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COSC: 360 Quiz 2

1) Find the phase angle: $\phi = \tan^{-1}(y/x)$

a) $z = -2 + 2j$

$\phi = \tan^{-1}(y/x) \Rightarrow \phi = \tan^{-1}(2j/-2)$

$= \tan^{-1}(-j)$

$\boxed{\tan^{-1}(-j) = \phi}$

b) $z = -2 - 2j$

$\phi = \tan^{-1}(y/x) \Rightarrow \phi = \tan^{-1}(-2j/-2)$

$= \tan^{-1}(j)$

$\boxed{\tan^{-1}(j) = \phi}$

c) $z = 2 - 2j$

$\phi = \tan^{-1}(y/x) \Rightarrow \phi = \tan^{-1}(-2j/2)$

$= \tan^{-1}(-j)$

$\boxed{\tan^{-1}(-j) = \phi}$

what phasor represents the sinusoids at $t=0$?

a) $v(t) = 4(\cos(2\pi(10.0)t + 30°)$

$= 4(\cos(2\pi(10.0)(0) + (\pi/6))$

$= 4(\cos(0.536))$

$= 4(0.8598) = 3.4392$

therefore, $z = -2 - 2j$ best represents

b) $v(t) = 2(\sin(2\pi(10.0)t - 60°)$

$2(\sin(2\pi(10.0)(0) - (\pi/3))$

$2(\sin(-1.0472)) = 2(-0.86602)$

$= -1.73204$

therefore, $z = 2 - 2j$ and $z = -2 + 2j$ best represent

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Fourier Analysis

2)

$$F(x) = \begin{cases} 1, & 0 \le x < \pi \\ 0, & \pi < x \le 2\pi \end{cases}$$

$$F(x) = \frac{A_0}{2} + \sum_{n+1}^{\infty} A_n \cos(nx) + B_n \sin(nx)$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} F(x) \cos(nx)\, dx \qquad a_n = \frac{1}{\pi} \int_0^{\pi} 1 \cdot \cos(nx)\, dx$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} F(x) \sin(nx)\, dx \qquad a_n = \frac{1}{\pi} \int_0^{\pi} \cos(nx)\, dx = 0$$

$$b_n = \frac{1}{\pi} \int_0^{\pi} 1 \cdot \sin(nx)\, dx = \frac{-1}{\pi} \left[\frac{\cos(nx)}{n} \right]_0^{\pi}$$

$$b_{odd} = \frac{1}{n\pi} (1 - (-1)) = \frac{2}{n\pi} \qquad f(x) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \cos(nx) \to Re$$

$$b_n = \frac{1}{\pi n} (1 - (-1)^n) \qquad f(w) = \int_{-\infty}^{\infty} f(x) e^{-jwt}\, dx \to Im$$

$$\phi(w) = \tan^{-1}(Im(f) / Re(f))$$

$$= \tan^{-1}\left(e^{-jwt} / \tfrac{1}{2} + \sum_{n=1}^{\infty} \left(\frac{2}{n\pi}\right) \cos(nx)\right) = \tan^{-1}\left(\frac{e^{-jwt}}{\frac{1}{2} + \sum_{n=1}^{\infty} (2/n\pi) \cos(nx)}\right)$$

therefore the phase of frequency,

$$\boxed{\phi(w) = \tan^{-1}\left(\frac{e^{-jwt}}{1/2 + \sum_{n=1}^{\infty} (2/n\pi) \cos(nx)}\right)}$$

When does magnitude equal zero?

$$|F(w)| = f(w) f^*(w) \Rightarrow \left(\int_{-\infty}^{\infty} 1/2 + \sum_{n=1}^{\infty} 2/n\pi \cos(nx) e^{-jwt}\, dx\right)$$

$$\left(\int_{-\infty}^{\infty} 1/2 - \sum_{n=1}^{\infty} 2/n\pi \cos(nx) e^{jwt}\, dx\right)$$

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2) when does magnitude equal zero?

$$= \left(\int_{-\infty}^{\infty} 1/2 + \sum_{n=1}^{\infty} 2/n\pi \cos(nx) e^{-j\omega t} dx\right)\left(\int_{-\infty}^{\infty} 1/2 - \sum_{n=1}^{\infty} 2/n\pi \cos(nx) e^{j\omega t} dx\right)$$

$F(w) = -\frac{1}{jw}\left[e^{-jwt}\right]_0^{\pi}$ *revised function.

$F(w) = -\frac{e^{-jwt}}{jw}$ $F^*(w) = \frac{e^{jwt}}{jw}$

$$\left(-\frac{e^{-jwt}}{jw}\right)\left(\frac{e^{jwt}}{jw}\right) = -\frac{e^{-jwt}e^{jwt}}{jw^2} = \frac{\sin(t)}{t}$$

magnitude $= \frac{\sin(t)}{t}$ sinc function

therefore when equaled to zero and find conclusion,

$0 = \frac{\sin(t)}{t} \Rightarrow \sin(t) = 0 \rightarrow t = 2\pi$ and $\pi + 2\pi$

→ Resulting in when magnitude is 2π and $\pi + 2\pi$ then it will be equal to zero.

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3) uniform distribution with range [0,1]

equation p(x) → normalized [a,b] ⟷ [0,1]

theoretical

mean $\mu = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{a+b}{2}$ a=0 b=1

standard deviation $\sigma = \sqrt{\frac{(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{(b-a)^2}{12}}$

$P(x) = \frac{1}{b-a}$ therefore, $\boxed{p(x) = 1}$ → probability density function

mean $= \frac{0+1}{2} = \boxed{\frac{1}{2} \text{ or } .5}$ μ

standard deviation $= \sqrt{\frac{(1-0)^2}{12}} = \boxed{\frac{1}{\sqrt{12}}}$ σ