Digital Signal Processing: COSC360

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Complex numbers 1: theory and examples

Complex numbers 1

Convert each of these complex numbers to polar form:

1. $z = 4 + 4j$ ... $z = \sqrt{32}\,\exp(j\pi/4)$
2. $z = 1$, $z = j$, $z = -1$, $z = -j$ ... All magitudes 1, the phases are 0 deg, 90 deg, 180 deg, and 270 deg, respectively.
3. In the previous problem, describe in words what is happening to the *phase angle* of each number. ... Rotating by 90 degrees each time.

Convert each of these complex numbers to rectangular form ($z = x + jy$).

1. $z = 2\exp(j\pi/4)$ ... $z = \sqrt{2} + j\sqrt{2}$
2. $z = 5\exp(j\pi)$ ... $z = -5 + 0j$.

Complex numbers 1

Suppose that $x_i = 2\pi ft + \phi_i$. The sum of two sinusoids *in the complex plane* with amplitudes a_1 and a_2 can then be written

$$V(t) = a_1 \exp(jx_1) + a_2 \exp(jx_2) \tag{1}$$

It is assumed that we would take the real part of $V(t)$ to be physical.

1. Compute $|V|^2 = V^*V$, and $\phi_2 - \phi_1 = \pi$, $\phi_2 - \phi_1 = 0$.
2. What is $\phi_V = \tan^{-1}(\operatorname{Im}\{V\}/\operatorname{Re}\{V\})$ in each case?

Why do these results make sense? Thus, the complex numbers encapsulate the concepts of *constructive* and *destructive* interference.

Complex numbers 1

1. $|V|^2 = (a_1 + a_2)^2$ for $\Delta\phi = 0$, and $|V|^2 = (a_1 - a_2)^2$ for $\Delta\phi = \pi$.
2. For the phase, note that

$$\phi_V = \tan^{-1}\left(\frac{a_1 \sin(x_1) + a_2 \sin(x_2)}{a_1 \cos(x_1) + a_2 \cos(x_2)}\right) \tag{2}$$

but to simplify, you can take $t = 0$ as an example, along with $a_1 = a_2$. For $\Delta\phi = 0$, that means

$$\phi_V = \phi_1 = \phi_2 \tag{3}$$

For the $\Delta\phi - \pi$ case, the functions in the numerator and denominator are out of phase, so they cancel and ϕ_V approaches 45 degrees.

Complex numbers 3: Application to AC cicuits

Recompute $h(\omega)$, but start with $L = 0$ $(Z_2 = 0)$. The answer is

$$h(\omega) = \frac{1}{1 + j\omega\tau} \tag{4}$$

where $\tau = RC$. Plotting this function shows that it is about 1 for low frequencies, but decreases to zero for high frequencies. Frequencies are relative to $1/\tau$.