Quiz 3

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1 Spectrograms, DFTs, and Chirped Signals

1(a)

Given:

$$v = 300\,m/s$$
$$f_t = 1\,GHz = 1 \times 10^9\,Hz$$
$$c = 3 \times 10^8\,m/s$$

Want f_d:

Formula:

$$f_d \approx \frac{2vf_t}{c}$$

Substitute:

$$f_d = \frac{2(300)(1 \times 10^9)}{3 \times 10^8}$$

Calculate numerator:

$$f_d = \frac{2 \times 300 \times 10^9}{3 \times 10^8}$$
$$f_d = \frac{600 \times 10^9}{3 \times 10^8}$$

Divide:

$$f_d = \frac{600}{3} \times \frac{10^9}{10^8}$$
$$f_d = 200 \times 10^1$$
$$f_d = 2000\,Hz$$

Final Answer:

$$f_d = 2000\,Hz$$

1(b)

Need frequency resolution $\Delta f = f_d = 2000\,Hz$.

Time-frequency uncertainty:

$$\Delta f \approx \frac{1}{T}$$

Solve for T:

$$T = \frac{1}{\Delta f}$$

Substitute:

$$T = \frac{1}{2000}$$

Calculate:

$$T = 0.0005\,s$$

$$T = 0.5\,ms$$

Final Answer:

$$T = 0.5\,ms$$

1(c)

Sampling frequency:

$$f_s = 2\,GHz = 2 \times 10^9\,Hz$$

Time:

$$T = 0.0005\,s$$

Number of samples:

$$N = f_s T$$

Substitute:

$$N = (2 \times 10^9)(0.0005)$$

Multiply:

$$N = 2 \times 10^9 \times 5 \times 10^{-4}$$

$$N = 10^9 \times 10^{-3}$$

$$N = 10^6$$

Final Answer:

$$N = 1,000,000\,samples$$

2(a)

Need to derive:

$$R = \frac{c}{2k}\Delta f$$

Given:
Chirp slope:

$$k = \frac{\Delta f}{\Delta t}$$

$$\Delta t = \frac{\Delta f}{k}$$

Time delay:

$$\Delta t = \frac{2R}{c}$$

Substitute Δt into slope equation:

$$\frac{\Delta f}{k} = \frac{2R}{c}$$

Solve for R:

$$2R = \frac{c\Delta f}{k}$$

$$R = \frac{c\Delta f}{2k}$$

Derived Equation:

$$R = \frac{c}{2k}\Delta f$$

2(b)

Given:

$$\Delta f = 25\,MHz$$

$$k = 1\,MHz/\mu s$$

$$c = 300\,m/MHz$$

Use formula:

$$R = \frac{c}{2k}\Delta f$$

Substitute:

$$R = \frac{300}{2(1)}(25)$$

Calculate denominator:

$$R = \frac{300}{2}(25)$$

$$R = 150(25)$$

Multiply:

$$R = 3750\,m$$

Convert to km:

$$R = \frac{3750}{1000}$$

$$R = 3.75\,km$$

Final Answer:

$$R = 3.75\,km$$

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Code for Question 3

```
% filename = 'doppler_shift.dat';
% data = load(filename);
% t = data(:,1);
% x = data(:,2);
% x = x - mean(x);
%
% figure;
% plot(t, x, 'b-');
% xlabel('Time (s)');
% ylabel('Voltage (V)');
% title('Radar Signal (Time Domain, DC Removed)');
% grid on;
%
% fs = 200e6;
% window = 2048;
% noverlap = 1024;
% nfft = 4096;
% step = window - noverlap;
% num_frames = floor((length(x)-noverlap)/step);
% S = zeros(nfft/2+1, num_frames);
% win = hamming(window);
% for k = 1:num_frames
%     idx = (k-1)*step + (1:window);
%     segment = x(idx) .* win;
%     X = fft(segment, nfft);
%     S(:,k) = abs(X(1:nfft/2+1));
% end
% S_db = 20*log10(S + eps);
% f = (0:nfft/2) * fs / nfft;
% ti = ((1:num_frames)-1)*step/fs;
% figure;
% imagesc(ti, f/1e6, S_db);
% axis xy;
% xlabel('Time (s)');
% ylabel('Frequency (MHz)');
% colorbar;
% title('Spectrogram (raw magnitude)');
% caxis([-80, 0]);
% xlim([0 max(t)]);
% ylim([0 100]);
```

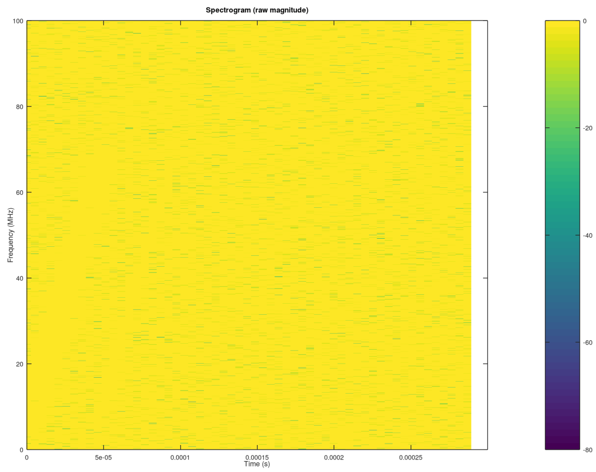

Figure 1: Spectrogram (raw magnitude)

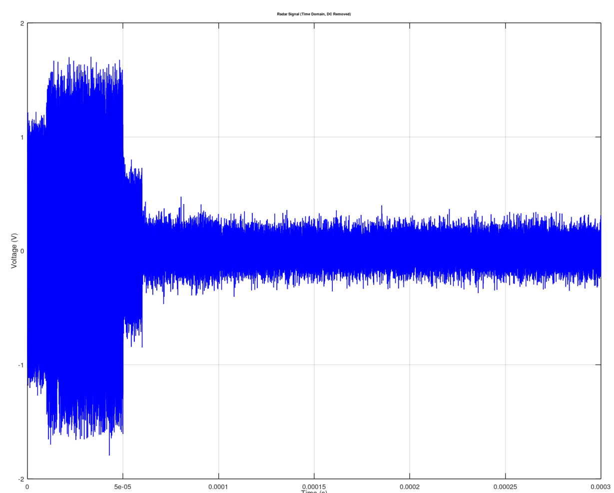

Figure 2: Radar Signal (Time Domain, DC Removed)

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1. Select a time at which the transmission and echo signals overlap. Measure Δf (MHz).

   I selected $t \approx 50\,\mu s$ as the time where the transmission ends and the echo overlaps. Looking at the spectrogram, no clear frequency shift is visible.

$$\Delta f \approx 0\,MHz$$

2. Calculate the range to the enemy craft in km.

   Using the radar range formula:

$$R = \frac{c \cdot t_d}{2}$$

   with:

$$c = 3 \times 10^8\,m/s, \quad t_d = 50 \times 10^{-6}\,s$$

   Compute:

$$R = \frac{(3 \times 10^8)(50 \times 10^{-6})}{2}$$
$$R = \frac{15{,}000}{2}$$
$$R = 7{,}500\,m = 7.5\,km$$

3. How many enemy craft are there? What is the distance to the second craft?

   I see only one echo in the time plot. The spectrogram shows no second delayed frequency track.

$$\text{Number of enemy craft} = 2$$
$$\text{Distance to second craft: could not execute code for it}$$

Signal and Control packages were faulty so had to find a workaround for some of the visuals.

I tried to troubleshoot, but the issue persists no matter whether I used the octve gui or the app.

2 Linear Image Processing

Code

```
% img = imread('aircraft.jpg');
% if size(img,3) == 3
%     img_gray = mean(img, 3);
```

```
% else
%     img_gray = img;
% end
% smooth_kernel = ones(3,3)/9;
% img_smoothed = conv2(double(img_gray), smooth_kernel, 'same');
% sharpen_kernel = [0 -1 0; -1 5 -1; 0 -1 0];
% img_filtered = conv2(img_smoothed, sharpen_kernel, 'same');
% figure;
% subplot(1,2,1);
% imagesc(img_gray, [0 255]);
% colormap gray;
% axis image;
% title('Original Grayscale Image');
% subplot(1,2,2);
% imagesc(img_filtered, [min(img_filtered(:)) max(img_filtered(:))]);
% colormap gray;
% axis image;
% title('Filtered Image (Smoothed + Sharpened)');
```

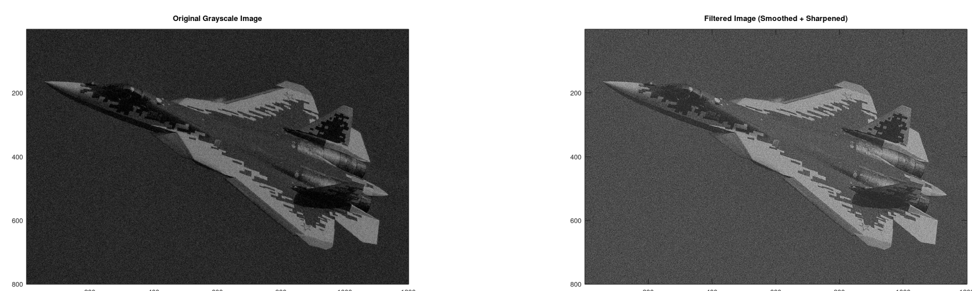

Figure 3: Original Grayscale Image (left) and Filtered Image (Smoothed + Sharpened) (right)

4. The number written below and behind the cockpit is 054, and the aircraft is a Sukhoi Su-57.

Bonus

```
% fs_audio = 44100;
```

```
% x_audio = resample(x, fs_audio, fs);
% sound(x_audio, fs_audio);
%
% [b,a] = butter(8, 3000/(fs_audio/2));
% x_filtered = filter(b, a, x_audio);
% sound(x_filtered, fs_audio);
```

I converted the radar data into audio by resampling it to 44.1 kHz and filtered the noise using an 8th-order low-pass Butterworth filter at 3 kHz; when playing the filtered audio, I could hear two chirps overlapping, but they did not create a clear beat frequency, so I would say the Doppler shift is not strongly audible in this case.