

# Complex analysis of Askaryan Radiation: UHECR and UHE- $\nu$ Reconstruction with Analytic Signals

---

Jordan Hanson

November 10, 2025

Whittier College Department of Physics and Astronomy

## Summary

---

# UHECR and UHE- $\nu$ Reconstruction with Analytic Signals

## 1. Introduction:

- A fully analytic Askaryan E-field model in the time-domain [4]
- Based on work begun with Prof. Amy Connolly [3]
- Based on work begun by J. Ralston and R. Buniy [2]
- Advantages: (i) extract UHE- $\nu$  cascade parameters by fitting model to raw voltage traces, (ii) fast and simple, (iii) analytic equations may be embedded as event filter

## 2. UHECR and UHE- $\nu$ identification:

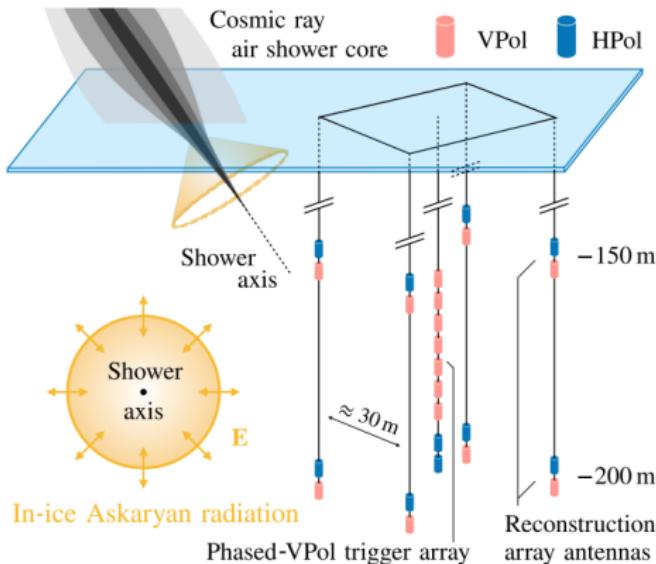
- A fully analytic Askaryan voltage model, time-domain [this work]
- Equations for both voltage trace, and envelope of voltage trace
- Verification with NuRadioMC: strong thermal background rejection, signal identification, and (rough)  $\log_{10}(E_\nu)$  estimate

## 3. Correlation with Recent ARA observations of UHECR

# UHECR Event Geometry for in the ARA Detector

---

# UHECR Event Geometry for in the ARA Detector



## ARA detector schematic

- Askaryan component of the radiation
- Vpol and Hpol channels are dipole antennas
- E-field components are radial to the cascade axis
- Reference:  
<https://arxiv.org/pdf/2510.21104>

Figure 1: (Top right) UHECR cascade core interacting in ice. (Bottom left) ARA RF detection channels (Vpol and Hpol).

# Notations, Definitions, and Analytic Signals

---

# Notations, Definitions, and Analytic Signals

Askaryan electric field,  $\vec{E}(r, t)$ , [V m<sup>-1</sup>]

$$r\vec{E}(t, \theta) = -\frac{E_0 \omega_0 \sin(\theta)}{8\pi p} t_r e^{-\frac{t_r^2}{4p} + p\omega_0^2} \operatorname{erfc}(\sqrt{p}\omega_0) \quad (1)$$

Variable	Definition	Units
$c$	speed of light in medium	m ns <sup>-1</sup>
$r$	distance to cascade peak	m
$t_r$	$t - r/c$	ns
$\theta_C$	Cherenkov angle	radians
$\theta$	viewing angle from cascade axis	radians
$a$	longitudinal cascade length (see [2, 3, 4])	m
$n_{max}$	max excess cascade particles (see [2, 3, 4])	none
$E_0$	$\propto n_{max} a$ (see [2, 3, 4])	V GHz <sup>-2</sup>
$p$	$\frac{1}{2}(a/c)^2 (\cos \theta - \cos \theta_C)^2$ (see [4])	ns <sup>2</sup>
$\omega_0$	$\sqrt{\frac{2}{3}}(c\sqrt{2\pi\rho_0})/(\sin \theta)$ (see [3, 4])	GHz
$\sqrt{2\pi}\rho_0$	lateral ICD width (see [3, 4])	m <sup>-1</sup>

Table 1: Parameters relevant for  $E(t)$ .

# Notations, Definitions, and Analytic Signals

Askaryan signal,  $\vec{s}(t)$ , [V m<sup>-1</sup>]

Let  $E_0$  represent all amplitude factors not dependent on time. Further, let  $\sigma_t = \sqrt{2p}$ . The Askaryan E-field is

$$s(t) = -E_0 t e^{-\frac{1}{2}(t/\sigma_t)^2} \quad (2)$$

Detector response,  $\vec{r}(t)$ , [m ns<sup>-1</sup>]

$$r(t) = R_0 e^{-2\pi\gamma t} \cos(2\pi f_0 t) \quad (3)$$

- Impulse response of a causal ( $t \geq 0$ ) damped, driven RLC circuit
- First used by the RICE collaboration [5]
- RF dipole channels are used in RICE, ARA, RNO-G, and the proposed IceCube Gen2 (radio) [5, 7, 6, 9, 8]
- RF channels must fit inside cylindrical ice boreholes

# Notations, Definitions, and Analytic Signals

Analytic signal,  $s_a(t)$ , of  $s(t)$ .

Let  $s_a(t)$  be the *analytic signal* of the signal  $s(t)$ . Further, let  $\hat{s}(t)$  be the Hilbert transform of  $s(t)$ . Finally, let  $\mathcal{E}_s(t)$  be the *envelope* of  $s(t)$ .

$$s_a(t) = s(t) + j\hat{s}(t) \quad (4)$$

$$\mathcal{E}_s(t) = |s_a(t)| \quad (5)$$

Analytic signals,  $s_a(t)$ , and  $r_a(t)$ .

Let  $x = t/(\sqrt{2}\sigma_t)$ , and let  $D(x)$  be the *Dawson function*. The analytic signals for  $s(t)$  ( $\text{V m}^{-1}$ ) and  $r(t)$  ( $\text{m ns}^{-1}$ ) are

$$s_a(t) = -E_0 \left( te^{-\frac{1}{2}(t/\sigma_t)^2} - \frac{2j\sigma_t}{\sqrt{2\pi}} \frac{dD(x)}{dx} \right) \quad (6)$$

$$r_a(t) = R_0 e^{-2\pi\gamma t} e^{2\pi j f_0 t} \quad (7)$$

# Notations, Definitions, and Analytic Signals

Detected signals,  $r(t) * s(t)$ , [V]

$$r(t) * s(t) = \int_{-\infty}^{\infty} r(\tau)s(t - \tau)d\tau \quad (8)$$

Theorem: the envelope of detected signal

Let  $\mathcal{E}_{r*s}(t)$  represent the *envelope* of the convolution of  $r(t)$  and  $s(t)$ . If  $s_a(t)$  and  $r_a(t)$  are the analytic signals of  $s(t)$  and  $r(t)$ , respectively, then

$$\mathcal{E}_{r*s}(t) = \frac{1}{2}|r_a(t) * s_a(t)| \quad (9)$$

# Notations, Definitions, and Analytic Signals

$\mathcal{E}_{r*s}(t)$ : part I

Let  $x = t/(\sqrt{2}\sigma_t)$ ,  $y = \tau/(\sqrt{2}\sigma_t)$ , and  $z = (2\pi j f_0 - 2\pi\gamma)\sqrt{2}\sigma_t$ .

Let  $w(q)$  be the *Faddeeva function*, with  $b = jq$ ,  $b = x + \frac{1}{2}z$ .

The convolution of  $r_a(t)$  with  $\Re\{s_a(t)\}$  is

$$r_a(t) * \Re\{s_a(t)\} =$$

$$-\sqrt{\pi}R_0E_0\sigma_t^2 \left( xe^{-x^2}w(q) + \left(\frac{j}{2}\right) e^{-x^2} \frac{dw(q)}{dq} \right) \quad (10)$$

# Notations, Definitions, and Analytic Signals

$\mathcal{E}_{r*s}(t)$ : part II

Let  $x = t/(\sqrt{2}\sigma_t)$ ,  $y = \tau/(\sqrt{2}\sigma_t)$ , and  $z = (2\pi j f_0 - 2\pi\gamma)\sqrt{2}\sigma_t$ . Let  $u = x - y$ ,  $z = -k$ , and let  $\mathcal{L}\{D(u - x)\}_k$  be the Laplace transform of the shifted Dawson function. The convolution of  $r_a(t)$  with  $\Im\{s_a(t)\}$  is

$$r_a(t) * \Im\{s_a(t)\} = \frac{2}{\sqrt{\pi}} R_0 E_0 \sigma_t^2 (D(x) + k \mathcal{L}\{D(u - x)\}_k) \quad (11)$$

## Notations, Definitions, and Analytic Signals

$\mathcal{E}_{r*s}(t)$ : part III

$$\mathcal{E}_{r*s}(t) = \frac{1}{2} |r_a * s_a| = \frac{1}{2} |r_a * (\Re\{s_a\} + j\Im\{r_a\})| \quad (12)$$

$$\mathcal{E}_{r*s}(t) = \frac{1}{2} |r_a * \Re\{s_a\} + j r_a * \Im\{r_a\}| \quad (13)$$

- We have calculated  $r_a(t) * \Re\{s_a(t)\}$  and  $r_a(t) * \Im\{s_a(t)\}$
- Combine to form the final result
- This result is used to predict the *envelope* of the voltage traces and CSWs
- Python3 code provided in the paper

# Notations, Definitions, and Analytic Signals

$\mathcal{E}_{r*s}(t)$ : part IV

Using prior definitions, it turns out that  $s * r$  is

$$s * r = -\sqrt{\pi} R_0 E_0 \sigma_t^2$$

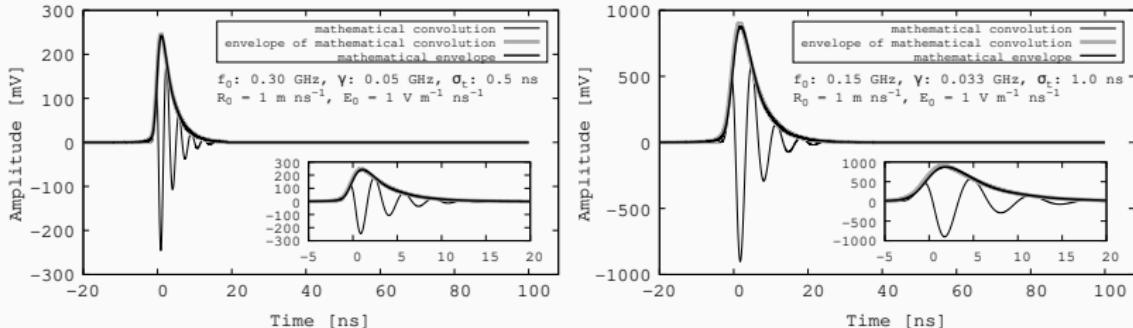
$$\Re \left\{ xe^{-x^2} w(q) - \frac{1}{2} e^{-x^2} \frac{dw(q)}{dx} \right\} \quad (14)$$

- This result is used to predict the voltage traces and CSWs
- Python3 code provided in the paper

## Results: Voltage Traces and Envelopes

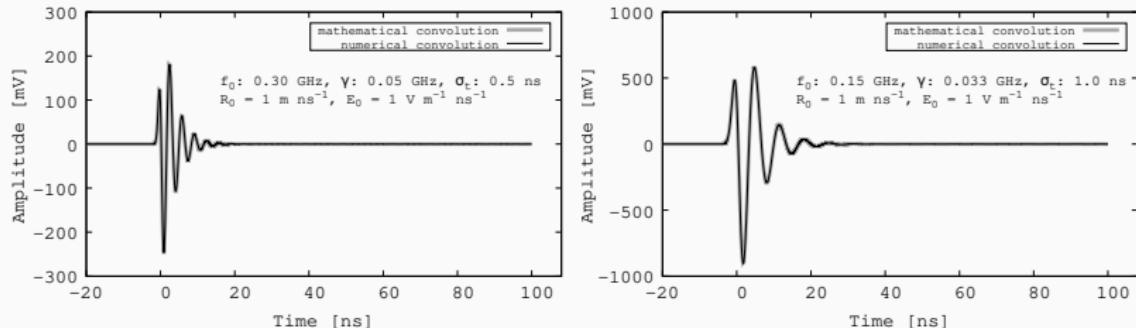
---

# Results: Voltage Traces and Envelopes



**Figure 2:** (Left) The thin black line represents  $s * r$ . The light gray envelope represents the envelope of  $s * r$  computed with the Python3 SciPy function `scipy.special.hilbert`. The black envelope represents  $\mathcal{E}_{r*s}(t)$ . (Right) Same as left, for different parameter values.

# Results: Voltage Traces and Envelopes



**Figure 3:** (Left) The thin black line represents  $s$  convolved with  $r$  using the Python3 SciPy function `scipy.signal.convolve`. The gray line represents  $s * r$ . (Right) Same as left, for different parameter values.

## Results: Comparisons to NuRadioMC

---

## Results: Comparisons to NuRadioMC

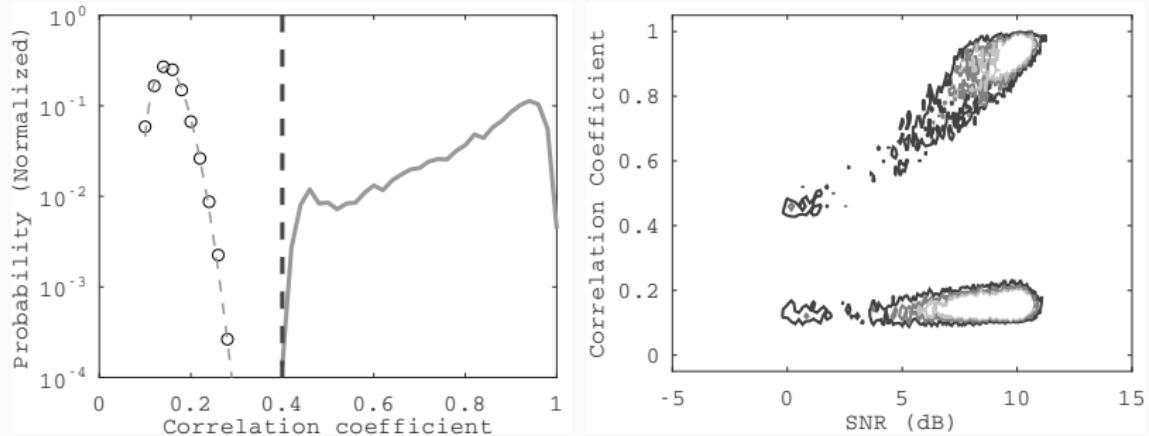
Parameter	Value	Note
Ice Model	South Pole	2015 measurements
Signal Model	AHRZ2020	(see [1])
Trigger	3 of 8 channels	$\pm 3v_{\text{rms}}$
RF channels	8	RF bicone (in firn)
Channel filters	[80-1000] MHz	Passband
Noise Temperature	233K	Sets $v_{\text{rms}}$
Sampling Rate	1 GHz	$f_c = 500$ MHz
Samples per channel	256	total time, 256 ns
Channel depths	[-4,-6,-8,...-18] m	cable delays included
RF cable type	LMR-400	$\approx -1$ dB at 20 m

Table 2: Important NuRadioMC parameters.

NuRadioMC was used to generate 15133 UHE- $\nu$  events that triggered the detector with  $E_C = 100$  PeV.

# Results: Comparisons to NuRadioMC

**Bottom line:** 0.2 noise events pass correlation threshold in 5 years.



**Figure 4:** (Left) The black circles represent the noise distribution. The gray dashed line is a fitting function to noise distribution. The solid gray line represents the correlation distribution for mathematical envelope to envelope of UHE- $\nu$  signals. The dashed black line is the correlation threshold. (Right) The correlation coefficient versus SNR.

## Results: ARA5 UHECR Results

---

# Results: ARA5 UHECR Results

Correlation coefficient for ARA5 UHECR candidate and model: 0.853.

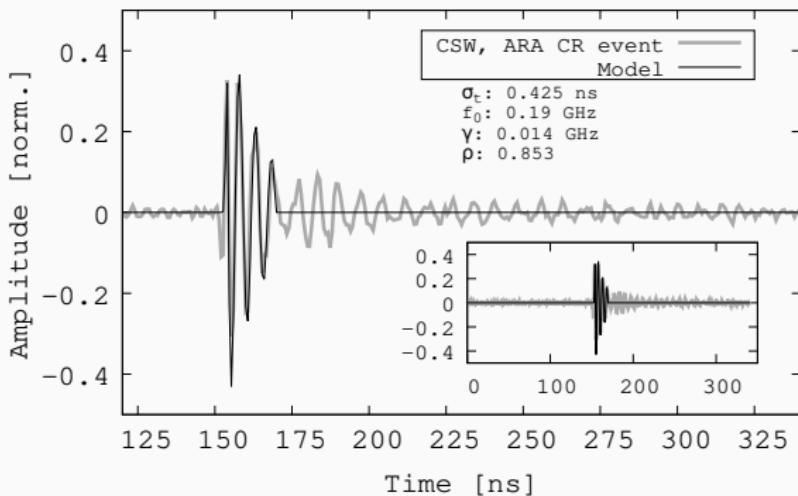


Figure 5: Fit for analytic  $s * r$  to an ARA5 UHECR candidate 1915-26288.

# Results: ARA5 UHECR Results

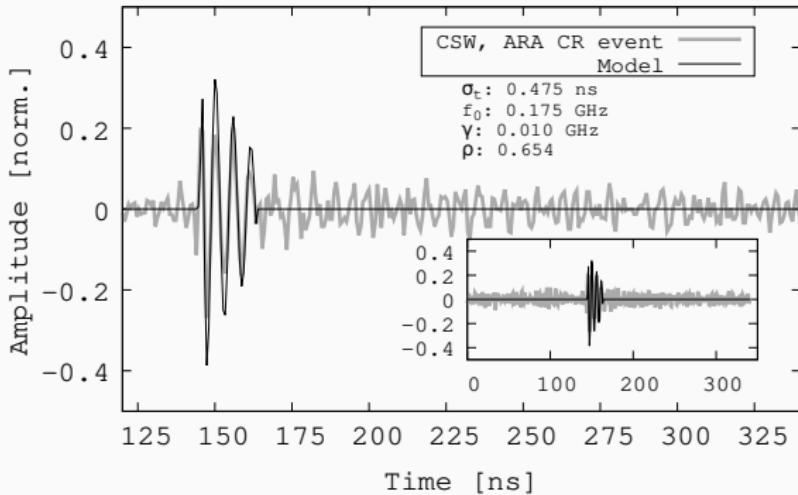


Figure 6: Fit for analytic  $s * r$  to an ARA5 UHECR candidate 1957-13330.

# Results: ARA5 UHECR Results

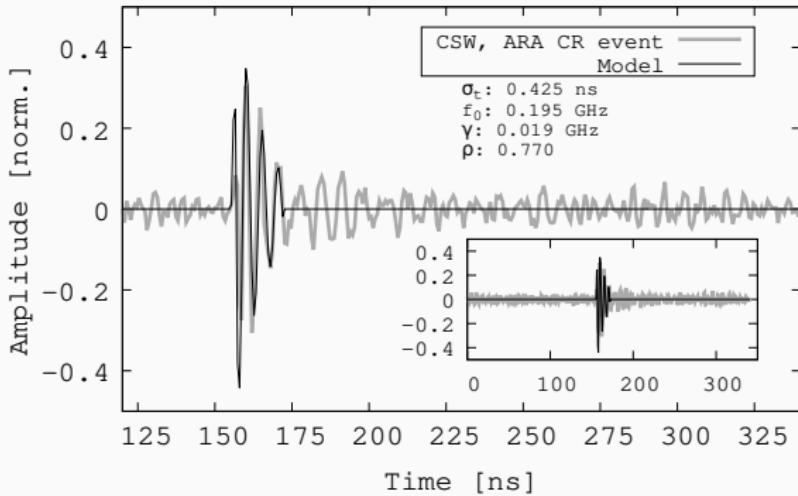


Figure 7: Fit for analytic  $s * r$  to an ARA5 UHECR candidate 2171-31805.

## Conclusion

---

# UHECR and UHE- $\nu$ Reconstruction with Analytic Signals

## 1. Introduction:

- A fully analytic Askaryan E-field model in the time-domain [4]
- Based on work begun with Prof. Amy Connolly [3]
- Based on work begun by J. Ralston and R. Buniy [2]

## 2. UHECR and UHE- $\nu$ identification:

- A fully analytic Askaryan voltage model, time-domain [this work]
- Equations for both voltage trace, and envelope of voltage trace
- Verification with NuRadioMC: strong thermal background rejection, signal identification, and (rough)  $\log_{10}(E_\nu)$  estimate

## 3. Correlation with Recent ARA observations of UHECR

- Three events reconstructed, with ten more to come
- Improvements: noise filter, channel selection
- **Proposal:** short manuscript with equations and fits
- Mathematical physics and NuRadioMC comparison has been submitted to PRD

# Bibliography

---

## Bibliography i

-  J. Alvarez-Muniz, P. M. Hansen, A. Romero-Wolf, and E. Zas.  
**Askaryan radiation from neutrino-induced showers in ice.**  
*Phys. Rev. D*, 101:083005, Apr 2020.
-  R. V. Buniy and J. P. Ralston.  
**Radio detection of high energy particles: Coherence versus multiple scales.**  
*Physical Review D*, 65(1), 2001.

## Bibliography ii

-  J. C. Hanson and A. L. Connolly.  
**Complex analysis of askaryan radiation: A fully analytic treatment including the LPM effect and cascade form factor.**  
*Astroparticle Physics*, 91:75–89, 2017.
-  J. C. Hanson and R. Hartig.  
**Complex analysis of askaryan radiation: A fully analytic model in the time domain.**  
*Physical Review D*, 105(12):123019, 2022.

## Bibliography iii

-  I. Kravchenko et al.  
Updated results from the RICE experiment and future  
prospects for ultra-high energy neutrino detection at the  
south pole.  
*Physical Review D*, 85(6):062004, 2012.
-  The ARA Collaboration.  
Design and initial performance of the Askaryan Radio  
Array prototype EeV neutrino detector at the South Pole.  
*Astroparticle Physics*, 35(7):457–477, 2012.

## Bibliography iv

-  The ARA Collaboration.  
**Constraints on the diffuse flux of ultrahigh energy neutrinos from four years of Askaryan Radio Array data in two stations.**  
*Physical Review D*, 102(4):043021, 2020.
-  The IceCube-Gen2 Collaboration.  
**IceCube-Gen2: The Window to the Extreme Universe.**  
*arXiv*, 2020.
-  The RNO-G Collaboration.  
**Design and sensitivity of the Radio Neutrino Observatory in Greenland (RNO-G).**  
*Journal of Instrumentation*, 16(03):P03025, 2021.