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Quiz 2

Review Material

1. Find the phase angle:

a) $z = -2+2j$ $\quad \phi = 3\pi/4$

b) $z = -2-2j$ $\quad \phi = 5\pi/4$

c) $z = 2-2j$ $\quad \phi = 7\pi/4$, or $-\pi/4$

2.

a) $v(t) = 4\cos(2\pi(10.0)t + \pi/6)$

$A = 4$

$\omega = 2\pi \times 10$ $\quad$ Phasor: $V(t) = 4e^{j\pi/6}$

$\phi = \pi/6$

b) $v(t) = 2\sin(2\pi(10)t - \pi/3)$

Phasor:

$V(t) = 2e^{-j\pi/3}$

Fourier analysis

$$f(x) = \begin{cases} 1, & 0 \le x \le \pi \\ 0, & \pi < x \le 2\pi \end{cases}$$

$$A_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)$$

$$B_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)\,dx$$

$$F(n) = A_n - jB_n = \int f(t)\cos(nt)\,dt - j\int f(t)\sin(nt)\,dt$$

so, $F(n) = \int f(t)\exp(-jnt)\,dt$

* t has to be unit-less. So lets use t/T.

$F(n) = \frac{1}{T}\int f(t)\exp(-jnt/T)\,dt$

- an integer n selects the lowest harmonic frequency f, which is $f = n(\frac{1}{T}) = n/T$, in angular frequency $\omega = 2\pi f = \; = n/T$

$F(n) = \frac{1}{T}\int_{-T}^{T} f(t)\exp(-2j\pi ft)\,dt = \frac{1}{T}\int_{-T}^{T} f(t)\exp(-j\omega t)\,dt$

$F(\omega) = \lim_{T\to\infty} \frac{1}{T}\int_{-T}^{T} f(t)\exp(-j\omega t)\,dt$

$F(\omega) = \int_{-\infty}^{\infty} f(t)\exp(-j\omega t)\,dt$

$f(t)=1$: $\frac{1}{2\pi}\int_0^{\pi} e^{-j\omega t}\,dt = \frac{1}{2\pi}\int_0^{\pi}\cos(\omega t)\,dt - j\frac{1}{2\pi}\int_0^{\pi}\sin(\omega t)\,dt$

$\Rightarrow \frac{1}{2\pi}\left[\frac{1}{\omega}\sin(\omega t)\right]_0^{\pi} - j\left(\frac{1}{2\pi}\right)\left[-\frac{1}{\omega}\cos(\omega t)\right]_0^{\pi}$

$0 \quad + j\frac{1}{2\pi}\left(\frac{1}{\omega} + \frac{1}{\omega}\right) = j\frac{1}{\pi\omega}$

$f(t)=0$: $\frac{1}{2\pi}\int_{\pi}^{2\pi}\cos(\omega t)\,dt - j\frac{1}{2\pi}\int_{\pi}^{2\pi}\sin(\omega t)\,dt$

$\Rightarrow \frac{1}{2\pi}\left[\frac{1}{\omega}\sin(\omega t)\right]_{\pi}^{2\pi} + j\frac{1}{2\pi}\left[\frac{1}{\omega}\cos(\omega t)\right]_{\pi}^{2\pi}$

$\Rightarrow \frac{1}{2\pi\omega}(0) + j\frac{1}{2\pi\omega}(1-1) = 0$

1. $\phi = -j\omega t$, $f = t^{-1}$

SO, $\frac{\phi}{f} = \frac{-j\omega t}{t^{-1}} = -j\omega t^2$

2. the magnitude $= 0$ when the function is between π & 2π.

Probability & statistics

range $[0, 1]$

$$\int_{-\infty}^{\infty} P(x)\, dx = 1$$ ensures normalized.

1. $P(x) = \sqrt{\frac{1}{2\pi \sigma_x^2}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma_s}\right)^2\right)$

2. the statistical mean is defined as $\underline{\mu}$:

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

3. The standard deviation described s σ:

$$\sigma^2 = \frac{1}{N-1}\sum_{i=1}^{N} (x_i - \mu)^2$$