Linear Systems

1.

Homework #3

Linear Systems

1) a) $A\{x[n]\} = A\{B\{x[n]\}\} = B\{A\{x[n]\}\}$

$A\{x[n]\} = 2x[n] - 1 \qquad B\{x[n]\} = 0.5x[n]$

linear if Ex: $A\{c \cdot x[n]\} = c \cdot A\{x[n]\}$

$A\{c \cdot x[n]\} = 2(c \cdot x[n]) - 1 = 2c \cdot x[n] - 1$

$c \cdot A\{x[n]\} = c \cdot (2x[n] - 1) = 2c \cdot x[n] - c$

System A not linear. Breaks scaling property homogenity

$B\{c \cdot x[n]\} = 0.5(c \cdot x[n]) = 0.5c \cdot x[n] = c \cdot (0.5x[n]) = c \cdot B\{x[n]\}$

System B linear

b) remove the constant for system A

$A\{x[n]\} = 2x[n]$

prove

$A\{B\{x[n]\}\} = B\{A\{x[n]\}\}$

$A\{B\{x[n]\}\}$

$= A\{0.5x[n]\} = 2(0.5x[n]) = x[n]$

$B\{A\{x[n]\}\}$

$= B\{2x[n]\} = 0.5(2x[n]) = x[n]$

2.

HW #3

linear Systems

2) $f(t) = a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t)$

$T_1 = 1/f_1$ $\quad f_2 = 2f_1$

$T_2 = 1/f_2$

• $\int_{-\infty}^{\infty} f(t)\,\delta(t-T_1)\,dt$

• $\int_{-\infty}^{\infty} f(t)\,\delta(t-T_2)\,dt$

$$\int_{-\infty}^{\infty} a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t)\,\delta(t-T_1)\,dt$$

$$= a_1 \cos(2\pi f_1 T) + a_2 \cos(2\pi f_2 T) = a_1 \cos(2\pi f_1 \tfrac{1}{f_1}) + a_2 (2\pi f_2 \tfrac{1}{f_2})$$

$$= a_1 \cos(2\pi) + a_2 \cos(2\pi \cdot 2) = a_1 \cdot 1 + a_2 \cdot 1 = \boxed{a_1 + a_2}$$

$$\int_{-\infty}^{\infty} a_1 \cos\ (2\pi f_1 t + a_2 \cos 2\pi f_2 t]\ \delta(t-T_2)\,dt$$

$$a_1 \cos(2\pi f_1 T_2) + a_2 \cos(2\pi f_2 T_2) = a_2 \cos(2\pi f_1 \tfrac{1}{2f_1}) + a_2 \cos(2\pi$$

$$= a_1 \cos(\pi) + a_2 \cos(2\pi) = \boxed{-a_1 + a_2}$$

3.

HW #3

Linear System

3)

a) $F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt$

$f(\omega) = \int_{-\infty}^{\infty} a\,\delta(t - t_0)\, e^{-j\omega t}\, dt$

sifting properties

$F(\omega) = a e^{-j\omega t_0}$

$$\boxed{|F(\omega)| = |a e^{-j\omega t_0}| = |a| \cdot |e^{-j\omega t_0}| = |a|}$$

b)

$\phi(\omega) = \arg(a e^{-j\omega t_0})$

$\phi(\omega) = -\omega t_0$

$$\boxed{\phi(f) = -2\pi f t_0}$$

c) $\tau_g = -\frac{d\phi(\omega)}{d\omega}$

$$\boxed{\tau_g = -\frac{d(-\omega t_0)}{d\omega} = t_0}$$

4.

Hw #3

Linear system

4) $\delta[n] = [1,0,0,0,0,0,0,0]$

$y[n] = S[x[n]] = 0.5x[n-2]$

a) $S[\delta[n]] = 0.5x[n-2$

impulse shift

$\delta[n-2] = [0,0,1,0,0,0,0,0]$

$S[\delta[n]] = 0.5[0,0,1,0,0,0,0,0] = \boxed{[0,0,0.5,0,0,0,0,0]}$

b) $s[n] = [0,1,1,1,1,1,1,1,1]$

$y_s[n]$

$h[n] = S[\delta[n]] = [0,0,0.5,0,0,0,0,0]$

$y_s[n] = [0,1,1,1,1,1,1,1] \cdot [0,0,0.5,0,0,0,0,0]$

$$\boxed{y_s[n] = [0,0,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0]}$$

Fourier Transforms & Basic Filters

1.

HW #3

Fourier Transforms & Basic Filters

1)

$$s(t) = a\,\delta(t - t_0)$$

a) $S(f) = a \cdot e^{-j2\pi f t_0}$

b) $H_{LPF}(f) = \frac{1}{1+j\omega\tau}$ where $\omega = 2\pi f$

$$Y_{LPF}(f) = S(f) \cdot H_{LPF}(f) = \frac{a \cdot e^{-j2\pi f t_0}}{1+j2\pi f\tau}$$

$$= |Y_{LPF}(f)| = \left| \frac{a \cdot e^{-j2\pi f t_0}}{1+j2\pi f\tau} \right|$$

$$e^{-j2\pi f T_0} = 1$$

$$|Y_{LPF}(f)| = \left| \frac{a}{1+j2\pi f\tau} \right|$$

Magnitude

$$1 + j2\pi f\tau \rightarrow \sqrt{1+(2\pi f\tau)^2}$$

$$\boxed{|Y_{LPF}(f)| = \frac{a}{\sqrt{1+(2\pi f\tau)^2}}}$$

c) $Y_{HPF}(f) = S(f) \cdot H_{HPF}(f) = \frac{a \cdot e^{j2\pi f t_0} \cdot j2\pi f\tau}{1+j2\pi f\tau}$

$$|Y_{HPF}(f)| = \left| \frac{a \cdot j2\pi f\tau \cdot e^{j2\pi f t_0}}{1+j2\pi f\tau} \right|$$

$$\boxed{|Y_{HPF}(f)| = \frac{a \cdot 2\pi f\tau}{\sqrt{1+(2\pi f\tau)^2}}}$$

2.

Fourier Transforms & Basic Filters

2) Low pass

$$H_{LPF} = \frac{1}{1+j\omega\tau}$$

$$\phi_{LPF}(f) = \arg\left(\frac{1}{1+j\omega\tau}\right) \Rightarrow \arg\left(\frac{1}{1+j2\pi f\tau}\right)$$

$$\phi_{LPF}(f) = -\tan^{-1}(2\pi f\tau)$$

$$\tau_{g,LPF}(f) = -\frac{d\left(-\tan^{-1}(2\pi f\tau)\right)}{d\omega}$$

$$\boxed{\tau_{g,LPF}(f) = \frac{\tau}{1+(2\pi f\tau)^2}}$$

High pass

$$H_{HPF}(f) = \frac{j\omega\tau}{1+j\omega\tau}$$

$$\phi_{HPF}(f) = \arg\frac{j\omega\tau}{1+j\omega\tau} \Rightarrow \arg\left(\frac{j2\pi f\tau}{1+j2\pi f\tau}\right)$$

$$\tau_{g,HPF}(f) = \frac{-d}{d\omega}\left(\pi/2 - \tan^{-1}(2\pi f\tau)\right)$$

$$\boxed{\tau_{g,LPF} = \frac{\tau}{1+(2\pi f\tau)^2}}$$

3.

Fourier Transforms & basic Filters

3) $S(f) = \frac{a}{2}(\delta(f-f_0) + \delta(f+f_0))$

a) $s(t) = \int_{-\infty}^{\infty} \frac{a}{2}(\delta(f-f_0)+\delta(f+f_0))e^{j2\pi f t}\,df$

$s(t) = \frac{a}{2}(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}) \rightarrow = 2\cos(2\pi f_0 t)$

$$\boxed{= s(t) = a\cos(2\pi f_0 t)}$$

b) $\frac{a}{2j}(\delta(f-f_0) - \delta(f+f_0))$

$s(t) = \int_{-\infty}^{\infty} S(f)e^{j2\pi f t}\,df$

$s(t) = \int_{-\infty}^{\infty} \frac{a}{2j}(\delta(f-f_0) - \delta(f+f_0))e^{j2\pi f t}\,df$

sift prop

$s(t) = \frac{a}{2j}(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}) \rightarrow = 2j\sin(2\pi f_0 t)$

$$\boxed{= s(t) = a\sin(2\pi f_0 t)}$$

Convolution & Octave Code

1.

4. Convolution & Octave Code

1) $y[n] = h[n] \cdot \delta[n]$

a)

$$y[n] = \sum_{k=-\infty}^{\infty} h[k] \cdot \delta[n-k]$$

$\delta[n-k]$ is 1 when $n=k$

$$\boxed{y[n] = h[n]}$$

b) $x[n] = \delta[n-n_0]$

$$y[n] = h[n] \cdot \delta[n-n_0]$$

$$y[n] = \sum_{k=\infty}^{\infty} h[k] \cdot \delta[n-n_0-k]$$

$\delta[n-n_0-k]$ is 1 when $k=n-n_0$

$$\boxed{y[n] = h[n-n_0]}$$