

Complex analysis of Askaryan Radiation: UHECR and UHE- ν Reconstruction with Analytic Signals

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Summary

UHECR and UHE- ν Reconstruction with Analytic Signals

1. Introduction:

- **A fully analytic Askaryan E-field** model in the time-domain [4]
- Based on work begun with Prof. Amy Connolly [3]
- Based on work begun by J. Ralston and R. Buniy [2]
- Advantages: (i) extract UHE- ν cascade parameters by fitting model to raw voltage traces, (ii) fast and simple, (iii) analytic equations may be embedded as event filter

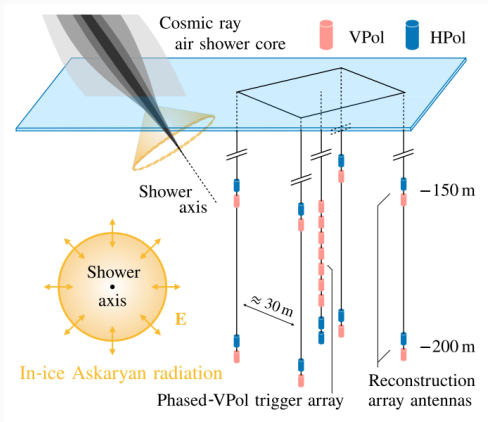
2. UHECR and UHE- ν identification:

- **A fully analytic Askaryan voltage** model, time-domain [this work]
- Equations for both voltage trace, and envelope of voltage trace
- Verification with NuRadioMC: strong thermal background rejection, signal identification, and (rough) $\log_{10}(E_\nu)$ estimate

3. Correlation with Recent ARA observations of UHECR

UHECR Event Geometry for in the ARA Detector

UHECR Event Geometry for in the ARA Detector



ARA detector schematic

- Askaryan component of the radiation
- Vpol and Hpol channels are dipole antennas
- E-field components are radial to the cascade axis
- Reference:
<https://arxiv.org/pdf/2510.21104>

Figure 1: (Top right) UHECR cascade core interacting in ice. (Bottom left) ARA RF detection channels (Vpol and Hpol).

Notations, Definitions, and Analytic Signals

Notations, Definitions, and Analytic Signals

Askaryan electric field, $\vec{E}(r, t)$, [V m⁻¹]

$$r\vec{E}(t, \theta) = -\frac{E_0\omega_0 \sin(\theta)}{8\pi p} t_r e^{-\frac{t_r^2}{4p} + p\omega_0^2} \operatorname{erfc}(\sqrt{p}\omega_0) \quad (1)$$

Variable	Definition	Units
c	speed of light in medium	m ns ⁻¹
r	distance to cascade peak	m
t_r	$t - r/c$	ns
θ_C	Cherenkov angle	radians
θ	viewing angle from cascade axis	radians
a	longitudinal cascade length (see [2, 3, 4])	m
n_{max}	max excess cascade particles (see [2, 3, 4])	none
E_0	$\propto n_{max}a$ (see [2, 3, 4])	V GHz ⁻²
p	$\frac{1}{2}(a/c)^2 (\cos \theta - \cos \theta_C)^2$ (see [4])	ns ²
ω_0	$\sqrt{\frac{2}{3}}(c\sqrt{2\pi\rho_0})/(\sin \theta)$ (see [3, 4])	GHz
$\sqrt{2\pi\rho_0}$	lateral ICD width (see [3, 4])	m ⁻¹

Table 1: Parameters relevant for $E(t)$.

Notations, Definitions, and Analytic Signals

Askaryan *signal*, $\vec{s}(t)$, [$V\ m^{-1}$]

Let E_0 represent all amplitude factors not dependent on time. Further, let $\sigma_t = \sqrt{2p}$. The Askaryan E-field is

$$s(t) = -E_0 t e^{-\frac{1}{2}(t/\sigma_t)^2} \quad (2)$$

Detector *response*, $\vec{r}(t)$, [$m\ ns^{-1}$]

$$r(t) = R_0 e^{-2\pi\gamma t} \cos(2\pi f_0 t) \quad (3)$$

- Impulse response of a causal ($t \geq 0$) damped, driven RLC circuit
- First used by the RICE collaboration [5]
- RF dipole channels are used in RICE, ARA, RNO-G, and the proposed IceCube Gen2 (radio) [5, 7, 6, 9, 8]
- RF channels must fit inside cylindrical ice boreholes

Notations, Definitions, and Analytic Signals

Analytic signal, $s_a(t)$, of $s(t)$.

Let $s_a(t)$ be the *analytic signal* of the signal $s(t)$. Further, let $\hat{s}(t)$ be the Hilbert transform of $s(t)$. Finally, let $\mathcal{E}_s(t)$ be the *envelope* of $s(t)$.

$$s_a(t) = s(t) + j\hat{s}(t) \quad (4)$$

$$\mathcal{E}_s(t) = |s_a(t)| \quad (5)$$

Analytic signals, $s_a(t)$, and $r_a(t)$.

Let $x = t/(\sqrt{2}\sigma_t)$, and let $D(x)$ be the *Dawson function*. The analytic signals for $s(t)$ (V m^{-1}) and $r(t)$ (m ns^{-1}) are

$$s_a(t) = -E_0 \left(te^{-\frac{1}{2}(t/\sigma_t)^2} - \frac{2j\sigma_t}{\sqrt{2\pi}} \frac{dD(x)}{dx} \right) \quad (6)$$

$$r_a(t) = R_0 e^{-2\pi\gamma t} e^{2\pi j f_0 t} \quad (7)$$

Notations, Definitions, and Analytic Signals

Detected signals, $r(t) * s(t)$, [V]

$$r(t) * s(t) = \int_{-\infty}^{\infty} r(\tau)s(t - \tau)d\tau \quad (8)$$

Theorem: the envelope of detected signal

Let $\mathcal{E}_{r*s}(t)$ represent the *envelope* of the convolution of $r(t)$ and $s(t)$. If $s_a(t)$ and $r_a(t)$ are the analytic signals of $s(t)$ and $r(t)$, respectively, then

$$\mathcal{E}_{r*s}(t) = \frac{1}{2}|r_a(t) * s_a(t)| \quad (9)$$

$\mathcal{E}_{r*s}(t)$: part I

Let $x = t/(\sqrt{2}\sigma_t)$, $y = \tau/(\sqrt{2}\sigma_t)$, and $z = (2\pi j f_0 - 2\pi\gamma)\sqrt{2}\sigma_t$.
Let $w(q)$ be the *Faddeeva function*, with $b = jq$, $b = x + \frac{1}{2}z$.
The convolution of $r_a(t)$ with $\Re\{s_a(t)\}$ is

$$r_a(t) * \Re\{s_a(t)\} = \\ -\sqrt{\pi}R_0E_0\sigma_t^2 \left(x e^{-x^2} w(q) + \left(\frac{j}{2} \right) e^{-x^2} \frac{dw(q)}{dq} \right) \quad (10)$$

$\mathcal{E}_{r*s}(t)$: part II

Let $x = t/(\sqrt{2}\sigma_t)$, $y = \tau/(\sqrt{2}\sigma_t)$, and $z = (2\pi j f_0 - 2\pi\gamma)\sqrt{2}\sigma_t$. Let $u = x - y$, $z = -k$, and let $\mathcal{L}\{D(u - x)\}_k$ be the Laplace transform of the shifted Dawson function. The convolution of $r_a(t)$ with $\Im\{s_a(t)\}$ is

$$r_a(t) * \Im\{s_a(t)\} = \frac{2}{\sqrt{\pi}} R_0 E_0 \sigma_t^2 (D(x) + k \mathcal{L}\{D(u - x)\}_k) \quad (11)$$

$\mathcal{E}_{r*s}(t)$: part III

$$\mathcal{E}_{r*s}(t) = \frac{1}{2} |r_a * s_a| = \frac{1}{2} |r_a * (\Re\{s_a\} + j\Im\{s_a\})| \quad (12)$$

$$\mathcal{E}_{r*s}(t) = \frac{1}{2} |r_a * \Re\{s_a\} + jr_a * \Im\{s_a\}| \quad (13)$$

- We have calculated $r_a(t) * \Re\{s_a(t)\}$ and $r_a(t) * \Im\{s_a(t)\}$
- Combine to form the final result
- This result is used to predict the *envelope* of the voltage traces and CSWs
- Python3 code provided in the paper

$\mathcal{E}_{r*s}(t)$: part IV

Using prior definitions, it turns out that $s * r$ is

$$s * r = -\sqrt{\pi}R_0E_0\sigma_t^2 \Re \left\{ x e^{-x^2} w(q) - \frac{1}{2} e^{-x^2} \frac{dw(q)}{dx} \right\} \quad (14)$$

- This result is used to predict the voltage traces and CSWs
- Python3 code provided in the paper

Results: Voltage Traces and Envelopes

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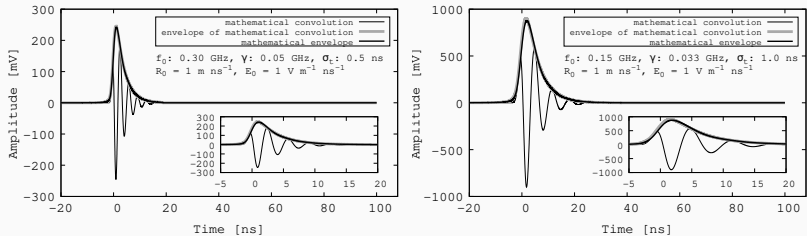


Figure 2: (Left) The thin black line represents $s * r$. The light gray envelope represents the envelope of $s * r$ computed with the Python3 SciPy function `scipy.special.hilbert`. The black envelope represents $\mathcal{E}_{r*s}(t)$. (Right) Same as left, for different parameter values.

Results: Voltage Traces and Envelopes

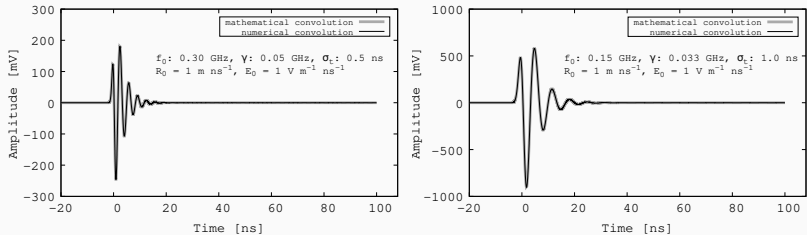


Figure 3: (Left) The thin black line represents s convolved with r using the Python3 SciPy function `scipy.signal.convolve`. The gray line represents $s * r$. (Right) Same as left, for different parameter values.

Results: Comparisons to NuRadioMC

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Parameter	Value	Note
Ice Model	South Pole	2015 measurements
Signal Model	AHRZ2020	(see [1])
Trigger	3 of 8 channels	$\pm 3v_{\text{rms}}$
RF channels	8	RF bicone (in firn)
Channel filters	[80-1000] MHz	Passband
Noise Temperature	233K	Sets v_{rms}
Sampling Rate	1 GHz	$f_c = 500$ MHz
Samples per channel	256	total time, 256 ns
Channel depths	[-4,-6,-8,...-18] m	cable delays included
RF cable type	LMR-400	≈ -1 dB at 20 m

Table 2: Important NuRadioMC parameters.

NuRadioMC was used to generate 15133 UHE- ν events that triggered the detector with $E_C = 100$ PeV.

Results: Comparisons to NuRadioMC

Bottom line: 0.2 noise events pass correlation threshold in 5 years.

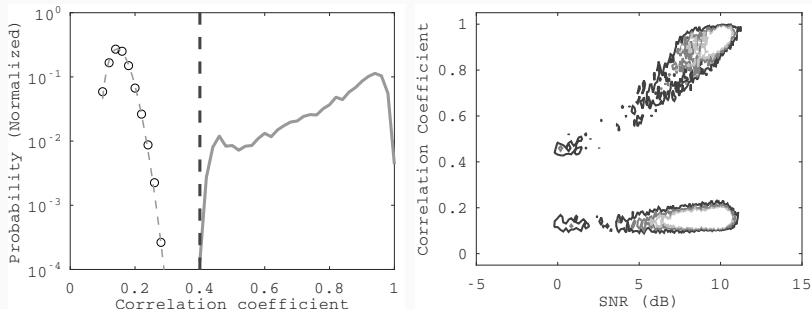


Figure 4: (Left) The black circles represent the noise distribution. The gray dashed line is a fitting function to noise distribution. The solid gray line represents the correlation distribution for mathematical envelope to envelope of UHE- ν signals. The dashed black line is the correlation threshold. (Right) The correlation coefficient versus SNR.

Results: ARA5 UHECR Results

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Correlation coefficient for ARA5 UHECR candidate and model: **0.853**.

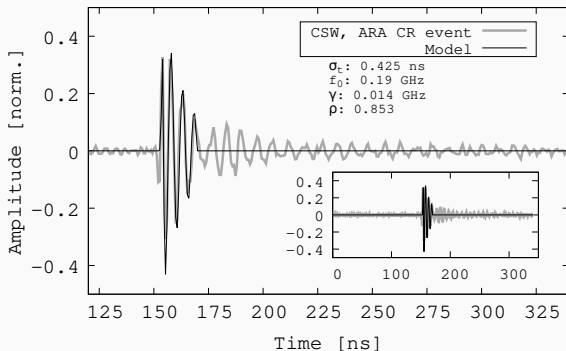


Figure 5: Fit for analytic $s * r$ to an ARA5 UHECR candidate 1915-26288.

Results: ARA5 UHECR Results

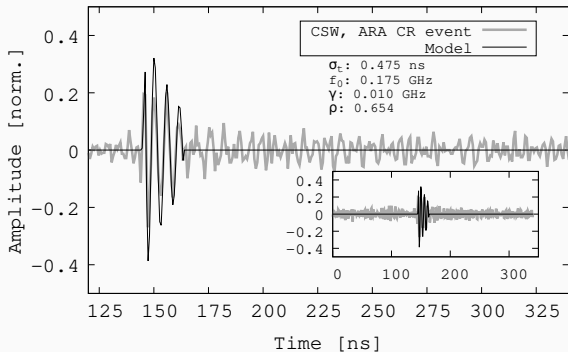


Figure 6: Fit for analytic $s * r$ to an ARA5 UHECR candidate 1957-13330.

Results: ARA5 UHECR Results

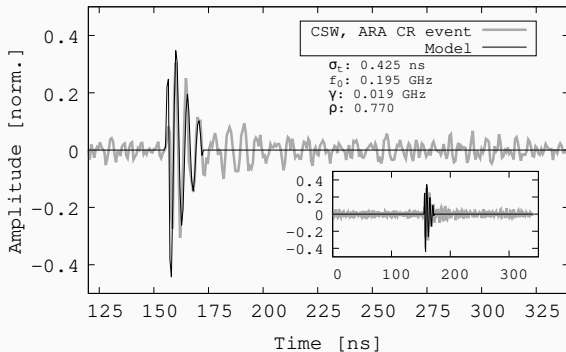


Figure 7: Fit for analytic $s * r$ to an ARA5 UHECR candidate 2171-31805.

Conclusion

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3. Correlation with Recent ARA observations of UHECR

- Three events reconstructed, with ten more to come
- Improvements: noise filter, channel selection
- **Proposal:** short manuscript with equations and fits
- Mathematical physics and NuRadioMC comparison has been submitted to PRD

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