Warm Up: Work and Energy

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1 Memory Bank

- $W = \vec{F} \cdot \Delta \vec{x}$ ... Definition of work
- $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$ ... The dot-product of $\vec{a}$ and $\vec{b}$. The components of $\vec{a}$ and $\vec{b}$ are a_x and a_y, and b_x and b_y, respectively.
- $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$ ... The dot-product of $\vec{a}$ and $\vec{b}$. The magnitude of $\vec{a}$ is $|\vec{a}|$, and the magnitude of $\vec{b}$ is $|\vec{b}|$.
- $KE = \frac{1}{2}mv^2$ ... Definition of Kinetic Energy
- $W = KE_f - KE_i$ ... Work-energy theorem.
- $\int k \ dx = kx + C$ ... The *integral* of a constant function, $f(x) = k$, where C is some constant.
- $\int kx \ dx = \frac{1}{2}kx^2 + C$ ... The *integral* of a linear function, $f(x) = kx$, where C is some constant.

2 Work and Energy

1. In Fig. 1 below, assume that the upper vector represents force and the lower vector represents displacement. (a) Which scenario (left, middle, or right) corresponds to the maximum amount of work done on the system? (b) In Fig. 1 (left), let $\vec{F} = 15\hat{i} + 15\hat{j}$ N and $\Delta\vec{x} = 15\hat{i}$ meters. What is the work done? (c) For Fig. 1 (middle), if $\Delta\vec{x} = 15\hat{i}$ meters, and $W = 0$, what is $\vec{F}$? (d) What is the ratio of the work in Fig. 1 (left) to the work in Fig. 1 (right)?

2. Suppose the system in Fig. 1 (left) has a mass of 10 kg, and the work done is 100 J through a displacement of 2 meters. (a) What is the magnitude of the force? (b) What is the force as a vector? (c) If the total work done is 100 J, what is the final velocity of the system?

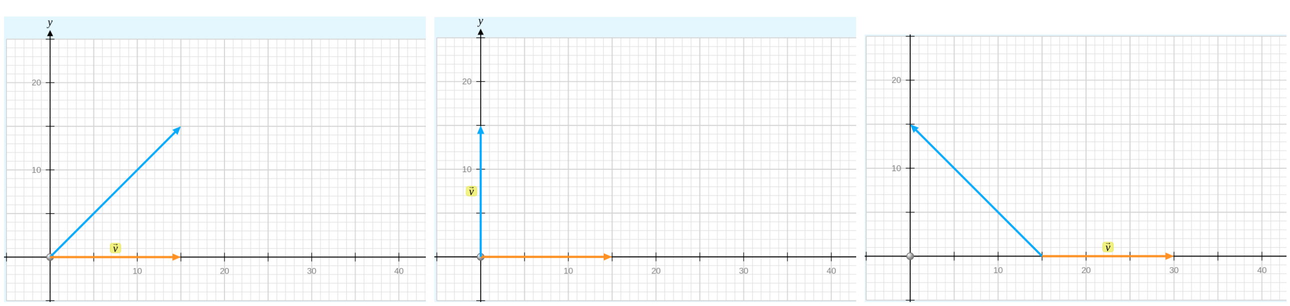

Figure 1: (Left) Two vectors separated by 45 degrees. (Middle) Two vectors separated by 90 degrees. (Right) Two vectors separated by 135 degrees.