DIGITAL SIGNAL PROCESSING: COSC390

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1. *What is digital signal processing?*
2. *COSC330: Computer Logic and Digital Circuit Design*
3. Read the syllabus for a roadmap
4. *This course can be fast.*
5. **Data science project and presentation**
6. Textbook: `http://dspguide.com`
7. Download and install octave: `https://www.gnu.org/software/octave`

LECTURE FORMAT, WITH MODIFICATIONS

- Theory and examples
- Programming with Octave
- Application
- Study hall
    1. Homework help
    2. Project and presentation development
    3. Special topics lectures

1. Complex numbers 1: Arithmetic and some calculus (continuous and discete)
2. Complex numbers 2: The Fourier series and Fourier transform (continuous and discrete)
3. *Time-permitting*: The Laplace transform (continuous and discrete)

COMPLEX NUMBERS 1: THEORY AND EXAMPLES

A complex number is an expression for which one term is proportional to $j = \sqrt{-1}$:

$$z = x + jy \tag{1}$$

To call the *complex unit* j is the convention in electrical engineering, and in physics it is often called i.

Example of complex numbers: $(3 + 4j)$, $(x_1 + x_2 j)$. Each number has a *real* part and an *imaginary* part.

Operations to learn:

1. Addition
2. Subtraction
3. Real part Re and Im
4. Multiplication
5. Conjugation
6. Magnitude/Norm
7. Division

Notations to learn:

1. Cartesian
2. Polar

Addition follows the pattern of two-dimensional vectors:

$$z_1 = 3 + 4j \tag{2}$$
$$z_2 = -2 + 5j \tag{3}$$
$$z_1 + z_2 = 1 + 9j \tag{4}$$

Subtraction follows the pattern of two-dimensional vectors:

$$z_1 = 3 + 4j \tag{5}$$
$$z_2 = -2 + 5j \tag{6}$$
$$z_1 - z_2 = 5 - 1j \tag{7}$$

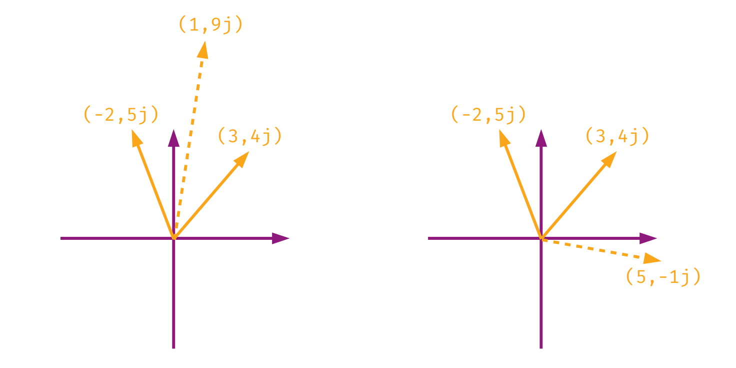

Figure 1: Complex addition and subtraction follows the pattern of two-dimensional vectors. (Left): Addition of z_1 and z_2. (Right): Subtraction of z_1 and z_2.

We also have the Re and Im operations:

$$z_1 = 3 + 4j \tag{8}$$

$$\mathrm{Re}\{z_1\} = 3 \tag{9}$$

$$\mathrm{Im}\{z_2\} = 4 \tag{10}$$

These are known as taking the *real*-part and the *imaginary*-part. The original complex number can be recovered by adding real and imaginary parts together:

$$z_1 = \mathrm{Re}\{z_1\} + j\,\mathrm{Im}\{z_1\} \tag{11}$$

When we add/subtract complex numbers, we combine the real parts and imaginary parts separately.

Add or subtract, then simplify:

1. $z_1 = 7 + 7j$, $z_2 = -6 + 3j$. $z_1 + z_2 =$
2. $z_1 = 2 + 2j$, $z_2 = 3 - 3j$. $z_1 - z_2 =$
3. $z_1 = 2x + 7j$, $z_2 = 2 + 4xj$. $z_1 + z_2 =$

Let $x = -1$ and $y = 1$. *Evaluate the following expressions:*

1. $z_1 = x + yj$, $z_2 = y + xj$. $z_1 + z_2 =$
2. $z_1 = x^2 + y^2 j$, $z_2 = 2y^2 + x^2 j$. $z_1 - z_2 =$

Multiplication: Recall that $j^2 = -1$.

$$z_1 = x_1 + jy_1 \tag{12}$$

$$z_2 = x_2 + jy_2 \tag{13}$$

$$z_1 \times z_2 = x_1x_2 - y_1y_2 + j(x_1y_2 + x_2y_1) \tag{14}$$

The cross-terms are straightforward, but remember the minus sign when multiplying the imaginary parts.

Another similarity with two-dimensional vectors?

$$z_1 = 4 - 1j \tag{15}$$

$$z_2 = 1 + 4j \tag{16}$$

$$z_1 \times z_2 = 8 + 15j \neq 0 \tag{17}$$

What would be the result if we were dealing with regular two-dimensional vectors?

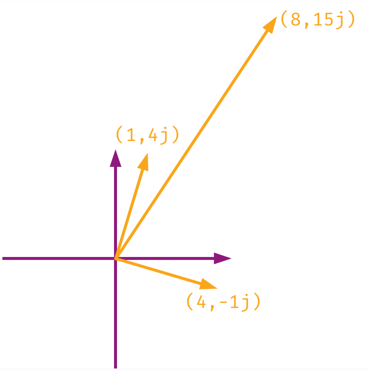

Figure 2: Complex multiplication resembles the *dot-product* for two-dimensional vectors, with key differences.

Complex conjugation: change the sign of the imaginary part.

$$z_1 = 4 - 1j \tag{18}$$

$$z_1^* = 4 + 1j \tag{19}$$

$$z_2 = = 2x + 1j \tag{20}$$

$$z_2^* = 2x - 1j \tag{21}$$

Is there a significance to $z_1 z_2^*$? What about $z_1 z_1^*$? What about $\sqrt{z_1 z_1^*}$?

Let $z = x + jy$. Compute the following:

1. $zz^* =$
2. $\sqrt{zz^*} =$

The second item on this list has a special name: the *magnitude* or *norm* of the complex number, $|z|$.

Compute the norm of the following complex numbers:

1. $2 + 2j$
2. $3 + 4j$

Division of complex numbers: remember that there are multiple steps.

$$z_1 = x_1 + jy_1 \tag{22}$$

$$z_2 = x_2 + jy_2 \tag{23}$$

$$\frac{z_2}{z_1} = \frac{x_2 + jy_2}{x_1 + jy_1} \tag{24}$$

$$\frac{z_2}{z_1} = \frac{z_2 z_1^*}{z_1 z_1^*} = \frac{z_2 z_1^*}{|z_1|^2} \tag{25}$$

$$\frac{z_2}{z_1} = \frac{\mathrm{Re}\{z_2 z_1^*\}}{|z_1|^2} + j\frac{\mathrm{Im}\{z_2 z_1^*\}}{|z_1|^2} \tag{26}$$

$$\frac{z_2}{z_1} = \frac{x_1 x_2 + y_1 y_2}{x_1^2 + y_1^2} + j\frac{x_1 y_2 - x_2 y_1}{x_1^2 + y_1^2} \tag{27}$$

Using Eq. 27, show that if $z_1 = z_2$, that $z_2/z_1 = 1$.

Evaluate the divisions below:

1. $z_1 = 1 + 4j$, $z_2 = 2 - 2j$. $z_2/z_1 =$
2. $z_1 = 1 + 1j$, $z_2 = -3 - 3j$. $z_2/z_1 =$

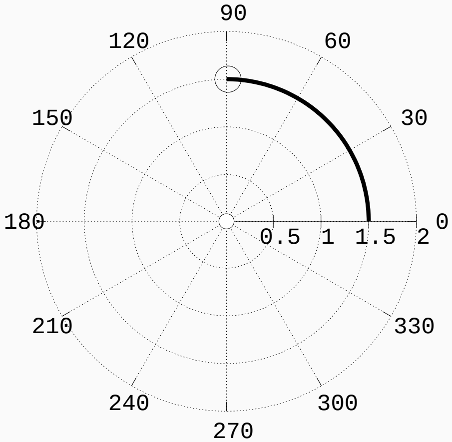

Figure 3: Polar coordinate systems rely on (ρ, ϕ) notation, rather than (x, y) notation.

COMPLEX NUMBERS 1: PROGRAMMING WITH OCTAVE

Let's take the time to get octave installed on your systems: `https://www.gnu.org/software/octave`. If we cannot get it installed on your systems, we can always run it on the local desktops.

Octave is a high-level *scripting* programming language. Although it is possible to write packages and compile code in octave, the most common application is executing a script that performs some analysis on digital data.

```
a = 1+1i;
b = conj(a);
a * b
```

The result of this code should be 2.0. Why? We are defining a complex number in the first line, computing the complex conjugate, and multiplying them.

Octave naturally handles vectors of numbers and matrices. Let's define a vector of complex numbers.

```
a = [1 2 3 5 7 11];
size(a)
ans = 1   6
a = a';
size(a)
```

The code in the fourth line *transposes* the vector. This means trading the rows for the columns of the vector. What begins as a 1×6 vector (one row by six columns) ends as a 6×1 vector (six rows by one column).

CONCLUSION

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