COSC: 360 Quiz 1

Matthew Townsend

Complex to polar form

1) $z = 4 + 4j$

$r = \sqrt{4^2 + 4^2} = \sqrt{16+16} = \sqrt{32}$ $\theta = \tan\left(\frac{4}{4}\right) = (\pi/4)$

$$\boxed{= \sqrt{32}\left(\cos(\pi/4) + j\sin(\pi/4)\right)}$$

2) $z = 1$

$r = \sqrt{1^2 + 0^2} = \sqrt{1} = 1$ $\theta = (2\pi)$ $0°$

$z = 1(\cos(2\pi)) = \boxed{z = 1(\cos(2\pi))}$

$z = j$

$r = \sqrt{0^2 + 1^2} = \sqrt{1} = 1$ $\theta = (\pi/2)$ $90°$

$z = 1(j\sin(\pi/2)) = \boxed{z = (j\sin(\pi/2))}$

$z = -1$

$r = \sqrt{-1^2 + 0^2} = \sqrt{-1^2} = 1$ $\theta = (\pi)$ $180°$

$z = 1(\cos(\pi)) = \boxed{z = (\cos(\pi))}$

$z = -j$

$r = \sqrt{0^2 - 1^2} = \sqrt{-1}$ $\theta = (3\pi/2)$ $270°$

$z = (j\sin 3\pi/2) = \boxed{z = \sqrt{-1}\,(j\sin 3\pi/2)}$

since value is negative

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3) Explain phase angle of each number

→ For the phase angle, the real part of the complex number can be looked at as the x value on the axis or $\cos(\theta)$. The imaginary part is the opposite so y-axis, $\sin(\theta)$. So with $z = 1$ and $z = -1$, only dealing with the x-axis So the phase angle will result in being 2π or π in the polar form (0°, 180). Therefore, the imaginary phase angle is only dealing with y-axis and have either $\pi/2$ or $3\pi/2$ depending if it is positive or negative value. Phase angle is reliant on the original values from the complex number and works from there.

Complex to rectangular form

1) $z = 2\exp(j\pi/4)$
$= 2(\cos(\pi/4) + j\sin(\pi/4))$
$= 2(0.7071 + j(0.7071))$
$= 1.4142 + j1.4142$

$\boxed{\approx 1.4142 + j1.4142}$

2) $z = 5\exp(j\pi)$
$= 5(\cos(\pi) + j\sin(\pi))$
$= 5(-1 + j(0))$
$= -5$

$\boxed{= -5}$

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$V(t) = a_1 \exp(jx_1) + a_2 \exp(jx_2)$ $\quad x_i = 2\pi f t + \phi_i$

$\rightarrow V^*V = a_1 e^{jx_1} + a_2 e^{jx_2}(a_1 e^{-jx_1} + a_2 e^{-jx_2})$

$= a_1^2 + a_1 a_2 e^{jx_1 - x_2} + a_1 a_2 e^{jx_2 - x_1} + a_2^2$

$\Rightarrow (a_1^2 + a_2^2) + [a_1 a_2 e^{jx_1 - x_2} + a_1 a_2 e^{-jx_1 - x_2}]$

$= (a_1^2 + a_2^2)[(\cos)(\sin)(e^{jx_2 - x_1}) + (\cos)(\sin)(e^{-j(x_2 - x_1)}) + 1]$

$\rightarrow \phi_2 - \phi_1 = \pi$

$= (a_1^2 + a_2^2)((\cos)(\sin)(e^{j\pi}) + (\cos)(\sin)e^{-j\pi} + 1)$

$(2(\frac{e^{j\pi} + e^{-j\pi}}{2}) + (\cos)(\sin) + 1)$

$(2(\cos)(\sin)(\cos(\pi) + 1)$

$(1 - 2(\cos)(\sin))$

$(1 - 2(\sin))$ $\quad \Rightarrow \boxed{(a_1^2 + a_2^2)(1 - 2(\sin))}$

$\rightarrow \phi_2 - \phi_1 = 0$

$= (a_1^2 + a_2^2)((\cos)(\sin)(e^{j0}) + (\cos)(\sin)e^{-j0} + 1)$

$(2(\cos)(\sin) + 1) \Rightarrow \boxed{(a_1^2 + a_2^2)(1 + 2(\sin))}$

$x_i = 2\pi f t + \phi_i$ $\quad V(t) = a_1 e^{j(2\pi f t + \phi_1)} + a_2 e^{j(2\pi f t + \phi_2)}$

$x_1 = 2\pi f t + \phi_1$ $\quad \sqrt{a_1^2 + a_2^2}\,(e^{j(2\pi f t)})(\cos \cdot e^{j\phi_2} + \sin \cdot e^{j\phi_1})$

$x_2 = 2\pi f t + \phi_2$ $\quad (e^{j(2\pi f t)})(\cos \cdot e^{j\pi} \cdot e^{j\phi_1} + \sin \cdot e^{j\phi_1})$

$(e^{j(2\pi f t)}) \cdot (e^{j\phi_1})(\sin - \cos)$

$e^{(2\pi f t + j\phi_1)}(\sin - \cos)$

$$\boxed{\phi_V = \tan^{-1}(\sin(2\pi f t + \phi_1)/(\cos(2\pi f t + \phi_1))}$$

$\sqrt{a_1^2 + a_2^2} \cdot e^{j2\pi f t}(\cos \cdot e^{j\phi_1} + \sin \cdot e^{j\phi_1})$

$'' \quad '' (\cos + \sin) \cdot e^{j2\pi f t + \phi_1}$

$\Rightarrow \boxed{\phi_V = \tan^{-1}(\sin(2\pi f t + \phi_1) / (\cos(2\pi f t + \phi_1))}$

Results of ϕ_V are the same with both of the presented cases. Was encapsulated so the phase is going to be the same.

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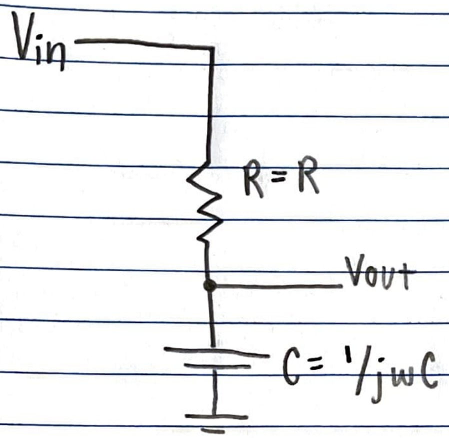
→ Recompute $h(w)$, but start with $L = 0$ ($Z_2 = 0$)

$Z_R = R \qquad T = RC$

$Z_C = 1/jwC$



$$\frac{V_{out}}{V_{in}} = \frac{1/jwC}{R + 1/jwC}$$

$$= \frac{1}{jwC(R + 1/jwC)} = \frac{1}{1 + (R)(jwC)}$$

$$= \frac{1}{1 + RC} = \frac{1}{1 + T}$$

$$\boxed{h(w) = \frac{1}{1 + T}}$$

→ Draw a graph of $|h(w)|$