HW2

Aman Kumpawat

2. Probability and Statistics, Noise

Problem 1

(a) Octave functions to generate the processes:

```
N = 100000;
% Square wave :
square_wave = zeros(1, N);
square_wave(1:2:end) = 0;
square_wave(2:2:end) = 1;
% Triangle wave:
half = N/2;
up = linspace(0, 1, half);
down = linspace(1, 0, half);
triangle_wave = [up, down];
% Gaussian noise:
gauss_noise = randn(1, N);
```

(b)

```
M = 50;
[sq_counts, sq_bins] = hist(square_wave, M);
[tr_counts, tr_bins] = hist(triangle_wave, M);
[gn_counts, gn_bins] = hist(gauss_noise, M);
```

(c)

```
sq_counts = sq_counts / N;
tr_counts = tr_counts / N;
gn_counts = gn_counts / N;
```

(d) shown below

(e) Compute the mean and standard deviation for each process:

$$\text{Square wave:} \quad \bar{x} = \frac{0+1}{2} = 0.5, \quad s = 0.5.$$

$$\text{Triangle wave (uniform on [0,1]):} \quad \bar{x} = 0.5, \quad s = \frac{1}{\sqrt{12}} \approx 0.289.$$

$$\text{Gaussian noise:} \quad \bar{x} \approx 0, \quad s \approx 1.$$

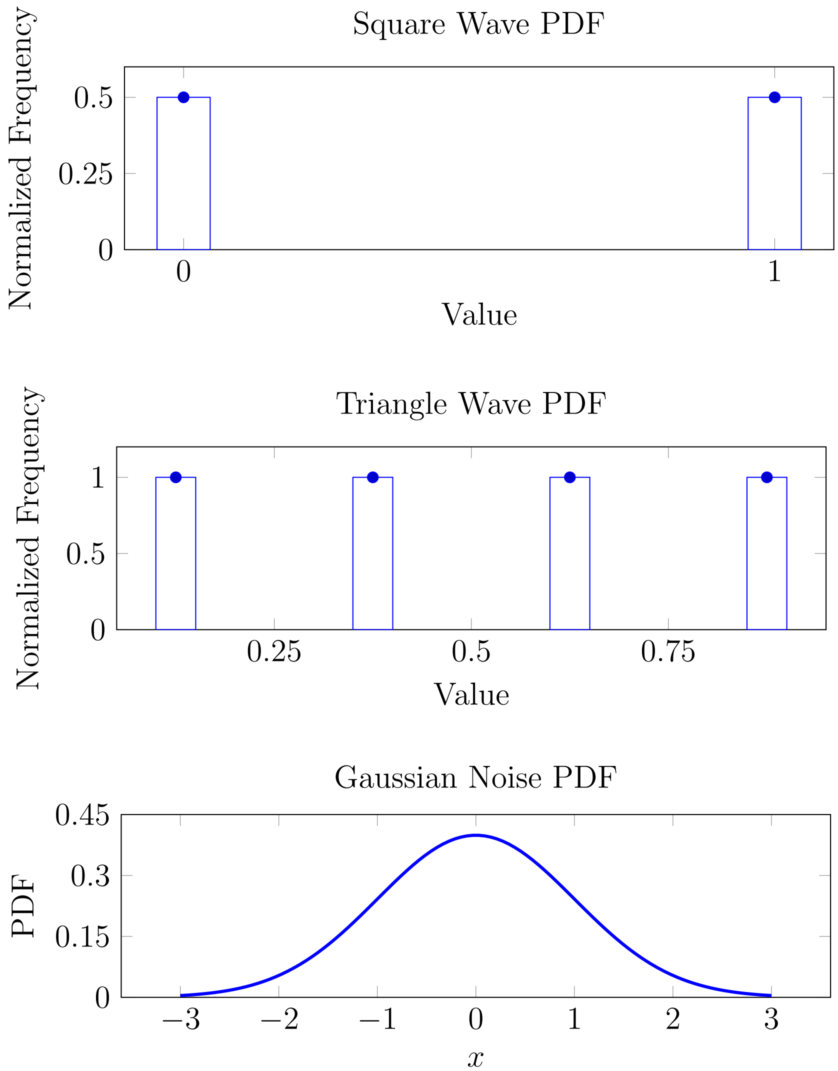

Figure 1: histograms for square wave, triangle wave, and Gaussian noise.

3. ADC and DAC

Problem 1
(a) $\Delta V = \frac{2.55\,\mathrm{V}}{255} = 0.01\,\mathrm{V}$ per level.
(b) $\Delta V = \frac{4.095\,\mathrm{V}}{4095} = 0.001\,\mathrm{V}$ per level.

(c) For $\Delta V < 1\,\text{mV}$ over 12 V:

$$2^n - 1 > \frac{12}{0.001} = 12000 \quad\Longrightarrow\quad n = 14\,(\text{since } 2^{14} - 1 = 16383).$$

(d) Digital amplitude for a 2.52 V signal (with 2048 levels over [0,5] V):

$$D \approx \frac{2.52}{5} \times 2047 \approx 1032\,\text{counts}.$$

DAC:
(a)

$$\text{Amplitude} = (256) \times (9.8 \times 10^{-3}\,\text{V}) \;=\; 2.51\,\text{V}.$$

(b)

$$\text{Amplitude} = 5\,\text{V} \times \frac{2048}{4095} \;\approx\; 2.50\,\text{V}.$$

(c)

$$0.25\,\text{V} \;=\; \left(\frac{128}{511}\right) V_{\max} \quad\Longrightarrow\quad V_{\max} \;=\; 0.25\,\text{V} \times \frac{511}{128} \;\approx\; 1.0\,\text{V}.$$

Problem 2 Given $f_s = 500\,\text{kHz}$:
(a) For $f = 50\,\text{kHz}$: $f_{\text{digital}} = 50\,\text{kHz}$.
(b) For $f = 250\,\text{kHz}$: $f_{\text{digital}} = 250\,\text{kHz}$.
(c) For $f = 750\,\text{kHz}$: $f_{\text{digital}} = 750 - 500 = 250\,\text{kHz}$.
(d) For $f = 1000\,\text{kHz}$: $f_{\text{digital}} = 1000 - 2 \times 500 = 0\,\text{kHz}$.
Problem 3 For a low-pass RC filter with

$$|H_{\text{LP}}(f)| = \frac{1}{\sqrt{1 + (2\pi f RC)^2}},$$

and $f = 25\,\text{MHz}$, $R = 10\,\text{k}\Omega$, $|H| = 0.1$:

$$0.1 = \frac{1}{\sqrt{1 + (2\pi f RC)^2}} \quad\Longrightarrow\quad (2\pi f RC)^2 = 99.$$

Thus,

$$C = \frac{\sqrt{99}}{2\pi f R} \approx \frac{9.95}{2\pi(25 \times 10^6)(10 \times 10^3)} \approx 6.3\,\text{pF}.$$

Problem 4 For a high-pass RC filter with

$$|H_{\mathrm{HP}}(f)| = \frac{2\pi fRC}{\sqrt{1+(2\pi fRC)^2}},$$

and $f = 10\,\mathrm{MHz}$, $R = 10\,\mathrm{k}\Omega$, $|H| = 0.1$:

$$0.1 = \frac{2\pi fRC}{\sqrt{1+(2\pi fRC)^2}}.$$

Squaring and rearranging,

$$0.99(2\pi fRC)^2 = 0.01 \quad\Longrightarrow\quad 2\pi fRC \approx 0.1005.$$

Thus,

$$C = \frac{0.1005}{2\pi(10\times10^6)(10\times10^3)} \approx 0.16\,\mathrm{pF}.$$

Problem 5

$$\phi_{\mathrm{LP}} = -\arctan(2\pi fRC), \quad \phi_{\mathrm{HP}} = \arctan\left(\frac{1}{2\pi fRC}\right).$$

For the low-pass filter (Problem 3) at $f = 25\,\mathrm{MHz}$:

$$\phi_{\mathrm{LP}} \approx -\arctan(9.95) \approx -84.3^\circ.$$

For the high-pass filter (Problem 4) at $f = 10\,\mathrm{MHz}$:

$$\phi_{\mathrm{HP}} \approx \arctan(9.95) \approx 84.3^\circ.$$