Error Analysis in PHYS180

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Abstract

Scientific measurements cannot be quoted without an assessment of the precision. We quantify this precision in the form of error analysis. Rather than originating from *human error*, *statistical errors* arrise from the intrinsic uncertainty encounted in measuring quantities with real instruments. Further, errors *propagate* through calculations. This document will explain how this works, and how to perform error analysis for final results.

1 Instrumental Precision

The instrumental precision of a piece of data is determined by the smallest division on the instrument. For example, suppose a length is measured with a meter stick, and the smallest divisions on the meter stick are 1 millimeter. If the length measured is 31.0 cm, then that result must be stated as 31.0 ± 0.1 cm.

2 Accuracy versus Precision

Accuracy and precision are not the same concept. Accuracy is how far the measured result is from the true value, whereas precision compares the statistical error of a measurement to the measured value itself. A measurement of 31.0 ± 0.1 cm is precise, because 0.1 cm is small compared to 31.0 cm. It is not *accurate* if the true length is 29.0 cm. Why? Because the numbers 31.0 cm and 29.0 cm are separated by $(31.0 - 29.0)/0.1 = 20$ factors of the error.

3 Statistical or Random Error

Suppose a measurement is made with a dial, where a needle is pointing to a number, but vibrating. When the dial is read one moment, it reads 101.3 kPa, and in another moment, 101.2 kPa. Which pressure measurement of the atmosphere is correct? It turns out that upon repeated measurements, the *distribution* of measurements resembles Fig. 1.

It's not that there isn't a true pressure, there is, and it is most likely 101.3 kPa. In reality, the dial needle is moving, making identification of the *exact* or true pressure impossible. Instead, the true value is ascertained from the statistical distribution.

4 Analysis of Data Subject to Random Error

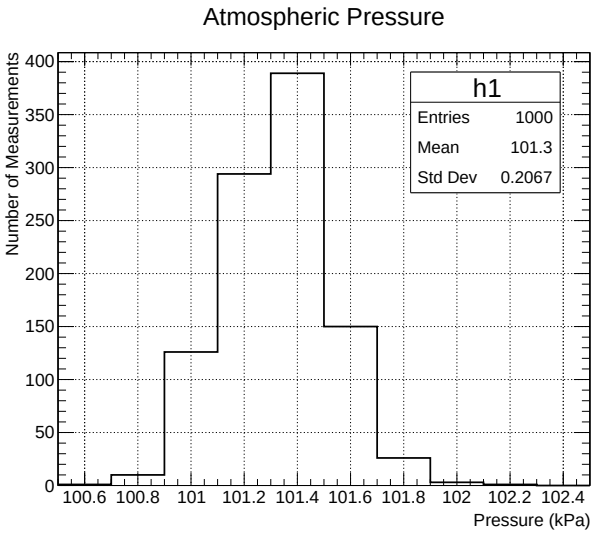

Figure 1: An example of a distribution of measurements from one instrument: a pressure gauge.