CALCULUS-BASED PHYSICS-1: MECHANICS (PHYS150-01): WEEK 2

Jordan Hanson

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Whittier College Department of Physics and Astronomy

WEEK 1 REVIEW

1. Methods of approximation
    - Estimating the correct order of magnitude
    - Function approximation
    - Unit analysis
2. Coordinates and vectors
    - Scalars and vectors
    - Cartesian (rectangular) coordinates, displacement
    - Vector addition, subtraction, and multiplication
3. Review of Calculus Techniques
    - Limits
    - Differentiation
    - Integration

WEEK 1 REVIEW PROBLEMS

Given the displacement vector $\vec{D} = (3\hat{i} - 4\hat{j})$ m, find the displacement vector $\vec{R}$ so that $\vec{D} + \vec{R} = -4D\hat{j}$.

- A: $\vec{R} = (-3\hat{i} - 16\hat{j})$ m
- B: $\vec{R} = (3\hat{i} + 16\hat{j})$ m
- C: $\vec{R} = (-3\hat{i} + 12\hat{j})$ m
- D: $\vec{R} = (-6\hat{i} + 6\hat{j})$ m

Estimate the surface area of a person.

- A: 0.2 m^2
- B: 2 m^2
- C: 5 m^2
- D: 10 m^2

WEEK 2 SUMMARY

1. Displacement, and instantaneous velocity and acceleration
    - *Mathematics review*: taking derivatives
    - Average velocity and average acceleration
2. The case of constant acceleration
    - Deriving an *an equation of motion* for constant acceleration
    - **Measuring acceleration of gravity: g**
3. Derivation and use of common equations of motion

DISPLACEMENT, AND INSTANTANEOUS VELOCITY AND ACCELERATION

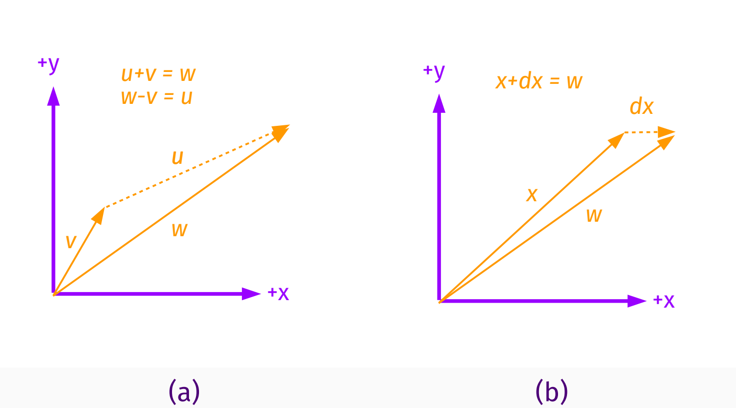

Figure 1: (Left): The displacement vector is $\vec{u}$. (Right) Treat displacement for a small change in time, dt, and call it $d\vec{x}$.

Let $f(t) = A\sin(Bt) + Ct^2$.
Compute f'.

- A: $f'(t) = AB\sin(Bt) + 2Ct$
- B: $f'(t) = AB\cos(Bt) + 2C$
- C: $f'(t) = AB\sin(Bt) + 2Ct$
- D: $f'(t) = AB\cos(Bt) + 2Ct$

Let $f(t) = (4t - 1)/(3t + 2)$.
Compute f'.

- A: $f'(t) = \frac{4}{3t+2}$
- B: $f'(t) = \frac{4}{(3t+2)^2} + \frac{12t-3}{(3t+2)^2}$
- C: $f'(t) = \frac{4}{3t+2} + \frac{12t-3}{(3t+2)^2}$
- D: $f'(t) = \frac{12t-3}{(3t+2)^2}$

Definition of instantaneous velocity vector:

$$\boxed{v(t) = \frac{d\vec{x}}{dt}} \tag{1}$$

Simple example: Let the vector position of an object be

$$\vec{x}(t) = (2t\hat{i} - 3t^2\hat{j}) \quad m \tag{2}$$

Then

$$\vec{v}(t) = (2\hat{i} - 6t\hat{j}) \quad m/s \tag{3}$$

Definition of instantaneous *acceleration* vector:

$$\boxed{a(t) = \frac{d\vec{v}}{dt} = \frac{d}{dt}\frac{d\vec{x}}{dt}} \tag{4}$$

Simple example: Let the vector position of an object be

$$\vec{x}(t) = (2t\hat{i} - 3t^2\hat{j}) \quad m \tag{5}$$

Then

$$\vec{v}(t) = (-6\hat{j}) \quad m/s^2 \tag{6}$$

Interesting... If the motion of an object is *quadratic* in time, then the acceleration is a constant.

Let the displacement versus time of an object be

$$\vec{y}(t) = (-\frac{1}{2}gt^2 + v_i t + y_0)\hat{j} \quad (m) \tag{7}$$

If Eq. 7 gives the displacement in the $\hat{j}$ direction, then what are the velocity and acceleration?

Using the definitions of instantaneous velocity and acceleration:

$$\frac{d\vec{y}}{dt} = (-gt + v_i)\hat{j} \quad (m/s) \tag{8}$$

$$\frac{d}{dt}\frac{d\vec{y}}{dt} = (-g)\hat{j} \quad (m/s^2) \tag{9}$$

The acceleration is just some constant, g, in the $-\hat{j}$ direction. This leads to a *linear* equation for the velocity, and a *quadratic* equation for the displacement.

So we have the following three equations for a system experiencing constant acceleration:

$$\vec{y}(t) = (-\frac{1}{2}gt^2 + v_{\mathrm{i}}t + y_0)\hat{j} \quad (m) \tag{10}$$

$$\vec{v}(t) = (-gt + v_{\mathrm{i}})\hat{j} \quad (m/s) \tag{11}$$

$$\vec{a}(t) = (-g)\hat{j} \quad (m/s^2) \tag{12}$$

What if we solve for time in Eq. 11, after taking the magnitude of the vector?

$$\frac{v - v_{\mathrm{i}}}{-g} = t \tag{13}$$

Now substitute Eq. 13 into Eq. 10:

$$y = -\frac{1}{2}g\left(\frac{v - v_\mathrm{i}}{-g}\right)^2 + v_\mathrm{i}\left(\frac{v - v_\mathrm{i}}{-g}\right) + y_0 \tag{14}$$

$$-2g(y - y_0) = (v - v_\mathrm{i})^2 + 2v_\mathrm{i}(v - v_\mathrm{i}) \tag{15}$$

$$-2g(y - y_0) = v^2 - v_\mathrm{i}^2 \tag{16}$$

$$-2g(y - y_0) + v_\mathrm{i}^2 = v^2 \tag{17}$$

Equation 17 provides a way to obtain the velocity of an accelerating system at some displacement without knowing the time.

A particle moves along the x-axis according to $x(t) = (10t - 2t^2)\hat{i} \quad (m)$. What is the instantaneous velocity at $t = 2$ seconds and $t = 3$ seconds? What is the average of these two numbers?

- A:
- B:
- C:
- D:

On February 15, 2013, a superbolide meteor (brighter than the Sun) entered Earth's atmosphere over Chelyabinsk, Russia, and exploded at an altitude of 23.5 km. Eyewitnesses could feel the intense heat from the fireball, and the blast wave from the explosion blew out windows in buildings. The blast wave took approximately 2 minutes 30 seconds to reach ground level. What was the average velocity of the blast wave? Compare this with the speed of sound, which is 343 m/s at sea level.

- A:
- B:
- C:
- D:

ANSWERS

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- $\vec{R} = (-3\hat{i} - 16\hat{j})$ m
- 2 m^2
- $f'(t) = AB\cos(Bt) + 2Ct$
- $f'(t) = \frac{4}{3t+2} + \frac{12t-3}{(3t+2)^2}$
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