QUIZ#1, NAME: Almas Waseem

Problem 1

(a) We use Euler's identity:

$\cos(\theta) = \mathrm{Re}\{e^{(j\theta)}\}$

So: $2.5 \cos(2\pi ft - \pi/4) = \mathrm{Re}\{2.5 e^{(j(2\pi ft - \pi/4))}\}$

(b) Since $\cos(\theta) = \mathrm{Im}\{e^{(j(\theta - \pi/2))}\}$, we get:

$2.5 \cos(2\pi ft - \pi/4) = \mathrm{Im}\{2.5 e^{(j(2\pi ft - \pi/2))}\}$

Problem 2

(a) 1 kHz = 1000 Hz ⇒ T = 1 / 1000 = 0.001 s = 1 ms

(b) T = 5 ns = 5×10^{-9} s ⇒ f = 1 / T = 200 MHz

(c) f = 5 kHz, f_s = 50 kHz ⇒ Samples per period = 50,000 / 5,000 = 10

(d) Δt = 0.002 s ⇒ f_s = 1 / 0.002 = 500 Hz

Samples per period = 500 / 5000 = 0.1 (undersampling)

Problem 3

(a) ΔV = 2.56 V / 256 levels = 0.01 V

(b) $256 = 2^8$ ⇒ 8 bits

(c) Doubling to 16 bits ⇒ 2^{16} = 65536 levels

New ΔV = 2.56 / 65536 ≈ 0.000039 V = 39 μV

Problem 4

(a) $s(t) = 2.5 \sin(2\pi ft) + 2.5$

(b) Add noise: z = s + randn(size(s))

(c) Plot z vs t in Octave

(d) SNR = 10 * log10(mean(s.^2) / mean(n.^2))

(e) Histogram shows Gaussian distribution

Problem 5

(a) $|R(f)| = x / \mathrm{sqrt}(1 + x^2)$, where $x = 2\pi fRC$

(b) $\angle R(f) = \pi/2 - \arctan(2\pi fRC)$

(c) Plot shows low $|R(f)|$ for small f, approaches 1 as $f \to \infty$

(d) $f = 500$ Hz, $R = 1\text{k}\Omega$, $C = 1\mu\text{F} \Rightarrow x = 2\pi(500)(1000)(1\text{e-}6) = \pi$

$|R(f)| = \pi / \mathrm{sqrt}(1 + \pi^2) \approx 0.953 \Rightarrow |R(f)|^2 \approx 0.908$

Problem 6

(a) 2.5 kHz < 5 kHz ⇒ no aliasing ⇒ 2.5 kHz

(b) 8 kHz alias: $|8 - 10| = 2$ kHz

(c) 5 kHz ⇒ right at Nyquist ⇒ 5 kHz

(d) 15 kHz alias: $|15 - 10| = 5$ kHz

(e) 20 kHz alias: $|20 - 20| = 0$ Hz

Problem 7

(a) $a(t) = 2 \sin(2\pi ft)$, $T/2 = 1/(2f)$

$S[a(t)] = a(t - T/2) = -2 \sin(2\pi ft)$

(b) Output is inverted sine wave

(c) $a(t) + S[a(t)] = 0$

Problem 8

(a) $x[2] = 2 \Rightarrow y[3] = -2 \Rightarrow y[n] = [0\ 0\ 0\ -2\ ...]$

(b) $y[n] = x[n]^2 \Rightarrow y[2] = 4$

(c) (a) is linear (scaling and shifting); (b) is nonlinear (squaring)

Problem 9

$\cos(2\pi ft) \rightarrow$ Even

$\exp(-(t/\sigma)^2) \rightarrow$ Even

$\exp(-\alpha t) \rightarrow$ Neither

$at^2 + bt + c \rightarrow$ Neither unless $b = 0$ (even) or $a=c=0$ (odd)

Problem 10

(a) Linearity: $F\{a\ f(t)\} = a\ F(\omega)$

(b) Additivity: $F\{f + g\} = F + G$

(c) Time shift: F{f(t - t0)} = e^{-jωt0} F(ω)

Problem 11

(a) F{δ(t - t0)} = ∫ δ(t - t0) e^{-jωt} dt = e^{-jωt0}

(b) Magnitude = 1

(c) Phase = -ωt0

Problem 12

(a) F(ω) = 1 ⇒ f(t) = δ(t)

(b) Flat magnitude and zero phase ⇒ δ(t)

Problem 13

(a) y(t) = x(t) * h(t)

(b) Low frequencies → filtered out (attenuated)

(c) High frequencies → preserved

(d) Input sine → output depends on frequency; low freq shrinks

Bonus

$\cos(2\pi f_0 t) = \frac{1}{2} [e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}]$

$FT\{\cos\} = \frac{1}{2} [\delta(f - f_0) + \delta(f + f_0)]$