2 Estimations and Unit Analysis

1. Suppose you are standing at the edge of a canyon. You clap, and here the sound of the echo off of the other side of the canyon wall about 1.5 seconds later. You estimate the canyon wall to be about 0.5 km away. a) What is the speed of sound in meters per second? b) What is it in kilometers per hour?

round trip = 1km total

displacement - 0.5km
time -1.5 s

speed is velocity velocity is $\frac{\Delta x}{\Delta t}$

a) $0.6\,km/s = \frac{1000\,m}{1\,km} = 0.6 \times 1000 \approx 600\,m/s$ b) $0.6\,\frac{km}{s} = \frac{3600}{1hr} = \frac{2160\,km}{h}$ $\frac{1\,km}{1.5\,s} = 0.6\,km/s$

2. a) What is 0.25 m^3 in cm^3? b) What is 100 km/hour in m/s? c) What is 2 kg m s^{-2} in gm cm ms^{-2}?

$.25\,m^3 \left(\frac{1\,m}{100\,cm}\right)^3 = \frac{.25\,m^3}{100^3\,cm^3}$

All same answer diff forms $= .25 \times 10^{-2}\,m^3 = 0.0025 = 2.5 \times 10^{-3}\,m^3$

b) $100\,\frac{km}{hr} = \left(\frac{1000\,m}{1\,km}\right)\left(\frac{1\,hr}{3600\,s}\right) = \frac{100{,}000}{3600} \approx 27.8\,m/s$

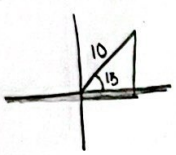
c) $2\,kg\,m/s^{-2}$ in $g/cm\,ms^{-2}$

$2\,kg\,\frac{m}{s^2} = \left(\frac{1000\,g}{1\,kg}\right)\left(\frac{1\,m}{100\,cm}\right) = \frac{2000}{100} = 20\,g/cm\,s^{-2}$

3 Vectors

1. Write the following vectors in component form: (a) $\vec{x}_1$ is a vector with a magnitude of 10 meters and that makes an angle of 15 degrees with respect to the x-axis. (b) $\vec{x}_2$ is a vector with magnitude 20 meters that makes an angle of 135.0 degrees with respect to the x-axis.

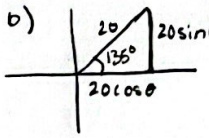
a)



$\vec{x}_1 = 10\cos 15\,\hat{i} + 10\sin 15\,\hat{j}$

$\vec{x}_1 =$

b)

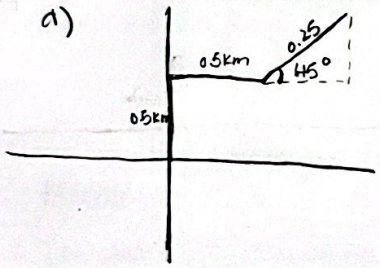


$\vec{x}_1 = 20\cos(135)\,\hat{i} + 20\sin(135)\,\hat{j}$

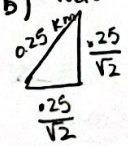
$\vec{x}_1 = -\frac{20}{\sqrt{2}}\,\hat{i} + \frac{20}{\sqrt{2}}\,\hat{j}$

2. A person goes for a walk. They head 0.5 km to the North, and then 0.5 km to the East. Finally, they head North-East at an angle of 45 degrees with respect to the x-axis for 0.25 km. a) Draw a diagram of their trajectory (East is x-axis, North is y-axis). b) What is the final location in x-y coordinates? c) What is the distance from the origin?

a)



b) vector form $= (.25\cos(45)\,\hat{i} + .25\sin(45)\,\hat{j})$ km



$\Delta x_2 = \left(\frac{.25}{\sqrt{2}}\,\hat{i} + \frac{.25}{\sqrt{2}}\,\hat{j}\right)$ km

c) $\Delta x = .5$ km

.5 + .25 = .75 (similar triangles, therefore final answer is correct)

$\vec{x}_f = \left(.5 + \frac{.25}{\sqrt{2}}\,\hat{i} + \frac{.25}{\sqrt{2}}\,\hat{j}\right)$ km

.85 km

4 Motion Along a Straight Line

1. The position of a particle moving along the x-axis is given by $x(t) = -1.0 - 4.0t$ m. (a) What is the displacement of the particle between $t = -2.0$ seconds and $t = 2.0$ seconds? (b) What is the velocity?

a) $\Delta x = x_f - x_i$

$= x(-2) - x(2)$

$-1 - 4(-2) - 1 + 4(2)$

$-1 + 8 - 1 + 8$

$-1+8$ $-1+8$

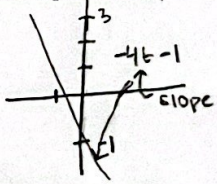
$7 - 1 + 8$

$6 + 8$

14m between -2 and 2 seconds

b) $-4.0t\,m - 1.0$ (y-int; slope = velocity)

b) $-4\,m/s$



2. A particle moves along the x-axis according to $x(t) = -2t + 7t^2$. (a) What is the average velocity between $t = 0$ and $t = 2$ seconds? (b) Draw a graph of the **velocity**. (c) What is the instantaneous velocity at $t = 1$ second? (d) What is the acceleration?

a) Avg Velo $\frac{x_f - x_i}{\Delta t}$ $-2t+7t^2$

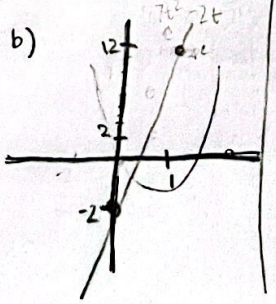
$\frac{-2(2)+7(2)^2 - -2(0)+7(0)^2}{2-0}$

$\frac{-4+28-0}{2}$

$\frac{24}{2} = 12\,m/s$

$-2+14t = velo$

b)



c) $\frac{dx}{dt}$ $-2+7t^2$

$-2+14t$ instantaneous velocity

$-2+14(1)$ at $t=1$

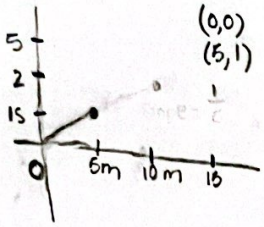
$12\,m/s$

$\frac{dv}{dx}$ $-2+14t$

$= 0+14$

d) $= 14\,m/s^2$

3. A sprinter has a constant acceleration of 5.0 m/s^2. Suppose she starts from rest. (a) How long does it take her to reach her top speed of 10.0 m/s? (b) What is her displacement at that time? (c) Suppose she is running the 100 meter sprint. If she continues at 10.0 m/s for the remainder of the race, what will be her total time?



a) $v(t) = v_i + at$

$10 = 0 + 5t$

$\frac{10}{5} = t$

$t = 2$ seconds

b) $x(t) = \frac{1}{2}at^2 + v_i t + x_i = \frac{1}{2}(5)(2)^2 + 0(2) + 0$

$\frac{5(4)}{2} = \frac{20}{2} =$ will reach at 10m at 2 sec

c) $\frac{90m}{10} = 10 \cdot t$

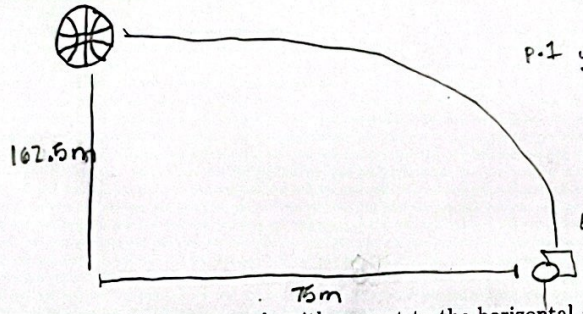
$t = 9$ + initial 2 seconds = $t = 11$ sec @ 100m

5 Motion in Two and Three Dimensions

1. The world record highest basketball shot was made from a height of 162.5 meters above the basketball hoop. The basketball hoop was placed 75 meters horizontally from the shooter. a) Draw a diagram of the situation. b) What is the horizontal velocity required to make the shot? That is, assume the shooter shoots the ball with no vertical velocity, only horizontal.

b) components needed = $\frac{\text{displacement}}{\text{time}}$

a)



p.1 $y(t) = -\frac{1}{2}gt^2 + v_{iy}t + y_i$

$y_i = -\frac{1}{2}gt^2$

$t = \sqrt{\frac{2y_i}{g}}$

$t = \sqrt{\frac{2(162.5)}{9.81}}$

$t = 5.76$ s

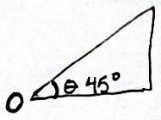
horizontal velo = $\frac{\text{disp}}{\text{time}}$

$v = \frac{75m}{5.76s}$

$v \approx 13$ m/s

2. A baseball is hit at a 45 degree angle with respect to the horizontal at 40 m/s. (a) How far away does it land? (b) How long is it in the air?

a) $v_0 = 40$m/s



$R = \frac{v^2 \sin(2\theta)}{g}$

$R = \frac{40^2 \sin(2(45))}{9.81}$

$R = \frac{40^2 \cdot \sin(90)}{9.81}$

$R = \frac{40^2 \cdot 1}{9.81}$

$R = 163.1$ m

b) $T = \frac{2 v_0 \sin(\theta)}{g}$

$T = \frac{2(40)}{9.81} \cdot \sin 45$

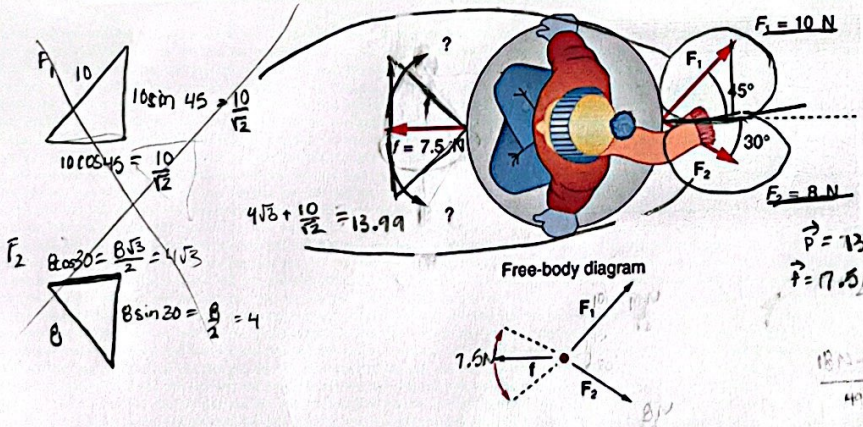
$T = 8.15 \cdot \sin 45$

$T = 8.15 \cdot 1/\sqrt{2}$

25.8 ft ≈ 5.76 s

6 Forces

1. Two children pull a third child on a snow saucer sled exerting forces $\vec{F}_1$ and $\vec{F}_2$ as shown from above in Fig. 1. Find the acceleration of the system if the mass of the child and sled together is 49.0 kg. Note that the direction of the frictional force is unspecified; it will be in the opposite direction of the sum of $\vec{F}_1$ and $\vec{F}_2$.



$10\sin 45 = \frac{10}{\sqrt{2}}$

$10\cos 45 = \frac{10}{\sqrt{2}}$

$8\cos 30 = \frac{8\sqrt{3}}{2} = 4\sqrt{3}$

$8\sin 30 = \frac{8}{2} = 4$

$4\sqrt{3} + \frac{10}{\sqrt{2}} = 13.99$

$\vec{F}_{NET} = \vec{P} + \vec{f} = m \cdot a$

$\frac{\vec{P} + \vec{f}}{m} = \vec{a}$

$\vec{P} = 13.99$

$\vec{f} = 7.5$N

$\frac{13.99 + 7.5\text{N}}{49\text{ kg}} = \vec{a}$

$\frac{26.49\text{ N}}{49\text{ kg}}$

$0.13\frac{N}{kg} = \vec{a}$

0.13 m/s^2

Figure 1: A child is pulled by two other children on a sled atop some ice.