CALCULUS-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND THERMODYNAMICS (PHYS180-02): UNIT 5

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UNIT 4 REVIEW

Suppose a bundle of wires is carrying current along what we call the $\hat{z}$ direction. Each wire runs along the z-axis and they are close enough to ignore the fact that the volume of each wire prevents it from being exactly on the z-axis. One wire carries +2.0 A, another carries +1.5 A, and a third carries -0.5 A. What is the B-field strength at a distance of 1 cm away in the x-y plane?

- A: 6 Gauss
- B: 0.6 Gauss
- C: 6 Tesla
- D: 0.6 Tesla

Suppose a loop of current exists in the x-y plane, and a uniform B-field is in the $\hat{z}$ direction. Which of the following will occur?

- A: The loop will not rotate - there is no torque.
- B: The loop will rotate 180 degrees - there is torque.
- C: The loop will rotate 90 degrees - there is torque.
- D: The loop will rotate -90 degrees - there is negative torque.

SUMMARY

Reading: Chapters 13 and 14

1. 13.1-2: Faraday's and Lenz's Law
2. 13.3: Motional EMF
3. 13.4: Induced E-fields

1. 14.1: Mutual inductance
2. 14.2: Self-inductance and inductors
3. 14.3: Energy in a magnetic field

FARADAY'S LAW AND LENZ'S LAW

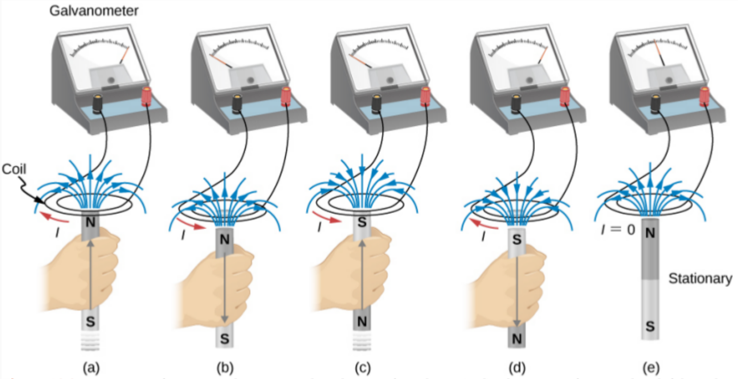

Figure 1: Not only does moving charge create B-fields, but B-fields can create moving charge. Study each of the cases above, and (Professor) define the concept of *magnetic flux*.

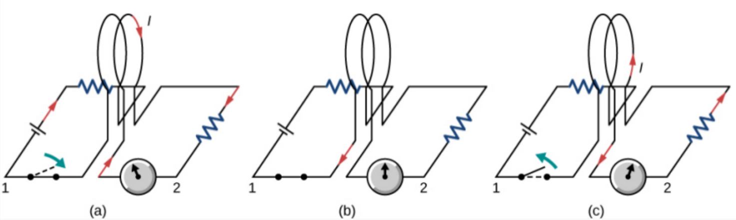

Figure 2: In addition to a moving magnetic field, *other circuits* can make current flow in a circuit. The effect must have something to do with *changing* magnetic fields.

Faraday's Law

The emf ϵ induced is the negative change in the magnetic flux Φ_m per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

$$\phi_m = \int_S \vec{B} \cdot d\vec{A} \tag{1}$$

$$\epsilon = -\frac{d\phi_m}{dt} \tag{2}$$

The unit of magnetic flux is the Webter, or 1 Wb = 1 T m^2.

Example: A square coil has sides 0.25 m long and is tightly wound with 200 turns of wire. The resistance of the coil 5.0 Ohms. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing by −0.040 T/s. (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?

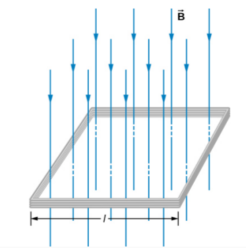

Figure 3: A 200 turn loop in a B-field.

Lenz's Law

The direction of the induced emf drives current around a wire loop to always oppose the change in magnetic flux that causes the emf.

Example: A magnetic field B is directed outward perpendicular to the plane of a circular coil of radius r = 0.50 m. The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to

$$B(t) = B_0 \exp(-at) \tag{3}$$

with $B_0 = 1.5$ T and $a = 5.0$ s^{-1}. (a) Calculate the emf induced in the coil at the times $t_0 = 0$, $t_1 = 0.05$, and $t_2 = 1.0$ seconds. (b) Determine the current in the coil if the resistance is 10 Ohms.

In the previous example, what would happen if the area A of the loop were increased?

- A: The current would decrease.
- B: The current would stay the same.
- C: The voltage would decrease.
- D: The voltage would increase.

In the previous example, what would happen if the sign of the exponent in $B(t)$ were flipped?

- A: The current would reverse direction and increase in magnitude.
- B: The current would reverse direction and decrease in magnitude.
- C: The current would keep the same direction and increase in magnitude.
- D: The current would keep the same direction and decrease in magnitude.

In the previous example, what would happen if α in the exponent in $B(t)$ were increased?

- A: The current would reverse direction and increase in magnitude.
- B: The current would reverse direction and decrease in magnitude.
- C: The current would keep the same direction and increase in magnitude.
- D: The current would keep the same direction and decrease in magnitude.

Example: The square coil of Figure 4 has sides l = 0.25 m long and is tightly wound with N = 200 turns of wire. The resistance of the coil is R = 5.0 Ω. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $dB/dt = 0.040t^2$. (a) Graph the magnitude of the emf induced in the coil. (b) What is the magnitude of the current through the coil at 100 ms?

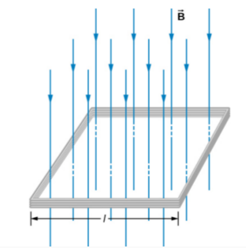

Figure 4: A 200 turn loop in a B-field.

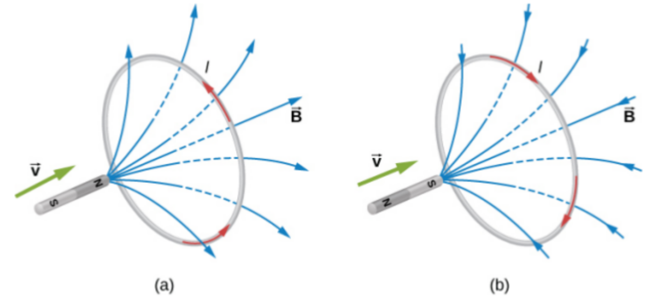

Figure 5: Lenz's Law relates sign of current to B-field.

MOTIONAL EMF

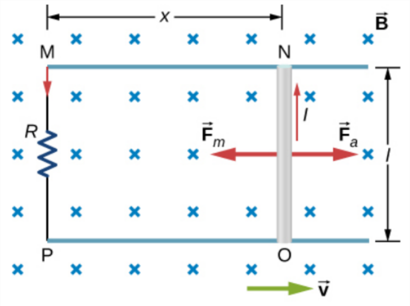

Figure 6: A system in which the magnetic flux depends on time.

1. Show that power is equal to $P = \vec{F} \cdot \vec{v}$ for constant acceleration.
2. Show that the emf is $\epsilon = Blv$, from Faraday's Law.
3. Show that power generated, $P = I^2R$, is equal to power injected.

In the previous example, what would happen if $\vec{F}_a$ was pointed to the left?

- A: The current would reverse direction.
- B: The current would keep the same direction.
- C: The magnetic flux due to the external field would decrease.
- D: A and C

In the previous example, what would happen if R were increased, but the magnitude of F_a were kept the same?

- A: The current would decrease.
- B: The current would increase.
- C: The current would remain constant.
- D: The power required would increase.

INDUCED ELECTRIC FIELDS

Recall that the relationship between voltage and electric field is

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z} \tag{4}$$

In one dimension, this becomes

$$\vec{E} = -\frac{dV}{dx}\hat{x} \tag{5}$$

If we take a dot product with $-d\vec{x} = -dx\,\hat{x}$ on each side, we find

$$-\vec{E} \cdot d\vec{x} = dV \tag{6}$$

Integrating, we have

$$V = -\int \vec{E} \cdot d\vec{x} \tag{7}$$

However, if the voltage is a result of a changing magnetic field, and Faraday's Law, then

$$\frac{d\phi_m}{dt} = \oint \vec{E} \cdot d\vec{x} \tag{8}$$

Recall that from *electrostatics*,

$$\oint \vec{E} \cdot d\vec{x} = 0 \tag{9}$$

Equation 9 is true for eletrostatics because the Coulomb force is **conservative**. But in a previous example we showed that power was being generated and *conserved*, despite the fact that magnetic flux is changing. What is happening?

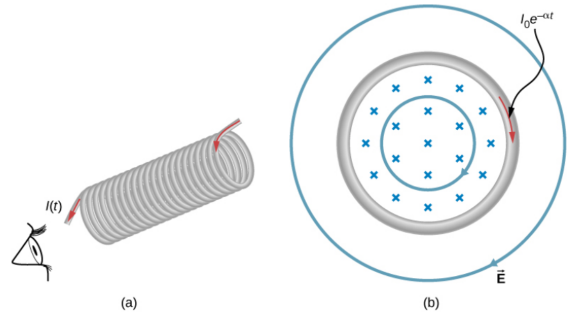

Figure 7: A solenoid with a changing current will induce an E-field. The solenoid has turn density *n*, and is long compared to the radius.

1. What is the E-field outside the solenoid?
2. What is the E-field inside the solenoid?
3. Create a graph of the E-field strength versus distance.

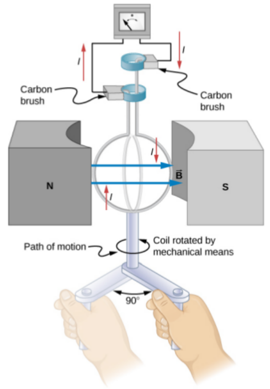

Figure 8: The basic concept behind an AC generator.

Start with Faraday's Law:

$$\epsilon = -\frac{d\phi_B}{dt} \tag{10}$$

The flux ϕ_B is changing and depends on time:

$$\phi_B = \vec{B} \cdot \vec{A}(t) = BA\cos(\theta(t)) \tag{11}$$

Let the *angular velocity* be constant: $\theta = \omega t$. Then we have

$$\phi_B = BA\cos(\omega t) \tag{12}$$

Thus the emf (with N loops) is

$$\epsilon = N\omega BA\sin(\omega t) = \epsilon_0 \sin(\omega t) \tag{13}$$

The generation of AC power stems from ω.
(Professor: diagram of $\epsilon(t)$).

$$\epsilon = N\omega BA \sin(\omega t) \tag{14}$$

The AC voltage equation above is a basic model for the voltage from a generator. Which of the following would increase the *amplitude* of the emf?

- A: Turning the area more slowly.
- B: Turning the area more quickly.
- C: Increasing the B-field.
- D: Both C and D.

$$\epsilon = N\omega BA\sin(\omega t) \tag{15}$$

The AC voltage equation above is a basic model for the voltage from a generator. Which of the following would increase the *frequency* of the emf?

- A: Turning the area more slowly.
- B: Turning the area more quickly.
- C: Increasing the B-field.
- D: Both C and D.

INDUCTANCE

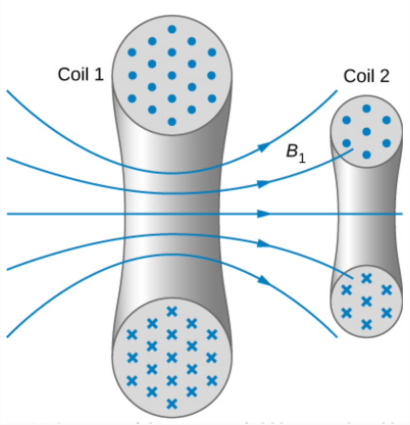

Figure 9: The concept of mutual inductance.

First, some notation:

- The flux through coil 2 by coil 1: ϕ_{21}
- The flux through coil 1 by coil 2: ϕ_{12}

Mutual inductance of coil 2 with respect to coil 1:

$$M_{21} = \phi_{21} \frac{N_2}{I_1} \tag{16}$$

Mutual inductance of coil 1 with respect to coil 2:

$$M_{12} = \phi_{12} \frac{N_1}{I_2} \tag{17}$$

It can be shown that

$$\boxed{M_{21} = M_{12}} \tag{18}$$

What are the units of mutual inductance? Consider the emf induced in loop 2 by loop 1:

$$\epsilon_2 = -\frac{d}{dt}\left(\phi_{21} N_2\right) \tag{19}$$

Substitution for the inductance gives

$$\epsilon_2 = -\frac{d}{dt}\left(\frac{M_{21} I_1}{N_2} N_2\right) \tag{20}$$

$$\epsilon_2 = -\frac{d}{dt}\left(I_1 M_{21}\right) \tag{21}$$

$$\epsilon_2 = -M\frac{dI_1}{dt} \tag{22}$$

$$\epsilon_1 = -M\frac{dI_2}{dt} \tag{23}$$

So inductance relates induced emf to current change, and has units of V s A^{-1}.

A coil of N_2 turns and radius R_2 surrounds a long solenoid of length l_1, radius R_1, and N_1 turns. (a) What is the mutual inductance of the two coils? (b) If $N_1 = 1000$, $N_2 = 20$, $R_1 = 3.0$ cm, $l_1 = 100.0$ cm, and $dI_1/dt = 150$ A/s, what is the induced emf in the surrounding coil?

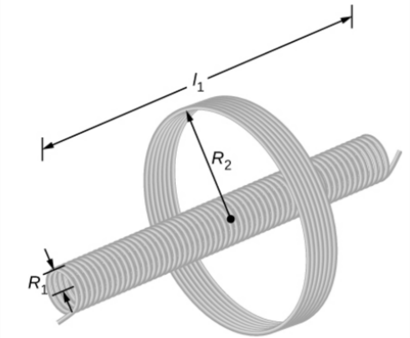

Figure 10: Example of mutual inductance.

A current $I(t) = I_0 sin(\omega t)$ flows through the solenoid. If $I_0 = 7.5$ A, and $\omega = 60\pi$ rad/sec, what is the maximum induced emf in the surrounding coil?

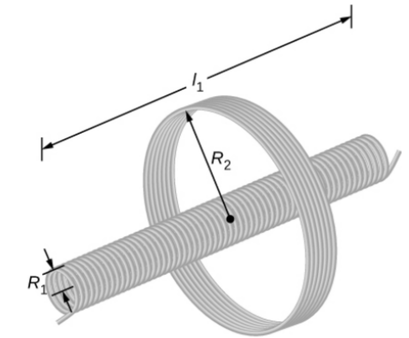

Figure 11: Example of mutual inductance.

SELF-INDUCTANCE AND INDUCTORS

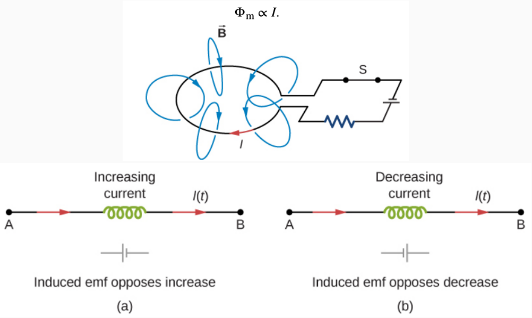

Figure 12: Self-inductance in a circuit, denoted L, rather than M.

Define

$$\epsilon = -L\frac{dI}{dt} \tag{24}$$

$$N\phi_m = LI \tag{25}$$

(Observe on board): Show that the inductance of a solenoid with volume V and turn density n is

$$L = \mu_0 n^2 V \tag{26}$$

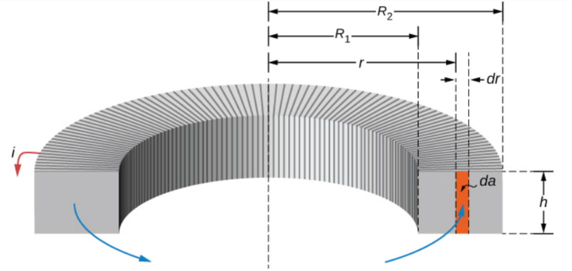

Figure 13: A rectangular toroid.

(Observe on board): Show that the inductance of a rectangular toroid as defined above is

$$L = \frac{\mu_0}{2\pi} N^2 h \ln\left(\frac{R_2}{R_1}\right) \tag{27}$$

Thus the two expressions have turn-density squared in common, and the volume comes into play.

Similar to calculating the capacitance in electrostatics.

THE RLC CIRCUIT

CONCLUSION

Reading: Chapters 13 and 14

This weekend:

1. 13.1-2: Faraday's and Lenz's Law
2. 13.3: Motional EMF
3. 13.4: Induced E-fields

Next week: Chapter 14.1-3

ANSWERS - CHAPTER 13 AND UNIT 4 REVIEW

ANSWERS

- B
- A
- D
- A
- D
- D
- A
- D
- B

ANSWERS - CHAPTER 14

ANSWERS

- ...

- ...