COMPUTER LOGIC AND DIGITAL CIRCUIT DESIGN (PHYS306/COSC330): UNIT 2.3

Jordan Hanson

January 30, 2018

Whittier College Department of Physics and Astronomy

SUMMARY

Reading: DF Chapter 3-4 (Moodle)

1. Logic Gates
    - Circuit diagram
    - Truth table
    - Timing diagram
    - Boolean logic
2. Boolean algebra I
3. IC Circuits, **data sheets**
4. Boolean algebra II

Homework: Chapter 3, ex. 1-22 (two weeks)

BOOLEAN ALGEBRA BASICS

Good paper topic: George Boole and Claude Shannon in the mid-19th century, early 20th century.

Boolean or logical algebra - *An Investigation of the Laws of Throught, on Which Are Founded the Mathematical Theories of Logic and Probabilities* (Boole's title)

What does *an algebra* contain?

1. Variables or *literals*
2. *complements* (resembles *identities, reciprocals, and nulls*)
3. Sums are ORs
4. Products are ANDs

An algebra has many objects, properties, and rules. An extensive list is outside our scope.

Boolean or logical algebra has these *objects*:

1. Variables: A is a variable, either TRUE or FALSE
2. complements: $\overline{A}$ is the *complement* of A, inverting the input

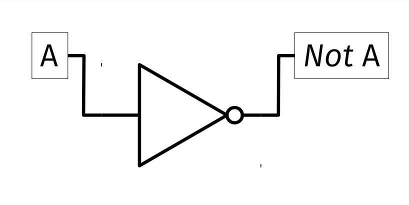

Figure 1: The inverter performs the complement operation at gate-level, $\overline{A} = NOT\ A$.

AND is multiplication, and OR is addition. Refer to Chapter 2 DF.

Boolean or logical algebra has these *properties*:

1. Property of commutativity
    - **Addition version:** $A + B = B + A$
    - (*an OR gate does not distinguish between inputs*)
    - **Mutliplication version:** $AB = BA$
    - (*an AND gate does not distinguish between inputs*)
2. Property of Associativity
    - **Addition version:** $(A + B) + C = A + (B + C)$
    - *OR gate order does not matter.*
    - **Mutliplication version:** $(AB)C = A(BC)$
    - *AND gate order does not matter.*
3. Property of Distribution
    - $A(B + C) = AB + BC$ (*gate input and gate order*)

Boolean or logical algebra has these *rules*:

1. $A + 0 = A$
2. $A + 1 = 1$
3. $A \cdot 0 = 0$
4. $A \cdot 1 = A$
5. $A + A = A$
6. $A + \overline{A} = 1$
7. $AA = A$
8. $A\overline{A} = 0$
9. $\overline{\overline{A}} = A$

Boolean or logical algebra has these *special rules*:

1. $A + AB = A$
2. $A + \overline{A}B = A + B$
3. $(A + B)(A + C) = A + BC$

Exercises:

- Prove special rule (1) with a truth table
- Why is special rule (1) intuitive?
- Prove special rule (2) with a truth table
- Why is special rule (2) intuitive?
- **Group board exercise:** Prove special rule (3) *Hint: this requires rules 2, 4, and 7. This is good practice at* ***gate simplification.***

De Morgan's Theorems

- **First theorem**: The complement of a product of literals is equal to the sum of the complements of the literals.
- **Second theorem**: The complement of a sum of literals is equal to the product of the complements of the literals.

- First theorem: $\overline{AB} = \overline{A} + \overline{B}$
- Second theorem: $\overline{A + B} = \overline{A}\,\overline{B}$

For example: $\overline{(AB + BC)} = \overline{AB}\;\overline{BC} = (\overline{A} + \overline{B})(\overline{B} + \overline{C})$

Group board exercise: Show that

$$\boxed{\overline{(A+B)(B+C)}+\overline{(A+\overline{A}B)(B+\overline{B}A)}=\overline{A}+\overline{B}+\overline{C}} \qquad (1)$$

- *Hint*: use De Morgan's two theorems
- *Hint*: use special rule 3
- *Hint*: consider the regular rules

Group board exercise: Create a circuit using logic gates that represents the left-hand side of Eq. 1, and do the same for the right-hand side. How many gates fewer are there in the right-hand solution?

Group board exercise: Show that

$$\boxed{\begin{aligned}(AB + C(B + A)) + \overline{(AB + C(B + A))}(AC + B(C + A)) \\ = AB + AC + BC\end{aligned}} \qquad (2)$$

- *Hint*: use special rule 2
- *Hint*: use De Morgan's two theorems

Group board exercise: Create a circuit using logic gates that represents the left-hand side of Eq. 2, and do the same for the right-hand side. How many gates fewer are there in the right-hand solution?

CONCLUSION

UNIT 2.3 SUMMARY - THEORETICAL LOGIC GATES, AND OPERATIONS

Reading: DF Chapter 3-4 (Moodle)

1. Logic Gates
    - Circuit diagram
    - Truth table
    - Timing diagram
    - Boolean logic
2. Boolean algebra I
3. IC Circuits, **data sheets**
4. Boolean algebra II

Homework: Chapter 3, ex. 1-22 (two weeks)