CALCULUS-BASED PHYSICS-1: MECHANICS (PHYS150-01): WEEK 8

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WEEK 7 REVIEW

WEEK 7 SUMMARY

1. Work and potential energy
    - **Lab activity:** Oscillator and gravity trading work and potential energy
2. Potential energy and **conservative forces**
3. Conservation of Energy
    - *Calculus review: the fundamental theorem of calculus*
    - Graphical representations of integrals and energy

WEEK 7 REVIEW PROBLEM

Suppose a particle moves in a potential energy surface $U(x) = U_0 \left(x^4 - x^2\right)$. What is the force at $x = 1$?

- A: $-4U_0 + 2U_1$
- B: $4U_0 - 2U_1$
- C: $-U_0 + U_1$
- D: 0

WEEK 8 SUMMARY

1. Definition of **momentum**
2. *Conservation of momentum*
    - The proof and the assumptions
    - Examples
3. *Classification of collisions*
    - Elastic
    - Inelastic
    - $1 \to 1$, $1 \to n$, $n \to 1$, $n \to n$
4. **Momentum in multiple dimensions**
5. **Center of mass**
    - Derivation of $\vec{F}_{\mathrm{Net}} = \frac{d\vec{P}_{\mathrm{CM}}}{dt}$
    - Center of mass motion

DEFINITION OF MOMENTUM

Ready to jump down the rabbit hole? Good. Momentum is defined as follows:

Definition of Momentum

A particle of mass m and velocity $\vec{v}$ has the vector *momentum*:

$$\vec{p} = m\vec{v}$$

There is a corollary:

Newton's Second Law with momentum

If a particle has acceleration $\vec{a} = \frac{d\vec{v}}{dt}$, then

$\vec{F}_{\mathrm{Net}} = \frac{d\vec{p}}{dt}$

An object that has a small mass and an object that has a large mass have the same momentum. Which mass has the largest kinetic energy?

- A: The one with the small mass
- B: The one with the large mass
- C: If the momentum is the same the kinetic energy is the same
- D: Cannot determine the answer

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The unit of linear momentum is kg m/s. Suppose that a raindrop reaches a terminal velocity of 1 m/s, and the density of water is 1 gram per cm^3. If a 1 cm^3 water droplet reaches terminal velocity, what is the momentum of the droplet?

- A: 10^{-3} kg m/s
- B: 10^{-2} kg m/s
- C: 10^{-1} kg m/s
- D: 1 kg m/s

If $\vec{F}_{\mathrm{Net}} = \frac{d\vec{p}}{dt}$, and an object is undergoing constant acceleration, which of the following is true of the momentum?

- A: It is constant in time.
- B: It is a linear function of time.
- C: It is a quadratic function of time.
- D: It is zero.

CONSERVATION OF MOMENTUM

...continuing to fall down the rabbit hole...

Conservation of Momentum

The momentum of a system of N particles undergoing no external forces is conserved.

$\frac{d\vec{P}}{dt} = 0$

Suppose two objects with momenta $\vec{p}_1 = m_1\vec{v}_1$ and $\vec{p}_2 = m_2\vec{v}_2$ collide. The new momenta after the collision are $\vec{p}'_1 = m_1\vec{v}'_1$ and $\vec{p}'_2 = m_2\vec{v}'_2$. If $m_2 = 2m_1$ and $\vec{v}_1 = 2\vec{v}_2$, and $\vec{v}'_2$ is observed to equal $\vec{v}_1$, what is $\vec{v}'_1$?

Solve together in groups on boards.

The proof of conservation of momentum is the combination of two concepts: Newton's 3rd Law and Newton's 2nd Law. The net forces on two particles by Newton's 3rd Law are

$$\vec{F}_{21} = -\vec{F}_{12} \tag{1}$$

Substituting Newton's 2nd Law for the forces,

$$m_1\vec{a}_1 = -m_2\vec{a}_2 \tag{2}$$

Acceleration is defined as the change in velocity, implying

$$m_1\frac{d\vec{v}_1}{dt} = -m_2\frac{d\vec{v}_2}{dt} \tag{3}$$

$$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt} \tag{4}$$

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \tag{5}$$

$$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = 0 \tag{6}$$

$$\boxed{\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = 0} \tag{7}$$

Equation 7 states that the total momentum does not change with time. The assumptions hold even if there are more than two particles, for every particle in the system exerts some force on every other particle, even if that force is zero.

$$\frac{d}{dt}\sum_{\mathrm{i}} \vec{p}_{\mathrm{i}} = \frac{d\vec{P}}{dt} = 0 \tag{8}$$

What two assumptions were necessary in the above proof?

- A: Each mass is constant in time, and the total velocity is zero.
- B: The total velocity is zero, and the net force is $\frac{dP}{dt}$.
- C: The net external force is zero, and the total velocity is zero.
- D: The net external force is zero, and the mass of each particle is zero.

CONCLUSION

ANSWERS

ANSWERS

- $-4U_0 + 2U_1$
- The one with the small mass
- The one with the large mass
- 10^{-2} kg m/s
- It is a linear function of time.
- The net external force is zero, and the mass of each particle is zero.
- ...