CALCULUS-BASED PHYSICS-1: MECHANICS (PHYS150-01): WEEK 6

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WEEK 5 REVIEW

1. Friction
    - Normal force and friction
    - Static, kinetic
2. Drag
    - Terminal velocity
3. Restoring Forces
    - Hooke's Law
    - Young's modulus
    - Shear modulus
    - Bulk modulus

WEEK 5 REVIEW PROBLEM

A car rests on four shock absorbers, and each is like a spring with a spring constant $k = 1000N/cm$. The car weighs 10000 N. By what distance is each spring compressed?

- A: 2.5 cm
- B: 10 cm
- C: 1 meter
- D: 0 cm

WEEK 6 SUMMARY

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1. Work has a scientifically precise definition
    - Units
    - As a product of force and displacement vectors
2. Kinetic Energy and the Work-Energy Theorem
3. Gravitational potential energy
    - Potential energy
    - *Simplifying otherwise complex calculations*
    - Potential energy near Earth's surface
    - ...in space
4. Definition of a **conservative force**
    - Relationship between conservative forces and potential energy
    - Conservation of energy for conservative forces

DEFINITIONS OF WORK

Physical Definition of Work

Let $\vec{F}$ be a force exerted on a system, which is displaced by a displacement $\vec{x}$. The **work** done on the system is $W = \vec{F} \cdot \vec{x}$

The units of work are N m = kg m/s^2, or *Joules*.

Extra credit opportunity: **Do you like beer**? Write a 10-page paper on the on the scientific challenge faced by James Prescott Joule, who began to formulate the modern view of energy in the 19th century, contrary to *caloric theory*. **Upon completion of this assignment I will change two homework scores to perfect scores.**

Let θ be the angle between the force and the displacement. Then this equation

$$W = \vec{F} \cdot \vec{x} \tag{1}$$

becomes

$$W = Fx \cos\theta \tag{2}$$

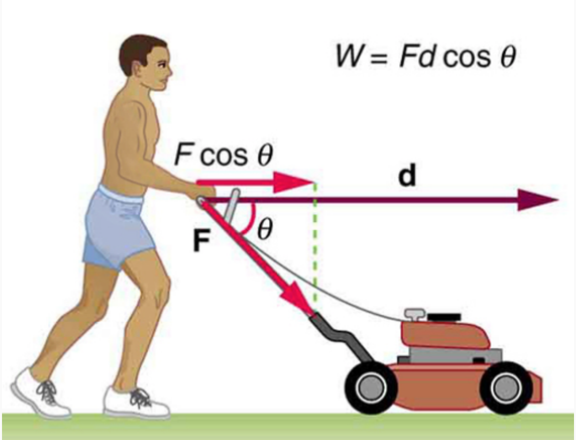

Figure 1: A case where $\theta \neq 0$.

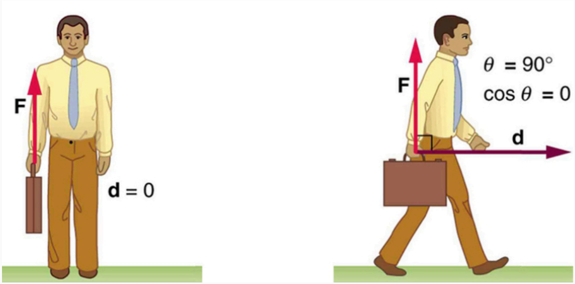

Figure 2: (Left): A case where $x = 0$, so $W = 0$. (Right): A case where $\theta = 90°$, so $W = 0$.

Just because an action requires *energy* doesn't mean we are performing *work*. It requires muscular energy to hold up a heavy briefcase but this is not what we mean by work. Work is about moving objects.

What about Newton's 3rd Law? If one system A exerts a force F_{AB} on a system B, then Newton's 3rd law states that system B exerts a force $-F_{\mathrm{AB}}$ on system A.

If the work done by A on B is $W = (F_{\mathrm{AB}})x\cos\theta$, then the work done by B on A is $W = -(F_{\mathrm{AB}})x\cos\theta$.

In Fig. 1, the work done by the man on the mower is positive, but the work done by the mower on the man is negative.

More units of energy:

Unit Name	Definition	Value
electron-volt (eV)	energy of 1 e^- through 1 V	1.60×10^{-19} J
1 Rydberg (Rd)	ionize 1 hydrogen atom	21.8×10^{-19} J
Joule	**1 N·m**	**1.0 J**
foot-pound	1 ft·lb	1.36 J
calorie	Raise 1 gram of water 1° C	4.184 J
British Thermal Unit	Raise 1 lb of ice to boil (°F)	1054.3 J
food calorie (kcal)	1000 calories	4184 J
kilowatt hours	1 kilowatt system for 1 hr	3.6×10^{6} J
gasoline galon equiv.	burning a galon of gas	$\approx 120 \times 10^{6}$ J
$E = mc^2$, 1 mole of H^+	Rest mass (fusion/fission)	9×10^{13} J

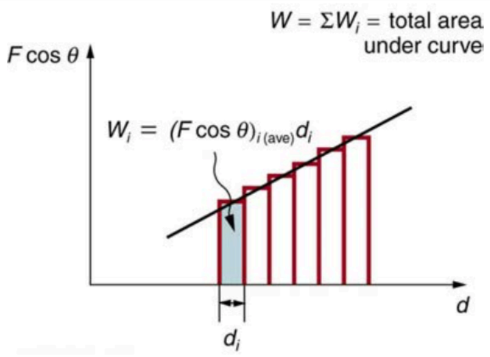

Figure 3: Breaking the displacement $\vec{x}$ into pieces, and summing them.

This interpretation naturally leads to the subject of *integration* in calculus.

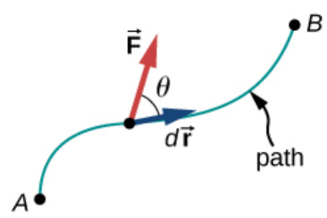

Figure 4: Summary of the concept of the work integral.

$$W = \int_{AB} \vec{F} \cdot d\vec{r} \tag{3}$$

SURVEY - THE INSIGHTS AND DATA (UNDER ANALYSIS)

- **Group problem solving is a boost**
    1. Must mix participation, someone knows how to start
    2. Prof. Hanson make rounds
    3. Show each and every step for iClicker and examples
- **Exams**
    1. Review day
    2. Practice problems and study guides are a boost
    3. Similarity to homework, lecture?
- **Lab activities** and Final Group Project
    1. Some want to drop them
    2. Scientific process is important (don't just *assume*)
    3. *The goal is to blend them*
- **Office hours are crucial**

SURVEY - THE ...INTERESTING BITS...

- ”You’re not a bad person, you just suck at explaining things.”
- ”The tests are not fair.”
- NO PARTIAL CREDIT??
- ”Everything should change except lab activities, and it’s good that those are not graded.”

My response: There is something called **The 80/20 Rule**... Bottom line: we are here to develop and grow into professionals.

Consider the case where the force doing the work on the system of mass m is friction:

$$W = \int_{AB} \vec{F} \cdot d\vec{r} = -\int_{AB} \mu_{\mathrm{k}} N dx = -\mu_{\mathrm{k}} mg \int_{AB} dx \qquad (4)$$

- Friction acts in opposite direction, so the dot product gives a minus sign
- Friction acts along path AB (whatever direction of motion is)

The driver of a 900 kg car slams on the breaks, and the tires slide on the pavement with $\mu_{\mathrm{k}} = 0.2$. The initial speed is 25 m/s. Assuming $g = 10$ m/s^2, how far does the car travel before coming to a stop?

- A: 312.5 m
- B: 625 m
- C: 31.25 m
- D: 62.5 m

What is the work done on the car?

- A: 280 J
- B: -280 J
- C: 280 kJ
- D: -280 kJ

Take your *algebraic* answer for the breaking distance (displacement from two slides ago), and substitute it into the expression for the work done on the car (force of friction times displacement). What do you get? (*Check your units*).

- A: mv_{i}^2
- B: $mg\frac{1}{2}mv_{\mathrm{i}}^2$
- C: $-\frac{1}{2}mv_{\mathrm{i}}^2$
- D: $\mu_{\mathrm{k}}mv_{\mathrm{i}}^2$

Keep this result in mind...

Suppose we raise the 900 kg car by a displacement of 10 meters. What is the work done on the car?

- A: 9 J
- B: 90 J
- C: 900 kJ
- D: 90 kJ

What if we drop the car from a height of 10 m? What is the final velocity of the car?

- A: $10\sqrt{2}$ m/s
- B: 20 m/s
- C: 10 m/s
- D: $10\sqrt{10}$ m/s

Stand up, and show in your groups that the work done on the falling car is equal to $\frac{1}{2}mv_{\mathrm{f}}^2$, both numerically and algebraically.

CONCLUSION

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ANSWERS

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- 2.5 cm
- 312.5 m
- -280 kJ
- $-\frac{1}{2}mv_{\mathrm{i}}^2$
- 90 kJ
- $10\sqrt{2}$ m/s

- ...