Quiz 1

Complex numbers 1:

1. $Z = 4 + 4j$

$|z| = \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

$Z = 4 + 4j$

$= 4\sqrt{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$

$\boxed{Z = 4\sqrt{2}(\cos\frac{\pi}{4} + i\sin\pi/4)}$

$Z = -1 = -1 + 0j$

$r = |z| = \sqrt{(-1)^2 + 0^2} = 1$

$z = 1(-1 + 0 \cdot j)$

$= 1(\cos\pi + 0j)$

$= (\cos\pi + j\sin\pi)$

2. $z = 1$

$= 1 + 0j$

$r = |z| = \sqrt{1^2 + 0^2} = 1$

$z = 1(1 + 0j)$

$= 1(\cos 0 + j\sin 0)$

$z = j = 0 + 1j$

$r = |z| = \sqrt{0^2 + 1^2} = 1$

$z = 1(0 + 1j)$

$= 1(\cos\pi/2 + j\sin\pi/2)$

$Z = -j = 0 + -1j$

$r = |z| = \sqrt{0^2 + (-1)^2} = \sqrt{1} = 1$

$z = 1(0 - j)$

$= 1\cos\pi/2 - i\sin\pi/2$

$= 1[\cos(-\pi/2) + i\sin(-\pi/2)]$

3. In the previous Problem, You can see that the Phase angle of each number rotates by $(\pi/2)$

Polar to rect:

1.) $Z = 2\exp(j\pi/4) \to Z = 2e^{j\pi/4}$

$= 2[\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}]$

$= 2[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}]$

$X = \sqrt{2},\ Y = \sqrt{2}$ $\quad = \sqrt{2} + j\sqrt{2}$

$x^2 + y^2 = |\sqrt{2} + j\sqrt{2}| = 2$

$x^2 + y^2 = 2$

2) $z = \exp(j\pi) \to z = 5e^{j\pi}$

$= 5[\cos\pi + j\sin\pi]$

$= 5\cos\pi + 0$

$z = -5$

$X = -5$

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1. Compute $|V|^2 = V^*V$, and $\phi_2 - \phi_1 = \pi,\ \phi_2 - \phi_1 = 0$

$V(t) = a_2 e^{jx_2} + a_1 e^{jx_1}$ $\qquad 2\pi f = w$

$x_i = 2\pi f t + \phi_i \Rightarrow 2wt + \phi_i$

$x_2 = wt + \phi_2 \Rightarrow wt + \phi_1$

$V(t) = a_2 e^{j(wt + \phi_2)} + a_1 e^{j(wt + \phi_1)}$

$\Rightarrow a_2 \cdot e^{jwt} e^{j\phi_2} + a_1 e^{jwt} \cdot e^{j\phi_1}$

$e^{jwt} \Rightarrow e^{jwt}[a_2 e^{j\phi_2} + a_1 e^{j\phi_1}]$

$= e^{jwt} \times \sqrt{a_1^2 + a_2^2}[\cos\alpha e^{j\phi_2} + \sin\alpha e^{j\phi_1}] \frac{a_i}{\sqrt{a_1^2 + a_2^2}} e^{j\phi_1})]$

$V(t) = \sqrt{a_1^2 + a_2^2}\, e^{jwt} [\cos\alpha + e^{j\phi_2} + \sin\alpha\, e^{j\phi}]$ — ①

let $\cos\alpha = \frac{a_2}{\sqrt{a_1^2 + a_2^2}}$, $\sin\alpha = \frac{a_1}{\sqrt{a_1^2 + a_2^2}}$

$V^*(t) = \sqrt{a_1^2 + a_2^2}\, e^{-jwt} [\cos\delta \cdot e^{-j\phi_2} + \sin\alpha\, e^{-j\phi_1}]$ — ②

$|V(t)|^2 = V^*(t) \times V(t)$

$= (a_1^2 + a_2^2) e^0 \cdot [\cos\alpha \cdot e^{j\phi_2} + \sin\alpha\, e^{j\phi_1}] \cdot [\cos\alpha\, e^{-j\phi_2} + \sin\alpha\, e^{-j\phi_1}]$

$\Rightarrow (a_1^2 + a_1^2)[1 + \cos\delta \cdot \sin\alpha \cdot e^{j(\phi_2 - \phi_1)} + \cos\alpha \cdot \sin\alpha\, e^{-j(\phi_2 - \phi_1)}$ — ③

Case 1 $\phi_2 - \phi_1 = \pi$

$(a_1^2 + a_2^2)[1 + \cos\alpha \cdot \sin\alpha \cdot e^{j\pi} + \cos\alpha \cdot \sin\delta \cdot e^{-j\pi}]$

$\Rightarrow (a_1^2 + a_2^2)[1 + \cos\alpha \cdot \sin\alpha\, \left(\frac{e^{j\pi} + e^{-j\pi}}{2}\right) \times 2]$

$\Rightarrow (a_1^2 + a_2^2)[1 - 2\cos\alpha \cdot \sin\alpha]$

$|V(t)|^2 = (a_1^2 + a_2^2)[1 - \sin 2\delta]$

Case-2 $\phi_2 - \phi_1 = 0$

$|V(t)|^2 = (a_1^2 + a_2^2)[1 + \cos\alpha \cdot \sin\alpha \cdot e^0 + \cos\alpha \cdot \sin\delta\, e^0]$

$= (a_1^2 + a_2^2)[1 + 2\cos\delta \cdot \sin\alpha]$

$= (a_1^2 + a_2^2)[1 + \sin 2\alpha$

from equation-①

$V(t) = \sqrt{a_1^2 + a_2^2}\, e^{jwt} [\cos\alpha \cdot e^{j\phi_2} + \sin\alpha\, e^{j\phi_1}]$

Case-1 $\phi_2 - \phi_1 = \pi \Rightarrow \phi_2 = \pi + \phi_1$

$\downarrow$

$$V(t) = k \cdot e^{j\omega t}\left[\cos\alpha \cdot e^{j\pi} \cdot e^{j\phi_1} + \sin\alpha \cdot e^{j\phi_1}\right]$$
$$= k \cdot e^{j\omega t} \cdot e^{j\phi_1}\left[\sin\alpha - \cos\alpha\right]$$
$$= k \cdot e^{j(\omega t + \phi_1)} \cdot \underbrace{(\cos\alpha - \sin\alpha)}_{\beta} \quad \text{—(4)}$$

$$= k \cdot \beta\left[\cos(\omega t + \phi_1) + j\sin(\omega t + \phi_1)\right]$$
$$\phi_v = \tan^{-1}\left[\frac{\sin(\omega t + \phi_1)}{\cos(\omega t + \phi_2)}\right] = \tan^{-1}\left[\tan^{-1}(\omega t + \phi_1)\right]$$
$$\boxed{\phi_v = \omega t + \phi_1}$$

Case-II $\phi_2 - \phi_1 = 0 \Rightarrow \phi_2 = \phi_1$

$$V(t) = k \cdot e^{j\omega t}\left[\cos\alpha \cdot e^{j\phi_1} + \sin\alpha \cdot e^{j\phi_1}\right]$$

$$V(t) = k(\cos\alpha + \sin\alpha)e^{j\omega t + \phi_1}$$

$$\phi_v = \tan^{-1}\left[\frac{\sin(\omega t + \phi_1)}{\cos(\omega t + \phi_1)}\right] = \tan^{-1}\left[\tan(\omega t + \phi_1)\right]$$

$$\boxed{\phi_v = \omega t = \phi_1}$$

Complex numbers 3.

RCL circuits

$L=0 \quad (z_2=0)$

$$h(\omega)=\frac{0+z_3}{z_1+0+z_3}=\frac{z_3}{z_1+z_3}$$

$$\omega_{LC}^{-2}=0$$

$$\tau=RC$$

$$R^2=1-\omega^2\cdot\omega_{LC}^{-2}=1-\omega^2\cdot 0=1$$

$$h(\omega)=\frac{1}{1+(\omega\tau)^2}-j\frac{\omega\tau}{1+(\omega\tau)^2}$$

$$*R^4=(R^2)^2=1^2=1$$

$$h(\omega)=\frac{1}{1+\omega^2+\tau^2}(1-j\omega\tau)$$

$$|h(\omega)|=\left|\frac{1}{1+\omega^2\tau^2}\right|\cdot|1-j\omega\tau|$$

$$=\frac{1}{1+\omega^2\tau^2}\cdot\sqrt{1^2+(\omega\tau)^2}$$

$\downarrow$

$$|h(\omega)| = \frac{\sqrt{(1+\omega^2\tau^2)}}{(1+\omega^2\tau^2)}$$

$$= \frac{1}{\sqrt{1+\omega^2\tau^2}}$$

Graph $|h(\omega)|$

$0 \le \omega \le \infty$

when $\omega = 0$

$$|h(\omega)| = \frac{1}{\sqrt{1+0^2\tau^2}} = \frac{1}{\sqrt{1}} = 1$$

when $\omega = \infty$

$$|h(\omega)| = \frac{1}{\sqrt{\infty^2}} = \frac{1}{\infty} = 0$$