Wednesday warm-up: Statics II and Gravitation

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November 19, 2025

1 Memory Bank

- Young's Modulus, Y, has units of N m^{-2}, and it relates the change in length ΔL of a system of original length L_0 and cross-sectional area A subject to a force F:

$$\frac{\Delta L}{L_0} = \frac{1}{Y}\frac{F}{A} \tag{1}$$

  Summarized as *stress*$= Y \times$*strain*.

- Shear Modulus, S, has units of N m^{-2}, and it relates the sideways change in length Δx of a system of length L_0 and cross-sectional area A subject to a force F:

$$\frac{\Delta x}{L_0} = \frac{1}{S}\frac{F}{A} \tag{2}$$

  Summarized as *stress*$= S \times$*sheer*.

- Bulk Modulus, B, has units of N m^{-2}, and it relates the fractional change in volume to changes in pressure:

$$\Delta p = -B\frac{\Delta V}{V_0} \tag{3}$$

  The factor $k = 1/B$ is called the *compressibility*.

- **Universal Law of Gravitation**:

$$\vec{F}_{\mathrm{G}} = G\frac{m_1 m_2 \hat{r}}{r^2} \tag{4}$$

  $G = 6.67 \times 10^{-11}$ N kg^{-2} m^2.

2 Stress, Strain, and Elastic Modulus

1. When water freezes, its volume increases by 9.05%. What force per unit area is water capable of exerting on a container when it freezes? The bulk modulus of water is 2.2×10^9 Pa.

2. The shear modulus of a wooden plank is 10×10^9 Pa. Suppose the plank dimensions are 5cm by 10 cm, and it is 5 meters long. (a) How much shear strain is caused by a 10 kg mass, 4.5 meters from the support point of the plank, if the 10 kg mass hangs freely?

3 Gravitation

1. Calculate the value of g, the gravitational acceleration near the surface of the Earth, from the Universal Law of Gravitation. Assume some *test mass* has a weight mg that is equal to the force of gravity one Earth radius from the center of the Earth, $R_{\mathrm{E}} = 6370$ km.

2. Note that the volume of a sphere is $V = (4/3)\pi r^3$, and the mass of an object of volume V with density ρ is $m = \rho V$. (a) Show that the gravitational acceleration near the surface of a spherical mass is $g = (4/3)\pi G\rho r$. (b) Suppose in the future, we land on a new planet, with radius $r = 0.9R_{\mathrm{E}}$, and find that $g = 0.67g_{\mathrm{E}}$. That is, the acceleration due to gravity is 67% as strong as that of Earth. What is the (average) density of the new planet, relative to Earth?

3. Calculate the velocity required to orbit the Earth. That is, at what speed will the gravitational attraction barely balance the centripetal force?