Thursday Warm Up, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- **Homogeneous system:** Let k be a constant, and let $s_{\text{in}}(t)$ and $s_{\text{out}}(t)$ be the input and output signals to a system S, respectively. S is *homogeneous* if:

$$s_{\text{out}}(t) = S[s_{\text{in}}(t)] \quad (1)$$
$$ks_{\text{out}}(t) = S[ks_{\text{in}}(t)] \quad (2)$$

- **Additive system:** Let $s_1(t)$ and $s_2(t)$ be two input signals to a system S, with outputs $s_1'(t)$ and $s_2'(t)$. S is *additive* if:

$$s_1'(t) = S[s_1(t)] \quad (3)$$
$$s_2'(t) = S[s_2(t)] \quad (4)$$
$$s_1'(t) + s_2'(t) = S[s_1(t) + s_2(t)] \quad (5)$$

- **Shift-invariant system:** Let $s_{\text{in}}(t)$ and $s_{\text{out}}(t)$ be input and output signals to a system S, and let t_0 be a constant. S is *shift invariant* if:

$$s_{\text{out}}(t) = S[s_{\text{in}}(t)] \quad (6)$$
$$s_{\text{out}}(t - t_0) = S[s_{\text{in}}(t - t_0)] \quad (7)$$
$$(8)$$

- **Synthesis:** combining input signal components together linearly to form an output signal.
- **Decomposition:** producing the output signal components linearly from an input signal.
- **Fundamental Concept of DSP:** Decomposing an input signal into components, passing them trough a linear system, and synthesizing the results produces the same output as passing the original signal through the system.
- **Impulse signal:** a single nonzero point in a string of zeros.
- **Impulse decomposition:** decomposing a digitized, sampled signal into a linear combination of impulse signals.
- **Even/Odd decomposition:** decomposing a digitized, sampled signal into even and odd signal components.
- $f(-t) = f(t)$ ... Even function. Even signals: $x_{\text{E}}[n] = (x[n] + X[N - n])/2$
- $f(-t) = -f(t)$ ... Odd function. Odd signals: $x_{\text{O}}[n] = (x[n] - X[N - n])/2$

2 Linear Systems

1. Develop an expression for $y_3[n]$ in Fig. 1. Which subsystems must be homogeneous, additive, and shift-invariant, so that $y_3[n]$ retains these properties?

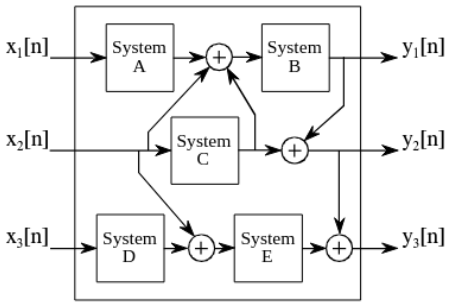

Figure 1: A DSP system with multiple inputs and outputs.

2. Determine if the following functions are even or odd:
   - $\sin(2\pi ft)$:
   - $\exp(-t^2)$:
   - $\exp(-t)$:
   - $at^2 + bt + c$:

3. Suppose a system S acts on a signal $x[n]$: $y[n] = S[x[n]]$. The result is that $x[n]$ is delayed (shifted to the right) by 10 samples, and reduced in amplitude by a factor of 2. (a) If $x[n] = [040000]$, what is $y[n]$? (b) If $x[n] = [000020]$, what is $y[n]$?

4. (a) Break the signal $x[n] = [010010]$ into component signals that are impulses. That is, perform an impulse deconposition on $x[n]$. (b) Pass each component through S from the previous exercise, and sum the output signal components.