1) COSC 360: HW1 Matthew Townsend

Show that if $z = \dfrac{x_1 + jy_1}{x_2 + jy_2}$ then $z^* = \dfrac{x_1 - jy_1}{x_2 - jy_2}$

$$z = \frac{x_1 + jy_1}{x_2 + jy_2}\left(\frac{x_2 - jy_2}{x_2 - jy_2}\right) = \frac{(x_1 + jy_1)(x_2 - jy_2)}{x_2^2 + y_2^2}$$

$$= \frac{x_1x_2 + y_1y_2 - x_1y_2j + x_2y_1j}{x_2^2 + y_2^2}$$

$$\Rightarrow z = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + j\,\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}$$

$$z^* = \frac{x_1 - jy_1}{x_2 - jy_2}\left(\frac{x_2 + jy_2}{x_2 + jy_2}\right) = \frac{(x_1 - jy_1)(x_2 + jy_2)}{x_2^2 + y_2^2}$$

$$= \frac{x_1x_2 + jy_2x_1 - jy_1x_2 + y_1y_2}{x_2^2 + y_2^2}$$

$$\Rightarrow z^* = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} - j\,\frac{y_2x_1 + y_1x_2}{x_2^2 + y_2^2}$$

Therefore the complex conjugate of z holds as z^* ■

2. (BONUS) COSC 360: HW1

Matt Townsend

Prove $e^{jx} = \cos(x) + j\sin(x)$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \cdots$ *Taylor's Series

$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots$

$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \cdots$

*Euler's formula

$$e^{jx} = 1 + (jx) + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} + \cdots$$

$$= 1 + jx + \frac{x^2}{2!} + j\frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\underbrace{\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!}\right)}_{\cos} + j\underbrace{\left(x + \frac{x^3}{3!} + \frac{x^5}{5!}\right)}_{\sin}$$

$$\boxed{e^{jx} = \cos(x) + \sin(x)}$$

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3) Prove $\cos(x) = \frac{1}{2}(e^{ix} + e^{-ix})$

$$e^{ix} = \sum_{i=0}^{\infty} \frac{(jx)^n}{n!} = \sum_{i=0}^{\infty} j^n \frac{x^n}{n!} = \sum_{even}^{\infty} j^{2n} \frac{x^{2n}}{(2n)!} + \sum_{odd}^{\infty} j^{2n+1} \frac{x^{2n+1}}{(2n+1)!}$$

$$e^{ix} = \sum_{i=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} + \sum_{i=0}^{\infty} j^{2n+1} \frac{x^{2n+1}}{(2n+1)!}$$

$$\left.\begin{array}{l} e^{ix} = \cos(x) + j\sin(x) \\ e^{-ix} = \cos(-x) + j\sin(-x) = \cos(x) - j\sin(x) \end{array}\right\} e^{ix} + e^{-ix} =$$

$$((\cos(x) + j\sin(x)) + (\cos(x) - j\sin(x))) = 2\cos(x)$$

$$e^{ix} - e^{-ix} = 2\cos(x)$$

$$= \cos(x) = \frac{1}{2}(e^{ix} - e^{-ix})$$