calculus-based physics-2: electricity, magnetism, and thermodynamics (phys180-02): unit 5

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UNIT 4 REVIEW

Reading: Chapters 7, 9, and 10

1. Voltage and Capacitance
2. Ohm’s Law
3. DC circuits

UNIT 4 REVIEW PROBLEMS

Which of the following would decrease the time required to charge the capacitor at right?

- A: Decreasing the capacitance
- B: Decreasing the resistance
- C: It already charges as fast as possible
- D: Both A and B

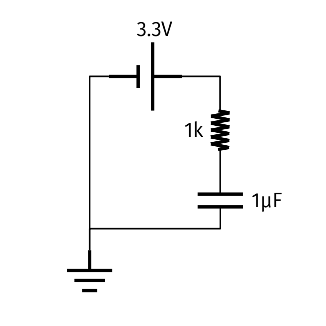


Figure 1: An RC circuit.

What is the RC time of the circuit?

- A: 1 μs
- B: 1 ms
- C: 1 s
- D: 10 s

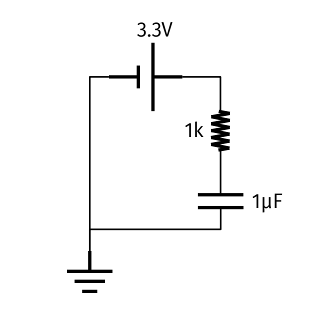


Figure 2: An RC circuit.

What is the maximum charge stored eventually in the capacitor? *Recall that* $Q = CV$.

- A: 3.3 μ C
- B: 1.5 μ C
- C: 3.3 mC
- D: 1.5 C

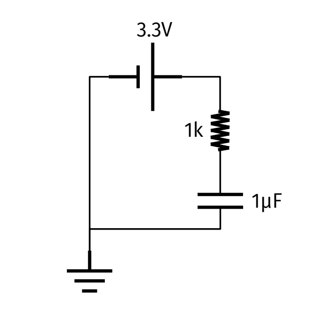


Figure 3: An RC circuit.

SUMMARY

Reading: Chapter 11

1. Magnetism and magnetic fields
2. Motion of a charged particle in a magnetic field
3. Other forces
4. Current loops

MAGNETISM AND MAGNETIC FIELDS

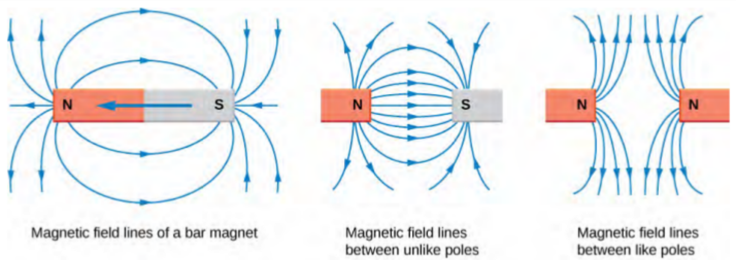


Figure 4: Various magnetic field line configurations.

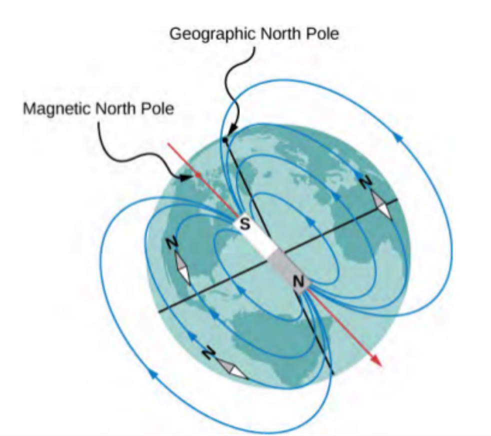


Figure 5: The magnetic and geographic poles are not the same.

It would be nice if we could say:

$$F = \mu_0 \frac{q_{m,1} g_{m,2}}{r^2} \tag{1}$$

But...we can't. Why? There's no such thing has magnetic charge:

$$\nabla \cdot \vec{E} = \rho/\epsilon_0 \tag{2}$$

$$\nabla \cdot \vec{B} = 0 \tag{3}$$

But there is a force associating charge and magnetic fields.

But first, let's review the cross-product.

What is a cross-product and how does it work?

Computing the cross product [edit]

Coordinate notation [edit]

The standard basis vectors $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ satisfy the following equalities in a right hand coordinate system:

$$\begin{aligned} \mathbf{i} \times \mathbf{j} &= \mathbf{k} \\ \mathbf{j} \times \mathbf{k} &= \mathbf{i} \\ \mathbf{k} \times \mathbf{i} &= \mathbf{j} \end{aligned}$$

which imply, by the anticommutativity of the cross product, that

$$\begin{aligned} \mathbf{j} \times \mathbf{i} &= -\mathbf{k} \\ \mathbf{k} \times \mathbf{j} &= -\mathbf{i} \\ \mathbf{i} \times \mathbf{k} &= -\mathbf{j} \end{aligned}$$

The definition of the cross product also implies that

$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0}$ (the zero vector).

Figure 6: The cross-product is a way of multiplying unit vectors.

Let $\vec{v} = 2\hat{i}$ and $w = -2\hat{j}$. What is $\vec{v} \times \vec{w}$?

- A: $-4\hat{k}$
- B: $4\hat{k}$
- C: $-2\hat{i}$
- D: $2\hat{j}$

Let $\vec{v} = 3\hat{j}$ and $w = 5\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $15\hat{i}$
- B: $5\hat{j}$
- C: $3\hat{i}$
- D: $15\hat{k}$

Let $\vec{v} = 3\hat{i} \times 3\hat{j}$ and $w = 2\hat{k}$. What is $\vec{v} \times \vec{w}$?

- A: $-6\hat{j} + 6\hat{k}$
- B: $-6\hat{j} + 6\hat{i}$
- C: $6\hat{j} + 6\hat{i}$
- D: $6\hat{k} + 6\hat{i}$

Group board exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} \tag{4}$$

$$\vec{w} = 4\hat{j} - 4\hat{i} \tag{5}$$

$$\vec{v} \times \vec{w} = ?? \tag{6}$$

Group board exercise: Compute the following cross product:

$$\vec{v} = 2\hat{i} - 2\hat{j} + \hat{k} \tag{7}$$

$$\vec{w} = 4\hat{j} - 4\hat{i} - \hat{k} \tag{8}$$

$$\vec{v} \times \vec{w} = ?? \tag{9}$$

The Lorentz Force

Let a particle with charge q and velocity $\vec{v}$ move through a magnetic field $\vec{B}$. The Lorentz force on the charged particle is

$$\vec{F}_{\mathrm{L}} = q\vec{v} \times \vec{B} \tag{10}$$

As a helpful memory tool, we have the right-hand rule to remember the direction of the cross-product. The units of the magnetic field are the Telsa, after Nikola Tesla. We also have the Gauss which is 10^{-4} Tesla.

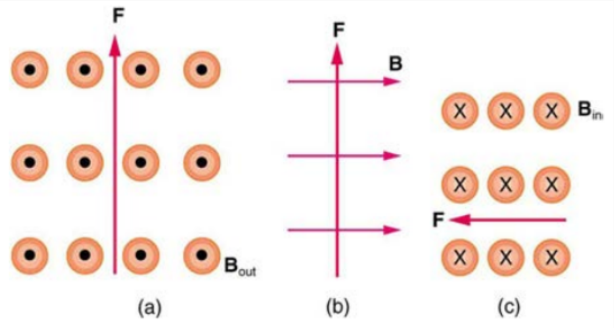


Figure 7: Three different magnetic field and charge scenarios. The vector $\vec{F}$ is the direction of the Lorentz force, and the magnetic field is uniform. A dot indicates that the magnetic field is coming out of the page, and an x indicates that the field is going into the page.

In which of the diagrams is a positively charged particle moving to the left?

- A: A
- B: B
- C: C
- D: WAT WAT WAT

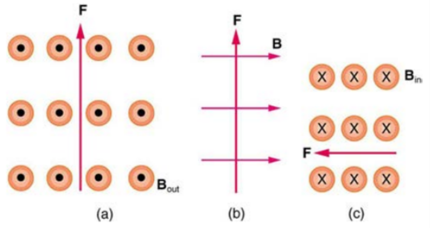


Figure 8: Three different magnetic field and charge scenarios.

In which of the diagrams is a positively charged particle moving upwards?

- A: A
- B: B
- C: C
- D: WAT WAT WAT

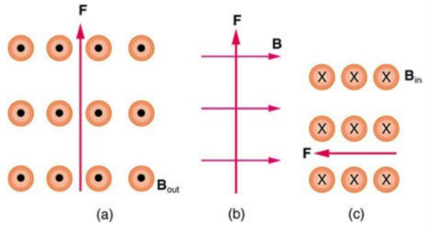


Figure 9: Three different magnetic field and charge scenarios.

In which of the diagrams is a negatively charged particle into the page?

- A: A
- B: B
- C: C
- D: WAT WAT WAT

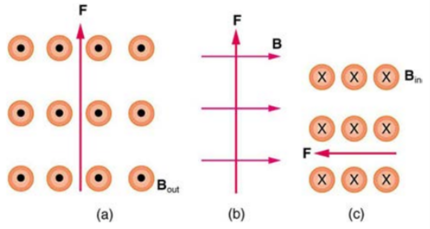


Figure 10: Three different magnetic field and charge scenarios.

In which of the diagrams is a negatively charged particle to the right?

- A: A
- B: B
- C: C
- D: WAT WAT WAT

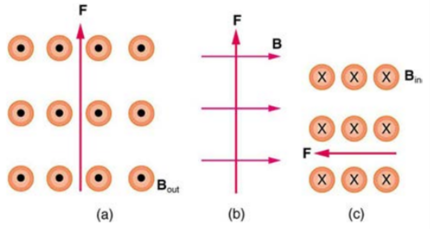


Figure 11: Three different magnetic field and charge scenarios.

A theorem for the magnitude of the cross-product: Let $\vec{a}$ and $\vec{b}$ be vectors and θ be the angle between them. The magnitude of the cross product is:

$$|\vec{a} \times \vec{b}| = ab \sin\theta \tag{11}$$

Thus, the magnitude of the Lorentz force is

$$F_{\mathrm{L}} = qvB \sin\theta \tag{12}$$

The angle θ is between the velocity and the magnetic field.

A cosmic ray proton moving toward the Earth at 3×10^6 m/s experiences a magnetic force of 2×10^{-17} N. What is the strength of the magnetic field of the Earth? (1 Gauss = 10^{-4} Tesla).

- A: 0.1 Gauss
- B: 0.6 Gauss
- C: 1 Gauss
- D: 6 Gauss

Figure 12: The aurora borealis, or northern lights.

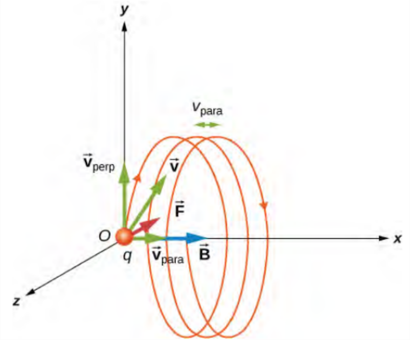


Figure 13: In three dimensions, charged particle motion in a $\vec{B}$-field can result in *helical motion*.

Suppose the velocity of a charged particle with mass m is $\vec{v} = v_x\hat{i} + v_z\hat{k}$ through a uniform field $\vec{B} = B\hat{k}$. The Lorentz force causes centripetal motion and the particle continues to have constant velocity in the $\hat{k}$ direction:

$$\vec{F} = q\vec{v} \times \vec{B} \tag{13}$$

$$\vec{F} = -qBv_x\hat{j} \tag{14}$$

$$mr\omega^2 = qBv_x \tag{15}$$

$$\omega = \sqrt{\frac{qBv_x}{mr}} \tag{16}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{qBv_x}{mr}}, \quad T = 2\pi\sqrt{\frac{mr}{qBv_x}} \tag{17}$$

$$\vec{v} \cdot \hat{k} = v_z \tag{18}$$

Which of the following is true of a charged particle moving in a helical fashion through a magnetic field?

- A: Raising the strength of the B-field increases the period
- B: Raising the strength of the B-field increases the frequency
- C: The particle has a constant velocity parallel to the field
- D: B and C

Two particles are moving in helixes through a region where there is a magnetic field. One moves clockwise as you observe it, and the other moves counter-clockwise, and the helices have about the same radius. Which of the following is true?

- A: The particles have identical charge.
- B: The particles have identical charge, and about the same mass.
- C: The particles have opposite charge, and about the same mass.
- D: The particles have different masses.

Same situation: if the B-field strength increases, and the frequency of the rotations stays constant, what happens to the radius of the helices?

- A: They increase
- B: They decrease
- C: The particles have opposite charge, and about the same mass.
- D: The particles have different masses.

Group board exercise: Recall that the *rotational kinetic energy* of a particle with mass m moving circularly with radius r is

$$K_{rot} = \frac{1}{2} I \omega^2 = \frac{1}{2} m r^2 \omega^2 \tag{19}$$

Show that the total energy, including translational and rotational, of the particle moving in the magnetic helix is

$$E_{tot} = \frac{1}{2} m v_z^2 + \frac{1}{2} q v_x r B \tag{20}$$

What is the total energy of a proton (mass of 1.6×10^{-27} kg), moving with $v_z = 0.1 \times 10^8$ m/s, $v_x = 1.0 \times 10^8$ m/s, and $r = 0.625$ m in a field with $B = 10^{-4}$ Tesla? What can we say about the field strength if we observe r decreasing and v_x to be constant?

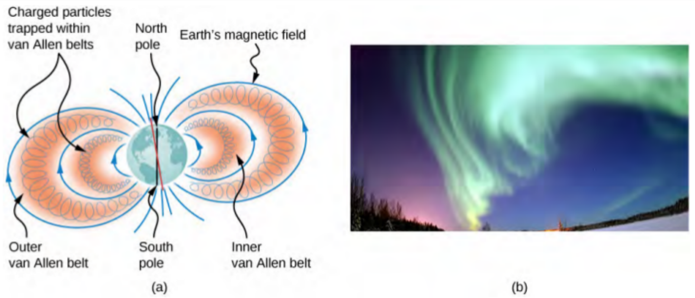
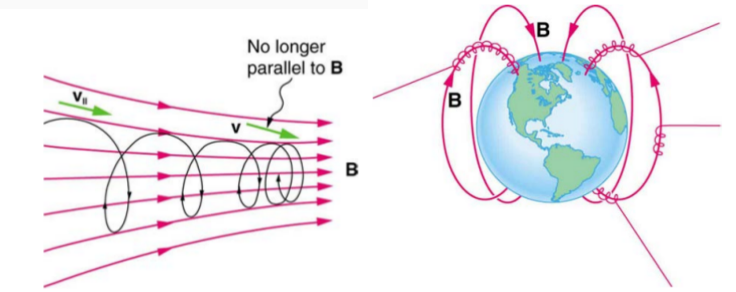


Figure 14: We observe this effect in the auroras, and the van Allen belts.

A cool talk on the aurora borealis:
`https://youtu.be/czMh3BnHFHQ`



One un-explained piece: what does it mean for the electrons and protons to *high-five* the neutral oxygen and nitrogen atoms?

OTHER FORCES

The Lorentz force, when applied to a section of current-carrying wire, becomes

$$d\vec{F} = I\vec{dl} \times \vec{B} \tag{21}$$

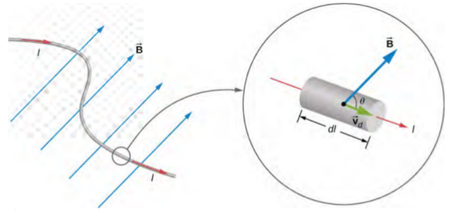


Figure 15: The magnetic force on a section of current.

If the field is uniform:

$$\vec{F} = I\vec{L} \times \vec{B} \tag{22}$$

Group board exercise: A wire of length 10 cm and mass 1 g is suspended in a horizontal plane by a pair of flexible leads. The wire is then subjected to a constant magnetic field of magnitude 0.1 T, which is directed into the board. What are the magnitude and direction of the current in the wire needed to remove the tension in the supporting leads?

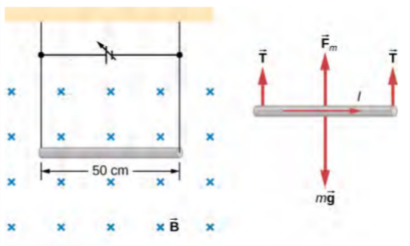


Figure 16: Current suspended by Lorentz force...?

Suppose a power supply provides the current in the previous example. What if the voltage is raised, and the resistance stays constant, so that the current is doubled. What will happen?

- A: The wire will rise.
- B: The wire will fall.
- C: The magentic field will decrease.
- D: Nothing.

If the wire is raised, what is doing the work to raise it?

- A: The wire will rise.
- B: The wire will fall.
- C: The magentic field will decrease.
- D: Nothing.

Group board exercise: Suppose the current is raised from 1 amp to 2 amps for 0.1 seconds. By how much will the wire be raised? What is doing the work to raise this object?

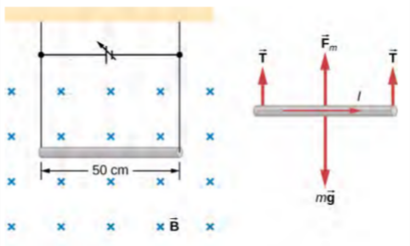


Figure 17: Current suspended by Lorentz force...?

Figure 18: An electromagnetic crane.

Observe on board. The force is $F = dlIB\sin\theta$, but $dl = Rd\theta$.

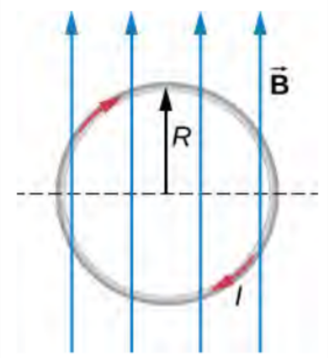


Figure 19: Lorentz force on a loop of wire.

CONCLUSION

Reading: Chapter 11

1. Magnetism and magnetic fields
2. Motion of a charged particle in a magnetic field
3. Other forces
4. Current loops

ANSWERS

ANSWERS

- Both A and B
- 1 ms
- 3.3 μ C
- B and C
- The particles have opposite charge, and about the same mass.
- The wire will rise.