# algebra-based physics-2: electricity, magnetism, and modern physics (phys135b-01): unit 1

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# UNIT 0 REVIEW

*Physics* - $\phi\upsilon\sigma\iota\kappa\acute{\eta}$ - "phusiké": *knowledge of nature*
from $\phi\acute{\upsilon}\sigma\iota\varsigma$ - "phúsis": *nature*

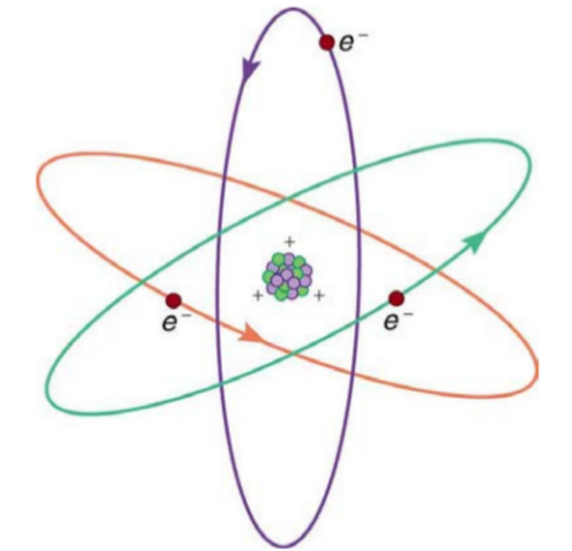
**Reading: Chapters 18 and 19 (for Unit 1)**

1. Estimation/Approximation
    - Estimating the correct order of magnitude
    - Building complex quantities
    - Unit analysis
2. Review of concepts from Newtonian mechanics
    - Kinematics and Newton's Laws
    - Work-energy theorem, energy conservation
    - Momentum, conservation of momentum

# Unit 0 Review Problems

# SUMMARY

# WELCOME TO ELECTROMAGNETISM AND MODERN PHYSICS!



# WELCOME TO ELECTROMAGNETISM AND MODERN PHYSICS!

# WELCOME TO ELECTROMAGNETISM AND MODERN PHYSICS!

**Reading: Chapters 18 and 19**

1. Charge, mass, the Coulomb force, and the gravitational force
2. Force fields
3. Electric potential and capacitance

# JITT - READING QUIZ RESULTS

# CHARGE, CONDUCTORS AND INSULATORS

Let's begin this topic in a special way: comparison to *gravity*. What do electricity and gravity have in common? The answer lies in a notion we call *charge*...

Charge: the constant of proportionality between the strength of a *field* and the force a field exerts on an *object*.

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Gravity

1. Force: $\vec{F} = G\frac{mM}{r^2}\hat{r}$
2. Parameters: $r$ is absolute distance between two objects with masses $m$ and $M$, and the direction is $\hat{r}$
3. *Charge* of one object: $m$
4. *Field felt by that object*: $\vec{G} = G\frac{M}{r^2}\hat{r}$
5. $\vec{F} = m\vec{G}$

Electricity

1. Force: $\vec{F} = k\frac{qQ}{r^2}\hat{r}$
2. Parameters: $r$ is absolute distance between two objects with electric charges $q$ and $Q$, and the direction is $\hat{r}$
3. *Charge* of one object: $q$
4. *Field felt by that object*: $\vec{E} = G\frac{Q}{r^2}\hat{r}$
5. $\vec{F} = q\vec{E}$

Charge: the constant of proportionality between the strength of a *field* and the force a field exerts on an *object*.

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In the field paradigm, objects with charges *emanate* fields, causing other objects with charge to experience force.

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Gravity

How many *types of charge*, or how many charges, exist under the force of gravity?

**One.** We call it mass.

Electricity

How many *types of charge*, or how many charges, exist under the force of electricity?

**Two.** We call one positive, and one negative.

Charge: the constant of proportionality between the strength of a *field* and the force a field exerts on an *object*.

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In the field paradigm, objects with charges *emanate* fields, causing other objects with charge to experience force.

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In the field paradigm, gravity has one charge (mass), and electricity has two charges (positive and negative).

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**There is one fundamental fact that is puzzling.** What about Newton's 2nd law? Acceleration is not a field, it is a kinematic function.

$$\vec{F}_{\mathrm{net}} = m\vec{a} \tag{1}$$

Aparently there are *two kinds of mass*: **inertial** and **gravitational**.

*Equivalence principle:*

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There are *two kinds of mass*: **inertial** and **gravitational**, with **equal value** for a given object.

`https://en.wikipedia.org/wiki/Equivalence_principle`

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There is no similar principle for charge. If the electric force on a charged object is calculated, that force must still be inserted into **Newton's 2nd Law** to obtain the acceleration, and the inertial mass must be known.

Charge has other properties, some similar to gravitational mass:

1. Charge is conserved globally (charge cannot be created nor destroyed). Mass has the same property.
2. Charge is conserved locally (if we pull charge out of the system, charge will flow into the system).
3. Charge is quantized, with an electron (for example) having the fundamental negative unit, and a proton (for example) having the fundamental positive unit.
4. The laws of physics are the same for positive and negative charges.
5. The two kinds of charge emit fields that attract each other; fields emitted by charges of the same type repel such charges.

**Benjamin Franklin and the Leyden Jar**. (Good paper topic).

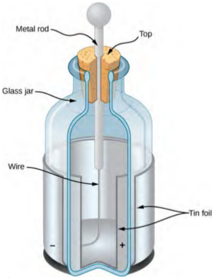


Figure 5.6 A Leyden jar (an early version of what is now called a capacitor) allowed experimenters to store large amounts of electric charge. Benjamin Franklin used such a jar to demonstrate that lightning behaved exactly like the electricity he got from the equipment in his laboratory.

**Figure 1:** A Leyden jar was an early version of a capacitor. Benjamin Franklin guessed that one type of charge moves and another remains stationary, explaining several behaviors of charged objects.

The rest of the properties of charge are connected to the development of the structure of the atom, and we will return to this topic at the end of the semeter.

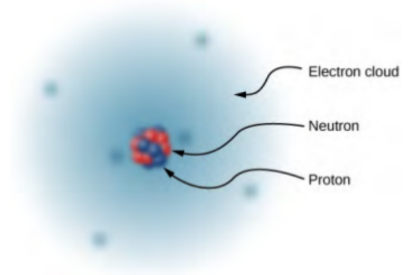


Figure 5.8 The nucleus of a carbon atom is composed of six protons and six neutrons. As in hydrogen, the surrounding six electrons do not have definite locations and so can be considered to be a sort of cloud surrounding the nucleus.

**Figure 2:** A sketch of our current atomic paradigm.

Suppose an ion is composed of six protons, eight neutrons, and five electrons. What is the net charge?

- A: +1
- B: 0
- C: -1
- D: -2

A rod with a positive charge is held next to a *conductor* (an object were charge can move around freely). Which of the following is true?

- A: The charges in the conductor all remain in place because charge is conserved.
- B: The negative charges in the conductor move toward the positive charges in the rod.
- C: The positive charges remain in place but the negative charges move away from the rod.
- D: The positive charges move toward the rod and the negative charges remain in place.

# COULOMB'S LAW AND ELECTRIC FIELDS

**Coulomb's Law** describes the force between charges.

### Coulomb's Law

The electric force, or **Coulomb force**, between two electrically charged systems with charges $q_1$ and $q_2$ separated by a distance $r$ is

$$\vec{F}_{\mathrm{C}} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \tag{2}$$

In Eq. 2, $\hat{r} = \vec{r}/|\vec{r}|$, and $\epsilon_0 = 8.85418782 \times 10^{-12} N^{-1} m^{-2} C^2$, called the *perimittivity of free space.*

### Coulomb Field

The electric field corresponding to Eq. 2, experienced by a charge $q$ and generated by a charge $Q$ is

$$\vec{E}_{\mathrm{C}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r} \tag{3}$$

In Eq. 3, $r$ remains the separation between $q$ and $Q$.

Thus we have: $\vec{F}_{\mathrm{C}} = q\vec{E}_{\mathrm{C}}$.

The SI Unit of charge is the Coulomb, which is equal to the amount of charge in a "current" of 1 amp for 1 second (more on this later). **The charge of an electron is** $1.6 \times 10^{-19}$ **Coulombs, or C.**

Suppose a charge $+q$ experiences the Coulomb field of another charge of $-2q$, separated by a distance $r$. Which of the following is true?

- A: The charge $+q$ accelerates towards the other charge, and the charge $-2q$ remains stationary, because opposite charges attract.
- B: The charge $-2q$ accelerates towards the other charge, and the charge $+q$ remains stationary, because opposite charges attract.
- C: No charges move; the force on one is equal to the force on the other.
- D: The charges accelerate towards each other.

The answer to the previous problem involves Newton's Third Law. (*Why did only the negative charges move*)?


**Figure 5.14** The electrostatic force $\vec{\mathbf{F}}$ between point charges $q_1$ and $q_2$ separated by a distance $r$ is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on $q_1$ is equal in magnitude and opposite in direction to the force it exerts on $q_2$. (a) Like charges; (b) unlike charges.

**Figure 3:** Newton's Third Law still applies.

Suppose a charge $+q$ experiences the Coulomb fields of two other charges of $-2q$, each located a distance $r$ from $+q$. The charges are all colinear (on the same line). Which of the following is true?

- A: The charge $+q$ accelerates towards one of the other charge, because opposite charges attract.
- B: The charges $-2q$ accelerate towards the charge $+q$.
- C: The charges $-2q$ accelerate towards the charge $+q$, but eventually repel each other back in the other direction.
- D: The charges $-2q$ repel each other from the start.

The Coulomb force equation gives a vector, and so does the corresponding electric field. Like a gravitational field, this effect has a vector at each point in space, so we refer to the Coulomb force and the Coulomb field as *vector fields*.

**Vector field**: An assignment of a vector to each point in a subset of space.

Which of the following is true of vectors $\vec{v}_{\mathrm{i}}$ in the lower left-hand corner of the figure at right?

- A: They are probably $\vec{v}_{\mathrm{i}} = -\hat{i} - \hat{j}$
- B: They are probably $\vec{v}_{\mathrm{i}} = \hat{i} + \hat{j}$
- C: They are probably $\vec{v}_{\mathrm{i}} = -\hat{i} + \hat{j}$
- D: They are probably $\vec{v}_{\mathrm{i}} = \hat{i} - \hat{j}$

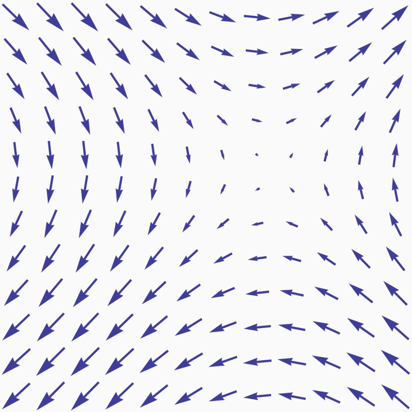

**Figure 4:** A vector field of vectors $\vec{v}_{\mathrm{i}}$. Let $\hat{j}$ represent up, and $\hat{i}$ represent right.

Which of the following is true of vectors $\vec{v}_{\mathrm{i}}$ in the upper left-hand corner of the figure at right?

- A: They are probably $\vec{v}_{\mathrm{i}} = -\hat{i} - \hat{j}$
- B: They are probably $\vec{v}_{\mathrm{i}} = \hat{i} + \hat{j}$
- C: They are probably $\vec{v}_{\mathrm{i}} = -\hat{i} + \hat{j}$
- D: They are probably $\vec{v}_{\mathrm{i}} = \hat{i} - \hat{j}$

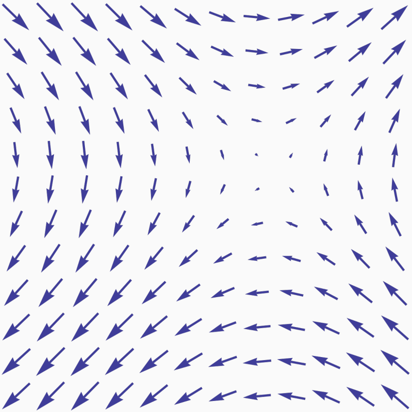

**Figure 5:** A vector field of vectors $\vec{v}_{\mathrm{i}}$. Let $\hat{j}$ represent up, and $\hat{i}$ represent right.

**Group board exercise**: What is the angle of the net electric field for the *test charge* at the point (1,1) in Fig. 6?

**Group board exercise**: What is the magnitude of the net electric field for the *test charge* at the point (1,1) in Fig. 6, if the distances have units of nanometers, and $q$ is the charge of an electron, $1.6 \times 10^{-19}$ C? (Let $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ N m$^2$ C$^{-2}$).

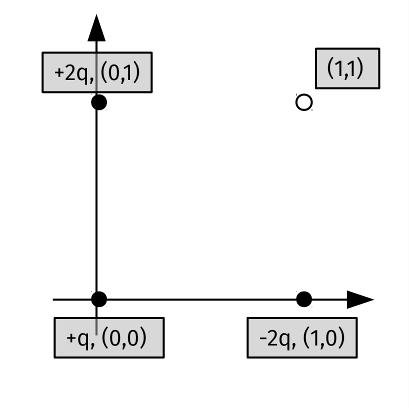


**Figure 6:** Three charges create a field for a hypothetical *test charge*.

What is the angle of the E-field at point (1,1) in Fig. 7 at right?

- A: 0 deg
- B: 45 deg
- C: 90 deg
- D: 135 deg

What is the fastest way to solve this problem?

- A: Blind luck
- B: Do the algebra
- C: Symmetry
- D: Numerical estimation

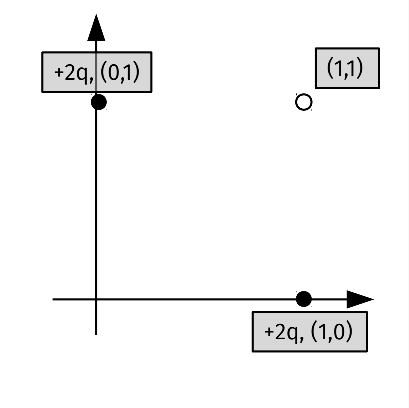


**Figure 7:** Two charges create a field for a hypothetical *test charge*.

What is the angle of the E-field at point (1,1) in Fig. 8 at right?

- A: About 0 deg
- B: About 25 deg
- C: About 45 deg
- D: About 60 deg

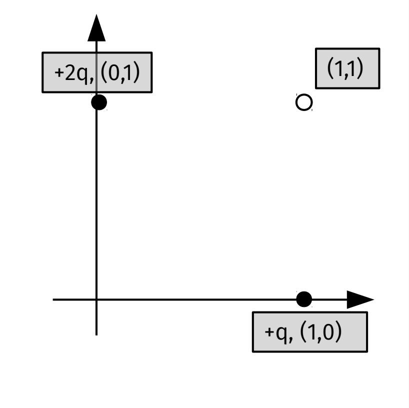


**Figure 8:** Two charges create a field for a hypothetical *test charge*.

The forces of $N$ fixed charges on a test charge $Q$ create a net force, where the individual forces simply add like vectors. This is known as the **superposition principle**.

$$\vec{F}_{\mathrm{C,Net}} = \frac{1}{4\pi\epsilon_0} Q \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i = Q\vec{E}_{\mathrm{C,Net}} \tag{4}$$

$$\vec{E}_{\mathrm{C,Net}} = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{N} \frac{q_i}{r_i^2} \hat{r}_i \tag{5}$$

For the expressions of fields built from the superposition principle, let's adopt a notation:

$$\vec{E}_{\mathrm{C,Net}}(P) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i^2}\hat{r}_i \tag{6}$$

Equation 6 represents the field at a *position* $P = P(x, y, z)$, relative to the positions $\vec{r}_i$ of the source charges.

**Table exercise:** Calculate $\vec{E}_{\mathrm{C,Net}}(P)$, if $P = (1, 1)$.

**Table exercise:** Calculate $\vec{E}_{\mathrm{C,Net}}(P)$, if $P = (-1, -1)$.

**Group discussion:** What does it mean if $P = (1, 0)$?

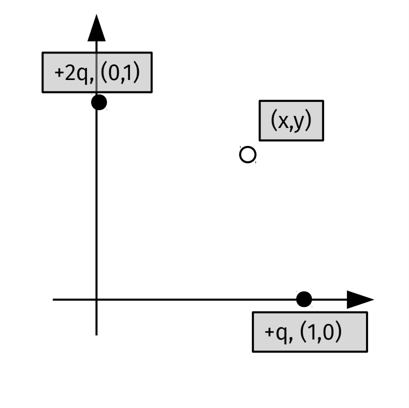


**Figure 9:** Two charges create a field for a hypothetical *test charge*.

Notice in the prior examples of *fixed* charges, we need an *explanation* for why the fixed charges remain fixed, even though they are obviously subject to Coulomb forces.

**Insulator**: A material in which there are no free charges available to conduct electricity. Charges may be fixed in position within an insulator.
**Conductor**: A material in which there are free charges available to conduct electricity. Charges may not be fixed in position within a conductor.
**Semi-conductor**: A material in which there are free charges available to conduct electricity if certain requirements are met.

The following problem is an example of solving for a field analytically, and *testing various limits*. Upon taking limits results are often simple and intuitive.

Two charges $+q$ are on the fixed in an insulator on the x-axis. Solve for the E-field at $P = (0, 0, z)$.

Show that the general solution is

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left(z^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}} \hat{k} \quad (7)$$

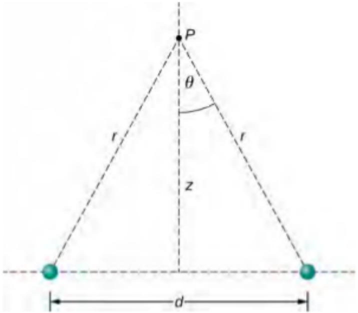


Figure 5.20 Finding the field of two identical source charges at the point $P$. Due to the symmetry, the net field at $P$ is entirely vertical. (Notice that this is *not* true away from the midline between the charges.)

**Figure 10:** Solve for the E-field as a function of $z$, $d$, and $q$.

Show that the general solution is

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left(z^2 + \left(\frac{d}{2}\right)^2\right)^{3/2}} \hat{k} \quad (8)$$

*Take the following two limits:*
1) $z \gg d$ and 2) $z = 0$. What are the results?

Keep these results in mind, because we are about to start drawing **vector fields,** in order to visualize the algebra.

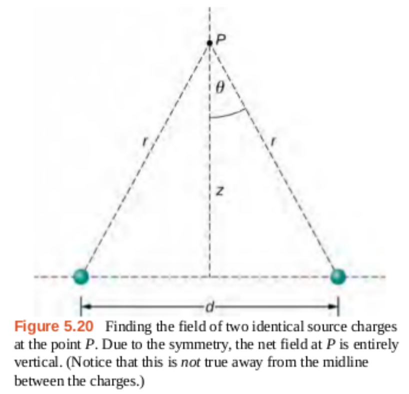


**Figure 11:** Solve for the E-field as a function of $z$, $d$, and $q$.

**PhET Simulation of E-fields from Charges**:

`https://phet.colorado.edu/en/simulation/charges-and-fields`

1. Create the situation in the prior problem, in Fig. 11.
2. Use the yellow sensor object to determine the local direction of the E-field at various points along the z-axis.
   - Do the results match the limit $z \gg d$?
   - Do the results match the limit $z = 0$, halfway between the charges?
   - Where is the field maximal?
3. Make sure you can see above and below the charges, and repeat steps 1 and 2 for negative z-values. What do you find?

**PhET Simulation of E-fields from Charges**:

Build E-fields with the following properties, by adding single charges. Let the *z-axis be upwards*, and let the *x-axis be to the right.*

1. Build an electric field that has **reflection symmetry** across the z-axis, with at least five charges.
2. Build an electric field that has *radial symmetry* about the origin, with at least six charges.
3. Build an electric field that would be the same if I rotated the picture by 90 degrees (**4-fold symmetry**) with at least four charges, some negative and some positive.
4. Build an electric field that would be the same if I rotated the picture by 45 degrees (**8-fold symmetry**) with at least eight charges, some negative and some positive.

PhET Simulation of E-fields from Charges:

The lesson is that the E-field has the *symmetry properties* of the *charge distribution*.

When we connect the vectors in a vector field, the results are figures like Fig. 12. Fields by convention originate from positive charges and terminate on negative ones.

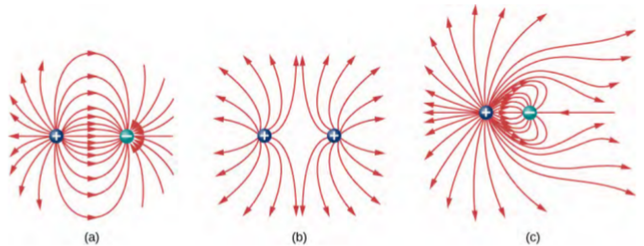


**Figure 5.31** Three typical electric field diagrams. (a) A dipole. (b) Two identical charges. (c) Two charges with opposite signs and different magnitudes. Can you tell from the diagram which charge has the larger magnitude?

**Figure 12:** Field-line diagrams. The density of lines indicates electric field strength.

# E-FIELDS OF CHARGE DISTRIBUTIONS

Welcome to calculus! Let $k = 1/(4\pi\epsilon_0)$.

$$\vec{E}(P) = k \sum_{i=1}^{N} \left( \frac{q_i}{r_i^2} \right) \hat{r} \tag{9}$$

$$\vec{E}(P) = k \int_{line} \left( \frac{\lambda dl}{r^2} \right) \hat{r} \tag{10}$$

$$\vec{E}(P) = k \int_{surface} \left( \frac{\sigma dA}{r^2} \right) \hat{r} \tag{11}$$

$$\vec{E}(P) = k \int_{volume} \left( \frac{\rho dV}{r^2} \right) \hat{r} \tag{12}$$

The functions $\lambda$, $\sigma$, and $\rho$ are just charge densities. They decribe where charge is, and how much there is.

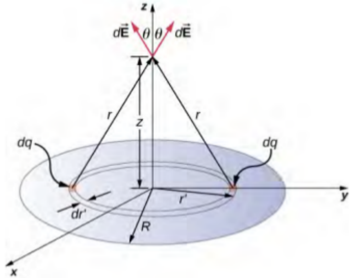


Figure 5.25 A uniformly charged disk. As in the line charge example, the field above the center of this disk can be calculated by taking advantage of the symmetry of the charge distribution.

**Figure 13:** We are going to work this example together, and other examples will be left to homework.

Observe on board.

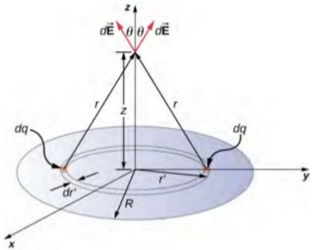


Figure 5.25 A uniformly charged disk. As in the line charge example, the field above the center of this disk can be calculated by taking advantage of the symmetry of the charge distribution.

**Figure 14:** We are going to work this example together, and other examples will be left to homework.

Result:

$$\boxed{\vec{E} = k\left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2+z^2}}\right)\hat{k}} \tag{13}$$

$$\boxed{\vec{E} = k\left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2 + z^2}}\right)\hat{k}} \tag{14}$$

Which of the following not true of Eq. 14?

- A: Taking the limit $R \to \infty$ yields a constant field.
- B: Taking the limit $z \to 0$ yields a constant field.
- C: The charge distribution has radial symmetry, so the field cannot have horizontal components.
- D: Taking the value $z = R$ represents a minimum in the field strength.

$$\boxed{\vec{E} = k\left(2\pi\sigma - \frac{2\pi\sigma z}{\sqrt{R^2+z^2}}\right)\hat{k}} \tag{15}$$

What happens to Eq. 15, in the limit that $R \to \infty$?

- A: The field decreases to zero.
- B: The field is constant.
- C: The field grows increasingly positive.
- D: The field grows increasingly negative.

In the limit that $R \to \infty$,

$$\vec{E} = 2\pi\sigma k\hat{k} = \frac{\sigma}{2\epsilon_0}\hat{k} \tag{16}$$

Equation for the electric field of a uniform infinite disk.

Imagine two infinite disks with equal uniform charge distributions, some distance apart. One has positive charge, the other negative charge. What is the E-field between them?

- A: 0
- B: $\frac{\sigma}{2\epsilon_0}$
- C: $\frac{\sigma}{\epsilon_0}$
- D: $\frac{\sigma}{4\epsilon_0}$

Imagine two infinite disks with equal uniform charge distributions, some distance apart. Both have positive charge. What is the E-field between them?

- A: 0
- B: $\frac{\sigma}{2\epsilon_0}$
- C: $\frac{\sigma}{\epsilon_0}$
- D: $\frac{\sigma}{4\epsilon_0}$

Other interesting charge distributions:

- A line of charge with length $L$ and total charge $Q = \lambda L$, where $P = (0, 0, z)$ above midpoint:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{\lambda L}{z\sqrt{z^2 + \frac{1}{4}L^2}} \hat{k} \tag{17}$$

- Equation 17, but with $L \to \infty$:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{z} \hat{k} \tag{18}$$

Other interesting charge distributions:

- A ring of radius $R$ and total charge $Q = 2\pi R\lambda$, where $P = (0, 0, z)$ above midpoint:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{2\pi R\lambda z}{\left(z^2 + R^2\right)^{3/2}}\hat{k} \tag{19}$$

- Equation 19, but with $z \gg R$:

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{2\pi R\lambda}{z^2}\hat{k} \tag{20}$$

In Eq. 19, what does the quantity $2\pi R\lambda$ represent?

- A: The total charge density on the ring
- B: The circumference of the ring
- C: The magnitude of the electric field from the ring
- D: The total charge on the ring

Let $Q_{\mathrm{tot}} = 2\pi R\lambda$. That makes Eq. 20

$$\vec{E}(z) = \frac{1}{4\pi\epsilon_0}\frac{Q_{\mathrm{tot}}}{z^2}\hat{k} \tag{21}$$

This is identical to the electric field of what charge distribution? (Think back to the definition of the electric field).

- A: A plane with charge density $Q_{\mathrm{tot}}/A$, where $A$ is the area
- B: A line with total charge $Q_{\mathrm{tot}}$
- C: A dipole of charge $\pm Q_{\mathrm{tot}}$
- D: A point charge $Q_{\mathrm{tot}}$

# ELECTRIC POTENTIAL, POTENTIAL ENERGY

Recall that the *change in potential energy* is force:

$$F = -\frac{\Delta U}{\Delta x} \tag{22}$$

- The units of $U$: Joules = Newtons per meter
- The units of $x$: meters
- The ratio: Newtons

An object of mass $m$ is a height $h$ above the ground, in Earth's gravity field. What is the *potential* energy of the object?

- A: $U = \frac{1}{2}mv^2$ ($v$ is the velocity)
- B: $U = \frac{1}{2}mv^2 + mgh$ ($v$ is the velocity)
- C: $U = mgh$
- D: $U$ is zero, because the object is at rest.

What is the expression for the force of gravity, if $\Delta U = mgh$, and $\Delta x = h$?

- A: $mg$
- B: $-mg$
- C: $g$
- D: $-g$

If the potential energy is a function of displacement, $U = U(\vec{x})$, it may be called a potential energy *surface*.

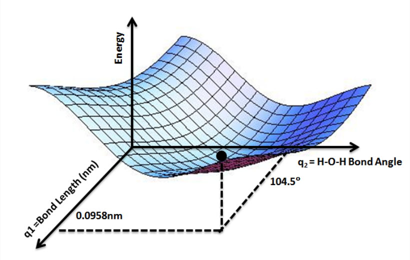


**Figure 15:** An example of a potential energy surface.

Considering *Newton's Second Law*, however, if $F = ma$ then $ma = -\frac{\Delta U}{\Delta x}$, and

$$a = -\frac{1}{m}\frac{\Delta U}{\Delta x} \tag{23}$$

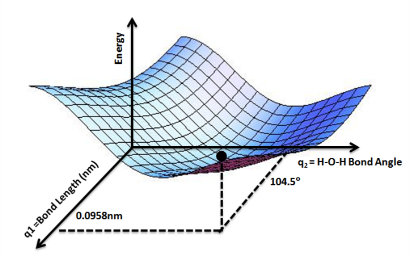


**Figure 16:** Imagine dividing the numbers on the z-axis by the mass, and treating this surface as the potential energy *per unit mass*.

Let's just scale the z-axis by the mass of the system:

$$a = -\frac{1}{m}\frac{\Delta U}{\Delta x} \tag{24}$$

$$\Delta V = \frac{\Delta U}{m} \tag{25}$$

$$a = -\frac{\Delta V}{\Delta x} \tag{26}$$

Instead of calling $\Delta V$ the potential energy, let's just call it *the potential.* Recall that if the force is a vector field, then acceleration is a vector field as well (the object has a given acceleration vector for all points in the space).

Newtonian mechanics

$$F = ma \tag{27}$$

$$a = -\frac{\Delta V}{\Delta x} \tag{28}$$

Electrostatics

$$F = qE \tag{29}$$

$$E = -\frac{\Delta V}{\Delta x} \tag{30}$$

In Eq. 30, we refer to $\Delta V$ as **voltage.**

Good paper topic:


**Figure 17:** (Lago di Como, Italia) Monument to Alessandro Volta, inventor of the electric battery. Debunked claim that electricity only generated by life.

**Voltage** is like a potential energy surface $\rightarrow$ *potential energy per unit charge.*

`https://phet.colorado.edu/en/simulation/charges-and-fields`

Using the PhET simulation about charges and fields:

1. Explore the voltage associated with fields generated by charges using the voltage button.
2. Add a single point charge, and use the ruler and voltmeter (potentiometer) to measure voltage versus distance, and plot it.
3. What function describes the relationship between voltage and distance?

https://phet.colorado.edu/en/simulation/charges-and-fields

Using the PhET simulation about charges and fields:

1. Note that the units of $\epsilon_0$ are N m$^2$ C$^{-2}$, and the value is $8.854 \times 10^{-12}$
2. We know from prior equations that the units of voltage are J C$^{-1}$
3. Using your measurements, show that the voltage due to a point charge is

$$\boxed{V = \pm \frac{1}{4\pi\epsilon_0}\frac{q}{r}} \tag{31}$$

(Where the sign depends on the charge, just like E-fields)

Voltage due to a point charge:

$$\boxed{V = \pm\frac{1}{4\pi\epsilon_0}\frac{q}{r}} \tag{32}$$

Equation 32 follows from the form of the electric field of a point charge, and $E = -\Delta V/\Delta x$ (derivative, calculus).

Voltage is an example of a **scalar field**, whereas the electric field is an example of a **vector field.** Recall the result for the electric field of a large plane of charge, with charge $\sigma$ per unit area:

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{k} \tag{33}$$

What should the voltage be, if the field is constant? (Think of the situation of gravity, with electric field standing in for acceleration).

https://phet.colorado.edu/en/simulation/charges-and-fields

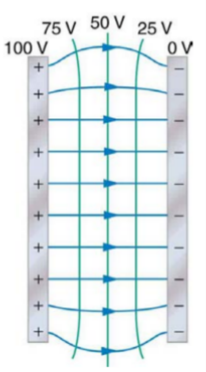


Figure 18: Parallel plates of charge, electric field, and potential. Notice the linear decrease in voltage. Did you see this in the PhET?

*Two parallel plates, opposite charge*:

$$V = -\frac{\sigma}{\epsilon_0} z + C \tag{34}$$

With the boundary condition that $V = V_0$ when $z = 0$, we have

$$V(z) - V_0 = -\frac{\sigma}{\epsilon_0} z \tag{35}$$

Let $\Delta V(z) = V(z) - V_0$, and $\Delta z = z$:

$$\frac{\Delta V}{\Delta z} = -\frac{\sigma}{\epsilon_0} = E \tag{36}$$

# USING PLATES AND LINES, CAPACITANCE AND BATTERIES, TIE BACK TO VOLTA

# CONCLUSION

# ANSWERS

# ANSWERS

- It is zero.
- Entropy has increased, but the internal energy returns to the original value
- 0.5
- The system does work equal to 0.5 liters times 1 atm
- +1
- The negative charges in the conductor move toward the positive charges in the rod.
- The charges accelerate towards each other.
- The charges $-2q$ accelerate towards the charge $+q$.
- They are probably $\vec{v}_{\mathrm{i}} = -\hat{i} - \hat{j}$
- They are probably $\vec{v}_{\mathrm{i}} = \hat{i} - \hat{j}$
- 45 deg
- Symmetry
- About 25 deg (26.56 degrees)
- Taking the value $z = R$ represents a minimum in the field strength.
- The field is constant.
- $\frac{\sigma}{\epsilon_0}$ N/C
- 0 N/C
- The total charge on the ring
- A point charge $Q_{\mathrm{tot}}$
- $mgh$
- $-mg$