COMPUTER LOGIC AND DIGITAL CIRCUIT DESIGN (PHYS306/COSC330): UNIT 0

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SUMMARY

Reading: Digital Fundamentals (DF) Ch. 1 (see Moodle)

1. Analog and Digital - RC filters and LC resonators
2. Introduction to digital concepts
3. Transistor radio...go!

Homework: exercises 1-20 Ch. 1 (DF)

Bonus Essay assignment: If you submit a 10-page paper on the history of computer science, including references from both online and library sources by the end of the semester, I will replace your lowest midterm score with the grade of the paper. Example topics:

- George Boole and the development of Boolean logic. What prompted this study?
- Bell Labs and the development of the transistor
- The first companies of silicon valley, the nobel prize, and the integrated circuit

Before beginning the essay, please make an appointment with me in office hours so that we may agree upon a topic.

ANALOG AND DIGITAL

The main topic of thise course is *digital* electronics. What is the distinction between *digital* and *analog*? It is helpful to have a lesson in analog circuits, and then a lesson in digital circuits, for comparison.

One set of examples of analog circuits is RC/LC filters. We must apply *Ohm's Law* (Eq. 1) and *Kirchhoff's Rules* (Eqs. 2-3) to understand these circuits, so let's review those.

$$V = IR \tag{1}$$

$$\sum_{i,node}^{N} j_i = 0 \tag{2}$$

$$\sum_{i,loop}^{N} v_i = 0 \tag{3}$$

A node: a point in a circuit where conductors meet.
A loop: a path in a circuit that returns to the same node.

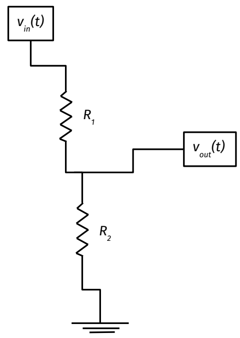

Figure 1: A two-resistor voltage divider.

- Using Ohm's Law, find expressions for $v_{\mathrm{in}}(t)$ and $v_{\mathrm{out}}(t)$ in terms of the resistances and the current.
- Take the ratio of the output voltage to the input voltage to find the **transfer function**, $H(t)$.
- Volunteer to board?

Answer:

$$H(t) = \frac{R_2}{R_1 + R_2} \tag{4}$$

In other words, the circuit does not change the time-domain properties of the signal, just the *amplitude*.

Can we replace a resistor with another component to *filter* signals based on time-domain properties?

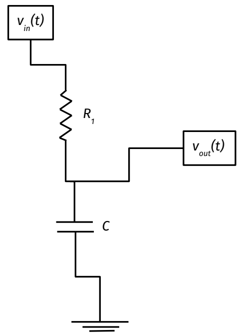

Figure 2: A single-pole low-pass RC filter. What is the resistance or impedance of a capacitor?

We need a math tool, the Fourier transform:

$$\mathcal{F}(f(t)) = \tilde{F}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \tag{5}$$

$$f(t) = g'(t) \tag{6}$$

$$\tilde{F}(\omega) = g(t)e^{-j\omega t}|_{-\infty}^{\infty} + j\omega \int_{\infty}^{-\infty} g(t)e^{-j\omega t}dt \tag{7}$$

$$\mathcal{F}(g'(t)) = j\omega\mathcal{F}(g(t)) \tag{8}$$

After integrating by parts we find that the Fourier transform of a derivative of a function is the imaginary unit times the angular frequency times the Fourier transform of the function. **Note that throughout this course j is the imaginary unit. This is common in engineering texts.**

Impedance of a capacitor:

$$C = \frac{q}{V} \tag{9}$$

$$VC = q \tag{10}$$

$$C\frac{dV}{dt} = \frac{dq}{dt} = i(t) \tag{11}$$

$$\tilde{i}(\omega) = j\omega C\tilde{V}(\omega) \tag{12}$$

$$\frac{\tilde{V}(\omega)}{\tilde{i}(\omega)} = \frac{1}{j\omega C} \tag{13}$$

Using Ohm's Law, we identify the impedance as

$$\boxed{Z_C = \frac{1}{j\omega C}} \tag{14}$$

Repeat the earlier exercise, but with the new circuit:

- Using Ohm's Law, find expressions for $v_{\mathrm{in}}(t)$ and $v_{\mathrm{out}}(t)$ in terms of the impedances and the current.
- Take the ratio of the output voltage to the input voltage to find the **transfer function**, $H(t)$.
- Volunteer to board?

Answer:

$$\boxed{M_{LP}(\omega) = \sqrt{\frac{\omega_0^2}{\omega^2 + \omega_0^2}}} \tag{15}$$

$$\boxed{\phi_{LP}(\omega) = -\tan^{-1}\left(\frac{\omega}{\omega_0}\right)} \tag{16}$$

The transfer function of this circuit depends on frequency in the following way:

1. $\omega \ll \omega_0$, $M_{\mathrm{LP}} \approx 1$, $\phi_{\mathrm{LP}} \approx 0$
2. $\omega \gg \omega_0$, $M_{\mathrm{LP}} \approx 0$, $\phi_{\mathrm{LP}} \approx -90°$

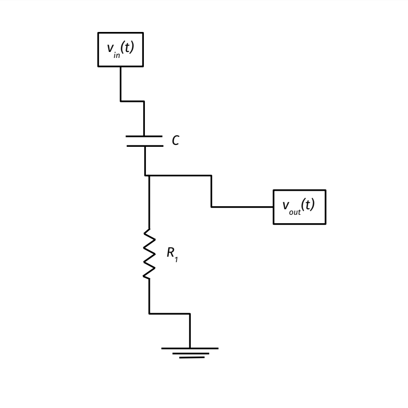

Figure 3: A single-pole high-pass RC filter. Can we repeat this process, only with the capacitor and resistor exchanged?

Answer:

$$\boxed{M_{HP}(\omega) = \sqrt{\frac{\omega^2}{\omega^2 + \omega_0^2}}} \tag{17}$$

$$\boxed{\phi_{HP}(\omega) = \tan^{-1}\left(\frac{\omega_0}{\omega}\right)} \tag{18}$$

The transfer function of this circuit depends on frequency in the following way:

1. $\omega \ll \omega_0$, $M_{\mathrm{HP}} \approx 0$, $\phi_{\mathrm{HP}} \approx 90^\circ$
2. $\omega \gg \omega_0$, $M_{\mathrm{HP}} \approx 1$, $\phi_{\mathrm{HP}} \approx 0$

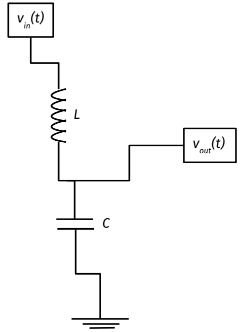

Figure 4: A simple LC-resonator (two-pole). What is the impedance of an inductor?

Impedance of an inductor:

$$L = \frac{d\phi}{di} \tag{19}$$

$$L = \frac{d\phi}{dt}\frac{dt}{di} = v/i(t) \tag{20}$$

$$\tilde{v}(\omega) = j\omega L\tilde{i}(\omega) \tag{21}$$

So we can read off the impedance of the inductor:

$$\boxed{Z_{\mathrm{L}} = j\omega L} \tag{22}$$

Following the same model, but with new impedances, we find Eq. 17 below by defining a *resonance frequency* ${\omega_R}^{-2} = LC$.

$$\boxed{M_{\mathrm{LC}}(\omega) = \frac{\omega_R^2}{\omega_R^2 - \omega^2} = \frac{\omega_R^2}{(\omega_R - \omega)(\omega_R + \omega)}} \tag{23}$$

ANALOG AND DIGITAL

Analog circuits

- Continuous functions of voltage and current representing continuous data
- Application of complex analysis

- Cannot be stored, copied or transmitted quickly and efficiently
- Requires knowledge of every frequency (bandwidth issues)
- Cannot be transmitted easily (noise issues)

Digital circuits

- Discrete functions of voltage and current representing coded data
- Application of Boolean logic

- Must sample enough information to reproduce analog signals

Music is an obvious example: storiing music on a vinyl record creates a groove in the material of the record that follows the pressure wave pattern of the sound. At no point are any frequencies lost, unless the record is scratched. When music is stored digitally, the sample rate affects quality, but with a large sample rate, the music is indistinguishable from the record player. Plus, the music can be perfectly copied or transmitted.

INTRODUCTION TO DIGITAL CONCEPTS

Recall from the low and high pass filter examples:

1. $\omega \ll \omega_0$, $M_{\mathrm{HP}} \approx 0$, $\phi_{\mathrm{HP}} \approx 90°$
2. $\omega \gg \omega_0$, $M_{\mathrm{HP}} \approx 1$, $\phi_{\mathrm{HP}} \approx 0$

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1. $\omega \ll \omega_0$, $M_{\mathrm{LP}} \approx 1$, $\phi_{\mathrm{LP}} \approx 0$
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What if instead of a frequency-based filter, we had an amplitude-based filter? (Also known as transistors...stay tuned).

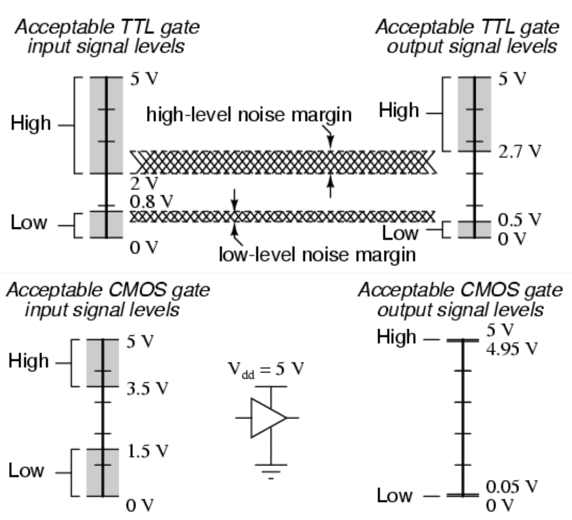

Figure 5: TTL and CMOS logic levels in integrated circuits today.

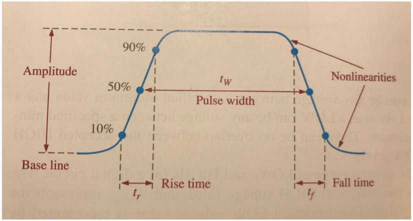

Figure 6: Properties of the logic pulse: rise and fall times, amplitude and non-linearities.

From where do the non-linearities arise? Turns out the Fourier transform of a square wave $s(t)$ with period T is a "sync" function ($2\pi\nu = \omega$):

$$\mathcal{F}(s(t)) = \tilde{S}(\nu) = \frac{\sin(\pi\nu T)}{\pi\nu} \tag{24}$$

From where do the non-linearities arise? Turns out the Fourier transform of a square wave $s(t)$ with period T is a "sync" function ($2\pi\nu = \omega$):

$$\mathcal{F}(s(t)) = \tilde{S}(\nu) = \frac{\sin(\pi\nu T)}{\pi\nu} \tag{25}$$

The **convolution theorem** states that if a signal is being filtered, the transfer function of that filter is being multiplied with the Fourier transform of the signal in the Fourier domain. Let's *low-pass filter* the sync function:

$$\tilde{S}(\nu) H_{\mathrm{LP}}(\omega) = \frac{\sin(\pi\nu T)}{\pi\nu} \sqrt{\frac{\omega_0^2}{\omega^2 + \omega_0^2}} \tag{26}$$

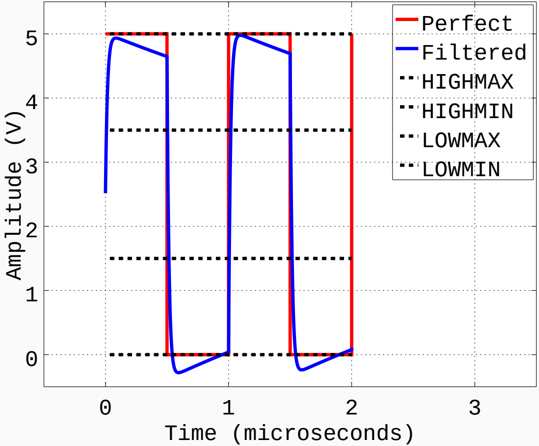

Figure 7: CMOS logic pulse filtered to remove frequencies above 0.01 MHz and below 2 MHz for a 1 MHz clock frequency.

CONCLUSION

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ANSWERS