calculus-based physics-1: mechanics (phys150-01): week 3

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Week 2 Review

1. Displacement, and instantaneous velocity and acceleration
    - *Mathematics review*: taking derivatives
    - Average velocity and average acceleration
2. The case of constant acceleration
    - An *an equation of motion* for constant acceleration
    - Derivation of common equations of motion
    - Average quantities and exercises
3. **Lab Activity: Measuring acceleration of gravity: g**
4. Exercises with vectors, graphs, and equations of motion

WEEK 2 REVIEW PROBLEMS

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

- A: To the right, positive
- B: To the right, negative
- C: To the left, positive
- D: To the left, negative

An object that is thrown straight up falls back to Earth. When is its velocity zero? Does its velocity change direction? Does the acceleration change sign?

- A: During flight, yes, no
- B: At the peak height, yes, yes
- C: At the peak height, yes, no
- D: During flight, no, no

WEEK 3 SUMMARY

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1. Displacement, velocity and acceleration vectors as functions of time
    - Breaking into components
    - Derivatives of components
2. Combining free-fall and vector components: projectile motion
    - The independence of velocity components
    - **Lab-activity: testing component independence**
3. Relative motion and reference frames
    - Relative motion in one-dimension
    - Relative motion in two-dimensions

VECTORS AS FUNCTIONS OF TIME

In general, the displacement of an object depends on time:

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} \tag{1}$$

- $x(t)$ is the displacement in the x-direction
- $y(t)$ is the displacement in the y-direction
- $z(t)$ is the displacement in the z-direction

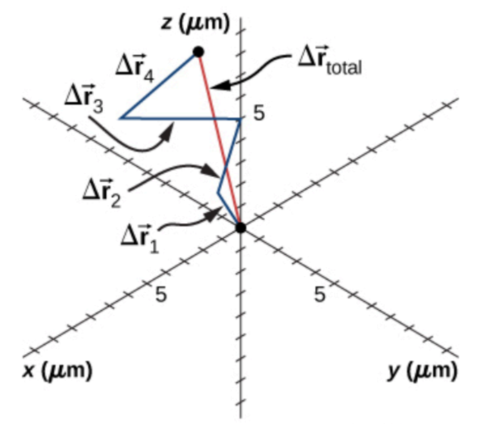

Figure 1: An example of a displacement vector at different moments in time.

The particle in Fig. 1 has four displacement vectors at four moments in time:

- $\vec{r}_1 = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k} \quad (\mu m)$ at t_1
- $\vec{r}_2 = -1.0\hat{i} + 0.0\hat{j} + 3.0\hat{k} \quad (\mu m)$ at t_2
- $\vec{r}_3 = 4.0\hat{i} + -2.0\hat{j} + 1.0\hat{k} \quad (\mu m)$ at t_3
- $\vec{r}_4 = -3.0\hat{i} + 1.0\hat{j} + 2.0\hat{k} \quad (\mu m)$ at t_4

What is the total displacement of the particle from the origin?

We can think of this type of problem as an accounting problem, lining up columns (units: μm):

t_{i}	$\vec{r}_{\mathrm{i}}(t_{\mathrm{i}})$	$x(t_{\mathrm{i}})$	$y(t_{\mathrm{i}})$	$y(t_{\mathrm{i}})$
t_1	$\vec{r}_1(t_1)$	2.0	1.0	3.0
t_2	$\vec{r}_2(t_2)$	-1.0	0.0	3.0
t_3	$\vec{r}_3(t_3)$	4.0	-2.0	1.0
t_4	$\vec{r}_4(t_4)$	-3.0	1.0	2.0
t_{total}	$\vec{r}_{\mathrm{total}}(t_{\mathrm{total}})$	2.0	0.0	9.0

Figure 2: Accounting for the different displacement components, in units of μm.

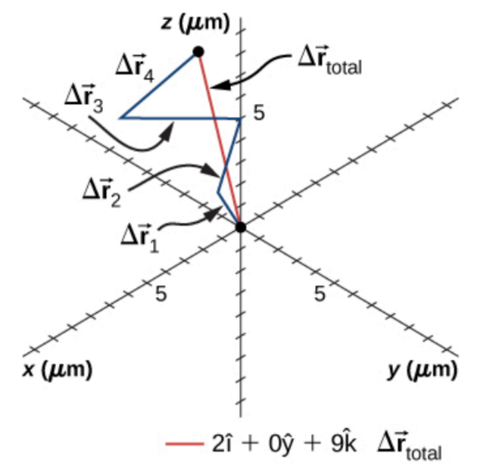

Figure 3: The total displacement of the particle is $\vec{r}_{\text{total}} = 2.0\hat{i} + 0.0\hat{k} + 9.0\hat{k} \quad (\mu m)$.

The 18th hole at Pebble Beach Golf Course is a dogleg to the left of length 496.0 meters. The fairway off the tee is taken to be the x direction. A golfer hits his tee shot a distance of 300 meters, corresponding to a displacement of $\vec{r}_1 = 300.0\hat{i} \quad (m)$, and then hits a second shot 189.0 meters with $\vec{r}_2 = 172.0\hat{i} + 80.3\hat{j} \quad m$. What is the final displacement from the tee?

- A: $\vec{r}_{\rm final} = 172.0\hat{i} + 80.3\hat{j} \quad (m)$
- B: $\vec{r}_{\rm final} = 172.0\hat{i} + 380.3\hat{j} \quad (m)$
- C: $\vec{r}_{\rm final} = 472.0\hat{i} + 0.0\hat{j} \quad (m)$
- D: $\vec{r}_{\rm final} = 472.0\hat{i} + 80.3\hat{j} \quad (m)$

If the first shot takes 5.0 seconds, the second shot takes 4.0 seconds, and the walking time in between the shots is 60.0 seconds, what is the average velocity vector for the ball after the two shots?

- A: $\vec{r}_{\text{final}} = 1.7\hat{i} + 8.3\hat{j} \quad (m/s)$
- B: $\vec{v}_{\text{final}} = 172.0\hat{i} + 80.3\hat{j} \quad (m/s)$
- C: $\vec{v}_{\text{final}} = 6.8\hat{i} + 1.2\hat{j} \quad (m)$
- D: $\vec{v}_{\text{final}} = 6.8\hat{i} + 1.2\hat{j} \quad (m/s)$

The prior problem indicates something you may already have guessed:

$$\vec{v}_{\mathrm{avg}}(t) = v_{\mathrm{x}}(t)\hat{i} + v_{\mathrm{y}}(t)\hat{j} + v_{\mathrm{z}}(t)\hat{k} = \frac{\Delta\vec{r}}{\Delta t} \tag{2}$$

- $v_{\mathrm{x}}(t)$ is the avg. velocity in the x-direction
- $v_{\mathrm{y}}(t)$ is the avg. velocity in the y-direction
- $v_{\mathrm{z}}(t)$ is the avg. velocity in the z-direction

In other words, we divide each displacement component by the time, to get a vector where each component is the average velocity in that direction. $\Delta\vec{r} = \vec{r}_{\mathrm{f}} - \vec{r}_{\mathrm{i}}$.

Instantaneously, Eq. 2 is true, if we take the limit $\Delta t \to 0$:

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} \tag{3}$$

- $\frac{dx}{dt}$ is the instantaneous velocity in the x-direction
- $\frac{dy}{dt}$ is the instantaneous velocity in the y-direction
- $\frac{dz}{dt}$ is the instantaneous velocity in the z-direction

The position of a particle is $\vec{r}(t) = 4.0t^2\hat{i} - 3.0\hat{j} + 2.0t^2\hat{k}$ (m). What is the velocity vector at $t = 2$ seconds? What is the average velocity between $t = 0$ and $t = 2$ seconds?

- A: $16\hat{x} + 8\hat{z}$ (m/s), $8\hat{x} + 4\hat{z}$ (m/s)
- B: $8\hat{x} + 4\hat{z}$ (m/s), $4\hat{x} + 2\hat{z}$ (m/s)
- C: $8\hat{x} + 8\hat{z}$ (m/s), $4\hat{x} + 4\hat{z}$ (m/s)
- D: $4\hat{x} + 2\hat{z}$ (m/s), $4\hat{x} + 2\hat{z}$ (m/s)

Instantaneously, from Eq. 3:

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{dv_{\mathrm{x}}}{dt}\hat{i} + \frac{dv_{\mathrm{y}}}{dt}\hat{j} + \frac{dv_{\mathrm{z}}}{dt}\hat{k} \tag{4}$$

- $\frac{dv_{\mathrm{x}}}{dt}$ is the instantaneous acceleration in the x-direction
- $\frac{dv_{\mathrm{y}}}{dt}$ is the instantaneous acceleration in the y-direction
- $\frac{dv_{\mathrm{z}}}{dt}$ is the instantaneous acceleration in the z-direction

The velocity of a particle is $\vec{v}(t) = 8.0t\hat{i} + 4.0t\hat{k}$ (m/s). What is the acceleration vector at $t = 2$ seconds? What is the average acceleration between $t = 0$ and $t = 2$ seconds?

- A: $4\hat{i} + 4\hat{k}$ (m/s^2), $2\hat{i} + 2\hat{k}$ (m/s^2)
- B: $8\hat{i} + 4\hat{k}$ (m/s^2), $8\hat{i} + 4\hat{k}$ (m/s^2)
- C: $8\hat{i} + 8\hat{k}$ (m/s^2), $4\hat{i} + 4\hat{k}$ (m/s^2)
- D: $4\hat{i} + 8\hat{k}$ (m/s^2), $2\hat{i} + 4\hat{k}$ (m/s^2)

The displacement of a particle is $\vec{x}(t) = (2t+3)\hat{i} + (\frac{3}{2}t^2 + 2t + 3.0)\hat{j}$ (m). What is the horizontal velocity (the $\hat{i}$-component of the velocity) at $t = 4$ seconds? At $t = 10$ seconds?

- A: 4 m/s, 4 m/s
- B: 2 m/s, 4 m/s
- C: 2 m/s, 2 m/s
- D: 4 m/s, 2 m/s

The displacement of a particle is $\vec{x}(t) = (2t + 3)\hat{i} + (\frac{3}{2}t^2 + 2t + 3.0)\hat{j}$ (m). What is the vertical velocity (the $\hat{j}$-component of the velocity) at $t = 4$ seconds? At $t = 10$ seconds?

- A: 14 m/s, 32 m/s
- B: 32 m/s, 14 m/s
- C: 12 m/s, 30 m/s
- D: 30 m/s, 12 m/s

Notice in the previous example, the x-velocity and y-velocity were not the same function.

In the kinematic description of motion, *we are able to treat the different components of motion separately.* In many cases, motion in the horizontal direction does not affect motion in the vertical direction, and vice versa.

Motions in displacement components are independent.

(Exception: non-conservative forces. More on this later.)

COMBINING FREE-FALL AND VECTOR COMPONENTS: PROJECTILE MOTION

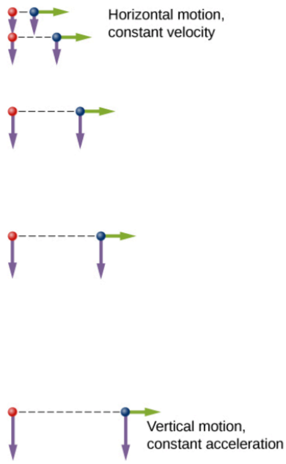

Figure 4: The red particle accelerates vertically, with no horizontal velocity. The blue particle acclerates vertically, with some horizontal velocity.

LAB ACTIVITY

Is this true? Figure 4 is testable by experiment.

Procedure:

1. Obtain two marbles, a meter stick, and a stopwatch.
2. Measure the height of the lab bench, Δx.
3. We are going to drop a marble from this height (Δx) and record the time. Show first algebraically that the predicted time for the marble to fall is $t = \sqrt{2\Delta x/g}$.
4. Measure t for several trials. Does it match the expected result $\sqrt{2\Delta x/g}$? What are sources of error?
5. Repeat the measurement, but **roll the marble off of the table instead of dropping it** from Δx. Does the average result for t change?

ANSWERS

- To the right, positive
- At the peak height, yes, yes
- $\vec{r}_{\text{final}} = 472.0\hat{i} + 80.3\hat{j} \quad (m)$
- $\vec{v}_{\text{final}} = 6.8\hat{i} + 1.2\hat{j} \quad (m/s)$
- $16\hat{x} + 8\hat{z}$ (m/s), $8\hat{x} + 4\hat{z}$ (m/s)
- $8\hat{i} + 4\hat{k}$ (m/s^2), $8\hat{i} + 4\hat{k}$ (m/s^2)
- 2 m/s, 2 m/s
- 14 m/s, 32 m/s