1.

$$f(t) = a_0 + a_1 \cos(t) + a_2 \cos(2t) + \dots$$
$$+ b_1 \sin(t) + b_2 \sin(2t) + \dots$$

$$a_0 = \frac{1}{2\pi} \int_0^{\pi} a \, dt = \frac{a}{2}$$

$$a_n = \frac{1}{\pi} \int_0^{\pi} a \cos(nt) \, dt = \frac{a}{\pi} \int_0^{\pi} \cos(nt) \, dt$$

$$= \frac{a}{n\pi} \int_0^{\pi} n \cos(nt) \, dt = \frac{a}{n\pi} \left(\sin(nt) \right) \Big|_0^{\pi}$$

$$= 0$$

$$b_n = \frac{-a}{n\pi} \int_0^{\pi} -n \sin(nt) \, dt = \frac{-a}{n\pi} \left(\cos(n\pi) - 1 \right)$$

$$b_n \begin{cases} 0 & n \text{ even} \\ \frac{2a}{n\pi} & n \text{ odd} \end{cases}$$

$$f(t) = \frac{a}{2} + \frac{2a}{\pi} \sin(t) + \frac{2\pi}{3} \sin(t) + \frac{2\pi}{5} \sin(t) + \dots$$

```
%question 1 and 2
clear;
close;
home;

f0 = 1.4e6;
T0 = 1 / f0;
T = 10 * T0;
N = 23;

fs = f0*100;
dt = 1/fs;
t = dt:dt:T;

x = zeros(size(t));
for n = 1:2:N
	x += (4)/(n*pi) * sin(2*pi*n*f0*t);
end


figure(1);
subplot(2,1,1);
plot(t,x);
xlabel(“time (s)”);
ylabel(“magnitude”);

X = fft(x);
X = X(1:end/2);
X = abs(X);
f = linspace(1,fs/2,length(X));
subplot(2,1,2);
plot(f,X);
xlabel(frequency (Hz)”);
ylabel(“magnitude”);

%question 2
%the gibbs effect is observed as at the changes in signals there is seen to be an overshoot. this overshoot becomes sharper as the value of N is increased.
```

```
%question 3
clear;
close;
home;
%define impulse
N = 1000;
%change to shift impulse
S = 100;
%S = 1;



t = 1:1:N;

amplitude = 10.0;
x = zeros(size(t));
x(S) = amplitude;

noise_sigma =  1e-3;
noise = randn(size(t))*noise_sigma;
x = x+noise;




figure(1);
plot(x);
title('raw data','fontsize',20);
xlabel('Time (s)');
ylabel('Magnitude (v)');

%computing dft phase and mag
X = fft(x);
X = X(1:end/2);
f = linspace(0,1/2,length(X));

Xp = angle(X);
Xm = abs(X);


figure(2);
plot(f,Xp);
title('fft phase','fontsize',20);
```

```
xlabel('Frequency (Hz)');
ylabel('Phase');
figure(3 );
plot(f,Xm);
title('fft mag','fontsize',20)
xlabel('Frequency (Hz)');
ylabel('Magnitude');

XpUnwrap = unwrap(Xp);

%they are linearly related with a slope
figure(4);
plot(f,XpUnwrap);
title('fft phase unwrapped','fontsize',20);
xlabel('Frequency (Hz)');
ylabel('Phase');

slope = (XpUnwrap(1)-XpUnwrap(2))/(f(1)-f(2));

%the slope of the unwrapped fft phase is the shift applied to the impulse multiplied by 2pi and
subtracted by 1 and inverted
recoveredShift = (-slope/(2*pi))+1


%(d) the phase is being changed as the impulse is being pushed back
%(e) the group delay should be zero when the impulse is not shifted which does happen with
some error for the final result. when the impulse is shifted the group delay should equal the shift
which it does.
%(f) the group delay stays the same despite the noise in the system
```

```
%question 4
clear;
close;
home;

fs = 1e6;
T = 6e-3;
dt = 1/fs;
t=  dt:dt:T;


f = 100e3;
a = 1;


x = a*sin(2*pi*f.*t);
%cliping
x(find(x>=0.75)) = 0.75;
x(find(x<=-0.75)) = -0.75;

X = fft(x);
X = X(1:end/2);
X = abs(X);
f = linspace(1,fs/2,length(X));
figure(1);
plot(f,X);
title('fft of x','fontsize',20);
xlabel('Frequency (Hz)');
ylabel('Magnitude');
```

%(d) while an unclipped sine wave would only result in a singular frequency comprising it due to the clipping higher harmonics occur as the clipped wave closer resembles a square wave.

```
%question 5
clear;
close;
home;

data = dlmread("nasdaq_2024_2025.dat",'\t',0,0);
days = data(:,1);
prices = data(:,2);


%filter
w = 7;
filter = ones(w)/(w);
filter = filter(1,:);
filterPrices = conv(prices,filter);

plot(days,prices);
hold on;
plot(filterPrices,'r');
xlabel("days");
ylabel("price(dollars)");


%(b) both occur within the last week of the data set
%(c) see (b)
%(d) no as these more aggressive changes are smoothed out via averaging making them less noticeable in the filtered data.
%(e) the lag should be about 3 days this is because the filter must fully encompass the feature to be included into the graph this means it must travel half its length from touching the feature to fully recognise it in this case it's about 3 days.
```

```
%question 6
clear;
close;
home;

fs = 10e7;
lpFc = 745e3;
hpFc = 735e3;
M = 101;

wcLp= 2 * pi * (lpFc / fs);
wcHp = 2 * pi * (hpFc / fs);



n = -(M-1)/2 : (M-1)/2;
h_lp = (sin(wcLp* n) ./ (pi * n));
h_lp((M+1)/2) = wcLp/ pi;  % handle division by zero
%apparently this apostrophe is key to the program somehow also apparently takes care of zero
frequency
w = blackman(M)';
h_lp = h_lp .* w;
h_lp = h_lp / sum(h_lp);


h_lpi = (sin(wcHp * n) ./ (pi * n));
h_lpi((M+1)/2) = wcHp / pi;
h_lpi = h_lpi .* w;
h_lpi = h_lpi / sum(h_lpi);


delta = zeros(1, M);
delta((M+1)/2) = 1;
h_hp = delta - h_lpi;

h_bp = conv(h_lp, h_hp);

T = 0.005;
t = 0:1/fs:T;
fc = 740e3;
f_audio = 2.5e3;
nScale = 10;

audio = sin(2*pi*f_audio*t);
```

```
noise = nScale * randn(size(t));
carrier = cos(2*pi*fc*t);
s = (1 + audio) .* carrier + noise;

S = fft(s);
S = S(1:end/2);
S = abs(S);
f = linspace(0,fs/2,length(S));


figure;
plot(f, S);
title('Data Without Filtering');
xlabel('Frequency (Hz)');
ylabel('Magnitude');

sf = filter(h_bp, 1, s);

Sf = fft(sf);
Sf = Sf(1:end/2);
Sf = abs(Sf);

figure;
plot(f, Sf);
title('After Filtering');
xlabel('Frequency (Hz)');
ylabel('Magnitude');
```

```
%question 7
clear;
close;
home;

fs = 128.0e3;
dt = 1/fs;
t = dt:dt:20;
nbits = 8;
nchannels = 1;

s1 = ones(200);
s2 = ones(200);
N = (length(s1)+length(s2))-1;
S1 = fft(s1,N);
S2 = fft(s2,N);
x = ifft(S1 .* S2);
figure(1);
plot(real(x));
xlabel("time (s)");
ylabe("magnitude");
%player1 = audioplayer(s1,fs,nbits);
%play(player1);


st1 = (0.01:0.01:200);
N = (2*length(st1))-1
x = fft(st1,N).^2;
x = ifft(x);
figure(2);
plot(real(x));
xlabel("time (s)");
ylabe("magnitude");
%player1 = audioplayer(st1,fs,nbits);
%play(player1);
```

```
%question 8 and 9
clear;
close;
home;

fs = 1e4; %Hz
fc = 0.5e2; %Hz
fc = fc/fs;
x = exp(-2*pi*fc);
a0 = 1-x;
b1 = x;


N=1000;
M=500;
s = [zeros(M,1); ones(N-M,1);];;



%lp filter rec
y = zeros(size(s));
y(1) = a0*s(1);
for i = 2:N
	y(i) = a0*s(i)+b1*y(i-1);
endfor

Y = fft(y);
Y = Y(1:end/2);
Y = angle(Y);
f = linspace(1,fs/2,length(Y));

figure(1);
plot(f,Y);
title("question #8 fft of ");
xlabel("Frequency(Hz)");
yLabel("phase");

%the results are not linear. It does converge to a linear form as frequency increases but as a
whole is not linear.



%9
s = [zeros(M,1); ones(N-M,1);];;
```

```
y = zeros(size(s));
y(1) = a0*s(1);
for i = 2:N
        y(i) = a0*s(i)+b1*y(i-1);
endfor


yr1 = fliplr(y);

yr2 = zeros(size(yr1));
y(1) = a0*yr1(1);
for i = 2:N
        yr2(i) = a0*yr1(i)+b1*yr2(i-1);
endfor

yr3 = fliplr(yr1);



Yr3 = fft(y);
Yr3 = Y(1:end/2);
Yr3 = angle(Y);
f = linspace(1,fs/2,length(Yr3));
figure(2);
plot(f,Yr3);
title("question #9");
xlabel("Frequency(Hz)");
yLabel("phase");
```

10

$$A\cos(2\pi f(t)t) \cdot A\cos(2\pi f(t-t_d)(t-t_d))$$

$$A(\cos(2\pi f(t)t) \cdot \cos(2\pi f(t-t_d)(t-t_d)))$$

$$\frac{A}{2}\left(\cos(2\pi(f(t)t + f(t-t_d)(t-t_d))) + \cos(2\pi(f(t)t - f(t-t_d)(t-t_d)))\right)$$

$$f(t) = f_0 - \beta t$$

$$\frac{A}{2}\left(\cos(2\pi((f_0-\beta t)t + (f_0 - \beta_t(t-t_d))(t-t_d))) + \cos(2\pi((f_0-\beta t)t - (f_0-\beta_t(t-t_d))(t-t_d)))\right)$$

$$f_0 t - \beta t^2 - f_0(t-t_d) - \beta(t-t_d)^2$$

$$f_0 t - \beta t^2 - f_0 t + f_0 t_d + \beta\ (t^2 - 2t t_d + t^2$$

$$= f_0 t_d - 2\beta t t_d + \beta t_d^2$$

$$f_{beat} = 2\beta t_d$$

```
%question 10
clear;
close;
home;
pkg load signal;

% Parameters
fs = 100e6;
dt = 1/fs;
t = 0:dt:10e-6;
f0 = 5e6;
b = 2e12;
td = 0.5e-6;

% Chirp signal

s = cos(2*pi*(f0*t - b*t.^2));
s_delayed = [zeros(1, round(td * fs)), s(1:end-round(td * fs))];

% Mix signals
m_s = s .* s_delayed;

% Add noise
noise_scale = 10;
noise = noise_scale * randn(size(m_s));
m_s = m_s + noise;

figure();
plot(t, m_s);

xlabel('Time (s)');
ylabel('Amplitude');



l_cf = 0.8e6;    % 0.8 MHz
h_cf = 1.2e6;   % 1.2 MHz

Wn = [l_cf h_cf] / (fs/2);

[b, a] = butter(4, Wn, "bandpass");
fm_s = filter(b,a,m_s);
plot(t,fm_s);
xlabel("time (s)");
```

```
ylabel("magnitude");
title("filtered signal");

fM_S = fft(fm_s);
fM_S = fM_S(1:end/2);
fM_S = abs(fM_S);
f = linspace(1,fs/2,length(fM_S));

plot(f,fM_S);
xlabel("frequency (Hz)");
ylabel("magnitude");
title("filtered signal fft");
```