CALCULUS-BASED PHYSICS-2: ELECTRICITY AND MAGNETISM (PHYS180-02): UNIT 2

Jordan Hanson

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Whittier College Department of Physics and Astronomy

UNIT 1 REVIEW

Reading: Chapters 5-6

1. Charge, Conductors and Insulators
2. Coulomb's Law and Electric Fields
3. E-fields of Charge Distributions
4. Gauss's Law

UNIT 1 REVIEW PROBLEMS

Suppose a charge q is located at the origin. Use Gauss' law to find the electric field. What is the electric field?

- A: $\vec{E} = \frac{kq}{r}\hat{r}$
- B: $\vec{E} = \frac{kq}{r^2}\hat{r}$
- C: $\vec{E} = \frac{kq}{r^3}\hat{r}$
- D: $\vec{E} = \frac{kq^2}{r^2}\hat{r}$

Suppose a line of charge runs up the z-axis. The charge per unit length is λ. Use Gauss' law to find the electric field. What is the electric field at a point $P = (x, 0, 0)$?

- A: $\vec{E} = \frac{2k\lambda}{r^2}\hat{x}$
- B: $\vec{E} = \frac{2k\lambda^2}{r^2}\hat{x}$
- C: $\vec{E} = \frac{2k\lambda}{r}\hat{x}$
- D: $\vec{E} = 0$

SUMMARY

Reading: Chapters 7-8

1. Voltage
    1.1 Review of work and energy
    1.2 Review of conservative forces
2. Capacitance

VOLTAGE

Voltage is analogous to potential energy in electrostatics. The negative derivative of potential energy U is the force F:

$$F = -\frac{dU}{dx} \tag{1}$$

For example, if the force is $F = -mg$, and the potential energy is $U = mgy$, then

$$\frac{dU}{dy} = -mg \tag{2}$$

Voltage is analogous to potential energy in electrostatics. The derivative of voltage V is the field E:

$$E = -\frac{dV}{dx} \tag{3}$$

For example, if the field is $E = \sigma/\epsilon_0$, and the potential energy is $V = \sigma/\epsilon_0 x + C$, then

$$\frac{dV}{dx} = \sigma/\epsilon_0 \tag{4}$$

Voltage is analogous to potential energy in electrostatics.

Potential energy is just an energy. A *potential* in mechanics is the potential energy per unit mass. *Voltage*, or *electrostatic potential* is **potential energy per unit charge.**

The Volt (V) is the unit of voltage, and it has units of 1 V = 1 J/C.

Group board exercise: Show that the units of electric field (normally Newtons per Coulomb) are also Volts per meter.

If the potential energy is a function of displacement, $U = U(\vec{x})$, it may be called a potential energy *surface*.

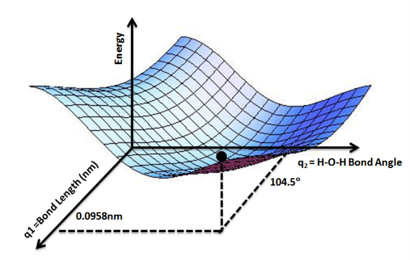

Figure 1: An example of a potential energy surface.

Considering *Newton's Second Law*, however, if $F = ma$ then $ma = -\frac{\Delta U}{\Delta x}$, and

$$a = -\frac{1}{m}\frac{\Delta U}{\Delta x} \tag{5}$$

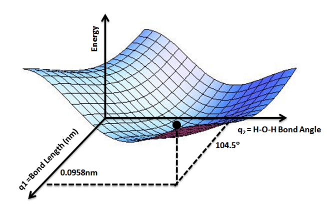

Figure 2: If we divide by the mass we have acceleration.

The derivative, or *gradient*, of a in Eq. 5 is analogous to the electric field. But the electric field is a *vector field*...

The gradient is like a derivative, but gives you the proper direction wherever you are.

$$\boxed{-\vec{\nabla}V = \vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}} \tag{6}$$

Suppose the voltage due to some charge distribution is $V(x, y, z) = ax + b$. What is the field?

- A: $\vec{E} = a\hat{j}$
- B: $\vec{E} = a\hat{k}$
- C: $\vec{E} = b\hat{i}$
- D: $\vec{E} = -a\hat{i}$

Suppose the voltage due to some charge distribution is $V(x, y, z) = ax + by$. What is the field?

- A: $\vec{E} = -a\hat{j} - b\hat{j}$
- B: $\vec{E} = a\hat{k} + b\hat{k}$
- C: $\vec{E} = -b\hat{i} - a\hat{j}$
- D: $\vec{E} = -a\hat{i} - b\hat{j}$

Suppose a charge distribution is made of two infinite plates of charge with charge per unit area $\pm\sigma$. We know that the field is $\sigma/\epsilon_0\hat{k}$ between them. **Group board exercise**: Draw the charge distribution, define a coordinate system, and write the function for the voltage.

Voltage is like a potential energy surface $\rightarrow$ *potential energy per unit charge.*

`https://phet.colorado.edu/en/simulation/charges-and-fields`

Using the PhET simulation about charges and fields:

1. Explore the voltage associated with fields generated by charges using the voltage button.
2. Add a single point charge, and use the ruler and voltmeter (potentiometer) to measure voltage versus distance, and plot it.
3. What function describes the relationship between voltage and distance?

https://phet.colorado.edu/en/simulation/charges-and-fields

Using the PhET simulation about charges and fields:

1. Note that the units of ϵ_0 are N m^2 C^{-2}, and the value is 8.854×10^{-12}
2. We know from prior equations that the units of voltage are J C^{-1}
3. Using your measurements, show that the voltage due to a point charge is

$$\boxed{V = \pm \frac{1}{4\pi\epsilon_0} \frac{q}{r}} \tag{7}$$

(Where the sign depends on the charge, just like E-fields)

Voltage due to a point charge:

$$\boxed{V = \pm\frac{1}{4\pi\epsilon_0}\frac{q}{r}} \tag{8}$$

Voltage is an example of a **scalar field**, whereas the electric field is an example of a **vector field.** Create two lines of charge, one positive and one negative in the PHeT simulator, and pretend they represent planes of charge coming out of the screen. The field should constant and uniform like

$$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{k} \tag{9}$$

1. Plot the voltage versus distance between the plates.
2. Calculate the slope in V/m, and the y-intercept in V.
3. Does the electric field depend on the y-intercept? Does the charge distribution?

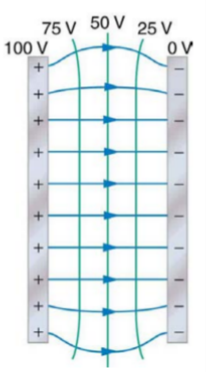

Figure 3: Parallel plates of charge, electric field, and potential. Notice the linear decrease in voltage.

How we define the y-intercept in voltage is analogous to the zero-point freedom of potential energy.

Two parallel plates, opposite charge:

$$V = -\frac{\sigma}{\epsilon_0} z + C \tag{10}$$

With the boundary condition that $V = V_0$ when $z = 0$, we have

$$V(z) - V_0 = -\frac{\sigma}{\epsilon_0} z \tag{11}$$

Let $\Delta V(z) = V(z) - V_0$, and $\Delta z = z$:

$$-\frac{\Delta V}{\Delta z} = \frac{\sigma}{\epsilon_0} = E \tag{12}$$

Continuing with PHeT: Make a ring of charge that has a radius of 100 cm, with the center at the origin.

1. Show that the voltage field is radially symmetric.
   - Place *equipotential lines* around the charge distribution and see that they are circular.
2. Show that at distances much larger than the radius, the voltage field looks like that of a point charge.
   - Record the voltage versus distance from the origin.
   - Plot the data.
   - Compare to the plot corresponding to a point charge.

UNITS OF ENERGY

The electron-volt: eV. This is the energy gained by an electron accelerated through a voltage of 1 V.

- How many Joules per electron volt?
- What is the mass of a proton in eV? ($E = mc^2$, where c is the speed of light.)

A proton is released into a 40 kV electric potential. What is the final kinetic energy of the proton?

- A: -20 kV
- B: 40 kV
- C: -40 keV
- D: 40 keV

An alpha particle is a helium nucleus with a charge of $+2q_e$. Suppose an alpha particle is accelerated and has a final energy of 2 MeV. How many volts were required to accelerated it?

- A: 4 MV
- B: 3 MV
- C: 2 MV
- D: 1 MV

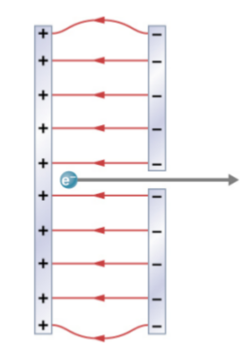

Figure 4: An electron accelerated by a parallel plate capacitor.

An electron has a charge $-q_e$. Suppose an electron is accelerated through a voltage of 1 V toward another electron at a fixed position. How close does the moving electron get to the stationary one?

- A: about 0.1 nm
- B: about 1 nm
- C: about 10 nm
- D: about 100 nm

A bare helium nucleus has two positive charges and a mass of 6.64×10^{-27} kg. What is the kinetic energy at 2 percent of the speed of light? The speed of light is $c = 3.0 \times 10^{8}$ m/s.

- A: 1.2×10^{-10} J
- B: 1.2×10^{-11} J
- C: 1.2×10^{-12} J
- D: 1.2×10^{-13} J

What is the previous energy in eV?

- A: 0.75 MeV
- B: 0.75 keV
- C: 0.75 GeV
- D: 0.75 eV

Remember that this is an alpha particle (helium nucleus) with a +2 charge. How many volts are required to give it an energy of 0.75 MeV?

- A: 0.375 MV
- B: 0.75 MV
- C: 1.5 MV
- D: 0.375 V

CAPACITANCE

Capacitance is the ability of an object to store charge. Let the voltage difference across an object be V, storing $-Q$ on one side and $+Q$ on another. The *capacitance* C is given by

$$Q = CV \tag{13}$$

Let the object be two parallel plates, and the charges be $\pm Q$.

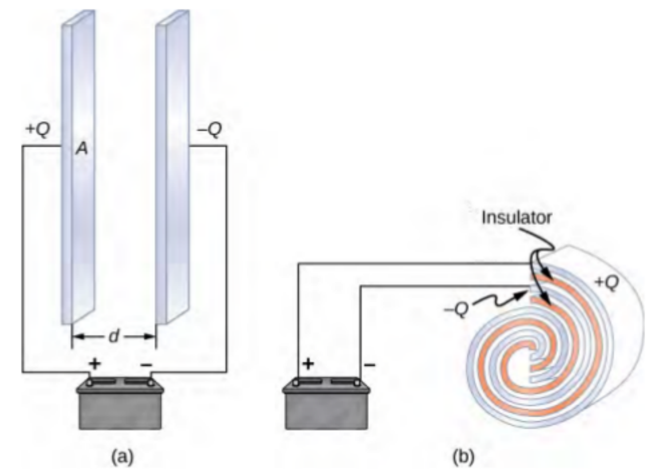

Figure 5: General scheme of a capacitor.

The electric field and voltage between two charged plates separated by a distance d is $V = Ed$. The field is $E = \sigma/\epsilon_0$, and $Q = \sigma/A$, where A is the plate area and σ is the surface charged density. Note that

$$
\begin{aligned}
C = Q/V &= (Q)/(Ed) && (14)\\
&= (Q\epsilon_0)/(\sigma d) = (QA\epsilon_0)/(Qd) && (15)\\
&= \frac{A\epsilon_0}{d} && (16)
\end{aligned}
$$

So the capacitance depends on the permittivity of free space, the area, and the distance. In other words, just the geometry of the system. The units of capcitance are **Farads** (F). This is a large unit. Typical capacitors have nF or pF.

Group board exercise: The permittivity of free space is $8.85 \times 10^{-12}\ \mathrm{N^{-1}\ C^2\ m^{-2}}$. A capacitor has an area of $1\ \mathrm{mm}^2$, and $d = 0.001$ mm. What is the capacitance?

Group board exercise: The permittivity of free space is $8.85 \times 10^{-12}\ \mathrm{N^{-1}\ C^2\ m^{-2}}$. A capacitor has an area of $10\ \mathrm{mm}^2$, and $d = 0.001$ mm. What is the capacitance?

Group board exercise: The permittivity of free space is $8.85 \times 10^{-12}\ \mathrm{N^{-1}\ C^2\ m^{-2}}$. A capacitor has an area of $1\ \mathrm{mm}^2$, and $d = 0.01$ mm. What is the capacitance?

CAPACITORS IN SERIES AND IN PARALLEL

Observe on board.

1. Consider two capacitors *in series*, with capacitance C_1 and C_2. What is the total capacitance?
2. Consider two capacitors *in parallel*, with capacitance C_1 and C_2. What is the total capacitance?

Create a simple circuit with capacitors, and a battery to charge them. How much total charge is stored? Exchange your design with another group, and solve each others' problem.

THE CYLINDRICAL CAPACITOR AND COAXIAL CABLES

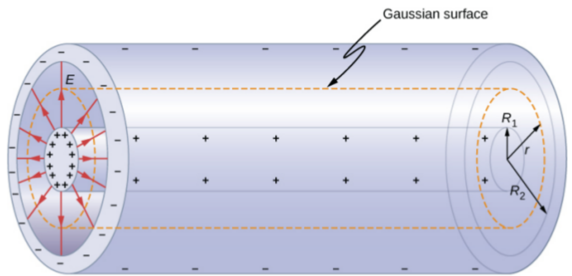

Figure 6: A cylindrical capacitor, as a model for a coaxial cable. There is an inner radius and an outer radius, with our Gaussian surface drawn in between the two conductors.

Recall that

$$Q = \Delta V C \tag{17}$$

$$\Delta V = -\int_{R_1}^{R_2} \vec{E} \cdot d\vec{r} \tag{18}$$

- What is the electric field $\vec{E}$ of a section of this coaxial cable of length l? (*Recall from warm-up.*).
- What if we integrate from positive charge (inner radius) to negative charge (outer)? **Observe on board.**

Thus we have the capacitance per unit length:

$$\boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(R_2/R_1)}} \tag{19}$$

What are the units of ϵ_0?

- A: F/m^2
- B: F/m
- C: F/ln(m)
- D: F

Thus we have the capacitance per unit length:

$$\boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(R_2/R_1)}} \tag{20}$$

Suppose the cap per unit length is 0.1 nF, and we *square* R_2/R_1. What is the new cap per unit length?

- A: 0.1 nF
- B: 0.05 nF
- C: 0.2 nF
- D: 0 nF

Thus we have the capacitance per unit length:

$$\boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(R_2/R_1)}} \tag{21}$$

Suppose the cap per unit length is 0.1 nF, and we *triple* ϵ_0 in some way. What is the new cap per unit length?

- A: 0.1 nF
- B: 0.2 nF
- C: 0.3 nF
- D: 0.5 nF

(Preview of Unit 3). Suppose we have a system that obeys

$$v(t) = R\frac{dQ}{dt} = i(t)R \tag{22}$$

This is called **Ohm's law.**

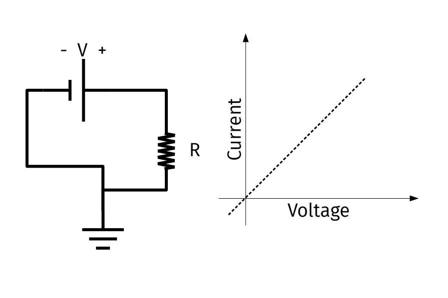

Figure 7: A simple circuit with a resistor element, some voltage, and *ground*. This just means that 0V is at the negative terminal.

(Preview of Unit 3). What if we add a capacitor?

$$V_0 = R\frac{dQ}{dt} + \frac{Q}{C} \tag{23}$$

This is called **Ohm's law.**

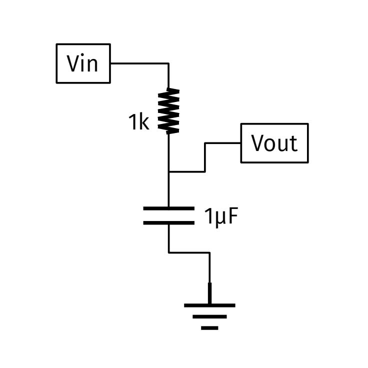

Figure 8: A simple circuit with a resistor and capacitor elements.

(Preview of Unit 3). What if we add a capacitor? We can show that

$$V_{out}(t) = V_0 \exp(-t/\tau) \tag{24}$$

with $\tau = RC$. Thus, if we send a signal V_{in} down a "very long" coaxial cable, with some capacitance per unit length, it will not exit the cable.

For example: `https://www.pasternack.com/images/ProductPDF/LMR-400.pdf`

What is the attenuation per 100 m of this cable at 150 MHz?

CONCLUSION

Reading: Chapters 7-8

1. Voltage
   1.1 Review of work and energy
   1.2 Review of conservative forces
2. Capacitance

ANSWERS

ANSWERS

- $\vec{E} = \frac{kq}{r^2}\hat{r}$
- $\vec{E} = \frac{2k\lambda}{r}$
- $\vec{E} = -a\hat{i}$
- $\vec{E} = -a\hat{i} - b\hat{j}$
- 40 keV
- 1 MV
- about 1 nm
- 1.2×10^{-13} J
- 0.75 MeV
- 0.375 MV
- F/m
- 0.05 nF
- 0.3 nF