

# CALCULUS-BASED PHYSICS-2: ELECTRICITY AND MAGNETISM (PHYS180-02): UNIT 2

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## UNIT 1 REVIEW

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### Reading: Chapters 5-6

1. Charge, Conductors and Insulators
2. Coulomb's Law and Electric Fields
3. E-fields of Charge Distributions
4. Gauss's Law

## UNIT 1 REVIEW PROBLEMS

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## UNIT 3 REVIEW PROBLEMS

Suppose a charge  $q$  is located at the origin. Use Gauss' law to find the electric field. What is the electric field?

- A:  $\vec{E} = \frac{kq}{r} \hat{r}$
- B:  $\vec{E} = \frac{kq}{r^2} \hat{r}$
- C:  $\vec{E} = \frac{kq}{r^3} \hat{r}$
- D:  $\vec{E} = \frac{kq^2}{r^2} \hat{r}$

Suppose a line of charge runs up the z-axis. The charge per unit length is  $\lambda$ . Use Gauss' law to find the electric field. What is the electric field at a point  $P = (x, 0, 0)$ ?

- A:  $\vec{E} = \frac{2k\lambda}{r^2} \hat{x}$
- B:  $\vec{E} = \frac{2k\lambda^2}{r^2} \hat{x}$
- C:  $\vec{E} = \frac{2k\lambda}{r} \hat{x}$
- D:  $\vec{E} = 0$

## SUMMARY

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### Reading: Chapters 7-8

#### 1. Voltage

- 1.1 Review of work and energy
- 1.2 Review of conservative forces

#### 2. Capacitance



## VOLTAGE

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Voltage is analogous to potential energy in electrostatics. The negative derivative of potential energy  $U$  is the force  $F$ :

$$F = -\frac{dU}{dx} \quad (1)$$

For example, if the force is  $F = -mg$ , and the potential energy is  $U = mgy$ , then

$$\frac{dU}{dy} = -mg \quad (2)$$

Voltage is analogous to potential energy in electrostatics. The derivative of voltage  $V$  is the field  $E$ :

$$E = -\frac{dV}{dx} \quad (3)$$

For example, if the field is  $E = \sigma/\epsilon_0$ , and the potential energy is  $V = \sigma/\epsilon_0 x + C$ , then

$$\frac{dV}{dx} = \sigma/\epsilon_0 \quad (4)$$

Voltage is analogous to potential energy in electrostatics.

Potential energy is just an energy. A *potential* in mechanics is the potential energy per unit mass. *Voltage*, or *electrostatic potential* is **potential energy per unit charge**.

The Volt (V) is the unit of voltage, and it has units of  $1 \text{ V} = 1 \text{ J/C}$ .

**Group board exercise:** Show that the units of electric field (normally Newtons per Coulomb) are also Volts per meter.

If the potential energy is a function of displacement,  $U = U(\vec{x})$ , it may be called a potential energy *surface*.

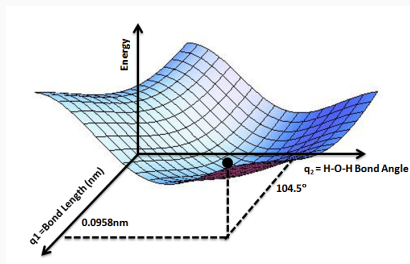
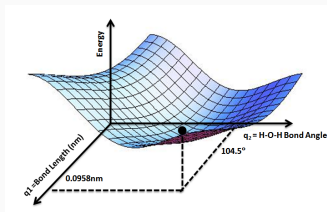


Figure 1: An example of a potential energy surface.

Considering *Newton's Second Law*, however, if  $F = ma$  then  $ma = -\frac{\Delta U}{\Delta x}$ , and

$$a = -\frac{1}{m} \frac{\Delta U}{\Delta x} \quad (5)$$



**Figure 2:** If we divide by the mass we have acceleration.

The derivative, or *gradient*, of  $a$  in Eq. 5 is analogous to the electric field. But the electric field is a *vector field*...

The gradient is like a derivative, but gives you the proper direction wherever you are.

$$-\vec{\nabla}V = \vec{E} = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k} \quad (6)$$



Suppose the voltage due to some charge distribution is  $V(x, y, z) = ax + b$ . What is the field?

- A:  $\vec{E} = a\hat{j}$
- B:  $\vec{E} = a\hat{k}$
- C:  $\vec{E} = b\hat{i}$
- D:  $\vec{E} = -a\hat{i}$

Suppose the voltage due to some charge distribution is  $V(x, y, z) = ax + by$ . What is the field?

- A:  $\vec{E} = -a\hat{j} - b\hat{j}$
- B:  $\vec{E} = a\hat{k} + b\hat{k}$
- C:  $\vec{E} = -b\hat{i} - a\hat{j}$
- D:  $\vec{E} = -a\hat{i} - b\hat{j}$

Suppose a charge distribution is made of two infinite plates of charge with charge per unit area  $\pm\sigma$ . We know that the field is  $\sigma/\epsilon_0 \hat{k}$  between them. **Group board exercise:** Draw the charge distribution, define a coordinate system, and write the function for the voltage.

**Voltage** is like a potential energy surface → *potential energy per unit charge*.

<https://phet.colorado.edu/en/simulation/charges-and-fields>

Using the PhET simulation about charges and fields:

1. Explore the voltage associated with fields generated by charges using the voltage button.
2. Add a single point charge, and use the ruler and voltmeter (potentiometer) to measure voltage versus distance, and plot it.
3. What function describes the relationship between voltage and distance?

<https://phet.colorado.edu/en/simulation/charges-and-fields>

Using the PhET simulation about charges and fields:

1. Note that the units of  $\epsilon_0$  are  $\text{N m}^2 \text{C}^{-2}$ , and the value is  $8.854 \times 10^{-12}$
2. We know from prior equations that the units of voltage are  $\text{J C}^{-1}$
3. Using your measurements, show that the voltage due to a point charge is

$$V = \pm \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (7)$$

(Where the sign depends on the charge, just like E-fields)

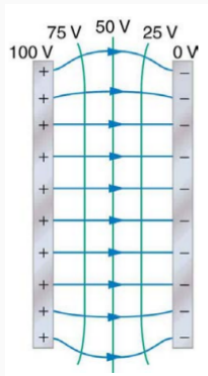
Voltage due to a point charge:

$$V = \pm \frac{1}{4\pi\epsilon_0} \frac{q}{r} \quad (8)$$

Voltage is an example of a **scalar field**, whereas the electric field is an example of a **vector field**. Create two lines of charge, one positive and one negative in the PHeT simulator, and pretend they represent planes of charge coming out of the screen. The field should be constant and uniform like

$$\vec{E} = \frac{\sigma}{\epsilon_0} \hat{k} \quad (9)$$

1. Plot the voltage versus distance between the plates.
2. Calculate the slope in V/m, and the y-intercept in V.
3. Does the electric field depend on the y-intercept? Does the charge distribution?



**Figure 3:** Parallel plates of charge, electric field, and potential. Notice the linear decrease in voltage.

How we define the y-intercept in voltage is analogous to the zero-point freedom of potential energy.



*Two parallel plates, opposite charge:*

$$V = -\frac{\sigma}{\epsilon_0}z + C \quad (10)$$

With the boundary condition that  $V = V_0$  when  $z = 0$ , we have

$$V(z) - V_0 = -\frac{\sigma}{\epsilon_0}z \quad (11)$$

Let  $\Delta V(z) = V(z) - V_0$ , and  $\Delta z = z$ :

$$-\frac{\Delta V}{\Delta z} = \frac{\sigma}{\epsilon_0} = E \quad (12)$$

**Continuing with PHeT:** Make a ring of charge that has a radius of 100 cm, with the center at the origin.

1. Show that the voltage field is radially symmetric.
  - Place *equipotential lines* around the charge distribution and see that they are circular.
2. Show that at distances much larger than the radius, the voltage field looks like that of a point charge.
  - Record the voltage versus distance from the origin.
  - Plot the data.
  - Compare to the plot corresponding to a point charge.

## UNITS OF ENERGY

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The electron-volt: eV. This is the energy gained by an electron accelerated through a voltage of 1 V.

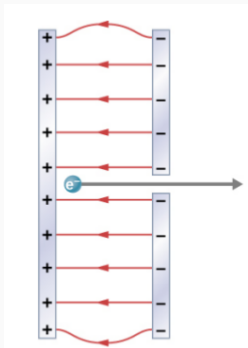
- How many Joules per electron volt?
- What is the mass of a proton in eV? ( $E = mc^2$ , where  $c$  is the speed of light.)

A proton is released into a 40 kV electric potential. What is the final kinetic energy of the proton?

- A: -20 kV
- B: 40 kV
- C: -40 keV
- D: 40 keV

An alpha particle is a helium nucleus with a charge of  $+2q_e$ . Suppose an alpha particle is accelerated and has a final energy of 2 MeV. How many volts were required to accelerated it?

- A: 4 MV
- B: 3 MV
- C: 2 MV
- D: 1 MV



**Figure 4:** An electron accelerated by a parallel plate capacitor.

An electron has a charge  $-q_e$ . Suppose an electron is accelerated through a voltage of 1 V toward another electron at a fixed position. How close does the moving electron get to the stationary one?

- A: about 0.1 nm
- B: about 1 nm
- C: about 10 nm
- D: about 100 nm



A bare helium nucleus has two positive charges and a mass of  $6.64 \times 10^{-27}$  kg. What is the kinetic energy at 2 percent of the speed of light? The speed of light is  $c = 3.0 \times 10^8$  m/s.

- A:  $1.2 \times 10^{-10}$  J
- B:  $1.2 \times 10^{-11}$  J
- C:  $1.2 \times 10^{-12}$  J
- D:  $1.2 \times 10^{-13}$  J

What is the previous energy in eV?

- A: 0.75 MeV
- B: 0.75 keV
- C: 0.75 GeV
- D: 0.75 eV

Remember that this is an alpha particle (helium nucleus) with a +2 charge. How many volts are required to give it an energy of 0.75 MeV?

- A: 0.375 MV
- B: 0.75 MV
- C: 1.5 MV
- D: 0.375 V

## CAPACITANCE

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**Capacitance** is the ability of an object to store charge. Let the voltage difference across an object be  $V$ , storing  $-Q$  on one side and  $+Q$  on another. The *capacitance*  $C$  is given by

$$Q = CV \quad (13)$$

Let the object be two parallel plates, and the charges be  $\pm Q$ .

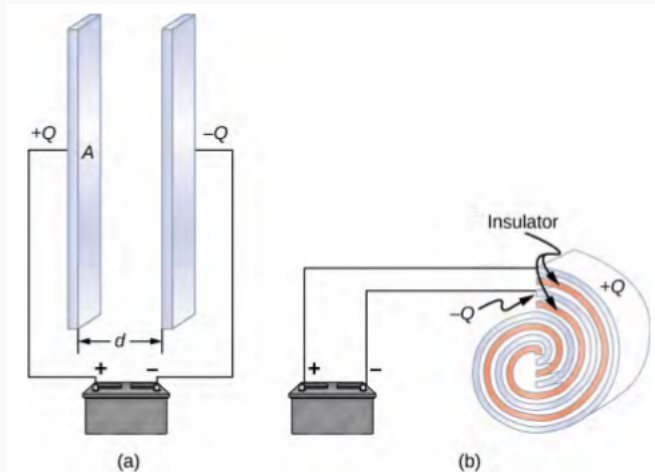


Figure 5: General scheme of a capacitor.

The electric field and voltage between two charged plates separated by a distance  $d$  is  $V = Ed$ . The field is  $E = \sigma/\epsilon_0$ , and  $Q = \sigma/A$ , where  $A$  is the plate area and  $\sigma$  is the surface charged density. Note that

$$C = Q/V = (Q)/(Ed) \quad (14)$$

$$= (Q\epsilon_0)/(\sigma d) = (QA\epsilon_0)/(Qd) \quad (15)$$

$$= \frac{A\epsilon_0}{d} \quad (16)$$

So the capacitance depends on the permittivity of free space, the area, and the distance. In other words, just the geometry of the system. The units of capacitance are **Farads** (F). This is a large unit. Typical capacitors have nF or pF.

**Group board exercise:** The permittivity of free space is  $8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$ . A capacitor has an area of  $1 \text{ mm}^2$ , and  $d = 0.001 \text{ mm}$ . What is the capacitance?

**Group board exercise:** The permittivity of free space is  $8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$ . A capacitor has an area of  $10 \text{ mm}^2$ , and  $d = 0.001 \text{ mm}$ . What is the capacitance?

**Group board exercise:** The permittivity of free space is  $8.85 \times 10^{-12} \text{ N}^{-1} \text{ C}^2 \text{ m}^{-2}$ . A capacitor has an area of  $1 \text{ mm}^2$ , and  $d = 0.01 \text{ mm}$ . What is the capacitance?



## CAPACITORS IN SERIES AND IN PARALLEL

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Observe on board.

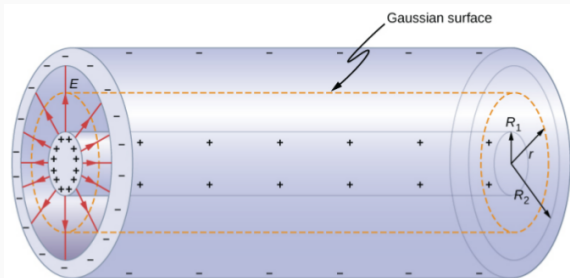
1. Consider two capacitors *in series*, with capacitance  $C_1$  and  $C_2$ . What is the total capacitance?
2. Consider two capacitors *in parallel*, with capacitance  $C_1$  and  $C_2$ . What is the total capacitance?

**Create a simple circuit with capacitors**, and a battery to charge them. How much total charge is stored? Exchange your design with another group, and solve each others' problem.

## THE CYLINDRICAL CAPACITOR AND COAXIAL CABLES

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## THE CYLINDRICAL CAPACITOR AND COAXIAL CABLES



**Figure 6:** A cylindrical capacitor, as a model for a coaxial cable. There is an inner radius and an outer radius, with our Gaussian surface drawn in between the two conductors.

Recall that

$$Q = \Delta VC \quad (17)$$

$$\Delta V = - \int_{R_1}^{R_2} \vec{E} \cdot d\vec{r} \quad (18)$$

- What is the electric field  $\vec{E}$  of a section of this coaxial cable of length  $l$ ? (*Recall from warm-up.*)
- What if we integrate from positive charge (inner radius) to negative charge (outer)? **Observe on board.**

Thus we have the capacitance per unit length:

$$\boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(R_2/R_1)}} \quad (19)$$

What are the units of  $\epsilon_0$ ?

- A: F/m<sup>2</sup>
- B: F/m
- C: F/ln(m)
- D: F

Thus we have the capacitance per unit length:

$$\boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(R_2/R_1)}} \quad (20)$$

Suppose the cap per unit length is 0.1 nF, and we *square*  $R_2/R_1$ .  
What is the new cap per unit length?

- A: 0.1 nF
- B: 0.05 nF
- C: 0.2 nF
- D: 0 nF

Thus we have the capacitance per unit length:

$$\boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln(R_2/R_1)}} \quad (21)$$

Suppose the cap per unit length is 0.1 nF, and we *triple*  $\epsilon_0$  in some way. What is the new cap per unit length?

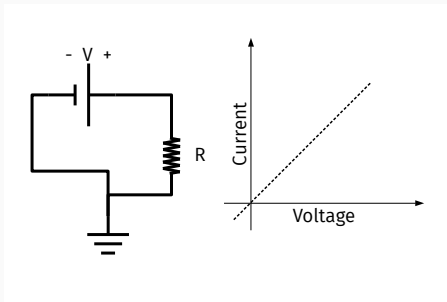
- A: 0.1 nF
- B: 0.2 nF
- C: 0.3 nF
- D: 0.5 nF



(Preview of Unit 3). Suppose we have a system that obeys

$$v(t) = R \frac{dQ}{dt} = i(t)R \quad (22)$$

This is called **Ohm's law**.

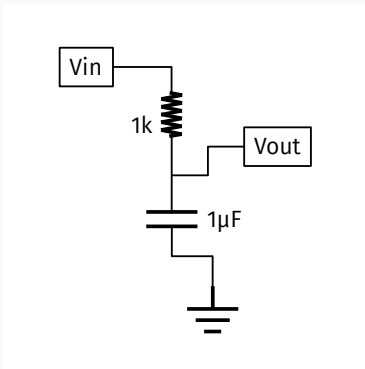


**Figure 7:** A simple circuit with a resistor element, some voltage, and *ground*. This just means that 0V is at the negative terminal.

(Preview of Unit 3). What if we add a capacitor?

$$V_0 = R \frac{dQ}{dt} + \frac{Q}{C} \quad (23)$$

This is called **Ohm's law**.



**Figure 8:** A simple circuit with a resistor and capacitor elements.

(Preview of Unit 3). What if we add a capacitor? We can show that

$$V_{out}(t) = V_0 \exp(-t/\tau) \quad (24)$$

with  $\tau = RC$ . Thus, if we send a signal  $V_{in}$  down a “very long” coaxial cable, with some capacitance per unit length, it will not exit the cable.

For example: <https://www.pasternack.com/images/ProductPDF/LMR-400.pdf>

What is the attenuation per 100 m of this cable at 150 MHz?

## OHM'S LAW, KIRCHHOFF'S RULES AND SIMPLE CIRCUITS

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Read chapters 9 and 10 for Monday. Voltage and capacitance can be combined with another concept, resistance, to build an understanding of circuits. Ohm's law is the relationship between *current* and *voltage*:

$$V = IR = \frac{dQ}{dt}R \quad (25)$$

Current is the *flow of charge*. The unit of resistance is called the Ohm, and the unit of current is called the amp, for Ampère.

A 9 V battery powers a smoke detector, and the current is 0.009 amps. What is the resistance of the circuit powering the smoke detector?

- A: 1000 Ohms
- B: 100 Ohms
- C: 10 Ohms
- D: 1 Ohm

A 3.3 V battery powers a christmas light, and the current is 0.33 amps. What is the resistance of the circuit powering the smoke detector?

- A: 1000 Ohms
- B: 100 Ohms
- C: 10 Ohms
- D: 1 Ohm

## OHM'S LAW, KIRCHHOFF'S RULES AND SIMPLE CIRCUITS

Resistors, capacitors, voltages, and current can be combined conceptually to form **DC Circuits** (direct-current). The following rules govern the interaction of resistors with each other, and capacitors with each other:

- For resistors *in series*:  $R = R_1 + R_2 + R_3 + \dots$
- For resistors *in parallel*:  $R^{-1} = R_1^{-1} + R_2^{-1} + R_3^{-1} + \dots$
- For capacitors *in parallel*:  $C = C_1 + C_2 + C_3 + \dots$
- For capacitors *in series*:  $C^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} + \dots$

**Observe on board:** difference between in series, and in parallel? (Hint, different voltage for in series, same voltage for in parallel).



Kirchhoff's rules:

1. Summing the voltage in a loop must equal zero ( $\vec{E}$ -fields are conservative).
2. Current entering an leaving a node is conserved.

Observe on board.

Two resistors have  $1\text{ k}\Omega$  each (1000 Ohms). What is the effective resistance if they are added *in parallel*?

- A: 1000 Ohms
- B: 500 Ohms
- C: 2000 Ohms
- D: 200 Ohms

Two capacitors have 1 nF each. What is the effective capacitance if they are added *in series*?

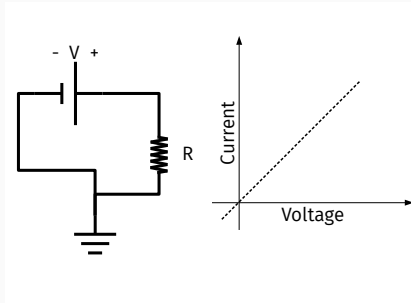
- A: 0.5 nF
- B: 5 nF
- C: 1 nF
- D: 2 nF

Two capacitors have 1 nF each. What is the effective capacitance if they are added *in parallel*?

- A: 0.5 nF
- B: 5 nF
- C: 1 nF
- D: 2 nF

# GRAPHICAL ANALYSIS OF SIMPLE CIRCUITS

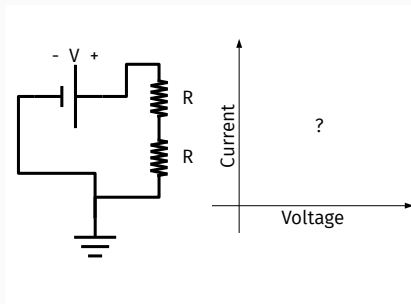
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**Figure 9:** Circuits components are represented graphically by  $iV$  curves.

If the resistance  $R$  is increased, what will happen?

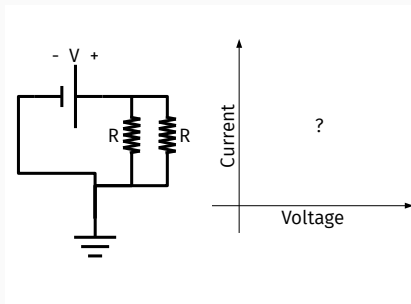
- A: The slope on the graph will increase
- B: The slope on the graph will decrease
- C: The slope will stay the same
- D: Cannot determine what will happen



**Figure 10:** Circuits components are represented graphically by  $iV$  curves.

Should the slope now be greater than, less than, or equal to the that of Fig. 9?

- A: Greater than Fig. 9
- B: Less than Fig. 9
- C: Equal to Fig. 9
- D: Cannot determine.

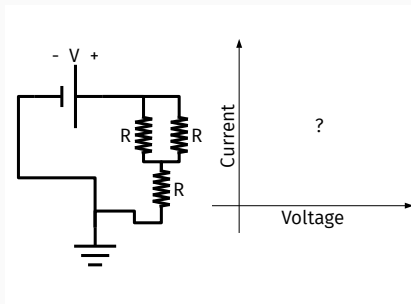


**Figure 11:** Circuits components are represented graphically by iV curves.

Should the slope now be greater than, less than, or equal to the that of Fig. 9?

- A: Greater than Fig. 9
- B: Less than Fig. 9
- C: Equal to Fig. 9
- D: Cannot determine.





**Figure 12:** Circuits components are represented graphically by  $iV$  curves.

Should the slope now be greater than, less than, or equal to the that of Fig. 9?

- A: Greater than Fig. 9
- B: Less than Fig. 9
- C: Equal to Fig. 9
- D: Cannot determine.

Group board exercise: Solve for the current.

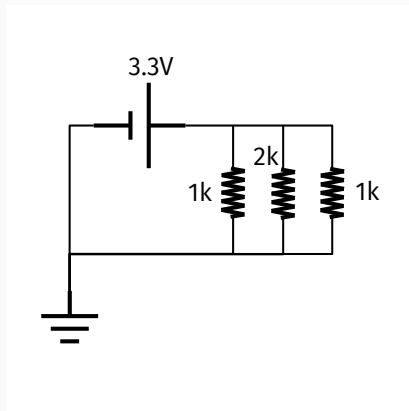


Figure 13: Three resistors in parallel.

Group board exercise: Solve for the current.

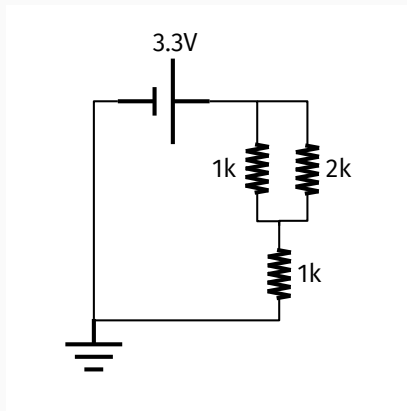


Figure 14: Two resistors in parallel, and in series with a third.

Group board exercise: Solve for the current.

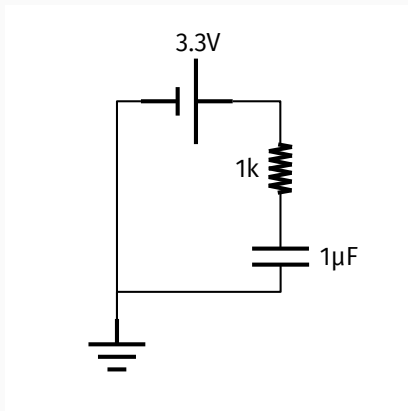


Figure 15: A resistor in series with a capacitor...

Using Kirchhoff's loop rule:

$$V - iR - QC^{-1} = 0 \quad (26)$$

$$Q_0 = CV \quad (27)$$

$$\tau = RC \quad (28)$$

$$\tau \dot{Q} + Q = Q_0 \quad (29)$$

**Group board exercise:** Show that the solution for the charge on the capacitor is

$$Q(t) = Q_0 (1 - \exp(-t/\tau)) \quad (30)$$

Take the derivative of both sides to find the current versus time. Graph the current.

So we see that the current is an exponential function:

$$i(t) = i_0 e^{-t/\tau} \quad (31)$$

( $i_0 = V/R$ , the initial current). What is the current in a circuit with a 1k resistor and a 1 nF capacitor after 3  $\mu\text{s}$ , if the voltage is 5 V?

- A: 0.25 mA
- B: 0.5 mA
- C: 1.0 mA
- D: 2.0 mA

So we see that the current is an exponential function:

$$i(t) = i_0 e^{-t/\tau} \quad (32)$$

( $i_0 = V/R$ , the initial current). Thinking of the same circuit:  
when is the current 0.0 mA?

- A: 3 microseconds
- B: 30 microseconds
- C: 300 microseconds
- D: never

CURRENT

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## Notions of current:

- $I = \frac{\Delta Q}{\Delta t}$  - The derivative of charge
- The *movement* of electrons
- The *flow* of charge
- Number of Coulombs per second (1 Amp = C/s)

There is an interesting problem with the notion of current as movement of charges.

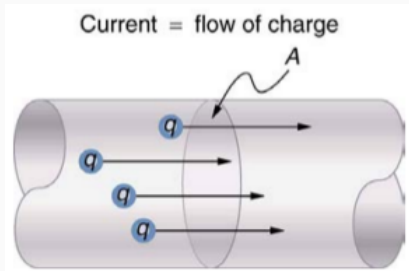
Speed of typical electronic signals:  $\approx 10^8$  m/s

Typical speed of actual charges passing through a conductor under voltage:  $\approx 10^{-4}$  m/s

Since there is a 12 order of magnitude range, it's probably a good idea to ponder...

# CURRENT

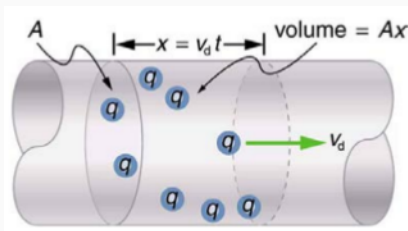
Are the electrons colliding/interacting to form electrical signals? Or just moving all together?



**Figure 16:** The *drift velocity* is the average velocity of an electron, and current is derived from this average velocity.

<https://youtu.be/8dgyPRA86K0>

So we see how electrical signals can move near the speed of light, but we measure the movements of electrons in circuits to be slow. Can we make a calculation to understand the speed of the electrons?



**Figure 17:** Consider the volume  $V$  of conductor with cross-sectional area  $A$  and length  $\Delta x$ , having  $n$  free electrons per unit volume.

An **amp** is one *Coulomb* per second. The definition of current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{qnA\Delta x}{\Delta t} = qnAv_d \quad (33)$$

Solving for drift velocity:

$$v_d = \frac{I}{qnA} \quad (34)$$

Suppose our conductor is a wire with radius  $r$  and  $A = \pi r^2$ .

Substituting,

$$v_d = \frac{I}{\pi qnr^2} \quad (35)$$

Remember that  $q = 1.6 \times 10^{-19}$  C, and  $n$  is the number of free electrons *per atom per unit volume*. How do we get this number?

**Number density:** The total number of objects in a system is equal to the *number density* times the volume of the system.

$$N = nV \quad (36)$$

- N: Total number
- n: *number density*
- V: Volume

**Example:** Number of Stars in the Milky Way. How many stars are in our galaxy? Assume the galaxy is a disk of height  $h$  and radius  $r$ . We observe  $n$  stars per unit volume.

- $r = 50 \times 10^3 \text{ light-years}$
- $h = 2 \times 10^3 \text{ light-years}$
- $n = 10^{-2} \text{ light-year}^{-3}$

1. Compute the volume in  $\text{light-years}^3$
2. Multiply the volume by the number density to obtain the total number.
3. Compare the result with others' results.

How many **conduction** electrons are there in a cube of copper that is 1 micron ( $1\ \mu\text{m} = 10^{-6}\ \text{m}$ ) on a side?

- Copper has a density of 8.8 grams per cubic centimeter.
- Copper has an atomic weight of 63.54 grams per mole. (*Do you remember what a mole is?*).
- There are  $N_A = 6.02 \times 10^{23}$  atoms per mole.
- Only one electron per atom of copper is a conduction electron.

- 
1. Divide the density by the atomic weight. What are the units?
  2. Multiply by  $N_A$  (Avogadro's number). What are the units?
  3. Convert the units from  $\text{cm}^{-3}$  to  $\mu\text{m}^{-3}$ .

**Number density:** Let's examine copper, a common wire material with one free electron per atom. Copper has a density of  $8800 \text{ kg/m}^3$ , and  $0.06354 \text{ kg/mol}$ . There are  $6.02 \times 10^{23}$  atoms/mol. How many free electrons per  $\text{m}^3$  of copper? (Remember that there is only one conduction electron per copper atom).

- A:  $10^{26}$  free electrons per  $\text{m}^3$
- B:  $10^{27}$  free electrons per  $\text{m}^3$
- C:  $10^{28}$  free electrons per  $\text{m}^3$
- D:  $10^{29}$  free electrons per  $\text{m}^3$



Consider a copper wire with radius  $r = 2.053 \text{ mm}$  that is carrying  $20.0 \text{ A}$  of current. Using  $q = 1.6 \times 10^{19}$ , and  $n = 8.34 \times 10^{28} \text{ electrons/m}^3$ , and  $v_d = I/(\pi q n r^2)$ , compute the drift velocity of charge in the wire. *This is a common situation in household wiring.*

- A:  $10^{-1} \text{ m/s}$
- B:  $10^{-2} \text{ m/s}$
- C:  $10^{-3} \text{ m/s}$
- D:  $10^{-4} \text{ m/s}$

**Drift speed vs. signal speed.** Given that the electrons move at 1 mm/s, how is it that electric signals move at  $10^8$  m/s?

Electrical signals are waves of charge:

<https://phet.colorado.edu/en/simulation/legacy/wave-on-a-string>

**Resistivity**  $\rho$  and **conductivity**  $\sigma$  are intrinsic properties of materials, and they are reciprocals of each other:

$$\rho = 1/\sigma \quad (37)$$

Let  $\vec{J}$  be the *current density*. The most general form of Ohm's law is

$$\boxed{\vec{J} = \sigma \vec{E}} \quad (38)$$

Let's assume that current is flowing down a conductor of length  $L$  and cross-sectional area  $\vec{A}$ , parallel to  $\vec{E}$ . Integrate both sides:

$$I = \sigma \int \vec{E} \cdot d\vec{A} = EA = \frac{V}{L}A \quad (39)$$

$$V = \frac{L\rho}{A}I = IR \quad (40)$$

Thus, the *resistance* of an object is

$$R = \frac{L\rho}{A} \quad (41)$$

The resistivity has units of Ohm meters. Which of the following is true of a conductor?

- A: Lengthening it increases the resistance, and widening it increases the resistance.
- B: Lengthening it decreases the resistance, and widening it increases the resistance.
- C: Lengthening it increases the resistance, and widening it decreases the resistance.
- D: Lengthening it decreases the resistance, and widening it decreases the resistance.

Thus, the *resistance* of an object is

$$R = \frac{L\rho}{A} \quad (42)$$

Suppose the length of a conductor is doubled, and the area is quadrupled. By how much does the resistance change?

- A: It decreases by a factor of two.
- B: It decreases by a factor of four.
- C: It increases by a factor of two.
- D: It increases by a factor of four.

What is the resistance of a 1 meter-long wire with area  $3 \text{ mm}^2$  that made from a metal that has  $\rho = 10^{-7} \Omega \text{ m}$ ?

- A: 0.01 Ohms
- B: 0.1 Ohms
- C: 1.0 Ohms
- D: 10.0 Ohms

Material	Conductivity, $\sigma$ ( $\Omega \cdot \text{m}$ ) <sup>-1</sup>	Resistivity, $\rho$ ( $\Omega \cdot \text{m}$ )	Temperature Coefficient, $\alpha$ ( $^{\circ}\text{C}$ ) <sup>-1</sup>
<i>Conductors</i>			
Silver	$6.29 \times 10^7$	$1.59 \times 10^{-8}$	0.0038
Copper	$5.95 \times 10^7$	$1.68 \times 10^{-8}$	0.0039
Gold	$4.10 \times 10^7$	$2.44 \times 10^{-8}$	0.0034
Aluminum	$3.77 \times 10^7$	$2.65 \times 10^{-8}$	0.0039
Tungsten	$1.79 \times 10^7$	$5.60 \times 10^{-8}$	0.0045
Iron	$1.03 \times 10^7$	$9.71 \times 10^{-8}$	0.0065
Platinum	$0.94 \times 10^7$	$10.60 \times 10^{-8}$	0.0039
Steel	$0.50 \times 10^7$	$20.00 \times 10^{-8}$	
Lead	$0.45 \times 10^7$	$22.00 \times 10^{-8}$	
Manganin (Cu, Mn, Ni alloy)	$0.21 \times 10^7$	$48.20 \times 10^{-8}$	0.000002
Constantan (Cu, Ni alloy)	$0.20 \times 10^7$	$49.00 \times 10^{-8}$	0.00003
Mercury	$0.10 \times 10^7$	$98.00 \times 10^{-8}$	0.0009
Nichrome (Ni, Fe, Cr alloy)	$0.10 \times 10^7$	$100.00 \times 10^{-8}$	0.0004

Figure 18: Resistivities of various metals.

Like the expansion of metals with increasing temperature, *resistivity has a temperature dependence*. Let  $\Delta T = T - T_0$ , with  $T_0 = 20.0$  C.

$$\rho = \rho_0 \exp(\alpha \Delta T) \quad (43)$$

Using a Taylor series around  $\Delta T = 0$ , we find

$$\rho(T) \approx \rho_0 (1 + \alpha \Delta T) \quad (44)$$



Material	Conductivity, $\sigma$ ( $\Omega \cdot \text{m}$ ) <sup>-1</sup>	Resistivity, $\rho$ ( $\Omega \cdot \text{m}$ )	Temperature Coefficient, $\alpha$ ( $^{\circ}\text{C}$ ) <sup>-1</sup>
<i>Conductors</i>			
Silver	$6.29 \times 10^7$	$1.59 \times 10^{-8}$	0.0038
Copper	$5.95 \times 10^7$	$1.68 \times 10^{-8}$	0.0039
Gold	$4.10 \times 10^7$	$2.44 \times 10^{-8}$	0.0034
Aluminum	$3.77 \times 10^7$	$2.65 \times 10^{-8}$	0.0039
Tungsten	$1.79 \times 10^7$	$5.60 \times 10^{-8}$	0.0045
Iron	$1.03 \times 10^7$	$9.71 \times 10^{-8}$	0.0065
Platinum	$0.94 \times 10^7$	$10.60 \times 10^{-8}$	0.0039
Steel	$0.50 \times 10^7$	$20.00 \times 10^{-8}$	
Lead	$0.45 \times 10^7$	$22.00 \times 10^{-8}$	
Manganin (Cu, Mn, Ni alloy)	$0.21 \times 10^7$	$48.20 \times 10^{-8}$	0.000002
Constantan (Cu, Ni alloy)	$0.20 \times 10^7$	$49.00 \times 10^{-8}$	0.00003
Mercury	$0.10 \times 10^7$	$98.00 \times 10^{-8}$	0.0009
Nichrome (Ni, Fe, Cr alloy)	$0.10 \times 10^7$	$100.00 \times 10^{-8}$	0.0004

**Figure 19:** Resistivities of various metals. Notice the temperature coefficients are small, so the Taylor series is justified.

Consider a copper wire that has a resistance of 0.1 Ohms at 20 degrees C. What will be the resistance of the wire at 40 degrees C, if the temperature coefficient of copper resistivity is 0.004?

- A: 0.104 Ohms
- B: 0.108 Ohms
- C: 0.112 Ohms
- D: 0.116 Ohms

Consider a copper wire that has a resistance of 5 Ohms at 20 degrees C. What will be the resistance of the wire at 50 degrees C, if the temperature coefficient of copper resistivity is 0.004?

- A: 4.8 Ohms
- B: 5.0 Ohms
- C: 5.6 Ohms
- D: 6.0 Ohms

Given that we found a 12% increase in the resistance of the wire in the prior example, what would happen to the voltage delivered to a device at the other end of that wire?

- A: It would be unaffected.
- B: It would decrease by 12 percent.
- C: It would increase by 12 percent.
- D: It would decrease by 6 percent.

*So you can see why we don't do DC power transmission over long distances.*

## CONCLUSION

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### Reading: Chapters 7, 9, and 10

1. Voltage and Capacitance
2. Ohm's Law
3. DC circuits

## ANSWERS

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## ANSWERS

- $\vec{E} = \frac{kq}{r^2} \hat{r}$
- $\vec{E} = \frac{2k\lambda}{r}$
- $\vec{E} = -a\hat{i}$
- $\vec{E} = -a\hat{i} - b\hat{j}$
- 40 keV
- 1 MV
- about 1 nm
- $1.2 \times 10^{-13}$  J
- 0.75 MeV
- 0.375 MV
- F/m
- 0.05 nF
- 0.3 nF