

COMPUTER LOGIC AND DIGITAL CIRCUIT DESIGN (PHYS306/COSC330): UNIT 2.4

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SUMMARY

Reading: DF Chapter 3-4 (Moodle)

1. Logic Gates
 - Circuit diagram
 - Truth table
 - Timing diagram
 - Boolean logic
2. Boolean algebra I
3. IC Circuits, data sheets
4. Boolean algebra II

Homework: Chapter 3, ex. 1-22 (two weeks)

KARNAUGH MAPS

KARNAUGH MAPS

A **Karnaugh Map**, or K-map, is a *cell-array*, with 2^N cells, where N is the number of logical inputs.

<div>AB \ C</div>	0	1
00	000	001
01	010	011
11	110	111
10	100	101

<div>AB \ C</div>	0	1
00	$\bar{A} \bar{B} \bar{C}$	$\bar{A} \bar{B} C$
01	$\bar{A} B \bar{C}$	$\bar{A} B C$
11	$A B \bar{C}$	$A B C$
10	$A \bar{B} \bar{C}$	$A \bar{B} C$

Table 1: The 3-input Karnaugh map, or K-map, lists all possible outcomes of a logic operation for all possible input combinations. (Left) Bit sequence representation of all input combinations. (Right) Symbolic representation of all input combinations.

KARNAUGH MAPS

CD \ AB	00	01	11	10
00	0000	0001	0011	0010
01	0100	0101	0111	0110
11	1100	1101	1111	1110
10	1000	1001	1011	1010

CD \ AB	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}C\bar{D}$	$\bar{A}\bar{B}CD$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BC\bar{D}$	$\bar{A}BCD$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABC\bar{D}$	$ABCD$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}C\bar{D}$	$A\bar{B}CD$

Table 2: The 4-input K-map, in the same notation as Tab. 1.

K-maps have several requirements for *adjacent cells*.

1. **Only one bit change** between adjacent cells.
2. Cells have *wrap-around* adjacency.
3. Diagonal cells are not adjacent.

KARNAGH MAPS: MAPPING S-SOP EXPRESSIONS

A S-SOP expression may be mapped to a K-map:

AB \ CD	00	01	11	10
00	1	1		
01		1	1	
11		1		
10		1		

Table 3: The 4-input K-map, with an S-SOP mapped.

S-SOP expression that is being mapped:

$$\bar{A} \bar{B} \bar{C} \bar{D} + \bar{A} \bar{B} \bar{C} D + \bar{A} B \bar{C} D + A B \bar{C} D + A \bar{B} \bar{C} D + \bar{A} B C D$$

AB \ CD	00	01	11	10
00		1		
01		1		
11		1		
10		1		

Table 4: The 4-input K-map, with an S-SOP mapped.

SOP expression that is being mapped:

$$\bar{A}\bar{B}\bar{C}D + \bar{A}B\bar{C}D + AB\bar{C}D + A\bar{B}\bar{C}D$$

KARNAGH MAPS: MAPPING NON-STANDARD SOP EXPRESSIONS

AB \ CD	00	01	11	10
00		1	1	
01		1	1	
11		1	1	
10		1	1	

Table 5: The 4-input K-map, with an S-SOP mapped.

S-SOP expression that is being mapped:

$$\bar{A} \bar{B} \bar{C} D + \bar{A} B \bar{C} D + D$$

(We must enumerate the non-standard term). We could convert to standard form, but this is faster.

S-SOP to K-map exercise:

AB \ CD	00	01	11	10
00				
01				
11				
10				

Table 6: Map the S-SOP expression below into the K-Map.

$$\bar{A} \bar{B} \bar{C} \bar{D} + ABC$$

S-SOP to K-map exercise:

AB \ CD	00	01	11	10
00				
01				
11				
10				

Table 7: Map the S-SOP expression below into the K-Map.

$$\bar{A} \bar{B} \bar{C} \bar{D} + ABCD + \bar{A} \bar{B} C D$$

APPLIED K-MAPS: LOGIC SIMPLIFICATION

The expression $\bar{A} \bar{B} \bar{C} D + \bar{A} B \bar{C} D + A B \bar{C} D + A \bar{B} \bar{C} D$ probably has a simpler form. How can we use the K-map to simplify?

1. *Group the 1's in adjacent cells*

- Group size must be equal to a power of 2
- Groups must be as large as possible

2. *Read simplified terms from map*

- One SOP term per group. **Exclude** contradictory variables.
- **3-variable maps:** 1-cell groups have 3-variable products, 2-cell groups with 2-variable products, 4-cell groups with 1-variable products.
- **4-variable maps:** 1-cell groups have 4-variable products, 2-cell groups with 3-variable products, 4-cell groups with 2-variable products, 8-cell groups with 1-variable products.

K-map to simplified SOP (*work several examples*):

AB \ CD	00	01	11	10
00				
01				
11				
10				

Table 8: Place 1's in the K-map to find the corresponding SOP expression.

K-map to simplified SOP (*use wrap-around adjacency*):

AB \ CD	00	01	11	10
00				
01				
11				
10				

Table 9: Place 1's in the K-map to find the corresponding SOP expression.

APPLIED K-MAPS: LOGIC SIMPLIFICATION

Simplify: $\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + A\bar{B}CD + \bar{A}\bar{B}C\bar{D} + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D}$

AB \ CD	00	01	11	10
00				
01				
11				
10				

Table 10: Place 1's in the K-map to find the corresponding SOP expression. *The final expression has only two terms.* Build the truth table from the final expression, and check against original expression.

TT to minimal SOP (*work several examples*).

A	B	C	D	X
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

CD \ AB	00	01	11	10
00				
01				
11				
10				

Table 11: The minimal SOP may be derived from a TT via the K-map.

TT to minimal SOP (utilize *don't care* conditions).

A	B	C	D	X
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	
1	1	1	0	
1	1	1	1	

CD \ AB	00	01	11	10
00				
01				
11				
10				

Table 12: The minimal SOP may be derived from a TT via the K-map.

CONCLUSION

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