Homework #1

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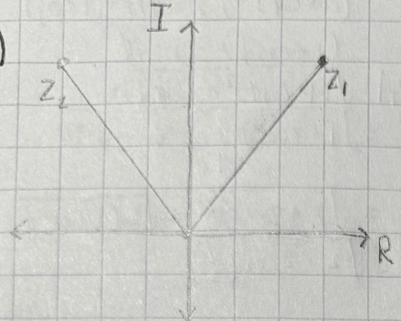
1/31

Complex Numbers and Signals

1. $z_1 = 3+4j \quad z_2 = -3+4j$

a)



b) $z_1 + z_2 : (3-3)+(4+4)j = 0+8j = \boxed{8j}$

c) $z_1 - z_2 : (3--3)+(4-4)j = 6+0j = \boxed{6}$

d) $z_1 \cdot z_2 : (3+4j)(-3+4j) = -9+12j-12j+16j^2 = -9-16 = \boxed{-25}$

e) $z_1/z_2 : \frac{3+4j}{-3+4j} \cdot \frac{-3-4j}{-3-4j} = \frac{-9-24j+16}{9+16} = \frac{7-24j}{25} = \boxed{\frac{7}{25} - \frac{24j}{25}}$

f) $|z_1| = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = \boxed{5}$

g) $|z_2| = \sqrt{-3^2+4^2} = \boxed{5}$

h) $\phi_1 : = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{0.93 \text{ rad}}$ i) $\tan^{-1}\left(\frac{4}{-3}\right) + \pi \approx \boxed{2.21 \text{ rad}}$

j) $z_1 = 5(\cos(0.93) + j\sin(0.93)) \quad z_2 = 5(\cos(2.2) + j\sin(2.2))$

$\boxed{z_1 = 5e^{j(0.93)} \quad z_2 = 5e^{j(2.2)}}$

2. Euler: $e^{j\phi} = \cos\phi + j\sin\phi \quad \phi = 2\pi ft$

a)

$e^{-j2\pi ft} = \cos(\underbrace{-2\pi ft}_{-\phi}) + j\sin(\underbrace{-2\pi ft}_{-\phi})$

$= \cos(2\pi ft) - j\sin(2\pi ft)$

$e^{j2\pi ft} + e^{-j2\pi ft} = (\cos(2\pi ft) \cancel{+ j\sin(2\pi ft)})$
$+ (\cos(2\pi ft) \cancel{- j\sin(2\pi ft)})$

$\frac{e^{j2\pi ft} + e^{-j2\pi ft}}{2} = \frac{\cancel{2}\cos(2\pi ft)}{\cancel{2}}$

$\boxed{\cos(2\pi ft) = \frac{e^{j2\pi ft} + e^{-j2\pi ft}}{2}}$

b)

$e^{-j2\pi ft} = \cos(2\pi ft) - j\sin(2\pi ft)$

$e^{j2\pi ft} - e^{-j2\pi ft} = (\cancel{\cos(2\pi ft)} + j\sin(2\pi ft))$
$- (\cancel{\cos(2\pi ft)} - j\sin(2\pi ft))$

$\frac{e^{j2\pi ft} - e^{-j2\pi ft}}{2j} = \frac{2j\sin(2\pi ft)}{2j}$

$\boxed{\sin(2\pi ft) = \frac{e^{j2\pi ft} - e^{-j2\pi ft}}{2j}}$

3. $v_1(t) = 4\cos(2\pi f_1 t) \quad v_2(t) = 4\cos(2\pi f_2 t - \phi)$

a) $P = v_1(t)v_2(t)$

$= (4\cos(2\pi f_1 t))(4\cos(2\pi f_2 t - \phi))$

$= 16\cos(2\pi f_1 t)\cos(2\pi f_2 t - \phi$

$P(t) = 16 \cdot \frac{1}{2}(\cos(2\pi f_1 t) - (2\pi f_2 t - \phi))$
$+ \cos(2\pi f_1 t) + (2\pi f_2 t - \phi))$

$\boxed{P(t) = 8(\cos(2\pi(f_1 - f_2)t + \phi) + \cos(2\pi(f_1 + f_2)t - \phi))}$

$f_+ = f_1 + f_2 \quad f_- = f_1 - f_2$

b) $\phi = 0 \quad f_1 = f_2$

$P(t) = 8(\cos(2\pi(0)t + 0) + \cos(2\pi(f_1+f_2)t - 0)$

$= 8(\cos(0) + \cos(4\pi ft))$

$= 8(1 + \cos(4\pi ft) \qquad -1 \leq 4\pi ft \leq 1$

$= 8(1+1)$

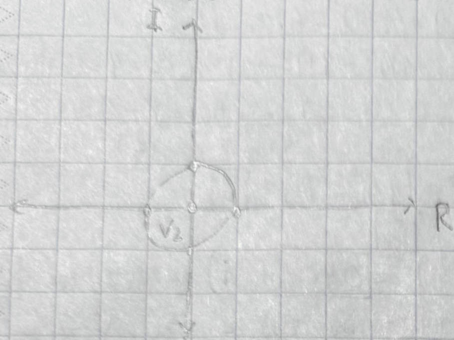
$= 8(2)$

$\boxed{P_{max} = 16}$

4. $v_1(t) = \Re\{e^{j(2\pi f t - \phi)}\}$
$v_2(t) = \Re\{e^{2\pi j f t}\}$

a) $v_1 = \Re(e^{j2\pi f t}) \cdot (e^{-j\phi})$ $\quad v_2 = \Re\{e^{(2\pi j f t)}\}$

$v_1 = \Re\{e^{-j\phi}\}$ $\quad v_2 = \Re(1)$



ϕ	v_1	v_2
0	$1+0j$	$0+0j$
$\frac{\pi}{2}$	$0+1j$	$0+0j$
π	$-1+0j$	$0,0$
$\frac{3\pi}{2}$	$0-1j$	$0,0$

b) $v_T = e^{-j\phi} + 1$

$= (\cos(\phi) - j\sin(\phi)) + (1+0j)$

$= (1+\cos\phi) - j\sin\phi$

$|v_T| = \sqrt{(1+\cos\phi)^2 + (\sin\phi)^2}$

$= \sqrt{1 + 2\cos\phi + \underbrace{\cos^2\phi + \sin^2\phi}_{1}}$

$= \sqrt{2 + 2\cos\phi} = \sqrt{2(1+\cos\phi)}$

$= \sqrt{2 \cdot 2\cos^2\left(\frac{\phi}{2}\right)}$

$|v_T| = 2\cos\left(\frac{\phi}{2}\right) \qquad \theta_T = \tan^{-1}\left(\frac{-\sin\phi}{1+\cos\phi}\right)$

$\theta = \tan^{-1}\left(\frac{-2\sin\left(\frac{\phi}{2}\right)\cos\left(\frac{\phi}{2}\right)}{2\cos^2\left(\frac{\phi}{2}\right)}\right) = \left(\frac{-\sin\left(\frac{\phi}{2}\right)}{\cos\left(\frac{\phi}{2}\right)}\right)$

$\boxed{\theta = \frac{-\phi}{2}}$

c) $\phi = 45°$

$v_T = 2\left(\cos\left(\frac{45}{2}\right)\right) \approx (0.92) \cdot 2 \qquad \theta = -\frac{45}{2} = \boxed{-22.5°}$

$|v_T| \approx \boxed{1.85}$

$\phi = 0°$

$|v_T| = 2\cos(0) = \boxed{2}$

$\theta = \frac{0}{2} = \boxed{0°}$

$\phi = 180°$

$|v_T| = 2\cos(90) = \boxed{0}$

$\theta = -\frac{180}{2} = \boxed{-90°}$

Probability and Statics, Noise

1. $p(x)\,dx = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{x-\mu}{\sigma}\right)^2} dx$

a) The graph of the histogram ressembles the probability dencity function for Gaussian distribution centred at 0.

$\mu = 0 \quad \sigma = 1$

$p(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

b) It appears as pure noise because its generated from digitized random values.

c) See other atatchment

2. $p(x)\,dx = \frac{dx}{b-a}, \quad a \le x \le b$

see atatchment

ADC and DAC

1. see atatchment

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// Homework1
// Probability and Statics
// Problem 1
fs = 1000;
t = 0:1/fs:1-1/fs;
f_sine = 50;
amplitude = 1;
SNR = 10;
sine_wave = amplitude * sin(2 * pi * f_sine * t);
noise = randn(size(t));
noisy_signal = sine_wave + noise;
figure;
subplot(3,1,1);
plot(t, noise);
title('Gaussian Noise');

// Problem 2
n_samples = 100000;
n_rands = 12;
sum_rands = sum(rand(n_samples, n_rands), 2);
sum_rands_normal = (sum_rands - n_rands / 2) / sqrt(n_rands / 12);

figure;
hist(sum_rands_normal, 50);
hold on;

x_vals = linspace(-5, 5, 100);
gauss_pdf = (1/sqrt(2*pi)) * exp(-0.5 * x_vals.^2);
plot(x_vals, gauss_pdf * n_samples * (x_vals(2) - x_vals(1)), 'r', 'LineWidth', 2);

title('Sum Random Variables');
xlabel('Normal Sum');
ylabel('Frequency');

//Problem 3
f = 10;
fs1 = 50;
fs2 = 2 * f;
t1 = 0:1/fs1:1;
t2 = 0:1/fs2:1;

sine_wave1 = sin(2 * pi * f * t1);
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sine_wave2 = sin(2 * pi * f * t2);


figure;
subplot(2,1,1);
plot(t1, sine_wave1);
ylabel('Amp');
xlabel('Time');

subplot(2,1,2);
plot(t2, sine_wave2);
xlabel('Time');
ylabel('Amp');
```