calculus-based physics-2: electricity, magnetism, and thermodynamics (phys180-02): unit 5

Jordan Hanson

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Whittier College Department of Physics and Astronomy

UNIT 4 REVIEW

Suppose a bundle of wires is carrying current along what we call the $\hat{z}$ direction. Each wire runs along the z-axis and they are close enough to ignore the fact that the volume of each wire prevents it from being exactly on the z-axis. One wire carries +2.0 A, another carries +1.5 A, and a third carries -0.5 A. What is the B-field strength at a distance of 1 cm away in the x-y plane?

- A: 6 Gauss
- B: 0.6 Gauss
- C: 6 Tesla
- D: 0.6 Tesla

Suppose a loop of current exists in the x-y plane, and a uniform B-field is in the $\hat{z}$ direction. Which of the following will occur?

- A: The loop will not rotate - there is no torque.
- B: The loop will rotate 180 degrees - there is torque.
- C: The loop will rotate 90 degrees - there is torque.
- D: The loop will rotate -90 degrees - there is negative torque.

SUMMARY

Reading: Chapters 13 and 14

1. 13.1-2: Faraday's and Lenz's Law
2. 13.3: Motional EMF
3. 13.4: Induced E-fields

1. 14.1: Mutual inductance
2. 14.2: Self-inductance and inductors
3. 14.3: Energy in a magnetic field

FARADAY'S LAW AND LENZ'S LAW

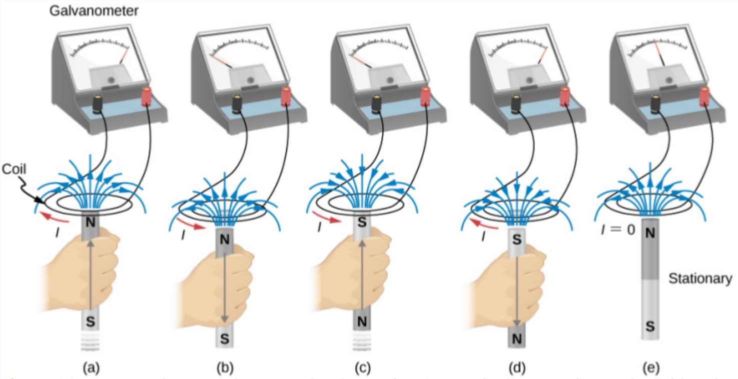

Figure 1: Not only does moving charge create B-fields, but B-fields can create moving charge. Study each of the cases above, and (Professor) define the concept of *magnetic flux*.

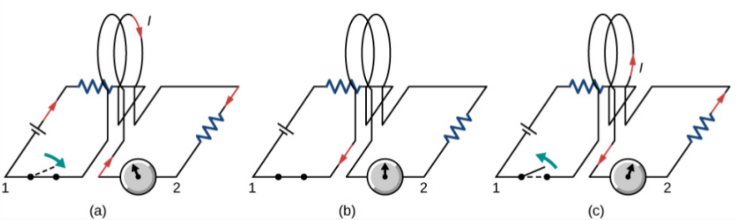

Figure 2: In addition to a moving magnetic field, *other circuits* can make current flow in a circuit. The effect must have something to do with *changing* magnetic fields.

Faraday's Law

The emf ϵ induced is the negative change in the magnetic flux Φ_m per unit time. Any change in the magnetic field or change in orientation of the area of the coil with respect to the magnetic field induces a voltage (emf).

$$\phi_m = \int_S \vec{B} \cdot d\vec{A} \tag{1}$$

$$\epsilon = -\frac{d\phi_m}{dt} \tag{2}$$

The unit of magnetic flux is the Webter, or 1 Wb = 1 T m^2.

Example: A square coil has sides 0.25 m long and is tightly wound with 200 turns of wire. The resistance of the coil 5.0 Ohms. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing by −0.040 T/s. (a) What is the magnitude of the emf induced in the coil? (b) What is the magnitude of the current circulating through the coil?

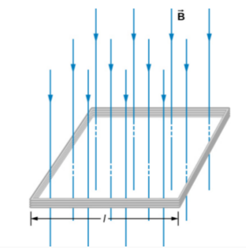

Figure 3: A 200 turn loop in a B-field.

Lenz's Law

The direction of the induced emf drives current around a wire loop to always oppose the change in magnetic flux that causes the emf.

Example: A magnetic field B is directed outward perpendicular to the plane of a circular coil of radius r = 0.50 m. The field is cylindrically symmetrical with respect to the center of the coil, and its magnitude decays exponentially according to

$$B(t) = B_0 \exp(-at) \tag{3}$$

with $B_0 = 1.5$ T and $a = 5.0\ \text{s}^{-1}$. (a) Calculate the emf induced in the coil at the times $t_0 = 0$, $t_1 = 0.05$, and $t_2 = 1.0$ seconds. (b) Determine the current in the coil if the resistance is 10 Ohms.

In the previous example, what would happen if the area A of the loop were increased?

- A: The current would decrease.
- B: The current would stay the same.
- C: The voltage would decrease.
- D: The voltage would increase.

In the previous example, what would happen if the sign of the exponent in $B(t)$ were flipped?

- A: The current would reverse direction and increase in magnitude.
- B: The current would reverse direction and decrease in magnitude.
- C: The current would keep the same direction and increase in magnitude.
- D: The current would keep the same direction and decrease in magnitude.

In the previous example, what would happen if α in the exponent in $B(t)$ were increased?

- A: The current would reverse direction and increase in magnitude.
- B: The current would reverse direction and decrease in magnitude.
- C: The current would keep the same direction and increase in magnitude.
- D: The current would keep the same direction and decrease in magnitude.

Example: The square coil of Figure 4 has sides l = 0.25 m long and is tightly wound with N = 200 turns of wire. The resistance of the coil is R = 5.0 Ω. The coil is placed in a spatially uniform magnetic field that is directed perpendicular to the face of the coil and whose magnitude is decreasing at a rate $dB/dt = 0.040t^2$. (a) Graph the magnitude of the emf induced in the coil. (b) What is the magnitude of the current through the coil at 100 ms?

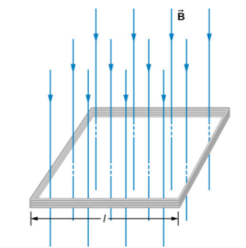

Figure 4: A 200 turn loop in a B-field.

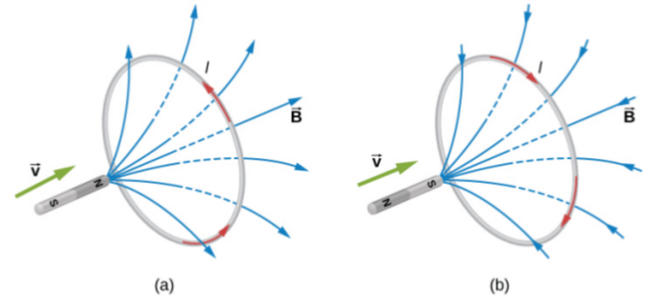

Figure 5: Lenz's Law relates sign of current to B-field.

MOTIONAL EMF

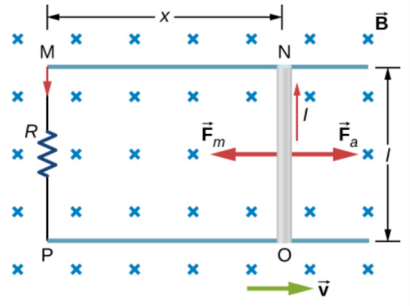

Figure 6: A system in which the magnetic flux depends on time.

1. Show that power is equal to $P = \vec{F} \cdot \vec{v}$ for constant acceleration.
2. Show that the emf is $\epsilon = Blv$, from Faraday's Law.
3. Show that power generated, $P = I^2R$, is equal to power injected.

In the previous example, what would happen if $\vec{F}_a$ was pointed to the left?

- A: The current would reverse direction.
- B: The current would keep the same direction.
- C: The magnetic flux due to the external field would decrease.
- D: A and C

In the previous example, what would happen if R were increased, but the magnitude of F_a were kept the same?

- A: The current would decrease.
- B: The current would increase.
- C: The current would remain constant.
- D: The power required would increase.

INDUCED ELECTRIC FIELDS

Recall that the relationship between voltage and electric field is

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{x} - \frac{\partial V}{\partial y}\hat{y} - \frac{\partial V}{\partial z}\hat{z} \tag{4}$$

In one dimension, this becomes

$$\vec{E} = -\frac{dV}{dx}\hat{x} \tag{5}$$

If we take a dot product with $-d\vec{x} = -dx\,\hat{x}$ on each side, we find

$$-\vec{E} \cdot d\vec{x} = dV \tag{6}$$

Integrating, we have

$$V = -\int \vec{E} \cdot d\vec{x} \tag{7}$$

However, if the voltage is a result of a changing magnetic field, and Faraday's Law, then

$$\frac{d\phi_m}{dt} = \oint \vec{E} \cdot d\vec{x} \tag{8}$$

Recall that from *electrostatics*,

$$\oint \vec{E} \cdot d\vec{x} = 0 \tag{9}$$

Equation 9 is true for eletrostatics because the Coulomb force is **conservative**. But in a previous example we showed that power was being generated and *conserved*, despite the fact that magnetic flux is changing. What is happening?

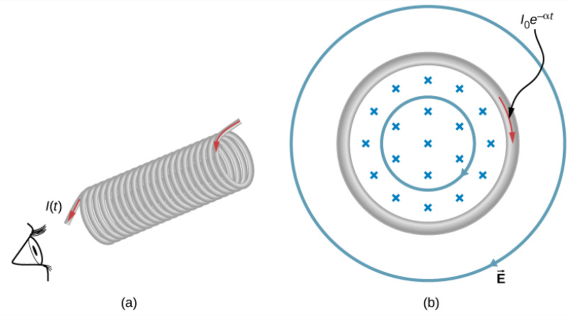

Figure 7: A solenoid with a changing current will induce an E-field. The solenoid has turn density *n*, and is long compared to the radius.

1. What is the E-field outside the solenoid?
2. What is the E-field inside the solenoid?
3. Create a graph of the E-field strength versus distance.

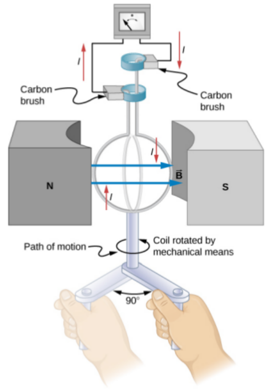

Figure 8: The basic concept behind an AC generator.

Start with Faraday's Law:

$$\epsilon = -\frac{d\phi_B}{dt} \tag{10}$$

The flux ϕ_B is changing and depends on time:

$$\phi_B = \vec{B} \cdot \vec{A}(t) = BA\cos(\theta(t)) \tag{11}$$

Let the *angular velocity* be constant: $\theta = \omega t$. Then we have

$$\phi_B = BA\cos(\omega t) \tag{12}$$

Thus the emf (with N loops) is

$$\epsilon = N\omega BA\sin(\omega t) = \epsilon_0\sin(\omega t) \tag{13}$$

The generation of AC power stems from ω.
(Professor: diagram of $\epsilon(t)$).

$$\epsilon = N\omega BA \sin(\omega t) \tag{14}$$

The AC voltage equation above is a basic model for the voltage from a generator. Which of the following would increase the *amplitude* of the emf?

- A: Turning the area more slowly.
- B: Turning the area more quickly.
- C: Increasing the B-field.
- D: Both B and C.

$$\epsilon = N\omega BA \sin(\omega t) \tag{15}$$

The AC voltage equation above is a basic model for the voltage from a generator. Which of the following would increase the *frequency* of the emf?

- A: Turning the area more slowly.
- B: Turning the area more quickly.
- C: Increasing the B-field.
- D: Both B and C.

INDUCTANCE

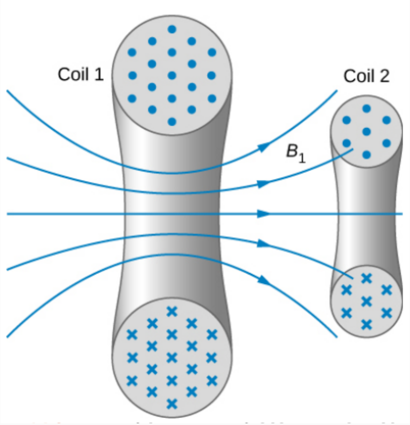

Figure 9: The concept of mutual inductance.

First, some notation:

- The flux through coil 2 by coil 1: ϕ_{21}
- The flux through coil 1 by coil 2: ϕ_{12}

Mutual inductance of coil 2 with respect to coil 1:

$$M_{21} = \phi_{21}\frac{N_2}{I_1} \tag{16}$$

Mutual inductance of coil 1 with respect to coil 2:

$$M_{12} = \phi_{12}\frac{N_1}{I_2} \tag{17}$$

It can be shown that

$$\boxed{M_{21} = M_{12}} \tag{18}$$

What are the units of mutual inductance? Consider the emf induced in loop 2 by loop 1:

$$\epsilon_2 = -\frac{d}{dt}\left(\phi_{21} N_2\right) \tag{19}$$

Substitution for the inductance gives

$$\epsilon_2 = -\frac{d}{dt}\left(\frac{M_{21} I_1}{N_2} N_2\right) \tag{20}$$

$$\epsilon_2 = -\frac{d}{dt}\left(I_1 M_{21}\right) \tag{21}$$

$$\epsilon_2 = -M\frac{dI_1}{dt} \tag{22}$$

$$\epsilon_1 = -M\frac{dI_2}{dt} \tag{23}$$

So inductance relates induced emf to current change, and has units of V s A^{-1}.

A coil of N_2 turns and radius R_2 surrounds a long solenoid of length l_1, radius R_1, and N_1 turns. (a) What is the mutual inductance of the two coils? (b) If $N_1 = 1000$, $N_2 = 20$, $R_1 = 3.0$ cm, $l_1 = 100.0$ cm, and $dI_1/dt = 150$ A/s, what is the induced emf in the surrounding coil?

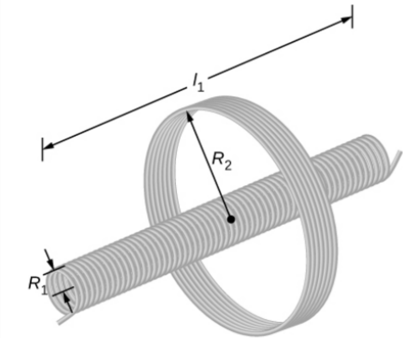

Figure 10: Example of mutual inductance.

A current $I(t) = I_0 sin(\omega t)$ flows through the solenoid. If $I_0 = 7.5$ A, and $\omega = 60\pi$ rad/sec, what is the maximum induced emf in the surrounding coil?

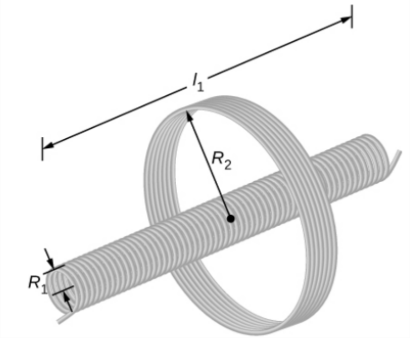

Figure 11: Example of mutual inductance.

SELF-INDUCTANCE AND INDUCTORS

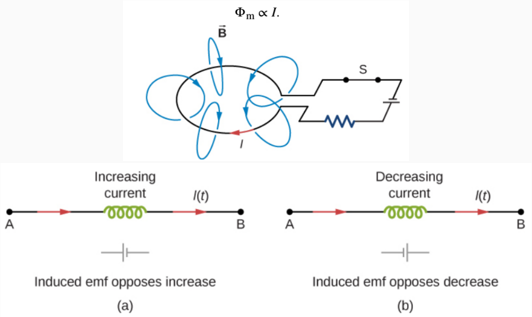

Figure 12: Self-inductance in a circuit, denoted L, rather than M.

Define

$$\epsilon = -L\frac{dI}{dt} \tag{24}$$

$$N\phi_m = LI \tag{25}$$

(Observe on board): Show that the inductance of a solenoid with volume V and turn density n is

$$L = \mu_0 n^2 V \tag{26}$$

Suppose we're making an inductor out of a coil of wire shaped like a solenoid. What happens to the inductance if we double the number of coils but keep the length and radius the same?

- A: L doubles.
- B: L quadruples.
- C: L gets divided by 2.
- D: L gets divided by 4.

Suppose we're making an inductor out of a coil of wire shaped like a solenoid. What happens to the inductance if we double the number of coils, but also double the length?

- A: L doubles.
- B: L quadruples.
- C: L gets divided by 2.
- D: L gets divided by 4.

We're almost finished with our design, which currently has an inductance L_0. We decide to double the number of turns, but keep the length the same. Also, we insert a type of iron into the solenoid that boosts μ_0 by a factor of 10^3, so the new inductance is L. What is L/L_0?

- A: 1000
- B: 2000
- C: 4000
- D: 8000

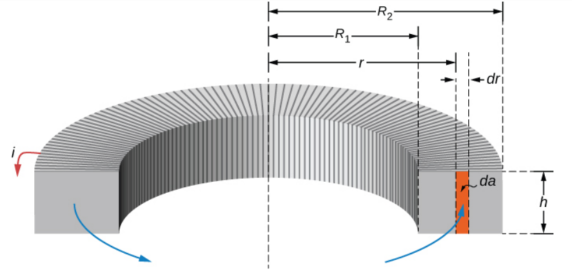

Figure 13: A rectangular toroid.

(Observe on board): Show that the inductance of a rectangular toroid as defined above is

$$L = \frac{\mu_0}{2\pi} N^2 h \ln\left(\frac{R_2}{R_1}\right) \tag{27}$$

Suppose we're making an inductor out of a toroid as in the previous slide. If we decrease the number of turns by a factor of 2, what happens to the inductance?

- A: L doubles.
- B: L quadruples.
- C: L gets divided by 2.
- D: L gets divided by 4.

Suppose we're making an inductor out of a toroid as in the previous slide. If we decrease the number of turns by a factor of 2 and double the height, what happens to the inductance?

- A: L doubles.
- B: L quadruples.
- C: L gets divided by 2.
- D: L gets divided by 4.

Suppose we're making an inductor out of a toroid as in the previous slide. If we quadruple the outer radius and only double the inner radius, what happens to the inductance?

- A: L increases by a factor of 2.
- B: L increases by a factor of ln(2).
- C: L increases by a factor of ln(4).
- D: L increases by a factor of 4.

ENERGY IN MAGNETIC FIELDS

The speed of light emerges in a most unexpected place.

PHET: THE RL CIRCUIT

Suppose we apply the loop rule to a circuit with a resistor and an inductor (observe on board).

We can show the current follows

$$I(t) = I_0 \left(1 - \exp(-t/\tau_l)\right) \tag{28}$$

Go to the link:

`https://phet.colorado.edu/en/simulation/circuit-construction-kit-ac-virtual-lab`

Instructions:

1. Select a battery at right and place it in the main area. Right click on it to set the voltage to 12.0 V.
2. Connect two wires to the battery, using the wire tool at right.
3. Connect a resistor to one of the wires (leaving the circuit open). Right click on it to set the resistance to 50 Ω.
4. Connect an inductor to one wire, using the inductor tool (bottom right), and right click it. Set the inductance to 100 Henries.
5. If you were to close this circuit by adding the final wire, how long would it take the current to reach the steady state?
6. What should the value of the steady state current be?
7. Select “current chart” at right and drag the reticule to measure the current in the circuit. Complete the circuit with a final wire.

Instructions:

1. How long before the current reaches the steady state?
2. What is the steady state current? Does it match your expectation?
3. Change the battery to an AC voltage source by right clicking the battery and selecting “remove,” and replacing with AC voltage object at right. You can control the properties of of the AC source by right clicking it.
4. Do a *frequency sweep*, by observing the current as you tune the AC voltage frequency from 0.0 to 2.0 Hz. What happens to the amplitude? Why does this make sense physically?
5. What do we call devices that change the output amplitude depending on frequency? (See COSC360: Digital Signal Processing).

CONCLUSION

Reading: Chapters 13 and 14

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ANSWERS - CHAPTER 13 AND UNIT 4 REVIEW

ANSWERS

- B
- A
- D
- A
- D
- D
- A
- D
- B

ANSWERS - CHAPTER 14

ANSWERS

- B
- A
- C
- D
- C
- C

- ...