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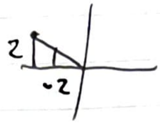
DSP Quiz 2.

1) Get the phase angle.

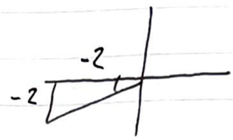
a) $z = -2 + 2i$



$\tan\theta = -1$

$\boxed{\theta = 135^\circ}$

b) $z = -2 - 2i$



$\boxed{\theta = 225^\circ}$

c) $z = 2 - 2i$

$\boxed{\theta = 315^\circ}$

2) At $t = 0$. a) $v(t) = 4\cos(30^\circ)$

This is real part of a signal, so

$Re\{v(t)\} = 4\cos(30^\circ)$

So full $v(t) = 4\cos(30^\circ) + 4i\sin(30^\circ)$

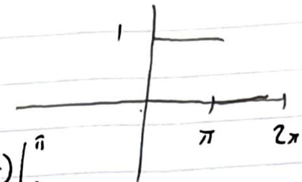
$\boxed{v(t) = 4\exp(i30^\circ)}$

$v(t) = 2\sin(-60^\circ)$.

Same as before. $\boxed{v(t) = 2\exp(-i60^\circ)}$

2) A Fourier Transform

$$f(x) = \begin{cases} 1 & 0 \le x < \pi \\ 0 & \pi < x \le 2\pi \end{cases}$$



And now get Fourier Transform.

$$F(t) = \int_0^{\pi} \exp(-i\omega t)\,dt = \frac{1}{-i\omega} \exp(-i\omega t)\Big|_0^{\pi}$$

$$= \frac{\exp(-i\omega t)}{-i\omega}\Big|_0^{\pi} = \frac{\exp(-i\omega t)}{i\omega}\Big|_{\pi}^{0} = \frac{\exp(0) - \exp(-i\omega\pi)}{i\omega}$$

$$= \frac{1-\exp(-i\pi\omega)}{i\omega} = \frac{1-(\cos(-\pi\omega) + i\sin(-\pi\omega))}{i\omega}$$

$$= \frac{1-\cos(-\pi\omega) - i\sin(-\pi\omega)}{i\omega} = \frac{1-\cos(\pi\omega) + i\sin(\pi\omega)}{i\omega}$$

$$\boxed{F(\omega) = \frac{1-\cos(\pi\omega) + i\sin(\pi\omega)}{i\omega}} \quad \left(\frac{i}{i}\right)$$

or $= \dfrac{i - i\cos(\pi\omega) - \sin(\pi\omega)}{-\omega} = \dfrac{\sin(\pi\omega) + i\cos(\pi\omega) - i}{\omega}$

$$\boxed{F(\omega) = \frac{\sin(\pi\omega)}{\omega} + \frac{\cos(\pi\omega)-1}{\omega} i}$$

Makes sense. Like a sync function w/ an imaginary component.

$$\phi(\omega) = \arctan\left(\frac{Im(F)}{Re(F)}\right) \qquad Im(F) = \frac{\cos(\pi\omega)-1}{\omega}$$

$$Re(F) = \frac{\sin(\pi\omega)}{\omega}$$

$$= \arctan\left(\frac{\frac{\cos(\pi\omega)-1}{\omega}}{\frac{\sin(\pi\omega)}{\omega}}\right) = \arctan\left(\frac{\cos(\pi\omega)-1}{\sin(\pi\omega)}\right) = \arctan\left(-\frac{1-\cos(\pi\omega)}{\sin(\pi\omega)}\right)$$

$$= \tan^{-1}\left(\tan\left(-\frac{\pi\omega}{2}\right)\right)$$ arctangent is odd.

$$\boxed{\phi = -\frac{\pi\omega}{2}}$$

$$|F(\omega)| = \sqrt{F(\omega)F^*(\omega)}$$

$$= \sqrt{\left(\frac{\sin(\pi\omega)}{\omega} + \frac{\cos(\pi\omega)-1}{\omega} i\right)\left(\frac{\sin(\pi\omega)}{\omega} - \frac{\cos(\pi\omega)-1}{\omega} i\right)}$$

$$= \frac{1}{\omega}\sqrt{\sin^2(\pi\omega) + (\cos(\pi\omega)-1)^2} = \frac{1}{\omega}\sqrt{\sin^2(\pi\omega) + \cos^2(\pi\omega) - 2\cos(\pi\omega) + 1}$$

$$= \frac{1}{\omega}\sqrt{2-2\cos(\pi\omega)}$$

$$\boxed{\text{Magnitude of } F(\omega) = \frac{1}{\omega}\sqrt{2-2\cos(\pi\omega)}}$$

* $|F(\omega)| \propto \frac{1}{\omega} = \text{"}a\text{"}$ so this makes sense.

$$0 = \frac{1}{\omega}\sqrt{2-2\cos(\pi\omega)}$$

$$2-2\cos(\pi\omega) = 0 \implies (\cos(\pi\omega) = 1)\ \text{arccos}$$

$$\pi\omega = 0 + \pi k,\ k \in 2, 4, \ldots$$

$$\omega = 0 + k,\ k \in 2, 4, \ldots$$

or $\boxed{\omega = 2k,\ k \in \mathbb{Z}.}$

3] $p(x) = 1.$ $\int_0^1 p(x) = 1.$

It is normalized.

The mean is $\int_0^1 x\,p(x)dx = \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}$

So $\boxed{\langle x \rangle = \frac{1}{2}}$ Makes sense, as random variables correlate w/ Gaussian, which has central value in the middle.

$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$ so

$$\langle x^2 \rangle = \int_0^1 x^2 p(x)dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$$

$$\sigma_x^2 = \frac{1}{3} - \frac{1}{4} = \frac{4-3}{12} = \frac{1}{12}$$

$$\boxed{\sigma_x = \frac{1}{\sqrt{12}} = \frac{1}{2\sqrt{3}}}$$