Wednesday warm-up: Forces III

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September 29, 2025

1 Memory Bank

- Force of drag, in air or other gas: $F_D = \frac{1}{2} C\rho A v^2$.
- In the above formula, C is an empirical constant, ρ is the density of the air or gas, A is the area of the object, and v is the object's velocity.
- The horizontal force of friction: $\vec{f} = -\mu N \hat{i}$, where μ can be either the *static* or *kinetic* coefficient of friction.

2 Forces III

1. Suppose a cyclist with $A = 0.5$ m^2, $C = 1.0$, and total mass $m = 70$ kg is pedalling at 20 m/s. Assume the density of air is $\rho = 1.2$ kg m^{-3}. (a) What is the drag force on the system? (b) What is the drag force if the speed drops to 10 m/s? (c) Suppose the speed is now 20 m/s again, but the cyclist ducks down to lower the area to $A = 0.25$ m^2. What is the new drag force?

2. Show that the acceleration of any object down a frictionless incline that makes an angle θ with the horizontal is $a = g \sin\theta$. (Note that this acceleration is independent of mass.)

3. (a) Show that the acceleration of any object down an incline where friction behaves simply (that is, where $f_k = \mu_k N$) is $a = g(\sin\theta - \mu_k \cos\theta)$. Show that the acceleration reduces to the expression found in the previous problem when friction becomes negligibly small.

4. Calculate the deceleration of a snow boarder going up a slope of 10 degrees, assuming the coefficient of friction for waxed wood on wet snow (0.1).

5. Sometimes, friction can be *static*. That is, there's enough friction to keep an object stuck in position. Suppose a sled is on a hill with incline angle θ, and the coefficient of static friction is μ_s. Show that the maximum θ, such that the sled does not move, satisfies

$$\mu_k = \tan\theta \tag{1}$$

6. Let the force of drag be $F_{\mathrm{D}} = (1/2)C\rho A v^2$, and the weight of a falling system be $w = mg$. Show that the maximum speed the system can achieve is

$$v = \sqrt{\frac{2mg}{C\rho A}} \tag{2}$$

Estimate this speed for a falcon diving to catch prey, letting $C = 0.1$.