dirac delta function

$$f(x) = \begin{cases} t = t_0; & \infty \\ t \neq t_0; & 0 \end{cases}$$

by def $F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x} dx$

$$F(f(x)) = \int_{-\infty}^{\infty} a \text{ delta}(t-t_0) e^{-i\omega t} dt$$

zero everywhere except when $t = t_0$

$$F(f(x)) = \frac{1}{\sqrt{2\pi}} \int_{t_0}^{t_0} a \infty e^{-i\omega t_0} dt$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{\infty e^{-i\omega t}}{-i\omega} \Bigg|_{t_0}^{t_0} \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{\infty e^{-i\omega t_0}}{-i\omega} - \frac{\infty e^{-i\omega t_0}}{-i\omega} \right]$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{\infty\left(e^{-2i\omega t_0}\right)}{-i\omega}\right)$$

$$|f(x)|^2 = \frac{1}{\sqrt{2\pi}}\left[\frac{\infty^2 e^{4i\omega t_0}}{\omega}\right]$$

$$\text{phase } \phi = \frac{b_n}{a_n} = \frac{\frac{e^{-2i\omega t_0}}{\omega}}{\frac{\infty}{\sqrt{2\pi}}}$$

$$= \frac{\sqrt{2\pi}\, e^{-2i\omega t_0}}{\omega\, \infty}$$

being divided by ∞
so the term approaches zero

$$\text{phase } \phi = 0^\circ$$

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