

Midterm 3

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1 Memory Bank

1. $\vec{E} = -\frac{\Delta V}{\Delta x}$... E-field is the slope or change in voltage with respect to distance
2. $V(x) = -Ex + V_0$... Voltage is linear between two charge planes
3. $Q = CV$... Definition of capacitance
4. $C = \frac{\epsilon_0 A}{d}$... Capacitance of a parallel plate capacitor
5. $C_{tot}^{-1} = C_1^{-1} + C_2^{-1}$... Adding two capacitors *in series*.
6. $C_{tot} = C_1 + C_2$... Adding two capacitors *in parallel*.
7. $i(t) = dQ/dt$... Definition of current.
8. $v_d = i/(nqA)$... Charge drift velocity in a current i in a conductor with number density n and area A .
9. $R_{tot}^{-1} = R_1^{-1} + R_2^{-1}$... Adding two capacitors *in parallel*.
10. $R_{tot} = R_1 + R_2$... Adding two capacitors *in series*.
11. $\Delta V = IR_{tot}$, $\vec{J} = \sigma \vec{E}$... Versions of Ohm's Law. (\vec{J} is the current density with units of Amps per meter-squared).
12. $P = IV$... Relationship between power, current, and voltage.
13. $V_C(t) = \epsilon_1 (1 - \exp(-t/\tau))$... voltage across the capacitor in an RC series circuit. The time constant $\tau = RC$.
14. $i(t) = \frac{\epsilon_1}{R} \exp(-t/\tau)$... Current in an RC series circuit.
15. $i_{in} = i_{out}$... Kirchhoff's junction rule.
16. $\epsilon_1 + \epsilon_2 + \epsilon_3 + \dots = 0$... Kirchhoff's loop rule.
17. $\vec{F} = q\vec{v} \times \vec{B}$... The Lorentz force on a charge q with velocity \vec{v} in a magnetic field \vec{B} .
18. $\vec{F} = I\vec{L} \times \vec{B}$... The Lorentz force on a conductor of length \vec{L} carrying a current I in a magnetic field \vec{B} .
19. $\int \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$... Ampère's Law.
20. $\epsilon = -Nd\phi/dt$... Faraday's Law.
21. $\phi = \vec{B} \cdot \vec{A}$... Definition of magnetic flux.
22. $-Nd\phi/dt = \oint \vec{E} \cdot d\vec{l}$... Induced E-field due to changing magnetic flux.
23. Faraday's Law using **Inductance**, M : $emf = -M \frac{dI}{dt}$.
24. Typically, we refer to *mutual inductance* between two objects as M , and *self inductance* as L . Self-inductance: $\Delta V = -L(dI/dt)$.
25. Units of inductance: $V \text{ s } A^{-1}$, which is called a Henry, or H.
26. $B = \mu_0 nI$... The B-field of a solenoid, $n = N/L$ is the turn density, and I is the current.

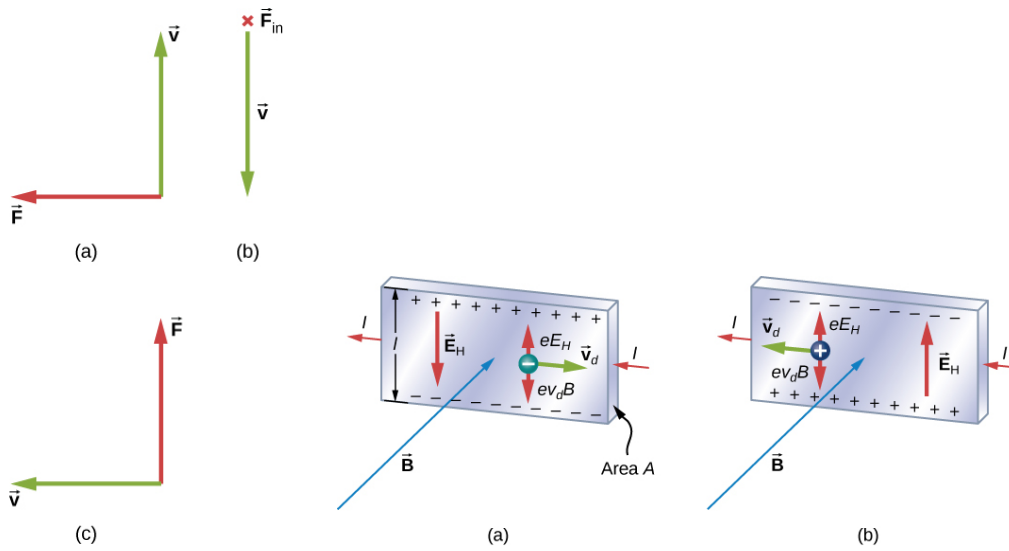


Figure 1: (Left) A current I experiences a force F in a B-field.

2 Chapter 11: Magnetic Forces and Fields

- Consider Fig. 1 (left). In each of the three cases, determine the direction of the B-field given that F is the Lorentz force.
 - a:
 - b:
 - c:
- Consider Fig. 1 (right). **The Hall Effect.** An E-field exists in the vertical direction and a B-field is perpendicular to the direction of charge velocity. (a) Show that if the E-field force on a charge balances the Lorentz force on a charge, that $v = E/B$. (b) If the E-field is constant, $E = \Delta V/\Delta x$. Show that

$$\Delta V = \frac{B\Delta x I}{nq_e A} \quad (1)$$

where n is the charge carrier density, q_e is the electron charge, A is the cross-sectional area of the conductor, and I is the current. Plug in $B = 1.33$ T, $\Delta x = 2$ cm, $I = 10$ A, $n = 2 \times 10^{28}$ m⁻³, $A = 1$ mm², and q_e is the charge of an electron.

- A proton has a magnetic field due to its spin. The field is similar to that created by a circular current loop 0.65×10^{-15} m in radius with a current of 1.05×10^4 A. Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

3 Chapter 12: Sources of Magnetic Fields

- (a) What is the B-field inside a solenoid with 500 turns per meter, carrying a current of 0.3 A? (b) Suppose we insert a piece of metal inside the solenoid, boosting μ_0 by a factor of 5000. What is the new B-field?

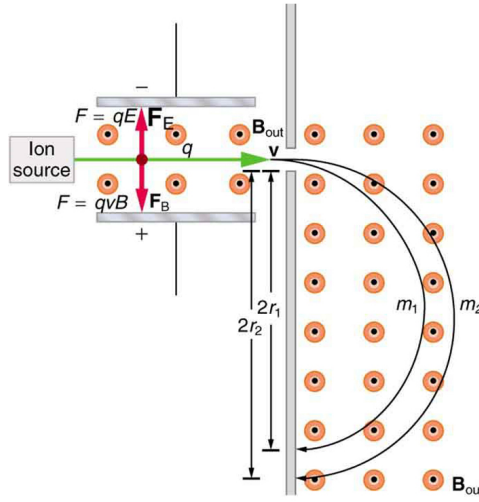


Figure 2: A basic diagram of a *toroid*, which is a solenoid wrapped into a circular tube.

2. Consider Fig. 2. **Mass spectrometer.** Suppose that the velocity of the charged particles moving to the right is $v = E/B$. (a) Show that if $v = E/B$, $F_{net} = 0$ in the region in the top left¹. (b) Recall that the centripetal force on a particle of mass m is mv^2/r . Set this equal to the magnitude of the Lorentz force to prove that

$$r = \frac{mE}{qB^2} \quad (2)$$

The mass of an oxygen nucleus is 16 times that of a proton (mass of proton: 1.67×10^{-27} kg). Suppose oxygen ions with the charge of 1 proton are sent through the mass-septometer. The E-field is 10 V/m, and the B-field is 0.01 T. What is the distance r ?

4 Chapter 13: Electromagnetic Induction

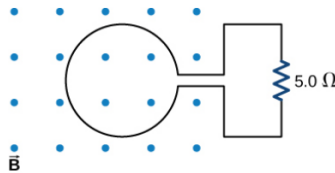


Figure 3: A voltage is induced on a loop by a changing B-field.

1. The magnetic field in Fig. 3 flows out of the page through a single ($N = 1$) loop, and is tuned to follow the form

$$B(t) = B_0 \left(\frac{1}{2} + \frac{2}{\pi} \sin(2\pi ft) + \frac{2}{3\pi} \sin(6\pi ft) + \frac{2}{5\pi} \sin(10\pi ft) \right) \quad (3)$$

The loop has a radius r . (a) In terms of the given variables, what is the induced voltage in the circuit? (b) If $B_0 = 0.1$ T, $r = 0.1$ m, and $f = 10^3$ Hz, what is the induced emf at $t = 0$? (c) What is the current through the resistor at $t = 1$ ms?

¹Molecules that do not have this velocity will hit the sides of this portion of the instrument.

5 Chapter 14: Inductance

1. What is (a) the rate at which the current through a 0.50-H coil is changing if an emf of 0.150 V is induced across the coil?
2. When a camera uses a flash, a fully charged capacitor discharges through an inductor. In what time must the 0.100-A current through a 2.00-mH inductor be switched on or off to induce a 500-V emf?