

CALCULUS-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND THERMODYNAMICS (PHYS180-02): UNIT 4

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UNIT 3 REVIEW

Reading: Chapters 5-7

1. Charge, Conductors and Insulators
2. Coulomb's Law and Electric Fields
3. E-fields of Charge Distributions
4. Gauss's Law

UNIT 3 REVIEW PROBLEMS

UNIT 3 REVIEW PROBLEMS

Suppose a charge q is located at the origin. Use Gauss' law to find the electric field. What is the electric field?

- A: $\vec{E} = \frac{kq}{r} \hat{r}$
- B: $\vec{E} = \frac{kq}{r^2} \hat{r}$
- C: $\vec{E} = \frac{kq}{r^3} \hat{r}$
- D: $\vec{E} = \frac{kq^2}{r^2} \hat{r}$

UNIT 3 REVIEW PROBLEMS

Suppose a line of charge runs up the z-axis. The charge per unit length is λ . Use Gauss' law to find the electric field. What is the electric field at a point $P = (x, 0, 0)$?

- A: $\vec{E} = \frac{2k\lambda}{r^2}$
- B: $\vec{E} = \frac{2k\lambda^2}{r^2}$
- C: $\vec{E} = \frac{2k\lambda}{r}$
- D: $\vec{E} = 0$

SUMMARY

Reading: Chapters 7, 9, and 10

1. Voltage and Capacitance
2. Ohm's Law
3. DC circuits

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1. Suppose we have two charged plates, one with $+Q$ and one with $-Q$, with a uniform electric field E in between them. If the distance between the plates is d , what is the voltage difference between the plates?
2. Form an analogy between voltage and potential energy from kinematics. In this analogy, what kinematic quantity is analogous to electric field?
3. Explain in your own words how the **gradient** of a **scalar field** is calculated, and how it relates to voltage and E-fields.

Suppose we have two charged plates, one with $+Q$ and one with $-Q$, with a uniform electric field E in between them. If the distance between the plates is d , what is the voltage difference between the plates?

“you would use the equation $Q=CV$ to find the voltage difference between the plates.”

“ $\Delta V = \Delta U / Q$ ”

“The voltage difference would be $+Q-Q$.”

Form an analogy between voltage and potential energy from kinematics. In this analogy, what kinematic quantity is analogous to electric field?

“Voltage is similar to potential energy in that voltage is the potential change in charge and potential energy is the potential change in velocity. The electric field of a voltage would be similar to the gravity of a potential energy.”

“voltage is the electric potential difference, or the possible difference between two charges. This is similar to potential energy because potential energy is the possible amount of energy of an object. Gravity would be the kinematic equivalent to electric fields.”

Explain in your own words how the **gradient** of a **scalar field** is calculated, and how it relates to voltage and E-fields.

“you differentiate the relationship between $(\Delta)V$ (this is Voltage) and $(\Delta)s$ (this is distance). $\Delta V / \Delta s = \text{Electric field}.$ ”

Let a scalar field be $U(x, y, z)$. The *gradient* of the scalar field is

$$\vec{\nabla} U = \frac{\partial U}{\partial x} \hat{i} + \frac{\partial U}{\partial y} \hat{j} + \frac{\partial U}{\partial z} \hat{k} \quad (1)$$

VOLTAGE

Voltage is analogous to potential energy in electrostatics. The negative derivative of potential energy U is the force F :

$$F = -\frac{dU}{dx} \quad (2)$$

For example, if the force is $F = -mg$, and the potential energy is $U = mgy$, then

$$\frac{dU}{dy} = -mg \quad (3)$$

Voltage is analogous to potential energy in electrostatics. The derivative of voltage V is the field E :

$$E = -\frac{dV}{dx} \quad (4)$$

For example, if the field is $E = \sigma/\epsilon_0$, and the potential energy is $V = \sigma/\epsilon_0 x + C$, then

$$\frac{dV}{dx} = \sigma/\epsilon_0 \quad (5)$$

Voltage is analogous to potential energy in electrostatics.

Potential energy is just an energy. A *potential* in mechanics is the potential energy per unit mass. *Voltage*, or *electrostatic potential* is **potential energy per unit charge**.

The Volt (V) is the unit of voltage, and it has units of $1 \text{ V} = 1 \text{ J/C}$.

Group board exercise: Show that the units of electric field (normally Newtons per Coulomb) are also Volts per meter.

If the potential energy is a function of displacement, $U = U(\vec{x})$, it may be called a potential energy *surface*.

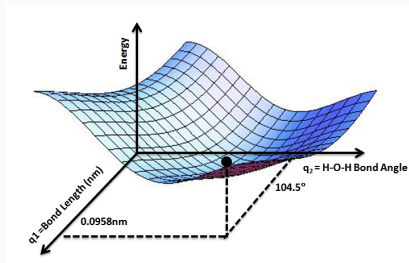


Figure 1: An example of a potential energy surface.

Considering *Newton's Second Law*, however, if $F = ma$ then $ma = -\frac{\Delta U}{\Delta x}$, and

$$a = -\frac{1}{m} \frac{\Delta U}{\Delta x} \quad (6)$$

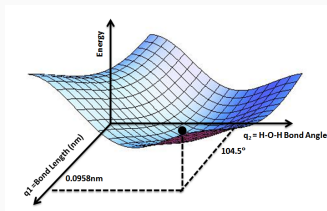


Figure 2: If we divide by the mass we have acceleration.

The derivative, or *gradient*, of a in Eq. 6 is analogous to the electric field. But the electric field is a *vector field*...

The gradient is like a derivative, but gives you the proper direction wherever you are.

$$\vec{\nabla}V = \vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} \quad (7)$$

Suppose the voltage due to some charge distribution is $V(x, y, z) = ax + b$. What is the field?

- A: $\vec{E} = a\hat{j}$
- B: $\vec{E} = a\hat{k}$
- C: $\vec{E} = b\hat{i}$
- D: $\vec{E} = a\hat{i}$

Suppose the voltage due to some charge distribution is $V(x, y, z) = ax + by$. What is the field?

- A: $\vec{E} = -a\hat{j} - b\hat{j}$
- B: $\vec{E} = a\hat{k} + b\hat{k}$
- C: $\vec{E} = -b\hat{i} - a\hat{j}$
- D: $\vec{E} = -a\hat{i} - b\hat{j}$

Suppose a charge distribution is made of two infinite plates of charge with charge per unit area $\pm\sigma$. We know that the field is $\sigma/\epsilon_0 \hat{k}$ between them. **Group board exercise:** Draw the charge distribution, define a coordinate system, and write the function for the voltage.

Voltage is like a potential energy surface → *potential energy per unit charge*.

<https://phet.colorado.edu/en/simulation/charges-and-fields>

Using the PhET simulation about charges and fields:

1. Explore the voltage associated with fields generated by charges using the voltage button.
2. Add a single point charge, and use the ruler and voltmeter (potentiometer) to measure voltage versus distance, and plot it.
3. What function describes the relationship between voltage and distance?

<https://phet.colorado.edu/en/simulation/charges-and-fields>

Using the PhET simulation about charges and fields:

1. Note that the units of ϵ_0 are $\text{N m}^2 \text{C}^{-2}$, and the value is 8.854×10^{-12}
2. We know from prior equations that the units of voltage are J C^{-1}
3. Using your measurements, show that the voltage due to a point charge is

$$V = \pm \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (8)$$

(Where the sign depends on the charge, just like E-fields)

Voltage due to a point charge:

$$V = \pm \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (9)$$

Voltage is an example of a **scalar field**, whereas the electric field is an example of a **vector field**. Create two lines of charge, one positive and one negative in the PHeT simulator, and pretend they represent planes of charge coming out of the screen. The field should be constant and uniform like

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{k} \quad (10)$$

1. Plot the voltage versus distance between the plates.
2. Calculate the slope in V/m, and the y-intercept in V.
3. Does the electric field depend on the y-intercept? Does the charge distribution?

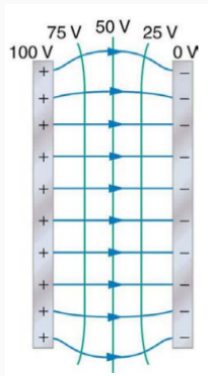


Figure 3: Parallel plates of charge, electric field, and potential. Notice the linear decrease in voltage.

How we define the y-intercept in voltage is analogous to the zero-point freedom of potential energy.

Two parallel plates, opposite charge:

$$V = -\frac{\sigma}{\epsilon_0}z + C \quad (11)$$

With the boundary condition that $V = V_0$ when $z = 0$, we have

$$V(z) - V_0 = -\frac{\sigma}{\epsilon_0}z \quad (12)$$

Let $\Delta V(z) = V(z) - V_0$, and $\Delta z = z$:

$$-\frac{\Delta V}{\Delta z} = \frac{\sigma}{\epsilon_0} = E \quad (13)$$

Continuing with PHeT: Make a ring of charge that has a radius of 100 cm, with the center at the origin.

1. Show that the voltage field is radially symmetric.
 - Place *equipotential lines* around the charge distribution and see that they are circular.
2. Show that at distances much larger than the radius, the voltage field looks like that of a point charge.
 - Record the voltage versus distance from the origin.
 - Plot the data.
 - Compare to the plot corresponding to a point charge.

Now that we know that only voltage *differences* matter:

$$\Delta V = - \int \vec{E} \cdot d\vec{l} \quad (14)$$

What else is analogous to potential energy?

Suppose the voltage due to some charge distribution is V .
What is the voltage difference if when we try to integrate like:

$$\Delta V = - \oint \vec{E} \cdot d\vec{l} \quad (15)$$

- A: $\Delta V = E \cdot L$
- B: $\Delta V = 0$
- C: $\Delta V = Edl$
- D: $\Delta V = L$

The previous question reminds us that electric fields are **conservative**.

The voltage of a point charge $+q$ is $V = \frac{kq}{r}$. What is the electric field?

- A: $\frac{kq}{r}$
- B: $\frac{kq}{r^2}$
- C: $\frac{kq^2}{r}$
- D: $\frac{kq^2}{r^2}$

What happened to the minus sign in the last problem?

- A: It disappeared...because.
- B: Only voltage differences matter.
- C: The zero-point of the voltage is at infinity.
- D: It's cancelled by the definition: negative derivative of voltage is E-field.

The electric field of a long line of charge with charge density λ is $\vec{E} = \frac{2k\lambda}{r} \hat{r}$. What is the voltage?

- A: $\frac{2k\lambda}{r^2}$
- B: $\frac{4k\lambda}{r^2}$
- C: $2k\lambda^2 \ln(r) + V_0$
- D: $-2k\lambda \ln(r) + V_0$

Line integral example: view on board.

$$\Delta V = - \int \vec{E} \cdot d\vec{l} \quad (16)$$

Once the path is specified, we can perform this integral:
https://en.wikipedia.org/wiki/Line_integral

UNITS OF ENERGY

The electron-volt: eV. This is the energy gained by an electron accelerated through a voltage of 1 V.

- How many Joules per electron volt?
- What is the mass of a proton in eV? ($E = mc^2$, where c is the speed of light.)

A proton is released into a 40 kV electric potential. What is the final kinetic energy of the proton?

- A: -20 kV
- B: 40 kV
- C: -40 keV
- D: 40 keV

CAPACITANCE

Capacitance is the ability of an object to store charge. Let the voltage difference across an object be V , storing $-Q$ on one side and $+Q$ on another. The *capacitance* C is given by

$$Q = CV \quad (17)$$

Let the object be two parallel plates, and the charges be $\pm Q$.

CAPACITANCE

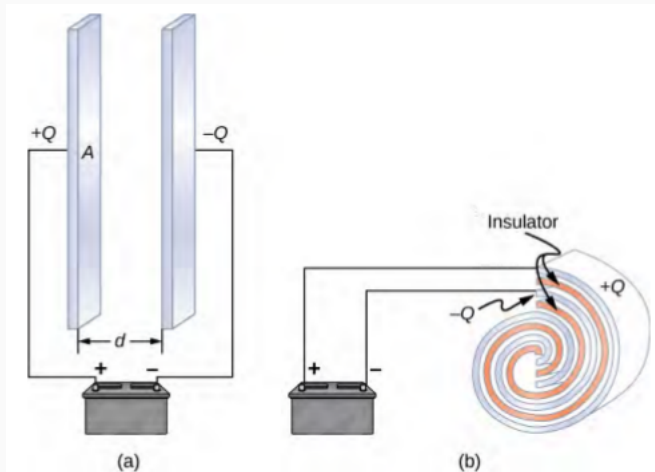


Figure 4: General scheme of a capacitor.

CAPACITANCE

The electric field and voltage between two charged plates separated by a distance d is $V = Ed$. The field is $E = \sigma/\epsilon_0$, and $Q = \sigma/A$, where A is the plate area and σ is the surface charged density. Note that

$$C = Q/V = (Q)/(Ed) \quad (18)$$

$$= (Q\epsilon_0)/(\sigma d) = (QA\epsilon_0)/(Qd) \quad (19)$$

$$= \frac{A\epsilon_0}{d} \quad (20)$$

So the capacitance depends on the permittivity of free space, the area, and the distance. In other words, just the geometry of the system. The units of capacitance are **Farads** (F). This is a large unit. Typical capacitors have nF or pF.

Group board exercise: The permittivity of free space is $8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$. A capacitor has an area of 1 mm^2 , and $d = 0.001 \text{ mm}$. What is the capacitance?

Group board exercise: The permittivity of free space is $8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$. A capacitor has an area of 10 mm^2 , and $d = 0.001 \text{ mm}$. What is the capacitance?

Group board exercise: The permittivity of free space is $8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$. A capacitor has an area of 1 mm^2 , and $d = 0.01 \text{ mm}$. What is the capacitance?

OHM'S LAW, KIRCHHOFF'S RULES AND SIMPLE CIRCUITS

Read chapters 9 and 10 for Monday. Voltage and capacitance can be combined with another concept, **resistance**, to build an understanding of **circuits**. Ohm's law is the relationship between *current* and *voltage*:

$$V = IR = \frac{dQ}{dt}R \quad (21)$$

Current is the *flow of charge*. The unit of resistance is called the Ohm, and the unit of current is called the amp, for Ampère.

A 9 V battery powers a smoke detector, and the current is 0.009 amps. What is the resistance of the circuit powering the smoke detector?

- A: 1000 Ohms
- B: 100 Ohms
- C: 10 Ohms
- D: 1 Ohm

A 3.3 V battery powers a christmas light, and the current is 0.33 amps. What is the resistance of the circuit powering the smoke detector?

- A: 1000 Ohms
- B: 100 Ohms
- C: 10 Ohms
- D: 1 Ohm

OHM'S LAW, KIRCHHOFF'S RULES AND SIMPLE CIRCUITS

Resistors, capacitors, voltages, and current can be combined conceptually to form **DC Circuits** (direct-current). The following rules govern the interaction of resistors with each other, and capacitors with each other:

- For resistors *in series*: $R = R_1 + R_2 + R_3 + \dots$
- For resistors *in parallel*: $R^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1} + \dots$
- For capacitors *in parallel*: $C = C_1 + C_2 + C_3 + \dots$
- For capacitors *in series*: $C^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} + \dots$

Observe on board: difference between in series, and in parallel? (Hint, different voltage for in series, same voltage for in parallel).

Kirchhoff's rules:

1. Summing the voltage in a loop must equal zero (\vec{E} -fields are conservative).
2. Current entering an leaving a node is conserved.

Observe on board.

Two resistors have $1\text{ k}\Omega$ each (1000 Ohms). What is the effective resistance if they are added *in parallel*?

- A: 1000 Ohms
- B: 500 Ohms
- C: 2000 Ohms
- D: 200 Ohms

Two capacitors have 1 nF each. What is the effective capacitance if they are added *in series*?

- A: 0.5 nF
- B: 5 nF
- C: 1 nF
- D: 2 nF

Two capacitors have 1 nF each. What is the effective capacitance if they are added *in parallel*?

- A: 0.5 nF
- B: 5 nF
- C: 1 nF
- D: 2 nF

GRAPHICAL ANALYSIS OF SIMPLE CIRCUITS

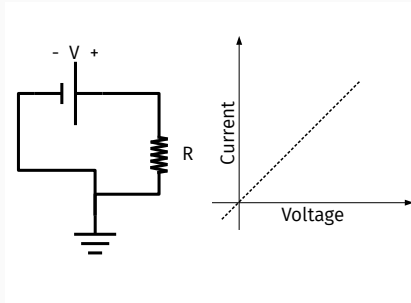


Figure 5: Circuits components are represented graphically by iV curves.

If the resistance R is increased, what will happen?

- A: The slope on the graph will increase
- B: The slope on the graph will decrease
- C: The slope will stay the same
- D: Cannot determine what will happen

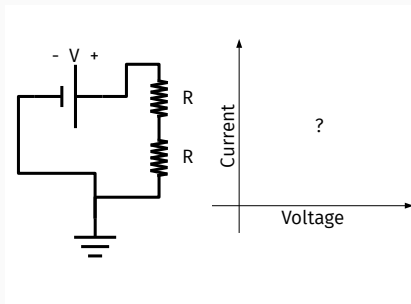


Figure 6: Circuits components are represented graphically by iV curves.

Should the slope now be greater than, less than, or equal to the that of Fig. 5?

- A: Greater than Fig. 5
- B: Less than Fig. 5
- C: Equal to Fig. 5
- D: Cannot determine.

GRAPHICAL ANALYSIS

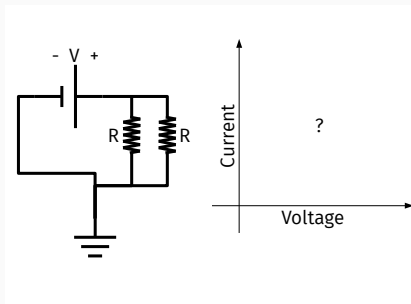


Figure 7: Circuits components are represented graphically by iV curves.

Should the slope now be greater than, less than, or equal to the that of Fig. 5?

- A: Greater than Fig. 5
- B: Less than Fig. 5
- C: Equal to Fig. 5
- D: Cannot determine.

GRAPHICAL ANALYSIS

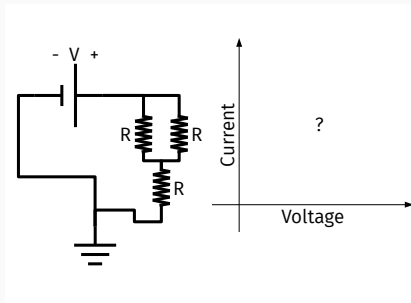


Figure 8: Circuits components are represented graphically by iV curves.

Should the slope now be greater than, less than, or equal to the that of Fig. 5?

- A: Greater than Fig. 5
- B: Less than Fig. 5
- C: Equal to Fig. 5
- D: Cannot determine.

Group board exercise: Solve for the current.

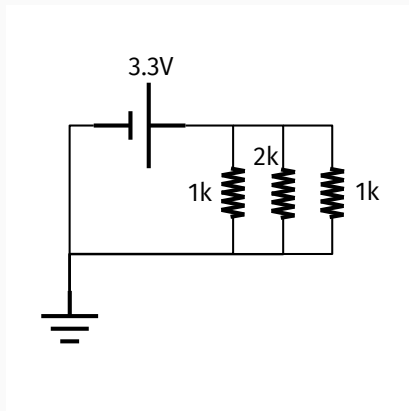


Figure 9: Three resistors in parallel.

Group board exercise: Solve for the current.

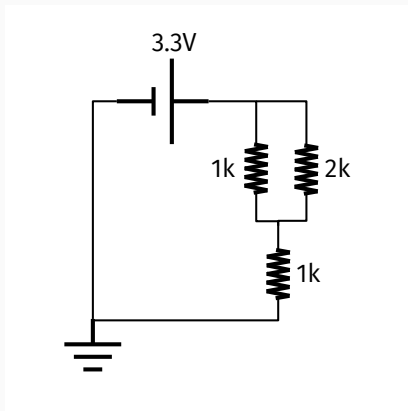


Figure 10: Two resistors in parallel, and in series with a third.

Group board exercise: Solve for the current.

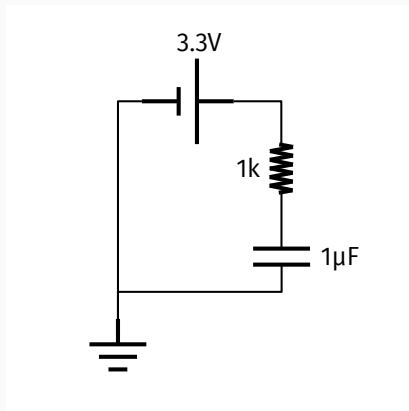


Figure 11: A resistor in series with a capacitor...

Using Kirchhoff's loop rule:

$$V - iR - QC^{-1} = 0 \quad (22)$$

$$Q_0 = CV \quad (23)$$

$$\tau = RC \quad (24)$$

$$\tau \dot{Q} + Q = Q_0 \quad (25)$$

Group board exercise: Show that the solution for the charge on the capacitor is

$$Q(t) = Q_0 (1 - \exp(-t/\tau)) \quad (26)$$

Take the derivative of both sides to find the current versus time. Graph the current.

So we see that the current is an exponential function:

$$i(t) = i_0 e^{-t/\tau} \quad (27)$$

($i_0 = V/R$, the initial current). What is the current in a circuit with a 1k resistor and a 1 nF capacitor after 3 μs , if the voltage is 5 V?

- A: 0.25 mA
- B: 0.5 mA
- C: 1.0 mA
- D: 2.0 mA

So we see that the current is an exponential function:

$$i(t) = i_0 e^{-t/\tau} \quad (28)$$

($i_0 = V/R$, the initial current). Thinking of the same circuit:
when is the current 0.0 mA?

- A: 3 microseconds
- B: 30 microseconds
- C: 300 microseconds
- D: never

CURRENT

1. Car batteries are rated in ampere-hours (A h). To what physical quantity do ampere-hours correspond (voltage, current, charge, energy, power,...)?
2. When working with high-power electric circuits, it is advised that whenever possible, you work “one-handed” or “keep one hand in your pocket.” Why is this a sensible suggestion?
3. Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width?
4. How many electrons flow through a point in a wire in 3.00 s if there is a constant current of $I = 4.00 \text{ A}$?

Car batteries are rated in ampere-hours (A h). To what physical quantity do ampere-hours correspond (voltage, current, charge, energy, power,...)?

“They are rated to charge.”

“1 ampere an hour is equal to 3600 coulomb.”

“Ampere hours equals voltage.”

When working with high-power electric circuits, it is advised that whenever possible, you work “one-handed” or “keep one hand in your pocket.” Why is this a sensible suggestion?

“Keep you grounded.”

“So you never complete the current your working with so you are not electrocuted.”

“to keep you grounded, your body has a lot more resistance, and you can’t complete the circuit when you only work with one hand.”

Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width?

“yes, the resistance along the length would be greater than that along the width.”

“resistance depends on the path you take through and object since the equation $R = \rho(L/A)$ depends on a length L to find Resistance. The resistance is not the same when you go through the length and width. The length would produce a higher resistance.” “Yes it still has the same resistance.”

How many electrons flow through a point in a wire in 3.00 s if there is a constant current of $I = 4.00 \text{ A}$?

- A: 6×10^{18}
- B: 6×10^{19}
- C: 6×10^{20}
- D: 6×10^{21}

Notions of current:

- $I = \frac{\Delta Q}{\Delta t}$ - The derivative of charge
- The *movement* of electrons
- The *flow* of charge
- Number of Coulombs per second (1 Amp = C/s)

There is an interesting problem with the notion of current as movement of charges.

Speed of typical electronic signals: $\approx 10^8$ m/s

Typical speed of actual charges passing through a conductor under voltage: $\approx 10^{-4}$ m/s

Since there is a 12 order of magnitude range, it's probably a good idea to ponder...

CURRENT

Are the electrons colliding/interacting to form electrical signals? Or just moving all together?

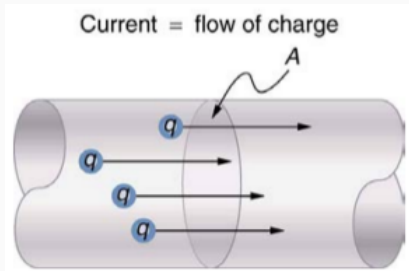


Figure 12: The *drift velocity* is the average velocity of an electron, and current is derived from this average velocity.

<https://youtu.be/8dgyPRA86K0>

So we see how electrical signals can move near the speed of light, but we measure the movements of electrons in circuits to be slow. Can we make a calculation to understand the speed of the electrons?

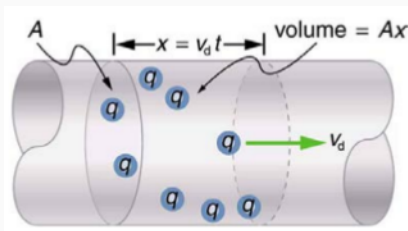


Figure 13: Consider the volume V of conductor with cross-sectional area A and length Δx , having n free electrons per unit volume.

An **amp** is one *Coulomb* per second. The definition of current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{qnA\Delta x}{\Delta t} = qnAv_d \quad (29)$$

Solving for drift velocity:

$$v_d = \frac{I}{qnA} \quad (30)$$

Suppose our conductor is a wire with radius r and $A = \pi r^2$.

Substituting,

$$v_d = \frac{I}{\pi qnr^2} \quad (31)$$

Remember that $q = 1.6 \times 10^{-19}$ C, and n is the number of free electrons *per atom per unit volume*. How do we get this number?

Number density: The total number of objects in a system is equal to the *number density* times the volume of the system.

$$N = nV \quad (32)$$

- N: Total number
- n: *number density*
- V: Volume

Example: *Number of Stars in the Milky Way*. How many stars are in our galaxy? Assume the galaxy is a disk of height h and radius r . We observe n stars per unit volume.

- $r = 50 \times 10^3$ *light-years*
- $h = 2 \times 10^3$ *light-years*
- $n = 10^{-2}$ *light-year*⁻³

1. Compute the volume in *light-years*³
2. Multiply the volume by the number density to obtain the total number.
3. Compare the result with others' results.

How many **conduction** electrons are there in a cube of copper that is 1 micron ($1\ \mu\text{m} = 10^{-6}\ \text{m}$) on a side?

- Copper has a density of 8.8 grams per cubic centimeter.
- Copper has an atomic weight of 63.54 grams per mole. (*Do you remember what a mole is?*).
- There are $N_A = 6.02 \times 10^{23}$ atoms per mole.
- Only one electron per atom of copper is a conduction electron.

-
1. Divide the density by the atomic weight. What are the units?
 2. Multiply by N_A (Avogadro's number). What are the units?
 3. Convert the units from cm^{-3} to μm^{-3} .

Number density: Let's examine copper, a common wire material with one free electron per atom. Copper has a density of 8800 kg/m^3 , and 0.06354 kg/mol . There are 6.02×10^{23} atoms/mol. How many free electrons per m^3 of copper? (Remember that there is only one conduction electron per copper atom).

- A: 10^{26} free electrons per m^3
- B: 10^{27} free electrons per m^3
- C: 10^{28} free electrons per m^3
- D: 10^{29} free electrons per m^3

Consider a copper wire with radius $r = 2.053$ mm that is carrying 20.0 A of current. Using $q = 1.6 \times 10^{19}$, and $n = 8.34 \times 10^{28}$ electrons/m³, and $v_d = I/(\pi q n r^2)$, compute the drift velocity of charge in the wire. *This is a common situation in household wiring.*

- A: 10^{-1} m/s
- B: 10^{-2} m/s
- C: 10^{-3} m/s
- D: 10^{-4} m/s

Drift speed vs. signal speed. Given that the electrons move at 1mm/s, how is it that electric signals move at 10^8 m/s?

Electrical signals are more like a **wave on a string**:
<https://phet.colorado.edu/en/simulation/legacy/wave-on-a-string>

CONCLUSION

Reading: Chapters 7, 9, and 10

1. Voltage and Capacitance
2. Ohm's Law
3. DC circuits

ANSWERS

ANSWERS

- $\vec{E} = \frac{kq}{r^2} \hat{r}$
- $\vec{E} = \frac{2k\lambda}{r} \hat{r}$
- $\vec{E} = -a\hat{i}$
- $\vec{E} = -a\hat{i} - b\hat{j}$
- $\Delta V = 0$
- $\frac{kq}{r^2}$
- D
- $-2k\lambda \ln(r) + V_0$
- 40 keV
- 1000 Ohms
- 10 Ohms
- 500 Ohms
- 0.5 nF
- 2 nF
- The slope on the graph will decrease
- Less than Fig. 5
- Greater than Fig. 5
- Less than Fig. 5
- 0.25 mA
- never
- 6×10^{19}