

Figure 1: (Left) A current I experiences a force F in a B -field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B -field given that F is the Lorentz force.

- a: Into the page
- b: Left
- c: Out of the page

2. Consider Fig. 1 (right). **The Hall Effect.** An E -field exists in the vertical direction and a B -field is perpendicular to the direction of charge velocity. (a) Show that if the E -field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E -field is constant, $E = \Delta V/\Delta x$. Show that

$$\Delta V = \frac{B \Delta x I}{n q_e A} \quad (1)$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33 \text{ T}$, $\Delta x = 2 \text{ cm}$, $I = 10 \text{ A}$, $n = 2 \times 10^{28} \text{ m}^{-3}$, $A = 1 \text{ mm}^2$, and q_e is the charge of an electron.

a) $F_1 = qv \times B$
 $F_2 = qE$
 if $F_1 = F_2$ then $qE = qvB \sin(\theta)$
 $E = vB$
 $v = E/B$

b) $v = I/nqA$
 $E = vB$
 $E = \Delta V/\Delta x$
 $\Delta V = E \Delta x$
 $\Delta V = \frac{B \Delta x I}{n q A}$
 $\Delta V = \frac{(1.33 \text{ T} \cdot 0.02 \text{ m} \cdot 10 \text{ A})}{(2 \cdot 10^{28} \text{ m}^{-3} \cdot 1.602 \cdot 10^{-19} \text{ C} \cdot 1 \cdot 10^{-6} \text{ m}^2)}$
 $\Delta V = 0.83 \text{ V}$

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop $0.65 \times 10^{-15} \text{ m}$ in radius with a current of $1.05 \times 10^4 \text{ A}$. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

$\Phi = B \cdot A$ $\tau = FL$ $\tau_{\text{max}} = B I (L \cdot L) = B I A$
 $F = B \times IL$
 $(1.05 \cdot 10^4) \cdot \pi \cdot (0.65 \cdot 10^{-15})^2 \cdot 2.5 = 3.484 \cdot 10^{-26} \text{ N}\cdot\text{m}$

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B -field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A ? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B -field?

a) $B = \mu_0 n I$
 $= (4\pi \cdot 10^{-7}) (500) (0.3)$
 $= 1.885 \cdot 10^{-4} \text{ T}$

b) $B = 5000 \mu_0 n I$
 $= 0.942 \text{ T}$

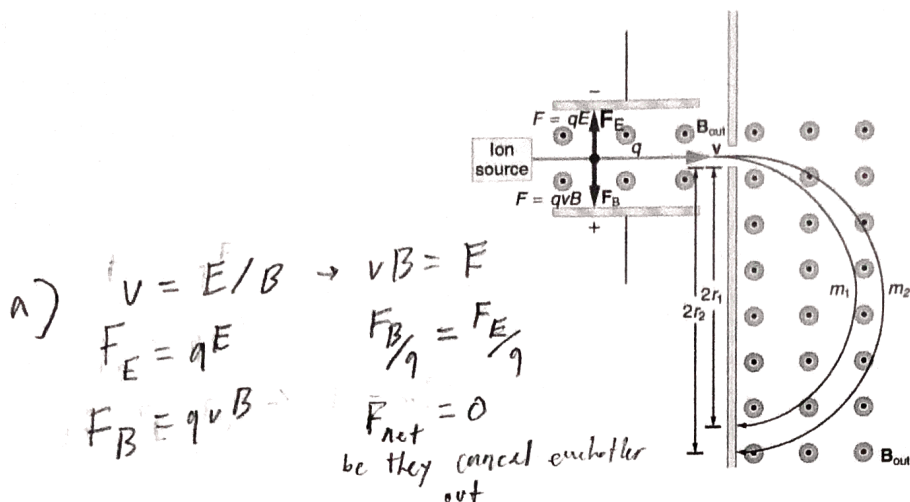


Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer.** Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left¹. (b) Recall that the centripetal force on a particle of mass m is mv^2/r . Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \quad (2)$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-septometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r ?

b) $F = qv \times B$
 $F = \frac{mv^2}{r}$
 $r = \frac{mv^2}{F}$
 $r = \frac{mv^2}{qvB}$
 $r = \frac{mE}{qB^2}$
 $= \frac{16(1.67 \cdot 10^{-27})(10)}{(1.602 \cdot 10^{-19})(0.01)^2} = 1.67 \text{ cm}$

4 Chapter 13: Electromagnetic Induction

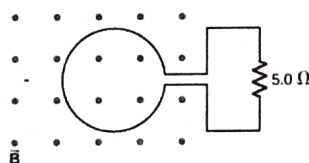


Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and is tuned to follow the form

$$B(t) = B_0 \left(\frac{1}{2} + \frac{2}{\pi} \sin(2\pi ft) + \frac{2}{3\pi} \sin(6\pi ft) + \frac{2}{5\pi} \sin(10\pi ft) \right) \quad (3)$$

The loop has a radius r . (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, and $f = 10^3$ Hz, what is the induced emf at $t = 0$? (c) What is the current through the resistor at $t = 1$ ms?

a) $\epsilon = -N \frac{d\Phi}{dt}$
 $\Phi = B \cdot A$
 $\epsilon = -N A \cdot B(t)$

b) $B(0) = .1 \left(\frac{1}{2} + 0 + 0 + 0 \right)$
 $\epsilon = (1) \cdot \pi \cdot (.1)^2 \cdot (.05)$
 $= .00157 \text{ V}$
 $@ t = 0$

c) $\Delta V = IR$
 $B(0.001) = .05$
 $\epsilon = (1) \cdot \pi \cdot (.1)^2 \cdot (.05)$
 $\epsilon = .00157$
 $\frac{.00157}{5} = 3.1416 \cdot 10^{-4} \text{ A}$
 $\text{or } \pi \cdot 10^{-4} \text{ A}$

¹Molecules that do not have this velocity will hit the sides of this portion of the instrument.

5 Chapter 14: Inductance

1. What is (a) the rate at which the current through a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

$$\Delta V = -L \frac{dI}{dt} \quad \frac{\Delta V}{-L} = \frac{0.15}{-0.5} = \frac{dI}{dt} = .3 \text{ A}$$

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

$$\text{emf} = -L \frac{dI}{dt}$$
$$\frac{500 \text{ V}}{-2 \cdot 10^{-3} \text{ H}} = 250,000 = \frac{0.1}{-4 \cdot 10^{-7}}$$
$$\boxed{4 \cdot 10^{-7} \text{ s}}$$