Homework #1

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1. Show that if z is $\frac{(x_1 + jy_1)}{(x_2 + jy_2)}$, that $z^* = \frac{(x_1 - jy_1)}{(x_2 - jy_2)}$

Since z^* is the complex conjugate of z, by the definition of a complex conjugat being to change the sign of the imaginary part,

$$\frac{(x_1 + jy_1)}{(x_2 + jy_2)} \cdot \frac{(x_1 - jy_2)}{(x_2 - jy_2)} = \frac{(2x_1 - 2jy_1y_2)}{(2x_2 - 2jy_2)}$$

$$= \frac{(x_1 - jy_1)}{(x_2 - jy_2)} \quad \square$$

2. Prove that $\exp(jx) = \cos(x) + j\sin(x)$

$$\exp(jx) = \sum_{i=0}^{\infty} \frac{(jx)^n}{n!} = \sum_{i=0}^{\infty} \frac{j^n x^n}{n!}$$

$$= \sum_{\text{even}}^{\infty} \frac{j^n x^n}{n!} + \sum_{\text{odd}}^{\infty} \frac{j^n x^n}{n!}$$

$$= \sum_{i=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} + j\sum_{i=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

$$= \cos x + j\sin x \quad \square$$

3. Prove $\cos(x) = \left(\frac{1}{2}\right)\left(\exp(jx) + \exp(-jx)\right)$

$$\exp(jx) = \cos(x) + j\sin(x)$$
$$\exp(-jx) = \cos(-x) + j\sin(-x)$$

$$\exp(jx) + \exp(-jx) = 2\cos(x)$$

$$\cos(x) = \frac{\exp(jx) + \exp(-jx)}{2} \quad \square$$