Homework 5, Unit 1: Filter Design, DFT Properties and Applications

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1 Memory Bank

- **Convolution**: this is an operation that characterizes the response $h[n]$ of a linear system.

$$y[i] = h[n] * x[n] = \sum_{j=0}^{M-1} h[j]x[i-j] \tag{1}$$

  In words, the output at sample i is equal to the produce of the system response h and the input signal x, summed over the proceeding M samples (from $j = 0$ to $j = M - 1$).

- **Discrete Delta Function**, $\delta[n]$: A standard impulse response that contains one non-zero sample. It has the following property:

$$x[n] = \delta[n] * x[n] \tag{2}$$

- **Discrete Fourier Transform**, for a sampled, digitized signal x_n:

$$X_{\mathrm{k}} = \sum_{n=0}^{N-1} x_n e^{-2\pi j(k/N)n} \tag{3}$$

- In DFT analysis, we often need to know the Δt, time duration for samples, and the sampling rate, f_{s}. Note that $1/f_{\mathrm{s}} = \Delta t$.
- For a sinusoid of frequency f (Hz), the period is $T = 1/f$ (seconds).
- **Inverse Discrete Fourier Transform**, for a sampled, digitized signal X_k in the frequency domain:

$$x_{\mathrm{n}} = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{2\pi j(k/N)n} \tag{4}$$

2 Discrete Fourier Transform, Applications

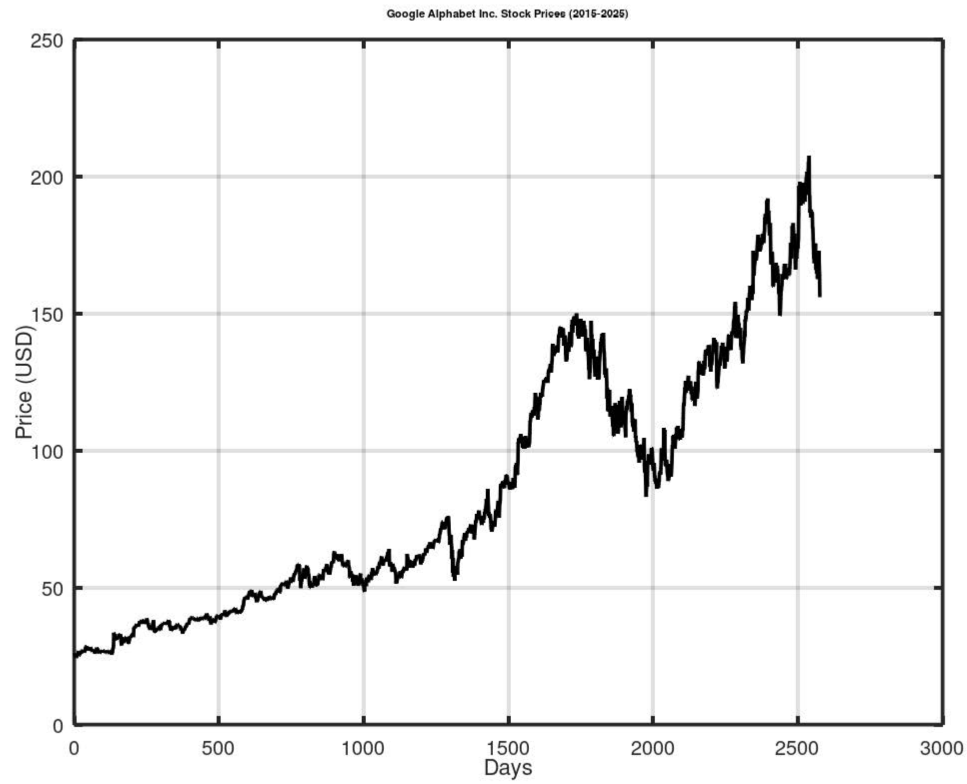
1. **Download and Graph Data.** On our Course Moodle page under Unit 1, download the file “Stock Data, Google Alphabet Inc., 2015-2025.” Move it into a folder accessible to `octave`. Use the `csvread` function to import the data into the `octave` workspace. Plot the data and label the x-axis “Days” and the y-axis “Price (USD).” For example, if the CSV data is stored in a variable `data`, then plot it via

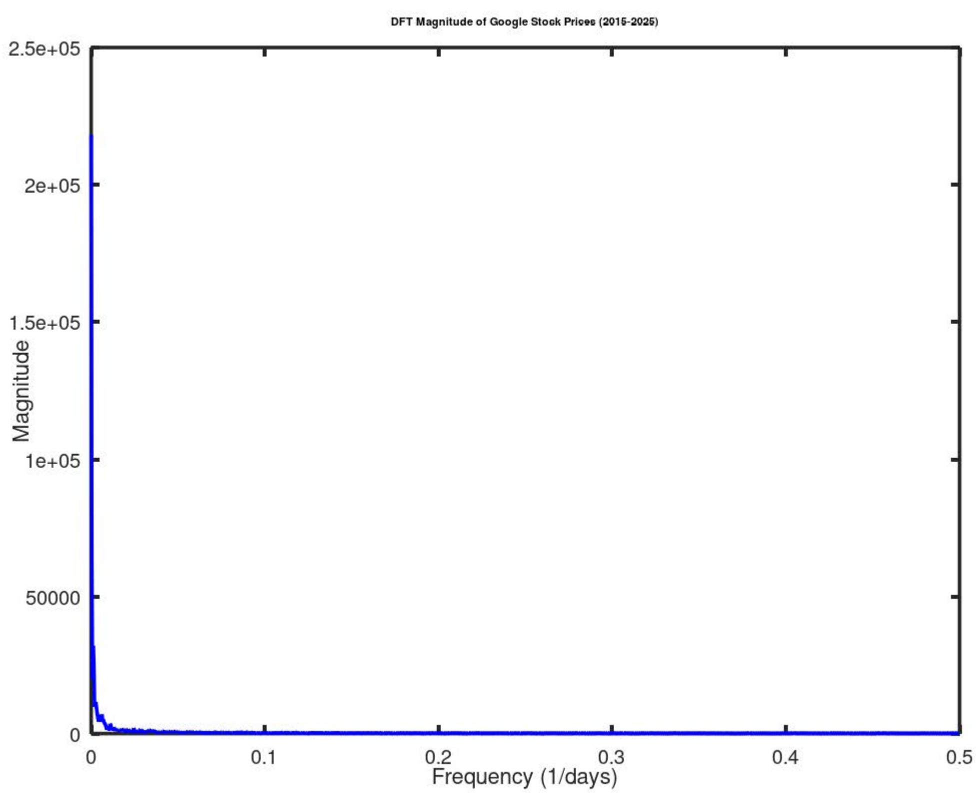
```
plot(data(:,1),data(:,2),'-','color','black')
```

2. **Create the discrete Fourier transform.** The units of the graph are stock price (closing) in USD, versus days. Day 1 corresponds to January 1st, 2015. Using techniques we covered in previous code labs, create and graph the magnitude of the DFT of the stock data.
3. **Identify Peaks and Frequencies**. What peaks, if any, do you observe? What are the corresponding frequencies?

3 Filter Design

1. **Smoothing the Time Series Data.** As in a previous code lab, implement a *running average filter* kernel, and convolve it with the time series data. Use this filter to smooth the data, and plot it with the original, unfiltered data.
2. **Graph the Filtered Spectrum**. Add the magnitude of the DFT of the filtered data to the graph of the magnitude of the DFT of the raw data. Does the result make sense?


Google Alphabet Inc. Stock Prices (2015-2025)
Price (USD)
Days
0
50
100
150
200
250
0
500
1000
1500
2000
2500
3000


DFT Magnitude of Google Stock Prices (2015-2025)
Magnitude
Frequency (1/days)
0
50000
1e+05
1.5e+05
2e+05
2.5e+05
0
0.1
0.2
0.3
0.4
0.5

```
Frequency: 0.00117 (1/days), Magnitude: 32002.85
Frequency: 0.00272 (1/days), Magnitude: 11488.48
Frequency: 0.00621 (1/days), Magnitude: 6892.35
Frequency: 0.00544 (1/days), Magnitude: 6608.66
Frequency: 0.00466 (1/days), Magnitude: 6322.87
>>
```