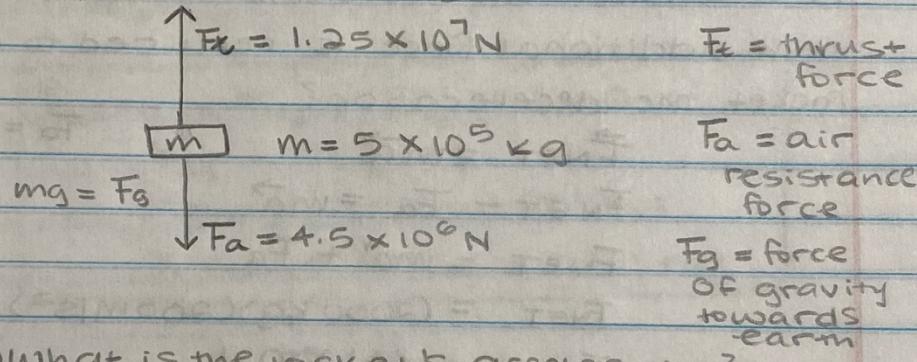


## MIDTERM #2

## 2 Chapter 4: Dynamics, Force, and Newton's Laws of Motion

① A  $5 \times 10^5$  kg rocket is accelerating straight up. The thrusters produce an upward force of  $1.25 \times 10^7$  N, and the force of air resistance is  $4.5 \times 10^6$  N downward.

(a) Draw a free body diagram including the weight of the rocket, the thrust, and air resistance.



(b) What is the rocket's acceleration?

$$\vec{F}_{NET} = m\vec{a}$$

$$\vec{F}_{NET} = F_t - F_a - F_g = ma$$

$$\vec{F}_{NET} = \frac{F_t - F_a - mg}{m} = \frac{ma}{m}$$

$$a = \frac{F_t - F_a - mg}{m}$$

$$= (1.25 \times 10^7 \text{ N}) - (4.5 \times 10^6 \text{ N}) - ((5 \times 10^5 \text{ kg})(9.8 \text{ m/s}^2))$$

$$5 \times 10^5 \text{ kg}$$

$$\approx [6.2 \text{ m/s}^2]$$

Great work =)

② A football player with mass 70kg pushes a player with mass 90kg.

(a) According to Newton's third law, if the first player exerts a force

OF 700 N on the second player, what is the force the second player exerts on the first player?

$$\vec{F}_{BA} = -\vec{F}_{AB}$$

$$= \boxed{-700\text{N}}$$

$$m_1 = 70\text{kg}$$

$$m_2 = 90\text{kg}$$

$$F_1 = 700\text{N}$$

③ A rocket sled is decelerated at a rate of  $200\text{ m/s}^2$ , and it has a mass of  $2000\text{ kg}$ . There is a constant air resistance force of  $1000\text{N}$ .

What additional force is required to give the rocket the deceleration?

$$\vec{F}_{NET} = m\vec{a}$$

$$F_a = \text{air resistance}$$

$$\vec{F}_{NET} - F_a = m\vec{a}$$

$$\vec{F}_{NET} = m\vec{a} + F_a$$

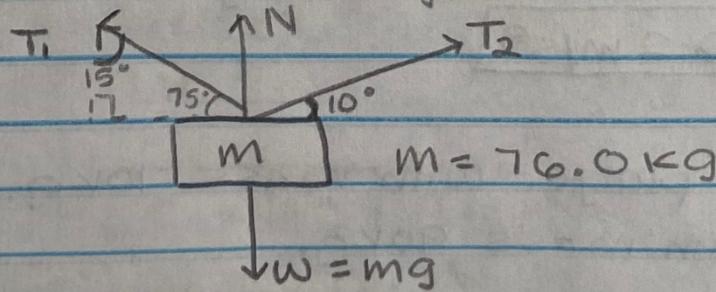
$$F_{NET} = (2000\text{ kg})(200\text{ m/s}^2) + (1000\text{N})$$

$$F_{NET} = 401000\text{N}$$

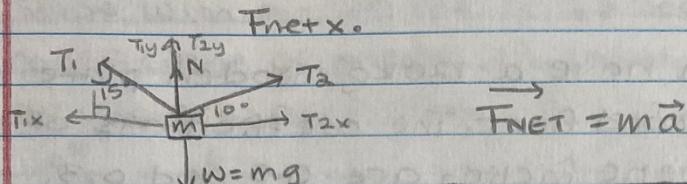
$$= \boxed{401000\text{N} \text{ or } 40.1 \times 10^4\text{N}}$$

④ A  $76.0\text{-kg}$  person is being pulled away from a burning building as shown in Fig.1.

(a) Draw a freebody diagram including the two tension vectors and the woman's weight.



(b) Write down an expression for  $F_{\text{net}x}$ .



$$\vec{F}_{\text{NET}} = m\vec{a}$$

Forces of x

$$= T_1 \cos 75^\circ$$

$$= T_2 \cos 10^\circ$$

$$F_{\text{NET}x} = T_{1x} - T_{2x} = 0$$

$$T_{1x} = T_{2x}$$

$$T_1 \cos 75^\circ = T_2 \cos 10^\circ$$

(c) Write down an expression for  $F_{\text{net}y}$ .

$$F_{\text{NET}y} = T_{1y} + T_{2y} - w = 0$$

$$T_{1y} + T_{2y} = w$$

$$T_1 \sin 15^\circ + T_2 \sin 80^\circ = w$$

$$T_1 \sin 15^\circ + T_2 \sin 80^\circ = mg$$

(d) Assuming  $\vec{F}_{\text{NET}} = 0$ , calculate the tension in the two ropes.

$$T_1 \cos 75^\circ = T_2 \cos 10^\circ$$

$$T_2 = 0.2628 T_1$$

$$T_1 \sin 15^\circ + T_2 \sin 80^\circ = mg$$

$$T_1 (0.25882) + (0.2628 T_1)(0.9848) =$$

$$(76.0 \text{ kg})(9.8 \frac{\text{m}}{\text{s}^2})$$

$$(0.25882)T_1 + (0.25881)T_1 = 744.8$$

$$0.517629 T_1 = 744.8$$

$$T_2 = 0.2628(1438.9)$$

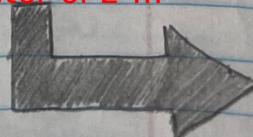
$$T_1 = 1438.9 \text{ N}$$

$$T_2 = 378.1 \text{ N}$$

(-1) The setup looks correct  
but I think the results are  
off by a factor of 2 ...

$$T_1 = 1438.9 \text{ N}$$

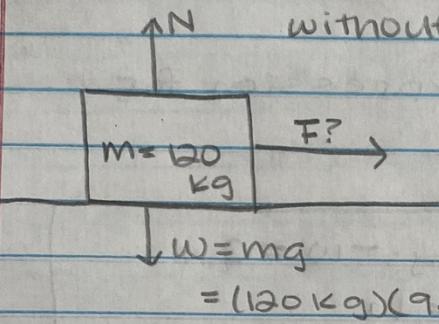
$$T_2 = 378.1 \text{ N}$$



### 3 Chapter 5: Friction, drag, and Elasticity

① suppose you have a 120kg wooden plate resting on a wood floor. The coefficients of static and kinetic friction are 0.5 and 0.3, respectively.

(a) what maximum force can you exert horizontally on the crate without moving it?



$$W = mg \\ = (120 \text{ kg})(9.8 \text{ m/s}^2)$$

$$= 1176 \text{ N}$$

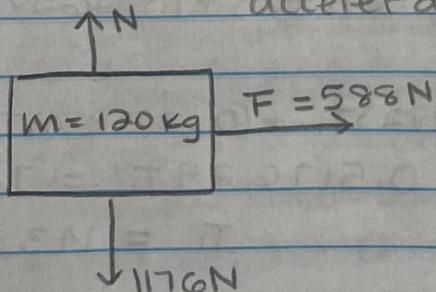
Static friction is greater than kinetic friction.

$$\mu_s = 0.5 \rightarrow \text{static}$$

$$\mu_k = 0.3 \rightarrow \text{kinetic}$$

$$f = \mu N \\ = (0.5)(1176 \text{ N}) \\ = 588 \text{ N}$$

(b) IF you continue to exert this force once the crate starts to slip, what will the magnitude of the acceleration then be?



\*Kinetic friction

$$\mu_k = 0.3$$

$$f = \mu N \\ = (0.3)(1176 \text{ N}) \\ = 352.8 \text{ N}$$

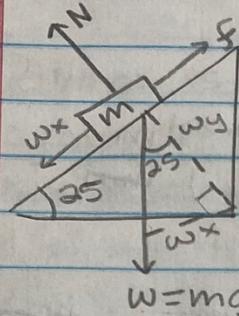
$$\sum F_x = ma$$

$$\frac{588 \text{ N} - 352.8 \text{ N}}{120 \text{ kg}} = \frac{(120 \text{ kg})a}{120}$$

$$a = 1.96 \text{ m/s}^2$$

Excellent

② Suppose a skier (Fig. 3) is sliding down a slope with an incline of 25 degrees. If the coefficient of kinetic friction is 0.1, what is the skier's acceleration?



$$\mu_k = 0.1$$

$$\sin 25^\circ = \frac{w_x}{w}$$

$$w = mg$$

$$w_x = w \sin 25^\circ$$

$$f = \mu N$$

$$= (0.1)(w_y)$$

$$= (0.1)(w \cos 25^\circ)$$

$$\cos 25^\circ = -\frac{w_y}{w}$$

$$w_y = -w \cos 25^\circ$$

$$\sum F_x = m \vec{a}$$

$$w_x - f = m \vec{a}$$

$$w \sin 25^\circ - (0.1(w \cos 25^\circ)) = m \vec{a}$$

$$w [\sin 25^\circ - 0.1 \cos 25^\circ] = m \vec{a}$$

$$\cancel{mg} [\sin 25^\circ - 0.1 \cos 25^\circ] = \frac{m \vec{a}}{m}$$

$$9.8 [\sin 25^\circ - 0.1 \cos 25^\circ] = \vec{a}$$

$$a = 3.25 \text{ m/s}^2$$

③ DRAG FORCE Suppose the skier reaches a top speed of 40 m/s. If his area is  $0.75 \text{ m}^2$ , the density of air is  $1.225 \text{ kg/m}^3$ , and  $C = 0.75$ , what is the magnitude of the drag force in Newtons?

$$F_D = \frac{1}{2} C_D A V^2$$

$$= \frac{1}{2} (0.75) (1.225 \text{ kg/m}^3) (0.75 \text{ m}^2) (40 \text{ m/s})^2$$

$$F_D = 551.25 \text{ N}$$

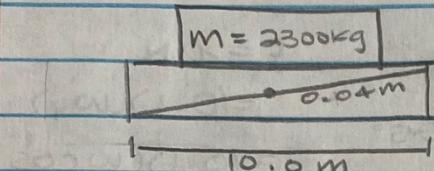
$$\begin{aligned} & \frac{\text{kg}}{\text{m}^3} \cdot \frac{\text{m}^2}{1} \cdot \frac{\text{m}^2}{\text{s}^2} \\ & = \text{kg m/s}^2 \end{aligned}$$

\* ④ A mass of 2300 kg is placed on top of a 10.0m long wooden beam with radius 4cm.

If the length of the beam decreases by 3mm, what is the Young's modulus of wood?

Pay attention to the units.

$$\frac{F}{A} = Y \left( \frac{\Delta x}{L} \right)$$



$$3\text{ mm} \cdot \frac{0.001\text{ m}}{1\text{ mm}} = 0.003\text{ m}$$

$$10.000\text{ m} - 0.003\text{ m} = 9.997\text{ m}$$

$$4\text{ cm} \cdot \frac{0.01\text{ m}}{1\text{ cm}} = 0.04\text{ m}$$

Young's modulus

of wood (hardwood)

$$= 1.5 \times 10^{10} \text{ Pa}$$

$$A = r^2 \pi$$

$$= (0.04\text{ m})^2 \pi$$

$$\frac{(2300\text{ kg})}{(0.0016\text{ m}^2 \pi)} = \frac{Y}{10.0\text{ m}} \quad = 0.0016\text{ m}^2 \pi$$

$$0.0016\text{ m}^2 \cdot \left( \frac{1\text{ mm}}{0.001\text{ m}} \right)^2$$

(-1) Although the correct result is listed above, the calculation is in N/mm<sup>2</sup>, so if I convert to m<sup>2</sup>, I don't get the listed result

#### 4 Chapter 6: Uniform Circular Motion and Gravitation

① A pitcher in baseball pitches a ball at 144 km/hr, and the ball rotates around his arm at a radius of 0.5 meters. What is the angular velocity of the ball as he throws it, in radians per second?

$$V = rw$$

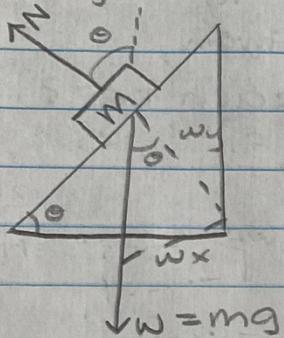
$$\frac{144\text{ km}}{\text{hr}} \cdot \frac{1\text{ hr}}{60\text{ min}} \cdot \frac{1\text{ min}}{60\text{ s}} \cdot \frac{1000\text{ m}}{1\text{ km}} = 144000\text{ m/s}$$

$$\frac{144000 \text{ m/s}}{0.5 \text{ m}} = \frac{(0.5 \text{ m})\omega}{0.5 \text{ m}}$$

(-1) The correct result was 80 rad/s, from  $v = r * \omega$

$$\omega = 288000 \text{ rad/s} \text{ or } 28.8 \times 10^4 \text{ rad/s}$$

- ② What is the ideal banking angle for a gentle turn of 0.9 km radius on a highway with a 120 km per hour speed limit, assuming everyone travels at the limit?



$$\frac{mv^2}{r} = N \sin \theta \quad \leftarrow \rightarrow \frac{mv^2}{r(mg)} = \tan \theta$$

$$mg = N \cos \theta \quad \frac{v^2}{rg} = \tan \theta$$

$$\theta = \tan^{-1} \left( \frac{v^2}{rg} \right)$$

$$= \tan^{-1} \left( \frac{(120 \text{ km/h})^2}{(900 \text{ m})(9.8 \text{ m/s}^2)} \right)$$

$$\frac{120 \text{ km}}{1 \text{ hr}} \cdot \frac{1 \text{ hr}}{60 \text{ min}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 12000 \text{ m/s}$$

$$0.9 \text{ km} \cdot \frac{1000 \text{ m}}{1 \text{ km}} = 900 \text{ m}$$

$$\theta \approx 90^\circ$$

(-1) This is not a reasonable result, because the driver would be sideways, right?

- ③ Two race car drivers routinely navigate a turn as shown in Fig. 3.

(a) which path may be taken at a higher speed, if both paths correspond to the same force of friction and centripetal force?

If both the force of friction and centripetal force were the same, Path 2 may be taken

at a higher speed. The turn would not be as extreme, as the turn angle is minimized by cutting the corner. By increasing the radius or curvature of the car, the amount of force the tyres exert on

the road is reduced, increasing velocity in the equation for centripetal force:

$$F_c = \frac{mv^2}{r}$$

(b) Suppose path 1 has a radius of curvature of 400m, and path 2 has a radius of curvature 800m. The coefficient of friction is 1.0. If the force of friction balances the centripetal force, what are the tangential velocities of each car?

$$\text{centripetal force} = \frac{mv^2}{r}$$

$$r \cdot \mu mg = \frac{mv^2}{r}$$

$$\frac{r \mu mg}{m} = \frac{mv^2}{r}$$

$$\sqrt{\mu g} = \sqrt{\frac{v^2}{r}}$$

$$v = \sqrt{\mu gr}$$

$$\text{path 1: } v = \sqrt{(400)(1.0)(9.8)} \\ = 62.6 \text{ m/s}$$

$$\text{path 2: } v = \sqrt{(800)(1.0)(9.8)} \\ = 88.5 \text{ m/s}$$

Well done ... (I put my answers in km / hour in the key)

Velocity of car on path 1 is 62.6 m/s.

Velocity of car on path 2 is 88.5 m/s.

④ TWO bonus points: The existence of the dwarf planet Pluto was proposed based on irregularities in Neptune's orbit. Pluto was subsequently discovered near its predicted position. But now it appears that the discovery was fortuitous, because Pluto is small and the irregularities in Neptune's Orbit were not well known. To illustrate that Pluto has a minor effect on the orbit of Neptune compared to the closest planet to Neptune,

(a) calculate the acceleration due to gravity at Neptune due to Pluto when they are  $4.5 \times 10^{12} \text{ m}$  apart, as they are now. The mass of Pluto is  $1.4 \times 10^{22} \text{ kg}$ .

Gravity above the earth's surface

$$a_c = \frac{Gm}{r^2} \quad F_G = G \frac{m_1 m_2}{r^2} \hat{r} \quad g = G \frac{M_E}{r^2}$$

$$a_c = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.4 \times 10^{22} \text{ kg})}{(4.5 \times 10^{12} \text{ m})^2}$$

Gravitational Constant =  $6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$

$$a_c = 4.01 \times 10^{-14} \text{ m/s}^2$$

(b) Now calculate the acceleration due to gravity at Neptune due to Uranus, presently about  $2.5 \times 10^{12} \text{ m}$  apart, and compare it with that due to Pluto. The mass of Uranus is  $8.63 \times 10^{25} \text{ kg}$ .

$$a_c = \frac{Gm}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(8.63 \times 10^{25} \text{ kg})}{(2.5 \times 10^{12} \text{ m})^2}$$

$$a_c = 9.2 \times 10^{-10} \text{ m/s}^2$$

(+2) Bonus!