Friday Warm Up: Unit 5: Momentum II

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1 Memory Bank

- Let M be the total mass of a system, and let m_j and $\vec{r}_j$ $(j = 1, ..., N)$ be the masses and positions of the constituent parts of the system. The position of the center of mass is

$$\vec{r}_{\text{CM}} = \frac{1}{M} \sum_{j=1}^{N} m_j \vec{r}_j \tag{1}$$

- The momentum of the center of mass $\vec{P}_{\text{CM}}$ is

$$\vec{P}_{\text{CM}} = M \frac{d\vec{r}_{\text{CM}}}{dt} = \sum_{j=1}^{N} \vec{p}_j \tag{2}$$

- The net external force on a system obeys

$$\vec{F} = \frac{d\vec{P}_{\text{CM}}}{dt} = M \frac{d^2 \vec{r}_{\text{CM}}}{dt^2} \tag{3}$$

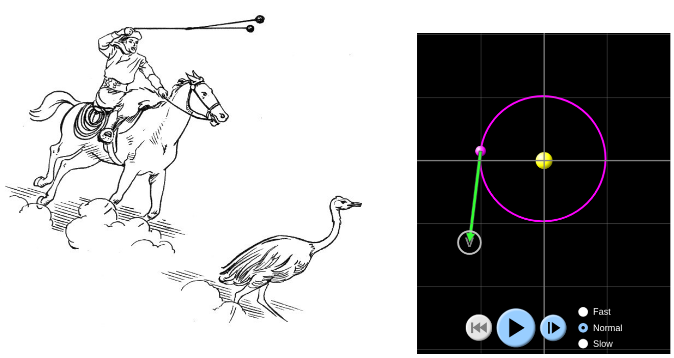

Figure 1: (Left) A *gaucho* using a bola weapon to hunt a rhea bird. (Right) A planet orbits a star.

2 Momentum II

1. In Pre-columbian and colonial period Latin America, *gauchos* would sometimes hunt with a weapons known as *bolas* (Fig. 1, left). The bolas were thrown, and would spin around the center of mass until they wrapped the limbs of the prey. (a) Suppose two masses m are separated by a diameter d. The masses orbit the center with frequency f. (a) Graph the positions in an x-y coordinate system, and (b) write down a system of equations describing the positions of the masses versus time. (c) Suppose $f = 5$ Hz, or 5 rotations per second. Locate the center of mass at $t = 0.2$ seconds. (d) If the bolas are each 1.2 kg, what is the magnitude of the momentum of each bola? (e) What is the *total momentum* P_{CM}?

2. Consider Fig. 1 (right), in which a single planet orbits a star located at the origin at $t = 0$. Let the star have mass M, the planet have mass m, and let the distance between them be r. Let the ratio of the masses be $\mu = m/M$. (a) Show that the center of mass is given by

$$\vec{r}_{\text{CM}} = \left(\frac{\mu}{\mu + 1}\right) \vec{r} \tag{4}$$

   (b) Show that $\vec{r} = 0$ in the limit that $\mu \ll 1$.

3. Assume there is no *net, external* force on the system. The center of mass will
   - A: Accelerate
   - B: Decelerate
   - C: Remain stationary
   - D: Remain at constant velocity