Tuesday Warm Up, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- **Convolution**: this is an operation that characterizes the response $h[n]$ of a linear system.

$$y[i] = h[n] * x[n] = \sum_{j=0}^{M-1} h[j]x[i-j] \quad (1)$$

  In words, the output at sample i is equal to the produce of the system response h and the input signal x, summed over the proceeding M samples (from $j = 0$ to $j = M - 1$).
- **Discrete Delta Function**, $\delta[n]$: A standard impulse response that contains one non-zero sample. It has the following property:

$$x[n] = \delta[n] * x[n] \quad (2)$$

- **Discrete Fourier Transform**, for a sampled, digitized signal x_n:

$$X_{\mathrm{k}} = \sum_{n=0}^{N-1} x_n e^{-2\pi j (k/N)n} \quad (3)$$

- In DFT analysis, we often need to know the Δt, time duration for samples, and the sampling rate, f_{s}. Note that $1/f_{\mathrm{s}} = \Delta t$.
- For a sinusoid of frequency f (Hz), the period is $T = 1/f$ (seconds).

2 Unit Conversions for Frequency, Period, and Sampling Rate

1. Convert the following:
   - 0.0005 seconds to ms
   - 2.5×10^{-8} seconds to ns
   - 6.25×10^{-7} seconds to μs
   - 0.0000125 seconds to ms
2. Convert the following:
   - 12500 Hz to kHz
   - 2.5×10^{8} Hz to GHz
   - 6.25×10^{7} Hz to MHz
   - 2500 Hz to kHz
3. Given the following *sampling rates,* give the time duration of samples:
   - 128 kHz
   - 64 kHz
   - 500 MHz
   - 50 MHz
4. Given the following *time duration of samples,* give the sampling rates:
   - 0.1 ms
   - 0.01 ms
   - 0.1 μs
   - 0.01 μs

3 The Discrete Fourier Transform

1. Type `help fft` in an `octave` command window. Read about the various ways to input data into this function that computes the "fast Fourier transform" of the data.
2. Write a brief `octave` script that defines a sampling rate, time samples, and a vector of data representing a unit impulse, $\delta[n]$.
3. Pass the data into the `fft()` function, and store the output in a variable, `X`.
4. Keep only the first half of the data output, `X = X(1:end/2)`.
5. Let f_{s} be the sampling rate. Define frequencies as `f = linspace(0,fs/2,N)`, where N is the current length of `X`.
6. Multiply the vector `X` by $1/f_{\mathrm{s}} = \Delta t$, then plot the absolute value of `X` versus the frequencies `f`. What result do you observe?
7. Repeat this process with a sine wave input.