Complex analysis of Askaryan radiation: energy and angular reconstruction of ultra-high energy neutrinos

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Outline: from mathematical physics to UHE-ν observations

Background

- UHE-ν flux
- The Askaryan effect
- RF UHE-ν detectors

The Askaryan signal

- Analytic radiation model
- Signal *envelopes*
- Uncertainty principles

Event reconstruction

- **NuRadioMC**: MC software
- Mathematics $\leftrightarrow$ signals
- Initial results

Figure 1: Whittier College.

Background

UHE-ν detection via the Askaryan Effect

- $E_\nu \geq 10^{16}$ eV
- **Flux is small**
- Cascades $\leftrightarrow$ RF
- Ice is transparent
- Station arrays

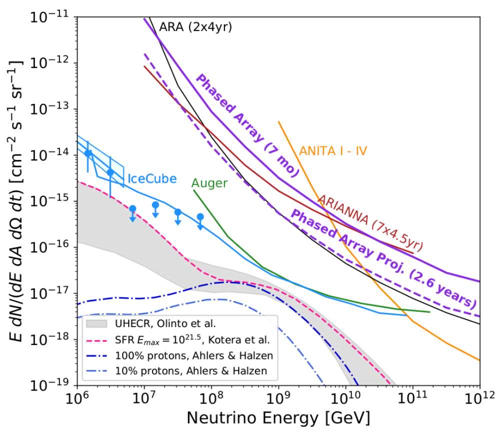

Figure 2: UHE-ν flux predictions and upper limits.

Background: in-ice UHE-ν observations

UHE-ν detection via the Askaryan Effect

- $E_\nu \geq 10^{16}$ eV
- Flux is small
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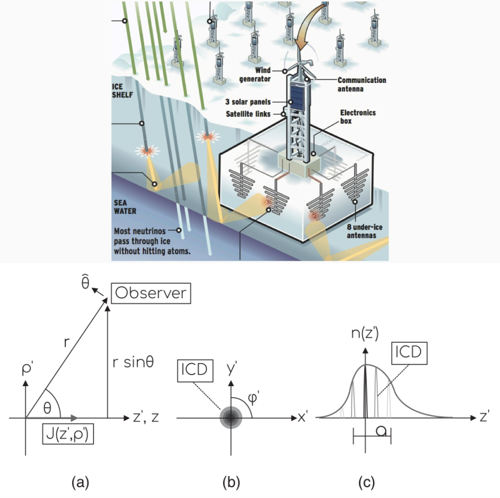

Figure 3: The Askaryan effect.

Background: in-ice UHE-ν observations

UHE-ν detection via the Askaryan Effect

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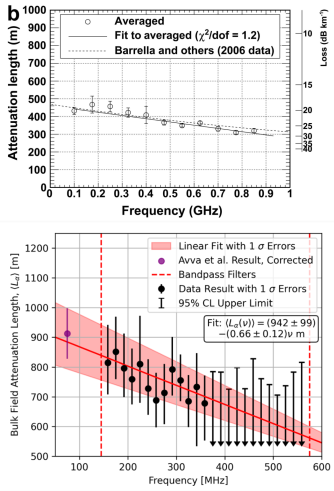

Figure 4: RF attenuation lengths in ice.

UHE-ν detection via the Askaryan Effect

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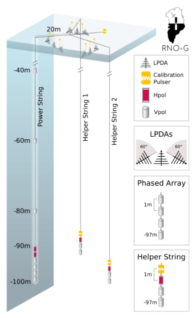

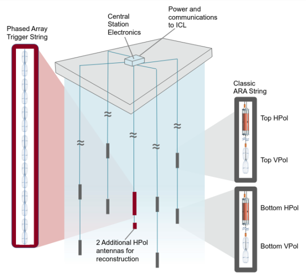

Figure 5: In-ice UHE-ν detectors. **Left:** RNO-G (Greenland). **Right:** ARA (Antarctica).

The Askaryan Signal

The Askaryan Signal: equation for $\vec{E}$-field

Askaryan electric field, $\vec{E}(r,t)$, [V m^{-1}]

$$r\vec{E}(t,\theta) = -\frac{E_0 \omega_0 \sin(\theta)}{8\pi p} t_r e^{-\frac{t_r^2}{4p}+p\omega_0^2} \operatorname{erfc}(\sqrt{p}\omega_0) \qquad (1)$$

Geometric parameters

r	vertex distance [m]
E_0	E-field amplitude [V GHz^{-2}]
t_r	Retarded time [ns][a]
θ	Observation angle [rad]
p	$\sigma_t = \sqrt{2p}$ (Eq. 2) [ns^2]

[a] Definition: $t_{\mathrm{r}} = t_{\mathrm{ref}} - nR/c$.

Particle physics parameters

ω_0	Form-factor frequency [GHz]
θ_{C}	Cherenkov angle [rad]
p	$\sigma_t = \sqrt{2p}$ (Eq. 2) [ns^2]
a	Cascade length (Eq. 2) [m]

$$p = \frac{1}{2}\left(\frac{a}{c}\right)^2 (\cos\theta - \cos\theta_C)^2 \qquad (2)$$

$$\sqrt{2p} \approx (a/c)|\theta - \theta_{\mathrm{C}}| \sin\theta_{\mathrm{C}} \qquad (3)$$

$$\sigma_t = \sqrt{2p} \propto \ a\Delta\theta \qquad (4)$$

The Askaryan Signal: $\vec{E}$-field time-dependence, analytic signal

Askaryan electric field, $\vec{E}(r,t)$, [V m^{-1}], time-dependence

$$s(t) = \vec{E}(t,\theta) = -E_0 t e^{-\frac{1}{2}\left(\frac{t}{\sigma_t}\right)^2} \tag{5}$$

Askaryan electric field *analytic signal*

$$s_a(t) = -E_0\left(te^{-\frac{1}{2}(t/\sigma_t)^2} - \frac{2j\sigma_t}{\sqrt{2\pi}}\frac{dD(x)}{dx}\right) \tag{6}$$

The signal envelope

- $s_a(t) = s(t) + j\,\widehat{s}(t)$
- $\widehat{s}(t)$, Hilbert transform
- $|s_a(t)|$, signal *envelope*

Special functions and variables

- $D(x)$, Dawson function
- $x = t/(\sqrt{2}\sigma_t)$, normalized time

The Askaryan Signal: detected signals

Common response function for RF channels $r(t)$, [m ns^{-1}]

$$r(t) = R_0 \cos(2\pi f_0 t) e^{-\gamma t} \tag{7}$$

Common analytic signal for RF channels $r_a(t)$, [m ns^{-1}]

$$r_a(t) = R_0 e^{2\pi j f_0 t - \gamma t} \tag{8}$$

The signal envelope

- $r_a(t) = r(t) + j\,\widehat{r}(t)$
- $\widehat{r}(t)$, Hilbert transform
- $|r_a(t)|$, signal *envelope*

RF channels: RLC circuits

- $|r_a(t)| = R_0 \exp(-\gamma t)$
- γ, damping coefficient [GHz]
- f_0, resonance frequency [GHz]

The Askaryan Signal: detected signals

Detected signals, $r(t) * s(t)$, [V]

$$r(t) * s(t) = \int_{-\infty}^{\infty} r(\tau)s(t-\tau)d\tau \rightarrow \int_{0}^{\infty} r(\tau)s(t-\tau)d\tau \quad (9)$$

Theorem: the envelope of detected signal

Let $\mathcal{E}_{r*s}(t)$ represent the *envelope* of the convolution of $r(t)$ and $s(t)$. If $s_a(t)$ and $r_a(t)$ are the analytic signals of $s(t)$ and $r(t)$, respectively, then

$$\mathcal{E}_{r*s}(t) = \frac{1}{2}|r_a(t) * s_a(t)| \quad (10)$$

The Askaryan Signal: uncertainty principles

Uncertainty principles within Askaryan field

Let $\Delta\theta = \theta - \theta_{\mathrm{C}}$. Let a be the cascade length, c be the speed of light, σ_t be the pulse width, and σ_f be the width of the Fourier spectrum.

$$a\Delta\theta = \frac{c\sigma_t}{\sin\theta_{\mathrm{C}}} \tag{11}$$

Further, $s(t)$, and $S(f)$ (the Fourier transform), have widths that satisfy

$$\sigma_t\sigma_f = \frac{1}{2\pi}\left(1+\eta^2\right)^{1/2} \tag{12}$$

In the far-field, $\eta \to 0$.

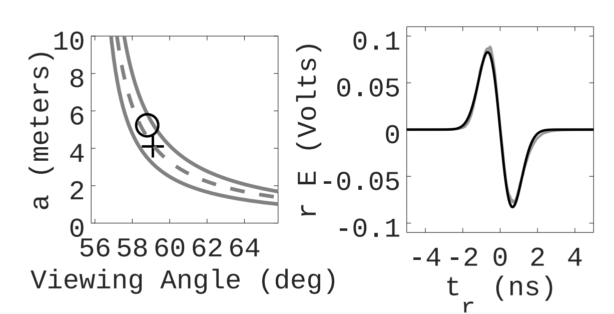

Figure 6: Scanning Askaryan field equation parameters to fit NuRadioMC output.

Energy reconstruction potential: $a^2 \propto \ln(E_{\mathrm{C}}/E_{\mathrm{crit}})$

Conclusion

Unit 1.1 Outline

Unit 1.1 covered: