

# CALCULUS-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND THERMODYNAMICS (PHYS180-02): UNIT 2

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Jordan Hanson

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Whittier College Department of Physics and Astronomy

## UNIT 1 REVIEW

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### Reading: Chapters 1 and 2

1. Temperature, Heat, and the 0th Law of Thermodynamics
2. Heat flow and transfer mechanisms
3. Kinetic Theory of Gases

## UNIT 1 REVIEW PROBLEMS

James Prescott Joule first published in December 1840, an abstract in the Proceedings of the Royal Society, suggesting that heat could be generated by an electrical current. Joule immersed a length of wire in a fixed mass of water and measured the temperature rise due to a known current flowing through the wire for a 30 minute period. By varying the current and the length of the wire he deduced that the heat produced was proportional to the square of the current multiplied by the electrical resistance of the immersed wire.

Wikipedia:

[https://en.wikipedia.org/wiki/Joule\\_heating](https://en.wikipedia.org/wiki/Joule_heating).

## UNIT 1 REVIEW PROBLEMS

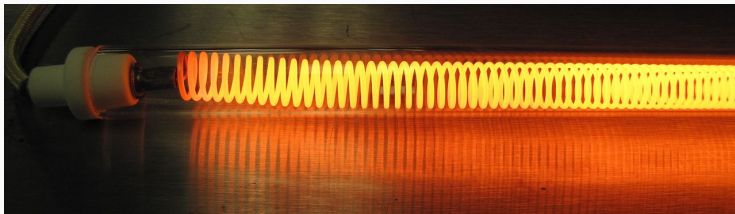
Suppose the wire of J. Joule is carrying 10 W of power. If the mass of the water in the container is 1 kg, how long will it take to raise the temperature of the water by 5 degrees? (The heat capacity of water is  $4186 \text{ J/kg/}^{\circ}\text{C}$ ).

- A: 10 seconds
- B: 80 seconds
- C: 420 seconds
- D: 3600 seconds

## UNIT 1 REVIEW PROBLEMS

Suppose the the water begins at  $95^{\circ}\text{C}$ , and ends at  $100^{\circ}\text{C}$ . How long will it take to boil away all of the water? (The latent heat of vaporization of water is  $2256 \text{ kJ/kg}$ ).

- A: About an hour
- B: About 6 hours
- C: About 60 hours
- D: Several days



**Figure 1:** An example of Joule heating in a toaster coupling.

## SUMMARY

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### Reading: Chapters 3 and 4

1. The First Law of Thermodynamics
2. The Second Law of Thermodynamics

## JITT - READING QUIZ RESULTS

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# THE FIRST LAW OF THERMODYNAMICS

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**Equation of state:** The equation of state of a system, in general, looks like this:

$$f(p, V, T) = 0 \quad (1)$$

- $V$  is an **extensive** variable.
- $p$  and  $T$  are **intensive** variables.
- Holding *intensive* variables constant, doubling the mass of a system doubles the volume. That is the relationship between intensive and extensive variables.

The equation of state of an ideal gas looks like this:

$$pV - nRT = 0 \quad (2)$$

Which of the following is correct?

- A: The extensive variables are  $p$ ,  $V$ , and  $R$
- B: The only extensive variable is  $n$ , and  $p$ ,  $V$ , and  $T$  are the intensive ones
- C: The only extensive variable is  $n$ , and  $p$  is the only intensive one
- D: There are no extensive variables because none of them is mass

Recall that the definition of work done on a system is:

$$W = \int \vec{F} \cdot d\vec{x} \quad (3)$$

- $\vec{F}$  is the net force acting on the system
- $d\vec{x}$  is the incremental displacement

# THE FIRST LAW OF THERMODYNAMICS

Example: suppose the force is gravity, near the surface of the Earth  $\vec{F} = -mg\hat{y}$ , and an object at height  $h$  is released.

$$W = \int \vec{F} \cdot d\vec{x} = -mg\hat{y} \cdot -h\hat{y} = mgh \quad (4)$$

- $\vec{F}$  is the net force acting on the system
- $d\vec{x}$  is the incremental displacement
- Work-energy theorem can give the final speed:  
 $\frac{1}{2}mv_f^2 = mgh.$

Definition of work, applied to a thermodynamic system *with an equation of state*:

$$W = \int \vec{F} \cdot d\vec{x} = \int p\vec{A} \cdot d\vec{x} = \int p dV \quad (5)$$

- $\vec{A}$  and  $d\vec{x}$  assumed to be parallel
- $p$  is constant over  $d\vec{x}$



# THE FIRST LAW OF THERMODYNAMICS

Definition of work, applied to a thermodynamic system *with an equation of state*:

$$W = \int p dV \quad (6)$$

**Group board exercise:** Substitute the Ideal Gas Law into Eq. 6, to derive the work done *when temperature is constant and  $V$  is the independent variable*.

*Hint: the integral of  $\frac{1}{x}$  is  $\ln x + C$ .*

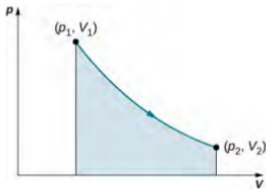
**Group board exercise:** Repeat, but at *constant pressure* instead of constant temperature, again with  $V$  as the independent variable.

The van der Waals *equation of state* is:

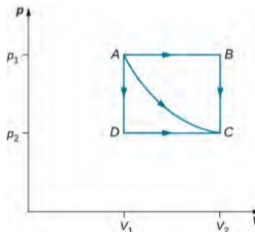
$$\left(p + \frac{a}{V^2}\right) (V - b) - nRT = 0 \quad (7)$$

**Group board exercise:** Substitute this equation of state into Eq. 6, to find the work done by isothermal expansion. Compare to the isothermal result from the Idea Gas Law, and recover the previous result for  $W$  by setting  $a$  and  $b$  to zero.

# THE FIRST LAW OF THERMODYNAMICS



**Figure 3.5** When a gas expands slowly from  $V_1$  to  $V_2$ , the work done by the system is represented by the shaded area under the  $pV$  curve.



**Figure 3.6** The paths ABC, AC, and ADC represent three different quasi-static transitions between the equilibrium states A and C.

**Figure 2:** On a  $pV$  diagram, work is the area under the curve, in the same fashion that it is the area under the curve on a force displacement diagram.

How much energy is available to do work, in the form of heat? (For example, if we expand a gas at constant pressure, by how much can we decrease the temperature?)

Recall from the prior chapter that

$$E_{\text{int}} = \frac{3}{2}nRT = \frac{3}{2}nN_A k_B T = \frac{3}{2}NkT \quad (8)$$

The *internal energy* of a gas must be comprised of the average kinetic energies of the molecules. Here we ignore molecular interactions.

## The First Law of Thermodynamics

Associated with every equilibrium state of a system is its internal energy  $E_{\text{int}}$ . The change in  $E_{\text{int}}$  for any transition between two equilibrium states is  $\Delta E_{\text{int}} = Q - W$ , where  $Q$  is the heat exchanged by the system and  $W$  is the work done on or by the system.

- If heat is added to the system:  $Q > 0$
- If heat is removed from the system:  $Q < 0$
- If work is done by the system:  $W > 0$
- If work is done on the system:  $W < 0$

# THE FIRST LAW OF THERMODYNAMICS

In each of the following scenarios, select the best answer:

- A:  $E_{\text{int}}$  increases
  - B:  $E_{\text{int}}$  decreases
  - C:  $E_{\text{int}}$  does not change
  - D: I am confused.
- 
- Scenario 1: A gas expands against a piston, doing some work, but the gas loses no heat.
  - Scenario 2: A gas expands against a piston, performing 10 J of work. A separate device adds 10 J of heat to the gas.
  - Scenario 3: A piston is pushed against a gas, requiring 10 J of work. A separate device adds 10 J of heat to the gas.

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# THE FIRST LAW OF THERMODYNAMICS

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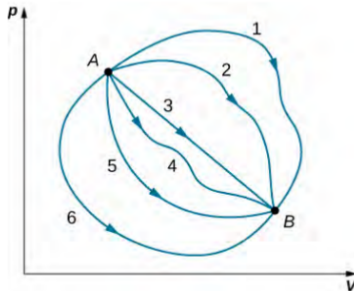
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  - D: I am confused.
- 
- Scenario 4: A cold scientist clutches a warm canteen of water to his chest (the system is the canteen).
  - Scenario 5: A cylinder of gas has a piston on one end. Someone pulls the piston slowly, doing 10 J of work.

# THE FIRST LAW OF THERMODYNAMICS

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# THE FIRST LAW OF THERMODYNAMICS



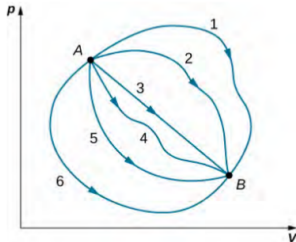
**Figure 3.7** Different thermodynamic paths taken by a system in going from state A to state B. For all transitions, the change in the internal energy of the system  $\Delta E_{\text{int}} = Q - W$  is the same.

**Figure 3:** The internal energy for a given state depends only on the temperature, and not the path taken to arrive at that state.

# THE FIRST LAW OF THERMODYNAMICS

Which of the following is true of the *work done by the system*?

- A: Path 2 corresponds to the most work done, and path 5 the least work done.
- B: Path 5 corresponds to the most work done, and path 2 the least work done.
- C: Path 3 corresponds to the most work done, and path 4 the least work done.
- D: Path 4 corresponds to the most work done, and path 3 the least work done.



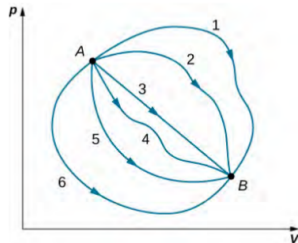
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**Figure 4:** Use this diagram to answer the questions at left

# THE FIRST LAW OF THERMODYNAMICS

Suppose  $\Delta E_{\text{int}} = 0$ . Which of the following is true?

- A: For path 1, the system gains the most heat.
- B: For path 5, the system loses the most heat.
- C: For path 3, the system loses the most heat.
- D: For path 1, the system loses the most heat.



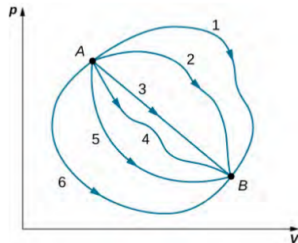
**Figure 3.7** Different thermodynamic paths taken by a system in going from state A to state B. For all transitions, the change in the internal energy of the system  $\Delta E_{\text{int}} = Q - W$  is the same.

**Figure 5:** Use this diagram to answer the questions at left

# THE FIRST LAW OF THERMODYNAMICS

Suppose the system in question is an ideal gas. Which path most likely represents a transition with constant temperature?

- A: Path 3
- B: Path 2
- C: Path 5
- D: Path 1



**Figure 3.7** Different thermodynamic paths taken by a system in going from state A to state B. For all transitions, the change in the internal energy of the system  $\Delta E_{\text{int}} = Q - W$  is the same.

**Figure 6:** Use this diagram to answer the questions at left

Recall that the work done by an  $n$  moles of an expanding ideal gas at constant temperature  $T$  is given by  $W = nRT \ln(V_f/V_i)$ , where  $R = 8.314 \text{ J/mole/Kelvin}$ . Calculate the work done if heat is added to 2 moles of an ideal gas at 300 K such that the volume of the gas grows by a factor of 2.

- A: 416 J
- B: 520 J
- C: 3460 J
- D: 4320 J

Same scenario, but calculate the work if the gas was at a constant temperature of 200 K.

- A: 2305 J
- B: 3410 J
- C: 280 J
- D: 320 J



If the temperature remains at 200K during the process, what is the heat transferred into or out of the system?

- A: -2305 J
- B: 2305 J
- C: -3410 J
- D: 3410 J

Suppose we have a system where we can add external heat into a rocket, and then 100 percent of that heat is converted to kinetic energy for the rocket. If the rocket has 0.1 kg of mass, and  $g = 10 \text{ m/s/s}$ , how much heat is required to launch the rocket to a height of 50 meters?

- A: 20 J
- B: 30 J
- C: 40 J
- D: 50 J

If in the heat-to-rocket system, we have to provide the heat first to 2 moles of an ideal gas, what would be the required temperature change?

- A: 2 degrees C
- B: 3 degrees C
- C: 20 degrees C
- D: 30 degrees C

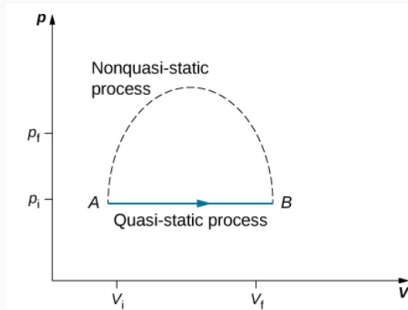
## THE FIRST LAW OF THERMODYNAMICS

In the previous two problems we equated heat energy with mechanical energy:  $mgh = Q$ , and then calculated  $Q$  with  $Q = \frac{3}{2}nRT$ . Thus, we used  $mgh = \frac{3}{2}nRT$ . Let's try one with kinetic energy instead of potential energy.

If we are using the heat stored in 2 moles of an ideal gas, what temperature change is required to get that heat to accelerate a 0.1 kg object to 10 m/s?

- A: 3 degrees C
- B: 2 degrees C
- C: 0.3 degrees C
- D: 0.2 degrees C

# THE FIRST LAW OF THERMODYNAMICS



**Figure 7:** Two processes: quasi-static and non-quasi-static. We study state transitions composed of many quasi-static transitions. The pictured non-quasi-static transition has two volumes with the same pressure, so how can we reasonably define the state of a system?

Words of the text: “A quasi-static transition is one in which the change in state is made infinitesimally slowly so that at each instant, the system can be assumed to be at a thermodynamic equilibrium with itself and with the environment.”

**What follows are a few examples of thermodynamic processes that may be classified as quasi-static.** In realistic systems, some amount of non-quasi-static behavior is normal.

A process is called **cyclic** if it returns a system to the same state with the same internal energy. We should know at least these types of quasi-static processes:

1. Isothermal ( $T$  is constant)
2. Isobaric ( $p$  is constant)
3. Adiabatic ( $Q$ , the heat added/subtracted, is zero)
4. Isochoric ( $V$  is constant)

Exercise: use the following simulation to create cyclic processes.

<https://www.geogebra.org/m/mXvUPjFz>

Exercise: use the following simulation to create cyclic processes: <https://www.geogebra.org/m/mXvUPjFz>.  
What do you notice about the total heat exchanged  $Q$ , and the total work done,  $W$ ?

1. Create four different cyclic processes that involve the four quasi-static process types, and explain quantitatively (in your lab notebook) why each fits the relevant criterion.
2. What is true about  $Q$  and  $W$  for each of these for processes you've created? Why is this the case?



## HEAT CAPACITIES OF AN IDEAL GAS

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What follows is a traditional proof of the following theorem:

$$C_P = C_V + R \quad (9)$$

*The molar heat capacity of a gas at constant pressure is always larger than at constant volume.*

- **Assume** an ideal gas
- **Apply** First Law of Thermodynamics
- Evidence for *independence of state function*
- Evidence for *kinetic theory of gases*

## HEAT CAPACITIES OF AN IDEA GAS

The first law of thermodynamics:  $dE = dQ - dW$ . Consider an ideal gas, in a vessel with fixed volume. Since  $dW = pdV$ ,  $dW = 0$ . If heat  $dQ$  is added to that vessel, the corresponding temperature increase is

$$dQ = nC_V dT \quad (10)$$

where  $C_V$  represents the *heat capacity at constant volume*<sup>1</sup>. Because  $dE = dQ$ ,

$$dE = nC_V dT \quad (11)$$

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<sup>1</sup>Recall that from kinetic theory, for ideal gases with no molecular interactions,  $C_V = \frac{3}{2}R$ .

## HEAT CAPACITIES OF AN IDEA GAS

Consider the same gas, but at fixed pressure. Beginning with first law of thermodynamics now accounts for work:

$dE = dQ - dW = dQ - pdV$ . If heat  $dQ$  is added to the vessel, the corresponding temperature change is

$$dQ = nC_P dT \quad (12)$$

where  $C_P$  is the *heat capacity at constant pressure*. In order to obtain an expression for  $dE$  involving only temperature, the ideal gas law must be differentiated:

$$d(pV) = d(nRT) \quad (13)$$

$$pdV = nRdT \quad (14)$$

Substituting the right-hand side of Eq. 14 for  $dW$  in the First Law, and the right-hand side of Eq. 12 for  $dQ$  in the first law, the result is

$$dE = n(C_P - R)dT \quad (15)$$

## HEAT CAPACITIES OF AN IDEA GAS

The expressions for  $dE$  under isobaric and isochoric conditions have been obtained:

$$dE = C_V dT \quad (16)$$

$$dE = n(C_P - R)dT \quad (17)$$

If the internal energy of a thermodynamic system depends only on the temperature, then these two expressions must be equal. Upon setting them equal, a fundamental relationship is revealed:

$$\boxed{C_P = C_V + R} \quad (18)$$

It has been shown that  $C_V = \frac{3}{2}R$  from the equipartition principle, so  $C_P = \frac{5}{2}R$  for an ideal gas.

## HEAT CAPACITIES OF AN IDEAL GAS

Jasmine, the heroine of *Artemis*, by Andy Weir, lives on the moon. Suppose she needs to power up a bulkhead in a compartment that has lost pressure, but the bulkhead won't work below a certain temperature. She devises a plan fill up a heated container with oxygen and open it in the compartment, both releasing oxygen and heat. How should she fill the container with heated air, to store the most heat in a given time limit?

- A: Fill a fixed-volume container with low-pressure air and heat it up.
- B: Fill an inflatable object with heated air at constant, low pressure.
- C: Fill a fixed-volume object with high-pressure air and heat it up.
- D: Fill an inflatable object with heated air at constant, high pressure.

## HEAT CAPACITIES OF AN IDEAL GAS

- A: Fill a fixed-volume container with low-pressure air and heat it up.
- B: Fill an inflatable object with heated air at constant, low pressure.
- C: Fill a fixed-volume object with high-pressure air and heat it up.
- D: Fill an inflatable object with heated air at constant, high pressure.

*I would say **D**, merely because we know that the heat capacity of the gas is higher at constant pressure rather than constant volume, and at high pressure we get more atoms of gas...*

Suppose a cylinder of 1 mole of an ideal gas at fixed volume is heated for 6 minutes with 10 W of power. What is the change in temperature? ( $R = 8.314 \text{ J/mol/Kelvin}$ ).

- A: About 10 degrees Kelvin
- B: About 50 degrees Kelvin
- C: About 100 degrees Kelvin
- D: About 150 degrees Kelvin



# ADIABATIC PROCESSES WITH AN IDEAL GAS

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**Adiabatic** describes a process in which no heat is exchanged,  $dQ = 0$ . From the first law of thermodynamics,

$$dE_{\text{int}} = -dW \quad (19)$$

*Free expansion* is used in the text to refer to the special adiabatic case in which  $dW = 0$ , so  $E_{\text{f,int}} = E_{\text{i,int}}$ . More interesting is to consider the two forms of heat capacity, of the previous section, corresponding to isobaric and isochoric processes.

What follows is a traditional proof of the following theorem:

Let  $\gamma = \frac{C_P}{C_V}$ . During an adiabatic process, the following quantity remains constant:

$$pV^\gamma = \text{const} \quad (20)$$

*The produce of the pressure and volume to a fixed power remains constant during an adiabatic process.*

- **Assume** an ideal gas
- **Apply** First Law of Thermodynamics
- Evidence for *independence of state function*
- Evidence for *kinetic theory of gases*

## ADIABATIC PROCESSES WITH AN IDEAL GAS

Consider an *adiabatic* process ( $dQ = 0$ ) in which the First Law of Thermodynamics becomes

$$dE_{\text{int}} = -dW = -pdV \quad (21)$$

For any ideal gas, the internal energy is proportional to the temperature, regardless of the process type. Thus,

$$dE_{\text{int}} = nC_V dT \quad (22)$$

Substituting into the First Law and rearranging,

$$dT = -\frac{pdV}{nC_V} \quad (23)$$

## ADIABATIC PROCESSES WITH AN IDEAL GAS

As before, differentiating the the idea gas law, assuming both pressure and volume may vary, yields

$$dT = \frac{dpV + pdV}{nR} \quad (24)$$

Equating these two expressions for  $dT$ , Eq. 23 and Eq. 24, gives

$$C_V V dp + (C_V + R) p dV = 0 \quad (25)$$

Recall that  $C_V + R = C_p$ . Using this, Eq. 25 becomes

$$C_V V dp + C_p p dV = 0 \quad (26)$$

Letting  $\gamma = \frac{C_p}{C_V} > 1$ , Eq. 26 reads

$$V dp + \gamma p dV = 0 \quad (27)$$

Rearranging Eq. 27 and integrating both sides,

$$\int \frac{dp}{p} + \gamma \int \frac{dV}{V} = 0 \quad (28)$$

$$\ln p + \ln V^\gamma = \text{const} \quad (29)$$

$$\ln pV^\gamma = \text{const} \quad (30)$$

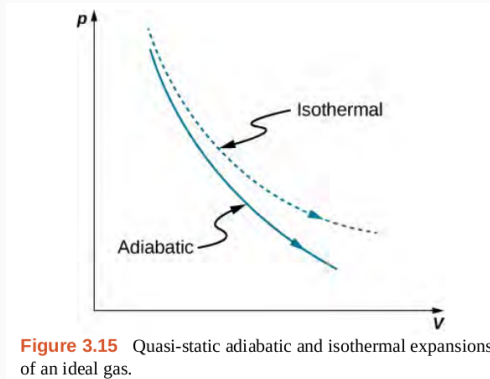
$$pV^\gamma = \text{const} \quad (31)$$

Thus, the theorem holds:

$$\boxed{pV^\gamma = \text{const}} \quad (32)$$

## ADIABATIC PROCESSES WITH AN IDEAL GAS

The reason this result is important is now we have a way to describe adiabatic transitions on  $pV$  diagrams.



**Figure 8:** Isothermal:  $pV = \text{const.}$  Adiabatic:  $pV^\gamma = \text{const.}$

## ADIABATIC PROCESSES WITH AN IDEAL GAS

With an understanding of  $pV$  behavior of different processes, and the First Law, we can begin to construct more complex processes, and predict the work done, and heat required.

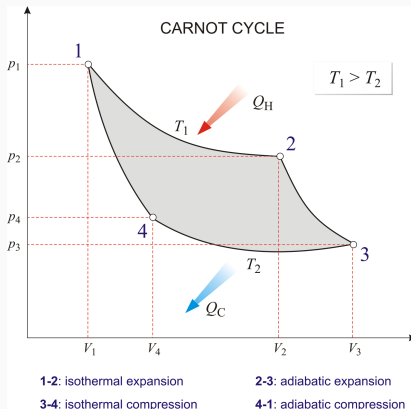


Figure 9: An example of a complex cycle: the Carnot cycle.



We have seen how **cyclic** processes have  $dE_{\text{int}} = 0$ , so  $dQ = dW$  by the First Law. However, we need to introduce *how* to calculate the  $dQ$  to perform a given process, so that we may know the predicted work to be done.

*Isothermal process, ideal gas:*

$$dQ = dW \quad (33)$$

$$dQ = pdV \quad (34)$$

$$dQ = \frac{nRTdV}{V} \quad (35)$$

$$\int dQ = \int \frac{nRTdV}{V} \quad (36)$$

$$Q = nRT \ln \left( \frac{V_f}{V_i} \right) \quad (37)$$

Heat required to change the volume of an idea gas isothermally:

$$Q = nRT \ln \left( \frac{V_f}{V_i} \right) \quad (38)$$

We can repeat this procedure for adiabatic processes:

$$Q = \frac{k}{1 - \gamma} \left( V_f^{1-\gamma} - V_i^{1-\gamma} \right) \quad (39)$$

In Eq. 39,  $k = p_i V_i^\gamma$ . The utility of these formulae: we can predict the heat required to go from one known state to another. The isobaric and isochoric processes are much simpler, but the derivation is similar.

One mole of an ideal gas in a piston is at a temperature of 600 K. How much heat is required to expand the volume of the piston isothermally by a factor of 2.718?

- A: 3 kJ
- B: 4 kJ
- C: 5 kJ
- D: 6 kJ

One mole of an ideal gas in a piston is at a temperature of 300 K. How much heat is required to expand the volume of the piston isothermally by a factor of 2.718?

- A: 1.5 kJ
- B: 2.5 kJ
- C: 3.5 kJ
- D: 4.5 kJ

## CONCLUSION

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## ANSWERS

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## ANSWERS

- 420 seconds
- About 60 hours
- The only extensive variable is  $n$ , and  $p$ ,  $V$ , and  $T$  are the intensive ones
- $E_{\text{int}}$  decreases
- $E_{\text{int}}$  does not change
- $E_{\text{int}}$  increases
- $E_{\text{int}}$  decreases
- Path 5
- 3460 J
- 2305 J
- -2305 J
- 50 J
- 0.2 degrees C
- Fill an inflatable object with heated air at high pressure.
- The piston
- About 100 degrees Kelvin
- 5 kJ
- 2.5 kJ