calculus-based physics-2: electricity, magnetism, and thermodynamics (phys180-02): unit 4

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UNIT 4 REVIEW

Reading: Chapters 9-10

1. Current
2. DC circuits

UNIT 4 REVIEW PROBLEMS

Which of the following would decrease the time required to charge the capacitor at right?

- A: Decreasing the capacitance
- B: Decreasing the resistance
- C: It already charges as fast as possible
- D: Both A and B

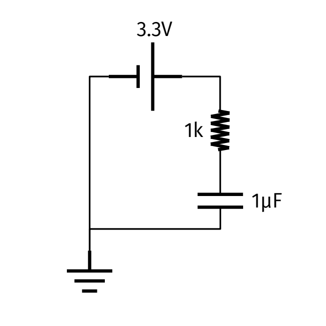


Figure 1: An RC circuit.

What is the RC time of the circuit?

- A: 1 μs
- B: 1 ms
- C: 1 s
- D: 10 s

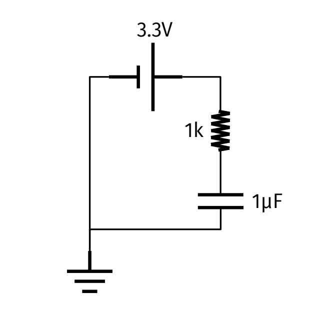


Figure 2: An RC circuit.

What is the maximum charge stored eventually in the capacitor? *Recall that $Q = CV$.*

- A: 3.3 μ C
- B: 1.5 μ C
- C: 3.3 mC
- D: 1.5 C

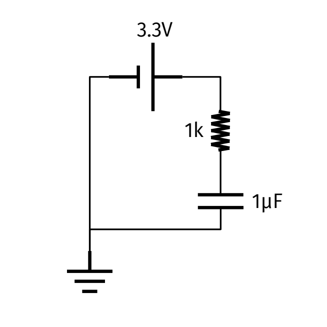


Figure 3: An RC circuit.

SUMMARY

Reading: Chapters 11-12

This class: 11.1-4

1. Magnetism and magnetic fields
2. Motion of a charged particle in a magnetic field
3. Forces on conductors carrying current

Next class: 11.5-7

1. Current loops
2. The Hall effect
3. Applications

Next week: Chapter 12

MAGNETISM AND MAGNETIC FIELDS

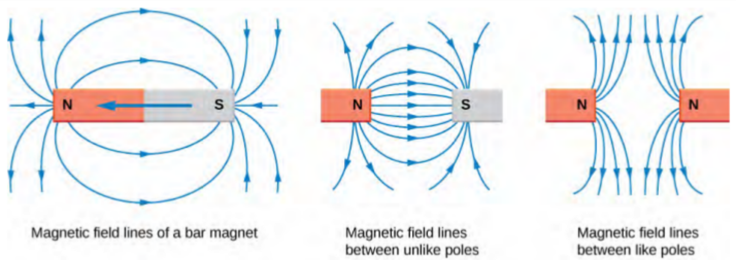


Figure 4: Various magnetic field line configurations.

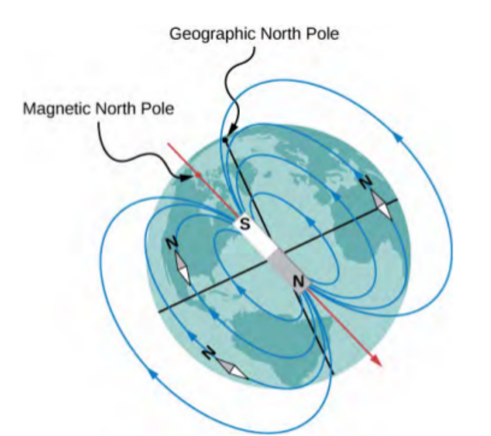


Figure 5: The magnetic and geographic poles are not the same.

It would be nice if we could say:

$$F = \mu_0 \frac{q_{m,1} q_{m,2}}{r^2} \tag{1}$$

But...we can't. Why? There's no such thing has magnetic charge:

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \tag{2}$$

$$\nabla \cdot \vec{B} = 0 \tag{3}$$

But there is a force associating charge and magnetic fields.

But first, let's review the cross-product.

What is a cross-product and how does it work?

Computing the cross product [edit]

Coordinate notation [edit]

The standard basis vectors **i**, **j**, and **k** satisfy the following equalities in a right hand coordinate system:

$$\begin{aligned} \mathbf{i} \times \mathbf{j} &= \mathbf{k} \\ \mathbf{j} \times \mathbf{k} &= \mathbf{i} \\ \mathbf{k} \times \mathbf{i} &= \mathbf{j} \end{aligned}$$

which imply, by the anticommutativity of the cross product, that

$$\begin{aligned} \mathbf{j} \times \mathbf{i} &= -\mathbf{k} \\ \mathbf{k} \times \mathbf{j} &= -\mathbf{i} \\ \mathbf{i} \times \mathbf{k} &= -\mathbf{j} \end{aligned}$$

The definition of the cross product also implies that

$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$ (the zero vector).

Figure 6: The cross-product is a way of multiplying unit vectors.

Let $\vec{v} = 2\hat{i}$ and $w = -2\hat{j}$. What is $\vec{v} \times \vec{w}$?

- A: $-4\hat{k}$
- B: $4\hat{k}$
- C: $-2\hat{i}$
- D: $2\hat{j}$

Let $\vec{v} = 3\hat{j}$ and $w = 5\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $15\hat{i}$
- B: $5\hat{j}$
- C: $3\hat{i}$
- D: $15\hat{k}$

Let $\vec{v} = 3\hat{i} + 3\hat{j}$ and $w = 2\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $-6\hat{j} + 6\hat{k}$
- B: $-6\hat{j} + 6\hat{i}$
- C: $6\hat{j} + 6\hat{i}$
- D: $6\hat{k} + 6\hat{i}$

Group board exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} \tag{4}$$

$$\vec{w} = 4\hat{j} - 4\hat{i} \tag{5}$$

$$\vec{v} \times \vec{w} = ?? \tag{6}$$

What happens when we draw these two vectors?

Group board exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} + \hat{k} \tag{7}$$

$$\vec{w} = 4\hat{j} - 4\hat{i} - \hat{k} \tag{8}$$

$$\vec{v} \times \vec{w} = ?? \tag{9}$$

Use your knowledge of unit vectors to skip the terms that are zero.

The Lorentz Force

Let a particle with charge q and velocity $\vec{v}$ move through a magnetic field $\vec{B}$. The Lorentz force on the charged particle is

$$\vec{F}_{\mathrm{L}} = q\vec{v} \times \vec{B} \tag{10}$$

As a helpful memory tool, we have the right-hand rule to remember the direction of the cross-product. The units of the magnetic field are the Telsa, after Nikola Tesla. We also have the Gauss which is 10^{-4} Tesla.

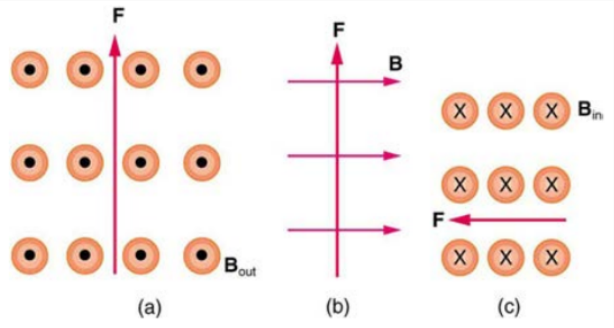


Figure 7: Three different magnetic field and charge scenarios. The vector $\vec{F}$ is the direction of the Lorentz force, and the magnetic field is uniform. A dot indicates that the magnetic field is coming out of the page, and an x indicates that the field is going into the page.

In which of the diagrams is a positively charged particle moving to the left?

- A: A
- B: B
- C: C
- D: WAT WAT WAT

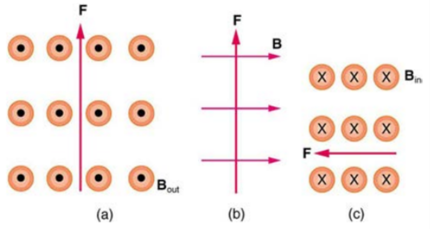


Figure 8: Three different magnetic field and charge scenarios.

In which of the diagrams is a positively charged particle moving upwards?

- A: A
- B: B
- C: C
- D: WAT WAT WAT

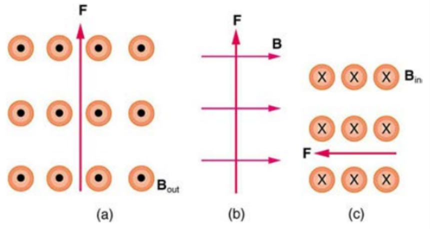


Figure 9: Three different magnetic field and charge scenarios.

In which of the diagrams is a negatively charged particle moving into the page?

- A: A
- B: B
- C: C
- D: WAT WAT WAT

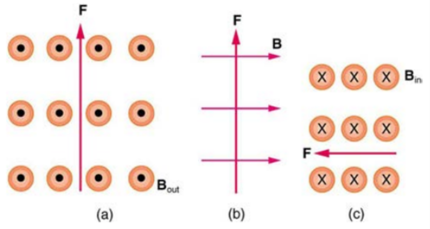


Figure 10: Three different magnetic field and charge scenarios.

In which of the diagrams is a negatively charged particle moving to the right?

- A: A
- B: B
- C: C
- D: WAT WAT WAT

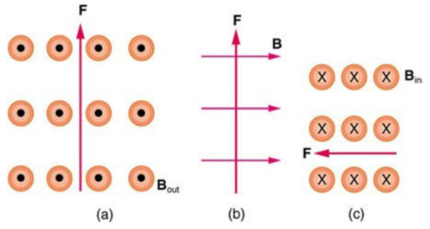


Figure 11: Three different magnetic field and charge scenarios.

A theorem for the magnitude of the cross-product: Let $\vec{a}$ and $\vec{b}$ be vectors and θ be the angle between them. The magnitude of the cross product is:

$$|\vec{a} \times \vec{b}| = ab \sin\theta \tag{11}$$

Thus, the magnitude of the Lorentz force is

$$F_{\mathrm{L}} = qvB \sin\theta \tag{12}$$

The angle θ is between the velocity and the magnetic field.

A cosmic ray proton moving toward the Earth at 3×10^6 m/s experiences a magnetic force of 2×10^{-17} N. What is the strength of the magnetic field of the Earth? (1 Gauss = 10^{-4} Tesla).

- A: 0.1 Gauss
- B: 0.6 Gauss
- C: 1 Gauss
- D: 6 Gauss

Figure 12: The aurora borealis, or northern lights.

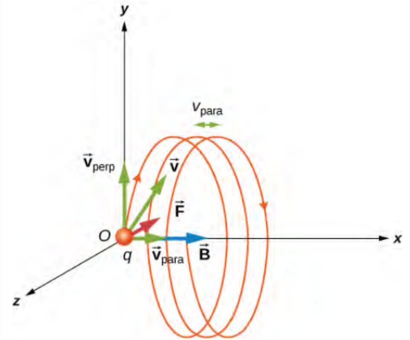


Figure 13: In three dimensions, charged particle motion in a $\vec{B}$-field can result in *helical motion*.

Suppose the velocity of a charged particle with mass m is $\vec{v} = v_x\hat{i} + v_z\hat{k}$ through a uniform field $\vec{B} = B\hat{k}$. The Lorentz force causes centripetal motion and the particle continues to have constant velocity in the $\hat{k}$ direction:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{13}$$

$$\vec{F} = -qBv_x\hat{j} \tag{14}$$

$$\frac{mv_x^2}{r} = qBv_x \tag{15}$$

$$\omega = \frac{v_x}{r} \tag{16}$$

$$\frac{q}{m} = \frac{\omega}{B} \tag{17}$$

Sub-atomic properties are isolated!

Which of the following is true of a charged particle moving in a helical fashion through a magnetic field?

- A: Raising the strength of the B-field increases the period
- B: Raising the strength of the B-field increases the frequency
- C: The particle has a constant velocity parallel to the field
- D: B and C

Two unknown particles are moving in helixes through a region where there is a magnetic field. One moves clockwise as you observe it, and the other moves counter-clockwise, and the helices have about the same radius. Which of the following is true?

- A: The particles have identical charge.
- B: The particles have identical charge, and the same mass.
- C: The particles have opposite charge, and the same mass.
- D: The particles have different masses.

Two unknown particles are moving in helixes through a region where there is a magnetic field. Both move clockwise as you observe them. One particle spins around the field line with higher frequency compared to the other. Which of the following is true?

- A: The particles are identical; they just had different initial conditions.
- B: The charge is smaller for the particle with the larger frequency.
- C: The mass is larger for the particle with the larger frequency.
- D: The q/m ratio is larger for the particle with the larger frequency.

Group exercise: Suppose we place a gass of unknown particles in the uniform magnetic field of Fig. 13 and get them moving in a circle. The angular frequency is 95.5788 MHz, and the B-field is exactly 1.0 T. (a) Show that the relationship between the angular frequency ω, the B-field strength B, and the q/m ratio is $q/m = \omega/B$. (b) With which particle are we dealing? Is it a proton, a neutron, an electron, or an alpha particle? (*Hint: use the angular frequency and magnetic field to obtain the q/m ratio, and then look up the masses and charges of these particles to make the determination*).

Other examples:

1. Magnetic fields do no work
2. Velocity selector
3. Mass spectrometer

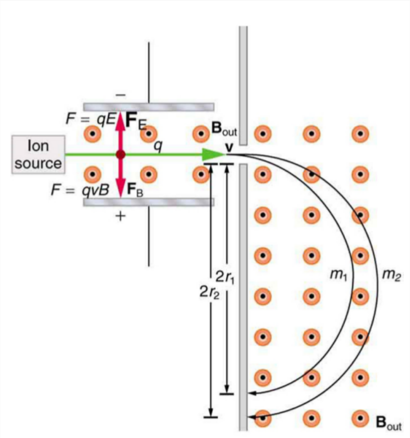


Figure 14: The basic ideas behind a mass spectrometer.

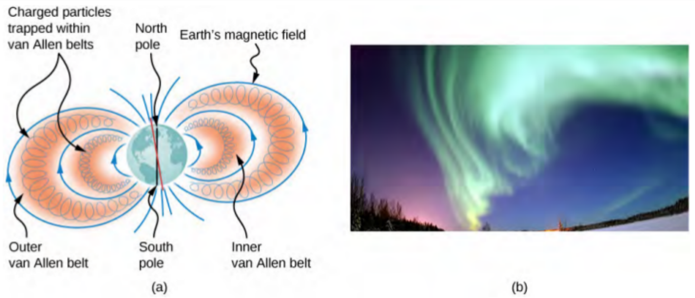
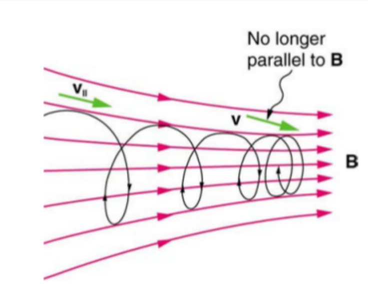
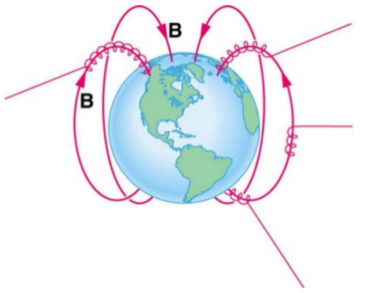


Figure 15: We observe this effect in the auroras, and the van Allen belts.

A cool talk on the aurora borealis:
`https://youtu.be/czMh3BnHFHQ`





One un-explained piece: what does it mean for the electrons and protons to *high-five* the neutral oxygen and nitrogen atoms?

FORCES ON CURRENT-CARRYING CONDUCTORS

Introductions to observable magnetic forces (PBS):

First connection between electricity and magnetism:
`https://youtu.be/s94suB5uLWw`

Further experiments, Ampère's Law:
`https://youtu.be/5fqwJyt4Lus`

The Lorentz force, when applied to a section of current-carrying wire, becomes

$$d\vec{F} = I\vec{dl} \times \vec{B} \tag{18}$$

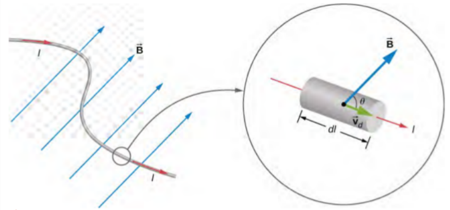


Figure 16: The magnetic force on a section of current.

If the field is uniform:

$$\vec{F} = I\vec{L} \times \vec{B} \tag{19}$$

Group board exercise: A wire of length 10 cm and mass 1 g is suspended in a horizontal plane by a pair of flexible leads. The wire is then subjected to a constant magnetic field of magnitude 0.1 T, which is directed into the board. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads?

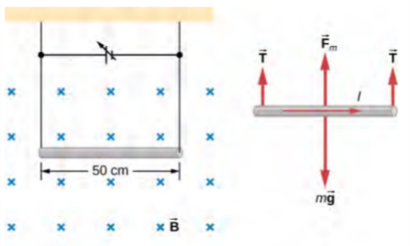


Figure 17: Current suspended by Lorentz force...?

Suppose a power supply provides the current in the previous example. What if the voltage is raised, and the resistance stays constant, so that the current is doubled. What will happen?

- A: The wire will rise.
- B: The wire will fall.
- C: The magentic field will decrease.
- D: Nothing.

If the wire rises, what is doing the work to raise it?

- A: The B-field
- B: The current
- C: The battery
- D: Gravity

Group board exercise: Suppose the current is raised from 1 amp to 2 amps for 0.1 seconds. By how much will the wire be raised? *Hint: you can obtain the acceleration from the* ***net*** *force, and then obtain the displacement.*

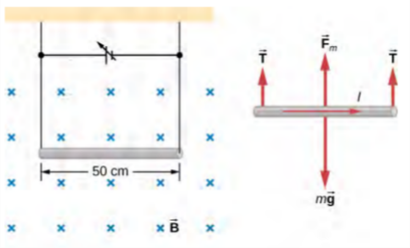


Figure 18: Current suspended by Lorentz force...?

Figure 19: An electromagnetic crane.

Observe on board. The force is $F = dlIB\sin\theta$, but $dl = Rd\theta$.

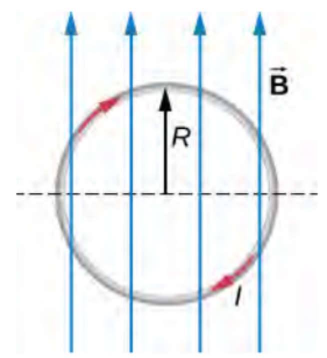


Figure 20: Lorentz force on a loop of wire. Think of (a) the net force, and (b) the torque. Which are non-zero?

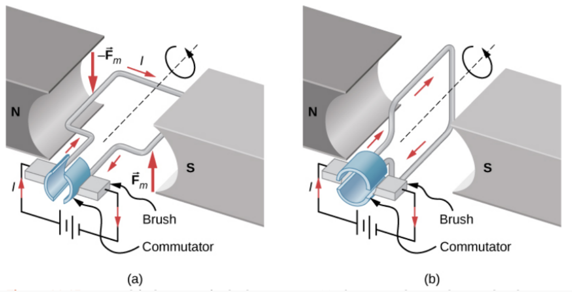


Figure 21: Lorentz force on a loop of wire. Think of (a) the net force, and (b) the torque. Which are non-zero?

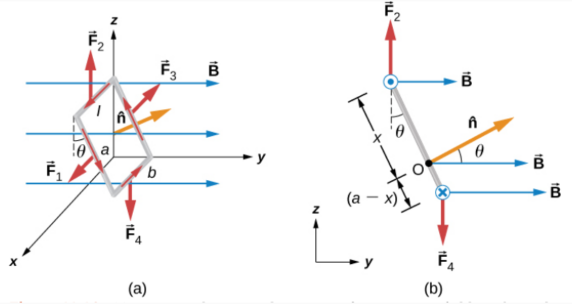


Figure 22: The B-field causes a torque on a loop of current just like an E-field causes a torque on a dipole. We like to think of the **magnetic dipole moment** as $\vec{\mu} = NIA\hat{n}$.

Let a single current loop of current I and area $\vec{A} = A\hat{n}$ exist in a uniform magnetic field $\vec{B}$. The torque τ on the loop is

$$\boxed{\tau = \vec{\mu} \times \vec{B}} \tag{20}$$

In Eq. 20, the quantity $\vec{\mu} = I\vec{A}$ is called the *magnetic dipole moment.*

- $\vec{p} = q\vec{d}$ ... $\vec{\mu} = I\vec{A}$.
- $\vec{\tau}_E = \vec{p} \times \vec{E}$ ... $\vec{\tau}_B = \vec{\mu} \times \vec{B}$

How do we make a uniform $\vec{B} - \mathit{field}$**?**...Postpone this to discuss one more effect: *The Hall Effect*

THE HALL EFFECT

Negative charge flows in conductors.

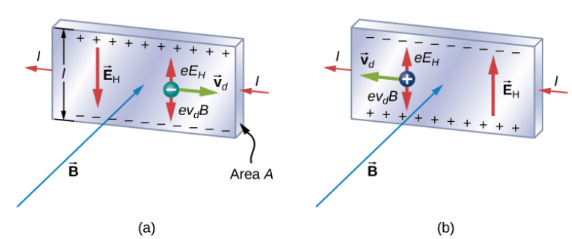


Figure 23: The Hall effect is an important way to establish that what we call *negative charge* is actually flowing in conductors.

Negative charge flows in conductors.

“Therefore, by simply measuring the sign of V, we can determine the sign of the majority charge carriers in a metal. Hall potential measurements show that electrons are the dominant charge carriers in most metals. However, Hall potentials indicate that for a few metals, such as tungsten, beryllium, and many semiconductors, the majority of charge carriers are positive. It turns out that conduction by positive charge is caused by the migration of missing electron sites (called holes) on ions.”

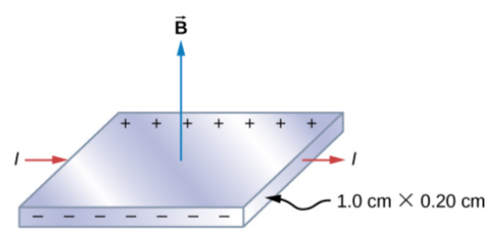


Figure 24: An example of a Hall measurement, with some typical numbers.

- $I = 100$ A
- $B = 1.5$ T
- $l = 1.0 \times 10^{-2}$ m
- $n = 5.9 \times 10^{28}$ m^{-3}, $e = 1.6 \times 10^{-19}$ C
- $A = 2.0 \times 10^{-5}$ m^2

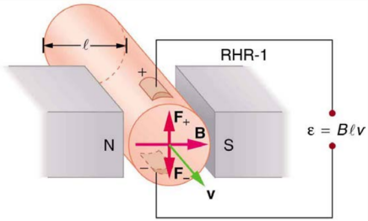


Figure 25: A Hall measurement that is used to measure fluid flow.

Group exercise: A Hall effect flow probe is placed on an artery, applying a 0.1 T magnetic field across it, in a setup similar to that in Fig. 25. What is the blood velocity, given the vessel's inside diameter is 4.00 mm and the Hall voltage is $0.8\mu V$?

Last week: Chapter 11

This week: Chapter 12

Next class: 12.1-12.4

1. 12.1: The Biot-Savart Law
2. 12.2: Thin straight wire
3. 12.3: Two parallel currents
4. 12.4: Current loops

Friday's class: read sections 12.5-7

PHET: ELECTROMAGNETS

Follow the link:

`https://phet.colorado.edu/en/simulation/magnets-and-electromagnets`

1. Click on the electromagnet tab, and hide the field and compass using the menu in the upper right. Also, display the magnetometer.
2. Place the magnetometer to one side of the *solenoid*. Work out the relationship between the magnetic field strength and voltage. Is it linear, quadratic, or something else?
3. Assuming the circuit has some fixed resistance, is the relationship between current and field strength linear? Why or why not?
4. Now fix the voltage and vary the number of loops. Work out the relationship between magnetic field strength and loop number. Is it linear, quadratic, or something else?
5. Propose an equation for $B_{solenoid}$ based on the prior measurements.

Electromagnets.

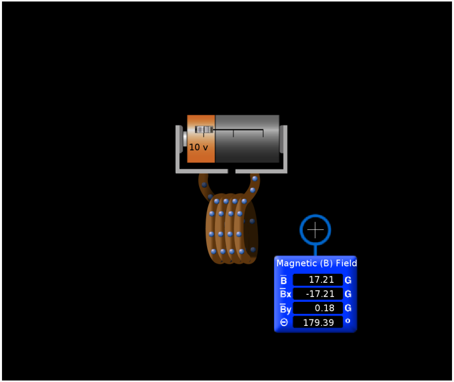


Figure 26: The electromagnet converts charge to magnetic field strength.

The result should be something like:

$$B \propto NI \tag{21}$$

$$B = \mu_0 n I \tag{22}$$

- n: Number of turns per unit length (because we can always change the density and get a different answer).
- I: Current
- μ_0: Magnetic permeability of free space (solenoid is empty).

THE BIOT-SAVART LAW

The Biot-Savart Law

Let a current I exist along a line segment $d\vec{l}$ located a displacement $\vec{r}$ from an observation point. The magnetic field contribution from this current element is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{l} \times \hat{r}}{r^2} \tag{23}$$

- Integrating this expression properly yields the total magnetic field at a given point
- We have to take advantage of symmetries just like Coulomb's law

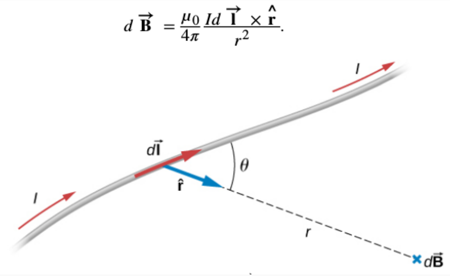


Figure 27: The angle θ is between $\hat{r}$ and $d\vec{l}$, as shown.

Which expression is equal to the magnitude of $Id\vec{l} \times \hat{r}$?

- A: $Idlr$
- B: $Idlr\sin(\theta)$
- C: $Idl\sin(\theta)$
- D: $I\sin(\theta)$

If the B-field due to a line-segment of current is 1.0 Gauss at 1cm, what is the value of the B-field 10 cm from the line-segment, for the same orientation?

- A: 0.1 Gauss
- B: 0.05 Gauss
- C: 1.0 Gauss
- D: 0.01 Gauss

If the B-field due to a line-segment of current is 1.0 Gauss at 1cm, and $\hat{r}$ is perpendicular to $d\vec{l}$, what is the B-field when $d\vec{l}$ is *parallel* to $d\vec{l}$?

- A: 0.0 Gauss
- B: 0.05 Gauss
- C: 1.0 Gauss
- D: 0.01 Gauss

If the B-field due to a line-segment of current is 1.0 Gauss for a current of 0.5 A, what is the B-field when the current is increased to 1.0 A?

- A: 0.0 Gauss
- B: 0.02 Gauss
- C: 2.0 Gauss
- D: 0.02 Gauss

If the B-field due to a line-segment of current is 1.0 Gauss for a current of 0.5 A at a distance of 10 cm, what is the B-field when the current is increased to 1.0 A, and the distance is decreased to 1 cm?

- A: 20 Gauss
- B: 200 Gauss
- C: 500 Gauss
- D: 1000 Gauss

BIOT-SAVART EXAMPLE: THE THIN STRAIGHT WIRE

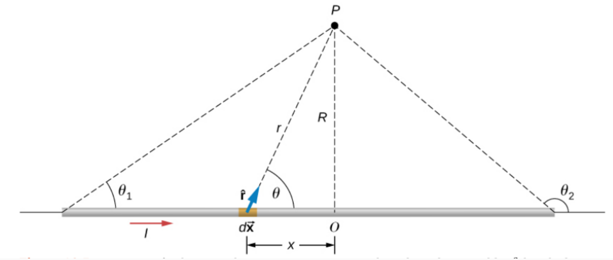


Figure 28: Observe on board the derivation of the formula for $\vec{B}$ at a point P.

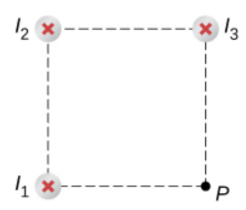


Figure 29: Derive the magnitude and direction for $\vec{B}$ at a point P, if the sides of the square are 1cm and the currents are each 2.0 A.

CONCLUSION

Reading: Chapter 11

1. Magnetism and magnetic fields
2. Motion of a charged particle in a magnetic field
3. Other forces
4. Current loops

ANSWERS - CHAPTER 11

ANSWERS

- Both A and B
- 1 ms
- 3.3μC
- $-4\hat{k}$
- $15\hat{i}$
- $-6\hat{j} + 6\hat{i}$
- A
- C
- B
- A
- B
- D
- C
- D
- A
- C

ANSWERS - CHAPTER 12

ANSWERS

- C
- D
- A
- C
- B

- ...