# COMPUTER LOGIC AND DIGITAL CIRCUIT DESIGN (PHYS306/COSC330): UNIT 2.1

Jordan Hanson January 25, 2018

Whittier College Department of Physics and Astronomy



### UNIT 2.1 SUMMARY - THEORETICAL LOGIC GATES, AND OPERATIONS

### Reading: DF Chapter 3-4 (Moodle)

- 1. Logic Gates
  - · Circuit diagram
  - Truth table
  - Timing diagram
  - · Boolean logic
- 2. Boolean algebra I
- 3. IC Circuits, data sheets
- 4. Boolean algebra II

Homework: Chapter 3, ex. 1-22 (two weeks)

A **logic gate** is a digital circuit component made of transistors, that performs a basic logic function with *n* inputs and *m* outputs.

### We will cover this basic set:

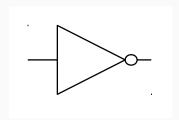
- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

Each gate has *n* inputs, and *m* outputs. We represent the inputs (HIGH/LOW) with *A*, *B*, ..., and the output usually with *X*.

- · Circuit diagram
- · Truth table
- Timing diagram
- Boolean logic

The NOT gate: flips the input from LOW/HIGH to HIGH/LOW.

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 1:** The NOT gate is represented with a triangle and a circle. It has one input and one output.

The NOT gate: flips the input from LOW/HIGH to HIGH/LOW.

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2, m = 1$$

Table 1: Truth table for NOT.

The NOT gate: flips the input from LOW/HIGH to HIGH/LOW.

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

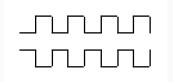


Figure 2: Example timing diagram for the NOT gate. (Top) example signal, s. (Bottom) NOT s, or  $\bar{s}$ .

The NOT gate: flips the input from LOW/HIGH to HIGH/LOW.

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2, m = 1$$

$$NOT(A) = \bar{A} \tag{1}$$

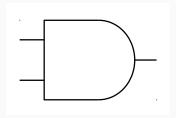
The NOT gate: flips the input from LOW/HIGH to HIGH/LOW.

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

### In class exercises:

- Using NOT gates, create a circuit that forms the 2's compliment of an 8 bit binary number.
- Once you have this, draw a timing diagram representing the conversion of 142 to the proper 2's compliment, followed by the conversion of 56 to the proper 2's compliment.

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 3:** The circuit representation of an AND gate. Two inputs, one output.

The AND gate: requires both inputs to be HIGH, for HIGH output.

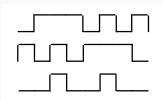
- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

Α	В	Χ
1	1	1
1	0	0
0	1	0
0	0	0

Table 2: Truth table for AND.

How many input combinations are there for *n*? Specifically, how many for two-input gates? Three-input?

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 4:** Example timing diagram for AND gate. (Top) Input *A*. (Middle) Input *B*. (Bottom) *AB* (A AND B).

• NOT, 
$$n = 1, m = 1$$

$$A AND B = AB$$
 (2)

- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

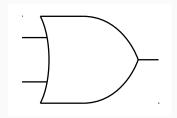
The AND gate: requires both inputs to be HIGH, for HIGH output.

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

### In-class exercises:

- Using AND as "enable:"
   Develop a circuit that flashes a light every time the clock signal is HIGH, but only if an enable signal is also raised to HIGH.
- Derive the truth table for a 3-input AND gate. How many input combinations should there be?

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 5:** The circuit representation of an OR gate. Two inputs, one output.

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

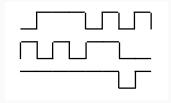
• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2$$
,  $m = 1$ 

Table 3: Truth table for OR.

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 6:** Example timing diagram for OR gate. (Top) Input A. (Middle) Input B. (Bottom) A + B (A OR B).

• NOT, 
$$n = 1$$
,  $m = 1$ 

$$A OR B = A + B \tag{3}$$

- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

### The OR gate: requires either input to be HIGH, for HIGH output.

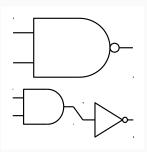
- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

#### In-class exercises:

- In physics experiments, sometimes we establish a combinatorial trigger, sometimes called a coincidence trigger. Suppose the task of three digital channels is to observe a high-energy particle pass through a detector. If there is an observation in a channel, it raises HIGH for a time called a gate time.
- Using OR gates, draw a circuit that triggers if two
  of the three channels observes the particle.
   Draw an example of a timing diagram
  corresponding to a trigger. (The gate time is up
  to you, it just means that if a channel is HIGH it
  stays high for at least one gate).

The NAND gate: requires neither input to be HIGH for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 7:** The circuit representation of an NAND gate. Two inputs, one output.

The NAND gate: requires neither input to be HIGH for the output to be HIGH

• NOT, 
$$n = 1, m = 1$$

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2, m = 1$$

**Table 4:** Truth table for NAND.

The NAND gate: requires neither input to be HIGH for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

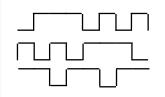


Figure 8: Example timing diagram for NAND gate. (Top) Input *A*. (Middle) Input *B*. (Bottom)  $\overline{AB}$  (A NAND B or NAND AB).

The NAND gate: requires neither input to be HIGH for the output to be HIGH

• NOT, 
$$n = 1, m = 1$$

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2, m = 1$$

$$A \ NAND \ B = \overline{AB} \tag{4}$$

Note: have you started to pick up the notation here? The bar is like complex conjugation, or the *compliment* of the signal.

The NAND gate: requires neither input to be HIGH for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

### In-class exercises:

 A certain fluid pump has a display that shows sensor information from two pairs of pipes, pair A and pair B. The sensors are raised HIGH if no fluid is flowing through a pipe, and there is one sensor per pipe.

The NAND gate: requires neither input to be HIGH for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

### In-class exercises:

 Design a system using NAND gates that produces three output signals: one LOW if neither pipe in pair A has fluid flowing, one LOW if neither pipe in pair B has fluid flowing, and one LOW if neither pair has fluid flowing. Create a timing diagram that shows pair A turning off, then pair B.

The NAND gate: requires neither input to be HIGH for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

### In-class exercises:

- Create an OR gate with NAND and NOT gates. a) 2-input version 3) 3-input version.
- Create an AND gate with NAND and NOT gates. a)
   2-input version 3) 3-input version.

The NOR gate: requires both inputs to be LOW for the output to be HIGH

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

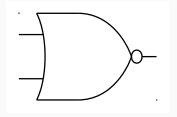
• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2$$
,  $m = 1$ 



**Figure 9:** The circuit representation of an NOR gate. Two inputs, one output.

The NOR gate: requires both inputs to be LOW for the output to be HIGH

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2, m = 1$$

**Table 5:** Truth table for NOR.

The NOR gate: requires both inputs to be LOW for the output to be HIGH

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

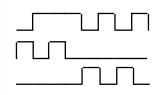
• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2, m = 1$$



**Figure 10:** Example timing diagram for NOR gate. (Top) Input *A*. (Middle) Input *B*. (Bottom)  $\overline{A+B}$  (A NOR B or NOR AB).

The NOR gate: requires both inputs to be LOW for the output to be HIGH

• NOT, 
$$n = 1$$
,  $m = 1$ 

$$A NOR B = \overline{A + B}$$
 (5)

- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

The NOR gate: requires both inputs to be LOW for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

### In-class exercises:

 Create a NOR gate from a NAND gate and inverters.
 a) 2-input version b)
 3-input version. Verify with either a timing diagram or truth table.

## The NOR gate: requires both inputs to be LOW for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

#### In-class exercise (from the text, this one is cool)

 As part of an aircraft's functional monitoring system, a circuit is required to indicate the status of the landing gears prior to landing. A green LED display turns on if all three gears are properly extended when the gear-down switch has been activated in preparation for landing. A red LED display turns on if any of the gears fail to extend properly prior to landing. When a landing gear is extended, its sensor produces a LOW voltage. When a landing gear is retracted, its sensor produces a HIGH voltage. Implement a circuit to meet this requirement.

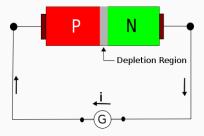
To solve this problem, let's have a quick aside as to how LED's (light emitting diodes) work...

**LED - light emitting diode.** What's a diode? What is a transistor? One kind of transistor is a p-n-p junction, or a n-p-n junction:



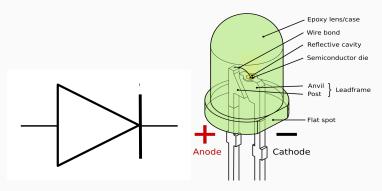
Figure 11: One kind of transistor is the sandwich of semiconductors.

**LED - light emitting diode.** What's a diode? Like a transistor, minus one layer. Diodes will only conduct in one direction.



**Figure 12:** A diode is a sandwich of only two layers of semiconductor rather than three.

**LED - light emitting diode.** What's a diode? Like a transistor, minus one layer. Diodes will only conduct in one direction.



**Figure 13:** (Left) The symbol for a diode. Current flows towards the flat line. (Right) Some diodes emit visible photons when conducting electricity.

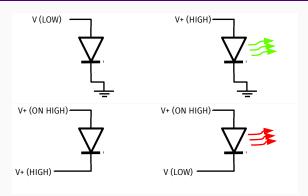


Figure 14: (Upper left): The green LED is held off by a LOW signal, no voltage gradient. (Upper right) The green LED is activated by a HIGH signal, a voltage gradient. (Lower left): The red LED is held off by a HIGH signal, no voltage gradient. (Lower right): The red LED is activated by a LOW signal, a voltage gradient.

# The NOR gate: requires both inputs to be LOW for the output to be HIGH

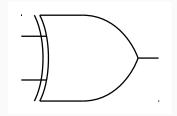
- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

#### So now we can solve this problem:

 As part of an aircraft's functional monitoring system, a circuit is required to indicate the status of the landing gears prior to landing. A green LED display turns on if all three gears are properly extended when the gear-down switch has been activated in preparation for landing. A red LED display turns on if any of the gears fail to extend properly prior to landing. When a landing gear is extended, its sensor produces a LOW voltage. When a landing gear is retracted, its sensor produces a HIGH voltage. Implement a circuit to meet this requirement.

The XOR gate: requires exactly one input to be HIGH for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 15:** The circuit representation of an XOR gate. Two inputs, one output.

The XOR gate: requires exactly one input to be HIGH for the output to be HIGH

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2$$
,  $m = 1$ 

• XNOR, 
$$n = 2, m = 1$$

Table 6: Truth table for XOR.

The XOR gate: requires exactly one input to be HIGH for the output to be HIGH

• NOT, 
$$n = 1, m = 1$$

• AND, 
$$n, m = 1$$

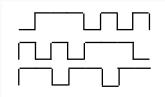
• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2, m = 1$$

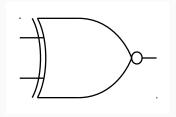
• XNOR, 
$$n = 2$$
,  $m = 1$ 



**Figure 16:** Example timing diagram for XOR gate. (Top) Input *A.* (Middle) Input *B.* (Bottom) Output

The XNOR gate: requires both inputs to be HIGH or LOW for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 17:** The circuit representation of an XNOR gate. Two inputs, one output.

The XNOR gate: requires both inputs to be HIGH or LOW for the output to be HIGH

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2, m = 1$$

• XOR, 
$$n = 2$$
,  $m = 1$ 

• XNOR, 
$$n = 2, m = 1$$

**Table 7:** Truth table for XNOR.

The XNOR gate: requires both inputs to be HIGH or LOW for the output to be HIGH

• NOT, 
$$n = 1$$
,  $m = 1$ 

• AND, 
$$n, m = 1$$

• OR, 
$$n, m = 1$$

• NAND, 
$$n, m = 1$$

• NOR, 
$$n = 2$$
,  $m = 1$ 

• XOR, 
$$n = 2, m = 1$$

• XNOR, 
$$n = 2$$
,  $m = 1$ 

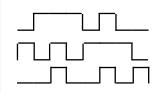


Figure 18: Example timing diagram for XNOR gate. (Top) Input A. (Middle) Input B. (Bottom) Output

# The NOR gate: requires both inputs to be LOW for the output to be HIGH

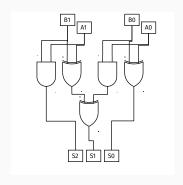
- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1

#### In-class exercises:

- Show that an XOR gate is a two-bit adder, neglecting the carry bit.
- · Create an 2-bit adder from XOR and AND gates.

The NOR gate: requires both inputs to be LOW for the output to be HIGH

- NOT, n = 1, m = 1
- AND, n, m = 1
- OR, n, m = 1
- NAND, n, m = 1
- NOR, n = 2, m = 1
- XOR, n = 2, m = 1
- XNOR, n = 2, m = 1



**Figure 19:** Example of a circuit that adds two-bit digital numbers.

# CONCLUSION

### UNIT 2.1 SUMMARY - THEORETICAL LOGIC GATES, AND OPERATIONS

## Reading: DF Chapter 3-4 (Moodle)

- 1. Logic Gates
  - · Circuit diagram
  - Truth table
  - · Timing diagram
  - · Boolean logic
- 2. Boolean algebra I
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Homework: Chapter 3, ex. 1-22 (two weeks)