Kai Stephens
COSC 360

Homework #4

Part 2

(1) let the sampling frequency be 20 kHz

a) start with a 2-second $\delta[n]$. How many samples should it contain?

it should contain $40 \times 10^3 + 1$ samples.

```
fs = 20E3;
dt = 1/fs;
t = 0.0 : dt : 2.0;
```

b) modify the $\delta[n]$ to create an echo every 0.2 seconds.

```
N = length(t);
d = zeros(N,1);
d(1) = 1;
```

the nonzero samples are at d(2000), 4000, 6000, 8000, 10000, and so on, increasing by 2000.

c) Modify the response function to make each echo half the amplitude of the previous echo.

d(2000) = 0.5;
d(4000) = 0.25;
d(6000) = 0.125;
d(8000) = 0.0625;
d(10000) = 0.03125;

d)

(2) $L[n]$ = impulse response of
Single-Pole lowpass filter

$h[n]$ = " " " " high pass
filter

Same cutoff frequency, f_c.

$\xrightarrow{\text{impulse}} \boxed{\text{low pass thing}} \longrightarrow$ stuff
low frequency

$\xrightarrow{\text{impulse}} \boxed{\text{high pass thing}} \longrightarrow$ stuff
high frequency

assume $h[n] = \delta[n] - l[n]$

$$h[n] * \delta[n] = \delta[n] * \delta[n] - l[n] * l[n]$$

$$= \cancel{\delta}[n] - l[n] * \cancel{\delta}[n]$$

$$h = \delta - l \quad , \quad \delta * (h + l) = (\delta) * \delta$$

$$\delta * (l + h] = \delta * \delta = \delta$$

basically assume Part b) is true! that $h[n] = \delta[n] - l[n]$

this can be rewritten as

$$h[n] + l[n] = \delta[n].$$

if we can volve both sides with the signal $s[n]$, Then

$$s[n] * (h[n] + l[n]) = \delta[n] * s[n]$$

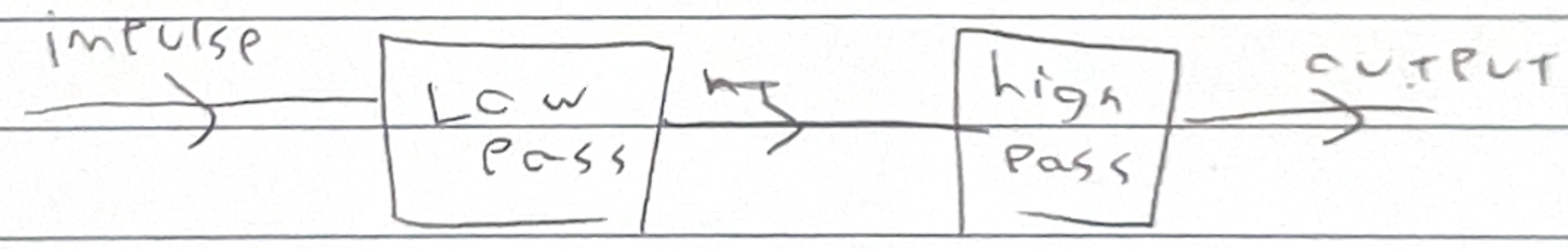
we know that $s[n] * \delta[n] = s[n]$.

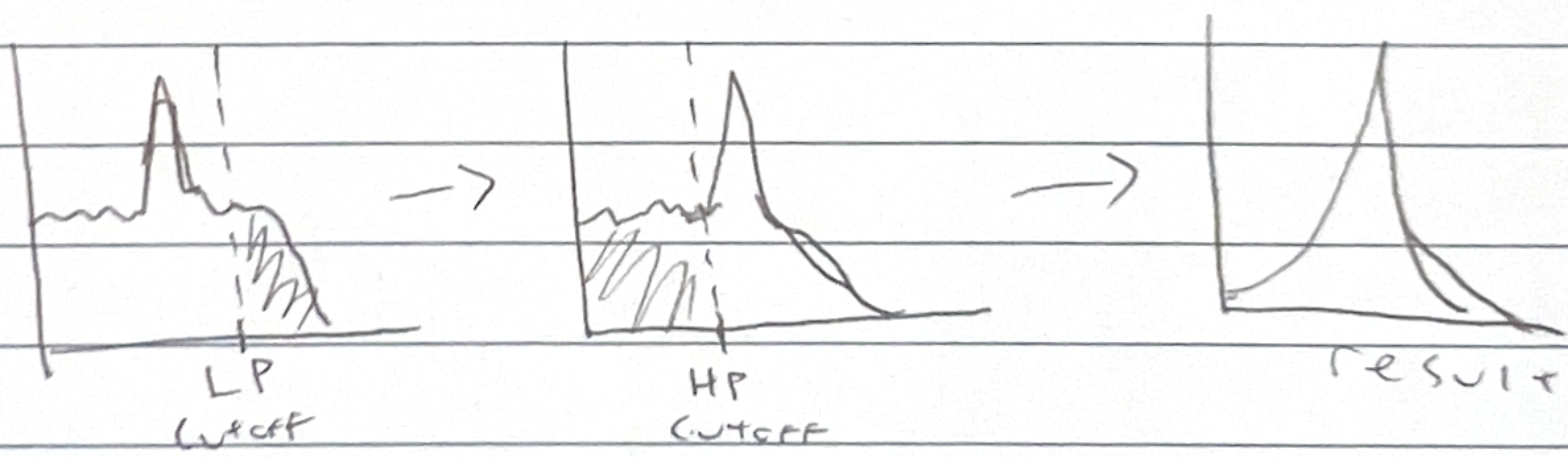
so

$$s[n] * (h[n] + l[n]) = s[n]$$

So the sum of the outputs of $s[n]$ fed into a high-pass and low-pass filter is just $s[n]$.

c)





B seems to be correct.

Part 3

① Discrete fourier tf

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi j (\frac{k}{N}) n}$$

$$X_{k1} = \sum_{n=0}^{N-1} x_n e^{-2\pi j\left(\frac{k_1}{N}\right)n}$$

$$X_{k2} = \sum_{n=0}^{N-1} x_n e^{-2\pi j\left(\frac{k_2}{N}\right)n}$$

$$X_{k_1} + X_{k_2} = \sum_{n=0}^{N-1} x_n e^{-2\pi j\left(\frac{k_1}{N}\right)n} + \sum_{n=0}^{N-1} x_n e^{-2\pi j\left(\frac{k_2}{N}\right)n}$$

$$= \sum_{n=0}^{N-1} x_n \left(e^{-2\pi j\left(\frac{k_1}{N}\right)n} + e^{-2\pi j\left(\frac{k_2}{N}\right)n}\right)$$

all sums are additive.

scalar J

$$J X_k = \sum_{n=0}^{N-1} J x_n e^{-2\pi j\left(\frac{k}{N}\right)n} = J \sum_{n=0}^{N-1} x_n e^{-2\pi j\left(\frac{k}{N}\right)n}$$

all sums are homogeneous.

b) $X_k = \delta[k]$

$k = k_0$

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{2\pi j\left(\frac{k}{N}\right)n} = \frac{1}{N}\sum_{k=0}^{N-1} \delta[k]\, e^{2\pi j\left(\frac{k}{N}\right)n}$$

$$= \frac{1}{N} e^{2\pi j\left(\frac{k_0}{N}\right)n} = \frac{1}{N}\left(\cos\left(2\pi\left(\frac{k_0}{N}\right)n\right) + j\sin\left(2\pi\left(\frac{k_0}{N}\right)n\right)\right)$$

(2) a)