Midterm #2

Part 1

1. $U(x) = k(x^4 - x^2)$ PE. func.

a. $U'(x) = k(4x^3 - 2x) = 0$

$4x^3 - 2x = 0$

$2x(2x^2 - 1) = 0$

$2x = 0$ → $x = 0$

$2x^2 - 1 = 0$ → $x = \pm\sqrt{\frac{1}{2}}$

where force is zero

b. $k(12x^2 - 2)$

$= k(12(0)^2 - 2)$

$= -2k$

$\sqrt{\frac{1}{2}}$ Largest Displacement

$k(12(\sqrt{\frac{1}{2}})^2 - 2)$

$k(6 - 2)$

$= 4k$

2. $\mu = 0.5$

a. $m = 1\,kg$

$F = 5\,N$

$\Delta x = 1\,m$

work = ?

$F_{friction} = 0.5 \cdot 9.81 = 4.905\,N$

$F = 5 - 4.905 = 0.095\,N$

$W = 0.095\,N \cdot 1\,m$

$= \boxed{0.095\,J}$

b. $\boxed{W = 0.095\,J}$

c. $\boxed{W = 0.095\,J}$

d. $\boxed{W = 0.095\,J}$

e. Total work = $\boxed{0.095\,J}$

f. Total work should be $\boxed{0\,J}$

Part 2

1. $\boxed{C.\ 0\ m/s}$

2. $V_f = \frac{V\sqrt{2+\sqrt{2}}}{2} = \boxed{323.36\ m/s}$

3.

a. $4000\,N$

$0.2\,sec$

$Impulse = F \cdot \Delta t$

$= 4000\,N \cdot 0.2\,s$

$= \boxed{800\,N_s}$

b. $V_i = 2.8\,m/s$

$m = 200\,kg$

$Impulse = m(V_f - V_i)$

$800 = 200(V_f - 2.8\,m/s)$

$4 = V_f - 2.8$

$\boxed{V_f = 6.8\,m/s}$

4. $\boxed{D.\ Totally\ elastic}$

5.

a. $m_i = 30{,}000\,kg$

$V_i = 0.85\,m/s$

$m_f = 140{,}000\,kg$

$V_f = \frac{(30{,}000 \cdot 0.85)}{(110{,}000 + 30{,}000)}$

$= \boxed{0.182\,m/s}$

b. $KE_i = \frac{1}{2}(30{,}000)(0.85)^2$

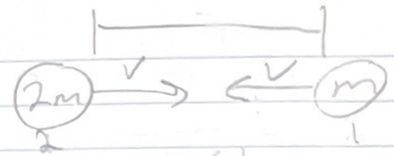
$= 10{,}837.5$

$KE_f = \frac{1}{2}(140{,}000)(0.182)^2$

$= 2318.68$

$10837.5 - 2318.68$

Energy lost = $\boxed{8518.82\,J}$

6. 

$$x_{cm}(t) = \frac{m \cdot x_1(t) + 2m \cdot x_2(t)}{m+2m}$$

Before $\boxed{x_{cm}(t) = \frac{x_1 + x_2}{3} + vt}$

After $v_f = -\frac{v}{3}$ $\boxed{x_{cm}(t) = \left(\frac{x_1+x_2}{3} + vt\right) - \frac{v}{3}(t-t_1)}$

$-mv = 3mv_f$

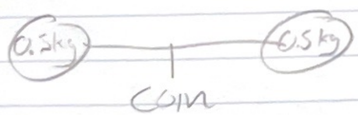
Part 3

1. $\frac{1 \text{ rot}}{1.33 \text{ sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} = \boxed{45 \text{ rpm}}$ B.

2. $a_c = \frac{v^2}{r}$ $\boxed{D.\ 2.2 \text{ m/s}^2}$

3. $\boxed{D.\ \omega \to 10\omega}$

4. $\frac{200 \text{ rot}}{\text{sec}} \cdot \frac{2\pi}{60} \text{ rad/sec}$ $.01 \cdot 20.94^2 \cdot .12 = \boxed{B.\ 0.53 \text{ N}}$

5. 

$\frac{4 \text{ rot}}{\text{sec}} \cdot \frac{60 \text{ sec}}{1 \text{ min}} = 240 \text{ rpm}$

a. COM $v = 8$ m/s $\quad KE = \frac{1}{2}(1)(8)^2 = \boxed{32 \text{ J}}$

b. $P_{ang} = 2mr^2\omega = \boxed{1r^2 8\pi \frac{\text{kg m}^2}{\text{sec}}}$

$I = mr^2_{(hid)}$

c. $h = \frac{8^2}{2(9.8)} = \boxed{3.27 \text{ m}}$

6. $\vec{r} = \underset{.05m}{5\hat{i}} + 5\hat{j}$ cm $\quad T = \hat{k}(.05)(10) - (.05)(-10)$

$T = \hat{k}(0.5 + 0.5)$

$\boxed{T = 1\hat{k} \text{ N·m}}$

a.

b. Torque doubles $\boxed{2\hat{k} \text{ N·m}}$

c. $30 \text{ N·cm} = 0.3 \text{ N·m}$ $\quad T = r \cdot F$

$F = \frac{T}{r}$

$\frac{0.3}{0.707}$ $\quad r = \sqrt{.05^2 + .05^2} = 0.0707$

$= \boxed{4.24 \text{ N}}\ \hat{k}$

7. $\omega(t) = 10t + 60 \text{ rpm} \times \frac{2\pi}{60} \frac{\text{rad}}{\text{sec}} = \frac{\pi}{30}(10t + 60) \frac{\text{rad}}{\text{sec}}$

$\alpha(t) = \frac{\pi}{30}(10) = \frac{\pi}{3} \frac{\text{rad}}{\text{sec}^2}$

$T = I \cdot \alpha = \frac{1}{2}mr^2 \cdot \frac{\pi}{3}$

$\boxed{T = \frac{\pi}{6}mr^2 \text{ N·m}}$

8. $30 \text{ rpm} = \pi \text{ rad/s} = \omega$ $\quad I = \frac{1}{2}mr^2 = 112.5$

$L = I \times \omega$ $\quad \frac{1}{2}(100)(1.5)^2\pi = \boxed{353.43 \text{ kg·m}^2/\text{s}}$

$(40)(1.5)^2 = 90 \text{ kg·m}^2 + 112.5 = 202.5 \text{ kg·m}^2$

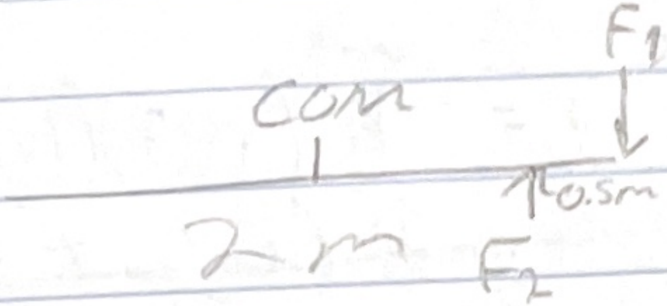
$\frac{112.5\pi}{202.5} = \omega = 1.745 \frac{\text{rad}}{\text{sec}} \cdot \frac{60 \text{ sec}}{2\pi} = \boxed{16.67 \text{ rpm}}$

Part 4

1. **A. motionless**

$40 \cdot 3 = 120\ V$
$60 \cdot 2 = 120\ V$

2.



$m = 20\ kg$

$20 \cdot 9.81 = 196.2$

$F_1 + F_2 = 196.2\ N$

$F_1 = 98.1\ N$
$F_2 = 98.1\ N$