

Calculus-Based Physics-1: Mechanics (PHYS150-02): Unit 0

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Course Introduction

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3. Syllabus: Moodle
4. Office hours: Discord 918particle#5083
5. Course pre-requisites: Calculus 1 (concurrently or previously, high-school equivalent).
6. Text: University Physics Volume 1 (see syllabus)
7. Homework: OpenStax Tutor (see syllabus)

Opening Remarks - Welcome!

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Figure 1: The usual look from a student taking physics for the first time.

Summary

Week 1 Summary

Physics - φυσική - "phusiké": knowledge of nature
from φύσις - "phúsis": nature

1. Methods of approximation
 - **Estimating** the correct order of magnitude
 - **Function** approximation
 - **Unit analysis**
2. Coordinates and vectors
 - **Scalars** and **vectors**
 - **Cartesian** (rectangular) coordinates, displacement
 - **Vector** addition, subtraction, and multiplication
3. Review of Calculus Techniques
 - Differentiation of power series.
 - Integration of power series.

Methods of approximation

Methods of approximation - Estimation: Chapter 1

In science and engineering, **estimation** is to obtain a quantity in the absence of precision, informed by rational constraints.

1. Define relevant **scales**
 - 1 *AU* for the solar system (distance from Sun to Earth)
 - 1 *angstrom* (10^{-10} meters) for cell membranes
2. Obtain **complex quantities** from simple ones
 - Obtain *areas* and *volumes* from *lengths*
 - Obtain *rates* from *numerators* and *denominators*
3. Constrain the unknown with **upper** and **lower** limits
 - The solar system is *less than one light-year* across
 - An insect is *at least one millimeter* long

Professor: work several examples.

Methods of approximation - Estimation: Chapter 1

Estimate the mass of ants in an ant colony. Assume that the colony is a species known to have 10^5 ants (roughly) per colony.

- A: 0.01 kg
- B: 0.1 kg
- C: 1 kg
- D: 10 kg

An adult blue whale is about 30 meters long. What is the mass of a blue whale calf? (1 tonne = 1000 kg).

- A: 300 kg
- B: 0.3 tonnes
- C: 3 tonnes
- D: 50 tonnes

Methods of approximation - Estimation: Chapter 1

How long does it take an airliner to fly across the Atlantic ocean? Assume the velocity is 500 mph, and the radius of the Earth is 7000 km.

- A: 10 hours
- B: 15 hours
- C: 2 hours
- D: 4 hours

A person has 150 kg of mass and is about 1.5 meters tall. What is his density (mass divided by volume)?

- A: 100 kg/m³
- B: 400 kg/m³
- C: 1000 kg/m³
- D: 10⁴ kg/m³

Methods of approximation - Estimation: Chapter 1

A jar of coffee beans sits on a counter at a cafe. If the jar is about 10 cm tall, and has a radius of about 5 cm, estimate the number of beans inside.

- A: 300 beans
- B: 3000 beans
- C: 30,000 beans
- D: 30 beans

What is the volume of a basketball?

- A: 700 cm^3
- B: 7000 cm^3
- C: 700 cm^2
- D: 7000 cm^2

Methods of approximation - Function approximation

In science and engineering, **function approximation** or **expanding a function** is a technique in which a simple function is used to obtain the value of a more complicated function near a point where they are approximately equal.

1. Memorizing **special cases**

- $\sin(x) \approx x$, when $|x| < 1$
- $\tan(x) \approx x$, when $|x| < 1$
- $(1+x)^{1/2} \approx 1 + \frac{1}{2}x$, when $|x| < 1$
- $\exp(x) \approx 1 + x$, when $|x| < 1$

2. Utilizing the **Taylor Series** (more on this later)

$$\bullet f(x) \approx f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

Methods of approximation - Function approximation

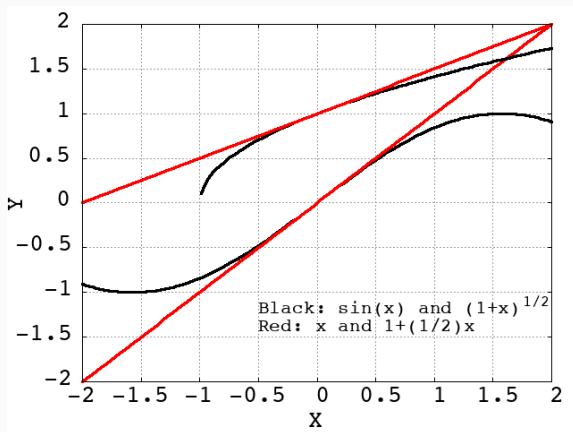


Figure 2: Certain functions may be approximated by simpler ones. In this case, $\sin(x)$ is approximated by x near $x = 0$, and $(1+x)^{1/2}$ is approximated by $1 + \frac{1}{2}x$ near $x = 0$.

Methods of approximation - Function approximation

The height in meters of a surfer above some average height as he bobs in the waves is described by $h(t) = \sin(\pi t/2)$. What is his height at 0.2 seconds? What is his height at -0.2 seconds?

- A: 10 meters, -10 meters
- B: $\pi/10$ meters, $-\pi/10$ meters
- C: -0.1 meters, 0.1 meters
- D: $-\pi$ meters, π meters

The value of an investment in dollars, v , versus time in years, t , follows the form $v(t) = P \exp(rt)$, where P is the value at $t = 0$, and $r = 1/3$. What is $v(1)$, the value after one year?

- A: $\approx 1/3P$
- B: $\approx 2/3P$
- C: $\approx 3/2P$
- D: $\approx 4/3P$

Methods of approximation - Units: Chapter 1

Physics requires **units** to relate ideas to the real world, and **unit analysis** is a powerful tool to eliminate incorrect results and to facilitate estimation.

1. SI units, and kilogram-meter-second unit set

- mass: **kilogram** (gram = 10^{-3} kg, milligram = 10^{-6} kg)
- length: **meter** (millimeter = 10^{-3} m, kilometer = 10^3 m)
- time: **second** (1 year $\approx \pi \times 10^7$ sec, 1 hour = 3600 sec)

2. Unit analysis

- If we are calculating a density, the units should work out to be kg/m^3
- Identifying the fundamental unit in a complex calculation often simplifies it (when done properly, this reveals the beauty of physics)

Professor: work several examples.

Methods of approximation - Units: Chapter 1

A millenium is 1000 years. If a glacier retreats at a pace of 10 cm per year, what is this rate in meters per millenium?

- A: 0.1 meter per millenium
- B: 1 meter per millenium
- C: 10 meters per millenium
- D: 100 meters per millenium

Ice has a density of 0.917 grams per centimeter cubed. What is this density in kilograms per meter cubed?

- A: 91.7 kg/m³
- B: 917 kg/m³
- C: 9170 kg/m³
- D: 9.17 kg/m³

Methods of approximation - Units: Chapter 1

Sometimes, the beauty of physics arises from choosing the right unit.

http://joshworth.com/dev/pixelspace/pixelspace_solarsystem.html

The Sun in this ruler is at 0 km, and Jupiter is at about 780,000,000 km (good luck finding it). Clearly, the kilometer is the wrong unit to choose for interplanetary distances. What if we defined a new unit, the [astronomical unit](#), as the distance between the Earth and the Sun?

Methods of approximation - Units: Chapter 1

Planetary orbital radii in AU (geometric means):

Mercury	0.379
Venus	0.722
Earth	1.00
Mars	1.52
Jupiter	5.20
Saturn	9.54
Uranus	19.2
Neptune	30.1

Figure 3: Why such simple numbers? There is a set of simple relationships between the *orbital period* and the *orbital radius* of planets called Kepler's Laws, which led to the discovery of [Newton's Law of Gravity](#).

Coordinates and Vectors

Introduction of the problem, and group activity:

Navigation in the film The Hunt for Red October.

<https://youtu.be/4unk6si0-tI>

Coordinates and Vectors - Scalars, Vectors (Chapters 2.1-2.3)

Physics requires **mathematical objects** to build equations that capture the behavior of nature. Two examples of such objects are **scalar** and **vector** quantities. Each type of object obeys similar but different rules.

1. Scalar quantities

- mass: $m_1 + (m_2 + m_3) = (m_1 + m_2) + m_3$
- speed: $v_1(v_2 + v_3) = v_1v_2 + v_1v_3$
- charge: $q_1\left(\frac{1}{q_1}\right) = 1, q_1(0) = 0$

2. Vector quantities

- velocity: $\vec{v}_1 + (\vec{v}_2 + \vec{v}_3) = (\vec{v}_1 + \vec{v}_2) + \vec{v}_3$
- tension: $\vec{t}_1 \cdot (\vec{t}_2 + \vec{t}_3) = \vec{t}_1 \cdot \vec{t}_2 + \vec{t}_1 \cdot \vec{t}_3$

Coordinates and Vectors - Scalars, Vectors (Chapters 2.1-2.3)

A vector may be expressed as *a list of scalars*: $\vec{v} = (4, 2)$ (a vector with two *components*), $\vec{u} = (3, 4, 5)$ (three *components*). Now, we know how to add and subtract scalars. How do we add and subtract vectors?

What is

$(1, 3, 8) +$

$(0, 2, 1)$?

Answer: $(1, 5, 9)$

In other words, when adding vectors, we add them component by component.

How do we subtract vectors? In the same fashion:

What is

$(1, 3, 8) -$

$(0, 2, 1)$?

Answer: $(1, 1, 7)$

In other words, when subtracting vectors, we subtract them component by component.

A HTML-based demonstration for adding vectors:

[https://phet.colorado.edu/en/simulations/
vector-addition](https://phet.colorado.edu/en/simulations/vector-addition)

Notice several things:

- Produce vectors that *cancel* each other.
- What happens when vectors are parallel and orthogonal?

How do we multiply vectors? In the same fashion, *for one kind of multiplication*:

What is

$$(1, 3, 8) \cdot (0, 2, 1)?$$

$$\text{Answer: } 1 \cdot 0 + 3 \cdot 2 + 8 \cdot 1 = 14$$

This kind of multiplication is known as the dot-product. There is also the *cross-product*, which we will save for later.

Coordinates and Vectors - Coordinates (Chapters 2.1-2.3)

The components of a vector may describe quantities in a **coordinate system**, such as *Cartesian coordinates* - after René Descartes.

Vectors in the 3D Cartesian coordinate system (x,y,z) may be written in the following notation:

$$\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}$$

- a: The amount in the +x-direction, \hat{i} : a vector of length 1, in the +x-direction
- b: The amount in the +y-direction, \hat{j} : a vector of length 1, in the +y-direction
- c: The amount in the +z-direction, \hat{k} : a vector of length 1, in the +z-direction

Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

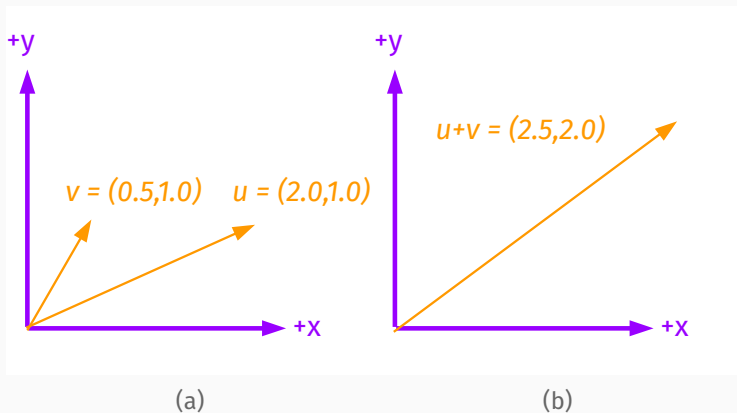


Figure 4: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} + \vec{v}$? Adding components: $\vec{u} + \vec{v} = 2.5\hat{i} + 2.0\hat{j}$.

Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

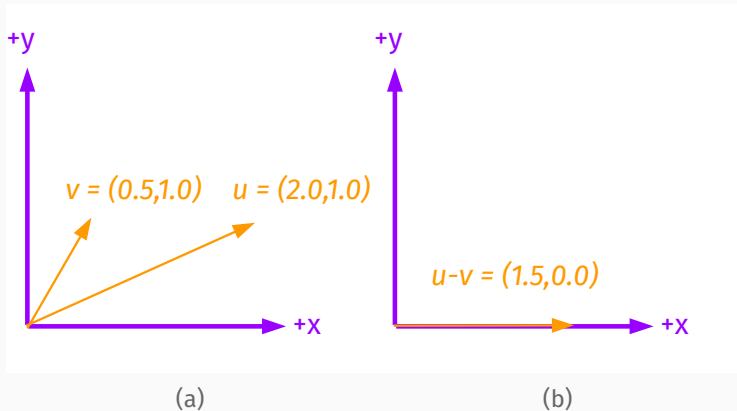


Figure 5: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} - \vec{v}$? Subtracting components: $\vec{u} - \vec{v} = 1.5\hat{i} + 0.0\hat{j}$.

Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

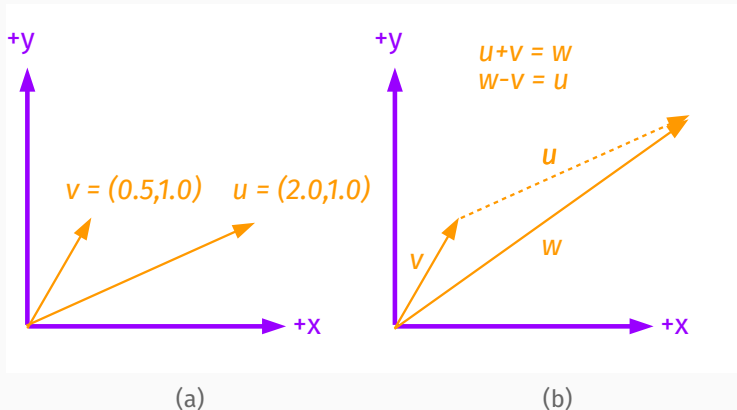


Figure 6: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) To compute $\vec{w} - \vec{v}$, arrange the vectors to get a sense of the result, \vec{u} .

Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

$$\vec{p} = 4\hat{i} + 2\hat{j}. \quad \vec{q} = -4\hat{i} + 2\hat{j}.$$

Compute $\vec{p} \cdot \vec{q}$.

- A: 12
- B: -12
- C: 4
- D: 8

$$\vec{p} = -1\hat{i} + 6\hat{j}. \quad \vec{q} = 3\hat{i} + 0.5\hat{j}.$$

Compute $\vec{p} \cdot \vec{q}$.

- A: -1
- B: 1
- C: 0
- D: 3

Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

Why was the last answer zero? Look at it graphically:

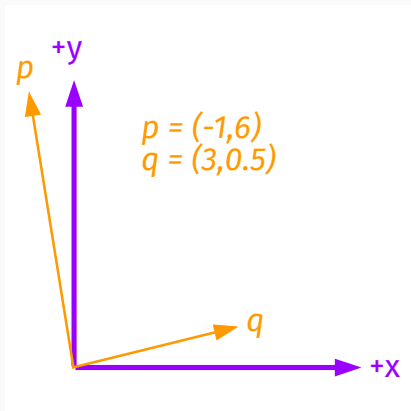


Figure 7: Two vectors \vec{p} and \vec{q} are orthogonal if $\vec{p} \cdot \vec{q} = 0$.

Coordinates and Vectors - Vectors (Chapters 2.1-2.3)

What if the vectors are parallel? Look at it graphically:

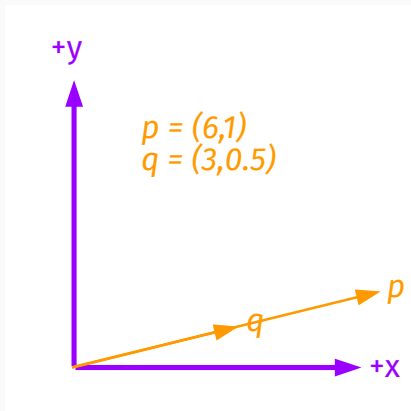


Figure 8: Two vectors \vec{p} and \vec{q} are *parallel* if $\vec{p} \cdot \vec{q}$ is maximal.

Coordinates and Vectors - Dot Product (Chapters 2.1-2.3)

The *length* or *norm* of a vector $\vec{v} = a\hat{i} + b\hat{j}$ is $|\vec{v}| = \sqrt{a^2 + b^2}$.

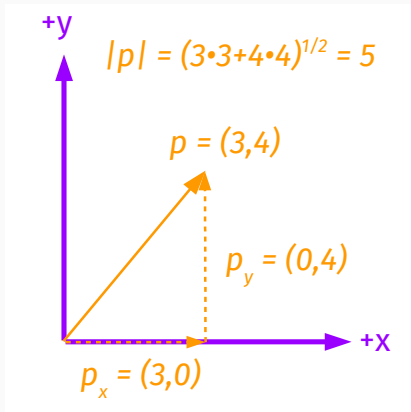


Figure 9: Computing the norm of a vector \vec{p} .

Coordinates and Vectors - Dot Product (Chapters 2.1-2.3)

Notice that $\sqrt{\vec{p} \cdot \vec{p}} = |\vec{p}|$.

Let θ_p be the angle between \vec{p} and the x-axis.

$$p_x = \vec{p} \cdot \hat{i} = |\vec{p}| \cos(\theta_p).$$

$$p_y = \vec{p} \cdot \hat{j} = |\vec{p}| \sin(\theta_p).$$

Theorem: The dot product of two vectors \vec{p} and \vec{q} is $|\vec{p}||\vec{q}| \cos(\theta)$, if θ is the angle between them.

$$\begin{aligned} \text{Proof: } \vec{p} \cdot \vec{q} &= p_x q_x + p_y q_y = |p||q| \cos \theta_p \cos \theta_q + |p||q| \sin \theta_p \sin \theta_q \\ &= |p||q| (\cos \theta_p \cos \theta_q + \sin \theta_p \sin \theta_q) = |p||q| \cos(\theta_p - \theta_q) \\ &= |p||q| \cos \theta. \end{aligned}$$

$$\boxed{\vec{p} \cdot \vec{q} = |p||q| \cos \theta}$$

Coordinates and Vectors - Dot Product (Chapters 2.1-2.3)

An object moves at 2 m/s at $\theta = 60^\circ$ with respect to the x-axis. What is the velocity of the object?

- A: $(1\hat{i} + 1\hat{j})$ m/s
- B: $(\sqrt{3}\hat{i} + 1\hat{j})$ m/s
- C: $(\sqrt{3}\hat{i} + \sqrt{3}\hat{j})$ m/s
- D: $(1\hat{i} + \sqrt{3}\hat{j})$ m/s

What is the dot product of this velocity with another velocity: 5 m/s along the x-axis?

- A: 1 (m/s)^2
- B: 5 (m/s)^2
- C: 10 (m/s)^2
- D: 5 (m/s)

Coordinates and Vectors - Scalars, Vectors (Chapters 2.1-2.3)

Is it possible to multiply vectors and scalars? Of course:

$$a_1 \vec{p} = a_1 p_x \hat{i} + a_1 p_y \hat{j}.$$

Also, multiplication properties still hold. For example:

$$(a_1 + a_2) \vec{p} = a_1 \vec{p} + a_2 \vec{p}.$$

A spacecraft moves at 400 m/s, at an angle of 30 degrees with respect to the x-axis. If it fires two thrusters that boost the x-component and y-component of the velocity by 25% and 50%, respectively, what is the final velocity?

- A: $(433\hat{i} + 300\hat{j})$ m/s
- B: $(300\hat{i} + 433\hat{j})$ m/s
- C: 400 m/s
- D: $(400\hat{i} + 433\hat{j})$ m/s

Coordinates and Vectors - Displacement (Chapters 2.1-2.3)

We define the *position* of an object as a vector locating it in a given coordinate system. The scalar *distance* is the norm of the position vector, that is, the distance to to the origin.

Now we can introduce the concept of **displacement**: a vector describing a movement of an object.

Coordinates and Vectors - Displacement (Chapters 2.1-2.3)

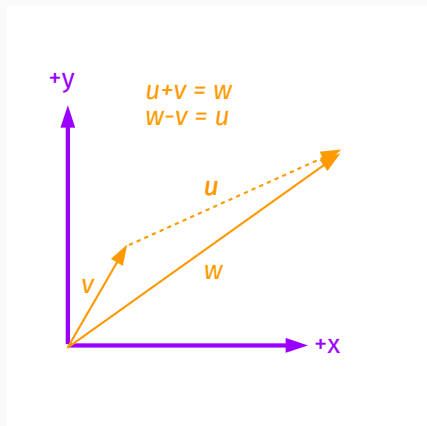


Figure 10: Suppose an object moves from position \vec{v} to \vec{w} . In this case, the displacement is \vec{u} . Thus, the final position is the initial position, plus the displacement.

Coordinates and Vectors - Displacement (Chapters 2.1-2.3)

It follows that the *displacement* is zero if the initial and final positions are the same, but the *distance travelled* is not.

Suppose a jet fighter travelling at 800 km per hour banks such that it flies in a circle of radius 0.5 km. How long does it take to complete the circle? What is the distance traveled, and what is the displacement?

- A: 2π km, 28 seconds, 2π km
- B: π km, 14 seconds, π km
- C: π km, 28 seconds, π km
- D: π km, 14 seconds, 0 km

Conclusion of Chapter 2

For a list of helpful equations and mathematical definitions, see p. 92 of the the text.

Coordinates and Vectors - Average Velocity (Chapter 3.1)

Average velocity is the ratio of the **displacement** to the elapsed time.

$$\boxed{\vec{v}_{\text{avg}} = \frac{\Delta \vec{x}}{\Delta t}} \quad (1)$$

The *average speed* is the norm of the average velocity:

$$\boxed{v_{\text{avg}} = \frac{|\Delta \vec{x}|}{\Delta t}} \quad (2)$$

If the motion is in one dimension, then the average speed is

$$v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} \quad (3)$$

Coordinates and Vectors - Average Velocity (Chapter 3.1)

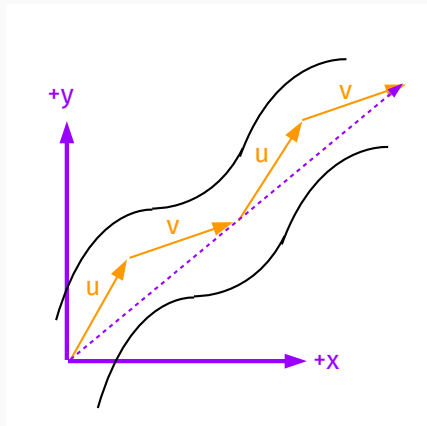


Figure 11: A Formula-1 driver keeps his car on the track by following a path approximated by the position vectors u , v , u , and v . The dashed arrow represents the total displacement.

Coordinates and Vectors - Average Velocity (Chapter 3.1)

If $\vec{u} = (20\hat{i} + 30\hat{j})$ m, and $\vec{v} = (30\hat{i} + 20\hat{j})$ m, what is the total displacement? If the elapsed time is 10 seconds, what is the average velocity?

- A: $(50\hat{i} + 50\hat{j})$ m, 14 m/s
- B: $(80\hat{i} + 100\hat{j})$ m, 10 m/s
- C: $(100\hat{i} + 100\hat{j})$ m, 14 m/s
- D: $(50\hat{i} + 150\hat{j})$ m, 10 m/s

Review of Calculus Techniques

Review of Calculus - Skill 1: Taking Limits

Taking a limit has a variety of uses in physics. Consider the following example:

The weight of an object sitting on the Earth's surface is given by the following equation, which we will encounter later in physics:

$$w = G \frac{mM}{r^2} \quad (4)$$

- w : weight
- G : a constant of nature
- m : the object's mass
- M : the mass of the Earth
- r : the distance between the center of the Earth and the object

Review of Calculus - Skill 1: Taking Limits

The weight of an object sitting on the Earth's surface is given by the following equation, which we will encounter later in physics:

$$w = G \frac{mM}{r^2} \quad (5)$$

Let R be the radius of the Earth, r_0 be the object's height above the Earth's surface, and $\epsilon = r_0/R \ll 1$. Also, let $g = GM/R^2$.

Rearranging the equation for weight:

$$w = mg(1 + \epsilon)^{-2} \quad (6)$$

Since $\epsilon \ll 1$, take the limit:

$$\lim_{\epsilon \rightarrow 0} w = \lim_{\epsilon \rightarrow 0} mg(1 + \epsilon)^{-2} = mg \quad (7)$$

Review of Calculus - Skill 1: Taking Limits

Thus, for practical calculations where the object is not far from the Earth's surface, the weight is $w = mg$, or the mass times some measurable constant, g .

Often in other branches of physics, we often encounter the *sinc function*:

$$s(t) = \frac{\sin(t)}{t} \quad (8)$$

What is $\lim_{t \rightarrow 0} s(t)$? It looks like $0/0$, but that is not defined mathematically.

Review of Calculus - Skill 1: Taking Limits

We can use **L'Hôpital's Rule**: take the *derivative* of the numerator and the denominator, then take the limit:

$$\lim_{t \rightarrow 0} s(t) = \lim_{t \rightarrow 0} \left| \frac{\frac{d}{dt} \sin(t)}{\frac{d}{dt} t} \right| = \lim_{t \rightarrow 0} \cos(t) = 1 \quad (9)$$

This approach is valid for cases like $0/0$ or ∞/∞ . But what is the **derivative**? How do we know that the derivative of $\sin(t)$ is $\cos(t)$, and the derivative of t is one? The definition of the derivative:

$$f'(t) = \dot{f} = \frac{df}{dt} \equiv \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h} \quad (10)$$

"The change in y over the change in x at h ."

Review of Calculus - Skill 2: Taking Derivatives

1. $f(t) = t^n$

2. $f(t) = \exp(t)$

3. $f(t) = \ln(t)$

4. $f(t) = \sin(t)$

5. $f(t) = \cos(t)$

6. $f(t) = \tan(t)$

7. $f(t) = a$

1. $f' = nt^{n-1}$

2. $f' = \exp(t)$

3. $f' = 1/t$

4. $f' = \cos(t)$

5. $f' = -\sin(t)$

6. $f' = \sec^2(x)$

7. $f' = 0$

Review of Calculus - Skill 2: Derivative Properties

1. $g(t) = af(t)$

2. $g(t) = a(t) + b(t)$

3. $g(t) = a(t)b(t)$

4. $g(t) = a(t)/b(t)$

5. $g(t) = a(b(t))$

1. $g' = af'$

2. $g' = a' + b'$

3. $g' = ab' + a'b$

4. $g' = \frac{ba' - ab'}{b^2}$

5. $g' = a'(b)b'$

Review of Calculus - Skill 3: Anti-derivatives and integrals

An *anti-derivative* just reverses the action of the derivative. Also called an indefinite integral. For example:

$$x(t) = at^n \quad (11)$$

$$x' = nat^{n-1} \quad (12)$$

$$\int nat^{n-1} dt = at^n + C \quad (13)$$

In Eq. 13, the \int symbol represents integration (a big S for "summation"). There is a constant C because, technically, if we take the derivative of Eq. 13, the result is Eq. 12 (derivative of a constant C is zero).

Review of Calculus - Skill 3: Anti-derivatives and integrals

An integral is the difference in the value of the anti-derivative evaluated at two points:

$$\int_c^d nat^{n-1}dt = a(d)^n - a(c)^n \quad (14)$$

"Raise the power by one, and divide the whole thing by that number." - Just like derivatives, there are many simple cases to memorize.

$$\int_a^b \cos(t)dt = \sin(b) - \sin(a) \quad (15)$$

Review of Calculus - Skill 3: Anti-derivatives and integrals

1. $f(t) = t^n$

2. $f(t) = \exp(t)$

3. $f(t) = 1/t$

4. $f(t) = \sin(t)$

5. $f(t) = \cos(t)$

6. $f(t) = a$

7. ...

1. $\int f(t)dt = (n+1)^{-1}t^{n+1} + C$

2. $\int f(t)dt = \exp(t) + C$

3. $\int f(t)dt = \ln(t) + C$

4. $\int \sin(t)dt = -\cos(t) + C$

5. $\int \cos(t)dt = \sin(t) + C$

6. $\int f(t) = at + C$

7. ...

A Few Calculus Problems

Let $x(t) = 5t^2 - 2t + 7$ in the \hat{x} -direction, where x is in meters and t is in seconds. What is the velocity (take the derivative) at $t = 1$ second?

- A: 8 m/s in the \hat{y} -direction
- B: 6 m/s in the \hat{x} -direction
- C: 8 m/s in the \hat{x} -direction
- D: 6 m/s in the \hat{y} -direction

Let $v(t) = 2t + 2$ in the \hat{x} -direction. What is the position versus time (take the integral)?

- A: $t^2 + 2t + C$
- B: $t^2 + 2t$
- C: $t + 2 + C$
- D: $t^3 2t^2 + C$

Conclusion

Week 1 Summary

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 - Limits
 - Differentiation
 - Integration