① Unit 4.

$U(x) = k(x^4 - x^2)$

ⓐ $F = 0$ ∴ $F(x) = \int k(x^4 - x^2)\,dx$

$x = ??$ $F(x) = k\left(\frac{x^5}{5} - \frac{x^3}{3}\right) + c$

$0 = k\left(\frac{x^5}{5} - \frac{x^3}{3}\right) + c$

Unit 4

① $U(x) = k(x^4 - x^2)$

ⓐ $F = 0$ $F(x) = -(U(x))'$

$x = ??$ $F(x) = -\frac{dU(x)}{d(x)}$

$F(x) = k(4x^3 - 2x) = 0$

$4x^3 - 2x = 0$

$2x(2x^2 - 1) = 0$

$x = 0$ & $x = \pm 1/\sqrt{2}$

∴ Force is zero when $x = 0$, $x = 1/\sqrt{2}$ & $x = -1/\sqrt{2}$.

ⓑ so critical no's are $0, \pm 1/\sqrt{2}$. checking on $U(x)$.

$U(0) = k(0^4 - 0^2) = 0$, $U(1/\sqrt{2}) = k\left(\frac{1}{(\sqrt{2})^4} - \frac{1}{(\sqrt{2})^2}\right) = k\left(\frac{1}{4} - \frac{1}{2}\right) = -\frac{1}{4}k$

$U(-1/\sqrt{2}) = k\left(\frac{1}{(-\sqrt{2})^4} - \frac{1}{(-\sqrt{2})^2}\right) = -\frac{1}{4}k$

∴ so the maximum displacement it could achieve is when $x = \frac{1}{\sqrt{2}}$ that is from the origin is $\frac{1}{\sqrt{2}}$ since the system has been perturbed to the right

① Unit 4: (part 2)

② $m = 1kg$ ⓐ $F = 5\hat{i}\,N$

$\mu = 0.5$

$\Delta x = (1-0)\hat{i} + (0-0)\hat{j} = 1\hat{i}\,m$

$W_a = F \cdot \Delta x = 5\hat{i}(1\hat{i}) = 5J.$

ⓑ $F = 5\hat{j}\,N$

$\Delta x = (1-1)\hat{i} + (1-0)\hat{j} = 1\hat{j}\,m$

$W_b = F \cdot \Delta x = (5\hat{j}) \cdot (1\hat{j}) = 5J.$

ⓒ $F = -5\hat{i}\,N$

$\Delta x = (0-1)\hat{i} + (1-1)\hat{j} = -1\hat{i}\,m$

$W_c = F \cdot \Delta x = (-5\hat{i})(-1\hat{i}) = 5J$

ⓓ $F = -5\hat{j}\,N$

$\Delta x = (0-0)\hat{i} + (0-1)\hat{j} = -1\hat{j}\,m$

$W_d = F \cdot \Delta x = (-5\hat{j}) \cdot (-1\hat{j}) = 5J$

ⓔ $W_{total} = W_a + W_b + W_c + W_d$

$W_{total} = 5 + 5 + 5 + 5 = \underline{20J}$

ⓕ If force of friction was conservative, because the path is closed.

$\underline{W_T = 0J}$

(2) Unit 5 : Linear Momentum.

(1) $m = 20 \times 10^{-25}$ kg.

$v_1 = 350$ m/s

$v_2 = -350$ m/s

$v = ??$

$P_i = P_f$

$m_1 v_1 + m_2 v_2 = MV$

$$V = \frac{m_1 v_1 + m_2 v_2}{M} = \frac{m(v_1 + v_2)}{M}$$

$$= \frac{20 \times 10^{-25} (0)}{40 \times 10^{-25}} = \underline{0} \text{ m/s}$$

$\therefore$ Answer is (C) : $0 \ ms^{-1}$

(2) $m_1 = m_2$

$\theta = 45^\circ$

$v_f = ??$

$u_1 = u_2 = v$

$P = mv,$

$$|P_{total}| = \sqrt{|P_1|^2 + |P_2|^2 + 2|P_1||P_2|\cos\theta}.$$

$$|P_{total}| = \sqrt{(mv)^2 + (mv)^2 + 2(mv)(mv)\frac{\sqrt{2}}{2}}$$

$$= \sqrt{2(mv)^2 + \sqrt{2}(mv)^2}$$

$$= \sqrt{(mv^2)(2 + \sqrt{2}}$$

$$|P_{total}| = mv\sqrt{2 + \sqrt{2}}$$

$$v_f = \frac{|P_{total}|}{m_{total}}, \quad m_{total} = m + m = 2m$$

$$v_f = \frac{\cancel{m}v\sqrt{2+\sqrt{2}}}{2\cancel{m}}$$

$\therefore$ final velocity $\rightarrow$ $$v_f = \frac{v\sqrt{2+\sqrt{2}}}{2}$$

(2) Unit 5 ! (part 2)

(3) $f = 4000N$    (a) $\Delta p = F\Delta t$

$t = 0.2s.$    $= 4000N \times 0.2s$

$\Delta p = ??$    $= \underline{800\,Ns}$

(b) $v = 2.8m/s$    $\frac{\Delta p}{m} = \frac{m(v_f - v_i)}{v}$

$m = 200kg.$    $v_f = \frac{\Delta p}{m} + v_i$

$v = ??$

$$= \frac{-800}{200} + 2.8 =$$

$$= \underline{-1.2m/s}$$

∴ The final velocity is 1.2m/s in the opposite direction.

(4) The Answer is (c) : Elastic

(5) $m_1 = 30,000kg$    (a) $m_1v_1 + m_2\overset{0}{v_2} = Mv_f$

$v_1 = 0.85m/s$    $30,000(0.85) = 140,000\, v_f.$

$m_2 = 110,000\,kg$    $v_f = \frac{25,500}{140,000} = 0.182m/s$

$v_f = ??$

∴ The final velocity is 0.182m/s.

(b) $KE_i = \frac{1}{2}m_1v_1^2 = \frac{1}{2}30,000(0.85)^2 = 10,837.5J.$

$KE_f = \frac{1}{2}m_Tv_f^2 = \frac{1}{2}(140,000)(0.182)^2 = 2,316.76J$

$\Delta KE = KE_i - KE_f$

$= 10,837.5 - 2,316.76J.$

$= \underline{8,520.74J}$

∴ the kinetic energy lost is 8,520.74J.

(2) Unit 5: (part 3).

(6) $m_1 = m$ before $x_1(t) = x_{1,0} - vt$, where $x_{1,0}$ is $m_1$ initial position

$m_2 = 2m$ $x_2(t) = x_{2,0} + vt$, where $x_{2,0}$ is $m_2$ initial position

$V_{\text{each}} = v$

$$x_{COM} = \frac{m x_1(t) + 2m x_2(t)}{m + 2m}$$

$\overset{\rightarrow}{2m}$ $\overset{\leftarrow}{m}$

$$x_{COM} = \frac{\cancel{m}(x_{1,0} - vt) + 2\cancel{m}(x_{2,0} + vt)}{3\cancel{m}}$$

$$x_{COM} = \frac{x_{1,0} + 2x_{2,0}}{3} + \frac{v}{3}t.$$

~~Although~~ After $m + 2m = 3m$ (sticking)

$P_{bef} = P_{aft}$

$P_{bef} = m(-v) + 2m(v) = mv$

$p_{aft} = 3m v_{COM}$

$v_{COM} = v/3$

∴ Although the collision is inelastic, momentum is conserved and the COM move with a velocity of $v/3$ and its position is found by $x_{COM}(t) = \frac{x_{1,0} + 2x_{2,0}}{3} + v/3\, t$.

③ Unit 6: fixed-axis rotation & angular momentum.

①. $t = 1.33$ s/rot $\quad \frac{1 rot}{1.33 s} \times \frac{60 s}{1 min} = 60/1.33$ rpm $= 45.1$ rpm

rpm = ??  $\approx 45$ rpm

$\omega$ = ?? $\quad \frac{45 rot}{1 min} \cdot \frac{2\pi rad}{1 rot} \cdot \frac{1 min}{60 s} = 1.5\pi$ rad/s

$= 1.5 \times 3.14$ rad/s

∴ Answer is Ⓑ $= 4.71$ rad/s.

② $a_c$ = ??

∴ Answer is Ⓓ $2.2\ m/s^2$.

③ $a_c = \omega^2 r$

when $\omega \to 10\omega$

$a_c = (10\omega)^2 r$

$a_c = 100\omega^2 r$

∴ Answer is Ⓓ: $\omega \to 10\omega$

④ $\omega = 200$ rpm $\quad f_c = ma_c$

$r = 12$ cm $\quad f_c = m\omega^2 r$

$m = 10$ g $\quad = \frac{10}{1000} \times \left(\frac{200\pi}{30}\right)^2 \times \frac{12}{100} = 0.53$ N

$f_c$ = ??

∴ Answer is Ⓑ: 0.53N.

⑤ $m = 0.5$ kg $\quad I = \sum m_i r_i^2 \Rightarrow I = mr^2 + mr^2 = 2mr^2$

ⓐ $\omega = 4$ rev/s. $\quad K.E_T = KE_R + K.E_T$

$V_c = 8$ m/s $\quad = \frac{1}{2} I\omega^2 + \frac{1}{2} m v_c^2$

$K.E_T$ = ?? $\quad = \frac{1}{2}(2mr^2)\omega^2 + \frac{1}{2} m v_c^2$

$= 0.5 r^2 (4(2\pi))^2 + \frac{1}{2}(1)(64)$

$= 32\pi^2 r^2 + 32$

$= 32(\pi^2 r^2 + 1)$ but $V = \omega r$

$r = V/\omega = \frac{8}{8\pi} = 1/\pi$

$= 32(\pi^2(\frac{1}{\pi^2}) + 1)$

$= 64$ J

3 Unit 6 (part 2)

(5)

(b) $L = ??$ $\quad L = I\omega$

$$= 2mr^2\omega$$

$$= 2(0.5)\left(\frac{1}{\pi}\right)^2 8\pi$$

$$= 8/\pi = \underline{2.55\ kgm^2/s}$$

(c) $h = ??$

• At the highest point $K.E = P.E$

$$62 = mgh$$

$$h = \frac{62}{(1)(9.8)} = \underline{6.53m}$$

(6) $\vec{r} = 5\hat{i} + 5\hat{j}$ cm $\quad$ (a) $\tau = r \times f$

$F = -10\hat{i} + 10\hat{j}$ N

$\tau = ??$

$$= \begin{vmatrix} 5/100 & 5/100 \\ -10 & 10 \end{vmatrix}$$

$$= 5/10 - 5\left(-\frac{1}{10}\right)$$

$$= 0.5 + 0.5 = \underline{1\ N.m\hat{k}}$$

(b) $\tau = 2r \times f = 2(r \times f) = 2(1) = \underline{2\ N.m\hat{k}}$

(c) $\tau = 30$ Ncm.

$\tau = r \times f = rF\sin\theta$ (assuming they're $\perp$)

$\tau = rF$

$r = \frac{5}{100}\hat{i} + \frac{5}{100}\hat{j}$ m. $\quad |r| = \sqrt{(5/100)^2 + (5/100)^2}$

$F = \tau/r = \frac{30/100}{0.0707} = \frac{3}{0.707} = \boxed{4.24N}$

$$= \sqrt{1/400 + 1/400}$$

$$= \sqrt{\frac{\cancel{800}}{\cancel{216 \times 10^4}}}$$

$$= \sqrt{\frac{1}{200}}$$

$$= \cancel{0.707}\ \underline{0.0707}$$

3: unit 6 (part 3).

(7) $I = \frac{1}{2}MR^2$

$\omega(t) = 10t + 60$ rpm

$\tau = ??$

$\tau = I\alpha \qquad \alpha = \frac{d\omega(t)}{dt}$

$\tau = I\frac{d\omega}{dt} \qquad \frac{d\omega}{dt} = 10$ rpm

$= \frac{1}{2}MR^2\left(\frac{\cancel{10}\pi}{3\cancel{0}}\right)$

$\underline{\tau = \frac{\pi}{6}MR^2\ N.m}$

(8) $m = 100$ kg

$r = 1.5$ m

$\omega = 30$ rpm

$L = ??$

(a) $L = I\omega$

$= \frac{1}{2}mr^2\omega$

$= \frac{1}{\cancel{2}_1}\,\cancel{100}^{50}\,(1.5)^2\,\frac{\cancel{30}(\pi)}{\cancel{30}}$

$= 50(2.25)\pi$

$= \underline{353.5\ kgm^2/s}$

(b) $m = 40$ kg.

$\omega_N = ??$

$L_i = L_f$

$353.5 = I_N\omega_N$

$\omega_N = \frac{353.5}{I_N}$

$\omega_N = \frac{353.5}{202.5} \times \frac{30}{\pi}$

$= \underline{16.67\ rpm}$

$I_N = \frac{1}{2}mr^2 + md^2$

$= \frac{1}{2}mr^2 + mr^2$

$= \frac{1}{2}(100)(1.5)^2 + 40(1.5)^2$

$= 50(1.5)^2 + 40(1.5)^2$

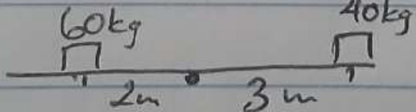
$= 1.5^2(90)$

$= 2.25(90)$

$= 202.5\ kgm^2/s$.

(4) Unit 7: Statics.

(1) $m_1 = 40kg$




$x_1 = 3m$

$m_2 = 60kg$

$x = 2m$

$\tau_1 = r_1 \times f_1$

$= 3 \times 40$

$\tau_1 = 120 N.m.$

$\tau_2 = r_2 \times f_2$

$= 2 \times 60$

$= 120 N.m$

$\tau_{net} = \Delta\tau = \tau_2 - \tau_1$

$= 120 - 120 = 0$

∴ there is no net torque

(A) the system will remain motionless.

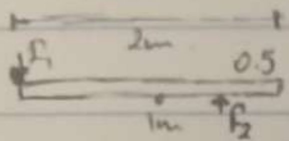
2) $l = 2m$

$x = 0.5m$

$m = 20kg$.

$F_1 = ??$

$F_2 = ??$




Criteria 1.

$F_{net} = 0$

$F_1 + F_2 = -W$

$F_1 + F_2 = -20(-9.8)$

$F_1 + F_2 = 196N$ -- (1)

$F_2 = 196 - 65.3 = 132.6$

$F_1 = 65.3N$ ~~upward~~ downward

$F_2 = 132.6$ upward.

Criteria 2.

$\tau_{net} = 0$ choosing $f_2$ as the pivot.

$\tau_1 = F_1 \cdot 1.5$

$\tau_W = W \cdot 0.5 = 20(9.8)(0.5)$

$\tau_W = -98 N.m.$

$\tau_1 + \tau_W = 0$ $\tau_1 = -\tau_W$.

$F_1(1.5) = -(-98)$

$F_1 = 98/1.5 = \underline{65.3}$