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3-26-20

Midterm 2

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March 26, 2020

1 Memory Bank

1. $\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$ ... Coulomb Force
2. $k = 9 \times 10^9$ N C^{-2} m^2 ... Remember $k = 1/(4\pi\epsilon_0)$.
3. $q_e = 1.6 \times 10^{-19}$ C ... Charge of an electron/proton
4. $\vec{F} = q\vec{E}$ ... Electric field and charge
5. $\vec{E}(z) = \frac{\sigma}{\epsilon_0}\hat{z}$ ... Electric field of two oppositely charge planes each with charge density σ
6. $\epsilon_0 \approx 8.85 \times 10^{-12}$ F/m
7. $dE = \int kdq/r^2$ ... Remember that dq takes the form below
8. $dq = \lambda dx$ ... Linear charge density (C/m)
9. $\vec{E} \cdot \vec{A} = Q_{enc}/\epsilon_0$ ... Gauss' Law, constant electric field over the surface area.
10. $U = q\Delta V$ ... Potential energy and voltage
11. 1 eV: an electron-Volt is the amount of energy one electron gains through 1 V.
12. $V(r) = k\frac{q}{r}$ ... Voltage of a point charge
13. $\vec{E} = -\frac{\Delta V}{\Delta x}$ ... E-field is the slope or change in voltage with respect to distance
14. $V(x) = -Ex + V_0$ ... Voltage is linear between two charge planes
15. $Q = CV$ ... Definition of capacitance
16. $C = \frac{\epsilon_0 A}{d}$ ... Capacitance of a parallel plate capacitor
17. $C_{tot}^{-1} = C_1^{-1} + C_2^{-1}$ ... Adding two capacitors *in series*.
18. $C_{tot} = C_1 + C_2$ ... Adding two capacitors *in parallel*.
19. $i(t) = dQ/dt$ ... Definition of current.
20. $v_d = i/(nqA)$ ... Charge drift velocity in a current i in a conductor with number density n and area A.
21. $R_{tot}^{-1} = R_1^{-1} + R_2^{-1}$ ... Adding two capacitors *in parallel*.
22. $R_{tot} = R_1 + R_2$ ... Adding two capacitors *in series*.
23. $\Delta V = IR_{\rm tot}$, $\vec{J} = \sigma\vec{E}$ ... Versions of Ohm's Law. ($\vec{J}$ is the current density with units of Amps per meter-squared).
24. $P = IV$ ... Relationship between power, current, and voltage.
25. $V_C(t) = \epsilon_1(1 - \exp(-t/\tau))$ ... voltage across the capacitor in an RC series circuit. The time constant $\tau = RC$.
26. $i(t) = \frac{\epsilon_1}{R}\exp(-t/\tau)$ ... Current in an RC series circuit.
27. $i_{\rm in} = i_{\rm out}$ ... Kirchhoff's junction rule.
28. $\epsilon_1 + \epsilon_2 + \epsilon_3 + ... = 0$ ... Kirchhoff's loop rule.
29. $\vec{F} = q\vec{v} \times \vec{B}$ ... The Lorentz force on a charge q with velocity $\vec{v}$ in a magnetic field $\vec{B}$.
30. $\vec{F} = I\vec{L} \times \vec{B}$ ... The Lorentz force on a conductor of length $\vec{L}$ carrying a current I in a magnetic field $\vec{B}$.

2 Chapter 9: Current and Resistance

1. An ECG monitor must have an RC time constant less than 100μs to be able to measure variations in voltage over small time intervals. (a) If the resistance of the circuit (due mostly to that of the patients chest) is 1.00 kΩ, what is the maximum capacitance of the circuit? (b) Would it be difficult in practice to limit the capacitance to less than the value found in (a)? (c) If the patient's resistance really is 1.00 kΩ, and the typical maximum amplitude of the patient's heartbeat is 60 mV, when does the voltage rise to 30 mV in the EKG monitor (using the C you found in (a))?

a) $\tau = RC$

$\frac{100 \mu s}{1000 \Omega} = \frac{1000 \Omega C}{1000 \Omega}$

$0.1 \frac{\mu s}{\Omega} \cdot \frac{10^{-6} s}{\mu s} = 10^{-7} F = \boxed{0.1 \mu F = C}$

b) No because it is typically normal for capacitors to have μF values. It's a big unit.

c) $V_c(t) = \mathcal{E}_1 (1 - e^{-t/\tau})$

$\frac{30 mV}{60 mV} = \frac{60 mV (1 - e^{-t/0.1 \mu F})}{60 mV}$

$\frac{1}{2} = 1 - e^{-t/0.1 \mu F}$

$\ln(+\tfrac{1}{2}) = \frac{-t}{0.1 \mu F} \ln(e)$ $\boxed{t = -0.1 \cdot \ln(1/2) s}$

$-0.1 \ln(1/2) \cdot \frac{1}{10^7} \frac{s}{\Omega}$

2. Imagine an *alternating current* (AC) system, as opposed to the DC systems we normally consider. In AC circuits, the voltage follows a form

$$V(t) = V_0 \sin(2\pi f t + \phi) \quad (1)$$

The wall outlets in the USA have $f = 60$ Hz and $V_0 = 120$ V. *We have the freedom to choose ϕ in this example, much like choosing the zero-point of voltage.* (a) Suppose $\phi = 0$. At what times will $V(t) = 0$? (b) What is the max power delivered to a 1kΩ resistor? (c) What is the *average* power delivered to a 1kΩ resistor?

a) $\sin(2\pi f(0)) \rightarrow 0$

$\frac{2\pi(60)t}{2\pi 60} = \frac{\pi}{2\pi 60}$ | $\frac{2\pi(60)t}{2\pi 60} = \frac{2\pi}{2\pi(60)}$

$t = 1/120$ sec $t = 0$ sec $t = 1/60$ sec

b) $P = \frac{V_0^2}{R} = \frac{14400 V^2}{10^3 \Omega}$

$= \boxed{14.4 W}$

$I_0 = \frac{P}{V_0} = \frac{14.4 W}{120 V} = 0.12 A$

c) $P_{avg} = \frac{1}{2} I_{rms}^2 R$

$I_{rms} = \frac{1}{\sqrt{2}} I_0 = \frac{0.12 A}{\sqrt{2}}$

$= 0.085 A$

$= \frac{0.007}{2} \cdot 1000 \Omega$

$= \boxed{3.5 W}$

3. **For those of us stuck at home!** A physics student has a single-occupancy dorm room. The student has a small refrigerator that runs with a current of 3.00 A and a voltage of 110 V, a lamp that contains a 100-W bulb, an overhead light with a 60-W bulb, and various other small devices adding up to 3.00 W. In Southern California, electricity costs about 0.2 dollars per kiloWatt-hour. How much money does this student spend if the total wattage is on for 12 hours per day for one month?

$P = IV$

$= 3A \cdot 110V$

$= 330 W$

$P_{tot} = 3W + 330W + 60W$

$= 393 W \times 10^{-3}$

$= 0.393 kW$

$0.393 kW \cdot 360 hrs$

$= 141.48 kW hrs \cdot \frac{0.2\$}{kW hrs}$

$= \boxed{\$28.30}$

12 hrs $\cdot$ 30

$= 360$ hrs

3 Chapter 10: Direct-Current (DC) Circuits

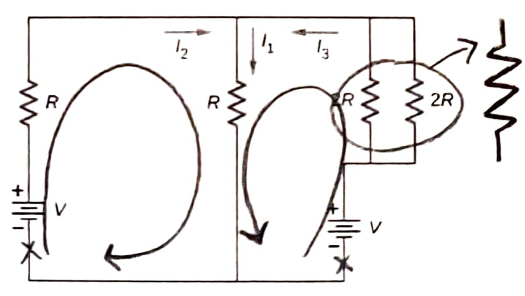
$I_2 + I_3 = I_1$

$-(-1000 \Omega I_2 - 1000 I_1 = -12 V)$

$-1000 I_3 - 1000 I_1 = -12 V$

$1000 \Omega I_2 - 1000 I_3 = 0$

$\Rightarrow I_2 = I_3 \Rightarrow I$



$\frac{4R^2}{4R} = R$

$-1000 I - 1000 I_1 = -12$

$\frac{-1000 I}{-1000} = \frac{-12 + 1000 I_1}{-1000}$

$I = 0.012 A - I_1$

$2(0.012 A - I_1) = I_1$

$0.024 A - 2 I_1 = I_1$

$\frac{0.024 A}{3} = \frac{3 I_1}{3}$

$I_1 = 0.008 A$

Figure 1: A circuit with two batteries and three resistors.

1. Solve for i_1, i_2, and i_3 in Fig. 1, if $R = 1k\Omega$, and $V = 12.0$ Volts. What power is consumed in the resistors?

So $I_1 = 2I$ a) $I_1 = 0.008 A$ $I_3 = 0.004 A$

$I_2 = 0.004 A$

$P_1 = R \times I_1^2 = \boxed{0.064 W}$

$P_2 = R \times I_2^2 = \boxed{0.016 W} = P_3$

2. Suppose an electronic device with resistance R needs between 1.4 and 2.0 volts to operate. Two AA batteries with $\varepsilon = 1.5$V and $r = 0.25\Omega$ are connected (Fig. 2) in parallel with the device. (a) If $R = 50\Omega$, what is the current flow? (b) If the batteries each have a charge $q = 2.5$ A hr, how long will the current flow?

a) $+1.5v - I_1 0.25\Omega - I\,50\Omega = 0$

$+1.5v - I_2 0.25\Omega - I\,50\Omega = 0$

$I = I_1 + I_2 \Rightarrow 0.015 + 0.015$

$\boxed{I = 0.03A}$

b) $\frac{Q}{I} = \frac{I \cdot t}{\cancel{I}}$

$t = \frac{Q}{I} = \frac{2.5 Ahr}{0.03A} = \boxed{83.3 \text{ hrs}}$

$1.5 = I_1 0.25 + I_1 50 + I_2 50$

$1.5 = I_1 50.25 + I_2 50$

$\frac{1.5 - I_2 50}{50.25} = \frac{I_1 \cancel{50.25}}{\cancel{50.25}}$

$I_1 = 0.03A - I_2 0.99$

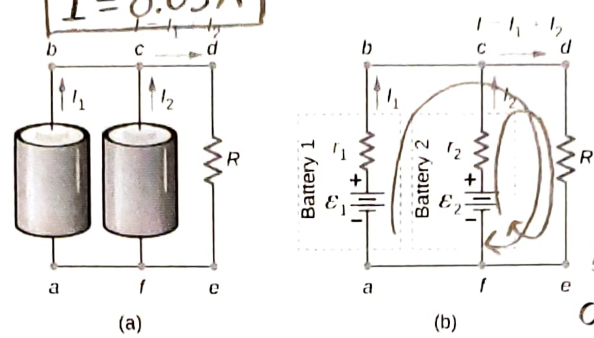


Figure 2: Two AA batteries are connected *in parallel* to power a calculator represented by R. (a) The batteries are connected in parallel. (b) A circuit diagram representing the circuit in (a).

$1.5 = I_2 0.25 + I_1 50 + I_2 50$

$1.5 = I_2 50.25 + I_1 50$

$\frac{1.5v - I_1 50}{50.25\Omega} = \frac{I_2 \cancel{50.25}}{\cancel{50.25}}$

$0.03A - I_1 0.99 = I_2$

$I_1 = 0.03A - (0.03A - I_1 0.99)0.99$

$I_1 = 0.03A - 0.0297A + I_1 0.98$

$\frac{\cancel{0.02} I_1}{\cancel{0.02}} = \frac{0.0003A}{0.02} = 0.015A$

Same battery so it makes sense to have same current.

4 Chapter 11: Magnetic Forces and Fields

$I_2 = 0.03A - 0.99(0.03A - I_2 0.99)$

$I_2 = 0.03A - 0.0297A + 0.98 I_2$

$\frac{0.02 \cancel{I_2}}{\cancel{0.02}} = \frac{0.0003A}{0.02}$

$I_2 = 0.015A$

Figure 3: The trajectory of a sub-atomic particle through a cloud chamber.

1. The experimental result depicted in Fig. 3 shows the trajectory of a sub-atomic particle that is revealed by a device called a *cloud chamber*. The particle bends to the *left* after passing through a lead plate. (a) The magnetic field is *into the page*. What is the sign of the charge of this particle? (b) It was later deduced that this particle had the mass of an elecron, from the radius of curvature. Why is that strange? (c) Imagine the B-field had a strength of 0.05 T and the velocity of the paricle was 10^6 m/s. What was the force on the particle, and in what direction was the force?

a) The charge of the particle is Positive.

b) Well it's strange because electrons are known to have a negative charge.

c) $\vec{F} = -q v_\perp B \hat{x}$

$= -(-1.6\times10^{-19}C)(10^6\frac{m}{s})(5\times10^{-2}T)\hat{x}$

$= 1.6\times10^{-19}C(10^6\frac{m}{s})5\times10^{-2}T\hat{x}$

$\boxed{\vec{F} = 8\times10^{-15}N\hat{x}}$

Positive in $\hat{x}$

I see what happened here.