2/24/25
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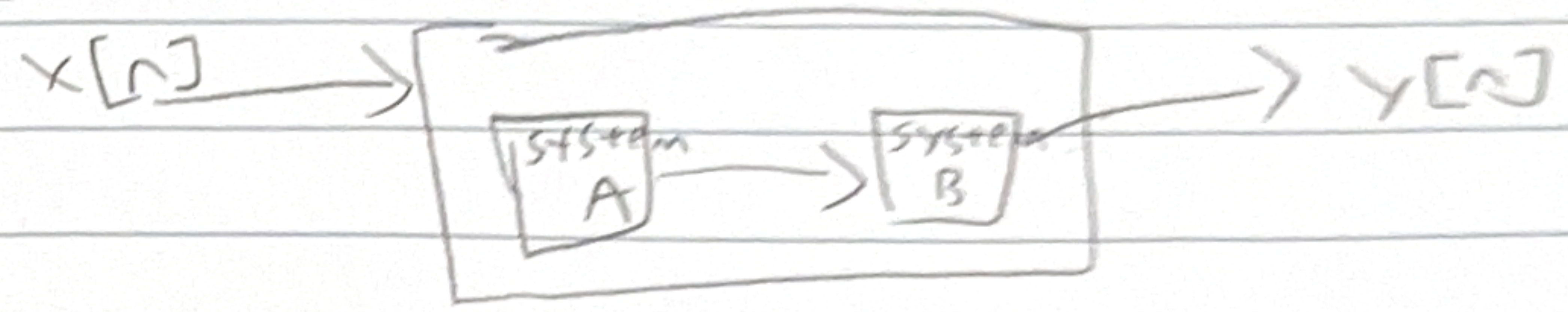
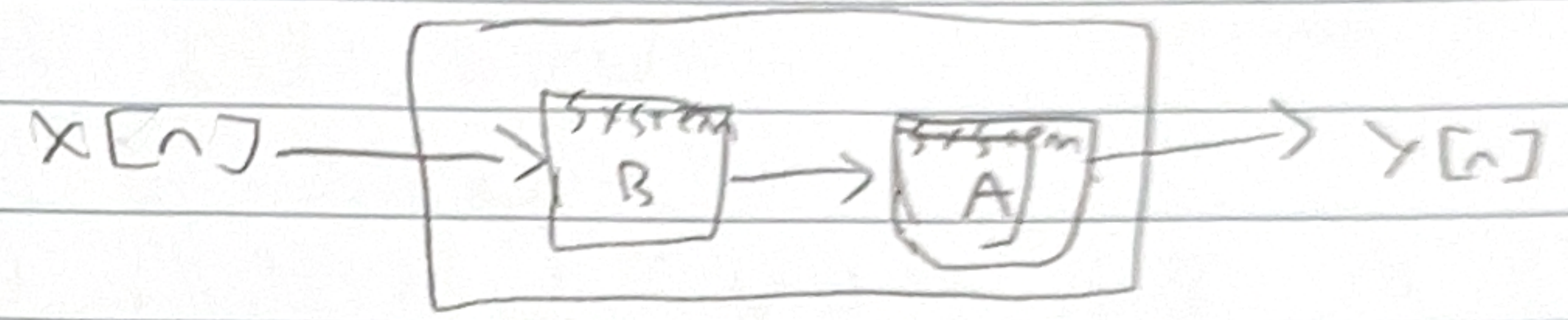
ECSC 360

homework #3

① If



then



linear systems commute.

a) let $A\{x[n]\} = 2x[n]-1$
$B\{x[n]\} = 0.5\,x[n]$

which system is linear?

are they homogeneous?

$A(k[x[n]]) \stackrel{?}{=} k(2x[n]-1)$

$2kx[n]-1 \neq 2kx[n]-k$. so no.

$B(k[x[n]]) \stackrel{?}{=} k(0.5\,x[n])$
$0.5k[x[n]] = 0.5kx[n]$ so yes

so B is the linear system because A is not additive.

b) $[2x[n]-1]+1$ add 1 to the system
$= 2x[n]$

now

$$A\{B\{x[n]\}\} = B\{A\{x[n]\}\}$$

$$2(0.5(x[n])) = 0.5(2(x[n]))$$

$$2\left(\frac{x[n]}{2}\right) = 0.\frac{5}{2}(2\,x[n])$$

$$x[n] = x[n] \checkmark$$

(2) eq (8): $f(t_0) = \int_{-\infty}^{\infty} f(t)\,\delta(t - t_0)\,dt$

$$f(t) = a_1\cos(2\pi f_1 t) + a_2\cos(2\pi f_2 t)$$

$$T_1 = \frac{1}{f_1} \qquad T_2 = \frac{1}{f_2} \qquad f_2 = 2f_1$$

a) find $\int_{-\infty}^{\infty} f(t)\,\delta(t - T_1)\,dt$

$$\int_{-\infty}^{\infty} a_1\cos(2\pi f_1 t) + a_2\cos(2\pi f_2 t)\,\delta\left(t - \frac{1}{f_1}\right)dx$$

$$= a_1\cos\left(2\pi \cancel{f_1}\frac{1}{\cancel{f_1}}\right) + a_2\cos\left(2\pi f_2\frac{1}{f_1}\right)$$

$$= a_1\cos(2\pi) + a_2\cos\left(2\pi f_2\frac{1}{f_1}\right)$$

$$= a_1 + a_2\cos\left(\frac{2\pi 2\cancel{f_1}}{\cancel{f_1}}\right)$$

$$= a_1 + a_2\cos(4\pi) = \boxed{a_1 + a_2}$$

b) find $\int_{-\infty}^{\infty} f(t)\,\delta(t - T_2)\,dt$

$$\int_{-\infty}^{\infty} a_1\cos(2\pi f_1 t) + a_2\cos(2\pi f_2 t)\,\delta\left(t - \frac{1}{f_2}\right)dx$$

$= a_1 \cos(2\pi f_1 \frac{1}{f_2}) + a_2 \cos(2\pi f_2 \frac{1}{f_2})$

$= a_1 \cos(2\pi f_1 \frac{1}{2f_1}) + a_2 \cos(2\pi)$

$= a_1 \cos(\pi) + a_2$

$= \boxed{a_2 - a_1}$

③ $f(t) = a\delta(t - t_0)$

a) Show that the magnitude of the Fourier transform of this impulse is a

the F.T of $f(t) \frac{1}{T}$

$$F(f) = \int_{-\infty}^{\infty} f(t) e^{-2\pi j f t} dt$$

$$F(f) = \int_{-\infty}^{\infty} a\delta(t - t_0) e^{-2\pi j f t} dt$$

$$= a e^{-2\pi j f t_0}$$

magnitude: $\sqrt{a^2(e^{2\pi j f t} + e^{-2\pi j f t})} = \sqrt{a^2} = a$

$\sqrt{F^* F}$

b) Show that the phase angle

$\phi = -2\pi f t_0$

well, the F.T is $a e^{-2\pi j f t}$

so $\phi = -2\pi f t_0$

and this is the phase angle

c) Show that the group delay $\tau_g = -\frac{d\phi}{d\omega}$ is t_0

$w = 2\pi f$

$$-\frac{d\phi}{d\omega} = -\frac{d}{d\omega}(-2\pi f t_0) = +\frac{d}{d\omega}(+2\pi t_0)$$

$$= t_0$$

④ $\delta[n] = [1000\ 0000]^T$

a) if $y[n] = S[x[n]] = 0.5\,x[n-2]$, what is $S[\delta[n]]$?

$$S[1000\ 0000] \xrightarrow{n-2} [0010\ 0000]$$

$$\xrightarrow{0.5n} \boxed{[00\tfrac{1}{2}0\ 0000]}$$

b) Step input $= s[n] = [0\,1111111]$

find y[the step response

$$[0111\ 1111] \xrightarrow{n-2} [0001111] \xrightarrow{\frac{1}{2}n} \boxed{[000\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}]}$$

Part 3

① We pass a signal $s(t)$ into a low-pass filter.

The signal as a function of frequency is $S(f)$, the Fourier transform of $s(t)$.

low pass output: $S(f)\left(\frac{1}{1+j\omega T}\right)$ $\quad \omega = 2\pi f$

$T = RC$

a) calculate $S(f)$. $\quad s(t) = a\,\delta(t-t_0)$

$$S(f) = \int_{-\infty}^{\infty} a\,\delta(t-t_0)\,e^{-2\pi j f t}\,dt$$

$$= \boxed{a e^{-2\pi j f t_0}}$$

b) suppose we pass $s(t)$ into a low pass filter. Find magnitude of output as a function of f.

$$a e^{-2\pi j f t_0} \left(\frac{1}{1 + j(2\pi f)(RC)} \right)$$

$$= \frac{a e^{-2\pi j f t_0}}{1 + j(2\pi f)(RC)}$$

magnitude: $|L| = \sqrt{\left(\frac{a e^{-2\pi j f t_0}}{1 - j(2\pi f)(RC)} \cdot \frac{a e^{2\pi j f t_0}}{1 + j(2\pi f)(RC)} \right)}$

$$= \sqrt{\frac{a^2 \left(e^{-2\pi j f t_0} + e^{2\pi j f t_0}\right)}{1 + (2\pi f RC)^2}} = \boxed{\frac{a}{\sqrt{1 + (\omega \tau)^2}}} = \boxed{\frac{a}{\sqrt{1 + (2\pi f \tau)^2}}}$$

c) repeat, but for a high-pass filter. $\frac{j\omega\tau}{1 + j\omega\tau}$

$$\frac{a e^{-2\pi j f t_0} \, j\omega\tau}{1 + j\omega\tau}$$

$$|z| = \sqrt{\left(\frac{-a e^{-2\pi j f t_0} \, j\omega\tau}{1 - j\omega\tau} \right) \left(\frac{a e^{2\pi j f t_0} \, j\omega\tau}{1 - j\omega\tau} \right)}$$

$$= \sqrt{\frac{a^2 (\omega\tau)^2}{1 + (\omega\tau)^2}} = \frac{a\omega\tau}{\sqrt{1 + (\omega\tau)^2}}$$

② Calculate group delays for previous exercise

$$\frac{a e^{-j\omega t_0}}{1+j\omega\tau} \Rightarrow \frac{a}{\sqrt{1+(\omega\tau)^2}} e^{\text{Phase}}$$

$$\text{Phase} = \tan^{-1}\Big($$

$$U + jV \quad \frac{a e^{-j\omega t_0}}{1+j\omega\tau} = \frac{a(\cos(\omega t_0) - j\sin(\omega t_0))}{1+j\omega\tau}$$

$$\frac{a(\cos(\omega t_0) - j\sin(\omega t_0))}{1 + j\omega\tau} = \frac{a\cos(\omega T_0) - ja\sin(\omega T_0)}{1 + j\omega\tau}$$

$$(a\cos(\omega t_0) - ja\sin(\omega t_0))(1 - j\omega\tau)$$

$$= a\cos(\omega t_0) - ja\sin(\omega t_0) - ja\omega\tau\cos(\omega t_0) + a\omega\tau\sin(\omega t_0)$$

$$\Rightarrow \frac{a\cos(\omega t_0) + a\omega\tau\sin(\omega t_0)}{1 + (\omega\tau)^2} - j\,\frac{a\sin(\omega t_0) + a\omega\tau\cos(\omega t_0)}{1 + (\omega\tau)^2}$$

$$\frac{a\sin(\omega t_0) + a\omega\tau\cos(\omega t_0)}{\cancel{1 + (\omega\tau)^2}} \cdot \frac{\cancel{1 + (\omega\tau)^2}}{a\cos(\omega t_0) + a\omega\tau\sin(\omega t_0)}$$

$$= \frac{a\sin(\omega t_0) + a\omega\tau\cos(\omega T_0) \div \cos(\omega t_0)}{a\cos(\omega t_0) + a\omega\tau\sin(\omega t_0) \div \cos(\omega t_0)}$$

$$\Rightarrow \frac{a\tan(\omega t_0) + a\omega\tau}{a + a\omega\tau\tan(\omega t_0)} = \tan\phi$$

$$\tan^{-1}\left(\frac{a\tan(\omega t_0) + a\omega\tau}{a + a\omega\tau\tan(\omega t_0)}\right) = \phi$$

$$= \tan^{-1}\left(\frac{\cancel{a}(\tan(\omega t_0) + \omega\tau)}{\cancel{a}(1 + \omega\tau\tan(\omega t_0))}\right)$$

$=$

(3)) $$S(f) = \frac{a}{2}(\delta(f - f_c) + \delta(f + f_c))$$

$$\int_{-\infty}^{\infty} \frac{a}{2}(\delta(f - f_c) + \delta(f + f_c))e^{2\pi j f t}\,df$$

$$= \int_{-\infty}^{\infty} \frac{a}{2}(\delta(f - f_c)e^{2\pi j f t}\,df + \int_{-\infty}^{\infty} \frac{a}{2}(\delta(f + f_c)e^{2\pi j f t}\,df$$

$$= \frac{a}{2}e^{2\pi j f_c t} + \frac{a}{2}e^{2\pi j(-f_c)t}$$

$$= \frac{a}{2}\left(e^{2\pi j f_0 t} + e^{-2\pi j f_0 t}\right) = \frac{a\left(e^{2\pi j f_0 t} + e^{-2\pi j f_0 t}\right)}{2}$$

$$= \boxed{a\cos(2\pi f_0 t)} \checkmark$$

b) $S(f) = \left(\frac{a}{2j}\right)\left(\delta(f-f_0) - \delta(f+f_0)\right)$

$$\int_{-\infty}^{\infty} \left[\frac{a}{2j}\left(\delta(f-f_0) - \delta(f+f_0)\right) e^{2\pi j f t} \, df\right.$$

$$= \int_{-\infty}^{\infty} \frac{a}{2j}\left(\delta(f-f_0)\right) e^{2\pi j f t} \, df - \int_{-\infty}^{\infty} \frac{a}{2j}\left(\delta(f+f_0)\right) e^{2\pi j f t} \, df$$

$$= \frac{a}{2j} e^{2\pi j f_0 t} - \frac{a}{2j} e^{-2\pi j f_0 t}$$

$$= \frac{a\left(e^{2\pi j f_0 t} - e^{-2\pi j f_0 t}\right)}{2j} = \boxed{a \sin(2\pi f t)} \checkmark$$

Part 4.

4 digital impulse

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

for example,

$$\delta[n-5] = 1, \quad n = 5$$

$$\delta[n] = x[n]$$

$$h[n] * x[n] = \sum_{j=0}^{m-1} h[j] x[i-j]$$

$$= \sum_{j=0}^{m-1} h[j] \delta[i-j]$$

now, the only signal that doesn't die (become 0) is the one where $j = i$.

so, $\sum_{j=0}^{m-1} h[j] \delta[i-j] = h[i] = h[n]$.

b) The same must be true even if

$x[n] = \delta[n-n_0] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$

again, when convolved:

$\sum_{j=0}^{M-1} h[j]\delta[i-j]$ the only signal that doesn't become ~~0~~ is when $j = i$.