

# Calculus-Based Physics-1: Mechanics (PHYS150-01): Unit 0

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## Opening Remarks - Welcome!

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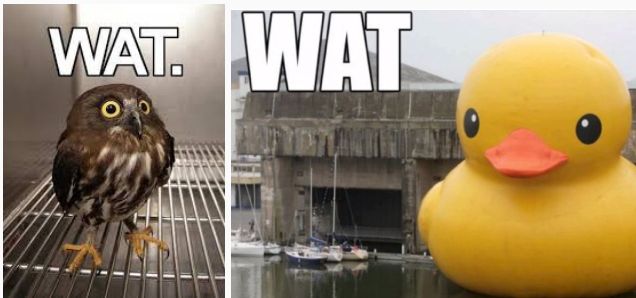


Figure 1: Taking physics for the first time.

## Summary

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# Week 1 Summary

*Physics* - φυσική - "phusiké": *knowledge of nature*  
from φύσις - "phúsis": *nature*

1. Estimation/Unit Analysis - Chapters 1.1 - 1.4
  - **Estimating** the correct order of magnitude
  - **Unit analysis** - dealing with the units of numbers
2. Coordinates and vectors - Chapters 2.1 - 2.4
  - **Scalars** and **vectors**
  - **Cartesian** (rectangular) coordinates, displacement
  - **Vector** addition, subtraction, and multiplication
3. Review of Calculus Techniques
  - The derivative, derivatives of elementary functions
  - **Function** approximation
  - Anti-derivatives and integration

# Estimation/Unit Analysis - Chapters 1.1 - 1.4

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# Estimation/Unit Analysis

In science and engineering, **estimation** is to obtain a quantity in the absence of precision, informed by rational constraints.

1. Define relevant **unit scales**: (mg, g, or kg), (m/s or km/hr)
2. Obtain **complex quantities** from simple ones
  - Obtain *areas* and *volumes* from *lengths*
  - Obtain *rates* from *numerators* and *denominators*
3. Taking advantage of **scaling problems**
  - Knowing *relationship* between variables
  - Using that *relationship* to obtain new information
4. Constrain the unknown with **upper** and **lower** limits

**Units:** Which of the following represents a *volume*?

- A: 10 gm
- B: 10 cm<sup>2</sup>
- C: 1 cm<sup>3</sup>
- D: 1 cm s<sup>-1</sup>



## Estimation/Unit Analysis

**Units:** If a grain of sand within a fluid sinks 15 cm in 5 seconds, what is the speed of the grain?

- A: 3 cm
- B: 3 s
- C: 3 s/cm
- D: 3 cm/s

**Unit conversion:** If a person weights 120 lbs, what is their weight in kilograms?<sup>1</sup>

- A: 54.5 kg
- B: 264 kg
- C: 54.5 lbs
- D: 264 lbs

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<sup>1</sup>One kilogram is 2.2 lbs.

**Unit conversion:** A **density** is a mass divided by a volume. For example, water has a density of  $1 \text{ gm cm}^{-3}$ . What is the density of water in  $\text{kg m}^{-3}$ ?

- A:  $1 \text{ kg m}^{-3}$
- B:  $10 \text{ kg m}^{-3}$
- C:  $100 \text{ kg m}^{-3}$
- D:  $1000 \text{ kg m}^{-3}$

## Estimation/Unit Analysis

**Group exercise on complex units:** A *vitrolero* is a classic container for serving *agua fresca*. It has a diameter of 20 cm, and a height of 30 cm. How many cups can we serve from the vitrolero if we put 0.25 liters of *agua fresca* in each cup?

- *Hint:* 1 liter is 1000 mL
- *Hint:* 1 mL is 1 cm<sup>3</sup>
- **Volume:** The volume of a cylinder is  $\pi$  times the radius of the base, squared, times the height:  $\pi r^2 h$ .

## Estimation/Unit Analysis

**Unit scale:** A generation is about one-third of a lifetime.  
Determine how many generations have passed since the year 0 AD<sup>2</sup>.

- A: 10
- B: 20
- C: 60
- D: 100

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<sup>2</sup>What is the appropriate scale here?

## Estimation/Unit Analysis

**Unit scale:** (a) What fraction of Earth's diameter<sup>3</sup> is the greatest ocean depth (11 km below sea level)? (b) The greatest mountain height (8.8 km above sea level)?

- A:  $8.6 \times 10^{-2}$ ,  $6.9 \times 10^{-2}$
- B:  $8.6 \times 10^{-3}$ ,  $6.9 \times 10^{-3}$
- C:  $8.6 \times 10^{-4}$ ,  $6.9 \times 10^{-4}$
- D:  $8.6 \times 10^{-5}$ ,  $6.9 \times 10^{-3}$

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<sup>3</sup>The diameter of the Earth is 12,800 km.

**Complex quantities:** If a Whittier College athlete ran the 5k race at a track meet in 35 minutes, what was her average speed?

- A: 0.3 meters per second
- B: 3 meters per second
- C: 30 meters per second
- D: 300 meters per second

**Complex quantities:** Suppose you won the lottery and received \$1 billion USD. Because your life is dope, you stack that paper over the Whittier College soccer field. Each stack contains 100 bills, and each bill is worth \$100. If you evenly cover the field, how high is the money level?

- A: 0.5 inch
- B: 1 inch
- C: 2 inches
- D: 1 foot



**Scaling problem:** Supposed you have an ideal gas in a cylinder of fixed volume. If the pressure begins as 100 kPa, and you *double* the temperature of the gas, what is the new pressure?

- A: 100 kPa
- B: 50 kPa
- C: 10 kPa
- D: 200 kPa

**Scaling problem:** Supposed you have an ideal gas in a cylinder of fixed volume. If the pressure begins as 100 kPa, and you *halve* the temperature of the gas, what is the new pressure?

- A: 100 kPa
- B: 50 kPa
- C: 10 kPa
- D: 200 kPa

**Upper/lower limits:** How many undergraduate students are there at Whittier College<sup>4</sup>?

- A: 5,000
- B: 1,000
- C: 1,250
- D: 500

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<sup>4</sup>What is the absolute lower limit, and what is the upper limit?

## Estimation/Unit Analysis

**Upper/lower limits:** What is the average yearly college tuition in the United States (before subtracting grants and scholarships)?

- A: \$5,000
- B: \$10,000
- C: \$25,000
- D: \$40,000

What information affects the **upper** and **lower** limits here?

## Coordinates and Vectors - Chapters 2.1 - 2.4

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### Activity Link

*Who understands coordinates and vectors better than anyone else?*

[https://youtu.be/0B7WL7nhIF4?si=\\_dl4t\\_GwL98aXWFa](https://youtu.be/0B7WL7nhIF4?si=_dl4t_GwL98aXWFa)

# Coordinates and Vectors - Scalars, Vectors (Chapters 2.1-2.3)

Physics requires **mathematical objects** to build equations that capture the behavior of nature. Two examples of such objects are **scalar** and **vector** quantities. Each type of object obeys similar but different rules.

## 1. Scalar quantities

- mass:  $m_1 + (m_2 + m_3) = (m_1 + m_2) + m_3$
- speed:  $v_1(v_2 + v_3) = v_1v_2 + v_1v_3$
- charge:  $q_1\left(\frac{1}{q_1}\right) = 1, q_1(0) = 0$

## 2. Vector quantities

- velocity:  $\vec{v}_1 + (\vec{v}_2 + \vec{v}_3) = (\vec{v}_1 + \vec{v}_2) + \vec{v}_3$
- tension:  $\vec{t}_1 \cdot (\vec{t}_2 + \vec{t}_3) = \vec{t}_1 \cdot \vec{t}_2 + \vec{t}_1 \cdot \vec{t}_3$

## Coordinates and Vectors - Scalars, Vectors (Chapters 2.1-2.3)

A vector may be expressed as *a list of scalars*:  $\vec{v} = (4, 2)$  (a vector with two *components*),  $\vec{u} = (3, 4, 5)$  (three *components*). Now, we know how to add and subtract scalars. How do we add and subtract vectors?

What is

$(1, 3, 8) +$

$(0, 2, 1)$ ?

Answer:  $(1, 5, 9)$

In other words, when adding vectors, we add them component by component.



## Coordinates and Vectors - Scalars, Vectors (Chapters 2.1-2.3)

How do we subtract vectors? In the same fashion:

What is

$(1, 3, 8) -$

$(0, 2, 1)$ ?

Answer:  $(1, 1, 7)$

In other words, when subtracting vectors, we subtract them component by component.

A HTML-based demonstration for adding vectors:

`https://phet.colorado.edu/en/simulations/  
vector-addition`

Notice several things:

- Produce vectors that *cancel* each other.
- What happens when vectors are parallel and orthogonal?

How do we multiply vectors? In the same fashion, *for one kind of multiplication*:

What is

$$(1, 3, 8) \cdot (0, 2, 1)?$$

$$\text{Answer: } 1 \cdot 0 + 3 \cdot 2 + 8 \cdot 1 = 14$$

*This kind of multiplication is known as the dot-product.* There is also the *cross-product*, which we will save for later.

## Coordinates and Vectors - Coordinates (Chapters 2.1-2.3)

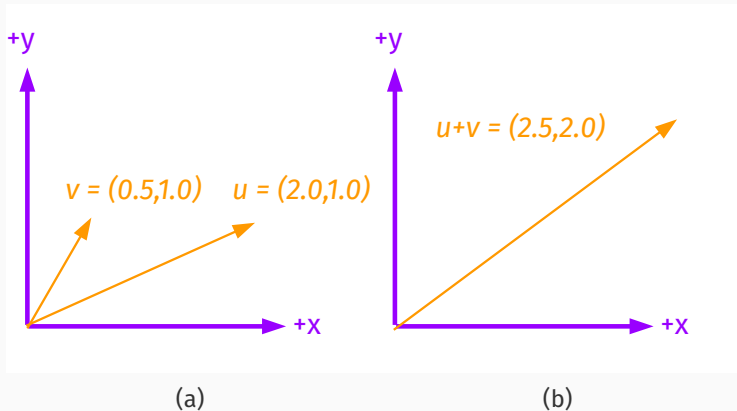
The components of a vector may describe quantities in a **coordinate system**, such as *Cartesian coordinates* - after René Descartes.

Vectors in the 3D Cartesian coordinate system (x,y,z) may be written in the following notation:

$$\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$$

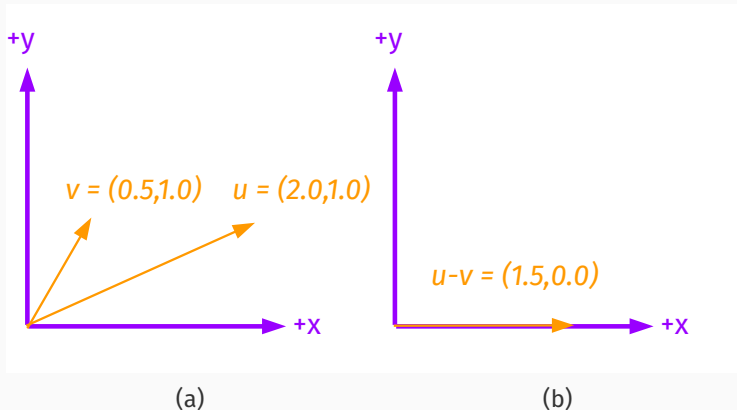
- a: The amount in the +x-direction,  $\hat{i}$ : a vector of length 1, in the +x-direction
- b: The amount in the +y-direction,  $\hat{j}$ : a vector of length 1, in the +y-direction
- c: The amount in the +z-direction,  $\hat{k}$ : a vector of length 1, in the +z-direction

## Coordinates and Vectors - Vectors (Chapters 2.1-2.3)



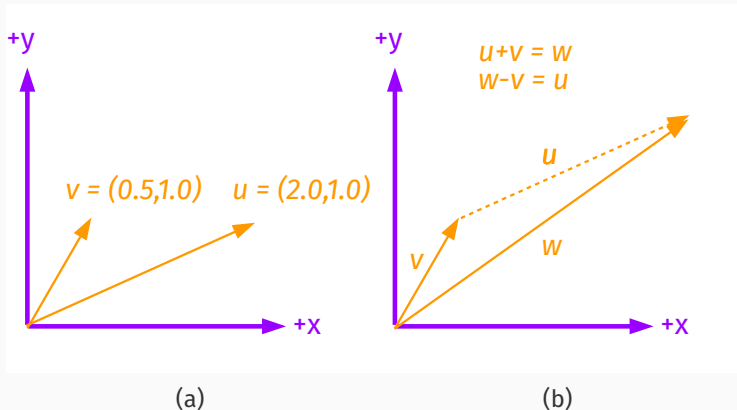
**Figure 2:** (a) Two vectors in a two-dimensional Cartesian coordinate system:  $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$  and  $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$ . (b) What is  $\vec{u} + \vec{v}$ ? Adding components:  $\vec{u} + \vec{v} = 2.5\hat{i} + 2.0\hat{j}$ .

## Coordinates and Vectors - Vectors (Chapters 2.1-2.3)



**Figure 3:** (a) Two vectors in a two-dimensional Cartesian coordinate system:  $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$  and  $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$ . (b) What is  $\vec{u} - \vec{v}$ ? Subtracting components:  $\vec{u} - \vec{v} = 1.5\hat{i} + 0.0\hat{j}$ .

## Coordinates and Vectors - Vectors (Chapters 2.1-2.3)



**Figure 4:** (a) Two vectors in a two-dimensional Cartesian coordinate system:  $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$  and  $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$ . (b) To compute  $\vec{w} - \vec{v}$ , arrange the vectors to get a sense of the result,  $\vec{u}$ .

## Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

$$\vec{p} = 4\hat{i} + 2\hat{j}. \quad \vec{q} = -4\hat{i} + 2\hat{j}.$$

Compute  $\vec{p} \cdot \vec{q}$ .

- A: 12
- B: -12
- C: 4
- D: 8

$$\vec{p} = -1\hat{i} + 6\hat{j}. \quad \vec{q} = 3\hat{i} + 0.5\hat{j}.$$

Compute  $\vec{p} \cdot \vec{q}$ .

- A: -1
- B: 1
- C: 0
- D: 3



## Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

Why was the last answer zero? Look at it graphically:

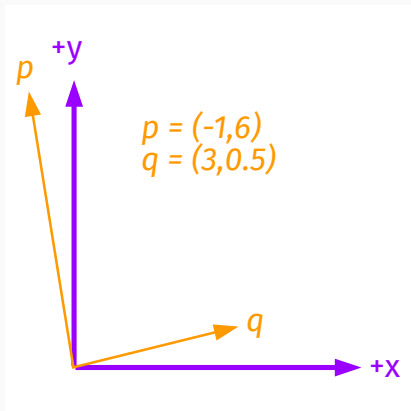


Figure 5: Two vectors  $\vec{p}$  and  $\vec{q}$  are *orthogonal* if  $\vec{p} \cdot \vec{q} = 0$ .

## Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

What if the vectors are parallel? Look at it graphically:

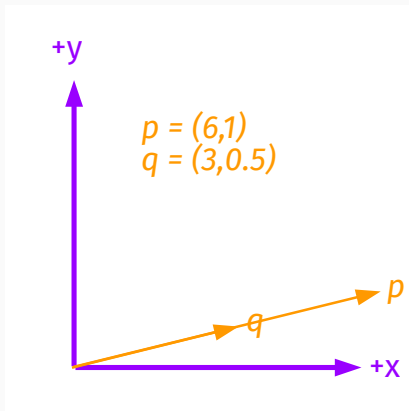


Figure 6: Two vectors  $\vec{p}$  and  $\vec{q}$  are *parallel* if  $\vec{p} \cdot \vec{q}$  is maximal.

## Coordinates and Vectors - Dot Product (Chapters 2.1-2.3)

The *length* or *norm* of a vector  $\vec{v} = a\hat{i} + b\hat{j}$  is  $|\vec{v}| = \sqrt{a^2 + b^2}$ .

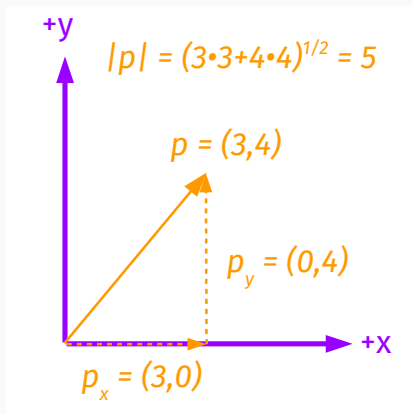


Figure 7: Computing the norm of a vector  $\vec{p}$ .

## Coordinates and Vectors - Dot Product (Chapters 2.1-2.3)

Notice that  $\sqrt{\vec{p} \cdot \vec{p}} = |\vec{p}|$ .

Let  $\theta_p$  be the angle between  $\vec{p}$  and the x-axis.

$$p_x = \vec{p} \cdot \hat{i} = |\vec{p}| \cos(\theta_p).$$

$$p_y = \vec{p} \cdot \hat{j} = |\vec{p}| \sin(\theta_p).$$

*Theorem:* The dot product of two vectors  $\vec{p}$  and  $\vec{q}$  is  $|\vec{p}||\vec{q}| \cos(\theta)$ , if  $\theta$  is the angle between them.

$$\begin{aligned} \text{Proof: } \vec{p} \cdot \vec{q} &= p_x q_x + p_y q_y = |p||q| \cos \theta_p \cos \theta_q + |p||q| \sin \theta_p \sin \theta_q \\ &= |p||q| (\cos \theta_p \cos \theta_q + \sin \theta_p \sin \theta_q) = |p||q| \cos(\theta_p - \theta_q) \\ &= |p||q| \cos \theta. \end{aligned}$$

$$\boxed{\vec{p} \cdot \vec{q} = |p||q| \cos \theta}$$

## Coordinates and Vectors - Dot Product (Chapters 2.1-2.3)

An object moves at 2 m/s at  $\theta = 60^\circ$  with respect to the x-axis. What is the velocity of the object?

- A:  $(1\hat{i} + 1\hat{j})$  m/s
- B:  $(\sqrt{3}\hat{i} + 1\hat{j})$  m/s
- C:  $(\sqrt{3}\hat{i} + \sqrt{3}\hat{j})$  m/s
- D:  $(1\hat{i} + \sqrt{3}\hat{j})$  m/s

What is the dot product of this velocity with another velocity: 5 m/s along the x-axis?

- A:  $1 \text{ (m/s)}^2$
- B:  $5 \text{ (m/s)}^2$
- C:  $10 \text{ (m/s)}^2$
- D:  $5 \text{ (m/s)}$

## Coordinates and Vectors - Scalars, Vectors (Chapters 2.1-2.3)

Is it possible to multiply vectors and scalars? Of course:

$$a_1\vec{p} = a_1p_x\hat{i} + a_1p_y\hat{j}.$$

Also, multiplication properties still hold. For example:

$$(a_1 + a_2)\vec{p} = a_1\vec{p} + a_2\vec{p}.$$

A spacecraft moves at 400 m/s, at an angle of 30 degrees with respect to the x-axis. If it fires two thrusters that boost the x-component and y-component of the velocity by 25% and 50%, respectively, what is the final velocity?

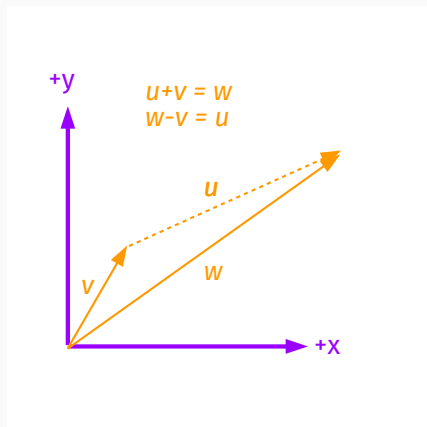
- A:  $(433\hat{i} + 300\hat{j})$  m/s
- B:  $(300\hat{i} + 433\hat{j})$  m/s
- C: 400 m/s
- D:  $(400\hat{i} + 433\hat{j})$  m/s

## Coordinates and Vectors - Displacement (Chapters 2.1-2.3)

We define the *position* of an object as a vector locating it in a given coordinate system. The scalar *distance* is the norm of the position vector, that is, the distance to to the origin.

Now we can introduce the concept of **displacement**: a vector describing a movement of an object.

## Coordinates and Vectors - Displacement (Chapters 2.1-2.3)



**Figure 8:** Suppose an object moves from position  $\vec{v}$  to  $\vec{w}$ . In this case, the displacement is  $\vec{u}$ . Thus, the final position is the initial position, plus the displacement.



## Coordinates and Vectors - Displacement (Chapters 2.1-2.3)

It follows that the *displacement* is zero if the initial and final positions are the same, but the *distance travelled* is not.

Suppose a jet fighter travelling at 800 km per hour banks such that it flies in a circle of radius 0.5 km. How long does it take to complete the circle? What is the distance traveled, and what is the displacement?

- A:  $2\pi$  km, 28 seconds,  $2\pi$  km
- B:  $\pi$  km, 14 seconds,  $\pi$  km
- C:  $\pi$  km, 28 seconds,  $\pi$  km
- D:  $\pi$  km, 14 seconds, 0 km

## Coordinates and Vectors - Average Velocity (Chapter 3.1)

**Average velocity** is the ratio of the **displacement** to the elapsed time.

$$\boxed{\vec{v}_{\text{avg}} = \frac{\Delta \vec{x}}{\Delta t}} \quad (1)$$

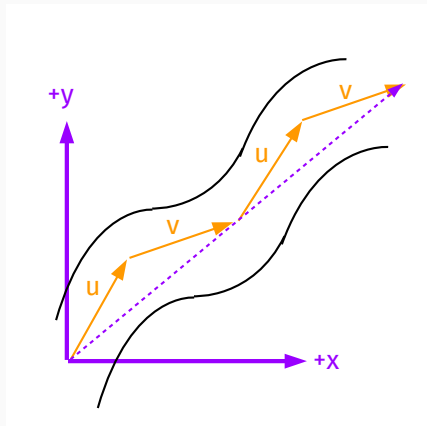
The *average speed* is the norm of the average velocity:

$$\boxed{v_{\text{avg}} = \frac{|\Delta \vec{x}|}{\Delta t}} \quad (2)$$

If the motion is in one dimension, then the average speed is

$$v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} \quad (3)$$

## Coordinates and Vectors - Average Velocity (Chapter 3.1)



**Figure 9:** A Formula-1 driver keeps his car on the track by following a path approximated by the position vectors  $u$ ,  $v$ ,  $u$ , and  $v$ . The dashed arrow represents the total displacement.

## Coordinates and Vectors - Average Velocity (Chapter 3.1)

If  $\vec{u} = (20\hat{i} + 30\hat{j})$  m, and  $\vec{v} = (30\hat{i} + 20\hat{j})$  m, what is the total displacement? If the elapsed time is 10 seconds, what is the average velocity?

- A:  $(50\hat{i} + 50\hat{j})$  m, 14 m/s
- B:  $(80\hat{i} + 100\hat{j})$  m, 10 m/s
- C:  $(100\hat{i} + 100\hat{j})$  m, 14 m/s
- D:  $(50\hat{i} + 150\hat{j})$  m, 10 m/s

# Review of Calculus Techniques

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# Review of Calculus Techniques

## 1. Computing limits

- Examples in mathematics
- Examples in physics

## 2. Differentiation

- Definition of the derivative
- Examples of derivatives

## 3. Integration

- Definition of the integral
- Examples of integrals

# Review of Calculus Techniques - Computing Limits

Consider the function  $f(x)$ , defined below:

$$f(x) = \frac{1}{1+x^2} \quad (4)$$

**Evaluate** the function at the following points:

- $x = 0$
- $x = 10$
- $x = 100$
- $x = 1000$

What is the *limiting value* of the function as  $x \rightarrow \infty$ ? What is the *limiting value* of the function as  $x \rightarrow -\infty$ ?

# Review of Calculus Techniques - Computing Limits

Consider the function  $f(x)$ , defined below:

$$f(x) = \exp(x) = e^x \quad (5)$$

**Evaluate** the function at the following points:

- $x = 0$
- $x = 10$
- $x = 100$
- $x = 1000$

What is the *limiting value* of the function as  $x \rightarrow \infty$ ? What is the *limiting value* of the function as  $x \rightarrow -\infty$ ?



# Review of Calculus Techniques - Computing Limits

Consider the function  $f(x)$ , defined below:

$$f(x) = \sin(x) \quad (6)$$

**Evaluate** the function at the following points:

- $x = \pi$
- $x = -\pi$
- $x = 0.1$
- $x = 0.01$

What is the *limiting value* of the function as  $x \rightarrow 0$ ? The procedure is straightforward if the function is *continuous* and *differentiable*.

# Review of Calculus Techniques - Differentiation

## Derivative of a Function

Let  $f(t)$  be a continuous function on an interval  $[a, b]$ , and  $a < t < b$ . The derivative of  $f(t)$  is

$$f'(t) = \frac{df}{dt} = \lim_{\Delta t \rightarrow 0} \frac{f(t + \Delta t) - f(t)}{\Delta t} \quad (7)$$

## List of Common Derivatives

Here is a link to lists of common derivatives: <https://en.wikipedia.org/wiki/Derivative>  
[https://en.wikipedia.org/wiki/Differentiation\\_rules](https://en.wikipedia.org/wiki/Differentiation_rules)

**Professor:** work some examples.

# Review of Calculus Techniques - Integration

## Integral of a Function

Let  $f(t)$  be a continuous function on an interval  $[a, b]$ , and  $a < t_i < b$ , where  $t_i$  are regular points within the interval. The Riemann integral is

$$I = \sum_i f(t_i) \Delta t_i \rightarrow \int_a^b f(t) dt \quad (8)$$

## List of Common Derivatives

Here is a link to a list of common integrals: [https://en.wikipedia.org/wiki/Lists\\_of\\_integrals](https://en.wikipedia.org/wiki/Lists_of_integrals)

**Professor:** work some examples.

# Conclusion

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# Week 1 Summary

1. Methods of approximation
  - **Estimating** the correct order of magnitude
  - **Function** approximation
  - **Unit analysis**
2. Coordinates and vectors
  - **Scalars** and **vectors**
  - **Cartesian** (rectangular) coordinates, displacement
  - **Vector** addition, subtraction, and multiplication
3. Review of Calculus Techniques
  - Limits
  - Differentiation
  - Integration