Time?

We use "time" a lot in physics. But let's be sure we know what we mean when we specify a "time".

<u>Without discussing with your group mates</u> write down a "time" in the space below. (Hint: use some numbers)

<u>Now show your answers to your group mates and discuss.</u>

Did you all interpret the word "time" in the same way?
<u>If not</u>, then, in the space below, comment on the different interpretations.

If you did, can your group come up with another interpretation

In this space, write down a couple of words that the instructor wants you to know the meaning of (Also write the meanings, of course.)

If you read sect. 2.1 in Giancoli then you should be able to answer these questions.

Three important words: position, displacement, distance.

In the last handout we discussed the difference between "clock reading" and "time interval".

Two of the three words (position, displacement, distance) are the spatial analogies of these two temporal phrases.

Discuss in your group which of the three words is analogous to "clock reading" and why.

Which of the three words is analogous to "time interval"?

What about the third word? Discuss why there is no analogous temporal term.

Name______________________________

Class______________________________

Lab Partners ______________________________

INTRODUCTION TO MOTION

Investigation 1: Distance(Position)-Time Graphs of Your Motion

To find out How you can measure your motion with a motion detector

How your motion looks as a distance (position)-time graph

Materials

motion detector Universal Lab Interface (ULI)

number line on floor in meters (optional)

Introduction In this investigation, you will use a motion detector to plot a distance (position)-time graph of your motion. As you walk (or jump, or run), the graph on the computer screen displays how far away from the detector you are.

- *"Distance" is short for "distance from the motion detector."*
- The motion detector is the *origin* from which distances are measured.
- It detects the closest object directly in front of it (including your arms if you swing them as you walk).
- It will not correctly measure anything closer than 1/2 meter. When making your graphs don't go closer than 1/2 meter from the motion detector.

Activity 1 **Making Distance-Time Graphs**

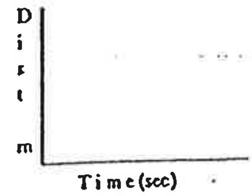
1. Double click on "distance-time". This opens the software file.

2. ~~When you are ready to start graphing distance, click once on the~~

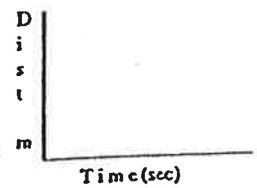
Collect button at the top of the screen.

4. <u>Make distance-time graphs for different walking speeds and directions</u>.

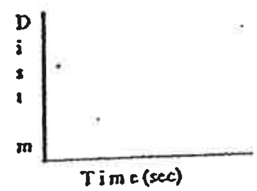
a. Start at the 1/2-meter mark and make a distance/time graph, walking away from the detector *slowly and steadily*. Sketch the graph on the right.



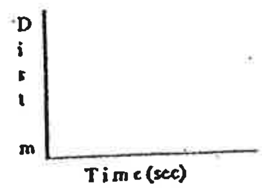
b. Make a distance/time graph, walking away from the detector *medium fast and steadily*. Sketch the graph.



c. Make a distance/time graph, walking toward the detector *slowly and steadily*. Sketch the graph.



d. Make a distance/time graph, walking toward the detector *medium fast and steadily.* Sketch the graph.



Questions Describe the difference between the graph you made by walking away slowly and the one made by walking away more quickly. (Q1)

____

____

Describe the difference between the graph made by walking toward and the one made walking away from the motion detector. (Q2)

____

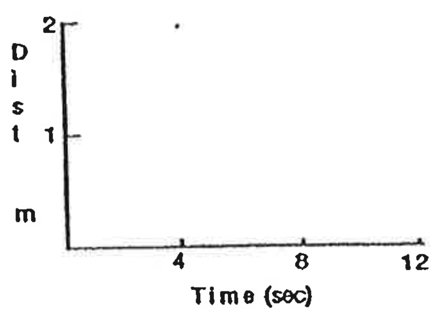
____

Prediction

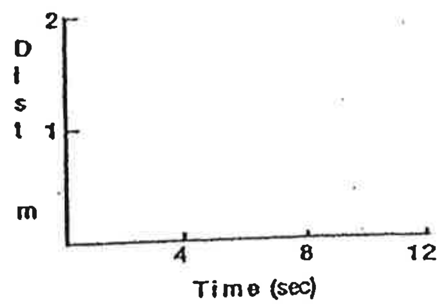
Predict the graph produced when a person starts at the 1-meter mark, walks away from the detector *slowly and steadily* for 4 seconds, stops for 4 seconds, and then walks toward the detector *quickly*. Draw your *prediction* on the left axes below using a *dotted line*.

Compare predictions with the rest of your group. See if you can all agree. Draw your group's prediction on the left hand axes using a *solid line*. (Do not erase your original prediction.)

PREDICTION



FINAL RESULT



5. Do the experiment. Move in the way described and graph your motion. When you are satisfied with your graph, draw your group's final result on the right axes.

Question

Is your prediction the same as the final result? If not, describe how you would move to make a graph that looks like your *prediction*. (Q3)

____

____

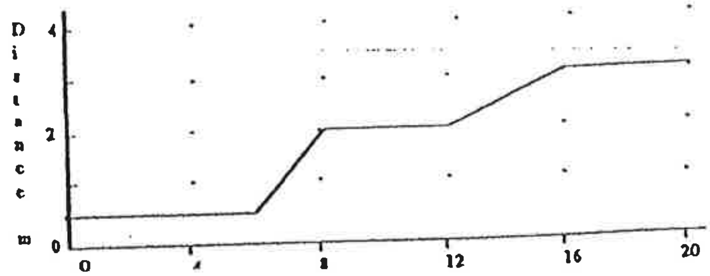
____

Activity 2 Matching a Distance Graph

In this activity you will match a distance graph shown on the computer screen.

1. Open the "distance match" file.

graph below will appear on the screen.



The graph is stored in the computer as **Run1**. New data from the motion detector are always stored as **Latest** and can therefore be collected without erasing the **Distance.Match** graph.

2. <u>Move to match the distance graph shown on the computer screen.</u> You must move to duplicate the Distance.Match graph. You may try a number of times. Work as a team. Get the times right. Get the distances right. Each person should take a turn.

Question What was the difference in the way you moved to produce the two differently sloped parts of the graph you just matched? (Q4)

____

____

Investigation 2: Velocity-Time Graphs of Your Motion

To find out The connection between velocity and your actual motion
How your motion looks as a velocity-time graph

Materials
motion detector
Universal Laboratory Interface (ULI)
number line on floor in meters (optional)

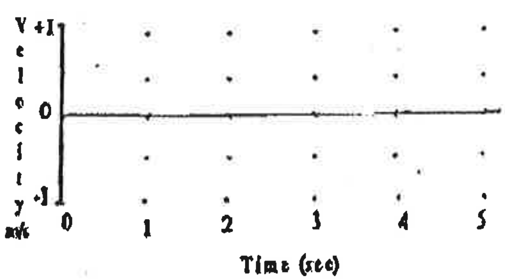
Introduction You have already plotted your distance (position) from the motion detector as a function of time. You can also plot how fast you are moving. How fast you move is your speed. It is the rate of change of distance with respect to time. *Velocity* takes into account your speed and the direction you are moving. When you measure motion along a line, velocity can be positive or negative

Activity 1 **Making Velocity Graphs**

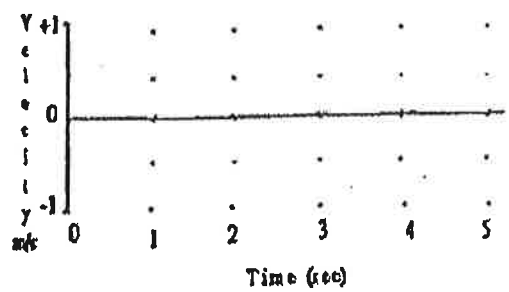
1. al

Open the "vel-time" file.

Make a velocity graph by standing still about 1 meter in front of the detector.



Now make a velocity graph by standing still about 2 meters in front of the detector.

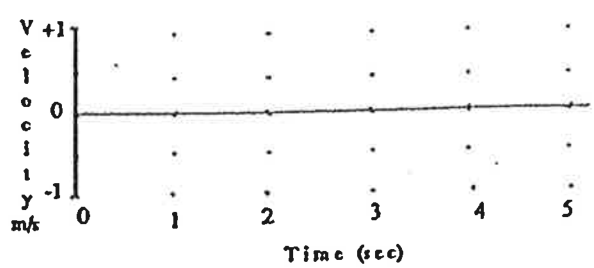


2. Graph your velocity for different walking speeds and directions.
   a. Make a velocity graph by walking *away* from the detector *slowly and steadily*. Try again until you get a graph you're satisfied with.

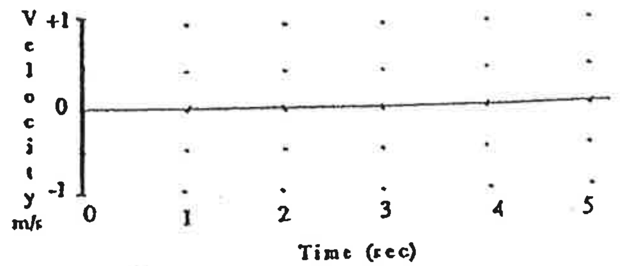
   You may want to change the velocity scale so that the graph fills more of the screen and is clearer. To do this, double click anywhere on the graph and change the velocity range.

   Sketch your result below. (Just draw *smooth* patterns; leave out smaller bumps that are mostly due to your steps.)

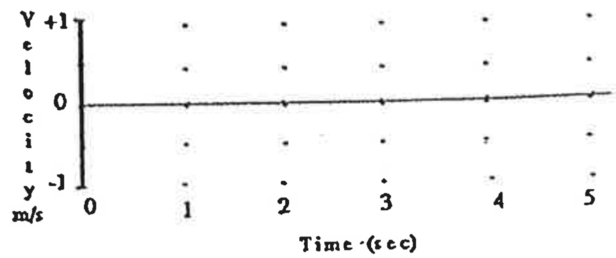
When you walk the velocity graphs, please stand still in front of the detector before you hit the collect button and make sure you see a zero velocity line start to appear before you start walking.



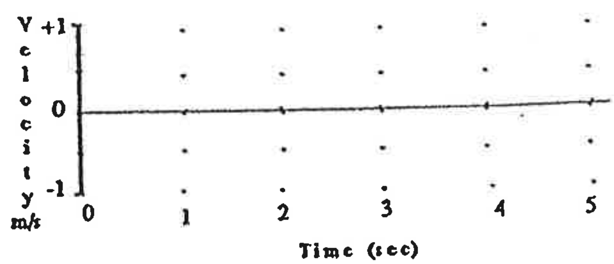
b. Make a velocity graph, walking *away* from the detector *medium fast and steadily.* Sketch your graph.



c. Make a velocity graph, walking *toward* the detector *slowly and steadily.* Sketch your graph.



d. Make a velocity graph, walking *toward* the detector *medium fast and steadily* . Sketch your graph.



Questions

What is the most important difference between the graph made by slowly walking away from the detector and the one made by walking away more quickly? (Q1)

How are the velocity-time graphs different for motion away and motion toward the detector? (Q2)

Comment

How fast you move is your speed, the rate of change of distance with respect to time. Velocity implies both speed and *direction.* As you have seen, for motion along a line (the positve x axis) the sign (+ or -) of the velocity indicates the direction. If you move away from the detector (origin), your velocity is positive, and if you move toward the detector, your velocity is negative.

The faster you move *away* from the origin, the larger positive number your velocity is. The faster you move *toward* the origin, the "larger" negative number your velocity is. That is -4 m/s is twice as fast as -2 m/s and both motions are toward the origin.

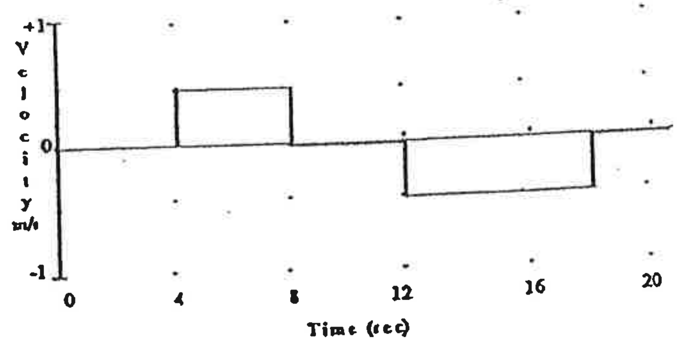
Activity 2

Matching a Velocity Graph

In this activity, you will move to match a velocity graph shown on the computer screen.

1. Display the velocity graph on the screen.

Open the "~~velo~~ velocity match" file.



2. Move so as to imitate this graph. You may try a number of times. Work as a team and plan your movements. Get the times right. Get the velocities right. Each person should take a turn.

   Draw in your group's best match on the axes above.

Questions

Describe how you moved to match each part of the graph. (Q3)

____

____

____

Is it possible for an object to move so that it produces an absolutely vertical line on a velocity time graph? Explain. (Q4)

____

____

INTRODUCTION TO MOTION
Investigation 3: Distance and Velocity Graphs

To find out The relationship between distance-time and velocity-time graphs.

Materials

Universal Laboratory Interface (ULI)
number line on floor in meters (optional)

Introduction You have looked at distance- and velocity-time graphs separately. Now you will see how they are related.

Activity 1 Predicting Velocity Graphs from Distance Graphs

1. Set up to graph Distance and Velocity.

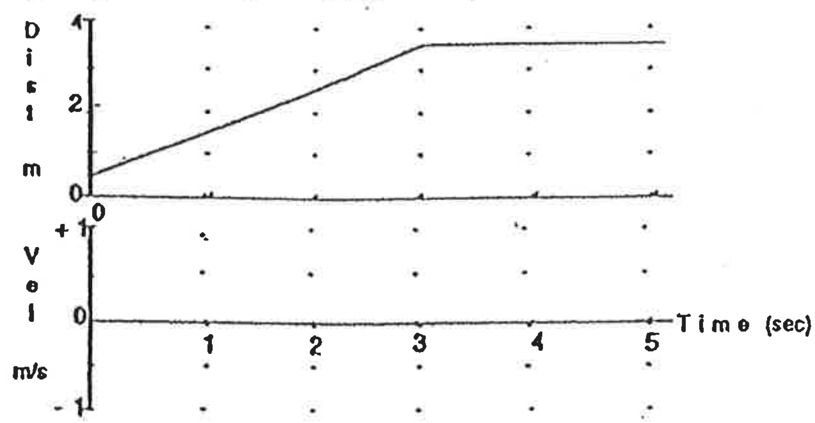
   Open the "vel and dist" file.

2. Predict a velocity graph from a distance graph. Carefully study the distance graph shown below and predict the velocity-time graph that would result from the motion. Using a *dotted line*, sketch your *prediction* of the corresponding velocity-time graph on the velocity axes.

3. Make the graphs. After each person has sketched a prediction, Start, and do your group's best to make a distance graph like the one shown below. Walk as smoothly as possible.

   When you have made a good duplicate of the distance graph, sketch your actual graph over the existing distance-time graph.

   Use a *solid line* to draw the actual velocity graph on the same graph with your prediction. (Do not erase your prediction).



Questions How would the distance graph be different if you moved faster? Slower? (Q1)

____

____

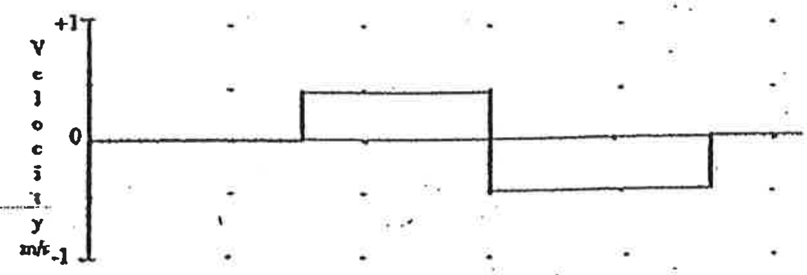
How would the velocity graph be different if you moved faster? Slower? (Q2)

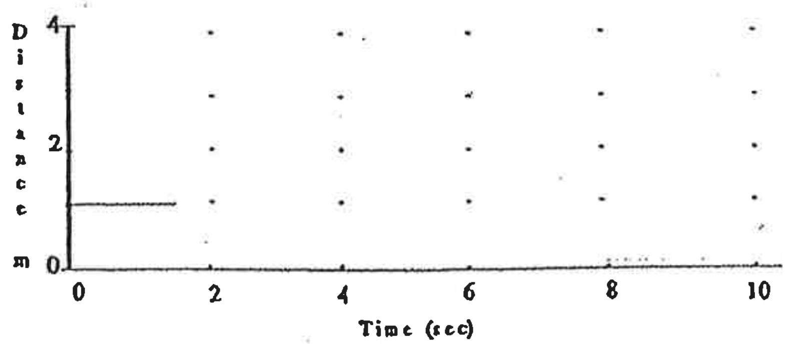
____

____

Activity 3 Predicting Distance Graphs from Velocity Graphs

1. Predict a distance(position)-time graph from a velocity-time graph. Carefully study the velocity graph below. Using a *dotted line*, sketch your *prediction* of the corresponding distance graph on the bottom set of axes. (Assume that you started at the 1-meter mark.)



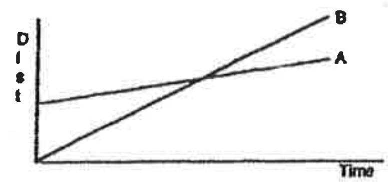


2. Make the graphs. After each person has sketched a prediction do your group's best to duplicate the top (velocity-time) graph by walking. (Reset the Time scale to 10 sec before you start.)

   When you have made a good duplicate of the velocity-time graph, draw your actual result over the existing velocity-time graph.

   Use a *solid line* to draw the actual distance-time graph on the same axes with your prediction. (Do not erase your prediction.)

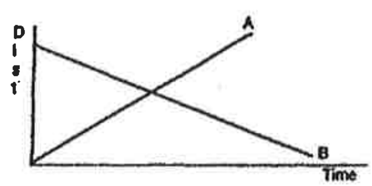
Interpretation of position graphs



a) Which object is moving faster--A or B?_____

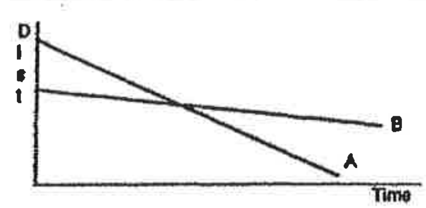
b) Which starts ahead? _____
Define what you mean by "ahead."

c) What does the intersection mean?



a) Which object is moving faster? _____

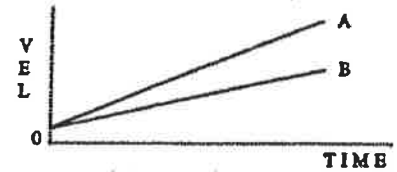
b) Which object has a negative velocity according to the convention we have established? _____



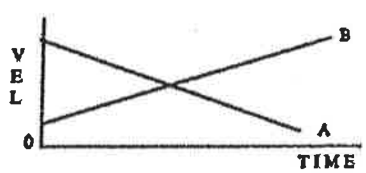
a) Which object is moving faster? _____

b) Which starts ahead? _____
Explain what you mean by "ahead."

Interpretation of velocity graphs



a) Is one faster than the other? If so, which one is faster? (A or B)
b) What does the intersection mean?
c) Can one tell which object is "ahead"? (define "ahead")
d) Does either object A or B turn around? Explain.



a) Is one faster than the other? If so, which one is faster? (A or B)
b) What does the intersection mean?
c) Can one tell which object is "ahead"? (define "ahead")
d) Does either object A or B turn around? Explain.

Rate of Change

When Fred was 12 years old he was 5 feet tall.

When Fred was 20 years old he was 6 feet 2 inches tall.

What was the value of the average rate of change of Fred's height during this period of his life.

In words write the instructions for calculating the rate of change of any quantity. Be totally explicit and do not use any numbers or equations.

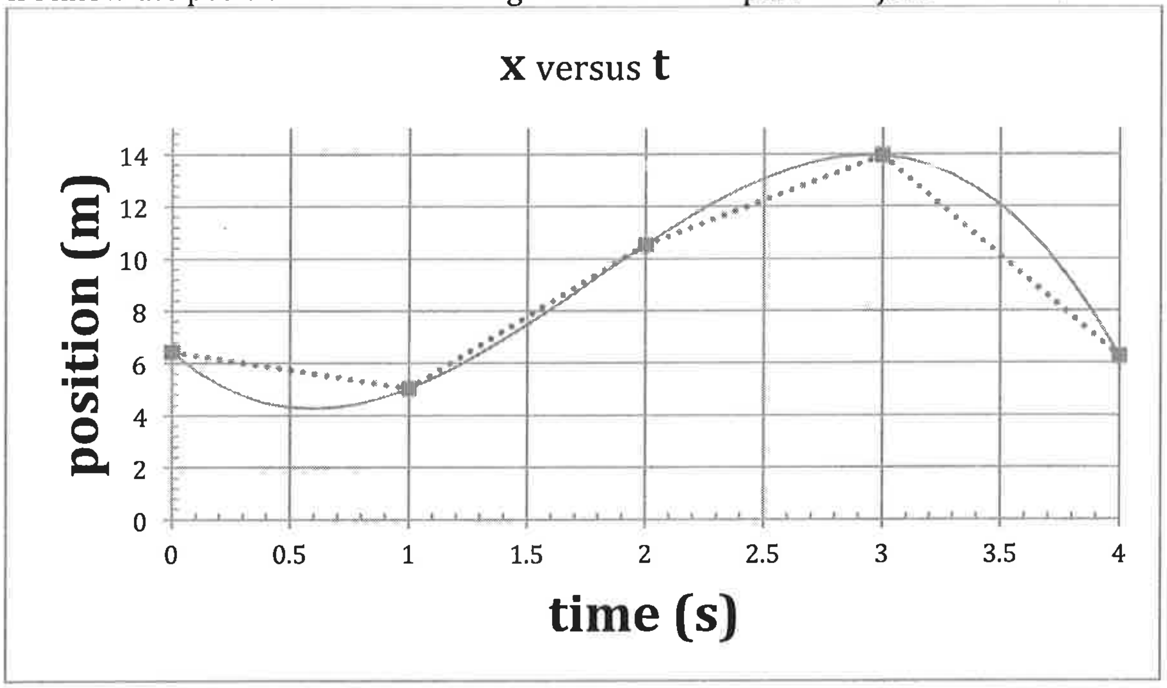
Now convert your instructions into an equation using appropriate notation.

What does the word "per" mean? (for instance, in the sentence "the disk rotated at 200 revolutions per minute")

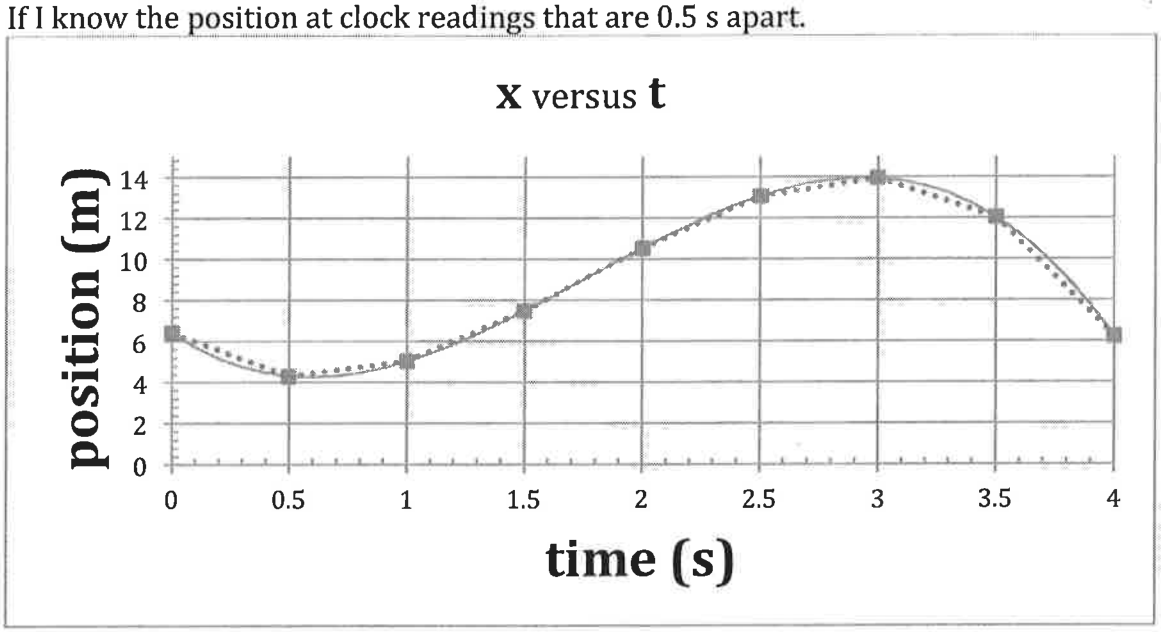
Calculating velocity from position data.

A car moves along the road according to the smooth curve in the graphs below.

If I know the position at clock readings that are 1.0 s apart and "join the dots".



If I know the position at clock readings that are 0.5 s apart.

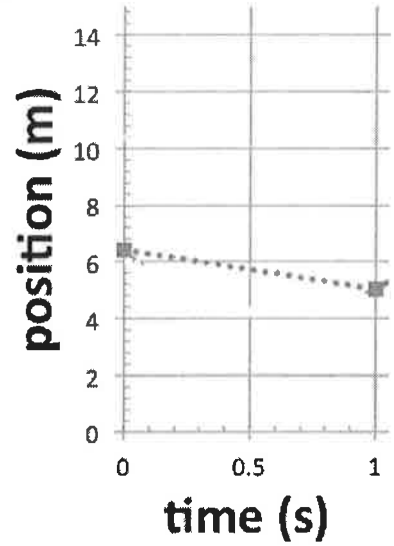


What is the physical meaning of the slopes of the dotted lines.

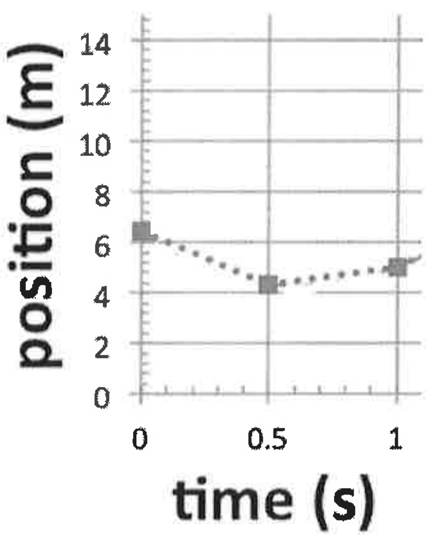
As the time intervals become smaller and smaller the dotted lines more closely match the true motion of the car.

In the following use the values (t,x)) = (0.0,6.5), (0.5,4.3), (1.0,5.1), (2.0,10.3), (3.0,14.0), (4.0,6.2) for the data points.

Suppose we ask "What is the velocity of the car at a clock-reading of 0.83 s?" and all we know are the slopes of the dotted lines in the first graph. What answer will we give?



Suppose we ask "What is the velocity of the car at a clock-reading of 0.83 s?" and all we know are the slopes of the dotted lines in the second graph. What answer will we give?



If we use smaller and smaller time intervals then we will get closer and closer to the true value of the **"instantaneous" velocity** at 0.83 s. As the time interval gets smaller the slope of the dotted line becomes closer and closer to the slope of the tangent line at clock-reading 0.83 s.

"Instantaneous velocity at clock-reading t" is found by evaluating (x2-x1)/(t2-t1) in the limit as t2 and t1 approach the clock-reading t. This value is the slope of the tangent line to the position versus time graph at clock-reading t.

In mathematical notation this is written

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \text{ (= slope of the tangent line)}$$

$\frac{dx}{dt}$ is called "the derivative of x with respect to t."

In Physics there are two situations that we encounter regarding the position of an object.

One situation (A) is where we have **experimental data** about the position of the object at a finite number of clock readings. The other situation (B) is when we have **a theoretical function** that represents the position of the object as a function of time x(t).

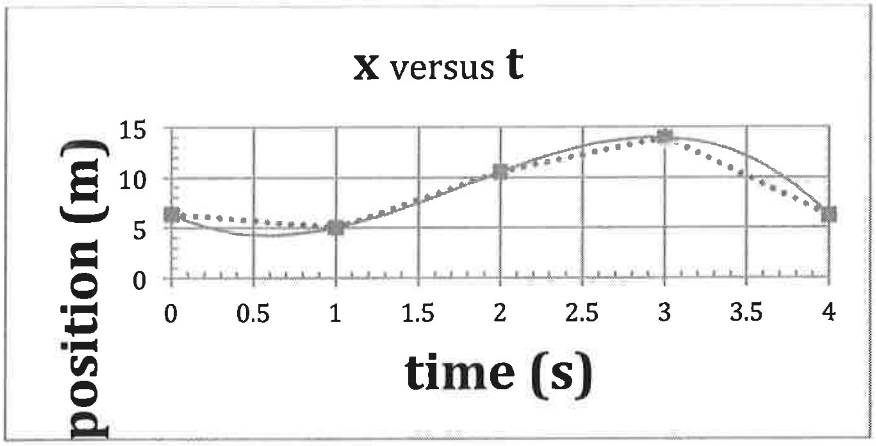
In situation A we cannot perform the limit indicated in the equation above. We are constrained to calculating average velocities over the time intervals between our given clock readings.

In situation B we basically know the position (theoretically) at every clock-reading and so we can perform the limit indicated in the equation above. In Calculus class you learn how to do this. It is called "taking the time derivative" of the function x(t).

Your instructor will give you more details about how to take a time derivative .

Let's concentrate on situation A (experimental data) for the moment.

In the following use the values (t,x)) = (0.0,6.5), (0.5,4.3), (1.0,5.1), (2.0,10.3), (3.0,14.0), (4.0,6.2) for the data points.



During the time interval between t = 0.0 and 1.0 s the average velocity is given by the slope of the dotted line.

At what clock-reading during the time interval is the slope of the tangent line equal to the slope of the dotted line. (Just eyeball the graph)

Consider this same question for the other three time intervals shown.

Using your observations, answer the following question.

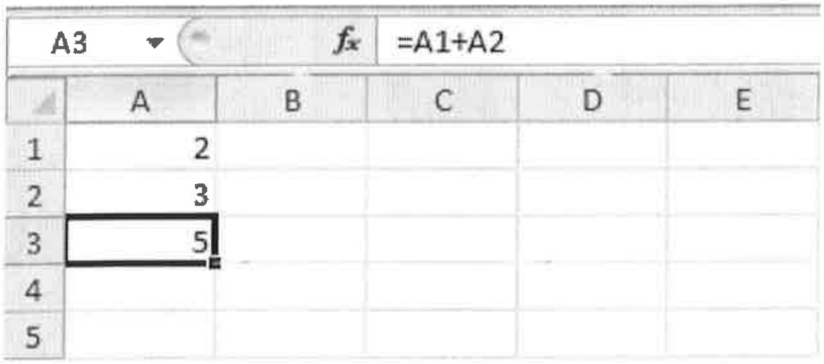
In a given time interval, the average velocity of the object is our best estimate of the instantaneous velocity of the object at :

(a) the start of the interval
(b) the end of the interval
(c) the mid-time of the interval

You will now apply this concept to some position data using an Excel spreadsheet.

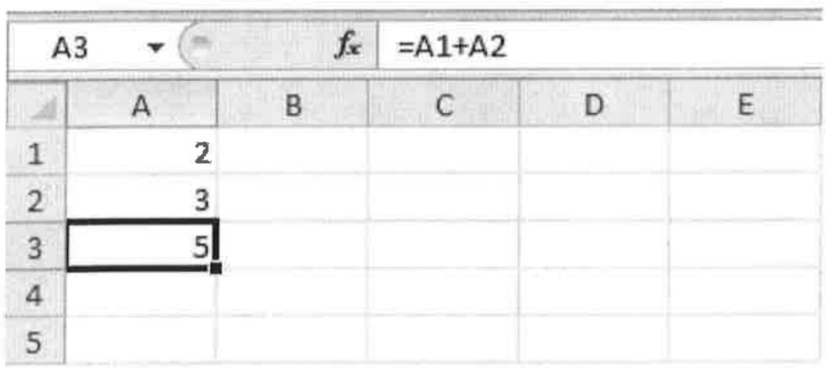
A **formula** is an expression which calculates the value of a cell. **Functions** are predefined formulas and are already available in **Excel**.

For example, cell A3 below contains a formula which adds the value of cell A2 to the value of cell A1.



To enter a formula, execute the following steps.

1. Select a cell.
2. To let Excel know that you want to enter a formula, type an equal sign (=).
3. For example, type the formula A1+A2.

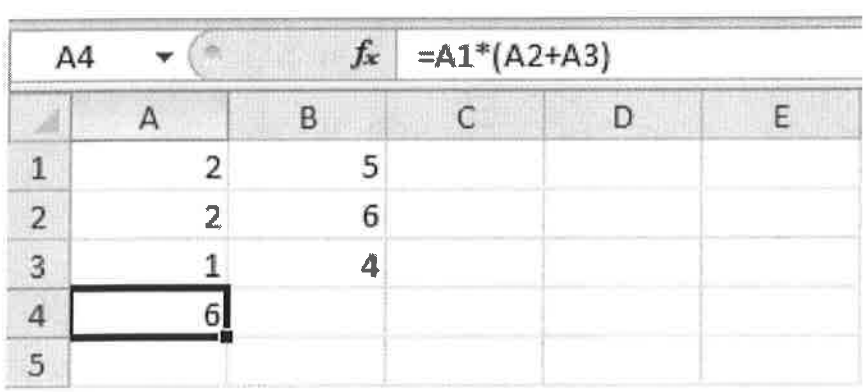


Tip: instead of typing A1 and A2, simply select cell A1 and cell A2.

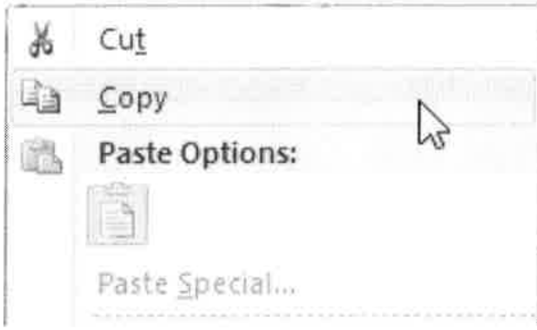
Copy/Paste a Formula

When you copy a formula, Excel automatically adjusts the cell references for each new cell the formula is copied to. To understand this, execute the following steps.

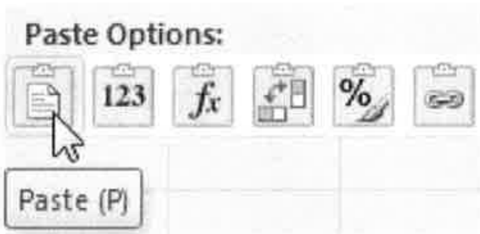
1. Enter the formula shown below into cell A4.



2a. Select cell A4, right click, and then click Copy (or press CTRL + c)...

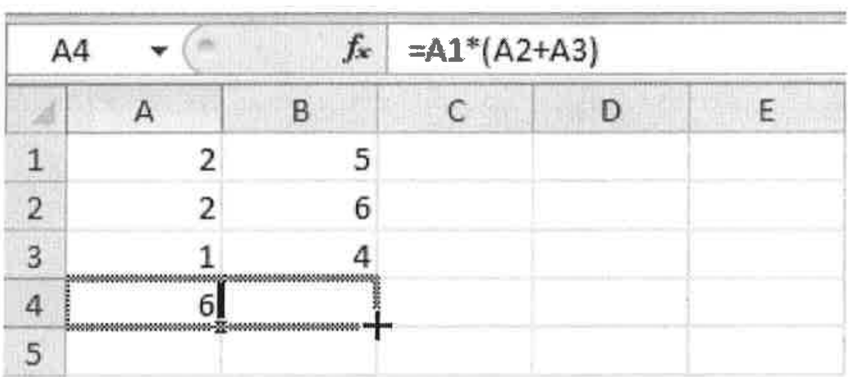


...next, select cell B4, right click, and then click Paste under 'Paste Options:' (or press CTRL + v).

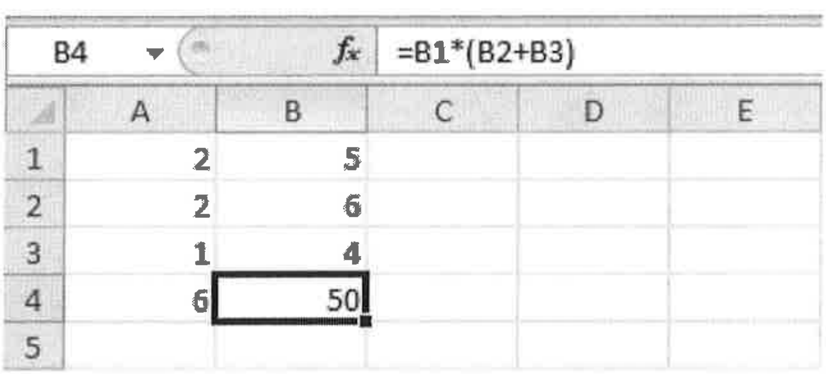


2b. You can also drag the formula to cell B4. Select cell A4, click on the lower right corner of cell (THE FILL HANDLE) A4 and drag it across to cell B4. This is much easier and gives the exact same result!

WE WILL CALL THIS ACTION "FILL ACROSS"



Result. The formula in cell B4 references the values in column B.



YOU CAN ALSO PERFORM "FILL DOWN" WHICH WOULD PRODUCE THE FORMULA =A2*(A3+A4) IN CELL A5.

Acceleration

The definition of average acceleration is $a_{ave} = \dfrac{v_2 - v_1}{t_2 - t_1}$

Consider the following situations and choose which of the answers (a) through (f) are true (there will be more than one true answer).

First the answers:

(a) v_1 and v_2 are both positive
(b) v_1 and v_2 are both negative
(c) $(v_2 - v_1)$ is a positive number
(d) $(v_2 - v_1)$ is a negative number
(e) a_{ave} is positive
(f) a_{ave} is negative

Now the situations:

(1) The object is moving in the positive direction and getting faster

(2) The object is moving in the positive direction and getting slower

(3) The object is moving in the negative direction and getting faster

(4) The object is moving in the negative direction and getting slower

Is it possible for an object to get faster but have a negative acceleration?

Is it possible for an object to get slower but have a positive acceleration?

Suppose an object is moving in the positive direction and then turns around so that it is moving in the negative direction. What is the sign of its acceleration.

At the bottom of each page describe in words what the actual motion of the man looked like.

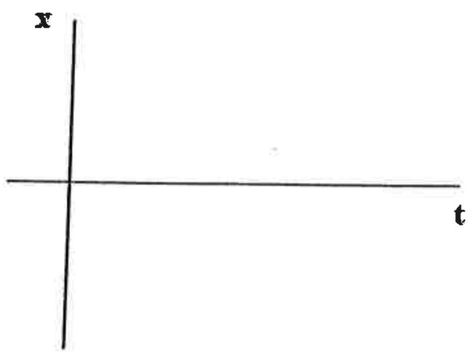
MOVING (WO)MAN **What does "constant acceleration" motion look like?**

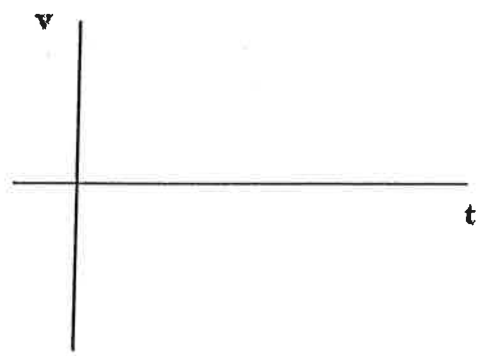
Open up the **Moving Man simulation.** (Go to the Home Page for the course, then to the Applets page and then to the simulation.) This simulation shows the motion of a man moving at a constant acceleration that you choose. *DO NOT CLICK Go! until you are told to.*

Set the initial values of position and velocity to the values **x = 5.0** and **v = -5.0**. This determines where the man starts and his velocity at the start.

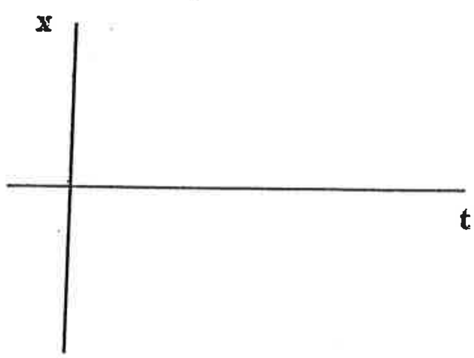
Now set the constant acceleration to a value of **a = 1.0**. *DO NOT CLICK Go! yet.*

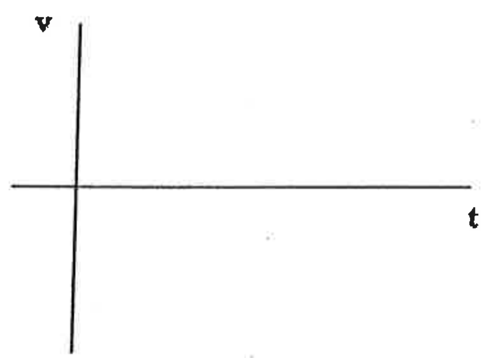
In the space below sketch your prediction for what the position and velocity graphs will look like for the man's motion.





Now you can click **Go!** and see the motion. Draw the correct graphs below.

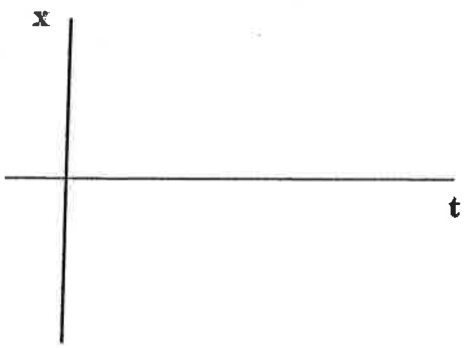


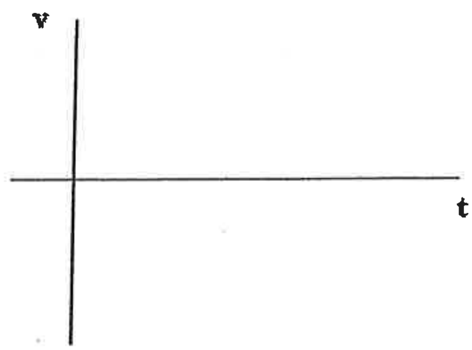


Now clear the graphs and repeat the exercise with the following values. Always make your prediction before running the simulation.

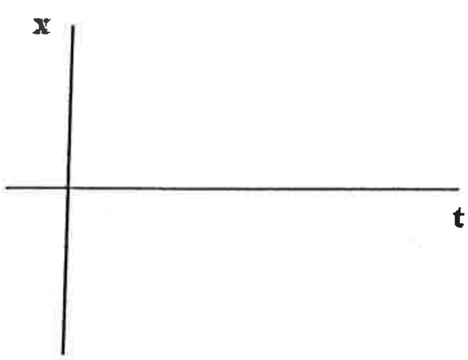
Initial position x = -2.0,   initial velocity = -5.0,   constant acceleration = 2.0

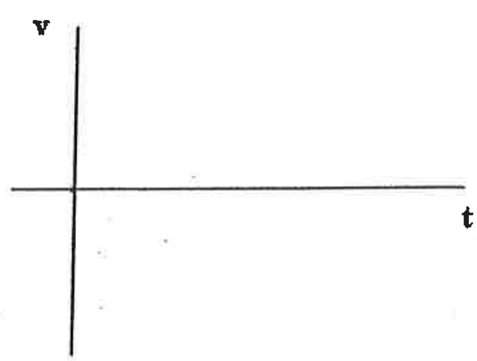
Prediction





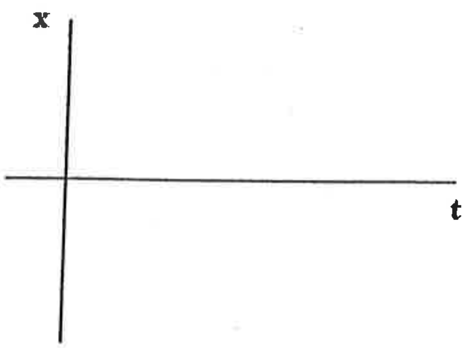
Correct graphs

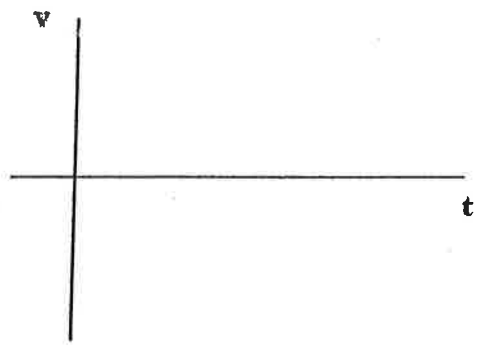




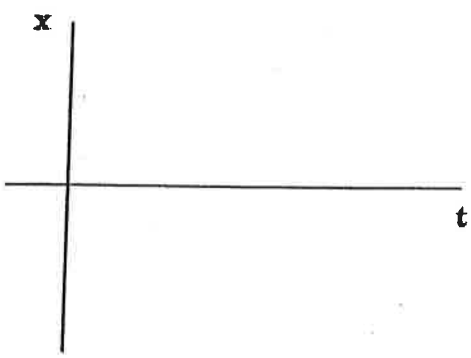
Initial position x = 0.0,   initial velocity = 2.0, constant acceleration = 2.0

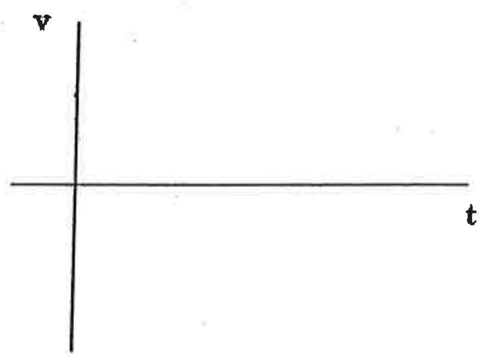
Prediction





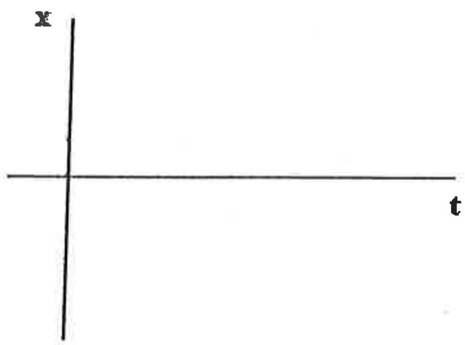
Correct graphs

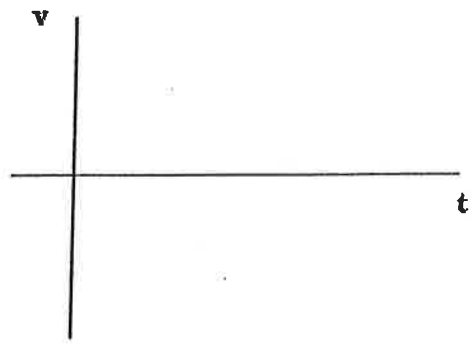




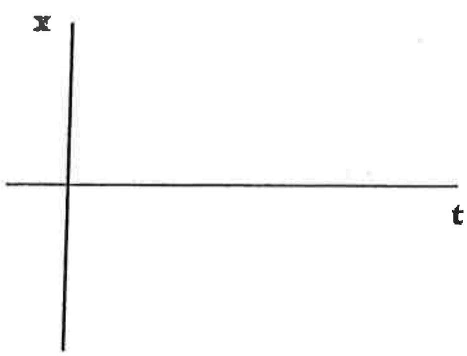
Initial position x = 5.0,   initial velocity = 2.0, constant acceleration = -0.5

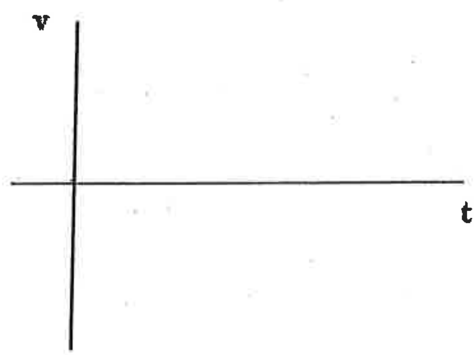
Prediction





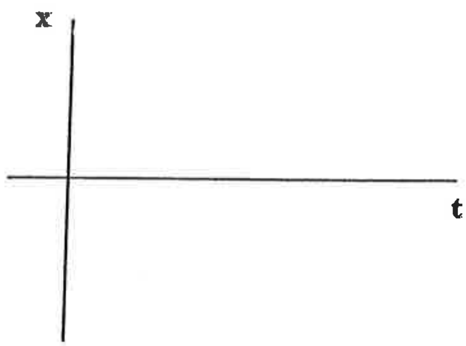
Correct graphs

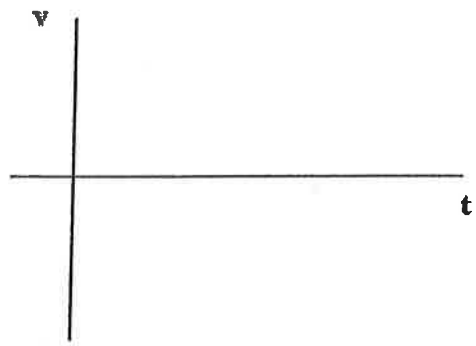




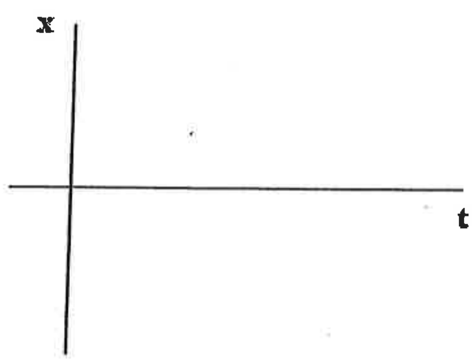
Initial position x = -8.0, initial velocity = 4.0, constant acceleration = -2.0

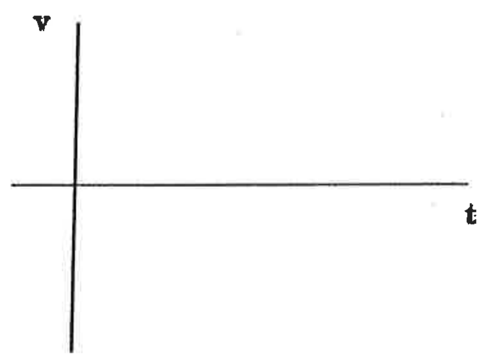
Prediction





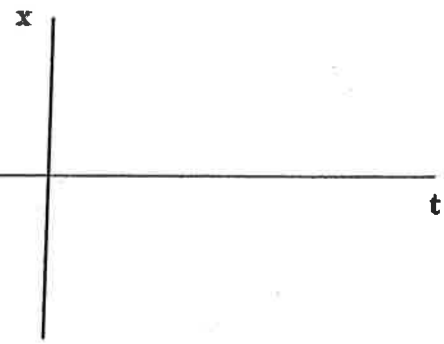
Correct graphs

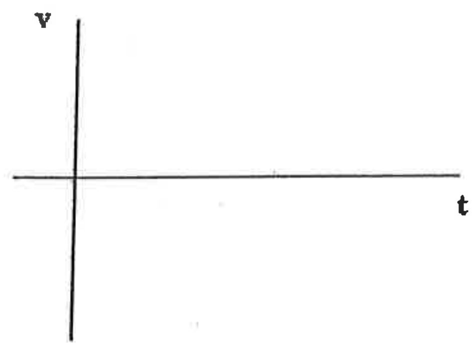




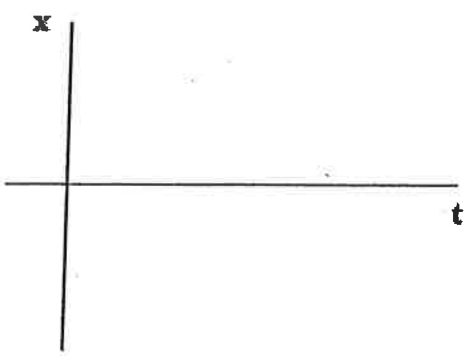
Initial position x = -4.0, initial velocity = 0.0, constant acceleration = 1.0

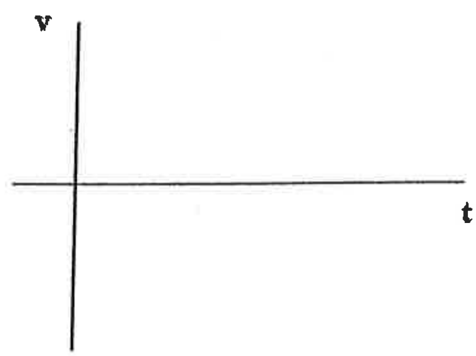
Prediction



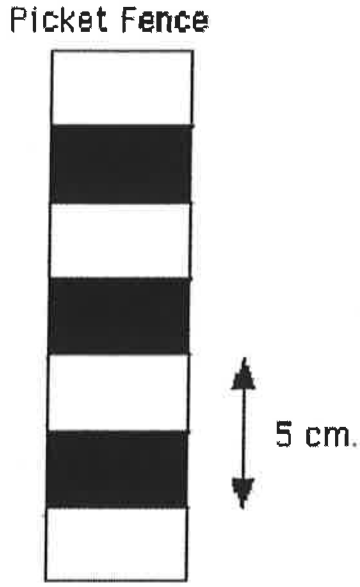


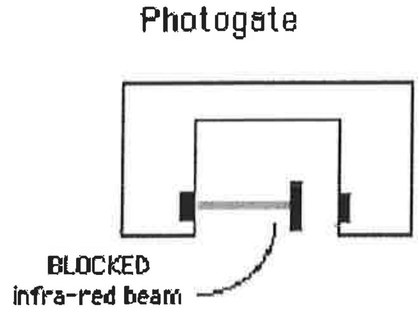
Correct graphs

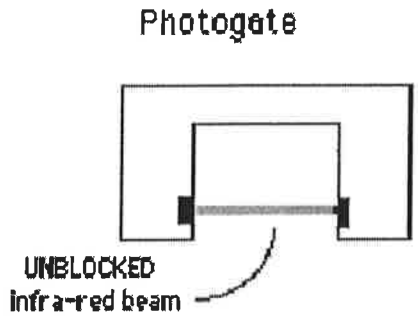




Acceleration due to gravity (g).







In this experiment we will use a photogate which is a device which emits an infra-red beam from one side and detects the beam at the other side.

Start up the program called **acc due to grav**. This program records the instants in time ("clock readings") when the photogate beam changes state (goes from <u>UNBLOCKED</u> to <u>BLOCKED</u> or vice versa).

(1) Click on Collect so that the program is ready to take the data. Then, ***making sure that someone catches the picket fence before it hits the ground***, drop the picket fence through the photogate. The data will appear in the table.

(2) Copy the data from the table and then paste it into an Excel spreadsheet. Delete the contents of the rows with no position data.

(3) Calculate the average velocity for each of the time intervals and assign this velocity to the mid-time of the time interval.

(4) Create a graph of velocity versus time (an x-y plot with no lines joining the dots).

(5) Fit a straight line to the data and note the slope of the line. The slope of the velocity graph is the acceleration of the falling object. SAVE THE SPREADSHEET

(6) Repeat the whole process two more times to get a total of three values for the acceleration due to gravity. (Save the spreadsheet each time.)

(7) Your picket fence had a weight attached to it. Now remove this weight and find the acceleration of this lighter picket fence as it falls. Do this a total of three times. (Save the spreadsheets.)

Heavy object		**Light object**	
RESULT 1	**g =**	**RESULT 1**	**g =**
RESULT 2	**g =**	**RESULT 2**	**g =**
RESULT 3	**g =**	**RESULT 3**	**g =**

Uncertainty in Experimental Data

Instrumental precision

The instrumental precision of a piece of data is determined by the smallest division on the instrument which you are using to make the measurement.

e.g. Suppose you are measuring a length using a meter stick which has smallest divisions of 1 mm. You may feel that you can mentally sub-divide those smallest divisions in half and so the instrumental precision of your readings will be 0.5 mm. A typical measurement would then be 2.35 cm with an uncertainty of $\pm$ 5 in the last digit. If you only made this one measurement of the length you would write your result as 2.35 $\pm$ .05 cm.

If you measured the length using a vernier calipers which has a smallest division of 0.1 mm then your result might be 2.37 $\pm$ .01 cm.

Accuracy

In the previous example you may have made an error in reading the vernier calipers and the correct length is really 3.37 $\pm$ .01 cm. The accuracy of your result is therefore lousy (you are off by 1 cm) even though you have an instrumental precision of .01 cm.

What are the factors which effect the accuracy of your data?

There are several sources of error which can effect the accuracy of your data. They include personal, instrumental and accidental error. The example given above is a case of personal error i.e. you make a mistake in taking the reading. Instrumental error is when the instrument you are using to make the measurement is flawed in some fashion. We will be most interested in the third case, **accidental error** (also known as **random error**).

Random error becomes apparent when you take the same measurement several times and find that your readings are not all the same - there is a spread in the values obtained. This spread in values is due to random processes beyond the control of the experimenter e.g. vibrations, noise, temperature fluctuations, changes in air pressure etc. any of which may affect your readings.

Analysis of data subject to random error.

If you have several readings for the same measurement then the best estimate for that measurement is the average (or mean) of your readings.

Since there is uncertainty associated with the measurement you must give some indication of the size of the uncertainty. The method for calculating uncertainty due to random error is best illustrated by an example. Suppose you have five readings for a length measurement taken using a vernier calipers (i.e. instrumental precision of $\pm$ .01 cm).

Analysis of the uncertainty uses the **"standard deviation"** and **"standard error"**.

To calculate the **standard deviation** take the sum of the squared deviations from the mean, divide by (number of readings - 1) and then take the square root.

To calculate the **standard error** divide the standard deviation by the square root of the number of readings.

For example, if we have the following five readings for the same measurement:

length readings (cms)	deviation from the mean	Squared deviation
2.3700	0.0240	0.0006
2.3000	-0.0460	0.0021
2.3300	-0.0160	0.0003
2.3900	0.0440	0.0019
2.3400	-0.0060	0.0000

Mean = 2.3460 cm standard deviation = 0.0351 cm

standard error = 0.0157 cm

The best estimate of your measurement is the mean value 2.346 cm .

The best estimate for the uncertainty is the standard error 0.016 cm.

We would therefore express our result for the length measurement as

2.346 ± .016 cm (We have taken the standard error to two significant figure.)

or

2.35 ± 0.02 cm (We have taken the standard error to one significant figure.)

Never quote your result to an accuracy better than your uncertainty allows.

E.g. if my uncertainty is ± 0.02 cm this means that I am not confident about the exact value of the second decimal place of my result. It makes no sense therefore to quote a value for the third decimal place. So I can only quote the result to the second decimal place i.e. 2.35 cm.

This is a situation where the uncertainty due to random error is larger than the instrumental instrumental precision of the instrument used. But what if the uncertainty due to random error is smaller than the instrumental precision of the instrument? Suppose the uncertainty due to random error is ± .02 cm but the instrumental precision of the instrument is ± .05 cm. (i.e. you used a meter stick instead of a vernier calipers). The result would be written 2.37 ± .05 cm. since the instrumental precision is a larger source of uncertainty than the random error.

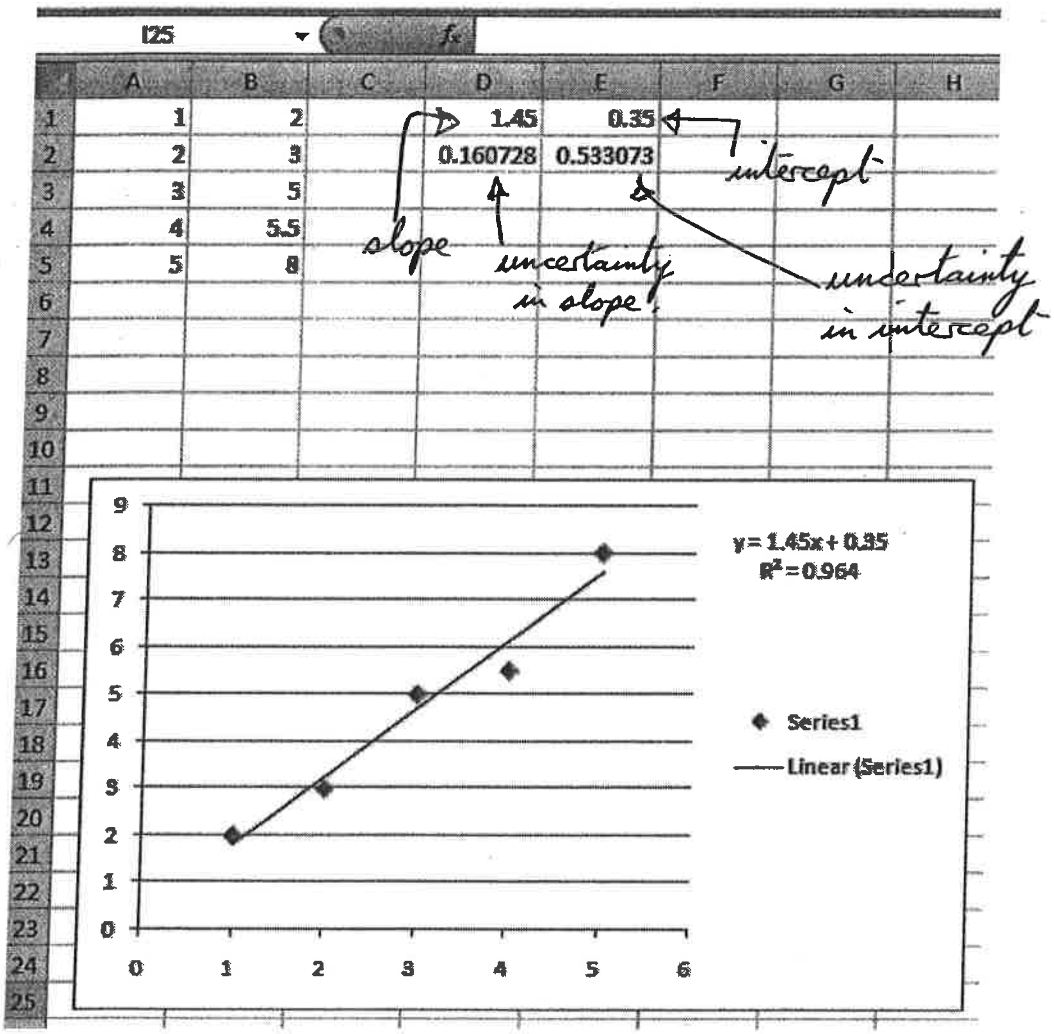
The rule is: of the two sources of uncertainty in your result (instrumental precision and random error) use the larger.

To get the uncertainty in the slope and intercept for a linear fit to data you can use the function LINEST.

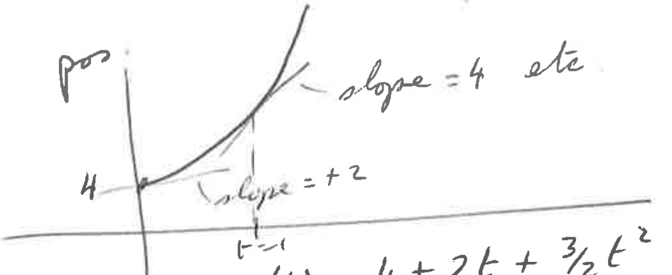
For instance, in the sheet below there is data in column A (x-values) and in column B (y-values). There is already a graph with a linear fit showing the equation of the fit. But I want to get the uncertainty in the slope and intercept.

First select cell D1 (say) and then click and drag to highlight the cells D1,E1,D2,E2.
Go up to the input line and type in =LINEST(B1:B5,A1:A5,,TRUE)
<u>Notice the double comma.</u>
Then hit CTRL/SHIFT/ENTER.

The four highlighted cells should now contain the following values:
D1 = slope D2 = uncertainty in slope
E1 = intercept E2 = uncertainty in intercept

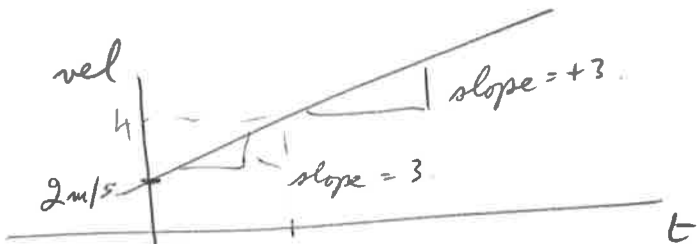


Derive const. accel. equations

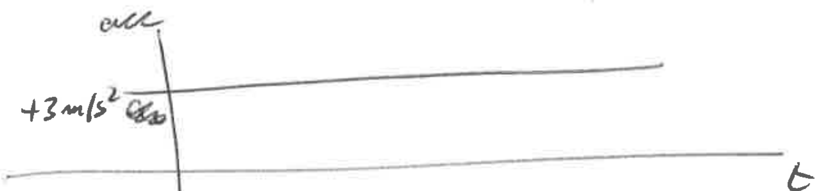


$x(t) = 4 + 2t + \frac{3}{2}t^2$

$\boxed{x = x_0 + v_0 t + \frac{1}{2} a_0 t^2}$



$\boxed{v(t) = 2 + 3t}$ $\boxed{v = v_0 + a_0 t}$



$\boxed{a(t) = 3}$ $\boxed{a = a_0}$

What would position graph look like? What extra information is needed.

① what would vel graph look like? (what extra information is needed)

② What is the slope of the line?

③ What is equation of this straight line?

start here

Supposedly they have already seen a lot a const. acc. motion graphs.

$$\boxed{x = x_0 + v_0 t + \frac{1}{2} a_0 t^2} \quad \text{—— (1)}$$

$$\boxed{v = v_0 + a_0 t} \quad \text{—— (2)}$$

$$t = \frac{v - v_0}{a_0}$$

$$\therefore \quad x = x_0 + v_0 \left(\frac{v - v_0}{a_0} \right) + \frac{1}{2} a_0 \left(\frac{v - v_0}{a_0} \right)^2$$

$$= x_0 + \frac{v v_0}{a_0} - \frac{v_0^2}{a_0} + \frac{1}{2} \cancel{a_0} \left(\frac{v^2 - 2 v v_0 + v_0^2}{a_0^{\cancel{2}}} \right)$$

$$= x_0 + \cancel{\frac{v v_0}{a_0}} - \frac{v_0^2}{a_0} + \frac{1}{2} \left(\frac{v^2 + v_0^2}{a_0} \right) - \cancel{\frac{v v_0}{a_0}}$$

$$\therefore \quad (x - x_0) = \frac{1}{2} \left(\frac{v^2 + v_0^2}{a_0} \right) - \frac{v_0^2}{a_0}$$

$$\therefore \quad 2 a_0 (x - x_0) = v^2 + v_0^2 - 2 v_0^2 = v^2 - v_0^2$$

$$\therefore \quad \boxed{v^2 = v_0^2 + 2 a_0 (x - x_0)} \quad \text{—— (3)}$$

(1), (2), (3) are not <u>independent</u>

So only can use two of them.

What have we been doing?

① Defining our terms rigorously

② Connecting physical reality to the mathematical tools we use to describe that reality.

③ Investigating the nature of the physical world by experimentation. (and exploring the limits of our experiments).

In particular, we have been focusing on <u>the motion of objects</u>. Experimentally determining what that motion looks like & how we can represent this motion mathematically.

Then we can use the mathematics to make predictions.

What have we <u>not</u> been doing? (I hope!)

④ Memorizing equations & plugging numbers into them.

⑤ Taking received knowledge & blindly believing it.

⑥ Using terms (words) ~~that~~ whose meaning we are not ~~so~~ ~~so~~ clear about.

I could have "covered" the material in 2 classes and many of you ~~all~~ would have been doing 4, 5 and 6. <u>THAT IS NOT SCIENCE!</u>

<u>Displacement Vectors Represented as Arrows</u>

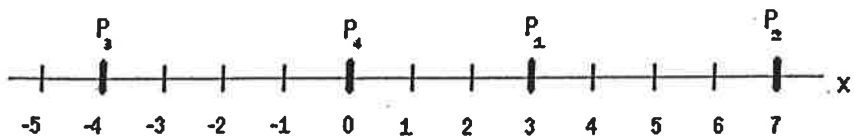
A <u>vector</u> is a quantity that has both magnitude and direction.

Consider two motions:

(A) Object starts at P1 and goes to P2.

(B) Object starts at P3 and goes to P4.

Represent motions A and B as arrows on the x-axis.



NOTE: The two "motions" are different but we consider the two "displacements" (or "displacement vectors") to be the same. In words they could both be described in terms of magnitude and direction as "*4 meters to the right*".

Now consider a more complicated motion:

(C) Object starts at P1, goes to P3 and then goes to P2.

Represent "motion" (C) using two "displacement" arrows.

We can think of the <u>total</u> displacement (start at P1, end at P2) as the <u>sum</u> of these two displacements.

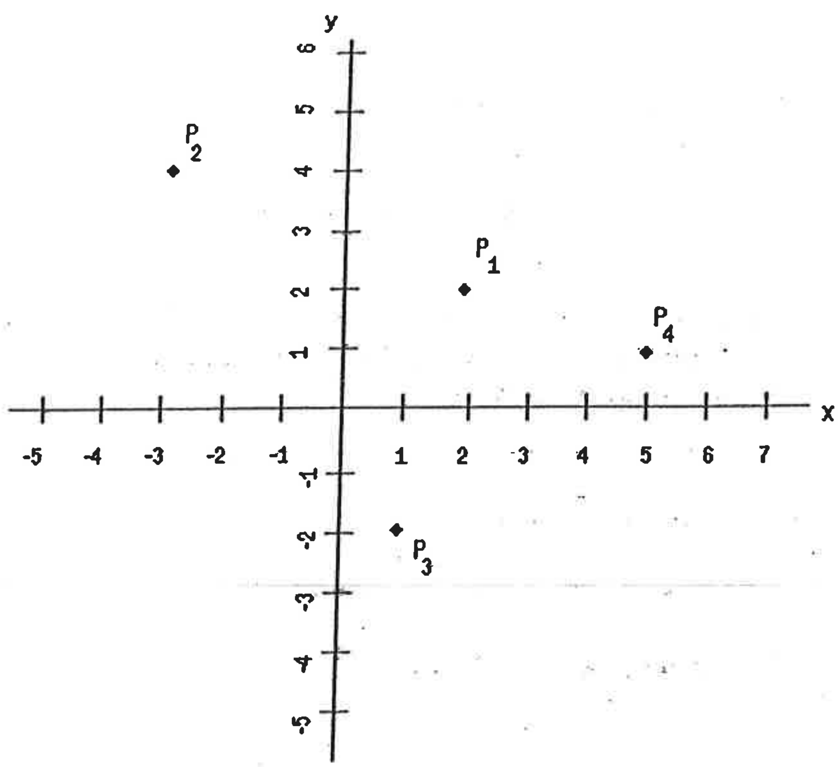
Draw a single arrow representing the <u>total</u> displacement.

Describe all three displacements in words.

Now let's look at displacements in two dimensions.

(D) Object starts at P1, goes to P3 and then goes to P2.

Represent "motion" (D) using two "displacement" arrows on the co-ordinate system below.



We can think of the <u>total</u> displacement (start at P1, end at P2) as the <u>sum</u> of these two displacements.

Draw a single arrow representing the <u>total</u> displacement.

In both the one-dimensional and two-dimensional case check to see if the following statement is true:

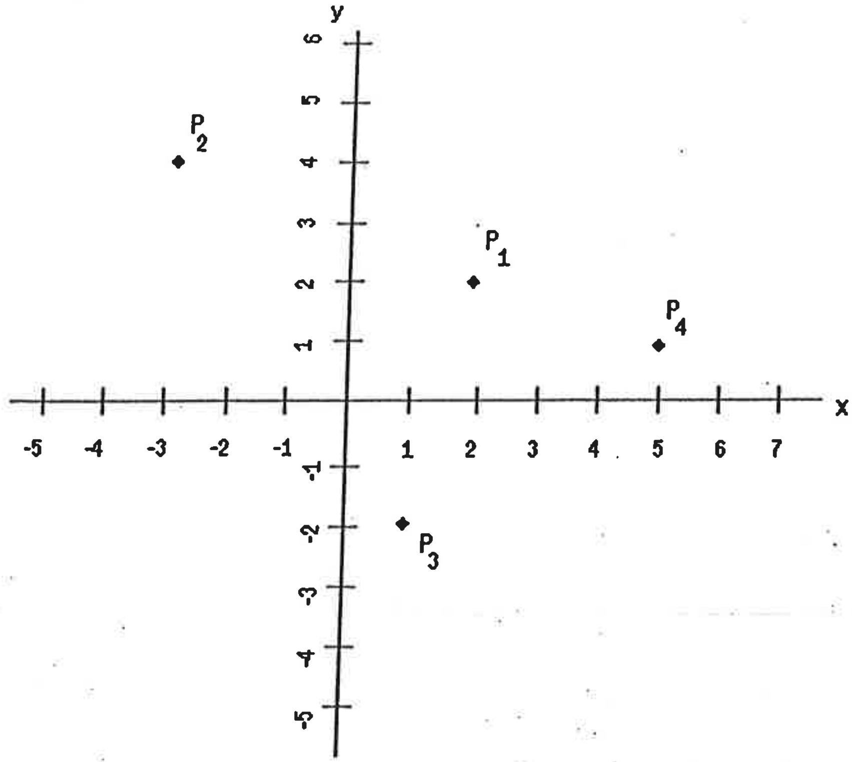
"To add two vectors together put the tail of one arrow touching the head of the other arrow. The vector sum is then a new arrow starting at the free tail and ending at the free head."

What is the negative of a vector?

Well for "scalars" (just regular numbers) <u>if we add a quantity to its negative we get zero</u>. The same is true for vectors.

On the diagram below draw the displacement vector representing the motion

(E) Object starts at P1 and goes to P4



Now draw the negative of this vector i.e. draw the vector that, when added to our first vector, gives zero displacement.

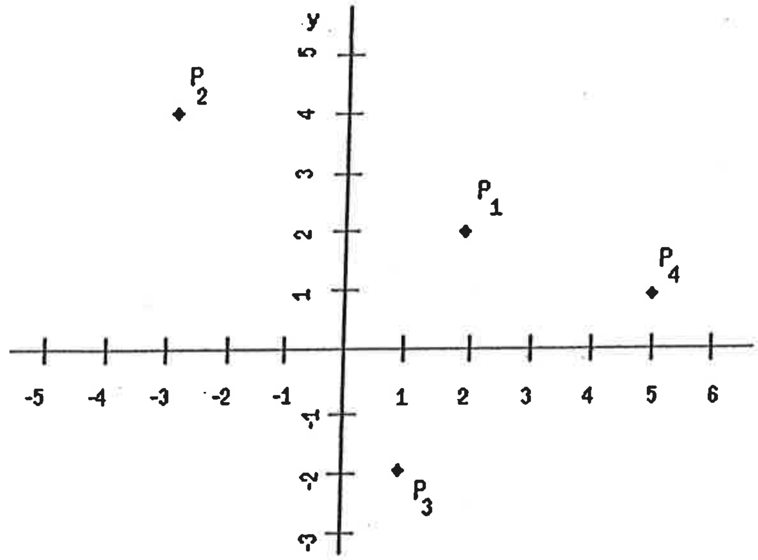
Does what you did correspond to the following statement?

"To get the negative of a vector simply reverse the direction of the arrow."

How to subtract two vectors.

With scalars we can think of subtraction as adding a negative   e.g. $6 - 2 = 6 + (-2) = 4$

We can do the same with vectors.

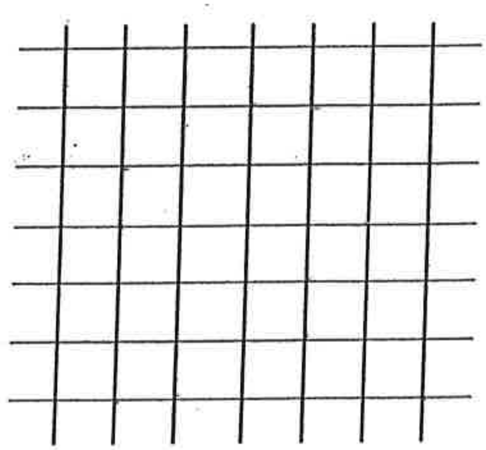


Consider the two vectors

D1 = displacement starting at P1 and ending at P3

D2 = displacement starting at P1 and ending at P4

Draw **D1** and **D2** as arrows on the sketch above and then construct the difference in the two vectors (**D2 – D1**) as an arrow on the grid below.
Use the idea that D2 – D1 = D2 + (-D1).



For the two-dimensional vectors it is trickier to describe them in words. Take, for instance, vector D1 on the last page. How would you describe it in words?

It's a bit longer than 4 meters and it's sort of down and to the left.

Not accurate enough!! Exactly how long is it? Exactly what direction is it pointing? (We will come back to those questions later!)

Might it be easier to just say: **"D1 is the same displacement that would result if the object had moved 4 meters downward and 1 meter to the left".**

This way of describing the vector is called "*component form*". Just break the vector down into two parts, one pointing along the x-axis and one pointing along the y-axis.

There is a notation we can use to make it easier to write a vector in component form.

The symbol that is used to mean "in the positive x-direction" is **i**.
[(- **i**) means "in the negative x-direction"]

The symbol that is used to mean "in the positive y-direction" is **j**.
[(- **j**) means "in the negative y-direction"]

Using this notation D1 can be written as

$$\mathbf{D1} = 1.0\,(-\,\mathbf{i}) + 4.0\,(-\,\mathbf{j})\ \text{meters} = -\,1.0\,\mathbf{i} - 4.0\,\mathbf{j}\ \text{m}$$

- 1.0 m is the x-component of **D1** and **- 4.0 m** is the y-component of **D1**

Write **D2** in component form.

$$\mathbf{D2} = \qquad \mathbf{i} + \qquad \mathbf{j}$$

What is the value of the x-component of **D2**?

What is the value of the y-component of **D2**?

Write **D2 - D1** in component form. (*Consider the arrow you already drew for this vector)*

$$\mathbf{D2} - \mathbf{D1} = \qquad \mathbf{i} + \qquad \mathbf{j}$$

What is the value of the x-component of **D2 - D1**?

What is the value of the y-component of **D2 - D1**?

Vectors

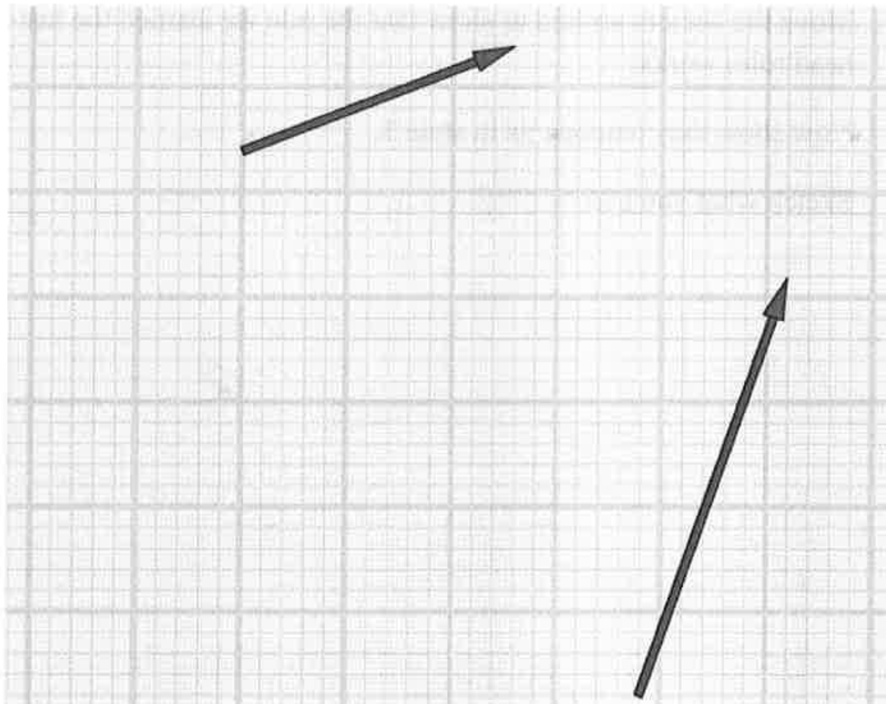
Open the **Vector Applet** in the applets window of the course web site. Run the applet.

Create the two vectors

(7 **i** + 20 **j**)and (13 **i** +5 **j**)

and place them on the screen as shown.

Look at the components of these vectors in the three different styles. Sketch the three styles below.



Now remove the component display and check the Show Sum box.

Move the vectors around to show that the rule we learned the last day for adding vectors (tail-to-head rule) works.

Now show the components in style 3.

Sketch what you see.

Notice that the x-component of the sum (green) is just the sum of the two x-components (red).

The y-component of the sum (green) is just the sum of the two y-components (red).

So this leads us to a new rule for adding vectors.

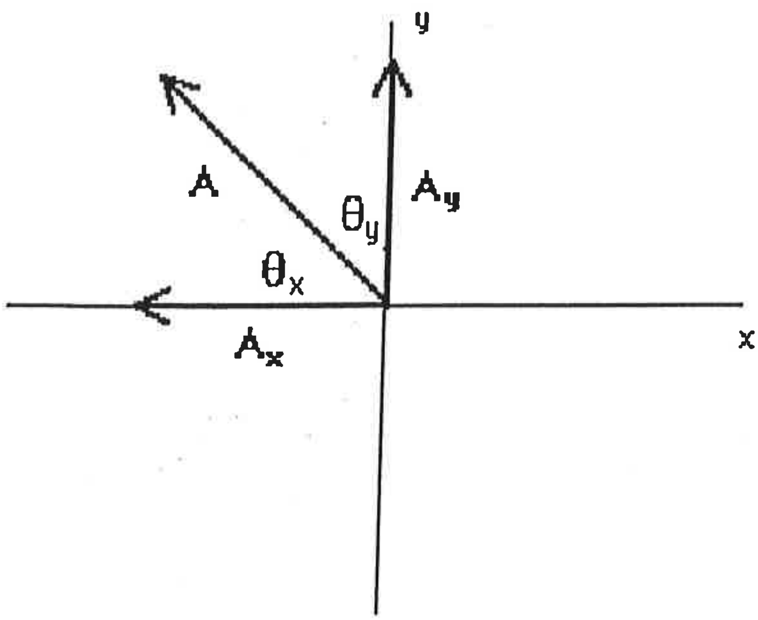
Add the x-components together to get the total x-component.

Add the y-components together to get the total y-component.

Two ways to take components

(Note that in the example below the x-component is negative and I have to know to put in the minus sign. This is always the case when you use angles that are less than 90 degrees.)

(A) **Always use the angle with the x-axis (θ_x).** In this method you use a Cosine function to get the x-component and a Sine function to get the y-component.



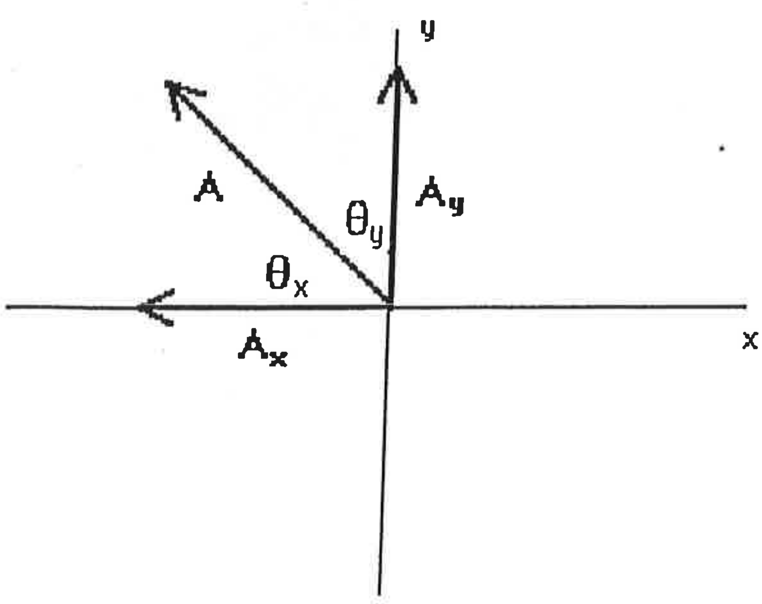
In this case

$$A_x = -A\cos\theta_x$$

and

$$A_y = A\sin\theta_x$$

(B) **Always use the Cosine function.** In this method you use the angle with the x-axis to get the x-component and the angle with the y-axis to get the y-component.

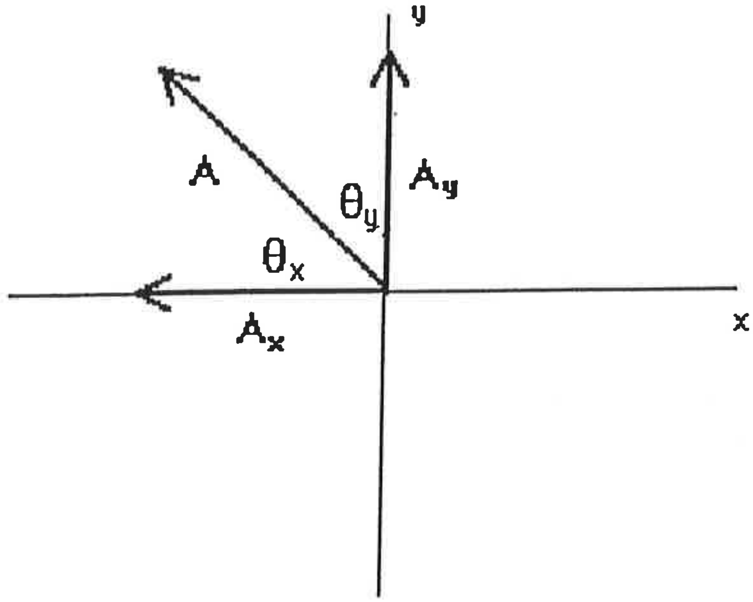


In this case

$$A_x = -A\cos\theta_x$$

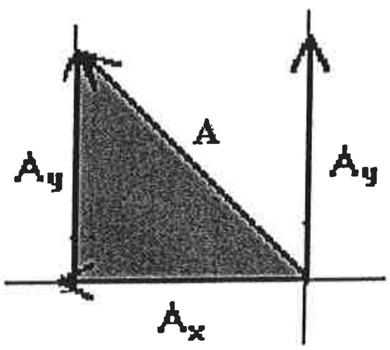
and

$$A_y = A\cos\theta_y$$



The magnitude of a vector is always given by the sum of the squares of its components.

$$A = \sqrt{A_x^2 + A_y^2}$$

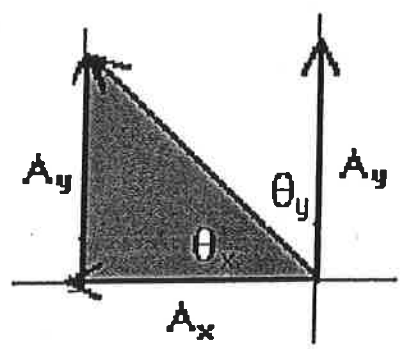


This is just **Pythagoras' Theorem** applied to the shaded triangle shown to the left.

The angles can be found using the inverse tangent function (also called arctan)

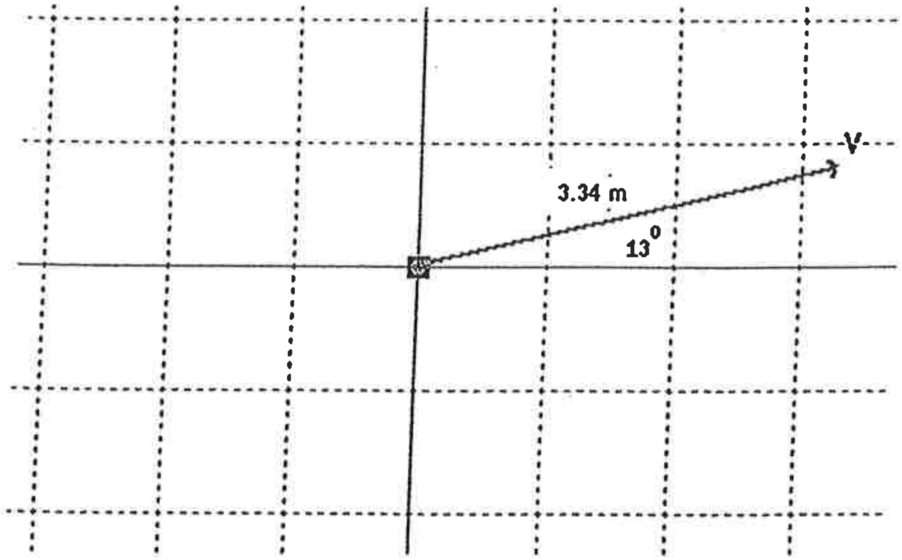
$$\theta_x = tan^{-1}\left(\frac{|A_y|}{|A_x|}\right)$$

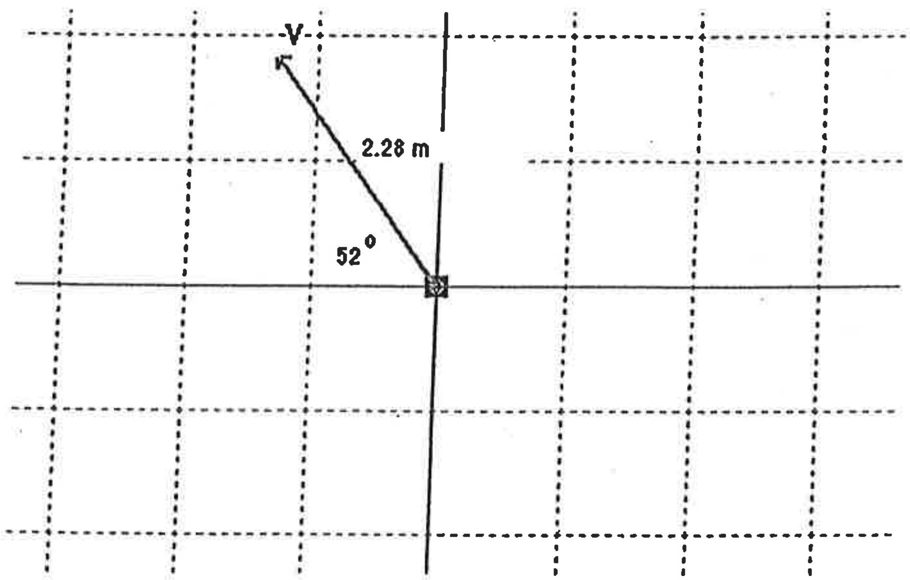
I remember this by looking at the shaded triangle and realizing that $\tan(\theta_x) = \frac{opposite}{adjacent} = \left(\frac{|A_y|}{|A_x|}\right)$



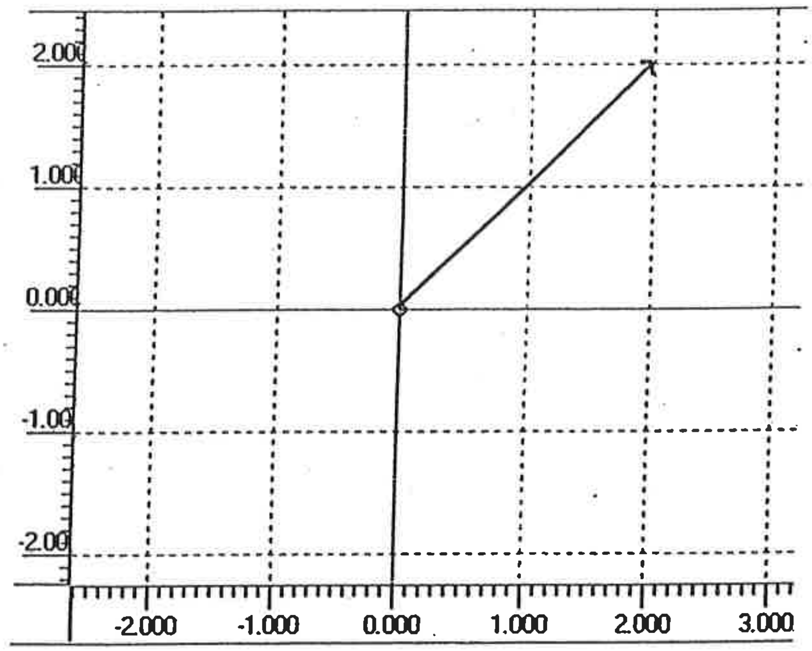
Then $\theta_y = 90 - \theta_x$ $\quad$ *or* $\quad$ $\theta_y = tan^{-1}\left(\frac{|A_x|}{|A_y|}\right)$

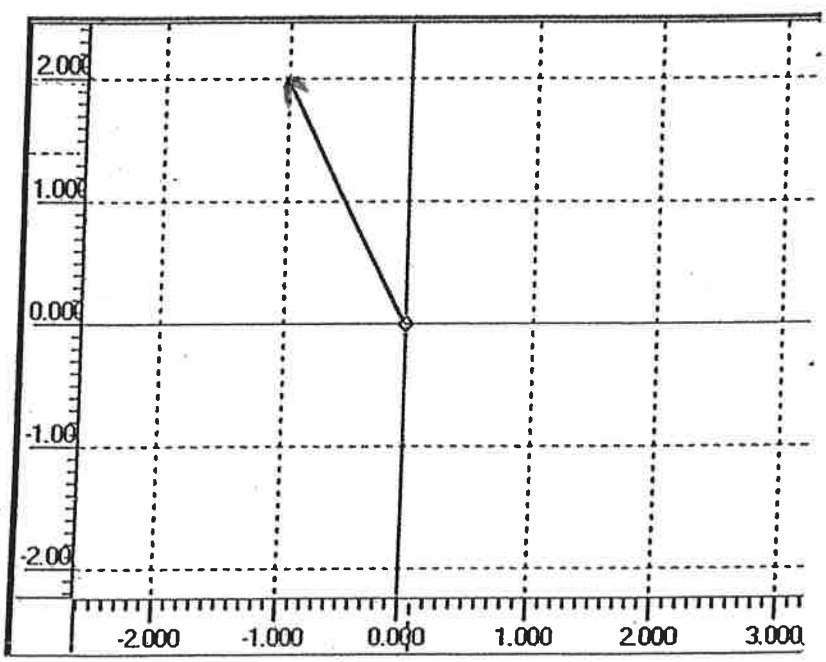
For each example write the vector in component form.





For each example, write the vector in component form. Then calculate the magnitude and the angle made with each axis (angles less than 90 degrees please!)





Prob. 3-23

Scale: 1 small square = 1 cm.

Specifying directions.

In the horizontal plane.

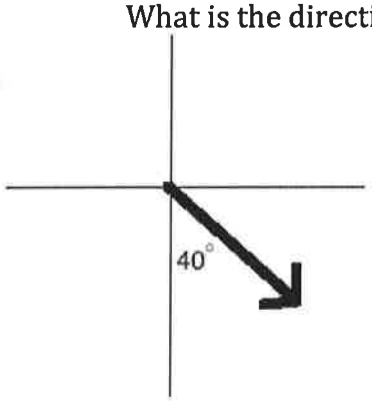
What do the following statements mean to you? (draw an arrow showing this direction and be as specific as possible)

"I am traveling SW."

"I am traveling in a direction 30 degrees SW."

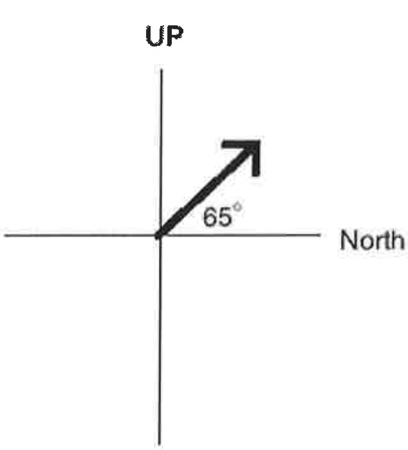
"I am traveling 30 degrees S of W."

What is the direction of this vector? (use the normal convention for NSEW)

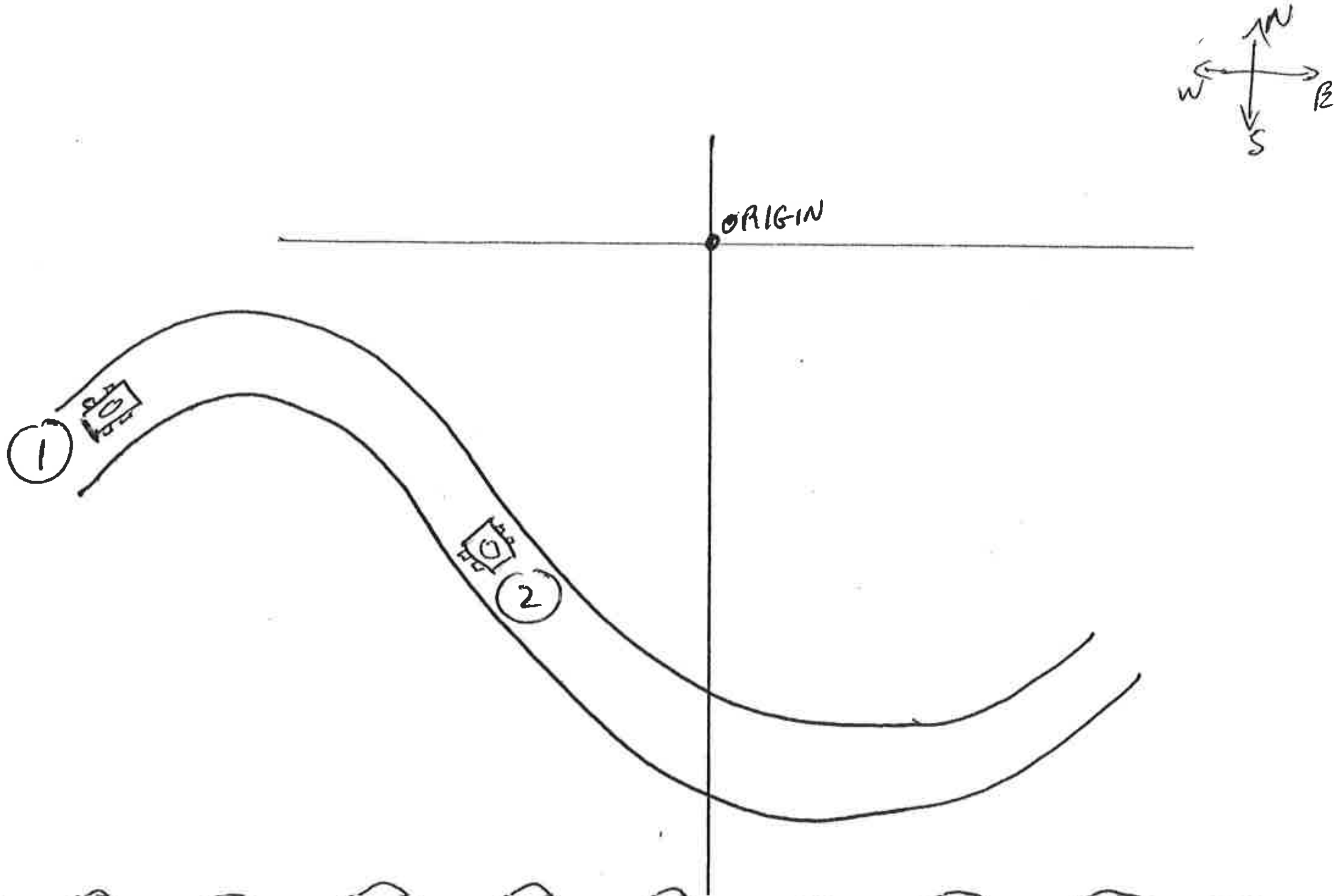


In the vertical plane.

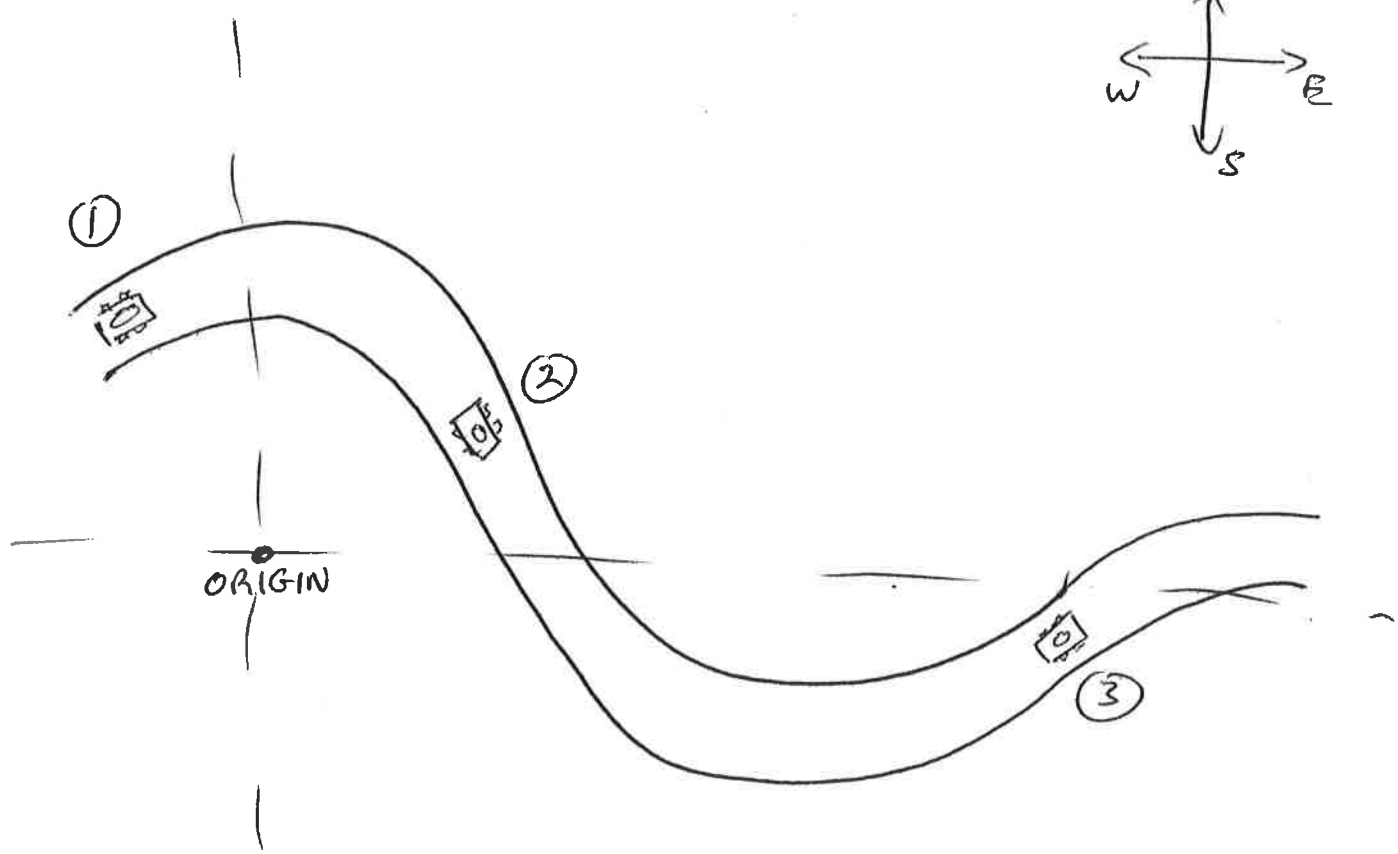
How would you describe this vector in words?



A



B



Draw the <u>position</u> vectors representing the position of the car at instants 1 and 2.

Call these vectors $\vec{r}_1$ and $\vec{r}_2$.

Draw the vector representing the displacement ($\Delta\vec{r}$) of the car during the time interval between instants 1 and 2.

Graphically construct the vector $(\vec{r}_2 - \vec{r}_1)$ in the space below.

($\vec{r}_1$ is the car's position vector at instant 1)

($\vec{r}_2$ is the car's position vector at instant 2)

OVER

What is the direction of the <u>average</u> <u>velocity</u> of the car during this time interval? (between 1 and 2)

What is the direction of the <u>instantaneous</u> velocity $\begin{cases} \text{at instant 1 ?} \\ \text{at instant 2 ?} \end{cases}$

I did this for them

2-Dimensional Projectile Motion

Go to the START menu button and from the list of applications open up VideoPoint 2.1

Click anywhere in the first window that comes up and then ask it to open a movie. The movie you want is called PASCO108 and is in the PHYS150 folder on the desktop. Tell the program that you want to follow the motion of one object.

The movie shows a small steel ball in free-fall moving in 2 dimensions. Click on the ball in all twenty frames of the movie (the movie will automatically forward to the next frame when you click on the ball). DON'T USE THE FIRST FRAME DATA.

Now set the scale of the movie by clicking on the meter stick icon on the left of the screen. You will be using a one-meter long vertical stick that appears in the movie as the object of known length. Follow the instructions on the screen.

Check that your data appears in the data table.

Now produce the following graphs

(i) x-position versus time (get a linear fit)
(ii) y-position versus time
(iii) y-velocity versus time (get a linear fit)

From these graphs what can you deduce about:

(a) the x-component of velocity of the ball
(b) the x-component of acceleration of the ball
(c) the y-component of acceleration of the ball

From the data figure out the initial velocity of the ball in component form and in magnitude and direction form.

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New Groups

Forces

Pushes & pulls. VECTORS

Force BY an object ON another object.

Most are "contact" forces.

Normal force. — a force BY a frictionless surface ON an object touching the surface. (Perpendicular to the surface)

Tension Force. — a force BY a rope or wire or string ON an object attached to the string

Gravitational Force of Earth (also called WEIGHT force) — a force BY the planet Earth ON an object in the vicinity of Earth

Not a contact force.

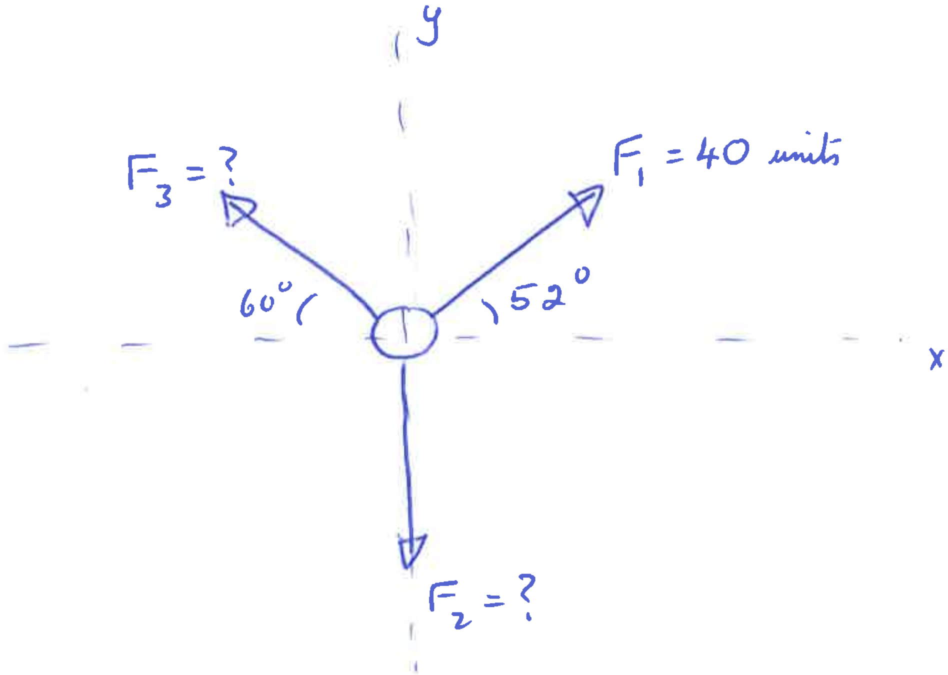
Friction force — a force BY a surface ON an object touching the surface (parallel to the surface).

Newtons Laws of Motion

Net force — vector sum of all the actual forces acting ON an object

Applied force — generic term for the force exerted BY someone or something ON an object.

Consider an object (a plastic ring) which has three pulling forces exerted <u>ON</u> it, as shown.



F_1, F_2 and F_3 are the magnitudes of the forces.

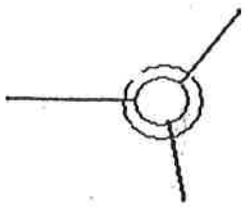
If the ring is at rest then Newton's 1st Law states that the net (total) force on the ring equals ZERO.

In terms of F_2 and F_3 find the net force in component form. Set each component equal to zero and solve for F_2 and F_3.

Reproduce this set-up on the tables.

Newton's First Law

Now apply three tension forces to the ring so that the ring is balanced and stationary at the center of the table. MAKE SURE THE STRINGS POINT DIRECTLY TO THE MIDDLE OF THE RING AND THAT THE RING IS EXACTLY IN THE CENTER OF THE TABLE.



Use the angular scale on the table to figure out the x- and y-components of the three tensions.

Add up all the x-components of the forces. This is the x-component of the total force on the ring.
Add up all the y-components of the forces. This is the y-component of the total force on the ring.

Show your work in the space below.

Are the components close to zero? How far off from zero are they? Are they <u>significantly</u> different from zero? Explain.

How to calculate the uncertainty in a result derived from several pieces of data.

If you have several pieces of data, each of which has an error associated with it, how do you calulate the error in a combination of these pieces of data? Here are the rules. (In general I will round the error off to one significant figure.)

(1) Multiplication of a piece of data by a constant: multiply the absolute error by the same constant

e.g. $3 * (6 \pm 1) = 18 \pm 3$

(2) Addition and subtraction of two or more independent pieces of data: add the squares of the **absolute** errors and then take the square root

e.g. $(6 \pm 1) + (3 \pm 2) = 9 \pm \text{sqrt}(1^2 + 2^2) = 9 \pm 2.2 = 9 \pm 2$

(taking the error to one significant figure)

e.g. $(5.0 \pm 0.1) - (4.0 \pm 0.3) = 1.0 \pm \text{sqrt}(0.1^2 + 0.3^2) = 1.0 \pm 0.32 = 1.0 \pm 0.3$

(the result should not contain more significant figures than the error warrants)

(3) Multiplication and division of two independent pieces of data: add the squares of the **percentage** errors and then take the square root. This gives the percentage error of the result (and you may then convert back to absolute error if necessary).

e.g. $(6 \pm 1) * (13 \pm 2) = (6 \pm 17\%) * (13 \pm 15\%)$

$= 78 \pm \text{sqrt}(17^2 + 15^2)\ \% = 78 \pm 23\ \%$

$= 78 \pm 18$

If we round off the error to one significant figure then this becomes 80 ± 20.

(4) Raising a piece of data to a power: multiply the **percentage** error by the power (and then convert back to absolute error if necessary)

e.g. $(5.0 \pm 0.1)^3 = (5.0 \pm 2\%)^3 = 125 \pm 6\% = 125 \pm 8$

Note: the error in squaring a piece of data is not calculated as if it were the multiplication of two *independent* pieces of data (since it is the multiplication of a piece of data by itself and not by another independent piece of data).

Example:

Suppose $y = x^2/z$ and $x = 5.0 \pm 0.2$ $z = 30.0 \pm 0.5$

Then $x = 5 \pm 4\%$ so $x^2 = 25 \pm 8\%$

and $z = 30 \pm 1.7\%$

Therefore $y = 25/30 \pm \text{sqrt}(8^2 + 1.7^2)\ \% = 0.83 \pm 8.2\% = 0.83 \pm 0.07$

Problems: $b = 3.0 \pm 0.1$ $c = 1.6 \pm 0.1$ $d = 7.2 \pm 0.1$ $a = ? \pm ?$

(1) $a = b - c$

(2) $a = b * c$

(3) $a = (b + c)/d$

(4) $a = b^2/(c - d)$

(5) $a = b/(c + d)^3$

(6) $a = b^2 + c^2$

(7) $a = (b^2 + c^2)^{1/2}$

Please give your answer in terms of absolute uncertainty. Remember, your final result in each case should only have as many decimal places as your uncertainty warrants.

$\boxed{b = 3.0 \pm \left(\frac{0.1}{3.0} \times 100\%\right) = 3.0 \pm 3.3\%.}$ $\boxed{c = 1.6 \pm \left(\frac{0.1}{1.6} \times 100\%\right) = 1.6 \pm 6.3\%}$

Problems: $b = 3.0 \pm 0.1$ $c = 1.6 \pm 0.1$ $d = 7.2 \pm 0.1$ $a = ? \pm ?$

(1) $a = b - c$ (2) $a = b * c$ (3) $a = (b + c)/d$ $d = 7.2 \pm \left(\frac{0.1}{7.2} \times 100\%\right)$

(4) $a = b^2/(c - d)$ (5) $a = b/(c + d)^3$ (6) $a = b^2 + c^2$ $\boxed{\therefore d = 7.2 \pm 1.4\%.}$

(7) $a = (b^2 + c^2)^{1/2}$

Please give your answer in terms of absolute uncertainty. Remember, your final result in each case should only have as many decimal places as your uncertainty warrants.

(1) $(3.0 \pm 0.1) - (1.6 \pm 0.1) = (3.0 - 1.6) \pm \sqrt{0.1^2 + 0.1^2}$

$= 1.4 \pm 0.14$ or $\boxed{1.4 \pm 0.1.}$

---

(2) $(3.0 \pm 0.1) * (1.6 \pm 0.1) = (3.0 \pm 3.3\%) * (1.6 \pm 6.3\%)$

$= (3.0 * 1.6) \pm \sqrt{3.3^2 + 6.3^2}\ \%.$

$= \boxed{4.8 \pm 7\%} = 4.8 \pm \left(\frac{4.8}{100} * 7\right)$

$= \boxed{4.8 \pm 0.3}$

---

(3) $(b + c) = (3.0 \pm 0.1) + (1.6 \pm 0.1) = 4.6 \pm \sqrt{0.1^2 + 0.1^2} = 4.6 \pm 0.14$

$= 4.6 \pm 3\%.$

NOTE: don't round off until the end!

$\therefore \frac{b+c}{d} = \frac{4.6 \pm 3\%}{7.2 \pm 1.4\%} = 0.64 \pm \sqrt{3^2 + 1.4^2}\ \% = \boxed{0.64 \pm 3.3\%}$

$= \boxed{0.64 \pm 0.02}$

---

(4) $b^2 = (3.0 \pm 3.3\%)^2 = (3.0)^2 \pm (2 * 3.3\%) = 9.0 \pm 6.6\%.$

$c - d = (1.6 \pm 0.1) - (7.2 \pm 0.1) = -5.6 \pm \sqrt{0.1^2 + 0.1^2}$

$= -5.6 \pm 0.14$

$= -5.6 \pm 2.5\%$

$\therefore \frac{b^2}{c-d} = \frac{9.0 \pm 6.6\%}{-5.6 \pm 2.5\%} = \left(\frac{9.0}{-5.6}\right) \pm \sqrt{6.6^2 + 2.5^2}\ \%.$

$= -1.61 \pm 7\%$

$= -1.61 \pm 0.11$

$= \boxed{-1.6 \pm 0.1}$

---

(5) $c + d = (1.6 \pm 0.1) + (7.2 \pm 0.1) = 8.8 \pm \sqrt{0.1^2 + 0.1^2} = 8.8 \pm 0.14$

$= 8.8 \pm 1.6\%.$

$\therefore \frac{b}{(c+d)^3} = \frac{3.0 \pm 3.3\%}{(8.8 \pm 1.6\%)^3} = \frac{3.0 \pm 3.3\%}{8.8^3 \pm 4.8\%} = \frac{3.0}{681.5} \pm \sqrt{3.3^2 + 4.8^2}\ \%.$

$= 0.0044 \pm 5.8\% = \boxed{0.0044 \pm 0.0003}$

---

(6) $b^2+c^2 = (3.0 \pm 3.3\%)^2 + (1.6 \pm 6.3\%)^2$

$= (9.0 \pm 6.6\%) + (2.56 \pm 12.6\%) = (9.0 \pm 0.6) + (2.56 \pm 0.32)$

$= 11.56 \pm \sqrt{0.6^2 + 0.3^2}$

$= 11.56 \pm 0.67$

$= \boxed{11.6 \pm 0.7}$

---

(7) $(b^2+c^2)^{\frac{1}{2}} = (11.56 \pm 0.67)^{\frac{1}{2}} = (11.56 \pm 5.8\%)^{\frac{1}{2}}$

$= 3.4 \pm 2.9\%$

= 3.4 ± 0.[redacted]

---

(1) The sketch is not the same as the free-body diagram.

(2) The force arrows should all have their "tails" at a central point representing the object.

(3) The x-y axes should be shown as dotted line passing through the central point. (One of the axes should be in the direction of the acceleration) Label the axes {with an x at the positive x-direction and a y at the positive y-direction}

PROBLEM SOLVING

Newton's Laws; Free-Body Diagrams

1. **Draw a sketch** of the situation.
2. Consider only one object (at a time), and draw a **free-body diagram** for that object, showing *all* the forces acting *on* that object. Include any unknown forces that you have to solve for. Do not show any forces that the chosen object exerts on other objects.

   Draw the arrow for each force vector reasonably accurately for direction and magnitude. Label each force acting on the object, including forces you must solve for, as to its source (gravity, person, friction, and so on).

   If several objects are involved, draw a free-body diagram for each object *separately*, showing all the forces acting *on that object* (and *only* forces acting on that object). For each (and every) force, you must be clear about: *on* what object that force acts, and *by* what object that force is exerted. Only forces acting *on* a given object can be included in $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ for that object.
3. Newton's second law involves vectors, and it is usually important to **resolve vectors** into components. **Choose** x **and** y **axes** in a way that simplifies the calculation. For example, it often saves work if you choose one coordinate axis to be in the direction of the acceleration.
4. For each object, **apply Newton's second law** to the x and y components separately. That is, the x component of the net force on that object is related to the x component of that object's acceleration: $\Sigma F_x = ma_x$, and similarly for the y direction.
5. **Solve** the equation or equations for the unknown(s).

Every force arrow should have a value or symbol for the magnitude of that force (in addition to a label as to its source)

Do problems 27, 31, 32(a), 33, 46

I do

Compound objects?

3rd Law Pairs

Look at questions 23

Prob. 18

Talk about strings & pulleys.

Drawing force diagrams (free-body diagrams)

For problems 10

What ~~forces~~ force causes the acceleration of an object in free-fall?

What is the magnitude of this force that ~~produces~~ causes an acc. of 9.8 m/s² downward?

If the object is being held stationary in my hand, is the force of gravity any different than before?

Is the object's acc. any different from before?

Testing Newton's Second Law

Giancoli states Newton's 2nd Law as follows:

The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

We want to design an experiment to test this Law. What are the essential features of such an experiment?

First we need an object.

(1) What do we need to know about the object?

(2) What do we need to do to the object?

(3) What do we need to measure?

(4) How can you use a graph to show that acceleration is directly proportional to net force?

(5) If the acceleration is inversely proportional to the object's mass, what effect would using a heavier object have on the graph referred to in (4)?

~~Newton's 3rd Law Pairs.~~

Go over 4-23, ~~[illegible]~~ 68

Then do Newton 2 expt

Have them draw force diagrams for Ch4 86, 87.

Finish 86 & 87.

Go over analysis of
Newton's 2nd law Expt

Talk about apparent weight.
Talk about friction.

What does it mean to be weightless?

How do we experience
the force of gravity on our ~~bodies~~?
What does it feel like?

Imagine yourself in free-fall on the Moon, which
has no atmosphere. Your eyes are closed.
What do you think you would feel?

Close your
eyes & think
about it.

Sitting
Standing

Physiologically, how do we experience the force of gravity?

---

(1) force on the soles of your feet (standing)
force on your butt (sitting)

Thinking of your body as a whole.

tension in your muscles (tendons?) holding your body parts stationary

(legs holding upper body in place).
(shoulders holding arms in place.)
(neck muscles holding head up)

Thinking of different parts of your body as separate objects.

---

(3) Now imagine you are on the Moon (no atmosphere)
& you jump out of a tall space ship and are in free fall.

What would it feel like physiologically? (eyes closed)

---

(2) If you were in deep space far from any other object (star, planet)
what would it feel like physiologically?

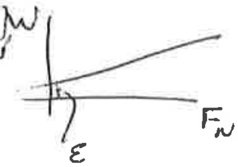
Are you "weightless" in either (2) or (3).

In an elevator in free fall, what would it feel like.

Note: if pulley has friction then tension at block is less than weight of hanging mass $\therefore F_f = F_T = W - \varepsilon$ where ε is small.

Static Friction

$\therefore W - \varepsilon = \mu_s F_N$ $\boxed{\therefore W = \mu_s F_N + \varepsilon}$



Friction is the force between surfaces that tends to prevent the surfaces from sliding across each other.

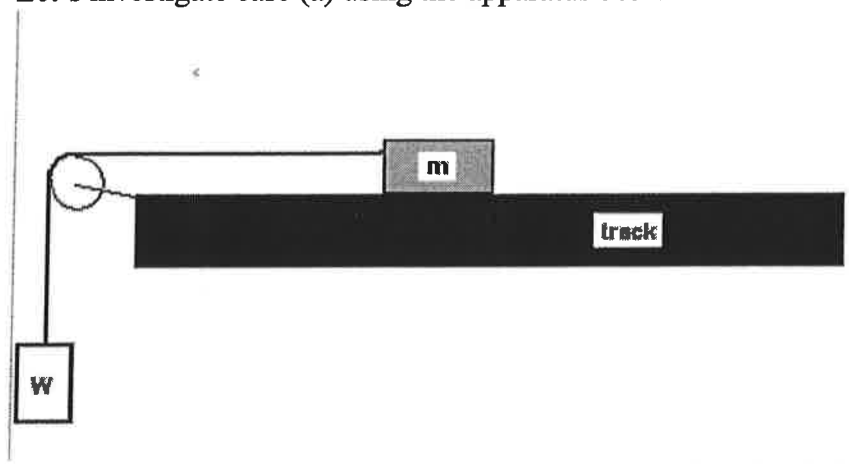
There are two situations that arise.

(a) Friction is successful at preventing the sliding of the surfaces across each other.

(b) Friction is not successful at preventing the sliding.

In case (a) we call the force a "static friction" force.
In case (b) we call the force a "kinetic friction" force.

Let's investigate case (a) using the apparatus shown below.



We can apply a known force to the block of mass **m (= 125 +/- 4 grams)** by adding weights to the end of the string hanging over the frictionless pulley. Because the hanging mass is stationary the sum of the forces on the hanging mass is zero. Therefore the tension in the string is equal in magnitude to the force of gravity on the hanging mass. (Newton's First Law)

(1) If you apply a small force to the block what do you think will happen? Explain in terms of the friction force between the block and the track.

(2) If you increase the force applied to the block little by little what do you think will happen?

(3) Find the largest force you can apply to the block without it sliding.

Because the block is stationary, the sum of the forces on the block is zero. Draw a force diagram for the block and convince yourself (using Newton's First Law) that the friction force equals the tension force and the normal force equals the force of gravity on the block.

Therefore you have just found the **maximum static friction force for a known normal force**. Record your results in the table below.

(4) Now place a mass of 100 grams onto the block and repeat the experiment. What is the maximum static friction force and what is the normal force on the block? Note: the normal force is now greater than the mass of the block because of the extra mass you placed on top. Draw a force diagram for the block to help you calculate the new normal force.
Record your results.

(5) Continue adding additional mass on top of the block and record the maximum friction force for a total of five different normal forces. Record your results.

(6) The textbook claims that $(F_{friction})_{max} = \mu_s F_N$. Prove or disprove this by plotting a graph. If it is true then find the value of μ_s

Normal force on block (N)	Max Friction force (N)

Physics Lab Write-up

A full lab write-up should follow the format described below. It should be typed, preferably on a computer.

Introduction:

Explain the purpose of the lab. Be brief and concise. A couple of sentences will do.

e.g. "This lab examines the motion of freely falling bodies. In particular, it finds the acceleration of such bodies."

Procedure:

Describe the apparatus and what it does. Be sure to include a diagram.

For instance if part of the apparatus is a photogate then you need to explain what it does and how. Include a drawing a diagram of what it looks like.

Then describe the steps necessary to carry out the experiment. Do not use bullets or numbered sentences. Use common sense to decide just how much detail to go into. The idea is to be clear enough so that someone else could carry out the experiment using this description. Too much detail is not good but you need to be complete. Do not say things like "switch on the computer"!!

Theory and Analysis:

This section will contain the ideas and equations that you will use to analyze and interpret the data you collect. When stating equations, be sure to explain the meaning of the symbols.

For instance, in the acceleration due to gravity experiment, this section would contain the definition of average velocity, the idea that the average velocity over a time interval is approximately equal to the instantaneous velocity at the mid-time of the interval, that the acceleration is the slope of the velocity graph and that the slope is found from the linear fit to the velocity data.

Data and Results:

This section contains the measured values (usually in the form of a table) and then any calculated values (e.g. average velocities calculated from position and time data). Be sure your tables have headings that include the units of the quantities. Any time you calculate values from raw data then you should give one explicit example of that calculation.
Also in this section will be any graphs that you produce from the tables. All graphs should have a title and labeled axes (including units).
Error analysis will also appear in this section.

Conclusion:

Summarize what you have accomplished in the lab. Did you verify some law of physics? If so, what results allow you to say that the law is verified? Have you calculated a value of some quantity? If so, what is the uncertainty in your value? Do you have an accepted value that you are comparing to? Is your result within range of the accepted value or not?

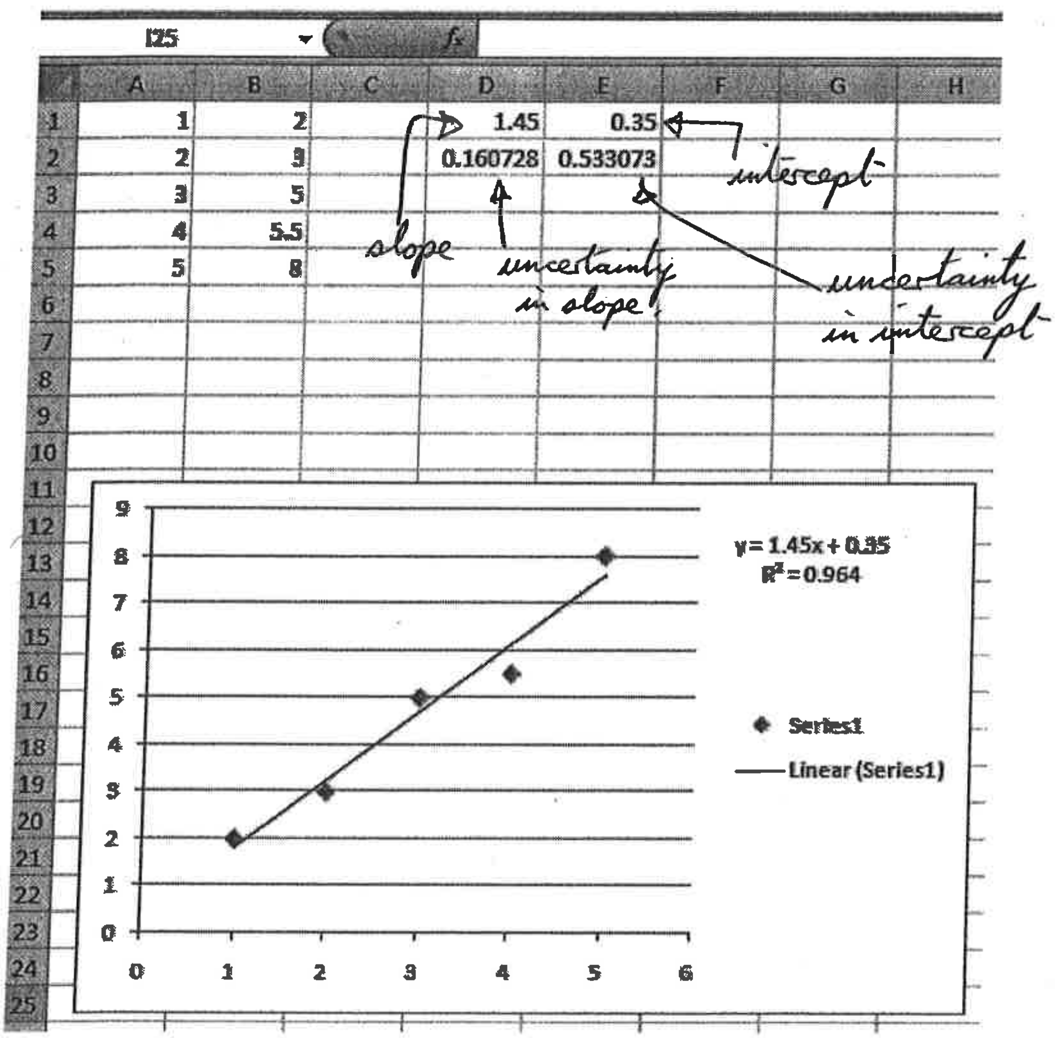
Whenever possible, comment on the sources of error in the experiment and on how the experiment might be improved.

To get the uncertainty in the slope and intercept for a linear fit to data you can use the function LINEST.

For instance, in the sheet below there is data in column A (x-values) and in column B (y-values). There is already a graph with a linear fit showing the equation of the fit. But I want to get the uncertainty in the slope and intercept.

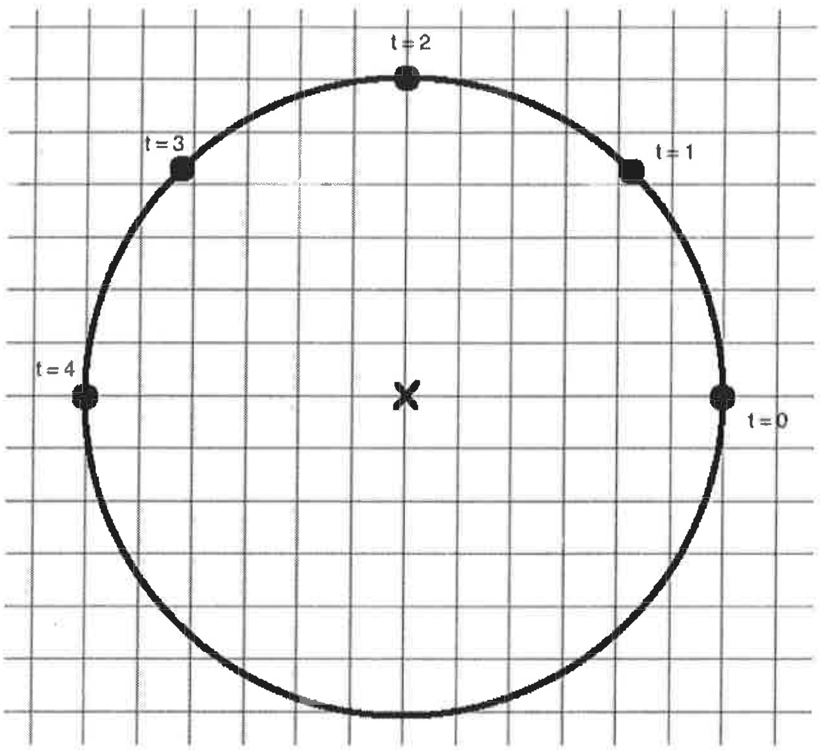
First select cell D1 (say) and then click and drag to highlight the cells D1,E1,D2,E2.
Go up to the input line and type in =LINEST(B1:B5,A1:A5,,TRUE)
<u>Notice the double comma.</u>
Then hit CTRL/SHIFT/ENTER.

The four highlighted cells should now contain the following values:
D1 = slope D2 = uncertainty in slope
E1 = intercept E2 = uncertainty in intercept



Uniform Circular Motion

An object is traveling in a circle of radius 6 m at a constant speed. Its position is shown below at several different clock readings.



(a) What is the speed of the object?

(b) On the picture draw the velocity arrows (tail coming out of the object) at times t = 0, t = 1 and t = 2.

(c) Write the velocity of the object at times t = 0 and t = 2 in vector form.

(d) Calculate the average acceleration of the object over the time interval between t = 0 and t = 2.

(e) This is an approximation to the instantaneous acceleration at the mid-time of the interval i.e. at t = 1. Draw the acceleration arrow (tail coming out of the object) at t = 1.

(1) Do without R & v values. $t = 0$ to $t = \left(\frac{\pi R}{2}\right)/v$.

(2) Repeat for interval $t = 0$ to $t = \pi R/v$

estimate $|a|$ each time

(2) $|a| = \frac{2}{\pi} \, v^2/R = 0.64 \, v^2/R$.

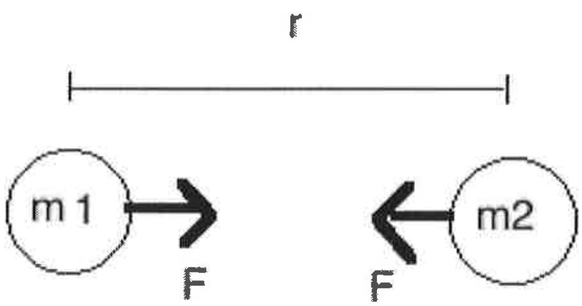
(1) $|a| = \frac{2\sqrt{2}}{\pi} \, v^2/R = 0.90 \, v^2/R$

For $1/8$ of a circle.

Newton's Law of Gravitation

Masses tend to attract each other. Newton formulated a law describing this effect and it was subsequently used by other scientists to explain the behavior of the planets orbiting around the Sun (among other things). The form of the law for the force of attraction between two masses m_1 and m_2 separated by a distance r is

$F = G\frac{m_1 m_2}{r^2}$ in a direction along the line of centers of the two objects.



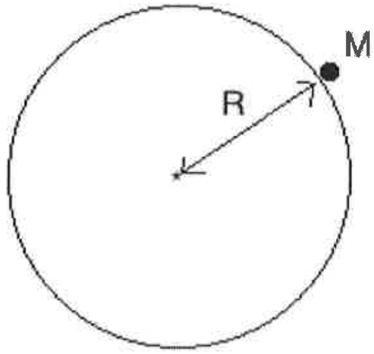
Note: the two forces shown above are a "Newton's 3rd Law pair".

For large spherical objects, such as planets and stars, the position of the object is taken as the position of its center.

Consider a mass **M** at the surface of the planet Earth. Using the above equation calculate the magnitude of the gravitational force on the mass M.

G = 6.67×10^{-11} N m^2/kg^2 Mass of the Earth = 5.98×10^{24} kg

Radius of the Earth = 6.38×10^6 m



Do the same thing for a mass M on the surface of the Moon.

Radius of the Moon = 1.74×10^6 m   Mass of the Moon = 7.35×10^{22} kg

A satellite in orbit around the Earth is kept in its circular path by the gravitational force (i.e. this force produces the acceleration v^2/R). Use this idea to calculate the speed of a satellite orbiting at a distance of 42×10^6 m from the center of the Earth. (shown approximately in the picture below).
Use your answer to find the period of the orbit in days (rather than seconds).

Derive a formula for the period of a satellite in terms of the mass of the planet and the radius of the satellite's orbit.

Start with F = m a where F is the gravitational force and $a = v^2/R$.
Then substitute $v = \frac{2\pi R}{T}$
Then solve for T.

Estimate the mass of the Sun using the fact that it takes a year for the Earth to orbit the Sun and the Earth is about 1.5×10^{11} m from the Sun.

Hooke's Law

What is the force that a spring exerts on an object at the end of the spring? It is easy to verify that stretched springs pull and compressed springs push and the bigger the stretch then the bigger the force. But is the force directly proportional to the stretch?

When a mass is hung on a spring what are the two forces on the mass?

Draw a force diagram and show that F(spring) = F(gravity) using Newton's 1st Law.

Hang a known mass on the spring and measure the stretch from its normal length.

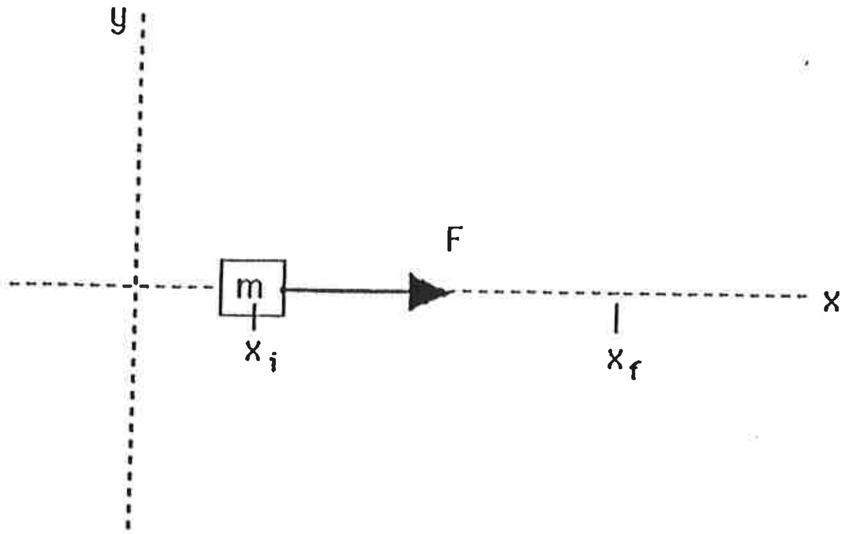
Increase the mass hung on the spring and again measure the stretch from its normal length. Do this several more times.

Fill in the table below. Then plot F versus x. Show that the force is directly proportional to the stretch. This is known as Hooke's Law.

Stretch (x)	Spring force (F)

Work and Kinetic Energy

Consider a constant force acting on an object causing it to accelerate along the x-axis.



During the time it travels between an initial position x_i and a final position x_f it experiences a constant acceleration. Therefore we can apply the constant acceleration equation

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Express the acceleration in terms of the force and the mass and substitute this into the above equation.

Move the expression for the work done by the force over to the left-hand-side of the equation and move everything else over to the right-hand-side.

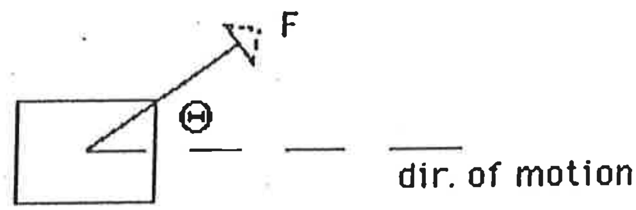
The right-hand-side can be written as the difference between the initial value of a quantity and the final value of the same quantity. What is this quantity?

We call this quantity the **Kinetic Energy** of the object.

Thus we see that the work done on an object is equal to the change in its kinetic energy. This is known as the **Work-Energy Theorem.**

Conservative forces

When a force is applied as an object moves through a distance then the force is said to perform work on the object. If the force is constant and is applied at an angle θ to the direction of motion



then the work done is given by the formula

$$W = F\, d\, \mathrm{Cos}(\Theta)$$

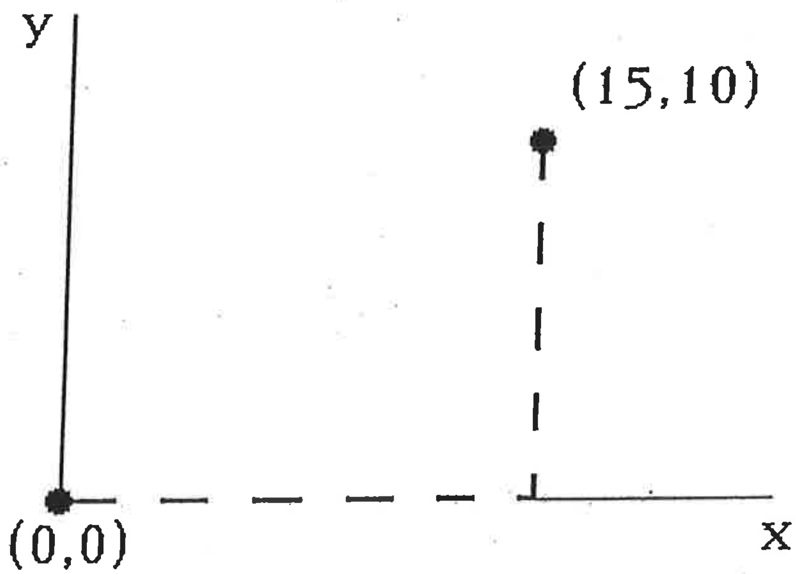
Note that in this definition of work it is possible for the angle Θ to be larger than 90° and therefore it is possible for the work done by the force to be negative.

Consider the work done by the force of gravity when a 10 kg block moves from the point (0,0) to (15,10) (both in meters) by the three paths shown below.

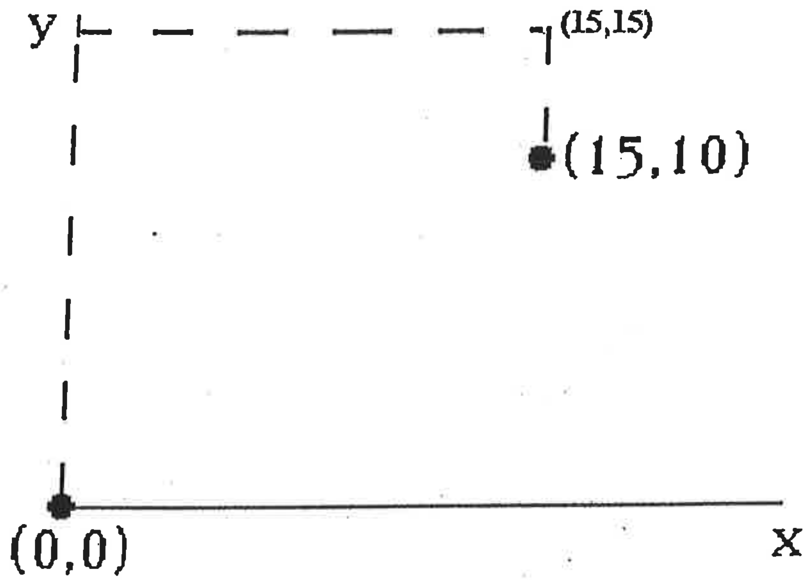
(Hints: The force of gravity is constant and points vertically downward. If the path consists of several straight-line segments then you must treat each segment separately since the angle θ will be different for each segment.)

Did the work done depend on the path? If not then the force of gravity is a **conservative force.**

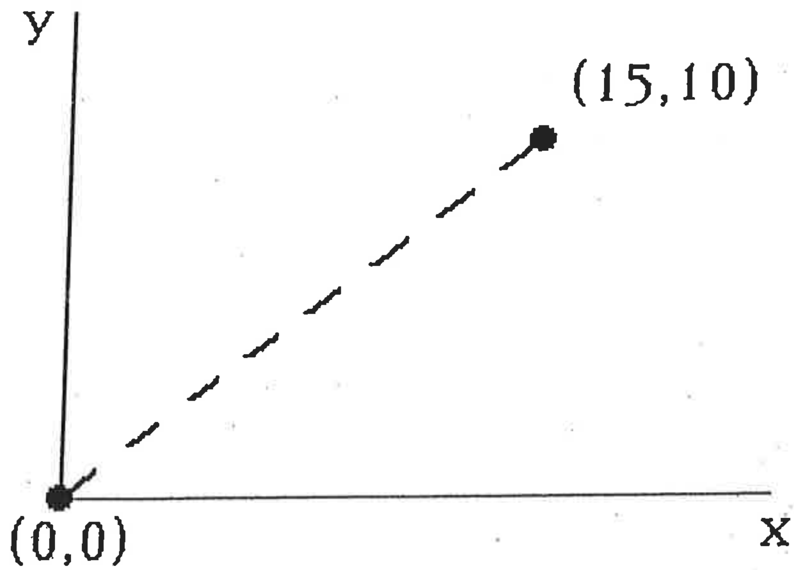
(a)



(b)

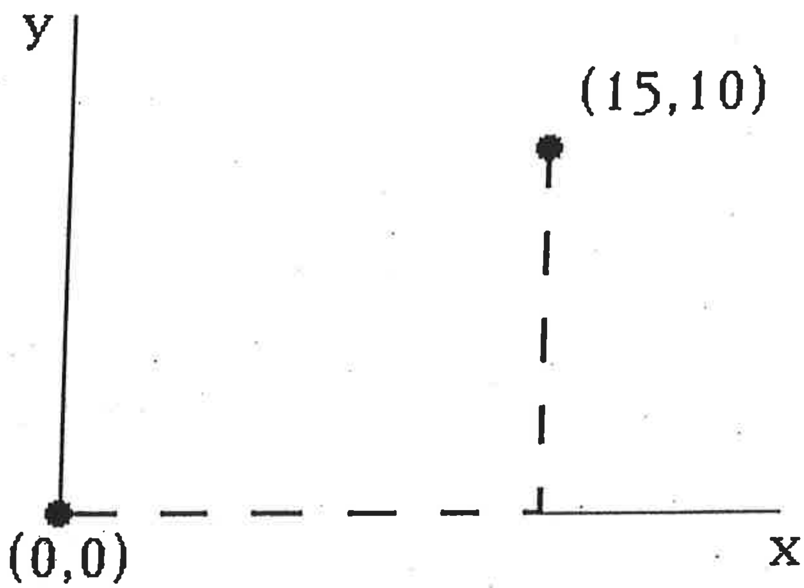


(c)

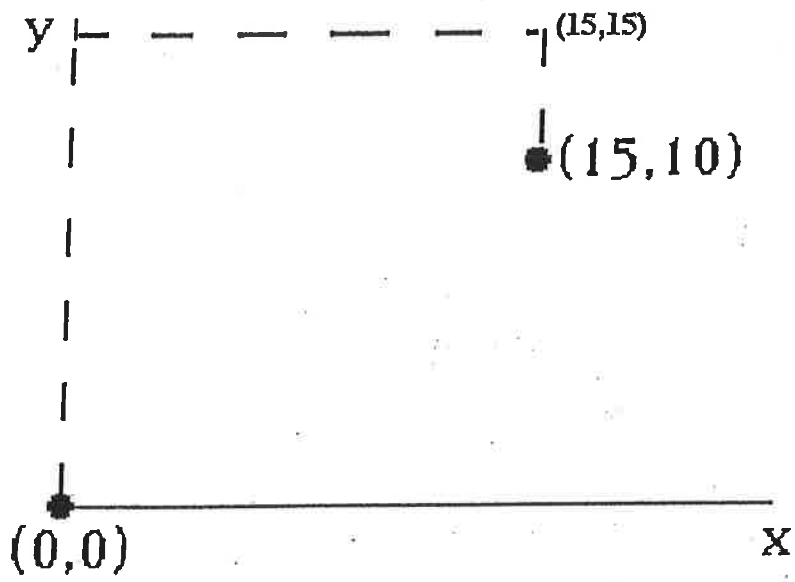


Now consider the following three diagrams as representing a "bird's-eye view" of a block being moved on a table-top along the dotted paths. There is a constant friction force of 5 N opposing the motion at all times. Calculate the work done by the friction force in each case.

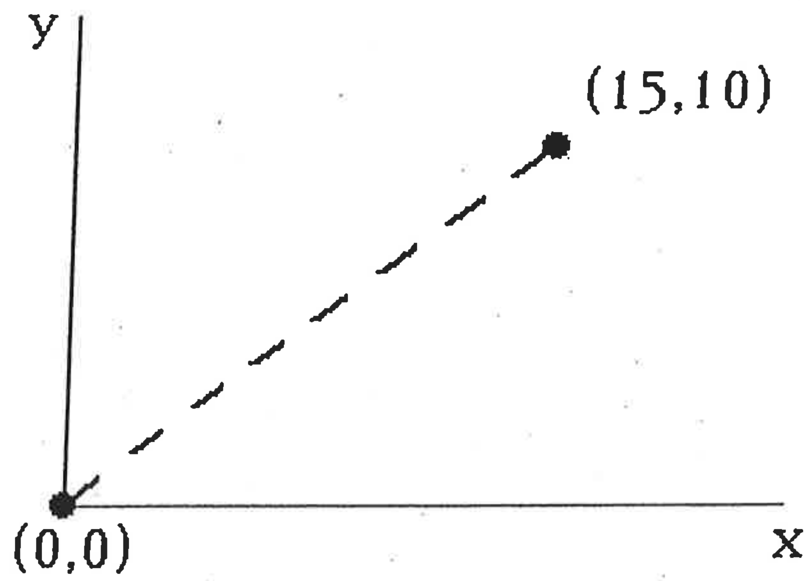
(a)



(b)



(c)

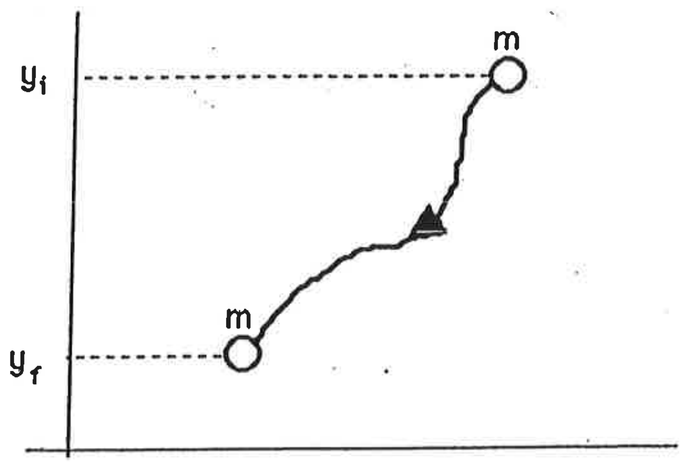


Based on your results answer the following question. Is friction a conservative or non-conservative force?

Potential Energy

Potential energy is an invented quantity associated with special kinds of forces called conservative forces (see previous activity on conservative forces). It is defined in such a way that we can use a *conservation principle* for total energy.

Let's see how potential energy is defined for the force of gravity.



(2) How much work is done by the force of gravity as the mass m travels from its initial position to its final position as shown in the diagram above?
(Hint: for a conservative force the work done only depends on the end-points not on the path taken. See previous activity on conservative forces.)

(3) Use the expression from (2) in the work-energy theorem

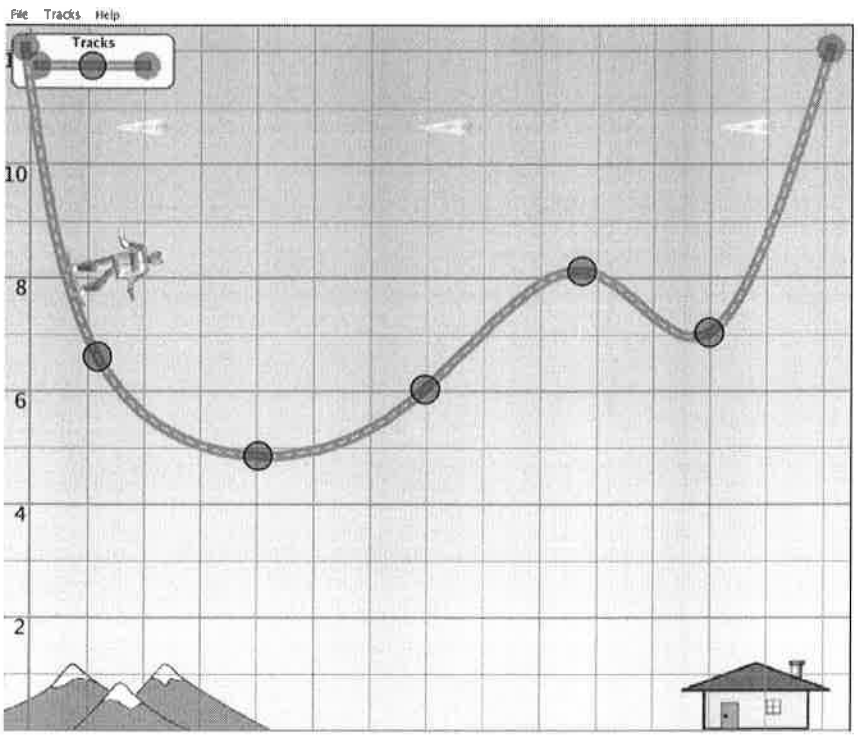
$$W = \Delta K = K_f - K_i$$

(4) Bring all quantities with subscripts i to the left hand side of the equation and all quantities with subscripts f to the right of the equation.

(5) Identify the combination of quantities which is conserved.

(6) We call this combination the total energy. It is the sum of the kinetic energy and the potential energy (associated with gravity). What is the expression for the potential energy due to gravity?

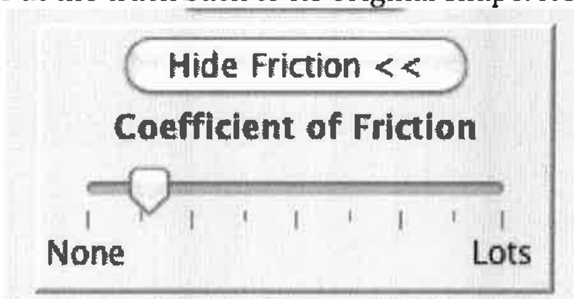
Open skater3.
The skater starts at a height of 8 meters.
Will the skater make it over the bump in the track?



Pull the second blue control point downward.
Now will the skater make it over the bump in the track?

Where should you place the skater so that he makes it over the bump? Why?
What happens to the skater after he makes it over the bump? How high will he go?
Will he make it over the bump on the way back? Explain.

Put the track back to its original shape. **Now add friction** as shown below.



Where should the skater be placed in order to make it over the bump?

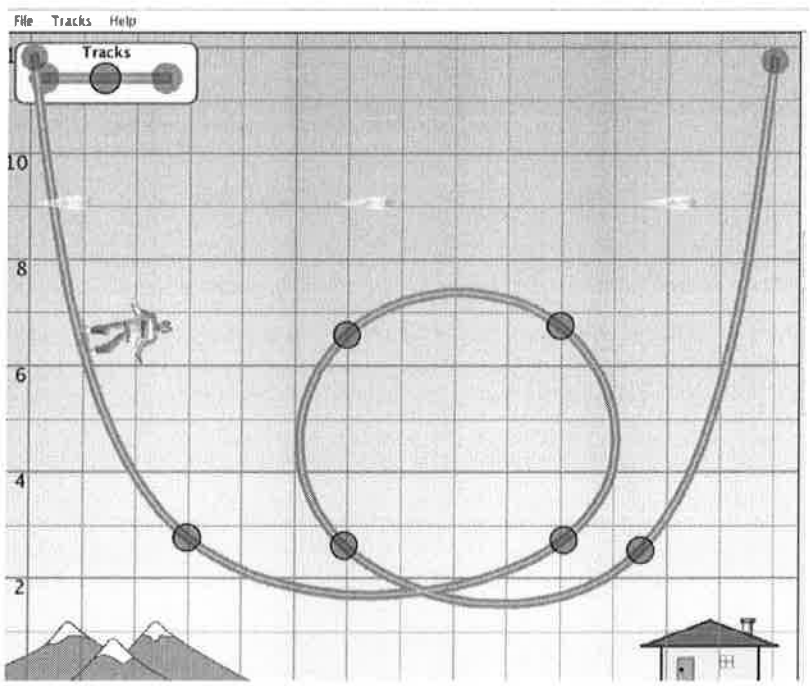
Pull the second blue control point downward as before.

Now where should the skater be placed in order to make it over the bump? Why?

Will the skater make it over the bump on the way back? Why or why not?

Right-click on the track and turn off roller-coaster mode. Now the skater is free to come off the track.
Where can the skater start in order to make it over the bump but not come off the track at any time?

Open skater2



The loop has a radius of about ___________________?

Where on the track can the skater start and not fall off the track?

(a) without going around the loop

(b) going around the loop

How much higher than the top of the loop must the skater be to make it around the loop?

Campare this to the radius of the loop.

Use conservation of energy and circular motion ideas to explain the answers to the last two questions.

① $x(t) = x_0 + v_{x0} t + \frac{1}{2} a_x t^2$

② $\sum F_x = m a_x$.

③ $W = F d \cos\theta$

④ $W_{total} = \Delta K$

⑤ $W_{fr} + W_{applied} = \Delta E$

⑥ $E_i = E_f$

⑦ $K = \frac{1}{2} m v^2$

⑧ $a_R = v^2/R$

⑨ $E = K + U_{grav} + U_{spring}$.

⑩ $U_{grav} = mgy$

⑪ $U_{spring} = \frac{1}{2} k x^2$

⑫ $F_{fr} = \mu_k F_N$

Which equations are definitions?

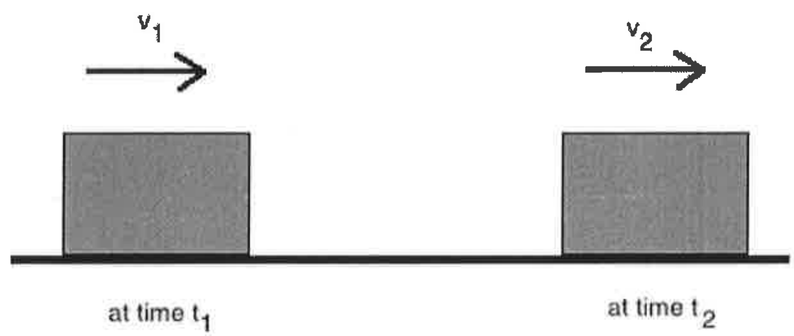
Which equations are "cause and effect" equations? (cause on the left, effect on the right).

Which are only true under special circumstances?

Which are "empirical" equations i.e. determined from experiment?

Momentum and Force

Consider a mass m moving under the influence of an average force $\vec{F}_{ave}$



Express Newton's 2nd Law in terms of the velocities and times shown above.

Rearrange the equation so that the left hand side is

$$\vec{F}_{ave}\,\Delta t \ =$$

Rewrite the equation using the definition of momentum $\vec{p} = m\vec{v}$

Center of Mass

Consider the simple example of a system which consists of only two masses. Then we define the Center of Mass as follows.

$$\vec{R} = \frac{m_1 \vec{r_1} + m_2 \vec{r_2}}{m_1 + m_2}$$

This is a vector equation and therefore it really is three equations (in 3 dimensions) one for each component of the vectors. The x-and y-component equations are

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \qquad Y = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2}$$

If the masses are moving around then the coordinates x_1, x_2, y_1, y_2 are functions of time and therefore the coordinates of the center of mass are functions of time also.

Videopoint allows you to calculate the C of M of a collection of objects of known mass.

Download **Collision Movie3, Collision Movie4,** and **2-dimensional collision movie** from the course web page. Open **Collision Movie3** using Videopoint (if you double click on it I think it will automatically open using Videopoint, if not then get instructions from Seamus).

Set the software to locate 2 objects. Set the scale of the movie using the ruler icon on the left hand side of the screen (it allows you to click on the two ends of a meter stick that appears in the movie).

Drag the slider to view the frames in the movie. Take data on the positions of the carts about 10 frames before the collision and continue until about 10 frames after the collision.

In the coordinate systems window input the masses for the two carts. Then in the Create menu choose Center of Mass and choose the two carts (points SJ and S2) as the objects you want to include. Then plot a graph of the x-position of the C of M. Comment on the graph.

SAVE THIS VIDEOPOINT FILE FOR LATER USE!

Repeat the above procedure for Collision Movie4 and save the videopoint file for later.

Now download 2-dimensional collision movie and analyze the motion of the Center of Mass of the three objects. Plot a graph of the x-position of the C ofM and another graph of the y-position of the C of M.

Comment on the graph.

Let's go back and consider the case of the colliding carts.

$$X = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

X is the x-coordinate of the Center of Mass.

In this equation which are the quantities that are changing with time?

Symbolically, take the time derivative of this equation.

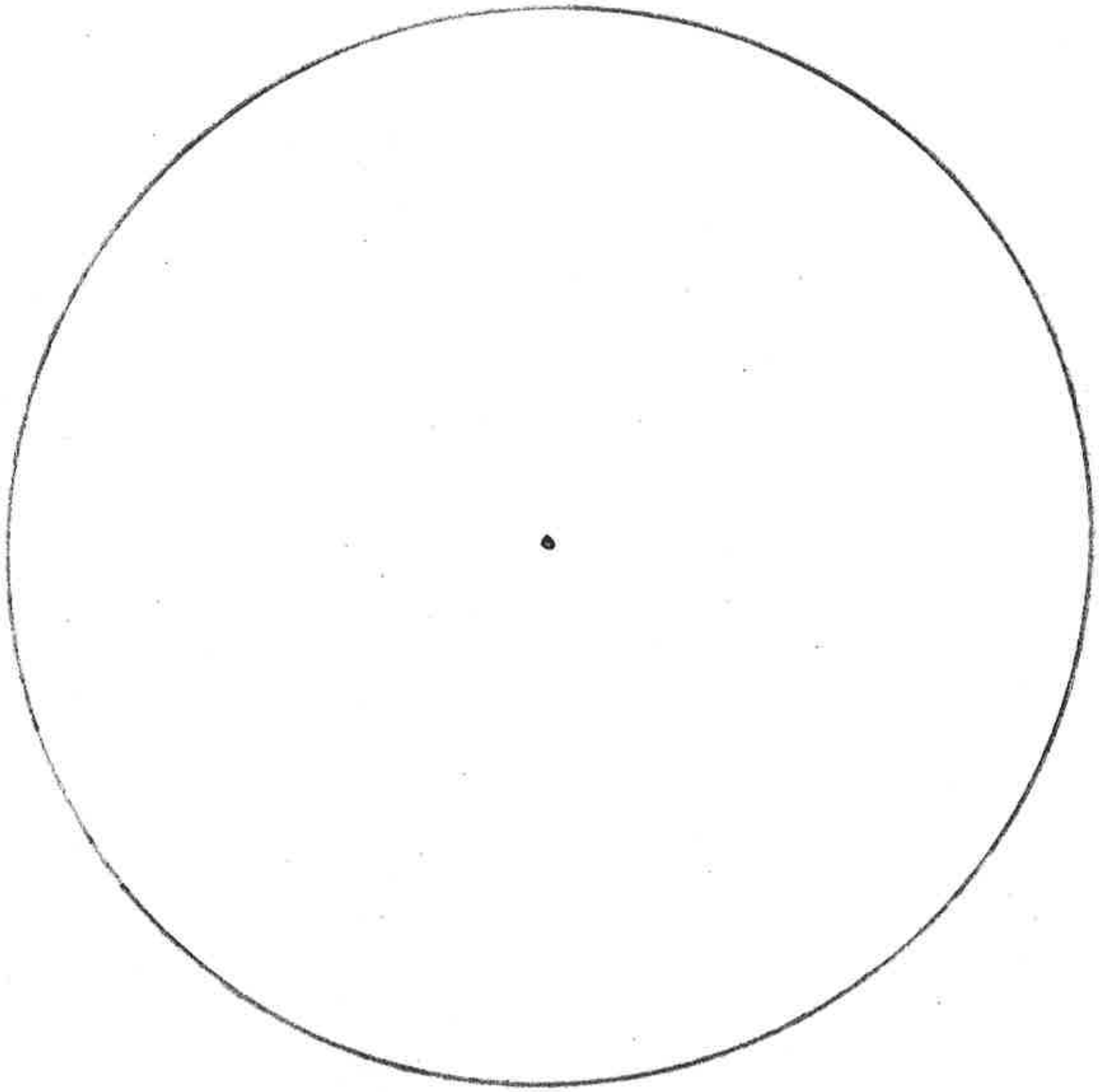
Some of the time derivatives are equal to velocities of objects. Which ones?

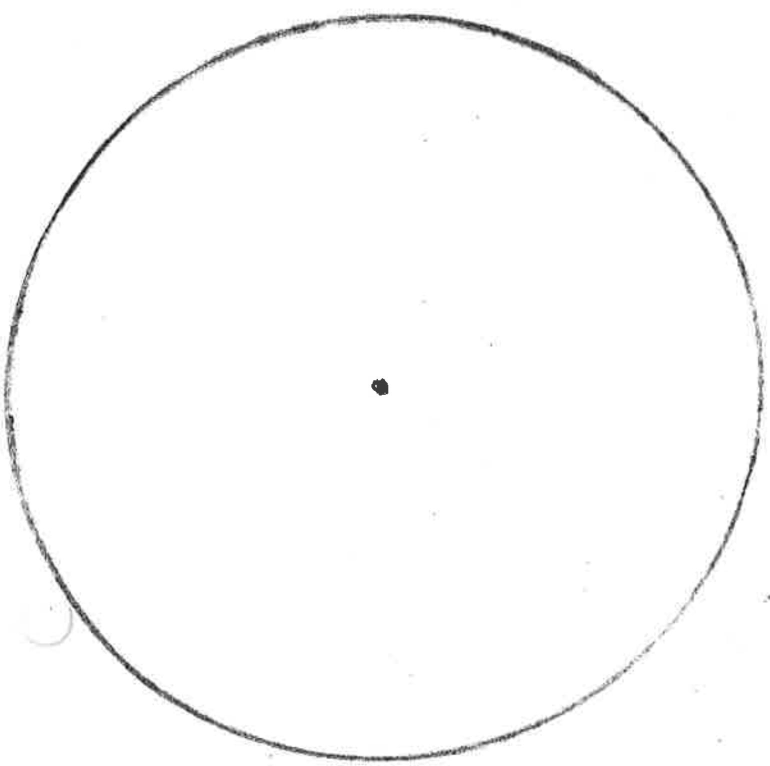
Go back to the VideoPoint file for the Collision Movie3 and create position versus time graphs for both objects separately. You should now have three graphs (X versus t, x1 versus t and x2 versus t.

Remember that the derivative of a function is the slope of the graph of the function. Which time derivative is constant throughout the collision?

From the considerations above, what can you say about the value of $(m_1v_1+m_2v_2)$? Does it change during the collision?

We define the momentum of an object as $p = mv$.





Angular Measurements

An understanding of the relationship between angles in radians, angles in degrees, and arc lengths is critical in the study of rotational motion. There are two common units used in the measurement of angles—degrees and radians.

1. A degree is defined as 1/360th of a rotation in a complete circle.

2. A radian is defined as the angle for which the arc along the circle is equal to its radius as shown in Figure 10-2 below.

In other words, **every arc of length equal to the radius subtends an angle of 1 radian**

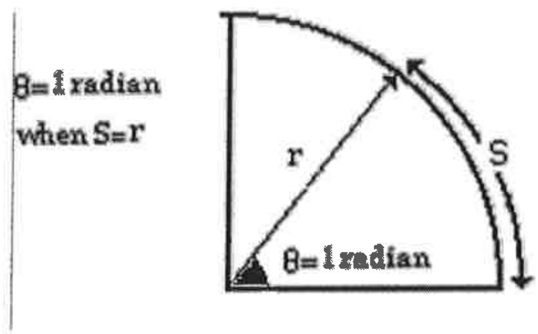


Figure 10-2: A diagram defining the radian

In the next series of activities you will be relating angles, arc lengths, and radii for a circle. To complete these activities you will need the following:

- A drawing compass
- A flexible ruler
- A protractor
- A pencil

(a) Approximately how many degrees are in one radian? Let's do this experimentally. Using the compass draw a circle and measure its radius. Then, using the flexible ruler trace out a length of arc, S, that has the same length as the radius. Next measure the angle (in degrees) that is subtended by the arc.

1.0 radian = ______________ degrees

How would you construct an angle of **one half of a radian** using just the ruler? Do it! Check your angle with the protractor to see if you did it correctly.

(b) Let's find out how many radians there are in a full circle.

(i) On a sheet of paper draw a circle of radius R and divide its circumference up into arcs of length R. Mark the "pieces of pie" in the circle. Remember each "piece of pie" has an angle of one radian at the center.

You will have a bit left over. What is the length of the left-over bit of arc (as a fraction of R)? What is the angle at the center of this left-over bit (as a fraction of a radian)?

(ii) Add up all the angles at the center to find how many radians there are in one full revolution.

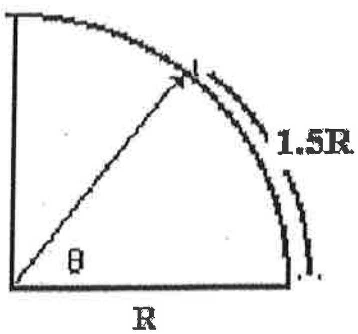
The exact answer is 6.28.... radians, commonly written as 2π radians where π=3.1415927. . . .

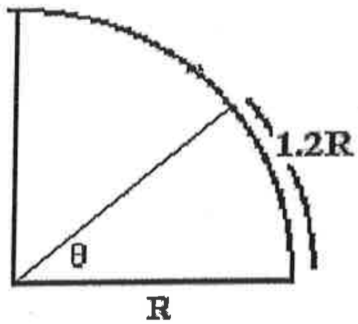
Each radian "subtends" an arc length of R. Hence the arc length subtended by a full revolution is 2π times R. This of course is just the formula for the circumference of a circle ($2\pi R$).

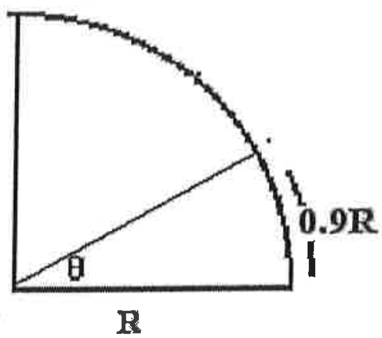
(iii) How many degrees does a full revolution correspond to?

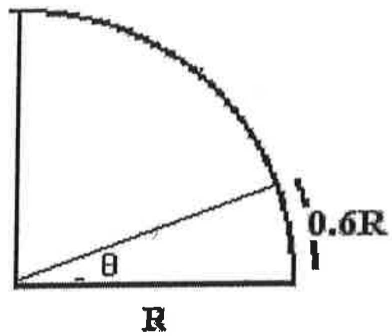
(iv) How many degrees, then, are in one radian? (Use the exact value of 2π for the number of radians in a full revolution.) How does this answer compare to your previous estimate.

For each of the arc lengths shown below, what is the angle θ in radians?

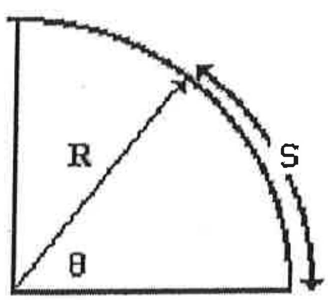








(c) Consider an arc length of arbitrary length S.

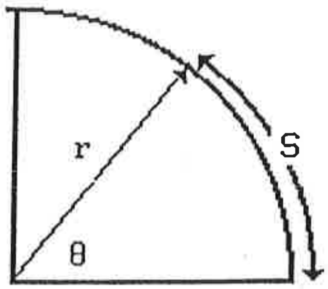


What is the angle θ? (in terms of R and S)
<u>Hint</u>: think about how you found θ in all the previous examples.

Rearrange this equation to get S in terms of R and θ. S = __________

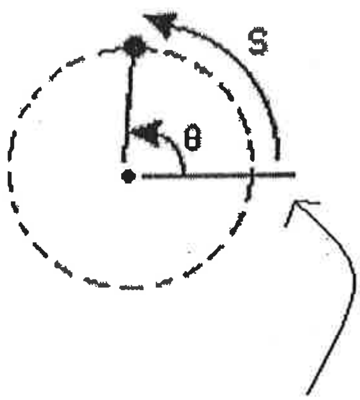
This is a very useful equation that we will use again.

Angular Measurements II



$$S = r\,\theta$$

This is a general relationship between a length of arc, S, on a circle, the radius r in meters and the angle θ (in radians) subtended by the arc.



$$v = \frac{\Delta S}{\Delta t}$$

$$\omega = \frac{\Delta \theta}{\Delta t}$$

Suppose an object is moving in a circle of constant radius r. (It might be attached to a rod or string that ispivoted at the other end, or it might be attached to a flat object that is pivoted about some point. In either case, the pivot point will be the center of the circle shown below.) The position of the object on the circle is described by the arc length S measured from some reference point on the circle to the position of the object. Since this arc length is changing as the object moves S is a function of time. Taking the change in S divided by the corresponding time interval gives us the linear speed (v) of the object as it moves around the circle.

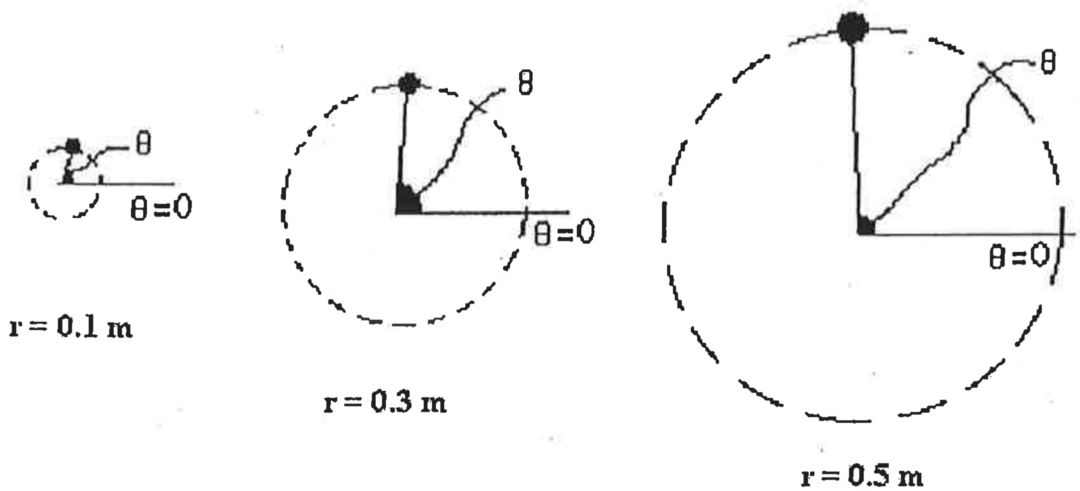
$$v = \frac{\Delta S}{\Delta t}$$

The position of the object can also be described by the angle θ which a line (from the center of the circle to the object) makes with a reference line (from the center of the circle to the reference point on the circle). Since this angle is changing as the object moves θ is a function of time. Taking the change in θ divided by the corresponding time interval gives us the angular speed (ω) of the object as it moves around the circle.

$$\omega = \frac{\Delta \theta}{\Delta t}$$

By using the relationship $S = r\,\theta$, show that the linear speed, v (in m/s), is related to the angular speed, ω (in radians/s), by the equation $v = r\,\omega$. (Note: **r** does not change as the object moves.)

Several objects move through complete circles in the same fixed amount of time (say 3.0 secs) but the circles have different radii.



What is the value of v and ω in each of theses cases?

Which quantity is the same in each case, v or ω?

The rate of change of angular speed (ω) is the angular acceleration (α).

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

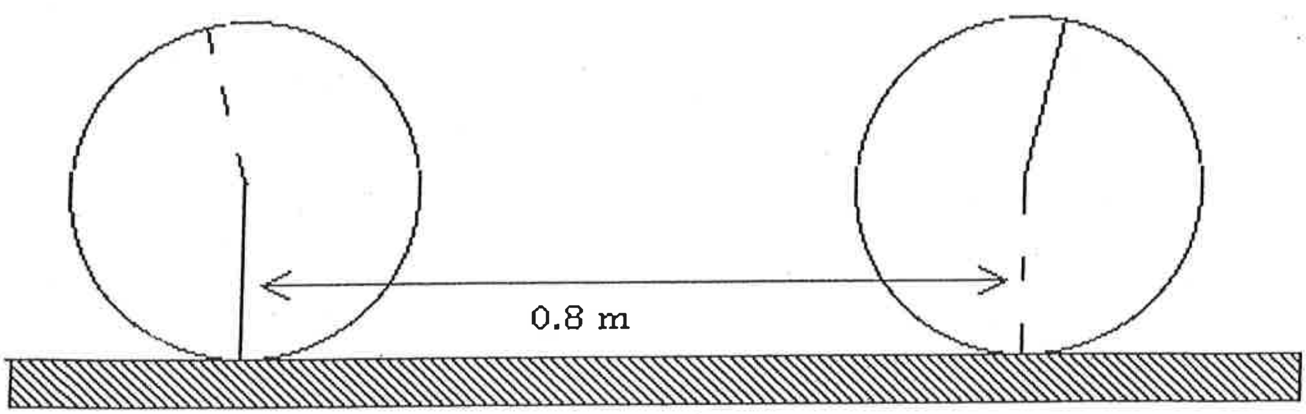
The tangential acceleration (a_{tan}) is the rate of change of speed. (not to be confused with the centripetal acceleration which points towards the center of the circle).

$$a_{tan} = \frac{\Delta v}{\Delta t}$$

What is the relationship between the tangential acceleration (a_{tan}) and the angular acceleration (α)?

Problem

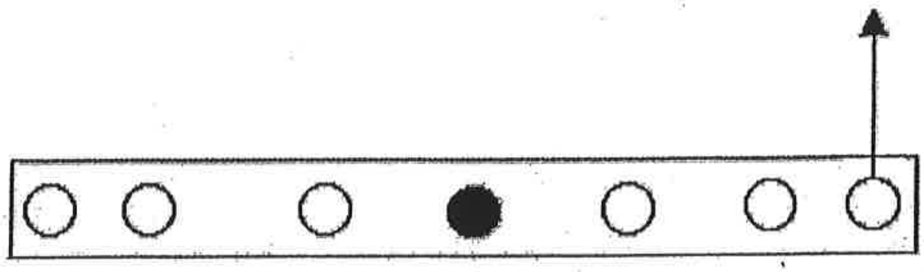
A wheel of radius .25 m is **rolling (without slipping)** on flat ground. If the center of the wheel travels 0.8 m to the right, how many radians has the wheel rotated?



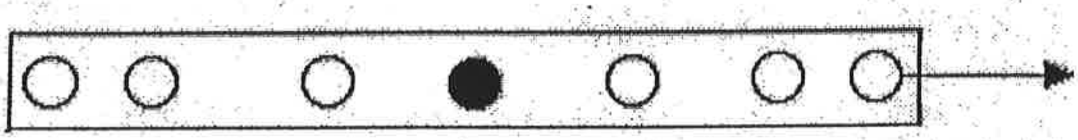
Hint: if there is no slipping then there is an arc length on the wheel which is also 0.8 m long.

If this movement occurred in 2 secs then what is the angular speed of the wheel about the center. What is the speed of the center of the wheel to the right?

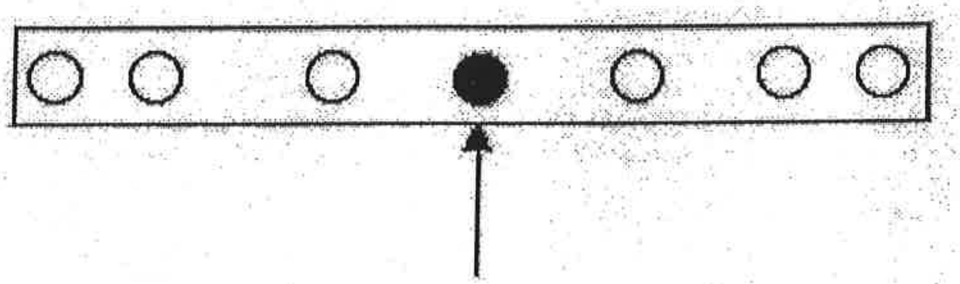
<u>The turning effect of a force on an object pivoted on an axis. (TORQUE)</u>



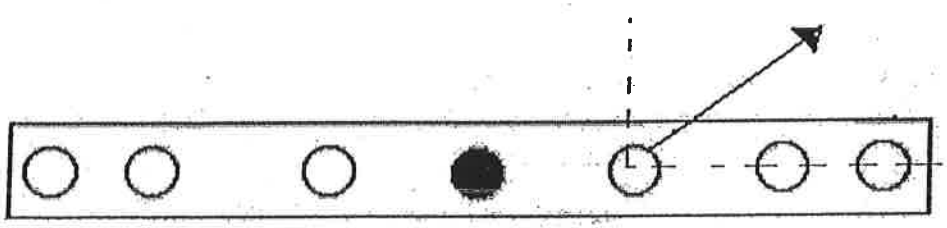
Will this force cause the rod to turn? The rod can pivot on an axis in the center (the black hole).



Will this force cause the rod to turn? The rod can pivot on an axis in the center (the black hole).



Will this force cause the rod to turn? The rod can pivot on an axis in the center (the black hole).



Will this force cause the rod to turn? The rod can pivot on an axis in the center (the black hole).

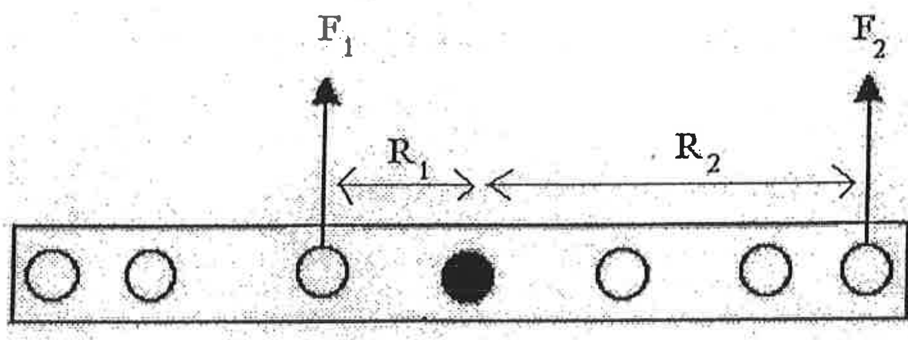
In the last example, which component of the force causes the rod to turn?

Let's hypothesize that the turning effect or "torque" of the force depends on the perpendicular component of the force and the distance of the point of application of the force from the pivot axis. The simplest assumption would be that the torque is directly proportional to both these quantities i.e.

Torque = $(F_{perp}) * (R)$

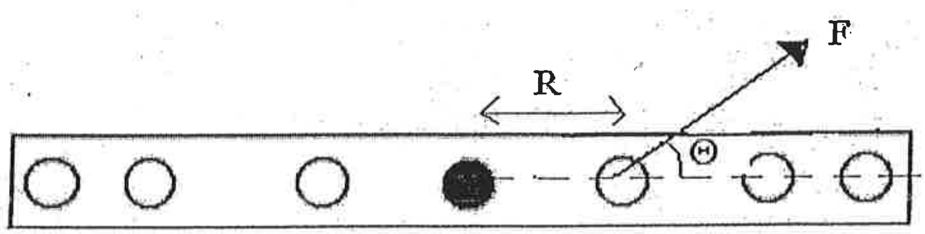
Now, if we apply two torques to the rod, one tending to turn it clockwise and the other tending to turn it counter-clockwise and <u>if the rod does not turn</u> then we would expect that the two torques were of equal magnitude.

Set up the situation below and check that our definition of torque satisfies the requirement that the two torques are of equal magnitude.



Noting that the perpendicular component of the force is F Sin(θ) , we can now agree on a definition of the torque due to an applied force as follows:

Torque = F R Sin(θ) where R = distance from the pivot axis.



Torque

Open up the Torque Simulation. You will see a beam pivoted about a point P.
A spring balance exerts a force on one side and a string with a mass on the end of it exerts a force (5.0 N) on the other side.

(1) Check the "Vertical spring balance, horizontal beam" box .
Put the mass and spring at opposite ends of the beam.
Put the pivot point in the middle. (the beam is 108 cm long)
Predict the spring balance reading for the following positions of the hanging mass and then check your answers.

(a) WP = 40 cm

(b) WP = 20 cm

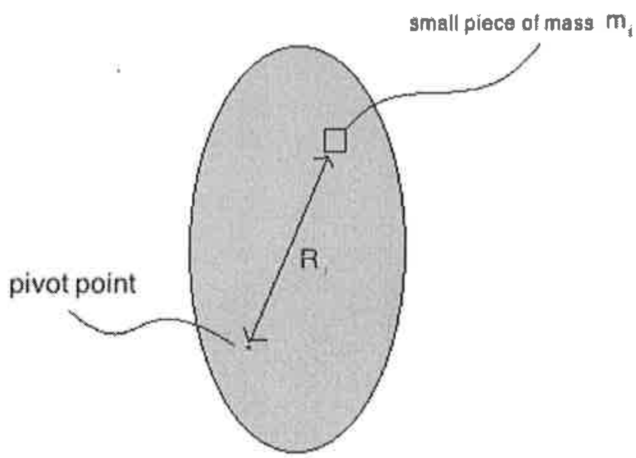
(2) Put the mass at WP = 30 cm and keep the spring balance at PS = 54 cm. If the angle $\varphi = 50^{\circ}$ what spring force will keep the beam horizontal?

Deselect the "Vertical spring balance, horizontal beam" box and check your answer.
When you have done so check the "Show forces and moment arms" box.
Sketch the picture with forces and moment arms in the space below.

Moment of Inertia

The moment of inertia of an object whose mass, M, is all concentrated at one point a distance R from the pivot point (axis of rotation) is given by $I = MR^2$.
(This would be the case for a massless rod of length R pivoted at one end with a mass M attached to the other end of it.)

For an object whose mass is spread out so that there is not a single value of R to use. Therefore we have to consider the object as made up of many small pieces of mass m_i each at a distance R_i from the pivot point.



Then the moment of inertia of the whole object is the sum of the moments of inertia of each piece

$$I = \sum_i m_i R_i^2$$

(a) Let's apply this idea to a rod of uniform density pivoted at one end.

Let the mass of the rod be 1.0 kg and the length of the rod be 0.2 m.

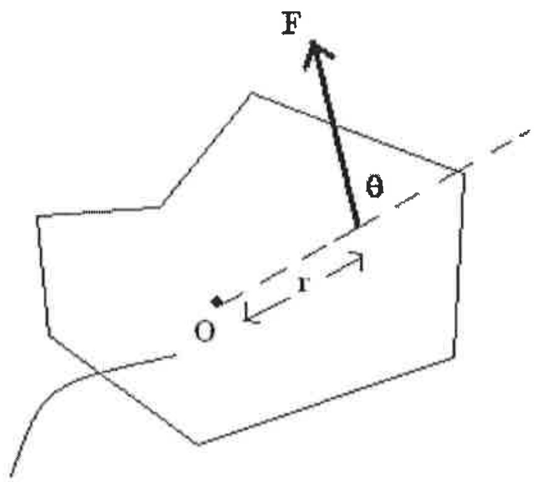
Split the rod into 20 pieces each of length 0.01 m and, **using Excel**, calculate $m_i R_i^2$ for each piece. Then sum the values.
(NOTE: R_i should be measured from the pivot point to the center of each piece.)

(b) Repeat this procedure for the case of the rod pivoted at the center of the rod.

The Effect of an applied torque.

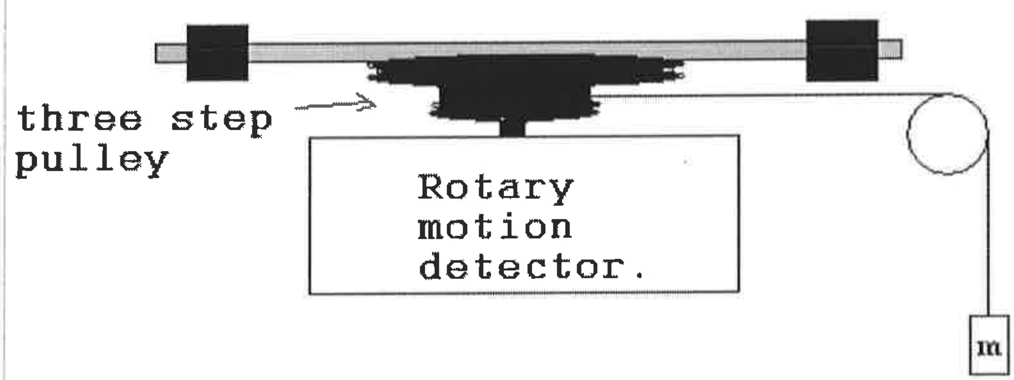
In a previous activity we came to the conclusion that when an object is free to rotate about an axis (pivot point) then what causes the object to rotate if it is initially at rest is not the force applied to the object but the *torque* due to the force. We defined the torque (τ) about the axis of rotation to be

$\tau = F\, r\, \mathrm{Sin}(\theta)$ where F is the magnitude of the force, r is the distance from the pivot point and θ is the angle as shown below



O indicates the axis of rotation (pivot point)

We want to explore in more detail the effect of a constant torque on an object. To do so we will apply a constant torque to an object and monitor its rotational speed (ω).



Set up the apparatus shown. A rod with masses attached is pivoted on an axis and is free to rotate under the action of the torque exerted by the tension force in the string wrapped around the "three step pulley". The rotary motion detector monitors the angular velocity of the rod.

The rod is 38 cm long. The masses are each 2.5 cm wide and 0.080 kg in mass. The middle portion of the three step pulley has a diameter of 2.85 cm.

Place the masses so that their outer edges are flush with the ends of the rod, one on either side. Arrange the vertical pulley so that the string runs over the pulley correctly when you hang a mass holder on one end.

(1) Apply a constant torque to the system by hanging a mass of 15 grams (this is the empty mass holder plus 10 grams) on the end of the string. Plot the angular speed versus time and determine the angular acceleration **(the slope)** produced by this constant torque.
(You will find the slope of the line by performing a linear fit to the data. When doing so, **always use the part of the data with angular velocity values between 4 rad/s and 8 rad/s.)**

Is the angular acceleration constant? If so what is its value?

NOTE: the sign of the acceleration will depend on how you have set up the apparatus. **Let's use the magnitude of the angular acceleration at all times.** This just means that we are taking the direction of rotation to be the positive direction.

Let's draw a force diagram for the 5 gram mass and use it to help us figure out the tension in the string.

$\Sigma F_y = m\, a_y$ Therefore $T - 9.8m = m(-a)$

where a is the magnitude of the linear acceleration of the hanging mass

But what is the value of the linear acceleration of the 5 gram mass?

The angular acceleration (α) of the three-step-pulley and the linear acceleration (a) of the hanging mass are related by the radius (R_3) of the three-step-pulley around which the string is wrapped

$$\mathbf{a = \alpha R_3}$$

Therefore $T - 9.8m = m(-\alpha R_3)$

And so finally, $\mathbf{T = m\,(9.8 - \alpha R_3)}$

Using your value for the tension in the string, calculate the torque that this tension exerts about the pivot point. Make sure to record these values in a safe place, you will need them later.

(2) Now let's see how the angular acceleration depends on the applied torque.

Find the angular acceleration for five different applied torques (you already have one of the five). **Remember to always use the part of the data with angular velocity values between 4 rad/s and 8 rad/s.)**

Plot the applied torque (on y-axis) versus the angular acceleration (on the x-axis).

Does your graph look like a straight line? Fit a straight line to your graph. Does it pass through the origin? Probably not because there is some friction torque at the axis tending to oppose the motion. We can estimate the torque due to friction at the axis from the y-intercept of the graph.

$$\tau_{\text{total}} = I\,\alpha$$

where **I** is the Moment of Inertia of the rotating object. But

$$\tau_{\text{total}} = \tau_{\text{applied}} + \tau_{\text{friction}}$$

The friction torque will be negative since it opposes the motion (which we take to be the positive direction) and so I will write this equation in terms of the magnitude of the friction torque.

$$\tau_{\text{total}} = \tau_{\text{applied}} - |\tau_{\text{friction}}|$$

Therefore

$$\tau_{\text{applied}} - |\tau_{\text{friction}}| = I\,\alpha$$

Or

$$\tau_{\text{applied}} = I\,\alpha + |\tau_{\text{friction}}|$$

This implies that the y-intercept of our graph is $|\tau_{\text{friction}}|$ and the slope is I.

Estimate the friction torque and the moment of inertia of the object. Be sure to include error estimates.
(Units of torque are N.m and units of moment of inertia are kg.m^2)

(3) In the textbook you will see that the moment of inertia of an object is the sum of MR^2 for every piece of mass on the object (R is the distance of the piece of mass from the axis of rotation).

If we ignore the mass of the rod and the "three-step-pulley" then what is the moment of inertia of our object (two masses at the ends of the rod).

How does this value compare to the value found from the slope of your graph?