Ampère's Law: $\vec{B} \cdot d\vec{l}$ measurement

April 10, 2019

1 Introduction

Let $\vec{B}$ represent magnetic field in a region of space containing a current I enclosed by a *closed path* comprised of line segments $d\vec{l}$, and let $\mu_0 = 4\pi \times 10^{-7}$ T m/A. Ampère's Law states that

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \tag{1}$$

The type of integral on the left hand side is called a *closed line-integral.* If the *closed path* is broken into segments, and θ is the angle between the line segment and $\vec{B}$ at each line segment position, the integral is a summation

$$\sum_i B dl_i \cos(\theta_i) = \mu_0 I \tag{2}$$

Further, if the line segments are all the same length, the dl does not depend on i and may be taken out of the summation:

$$dl \sum_i B \cos(\theta_i) = \mu_0 I \tag{3}$$

Finally, $B\cos(\theta_i)$ is the component of the B-field that is parallel to the direction of the local line segment. Labelling it $B_{||}$ gives

$$dl \sum_i B_{||} = \mu_0 I \tag{4}$$

If measurements of $B_{||}$ are taken around a closed loop and summed, the result should be proportional to the enclosed current.

2 Wire coils

We have measured the number of wires in the coils to be $N = 90 \pm 5$. Let us take this to be a given for this lab exercise. Connect a power supply to the wire coil terminals with the standard red and black wires with banana connectors. Using the current limiting knob, place a current of 0.75 A through the coils. The Pasco magnetic field probe should detect a current significantly higher than the background field due to the Earth.

Turn off the current. Insert the yellow page marked with 1 cm line segments onto the coil platform. Each line segment is 1 cm and the gap is 2 cm. At each 1 cm line segment, measure $B_{||}$ and record the result in a list of numbers. Pay attention to units. Record the sum of the $B_{||}$. This forms a *systematic* *bias* in the data, representing the calibration offset of the magnetic probe.

3 The Measurement

With the 0.75 A current activated, repeat the procedure of the prior section by recording the $B_{||}$ measurements. Record a **statistical error** for each measurement. Finally, record the sum of the measurements, **including the associated error.** Subtract the **systematic bias** from the total.

4 The Interpretation

Solving Eq. 4 for the current (accounting for the number of turns in the coil) gives

$$I_{pred} = \frac{dl}{\mu_0} \sum_i B_{||} \tag{5}$$

Assume no error for dl, only account for the error in the B-field. Make sure to first convert the sum and the error in the sum to Tesla before using Eq. 5. The result is the *predicted current* I_{pred}. How do the predicted and measured current compare? Compute the percent error between the predicted current and measured current below. Do we observe a systematic offset? **Bonus: with the current on, try measuring around a loop that does not enclose the current. What happens?**