Unit 7 Lab Activity Conservation of Energy

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1 Introduction

Imagine a weight hanging on a spring, subject to two forces: the force of gravity and the spring force (see Fig. 1).

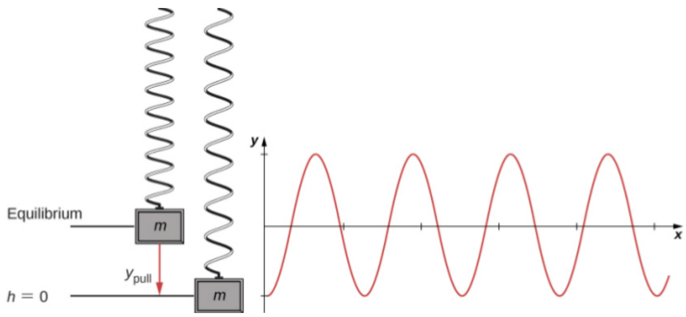

Figure 1: A weight oscillates on a spring.

At equilibrium, the spring is exactly the length required to balance gravity with the spring force. The spring is stretched by an amount Δy that depends on the k-value. The k-value can be measured by hanging a known weight on the spring and measuring Δy:

$$k(\Delta y) = mg \tag{1}$$

$$k = \frac{mg}{\Delta y} \tag{2}$$

The net force is

$$F_{\mathrm{net}} = k y_{\mathrm{pull}} - mg \tag{3}$$

Substituting for k:

$$F_{\mathrm{net}} = \left(\frac{mg}{\Delta y}\right) y_{\mathrm{pull}} - mg \tag{4}$$

Because $|\Delta y| < |y_{\mathrm{pull}}|$, the net force is upwards. Using Newton's Second Law, we find that

$$a = g\left(\frac{y_{\mathrm{pull}}}{\Delta y} - 1\right) \tag{5}$$

The acceleration would have the same value but be oriented downward if the spring had been pushed upward by a distance y_{pull} from equilibrium, by symmetry. In general, the acceleration will always point towards the equilibrium position, and have a value

$$a = g\left(\frac{y}{\Delta y} - 1\right) \tag{6}$$

2 Conservation of Energy

Equation 6 implies that if the mass is lowered by y_{pull}, when it returns to equilibrium, it will have been accelerated to a velocity v, and thus have a kinetic energy of $\frac{1}{2}mv^2$, despite having a net force of zero. The acceleration, however, depends on the position y above the zero-point, and is therefore not constant. Thus, constant acceleration equations may not be used to predict the velocity. Energy conservation, however, may be used to predict it.

Let the initial potential and kinetic energy of a system be U_i and K_i respectively. Let U_f and K_f represent the final potential and kinetic energies. Conservation of energy states that

$$U_{\text{i}} + K_{\text{i}} = U_{\text{f}} + K_{\text{f}} \tag{7}$$

Assume that the initial state is when the spring is pulled down by y_{pull}, and the final state is equilibrium. The initial energies are

$$U_{\text{i}} = mg(0) + \frac{1}{2}ky_{\text{pull}}^2 \tag{8}$$

$$KE_{\text{i}} = 0 \tag{9}$$

The final energies are

$$U_{\text{f}} = mg(y_{\text{pull}}) + \frac{1}{2}k(0)^2 \tag{10}$$

$$KE_{\text{f}} = \frac{1}{2}mv^2 \tag{11}$$

1. Apply energy conservation to show that

$$v = \left(\frac{k}{m}y_{\text{pull}}^2 - 2gy_{\text{pull}}\right)^{1/2} \tag{12}$$

2. Use the LabPro system with motion detector, along with the hanging mass and spring to measure the maximum velocity of the mass. In a prior lab experiment, we measured the k value. Insert this value of k into Eq. 12, along with g and the value used for y_{pull}. Does the measured velocity agree with the predicted velocity? Why or why not?

3. Why does the mass rise to a height of $2y_{\text{pull}}$? Can you justify this based on energy conservation? Explain in your own words.