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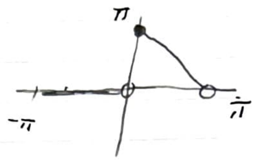
1]

The Fourier Series.

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \pi - x & 0 \le x < \pi \end{cases}$$



$$\frac{1}{\pi}\int_0^\pi (\pi-x)\cos(nx)\,dx = \int_0^\pi \cos(nx)\,dx - \frac{1}{\pi}\int_0^\pi x\cos(nx)\,dx$$

$\frac{d}{dx}$	$\int$
$+\ x$	$\cos(nx)$
$-\ 1$	$\frac{1}{n}\sin(nx)$
0	$-\frac{1}{n^2}\cos(nx)$

$$= \underbrace{\frac{1}{n}\sin(nx)\Big|_0^\pi}_{0} - \frac{1}{\pi}\left(+\frac{x}{n}\sin(nx) + \frac{1}{n^2}\cos(nx)\right)\Big|_0^\pi$$

$$= -\frac{1}{\pi}\left(\frac{x}{n}\sin(nx) + \frac{1}{n^2}\cos(nx)\right)\Big|_0^\pi = -\frac{1}{\pi}\left(\frac{\cos(n\pi)}{n^2} - \frac{\cos(0)}{n^2}\right)$$

$$= -\frac{1}{\pi}\left(\frac{(-1)^n - 1}{n^2}\right)$$

$$= \frac{1}{\pi}\left(\frac{1-(-1)^n}{n^2}\right) = \boxed{\frac{1-(-1)^n}{\pi n^2}}$$

Sine part is $\frac{1}{\pi}\int_0^\pi (\pi-x)\sin(nx)\,dx = \int_0^\pi \sin(nx)\,dx - \frac{1}{\pi}\int_0^\pi x\sin(nx)\,dx$

$\frac{d}{dx}$	$\int$
$+\ x$	$\sin(nx)$
$-\ 1$	$-\frac{1}{n}\cos(nx)$
0	$-\frac{1}{n^2}\sin(nx)$

$$= -\frac{1}{n}\cos(nx)\Big|_0^\pi - \frac{1}{\pi}\left(-\frac{x\cos(nx)}{n} + \cancel{\frac{\sin(nx)}{n^2}}\right)\Big|_0^\pi$$

$$= -\frac{\cos(n\pi)-\cos(0)}{n} + \frac{1}{\cancel{\pi}}\left(\frac{\cancel{\pi}\cos(n\pi)}{n}\right)$$

$$= \frac{1-(-1)^n}{n} + \frac{(-1)^n}{n} = \boxed{\frac{1}{n}}$$

Now for leading constant.

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{0} 0\,dx + \frac{1}{2\pi}\int_0^\pi(\pi-x)\,dx = \frac{1}{2\pi}\int_0^\pi (\pi-x)\,dx = \frac{1}{2}\int_0^\pi dx - \frac{1}{2\pi}\int_0^\pi x\,dx$$

$$= \frac{\pi}{2} - \frac{\pi^2}{4\pi} = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\boxed{\text{So } f(x) = \frac{\pi}{4} + \sum_{n=1}^{\infty}\left(\frac{1-(-1)^n}{\pi n^2}\right)\cos(nx) + \sum_{n=1}^{\infty}\left(\frac{1}{n}\right)\sin(nx)}$$

The Fourier transform of $\delta(t) \equiv f(t) = a\delta(t-t_0)$

$$F(\omega) = \int_{-\infty}^{\infty} a\delta(t-t_0)e^{-i\omega t}\,dt = ae^{-i\omega t_0}$$

So $F(\omega) = a\left(\cos(-\omega t_0) + i\sin(-\omega t_0)\right)$

$$\boxed{F(\omega) = a\left(\cos(\omega t_0) - i\sin(\omega t_0)\right)}$$

$$\boxed{F(\omega) = a\cos(\omega t_0) - ai\sin(\omega t_0)}$$

$$|F(\omega)|^2 = ZZ^* \text{ so } = \left(a\cos(\omega t_0) - ai\sin(\omega t_0)\right)\left(a\cos(\omega t_0) + ai\sin(\omega t_0)\right)$$

$$= a^2\cos^2(\omega t_0) + a^2\sin^2(\omega t_0) = a^2\left(\cos^2(\omega t_0) + \sin^2(\omega t_0)\right)$$

$$\boxed{|F(\omega)|^2 = a^2}$$

$$\boxed{\phi = -\omega t_0}$$

So $\boxed{\frac{-d\phi}{d\omega} = +t_0 = t_0}$