Quiz 1

1. $v(t) = 2.5\cos(2\pi ft - \frac{\pi}{4})$ $f = 1\,kHz$

   let $\phi = 2\pi ft - \frac{\pi}{4}$

a) $v(t) = \Re\{2.5e^{j\phi}\}$

$2.5 \cdot e^{j\theta} = \cos\phi + j\sin\phi \cdot 2.5$

$2.5e^{j\theta} = 2.5\cos\phi + 2.5\sin\phi$

$\Re\{2.5e^{j\phi}\} = 2.5\cos(2\pi ft - \frac{\pi}{4})$

$\boxed{\Re\{2.5e^{j\phi}\} = v(t)}$

b) $v(t) = \Im\{2.5e^{j(\phi - \frac{\pi}{2})}$

$e^{j(\phi - \frac{\pi}{2})} = e^{j\phi}e^{-j\frac{\pi}{2}}$

$\cdot 2.5 \quad = -je^{j\phi} \cdot 2.5$

$2.5e^{j(\phi - \frac{\pi}{2})} = -2.5je^{j\phi}$

$j^2 = -1 \quad = -2.5j\cos\phi - 2.5j\sin\phi$

$\Im\{2.5e^{j(\phi - \frac{\pi}{2})}\} = -2.5\cos\phi$

$= 2.5\cos(\phi + \pi)$

$= 2.5\cos\phi$

$\Im\{2.5e^{j(\phi - \frac{\pi}{2})} = 2.5\cos(2\pi ft - \frac{\pi}{4})$

$\boxed{\Im\{2.5e^{j(\phi - \frac{\pi}{2})} = v(t)}$

2. $T = \frac{1}{f}$

a) $1kHz^{-1} = \frac{1}{1kHz} \cdot \frac{1kHz}{1000Hz}$

$= \frac{1}{1000s} = 0.001s$

$\boxed{1kHz^{-1} = 1ms}$

b) $T = 5ns$

$\frac{f}{T} \cdot T = \frac{1}{f} \cdot \frac{f}{T} \rightarrow f = \frac{1}{T}$

$= \frac{1}{5 \times 10^{-9}s}$

$= 2 \times 10^{8}s$

$\boxed{f = 200MHz}$

c) $f = 5kHz$ $f_s = 50kHz$

$\frac{f_s}{f} = \frac{50kHz}{5kHz}$

$= \boxed{10 \text{ samples}}$

d) $\Delta t = \frac{1}{f_s} = 0.002\,ms$

$T = \frac{1}{5kHz} = \frac{1}{5000s} = 0.2ms$

$\frac{T}{\Delta t} = \frac{0.2}{0.002} = \boxed{100 \text{ samples}}$

3.

a) 256 steps 2.56V

$\Delta V = \frac{2.56V}{256} = 0.01V$

$\boxed{\Delta V = 10 mV}$

b) 11111111

$256 = 2^8$

$\boxed{8}$

c) $2^{16} = 65536$

$\Delta V = \frac{2.56}{65536} = 0.0000391V$

$\boxed{\Delta V = 39.1 \mu V}$

4.

a)

```
f = 10;
A = 2.5;
offset = 2.5;
dt = 1e-3;
t = 0:dt:1;
s = A * sin(2 * pi * f * t) + offset;
figure;
plot(t, s);
xlabel('Time (seconds)');
ylabel('Signal amplitude (V)');
title('Signal s(t) = 2.5 sin(2πft) + 2.5');
grid on;
```

b)

```
f = 10;
A = 2.5;
offset = 2.5;
dt = 1e-3;
t = 0:dt:1;
s = A * sin(2 * pi * f * t) + offset;
n = randn(size(t));
s_noisy = s + n;
figure;
plot(t, s_noisy);
xlabel('Time (seconds)');
ylabel('Signal amplitude (V)');
title('Noisy Signal: s(t) + Noise');
grid on;
```

c)

```
z = s + n;          figure(2);
plot(t, s, 'b-', t, z, 'r-');
```

```
xlabel('Time (s)');
ylabel('Amplitude (V)');
title('Signal and Signal plus Noise');
```

d)

```
signal_power = mean(s.^2);   % Power of the signal
noise_power = mean(n.^2);    % Power of the noise
SNR = 10 * log10(signal_power / noise_power);
```

e)

```
figure(2);
hist(z, 50);  xlabel('Amplitude (V)');
ylabel('Frequency');
title('Histogram ');
```

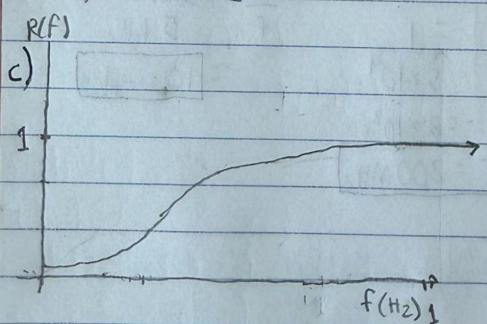
5. $R(f) = \frac{j\omega\tau}{(1 + j\omega\tau)}$ $\tau = RC$ $w = 2\pi f$

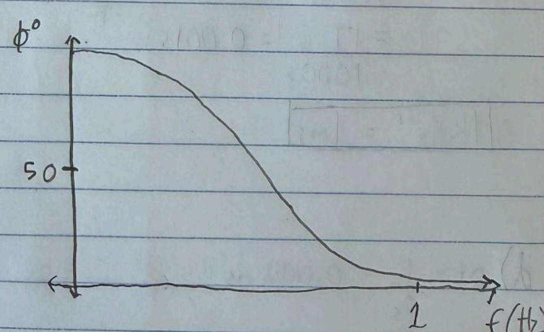
a) $\left| \frac{j\omega\tau}{1 + j\omega\tau} \right| = \frac{|\omega\tau|}{\sqrt{1 + \omega\tau^2}} = \boxed{\frac{\omega\tau}{\sqrt{(1 + (\omega\tau)^2)}}}$

b) $(1 + j\omega\tau) = \tan^{-1}(\omega\tau)$ $\boxed{\phi = \tan^{-1}(\omega\tau) - \frac{\pi}{2}}$

$(j\omega\tau) = \frac{\pi}{2}$ or $90°$

c)





d) f: A(f)   A = 1 at f = 0.5kHz   R = 1kΩ   C = 1μF

f = 0.5kHz

$w = 2\pi f = 2\pi(500)$

$= 1000\pi$

6.

a) $f_a = 2.5\text{ kHz}$ $s = 10\text{ kHz}$

$f_s = 2.5 - 0 \times 10 = \boxed{2.5\text{ kHz}}$

b) $f_a = 5\text{ kHz}$ $s = 10\text{ kHz}$

$f_s = 5 - 0 = 5 \times 10 = \boxed{5\text{ kHz}}$

c) $f_a = 15\text{ KHz}$ $s = 10\text{ kHz}$

$f_s = 15 - 10 = \boxed{5\text{ kHz}}$

d) $f_a = 20\text{ kHz}$ $s = 10\text{ kHz}$

$f_s = 20 - 2 \times 10 = 20 - 20 = \boxed{0\text{ kHz}}$

7. $S[s(t)] = s(t - \frac{T}{2})$

a) $s(t) = 2\sin(2\pi f t)$ $T = \frac{1}{f}$

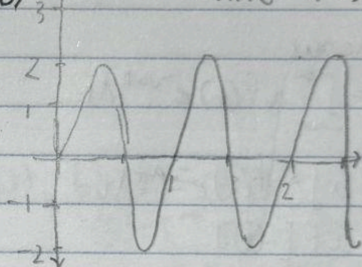
$S[s(t)] = s(t - \frac{T}{2}) = 2\sin(2\pi f(t - \frac{1}{2f}))$

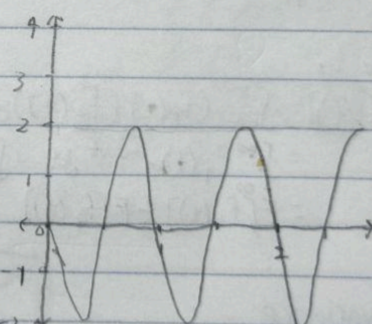
$= 2\sin(2\pi f t - \frac{2\pi f}{2f})$

$= 2\sin(2\pi f t - \pi)$

$\boxed{S[s(t)] = -2\sin(2\pi f t)}$

b) $s(t) = 2\sin(2\pi f t)$





c) $s(t) + S[s(t)]$

$= 2\sin(2\pi f t) + (-2\sin(2\pi f t))$

$= \boxed{0}$

8. $x[n] = [000200\ldots]$ samples = 100

a) $y[n] = S(x[n]) = -x[n-1]$

$n = k \rightarrow n = k+1$

$\boxed{y[n] = [0000\ -200\ldots]}$

b) $y[n] = S(x[n]) = (x[n])^2$

$n = k$ $y[k] = 2^2 = 4$

$\boxed{y[n] = [000400\ldots]}$

c) <u>System a</u>: $S(x_1[n] + x_2[n]) = -(x_1[n-1] + x_2[n-1])$ ✓

$\boxed{\text{linear}}$

$= -x_1[n-1] - x_2[n-1]$

$S(ax[n]) = -(ax[n-1]) = -ax[n]$ ✓

system b: $S(x_1[n] + x_2[n]) = (x_1[n])^2 + (x_2[n])^2$ ✓

$\boxed{\text{non-linear}}$ $S(ax[n]) = (ax[n])^2 = a^2(x[n])^2$ ×

9. $f(-t)=f(t) \rightarrow$ even $\quad f(-t)=-f(t) \rightarrow$ odd

- $\cos(2\pi ft) \rightarrow \cos(2\pi f(-t)) = \cos(-2\pi ft) = \cos(2\pi ft)$ $\boxed{\text{even}}$

- $\exp(-(\frac{t}{\sigma})^2) \rightarrow \exp(-(\frac{(-t)}{\sigma})^2) = \exp(-(\frac{t}{\sigma})^2)$ $\boxed{\text{even}}$

- $\exp(-\alpha t) \rightarrow \exp(-\alpha(-t)) = \exp(\alpha t) \neq \exp(-\alpha t)$ $\boxed{\text{not even or odd}}$

- $at^2+bt+c \rightarrow a(-t)^2+b(-t)+c = at^2-bt+c$ $\boxed{\text{not even or odd}}$

10.

a) homogeneous

$$F[af(t)] = \int_{-\infty}^{\infty} af(t)e^{-j\omega t}dt$$
$$= a\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$
$$= aF[f(t)]$$

$\boxed{\text{True}}$

b) additivity

$$F[f_1(t)+f_2(t)] = \int_{-\infty}^{\infty}(f_1(t)+f_2(t))e^{-j\omega t}dt$$
$$= \int_{-\infty}^{\infty} f_1(t)e^{-j\omega t}dt + \int_{-\infty}^{\infty} f_2(t)e^{-j\omega t}dt$$
$$= F[f_1(t)] + F[f_2(t)]$$

$\boxed{\text{True}}$

c) shift invariance

$$F[f(t-t_0)] = \int_{-\infty}^{\infty} f(t-t_0)e^{-j\omega t}dt$$

$u = t-t_0$ $\quad = \int_{-\infty}^{\infty} f(u) = e^{-j(u+t_0)}du$

$du = dt$ $\quad = e^{-j\omega t_0}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}du$

$$= e^{-j\omega t_0}F[f(t)]$$

$\boxed{\text{True. Complex constant: } e^{-j\omega t_0}}$

11. $f(t_0) = \int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt$

a) $\mathcal{F}[\delta(t-t_0)] = \int_{-\infty}^{\infty} \delta(t-t_0) e^{-j\omega t} dt$

$\mathcal{F}[a\delta(t-t_0)] = e^{-j\omega t_0} \cdot a$

$= \boxed{ae^{-j\omega t_0}}$

b) $|ae^{-j\omega t_0}|$

$= |a| \cdot |e^{-j\omega t_0}|$

$= |a| \cdot 1$

$= \boxed{|a|}$

c) $ae^{-j\omega t_0} = a + e^{-j\omega t_0}$

$\phi(\omega) = \boxed{-\omega t_0}$

12.

a) $F(f) = \left(\frac{a}{2}\right)\left(\delta(f-f_0) + a\delta(f+f_0)\right)$

$f(t) = \int_{-\infty}^{\infty} F(f) e^{j2\pi ft} df$

$= \frac{a}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)$

$= \frac{a}{2}\left(2\cos(2\pi f_0 t)\right)$

$\boxed{f(t) = a\cos(2\pi f_0 t)}$

b) $F(f) = \left(\frac{a}{2j}\right)\left(\delta(f-f_0) - a\delta(f+f_0)\right)$

$f(t) = \int_{-\infty}^{\infty} F(f) e^{j2\pi ft} df$

$= \frac{a}{2j}\left(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right)$

$= \frac{a}{2j}\left(2\sin(2\pi f_0 t)\right)$

$\boxed{f(t) = a\sin(2\pi f_0 t)}$

13.

a) $\boxed{\begin{aligned} A\cos(2\pi f_{LO} t) &= \frac{A}{2}\left(e^{j2\pi f_{LO} t} + e^{-j2\pi f_{LO} t}\right) \\ \frac{m}{A}\cos(2\pi f_A t) &= \frac{m}{2A}\left(e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right) \end{aligned}}$

b) $\left(\frac{A}{2}\left(e^{j2\pi f_{LO} t} + e^{-j2\pi f_{LO} t}\right)\right) \cdot \left(\frac{m}{2A}\left(e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right)\right)$

$= \frac{m}{4}\left(e^{j2\pi(f_{LO}+f_A)t} + e^{j2\pi(f_{LO}-f_A)t} + e^{-j2\pi(f_{LO}-f_A)t} + e^{-j2\pi(f_{LO}+f_A)t}\right)$

$= \frac{m}{2}\cos(2\pi(f_{LO}+f_A)t + \frac{m}{2}\cos(2\pi(f_{LO}-f_A)t)$

$\boxed{\begin{aligned} &f_{LO} + f_A \\ &f_{LO} - f_A \end{aligned}}$

Code Problem Audio Echo

```
Fs = 20000;
duration = 2;
```

Part a

```
N = Fs * duration;
delta = zeros(1, N);
delta(1) = 1;
fprintf('The Delta function is created with %d samples (Fs = %d Hz, duration = %d seconds)\n',
N, Fs, duration);
```

Part b

```
echo_interval = 0.25;
echo_samples = round(echo_interval * Fs);
num_echoes = floor(duration / echo_interval);
echo_response = zeros(1, N);
echo_response(1) = 1;
for i = 1:num_echoes
    echo_idx = i * echo_samples + 1;
    if echo_idx <= N
        echo_response(echo_idx) = 1;
    end
end
non_zero_indices = find(echo_response);
fprintf('\n(b) Non-zero sample locations:\n');
disp(non_zero_indices);
```

Part c

```
decaying_echo = zeros(1, N);
decaying_echo(1) = 1;

for i = 1:num_echoes
    echo_idx = i * echo_samples + 1;
    if echo_idx <= N
        decaying_echo(echo_idx) = (0.5)^i;
    end
end
```

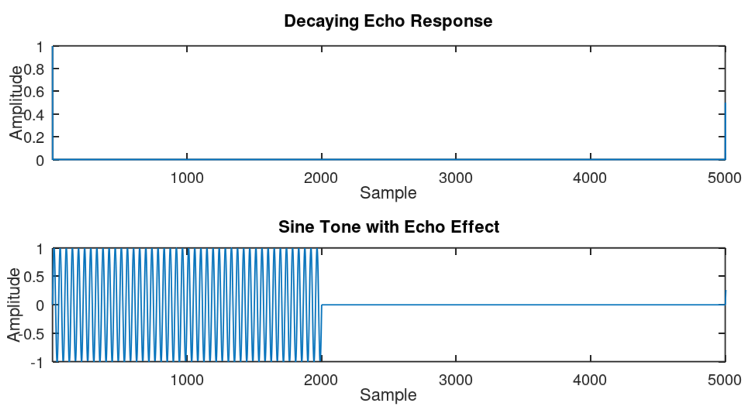
Part d

```
tone_duration = 0.1;
tone_freq = 440;
t = (0:1/Fs:tone_duration-1/Fs);
sine_tone = sin(2 * pi * tone_freq * t);
sine_padded = [sine_tone, zeros(1, N - length(sine_tone))];
```

Part e

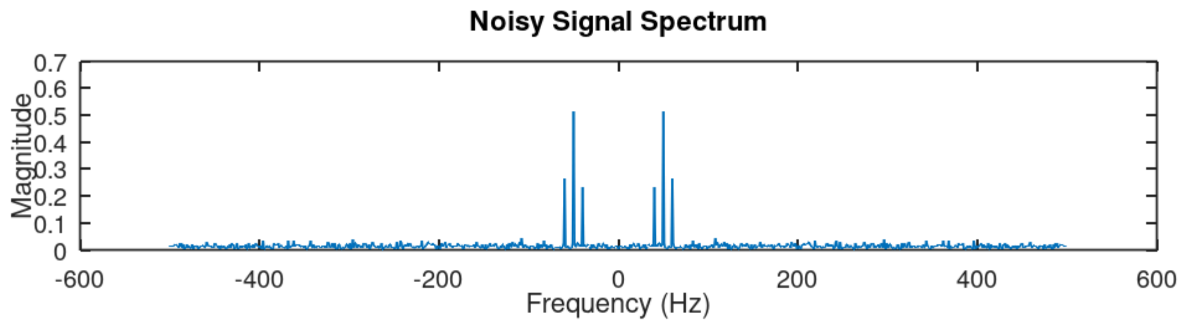
```
echo_sine = conv(sine_padded, decaying_echo);
echo_sine = echo_sine(1:N);  % Truncate to original length

echo_sine = echo_sine / max(abs(echo_sine));
soundsc(echo_sine, Fs);

figure;
subplot(3,1,1);
plot(decaying_echo);
title('Decaying Echo Response');
xlabel('Sample');
ylabel('Amplitude');
xlim([1, 5000]);

subplot(3,1,2);
plot(echo_sine);
title('Sine Tone with Echo Effect');
xlabel('Sample');
ylabel('Amplitude');
xlim([1, 5000]);
```

```
Non-zero sample locations:
   1     5001    10001    15001    20001    25001    30001    35001
```



Bonus:

```
fs = 1000
t = 0:1/fs:1-1/fs;
fc = 50;
fm = 10;
Amod = 1;
Acar = 1;

m = Amod * cos(2*pi*fm*t);
s = (1 + m) .* (Acar .* cos(2*pi*fc*t));
noise = 0.5 * randn(size(t));
noisy_signal = s + noise;
N = length(t);
frequencies = (-N/2:N/2-1) * (fs/N);
S_fft = abs(fftshift(fft(noisy_signal)))/N;

% low-pass filter
low_cutoff = 60;
[b, a] = butter(6, low_cutoff/(fs/2), 'low');
low_passed = filter(b, a, noisy_signal);

% high-pass filter
high_cutoff = 5;
[b, a] = butter(6, high_cutoff/(fs/2), 'high');
high_passed = filter(b, a, noisy_signal);

S_lp_fft = abs(fftshift(fft(low_passed)))/N;
S_hp_fft = abs(fftshift(fft(high_passed)))/N;

figure;
subplot(3,1,1);
plot(frequencies, S_fft);
title('Noisy Signal Spectrum'); xlabel('Frequency (Hz)'); ylabel('Magnitude');
subplot(3,1,2);
plot(frequencies, S_lp_fft);
title('Low-Pass Filtered Spectrum'); xlabel('Frequency (Hz)'); ylabel('Magnitude');
subplot(3,1,3);
plot(frequencies, S_hp_fft);
title('High-Pass Filtered Spectrum'); xlabel('Frequency (Hz)'); ylabel('Magnitude');
```


Noisy Signal Spectrum
Magnitude
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-600
-400
-200
0
200
400
600
Frequency (Hz)