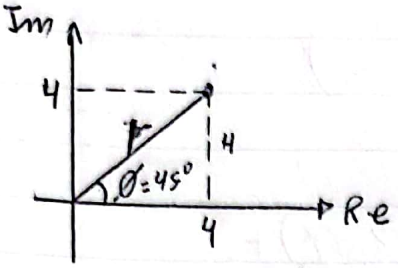
Quiz_1 Pablo Alvarado

1.- $Z = 4 + 4j$

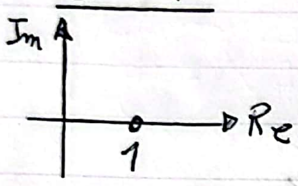


$r = \sqrt{4^2 + 4^2} \Rightarrow r = 4\sqrt{2}$

$z = r e^{j\emptyset}$

$\emptyset = 45° = \pi/4 \Rightarrow$ $Z = 4\sqrt{2}\, e^{j(\pi/4)}$

2.- $Z = 1$:



$Z = r e^{j\emptyset}$

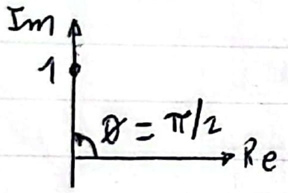
$Z = 1 e^{j(0)}$

$\Rightarrow \emptyset = 0° = 0\,rad$

$r = 1$

$Z = 1$ ← polar form

$Z = j$:



$r = 1$

$\emptyset = \pi/2$

$Z = (1) e^{j(\pi/2)} \Rightarrow Z = e^{\frac{j\pi}{2}}$

$Z = -1$: $r = 1$

$\emptyset = \pi$

$\Rightarrow z = (1)\, e^{j\pi} \Rightarrow Z = e^{j\pi}$

$Z = -j$:

$r = 1$

$\emptyset = 270° = 3\pi/2$

$\Rightarrow z = (1)\, e^{j(3\pi/2)} \Rightarrow Z = e^{\frac{3j\pi}{2}}$

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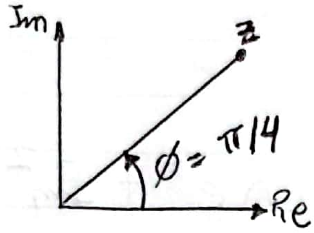
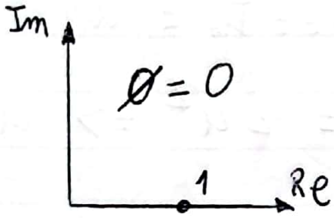
3.- $z = 4 + 4j$



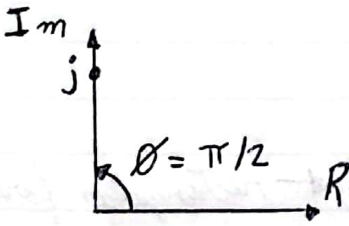
The angle in this case is $\pi/4$ rad because the real part and imaginary part have the same magnitud.

$z = 1$:



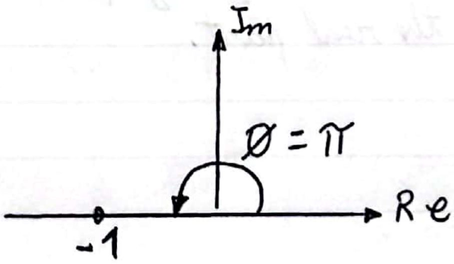
In this case there is only a real part and it is positive, so the phase angle can be (0) or (2π).

$z = j$:



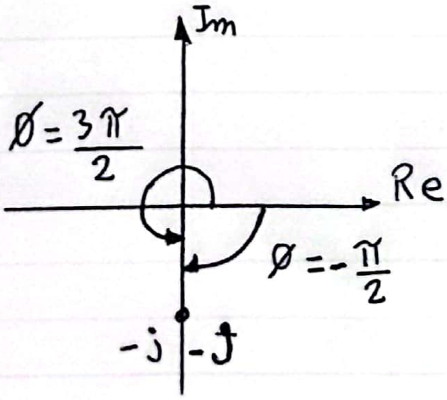
Here this problem has only imaginary part and it is positive, so the phase angle can be $(\pi/2)$ or $(-3\pi/2)$.

$z = -1$:



This example has only a real part and it is negative (-1), so the phase angle can be (π) or $(-\pi)$.

$z = -j$:



In this final example there is just imaginary part and it is negative then the phase angle is gonna be $(3\pi/2)$ or it can be $(-\pi/2)$.

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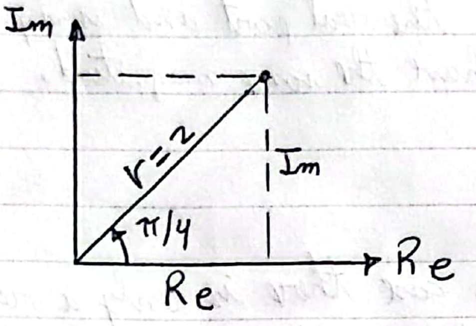
1.- $z = 2e^{j\pi/4}$

So: $r = 2$ and $\phi = \pi/4$



$r^2 = Re^2 + Im^2$

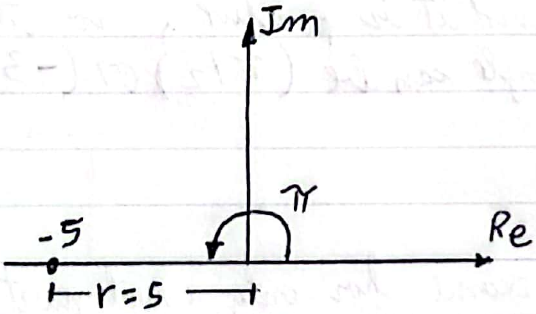
But $\phi = \pi/4$

so $Re = Im =$ "a"

$r^2 = 4 = 2a^2 \Rightarrow a = \sqrt{2}$

So: $z = \sqrt{2} + \sqrt{2}j$

2.- $z = 5e^{j\pi}$



$z = -5$ ← rectangular form

There is not imaginary part.
Just the real part.

01/ 11 /22

Quiz 1:

1. $v(t) = a_1 e^{jx_1} + a_2 e^{jx_2}$

$v^*(t) = a_1 e^{-jx_1} + a_2 e^{-jx_2}$ the conjugate

$v^* v = (a_1 e^{-jx_1} + a_2 e^{-jx_2})(a_1 e^{jx_1} + a_2 e^{jx_2})$

$v^* v = a_1^2 + a_2^2 + a_1 a_2 e^{-jx_1 + jx_2} + a_1 a_2 e^{-jx_2 + jx_1}$

$|v|^2 = a_1^2 + a_2^2 + a_1 a_2 (e^{j(x_2 - x_1)} + e^{-j(x_2 - x_1)})$

$\Delta x = x_2 - x_1 = \cancel{2\pi f t} + \theta_2 - (\cancel{2\pi f t} + \theta_1)$

$\Rightarrow |v|^2 = a_1^2 + a_2^2 + a_1 a_2 (e^{j\Delta x} + e^{-j\Delta x})$

$|v|^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos(\Delta x)$

$\Delta x = \Delta\theta = \theta_2 - \theta_1$

$\therefore |v|^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos(\Delta\theta)$

If: $\Delta\theta = 0$:

$|v|^2 = a_1^2 + a_2^2 + 2 a_1 a_2 = (a_1 + a_2)^2$

If $\Delta\theta = \pi \Rightarrow \cos(\Delta\theta) = -1$

$|v|^2 = a_1^2 + a_2^2 - 2 a_1 a_2 = (a_1 - a_2)^2$

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2. $\phi_V = \tan^{-1}(Im\{V\}/Re\{V\})$

$$V = a_1 e^{jx_1} + a_2 e^{jx_2}$$

$$V = a_1 \cos(x_1) + a_1 \sin(x_1) j + a_2 \cos(x_2) + a_2 \sin(x_2) j$$

$$Im\{V\} = Im = a_1 \sin(x_1) + a_2 \sin(x_2)$$

$$Re\{V\} = Re = a_1 \cos(x_1) + a_2 \cos(x_2)$$

$$\Rightarrow \phi_v = \tan^{-1}\left(\frac{a_1 \sin(x_1) + a_2 \sin(x_2)}{a_1 \cos(x_1) + a_2 \cos(x_2)}\right)$$

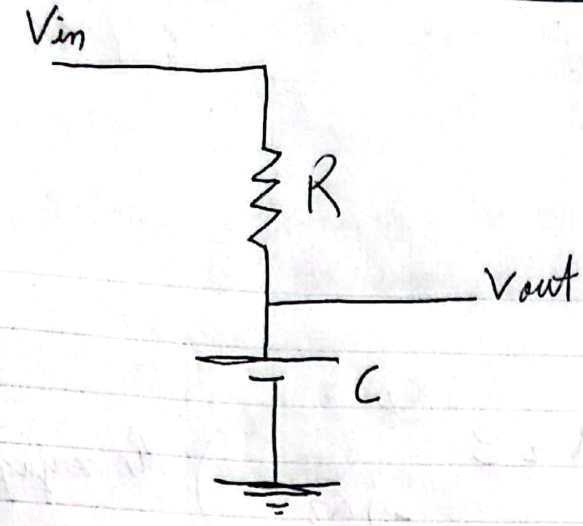
But: $x_i = 2\pi f t + \phi_i$

$$x_1 = 2\pi f t + \phi_1 \quad \wedge \quad x_2 = 2\pi f t + \phi_2$$

$$\Rightarrow \phi_v = \tan^{-1}\left(\frac{a_1 \sin(2\pi f t + \phi_1) + a_2 \sin(2\pi f t + \phi_2)}{a_1 \cos(2\pi f t + \phi_1) + a_2 \cos(2\pi f t + \phi_2)}\right)$$

When: $t = 0 \rightarrow x_1 = \phi_1 \quad \wedge \quad x_2 = \phi_2$

$$\phi_v = \tan^{-1}\left(\frac{a_1 \sin(\phi_1) + a_2 \sin(\phi_2)}{a_1 \cos(\phi_1) + a_2 \cos(\phi_2)}\right)$$



$$V_{in} = I\,Z(w)$$

$$Z = \frac{1}{jwC}$$

$$V_{in} - IR - IZ_c = 0$$

$$V_{in} = I(R + Z_c)$$

$$V_{out} = I Z_c$$

$$h(w) = \frac{V_{out}}{V_{in}}$$

$$h(w) = \frac{\cancel{I} Z_c}{(R+Z_c)\cancel{I}} = \frac{\frac{1}{jwC}}{R + \frac{1}{jwC}} * \frac{jwC}{jwC}$$

$$h(w) = \frac{1}{jwRC + 1}, \quad RC = \tau \rightarrow h(w) = \frac{1}{1 + jw\tau}$$

$$h^2 = \frac{1}{1+jw\tau}\,\frac{1}{1-jw\tau} = \frac{1}{1+(w\tau)^2}$$

$$|h| = \frac{1}{(1+(w\tau)^2)^{1/2}} \qquad x = w\tau$$

If $1 + jw\tau = 0$, $jw\tau = -1$

$$w = \frac{-j}{j^2\tau}, \quad j^2 = -1$$

$$w = j/\tau$$