

Answer-key 2 for Calculus-Based Physics-1: Mechanics (PHYS150-01)

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October 16th, 2017

1 Vectors and Newton's Laws

- Let $\vec{F}_1 = -\frac{3}{2}\hat{x} + 2\hat{y}$ N, and $\vec{F}_2 = -2\hat{x} + \frac{3}{2}\hat{y}$ N. a) Give the magnitude of each force. b) What is the net force? c) What is the angle between these two forces?
a) $|\vec{F}_1| = \sqrt{\left(\frac{3}{2}\right)^2 + (2)^2} = \sqrt{\frac{25}{4}} = \frac{5}{2}$. The magnitude of the forces are equal.
b) $\vec{F}_1 + \vec{F}_2 = \left(\frac{3}{2} - 2\right)\hat{i} + \left(2 + \frac{3}{2}\right)\hat{j} = -\frac{1}{2}\hat{i} + \frac{7}{2}\hat{j}$.
c) $\vec{F}_1 \cdot \vec{F}_2 = 0$, so the cosine of the angle is 0, meaning $\theta = 90$ degrees.
- Imagine you are sitting in an airplane that has just lifted off with an acceleration vector 45 degrees with respect to horizontal. Draw a free-body diagram corresponding to you, showing all forces acting on you.

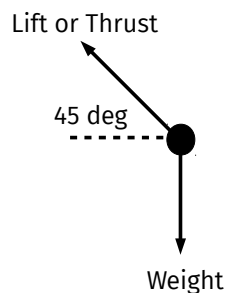


Figure 1: Answer to number 2.

- Imagine you are riding a skateboard down a hill (no friction) and the incline angle is 45 degrees. Draw a free-body diagram corresponding to you, showing all forces acting on you.

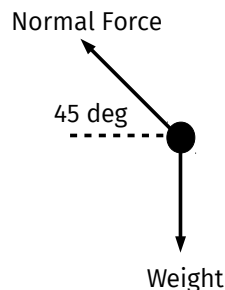


Figure 2: Answer to number 3, which turns out to be similar to number 2.

2 Newton's Laws, and Circular Motion

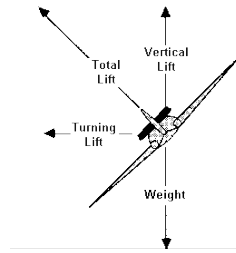


Figure 3: Let the weight be \vec{w} , and the total lift be \vec{L} , which may be broken into two components: the turning force (equal to centripetal force \vec{f}_C) and vertical lift (which balances weight).

- When banking, the free-body diagram of a jet-fighter resembles Fig. 3. To bank while maintaining altitude, the lift force \vec{L} must *both* balance the weight \vec{w} and provide the centripetal force \vec{f}_C . Let the mass of the aircraft be m , the radius of the turn be r , and the angle between \vec{L} and horizontal be θ .
 - Show that the angular velocity of the turn, ω , is $\omega = \sqrt{\frac{L \cos \theta}{rm}}$
 Turning force provides centripetal force: $L \cos \theta = mr\omega^2$. Solve for ω : $\omega = \sqrt{\frac{L \cos \theta}{rm}}$
 - If ω is the angular velocity, then the *period* is $T = 2\pi/\omega$. This is the time required to fly in a complete circle. Show that one-half period is $\frac{T}{2} = \pi \sqrt{\frac{rm}{L \cos \theta}}$. This is the time required to turn.
 If $\omega = \sqrt{\frac{L \cos \theta}{rm}}$, then $2\pi/\omega = 2\pi \sqrt{\frac{rm}{L \cos \theta}}$, and half of that is $\frac{T}{2} = \pi \sqrt{\frac{rm}{L \cos \theta}}$. This takes the aircraft half-way around a circle, or reversing course.
 - Let $L = 8 \times 10^5$ N, $m = 2 \times 10^4$ kg, $r = \frac{1}{2}$ km, and $\theta = 60$ degrees. How long does it take the jet fighter to turn?
 Plugging in the numbers, we get about 5π seconds, or about 15.7 seconds.
 - What is the speed of the jet fighter?
 If $v = r\omega$, then $v = \sqrt{\frac{rL \cos \theta}{m}}$. Putting in numbers, we have ≈ 100 m/s.
 - A: 10 m/s
 - B: 20 m/s
 - C: **100 m/s** (this is also a good estimate)
 - D: 120 m/s (this is a good estimate, but off by 20%).

3 Frictional Forces

- There is a spill of a mystery toxic liquid on a shop floor, and no one wants to touch it. Someone gets the bright idea that they can identify it by the coefficient of kinetic friction and a steel plate. Draw a free body diagram corresponding to a steel plate sliding along the liquid/floor, with friction decelerating it.

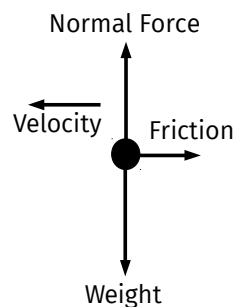


Figure 4: Answer to number 1.

- What is the coefficient of kinetic friction, μ_k , if a steel plate with an initial speed of 5 m/s comes to a stop after 2.5 seconds, assuming $g = 10$ m/s²? (Use the definition of acceleration $\Delta v/\Delta t = a$).

The net force is the frictional force, $\vec{F}_{\text{Net}} = m\vec{a} = \mu mg\hat{x}$, so $a = \mu g$. Putting this into the definition of acceleration, we have $g^{-1}\Delta v/\Delta t = \mu$. That's $10^{-1}(5/2.5) = 0.2$, so

- 0.1
- **0.2**
- 0.5
- 1.2 This answer is larger than 1, so forbidden.

3. Suppose they get a sample of the mystery liquid in a vile. They assume the drag force is given by Stoke's Law, $F_D = 6\pi r\eta v$, where v is the velocity of a particle moving through the fluid, r is the radius of the particle, and η is the *viscosity*. They drop a bead with $r = 1$ mm and a mass of one gram into the fluid, and observe the bead sink with a constant (terminal) velocity of 1 m/s. What is the viscosity of the fluid? Units: kg/(m s).

- $5/(3\pi)$ kg/(m s)
- $5/(30\pi)$ kg/(m s)
- 10 kg/(m s)
- 5 kg/(m s)