

CALCULUS-BASED PHYSICS-1: MECHANICS (PHYS150-01): WEEK 7

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WEEK 6 REVIEW

1. **Work** has a scientifically precise definition
 - Units
 - As a product of force and displacement vectors
2. Kinetic Energy and the **Work-Energy Theorem**
3. Gravitational potential energy
 - Potential energy
 - *Simplifying otherwise complex calculations*
 - Potential energy near Earth's surface
 - ...in space
4. Definition of a **conservative force**
 - Relationship between conservative forces and potential energy
 - Conservation of energy for conservative forces

WEEK 6 REVIEW PROBLEMS

WEEK 6 REVIEW PROBLEM

Recall that the *gravitational potential energy* associated with an object of mass m located a height y above ground level is $U = mgy$. What is $-dU/dy$?

- A: mg
- B: $-mg$
- C: g
- D: $-g$

What is the physical meaning of the quantity $-mg = -dU/dy$?

- A: The force of gravity
- B: The acceleration due to gravity
- C: The mass
- D: Change in kinetic energy

WEEK 7 SUMMARY

1. **Work** and **potential energy**
 - **Lab activity:** Oscillator and gravity trading work and potential energy
2. Potential energy and **conservative forces**
3. **Conservation of Energy**
 - *Calculus review: the fundamental theorem of calculus*
 - Graphical representations of integrals and energy

WORK AND POTENTIAL ENERGY

WORK AND POTENTIAL ENERGY

When we do work on a system, even if in the final state the system has no velocity, it can still have energy. The concept of *potential energy* is **like thinking of work backwards**:

- If we compress an oscillator (a spring), and keep compressing it, it has no kinetic energy but it will have kinetic energy if we release it.
- If we raise an object against the force of gravity to a certain height, it has no kinetic energy but it will have kinetic energy if we drop it.
- But it took work to create this change in kinetic energy, from the work-energy theorem.

*Systems are not necessarily **required** to operate like this...Think of stretching plastic or dough. Where does the work go?*

The relationship between work and potential energy therefore resembles (by definition) the opposite of the work-energy theorem:

$$\Delta U_{AB} = U_B - U_A = -W_{AB} \quad (1)$$

WORK AND POTENTIAL ENERGY

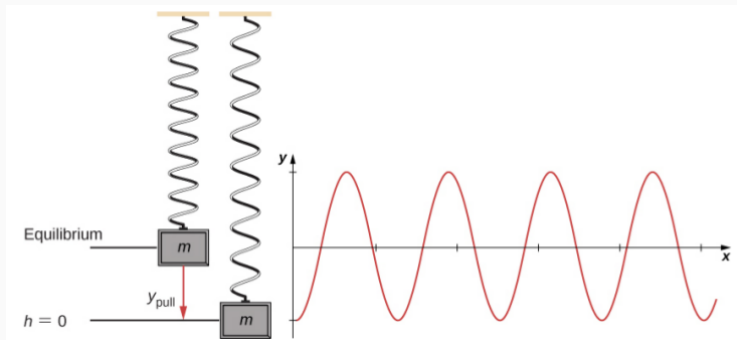


Figure 1: An oscillator is stretched to a new equilibrium point by gravity. When pulled a certain distance down, or compressed a certain distance upwards, y_{pull} , we observe oscillation.

WORK AND POTENTIAL ENERGY

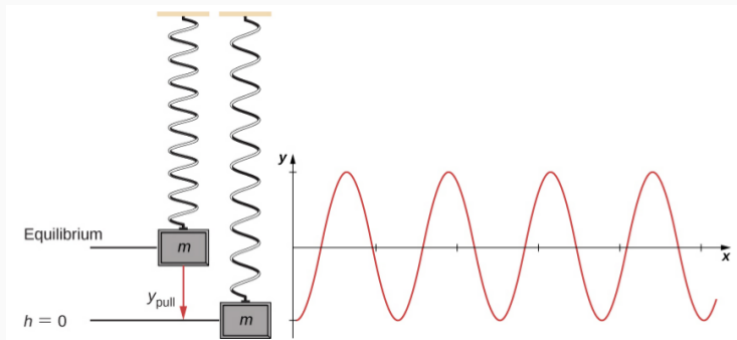


Figure 2: Notice that it does not matter what the observer defines as zero potential energy, since work is required to perform *changes* in potential energy.

WORK AND POTENTIAL ENERGY

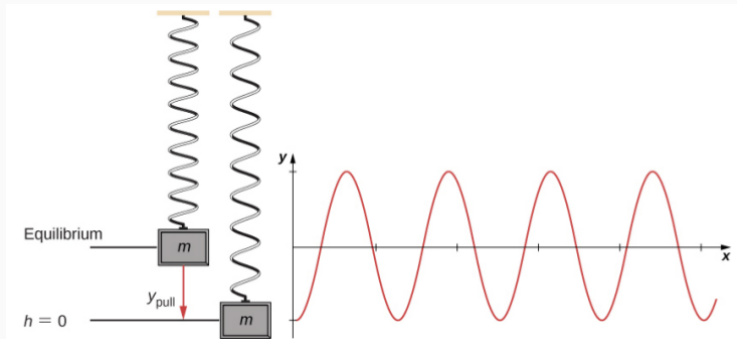


Figure 3: The fact that work is required only to perform changes in potential energy, but not does not determine the *absolute* scale of potential energy, means the observer may choose the location of zero potential energy, in the same fashion as choosing a coordinate system.

	Gravitational P.E.	Elastic P.E.	Kinetic E.
(3) Highest Point	$2mgy_{\text{pull}}$	$\frac{1}{2}ky_{\text{pull}}^2$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky_{\text{pull}}^2$	0

Figure 4: If a mass m is connected to the oscillator and we choose the potential energy zero-point to be **the low point of oscillation**, the values listed in this table correspond to the energies at various states.

LAB ACTIVITY - GRAVITY AND THE OSCIL- LATOR

LAB ACTIVITY: WORK AND POTENTIAL ENERGY

- Using the springs, weights, hooks, and system of clamps and grips, build a vertical oscillating system.
- Diagram the system, showing a clearly defined value for y_{pull} , and a clearly defined choice for the potential energy zero-point.
- Measure the *unstretched* spring length, and the *equilibrium length* caused by gravity, to **derive the spring constant k** . Quote the value of k in N/m.
- Pull the spring downwards by y_{pull} , and record the maximum and minimum heights of the weight as the spring oscillates it.
- Create a table like Tab. 4, and fill in the actual energy values in Joules. What is your predicted value for v , the speed at which the weight moves when the oscillator is at the equilibrium position?

	Gravitational P.E.	Elastic P.E.	Kinetic E.
(3) Highest Point	$2mgy_{\text{pull}}$	$\frac{1}{2}ky_{\text{pull}}^2$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky_{\text{pull}}^2$	0

Figure 5: If a mass m is connected to the oscillator and we choose the potential energy zero-point to be **the low point of oscillation**, the values listed in this table correspond to the energies at various states.

LAB ACTIVITY - GRAVITY AND THE OSCIL- LATOR, PART 2

LAB ACTIVITY: WORK AND POTENTIAL ENERGY

- Measure the *unstretched* spring length, and the *equilibrium length* caused by gravity, to **derive the spring constant k** . Quote the value of k in N/m.
- Pull the spring downwards by y_{pull} , and record the maximum and minimum heights of the weight.
- Create a table like Tab. 4, and fill in the actual energy values in Joules. What is your predicted value for v ?
- Using the **Vernier LabPro** and the *motion detector attachment*, measure v when the spring is at equilibrium position, and quote the value in m/s. Does it agree with your prediction based on energy conservation? Why or why not?

	Gravitational P.E.	Elastic P.E.	Kinetic E.
(3) Highest Point	$2mgy_{\text{pull}}$	$\frac{1}{2}ky_{\text{pull}}^2$	0
(2) Equilibrium	mgy_{pull}	0	$\frac{1}{2}mv^2$
(1) Lowest Point	0	$\frac{1}{2}ky_{\text{pull}}^2$	0

Figure 6: The value for v is predicted by energy conservation. What do you measure?

POTENTIAL ENERGY AND CONSERVATIVE FORCES

POTENTIAL ENERGY AND CONSERVATIVE FORCES

Let path 1 be through a force field that does work W_1 on a system, and path 2 be a different path that does work W_2 on a system.

$$W_1 = \int_{\text{Path1}} \vec{F} \cdot d\vec{r} = \int_{\text{Path2}} \vec{F} \cdot d\vec{r} = W_2 \quad (2)$$

A force is conservative if

$$W_1 = W_2 \quad (3)$$

Suppose path 1 goes from point A to B, and path 2 returns from B to A. If the force remains constant, but the path is reversed, then $W_1 = -W_2$. But this means the path is *closed*, so

$$\oint \vec{F} \cdot d\vec{r} = W_1 + W_2 = W_1 - W_1 = 0 \quad (4)$$

CONSERVATION OF ENERGY

Bringing all these concepts together: **Conservation of Energy**.

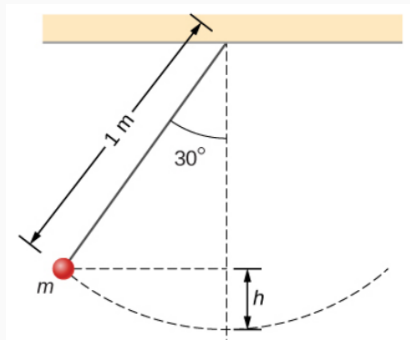
Conservation of Energy

$$\Delta(K E + P E) = 0$$

Recall the notion of a conservative force. In your own words, write down the defining characteristics of a conservative force. Which of these forces is NOT conservative?

- A: Stoke's Law of viscous drag
- B: Gravity near Earth's surface
- C: Hooke's Law
- D: All are conservative

CONSERVATION OF ENERGY



CONSERVATION OF ENERGY

We can show that a *pendulum* exhibits the same properties as a *spring*. A pendulum is like a spring, where the restoring force is determined by g , the gravitational acceleration, and L , the length of the pendulum. That is, $a_p = \frac{g}{L}x$, where x is the displacement from equilibrium, and a is the acceleration corresponding to x . (Derivation to follow). What is the force on a 1 meter-long pendulum suspending a mass of 100 grams if $x = 10$ cm?

- A: 10 N
- B: 1 N
- C: 0.1 N
- D: 0.01 N

The pendulum stores gravitational potential energy $U = mgh$. Set the zero point of gravitational potential energy to the lowest point of the pendulum. Show that the potential energy as a function of the angle θ is

$$U = mgL(1 - \cos \theta) \quad (5)$$

At which point in the trajectory of the mass on the end of the pendulum is the velocity the highest?

- The starting point at the top
- The lowest point
- The top point at the side opposite from release point
- In between the highest and lowest points

By setting pendulum potential energy equal to kinetic energy, show that the maximum velocity achieved by the pendulum mass is

$$v_{\max} = \sqrt{2Lg(1 - \cos \theta)}^{1/2} \quad (6)$$

If the length of the pendulum is 10 cm, and $g = 10$, and the initial angle is 30 degrees, what is the maximum speed of the pendulum?

- 0.1 m/s
- 0.5 m/s
- 1 m/s
- 5 m/s

We can show that the *period* of the pendulum is related to other quantities as follows:

$$T^2 = 4\pi^2 \left(\frac{L}{g} \right) \quad (7)$$

Using the PhET simulation below, measure the gravitational acceleration on the moon. Record your reasoning in lab notebooks.

https://phet.colorado.edu/sims/html/pendulum-lab/latest/pendulum-lab_en.html

CONSERVATION OF ENERGY

Conservative forces may be found by taking the negative derivative of the potential energies we've found through the work done:

Potential energy:

- $U_s = \frac{1}{2}kx^2$
- $U_g = mgy$
- $U_p = mgL(1 - \cos \theta)$

Corresponding force:

- $\vec{F} = -kx\hat{i}$
- $\vec{F} = -mg\hat{j}$
- $\vec{F} = -\left(\frac{mg}{L}\right)x\hat{i}$

Forces that follow Eq. 4 have to obey

$$\vec{F}(x) = -\frac{dU}{dx}\hat{i} \quad (8)$$

We may use conservation of energy to predict the behavior of systems, even when we don't know actual trajectory versus time...

CONSERVATION OF ENERGY

Suppose we encounter a particle with potential energy $U(x) = \frac{1}{2}kx^2 + Ae^{-\frac{1}{2}\left(\frac{x}{\sigma}\right)^2}$. If the particle has a mass m moving with this potential has a velocity v_b when located at $x = b$. Show that the particle will not cross the origin unless

$$A \leq \frac{kb^2 + mv_b^2}{2\left(1 - e^{-\frac{1}{2}\left(\frac{b}{\sigma}\right)^2}\right)} \quad (9)$$

Solve in groups at the boards. *Hint: Use conservation of energy, assuming the final energy is equal to the potential energy at the origin.*

Notice that if $\frac{1}{2} \left(\frac{b}{\sigma}\right)^2 \ll 1$, then

$$A \lesssim \frac{\frac{1}{2}kb^2 + \frac{1}{2}mv_b^2}{\frac{1}{2} \left(\frac{b}{\sigma}\right)^2} = \frac{E_i^0}{\frac{1}{2} \left(\frac{b}{\sigma}\right)^2} \quad (10)$$

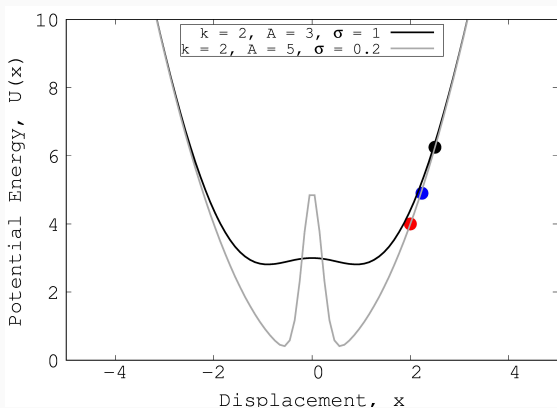
$$\left(\frac{A}{2}\right) \left(\frac{b}{\sigma}\right)^2 \lesssim E_i^0 \quad (11)$$

This is odd; why does the initial energy **appear to be less than A?**
 The answer is that, in order to satisfy the limit, $b \ll \sqrt{2}\sigma$. Thus, the particle has to be close to the origin to cross it, experiencing a gradual wall rather than a sharp one. **The corresponding problem in the *quantum limit* leads to quantum tunneling...**

CONSERVATION OF ENERGY

Which particle or particles will cross the origin, if each starts at rest?

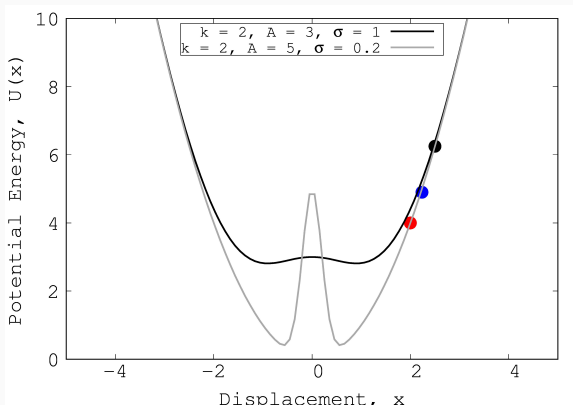
- A: Black only
- B: Black and Blue
- C: Blue only
- D: Red only



CONSERVATION OF ENERGY

Suppose each particle starts with sufficient negative velocity to cross the origin. If the velocities were instead positive, which would cross the origin?

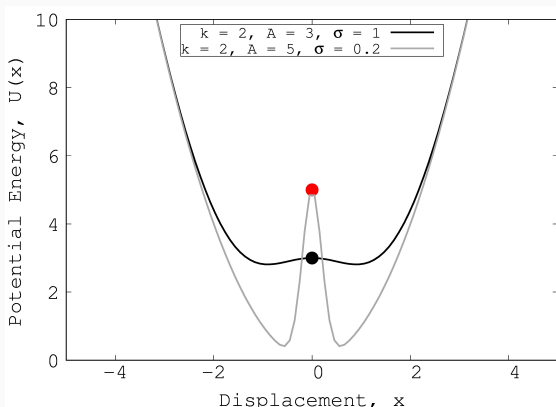
- A: Black only
- B: Black and Blue
- C: All
- D: None



CONSERVATION OF ENERGY

Suppose Black and Red particles begin at $x = 0$. Which particle or particles is displaced?

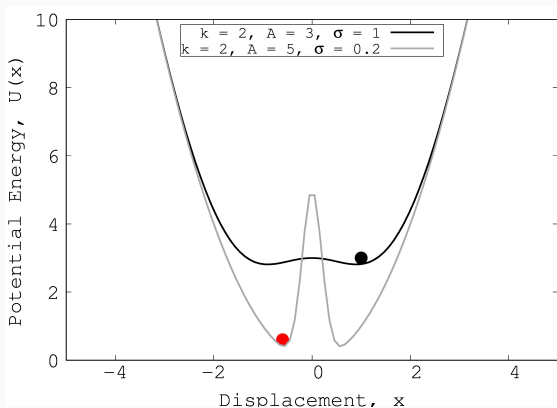
- A: Black only
- B: Red and Black
- C: Red only
- D: Neither



CONSERVATION OF ENERGY

Assuming each particle starts with no velocity, which particle or particles is displaced?

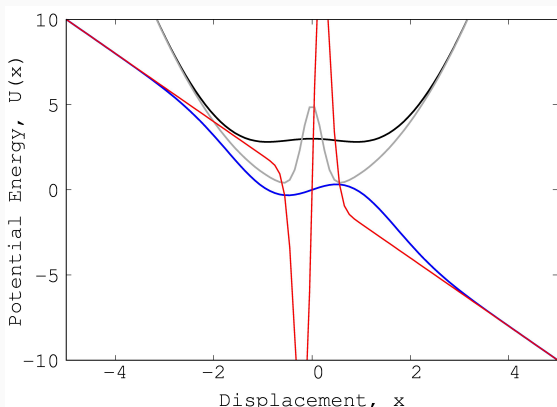
- A: Black only
- B: Red and Black
- C: Red only
- D: Neither



CONSERVATION OF ENERGY

Knowing that $F = -U'$, which of the following is true?

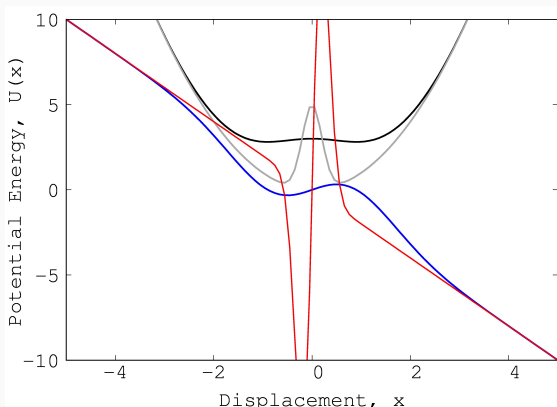
- A: Red is F of black U
- B: Blue is F of gray U
- C: Red is F of gray U
- D: Blue is F of red U



CONSERVATION OF ENERGY

Knowing that $F = -U'$, which of the following is true?

- A: Blue is F of black U
- B: Red is F of blue U
- C: Red is F of black U
- D: Blue is F of red U



CONSERVATION OF ENERGY

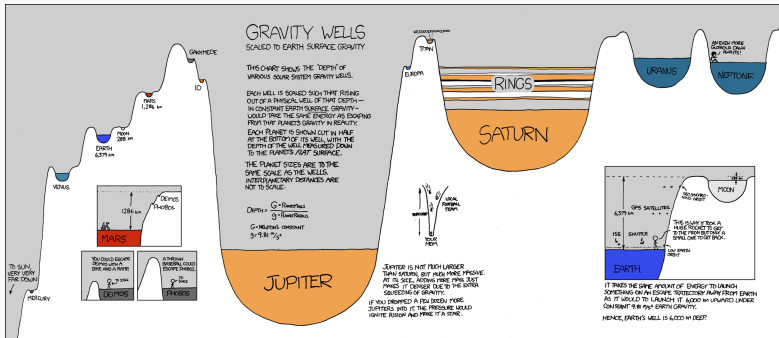


Figure 7: Potential energy surfaces in the solar system (xkcd.com).

CONCLUSION

1. **Work** and **potential energy**
 - **Lab activity:** Oscillator and gravity trading work and potential energy
2. Potential energy and **conservative forces**
3. **Conservation of Energy**
 - *Calculus review: the fundamental theorem of calculus*
 - Graphical representations of integrals and energy

ANSWERS

- $-mg$
- The force of gravity
- Stoke's Law of viscous drag
- 0.1 N
- The lowest point
- 0.5 m/s
- Black only
- All
- Neither
- Neither
- Red is force of gray U potential
- Red is F of gray U
- Blue is F of black U