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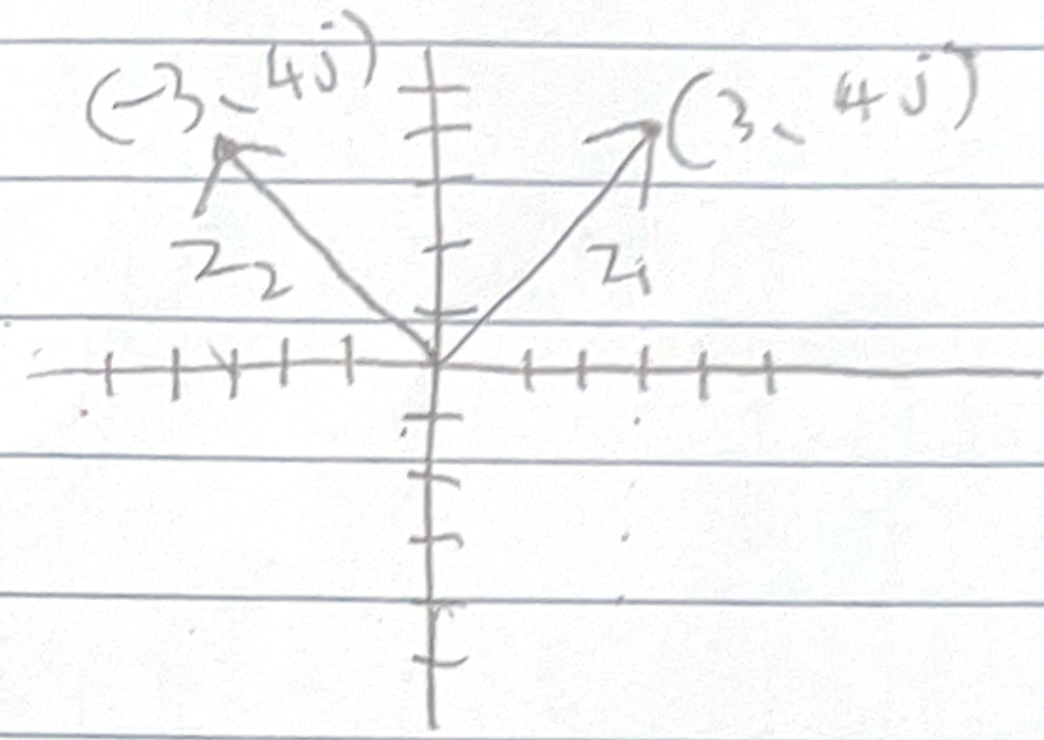
CoSC 360
1/26/25

Homework #2

① let $z_1 = 3 + 4j$ $\quad z_2 = -3 + 4j$

j = imaginary unit

a)



b) $z_1 + z_2$

$$(3 + 4j) + (-3 + 4j) = (3 + -3) + (4j + 4j)$$
$$= \boxed{8j}$$

c) $z_1 - z_2$ $\quad (3 + 4j) - (-3 + 4j)$

$$= (3 - -3) + (4j - 4j)$$
$$= \boxed{6}$$

d) $z_1 * z_2$

$$(3 + 4j)(-3 + 4j)$$
$$= -9 \cancel{-12j} \cancel{+12j} + 16j^2$$
$$= -9 + 16(-1) = \boxed{-25}$$

e) $\dfrac{z_1}{z_2}$ $\quad \dfrac{(3 + 4j)}{-3 + 4j} = \dfrac{(3 + 4j)(-3 - 4j)}{(-3 + 4j)(-3 - 4j)}$

$$z_2^* = -3 - 4j$$

$$= \frac{-9 - 12j - 12j - 16j^2}{9 \cancel{-12j} \cancel{+12j} - 16j^2}$$

$$= \frac{-9 - 24j + 16}{9 + 16} = \boxed{\frac{7 - 24j}{25}}$$

f) $|z_1| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \boxed{5}$

g) $|z_2| = \sqrt{(-3)^2 + 4^2} = \sqrt{9 + 16} = \boxed{5}$

h) ϕ_1

$$\tan\phi_1 = \frac{4}{3}$$

$$\tan^{-1}\left(\frac{4}{3}\right) = 0.927 = \phi_1$$

$$\boxed{\phi_1 = 53.1^\circ}$$

i) ϕ_2

$$\tan\phi_2 = \frac{4}{-3}$$

$$\tan^{-1}\left(-\frac{4}{3}\right) = -0.927 = \phi_2$$

$$\boxed{\phi_2 = -53.1^\circ}$$

j) $z_1 = 3 + 4j$ $\quad$ $z_2 = -3 + 4j$

$r_1 = 5$ $\quad$ $r_2 = 5$

$\phi_1 = 53.1^\circ = 0.927 \text{ rads}$ $\quad$ $\phi_2 = -53.1^\circ = -0.927 \text{ rad}$

$z_1 = 5\exp(0.927j)$ $\quad$ $z_2 = 5\exp(-0.927j)$

(2) $$\cos(2\pi f t) = \frac{e^{2\pi j f t} + e^{-2\pi j f t}}{2}$$

$$\sin(2\pi f t) = \frac{e^{2\pi j f t} - e^{-2\pi j f t}}{2j}$$

euler's identity: $e^{j\phi} = \cos\phi + j\sin\phi$

$$\cos(2\pi f t) = \frac{e^{2\pi j f t} + e^{-2\pi j f t}}{2}$$

Suppose $2\pi f t = \phi$

$$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2} = \frac{(\cos\phi + \cancel{j\sin\phi}) + (\cos\phi - \cancel{j\sin\phi})}{2}$$

$$= \frac{2\cos\phi}{2} = \boxed{\cos\phi}$$ ✓

$$\sin\phi = \frac{e^{j\phi} - e^{-j\phi}}{2j} = \frac{(\cancel{\cos\phi} + j\sin\phi) - (\cancel{\cos\phi} + j\sin\phi)}{2j}$$

$$= \frac{2j\sin\phi}{2j} = \boxed{\sin\phi}$$ ✓

(3) let $V_1(t) = 4\cos(2\pi f_1 t)$

$V_2(t) = 4\cos(2\pi f_2 t - \phi)$

a) Show that $P = V_1(t)\,V_2(t)$ is a pair of sinusoids with frequencies $f_+ = f_1 + f_2$ and $f_- = f_1 - f_2$, offset by a total phase shift of 2ϕ

$$P = 4\cos(2\pi f_1 t)(4\cos(2\pi f_2 t - \phi))$$

$$P = 16\cos(2\pi f_1 t)\cos(2\pi f_2 t - \phi)$$

b) if $\phi = 0$ and $f_1 = f_2$

$$P = 16\cos(2\pi f t)\cos(2\pi f t + 0)$$

$$P = 16\cos^2(2\pi f t)$$

a) as $\cos(2\pi f_1 t) = \frac{1}{2}(e^{2\pi f j t} + e^{-2\pi j f t})$

$$V_1(t) = 2(e^{2\pi j f_1 t} + e^{-2\pi j f_1 t})$$

$$V_2(t) = 2(e^{2\pi j f_2 t} e^{-j\phi} + e^{-2\pi j f_2 t} e^{-j\phi})$$

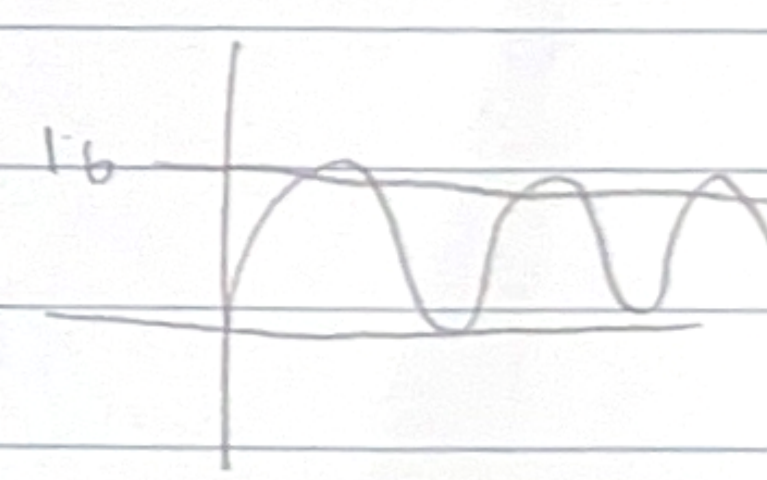
$$V_1 V_2 = 4(e^{2\pi j(f_1+f_2)t - j\phi} + e^{-2\pi j(f_1-f_2)t - j\phi} + e^{-j\phi} + e^{-j\phi})$$

$$= 4(e^{2\pi j(f_1+f_2)t - j\phi} + e^{-2\pi j(f_1-f_2)t - j\phi} + e^{-2j\phi}$$

So $f_+ = f_1 + f_2$ $\quad f_- = f_1 - f_2$

with an offset of 2ϕ

b) $P = 16\left(\left(\frac{1}{2}(e^{2\pi j f t} + e^{-2\pi j f t})\right)^2\right)$



$= 16(\ (1$

④ $v_1(t) = \text{imaginary}\{\exp(j(2\pi f t - \phi))\}$

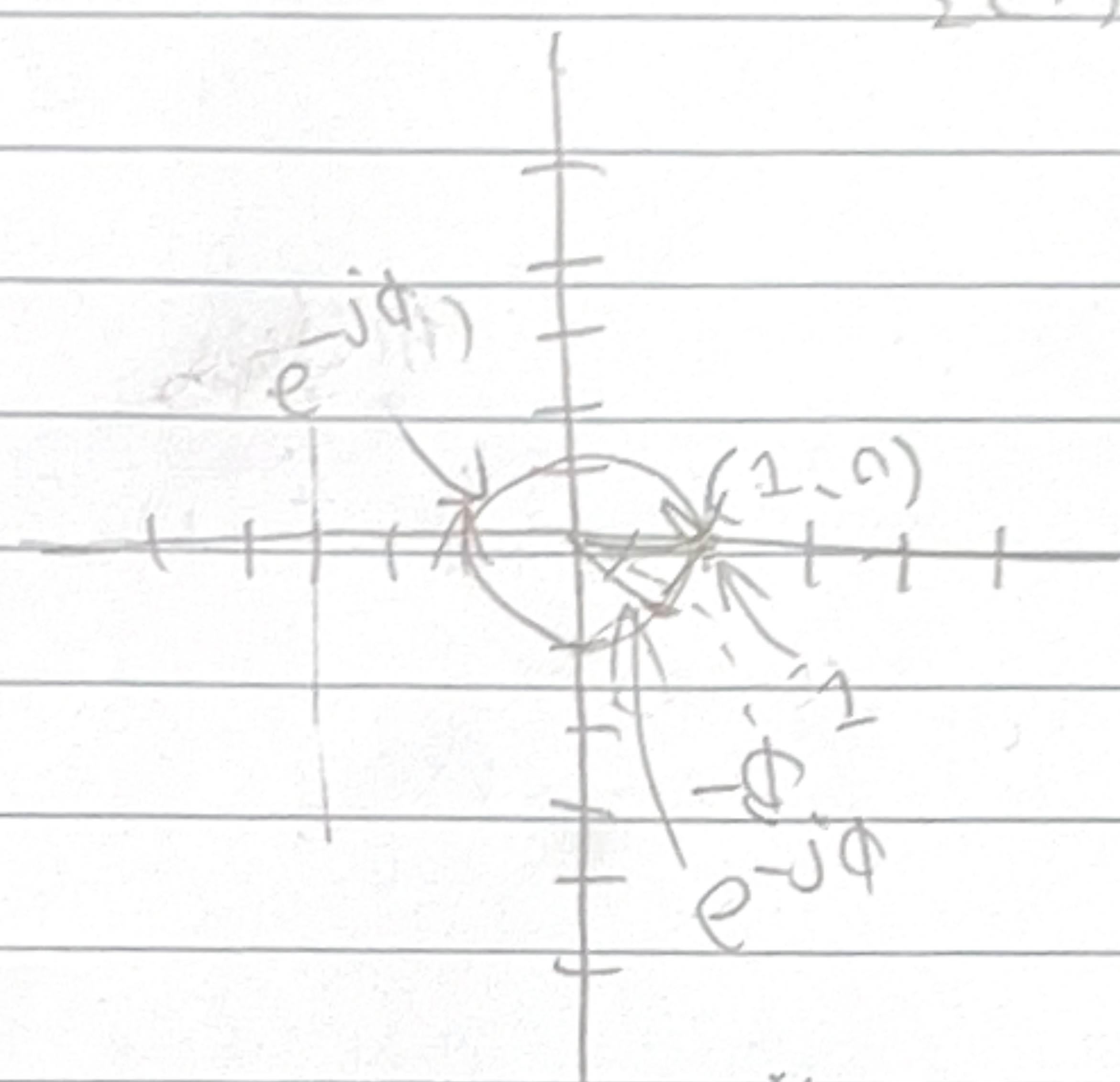
$v_2(t) = \text{imaginary}\{\exp(2\pi j f t)\}$

represent them as just

$v_1(t) = e^{-j\phi}$

$v_2(t) = 1$

a)



b) $1 + e^{-j\phi} = v_3 \quad |v_3| = \sqrt{v_3^* v_3}$

magnitude: $|v^3| = \sqrt{(1+e^{j\phi})(1+e^{-j\phi})} = \sqrt{1 + e^{j\phi} + e^{-j\phi} + 1}$

$= \sqrt{2 + e^{j\phi} + e^{-j\phi}}$

$e^{j\phi} + e^{-j\phi} = ((\cos\phi + j\sin\phi) + (\cos(-\phi) + j\sin(-\phi)))$

$\cos(-\phi) = \cos\phi$

$\sin(-\phi) = -\sin\phi$

$\cos\phi + \cancel{j\sin\phi} + \cos\phi - \cancel{j\sin\phi}$

so $\sqrt{2 + e^{j\phi} + e^{-j\phi}} = \boxed{\sqrt{2 + 2\cos\phi}}$

Phase angle: $\tan\phi = \dfrac{im(1+e^{-j\phi})}{re(1+e^{-j\phi})}$

$\tan\phi = \dfrac{-\sin(\phi)}{\cos(\phi)}$

$\boxed{\phi = \tan^{-1}\left(\dfrac{-\sin\phi}{\cos\phi}\right)} =$