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Physics 150 Midterm 2

1. Unit 4

2. a. $U(x) = k(x^4 - x^2)$ | force = 0 at $0, \pm\sqrt{1/2}$

$k(4x^3 + 2x) = 0$

$2xk(2x^2 - 1) = 0$

$2xk = 0$ $\quad 2x^2 - 1 = 0$

$\pm 1 \quad \pm 1$

$\sqrt{x^2} = \sqrt{1/2}$

$x = \pm\sqrt{1/2}$

b. $U''(x) = k(12x^2 - 2)$

$k(12(\sqrt{1/2})^2 - 2)$ ← maximum displacement

$k(12(1/2) - 2$

$k(6 - 2) = \boxed{4k}$

2. $\vec{F} = 5\hat{i}\,N$, $m = 1\,kg$, $\mu = 0.5$, $\Delta x = 1\,m$

a. $f_s = \mu \cdot (mg) = 0.5 \cdot (1\,kg \cdot 9.81) = -4.905\,N$ ← force of friction

$5\hat{i}\,N - 4.905\hat{i}\,N = 0.095\,N$

$W = 0.095\,N \cdot 1\,m = \boxed{0.095\,N/m \text{ or } J}$

b. When reaching $\vec{x} = 1\hat{i} + 0\hat{j}$, $F = 5\hat{j}\,N$, $\Delta x = 1\hat{i} + 1\hat{j}$

$W = d \cdot f$ $\quad W = ?$

$W = 1\hat{j}\,m \cdot 0.095\,N\hat{j} = \boxed{0.095\,N\,m\hat{j}}$

2. c. $\vec{F} = -5\hat{i}\ N$ system $x = 0\hat{i} + 1\hat{j}$. W done?

$\Delta X = -1\hat{i}$

$F = -0.095N$

$W = -1\hat{i}m \cdot -0.095N\hat{i} =$ **$0.095 Nm\hat{i}$**

d. $\vec{F} = -5\hat{j}\ N$, System returns to $x = 0\hat{i} + 0\hat{j}$

$F = -0.095N$, $\Delta X = -1\hat{j}$

$W = -1\hat{j}m \cdot -0.095N = 0.095 Nm\hat{j}$

E. $W_t = W_a + W_b + W_c + W_d$

$W_t = 0.095Nm + 0.095Nm + 0.095Nm + 0.095Nm$

$W_t = 0.38$

F. if the frictional force was conserved, then the work done would be zero.

2. Unit 5

1. $m_{1,2} = 20 \cdot 10^{-26}$ kg, $V_1 = 350$ m/s, $V_2 = -350$ m/s

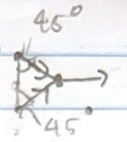
$P_1 + P_2 = (20 \cdot 10^{-26}\ kg \cdot 350\ m/s) + (20 \cdot 10^{-25}\ kg \cdot -350\ m/s)$

$7 \cdot 10^{-22} + (-7 \cdot 10^{-22}) = 0$ m/s

$350\ m/s - 350\ m/s = 0\ m/s$

C

2. m_1 and $m_2 = 20 \cdot 10^{-25}$ kg, $V_1 = 350$ m/s



$V_2 = -350$ m/s

$m_1 V_1 \cos\theta$	$m_2 V_2 \cos\theta$	$20 \cdot 10^{-25} \cdot 350 (\cos(45°)) = 4.95 \cdot 10^{-22}$
$m_2 V_2 \sin\theta$	$m_2 V_2 \sin\theta$	$20 \cdot 10^{-25} \cdot 350 (\sin(45°)) = 4.95 \cdot 10^{-22}$
$20 \cdot 10^{-25} \cdot -350 (\cos(45°))$		$F = -4.95 \cdot 10^{-22}$
$20 \cdot 10^{-25} \cdot -350 (\sin(45°))$		$F = -4.95 \cdot 10^{-22}$

$V_s = V_x$ direction $\quad P = mV \quad \frac{P}{m} = V / \frac{4.95 \cdot 10^{-22}}{20 \cdot 10^{-25}} = \boxed{247.5 \text{ m/s}}$

3. $F = 4000$ N. $\quad t = 0.2$ s

a. impulse $= F \cdot t$

$4000 \text{ N} \cdot 0.2 \text{ s} = \boxed{800 \text{ Ns}}$

b. $V_s = ?$ $\quad V_i = 2.8$ m/s, $m = 200$ kg

$V_s = (\text{impulse}/m) + V_i$

$V_s = 800 \text{ Ns} / 200 \text{ kg} + 2.8 \text{ m/s}$

$\boxed{V_s = 6.8 \text{ m/s}}$

4. D: totally elastic

$V > 0$ $\quad V = 0$ | $V = 0$ $\quad V > 0$

$0 \rightarrow \quad 0$ | $0 \quad 0 \rightarrow$

5. m_1 (car) $= 30{,}000\ kg$ $\quad v = 0.85\ m/s$ $\quad m_2$ (scrap) $= 110{,}000\ kg$

a. $m v_{1i} + m v_{2i} = m v_{1f} + m v_{2f}$

$30{,}000\ kg\,(0.85\ m/s) + (110{,}000\ kg \cdot 0\ m/s) =$

$= (30{,}000\ kg + 110{,}000\ kg)\, v_f$

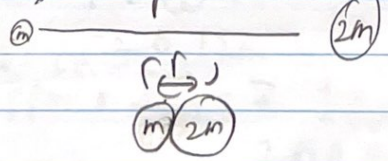
$\frac{25500}{140{,}000} = \frac{140{,}000 \cdot v_f}{140{,}000}$ $\quad$ **$v_f = 0.182\ m/s$**

b. $\frac{1}{2}(30{,}000)(0.85)^2 \;/\; \frac{1}{2}(30{,}000 + 110{,}000)(0.182)^2$

$KE_i = 10833.5\ J$, $\quad KE_f = 2318.68\ J$

$\Delta KE = 8518.82\ J$ ← lost

6.



$r_{cm} = \frac{1}{M_t} \sum_{j=1}^{N} m_j \vec{r}_j$

3 Unit 6: Fixed-axis rotation and angular momentum

1. 1 rot / 1.33 s

$\frac{1\text{ rot}}{1.33\text{ s}} \cdot \frac{60\text{ s}}{1\text{ m}} = \frac{60}{1.33} = 45.11 \approx 45\text{ rpm}$

4.7 rad/s

B: 45 rpm, 4.7 rad/s

2. D: 2.2 m/s^2

3. D: $\omega \Rightarrow 10\ \omega$

$a_c = r \cdot \omega^2 \quad \Leftarrow (10\ \omega)^2 = 100\ \omega$

$r = 62\text{ cm} = 0.12\text{ m}$

4. $200\text{ rpm} \cdot \frac{2\pi}{60} = 20.94\text{ rad/s} \quad m = 10\text{g} = 0.01\text{ kg}$

$20.94 \cdot 0.12 = 2.5128$

$F_c = \frac{0.01\text{ kg} \cdot 2.5128^2}{0.12} = 0.526 \approx 0.53\text{ N}$

B: 0.53N

5. $V = 8\,m/s \quad \omega = 8\pi \approx 25.13\,rad/s \quad m_t = 1\,kg$

a. $KE_{rot} = \frac{1}{2} I\omega^2 + \frac{1}{2} m V^2$

$r = V/\omega \rightarrow r = 8/25.13$

$r = 0.318 \approx 0.32\,m$

$\frac{1}{2}\left(\frac{1}{2}\cdot 0.32^2\right)(25.13^2) + \frac{1}{2}(1)(8)^2$

$KE_{rot} = 34.69 \approx \boxed{34.7\,J}$

b. $a_c = v^2/r \rightarrow \frac{(8\,m/s)^2}{0.32\,m} \quad \boxed{a_c = 200\,m/s^2}$

c. $PE = KE + KE_{rot}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$\rightarrow h = \frac{\frac{1}{2}(1)(8\,m/s)^2 + \frac{1}{2}\left(\frac{1}{2}(0.32\,m)^2 \cdot (25.13\,rad/s)^2\right)}{9.81\,m/s^2}$

$h = 3.5366 \approx \boxed{3.54\,m}$

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b. $r = 5\hat{i} + 5\hat{j}\,cm \quad F = -10\hat{i} + 10\hat{j}\,N$

a. $r = 0.05\hat{i} + 0.05\hat{j}\,m$

$t = F \cdot d \rightarrow (-10\hat{i} + 10\hat{j})N \cdot (0.05\hat{i} + 0.05\hat{j})m$

$\begin{vmatrix} \hat{i} & \hat{j} \\ -10 & 10 \\ 0.05 & 0.05 \end{vmatrix} \rightarrow (0.05)(10) - (0.05)(-10)$

$= 0.5 + 0.5$

$= 1$

$\boxed{t = 1\,Nm}$

b. $r \rightarrow 2r \rightarrow 2(0.05\hat{i} + 0.05\hat{j})m$

$r = (0.1\hat{i} + 0.1\hat{j})m$

$\begin{vmatrix} 0.1 & 0.1 \\ -10 & 10 \end{vmatrix} \rightarrow (0.1)(10) - (0.1)(-10)$

$= 1 + 1$

$= 2$

$\boxed{t = 2\,Nm}$

c. $T = 30\,N\,cm, \quad F = ? \qquad \frac{t}{d} = \frac{F \cdot d}{d}$

$F = \frac{t}{d} \rightarrow \begin{vmatrix} 5 & 5 \\ -3 & 3 \end{vmatrix} \rightarrow (5)(3) - (5)(-3)$

$15 + 15 = 30$

$\boxed{F = (3\hat{i} - 3\hat{j})}$

7. $I = \frac{1}{2}MR^2$. $\omega(t) = (10t + 60\text{rpm})\frac{2\pi}{60}$

$\omega(t) = \frac{\pi}{30}(10t + 60)$

$\alpha = \frac{\pi}{30}(10) = \frac{\pi}{3}$

$t = \left(\frac{1}{2}Mr^2\right)\frac{\pi}{3} \rightarrow \boxed{t = \frac{\pi}{6}Mr^2}$

8. $m = 100\text{ kg}$, $r = 1.5\text{ m}$, $\text{rpm} = 30$

a. $L = I \cdot \omega$

$30\text{rpm} \cdot \frac{2\pi}{60} = \frac{60\pi}{60} = \pi$ / $V = \pi$

$I = \frac{1}{2}mr^2 \rightarrow \frac{1}{2}(100)(1.5^2) = 112.5$

$L = 112.5 \cdot \pi \approx \boxed{353.43\text{ kgm}^2/\text{s}}$

b. $\Delta\omega = \Delta L / I$

$I_{\text{total}} = \frac{1}{2}(100)(1.5^2) + (40)(1.5^2) =$

$112.5 + 90 = 202.5$ $I = 202.5$

$\Delta\omega = 353.43\text{ kgm}^2/\text{s} / 202.5$

$1.745\text{ rad/s} \cdot \frac{60}{2\pi} \approx \boxed{16.7\text{ rpm}}$

4. Unit 7: Statics

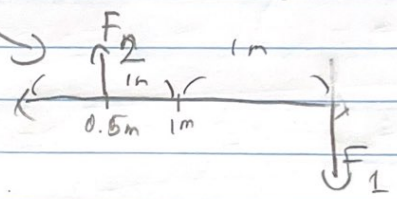
d. $m_1 = 40kg$, $m_2 = 60kg$, $d_1 = 3m$, $d_2 = 2m$

$(40 \cdot 9.81) \cdot 3 = (60 \cdot 9.81) \cdot 2$

$1177.2 = 1177.2$

A: Remain motionless

2. wood $d = 2.0m$ $m = 20kg$



$F_2 = F_n$

$t_2 = t_1$

$F_2 \cdot d_2 = F_1 \cdot d_1$

$20kg \cdot 9.81 \cdot 0.5m = F_1 \cdot 1m$

$\frac{98.1 Nm}{1m} = F_1 \cdot \frac{1m}{1m}$ $F_1 = 98.1N$

$F_2 = 196.2N$ $F_1 = 98.1N$

$t_2 = 98.1Nm$, $t_1 = 98.1Nm$

$F_t = 196.2N + 98.1N$

$F_t = 294.3N$