DIGITAL SIGNAL PROCESSING: COSC390

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Previous lectures covered:

- Complex numbers 2: The Fourier series and Fourier transform (continuous and discrete)
- *Time-permitting*: The Laplace transform (continuous and discrete)

This lecture will cover: (Reading: **Chapter 2**)

- Statistics and probability: the normal distribution and other useful distributions
- Noise: digitization and sampling
- Noise: Spectral properties of noise, ADC and DAC

STATISTICS AND PROBABILITY: THE NORMAL DISTRIBUTION

The *mean*, μ, and *standard deviation*, σ, of a data set $\{x_i\}$ are defined as

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{1}$$

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N} (x_i - \mu)^2 \tag{2}$$

Octave commands:

```
x = randn(100,1);
mean(x)
std(x)
```

One nice theorem: *The variance is the average of the squares minus the square of the average.* Let $\langle x \rangle$ represent the average of the quantity or expression x. We have

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 \tag{3}$$

Note: *process or signal process* versus *the data*. Just because the data has a given μ and σ does not imply that the signal process has or will continue to have the exact same values of μ and σ. The underlying process could be *non-stationary*.

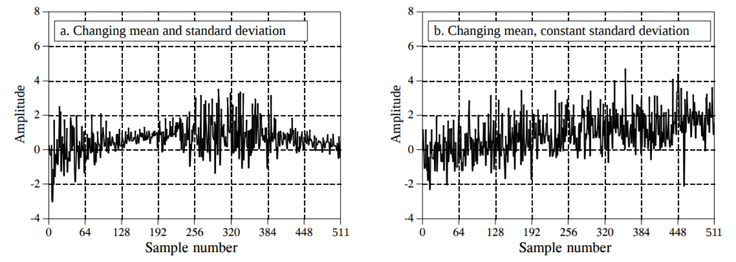

Figure 1: Signal processes in (a) and (b) are considered non-stationary because one or both of μ and σ depend on time.

A histogram is an object that represents the frequency of particular values in a signal. For example, below is a histogram of 256,000 numbers drawn from a probability distribution:

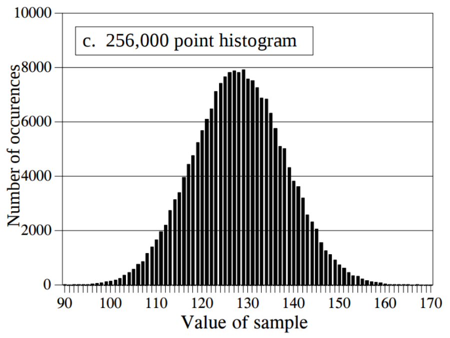

Figure 2: The histogram contains counts versus sample values.

The following octave code should reproduce something like Fig. 2 from the textbook:

```
x = randn(256000,1)*10.0+130.0;
[b,a] = hist(x,100);
plot(a,b,'o');
```

The function *randn(N,M)* draws $N \times M$ numbers from a normal distribution and returns them in the size the user desires. The function *hist(x,N)* creates N bins and sorts the data x_i into them.

For data that is appropriately stationary, we can use histograms to estimate μ and σ faster, since we only have to loop over bins rather than every data sample. Let H_i represent the counts in a given bin, and i represent the bin sample. We have:

$$\mu = \frac{1}{N} \sum_{i=1}^{M} i H_i \tag{4}$$

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^{M} (i - \mu)^2 H_i \tag{5}$$

(To obtain the mean in signal *amplitude*, you'd have to convert bin number to amplitude - more on that in a moment).

Some vocabulary:

- **normalization** - Total probability is 1.0. For pdf - the integral from $[-\infty, \infty]$ is 1.0. For pmf - the sum from $[-\infty, \infty]$ is 1.0.
- **pmf** - Probability mass function: A *normalized continuous function* that gives the probability of a value, given the value.
- **histogram** - Histograms are an attempted measurement of the pmf by breaking the data into discrete bins. Histograms can be *normalized* as well.
- **pdf** - Probability density function: A *normalized continuous function* that gives the probability density of a value, given the value. Integrating the *normalized* pdf between two values gives the probability of observing data between the given values.

STATISTICS AND PROBABILITY: OTHER USEFUL DISTRIBUTIONS

CONCLUSION

UNIT 1.3 OUTLINE

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