Digital Signal Processing: COSC390

Jordan Hanson

January 8, 2019

Whittier College Department of Physics and Astronomy

Normal distribution

Normally distributed data decreases in probability at a rate that is proportional (1) to the *distance from the mean*, and that is proportional (2) to the *probability itself.*

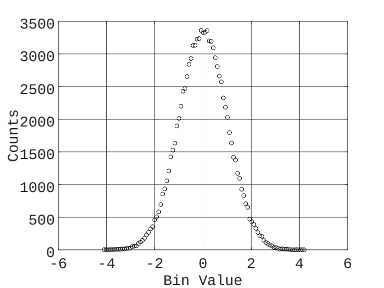

Figure 1: Normally distributed data counts decrease as measured further from the mean for *two reasons.*

Normal distribution

Normal Distribution PDF

Let $p(x)$ be the PDF of normally distributed data x with mean μ. In order to obey conditions (1) and (2), the function $p(x)$ must be described by the following differential equation, where k is some constant.

$$\frac{dp}{dx} = -k(x - \mu)p(x) \tag{1}$$

Normal distribution

Rearranging Eq. 1, we have

$$\frac{dp}{p} = -k(x-\mu)dx \tag{2}$$

Integrating both sides gives

$$\ln(p) = -\frac{1}{2}k(x-\mu)^2 + C_0 \tag{3}$$

Exponentiating,

$$p(x) = C_1 \exp\left(-\frac{1}{2}k(x-\mu)^2\right) \tag{4}$$

Ensuring that the PDF is *normalized* requires

$$\int_{-\infty}^{\infty} p(x)dx = 1 \tag{5}$$

Normal distribution

But how do we integrate Eq. ??? First, a change of variables. Let $s = x - \mu$