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3/1/25
COSC 360

Quiz # 1

① $V(t) = 2.5\cos(2\pi ft - \frac{\pi}{4})$ voltage signal

amplitude $A = 2.5$ volts
frequency $f = 1$ kHz
phase shift: $\frac{\pi}{4}$ (45°)
let $\phi = 2\pi ft - \frac{\pi}{4}$

a) Show that $V(t) = \mathrm{Real}(2.5e^{j\phi})$

$$2.5e^{j\phi} = 2.5e^{2\pi jft - \frac{\pi}{4}}$$

thanks to euler's identity, $e^{j\phi} = \cos\phi + j\sin\phi$

$$2.5e^{j\phi} = 2.5(\cos\phi + j\sin\phi)$$

$$R(2.5e^{j\phi}) = 2.5(\cos(2\pi ft - \frac{\pi}{4}) = V(t) \checkmark$$

b) show that $V(t) = I_{maginary}(2.5e^{j(\phi - \frac{\pi}{2})})$

$$I(2.5e^{j(\phi - \frac{\pi}{2})}) = 2.5\cos(\cancel{\phi} - \frac{\pi}{2}) + 2.5j\sin(\phi - \frac{\pi}{2})$$

$$= 2.5\cos(2\pi ft - \underbrace{\frac{\pi}{4}}_{-\frac{\pi}{2}}) + 2.5j\sin(2\pi ft - \underbrace{\frac{\pi}{4}}_{-\frac{\pi}{2}})$$

$$= 2.5\cos(2\pi ft - \frac{3\pi}{4}) + 2.5j\sin(2\pi ft - \frac{3\pi}{4})$$

$$I(2.5e^{j(\phi - \frac{\pi}{2})}) = 2.5j\sin(2\pi ft - \frac{\pi}{4}) - j\sin(\frac{\pi}{2})$$

$$= 2.5\cos(2\pi ft - \frac{\pi}{4})$$

② let a set of sample times be
$0, \Delta t, 2\Delta t, \ldots, n\Delta t$.

$f = f$ $\quad T = \frac{1}{f}$

a) show that $KHz^{-1} = 1$ milli sec.

kilo $= 10^3$ $1\,kHz = 1000\,Hz$

$$1kHz^{-1} = \frac{1}{1000} = 1\times10^{-3} = 1 \text{ milli sec}$$

b) if $T = 5ns$ nano $= 10^{-9}$
what is f?

$$T = \frac{1}{f} \quad f = \frac{1}{T}$$

$$f = \frac{1}{5\times10^{-9}} = \boxed{2\times10^{8}\ Hz}$$

c) suppose we are sampling a sinusoidal signal with $f = 5kHz = 5000\,Hz$
if sampling f, $f_s = 50kHz = 50000$ samples/sec
how many samples per period?

$T = \frac{1}{50}$ ms $=$ ~~5~~ 50000 samples per second

d) if $\Delta T = \frac{1}{f_s} = 0.002ms$, how many samples per period?

$$f_s = \frac{1}{\Delta T} = \frac{1}{0.002} = 500\,kHz$$

so, 500 ~~000 ms~~ samples per second.

3) A C circuit output runs from 0 - 2.56 volts

a) if we can digitize the new voltage range into 256 steps. what is the range between steps, ΔV?

$$\frac{2.56}{256} = \boxed{0.01}$$

b) what power of 2 gives 256?

8

c) if we double the # of bits, what is the new ΔV?

$$8 \to 256 \times 2 = 512$$

$$\frac{2.56}{512} = \boxed{0.005}$$

④ a signal with

$A = 2.5v$ DC offset $= 2.5$ V

$$s(t) = 2.5 \sin(2\pi f t) + 2.5$$

a)
```
f = 10;
dt = 0.001;
t = dT:dt:0.1;
s = 2.5 sin(2*pi*f*t) + 2.5
plot(t, y)
```

⑤ $$R(f) = \frac{j\omega\tau}{(1+j\omega\tau)}$$

a) find $\|R\|$

$$\|R\| = \sqrt{\frac{j\omega\tau}{(1+j\omega\tau)} \times \frac{-(j\omega\tau)}{(1-(j\omega\tau))}}$$

$$= \sqrt{\frac{-(\omega\tau)^2}{1+(\omega\tau)^2}} = \boxed{\frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}}$$

b) find ϕ

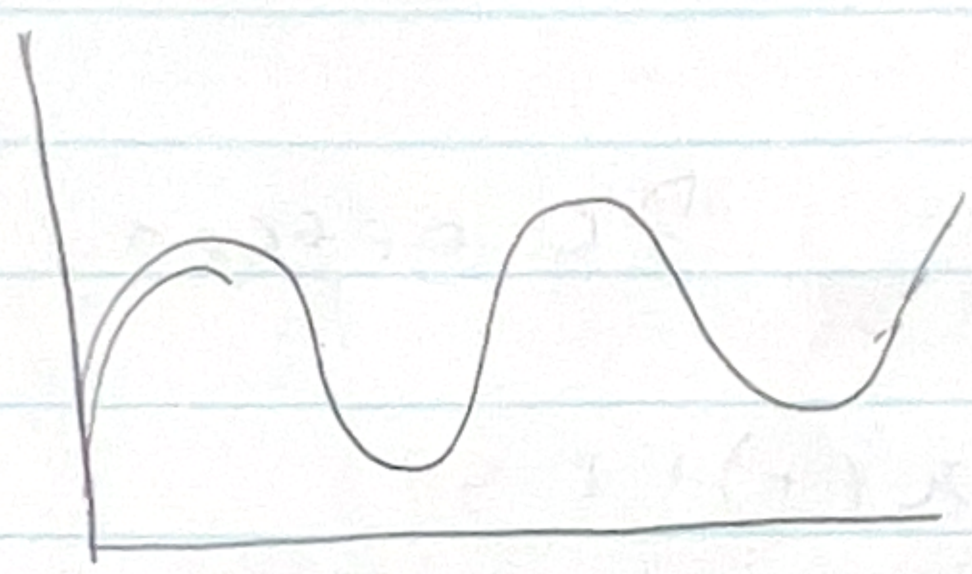
$$\frac{j\omega\tau}{(1+j\omega\tau)}\frac{(1-j\omega\tau)}{(1-j\omega\tau)} = \frac{j\omega\tau+(\omega\tau)^2}{1+(\omega\tau)^2}$$

$$\Rightarrow \frac{(\omega\tau)^2}{1+(\omega\tau)^2} - j\omega\tau$$

$$\phi = \tan^{-1}\left(\frac{\omega\tau}{\frac{(\omega\tau)^2}{1+(\omega\tau)^2}}\right) = \tan^{-1}\left(\frac{\omega\tau(1+(\omega\tau)^2)}{(\omega\tau)^2}\right)$$

$$= \boxed{\tan^{-1}\left(\frac{1+(\omega\tau)^2}{(\omega\tau)}\right)}$$

c)



d) $A(f)$ filtered A: $R(f)\,A(f)$

$\uparrow$ amp $\uparrow$ freq

$A=2$, $f=0.5\,kHz$, $R=1\,k\Omega$, $C=1\,\mu F$

what is $A(f)R(f)$?

4) b) `n = randn(size(t));`

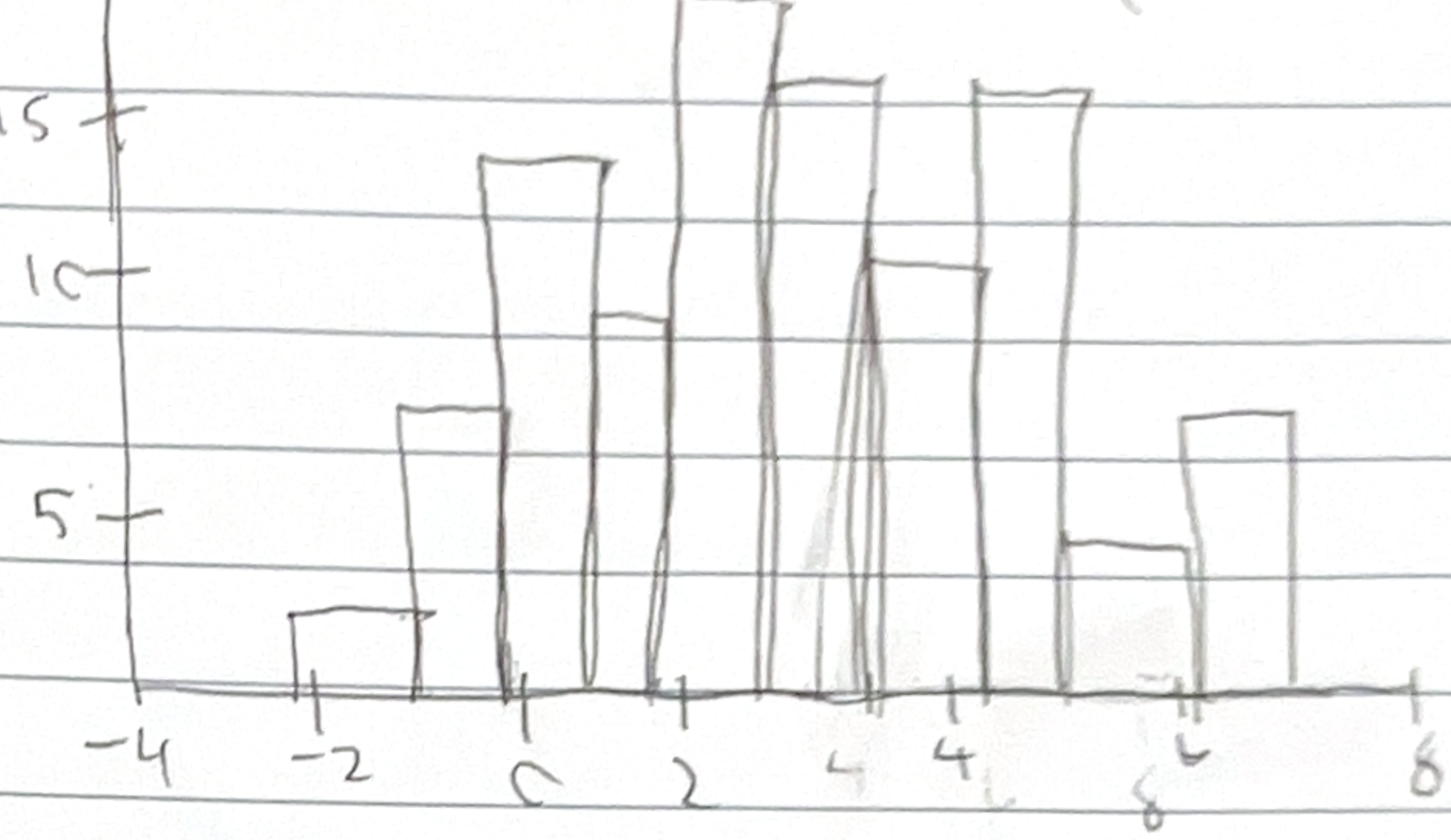
c) `z= y+n;`

`plot(t,z)`

d) $SNR = \frac{y}{n} = 0.4722$ from octave.

e)

e) using hist(z), I got:



(5) d) the filtered amplitude is $A(f)\,R(f)$

$R(f) =$

$A = 1 \qquad f = 0.5\,\text{kHz}$

$R = 1\,\text{k}\Omega \qquad C = 1\,\mu\text{F}$

find $A(f)\,R(f)$

$$A(f)\left(\frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}\right) = \frac{1(2\pi(0.5\times10^3)(1\times10^3)(1\times10^{-6}))}{\sqrt{1+(2.5(0.5\times10^3)(1\times10^3)(1\times10^{-6}))^2}}$$

$$= \frac{2\pi(0.5)}{\sqrt{1+(2\pi(0.5))^2}} \quad \frac{\pi}{\sqrt{1+\pi^2}}$$

negligible

$$\approx \frac{\pi}{\pi} = \boxed{1\ \text{volt}}$$

(6a) sampling rate = 10 kHz

analog $f = 2.5$ kHz

what is f_s?

$$\frac{2.5\,\text{kHz}}{10\,\text{kHz}} = \frac{1}{4} \Rightarrow \frac{1}{4}(10) = \boxed{2.5\,\text{kHz}}$$

b) Sampling rate = 10 kHz

$f_a = 5$ kHz

$$f_s = \frac{5\ \text{kHz}}{10\ \text{kHz}} = \frac{1}{2} \Rightarrow \frac{1}{2}(10) = \boxed{5\ \text{kHz}}$$

c) Sampling rate = 10 kHz

$f_a = 15$ kHz

$$f_s = \frac{15}{10} = 1.5 \Rightarrow 1.5(10) = \boxed{15\ \text{kHz}}$$

d) Sampling rate = 10 kHz

$f_a = 20$ kHz

$$f_s = \frac{20}{10} = 2 \Rightarrow 2(10) = \boxed{20\ \text{kHz}}$$

7) $S[s(t)] = s(t - \frac{T}{2})$

a) $s(t) = 2\sin(2\pi f(t))$ $T = \frac{1}{f}$

find $S[s(t)]$.

$$S[s(t)] = 2\sin(2\pi f(t - \tfrac{T}{2}))$$

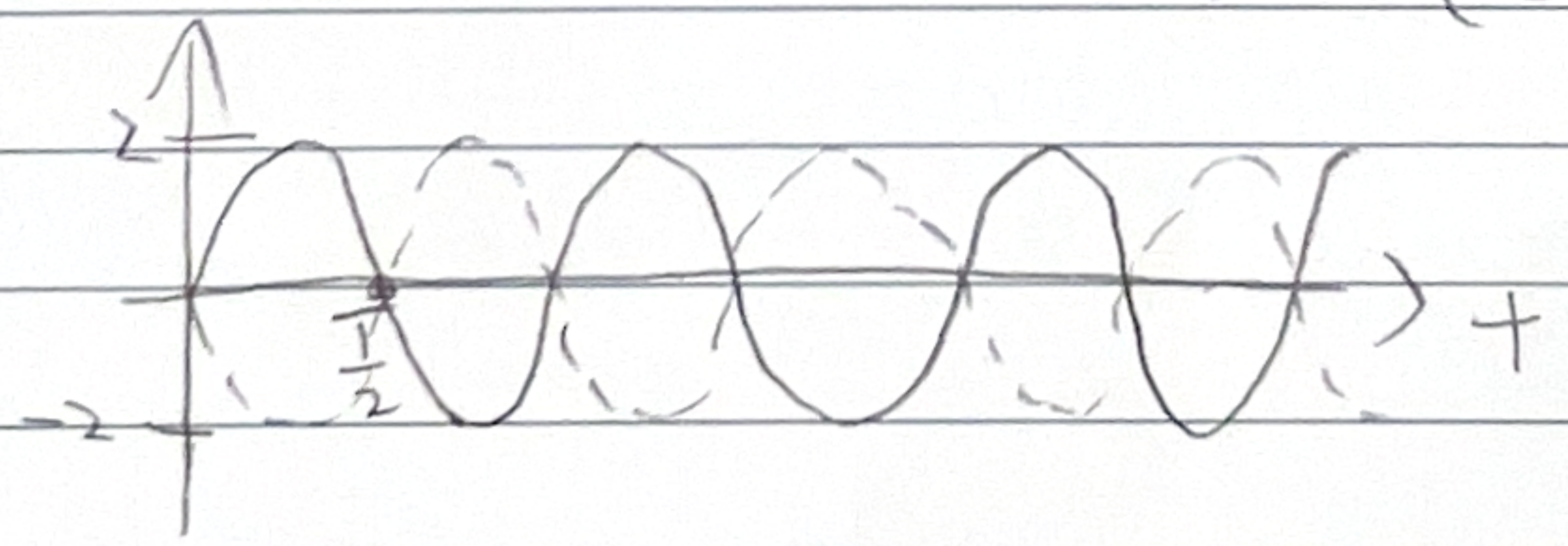
$\frac{\frac{1}{f}}{2} = \frac{2}{f}$

$$= 2\sin(2\pi f(t - \tfrac{2}{f}))$$

$$= 2\sin(2\pi ft - 2\pi \cancel{f}\tfrac{2}{\cancel{f}})\pi)$$

$$= 2\sin(2\pi ft - 4\pi)$$

b)



c) $s(t) + S[s(t)] = 2\sin(2\pi ft) + 2\sin(2\pi ft - 4\pi)$

$$= 2(\sin(2\pi ft) + \sin(2\pi ft - 4\pi))$$

(8) signal component impulse
$x[n] = [0\,0\,0\,2\,0\,0,\ldots]$
with 100 samples

a) $y[n] = S(x[n]) = -x[n-1]$
find $y[n]$

$$[0\,0\,0\,2\,0\,0\,0,\ldots] \xrightarrow{n-1} [0\,0\,0\,0\,2\,0\,0\,0,\ldots$$

$$\xrightarrow{-x} [0\,0\,0\,0\,\cancel{2}\,-2\,0\,0,\ldots] = y[n]$$

b) $y[n] = (x[n])^2 = [0\,0\,0\,4\,0\,0,\ldots]$

c) the system S in b) is linear, the system S in a) is non-linear.

(9) determine if they are even or odd.

a) $\cos(2\pi f t)$

$$\cos(2\pi f(-t)) \stackrel{?}{=} \cos(2\pi f t)$$

$$\cos(-2\pi f t) \stackrel{?}{=} \cos(2\pi f t)$$

$$\cos(2\pi) = 1, \quad \cos(-2\pi) = 1$$

this is <u>even</u>.

b) $e^{-\left(\frac{t}{\sigma}\right)^2}$ <u>even</u>

$$e^{-\left(\frac{t}{\sigma}\right)^2} \stackrel{?}{=} e^{-\left(\frac{-t}{\sigma}\right)^2}$$

$$e^{-\left(\frac{t^2}{\sigma^2}\right)} = e^{-\left(\frac{t^2}{\sigma^2}\right)}$$

c) $e^{-\alpha t}$ $\quad e^{-\alpha t} \overset{?}{=} e^{-\alpha(-t)}$

$$e^{-\alpha t} = e^{+\alpha t} \qquad e^{-\alpha t} \neq e^{\alpha t}$$

$$-e^{-\alpha t} = e^{-\alpha(-t)} \qquad -e^{-\alpha t} \neq e^{\alpha t}$$

neither

d) $at^2 + bt + c$

$$a(-t)^2 + b(-t) + c \overset{?}{=} at^2 + bt + c$$

$$a(t) - bt + c \overset{?}{=} at^2 + bt + c$$

even.

(10) Show that Fourier transform is:

a) homogeneous

$$S_{out}(t) = S[S_{in}(t)] \qquad kS_{out}(t) = S[kS_{in}(t)]$$

$$\mathcal{F}\{kf(t)\} = \int_{-\infty}^{\infty} k\, f(t)\, e^{-2\pi j f t}\, dt$$

$$= k\int_{-\infty}^{\infty} f(t)\, e^{-2\pi j f t}\, dt = k\,\mathcal{F}\{F(t)\}$$

b) additive

$$S'_1(t) = S[S_1(t)]$$

$$S'_2(t) = S[S_2(t)]$$

$$S'_1(t) + S'_2(t) = S[S_1(t) + S_2(t)]$$

$$\int_{-\infty}^{\infty} (f(t) + g(t))\, e^{-2\pi j f t}\, dt =$$

$$= \int_{-\infty}^{\infty} f(t)\, e^{2\pi j f t}\, dt + \int_{-\infty}^{\infty} g(t)\, e^{-2\pi j f t}\, dt$$

c) Shift-invariant

$$S_{out}(t) = S[S_{in}(t)] \qquad S_{out}(t - t_0) = S[S_{in}(t - t_0)]$$

$$\int_{-\infty}^{\infty} f(t-t_0)e^{-2\pi j f t}\,dt \qquad u = t-t_0 \quad t = u+t \qquad du = dt$$

$$= \int_{-\infty}^{\infty} f(u)e^{-2\pi j f(u+t_0)}\,du = \int_{-\infty}^{\infty} f(u)e^{-2\pi j f u}e^{-2\pi j f t_0})\,du$$

$$= e^{-2\pi j f t_0}\int_{-\infty}^{\infty} f(u)e^{-2\pi j f u}\,du$$

$$|z| = \sqrt{z^* z} = \sqrt{(e^{2\pi j f_0 t}e^{-2\pi j f_0 t})}$$

(11) dirac δ-fctn:

$$f(t_0) = \int_{-\infty}^{\infty} f(t)\delta(t-t_0)\,dt$$

a) find fourier tf of $a\delta(t-t_0)$

$$\int_{-\infty}^{\infty} a\delta(t-t_0)e^{-2\pi j f_0 t}\,dt = \boxed{a\,e^{-2\pi j f_0 t_0}}$$

b) a is the magnitude

c) the phase angle $\phi = -2\pi f_0 t_0$

(12) $F(f) = \frac{a}{2}(\delta(f-f_0) + a\delta(f+f_0))$

take the inverse fourier tf.

$$F^{-1} = \int_{-\infty}^{\infty} \frac{a}{2}(\delta(f-f_0) + \delta(f+f_0)e^{2\pi j f t}\,df$$

$$= \int_{-\infty}^{\infty} \frac{a}{2}\delta(f-f_0)e^{2\pi j f t}\,df + \int_{-\infty}^{\infty} \frac{a}{2}\delta(f+f_0)e^{2\pi j f t}\,df$$

$$= \frac{a e^{-2\pi j f_0 t}}{2} + \frac{a e^{-2\pi j f_0 t}}{2} = \boxed{a\cos(2\pi f_0 t)}$$

b) suppose the signal was:

$$F(f) = \frac{a}{2j}\left(\delta(f-f_a) - \delta(f+f_a)\right).$$

take the inverse fourier tf of this.

$$F^{-1} = \frac{a}{2j}\left[\int_{-\infty}^{\infty} \delta(f-f_a)\, e^{2\pi j f t}\, df + \int_{-\infty}^{\infty} \delta(f+f_a)\, e^{2\pi j f t}\, df\right]$$

$$= \frac{a}{2j}\left[e^{2\pi j f_a t} + e^{-2\pi j f_a t}\right] = \boxed{a \sin(2\pi f_a t)}$$

13) a) express the following as complex exponentials

I) $$A\cos(2\pi f_{LO} t) = \frac{A e^{2\pi j f_{LO} t} + A e^{-2\pi j f_{LO} t}}{2}$$

f_{LO} = local oscillator frequency

II) $$\frac{m}{A}\cos(2\pi f_A t) = \frac{m e^{2\pi j f_A t} + m e^{-2\pi j f_A t}}{2A}$$

f_A = audio frequency

b) $$\cancel{A}\cos(2\pi f_{LO} t)\left(\frac{m}{\cancel{A}}\cos(2\pi f_A t)\right)$$

$$= m\left(\frac{e^{2\pi j f_A t} + e^{-2\pi j f_A t}}{2}\right)\left(\frac{e^{2\pi j f_{LO} t} + e^{-2\pi j f_{LO} t}}{2}\right)$$

the new f_s are $f_{LO} + f_A$ and $f_{LO} - f_A$

$$= \frac{m}{4}\left(e^{2\pi j (f_{LO}+f_A)t} + e^{-2\pi j (f_{LO}+f_A)t} + e^{2\pi j (f_{LO}-f_A)t} + e^{-2\pi j (f_{LO}-f_A)t}\right)$$

$$= \frac{m}{4}\left(2\cos(2\pi(f_{LO}+f_A)t) + 2\cos(2\pi(f_{LO}-f_A)t)\right)$$

$$= m\left[\frac{1}{2}\cos(+) + \frac{1}{2}\cos(-)\right]$$

Then

Code projects

① a)
```
fs = 20x10^3;
T = 2.0;
N = T.fs;
```
b)
```
d= zeros(N,1)
d(1) = 1;
d(5000)=0.5;
d(10000)=0.25;
d(15000) = 0.125;
d(20000) = 0.125/2;
```
c)
```
f =440.0;
x= zeros(size(d));
dt= 1/fs;
t = dT:dt:T;
```
d)
```
x(1:5000) = sin(2π ft(1:5000));
Plot(t, x)
```
e)
```
player1= audioPlayer(x, fs, 8);
y = conv(x, d);
Player2= audioPlayer(y, fs, 8);
Play(Player2)
```