

# Algebra-Based Physics-1: Midterm 1

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## 1 Unit 0: Estimation, Unit Analysis, Vectors, and Kinematics I

1. Which of the following represents the density of lead?

- ~~A: 0.11 g cm<sup>-3</sup>~~
- ~~B: 1.10 g cm<sup>-3</sup>~~
- C: 11.0 g cm<sup>-3</sup>
- ~~D: 111 g cm<sup>-3</sup>~~

↓  
heavier  
than water  
water = 1g

2. A train leaves Los Angeles Union Station for the Bay Area (Emeryville) at 60 km/hr. If the destination is 600 km to the North, how long before the train reaches the destination?

- ~~A: 0.50 hours~~
- ~~B: 5.00 hours~~
- C: 10.0 hours
- ~~D: 24.0 hours~~

LA → BA 10.0 hrs  
 $\frac{60 \text{ km}}{\text{hr}} = \frac{600 \text{ km}}{\text{hr}}$

3. What is 25 m s<sup>-1</sup> in km hr<sup>-1</sup>?

- ~~A: 15 km hr<sup>-1</sup>~~
- ~~B: 25 km hr<sup>-1</sup>~~
- ~~C: 60 km hr<sup>-1</sup>~~
- D: 90 km hr<sup>-1</sup>

$1 \frac{\text{m}}{\text{s}} = 3.6 \frac{\text{km}}{\text{hr}}$   
 $25 \text{ m s}^{-1} \times 3.6 \frac{\text{km}}{\text{hr}} = 90 \text{ km/hr}$

4. Suppose a ship accelerates from 0 km hr<sup>-1</sup> to 10 km hr<sup>-1</sup> in 60 seconds. What is the acceleration?

- ~~A: 60 km hr<sup>-1</sup> s<sup>-1</sup>~~
- ~~B: 6 km hr<sup>-1</sup> s<sup>-1</sup>~~
- C: 1/6 km hr<sup>-1</sup> s<sup>-1</sup>
- ~~D: 1/60 km hr<sup>-1</sup> s<sup>-1</sup>~~

$\vec{a} = \Delta \vec{v} / \Delta t$   
 $\vec{a} = 10 \frac{\text{km}}{\text{hr}} \times \frac{60 \text{ secs}}{1} = \frac{1}{6} \text{ km/hr s}^{-1}$

5. Estimate the area of the North Quad of Whittier College (the open space outside the SLC):

- A: 5000 m<sup>2</sup>
- ~~B: 5000 cm<sup>2</sup>~~
- ~~C: 500 m<sup>2</sup>~~
- ~~D: 500 cm<sup>2</sup>~~

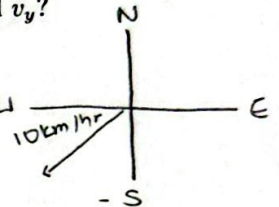
a)  $v = u + at$   
 $= 15 + (3)(4)$   
 $= 27 \frac{\text{m}}{\text{s}}$   
b)  $s = ut + \frac{1}{2}at^2$   
 $= (15)(4) + (\frac{1}{2})(3)(4^2)$   
 $= 60 + 24$   
 $= 84 \text{ m}$

6. A coffee bean is about 0.5 cm<sup>3</sup> in volume. How many could fit in a 2 liter bottle?

$2 \text{ liters} = 2000 \text{ cm}^3$   
 $\frac{2000 \text{ cm}^3}{0.5 \text{ cm}^3} = 4000 = 4 \times 10^3$   
• ~~A: 4 × 10<sup>1</sup>~~  
• ~~B: 4 × 10<sup>2</sup>~~  
• C: 4 × 10<sup>3</sup>  
• ~~D: 4 × 10<sup>4</sup>~~

7. Let  $\vec{v} = v_x \hat{i} + v_y \hat{j}$  represent a velocity vector. The wind velocity is 10 km/hr, Southwest. North and East vector components are positive, while South and West are negative. What are  $v_x$  and  $v_y$ ?

- ~~A: 7.1 and 7.1 km/hr~~
- ~~B: -7.1 and 7.1 km/hr~~
- ~~C: 7.1 and -7.1 km/hr~~
- D: -7.1 and -7.1 km/hr



8. What is the angle the velocity makes with the x-axis, in the previous exercise?

- A: 225 degrees
- ~~B: 180 degrees~~
- ~~C: 135 degrees~~
- ~~D: 90 degrees~~

$180^\circ + 45^\circ = 225^\circ$

9. (a) Let  $\vec{v} = -2\hat{i} + 2\hat{j}$ , and  $\vec{w} = 2\hat{i} - 2\hat{j}$ . Draw each in a 2D coordinate system below. (b) What is  $\vec{v} + \vec{w}$ ? (c) What is  $\vec{v} - \vec{w}$ ? (d) Add  $\vec{v} + \vec{w}$  and  $\vec{v} - \vec{w}$  to your coordinate system. (e) What is  $\vec{v} \cdot \vec{w}$ ?

a) e)  $\vec{v} \cdot \vec{w} = (-2 \cdot 2) + (2 \cdot -2) = -4 - 4 = -8$

b)  $\vec{v} + \vec{w} = (-2\hat{i} + 2\hat{j}) + (2\hat{i} - 2\hat{j})$   
 $= (-2\hat{i} + 2\hat{i}) + (2\hat{j} - 2\hat{j})$   
 $= 0\hat{i} + 0\hat{j}$

c)  $\vec{v} - \vec{w} = (-2\hat{i} + 2\hat{j}) - (2\hat{i} - 2\hat{j})$   
 $= (-2\hat{i} - 2\hat{i}) + (2\hat{j} + 2\hat{j})$   
 $= -4\hat{i} + 4\hat{j}$

## 2 Unit 1: Kinematics II and III

1. Suppose a cyclist has a velocity of 15 m s<sup>-1</sup> at  $t = 0$ . If the acceleration is 3 m s<sup>-2</sup>, (a) what is the velocity at  $t = 4$  seconds? (b) What is the displacement of the cyclist at  $t = 4$  seconds? (c) Are the average and instantaneous velocities different at  $t = 0$  or  $t = 4$  seconds?

	t	0	4
Inst v <sub>elo</sub>		15 m/s	27 m/s
Aver v <sub>elo</sub>		0	21 m/s

$\frac{84 \text{ m}}{4}$



2. Consider the motion of the system depicted in Fig. 1.

(a) From the given data, calculate the speed of the system at points P and Q. (b) Is the acceleration of the system positive or negative? Estimate the acceleration.

$t(s) = 5 \quad x(m) = 338$   
 $t(s) = 10 \quad x(m) = 600$   
 $600 - 338 = 262m$   
 $10 - 5 = 5s$   
 $v_p = \frac{\Delta x}{\Delta t} = \frac{262m}{5s}$   
 $v_p = 52.4 m/s$   
 $t(s) = 20 \quad x(m) = 1500$   
 $t(s) = 25 \quad x(m) = 2138$   
 $2138 - 1500 = 638m$   
 $25 - 20 = 5s$   
 $v_q = \frac{\Delta x}{\Delta t} = \frac{638m}{5s}$   
 $v_q = 127.6 m/s$   
 $a = \frac{v_q - v_p}{\Delta t} = \frac{127.6 - 52.4}{5} = 15.04 m/s^2$   
 $a = 15.04 m/s^2$

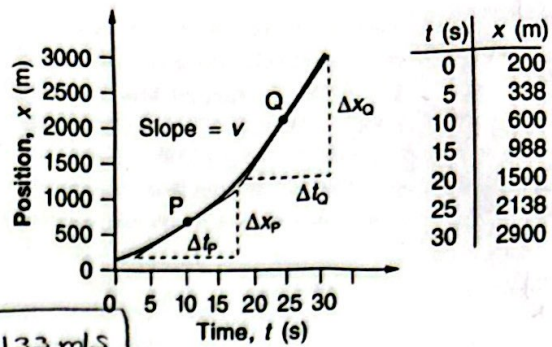


Figure 1: A system moves with non-constant velocity.

3. A swan on a lake gets airborne by flapping its wings and running on top of the water. (a) If the swan must reach a velocity of  $6.00 m/s$  to take off and it accelerates from rest at an average rate of  $0.8 m/s^2$ , how far will it travel before becoming airborne? (b) How long does this take?

$s = \frac{v^2 - u^2}{2a}$   
 $s = \frac{(6)^2 - (0)^2}{(2)(0.8)}$   
 $s = 22.5 meters$   
 $t = \frac{v - u}{a}$   
 $t = \frac{(6) - 0}{0.8}$   
 $t = 7.5 seconds$

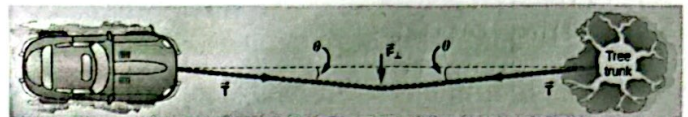


Figure 2: The net force is zero, just as the vehicle begins to move.

### 3 Unit 2: Forces I and II

1. Consider the effort to pull a vehicle from a ditch in Fig.

2. (a) If we can pull with  $F_{\perp} = 1000 N$ , and observe that the rope makes a  $7$  degree angle with respect to the line between the vehicle and the tree, what is the tension in the rope? (b) If the vehicle has  $900 kg$ , and the coefficient of kinetic friction is  $0.05$ , what is the acceleration of the vehicle as it starts to move?

$T = \frac{F}{\sin \theta}$   
 $T = \frac{1000}{(\sin 7)}$   
 $T = 8192.8 N$   
 $T = T \cos(\theta)$   
 $T = (8192.8)(\cos(7))$   
 $T = 8138.6 N$   
 $N = W - T$   
 $= 8829 - 8138.6 = 690.4$   
 $F_f = \mu_k \cdot N$   
 $= (0.05)(690.4)$   
 $= 34.52 N$   
 $F_{net} = 8192.8 - 34.52 = 8158.28 N$   
 $a = \frac{F_{net}}{m}$   
 $a = \frac{8158.28}{900}$   
 $a = 9.06 m/s^2$

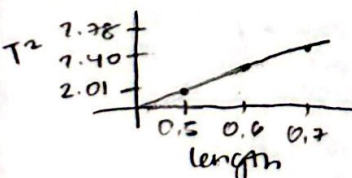
2. A  $20,000 kg$  jet fighter lands on an aircraft carrier, moving at  $120 km/hr$ . A tow cable grabs the aircraft and pulls it to a stop in  $100 meters$ . (a) What is the average acceleration? (b) What force does the tow cable exert to stop the jet?

$a = \frac{v_f^2 - v_i^2}{2\Delta x}$   
 $a = \frac{0 - (33.33)^2}{(2)(100)}$   
 $a = -5.55 m/s^2$   
 $F = m \cdot a$   
 $F = (20000)(-5.55)$   
 $F = -111000 N$

5. Design problem. In our lab, we used a pendulum to measure  $g$ , the gravitational constant. Use the following simulation to repeat that process: <https://phet.colorado.edu/en/simulations/pendulum-lab>. Recall that the formula relating period  $T$  to  $g$  is  $T = 2\pi\sqrt{L/g}$ , where  $L$  is the pendulum length. Show your work in the form of a (handwritten) graph and relevant calculations.

$T = 2\pi\sqrt{\frac{L}{g}}$   
 $g = \frac{4\pi^2 L}{T^2}$

L(m)	T(s)	T <sup>2</sup> (s <sup>2</sup> )	(m/s <sup>2</sup> )
0.5	1.42	2.01	9.81
0.6	1.55	2.40	9.82
0.7	1.67	2.78	9.81



$F = (10)(\cos 45) + (8)(\cos 30)$   
 $F = 14.0 N$   
 $F = 10(\sin 45) - (8)(\sin 30)$   
 $F = 3.07 N$   
 $|F| = \sqrt{(14)^2 + (3.07)^2}$   
 $|F| = 14.3326$

3. Two children pull a third child on a snow saucer sled exerting forces  $\vec{F}_1$  and  $\vec{F}_2$  as shown from above in Fig. 3. Find the acceleration of the  $50 kg$  sled and child

$a = \frac{F_{net}}{m}$   
 $a = \frac{14.3326 + 7.5}{50}$   
 $a = 0.43664 m/s^2$



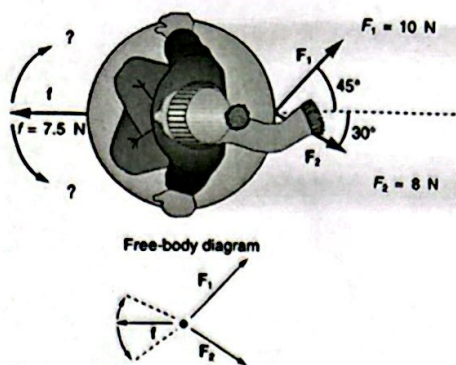


Figure 3: Two people pull on a third person on a sled, on an icy surface.

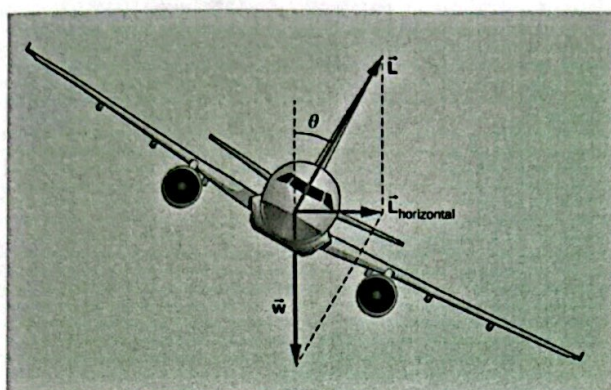


Figure 4: A plane banks into a circular turn.

system. Note that the direction of friction will be in the opposite direction of the sum of  $\vec{F}_1$  and  $\vec{F}_2$ .

#### 4 Unit 3: Forces III and IV

1. (a) Show that the acceleration of any object down an incline with friction is  $a = g(\sin \theta - \mu \cos \theta)$ . (b) What expression do you get as  $\mu \rightarrow 0$ ?

$$F = mg \cos \theta$$

$$F = mg \sin \theta$$

$$\mu N = \mu (mg \cos \theta)$$

$$F_{\text{net}} = mg \sin \theta - \mu (mg \cos \theta)$$

$$F_{\text{net}} = ma = mg \sin \theta - \mu (mg \cos \theta)$$

$$a = g \sin \theta - \mu g \cos \theta$$

$$\lim_{\mu \rightarrow 0} a = g \sin \theta$$

2. (a) Use the expression derived in the previous exercise to calculate the acceleration of a snowboarder traveling down a 10 degree incline. Use the standard values of  $g$  and coefficient of kinetic friction between waxed wood and snow. (b) How far down the slope will the person travel after 30 seconds, and what is their speed?

$$a = 9.8(\sin(10) - \mu \cos(10))$$

$$a = 9.8(0.1736 - 0.12436)$$

$$a = 1.218 \text{ m/s}^2$$

$$s = ut + \frac{1}{2}at^2$$

$$s = (0)(30) + \frac{1}{2}(1.218)(30^2)$$

$$s = 548.1 \text{ m}$$

3. Consider Fig 4, in which a plane flies in a circular trajectory. Suppose the total mass is 6000 kg,  $\theta = 30$  degrees, and magnitude of the lift force in Fig. 4 is 80,000 N. (a) What is the centripetal force? (b) If the speed is  $600 \text{ km hr}^{-1}$ , what is the turn radius? (c) What time will pass before the plane has gone halfway around the circle (to turn around)?

$$F_c = \frac{mv^2}{r}$$

$$F_L = F_L \sin \theta$$

$$F_c = F_L \sin \theta$$

$$F_c = (80000)(\sin(30))$$

$$F_c = 40000 \text{ N}$$

$$v = \frac{mv^2}{F_c}$$

$$r = \frac{(6000)(166.67)^2}{(40000)}$$

$$r = 4166.83 \text{ m}$$

$$t = \frac{\pi r}{v}$$

$$t = \frac{(\pi)(4166.83)}{166.67}$$

$$t = 78.541 \text{ s}$$

4. Consider three springs connected in parallel to an object of mass  $m$ . Each spring has a spring constant  $k$ , and each spring is attached to the floor and the object. (a) Draw a free-body diagram. (b) Derive an expression for the displacement of the springs. (c) Show that, in the limit that  $k \rightarrow \infty$ , the displacement goes to zero. This is a basic model for the suspension of a vehicle or cart.

$$F = 3kx$$

$$F_2 = mg$$

$$3kx - mg = 0$$

$$3kx = mg$$

$$x = \frac{mg}{3k}$$

$$\lim_{k \rightarrow \infty} \frac{mg}{3k} = 0$$

5. What is the terminal velocity of a 60 kg skydiver with area  $A = 0.25 \text{ m}^2$ , and drag coefficient  $C = 0.5$ ? Use the standard density of air:  $\rho = 1.2 \text{ kg m}^{-3}$ . (b) What is the terminal velocity if she opens the parachute, increasing the cross-sectional area by a factor of 100?

$$v_t = \sqrt{\frac{2mg}{\rho A C_d}}$$

$$v_t = \sqrt{\frac{(2)(60)(9.8)}{(1.2)(0.25)(0.5)}}$$

$$v_t = 88.543 \text{ m/s}$$

$$v_t = \sqrt{\frac{(2)(60)(9.8)}{(1.2)(25)(0.5)}}$$

$$v_t = 8.8543 \text{ m/s}$$

6. (a) Granite has a standard Young's modulus of about  $45 \times 10^9 \text{ N m}^{-2}$ . Calculate the change in length of a granite column supporting 10,000 N of weight. The column has a diameter of 20 cm, and is 10 meters tall. (b) Suppose the granite column was replaced with a new material with half the Young's modulus. What would the new change in length be?

$$A = \pi \left(\frac{0.2}{2}\right)^2$$

$$A = 0.03141 \text{ m}^2$$

$$\Delta L = \frac{(10,000)(10)}{(0.03141)(45 \times 10^9)}$$

$$\Delta L = 1.09200 \times 10^{-4} \text{ m}$$

$$v = ua +$$

$$v = 0 + 1.218 \cdot 30$$

$$v = 36.54 \text{ m/s}$$