CALCULUS-BASED PHYSICS-1: MECHANICS (PHYS150-01): WEEK 3

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WEEK 2 REVIEW

1. Displacement, and instantaneous velocity and acceleration
    - *Mathematics review*: taking derivatives
    - Average velocity and average acceleration
2. The case of constant acceleration
    - An *an equation of motion* for constant acceleration
    - Derivation of common equations of motion
    - Average quantities and exercises
3. **Lab Activity: Measuring acceleration of gravity: g**
4. Exercises with vectors, graphs, and equations of motion

WEEK 2 REVIEW PROBLEMS

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

- A: To the right, positive
- B: To the right, negative
- C: To the left, positive
- D: To the left, negative

An object that is thrown straight up falls back to Earth. When is its velocity zero? Does its velocity change direction? Does the acceleration change sign?

- During flight, yes, no
- At the peak height, yes, yes
- At the peak height, yes, no
- During flight, no, no

WEEK 3 SUMMARY

1. Displacement, velocity and acceleration vectors as functions of time
    - Breaking into components
    - Derivatives of components
2. Combining free-fall and vector components: projectile motion
    - The independence of velocity components
    - **Lab-activity: testing component independence**
3. Relative motion and reference frames
    - Relative motion in one-dimension
    - Relative motion in two-dimensions

VECTORS AS FUNCTIONS OF TIME

In general, the displacement of an object depends on time:

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} \tag{1}$$

- $x(t)$ is the displacement in the x-direction
- $y(t)$ is the displacement in the y-direction
- $z(t)$ is the displacement in the z-direction

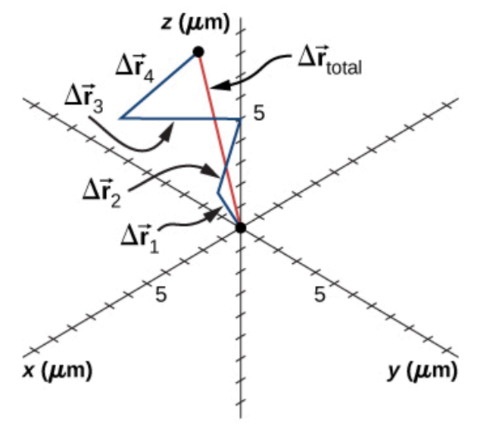

Figure 1: An example of a displacement vector at different moments in time.

The particle in Fig. 1 has four displacement vectors at four moments in time:

- $\vec{r}_1 = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k} \quad (\mu m)$ at t_1
- $\vec{r}_2 = -1.0\hat{i} + 0.0\hat{j} + 3.0\hat{k} \quad (\mu m)$ at t_2
- $\vec{r}_3 = 4.0\hat{i} + -2.0\hat{j} + 1.0\hat{k} \quad (\mu m)$ at t_3
- $\vec{r}_4 = -3.0\hat{i} + 1.0\hat{j} + 2.0\hat{k} \quad (\mu m)$ at t_4

What is the total displacement of the particle from the origin?

We can think of this type of problem as an accounting problem, lining up columns (units: μm):

t_i	$\vec{r}_\mathrm{i}(t_\mathrm{i})$	$x(t_\mathrm{i})$	$y(t_\mathrm{i})$	$y(t_\mathrm{i})$
t_1	$\vec{r}_1(t_1)$	2.0	1.0	3.0
t_2	$\vec{r}_2(t_2)$	-1.0	0.0	3.0
t_3	$\vec{r}_3(t_3)$	4.0	-2.0	1.0
t_4	$\vec{r}_4(t_4)$	-3.0	1.0	2.0
t_total	$\vec{r}_\mathrm{total}(t_\mathrm{total})$	2.0	0.0	9.0

Figure 2: Accounting for the different displacement components, in units of μm.

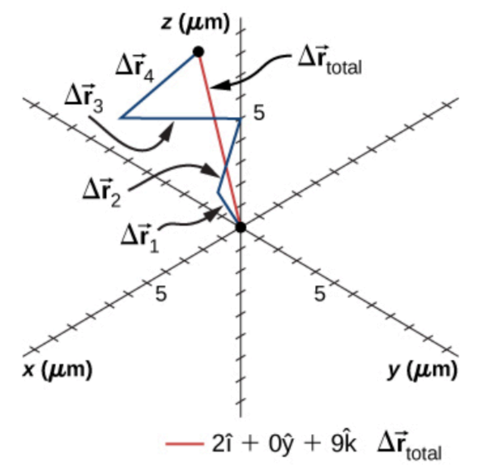

Figure 3: The total displacement of the particle is $\vec{r}_{\text{total}} = 2.0\hat{i} + 0.0\hat{k} + 9.0\hat{k} \quad (\mu m)$.

The 18th hole at Pebble Beach Golf Course is a dogleg to the left of length 496.0 meters. The fairway off the tee is taken to be the x direction. A golfer hits his tee shot a distance of 300 meters, corresponding to a displacement of $\vec{r}_1 = 300.0\hat{i} \quad (m)$, and then hits a second shot 189.0 meters with $\vec{r}_2 = 172.0\hat{i} + 80.3\hat{j} \quad m$. What is the final displacement from the tee?

- $\vec{r}_{\text{final}} = 172.0\hat{i} + 80.3\hat{j} \quad (m)$
- $\vec{r}_{\text{final}} = 172.0\hat{i} + 380.3\hat{j} \quad (m)$
- $\vec{r}_{\text{final}} = 472.0\hat{i} + 0.0\hat{j} \quad (m)$
- $\vec{r}_{\text{final}} = 472.0\hat{i} + 80.3\hat{j} \quad (m)$

If the first shot takes 5.0 seconds, the second shot takes 4.0 seconds, and the walking time in between the shots is 60.0 seconds, what is the average velocity vector for the ball after the two shots?

- $\vec{r}_{\text{final}} = 1.7\hat{i} + 8.3\hat{j} \quad (m/s)$
- $\vec{v}_{\text{final}} = 172.0\hat{i} + 80.3\hat{j} \quad (m/s)$
- $\vec{v}_{\text{final}} = 6.8\hat{i} + 1.2\hat{j} \quad (m)$
- $\vec{v}_{\text{final}} = 6.8\hat{i} + 1.2\hat{j} \quad (m/s)$

The prior problem indicates something you may already have guessed:

$$\vec{v}_{\mathrm{avg}}(t) = v_{\mathrm{x}}(t)\hat{i} + v_{\mathrm{y}}(t)\hat{j} + v_{\mathrm{z}}(t)\hat{k} = \frac{\Delta\vec{r}}{\Delta t} \tag{2}$$

- $v_{\mathrm{x}}(t)$ is the velocity in the x-direction
- $v_{\mathrm{y}}(t)$ is the velocity in the y-direction
- $v_{\mathrm{z}}(t)$ is the velocity in the z-direction

In other words, we divide each displacement component by the time, to get a vector where each component is the average velocity in that direction. $\Delta\vec{r} = \vec{r}_{\mathrm{f}} - \vec{r}_{\mathrm{i}}$.

ANSWERS

- To the right, positive
- At the peak height, yes, yes
- $\vec{r}_{\mathrm{final}} = 472.0\hat{i} + 80.3\hat{j} \quad (m)$
- $\vec{v}_{\mathrm{final}} = 6.8\hat{i} + 1.2\hat{j} \quad (m/s)$

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