CALCULUS-BASED PHYSICS-2: ELECTRICITY, MAGNETISM, AND THERMODYNAMICS (PHYS180-02): UNIT 2

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UNIT 1 REVIEW

Reading: Chapters 1 and 2

1. Temperature, Heat, and the 0th Law of Thermodynamics
2. Heat flow and transfer mechanisms
3. Kinetic Theory of Gases

James Prescott Joule first published in December 1840, an abstract in the Proceedings of the Royal Society, suggesting that heat could be generated by an electrical current. Joule immersed a length of wire in a fixed mass of water and measured the temperature rise due to a known current flowing through the wire for a 30 minute period. By varying the current and the length of the wire he deduced that the heat produced was proportional to the square of the current multiplied by the electrical resistance of the immersed wire.

Wikipedia:
`https://en.wikipedia.org/wiki/Joule_heating`.

Suppose the wire of J. Joule is carrying 10 W of power. If the mass of the water in the container is 1 kg, how long will it take to raise the temperature of the water by 5 degrees? (The heat capacity of water is 4186 J/kg/°C).

- A: 10 seconds
- B: 80 seconds
- C: 420 seconds
- D: 3600 seconds

Suppose the the water begins at 95 °C, and ends at 100 °C. How long will it take to boil away all of the water? (The latent heat of vaporization of water is 2256 kJ/kg).

- A: About an hour
- B: About 6 hours
- C: About 60 hours
- D: Several days

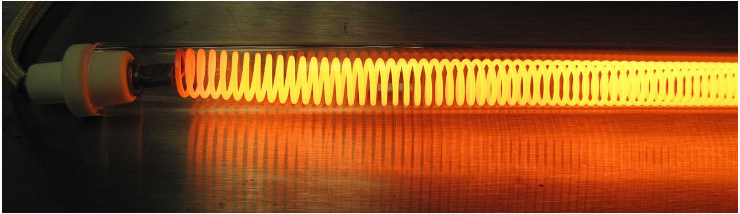

Figure 1: An example of Joule heating in a toaster coupling.

SUMMARY

Reading: Chapters 3 and 4

1. The First Law of Thermodynamics
2. The Second Law of Thermodynamics

JITT - READING QUIZ RESULTS

THE FIRST LAW OF THERMODYNAMICS

Equation of state: The equation of state of a system, in general, looks like this:

$$f(p, V, T) = 0 \tag{1}$$

- V is an **extensive** variable.
- p and V are **intensive** variables.
- Holding *intensive* variables constant, doubling the mass of a system doubles the volume. That is the relationship between intensive and extensive variables.

The equation of state of an ideal gas looks like this:

$$pV - nRT = 0 \tag{2}$$

Which of the following is correct?

- A: The extensive variables are p, V, and R
- B: The only extensive variable is n, and p, V, and T are the intensive ones
- C: The only extensive variable is n, and p is the only intensive one
- D: There are no extensive variables because none of them is mass

Recall that the definition of work done on a system is:

$$W = \int \vec{F} \cdot d\vec{x} \tag{3}$$

- $\vec{F}$ is the net force acting on the system
- $d\vec{x}$ is the incremental displacement

Example: suppose the force is gravity, near the surface of the Earth $\vec{F} = -mg\hat{y}$, and an object at height h is released.

$$W = \int \vec{F} \cdot d\vec{x} = -mg\hat{y} \cdot -h\hat{y} = mgh \tag{4}$$

- $\vec{F}$ is the net force acting on the system
- $d\vec{x}$ is the incremental displacement
- Work-energy theorem can give the final speed: $\frac{1}{2}mv_{\mathrm{f}}^2 = mgh$.

Definition of work, applied to a thermodynamic system *with an equation of state*:

$$W = \int \vec{F} \cdot d\vec{x} = \int p\vec{A} \cdot d\vec{x} = \int pdV \tag{5}$$

- $\vec{A}$ and $d\vec{x}$ assumed to be parallel
- p is constant over $d\vec{x}$

Definition of work, applied to a thermodynamic system *with an equation of state*:

$$W = \int p dV \tag{6}$$

Group board exercise: Substitute the Ideal Gas Law into Eq. 6, to derive the work done *when temperature is constant and V is the independent variable*.

Hint: the integral of $\frac{1}{x}$ *is* $\ln x + C$.

Group board exercise: Repeat, but at *constant pressure* instead of constant temperature, again with V as the independent variable.

The van der Waals *equation of state* is:

$$\left(p + \frac{a}{V^2}\right)(V - b) - nRT = 0 \tag{7}$$

Group board exercise: Substitute this equation of state into Eq. 6, to find the work done by isothermal expansion. Compare to the isothermal result from the Idea Gas Law, and recover the previous result for W by setting a and b to zero.

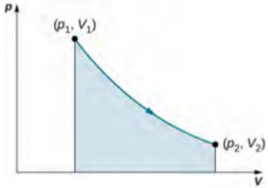

Figure 3.5 When a gas expands slowly from V_1 to V_2, the work done by the system is represented by the shaded area under the *pV* curve.

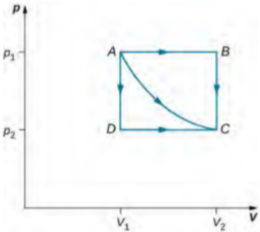

Figure 3.6 The paths *ABC*, *AC*, and *ADC* represent three different quasi-static transitions between the equilibrium states *A* and *C*.

Figure 2: On a *pV* diagram, work is the area under the curve, in the same fashion that it is the area under the curve on a force displacement diagram.

How much energy is available to do work, in the form of heat? (For example, if we expand a gas at constant pressure, by how much can we decrease the temperature?)

Recall from the prior chapter that

$$\boxed{E_{\mathrm{int}} = \frac{3}{2}nRT = \frac{3}{2}nN_{\mathrm{A}}k_{\mathrm{B}}T = \frac{3}{2}NkT} \tag{8}$$

The *internal energy* of a gas must be comprised of the average kinetic energies of the molecules. Here we ignore molecular interactions.

The First Law of Thermodynamics

Associated with every equilibrium state of a system is its internal energy E_{int}. The change in E_{int} for any transition between two equilibrium states is $\Delta E_{\mathrm{int}} = Q - W$, where Q is the heat exchanged by the system and W is the work done on or by the system.

- If heat is added to the system: $Q > 0$
- If heat is removed from the system: $Q < 0$
- If work is done by the system: $W > 0$
- If work is done on the system: $W < 0$

In each of the following scenarios, select the best answer:

- A: E_{int} increases
- B: E_{int} decreases
- C: E_{int} does not change
- D: I am confused.

- Scenario 1: A gas expands against a piston, doing some work, but the gas loses no heat.
- Scenario 2: A gas expands against a piston, performing 10 J of work. A separate device adds 10 J of heat to the gas.
- Scenario 3: A piston is pushed against a gas, requiring 10 J of work. A separate device adds 10 J of heat to the gas.

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- Scenario 4: A cold scientist clutches a warm canteen of water to his chest (the system is the canteen).
- Scenario 5: A cylinder of gas has a piston on one end. Someone pulls the piston slowly, doing 10 J of work.

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THE FIRST LAW OF THERMODYNAMICS

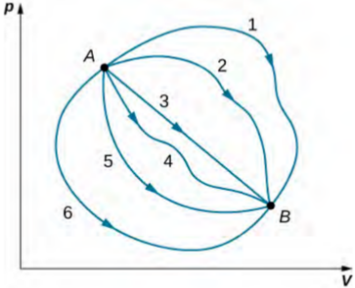

Figure 3.7 Different thermodynamic paths taken by a system in going from state A to state B. For all transitions, the change in the internal energy of the system $\Delta E_{\text{int}} = Q - W$ is the same.

Figure 3: The internal energy for a given state depends only on the temperature, and not the path taken to arrive at that state.

Which of the following is true of the *work done by the system*?

- A: Path 2 corresponds to the most work done, and path 5 the least work done.
- B: Path 5 corresponds to the most work done, and path 2 the least work done.
- C: Path 3 corresponds to the most work done, and path 4 the least work done.
- D: Path 4 corresponds to the most work done, and path 3 the least work done.

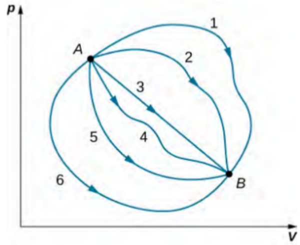

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Figure 4: Use this diagram to answer the questions at left

Suppose $\Delta E_{\text{int}} = 0$. Which of the following is true?

- A: For path 1, the system gains the most heat.
- B: For path 5, the system loses the most heat.
- C: For path 3, the system loses the most heat.
- D: For path 1, the system loses the most heat.

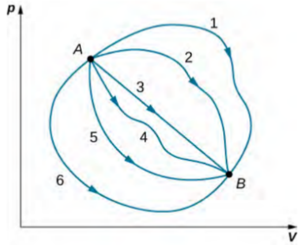

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Figure 5: Use this diagram to answer the questions at left

Suppose the system in question is an ideal gas. Which path most likely represents a transition with constant temperature?

- A: Path 3
- B: Path 2
- C: Path 5
- D: Path 1

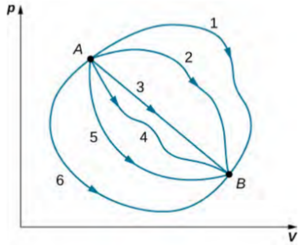

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Figure 6: Use this diagram to answer the questions at left

Recall that the work done by an n moles of an expanding ideal gas at constant temperature T is given by $W = nRT \ln(V_{\mathrm{f}}/V_{\mathrm{i}})$, where $R = 8.314$ J/mole/Kelvin. Calculate the work done if heat is added to 2 moles of an ideal gas at 300 K such that the volume of the gas grows by a factor of 2.

- A: 416 J
- B: 520 J
- C: 3460 J
- D: 4320 J

Same scenario, but calculate the work if the gas was at a constant temperature of 200 K.

- A: 2305 J
- B: 3410 J
- C: 280 J
- D: 320 J

If the temperature remains at 200K during the process, what is the heat transferred into or out of the system?

- A: -2305 J
- B: 2305 J
- C: -3410 J
- D: 3410 J

Suppose we have a system where we can add external heat into a rocket, and then 100 percent of that heat is converted to kinetic energy for the rocket. If the rocket has 0.1 kg of mass, and $g = 10$ m/s/s, how much heat is required to launch the rocket to a height of 50 meters?

- A: 20 J
- B: 30 J
- C: 40 J
- D: 50 J

If in the heat-to-rocket system, we have to provide the heat first to 2 moles of an ideal gas, what would be the required temperature change?

- A: 2 degrees C
- B: 3 degrees C
- C: 20 degrees C
- D: 30 degrees C

In the previous two problems we equated heat energy with mechanical energy: $mgh = Q$, and then calculated Q with $Q = \frac{3}{2}nRT$. Thus, we used $mgh = \frac{3}{2}nRT$. Let's try one with kinetic energy instead of potential energy.

If we are using the heat stored in 2 moles of an ideal gas, what temperature change is required to get that heat to accelerate a 0.1 kg object to 10 m/s?

- A: 3 degrees C
- B: 2 degrees C
- C: 0.3 degrees C
- D: 0.2 degrees C

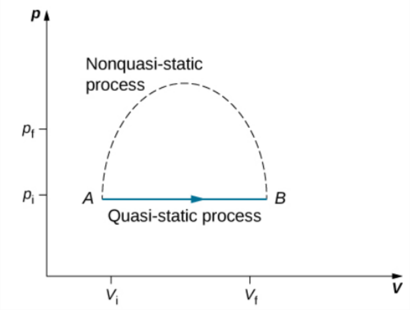

Figure 7: Two processes: quasi-static and non-quasi-static. We study state transitions composed of many quasi-static transitions. The pictured non-quasi-static transition has two volumes with the same pressure, so how can we reasonably define the state of a system?

Words of the text: "A quasi-static transition is one in which the change in state is made infinitesimally slowly so that at each instant, the system can be assumed to be at a thermodynamic equilibrium with itself and with the environment."

What follows are a few examples of thermodynamic processes that may be classified as quasi-static. In realistic systems, some amount of non-quasi-static behavior is normal.

A process is called **cyclic** if it returns a system to the same state with the same internal energy. We should know at least these types of quasi-static processes:

1. Isothermal (T is constant)
2. Isobaric (p is constant)
3. Adiabatic (Q, the head added/subtracted, is zero)
4. Isochoric (V is constant)

Exercise: use the following simulation to create cyclic processes.
`https://www.geogebra.org/m/mXvUPjFz`

Exercise: use the following simulation to create cyclic processes: `https://www.geogebra.org/m/mXvUPjFz`. What do you notice about the total heat exchanged Q, and the total work done, W?

1. Create four different cyclic processes that involve the four quasi-static process types, and explain quantitatively (in your lab notebook) why each fits the relevant criterion.
2. What is true about Q and W for each of these for processes you've created? Why is this the case?

HEAT CAPACITIES OF AN IDEAL GAS

What follows is a traditional proof of the following theorem:

$$C_{\mathrm{P}} = C_{\mathrm{V}} + R \tag{9}$$

The molar heat capacity of a gas at constant pressure is always larger than at constant volume.

- **Assume** an ideal gas
- **Apply** First Law of Thermodynamics
- Evidence for *independence of state function*
- Evidence for *kinetic theory of gases*

The first law of thermodynamics: $dE = dQ - dW$. Consider an ideal gas, in a vessel with fixed volume. Since $dW = pdV$, $dW = 0$. If heat dQ is added to that vessel, the corresponding temperature in increase is

$$dQ = nC_{\mathrm{V}}dT \tag{10}$$

where C_{V} represents the *heat capacity at constant volume* [1]. Because $dE = dQ$,

$$dE = nC_{\mathrm{V}}dT \tag{11}$$

[1] Recall that from kinetic theory, for ideal gases with no molecular interactions, $C_{\mathrm{V}} = \frac{3}{2}R$.

Consider the same gas, but at fixed pressure. Beginning with first law of thermodynamics now accounts for work:
$dE = dQ - dW = dQ - pdV$. If heat dQ is added to the vessel, the corresponding temperature change is

$$dQ = nC_{\mathrm{P}}dT \tag{12}$$

where C_{P} is the *heat capacity at constant pressure*. In order to obtain an expression for dE involving only temperature, the ideal gas law must be differentiated:

$$d(pV) = d(nRT) \tag{13}$$

$$pdV = nRdT \tag{14}$$

Substituting the right-hand side of Eq. 14 for dW in the First Law, and the right-hand side of Eq. 12 for dQ in the first law, the result is

$$dE = n(C_{\mathrm{P}} - R)dT \tag{15}$$

The expressions for dE under isobaric and isochoric conditions have been obtained:

$$dE = C_{\mathrm{V}} dT \tag{16}$$

$$dE = n(C_{\mathrm{P}} - R)dT \tag{17}$$

If the internal energy of a thermodynamic system depends only on the temperature, then these two expressions must be equal. Upon setting them equal, a fundamental relationship is revealed:

$$\boxed{C_{\mathrm{P}} = C_{\mathrm{V}} + R} \tag{18}$$

It has been shown that $C_{\mathrm{V}} = \frac{3}{2}R$ from the equipartition principle, so $C_{\mathrm{P}} = \frac{5}{2}R$ for an ideal gas.

Jasmine, the heroine of *Artemis*, by Andy Weir, lives on the moon. Suppose she needs to power up a bulkhead in a compartment that has lost pressure, but the bulkhead won't work below a certain temperature. She devises a plan fill up a heated container with oxygen and open it in the compartment, both releasing oxygen and heat. How should she fill the container with heated air, to store the most heat in a given time limit?

- A: Fill a fixed-volume container with low-pressure air and heat it up.
- B: Fill an inflatable object with heated air at constant, low pressure.
- C: Fill a fixed-volume object with high-pressure air and heat it up.
- D: Fill an inflatable object with heated air at high pressure.

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- D: Fill an inflatable object with heated air at high pressure.

*I would say **D**, merely because we know that the heat capacity of the gas is higher at constant pressure rather than constant volume, and at high pressure we get more atoms of gas...*

ADIABATIC PROCESSES WITH AN IDEAL GAS

CONCLUSION

ANSWERS

ANSWERS

- 420 seconds
- About 60 hours
- The only extensive variable is *n*, and *p*, *V*, and *T* are the intensive ones
- E_{int} decreases
- E_{int} does not change
- E_{int} increases
- E_{int} decreases
- Path 5
- 3460 J
- 2305 J
- -2305 J
- 50 J
- 0.2 degrees C
- Fill an inflatable object with heated air at high pressure.