Monday Warm Up: Unit 5: Momentum II

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1 Memory Bank

- $v = r\omega$ ... Relationship between tangential velocity and angular velocity.
- $\omega = 2\pi f = 2\pi/T$ ... Relationship between angular velocity (ω), frequency (f), and period (T).
- $\vec{p} = m\vec{v}$ ... Definition of momentum.
- $\vec{F}_{\rm Net} = \frac{d\vec{p}}{dt}$ ... Force and momentum
- Let M be the total mass of a system, and let m_j and $\vec{r}_j$ ($j = 1, ..., N$) be the masses and positions of the constituent parts of the system. The position of the center of mass is

$$\vec{r}_{\rm CM} = \frac{1}{M}\sum_{j=1}^{N} m_j \vec{r}_j \tag{1}$$

- The momentum of the center of mass $\vec{P}_{\rm CM}$ is

$$\vec{P}_{\rm CM} = \sum_{j=1}^{N} \vec{p}_j \tag{2}$$

- The net external force on a system obeys

$$\vec{F} = \frac{d\vec{P}_{\rm CM}}{dt} \tag{3}$$

2 Momentum II

1. In Pre-columbian and colonial period Latin America, *gauchos* would sometimes hunt with a weapons known as *bolas* (Fig. 1). The bolas were thrown, and would spin around the center of mass until they wrapped the limbs of the prey. (a) Suppose two masses m are separated by a diameter d. The positions of the masses are $r_1(t) = \frac{1}{2}d\cos(2\pi ft)$ and $r_2(t) = \frac{1}{2}d\cos(2\pi ft - \pi)$. (a) Graph the positions in an x-y coordinate system. (b) Locate the center of mass at $t = 0$. (c) Suppose $f = 5$ Hz, or 5 rotations per second. Locate the center of mass at $t = 0.2$ seconds.
2. Consider the same bola system as the previous exercise. (a) If the frequency f is 5 Hz, what is the angular velocity? (b) What is the tangential velocity of each bola, if $d = 0.7$ m? (c) If the mass of each bola is 1.2 kg, and the bolas are following $r_1(t)$ and $r_2(t)$, what is the magnitude of the momentum of each bola? (d) What is the *total momentum* $P_{\rm CM}$?
3. Now suppose the gaucho throws the bola (Fig. 1), and the *center of mass* has an initial velocity of 20 m s^{-1} at a 45 degree angle. (a) How far does it travel before it lands? (b) Does the rotation of the bolas around the center of mass affect how far it goes?

Figure 1: A *gaucho* using a bola weapon to hunt a rhea bird.