Warm Up: Conservative Forces

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1 Memory Bank

- $KE = \frac{1}{2}mv^2$ ... Definition of Kinetic Energy
- $W = KE_f - KE_i$ ... Work-energy theorem.
- $U = mgy$ ... Gravitational potential energy, corresponding to $\vec{F} = -mg\hat{j}$.
- $\vec{F}(x) = -dU/dx\ \hat{i}$ ... A *conservative force* can be written as the derivative of the potential energy function. This is the one-dimensional case, where the force only depends on x.
- Let $\oint \vec{F} \cdot d\vec{r}$ represent the integral of $\vec{F} \cdot d\vec{r}$ around a closed path. A force is *conservative* if

$$\oint \vec{F} \cdot d\vec{r} = 0 \tag{1}$$

  Let F_x represent the x-component of a force $\vec{F}$, and F_y the y-component. Equation 1 implies that, for a force in the x-y plane,

$$\frac{dF_x}{dy} = \frac{dF_y}{dx} \tag{2}$$

- $KE_i + PE_i = KE_f + PE_f$ ... One form of energy conservation. Consider that potential energy is just stored energy created by performing work, so this statement is not that different from the official work-energy theorem.

2 Conservative Forces

1. In Fig. 1, we find a potential energy function $U(x) = 2(x^4 - x^2)$. (a) What is the associated force, assuming it is a conservative force? (b) Set the derivative of U equal to zero to locate the points $\pm Q$. (c) What is the potential energy at the points $\pm Q$? (d) If a system with mass 0.1 kg was released from a distance dx to the right of the origin, what would be the velocity of the system at point Q? (e) Would that system ever reach $-Q$?

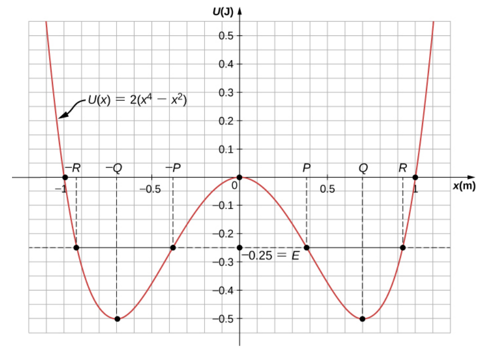

Figure 1: A function $U(x)$ that describes the potential energy as a function of position.

2. Suppose a system of mass 0.1 kg is at the origin, on a surface with coefficient of kinetic friction $\mu = 0.05$. The system is pushed from $\vec{x} = 0\hat{i} + 0\hat{j}$ m, to $\vec{x} = 1\hat{i} + 0\hat{j}$ m, then to $\vec{x} = 1\hat{i} + 1\hat{j}$ m, then to $\vec{x} = 0\hat{i} + 1\hat{j}$ m, and finally to $\vec{x} = 0\hat{i} + 0\hat{j}$ m. (a) What is the total work by the pushing force done against friction? (b) Note that work has been done despite returning to the original position. Is the friction force conservative?