

# Calculus - Based Physics - 1: Mechanics: Midterm 1

## Estimations and Unit Analysis

- 1 Suppose you are standing at the edge of a canyon. You clap and hear the sound echo off the other side of the canyon wall about 1.5 seconds later. You estimate the canyon wall to be about 0.5 km away. a) What is the speed of sound in meters per second? b) What is it in kilometers per hour?

$$a) \frac{0.5 \text{ km}}{1.5 \text{ sec}} \left| \begin{array}{c} 1000 \text{ m} \\ 1 \text{ km} \end{array} \right| = \frac{500 \text{ m/s}}{1.5}$$

$$b) \frac{0.5 \text{ km}}{1.5 \text{ sec}} \left| \begin{array}{c} 3600 \text{ sec} \\ 1 \text{ hr} \end{array} \right| = 1200 \text{ km/h}$$

- 2 a) What is  $0.25 \text{ m}^3$  in  $\text{cm}^3$ ? b) What is  $100 \text{ km/hour}$  in  $\text{m/s}$ ? c) What is  $2 \text{ km m s}^{-2}$  in  $\text{gm cm ms}^{-2}$

$$a) \frac{0.25 \text{ m}^3}{1 \text{ m}} \left| \begin{array}{c} 100 \text{ cm} \\ 1 \text{ m} \end{array} \right| \left| \begin{array}{c} 100 \text{ cm} \\ 1 \text{ m} \end{array} \right| \left| \begin{array}{c} 100 \text{ cm} \\ 1 \text{ m} \end{array} \right| = 250000 \text{ cm}^3$$

$$b) \frac{100 \text{ km}}{1 \text{ hr}} \left| \begin{array}{c} 1000 \text{ m} \\ 1 \text{ km} \end{array} \right| \left| \begin{array}{c} 1 \text{ m} \\ 1 \text{ m} \end{array} \right| \left| \begin{array}{c} 1 \text{ sec} \\ 3600 \text{ s} \end{array} \right| = \frac{250 \text{ m/s}}{9}$$

$$c) \frac{2 \text{ kg m}}{\text{s}^2} \left| \begin{array}{c} 1000 \text{ g} \\ 1 \text{ kg} \end{array} \right| \left| \begin{array}{c} 100 \text{ cm} \\ 1 \text{ m} \end{array} \right| \left( \frac{15}{1000 \text{ ms}} \right)^2 = \frac{1}{5} \frac{\text{g cm}}{\text{ms}^2}$$

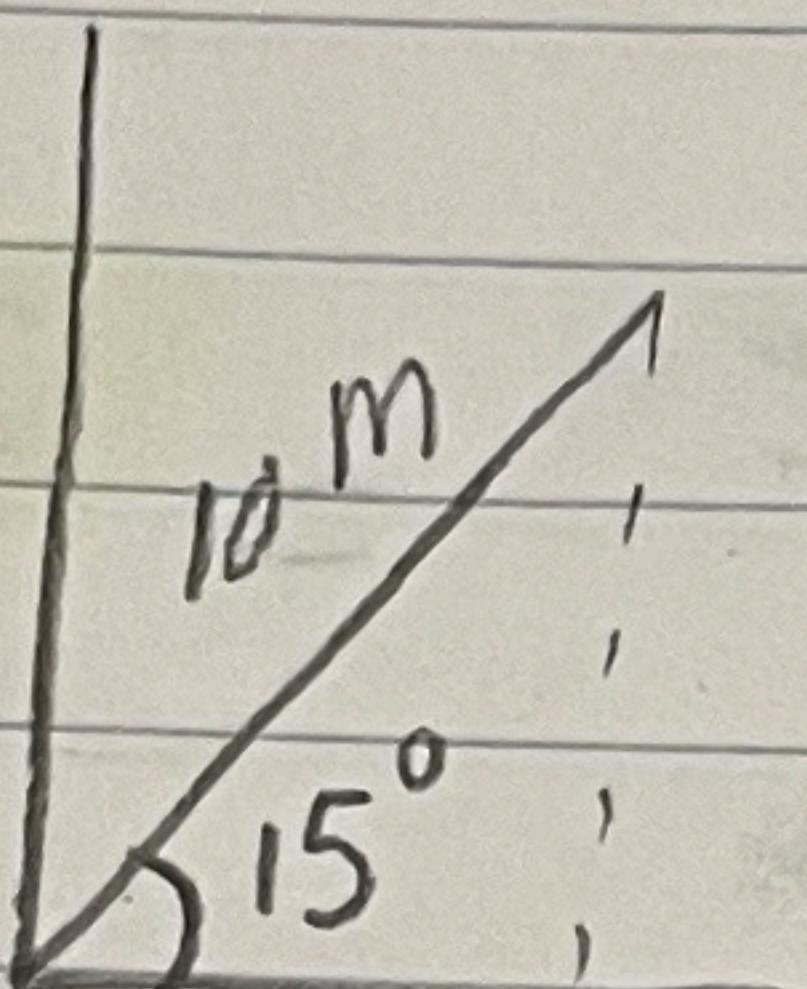
## Vectors

1 Write the following vectors in component form:

a)  $\vec{x}_1$  is a vector with a magnitude of 10 meters and that makes an angle of 15 degrees with respect to the x-axis

b)  $\vec{x}_2$  is a vector with a magnitude 20 meters that makes an angle of 135.0 degrees with respect to the x-axis

a)

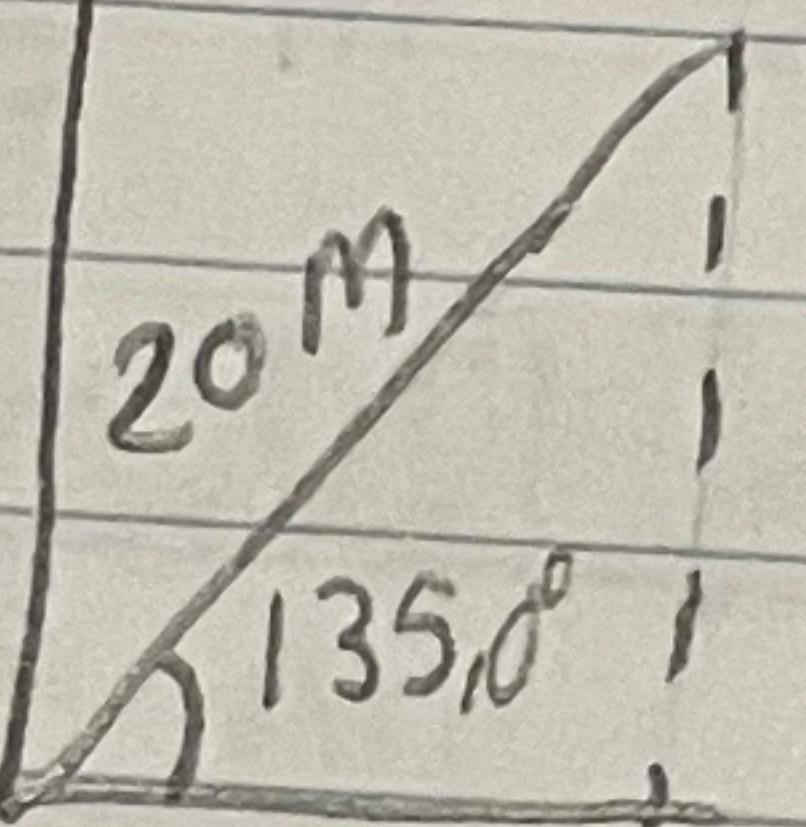


$$\sin(15) = \frac{y}{10}$$

$$\cos(15) = \frac{x}{10}$$

$$\vec{x}_1 = (9.65\hat{i} + 2.58\hat{j}) \text{ m}$$

b)



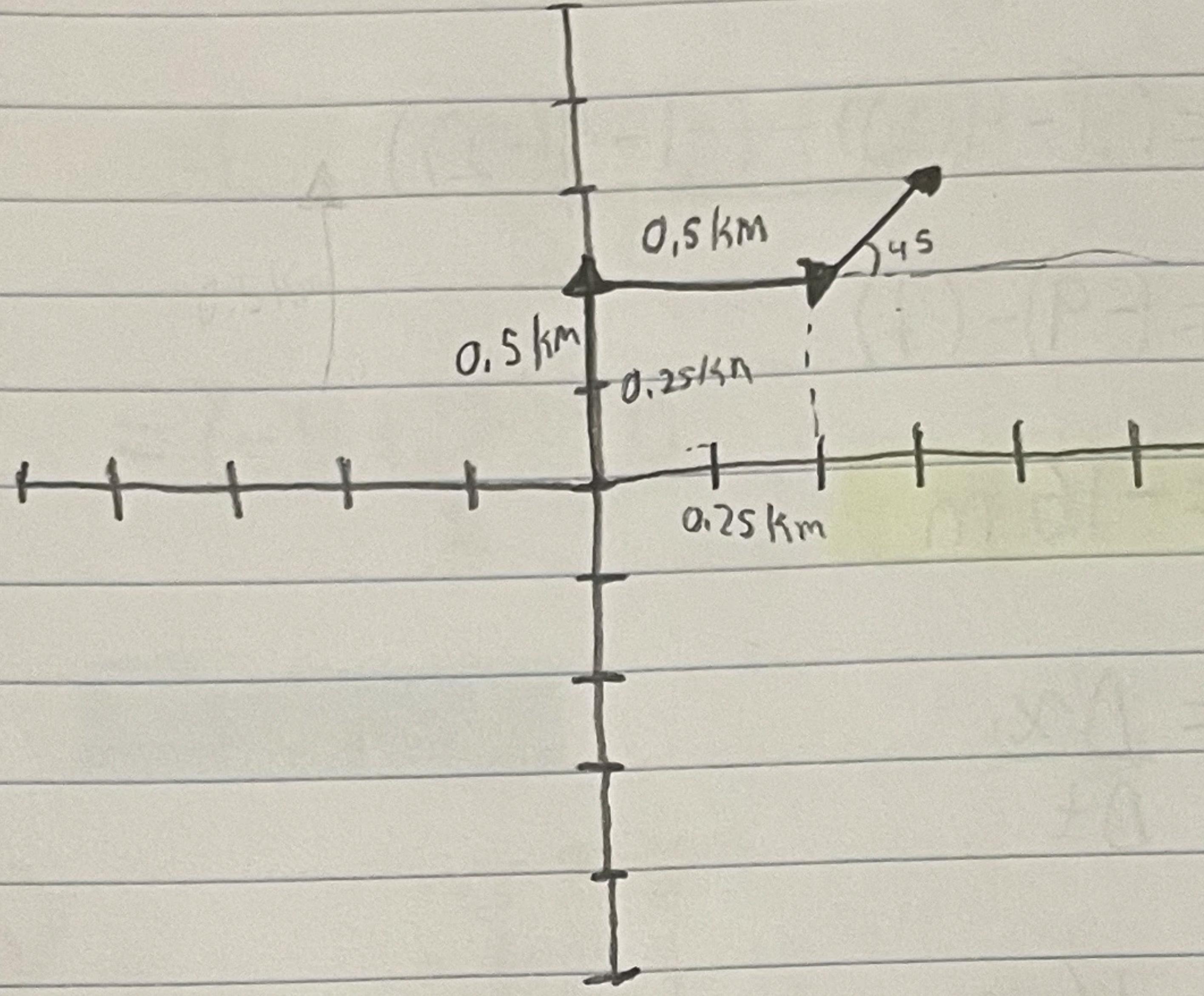
$$\sin(135) = \frac{y}{20}$$

$$\cos(135) = \frac{x}{20}$$

$$\vec{x}_2 = (-10\sqrt{2}\hat{i} + 10\sqrt{2}\hat{j}) \text{ m}$$

2 A person goes for a walk. They head 0.5 km to the north, and then 0.5 km to the east. Finally they head North-East at an angle of 45 degrees with respect to the x-axis for 0.25 km. a) Draw a diagram of their trajectory (East is x-axis, North is y-axis) b) what is the final location in x-y coordinates? c) what is the distance from the origin?

a)



$$b) 0.5 \text{ km} \hat{i} + 0.5 \text{ km} \hat{j} + (0.25 \cos(45) \text{ km} \hat{i}) + (0.25 \sin(45) \text{ km} \hat{j})$$

$$= \frac{4+\sqrt{2}}{8} \text{ km} \hat{i} + \frac{4+\sqrt{2}}{8} \text{ km} \hat{j}$$

$$c) \vec{x} = \sqrt{a^2 + b^2}$$

$$\vec{x} = \sqrt{\left(\frac{4+\sqrt{2}}{8}\right)^2 + \left(\frac{4+\sqrt{2}}{8}\right)^2}$$

$$\vec{x} = 0.95 \text{ km}$$

## Motion Along a straight line

- 1 The position of a particle moving along the  $x$ -axis is given by  $x(t) = -1.0 - 4.0t$  m. a) what is the displacement of the particle between  $t = -2.0$  seconds and  $t = 2.0$  seconds?  
b) what is the velocity?

$$\begin{aligned} \text{a) } \Delta x &= x(2.0) - (-2.0) \\ &= (-1 - 4(2)) - (-1 - 4(-2)) \\ &= (-9) - (7) \\ &= -16 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{b) } \vec{v} &= \frac{\Delta x}{\Delta t} \\ &= \frac{-16}{2.0 - (-2.0)} \\ &= -\frac{16}{4} \\ &= -4 \text{ m/s} \end{aligned}$$

2 A particle moves along the  $x$ -axis according to  $x(t) = -2t + 7t^2$

- What is the average velocity between  $t=0$  and  $t=2$  seconds?
- draw a graph of the velocity
- What is the instantaneous velocity at  $t=1$  second
- what is the acceleration?

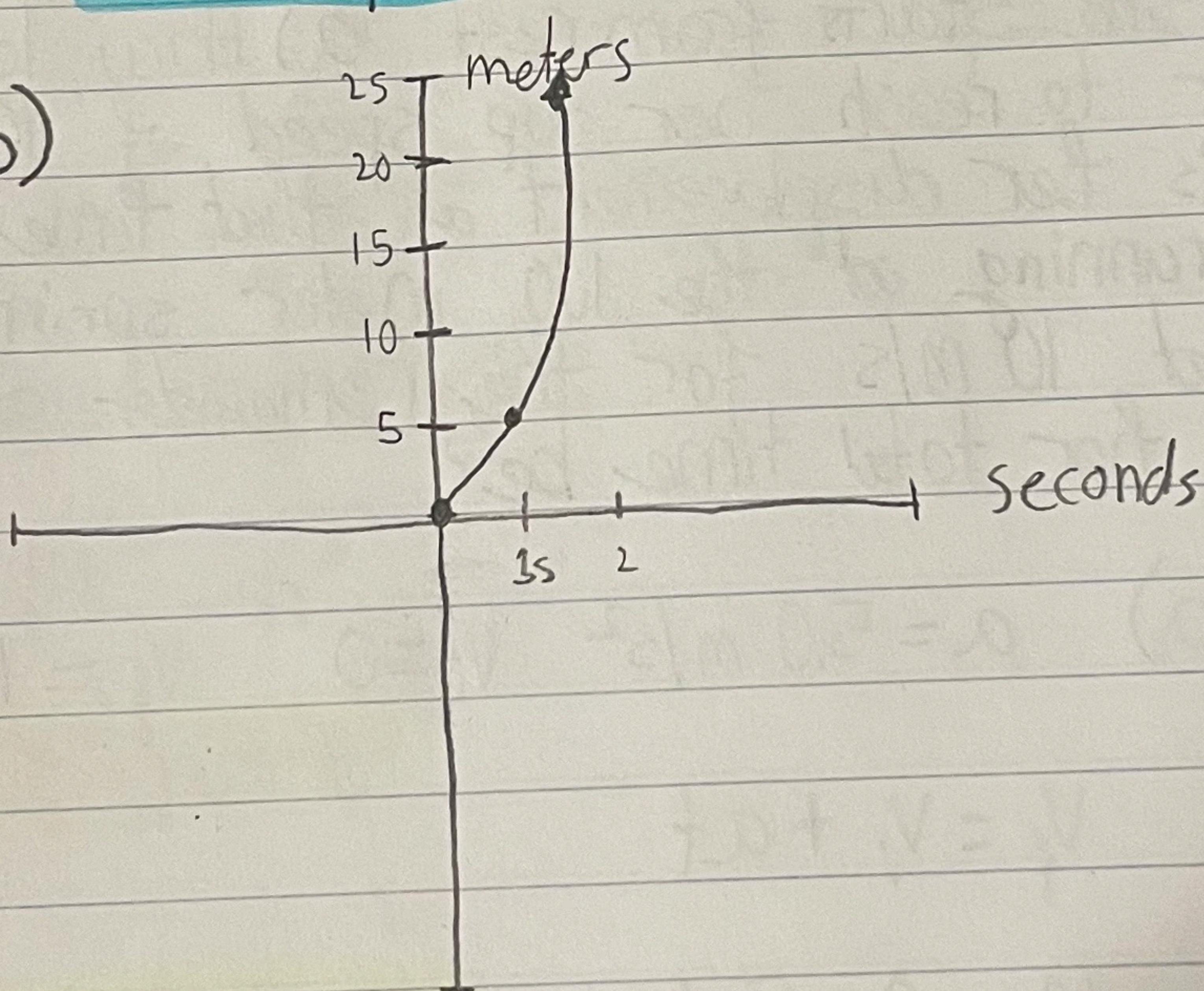
$$a) v = \frac{x(2) - x(0)}{2 - 0}$$

$$= \frac{(-2(2) + 7(2)^2) - (-2(0) + 7(0))}{2 - 0}$$

$$= \frac{(-4 + 28) - 0}{2}$$

$$= 12 \text{ m/s}$$

b)



$t$	$x(t)$
0	0
1	5
2	24
3	57

$$c) x'(t) = -2 + 14t$$

$$\begin{aligned}x'(1) &= -2 + 14(1) \\&= -2 + 14 \\&= 12 \text{ m/s}\end{aligned}$$

$$d) \ddot{a} = \frac{\Delta v}{\Delta t}$$

$$a = \frac{12 \text{ m/s}}{2 \text{ s}}$$

$$a = 6 \text{ m/s}^2$$

3 A sprinter has a constant acceleration of  $5.0 \text{ m/s}^2$ . Suppose she starts from rest.

- How long does it take her to reach her top speed of  $10 \text{ m/s}$ ?
- What is her displacement at that time?
- Suppose she is running at the 100 meter sprint. If she continues at  $10 \text{ m/s}$  for the remainder of the race, what will her total time be?

$$a = 5.0 \text{ m/s}^2 \quad v_i = 0 \quad v_f = 10 \quad t = ?$$

$$v_f = v_i + at$$

$$\frac{10}{5} = 0 + 5t \quad 2 \text{ seconds}$$

$$2 = t$$

$$b) x_f - x_i = \frac{1}{2} a t^2 + v_i t \neq x_i$$

$$x_f - x_i = \frac{1}{2} a t^2 + v_i t$$

$$\Delta x = \frac{1}{2} a t^2 + v_i t$$

$$\Delta x = \frac{1}{2} (0.0) (2)^2 + 0 (\pm)$$

$$\Delta x = 10 \text{ m}$$

$$c) DV = 10 \text{ m/s} \quad \Delta x = 100 \text{ m} \quad t = ?$$

$$\Delta x = \frac{1}{2} a t^2 + v_i t \quad v_i = 0$$

$$100 = \frac{1}{2} (5) t^2 + 0 t \quad v_f = 10$$

$$100 = \frac{1}{2} (5) t^2$$

$$50 \cdot 100 = \frac{5}{2} t^2$$

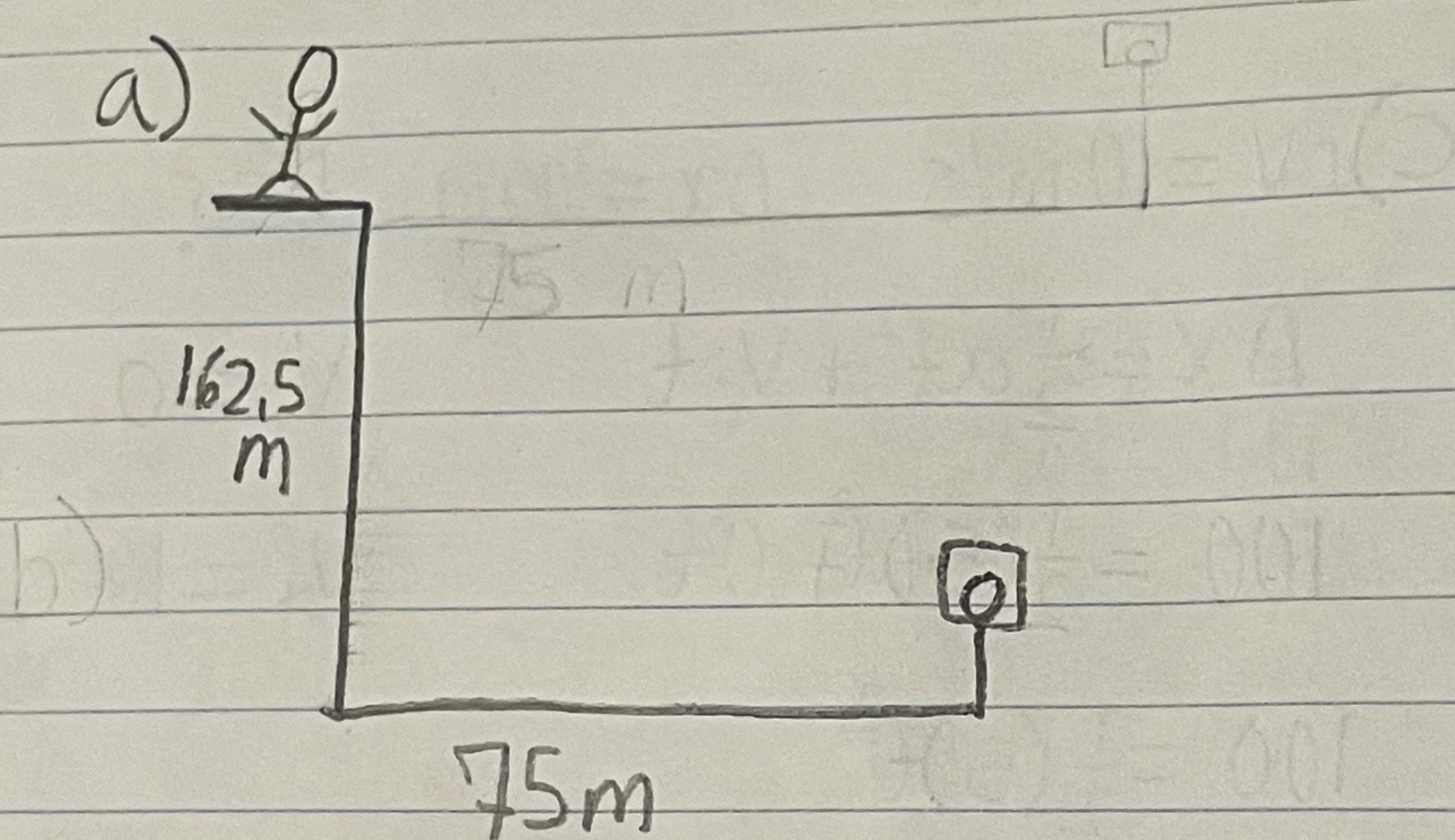
$$\sqrt{40} = \sqrt{t^2}$$

$$2\sqrt{10} \text{ s} = t$$

## Motion in two and three Dimensions

1 The world record highest basketball shot was made from a height of 162,5 meters above the basketball hoop. the basketball hoop was placed 75 meters horizontally from the shooter. a) Draw a diagram of the situation b) What is the horizontal velocity required to make the shot. That is assume the shooter shoots the ball with no vertical velocity, only horizontal

a)



b)  $\Delta x = v_x \Delta t$  need time

$$\Delta y = -\frac{1}{2} g t^2$$

$$\frac{2}{9.8} \cdot -162.5 = -\frac{1}{2}(9.8) t^2 \div 2$$

$$\sqrt{\frac{1625}{519}} = \sqrt{t^2}$$

$$t = \frac{5\sqrt{65}}{7}$$

$$\Delta x = v_x \Delta t$$

$$\frac{75}{\Delta t} = v_x \left( \frac{5\sqrt{65}}{7} \right)$$

$$\frac{75}{\Delta t} = v_x$$

$$v_x = \frac{21\sqrt{65}}{13} \quad 13.02 \text{ m/s}$$

2 A baseball is hit at a 45 degree angle with respect to the horizontal at 40 m/s. a) how far away does it land? b) How long is it in the air?

$$a) R = \frac{v^2 \sin(\theta)}{g}$$

$$= \frac{40^2 \sin(2(45))}{9.81}$$

$$= 163.09 \text{ m.}$$

$$b) T = \frac{2v \sin(\theta)}{g}$$

$$= \frac{2(40) \sin(45)}{9.81}$$

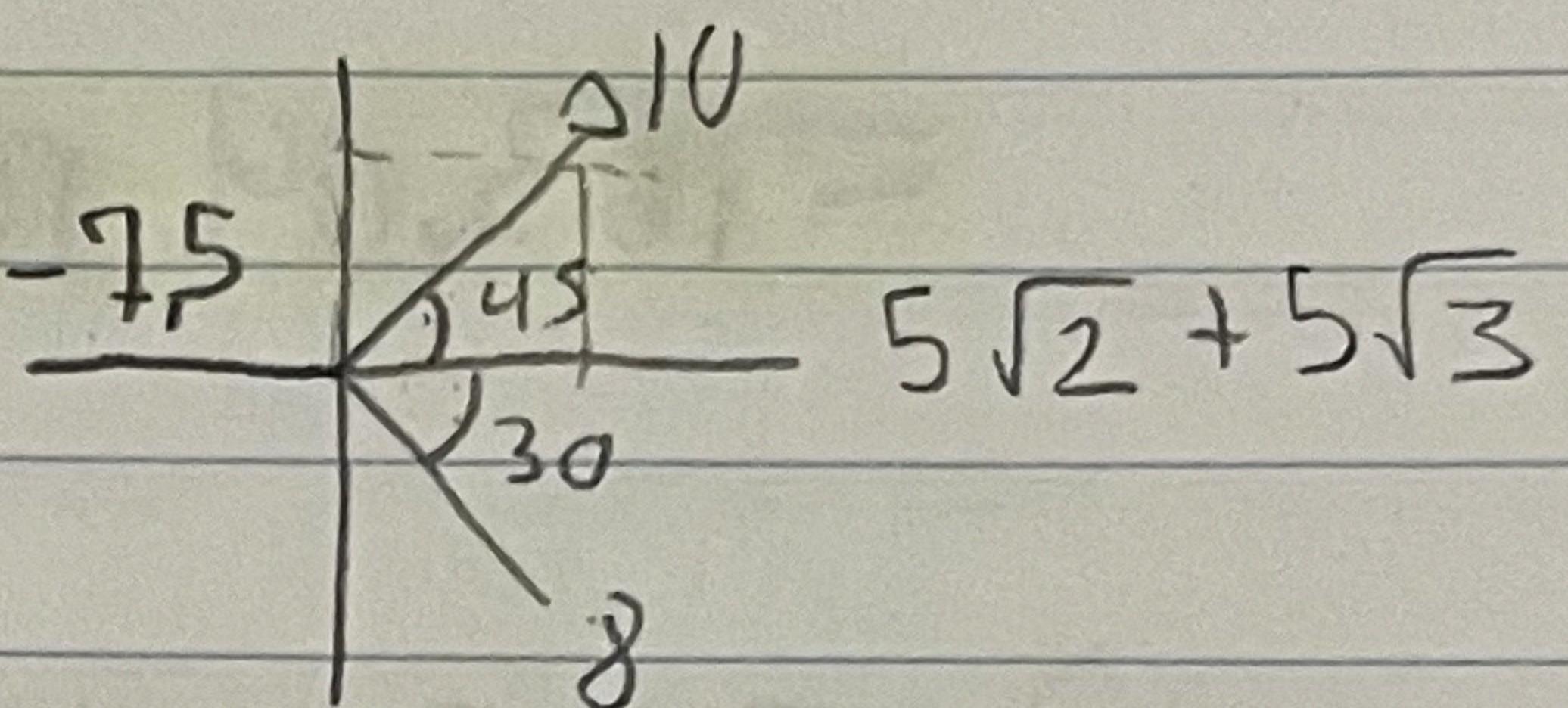
$$= 5.76 \text{ seconds}$$

## Forces

Two children pull a third child on a snow saucer sled exerting forces  $F_1$  and  $F_2$  as shown from above in fig 1. Find the acceleration of the system if the mass of the child and sled together is 49.0 kg. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of  $F_1$  and  $F_2$

$$\frac{F_{\text{net}}}{m} = m\ddot{a}$$

$$\frac{F}{m} = a$$



$$F_{\text{Net}} = 8, 23$$

$$a = \frac{8.23}{49.0}$$

$$a = 0.16 \text{ m/s}^2$$