Study Guide for Midterm 3 for Calculus-Based Physics: Electricity and Magnetism, with Answers

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1 Equations and constants

1. Kirchhoff's Rules: 1) $I_{in} + I_{out} = 0$ (Junction Rule) 2) $\sum_{loop} V_i = 0$ (Loop Rule)
2. Power from current and voltage: $P = iV$
3. Power from currenta and resistance: $P = I^2 R$
4. Definition of magnetic flux: $\phi = \vec{B} \cdot \vec{A}$. The units are T m^2, which is called a Weber, or Wb.
5. Faraday's Law: $emf = -N\frac{\Delta\phi}{\Delta t}$
6. Faraday's Law using **Inductance**, M: $emf = -M\Delta I/\Delta t$.
7. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L.
8. Magnetic permeability: $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}
9. Units of inductance: V s A^{-1}, which is called a Henry, or H.

2 Exercises - Solve them first, then read answers

1. **Review Problem (similar exercise on the final)**

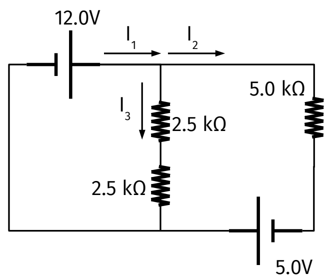

Figure 1: A circuit with three resistors.

(a) Solve for the currents I_1-I_3 in Fig. 1.

$I_3 = 12/5$ mA, $I_2 = 17/5$ mA, and $I_1 = 29/5$ mA.

(b) What is the power consumed by each resistor in Fig. 1?

Two of the resistors (the ones with 2.5 kΩ) each consume $P = IV = I_3 \times V_1 = 2.5 \times 10^{-3} \times 12.0$ W which is 30 mW. The other resistor consumes $P = I^2R = I_2^2 \times 5.0\text{k}\ \Omega$, so $(17/5 \times 10^{-3})^2 5.0 \times 10^3 = 57.8$ mW.

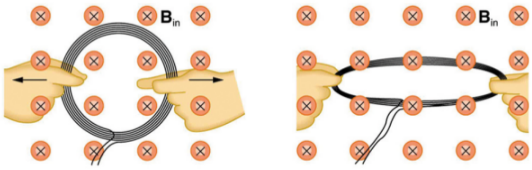

Figure 2: (Left) A magnetic field passes through loops of wire. (Right) The loops are stretched, reducing the area.

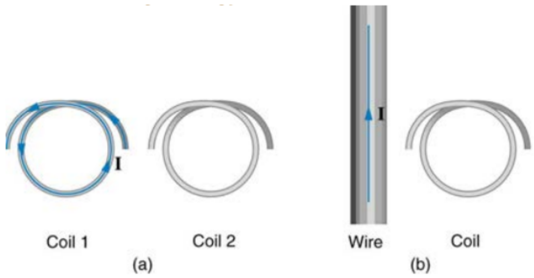

Figure 3: (a) The coils lie in the same plane. (b) The wire is in the plane of the coil.

2. **Chapter 13: Electromagnetic Induction**

   (a) In Fig. 2 (left) a uniform magnetic field passes through loops of wire. In Fig. 2 (right) the **area** of the loops is reduced by stretching the loops. Which of the following is true?

   - A: No current flows through the wires.
   - B: Current does flow through the wires, but there is no induced emf in the wires.
   - C: Current flows through the wires, because the induced emf is caused by a change in electric flux.
   - D: Current flows through the wires, because the induced emf is caused by a change in magnetic flux.

   (*The answer is D. Magnetic flux $\phi = \vec{B} \cdot \vec{A} = BA$ in this case. If the area changes, so does the flux.*

   (b) Consider again the system in Fig. 2. The initial area is $A_i = 0.02$ m^2, the final area is one-half, or $A_f = 0.01$ m^2, and the transition from A_i to A_f takes 0.1 seconds. The B-field strength is 0.1 T. What is $\Delta\phi/\Delta t$?

   The change in flux is $\Delta\phi = B\Delta A$, because the B-field is not changing. Thus, $\phi_f = BA_f$ and $\phi_i = BA_i$. Subtracting, we get the change in flux: $\Delta\phi = B(A_f - A_i) = 0.1 \times (0.02 - 0.01) = 0.1 \times 0.01 = 10^{-3}$ Wb. That makes $\Delta\phi/\Delta t = 10^{-3}/0.1 = 10^{-2}$ Wb/s, or 10^{-2} V (10 mV).

   (c) Continuing with Fig. 2, if $\Delta\phi/\Delta t$ gives 10 mV, and the coil of wire has 100 turns, what is the induced emf in the coil?

   Apply Faraday's Law: $-N\Delta\phi/\Delta t = -100 * 10$ mV, or -1.0 V.

   (d) Consider Fig. 3 (left). In which direction is the current in the right-hand coil induced, if the current in the left-hand coil (a) increases? (b) decreases?

   (a) If the counter-clockwise current on the left *increases*, the corresponding flux *into the page* in the right-hand coil will increase. Therefore, the right-hand coil will resist the change. That means that the B-field generated will have to be *out of the page* and therefore we get a **counter-clockwise** current. (b) The exact opposite will happen if the current at left decreases, so we get a **clockwise** current.

3. **Chapter 14: Inductance**

   (a) Assume that the current in the wire in Fig. 3 (right) is not changing. (a) Is there any induced emf in the coil at right? (b) Suppose the current changes at a rate of 25.0 A/s, and the mutual inductance M in the system is 0.1 mH. What is the induced emf in the coil?

   (a) No - only if the current changes will there be any induced emf. (b) Applying Faraday's law with inductance, we have $emf = -M\Delta I/\Delta t = 0.1 \times 10^{-3} \times 25 = 2.5 \times 10^{-3} = 2.5$ mV.

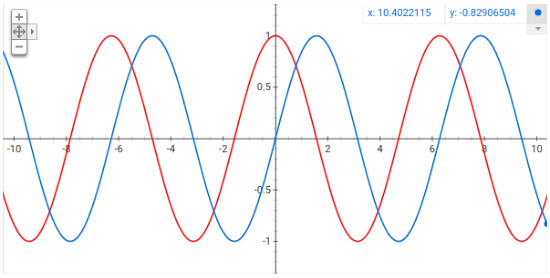

Figure 4: When you graph a cosine and a sine, make sure the sine is zero at the origin and cosine is maximal.

(b) A coil with a self-inductance L carries a current $I(t) = -at^2 + b$ A. (a) What is the emf induced in the coil as a function of time? (b) If $L = 3.0$ H, $a = 2.0$ A s^{-2}, and $b = 2.0$ A, what is the voltage after 0.1 seconds?

First we have to identify the correct relationship between emf and current: $emf = -LdI/dt$ (Faraday's law with self-inductance). Next, $dI/dt = -2at$, so $emf(t) = 2aLt$. (b) Using the given values, we have $emf = 2(2.0)(3.0)(0.1) = 1.2$ V.

(c) A coil with a self-inductance of 2.0 H carries a current that varies with time according to $I(t) = (2.0A)\sin(120\pi t)$. (a) Find an expression for the emf induced in the coil. (b) Graph the current and induced voltage, showing how they are related.

(a) $emf = -LdI/dt$, so we need $dI/dt = (2.0A)(120\pi)\cos(120\pi t)$. Since $L = 2.0$ H, we have $emf = -480\pi\cos(120\pi t)$ V. (b) A graph that is a cosine with the amplitude of 480π and period of $T = 2\pi/(120\pi) = 1/60.0$ is sufficient for the voltage, and note that the current is a sine function. This means that at the origin sine is zero and cosine is maximal. See Fig. 4.