

Score: 20/25

Susana
20643204

2 Estimations and Unit Analysis

(Technically, there is a factor of 2 from the sound going "there and back")

1. Suppose you are standing at the edge of a canyon. You clap, and here the sound of the echo off of the other side of the canyon wall about 1.5 seconds later. You estimate the canyon wall to be about 0.5 km away. a) What is the speed of sound in meters per second? b) What is it in kilometers per hour?

Speed = $\frac{d}{t}$; a) $\frac{(0.5 \text{ km})}{(1.5 \text{ sec})} \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) = \frac{1000 \text{ m}}{3 \text{ s}} \approx 333.34 \text{ m/s}$ b) $\left(\frac{0.5 \text{ km}}{1.5 \text{ sec}} \right) \left(\frac{3600 \text{ s}}{1 \text{ hr}} \right) = \left(\frac{3600 \text{ km}}{3 \text{ h}} \right) = 1200 \text{ km/hr}$

2. a) What is 0.25 m^3 in cm^3 ? b) What is 100 km/hour in m/s ? c) What is 2 kg m s^{-2} in gm cm ms^{-2} ?

a) $(0.25 \text{ m}^3) \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^3 = 0.25 \times 100^3 \text{ cm}^3 = 2.5 \times 10^5 \text{ cm}^3$
b) $\left(\frac{100 \text{ km}}{1 \text{ hr}} \right) \left(\frac{1000 \text{ m}}{1 \text{ km}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) = \frac{100 \times 1000 \text{ m}}{3600 \text{ s}} = \frac{250 \text{ m}}{9 \text{ s}} \approx 27.78 \text{ m/s}$
c) $\left(\frac{2 \text{ kg}}{1 \text{ s}^2} \right) \left(\frac{1000 \text{ g}}{1 \text{ kg}} \right) \left(\frac{100 \text{ cm}}{1 \text{ m}} \right) \left(\frac{1 \text{ s}}{1000 \text{ ms}} \right)^2 = \frac{1}{5} \frac{\text{gcm}}{\text{ms}^2} = 0.2 \text{ gcm ms}^{-2}$

3 Vectors

1. Write the following vectors in component form: (a) \vec{x}_1 is a vector with a magnitude of 10 meters and that makes an angle of 15 degrees with respect to the x-axis. (b) \vec{x}_2 is a vector with magnitude 20 meters that makes an angle of 135.0 degrees with respect to the x-axis.

a) $\vec{x}_1 = 10 \cos(15^\circ) \hat{i} + 10 \sin(15^\circ) \hat{j} = 9.659 \hat{i} + 2.588 \hat{j}$
b) $\vec{x}_2 = 20 \cos(135^\circ) \hat{i} + 20 \sin(135^\circ) \hat{j} = 20 \cos(45^\circ) \hat{i} + 20 \sin(45^\circ) \hat{j} = 10\sqrt{2} \hat{i} + 10\sqrt{2} \hat{j}$

2. A person goes for a walk. They head 0.5 km to the North, and then 0.5 km to the East. Finally, they head North-East at an angle of 45 degrees with respect to the x-axis for 0.25 km. a) Draw a diagram of their trajectory (East is x-axis, North is y-axis). b) What is the final location in x-y coordinates? c) What is the distance from the origin?

a) $\vec{W}_N + \vec{E} + \vec{NE}$
b) $\vec{v}_1 = (0 \text{ km})\hat{i} + (0.5 \text{ km})\hat{j}$
 $\vec{v}_2 = (0.5 \text{ km})\hat{i} + (0) \text{ km}\hat{j}$
 $\vec{v}_3 = \left(\frac{\sqrt{2}}{8} \text{ km} \right) \hat{i} + \left(\frac{\sqrt{2}}{8} \text{ km} \right) \hat{j}$
 $\vec{v}_f = \left(\frac{4 + \sqrt{2}}{8} \text{ km} \right) \hat{i} + \left(\frac{4 + \sqrt{2}}{8} \text{ km} \right) \hat{j}$
c) $\sqrt{c^2} = \sqrt{\frac{9 + 4\sqrt{2}}{16} \text{ km}^2} = \frac{\sqrt{9 + 4\sqrt{2}}}{4} \text{ km}$

4 Motion Along a Straight Line

1. The position of a particle moving along the x-axis is given by $x(t) = -1.0 - 4.0t$ m. (a) What is the displacement of the particle between $t = -2.0$ seconds and $t = 2.0$ seconds? (b) What is the velocity?

a) $x(-2) = -1 - 4(-2) \text{ m} = 7 \text{ m}$
 $x(2) = -1 - 4(2) \text{ m} = -9 \text{ m}$
Displacement = $7 \text{ m} - (-9 \text{ m}) = 16 \text{ m}$
b) Velocity = slope = -4.0 m/s

(-1) Sign mistake in the subtraction:
-9 minus 7 meters (final minus initial)

2. A particle moves along the x-axis according to $x(t) = -2t + 7t^2$. (a) What is the average velocity between $t = 0$ and $t = 2$ seconds? (b) Draw a graph of the velocity. (c) What is the instantaneous velocity at $t = 1$ second? (d) What is the acceleration?

a) Average velocity = $\frac{\Delta x}{\Delta t}$
 $x(0) = -2(0) + 7(0)^2 = 0 \text{ m}$
 $x(2) = -2(2) + 7(2)^2 = 24 \text{ m}$
 $\frac{24 \text{ m} - 0 \text{ m}}{2 \text{ s} - 0 \text{ s}} = 12 \text{ m/s}$
b) $v(t) = -2 + 14t$
c) $v(t) = (-2 + 14t) \text{ m/s}$
 $v(1) = -2 + 14(1) = 12 \text{ m/s}$
d) $a(t) = v'(t) = 14 \text{ m/s}^2$

(-1) acceleration has units of m/s^2 , not meters

3. A sprinter has a constant acceleration of 5.0 m/s^2 . Suppose she starts from rest. (a) How long does it take her to reach her top speed of 10.0 m/s ? (b) What is her displacement at that time? (c) Suppose she is running the 100 meter sprint. If she continues at 10.0 m/s for the remainder of the race, what will be her total time?

Velocity
 v_i

a) $a = \frac{v}{t}$

$t = 2.0 \text{ s}$

b) $\Delta x = x_f - x_i \Rightarrow 30 \text{ m} - 0 \text{ m} = 30 \text{ m}$

$x_f = \frac{1}{2}at^2 + v_i(t) + x_i$
 $= \frac{1}{2}(5 \text{ m/s}^2)(2 \text{ s})^2 + (10 \text{ m/s})(2 \text{ s}) + 0 \text{ m}$
 $= \frac{5 \text{ m/s}^2 \cdot 4 \text{ s}^2}{2} + \frac{10 \text{ m/s} \cdot 2 \text{ s}}{1}$
 $= 10 \text{ m} + 20 \text{ m} = 30 \text{ m}$

c) 100 m
 30 m
 $70 \text{ m} = 10 \text{ m/s} \cdot t$
 $t = \frac{70 \text{ m} - 10 \text{ m}}{10 \text{ m/s}} = 6 \text{ s}$

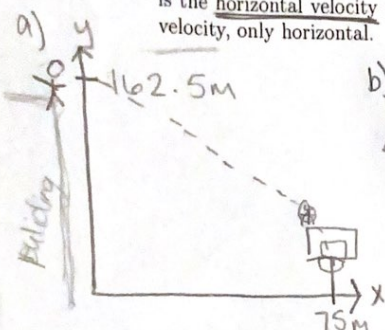
$v(t) = v_i + at$
 $70 \text{ m} = 10 \text{ m/s} + 5 \text{ m/s}^2(t)$
 $t = \frac{70 \text{ m} - 10 \text{ m}}{5 \text{ m/s}^2} = 12 \text{ s}$

$t = \frac{100 \text{ m}}{10 \text{ m/s}} = 10 \text{ s}$
 $t = 10 \text{ s} + 12 \text{ s} = 22 \text{ s}$

(-1) Everything is right except the part (b), initial velocity is supposed to be zero because it corresponds to the start of the race. This would have given a total time of 11 seconds.

5 Motion in Two and Three Dimensions

1. The world record highest basketball shot was made from a height of 162.5 meters above the basketball hoop. The basketball hoop was placed 75 meters horizontally from the shooter. a) Draw a diagram of the situation. b) What is the horizontal velocity required to make the shot? That is, assume the shooter shoots the ball with no vertical velocity, only horizontal.



b) $v_y = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$

$\Delta t = \sqrt{\frac{2\Delta x_i}{g}}$

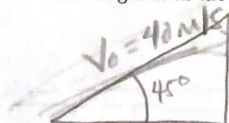
$v_x = \frac{162.5 \text{ m}}{\frac{5\sqrt{65}}{7} \text{ sec}} = \frac{7\sqrt{65}}{2} \text{ sec}$

$\approx 28.22 \text{ sec}$

$\Delta t = \sqrt{\frac{2 \cdot 162.5 \text{ m}}{9.8 \text{ m/s}^2}} = \frac{5\sqrt{65}}{7} \approx 5.76 \text{ s}$

(-1) horizontal distance in the numerator for this part, not vertical

2. A baseball is hit at a 45° angle with respect to the horizontal at 40 m/s . (a) How far away does it land? (b) How long is it in the air?



a) $R = \frac{v^2 \sin(2\theta)}{g}$

$= \frac{(40 \text{ m/s})^2 \sin(2 \cdot 45^\circ)}{9.81 \text{ m/s}^2}$
 $= 163.09 \text{ m}$

b) $T = \frac{2v \sin(\theta)}{g}$

$= \frac{2(40 \text{ m/s}) \sin(45^\circ)}{9.81 \text{ m/s}^2}$
 $\approx 5.77 \text{ sec}$

6 Forces

1. Two children pull a third child on a snow saucer sled exerting forces \vec{F}_1 and \vec{F}_2 as shown from above in Fig. 1. Find the acceleration of the system if the mass of the child and sled together is 49.0 kg . Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of \vec{F}_1 and \vec{F}_2 .

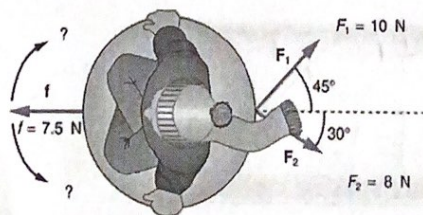
$a = \frac{\text{Pull} - \text{friction}}{\text{Mass}}$

$F = (10 \text{ N} + 8 \text{ N}) - (1.5 \text{ N})$

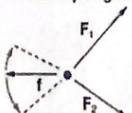
$M = 49 \text{ kg}$

$= \frac{10.5 \text{ N}}{49 \text{ kg}}$

$\Rightarrow \frac{3 \text{ kg m}}{14 \text{ kg s}^2} =$



Free-body diagram



$\frac{F}{M} = \frac{ma}{M}$

$\Rightarrow a = \frac{F}{M}$

$N = \frac{\text{kg m}}{\text{s}^2}$

(-1) Almost! You have to add the 10 N and the 8 N like vectors, not numbers.

Figure 1: A child is pulled by two other children on a sled atop some ice.

$0.2143 \text{ m/s}^2 = \text{acceleration}$