Warm-Up: Unit 5 Special Topic: Kepler's Laws and Orbits

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1 Memory Bank

- $T^2 \propto r^3$ ... Kepler's Third Law
- Given two planets, we can use this like:

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3 \tag{1}$$

- Let two masses be m_1 and m_2, separated by a distance r. Newton's Universal Law of Gravity says that the force of gravity between them is:

$$F_G = G\frac{m_1 m_2}{r^2}\hat{r} \tag{2}$$

- $F_C = mr\omega^2$ ... Centripetal force.

2 Newton's Law of Gravity

We will now derive Kepler's 3rd Law, which relates orbital period of a planet to the orbital radius. (a) Set centripetal force equal to the gravitational force from the memory bank. Substitute $\omega = (2\pi)/T$, where T is the orbital period. Solve for T in terms of r. (b) Consider this result for planet 1 with r_1 and T_1, and planet 2 with r_2 and T_2. Divide version 1 of the equation by version 2 of the equation to find Eq. 1 from the memory bank.

3 Kepler's Laws

1. Suppose we define a unit called an "Astronomical Unit" that is equal to 1.496×10^8 km. This is the distance between the Earth and the Sun. So we can say that the Earth is 1 AU from the Sun. It turns out that Venus is 0.72 AU from the Sun (it's closer). The orbit of the Earth is 1 year. Let $T_1 = 1$ year, $r_1 = 1.0$ AU for the Earth, and $r_2 = 0.72$ AU for Venus. Use Eq. 1. to find the orbital period of Venus, T_2.

2. The orbital period of Jupiter is observed to be 11.8 years. How far in AU is Jupiter from the Sun? (*Hint: it's the same procedure as the prior problem using Earth's numbers, but solving for* T_2).