Alyssa Rubalcava
Professor Hanson
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Quiz #1

1. Convert the complex #'s to polar form

a) $z = 4 + 4j \longrightarrow z = re^{j\varphi}$

$r = \sqrt{4^2 + 4^2}$

$r = 4\sqrt{2}$

$\varphi = \tan^{-1}(4/4)$

$\varphi = \frac{\pi}{4}$

$z = 4\sqrt{2}\, e^{j(\frac{\pi}{4})}$

b) $z = 1,\ z = j,\ z = -1,\ z = -j \longrightarrow z = re^{j\varphi}$

$z = 1e^{j0} \rightarrow z = 1$

$z = 1e^{j(\frac{\pi}{2})} \rightarrow z = e^{j\frac{\pi}{2}}$

$z = 1e^{j\pi} \rightarrow z = e^{j\pi}$

$z = 1e^{j(\frac{3\pi}{2})} \rightarrow z = e^{j\frac{3\pi}{2}}$

c) The phase angle of each number is increasing as it goes from 0 to $\frac{\pi}{2}$ to π to $\frac{3\pi}{2}$.

d) Convert to rectangular form

$$z = 2\exp\left(\frac{j\pi}{4}\right) \longrightarrow 2e^{j\frac{\pi}{4}}$$

$$x = 2\cos\left(\frac{\pi}{4}\right) = 1.41$$

$$y = 2\sin\left(\frac{\pi}{4}\right) = 1.41$$

$$1.41 + j1.41$$

e) $z = 5\exp(j\pi) \longrightarrow 5e^{j\pi}$

$$x = 5\cos(\pi) = -5$$

$$y = 5\sin(\pi) = 0$$

$$-5 + j0$$

2. compute $|v|^2 = v^* v$, & $\varphi_2 - \varphi_1 = \pi$, $\varphi_2 - \varphi_1 = 0$

a) $v(t) = a_1 e^{jx_1} + a_2 e^{jx_2}$

$v^*(t) = a_1 e^{-jx_1} + a_2 e^{-jx_2}$

$v^* v = (a_1 e^{-jx_1} + a_2 e^{-jx_2})(a_1 e^{jx_1} + a_2 e^{jx_2})$

$= a_1^2 + a_2^2 + a_1 a_2 e^{-jx_1 + jx_2}$

$+ a_1 a_2 e^{-jx_2 + jx_1}$

$|v|^2 = a_1^2 + a_2^2 + a_1 a_2 (e^{j(x_2 - x_1)} + e^{-j(x_2 - x_1)})$

$|v|^2 = a_1^2 + a_2^2 + a_1 a_2 (e^{j\Delta x} + e^{-j\Delta x})$

$|v|^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos(\Delta x)$

$\Delta x = \Delta\varphi = \phi_2 - \phi_2$, $\varphi = 0, \pi$

$|v|^2 = a_1^2 + a_2^2 + 2 a_1 a_2 \cos(\Delta\varphi)$

$|v|^2 = a_1^2 + a_2^2 + 2 a_1 a_2 = (a_1 + a_2)^2 + 2 a_1 a_2$

$|v|^2 = (a_1 + a_2)^2 + 2 a_1 a_2$

b) what is $\phi v = \tan^{-1}(\text{Im}\{v\}/\text{Re}\{v\})$ in each case?

$$\phi v = \tan^{-1}(\text{Im}\{v\}/\text{Re}\{v\})$$

$$v = a_1 e^{jx_1} + a_2 e^{jx_2}$$

$$= a_1\cos(x_1) + a_1\sin(x_1)j + a_2\cos(x_2) + a_2\sin(x_2)j$$

$$\phi v = \tan^{-1}\left(\frac{a_1\sin(x_1) + a_2\sin(x_2)}{a_1\cos(x_1) + a_2\cos(x_2)}\right)$$

3) Recompute $h(\omega)$, but start with $L=0\ (z_2=0)$.
Draw graph of $|h(\omega)|$.

$$L=0\ (z_2=0) \qquad V_c = \frac{Q}{C} = V_c(0) = \frac{0}{C} = 0$$

$$h(\omega) = \frac{0 + z_3}{z_1 + 0 + z_3} \quad ?$$

$$\omega_{LC}^{-2}$$