CALCULUS-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND THERMODYNAMICS (PHYS180-02): UNIT 5

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April 17, 2019

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UNIT 4 REVIEW

Suppose a bundle of wires is carrying current along what we call the $\hat{z}$ direction. Each wire runs along the z-axis and they are close enough to ignore the fact that the volume of each wire prevents it from being exactly on the z-axis. One wire carries +2.0 A, another carries +1.5 A, and a third carries -0.5 A. What is the B-field strength at a distance of 1 cm away in the x-y plane?

- A: 6 Gauss
- B: 0.6 Gauss
- C: 6 Tesla
- D: 0.6 Tesla

Suppose a loop of current exists in the x-y plane, and a uniform B-field is in the $\hat{z}$ direction. Which of the following will occur?

- A: The loop will not rotate - there is no torque.
- B: The loop will rotate 180 degrees - there is torque.
- C: The loop will rotate 90 degrees - there is torque.
- D: The loop will rotate -90 degrees - there is negative torque.

SUMMARY

Reading: Chapters 13 and 14

This weekend:

1. 13.1-2: Faraday's and Lenz's Law
2. 13.3: Motional EMF
3. 13.4: Induced E-fields

Next week: Chapter 14.1-3

FARADAY'S LAW AND LENZ'S LAW

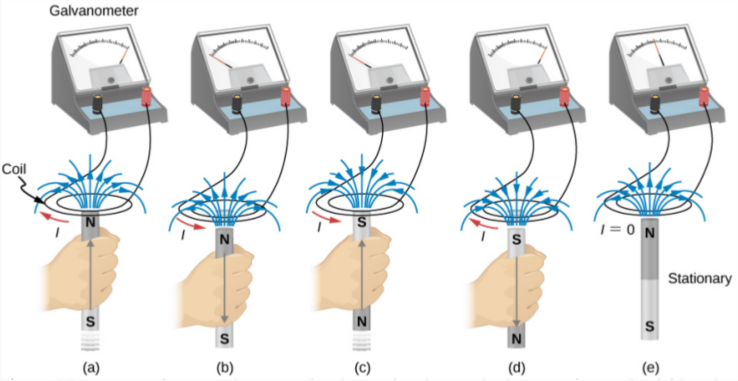

Figure 1: Not only does moving charge create B-fields, but B-fields can create moving charge. Study each of the cases above, and (Professor) define the concept of *magnetic flux*.

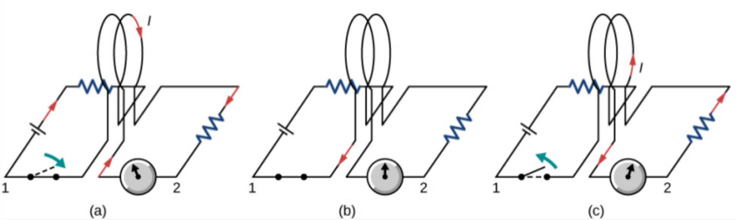

Figure 2: In addition to a moving magnetic field, *other circuits* can make current flow in a circuit. The effect must have something to do with *changing* magnetic fields.

Faraday's Law

The emf ϵ induced is the negative change in the magnetic flux Φ_m per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

$$\phi_m = \int_S \vec{B} \cdot d\vec{A} \tag{1}$$

$$\epsilon = -\frac{d\phi_m}{dt} \tag{2}$$

The unit of magnetic flux is the Webter, or 1 Wb = 1 T m^2.

Example: A square coil has sides 0.25 m long and is tightly wound with 200 turns of wire. The resistance of the coil 5.0 Ohms. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing by −0.040 T/s. (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?

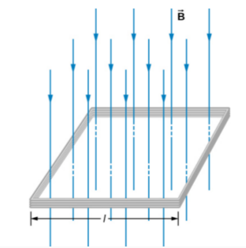

Figure 3: A 200 turn loop in a B-field.

Lenz's Law

The direction of the induced emf drives current around a wire loop to always oppose the change in magnetic flux that causes the emf.

Example: A magnetic field B is directed outward perpendicular to the plane of a circular coil of radius r = 0.50 m. The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to

$$B(t) = B_0 \exp(-at) \tag{3}$$

with $B_0 = 1.5$ T and $a = 5.0\ \mathrm{s}^{-1}$. (a) Calculate the emf induced in the coil at the times $t_0 = 0$, $t_1 = 0.05$, and $t_2 = 1.0$ seconds. (b) Determine the current in the coil if the resistance is 10 Ohms.

In the previous example, what would happen if the area A of the loop were increased?

- A: The current would decrease.
- B: The current would stay the same.
- C: The voltage would decrease.
- D: The voltage would increase.

In the previous example, what would happen if the sign of the exponent in $B(t)$ were flipped?

- A: The current would reverse direction and increase in magnitude.
- B: The current would reverse direction and decrease in magnitude.
- C: The current would keep the same direction and increase in magnitude.
- D: The current would keep the same direction and decrease in magnitude.

In the previous example, what would happen if α in the exponent in $B(t)$ were increased?

- A: The current would reverse direction and increase in magnitude.
- B: The current would reverse direction and decrease in magnitude.
- C: The current would keep the same direction and increase in magnitude.
- D: The current would keep the same direction and decrease in magnitude.

Example: The square coil of Figure 4 has sides l = 0.25 m long and is tightly wound with N = 200 turns of wire. The resistance of the coil is R = 5.0 Ω. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $dB/dt = 0.040t$. (a) Graph the magnitude of the emf induced in the coil. (b) What is the magnitude of the current through the coil at 100 ms?

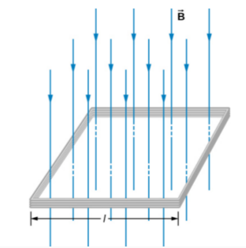

Figure 4: A 200 turn loop in a B-field.

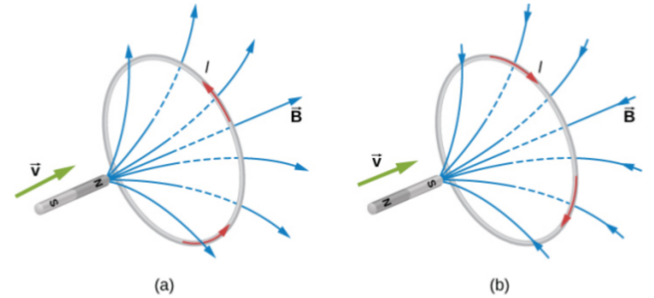

Figure 5: Lenz's Law relates sign of current to B-field.

MOTIONAL EMF

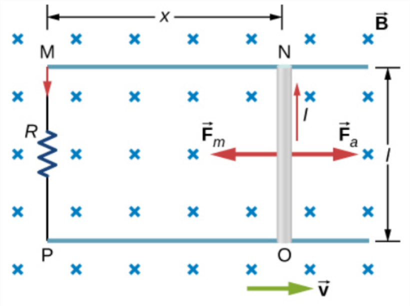

Figure 6: A system in which the magnetic flux depends on time.

1. Show that power is equal to $P = \vec{F} \cdot \vec{v}$ for constant acceleration.
2. Show that the emf is $\epsilon = Blv$, from Faraday's Law.
3. Show that power generated, $P = I^2R$, is equal to power injected.

In the previous example, what would happen if $\vec{F}_a$ was pointed to the left?

- A: The current would reverse direction.
- B: The current would keep the same direction.
- C: The magnetic flux due to the external field would decrease.
- D: A and C

In the previous example, what would happen if R were increased, but the magnitude of F_a were kept the same?

- A: The current would decrease.
- B: The current would increase.
- C: The current would remain constant.
- D: The power required would increase.

INDUCED ELECTRIC FIELDS

Recall that the relationship between voltage and electric field is

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z} \tag{4}$$

In one dimension, this becomes

$$\vec{E} = -\frac{dV}{dx}\hat{x} \tag{5}$$

If we take a dot product with $-d\vec{x} = -dx\,\hat{x}$ on each side, we find

$$-\vec{E} \cdot d\vec{x} = dV \tag{6}$$

Integrating, we have

$$V = -\int \vec{E} \cdot d\vec{x} \tag{7}$$

However, if the voltage is a result of a changing magnetic field, and Faraday's Law, then

$$\frac{d\phi_m}{dt} = \oint \vec{E} \cdot d\vec{x} \tag{8}$$

Recall that from *electrostatics*,

$$\oint \vec{E} \cdot d\vec{x} = 0 \tag{9}$$

Equation 9 is true for eletrostatics because the Coulomb force is **conservative**. But in a previous example we showed that power was being generated and *conserved*, despite the fact that magnetic flux is changing. What is happening?

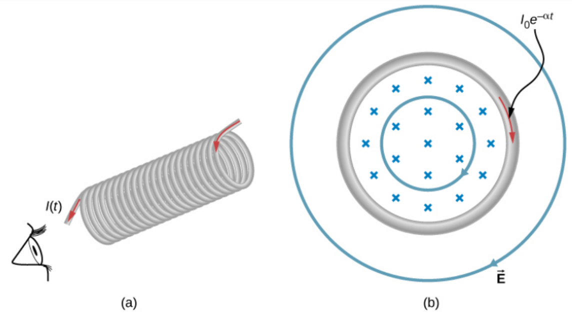

Figure 7: A solenoid with a changing current will induce an E-field. The solenoid has turn density *n*, and is long compared to the radius.

1. What is the E-field outside the solenoid?
2. What is the E-field inside the solenoid?
3. Create a graph of the E-field strength versus distance.

CONCLUSION

Reading: Chapters 13 and 14

This weekend:

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3. 13.4: Induced E-fields

Next week: Chapter 14.1-3

ANSWERS - CHAPTER 13 AND UNIT 4 REVIEW

ANSWERS

- B
- A
- D
- A
- D
- D
- A

- ...

ANSWERS - CHAPTER 14

ANSWERS

- ...

- ...