Midterm 3

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April 27, 2020

1 Memory Bank

1. $\vec{E} = -\frac{\Delta V}{\Delta x}$ ... E-field is the slope or change in voltage with respect to distance
2. $V(x) = -Ex + V_0$ ... Voltage is linear between two charge planes
3. $Q = CV$ ... Definition of capacitance
4. $C = \frac{\epsilon_0 A}{d}$ ... Capacitance of a parallel plate capacitor
5. $C_{tot}^{-1} = C_1^{-1} + C_2^{-1}$ ... Adding two capacitors *in series.*
6. $C_{tot} = C_1 + C_2$ ... Adding two capacitors *in parallel.*
7. $i(t) = dQ/dt$ ... Definition of current.
8. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
9. $R_{tot}^{-1} = R_1^{-1} + R_2^{-1}$ ... Adding two capacitors *in parallel.*
10. $R_{tot} = R_1 + R_2$ ... Adding two capacitors *in series.*
11. $\Delta V = IR_{\text{tot}}$, $\vec{J} = \sigma \vec{E}$ ... Versions of Ohm's Law. ($\vec{J}$ is the current density with units of Amps per meter-squared).
12. $P = IV$ ... Relationship between power, current, and voltage.
13. $V_{\text{C}}(t) = \epsilon_1 (1 - \exp(-t/\tau))$ ... voltage across the capacitor in an RC series circuit. The time constant $\tau = RC$.
14. $i(t) = \frac{\epsilon_1}{R} \exp(-t/\tau)$ ... Current in an RC series circuit.
15. $i_{\text{in}} = i_{\text{out}}$ ... Kirchhoff's junction rule.
16. $\epsilon_1 + \epsilon_2 + \epsilon_3 + ... = 0$ ... Kirchhoff's loop rule.
17. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
18. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.
19. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$ ... Ampère's Law.
20. $\epsilon = -N d\phi/dt$ ... Faraday's Law.
21. $\phi = \vec{B} \cdot \vec{A}$ ... Definition of magnetic flux.
22. $-N d\phi/dt = \oint \vec{E} \cdot d\vec{l}$ ... Induced E-field due to changing magnetic flux.
23. Faraday's Law using **Inductance**, M: $emf = -M\frac{dI}{dt}$.
24. Typically, we refer to *mutual inductance* between two objects as M, and *self inductance* as L. Self-inductance: $\Delta V = -L(dI/dt)$.
25. Units of inductance: V s A^{-1}, which is called a Henry, or H.
26. $B = \mu_0 n I$ ... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

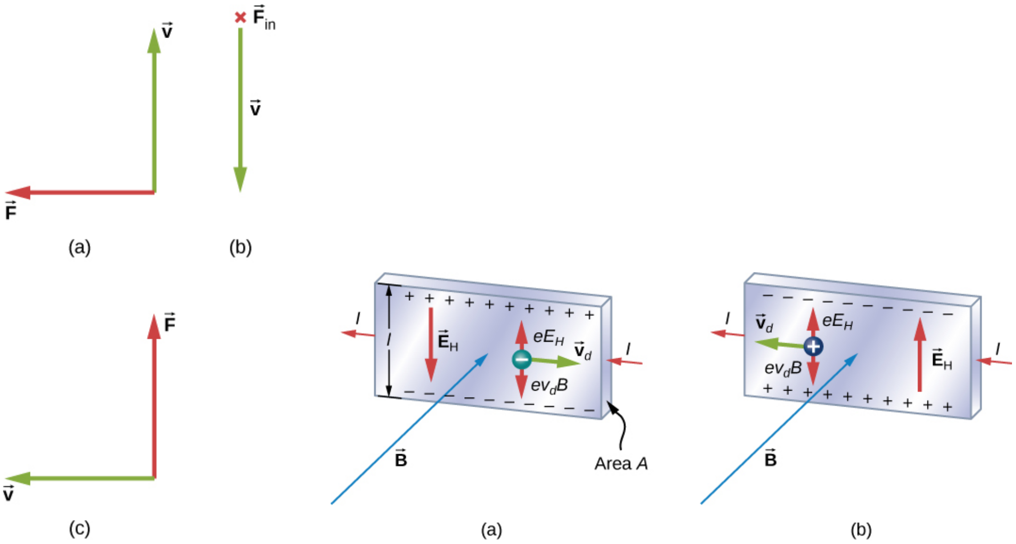

Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

1. Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.

   - a: **Into the page**
   - b: **To the left**
   - c: **Out of the page**

2. Consider Fig. 1 (right). **The Hall Effect**. An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that

$$\Delta V = \frac{B \Delta x I}{n q_e A} \tag{1}$$

   where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28}$ m^{-3}, $A = 1$ mm^2, and q_e is the charge of an electron.

   (a) Setting $F = qE$ equal to $F = qvB$ gives $qE = qvB \rightarrow v = E/B$. (b) Using the formula for the drift velocity in the memory bank, along with $\Delta V/\Delta x = E$, gives the correct result. (c) We get about 83 μV

3. A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^4 A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

   The magnitude of the magnetic moment is $\mu = NIA$, and $N = 1$ loop for the atom. The area is πr^2, and we're given the current and radius. The torque is then $\tau = \mu B = 3.5 \times 10^{-26}$ N m.

3 Chapter 12: Sources of Magnetic Fields

1. (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

   (a) $B = \mu_0 n I = 4\pi \times 10^{-7} \times 500 \times 0.3 = 0.19$ mT. (b) Thos is a scaling problem: $B = 0.94$ T.

2. Consider Fig. 2. **Mass spectrometer**. Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left[1]. (b) Recall that the centripetal force on a particle of mass m is mv^2/r. Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \tag{2}$$

[1] Molecules that do not have this velocity will hit the sides of this portion of the instrument.

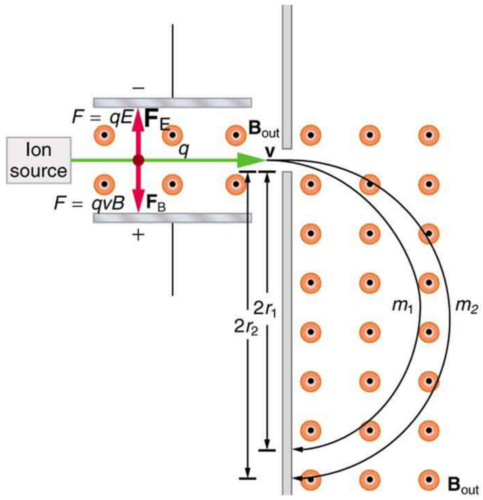

Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-sepctrometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r?

(a) Recall the argument from the Hall Effect problem that if $qE = qvB$, $v = E/B$. (b)

$$\frac{mv^2}{r} = qvB \tag{3}$$
$$\frac{mv}{r} = qB \tag{4}$$
$$\frac{r}{mv} = \frac{1}{qB} \tag{5}$$
$$r = \frac{mv}{qB} \tag{6}$$
$$r = \frac{mE}{qB^2} \tag{7}$$

(c) Insert the correct numbers into the formula:

$$r = \frac{16 * 1.66 \times 10^{-27} * 10}{1.6 \times 10^{-19} 10^{-4}} \approx 1.66\ cm \tag{8}$$

4 Chapter 13: Electromagnetic Induction

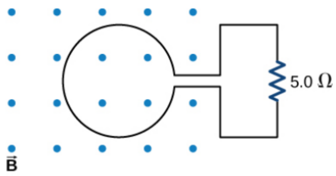

Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and is tuned to follow the form

$$B(t) = B_0 \left(\frac{1}{2} + \frac{2}{\pi} \sin(2\pi f t) + \frac{2}{3\pi} \sin(6\pi f t) + \frac{2}{5\pi} \sin(10\pi f t) \right) \tag{9}$$

The loop has a radius r. (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, and $f = 10^3$ Hz, what is the induced emf at $t = 0$? (c) What is the current through the resistor at $t = 1$ ms?

(a)

$$\epsilon(t) = -4B_0 \pi r^2 f \left(\cos(2\pi f t) + \cos(6\pi f t) + \cos(10\pi f t)\right) \tag{10}$$

(b) At time $t = 0$,

$$\epsilon(0) = -12B_0\pi r^2 f = -37.7\ V \tag{11}$$

(c) At time $t = 1$ *ms,* $ft = 1$, *so the cosines are all 1, as in part (b).*

$$\epsilon(t) = -12B_0\pi r^2 f = -37.7\ V \tag{12}$$

Using Ohm's law, we have $i = -7.54\ A$.

5 Chapter 14: Inductance

1. What is (a) the rate at which the current though a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?

   0.3 Amps per second, using Faraday's Law with inductance.

2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?

   $|\epsilon| = LdI/dt$, *so* $dt = LdI/|\epsilon| = 0.4\ \mu s$