Jackson
Dramé HW 1 Cosc 350

1. $z = \frac{x_1 + jy_1}{x_2 + jy_2}$, then $z^* = \frac{x_1 - jy_1}{x_2 - jy_2}$

$\frac{x_1 - jy_1}{x_2 - jy_2} \times \frac{x_1 + jy_2}{x_2 + jy_2}$ z^* multiplied by its complex conjugate of the denominator

$\Rightarrow \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} - j\left(\frac{x_2y_1 - x_1y_2}{x_2^2 + y_2^2}\right)$

2. $e^{jx} = \cos x + j\sin x$

Let $z = re^{jx}$ & $\sqrt{zz^*} = \sqrt{\cos^2 x + \sin^2 x} = 1$

Polar coordinates $z = r(\cos x + j\sin x)$

$z = re^{jx} = r(\cos x + j\sin x)$

3. $\cos x = \frac{1}{2}(e^{jx} + e^{-jx})$

Since $z = x + jy = re^{jx}$ & $z = r(\cos x + j\sin x)$

$\frac{z}{r} - j\sin x = \frac{1}{2}(e^{jx} + e^{-jx}) \Rightarrow e^{jx} - j\sin x = \frac{1}{2}(e^{jx} + e^{-jx})$

$\Rightarrow 2e^{jx} - j2\sin x = e^{jx} + e^{-jx} \Rightarrow -2j\sin x = -e^{jx} + e^{-jx}$

$\Rightarrow j\sin x = \frac{1}{2}(e^{jx} - e^{-jx})$