Calculus-Based Physics-1: Mechanics (PHYS150-02): Unit 7

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Unit 7 Summary

1. Work and potential energy
   - **Lab activity:** Oscillator and gravity trading work and potential energy
2. Potential energy and **conservative forces**
3. **Conservation of Energy**
   - *Calculus review: the fundamental theorem of calculus*
   - Graphical representations of integrals and energy

Work and Potential Energy

Work and potential energy

When we do work on a system, even if in the final state the system has no velocity, it can still have energy. The concept of *potential energy* is **like thinking of work backwards**:

- If we compress an oscillator (a spring), and keep compressing it, it has no kinetic energy but it will have kinetic energy if we release it.
- If we raise an object against the force of gravity to a certain height, it has no kinetic energy but it will have kinetic energy if we drop it.
- But it took work to create this change in kinetic energy, from the work-energy theorem.

*Systems are not necessarily **required** to operate like this...Think of stretching plastic or dough. Where does the work go?*

Work and potential energy

The relationship between work and potential energy therefore resembles (by definition) the opposite of the work-energy theorem:

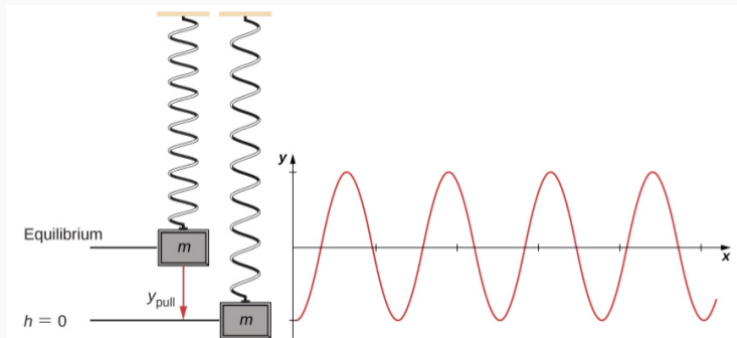$$\Delta U_{\mathrm{AB}} = U_{\mathrm{B}} - U_{\mathrm{A}} = -W_{\mathrm{AB}} \tag{1}$$

Figure 1: An oscillator is stretched to a new equilibrium point by gravity. When pulled a certain distance down, or compressed a certain distance upwards, y_{pull}, we observe oscillation.
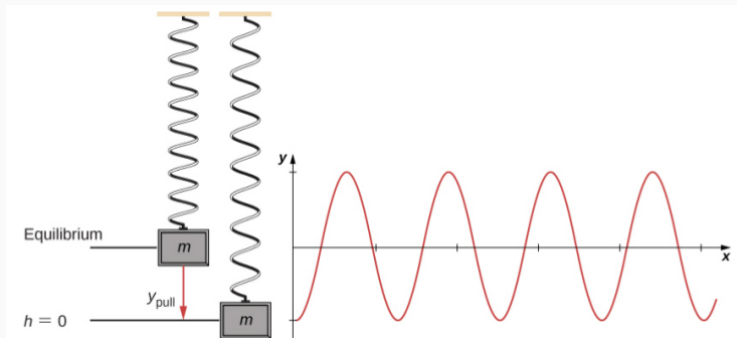
Figure 2: Notice that it does not matter what the observer defines as *zero* potential energy, since work is required to perform *changes* in potential energy.
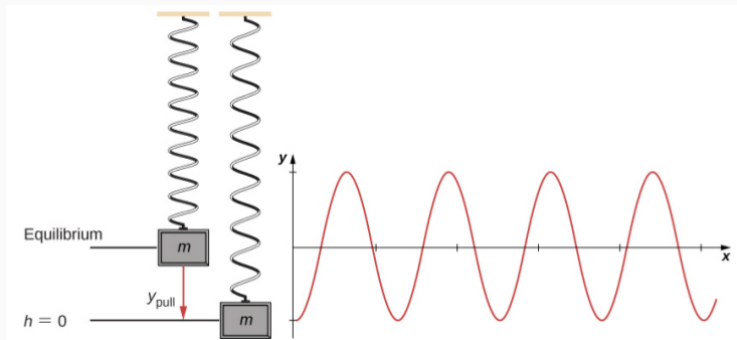
Figure 3: The fact that work is required only to perform changes in potential energy, but not does not determine the *absolute scale* of potential energy, means the observer may choose the location of zero potential energy, in the same fashion as choosing a coordinate system.

	Gravitational P.E.	Elastic P.E.	Kinetic E.
(3) Highest Point	$2mgy_{pull}$	$\frac{1}{2}ky^2_{pull}$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky^2_{pull}$	0

Figure 4: If a mass m is connected to the oscillator and *we choose* the potential energy zero-point to be the low point of oscillation, the values listed in this table correspond to the energies at various states.

Lab Activity - Gravity and the Oscillator

Lab Activity: Work and Potential Energy

- Using the springs, weights, hooks, and system of clamps and grips, build a vertical oscillating system.

- Diagram the system, showing a clearly defined value for y_{pull}, and a clearly defined choice for the potential energy zero-point.

- Measure the *unstretched* spring length, and the *equilibrium length* caused by gravity, to **derive the spring constant** k. Quote the value of k in N/m.

- Pull the spring downwards by y_{pull}, and record the maximum and minimum heights of the weight as the spring oscillates it.

- Create a table like Tab. 4, and fill in the actual energy values in Joules. What is your predicted value for *v*, the speed at which the weight moves when the oscillator is at the equilibrium position?

	Gravitational P.E.	**Elastic P.E.**	**Kinetic E.**
(3) Highest Point	$2mgy_{\text{pull}}$	$\frac{1}{2}ky^2{}_{\text{pull}}$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky^2{}_{\text{pull}}$	0

Figure 5: If a mass m is connected to the oscillator and *we choose* the potential energy zero-point to be the low point of oscillation, the values listed in this table correspond to the energies at various states.

Lab Activity - Gravity and the Oscillator, Part 2

Lab Activity: Work and Potential Energy

- Measure the *unstretched* spring length, and the *equilibrium length* caused by gravity, to **derive the spring constant** k. Quote the value of k in N/m.

- Pull the spring downwards by y_{pull}, and record the maximum and minimum heights of the weight.

- Create a table like Tab. 4, and fill in the actual energy values in Joules. What is your predicted value for v?

- Using the **Vernier LabPro** and the *motion detector attachment*, measure v when the spring is at equilibrium position, and quote the value in m/s. Does it agree with your prediction based on energy conservation? Why or why not?

	Gravitational P.E.	Elastic P.E.	Kinetic E.
(3) Highest Point	$2mgy_{\text{pull}}$	$\frac{1}{2}ky^2_{\text{pull}}$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky^2_{\text{pull}}$	0

Figure 6: The value for *v* is predicted by energy conservation. What do you measure?

Potential Energy and Conservative Forces

Potential Energy and Conservative Forces

Let path 1 be be through a force field that does work W_1 on a system, and path 2 be a different path that does work W_2 on a system.

$$W_1 = \int_{\mathrm{Path1}} \vec{F} \cdot d\vec{r} = \int_{\mathrm{Path2}} \vec{F} \cdot d\vec{r} = W_2 \qquad (2)$$

A force is conservative if

$$W_1 = W_2 \qquad (3)$$

Suppose path 1 goes from point A to B, and path 2 returns from B to A. If the force remains constant, but the path is reversed, then $W_1 = -W_2$. But this means the path is *closed*, so

$$\oint \vec{F} \cdot d\vec{r} = W_1 + W_2 = W_1 - W_1 = 0 \qquad (4)$$

Conservation of Energy

Bringing all these concepts together: **Conservation of Energy**.
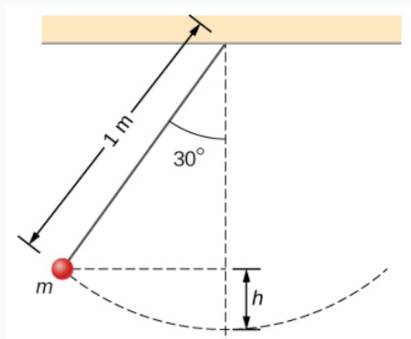

Conservation of Energy

$\Delta(KE + PE) = 0$

Recall the notion of a conservative force. In your own words, write down the defining characteristics of a conservative force. Which of these forces is NOT conservative?

- A: Stoke's Law of viscous drag
- B: Gravity near Earth's surface
- C: Hooke's Law
- D: All are conservative

Conservation of Energy

We can show that a *pendulum* exhibits the same properties as a *spring*. A pendulum is like a spring, where the restoring force is determined by *g*, the gravitational acceleration, and *L*, the length of the pendulum. That is, $a_{\mathrm{p}} = \frac{g}{L}x$, where *x* is the displacement from equilibrium, and *a* is the acceleration corresponding to *x*. (Derivation to follow). What is the force on a 1 meter-long pendulum suspending a mass of 100 grams if $x = 10$ cm?

- A: 10 N
- B: 1 N
- C: 0.1 N
- D: 0.01 N

The pendulum stores gravitational potential energy $U = mgh$. Set the zero point of gravitational potential energy to the lowest point of the pendulum. Show that the potential energy as a function of the angle θ is

$$U = mgL(1 - \cos\theta) \tag{5}$$

At which point in the trajectory of the mass on the end of the pendulum is the velocity the highest?

- The starting point at the top
- The lowest point
- The top point at the side opposite from release point
- In between the highest and lowest points

By setting pendulum potential energy equal to kinetic energy, show that the maximum velocity achieved by the pendulum mass is

$$v_{\max} = \sqrt{2Lg}(1 - \cos\theta)^{1/2} \tag{6}$$

If the length of the pendulum is 10 cm, and $g = 10$, and the initial angle is 30 degrees, what is the maximum speed of the pendulum?

- 0.1 m/s
- 0.5 m/s
- 1 m/s
- 5 m/s

Conservation of Energy

We can show that the *period* of the pendulum is related to other quantities as follows:

$$T^2 = 4\pi^2 \left(\frac{L}{g} \right) \tag{7}$$

Using the PhET simulation below, measure the gravitational acceleration on the moon. Record your reasoning in lab notebooks.

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https://phet.colorado.edu/sims/html/
pendulum-lab/latest/pendulum-lab_en.html
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Conservation of Energy

Conservative forces may be found by taking the negative derivative of the potential energies we've found through the work done:

Potential energy:

- $U_s = \frac{1}{2}kx^2$

- $U_g = mgy$

- $U_p = mgL(1 - \cos\theta)$

Corresponding force:

- $\vec{F} = -kx\hat{i}$

- $\vec{F} = -mg\hat{j}$

- $\vec{F} = -\left(\frac{mg}{L}\right)x\hat{i}$

Forces that follow Eq. 4 have to obey

$$\vec{F}(x) = -\frac{dU}{dx}\hat{i} \tag{8}$$

We may use conservation of energy to predict the behavior of systems, even when we don't know actual trajectory versus time...

Suppose we encounter a particle with potential energy $U(x) = \frac{1}{2}kx^2 + Ae^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$. If the particle has a mass m moving with this potential has a velocity v_b when located at $x = b$. Show that the particle will not cross the origin unless

$$A \leq \frac{kb^2 + mv_b^2}{2\left(1 - e^{-\frac{1}{2}\left(\frac{b}{\sigma}\right)^2}\right)} \tag{9}$$

Solve in groups at the boards. *Hint: Use conservation of energy, assuming the final energy is equal to the potential energy at the origin.*

Conservation of Energy

Notice that if $\frac{1}{2}\left(\frac{b}{\sigma}\right)^2 \ll 1$, then

$$A \lesssim \frac{\frac{1}{2}kb^2 + \frac{1}{2}mv_{\mathrm{b}}^2}{\frac{1}{2}\left(\frac{b}{\sigma}\right)^2} = \frac{E_{\mathrm{i}}^0}{\frac{1}{2}\left(\frac{b}{\sigma}\right)^2} \tag{10}$$
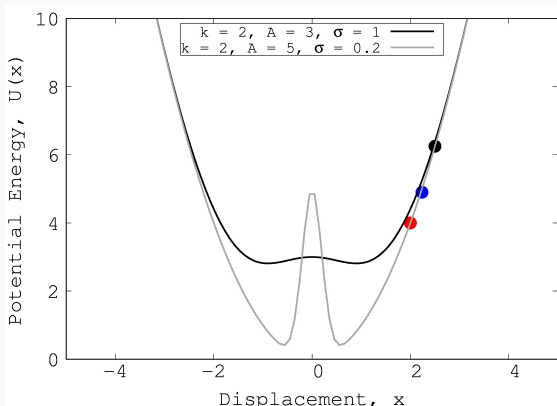
$$\left(\frac{A}{2}\right)\left(\frac{b}{\sigma}\right)^2 \lesssim E_{\mathrm{i}}^0 \tag{11}$$

This is odd; why does the initial energy appear to be *less than A*? The answer is that, in order to satisfy the limit, $b \ll \sqrt{2}\sigma$. Thus, the particle has to be close to the origin to cross it, experiencing a gradual wall rather than a sharp one. The corresponding problem in the *quantum limit* leads to quantum tunneling...

Which particle or particles will cross the origin, if each starts at rest?
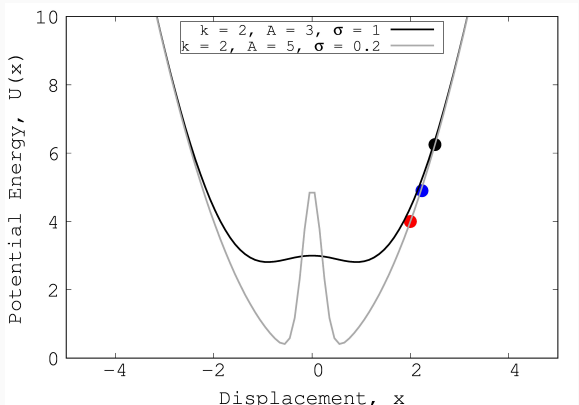
- A: Black only
- B: Black and Blue
- C: Blue only
- D: Red only

Conservation of Energy

Suppose each particle starts with sufficient negative velocity to cross the origin. If the velocities were instead positive, which would cross the origin?
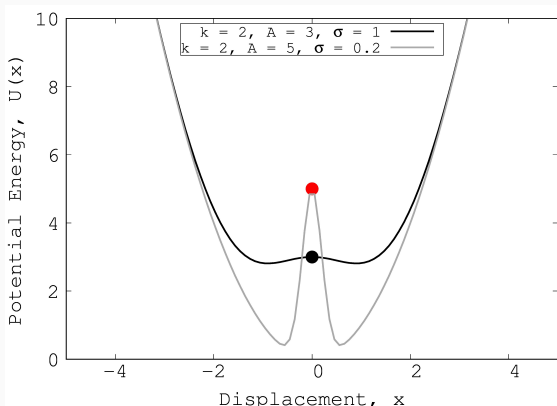
- A: Black only
- B: Black and Blue
- C: All
- D: None

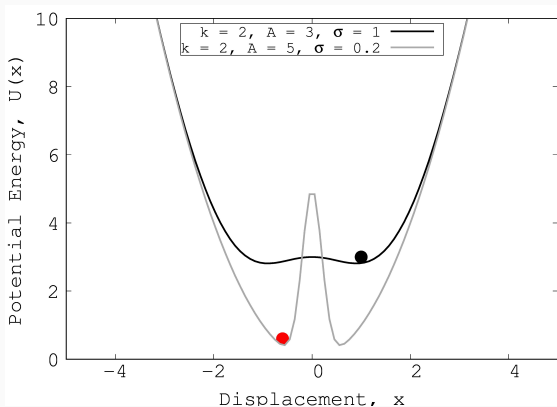Suppose Black and Red particles begin at $x = 0$. Which particle or particles is displaced?

- A: Black only
- B: Red and Black
- C: Red only
- D: Neither

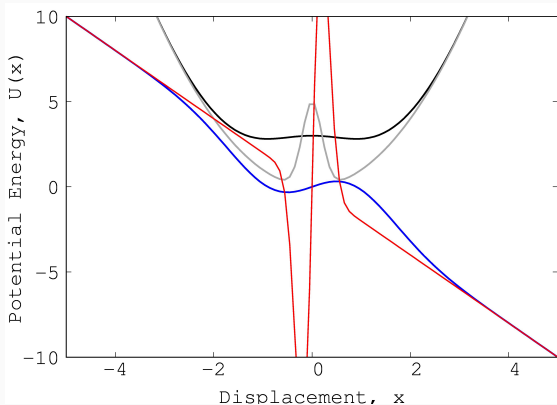Assuming each particle starts with no velocity, which particle or particles is displaced?

- A: Black only
- B: Red and Black
- C: Red only
- D: Neither

Knowing that $F = -U'$, which of the following is true?

- A: Red is F of black U
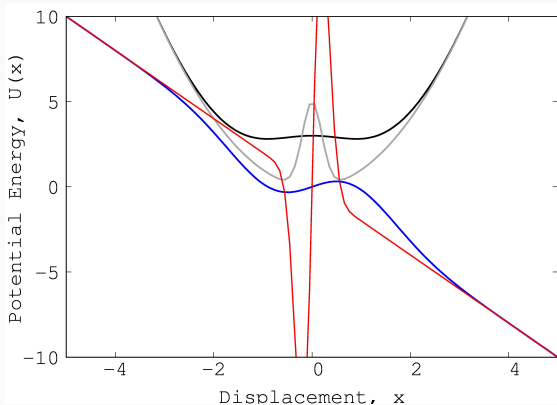
- B: Blue is F of gray U

- C: Red is F of gray U

- D: Blue is F of red U

Knowing that
$F = -U'$, which of
the following is
true?

- A: Blue is F of
  black U

- B: Red is F of
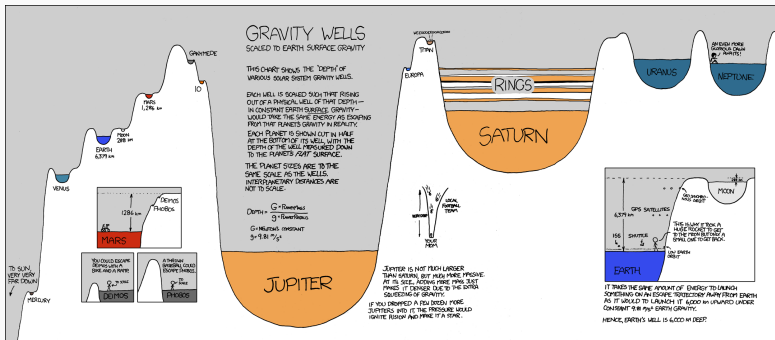 blue U

- C: Red is F of
  black U

- D: Blue is F of
  red U

Figure 7: Potential energy surfaces in the solar system (xkcd.com).

Conclusion

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   - **Lab activity:** Oscillator and gravity trading work and potential energy
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   - *Calculus review: the fundamental theorem of calculus*
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