Computer Logic and Digital Circuit Design (PHYS306/COSC330): Unit 3

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Summary

Unit 3 Summary

Functions of Combinatorial Logic

Reading: 6-1 - 6-6 (Tuesday)

Reading: 6-7 - 6-11 (Thursday)

1. Half-Adders and Full-Adders
   - Example from study guide
   - Propagation delays
2. Comparators
   - The XNOR gate
   - Multi-bit comparators
   - Inequalities
3. Decoders/Encoders
   - Binary to decimal circuits
   - Decimal to binary circuits

Half-Adders and Full-Adders, Ripple-Carry

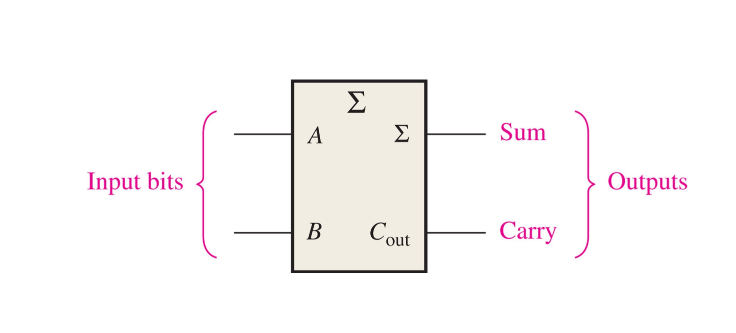


Figure 1: The desired inputs and outputs of the half-adder. There is no carry-input.

Half-Adders and Full-Adders, Ripple-Carry

TABLE 6–1

Half-adder truth table.

A	B	C_{out}	Σ
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Σ = sum
C_{out} = output carry
A and B = input variables (operands)

Figure 2: The truth table of the half-adder for 2-bits. What gate action does this match?

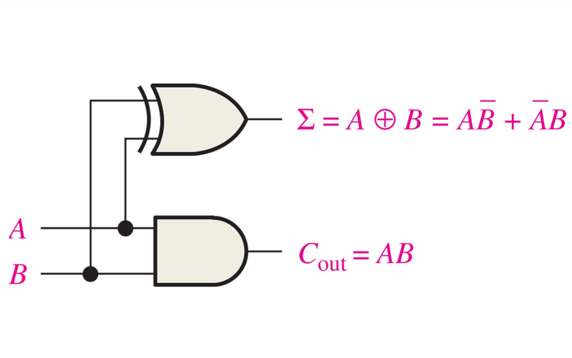


Figure 3: The logic function circuit diagram for the half-adder.

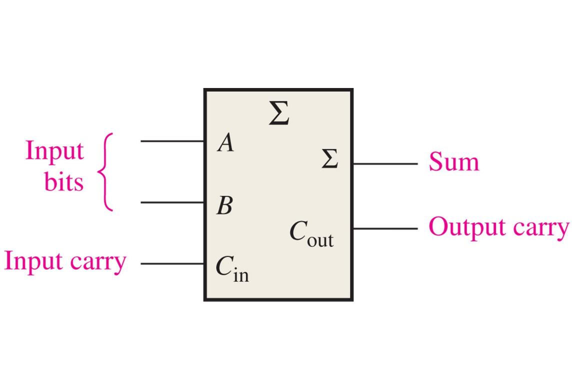


Figure 4: The desired inputs and outputs for the full-adder, with carry-input and carry-output.

Half-Adders and Full-Adders, Ripple-Carry

TABLE 6–2

Full-adder truth table.

A	B	C_{in}	C_{out}	Σ
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

C_{in} = input carry, sometimes designated as *CI*
C_{out} = output carry, sometimes designated as *CO*
Σ = sum
A and *B* = input variables (operands)

Figure 5: The truth table for the full-adder is more complex due to the increased number of inputs.

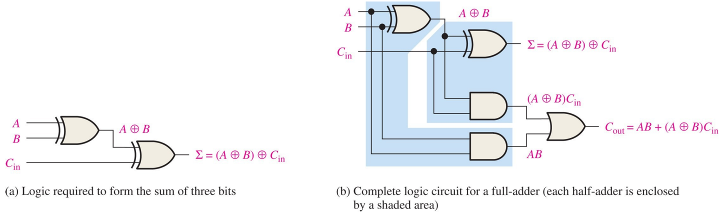


Figure 6: Circuit diagrams for the half-adder (left) and full-adder (right).

Half-Adders and Full-Adders, Ripple-Carry

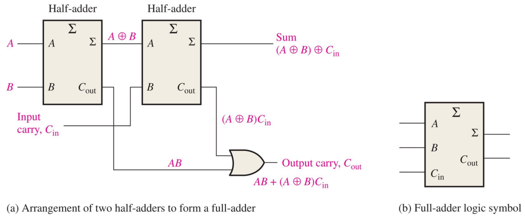


Figure 7: Two half-adders to form a full-adder.

Half-Adders and Full-Adders, Ripple-Carry

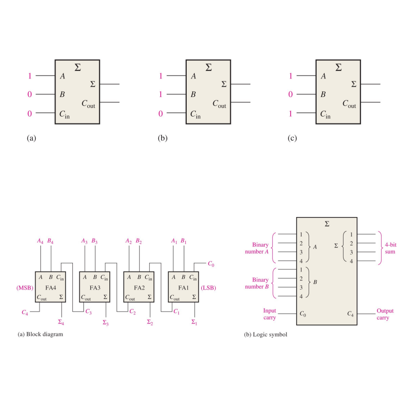


Figure 8: Four FA (full-adders) to add bits to the numbers being added.

Half-Adders and Full-Adders, Ripple-Carry

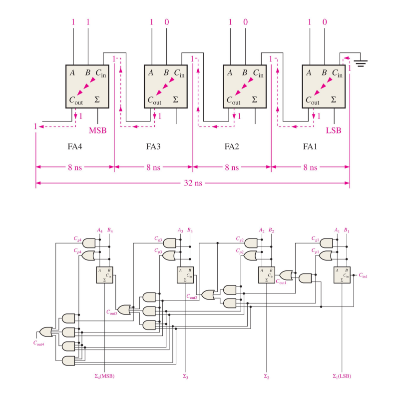


Figure 9: Propagation delays add serially in a full-adder with ripple carry topology.

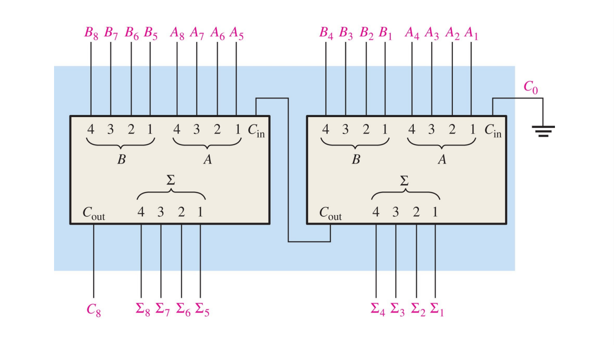


Figure 10: Two 4-bit FA connected to form an 8-bit FA, accounting for carries.

Comparators

Comparators

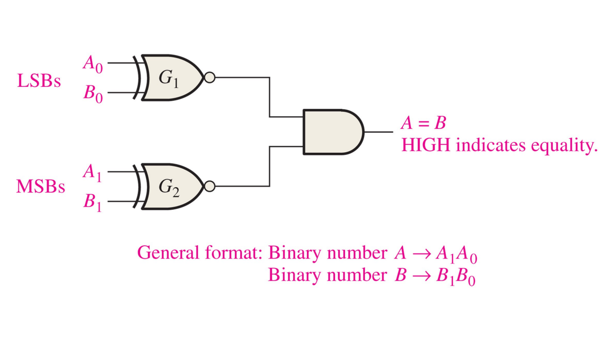


Figure 11: The basic idea behind the comparator. (a) Review the truth table for the XNOR-gate, which is the conjugate of the XOR gate. (b) What is the function of the AND gate?

Comparators

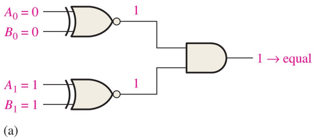
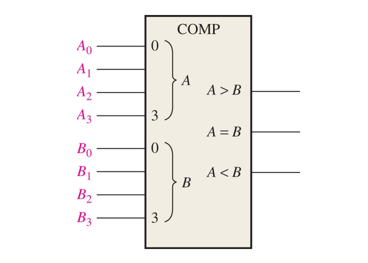


Figure 12: (a) Example of comparison of 2-bit binary numbers. (b) What is the truth table? (c) What is the logical representation of the function? (c) What is a logical representation for the inequality circuit?

Comparators



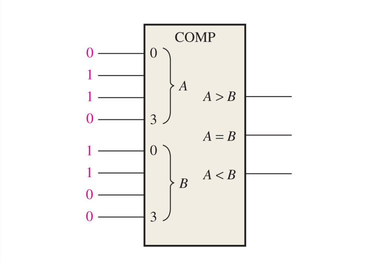


Figure 13: (Left) One *component* that is really 8 comparators linked to the same AND gate. (Right) What is the correct ouput? How to determine the inequality functions?

Comparators

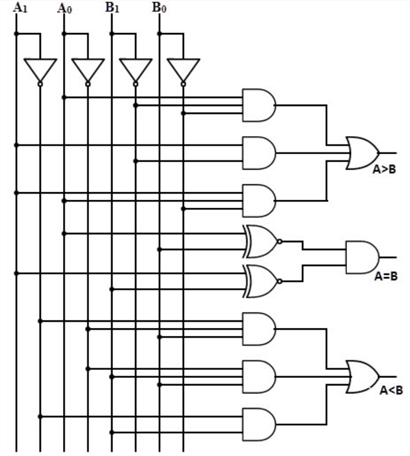


Figure 14: Try some simple cases: (a) A = 00 and B = 01, (b) A = 10 and B = 01.

Decoders

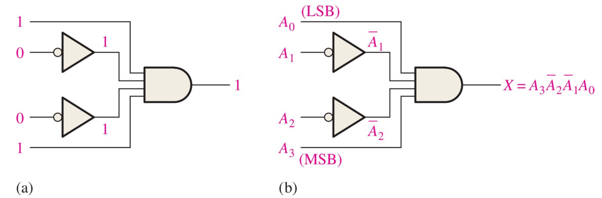


Figure 15: The binary decoder circuit for 9. This circuit is true if the binary number is 1001. (Pay attention to the order of MSB and LSB).

Decoders

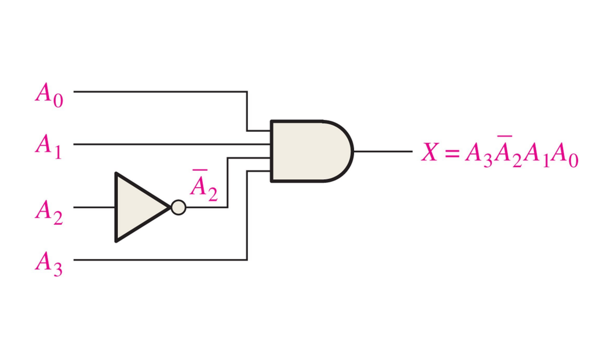


Figure 16: (a) Which binary number is being decoded here? (b) What would it take to decode all binary numbers of *n* bits?

Decoders

TABLE 6–4

Decoding functions and truth table for a 4-line-to-16-line (1-of-16) decoder with active-LOW outputs.

Decimal Digit	Binary Inputs				Decoding Function	Outputs															
	A_3	A_2	A_1	A_0		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	0	0	0	$\overline{A}_3\overline{A}_2\overline{A}_1\overline{A}_0$	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
1	0	0	0	1	$\overline{A}_3\overline{A}_2\overline{A}_1A_0$	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	0	0	1	0	$\overline{A}_3\overline{A}_2A_1\overline{A}_0$	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
3	0	0	1	1	$\overline{A}_3\overline{A}_2A_1A_0$	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1
4	0	1	0	0	$\overline{A}_3A_2\overline{A}_1\overline{A}_0$	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1
5	0	1	0	1	$\overline{A}_3A_2\overline{A}_1A_0$	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
6	0	1	1	0	$\overline{A}_3A_2A_1\overline{A}_0$	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
7	0	1	1	1	$\overline{A}_3A_2A_1A_0$	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
8	1	0	0	0	$A_3\overline{A}_2\overline{A}_1\overline{A}_0$	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
9	1	0	0	1	$A_3\overline{A}_2\overline{A}_1A_0$	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1
10	1	0	1	0	$A_3\overline{A}_2A_1\overline{A}_0$	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1
11	1	0	1	1	$A_3\overline{A}_2A_1A_0$	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1
12	1	1	0	0	$A_3A_2\overline{A}_1\overline{A}_0$	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
13	1	1	0	1	$A_3A_2\overline{A}_1A_0$	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
14	1	1	1	0	$A_3A_2A_1\overline{A}_0$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
15	1	1	1	1	$A_3A_2A_1A_0$	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0

Figure 17: The decoding table for ACTIVE LOW 4-bit binary. Terms from linear algebra: this matrix has 0 trace, and is symmetric (we can exchange rows and columns).

Encoders

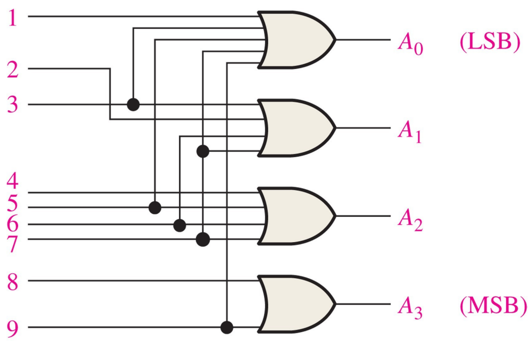


Figure 18: Recall the OR TT, and remember that this is a system in which there are forbidden or *don't care* states. Only one input line can be active at once.

Encoders

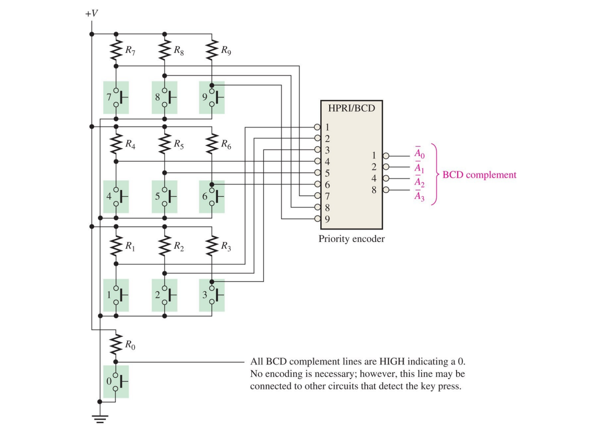


Figure 19: A keypad system. The binary encoder accepts only 9 digits because 0 is redundant. Note the use of *pull-up* resistors (recall LEDs operations).

Conclusion

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