
- C: 6 m/s
- D: 7 m/s

Newton's Second Law: Suppose there is no pushing force, but the crate moves at 5 m/s through an area with a frictional force that has a magnitude of 5 N. If the crate still weighs 50 kg, what is the acceleration?

- A: 0.2 m/s^2
- B: -0.1 m/s^2
- C: 1 m/s^2
- D: -2 m/s^2

Newton's Third Law: If a person hangs from a horizontal rope (with the ends tied to two walls), and the person has a weight $\vec{w} = -600N$, what is the total upward component of the tension in the rope?

- A: -600 N
- B: 60 N
- C: 600 N
- D: -60 N

Newton's Third Law: If a heavy truck and a light car collide, which exerts the larger force on the other?

- A: The heavy truck exerts a larger force on the car.
- B: The light car exerts a larger force on the heavy truck.
- C: They exert the same force on each other.
- D: Cannot determine.

Work-Energy Theorem and Conservation of Energy

Group board exercise: A firework of mass 1 kg is launched straight upwards. The gunpowder releases 500 J of energy. What is the velocity of the shell as it leaves the launcher? How high does it fly straight upwards?

Three useful concepts: 1) Work equation 2) Work-energy theorem 3) gravitational potential energy.

Work-energy theorem: How high in the air would a 0.1 kg rock go if it was launched straight upward by a spring with $k = 1000$ N/m, if the spring was compressed 0.1 m?

- A: 0.5 m
- B: 5 m
- C: 50 m
- D: 500 m

Note: the potential energy of a spring with spring constant k and displacement x is $U = \frac{1}{2}kx^2$.

Work-energy theorem: How high would it go if the spring was compressed 0.2 m?

- A: 100 m
- B: 200 m
- C: 500 m
- D: 50 m

Note: think of this exercise as a scaling problem.

Momentum

Momentum

A ball with mass 0.1 kg moves at 1 m/s. It strikes a stationary ball with twice the mass and stops. The heavier ball moves with a velocity of

- A: 0.1 m/s
- B: 1 m/s
- C: 5 m/s
- D: 0.5 m/s

A ball with mass 0.1 kg moves at 1 m/s. It strikes a stationary ball with the same mass and they stick together. What is the final velocity of the object?

- A: 0.1 m/s
- B: 1 m/s
- C: 5 m/s
- D: 0.5 m/s

Momentum

If the mass of an object that is rotating around an origin with angular velocity ω decreases by a factor of 2, the new angular velocity will be:

- A: $-\omega$
- B: -3ω
- C: 2ω
- D: ω

Electrostatics I

Electrostatics II

Conclusion

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