Warm Up: Energy I

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1 Memory Bank

- $W = \vec{F} \cdot \Delta\vec{x}$ ... Definition of work
- Unit of work: 1 Joule, or 1 N m
- $v_f^2 = v_i^2 + 2a\Delta x$ ... Kinematic equation with constant acceleration
- $KE = \frac{1}{2}mv^2$ ... Definition of Kinetic Energy
- $W = KE_f - KE_i$ ... Work-energy theorem
- $\vec{F} = k\Delta\vec{x}$ ... Hooke's Law, or the force of a spring
- $W = \frac{1}{2}k(\Delta x)^2$ ... Work done to compress or stretch a spring by Δx.
- $U = mgy$ ... Gravitational potential energy a distance y above the zero point

2 Work and Energy

1. (a) Suppose a force $\vec{F} = 3\hat{i} + 4\hat{j}$ N accelerates a system restrained to a track along the y-direction by 40 cm. What work is done on the system? (b) If the same force acts on a system restricted to move on a track aligned 45 degrees with respect to the x-axis, what work is done if the object is displaced by 40 cm?

2. Suppose a small amount of work done on a system dW by a force F causes a displacement of the system dx. If $dW = Fdx$, then $F = dW/dx$. (a) If the force of a spring is $F = -k\Delta x$, what is a function that describes the work done on a spring when it is compressed by Δx? (b) If the spring constant is 50 N m^{-1}, how much work is required to squeeze the spring by 5 cm?

3. Suppose a system is accelerated by a force F. (a) Write down (i) Newton's 2nd law, (ii) a kinematic equation for constant acceleration that does not involve time, and (iii) the definition of work for a force that is parallel to the displacement. (b) Solve the kinematic equation for $a\Delta x$, and substitute it into the definition of work. The result is known as the **work-energy theorem.** The quantity $(1/2)mv^2$ is known as the *kinetic energy.* (c) How much work is required to slow a 1000 kg vehicle to a stop, if the initial speed is 30 m/s?

4. Suppose a ball of mass m is loaded onto a spring, and the spring is compressed by Δx. (a) Derive an expression for the initial speed of the ball if it is launched by the spring. (b) How high will the ball go above the launch point, if it has a mass of 50 grams?