Digital Signal Processing: COSC360

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Complex numbers 1: theory and examples

Complex numbers 1

Convert each of these complex numbers to polar form:

1. $z = 4 + 4j$
2. $z = 1$, $z = j$, $z = -1$, $z = -j$
3. In the previous problem, describe in words what is happening to the *phase angle* of each number.

Convert each of these complex numbers to rectangular form ($z = x + jy$).

1. $z = 2\exp(j\pi/4)$
2. $z = 5\exp(j\pi)$

Complex numbers 1

Suppose that $x_i = 2\pi f t + \phi_i$. The sum of two sinusoids *in the complex plane* with amplitudes a_1 and a_2 can then be written

$$V(t) = a_1 \exp(jx_1) + a_2 \exp(jx_2) \tag{1}$$

It is assumed that we would take the real part of $V(t)$ to be physical.

1. Compute $|V|^2 = V^*V$, and $\phi_2 - \phi_1 = \pi$, $\phi_2 - \phi_1 = 0$.
2. What is $\phi_V = \tan^{-1}(\operatorname{Im}\{V\}/\operatorname{Re}\{V\})$ in each case?

Why do these results make sense? Thus, the complex numbers encapsulate the concepts of *constructive* and *destructive* interference.

Complex numbers 3: Application to AC cicuits

Complex numbers 1: application to AC circuits

Recall the RLC circuit transfer function looks like

$$h(\omega) = \frac{Z_2 + Z_3}{Z_1 + Z_2 + Z_3} \tag{2}$$

$$\omega_{LC}^{-2} = LC \tag{3}$$

$$\tau = RC \tag{4}$$

$$k^2 = 1 - \left(\frac{\omega}{\omega_{LC}}\right)^2 \tag{5}$$

$$h(\omega) = \frac{k^4}{k^4 + (\omega\tau)^2} - j\frac{k^2\omega\tau}{k^4 + (\omega\tau)^2} \tag{6}$$

and

$$Z_R = R + 0i \tag{7}$$

$$Z_C = 0 + \frac{1}{j\omega C} \tag{8}$$

$$Z_L = 0 + j\omega L \tag{9}$$

Recompute $h(\omega)$, but start with $L = 0$ ($Z_2 = 0$). This reduces the circuit to an RC circuit. It is still useful to have $\tau = RC$. Draw a graph of $|h(\omega)|$.