Grace Gronnis

Midterm 3

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April 24, 2020

1 Memory Bank

1. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
2. $P = IV$ ... Relationship between power, current, and voltage.
3. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
4. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.
5. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ... Ampère's Law.
6. $\epsilon = -Nd\phi/dt$ ... Faraday's Law.
7. $\phi = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux.
8. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
9. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L. Self-inductance: $\Delta V = -L(dI/dt)$.
10. Units of inductance: V s A^{-1}, which is called a Henry, or H.
11. $B = \mu_0 nI$ ... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

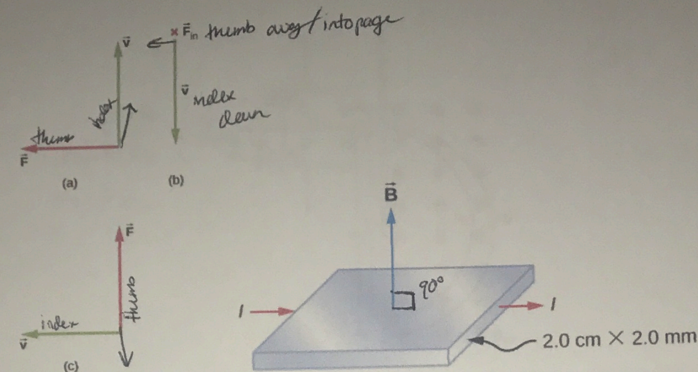


Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.
   - a: into the page
   - b: to the left
   - c: it would be coming out of the page

2. Consider Fig. 1 (right). **The Hall Effect**. An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that $F_E = F_B$

$$\Delta V = \frac{B \Delta x I}{n q_e A} \tag{1}$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28}$ m^{-3}, $A = 1$ mm^2 ($1 \times 10^{-6}\ m^2$), and q_e is the (1.6×10^{-19}) charge of an electron.

a) $\vec{F} = q\vec{v} \times B$

$F_B = qvB$

$F_E = qE$

$F_E = F_B$

$qvB = qE$

$E = vB$

$v = E/B$

b) $E = \Delta V/\Delta x$ $\quad E = vB$

$\Delta V = E\Delta x$ $\quad v = I/nqA$

$\Delta V = vB\Delta x$

$\Delta V = \frac{B\Delta x I}{nqA}$

$\Delta V = \frac{(1.33)(2\times10^{-2})(10)}{(2\times10^{28})(1\times10^{-6})(1.6\times10^{-19})} = 8.3 \times 10^{-5} V$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^4 A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.) $A = \pi r^2 \rightarrow \pi(0.65\times10^{-15})^2 = 1.33 \times 10^{-30} m^2$

$A = 1.33 \times 10^{-30} m^2$

$I = 1.05 \times 10^4 A$

$B = 2.50T$

$\theta = 90?$ perpendicular

$\tau = NIAB \sin\theta \rightarrow (1.05\times10^4)(1.33\times10^{-30})(2.50T) \sin 90$

$\tau = 3.49 \times 10^{-26} N\cdot m$

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter (1m?), carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

a) $B = \mu_0 n I$

$n = 500$ turns/m

$I = 0.3A$

$\mu_0 = 4\pi \times 10^{-7} H/m$

$B = (4\pi\times10^{-7})(500)(0.3)$

$= 1.88 \times 10^{-4} T$

b) $B = \mu_0 n I$

$n = 500$

$I = 0.3$

$\mu_0 = 4\pi \times 10^{-7} \times 5000 = 0.0063$

$B = (0.0063)(500)(0.3) = 0.95T$

$(0.945T)$

$\checkmark - R$

$F = qE = F_E$

$F = qvB = F_B$

$F_B = F_E$

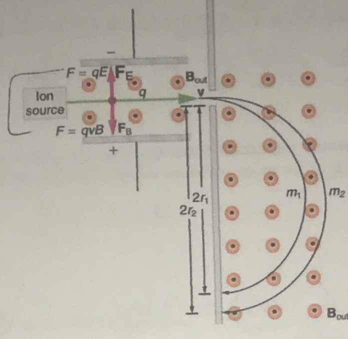


Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$F = \frac{mv^2}{r}$ $\qquad$ $F = qvB$

$$r = \frac{mE}{qB^2} \tag{2}$$

c) ?

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m and the B-field is 0.01 T. What is the distance r?

a) $v = E/B \rightarrow E = vB$

$F_{net} = qE - qvB \rightarrow E - vB$

if $E = vB$ then $E - vB = 0 \rightarrow F_{net} = 0$

b) $qvB = \frac{mv^2}{r}$

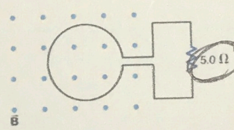
$rqB = mv$

$r = \frac{mv}{qB} \rightarrow \frac{mE}{qB^2}$ $\qquad$ $\frac{m(E/B)}{qB}$

c) $m = 16 \times (1.67 \times 10^{-27}) = 2.67 \times 10^{-26}$ kg

$r = \frac{(2.67 \times 10^{-26})(10)}{(1.6 \times 10^{-19})(0.01^2)} = 0.0167\,m$

4 Chapter 13: Electromagnetic Induction



$V = I \cdot R$?

$R = 5.0\,\Omega$ $\qquad$ @ $t_1 = 0.055$ ✓

d) $I = \frac{5.5 \times 10^{-2}}{5.0} = 1.1 \times 10^{-2}$ A

or 0.011 A

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and changes in magnitude according to

$$\frac{\Delta B}{\Delta t} = \frac{B_0}{T_0}(\sin(2\pi ft)) \tag{3}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage (emf) in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, $f = 10^3$ Hz, and $T = 1$ ms, what is the induced emf at $t = 0$? (c) What about $t_1 = 0.16$ ms? (d) What is the current through the resistor at t_1?

a) $emf = -N\, d\phi/dt$ $\quad$ ($N = 1$)

$emf = \frac{d\phi}{dt} \rightarrow \frac{d(BA)}{dt}$

$A = \pi r^2$

$emf = \frac{dB}{dt}(\pi r^2)$ ~~(πr^2)~~ $\left(\frac{B_0}{T_0}\right)(\sin(2\pi ft))$

$emf = (\pi r^2)\left(\frac{B_0}{T_0}\right)(\sin(2\pi ft))$

b) $T = 1 \times 10^{-3}$ s

$emf = (\pi \cdot 0.1^2)\left(\frac{0.1}{1 \times 10^{-3}}\right) \underbrace{\sin(2\pi \cdot 10^3 \cdot 0)}_{0}$

$(\pi \cdot 0.1^2)\left(\frac{0.1}{1 \times 10^{-3}}\right) \times 0 = 0$

c) $t_1 = 0.16 \times 10^{-3}$ s

$emf = (\pi \cdot 0.1^2)\left(\frac{0.1}{1 \times 10^{-3}}\right)\sin(2\pi \cdot 10^3 \cdot 0.16 \times 10^{-3})$

$= 5.5 \times 10^{-2}$ V or 0.055 V ✓

[1] Molecules that do not have this velocity will hit the sides of this portion of the instrument.

sorry messy

5

5 Chapter 14: Inductance

$H = V \cdot s \cdot A^{-1}$ $\rightarrow A/s$

L ΔV?

1. What is (a) the rate at which the current though a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

$\Delta V = -L(dI/dt)$ $\quad \frac{\Delta V}{L} = \frac{dI}{dt}$] rate $\rightarrow \frac{0.150V}{0.50H} = \frac{dI}{dt} = 0.3 A/s$

L

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time (dt) must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$2\times10^{-3} H = L$ $\quad \Delta V$

$\Delta V = -L(dI/dt)$

$dt = \frac{L}{\Delta V}(dI) \rightarrow \frac{(2\times10^{-3})(0.100)}{500} = \frac{(2\times10^{-3})(0.100)}{500} = 4.0\times10^{-7} s$