Thursday Warm Up, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- **Convolution**: this is an operation that characterizes the response $h[n]$ of a linear system.

$$y[i] = h[n] * x[n] = \sum_{j=0}^{M-1} h[j]x[i-j] \tag{1}$$

  In words, the output at sample i is equal to the produce of the system response h and the input signal x, summed over the proceeding M samples (from $j = 0$ to $j = M - 1$).

- **Discrete Delta Function**, $\delta[n]$: A standard impulse response that contains one non-zero sample. It has the following property:

$$x[n] = \delta[n] * x[n] \tag{2}$$

2 Convolution, Properties of Convolution, and Impulse Response

1. Let's design a DSP system that replicates *echo* in audio signals. Let the sampling frequency be 20 kHz. (a) Start with an delta function, $\delta[n]$, that is 2 seconds long. How many samples should it contain, given the sampling rate? (b) Modify the $\delta[n]$ to create an echo every 0.25 seconds. Give the locations of the non-zero samples only (instead of writing a huge list of numbers).[1] (c) Further modify the response function to make each echo have half the amplitude as the instance before it.

2. Let an impulse response be given by $h[0] = 1$, $h[1] = -1$, and zero for all other samples. (a) Write a quick bit of `octave` code that creates a $N = 10$ sample version of h. (b) Create a vector of data that increases linearly, with the same N. (c) Convolve the two, and show that (at least part) of the output corresponds to the slope of the linearly increasing data. (d) Why is this the case?

3. Suppose we have a response function $h[n]$ that is 1, if $n \geq 0$, and 0 otherwise. (a) Using Eq. 1, show that $h[n] * x[n]$ produces a cumulative sum of $x[n]$. (b) Suppose $x[n]$ was a square pulse with a width of $N = 10$ samples, and an amplitude of 1.0. What is the output of $h[n] * x[n]$? (c) Verify your calculations with a bit of `octave` code.

4. Our our reading, we encounter impulse responses corresponding to low-pass filters. Next, we encounter impulse responses corresponding to high-pass filters. The high-pass versions are equal to the low-pass versions subtracted from a $\delta[n]$. Use Eq. 2 to justify this design strategy. Why is a high-pass response simply "1" minus a low-pass response?

[1] *Hint: recall that we can break a complex response function into signal components, give them the right properties, then synthesize them into the correct response.*