

Quiz #2

1) Phase angle

a) $z = -2 + 2j$

Phase angle = $\phi = \tan^{-1}(b/a)$

$a = -2$ $b = 2$

Phase angle, $\phi = \tan^{-1}\left(\frac{2}{-2}\right)$

$= \tan^{-1}(-1)$

$= \pi - \tan^{-1}(1)$

$= \pi - \frac{\pi}{4} = \boxed{\frac{3\pi}{4}}$

b) $z = -2 - 2j$

$a = -2$, $b = -2$

$\phi = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left(\frac{-2}{-2}\right)$

$= \pi + \tan^{-1}(1) = \pi + \frac{\pi}{4} = \boxed{\frac{5\pi}{4}}$

c) $z = 2 - 2j$

$a = 2$, $b = -2$

$\tan^{-1}\left(\frac{-2}{2}\right) = \tan^{-1}(-1)$

$= 2\pi - \tan^{-1}(1)$

$= 2\pi - \frac{\pi}{4} = \boxed{\frac{7\pi}{4}}$

2) Phasor @ $t=0$

a) $V(t) = 4\cos(2\pi(10^6)t + 30^\circ)$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Here $\theta = 30^\circ = \frac{\pi}{6}$, $A = 4$

$$F = Ae^{j\theta} = 4e^{j(\pi/6)}$$

b) $V(t) = 2\sin(2\pi(10^6)t - 60^\circ)$

$$\begin{aligned} V(t) &= 2\sin(2\pi(10^6)t - 60^\circ) \\ &= 2\cos(90^\circ - (2\pi(10^6)t - 60^\circ)) \\ &= 2\cos(150^\circ - 2\pi(10^6)t + 60^\circ) \\ &= 2\cos(150^\circ - 2\pi(10^6)t) \\ &= 2\cos(30^\circ + 2\pi(10^6)t) \end{aligned}$$

$$V(t) = 2\cos(2\pi(10^6)t + 30^\circ)$$

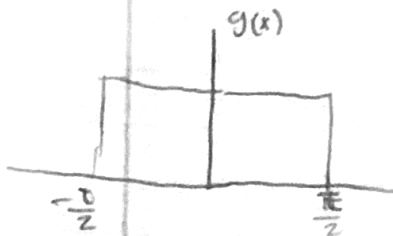
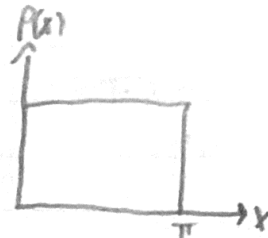
$$F = Ae^{j\theta}$$

$\theta = 30^\circ = \pi/6$, $A = 2$

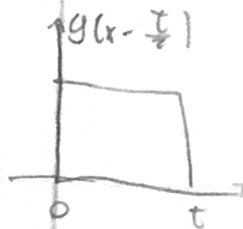
$$F = 2e^{j\pi/6}$$

Fourier Series

$$f(x) = \begin{cases} 1 & 0 \leq x \leq \pi \\ 0 & \pi < x \leq 2\pi \end{cases}$$



$$\rightarrow A \tau \left(\sin \frac{\omega \tau}{2} \right) / \frac{\omega \tau}{2}$$



$$j\omega e^{-j\omega \frac{\tau}{2}}$$

$$P(\omega) = e^{-j\omega(\frac{\tau}{2})} \\ = e^{-j\omega \frac{\tau}{2}}$$

$$P(\omega) = \frac{\tau}{2} \left(\frac{\omega \tau}{2} \right) \left[\cos \frac{\omega \tau}{2} - j \sin \frac{\omega \tau}{2} \right]$$

1) Where Phases versus frequency is
Phase = $\frac{\pi \omega}{2}$

2) Mag = $\pi/2$

$$\frac{\pi}{2} \cdot \frac{2\omega}{\tau/2}$$

$$\omega = \frac{\pi}{\tau/2} = 2$$

$$\omega = 2.416$$

Probability & Statistics

If X is uniformly distributed over $[a, b]$ then,

$$P(X) = \frac{1}{b-a}, \quad a \leq X \leq b$$

$$\text{Statistical mean } \mu = \frac{a+b}{2}$$

$$\text{STD} = \sqrt{\frac{(b-a)^2}{12}}$$

Here random variable is uniformly distributed over $[0, 1]$

$$P(X) = \frac{1}{1-0}$$

$$P(X) = 1$$

$$\mu = \frac{1+0}{2} = \left[\frac{1}{2} \right]$$

$$\text{STD} = \sqrt{\frac{(1-0)^2}{12}} = \left[\frac{1}{\sqrt{12}} \right]$$