James Saw

1.

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Quiz 1: Digital Signal Processing

1) $v(t) = 2.5\cos(2\pi f t - \pi/4)$ $\quad V = 2.5$ volts $\quad$ phase shift $= \pi/4$

$f = 1$ kHz $\quad \phi = 2\pi f t - \pi/4$

a) $\underline{v(t) = \mathbb{R}\{2.5e^{j\phi}\}}$

$\cos\phi = \mathbb{R}\{e^{j\phi}\}$

so $2.5\cos(2\pi f t - \pi/4) \Rightarrow 2.5\,\mathbb{R}\{e^{j(2\pi f t - \pi/4)}\}$

since ϕ $2\pi f t - \pi/4$

$$\boxed{\mathbb{R}\{2.5e^{j\phi}\}}$$

b) $v(t) = \mathbb{I}\{2.5e^{j(\phi - \pi/2)}\}$

$\sin(\phi) = \mathbb{I}\{e^{j\phi}\}$ imaginary part of eulers

$\cos(\phi) = \sin(\phi + \pi/2)$ since sin is a $\pi/2$ phase shift of $\cos\phi$

$v(t) = 2.5\cos(2\pi f t - \pi/4) \Rightarrow 2.5\sin(2\pi f t - \pi/4 + \pi/2)$

$= 2.5\sin(2\pi f t + \pi/4)$

$2.5\ \mathbb{I}\{e^{j(2\pi f t - \pi/4 + 2\pi)}\}$

$$\boxed{= \mathbb{I}\{2.5e^{j(\phi + \pi/2)}\}}$$

2.

Quiz #1

2) $T = 1/f$

a) $f = 1\ kHz$

$$T = \frac{1}{f} \Rightarrow \frac{1}{1kHz} = \frac{1\ Hz}{1000\ Hz} \boxed{= 0.001 \text{ second}}$$

b) $T = \frac{1}{f} \Rightarrow f = \frac{1}{T}$

$$\frac{1}{0.5ns} \rightarrow \frac{1}{5\cdot10^{-9}} \boxed{= 2.0\cdot10^{8}\ Hz}$$

c) $$\frac{f_s}{f} = \frac{50kHz}{5kHz} \boxed{= 10 \text{ samples per period}}$$

d) $\Delta t = 1/f_s = 0.002\ ms$

$$f_s = \frac{1}{\Delta t} \Rightarrow \frac{1}{0.002ms} = \frac{1}{2\cdot10^{-6}s} = 500{,}000\ Hz \Rightarrow 500k\ H$$

$$\frac{500\ kHz}{5\ kHz} \boxed{= 100 \text{ samples per period}}$$

3.

Quiz #1

3) 0 - 2,56 volts   digitize to 256 steps

a) $\frac{2,56}{256} = \boxed{0.01\,V = 10\,mV}$

b) $\log_2(256) = \boxed{8}$

c) $2^{16} = 65536$

$\frac{2,56}{65536} = \boxed{3,906 \cdot 10^{-5}\,V \text{ or } 3,906 \cdot 10^{-2}\,mV}$

4a)

Command Window

```
>> f = 10

f = 10
>> A = 2.5

A = 2.5000
>> DC = 2.5

DC = 2.5000
>> dt = 0.001

dt = 1.0000e-03
>> t = 0:dt:2;

>> s = A * sin(2*pi*f*t) + DC;

>> figure;

>> plot(t, s);

>> title('Signal s(t) = 2.5 sin(2\pi f t) + 2.5');

>> xlabel('Time (s)');

>> ylabel('Amplitude (V)');

>> grid on;
```

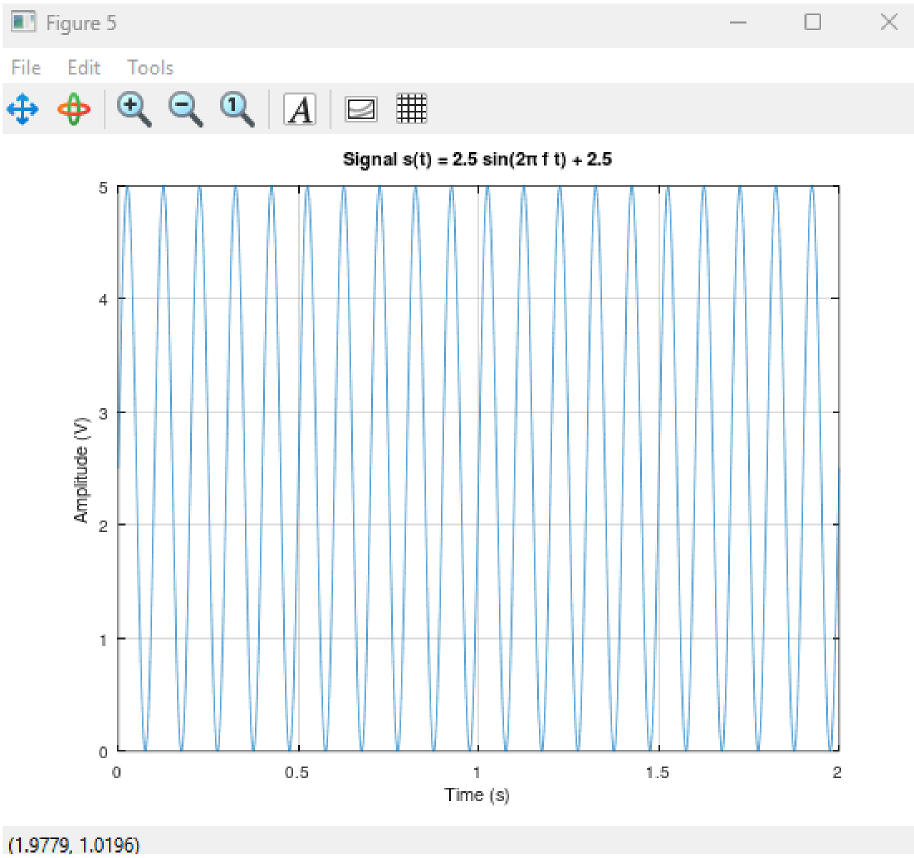
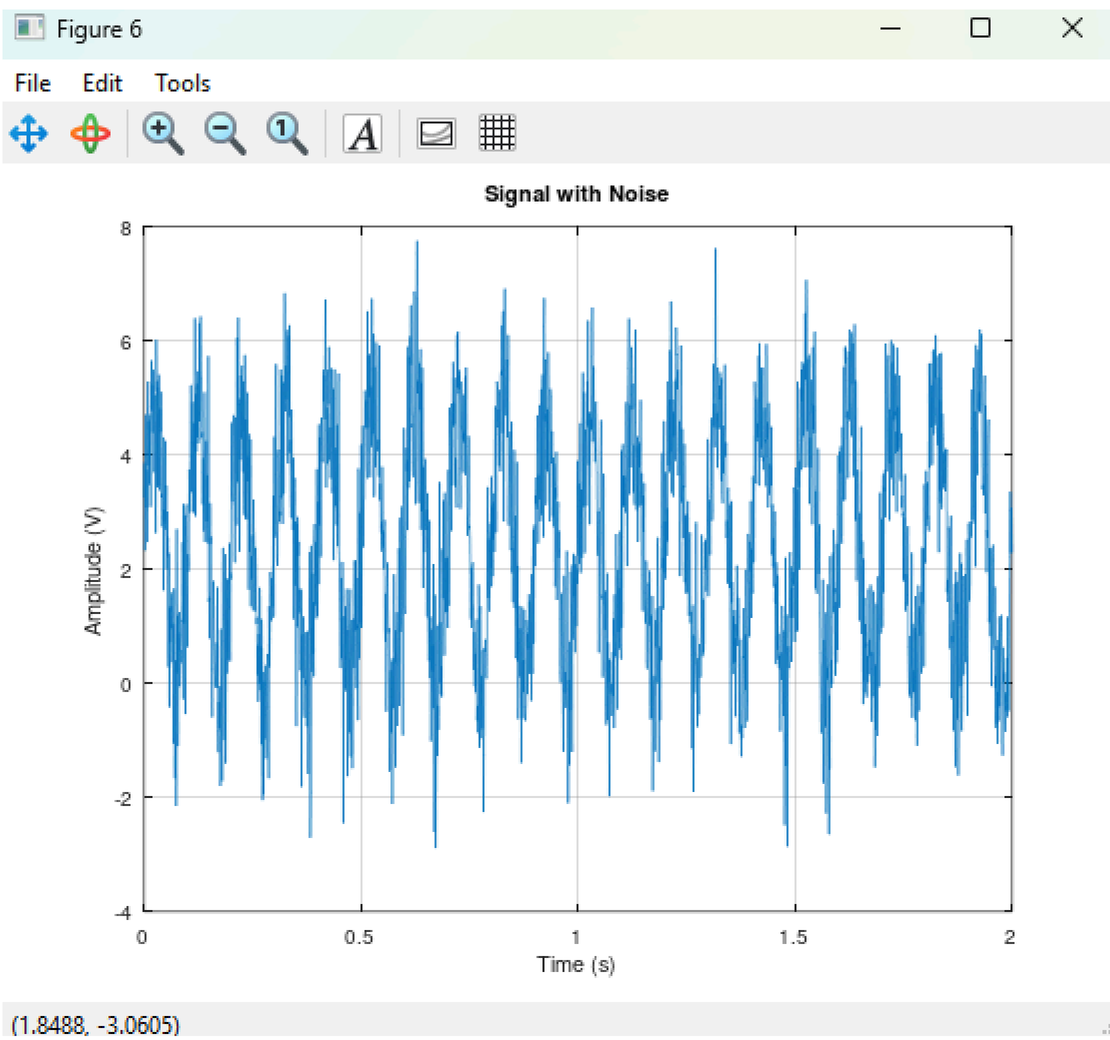


Figure 5
File
Edit
Tools
Signal s(t) = 2.5 sin(2π f t) + 2.5
Amplitude (V)
Time (s)
0
0.5
1
1.5
2
0
1
2
3
4
5
(1.9779, 1.0196)

4 b & c)

```
>> n = randn(size(t));

>> z = s + n;

>> figure;

>> plot(t, z);

>> title('Signal with Noise');

>> xlabel('Time (s)');

>> ylabel('Amplitude (V)');

>> grid on;
```



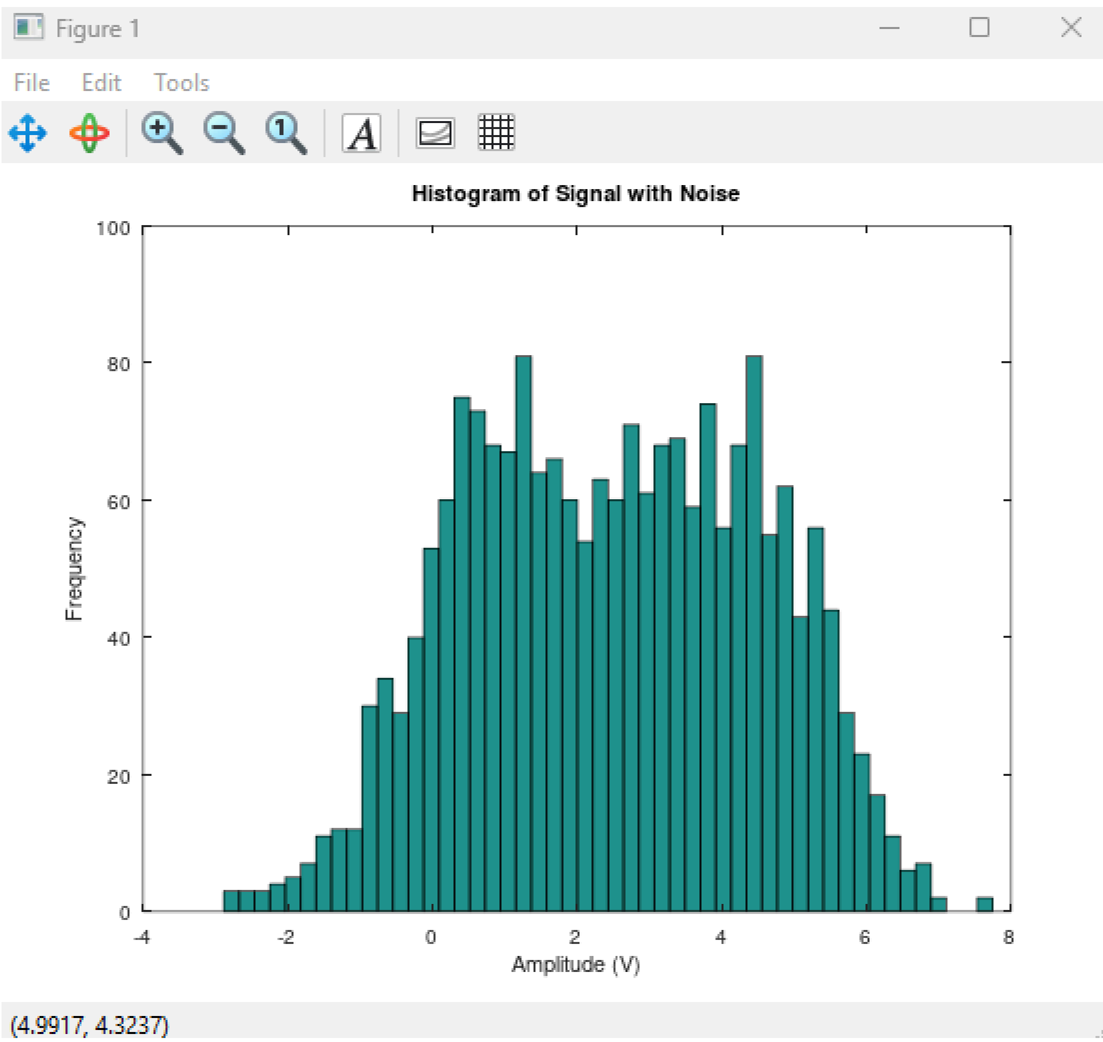
4d)

Command Window

```
>> signal_power = mean(s.^2);  % Power of the signal

>> noise_power = mean(n.^2);   % Power of the noise

>> SNR = signal_power / noise_power;

>> disp(['Signal-to-Noise Ratio (SNR): ', num2str(SN
R)]);
Signal-to-Noise Ratio (SNR): 9.674
>>
```

4e)

```
>> figure;

>> hist(z, 50);   % 50 bins for the histogram

>> title('Histogram of Signal with Noise');

>> xlabel('Amplitude (V)');

>> ylabel('Frequency');

>> |
```



5)

5) $R(f) = j\omega\tau/(1+j\omega\tau)$

a) $|R(f)| = RR^*$

$$= \sqrt{\left(\frac{-j\omega\tau}{1-j\omega\tau}\right)\left(\frac{j\omega\tau}{1+j\omega\tau}\right)} = \sqrt{\frac{(\omega\tau)^2}{1+(\omega\tau)^2}}$$

$$\boxed{= \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}}$$

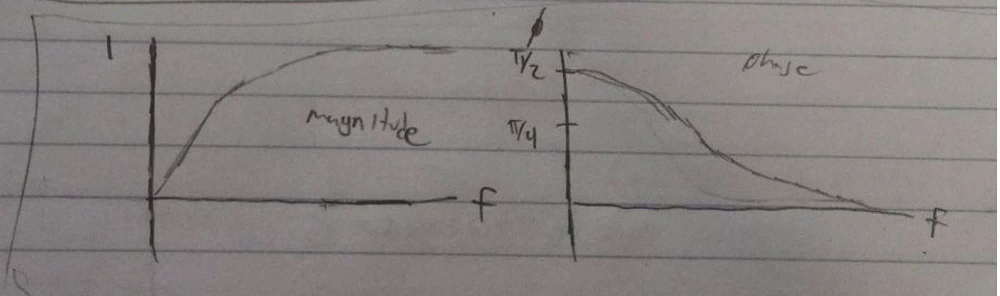
b) $\frac{\text{angle top}}{\text{angle bottom}}$ $\quad \angle H = \angle_{top} - \angle_{bottom}$

$\angle_{top} = 90° = \pi/2$

$\angle_{bottom} = \tan^{-1}(\omega\tau)$

$$\boxed{\phi = \frac{\pi}{2} - \tan^{-1}(\omega\tau)}$$

c)



d) $A(0.5\,kHz) = 1$ $\quad R = 1\,k\Omega = 1000\,\Omega$ $\quad C = 1\,\mu F$ $\quad \tau = R($

$f = 0.5\,kHz = 500\,Hz$ $\quad w = 2\pi$

$\omega\tau = 2\pi(500)(1000)(1\cdot10^{-6}) = 3.14$

$$|R(0.5)| = \frac{3.14159}{\sqrt{1+(3.14159)^2}} = 0.95$$

$$A(f)R(f) = 1 \cdot 0.9529 \boxed{= 0.9529}$$

6)

Quiz #1

6)

a) $\frac{2{,}5\,kHz}{10\,kHz} = 0{,}25 \boxed{= 2{,}5k\,Hz}$

b) $\frac{5\,kHz}{10\,kHz} = 0{,}5 \boxed{= 5k\,Hz}$

c) $\frac{15\,kHz}{10\,kHz} = 1.5 = \boxed{5k\,Hz}$

d) $\frac{20\,kHz}{10\,kHz} = 2 \boxed{= 0\,kHz}$

7)

7) $S[s(t)] = s(t - T/2)$

$s(t) = 2\sin(2\pi f t$ $\quad T = 1/f$

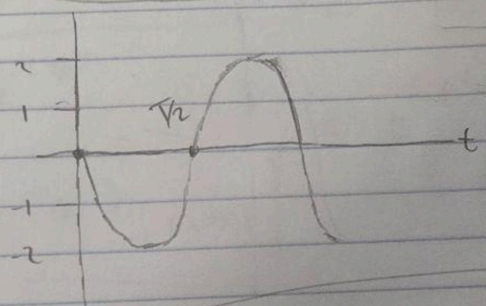
$S = (t - T/2) = s(t - \frac{1}{2f})$

$2\sin(2\pi f t - 2\pi f \frac{1}{2f}) = 2\sin(2\pi f t - \pi)$

since $\sin(x - \pi) = -\sin(x)$

$$\boxed{S[s(t)] = -2\sin(2\pi f t)}$$

b)



$S[s(t)]$

c) $s(t) + S[s(t)]$

$2\sin(2\pi f t) - 2\sin(2\pi f t)$

$$\boxed{s(t) + S[s(t)] = 0}$$

8)

8) $x[n] = [0\,0\,0\,2\,0\,0, \ldots, 0]$

a) $y[n] = S(x[n]) = -x[n-1]$

$y[n] = -x[n-1]$

since $x[3] = 2$

$x[n-1] = 2$ when $n = 4$

$y[4] = -2$

$\boxed{y[n] = [0\,0\,0\,-2, \ldots\, 0]}$

b) $y[n] = S(x[n]) = (x[n])^2$

$x[3] = 2$

$y[3] = (x[3])^2 = 4$

$\boxed{y[n] = [0\,0\,0\,4, \ldots, 0]}$

c) The system in a) is linear

The system in b) is non-linear

9)

9) even if $f(t) = f(-t)$ all t

odd if $f(t) = -f(-t)$ all t

- $\cos(2\pi f t)$
- $e^{-(t/\phi)^2}$
- $e^{-\alpha t}$
- $at^2 + bt + c$

- $\cos(2\pi f t) = \cos(-2\pi f t)$

$\hookrightarrow = \cos(2\pi f t)$

$\boxed{\cos 2\pi f t : \text{even}}$

- $e^{-(t/\phi)^2}$

$e^{(-(-t/\phi)^2)} \Rightarrow e^{(-(t^2/\phi^2))} \Rightarrow e^{(-(t/\phi)^2)}$

$\boxed{e^{(-(t/\phi)^2)} ; \text{even}}$

- $e^{-\alpha t}$

$e^{-\alpha(-t)} \Rightarrow e^{\alpha t}$

since $-f(-t) \neq e^{\alpha t}$ and

- $at^2 + bt + c$

$a(-t)^2 + b(-t) + c \Rightarrow at^2 - bt + c$

$-f(-t) \neq at^2 - bt + c$

and

$f(t) \neq f(-t)$

at^2+bt+c is neither

10)

Quiz #1

10) $F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$

a)

$F\{\alpha x(t)\} = \alpha F\{x(t)\}$

$F\{\alpha x(t)\} = \alpha \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$

$\boxed{F\{\alpha x(t)\} = \alpha X(f) \quad \text{homogenous}}$

b) $F\{x_1(t) + x_2(t)\} = F\{x_1(t)\} + F\{x_2(t)\}$

$F\{x_1(t) + x_2(t)\} = \int_{-\infty}^{\infty} x_1(t) e^{-j2\pi f t} dt + \int_{-\infty}^{\infty} x_2(t) e^{-j2\pi f t} dt$

$\boxed{F\{x_1(t) + x_2(t)\} = X_1(f) + X_2(f)}$

c) $F\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(u) e^{-j2\pi f(u+t_0)} du \qquad u = t - t_0$

$= \int_{-\infty}^{\infty} x(u) e^{-j2\pi f u} e^{-j2\pi f t_0} du$

$\boxed{F\{x(t-t_0)\} = e^{-j2\pi f t_0} X(f)}$

11)

11) $f(t_0) = \int_{-\infty}^{\infty} f(t)\delta(t - t_0)\,dt$

a) $\int_{-\infty}^{\infty} f(t - t_0) e^{-2\pi f t}\,dt$

$u = t - t_0 \qquad t = u + t_0$

$du = dt$

$\int_{-\infty}^{\infty} f(u) e^{-2\pi j f(u+t_0)}\,du$

$= \int_{-\infty}^{\infty} f(u) e^{-2\pi j f(u+t_0)}\,du$

$= \int_{-\infty}^{\infty} f(u) e^{-2\pi j f u} (e^{-2\pi j f t_0})\,du$

$\boxed{= e^{-2\pi j f t_0}}$

b) $|e^{-j 2\pi f t_0}| = 1$

$\boxed{|F\{\delta(t-t_0)\}| = 1}$

c) phase $e^{-j 2\pi f t_0} = -2\pi f t_0$

$\boxed{\text{phase}\left(F\{\delta(t-t_0)\}\right) = -2\pi f t_0}$

12)

12) $F(f) = (a/2)(\delta(f-f_0) + a\delta(f+f_0))$

a) $\int_{-\infty}^{\infty} a\,\delta(t-t_0)\,e^{-2\pi j f_0 t}\,dt$

$a\,e^{-2\pi j f_0 t_0}$ ← angle

↑ mag

$$F(f) = \frac{a}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$

$$F^{-1}\{F(f)\} = \frac{a}{2}\int_{-\infty}^{\infty} \delta(f-f_0)\,e^{2\pi j f t}\,df + \int_{-\infty}^{\infty} \delta(f+f_0)\,e^{2\pi j f t}\,df$$

$$\boxed{\frac{a}{2}\left[e^{2\pi j f_0 t} + e^{-2\pi j f_0 t}\right]}$$

b) $F(f) = \frac{a}{2j}(\delta(f-f_0) - \delta(f+f_0))$

$$\frac{a}{2j}\int_{-\infty}^{\infty} \delta(f-f_0)\,e^{2\pi j f t}\,df - \frac{a^2}{2j}\int_{-\infty}^{\infty} \delta(f+f_0)\,e^{j 2\pi f t}\,df$$

$$\boxed{= \frac{a}{2j}e^{j2\pi f_0 t} - \frac{a^2}{2j}e^{-j2\pi f_0 t}}$$

13)

13)

a) $A\cos(2\pi f_{LO} t)$; $(M/A)\cos(2\pi f_A t)$

$$\boxed{A\cos(2\pi f_{LO}t) = \frac{A}{2}\left(e^{j2\pi f_{LO}t} + e^{-j2\pi f_{LO}t}\right)}$$

$$\frac{M}{A}\cos(2\pi f_A t) = \frac{M}{A}\cdot\frac{1}{2}\left(e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right) = \frac{M}{A2}\left(e^{j2\pi f_A t} + e^{-j2\pi f_A}\right.$$

b) $$\left[\frac{A}{2}\left(e^{j2\pi f_{LO}t} + e^{-j2\pi f_{LO}t}\right) \cdot \frac{M}{\cancel{A}2}\left(e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right)\right]$$

$$\frac{M}{4}\left[\left(e^{j2\pi f_{LO}t} + e^{-j2\pi f_{LO}t}\right)\left(e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right)\right]$$

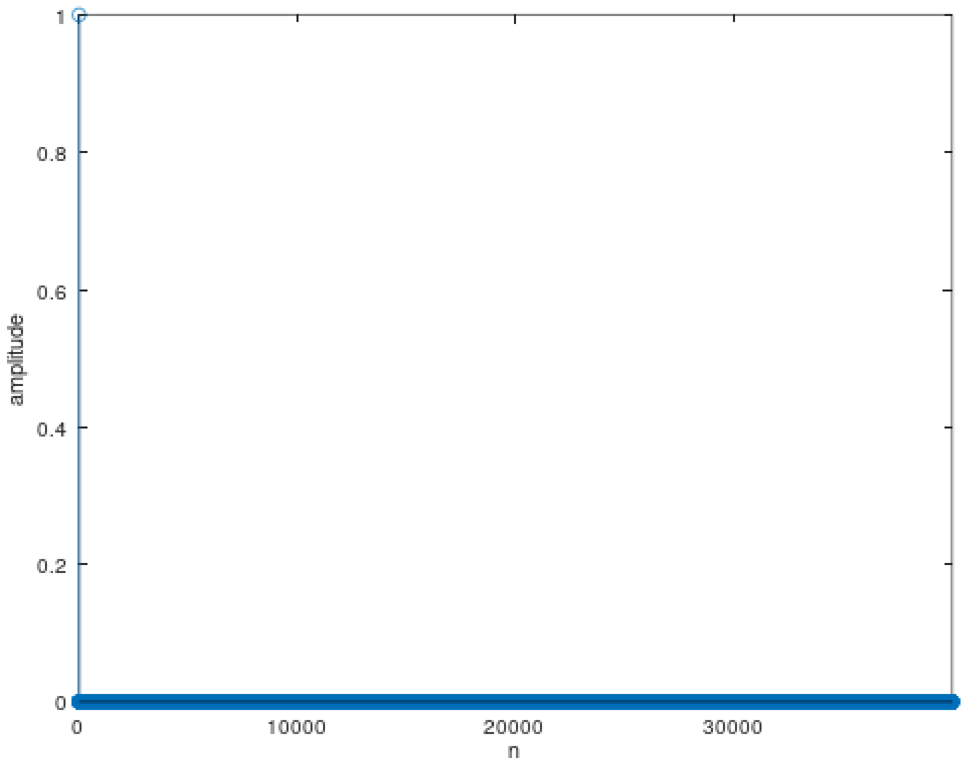
$$\frac{M}{4}\left[e^{j2\pi t(f_{LO}+f_A)} + e^{j2\pi t(f_{LO}+f_A)} + e^{-j2\pi t(f_{LO}+f_A)} + e^{-j2\pi t(}\right.$$

new frequencies

- $f_{LO} + f_A$
- $f_{LO} - f_A$

OCTAVE CODE PROJECT

1a)



(19373, 0.026092)

```
>> fs = 20000; % Hz
>> t = 2; % 2 seconds
>> N = fs * t; % number of samples
>>
>> delta_function = zeros(1,N);
>> delta_function(1) = 1;
>> figure;
>> stem(0:N-1, delta_function);
>> xlabel('n');
>> ylabel('amplitude');
>> xlim([0 N-1]);
>>
>> % 1a number of sampls should be 40000
>>
>>
>> |
```

1b)

```
>>
>> % part b
>>
>> echo_interval = 0.25;
>> echo_sample = round(echo_interval * fs);
>>
>> echo_function = zeros(1, N);
>> echo_function(1) = 1;
>> for i = echo_samples+1:echo_sample:N

Display all 1854 possibilities? (y or n)
echo_function(i) = 0.5;
end
error: 'echo_samples' undefined near line 1, c
olumn 9
>>
>> for i = echo_sample+1:echo_sample:N
echo_function(i) = 0.5;
end
>>
>> figure;
>> stem(0:N-1, echo_function);
>> xlabel('n');
>> ylabel('amplitude');
>> xlim([0 N-1]);
>> |
```

1b)

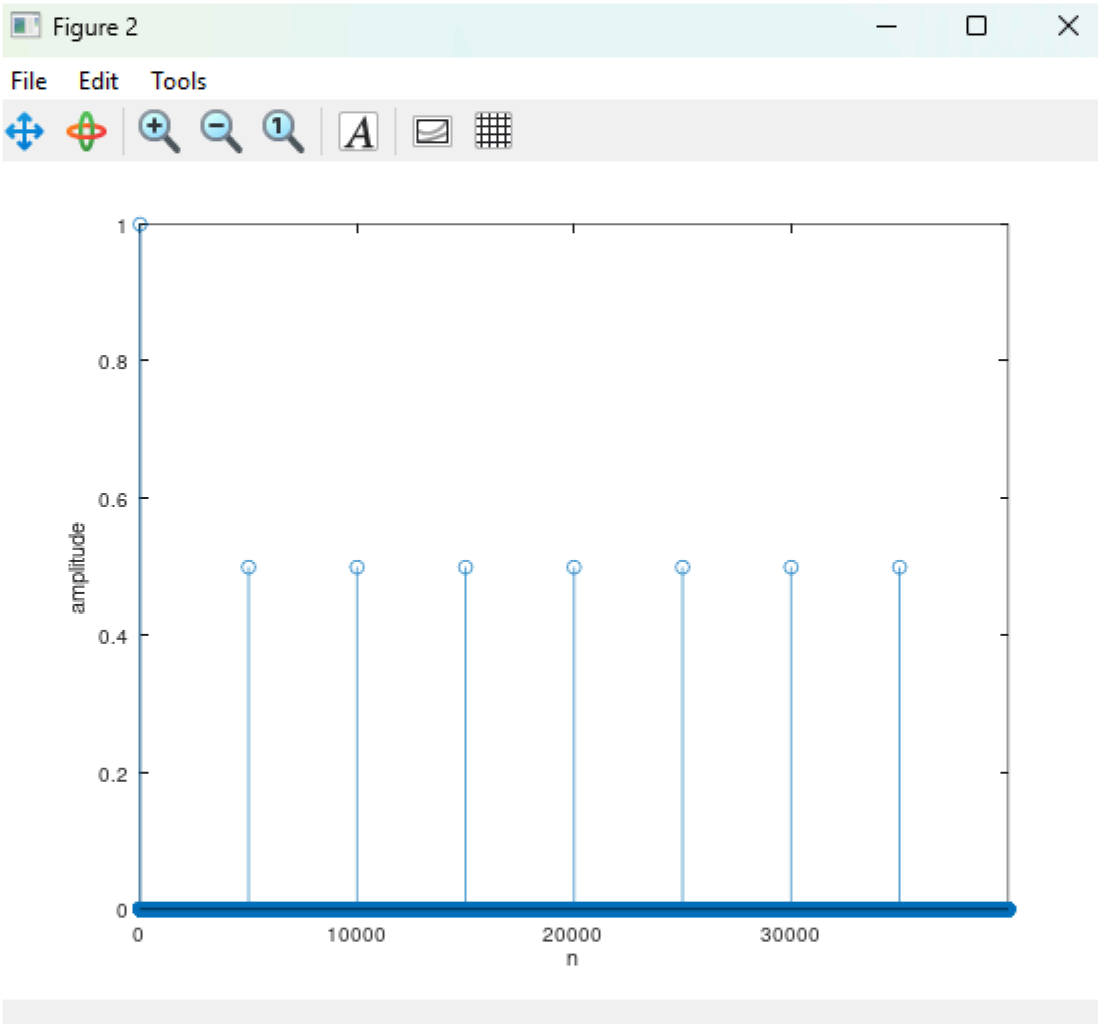
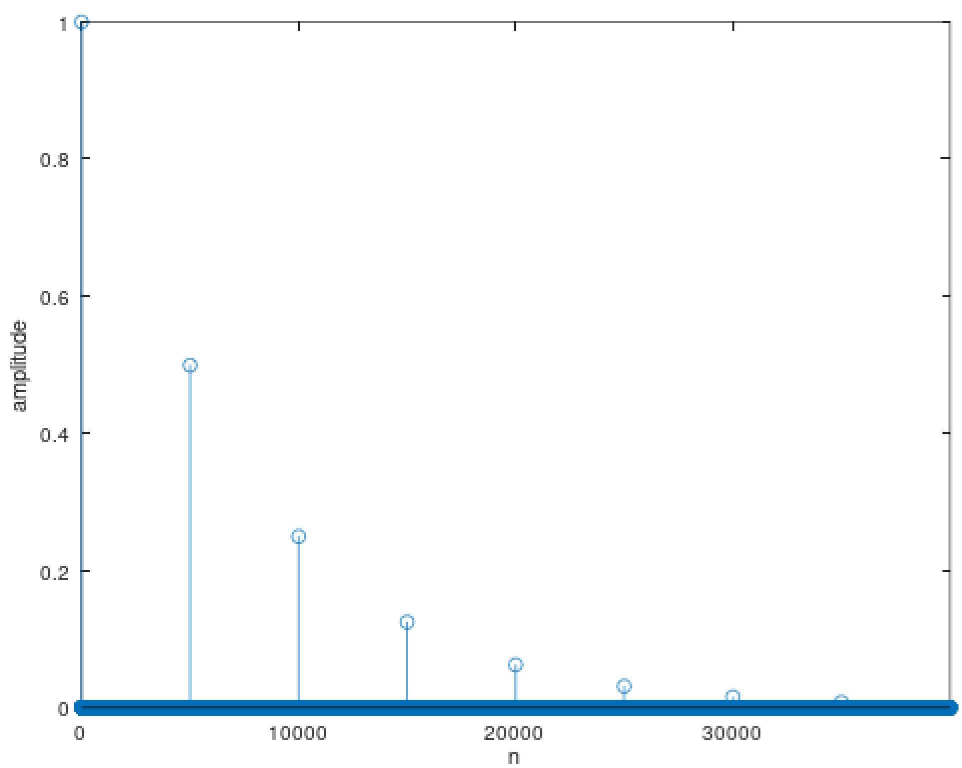


Figure 2
File
Edit
Tools
1
0.8
0.6
0.4
0.2
0
amplitude
0
10000
20000
30000
n

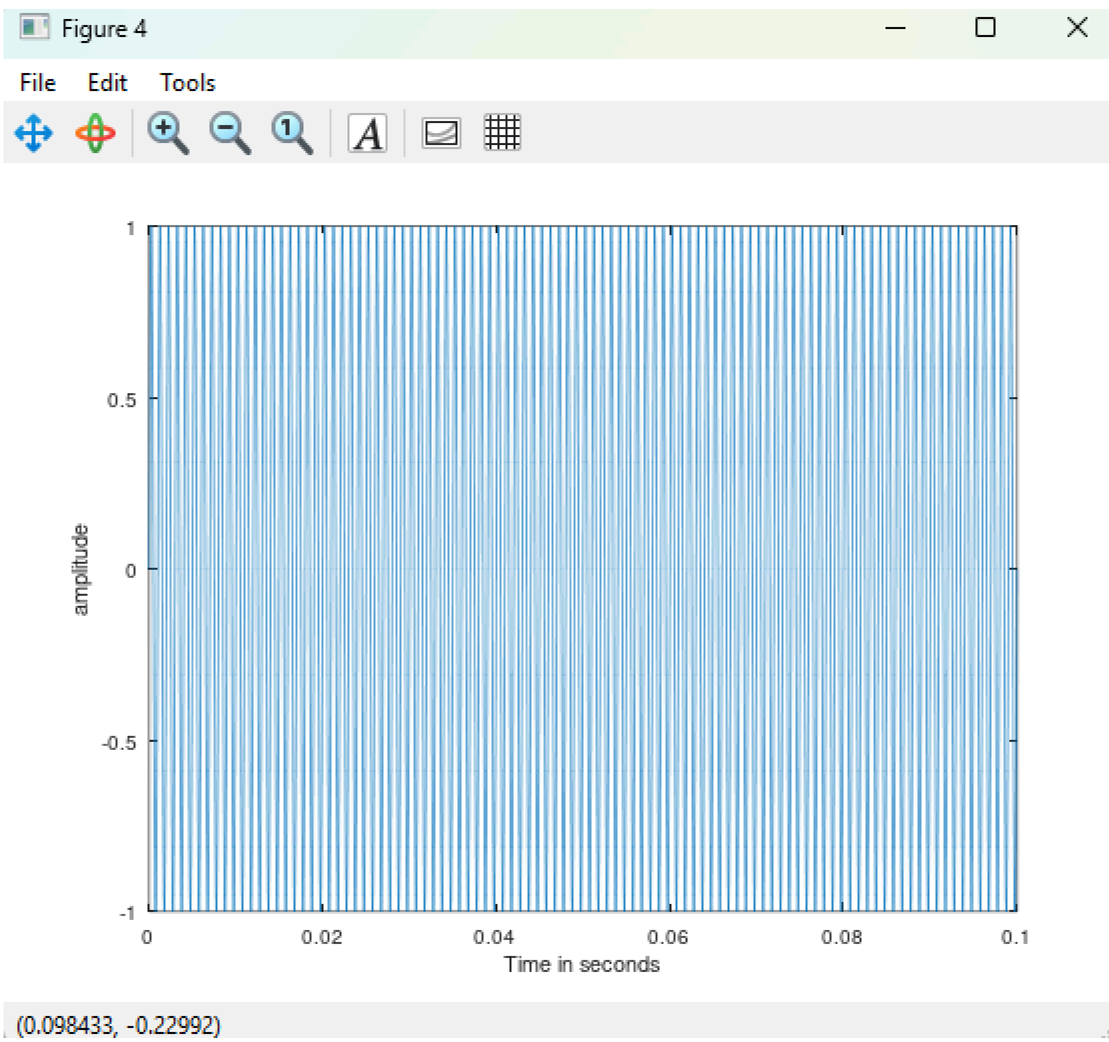
1c)

```
>> % part c
>>
>> amp_decay = 0.5;
>> echo_function = zeros(1, N);
>> echo_function(1) = 1;
>>
>> for i = echo_sample+1:echo_sample:N
echo_function(i) = amp_decay^(i / echo_sample
) * 1;
end
>>
>> figure;
>> stem(0:N-1, echo_function);
>> xlabel('n');
>> ylabel('amplitude');
>> xlim([0 N-1]);
>> |
```



1d)

```
>> % part d
>>
>> f = 1000; %frequency
>> t_sine = 0:1/fs:0.1;
>> sine_wave = sin(2 * pi * f * t_sine);
>> sine_wave = [sine_wave zeros(1, N - length
(sine_wave))];
>>
>> figure;
>> plot(t_sine, sine_wave(1:length(t_sine)));
>> xlabel('Time in seconds')
>> ylabel('amplitude');
>> |
```



1e)

```
>>
>> %part e
>>
>> output_signal = conv(sine_wave, echo_function);
>>
>> volume = 0.4; reduce volume
error: 'reduce' undefined near line 1, column 15
>> volume = 0.4; %reduce volume
>>
>> output_signal = output_signal * volume;
>>
>> sound(output_signal, fs);
>> |
```