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HW #1

1. Prove $z^* = \dfrac{x_1 - jy_1}{x_2 - jy_2}$

$$z = \frac{x_1 + jy_1}{x_2 + jy_2}\left(\frac{x_2 - jy_2}{x_2 - jy_2}\right) = \frac{(x_1 + jy_1)(x_2 - jy_2)}{x_2^2 + y_2^2}$$

$$= \frac{x_1x_2 + y_1y_2 - x_1y_2j + x_2y_1j}{x_2^2 + y_2^2}$$

$$z = \frac{x_1x_2 + y_1y_2}{x_2^2 + y_2^2} + j\,\frac{x^2y' - x_1y_2}{x_2^2 + y_2^2} \quad ✓$$

$$z^* = \frac{x_1 - jy_1}{x_2 - jy_2}\left(\frac{x_2 + jy_2}{x_2 + jy_2}\right) = \frac{(x_1 - jy_1)(x^2 + jy_2)}{x_2^2 + y_2^2}$$

$$= \frac{x_1 x_2 + j y_2 x_1 - j y_1 x_2 + y_1 y_2}{x_2^2 + y_2^2}$$

$$z^* = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} - j\,\frac{y_2 x_1 + y_1 x_2}{x_2^2 + y_2^2} \quad \checkmark$$

2. (Bonus) Prove $ej^{\phi} = \cos^{\phi} + j\sin\phi$ (Euler's formula)

$$f(\varphi) = e^{-j\phi}(\cos\varphi + j\sin\varphi) \qquad f(\varphi) = g(\varphi)\,h(\varphi)$$

$$f'(\phi) = g'(\varphi)\,h(\varphi) + g(\varphi)\,h'(\varphi)$$

$$f'(\varphi) = -je^{-j\varphi}(\cos\varphi + j\sin\varphi) + e^{-j\phi}(-\sin\varphi + j\cos\varphi)$$

$$f'(\varphi) = e^{-j\phi}[-j(\cos\varphi + j\sin\varphi) + (-\sin\varphi + j\cos\varphi)]$$

$$f'(\varphi) = e^{-j\phi}[-j\cos\phi - j^2\sin\varphi + (-\sin\varphi) + j\cos\varphi)]$$

$$j^2 = -1 \qquad -(-1) = 1$$

$$f'(\varphi) = e^{-j\phi}[\underbrace{-j\cos\varphi + \sin\varphi - \sin\varphi + j\cos\varphi}_{=0}]$$

$$f'(\varphi) = e^{-j\phi}[0] = 0 \qquad e^{-j\phi}(\cos\varphi + j\sin\varphi) = k$$

$$f'(\varphi) = 0 \qquad e^{-j(0)}(\cos(0) + j\sin(0)) = k$$

$$f(0) = k \qquad e^0(1+0) = k \quad k = 1$$

$$f(\phi) = e^{-j\varphi}(\cos\phi + j\sin\varphi) = 1$$

$$\longrightarrow e^{j\varphi} = \cos\varphi + j\sin\varphi \checkmark$$

3. Prove $\cos(x) = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$

$e^{jx} = \underline{\cos x} + j\sin x \qquad j = \sqrt{-1}$

$\cos x = e^{jx} - j\sin x \qquad e^{-jx} = \cos(-x) + j\sin(-x)$

$\qquad e^{-jx} = \cos(x) - j\sin(x)$

$$\frac{e^{jx} + e^{-jx}}{2} = \frac{(\cos x + j\sin x) + (\cos x - j\sin x)}{2}$$

$$= \frac{2\cos x}{2} = \cos x \checkmark$$

$$\downarrow$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2} \checkmark$$