CALCULUS-BASED PHYSICS-1: MECHANICS (PHYS150-01): WEEK 8

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Week 7 Review

1. Work and potential energy
    - **Lab activity:** Oscillator and gravity trading work and potential energy
2. Potential energy and **conservative forces**
3. Conservation of Energy
    - *Calculus review: the fundamental theorem of calculus*
    - Graphical representations of integrals and energy

WEEK 7 REVIEW PROBLEM

Suppose a particle moves in a potential energy surface $U(x) = U_0 \left(x^4 - x^2 \right)$. What is the force at $x = 1$?

- A: $-4U_0 + 2U_1$
- B: $4U_0 - 2U_1$
- C: $-U_0 + U_1$
- D: 0

WEEK 8 SUMMARY

1. Definition of **momentum**
2. *Conservation of momentum*
    - The proof and the assumptions
    - Examples
3. *Classification of collisions*
    - Elastic
    - Inelastic
    - $1 \to 1$, $1 \to n$, $n \to 1$, $n \to n$
    - **Lab activity**
4. **Momentum in multiple dimensions**
5. **Center of mass**
    - Derivation of $\vec{F}_{\mathrm{Net}} = \frac{d\vec{P}_{\mathrm{CM}}}{dt}$
    - Center of mass motion

DEFINITION OF MOMENTUM

Ready to jump down the rabbit hole? Good. Momentum is defined as follows:

Definition of Momentum

A particle of mass m and velocity $\vec{v}$ has the vector *momentum*:

$$\vec{p} = m\vec{v}$$

There is a corollary:

Newton's Second Law with momentum

If a particle has acceleration $\vec{a} = \frac{d\vec{v}}{dt}$, then

$\vec{F}_{\mathrm{Net}} = \frac{d\vec{p}}{dt}$

An object that has a small mass and an object that has a large mass have the same momentum. Which mass has the largest kinetic energy?

- A: The one with the small mass
- B: The one with the large mass
- C: If the momentum is the same the kinetic energy is the same
- D: Cannot determine the answer

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The unit of linear momentum is kg m/s. Suppose that a raindrop reaches a terminal velocity of 1 m/s, and the density of water is 1 gram per cm^3. If a 1 cm^3 water droplet reaches terminal velocity, what is the momentum of the droplet?

- A: 10^{-3} kg m/s
- B: 10^{-2} kg m/s
- C: 10^{-1} kg m/s
- D: 1 kg m/s

If $\vec{F}_{\mathrm{Net}} = \frac{d\vec{p}}{dt}$, and an object is undergoing constant acceleration, which of the following is true of the momentum?

- A: It is constant in time.
- B: It is a linear function of time.
- C: It is a quadratic function of time.
- D: It is zero.

CONSERVATION OF MOMENTUM

...continuing to fall down the rabbit hole...

Conservation of Momentum

The momentum of a system of N particles undergoing no external forces is conserved.

$\frac{d\vec{P}}{dt} = 0$

Suppose two objects with momenta $\vec{p}_1 = m_1\vec{v}_1$ and $\vec{p}_2 = m_2\vec{v}_2$ collide. The new momenta after the collision are $\vec{p}_1' = m_1\vec{v}_1'$ and $\vec{p}_2' = m_2\vec{v}_2'$. If $m_2 = 2m_1$ and $\vec{v}_1 = 2\vec{v}_2$, and $\vec{v}_2'$ is observed to equal $\vec{v}_1$, what is $\vec{v}_1'$?

Solve together in groups on boards.

The proof of conservation of momentum is the combination of two concepts: Newton's 3rd Law and Newton's 2nd Law. The net forces on two particles by Newton's 3rd Law are

$$\vec{F}_{21} = -\vec{F}_{12} \tag{1}$$

Substituting Newton's 2nd Law for the forces,

$$m_1\vec{a}_1 = -m_2\vec{a}_2 \tag{2}$$

Acceleration is defined as the change in velocity, implying

$$m_1\frac{d\vec{v}_1}{dt} = -m_2\frac{d\vec{v}_2}{dt} \tag{3}$$

$$\frac{d\vec{p}_1}{dt} = -\frac{d\vec{p}_2}{dt} \tag{4}$$

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \tag{5}$$

$$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = 0 \tag{6}$$

$$\boxed{\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = 0} \tag{7}$$

Equation 7 states that the total momentum does not change with time. The assumptions hold even if there are more than two particles, for every particle in the system exerts some force on every other particle, even if that force is zero.

$$\frac{d}{dt}\sum_{\mathrm{i}} \vec{p}_{\mathrm{i}} = \frac{d\vec{P}}{dt} = 0 \tag{8}$$

What two assumptions were necessary in the above proof?

- A: Each mass is constant in time, and the total velocity is zero.
- B: The total velocity is zero, and the net force is $\frac{dP}{dt}$.
- C: The net external force is zero, and the total velocity is zero.
- D: The net external force is zero, and the mass of each particle is zero.

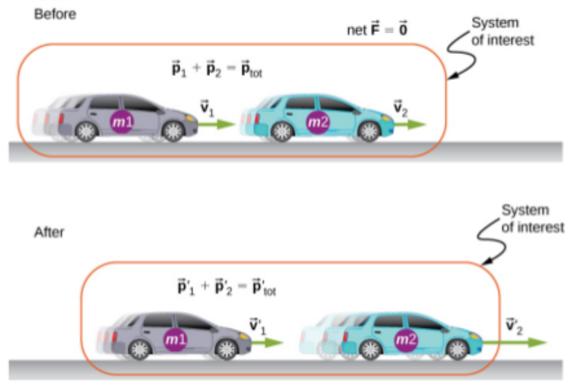

Figure 1: The assumptions for momentum conservation. (a) Which car exerts more force? (b) At which point are the cars accelerating?

Colliding Carts

Two carts in a physics lab roll on a level track, with negligible friction. These carts have small magnets at their ends, so that when they collide, they stick together (Figure 9.16). The first cart has a mass of 675 grams and is rolling at 0.75 m/s to the right; the second has a mass of 500 grams and is rolling at 1.33 m/s, also to the right. After the collision, what is the velocity of the two joined carts?

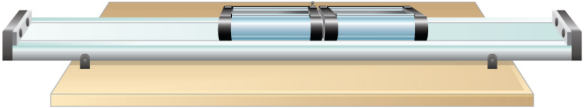

Figure 9.16 Two lab carts collide and stick together after the collision.

Figure 2: The laboratory activity for today.

- One cart still, other moving (magnet side)
- One cart still, other moving (velcro side)
- Both carts moving (magnet side)
- Both carts moving (velcro side)

Test cases from lab:

- One cart still, other moving (magnet side)
- One cart still, other moving (velcro side)
- Both carts moving (magnet side)
- Both carts moving (velcro side)

Answer the following questions:

- In which of the above is the momentum conserved?
- In which of the above is the kinetic energy conserved?
- In which of the above is both the kinetic energy and momentum conserved?
- In which of the above is the final kinetic energy zero?

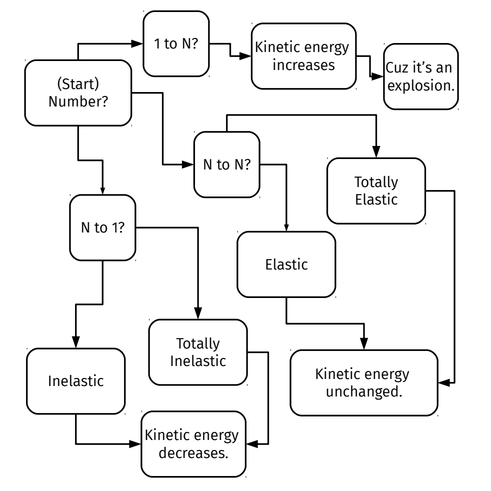

Figure 3: Classification of momentum interactions.

Special case of an ***explosion****...*

`https://www.youtube.com/watch?v=5zxVQBnmyDA`

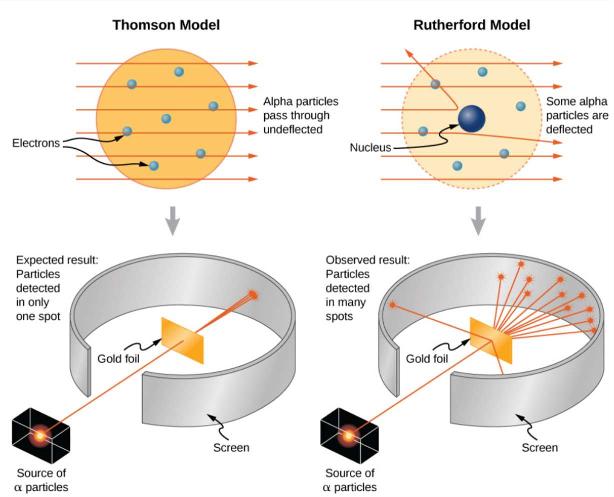

Figure 4: Knowledge of the atom via momentum conservation!

ELASTIC COLLISIONS

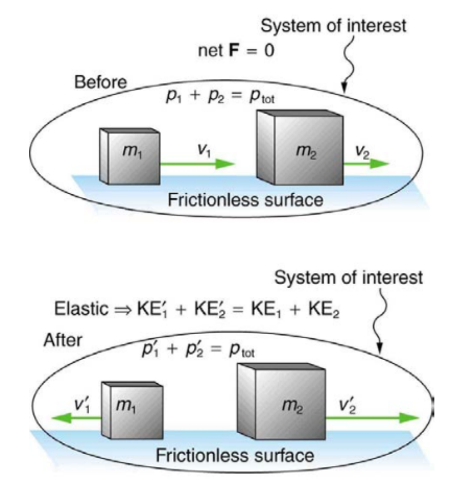

Figure 5: Internal kinetic energy and momentum are conserved if the collision is *elastic*.

Suppose two objects undero an elastic collision. Given the conditions below, find the quadratic equation for v_1'.

$$m_1 = 0.5kg \tag{9}$$

$$m_2 = 1.0kg \tag{10}$$

$$v_1 = 2m/s \tag{11}$$

$$v_2 = 0m/s \tag{12}$$

Solve in groups on boards.

Suppose two objects undero an elastic collision. Given the conditions below, find the quadratic equation for v_1'.

Answer: $\frac{3}{2}v_1'^2 - 2v_1' - 2 = 0$

Which root of this equation is correct, and why?

INLASTIC COLLISIONS

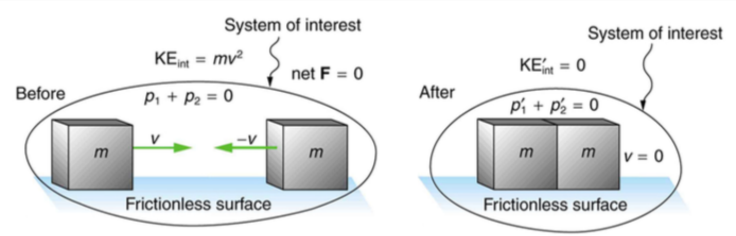

Figure 6: Internal kinetic energy is not conserved, and momentum is conserved if the collision is *inelastic*.

Suppose two objects undero a *perfectly* inelastic collision (also: *totally* inelastic). Given the conditions below,

$$m_1 = 0.5kg \tag{13}$$

$$m_2 = 1.0kg \tag{14}$$

$$v_1 = 2m/s \tag{15}$$

$$v_2 = 0m/s \tag{16}$$

Solve in groups on boards.

What is the right answer?

$v' = \frac{2}{3}$ m/s. Should it be positive or negative?

MOMENTUM CONSERVATION IN TWO DIMENSIONS

A particle interacts with another at rest. The incoming particle has mass 1 kg, and moves parallel to the x-axis at 1 m/s. In the final state, it moves at $\theta_1 = 60$ degrees with respect to the x-axis. The second particle, with mass 0.5 kg, moves at $\theta_2 = 30$ degrees with respect to the x-axis. Solve for the final velocities.

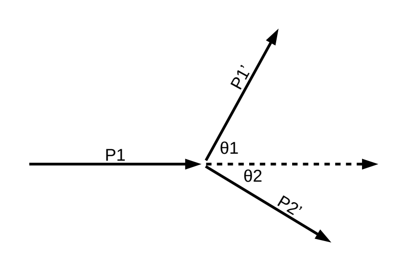

Figure 7: Solve as a group on the board. There are two unknowns, *but also two equations.*

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Answers: $v_1' = \frac{1}{\sqrt{3}}$ m/s, and $v_2' = 2$ m/s.

Does this answer make sense? Why should the second particle move faster than the first?

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Answers: $v_1' = \frac{1}{\sqrt{3}}$ m/s, and $v_2' = 2$ m/s.

Was this interaction elastic or not? Why?

A particle interacts with another. The first has mass 1 kg, and moves at 60 degrees with respect to the x-axis at $\frac{1}{\sqrt{3}}$ m/s. The second moves at 2 m/s, at 30 degrees with respect to the x-axis. Solve for the final velocity of the combined object if they stick together.

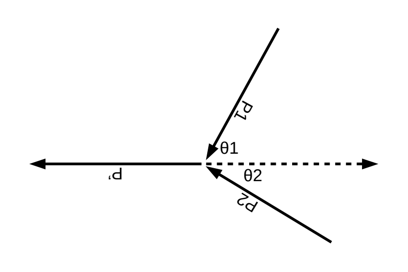

Figure 8: Solve as a group on the board.

A particle interacts with another. The first has mass 1 kg, and moves at 60 degrees with respect to the x-axis at $\frac{1}{\sqrt{3}}$ m/s. The second moves at 2 m/s, at 30 degrees with respect to the x-axis. Solve for the final velocity of the combined object if they stick together.

Answer: $v_1' = \frac{4}{3\sqrt{3}}$ m/s.

Does this answer make sense? Should it be larger or smaller than the first two velocities?

A particle interacts with another. The first has mass 1 kg, and moves at 60 degrees with respect to the x-axis at $\frac{1}{\sqrt{3}}$ m/s. The second moves at 2 m/s, at 30 degrees with respect to the x-axis. Solve for the final velocity of the combined object if they stick together.

Answer: $v_1' = \frac{4}{3\sqrt{3}}$ m/s.

Was this an inelastic or elastic collision? Why?

A particle interacts with another. The first has mass 1 kg, and moves at 60 degrees with respect to the x-axis at $\frac{1}{\sqrt{3}}$ m/s. The second moves at 2 m/s, at 30 degrees with respect to the x-axis. Solve for the final velocity of the combined object if they stick together.

Answer: $v_1' = \frac{4}{3\sqrt{3}}$ m/s.

Why does this final velocity not equal the initial velocity of Fig. 7?

CONCLUSION

1. Definition of momentum
2. *Conservation of momentum*
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3. *Classification of collisions*
    - Elastic
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    - $1 \to 1$, $1 \to n$, $n \to 1$, $n \to n$
    - Lab activity
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ANSWERS

- $-4U_0 + 2U_1$
- The one with the small mass
- The one with the large mass
- 10^{-2} kg m/s
- It is a linear function of time.
- The net external force is zero, and the mass of each particle is zero.
- ...