Discrete Fourier

①

$$X_k = \sum_{n=0}^{N-1} x_n \cdot e^{-j2\pi kn/N}$$

$$Z_k = \sum_{n=0}^{N-1} (x[n] + y[n]) \cdot e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} y[n] \cdot e^{-j2\pi kn/N}$$

$$= X_k + Y_k$$

$$Z_k = \sum_{n=0}^{N-1} (a \cdot x[n]) \cdot e^{-j2\pi kn/N}$$

$$= a \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N} = a \cdot X_k$$

DFT are additive & homogeneus because its a linear sum

b) $X_k = \delta[k - k_0]$

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$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k \cdot e^{j2\pi kn/N}$$

$X_k = 0$ except at $k = k_0$

so $\boxed{x_n = \frac{1}{N} \cdot e^{j2\pi k_0 n/N}}$

② a more narrow pulse in time has more abrupt changes meaning it contains higher frequencies

★ short pulse → wide spectrum

★ long pulse → narrow spectrum

b) $\Delta t \cdot \Delta f$

row	Δt	Δf	$\Delta t \cdot \Delta f$
1	0.01	100	1
2	0.02	50	1
3	0.04	25	1
4	0.08	12.5	1

$\Delta t \cdot \Delta f \approx 1$

confirms Uncertainty principle because it is constant

HW #4

① $f_s = 20kHz$

a) $N = f_s \cdot T = 20000 \cdot 2 = \boxed{40000}$

b) $0.2s \rightarrow 0.2 \cdot 20000 = 4000$

$n=0$

echoes at $n = 4000, 8000, 12000, \ldots, 36000$

$\boxed{n = 0, 4000, 8000, \ldots, 36000}$

c) $\delta[n] = \delta[n] + 0.5s[n-4000] + 0.25\delta[n-8000] + \ldots$

$$h[n] = \sum_{k=0}^{9} \left(\tfrac{1}{2}\right) \delta[n - 4000k]$$

d) $x[n] = \sin(2\pi f t)$

$N = 0.1 \cdot 20000 = 2000$ samples

② $y[n] = s[n] \cdot l[n] + s[n] \cdot h[n]$

$= s[n] \cdot (l[n] + h[n])$

$l[n] + h[n] = \delta[n] \rightarrow y[n] = s[n]$

$l[n] + h[n] = \delta[n]$

b) $y[n] = s[n] \cdot l[n] + s[n] \cdot h[n]$

$l[n] + h[n] = \delta[n]$

$s[n] \cdot \delta[n] = s[n]$

$l[n] + h[n] = \delta[n]$

$\hookrightarrow h[n] = \delta[n] - l[n]$

c) option A

$l[n]$ = low-pass : passes f below its f_c

$h[n]$ = high-pass: passes f above its f_c

if $l[n]$ passes up to f_{cL} & high-pass passes from f_{cH} onward no freq gets through both filters

Option B:

High-pass passes f above f_{cH}

low-pass passes f below f_{cL}

if $f_{cH} < f_{cL}$ then $f_{cH} < f < f_{cL}$

This is band pass filter behavior

B is correct Filter