Thursday Warm Up, Unit 0: Foundations and Fundamentals

Prof. Jordan C. Hanson

January 23, 2025

1 The Breadth and Depth of DSP

1. **Echo control**: Suppose a sinusoidal signal with frequency f is sent through a communication channel, and causes and echo. Let $v(t)$ be the original signal, and $w(t)$ be the echo:

$$v(t) = v_0 \sin(2\pi f t) \tag{1}$$
$$w(t) = a v_0 \sin(2\pi f t - \phi) \tag{2}$$

The echo signal $w(t)$ has a phase shift $\phi = \pi/4$, or 45 degrees. (a) As accurately as you can, graph $v(t)$ and $w(t)$ on the same plot, with $a = 0.1$. (b) Graph the total signal: $v(t) + w(t)$. (c) Add one final signal that *cancels* $w(t)$, but *does not cancel* $v(t)$.[1]

2. **Sampling a sine wave**: Let a set of sample times be 0, Δt, $2\Delta t$, ... $n\Delta t$. The period of $v(t)$ in the previous exercise is $T = 1/f$. That is, if $f = 4$ kHz, then $T = 1/4$ kHz^{-1}. (a) Show that kHz^{-1} is 1 millisecond. (b) Show that if $f = 10$ kHz, then $T = 0.1$ ms. (c) If we want to sample $v(t)$ and transmit it digitally, then we should probably have $\Delta \ll T$. Let $f = 5$ kHz for $v(t)$, and $\Delta = 0.02$ ms. How many samples per period? (d) If instead $\Delta t = 0.002$ ms, how many samples per period? (e) If $\Delta t = 0.002$ ms, what is $1/\Delta t = f_s$, the *sampling frequency*?

3. **Signal envelopes**: Suppose we send a *gaussian pulse* through a DSP system, with signal shape

$$p(t) = A e^{-\frac{1}{2}(t-t_0)^2/\sigma^2} \sin(2\pi f t) \tag{3}$$

This is a signal with sinusoidal behavior and a gaussian *envelope*. The envelope is the part of the function multiplying the sine wave. At $t = t_0$ the envelope has a maximum amplitude of $A \sin(2\pi f t_0)$. (a) Graph the *envelope* of the signal. (b) Add the sinusoidal oscillation to your graph. (c) If we are going to sample this signal envelope, which of the following should be true:

- A: $\Delta t = \sigma$
- B: $\Delta t > \sigma$
- C: $\Delta t < \sigma$
- D: $\Delta t \ll \sigma$

4. **Digitizing voltages**: Suppose we are dealing with an AC circuit that produces waveforms for audio systems. The output runs from -2.5 to 2.5 Volts. (a) What is the range if we add an offset of +2.5 V to the output signals? (b) If we can *digitize* the new voltage range into 256 steps, what is the voltage range between steps?

[1] Now imagine doing this for all frequencies, simultaneously.