Introduction to Vectors: Displacement and Velocity

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Abstract

The purpose of this group activity is to practice problem-solving with vectors. *Displacement* is a vector that represents translational motion. This is distinct from *distance*, which is a scalar quantity representing the total amount of length an object has traversed. *Velocity* is displacement divided by time duration. In this activity, we will attempt to navigate a submarine such that it is not eliminated by either a torpedo or an undersea wall of rock.

1 The Problem Statement - The Hunt for Red October

The Red October is the name of a Russian nuclear submarine (from the novel by Tom Clancy, 1984), captained by Cpt. Marco Rameus. Red October is being pursued by a torpedo in a large undersea canyon. The torpedo has a higher speed than the submarine. Both are traveling in a straight line initially, meaning at some time the torpedo will hit the submarine.

Let a two-dimensional x-y coordinate system describe the area. The appropriate unit of distance for the area is the kilometer, or 1000.0 m. The initial position of the submarine is $\vec{x}_{i,s} = (-1.0, 0.0)$ km. The initial position of the torpedo is $\vec{x}_{i,t} = (-2.0, 0.0)$ km. The initial velocity of the submarine is $\vec{v}_{i,s} = (v_x, 0.0)$ km/h. The initial velocity of the torpedo is $\vec{v}_{i,t} = (50.0, 0.0)$ km/h. The situation is depicted in Fig. 1.

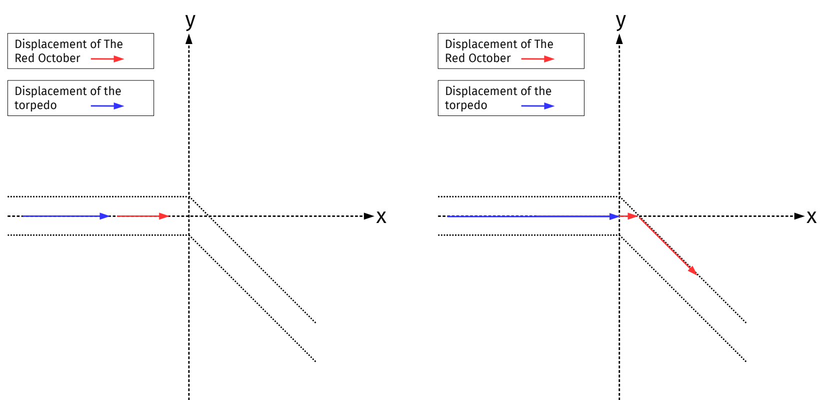

Figure 1: (Left) The submarine prepares for a right turn at an angle of 45 degrees. The lane (fine-dashed lines) is 0.5 km wide. (Right) The submarine captain sails up to the canyon wall, then turns South (downward in the diagram) at an angle of -45.0 degrees.

2 Initial Displacements

Displacement is formally defined as the difference between the final position and the initial position:

$$\Delta \vec{x} = \vec{x}_f - \vec{x}_i \tag{1}$$

What is the displacement of the submarine from the initial position $\vec{x}_{i,s}$ as it reaches the canyon wall? What is the displacement of the torpedo as it reaches the canyon wall?

$\Delta \vec{x}_s = (?, ?)$ $\Delta \vec{x}_t = (?, ?)$

3 Initial Velocities

Average *velocity* is formally defined as the ratio between displacement and some time duration $\Delta t = t_f - t_i$.

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t} \tag{2}$$

Multiplying both sides by Δt yields

$$\vec{v}\Delta t = \Delta \vec{x} \tag{3}$$

Taking the **absolute value** or **magnitude** of a vector is the same as using the Pythagorean theorem to determine the length of the vector. Taking the magnitude of both sides of Eq. 3 is written like

$$|\vec{v}\Delta t| = |\Delta \vec{x}| \tag{4}$$

$$v\Delta t = \Delta x \tag{5}$$

Assume the situation begins with $t_i = 0$. Using Eq. 5, determine when the torpedo hits the canyon wall, if the velocity remains constant.

$t_f = ?$

What speed (v_x) must Captain Rameus choose, if he is to turn at the moment the torpedo hits the canyon wall?

$v_x = ?$

Note that he **deliberately chooses a lower speed** to allow the torpedo to reach the wall at the same time as the submarine!

4 Final Displacement and Average Velocity

Captain Rameus continues at the same speed, but the boat turns at an angle of -45 degrees with respect to the x-axis. After 3 minutes, what is the final location of the submarine?

$\vec{x}_{f,s} = (?, ?)$

What is the final *displacement* of the submarine? What is the average velocity of the submarine ?

$\Delta \vec{x}_s = \vec{x}_{f,s} - \vec{x}_{i,s} = (?, ?)$ $\vec{v}_s = \frac{\Delta \vec{x}_s}{\Delta t} = (?, ?)$