PhET Activity: Work and Energy with the Pendulum

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1 Memory Bank

- Derivative of sine: $\frac{d}{dx}\sin(kx) = k\cos(kx)$
- Derivative of cosine: $\frac{d}{dx}\cos(kx) = -k\sin(kx)$

2 Introduction

Let the angle a pendulum makes with the vertical line be θ. If $\theta \ll 1$, the position of the mass at the end of pendulum is

$$x = L\theta \tag{1}$$
$$y = \frac{1}{2}L\theta^2 \tag{2}$$

The gravitational potential energy of the pendulum is

$$U(y) = mgy \tag{3}$$
$$U(\theta) = \frac{1}{2}mgL\theta^2 \tag{4}$$
$$k = mgL \tag{5}$$
$$U(\theta) = \frac{1}{2}k\theta^2 \tag{6}$$

Notice that the potential energy is a quadratic function, like the potential energy of the spring ($U(x) = \frac{1}{2}kx^2$). Since $x = L\theta$, $dx = Ld\theta$. This makes the derivative of $-U$ become

$$F = -\frac{dU}{dx} = -\frac{dU}{Ld\theta} = -mg\theta \tag{7}$$

Using Newton's 2nd Law,

$$m\frac{d^2x}{dt^2} = -mg\theta \tag{8}$$
$$\frac{d^2x}{dt^2} = -g\theta \tag{9}$$
$$L\frac{d^2\theta}{dt^2} = -g\theta \tag{10}$$
$$\frac{d^2\theta}{dt^2} = -\left(\frac{g}{L}\right)\theta \tag{11}$$

Let the *angular frequency* ω be defined by $\omega^2 = g/L$. Equation 11 becomes

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta \tag{12}$$

Equation 12 produces a sum of sines and cosines.

3 Physical Pendulum Behavior

Notice the form of the physical pendulum constructed in the lab. A ruler is suspended with a string, clamps, and rods. The ruler has a mass of 87 grams. We can tape the 50 gram weight to various locations along the ruler.

1. Create a graph below of the *period* of the pendulum versus the location of the 50 gram weight.

2. Save a table of the *period* of the pendulum versus the location of the 50 gram weight below.

3. How can you explain the trend in the data using torque, moment of inertia, and Eq. 12?