Digital Signal Processing HW2

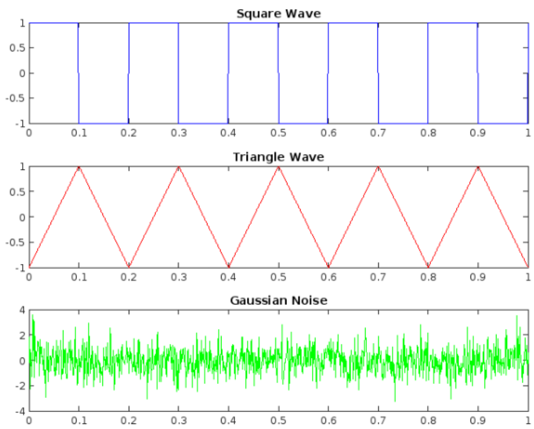
1 Probability and Statistics, Noise

Problem 1: Histogram Analysis

This problem involves generating and analyzing three different signals: a square wave, a triangle wave, and Gaussian noise.

(a) Generating the Signals

The following three signals were generated: - A square wave with amplitude levels of -1 and 1. - A triangle wave with linearly increasing and decreasing values. - A Gaussian noise signal with a mean of zero and unit variance.



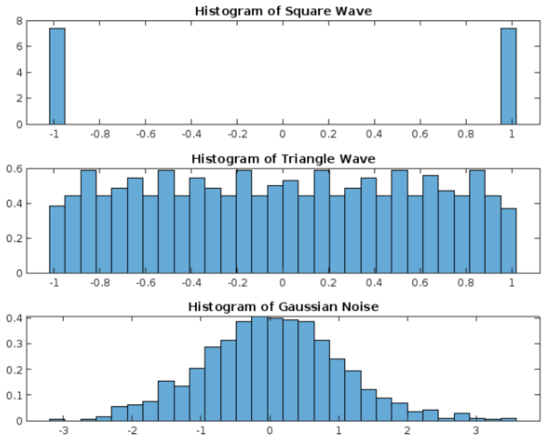
Octave Code:

```
clc; clear; close all;
fs = 1000;
t = 0:1/fs:1;

square_wave = square(2 * pi * 5 * t);
triangle_wave = sawtooth(2 * pi * 5 * t, 0.5);
```

```
gaussian_noise = randn(size(t));

figure;
subplot(3,1,1); plot(t, square_wave, 'b'); title('Square Wave');
subplot(3,1,2); plot(t, triangle_wave, 'r'); title('Triangle Wave');
subplot(3,1,3); plot(t, gaussian_noise, 'g'); title('Gaussian Noise');
saveas(gcf, 'waveforms.png');
```

(b) Creating Histograms

The histograms for each signal were created:



Octave Code:

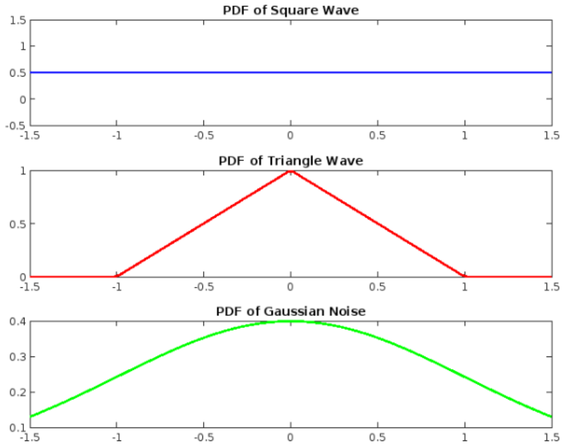
```
figure;
subplot(3,1,1);
histogram(square_wave, 30, 'Normalization', 'pdf');
title('Histogram of Square Wave');

subplot(3,1,2);
histogram(triangle_wave, 30, 'Normalization', 'pdf');
title('Histogram of Triangle Wave');

subplot(3,1,3);
histogram(gaussian_noise, 30, 'Normalization', 'pdf');
title('Histogram of Gaussian Noise');

saveas(gcf, 'histograms.png');
```

(c) Normalization

The histograms are normalized by dividing by the total sample size N.

(d) Comparing to PDFs

Theoretical probability density functions (PDFs) were plotted for each signal:



Octave Code:

```
x = linspace(-1.5, 1.5, 100);

square_pdf = ones(size(x)) / 2;
triangle_pdf = (1 - abs(x)) .* (x >= -1 & x <= 1);

mu = 0; sigma = 1;
gaussian_pdf = (1 / (sqrt(2 * pi) * sigma)) * exp(-x.^2 / (2 * sigma^2));

figure;
subplot(3,1,1);
plot(x, square_pdf, 'b', 'LineWidth', 2);
title('PDF of Square Wave');

subplot(3,1,2);
plot(x, triangle_pdf, 'r', 'LineWidth', 2);
title('PDF of Triangle Wave');

subplot(3,1,3);
plot(x, gaussian_pdf, 'g', 'LineWidth', 2);
title('PDF of Gaussian Noise');
```

```
saveas(gcf, 'pdfs.png');
```

(e) Compute Mean and Standard Deviation

The computed mean and standard deviation for each signal:

$$\text{Square Wave: Mean} = 0.000999, \quad \text{Standard Deviation} = 1.0005$$

$$\text{Triangle Wave: Mean} = -0.000999, \quad \text{Standard Deviation} = 0.57827$$

$$\text{Gaussian Noise: Mean} = 0.028306, \quad \text{Standard Deviation} = 0.99563$$

Octave Code:

```
square_mean = mean(square_wave);
square_std = std(square_wave);

triangle_mean = mean(triangle_wave);
triangle_std = std(triangle_wave);

gaussian_mean = mean(gaussian_noise);
gaussian_std = std(gaussian_noise);

disp(['Square Wave: Mean = ', num2str(square_mean), ', Std = ',
num2str(square_std)]);
disp(['Triangle Wave: Mean = ', num2str(triangle_mean), ', Std = ',
num2str(triangle_std)]);
disp(['Gaussian Noise: Mean = ', num2str(gaussian_mean), ', Std = ',
num2str(gaussian_std)]);
```

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2 ADC and DAC Precision

Problem 1: ADC and DAC Calculations

This problem involves calculating voltage precision in ADC and DAC components.

(A) ADC Calculations

The general formula for voltage per level in an ADC is:

$$\Delta V = \frac{\text{Voltage Range}}{\text{Number of Levels} - 1}$$

where the number of levels is 2^{bits}.

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(a) 8-bit ADC with range [0,2.55]V and 255 levels

$$\Delta V = \frac{2.55}{255} = 10mV$$

Answer: $\Delta V = 10mV$ per level

—

(b) 12-bit ADC with range [0,4.095]V and 4095 levels

$$\Delta V = \frac{4.095}{4095} = 1mV$$

Answer: $\Delta V = 1mV$ per level

—

(c) Required bits for $\Delta V < 1mV$ with range [0,12]V

$$\Delta V = \frac{12}{2^N - 1} < 1mV$$

Solving for N:

$$N > \log_2(12001) \approx 13.55$$

Answer: $N = 14$ bits

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(d) Digital amplitude for 2.52V signal in [0,5]V range with 2048 levels

$$\Delta V = \frac{5}{2047} \approx 2.44mV$$

$$\text{Digital Amplitude} = \frac{2.52}{0.00244} \approx 1034 \text{ counts}$$

Answer: 1034 counts

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(B) DAC Calculations

The analog voltage output of a DAC is given by:

$$V_{\text{out}} = \frac{\text{Digital Counts}}{\text{Max Counts}} \times \text{Voltage Range}$$

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(a) DAC with 256 counts, [0,5]V range, $\Delta V = 9.8mV$

$$V_{\text{out}} = 256 \times 9.8mV = 2.5088V$$

Answer: $V_{\text{out}} = 2.5088V$

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(b) DAC with 2048 counts, [0,5]V range, max counts = 4095

$$V_{\text{out}} = \frac{2048}{4095} \times 5V = 2.502V$$

Answer: $V_{\text{out}} = 2.502V$

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(c) DAC with 128 counts, max counts = 511, output = 0.25V

$$V_{\text{max}} = \frac{0.25V \times 511}{128} = 0.998V$$

Answer: $V_{\text{max}} = 0.998V$

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Problem 2: Digital Signal Frequency

The sampled frequency f_s' due to aliasing is given by:

$$f_s' = |f - Nf_s|$$

where: - f is the original analog signal frequency. - f_s is the sampling rate. - N is computed as:

$$N = \text{round}\left(\frac{f}{f_s}\right)$$

This ensures the observed frequency remains within $[0, f_s/2]$.

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(a) Given $f_s = 500$ kHz, $f = 50$ kHz

$$N = \text{round}\left(\frac{50}{500}\right) = 0$$

$$f_s' = |50 - (0 \times 500)| = 50 \text{ kHz}$$

Answer: $f_s' = 50$ kHz

—

(b) Given $f_s = 500$ kHz, $f = 250$ kHz

$$N = \text{round}\left(\frac{250}{500}\right) = 1$$

$$f_s' = |250 - (1 \times 500)| = 250 \text{ kHz}$$

Answer: $f_s' = 250$ kHz

—

(c) Given $f_s = 500$ kHz, $f = 750$ kHz

$$N = \text{round}\left(\frac{750}{500}\right) = 2$$

$$f_s' = |750 - (2 \times 500)| = 250 \text{ kHz}$$

Answer: $f_s' = 250$ kHz

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(d) Given $f_s = 500$ kHz, $f = 1000$ kHz

$$N = \text{round}\left(\frac{1000}{500}\right) = 2$$

$$f_s' = |1000 - (2 \times 500)| = 0 \text{ kHz}$$

Answer: $f_s' = 0$ kHz (Appears as DC)

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Problem 3: Determining Capacitance for Desired Attenuation

A low-pass RC filter follows the transfer function:

$$R(f) = \frac{1}{1 + j2\pi fRC}$$

where: - $f = 25$ MHz, - $R = 10$ k, - C (to be determined), - $A_{\text{in}} = 3.3$ V, - $A_{\text{out}} = 0.33$ V.

The magnitude is given by:

$$\frac{A_{\text{out}}}{A_{\text{in}}} = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$$

Substituting values:

$$0.1 = \frac{1}{\sqrt{1 + (2\pi \times 25 \times 10^6 \times 10^4 \times C)^2}}$$

$$(2\pi \times 250 \times 10^6 C)^2 = 99$$

$$C = \frac{\sqrt{99}}{2\pi \times 250 \times 10^6}$$

$$C \approx 6.34 \times 10^{-9} \text{ F} = 6.34 \text{ nF}$$

Answer: $C = 6.34$ nF
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Problem 4: Determining Capacitance for Desired Attenuation

A high-pass RC filter follows the transfer function:

$$R(f) = \frac{j2\pi fRC}{1 + j2\pi fRC}$$

The magnitude is:

$$|R(f)| = \frac{2\pi fRC}{\sqrt{1 + (2\pi fRC)^2}}$$

where: - $f = 10$ MHz, - $R = 10$ k, - C (to be determined), - $A_{\text{in}} = 3.3$ V, - $A_{\text{out}} = 0.33$ V.

Solving for C:

$$\frac{0.33}{3.3} = \frac{2\pi \times 10^7 \times 10^4 \times C}{\sqrt{1 + (2\pi \times 10^7 \times 10^4 \times C)^2}}$$

$$0.1 = \frac{2\pi \times 10^{11} C}{\sqrt{1 + (2\pi \times 10^{11} C)^2}}$$

Squaring both sides:

$$(2\pi \times 10^{11} C)^2 - 0.01(2\pi \times 10^{11} C)^2 = 0.01$$

$$(0.99)(2\pi \times 10^{11} C)^2 = 0.01$$

$$C = \frac{\sqrt{0.01/0.99}}{2\pi \times 10^{11}}$$

$$C \approx 1.6 \times 10^{-13} \text{ F} = 160 \text{ pF}$$

Answer: $C = 160$ pF

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