1

a) $A\{x[n]\} = 2x[n] - 1$ not linear not homogeneous

$B\{x[n]\} = 0.5\,x[n]$ linear

b) $A_{linear}\{x[n]\} = 2x[n]$

$$A\{B\{x[n]\}\} = 2(0.5x[n])$$
$$= 1x[n] = x[n]$$

$$B\{A\{x[n]\}\} = 0.5(2x[n]) = 1x[n] = x[n]$$

2

$$f(t) = a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t)$$

$$T_1 = \frac{1}{f_1}$$

$$T_2 = \frac{1}{f_2} = \frac{1}{2} T_1$$

$$\cos(2\pi) = \cos(4\pi) = 1$$

$$f_2 = 2f_1$$

$$\int_{-\infty}^{\infty} f(t)\delta(t - T_1)\,dt = f(T_1) = a_1 \cos(2\pi f_1 T_1) + a_2 \cos(2\pi f_2 T_1)$$
$$= a_1 \cos(2\pi) + a_2 \cos(4\pi)$$
$$= a_1 + a_2$$

$$\int_{-\infty}^{\infty} f(t)\delta(t - T_2)\,dt = f(T_2) = a_1 \cos(2\pi f_1 T_2) + a_2 \cos(2\pi f_2 T_2)$$
$$= a_1 \cos\left(2\pi f_1 \frac{1}{2f_1}\right) + a_2 \cos(2\pi)$$
$$= a_1 \cos(\pi) + a_2 \cos(2\pi)$$
$$= a_1 0 + a_2 1 = a_2$$

3

a) $f(t) = a\delta(t-t_0)$

$$F(f) = \int_{-\infty}^{\infty} a\delta(t-t_0)e^{-2\pi j f t} = \underset{\uparrow \\ \text{amp}}{a}e^{-2\pi j f t_0}$$

c) $\frac{-d\phi}{d\omega} = T_0 = 0$

$\phi = -2\pi f T_0$

4

$\delta[n] = [1000\ 0000]$

a) $S[\delta[n]] = 0.5\delta[n-2] = [0,0,0.5,0,0,0,0,0] = y[n]$

$s[n] = [0\ 111\ 1111]$

b) $S[s[n]] = 0.5\, s[n-2] = [0.5, 0.5, 0, 0.5, 0.5, 0.5, 0.5, 0.5]$

3

1 a) $s(t) = a\delta(t-t_0)$

$$S(f) = \int_{-\infty}^{\infty} a\delta(t-t_0)e^{-2\pi j f t} = ae^{-2\pi j f t_0}$$

$$S(f) = a\cos(2\pi f t_0) - aj\sin(2\pi f t_0)$$

b) $$\frac{|S(f)|}{|(1+j\omega T)|} = \frac{|ae^{-2\pi j f T_0}|}{|1+j\omega T|}$$

$$|a e^{-2\pi j f t_0}| = |a\cos(2\pi f t_0) - a j \sin(2\pi f t_0)|$$

$$= \sqrt{a^2(\cos^2(2\pi f t_0) + \sin^2(2\pi f t_0)} = |a|$$

$$|S(f)| = |a|$$

$$|(1+j\omega T)| = \sqrt{1^2 + (\omega T)^2} = \sqrt{1+(\omega T)^2}$$

$$= \sqrt{1 + 4(\pi f RC)^2}$$

$$\frac{|a|}{\sqrt{1+4(\pi f RC)^2}}$$

c) $$\left|\frac{j\omega T}{1+j\omega T}\right| = \frac{|j\omega T|}{|1+j\omega T|} = \frac{|\omega T|}{\sqrt{1+4(\pi f RC)^2}}$$

$$= \frac{|a||2\pi f RC|}{\sqrt{1+4.(\pi f RC)^2}}$$

2 a) $S(f) = a e^{-j2\pi f t_0}$

$\phi_{S(f)} = -2\pi f t_0$

$L(f) = \frac{1}{1+j2\pi f T} = \frac{1}{1+j2\pi f T} \frac{1-j2\pi f T}{1-j2\pi f T}$

$\phi_{L(f)} = \tan^{-1}\left(\frac{\frac{2\pi f T}{1+4(\pi f T)^2}}{\frac{1}{1+4(\pi f T)^2}}\right) = \frac{1-j2\pi f T}{1+4(\pi f T)^2}$

$\phi_{L(f)} = \tan^{-1}(2\pi f T) = \frac{1}{1+4(\pi f T)^2} - \frac{j2\pi f T}{1+4(\pi f T)^2}$

$T_g(f) = -\frac{d}{df}(\phi_{S(f)} + \phi_{L(f)})$

$= \frac{-d}{df}(-2\pi f t_0 + \tan^{-1}(2\pi f T))$

$= 2\pi t_0 - 2\pi T \frac{1}{1+(2\pi f T)^2}$

b) $H(f) = \frac{j2\pi f T}{1+j2\pi f T} = \frac{j2\pi f T}{1+j2\pi f T} \frac{1-j2\pi f T}{1-j2\pi f T}$

$= \frac{j2\pi f T + (2\pi f T)^2}{1+(2\pi f T)^2}$

$= \frac{j2\pi f T}{1+(2\pi f T)^2} + \frac{(2\pi f T)^2}{1+(2\pi f T)^2}$

$\phi_{H(f)} = \tan^{-1}\left(\frac{\frac{2\pi f T}{1+(2\pi f T)^2}}{\frac{(2\pi f T)^2}{1+(2\pi f T)^2}}\right) = \tan^{-1}\left(\frac{2\pi f T}{(2\pi f T)^2}\right)$

$= \tan^{-1}\left(\frac{1}{2\pi f T}\right)$

$T_g(f) = -\frac{d}{df}\left(-2\pi f t_0 + \tan^{-1}\left(\frac{1}{2\pi f T}\right)\right)$

$$T_g(f) = 2\pi t_0 + \frac{2\pi T}{(2\pi f T)^2} \cdot \frac{1}{1+\left(\frac{1}{2\pi f T}\right)^2}$$

$$= 2\pi t_0 + \frac{2\pi T}{(2\pi f T)^2 + 1}$$

3a)

$$s(f) = \left(\frac{a}{2}\right)\left(\delta(f-f_0)+\delta(f+f_0)\right)$$

$$F^{-1}\{s(f)\} = \frac{a}{2}\left(\int_{-\infty}^{\infty} \delta(f-f_0)e^{2\pi jft}df + \int_{-\infty}^{\infty} \delta(f+f_0)e^{2\pi jft}df\right)$$

$$= \frac{a}{2}\left(e^{2\pi jft_0} + e^{-2\pi jft_0}\right)$$

$$= \frac{a}{2}\left(\cos(2\pi ft_0) - j\sin(2\pi ft_0) + \cos(-2\pi ft_0) - j\sin(-2\pi ft_0)\right)$$

$$= \frac{a}{2}\left(2\cos(2\pi ft_0) - \cancel{j\sin(2\pi ft_0)} + \cancel{j\sin(2\pi ft_0)}\right)$$

$$= a\cos(2\pi ft_0)$$

b) $$s(f) = \frac{a}{2j}\left(\delta(f-f_0) - \delta(f+f_0)\right)$$

$$F^{-1}\{s(f)\} = \frac{a}{2j}\left(e^{2\pi jft_0} - e^{-2\pi jft_0}\right)$$

$$= \frac{a}{2j}\left(\cos(2\pi ft_0) - j\sin(2\pi ft_0) - \cos(-2\pi ft_0) + j\sin(-2\pi ft_0)\right)$$

$$= \frac{a}{2j}\left(\cos(2\pi ft_0) - \cos(2\pi ft_0) - j\sin(2\pi ft_0) - j\sin(2\pi ft_0)\right)$$

$$= \frac{a}{2j}\left(-2j\sin(2\pi ft_0)\right) = -a\sin(2\pi ft_0)$$

4

1.

a)

$x[n] = \delta[n] \quad y[n] = h[n] * x[n] = h[n]$

$$y[i] = \sum_{j=0}^{M-1} h[j]x[i-j]$$

$x[0] = 1$ $\quad \sum_{j=0}^{M-1} h[j]x[i-j] = h[i]$

$x[1] = 0$

$$y[i] = h[i]$$

$$y[n] = h[n]$$

b) $x[n] = \delta[n - n_0]$

$x[n_0] = 1$

$$y[i] = \sum_{j=0}^{M-1} h[j]x[i-j]$$

$i - j = n_0$

$i = n_0 + j$

$i - n_0 = j$

$$\sum_{j=0}^{M-1} h[j]x[i-j] = h[i - n_0]$$

$$y[i] = h[i - n_0]$$

```
clear;
close;
home;
f = 440.0;
dt = 1/f;
t = dt:dt:f/2;

n0 = 10;

s = sin(f*2*pi*t);
d = zeros(size(s));
d(n0) = 1;

plot(conv(s,d));
```