Tuesday Warm Up, Unit 1: Filter Design

Prof. Jordan C. Hanson

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1 Memory Bank

- **Convolution**: this is an operation that characterizes the response $h[n]$ of a linear system.

$$y[i] = h[n] * x[n] = \sum_{j=0}^{M-1} h[j]x[i-j] \quad (1)$$

  In words, the output at sample i is equal to the produce of the system response h and the input signal x, summed over the proceeding M samples (from $j = 0$ to $j = M - 1$).

- **Discrete Delta Function**, $\delta[n]$: A standard impulse response that contains one non-zero sample. It has the following property:

$$x[n] = \delta[n] * x[n] \quad (2)$$

- The **filter kernel** is the impulse response of a digital filter. Convolve this with the signal to obtain the output.
- **IIR** - infinite impulse response. A filter response can decay exponentially, never reaching zero.
- **FIR** - finite impulse response. A digital filter can be implemented via a kernel with a finite set of coefficients.
- **Decibels** - $dB = 10 \log_{10}(P_2/P_1) = 20 \log_{10}(A_2/A_1)$ ... A decibel unit of attenuation or amplification is a logarithmic unit of amplitude ratios A_i or power ratios P_i.

2 Basic Filter Concepts

1. (a) Suppose a filter lowers a signal amplitude from 5 V to 5 mV at 15 kHz. What is this attenuation in dB? (b) Suppose a filter lowers signal amplitude by 40 dB in the stop band. If the input signal amplitude is 12 V, what is the output amplitude? (c) We need to amplify a signal from 10 μW to 100 mW. What power amplification is required, in dB? (d) A **dBm** is a decibel unit in which the power in the denominator is set to 1 mW exactly. RF thermal noise in amplified RF antennas is usually observed to be -174 dBm. What is this power in regular units?

2. Consider Fig. 1, which contains the step response of a filter. (a) Assume the sampling frequency is 2 MHz. What is the time duration from sample 0 to sample 64? (b) Estimate the *rise-time* of the step response.

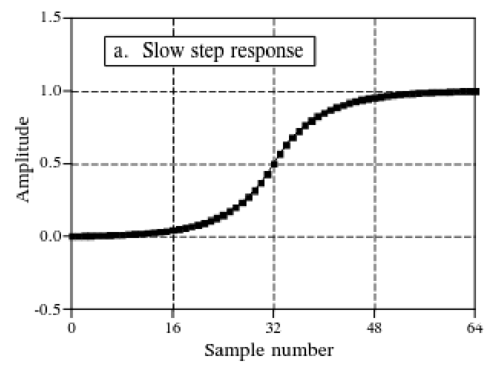

Figure 1: Example of a filter kernel with a slow rise-time.

3 Mathematics of Filter Response

1. Let $y[n]$ be the output of a filter. The output $y[n]$ is equal to $h[n]$ when the input is $\delta[n]$. (a) Prove that the *step response* is the running sum of $h[n]$. (b) Prove that the impulse is the discrete derivative (first-difference) of the step response.

2. Using `octave`, create a low-pass filter kernel with 16 samples, called `k`. To ensure it is low-pass, make it a smoothly rising and falling function. Compute the following: (a) the impulse response of `k`, and (b) the step response of `k`.

3. **Spectral inversion** of a filter kernel changes its behavior from low-pass to high-pass. Using your kernel from the previous exercise, create a high-pass kernel by subtracting it from an impulse at sample zero. Compute the following: (a) the new impulse response, and (b) the new step response. **Graph the responses from this and the previous exercise below.**