Dhiraj Kameti

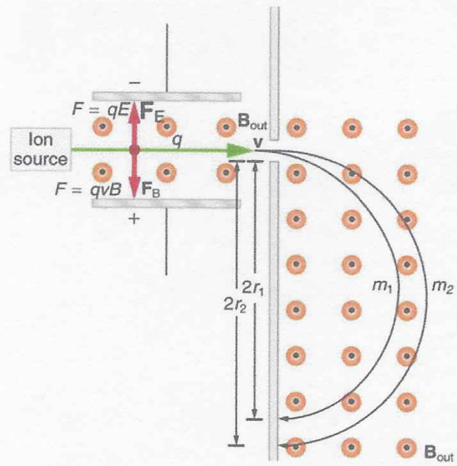

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \tag{2}$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r?

a. $\sum F = F_E - F_B = 0 \quad qE - q\left(\frac{E}{B}\right)B = 0$

$qE - qvB = 0 \nearrow \quad qE - qE = 0$

b. $(qvB)r = \left(\frac{mv^2}{r}\right)r \qquad \frac{rqvB}{qvB} = \frac{mv^2}{qvB} \nearrow \quad r = \frac{mv^2}{qvB} \quad r = \frac{mv}{qB} \nearrow \quad r = \frac{mE}{qB^2}$

$r = \frac{16(1.6\times10^{-27})(10)}{(1.67\times10^{-19})(0.01)^2}$

$= 1.6\times10^{-2}$ m

4 Chapter 13: Electromagnetic Induction

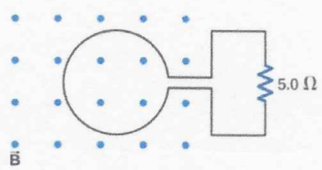

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and is tuned to follow the form

$$B(t) = B_0\left(\frac{1}{2} + \frac{2}{\pi}\sin(2\pi ft) + \frac{2}{3\pi}\sin(6\pi ft) + \frac{2}{5\pi}\sin(10\pi ft)\right) \tag{3}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, and $f = 10^3$ Hz, what is the induced emf at $t = 0$? (c) What is the current through the resistor at $t = 1$ ms?

a. $B(t) = B_0\left(\frac{1}{2} + \frac{2}{\pi}\sin(2\pi ft) + \frac{2}{3\pi}\sin(6\pi ft) + \frac{2}{5\pi}\sin(10\pi ft)\right)$

$\frac{dB}{dt} = B_0\left(\frac{2}{\pi}\cos(2\pi ft)2\pi f + \frac{2}{3\pi}\cos(6\pi ft)6\pi f + \frac{2}{5\pi}\cos(10\pi ft)10\pi f\right)$

$= 4fB_0(\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft))$

$\frac{d\phi}{dt} = 4\pi r^2 f B_0(\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft))$

$\mathcal{E} = -4\pi r^2 B_0(\cos(2\pi ft) + \cos(6\pi ft) + \cos(10\pi ft))$

b. $\mathcal{E} = -4\pi(3)$

$\mathcal{E} = -12\pi$ volts

c. $\mathcal{E} = 4\pi(3)$

$= -12\pi$ volts

$1ms = 0.001s$

[1] Molecules that do not have this velocity will hit the sides of this portion of the instrument.