

Figure 1: (Left) A current  $I$  experiences a force  $F$  in a  $B$ -field.

## 2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the  $B$ -field given that  $F$  is the Lorentz force.

- a:  $\vec{B} = \vec{F} \times \vec{v} = (-\hat{i} \times \hat{j}) = -\hat{k}$ , Into page
- b: Toward left
- c:  $\vec{B} = \vec{F} \times \vec{v} = (\hat{j} \times (-\hat{j})) = \hat{k}$ , Out of page

2. Consider Fig. 1 (right). **The Hall Effect.** An  $E$ -field exists in the vertical direction and a  $B$ -field is perpendicular to the direction of charge velocity. (a) Show that if the  $E$ -field force on a charge balances the Lorentz force on a charge, that  $v = E/B$ . (b) If the  $E$ -field is constant,  $E = \Delta V/\Delta x$ . Show that

$$\Delta V = \frac{B \Delta x I}{n q_e A} \quad (1)$$

where  $n$  is the charge carrier density,  $q_e$  is the electron charge,  $A$  is the cross-sectional area of the conductor, and  $I$  is the current. Plug in  $B = 1.33 \text{ T}$ ,  $\Delta x = 2 \text{ cm}$ ,  $I = 10 \text{ A}$ ,  $n = 2 \times 10^{28} \text{ m}^{-3}$ ,  $A = 1 \text{ mm}^2$ , and  $q_e$  is the charge of an electron.

a)  $F_E = F_B$   
 $qE = qvB \sin \theta$   
 $qE = qvB$   
 $E = vB$   
 $v = E/B$

b)  $E = \frac{\Delta V}{\Delta x}$   
 $\Delta V = vB \Delta x$   
 $\Delta V = B \Delta x v$   
 $\Delta V = B \Delta x \cdot \frac{I}{nq_e A}$   
 $\Delta V = \frac{B \Delta x I}{nq_e A}$

$\Delta V = \frac{1.33(2)(10)}{(2 \times 10^{28})(1.6 \times 10^{-19})(1 \times 10^{-6})} = 8.3125 \times 10^5 \text{ V}$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop  $0.65 \times 10^{-15} \text{ m}$  in radius with a current of  $1.05 \times 10^4 \text{ A}$ . Find the maximum torque on a proton in a  $2.50\text{-T}$  field. (This is a significant torque on a small particle.)

$$\tau = NIA B \sin \theta, \quad n=1, \quad \theta=90$$

$$A = \pi r^2 = \pi (0.65 \times 10^{-15})^2 = 1.33 \times 10^{-30} \text{ m}^2$$

$$\tau = (1)(1.33 \times 10^{-30})(2.50)(\sin 90)$$

$$= 3.48 \times 10^{-26} \text{ Nm}$$

## 3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the  $B$ -field inside a solenoid with 500 turns per meter, carrying a current of  $0.3 \text{ A}$ ? (b) Suppose we insert a piece of metal inside the solenoid, boosting  $\mu_0$  by a factor of 5000. What is the new  $B$ -field?

a)  $n = 500 \text{ turns/m}$ ,  $I = 0.3 \text{ A}$

$$B = \mu_0 n I$$

$$= (4\pi \times 10^{-7})(500)(0.3) = 1.885 \times 10^{-4} \text{ T}$$

b)  $B = (5000)(4\pi \times 10^{-7})(5000)(0.3)$

$$= 0.942476 \text{ T}$$

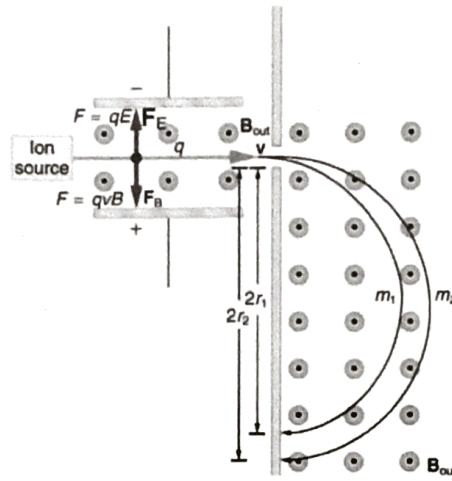


Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer.** Suppose that the velocity of the charged particles moving to the right is  $v = E/B$ . (a) Show that if  $v = E/B$ ,  $F_{net} = 0$  in the region in the top left<sup>1</sup>. (b) Recall that the centripetal force on a particle of mass  $m$  is  $mv^2/r$ . Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \quad (2)$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton:  $1.67 \times 10^{-27}$  kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-septometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance  $r$ ?

$a) F_e = qE$   
 $F_n = q(\vec{v} \times \vec{B})$   
 $F_{tot} = F_e + F_n = 0$   
 $q(\vec{E} + \vec{v} \times \vec{B}) = 0$   
 $|\vec{E}| = v|\vec{B}|$   
 $v = \frac{E}{B}$

$b) \frac{mE}{B^2} = r \frac{E}{B}$   
 $r = \frac{mE}{qB^2}$

$$= \frac{16 \cdot (1.67 \cdot 10^{-27}) \cdot 10}{1.602 \times 10^{-19} (0.01)^2} = 1.67 \text{ cm}$$

#### 4 Chapter 13: Electromagnetic Induction

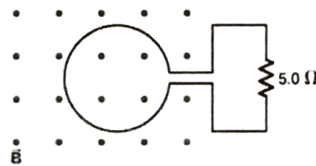


Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ( $N = 1$ ) loop, and is tuned to follow the form

$$B(t) = B_0 \left( \frac{1}{2} + \frac{2}{\pi} \sin(2\pi ft) + \frac{2}{3\pi} \sin(6\pi ft) + \frac{2}{5\pi} \sin(10\pi ft) \right) \quad (3)$$

The loop has a radius  $r$ . (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If  $B_0 = 0.1$  T,  $r = 0.1$  m, and  $f = 10^3$  Hz, what is the induced emf at  $t = 0$ ? (c) What is the current through the resistor at  $t = 1$  ms?

$a) e = \left| \frac{d\phi}{dt} \right| = \frac{d(BA)}{dt}$

$$e = A \frac{dB}{dt} = (\pi r^2) \left( \frac{B_0}{T_0} \right) \sin(2\pi ft)$$

$b) B_0 = 0.1, r = 0.1, f = 10^3$

$\text{will be } 0, \text{ since } \sin(2\pi ft) = 0$

$c) \text{ current at } t, I = \frac{e}{R} = \frac{0.05}{5} = 0.01 \text{ A}$

<sup>1</sup>Molecules that do not have this velocity will hit the sides of this portion of the instrument.

## 5 Chapter 14: Inductance

1. What is (a) the rate at which the current through a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?  $\text{emf} = 0.150 \text{ V}$   $\frac{dI}{dt} = ?$

$$\text{emf} = -L \frac{dI}{dt} \quad \frac{dI}{dt} = \frac{-\text{emf}}{L} = -\frac{0.15}{0.50} = \boxed{-0.3}$$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$$\mathcal{E} = -L \frac{dI}{dt}$$

$$\mathcal{E} = L \frac{dI}{dt}$$

$$500 = 2 \times 10^{-3} \text{ H} \frac{dI}{dt}$$

$$2.5 \times 10^5 = \frac{dI}{dt}$$

$$dI = 2.5 \times 10^5 dt$$

$$\int dI = \int 2.5 \times 10^5 dt$$

$$I = 2.5 \times 10^5 t$$

$$t = \frac{I}{2.5 \times 10^5} = \frac{0.1000}{2.5 \times 10^5} = \boxed{4 \times 10^{-7} \text{ sec}}$$