Name: Almas Waseem

H.w#3

Section 2: Linear Systems

1. Linearity and Commutativity of Systems A and B

System A: $A[x(t)] = 2x(t) - 1$
System B: $B[x(t)] = 0.5x(t)$

(a) System A is non-linear due to the constant term -1, which breaks the property of additivity and homogeneity.

(b) To make A linear, remove the -1: $A[x(t)] = 2x(t)$.

(c) With the modification:
$A[B[x(t)]] = A[0.5x(t)] = x(t)$
$B[A[x(t)]] = B[2x(t)] = x(t)$
✅ A and B commute.

2. Integrals with Cosine Components

Given $f(t) = a_0 \cos(2\pi ft) + a_1 \cos(2\pi f_0 t)$, and T_1, T_2 are constants.
Both integrals:
$\int f(t)(t - T_1)\, dt$ and $\int f(t)(t - T_2)\, dt$ over $(-\infty, \infty)$
are zero because the integrands are odd functions and the limits are symmetric.
✅ Final answer: 0 for both.

Section 3: Fourier Transforms and Basic Filters

1. Low-pass and High-pass Filtering

Input: $s(t) = a\delta(t - t_0)$
Fourier Transform: $S(\omega) = a\, e^{-j\omega t_0}$

Low-pass filter: $H(\omega) = 1 / (1 + j\omega\tau)$
Output magnitude: $|Y(\omega)| = |a| / \sqrt{1 + (\omega\tau)^2}$

High-pass filter: $H(\omega) = j\omega\tau / (1 + j\omega\tau)$
Output magnitude: $|Y(\omega)| = |a\omega\tau| / \mathrm{sqrt}(1 + (\omega\tau)^2)$

2. Group Delays

For both filters, the group delay τ_g is the same:
$\tau_g = \tau / (1 + (\omega\tau)^2)$

3. Inverse Fourier Transforms

(a) $S(f) = (a/2)[\delta(f - f_0) + \delta(f + f_0)] \rightarrow s(t) = a \cos(2\pi f_0 t)$
(b) $S(f) = (a/2j)[\delta(f - f_0) - \delta(f + f_0)] \rightarrow s(t) = a \sin(2\pi f_0 t)$

Section 4: Convolution and Octave Code

1. Discrete Convolution Properties

(a) If input is $\delta[n]$, output is $h[n]$ (the impulse response).

(b) If input is $\delta[n - n_0]$, output is $h[n - n_0]$ (shifted by n_0).

2. Octave Code to Convolve Sine Wave with Impulse

Use the conv function in Octave to shift a 440 Hz sine wave by convolving with $\delta[n - n_0]$.

Code:

```
fs = 8000;
f = 440;
duration = 0.01;
n = 0:round(duration*fs);
n0 = 40;
x = sin(2*pi*f*n/fs);
delta = zeros(1, length(n));
delta(n0+1) = 1;
y = conv(x, delta);

subplot(2,1,1);
plot(n/fs, x);
```

```
title('Original Sine Wave');
subplot(2,1,2);
plot((0:length(y)-1)/fs, y);
title('Shifted Sine Wave');
```

✅ This shifts the phase of the sine wave by n_0 samples.