(1a)

definition of v(t):

$$v(t) = 2.5\cos(\phi)$$

Eulers formula:

$$e^{j\phi} = \cos(\phi) + j\sin(\phi)$$

Real part:

$$\mathrm{Re}\{e^{j\phi}\} = \cos(\phi)$$

Algebra:

$$2.5\cos(\phi) = 2.5\,\mathrm{Re}\{e^{j\phi}\} = \mathrm{Re}\{2.5e^{j\phi}\}$$

$$v(t) = 2.5\cos(\phi) = \mathcal{R}\{2.5e^{j\phi}\}$$

(1b)

trig identity:

$$\cos(\phi) = \sin(\phi + \pi/2)$$

Then:

$$v(t) = 2.5\cos(\phi) = 2.5\sin(\phi + \pi/2)$$

recall:

$$\sin(\theta) = \mathrm{Im}\{e^{j\theta}\}$$

hence:

$$2.5\sin(\phi + \pi/2) = 2.5\,\mathrm{Im}\{e^{j(\phi+\pi/2)}\}$$

$$e^{j(\phi+\pi/2)} = e^{j(\phi-\pi/2)}e^{j\pi} = -e^{j(\phi-\pi/2)}$$

so:

$$\mathrm{Im}\{e^{j(\phi+\pi/2)}\} = -\mathrm{Im}\{e^{j(\phi-\pi/2)}\}$$

$$v(t) = 2.5\cos(\phi) = \mathcal{J}\{2.5e^{j(\phi-\pi/2)}\}$$

(2a) 1 KHz = 1000 cycles/second (Hz)

$$T = \frac{1}{f} = \frac{1}{1000\ \mathrm{Hz}} = 0.001\ \text{seconds} = 1\ \mathrm{ms}$$

$$1\ \mathrm{KHz}^{-1} = 1\ \mathrm{ms}$$

(2b) $T = 5\ \mathrm{ns} = 5\times10^{-9}\ \mathrm{s}$, $f = 1/T$

$$f = \frac{1}{5\times10^{-9}\ \mathrm{s}} = \frac{1}{5}\times10^{9}\ \mathrm{Hz} = 0.2\times10^{9}\ \mathrm{Hz}$$

$$= 200\ \mathrm{MHz}$$

(2c)

$$T = \frac{1}{f} = \frac{1}{5000\ \mathrm{Hz}} = 0.0002\ \mathrm{s} = 200\ \mu\mathrm{s}$$

$f_s = 50000\ \mathrm{Hz}$

$$\Delta t = \frac{1}{f_s} = \frac{1}{50000\ \mathrm{Hz}} = 0.00002\ \mathrm{s} = 20\ \mu\mathrm{s}$$

$$\text{samples/period} = \frac{T}{\Delta t} = \frac{200\ \mu\mathrm{s}}{20\ \mu\mathrm{s}} = 10$$

(2d)

$$\Delta t = 0.002\ \mathrm{ms} = 0.002\times10^{-3}\ \mathrm{s} = 2\times10^{-6}\ \mathrm{s} = 2\ \mu\mathrm{s}$$

$$\text{samples/period} = \frac{T}{\Delta t} = \frac{0.2\ \mathrm{ms}}{0.002\ \mathrm{ms}} = 100$$

(3a)

Total voltage range:

$$\text{range} = 2.56\mathrm{V} - 0\mathrm{V} = 2.56\mathrm{V}$$

# of steps = 256

$$\Delta V = \frac{\text{total range}}{\text{number of steps}} = \frac{2.56\mathrm{V}}{256} = 0.01\ \mathrm{V} = 10\mathrm{mV}$$

(3b)

$2^8 = 256$

= power of 8

(3c)

current # of bits = 8

doubled = $8 \times 2 = 16$

$2^{16} = 65{,}536$

new step size = $\dfrac{2.56V}{65{,}536} = 3.90625 \times 10^{-5} V = 0.000039062 5 V$

$= 39 \mu V$

(4a) on other pdf

(4b) on other pdf

(4c) on other pdf

(4d) $P_{signal} = \dfrac{A^2}{2}$

$P_{signal} = \dfrac{2.5^2}{2} = \dfrac{6.25}{2} = 3.125 (V^2)$

$P_{noise} = \sigma^2 = (1)^2 = 1 (V^2)$

$SNR = \dfrac{P_{signal}}{P_{noise}} = \dfrac{3.125}{1} = 3.125$

$SNR_{dB} = 10 \log_{10}(SNR) = 10 \log_{10}(3.125)$

$= 4.9485 \, dB$

(4e) on other pdf

(5a) $R(f) = \dfrac{j\omega\tau}{1 + j\omega\tau}$

$|R(f)| = \left| \dfrac{j\omega\tau}{1+j\omega\tau} \right| = \dfrac{|j\omega\tau|}{|1+j\omega\tau|} = \dfrac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$

(5b)

$R(f) = \dfrac{j\omega\tau}{1+j\omega\tau}$

$\angle(j\omega\tau) = \angle(j) = \pi/2$

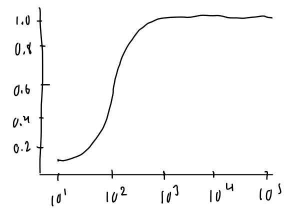
$\angle(1 + j\omega\tau) = \tan^{-1}(\omega\tau)$

$\angle R(f) = \pi/2 - \tan^{-1}(\omega\tau)$

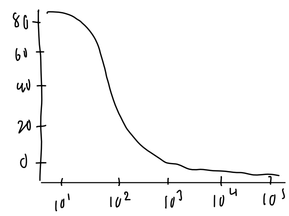
$\angle R(f) = \pi/2 - \tan^{-1}(\omega\tau)$

(5c)

Magnitude of R(F)



Phase of R(F)



(5d) $A(f) = 1$ at $f = 0.5 kHz = 500 Hz$

$R = 1k\Omega, C = 1\mu F$

$\tau = RC = (1000)(1 \times 10^{-6}) = 10^{-3} s$

$\omega = 2\pi f = 2\pi \cdot 500 = 1000 \, rad/s$

$\omega\tau = (1000\pi)(10^{-3}) = \pi \approx 3.14159$

$|R(f)| = \dfrac{\omega\tau}{\sqrt{1+(\omega\tau)^2}} = \dfrac{\pi}{\sqrt{1+\pi^2}} = \dfrac{3.14159}{\sqrt{1+9.8698}}$

$= \dfrac{3.14159}{\sqrt{10.8696}} = \dfrac{3.14159}{3.296} = 0.952$

$A_{filtered}(f) = A(f)\,|R(f)| = 1 \times 0.952 = 0.952$

(6a)

$f_{sampled} = |f_a - N \cdot f_s|$

$f_{sampled} = 2.5 kHz$ as it is under 5 kHz

no folding or aliasing

(6b)

$f_{sampled} = 5 kHz$

exact Nyquist frequency

(6c)

$15 kHz - 10 kHz = 5 kHz$

$f_{sampled} = 5 kHz$

(6d)

$20\,KHz - 2 \times (10\,KHz) = 0\,KHz$

$F_{sampled} = 0\,KHz$

(7a)

$s(t) = 2\sin(2\pi f t)$, $T = 1/f$

$s(t - T/2) = 2\sin(2\pi f(t - T/2))$

$2\pi f(t - \frac{1}{2f}) = 2\pi f t - 2\pi f(\frac{1}{2f}) = 2\pi f t - \pi$

$2\sin(2\pi f t - \pi) = 2[-\sin(2\pi f t)] = -2\sin(2\pi f t) = -s(t)$

$S[s(t)] = s(t - T/2) = -s(t)$

(7b) on other pdf

(7c)

Since $S[s(t)] = -s(t)$

$s(t) + S[s(t)] = s(t) + [-s(t)] = 0$

(8a)

Since $x[n] = 2$ at $n = 3$, $n-1 = 3$, $n = 4$

at $n = 4$:

$x[4-1] = x[3] = 2$

$y[4] = -x[3] = -2$

everywhere else:

$x[n-1] = 0$, $y[n] = 0$

hence:

$$y[n] = \begin{cases} -2, & n = 4 \\ 0, & \text{otherwise} \end{cases}$$

(8b)

$x[n]$ is 0 for all $n \neq 3$ and 2 at $n = 3$

$$x[n]^2 = \begin{cases} (2)^2 = 4, & n = 3 \\ 0, & \text{otherwise} \end{cases}$$

hence:

$$y[n] = (x[n])^2 = \begin{cases} 4, & n = 3 \\ 0, & \text{otherwise} \end{cases}$$

(8c)

System part (a): $y[n] = -x[n-1]$

$$S(ax_1[n] + bx_2[n]) = -(ax_1[n-1] + bx_2[n-1])$$
$$= a(-x_1[n-1]) + b(-x_2[n-1])$$
$$= ay_1[n] + by_2[n]$$

= linear

System part b: $y[n] = (x[n])^2$

$(x_1[n] + x_2[n])^2 = x_1[n]^2 + x_2[n]^2 + 2x_1[n]x_2[n]$

= not a linear combination, so non-linear

(9a) $* \cos(-x) = \cos(x)$

$x = 2\pi f t$

$\cos(2\pi f(-t)) = \cos(-2\pi f t) = \cos(2\pi f t)$

= even

(9b) $* (-t/\sigma)^2 = (t/\sigma)^2$

$\exp\left[-\left(\frac{t}{\sigma}\right)^2\right] = \exp\left[-\left(\frac{t}{\sigma}\right)^2\right]$

= even

(9c) $* \exp(-\alpha(-t)) = \exp(\alpha t)$

- They are not equal, so it is not even.
- They are not negatives of each other, so it is not odd

= neither even nor odd for real $\alpha \neq 0$

(9d) $f(t) = at^2 + bt + c$

$f(-t) = a(-t)^2 + b(-t) + c = at^2 - bt + c$

This is not equal to $f(t)$ nor the negative of $f(t)$

= neither even nor odd

(10a)

Let a be constant:

$$\mathcal{F}\{a f(t)\} = \int_{-\infty}^{\infty} [a f(t)] e^{-j2\pi v t} dt$$

$$= a \int_{-\infty}^{\infty} f(t) e^{-j2\pi v t} dt = a F(v)$$

$$\mathcal{F}\{a f(t)\} = a \mathcal{F}\{f(t)\}$$

(10b)

Let $f(t)$ and $g(t)$ be two signals

$$\mathcal{F}\{f(t) + g(t)\} = \int_{-\infty}^{\infty} [f(t) + g(t)] e^{-j2\pi v t} dt$$

$$= \int_{-\infty}^{\infty} f(t) e^{-j2\pi v t} dt + \int_{-\infty}^{\infty} g(t) e^{-j2\pi v t} dt$$

$$= F(v) + G(v)$$

$$\mathcal{F}\{f(t) + g(t)\} = \mathcal{F}\{f(t)\} + \mathcal{F}\{g(t)\}$$

(10c)

Consider shifting function $f(t)$ by t_0.

You want $\mathcal{F}\{f(t - t_0)\}$:

$$\mathcal{F}\{f(t - t_0)\} = \int_{-\infty}^{\infty} f(t - t_0) e^{-j2\pi v t} dt$$

Let $\tau = t - t_0$

$t = \tau + t_0$

$dt = d\tau$

$$= \int_{-\infty}^{\infty} f(\tau) e^{-j2\pi v(\tau + t_0)}$$

$$= e^{-j2\pi v t_0} \int_{-\infty}^{\infty} f(\tau) e^{-j2\pi v \tau} d\tau$$

$$= e^{-j2\pi v t_0} F(v)$$

Thus:

$$\mathcal{F}\{f(t - t_0)\} = e^{-j2\pi v t_0} \mathcal{F}\{f(t)\}$$

(11a)

integral:

$$\mathcal{F}\{a\,\delta(t - t_0)\} = \int_{-\infty}^{\infty} a\,\delta(t - t_0) e^{-j2\pi v t} dt$$

factor out a:

$$= a \int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi v t} dt$$

sifting property:

$$\int_{-\infty}^{\infty} \delta(t - t_0) e^{-j2\pi v t} dt = e^{-j2\pi v t_0}$$

hence:

$$\mathcal{F}\{a\,\delta(t - t_0)\} = a e^{-j2\pi v t_0}$$

(11b)

$$|a e^{-j2\pi v t_0}| = |a|\,|e^{-j2\pi v t_0}|$$

Since $|e^{-j\theta}| = 1$ for any real θ, Magnitude $= |a|$

(11c) $* a e^{-j2\pi v t_0}$

assuming a is real and positive:

Phase angle $= -2\pi v t_0$

(12a) $F(v) = \frac{a}{2}[\delta(v - f_0) + \delta(v + f_0)]$

inverse fourier transform:

$$f(t) = \int_{-\infty}^{\infty} \frac{a}{2}[\delta(v - f_0) + \delta(v + f_0)] e^{j2\pi v t} dv$$

hence:

$$f(t) = \frac{a}{2}\left[e^{j2\pi f_0 t} + e^{j2\pi(-f_0)t}\right]$$

$$= \frac{a}{2}\left(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right)$$

$$\cos(\theta) = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$f(t) = a\cos(2\pi f_0 t)$$

(12b) $F(v) = \frac{a}{2j}\left[\delta(v-f_0) - \delta(v+f_0)\right]$

inverse fourier transform:

$$F(t) = \int_{-\infty}^{\infty} \frac{a}{2j}\left[\delta(v-f_0) - \delta(v+f_0)\right] e^{j2\pi vt} dv$$

$$f(t) = \frac{a}{2j}\left[\int_{-\infty}^{\infty} \delta(v-f_0) e^{j2\pi vt} dv - \int_{-\infty}^{\infty} \delta(v+f_0) e^{j2\pi vt} dv\right.$$

Thus:

$$f(t) = \frac{a}{2j}\left[e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right]$$

$$\sin(\theta) = \frac{1}{2j}(e^{j\theta} - e^{-j\theta})$$

$$f(t) = a\sin(2\pi f_0 t)$$

(13a) $*\ \cos(wt) = \frac{1}{2}(e^{jwt} + e^{-jwt})$

carrier:

$$A\cos(2\pi f_{LO}t) = A\frac{1}{2}\left[e^{j2\pi f_{LO}t} + e^{-j2\pi f_{LO}t}\right]$$

audio:

$$\frac{m}{A}\cos(2\pi f_A t) = \frac{m}{A}\frac{1}{2}\left[e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right]$$

(13b)

Let:

$$x_{LO}(t) = A\cos(2\pi f_{LO}t)$$

$$x_A(t) = \frac{m}{A}\cos(2\pi f_A t)$$

- $y(t) = x_{LO}(t) \times x_A(t)$

$$x_{LO}(t) = \frac{A}{2}(e^{j2\pi f_{LO}t} + e^{-j2\pi f_{LO}t})$$

$$x_A(t) = \frac{m}{A}\frac{1}{2}(e^{j2\pi f_A t} + e^{-j2\pi f_A t})$$

$$y(t) = \frac{A}{2}(e^{j2\pi f_{LO}t} + e^{-j2\pi f_{LO}t}) \times \frac{m}{A}\frac{1}{2}(e^{j2\pi f_A t} + e^{-j2\pi f_A t})$$

$$y(t) = \frac{A}{2} \times \frac{m}{A}\frac{1}{2}(\ldots) = \frac{m}{4}[\ldots]$$

in the brackets:

$$(e^{j2\pi f_{LO}t} + e^{-j2\pi f_{LO}t}) \times (e^{j2\pi f_A t} + e^{-j2\pi f_A t})$$

distribute:

$$= (e^{j2\pi f_{LO}t} e^{j2\pi f_A t}) + (e^{j2\pi f_{LO}t} e^{-j2\pi f_A t}) + (e^{-j2\pi f_{LO}t} e^{j2\pi f_A t})$$
$$+ (e^{-j2\pi f_{LO}t} e^{-j2\pi f_A t})$$

$$y(t) = \frac{m}{4}\left[e^{j2\pi(f_{LO}+f_A)t} + e^{j2\pi(f_{LO}-f_A)t} + e^{j2\pi(f_A-f_{LO})t} + e^{-j2\pi(f_{LO}+f_A)t}\right]$$

sum frequency: $f_{LO} + f_A = 2\cos(2\pi(f_{LO}+f_A)t)$

difference frequency: $f_{LO} - f_A$, $(f_A - f_{LO}) = (-f_{LO} - f_A)$

$= 2\cos(2\pi(f_{LO} - f_A)t)$

Thus:

$$y(t) = \frac{m}{4}\left[2\cos(2\pi(f_{LO}+f_A)t) + 2\cos(2\pi(f_{LO}-f_A)t)\right]$$

$$y(t) = \frac{m}{2}\left[\cos(2\pi(f_{LO}+f_A)t) + \cos(2\pi(f_{LO}-f_A)t)\right]$$

New frequencies: $f_{LO} + f_A$ and $f_{LO} - f_A$

(4)

```
% (a)
f = 10;
dt = 0.001;
t = 0:dt:1;
s = 2.5 * sin(2*pi*f*t) + 2.5;

% (b)
n = randn(size(t));

% (c)
z = s + n;
figure; plot(t, s); title('s(t)');
xlabel('Time (s)'); ylabel('Voltage (V)');
figure; plot(t, z); title('z(t) = s(t) + n(t)');
xlabel('Time (s)'); ylabel('Voltage (V)');

% (d)
signal_power = (2.5^2)/2;
noise_power = 1;
SNR = signal_power / noise_power;
SNR_dB = 10 * log10(SNR);
disp(['SNR ratio = ', num2str(SNR)]);
disp(['SNR (dB)  = ', num2str(SNR_dB)]);

% (e)
figure;
hist(z, 50);
title('Histogram of z(t) = s(t) + n(t)');
xlabel('Voltage (V)');
ylabel('Counts');
```

(7)

```
% (b)
f = 10;
t = 0:0.001:0.1;
s = 2*sin(2*pi*f*t);
T = 1/f;
s_out = 2*sin(2*pi*f*(t - T/2));
figure; plot(t, s, t, s_out); title('Input and Output'); xlabel('Time (s)'); ylabel('Amplitude');
```

(code project)

```
% (a)
fs = 20000;      % 20 kHz
N = fs*2;        % 2 seconds -> number of samples
disp(['(a) Number of samples: ', num2str(N)]);

% (b)
h = zeros(1, N);
step1 = fs*0.25;   % echo every 0.25 s
for i = 0:7
  idx = 1 + round(i*step1);
  if idx <= N
    h(idx) = h(idx) + 1/(2^i);
  end
end
step2 = fs*0.27;   % second echo train every 0.27 s
for i = 0:5
  idx = 1 + round(i*step2);
  if idx <= N
    h(idx) = h(idx) + 0.5/(2^i);
  end
end
disp('(b) Non-zero sample locations in h:');
disp(find(h~=0));

% (c)
% already implemented the half-amplitude echoes above
```

```
% (d)
Ttone = 0.5;      % tone duration in seconds
Ntone = floor(Ttone*fs);
x = zeros(1, N);
fStart = 220;     % start freq
fEnd = 880;       % end freq
t_tone = (0:Ntone-1)/fs;
fSweep = fStart + (fEnd - fStart)*(t_tone / Ttone);
x(1:Ntone) = sin(2*pi.*fSweep.*t_tone);

% (e)
y = conv(x, h);
soundsc(y, fs);
disp('(e) Enjoy the echo with a frequency sweep!');
```

(BONUS)

```
% create amplitude modulated signal
fs = 20000; T = 1; t = 0:1/fs:T-1/fs;
fm = 200; fc = 5000;
m = 0.5 * sin(2*pi*fm*t);
carrier = cos(2*pi*fc*t);
x = (1 + m) .* carrier;

% add noise to the signal
noise = 0.5 * randn(size(t));
x_noisy = x + noise;

% design and apply low pass and high pass filters
[b_lp, a_lp] = butter(4, 1000/(fs/2), 'low');
x_lp = filtfilt(b_lp, a_lp, x_noisy);
[b_hp, a_hp] = butter(4, 4000/(fs/2), 'high');
x_hp = filtfilt(b_hp, a_hp, x_noisy);

% compute and plot FFT spectra of unfiltered and filtered signals
NFFT = 2^nextpow2(length(x_noisy));
f_axis = fs/2 * linspace(0, 1, NFFT/2+1);
X_noisy = fft(x_noisy, NFFT)/length(x_noisy);
X_lp = fft(x_lp, NFFT)/length(x_lp);
```

```
X_hp = fft(x_hp, NFFT)/length(x_hp);
figure;
subplot(3,1,1);
plot(f_axis, 2*abs(X_noisy(1:NFFT/2+1)));
title('Spectrum of Unfiltered Signal');
xlabel('Frequency (Hz)'); ylabel('Amplitude');
subplot(3,1,2);
plot(f_axis, 2*abs(X_lp(1:NFFT/2+1)));
title('Spectrum of Low Pass Filtered Signal');
xlabel('Frequency (Hz)'); ylabel('Amplitude');
subplot(3,1,3);
plot(f_axis, 2*abs(X_hp(1:NFFT/2+1)));
title('Spectrum of High Pass Filtered Signal');
xlabel('Frequency (Hz)'); ylabel('Amplitude');
```