Friday Reading Assessment: Unit 4, Power and Conservation of Energy

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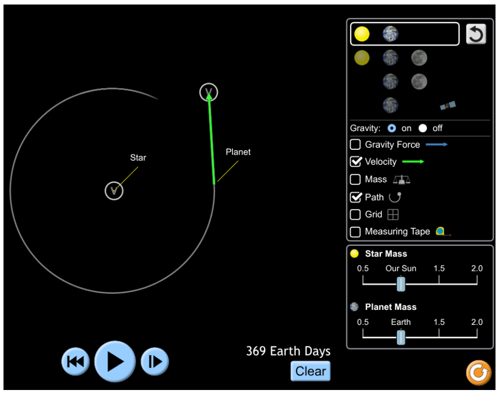

Figure 1: Gravity and orbits PhET.

1 Memory Bank

- **Conservative force:**

$$\vec{F} = -\nabla U(x,y,z) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k} \quad (1)$$

- The *curl* of a vector field:

$$\nabla \times \vec{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{i} - \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z}\right)\hat{j} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k} \quad (2)$$

- **Conservative force:**

$$\nabla \times \vec{F} = 0 \quad (3)$$

- **Gravitational force:**

$$\vec{F}_{\mathrm{G}} = G\frac{m_1 m_2}{r^2}\hat{r} \quad (4)$$

- **Gravitational potential energy:**

$$U(r) = -G\frac{m_1 m_2}{r} \quad (5)$$

2 Conservative Forces and Gravity

1. The *partial derivative* $\partial U/\partial x$ is the derivative with respect to one variable while holding the others constant. (a) Suppose $U(x,y) = \frac{1}{2}k(x^2+y^2)$. What is $-\nabla U$? (b) Suppose $U(x) = -kx^{-1}$. What is $-dU/dx$?

2. A *conservative force* obeys the following rule:

$$\nabla \times \vec{F} = 0 \quad (6)$$

This is called the *curl* of the force. Suppose the force $\vec{F}$ is restricted to the xy-plane, meaning neither the x-component nor the y-component depends on z, and there is no z component. Suppose $\nabla \times \vec{F} = 0$. Using the definition of the curl, show that

$$\frac{dF_x}{dy} = \frac{dF_y}{dx} \quad (7)$$

Suppose $\vec{F} = -k(x\hat{i} + y\hat{j})$. Does this force conserve energy? What type of system is described by such a force?

3. The force of gravity between objects of masses m_1 and m_2, separated by r, is given by

$$\vec{F}_{\mathrm{G}} = G\frac{m_1 m_2}{r^2}\hat{r} \quad (8)$$

The force is interpreted as *attractive* if it is positive, and $G = 6.674 \times 10^{-11}$ N kg^{-2} m^2. (a) What is the force of gravity between the Earth and Moon? Look up the masses in kg, and let $r = 384400$ km. (b) Let $F_{\mathrm{C}} = F_{\mathrm{G}}$. Show that $r^3 \propto T^2$. (c) Work out the period of the Moon around the Earth.

4. (a) Show that, if $F = -dU(r)/dr$, that the potential energy of the gravitational force is given by Eq. 5. (b) What is the change in potential energy if an object goes from orbital radius r_A to r_B? (c) For a circular orbit, prove that total energy is constant.