Wednesday Warm Up: Unit 5: Momentum II

Prof. Jordan C. Hanson

November 6, 2024

1 Memory Bank

- $v = r\omega$ ... Relationship between tangential velocity and angular velocity.
- $\omega = 2\pi f = 2\pi/T$ ... Relationship between angular velocity (ω), frequency (f), and period (T).
- $\vec{p} = m\vec{v}$ ... Definition of momentum.
- $\vec{F}_{\text{Net}} = \frac{d\vec{p}}{dt}$ ... Force and momentum
- Let M be the total mass of a system, and let m_j and $\vec{r}_j$ ($j = 1, ..., N$) be the masses and positions of the constituent parts of the system. The position of the center of mass is

$$\vec{r}_{\text{CM}} = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{r}_j \quad (1)$$

- The momentum of the center of mass $\vec{P}_{\text{CM}}$ is

$$\vec{P}_{\text{CM}} = \sum_{j=1}^{N} \vec{p}_j \quad (2)$$

- The net external force on a system obeys

$$\vec{F} = \frac{d\vec{P}_{\text{CM}}}{dt} \quad (3)$$

2 Momentum II

1. Consider Fig. 1, in which a single planet orbits a star located at the origin at $t = 0$. Let the star have mass M, the planet have mass m, and let the distance between them be r. Let the ratio of the masses be $\mu = m/M$. (a) Show that the center of mass is given by

$$\vec{r}_{\text{CM}} = \left(\frac{\mu}{\mu + 1}\right)\vec{r} \quad (4)$$

(b) Show that $\vec{r} = 0$ in the limit that $\mu \ll 1$.

2. Where is the center of mass if the star and the planet have the same mass?

3. Assume there is no *net, external* force on the system. The center of mass will
   - A: Accelerate
   - B: Decelerate
   - C: Remain stationary
   - D: Remain at constant velocity

4. Given the answer to the previous exercise, what do you conclude about the orbits of the star and planet?

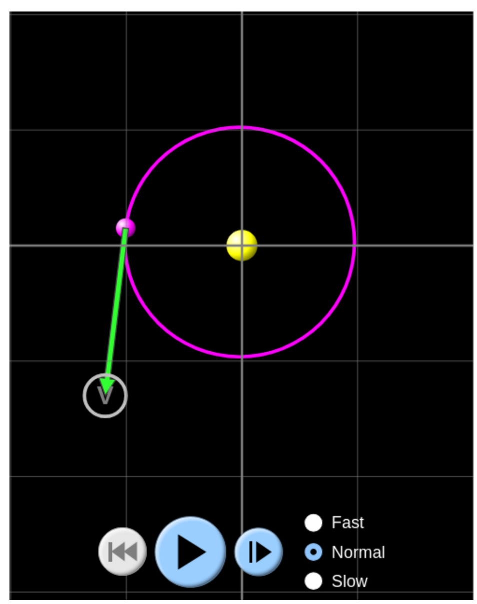

Figure 1: A planet orbits a star.