HW4 : Aman Kumpawat

Problem 1:

a)

sampling frequency: $f_s = 20{,}000$ Hz

time duration: $t = 2$ seconds

number of samples:

$$N = f_s \times T = 20{,}000 \times 2 = 40{,}000$$

b)

sample delay for each echo:

$$N_{echo} = f_s \times 0.2 = 20{,}000 \times 0.2 = 4{,}000$$

nonzero sample locations for echoes:

$$n = 0, 4000, 8000, 12000, 16000, 20000, 24000, 28000, 32000, 36000$$

c)

Let $h[n]$ be the impulse response

each echo decreases by half in amplitude compared to the last one:

$$h[n] = \delta[n] + \frac{1}{2}\delta[n-4000] + \frac{1}{4}\delta[n-8000] + \frac{1}{8}\delta[n-12000] + \ldots$$

d)

0.1s sine wave:

$$N = f_s \times 0.1 = 20{,}000 \times 0.1 = 2000$$

Convolve $h[n]$ with sin wave $x[n]$ to apply echo:

$$y[n] = h[n] * x[n]$$

Problem 2:

a) Show $S_L[n] + S_H[n] = S[n]$

signal S_N is split into low-pass $l[n]$ and high-pass $h[n]$ parts:

$$S_L[n] = l[n] * s[n],$$

$$S_H[n] = h[n] * s[n]$$

complementary filters:

$$l[n] + h[n] = \delta[n]$$

summing outputs:

$$S_L[n] + S_H[n] = (l[n] * s[n]) + (h[n] * s[n])$$

use convolution properties:

$$(l[n] + h[n]) * s[n] = \delta[n] * s[n] = s[n]$$

b) show $h[n] = \delta[n] - l[n]$

since:

$$l[n] + h[n] = \delta[n]$$

rearrange:

$$h[n] = \delta[n] - l[n]$$

c)

- A band-pass filter results from applying a low-pass and a high-pass filter in series
- The low-pass filter must have a lower cutoff frequency than the high-pass filter

A: f_c of $l[n]$ is lower than that of $h[n]$

Problem 3:

a) Prove additivity and homogeneity

DFT definition:

$$X_k = \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N}$$

Additivity:

$$DFT(x_n + y_n) = \sum_{n=0}^{N-1} (x_n + y_n) e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N} + \sum_{n=0}^{N-1} y_n e^{-j2\pi kn/N}$$

$$= X_k + Y_k$$

Homogeneity:

$$DFT(ax_n) = \sum_{n=0}^{N-1} a x_n e^{-j2\pi kn/N}$$

$$= a\sum_{n=0}^{N-1} x_n e^{-j2\pi kn/N} = aX_k$$

b)

DFT of $\delta[k]$:

Given:

$$X_k = \delta[k - k_0]$$

inverse DFT:

$$x_n = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}$$

Since X_k is nonzero only at k_0:

$$x_n = \frac{1}{N} e^{j2\pi k_0 n/N}$$

Problem 4:

a)

- The plots visually demonstrate that as the pulse width in the time domain decreases, the Fourier spectrum in the frequency domain widens.
- The inverse relationship follows the uncertainty principle, where the time-domain and frequency-domain widths are inversely proportional.

b) $\Delta t \cdot \Delta f \approx 0.5$

- $\Delta t = T$
- $\pi f T = \pm\pi$

$$f = \pm\frac{1}{T}$$

So, main lobe width:

$$\Delta f \approx \frac{1}{T}$$

Product of $\Delta t \cdot \Delta f$

$$\Delta t = T, \ \Delta f = \frac{1}{T}$$

$$\Delta t \cdot \Delta f = T \cdot \frac{1}{T} = 1$$

Since the function spreads energy across frequencies, the real bound is: $\Delta t \cdot \Delta f \approx 0.5$