Final for Calculus-Based Physics: Electricity and Magnetism

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1 Equations and constants

1. Volume of a sphere: $V_s = \frac{4}{3}\pi r^3$.
2. Density, mass and volume: $m = \rho V$.
3. Charge density, charge and volume: $Q = \rho V$.
4. Coulomb force: $\vec{F}_C = k\frac{q_1 q_2}{r^2}\hat{r}$.
5. Definition of electric field: $\vec{F}_C = q\vec{E}$.
6. Definition of electric flux: $\phi_E = \vec{E} \cdot \vec{A}$.
7. Gauss' Law: $\phi_E = q_{enc}/\epsilon_0$. (Assumes field is uniform over the surface).
8. Voltage and electric field, one dimension, uniform field: $|E| = -\frac{\Delta V}{\Delta x}$.
9. Voltage and electric field, general case: $\vec{E} = -\nabla V$.
10. Ohm's Law: $V = IR$.
11. Electrical power: $P = IV = I^2 R = V^2/R$.
12. Magnetic dipole moment: $\vec{\mu} = I\vec{A}$, where $\vec{A}$ is the area vector.
13. Torque on a magnetic dipole: $\tau = \vec{\mu} \times \vec{B}$.
14. Definition of magnetic flux: $\phi_m = \vec{B} \cdot \vec{A}$. The units are T m^2, which is called a Weber, or Wb.
15. Faraday's Law: $emf = -N\frac{d\phi}{dt}$
16. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
17. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L.
18. Inductance of a solenoid: $L = \mu_0 n^2 V$
19. Magnetic permeability: $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}
20. Units of inductance: V s A^{-1}, which is called a Henry, or H.
21. Coulomb constant: $k = 8.9876 \times 10^9$ N m^2 C^{-2}.
22. Fundamental charge: $q_e = 1.602 \times 10^{-19}$ C.

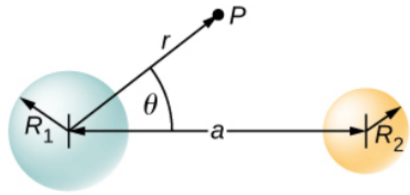

Figure 1: Two charged spheres with radii R_1 and R_2, separated by a distance a, observed at a point P at a distance r from the center of sphere 1, at an angle θ with respect to the line between the spheres. Assume that P is also a distance r from sphere 2.

2 Exercises

1. **Chapters 5-6: Electrostatics and Gauss' Law**

   (a) A spherical water droplet of radius 25×10^{-6} m carries an excess 250 electrons. If the density of water is 997 kg m^{-3}, what vertical electric field is needed to balance the gravitational force on the droplet at the surface of the earth? (a) First, calculate the mass of the water droplet. (b) Second, work out the total charge of the water droplet. (c) What is the Coulomb force required to balance gravity?

   (b) Two non-conducting spheres are uniformly charged with charge densities ρ_1 and ρ_2, respectively. The geometry of the system is depicted in Fig. 1. (a) Write an expression for the total charge of each sphere separately. (b) Using Gauss' law, write the expression for the electric field of each sphere separately, observed at P for each. (c) Break each electric field into x and y components. (d) Sum the total electric field due to spheres 1 and 2, observed at point P.

2. **Chapters 7-8: Voltage and Capacitance**

   (a) The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct.) You may assume a uniform electric field.

   (b) An electric potential is defined by $V(x, y, z) = ax^2 + by^2 + cz^2$, with $a = 2.0$ V m^{-2}, $b = 1.0$ V m^{-2}, and $c = 4.0$ V m^{-2}. What is the corresponding electric field?

I(A)	V(V)
0	3
2	23
4	39
6	58
8	77
10	100
12	119
14	142
16	162

Figure 2: A table of current (left column) and voltage (right column) data for a sample of material.

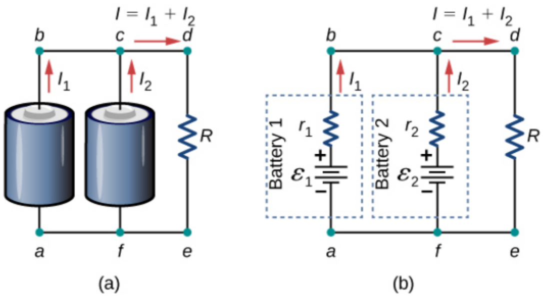

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Figure 3: Two batteries connected in parallel with internal resistances (left) can be modelled by a circuit (right).

3. **Chapters 9-10: Current, Resistance, and DC Circuits**

   (a) Figure 2 contains the measurements of a current through and the voltage across a sample of material. Plot the data, and assuming the object is an ohmic device, estimate the resistance.

   (b) An alternative to CFL bulbs and incandescent bulbs are light-emitting diode (LED) bulbs. A 100-W incandescent bulb can be replaced by a 16-W LED bulb. Both produce 1600 lumens of light. Assuming the cost of electricity is \$0.10 per kilowatt-hour, how much does it cost to run the bulb for one year if it runs for four hours a day?

   (c) Two AA batteries are connected *in parallel* with a load resistor R, as shown in Fig. 3. The two internal resistances are r_1 and r_2, and the two emf's are $\mathcal{E}_1$ and $\mathcal{E}_2$. (a) Using the junction rule once, and the loop rule twice, solve for the current through the load resistor algebraically. (b) If $\mathcal{E}_1 = \mathcal{E}_2 = 1.5$ V, $r_1 = r_2 = 0.1\Omega$, and $R = 100\Omega$, what is the current through R?

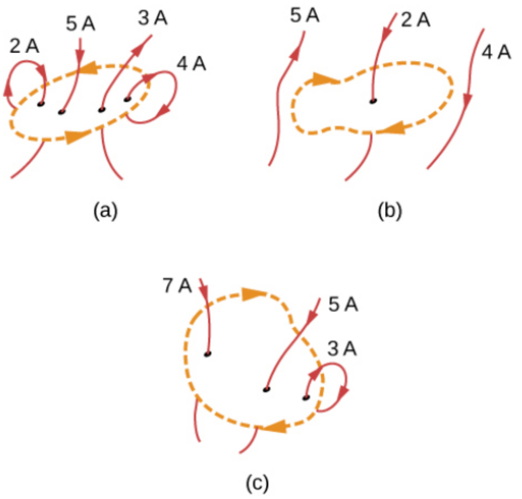

Figure 4: Two batteries connected in parallel with internal resistances (left) can be modelled by a circuit (right).

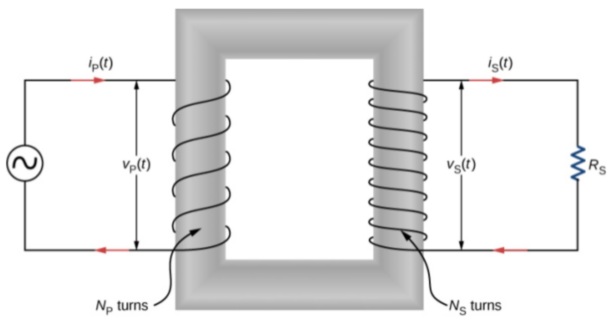

Figure 5: (Left) A magnetic field passes through loops of wire. (Right) The loops are stretched, reducing the area.

4. **Chapters 11-12: Magnetic fields and Sources of Magnetic Fields**

   (a) A circular loop of wire of area 10^{-2} m^2 carries a current of 20.0 A. At a particular instant, the loop lies in the xy-plane and is subjected to a magnetic field $\vec{B} = \left(3.0\hat{i} + 6.0\hat{j} + 3.0\hat{k}\right) \times 10^{-2}$ T. As viewed from above the xy-plane, the current is circulating clockwise. (a) What is the magnetic dipole moment of the current loop? (b) At this instant, what is the magnetic torque on the loop?

   (b) Use Ampère's Law to evaluate $\oint \vec{B} \cdot d\vec{l}$ for the current configurations and paths (a)-(c) in Fig. 4.

5. **Chapters 13-14: Electromagnetic Induction and Inductance**

   (a) In Fig. 5 (left) a *transformer* is depicted. The gray square represents an iron core which ensures that the magnetic flux through the left solenoid **is identical to** the magnetic flux on the right solenoid. Both solenoids are $L = 5$ cm long. Suppose the left solenoid has $N_L = 500$ turns, and the right solenoid has $N_R = 1000$ turns. Let the induced emf in the left solenoid be v_L, and the induced emf in the right solenoid be v_R. Show that

$$\frac{v_L}{N_L} = \frac{v_R}{N_R} \tag{1}$$

(b) The two solenoids each have volume $V = 5 \times 10^{-6}$ m^3. What is the inductance of each, in Henries?

(c) Suppose the current changes in the left solenoid: $\frac{dI}{dt} = 100$ A s^{-1}. (a) Using the inductance of the left solenoid, what is the induced emf in the left solenoid? (b) What is the induced emf in the right solenoid?