CALCULUS-BASED PHYSICS-1: MECHANICS (PHYS150-01): WEEK 7

Jordan Hanson

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Whittier College Department of Physics and Astronomy

WEEK 6 REVIEW

1. Work has a scientifically precise definition
    - Units
    - As a product of force and displacement vectors
2. Kinetic Energy and the Work-Energy Theorem
3. Gravitational potential energy
    - Potential energy
    - *Simplifying otherwise complex calculations*
    - Potential energy near Earth's surface
    - …in space
4. Definition of a **conservative force**
    - Relationship between conservative forces and potential energy
    - Conservation of energy for conservative forces

WEEK 6 REVIEW PROBLEMS

Recall that the *gravitational potential energy* associated with an object of mass m located a height y above ground level is $U = mgy$. What is $-dU/dy$?

- A: mg
- B: $-mg$
- C: g
- D: $-g$

What is the physical meaning of the quantity $-mg = -dU/dy$?

- A: The force of gravity
- B: The acceleration due to gravity
- C: The mass
- D: Change in kinetic energy

WEEK 7 SUMMARY

1. Work and potential energy
    - **Lab activity:** Oscillator and gravity trading work and potential energy
2. Potential energy and **conservative forces**
3. Conservation of Energy
    - *Calculus review: the fundamental theorem of calculus*
    - Graphical representations of integrals and energy

WORK AND POTENTIAL ENERGY

When we do work on a system, even if in the final state the system has no velocity, it can still have energy. The concept of *potential energy* is **like thinking of work backwards**:

- If we compress an oscillator (a spring), and keep compressing it, it has no kinetic energy but it will have kinetic energy if we release it.
- If we raise an object against the force of gravity to a certain height, it has no kinetic energy but it will have kinetic energy if we drop it.
- But it took work to create this change in kinetic energy, from the work-energy theorem.

*Systems are not necessarily **required** to operate like this...Think of stretching plastic or dough. Where does the work go?*

The relationship between work and potential energy therefore resembles (by definition) the opposite of the work-energy theorem:

$$\Delta U_{\mathrm{AB}} = U_{\mathrm{B}} - U_{\mathrm{A}} = -W_{\mathrm{AB}} \tag{1}$$

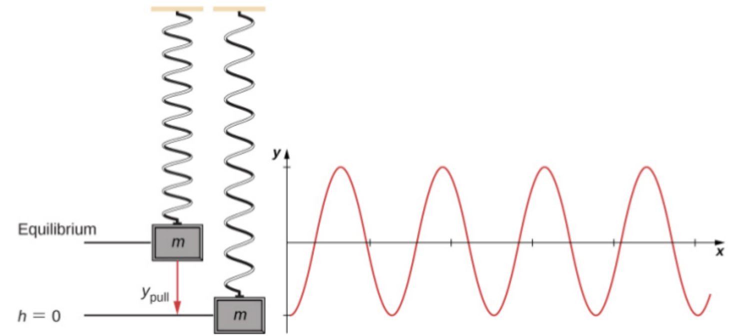

Figure 1: An oscillator is stretched to a new equilibrium point by gravity. When pulled a certain distance down, or compressed a certain distance upwards, y_{pull}, we observe oscillation.

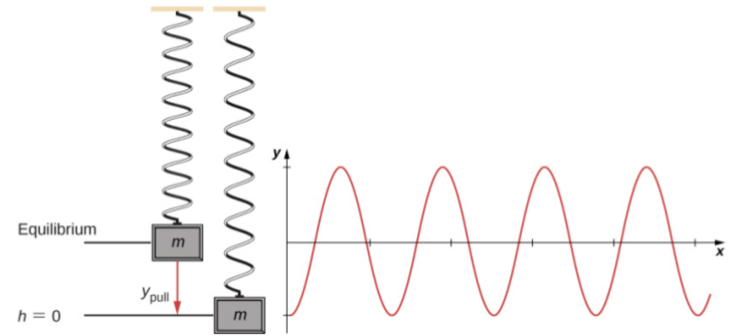

Figure 2: Notice that it does not matter what the observer defines as *zero* potential energy, since work is required to perform *changes* in potential energy.

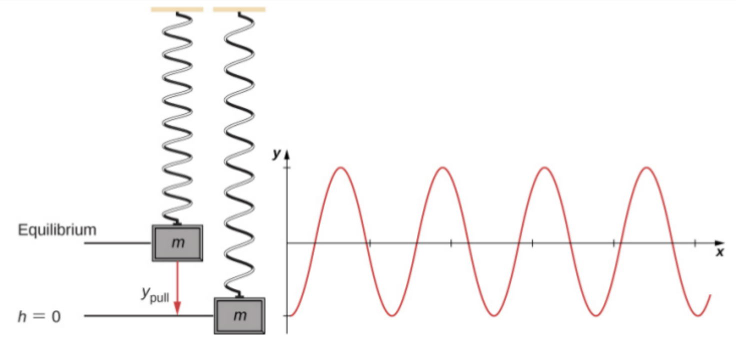

Figure 3: The fact that work is required only to perform changes in potential energy, but not does not determine the *absolute scale* of potential energy, means the observer may choose the location of zero potential energy, in the same fashion as choosing a coordinate system.

	Gravitational P.E.	Elastic P.E.	Kinetic E.
(3) Highest Point	$2mgy_{\text{pull}}$	$\frac{1}{2}ky^2{}_{\text{pull}}$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky^2{}_{\text{pull}}$	0

Figure 4: If a mass *m* is connected to the oscillator and *we choose* the potential energy zero-point to be **the low point of oscillation**, the values listed in this table correspond to the energies at various states.

LAB ACTIVITY - GRAVITY AND THE OSCILLATOR

- Using the springs, weights, hooks, and system of clamps and grips, build a vertical oscillating system.
- Diagram the system, showing a clearly defined value for y_{pull}, and a clearly defined choice for the potential energy zero-point.
- Measure the *unstretched* spring length, and the *equilibrium length* caused by gravity, to **derive the spring constant** k. Quote the value of k in N/m.
- Pull the spring downwards by y_{pull}, and record the maximum and minimum heights of the weight as the spring oscillates it.
- Create a table like Tab. 4, and fill in the actual energy values in Joules. What is your predicted value for v, the speed at which the weight moves when the oscillator is at the equilibrium position?

	Gravitational P.E.	Elastic P.E.	Kinetic E.
(3) Highest Point	$2mgy_{\text{pull}}$	$\frac{1}{2}ky^2{}_{\text{pull}}$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky^2{}_{\text{pull}}$	0

Figure 5: If a mass m is connected to the oscillator and *we choose* the potential energy zero-point to be **the low point of oscillation**, the values listed in this table correspond to the energies at various states.

Lab Activity - Gravity and the Oscillator, Part 2

- Measure the *unstretched* spring length, and the *equilibrium length* caused by gravity, to **derive the spring constant** k. Quote the value of k in N/m.
- Pull the spring downwards by y_{pull}, and record the maximum and minimum heights of the weight.
- Create a table like Tab. 4, and fill in the actual energy values in Joules. What is your predicted value for v?
- Using the **Vernier LabPro** and the *motion detector attachment*, measure v when the spring is at equilibrium position, and quote the value in m/s. Does it agree with your prediction based on energy conservation? Why or why not?

	Gravitational P.E.	Elastic P.E.	Kinetic E.
(3) Highest Point	$2mgy_{\text{pull}}$	$\frac{1}{2}ky^2{}_{\text{pull}}$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky^2{}_{\text{pull}}$	0

Figure 6: The value for v is predicted by energy conservation. What do you measure?

POTENTIAL ENERGY AND CONSERVATIVE FORCES

Let path 1 be be through a force field that does work W_1 on a system, and path 2 be a different path that does work W_2 on a system.

$$W_1 = \int_{\mathrm{Path1}} \vec{F} \cdot d\vec{r} = \int_{\mathrm{Path2}} \vec{F} \cdot d\vec{r} = W_2 \tag{2}$$

A force is conservative if

$$W_1 = W_2 \tag{3}$$

Suppose path 1 goes from point A to B, and path 2 returns from B to A. If the force remains constant, but the path is reversed, then $W_1 = -W_2$. But this means the path is *closed*, so

$$\oint \vec{F} \cdot d\vec{r} = W_1 + W_2 = W_1 - W_1 = 0 \tag{4}$$

How do you show that a force is conservative? We cannot test **all possible paths**. This is a question for *vector calculus*.

CONCLUSION

ANSWERS

- $-mg$
- The force of gravity
- ...