Calculus-Based Physics-2: Electricity, Magnetism, and Thermodynamics (PHYS180-02): Unit 6

Jordan Hanson

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Whittier College Department of Physics and Astronomy

Summary

Summary

Reading: Chapter 16.1 - 16.3

Resolving an issue with Ampère's Law

1. The Maxwell-Ampère Law
2. Maxwell's Equations

E-field $\rightarrow$ *B-field* $\rightarrow$ *E-field* $\rightarrow$ *...*

1. Electromagnetic wave equation
2. Energy density and radiation pressure

Resolving an issue with Ampère's Law

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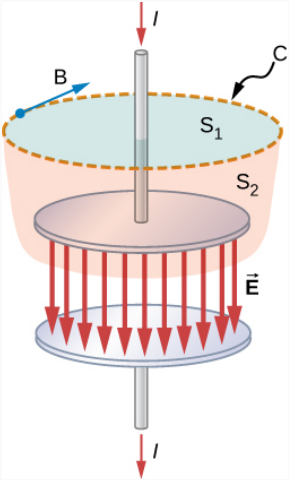

Figure 1: Two surfaces S1 and S2, for application of Ampère's Law.

Resolving an issue with Ampère's Law

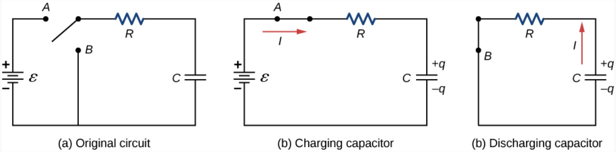

Figure 2: Recall how we obtain the voltage of a charging capacitor.

Resolving an issue with Ampère's Law

The voltage of a charging capacitor in *RC* circuit:

$$V_{\mathrm{C}}(t) = \epsilon\,(1 - \exp(-t/\tau)) \tag{1}$$

Let $\tau = RC$. But what happens when we think more carefully about Fig. 1?

- Isn't I = 0 if you use surface 2 for Ampère's Law?
- What about the changing electric field? Might there be a magnetic field? (Think of Faraday's law...)

Resolving an issue with Ampère's Law

Surface 1 versus surface 2:

$$\oint_{S1} \vec{B} \cdot d\vec{s} = \mu_0 I_{in} \tag{2}$$

$$\oint_{S2} \vec{B} \cdot d\vec{s} = 0 \tag{3}$$

Maxwell added a *displacement current:*

$$I_d = \epsilon_0 \frac{d\phi_E}{dt} \tag{4}$$

so that

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d) \tag{5}$$

Both surfaces should now be equivalent (verify that $I(t) = I_d(t)$).

Resolving an issue with Ampère's Law

The Maxwell-Ampère Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 (I + I_d) \tag{6}$$

- Resolves displacement current issue
- Relates integral of B-field to changing E-field

Maxwell’s Equations - All of Electromagnetism

Maxwell's Equations

Maxwell's Equations

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{in}}{\epsilon_0} \tag{7}$$

$$\oint \vec{B} \cdot d\vec{A} = 0 \tag{8}$$

$$\oint \vec{E} \cdot d\vec{s} = -\mu_0 \frac{d\phi_m}{dt} \tag{9}$$

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I + \epsilon_o \mu_0 \frac{d\phi_E}{dt} \tag{10}$$

Forces:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \tag{11}$$

Electromagnetic Wave Equation

Electromagnetic Wave Equation

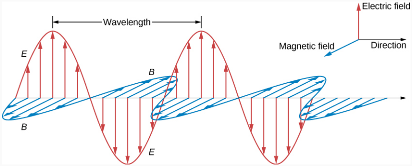

Figure 3: Consider a slice of volume with a 3D electric field *propagating* in the x-direction.

Electromagnetic Wave Equation

1. Define box, and show that the flux from E_y is zero
2. Same for E_z.
3. Net flux is from E_x, but $Q_{in} = 0$. What does this imply?
4. Integrate E_y around side 3, assuming Δx is small
5. Consider side 3 magnetic flux...

Electromagnetic Wave Equation

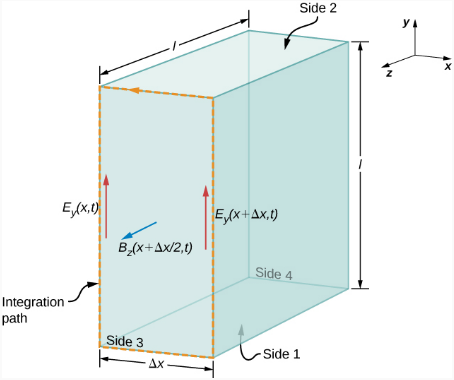

Figure 4: Consider a slice of volume with a 3D electric field *propagating* in the x-direction.

Electromagnetic Wave Equation

1. Apply Faraday's law to side 3.
2. Repeat this combination for side 2.
3. Apply Maxwell-Ampère's Law to sides 3 and 2.
4. Summarize four results.
5. Combine them to obtain **the wave equation.**
6. Solve wave equation...what is implied about ϵ_0 and μ_0?

Figure 5: Welcome to physics.

Electromagnetic Wave Equation

Figure 6: Welcome to physics.

Energy Density and Radiation Pressure

Energy Density and Radiation Pressure

Electromagnetic wave:

$$\vec{E}(x,t) = E_0 \cos(k(x - ct))\hat{x} \tag{12}$$

Let $\omega = ck$ so that

$$\vec{E}(x,t) = E_0 \cos(kx - \omega t)\hat{x} \tag{13}$$

What is k? What does this *propagator* function do?

Energy Density and Radiation Pressure

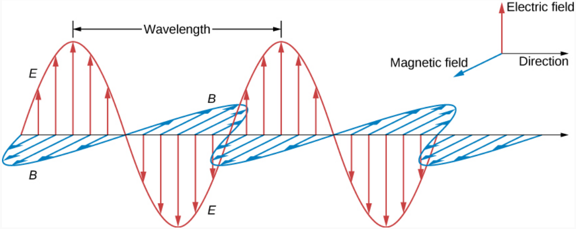

Figure 7: The E and B-fields of an electromagnetic wave.

Energy Density and Radiation Pressure

But haven't we shown that E and B-fields contain energy? For capacitors:

$$U = \frac{1}{2}\frac{Q^2}{C} \tag{14}$$

For inductors:

$$U = \frac{1}{2}LI^2 \tag{15}$$

- Re-derive each of these on the board.
- Derive U for a *parallel-plate capacitor*, and *solenoid.*

Energy Density and Radiation Pressure

Use these to obtain U:

$$C = \frac{\epsilon_0 A}{d} \tag{16}$$

$$L = \mu_0 n^2 V \tag{17}$$

- Obtain u_E and u_B, the energy densities for each.
- Get the total u_{EM}.

Energy Density and Radiation Pressure

The total energy density of an electromagnetic wave:

$$u_{EM} = \epsilon_0 E^2 \tag{18}$$

Radiation pressure: show that the units of the energy density are that of a *pressure.*

Energy Density and Radiation Pressure

Radiation pressure: energy carried by electromagnetic waves can move objects...

$$p = I/c \tag{19}$$

$$p = 2I/c \tag{20}$$

I is the intensity in Watts per meter squared, c is the speed of light, and p is the energy density or radiation pressure. Example: sunlight intensity is 1370 Watts per meter squared in space. What about a laser?

Examples.

The Electromagnetic Spectrum

Energy Density and Radiation Pressure

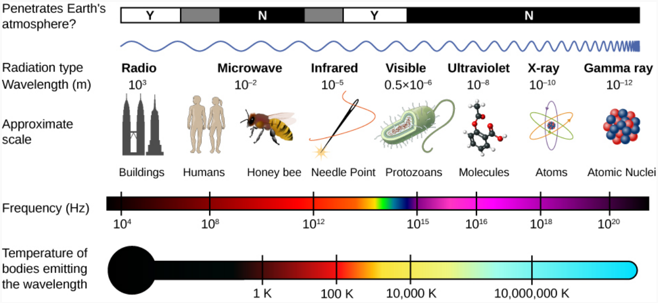

Figure 8: The electromagnetic spectrum.

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