

# CALCULUS-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND THERMODYNAMICS (PHYS180-02): UNIT 5

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## UNIT 4 REVIEW

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Suppose a bundle of wires is carrying current along what we call the  $\hat{z}$  direction. Each wire runs along the z-axis and they are close enough to ignore the fact that the volume of each wire prevents it from being exactly on the z-axis. One wire carries +2.0 A, another carries +1.5 A, and a third carries -0.5 A. What is the B-field strength at a distance of 1 cm away in the x-y plane?

- A: 6 Gauss
- B: 0.6 Gauss
- C: 6 Tesla
- D: 0.6 Tesla

Suppose a loop of current exists in the x-y plane, and a uniform B-field is in the  $\hat{z}$  direction. Which of the following will occur?

- A: The loop will not rotate - there is no torque.
- B: The loop will rotate 180 degrees - there is torque.
- C: The loop will rotate 90 degrees - there is torque.
- D: The loop will rotate -90 degrees - there is negative torque.

## SUMMARY

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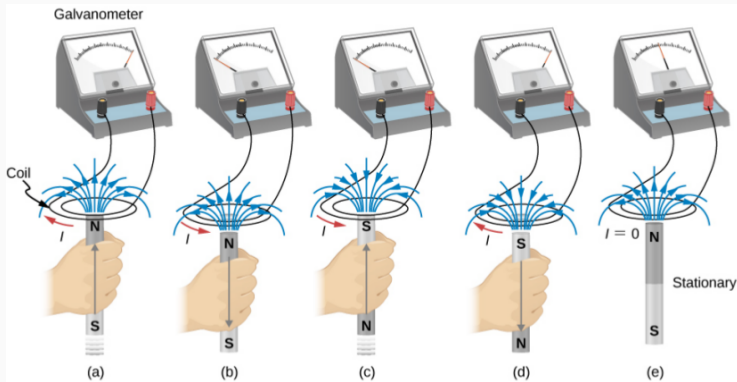
## Reading: Chapters 13 and 14

1. 13.1-2: Faraday's and Lenz's Law
  2. 13.3: Motional EMF
  3. 13.4: Induced E-fields
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1. 14.1: Mutual inductance
  2. 14.2: Self-inductance and inductors
  3. 14.3: Energy in a magnetic field

## FARADAY'S LAW AND LENZ'S LAW

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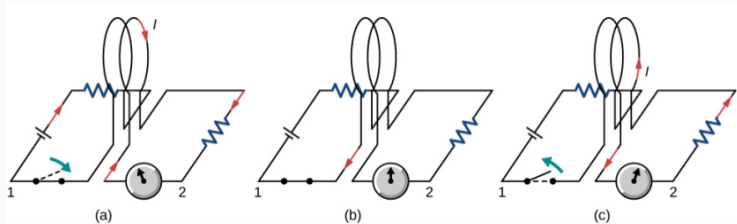
# FARADAY'S LAW



**Figure 1:** Not only does moving charge create B-fields, but B-fields can create moving charge. Study each of the cases above, and (Professor) define the concept of *magnetic flux*.



# FARADAY'S LAW



**Figure 2:** In addition to a moving magnetic field, *other circuits* can make current flow in a circuit. The effect must have something to do with *changing* magnetic fields.

### Faraday's Law

The emf  $\epsilon$  induced is the negative change in the magnetic flux  $\Phi_m$  per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

$$\phi_m = \int_S \vec{B} \cdot d\vec{A} \quad (1)$$

$$\epsilon = -\frac{d\phi_m}{dt} \quad (2)$$

*The unit of magnetic flux is the Webter, or  $1 \text{ Wb} = 1 \text{ T m}^2$ .*

## FARADAY'S LAW

**Example:** A square coil has sides 0.25 m long and is tightly wound with 200 turns of wire. The resistance of the coil 5.0 Ohms. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing by  $-0.040 \text{ T/s}$ . (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?

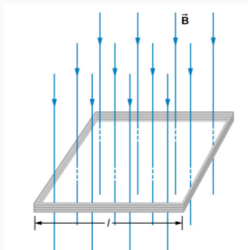


Figure 3: A 200 turn loop in a B-field.

### Lenz's Law

The direction of the induced emf drives current around a wire loop to always oppose the change in magnetic flux that causes the emf.

**Example:** A magnetic field  $B$  is directed outward perpendicular to the plane of a circular coil of radius  $r = 0.50$  m. The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to

$$B(t) = B_0 \exp(-at) \quad (3)$$

with  $B_0 = 1.5$  T and  $a = 5.0 \text{ s}^{-1}$ . (a) Calculate the emf induced in the coil at the times  $t_0 = 0$ ,  $t_1 = 0.05$ , and  $t_2 = 1.0$  seconds. (b) Determine the current in the coil if the resistance is 10 Ohms.

In the previous example, what would happen if the area  $A$  of the loop were increased?

- A: The current would decrease.
- B: The current would stay the same.
- C: The voltage would decrease.
- D: The voltage would increase.

In the previous example, what would happen if the sign of the exponent in  $B(t)$  were flipped?

- A: The current would reverse direction and increase in magnitude.
- B: The current would reverse direction and decrease in magnitude.
- C: The current would keep the same direction and increase in magnitude.
- D: The current would keep the same direction and decrease in magnitude.

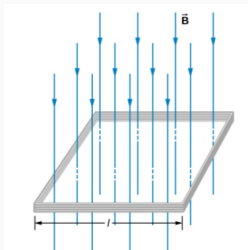
In the previous example, what would happen if  $\alpha$  in the exponent in  $B(t)$  were increased?

- A: The current would reverse direction and increase in magnitude.
- B: The current would reverse direction and decrease in magnitude.
- C: The current would keep the same direction and increase in magnitude.
- D: The current would keep the same direction and decrease in magnitude.



## FARADAY'S LAW

**Example:** The square coil of Figure 4 has sides  $l = 0.25$  m long and is tightly wound with  $N = 200$  turns of wire. The resistance of the coil is  $R = 5.0 \, \Omega$ . The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate  $dB/dt = 0.040t^2$ . (a) Graph the magnitude of the emf induced in the coil. (b) What is the magnitude of the current through the coil at 100 ms?



**Figure 4:** A 200 turn loop in a B-field.

# LENZ'S LAW

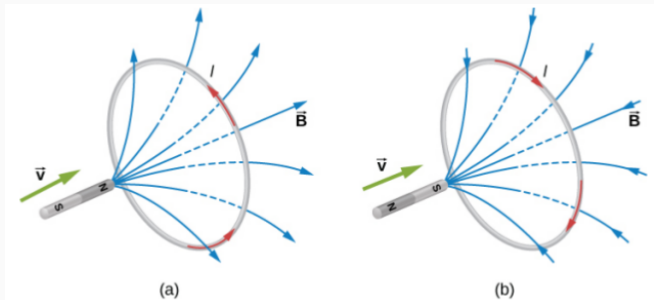


Figure 5: Lenz's Law relates sign of current to B-field.

## MOTIONAL EMF

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## LENZ'S LAW

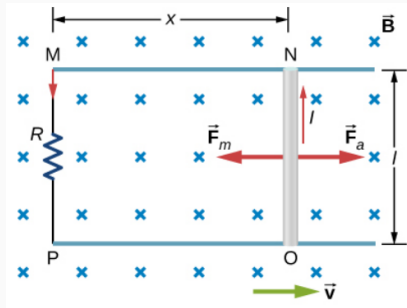


Figure 6: A system in which the magnetic flux depends on time.

1. Show that power is equal to  $P = \vec{F} \cdot \vec{v}$  for constant acceleration.
2. Show that the emf is  $\epsilon = Blv$ , from Faraday's Law.
3. Show that power generated,  $P = I^2R$ , is equal to power injected.

In the previous example, what would happen if  $\vec{F}_a$  was pointed to the left?

- A: The current would reverse direction.
- B: The current would keep the same direction.
- C: The magnetic flux due to the external field would decrease.
- D: A and C

In the previous example, what would happen if  $R$  were increased, but the magnitude of  $F_a$  were kept the same?

- A: The current would decrease.
- B: The current would increase.
- C: The current would remain constant.
- D: The power required would increase.

## INDUCED ELECTRIC FIELDS

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Recall that the relationship between voltage and electric field is

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z} \quad (4)$$

In one dimension, this becomes

$$\vec{E} = -\frac{dV}{dx}\hat{x} \quad (5)$$

If we take a dot product with  $-d\vec{x} = -dx \hat{x}$  on each side, we find

$$-\vec{E} \cdot d\vec{x} = dV \quad (6)$$

Integrating, we have

$$V = -\int \vec{E} \cdot d\vec{x} \quad (7)$$



However, if the voltage is a result of a changing magnetic field, and Faraday's Law, then

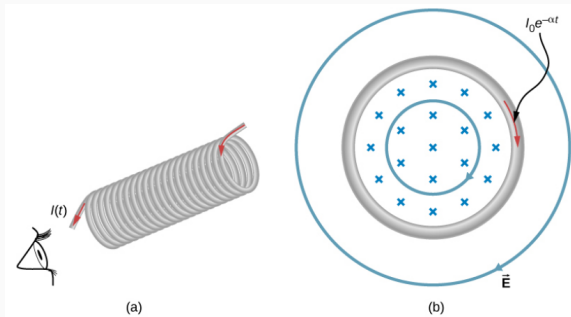
$$\frac{d\phi_m}{dt} = \oint \vec{E} \cdot d\vec{x} \quad (8)$$

Recall that from *electrostatics*,

$$\oint \vec{E} \cdot d\vec{x} = 0 \quad (9)$$

Equation 9 is true for electrostatics because the Coulomb force is **conservative**. But in a previous example we showed that power was being generated and *conserved*, despite the fact that magnetic flux is changing. What is happening?

# LENZ'S LAW



**Figure 7:** A solenoid with a changing current will induce an E-field. The solenoid has turn density  $n$ , and is long compared to the radius.

1. What is the E-field outside the solenoid?
2. What is the E-field inside the solenoid?
3. Create a graph of the E-field strength versus distance.

# FARADAY'S LAW: AN APPLICATION

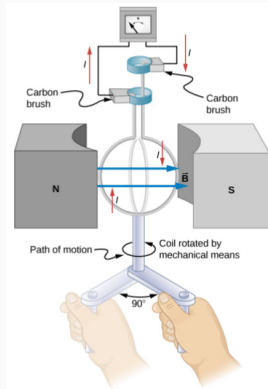


Figure 8: The basic concept behind an AC generator.

## FARADAY'S LAW: AN APPLICATION

Start with Faraday's Law:

$$\epsilon = -\frac{d\phi_B}{dt} \quad (10)$$

The flux  $\phi_B$  is changing and depends on time:

$$\phi_B = \vec{B} \cdot \vec{A}(t) = BA \cos(\theta(t)) \quad (11)$$

Let the *angular velocity* be constant:  $\theta = \omega t$ . Then we have

$$\phi_B = BA \cos(\omega t) \quad (12)$$

Thus the emf (with  $N$  loops) is

$$\epsilon = N\omega BA \sin(\omega t) = \epsilon_0 \sin(\omega t) \quad (13)$$

**The generation of AC power stems from  $\omega$ .**

(Professor: diagram of  $\epsilon(t)$ ).

$$\epsilon = N\omega BA \sin(\omega t) \quad (14)$$

The AC voltage equation above is a basic model for the voltage from a generator. Which of the following would increase the *amplitude* of the emf?

- A: Turning the area more slowly.
- B: Turning the area more quickly.
- C: Increasing the B-field.
- D: Both C and D.

$$\epsilon = N\omega BA \sin(\omega t) \quad (15)$$

The AC voltage equation above is a basic model for the voltage from a generator. Which of the following would increase the *frequency* of the emf?

- A: Turning the area more slowly.
- B: Turning the area more quickly.
- C: Increasing the B-field.
- D: Both C and D.

## INDUCTANCE

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# MUTUAL INDUCTANCE

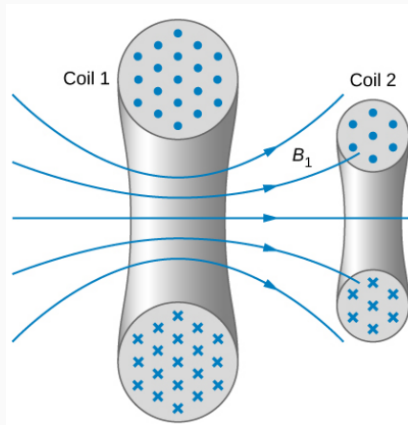


Figure 9: The concept of mutual inductance.



First, some notation:

- The flux through coil 2 by coil 1:  $\phi_{21}$
- The flux through coil 1 by coil 2:  $\phi_{12}$

Mutual inductance of coil 2 with respect to coil 1:

$$M_{21} = \phi_{21} \frac{N_2}{I_1} \quad (16)$$

Mutual inductance of coil 1 with respect to coil 2:

$$M_{12} = \phi_{12} \frac{N_1}{I_2} \quad (17)$$

It can be shown that

$$\boxed{M_{21} = M_{12}} \quad (18)$$

## MUTUAL INDUCTANCE

What are the units of mutual inductance? Consider the emf induced in loop 2 by loop 1:

$$\epsilon_2 = -\frac{d}{dt} (\phi_{21} N_2) \quad (19)$$

Substitution for the inductance gives

$$\epsilon_2 = -\frac{d}{dt} \left( \frac{M_{21} I_1}{N_2} N_2 \right) \quad (20)$$

$$\epsilon_2 = -\frac{d}{dt} (I_1 M_{21}) \quad (21)$$

$$\epsilon_2 = -M \frac{dI_1}{dt} \quad (22)$$

$$\epsilon_1 = -M \frac{dI_2}{dt} \quad (23)$$

So inductance relates induced emf to current change, and has units of  $\text{V s A}^{-1}$ .

## MUTUAL INDUCTANCE

A coil of  $N_2$  turns and radius  $R_2$  surrounds a long solenoid of length  $l_1$ , radius  $R_1$ , and  $N_1$  turns. (a) What is the mutual inductance of the two coils? (b) If  $N_1 = 1000$ ,  $N_2 = 20$ ,  $R_1 = 3.0$  cm,  $l_1 = 100.0$  cm, and  $dI_1/dt = 150$  A/s, what is the induced emf in the surrounding coil?

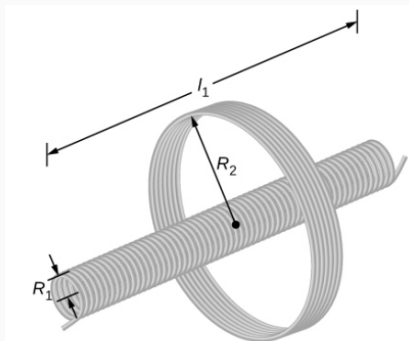


Figure 10: Example of mutual inductance.

## MUTUAL INDUCTANCE

A current  $I(t) = I_0 \sin(\omega t)$  flows through the solenoid. If  $I_0 = 7.5$  A, and  $\omega = 60\pi$  rad/sec, what is the maximum induced emf in the surrounding coil?

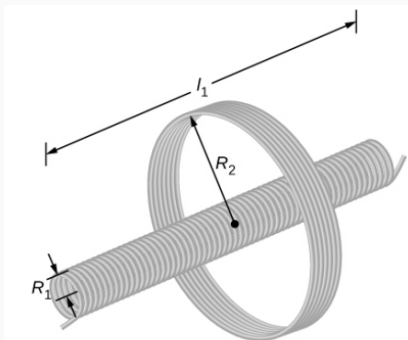


Figure 11: Example of mutual inductance.

## SELF-INDUCTANCE AND INDUCTORS

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## CONCLUSION

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## Reading: Chapters 13 and 14

*This weekend:*

1. 13.1-2: Faraday's and Lenz's Law
2. 13.3: Motional EMF
3. 13.4: Induced E-fields

*Next week:* Chapter 14.1-3

# ANSWERS - CHAPTER 13 AND UNIT 4 REVIEW

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• B

• A

• D

• A

• D

• D

• A

• D

• B

## ANSWERS - CHAPTER 14

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• ...

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