Homework 2, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- $\bar{x} = \frac{1}{N}\sum_{i=0}^{N-1} x_i$ ... Sample mean.
- $\overline{x^2} = \frac{1}{N}\sum_{i=0}^{N-1} x_i^2$ ... Sample mean of the square.
- $s = \frac{1}{N-1}\sum_{i=0}^{N-1}(x_i - \bar{x})^2$ ... Sample std. deviation.
- $s^2 = \overline{x^2} - \bar{x}^2$ ... Formula for the variance.
- Let a **histogram** be defined by M bins i, with the data organized into M *frequencies* H_i.
- Total number of data points in a histogram: $N = \sum_{i=0}^{M-1} H_i$
- (1) Sample mean and (2) variance from histograms:
  1. $\bar{x} = \frac{1}{N}\sum_{i=0}^{M-1} iH_i$
  2. $s = \frac{1}{N-1}\sum_{i=0}^{M-1}(i - \bar{x})^2 H_i$
- For the following two formulas: $\omega = 2\pi f$, $\tau = RC$.
- **Low-pass filter response**, as a function of frequency:

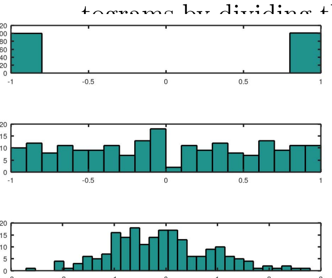
$$R(f) = \frac{1}{1 + j\omega\tau} \quad (1)$$

- **High-pass filter response**, as a function of frequency:

$$R(f) = \frac{j\omega\tau}{1 + j\omega\tau} \quad (2)$$

2 Probability and Statistics, Noise

1. Consult Fig. 2-6 in Ch. 2 of the course text. (a) Write three functions in `octave` that produce the following: a square wave, a triangle wave, and gaussian noise. (b) Write code that creates histograms of the outputs of the three functions. (c) **Normalize** the histograms by dividing the frequencies by the total num- ... V. (d) Graph the histograms to ... process matches the PDFs in ... the mean and standard devia-



e) Square : mean = .0050, std Dev = 1.6025
Triangle : mean = .0050, std Dev = 0.5873
Guassian : mean = −.0930, Std Dev = 0.9833

[1] *Hint: (1) square waves with amplitudes of 0 and 1 should have a mean of 0.5, (2) this is also true of flat PDFs, which also have a standard deviation of $1/\sqrt{12}$, and (3) Eq. 2-6 in the course text gives the Gaussian PDF, which has a std. dev. of σ.*

3 ADC and DAC

1. Complete the following exercises about the precision of ADC and DAC components:

- ADC:

  (a) What is the ΔV (voltage per level) of an ADC with signals in the [0,2.55] V range with 255 levels, plus zero (8-bit precision)?

  Vrange = 2.55−0 = 2.55V
  N = 255
  $\Delta V = \frac{2.55V}{255} = 0.01V = 10mV$ per level

  (b) What is the ΔV (voltage per level) of an ADC with signals in the [0,4.095] V range with 4095 levels, plus zero (12-bit precision)?

  Vrange = 4.095−0 = 4.095V
  N = 4095
  $\Delta V = \frac{4.095}{4095} = 0.001V = 1mV$ per level

  (c) How many bits of precision, or how many voltage levels, are required for $\Delta V < 1$ mV, if signals are in the [0,12] V range?

  Vrange = 12V
  $\Delta V = 0.001V$
  $N = \frac{12}{0.001V} = 12000$
  $2^b - 1 = N$
  so
  $2^b - 1 \geq 12000$
  $2^b \geq 12001$
  $b \geq \log_2(12001)$
  $b \approx 13.53$
  $\boxed{b = 14 \text{ bits}}$

  (d) What is the digital amplitude (in counts) of a 2.52 V signal, if signals are in the [0,5] V range, and there are 2048 levels?

  Vrange = 5V
  N = 2048−1 = 2047
  $\Delta V = \frac{5V}{2047} \approx 0.00244V$
  $\text{Counts} = \frac{2.52}{0.00244V} \approx 1033$
  $\boxed{1033 \text{ Counts}}$

- DAC:

  (a) If the digital amplitude of a signal is 256 counts, and signals are in the [0,5] V range with 9.8 mV per level, what is the signal amplitude in volts?

  Counts = 256
  $\Delta V = 9.8mV = 0.0098V$
  $V_{out} = 256 * 0.0098V$
  $\boxed{= 2.5088V}$

  (b) If the digital amplitude of a signal is 2048 counts, and signals are in the [0,5] V range with max counts 4095, what is the signal amplitude in volts?

  Counts = 2048
  $V_{max} = 5V$
  max count = 4095
  $\Delta V = \frac{5V}{4095} \approx 0.00122V$
  $V_{out} = 2048 * 0.00122V \approx \boxed{2.5V}$

  (c) If the digital amplitude of a signal is 128 counts, the max counts is 511, and the analog output is 0.25 V, what is the maximum voltage?

  Counts = 128
  max Count = 511
  $V_{out} = .25V$
  $V_{max} = \frac{.25V * 511}{128} \approx \boxed{.998V}$

2. For the following exercises, refer to Fig. 3-4 in Ch. 3 of the course text.

   (a) If the sampling rate is 500 kHz, and the analog signal frequency is 50 kHz, what is the digital signal frequency?

   $f_s = 500$ kHz

   nyquist frequency $= \frac{500}{2} = 250$ kHz

   50 kHz $\leq$ 250 kHz so digital signal = analog $\boxed{= 50 \text{ kHz}}$

   (b) If the sampling rate is 500 kHz, and the analog signal frequency is 250 kHz, what is the digital signal frequency?

   $f_s = 500$ kHz

   nyquist $= \frac{500}{2} = 250$ kHz

   250 kHz = 250 kHz

   digital = f_s - analog $= 500 - 250 = 250$ kHz $\boxed{250 \text{ kHz}}$

   (c) If the sampling rate is 500 kHz, and the analog signal frequency is 750 kHz, what is the digital signal frequency?

   $n = \left(\frac{750}{500}\right) = 1.5 \approx 2$

   $f_{alias} = |750 - (2 \times 500)| = |750 - 1000| = \boxed{250 \text{ kHz}}$

   (d) If the sampling rate is 500 kHz, and the analog signal frequency is 1000 kHz, what is the digital signal frequency?

   $n = \left(\frac{1000}{500}\right) = 2$

   $f_{alias} = |1000 - (2 \times 500)| = |1000 - 1000| = \boxed{0 \text{ kHz}}$

3. Consider Fig. 3-10 in the course text. The single-pole, low-pass RC filter is depicted in the top middle section of Fig. 3-10. (a) Suppose a signal has an amplitude of 3.3 V and a frequency of 25 MHz, while $R = 10$ kΩ. What value of C is necessary to filter the signal to 0.33 V?

   $V_{in} = 3.3V$

   $V_{out} = .33V$

   $f = 25MHz = 25 \times 10^6 Hz$

   $R = 10k\Omega = 10 \times 10^3 \Omega$

   $C = ?$

   $A = \frac{.33V}{3.3V} = .1$

   $0.1 = \frac{1}{\sqrt{1 + (2\pi fRC)^2}}$

   $0.01 = \frac{1}{1 + (2\pi fRC)^2}$

   $0.01(1 + (2\pi fRC)^2) = \frac{(1 + (2\pi fRC)^2)}{1 + (2\pi fRC)^2} = 1$

   $1 + (2\pi fRC)^2 = \frac{1}{0.01} = 100$

   $(2\pi fRC)^2 = 99$

   $2\pi fRC = \sqrt{99}$

   then

   $C = \frac{\sqrt{99}}{2\pi fRC}$

   $C = \frac{\sqrt{99}}{2\pi(25 \times 10^6)(10 \times 10^3)}$

   $C = \frac{9.95}{1.57 \times 10^{12}}$

   $C \approx 6.34 \times 10^{-12} F$

   $\boxed{C \approx 6.34 pF}$

4. Consider again Fig. 3-10. The single-pole, high-pass RC filter is similar to the depiction in the top middle section of Fig. 3-10, but with the C and R switched. (a) Suppose a signal has an amplitude of 3.3 V and a frequency of 10 MHz, while $R = 10$ kΩ. What value of C is necessary to filter the signal to 0.33 V?

   $V_{in} = 3.3V$

   $V_{out} = .33V$

   $f = 10MHz = 10 \times 10^6 Hz$

   $R = 10k\Omega = 10 \times 10^3 \Omega$

   $C = ?$

   $A = 0.1$

   $0.1 = \frac{2\pi fRC}{\sqrt{1 + (2\pi fRC)^2}}$

   $0.01 = \frac{(2\pi fRC)^2}{1 + (2\pi fRC)^2}$

   $0.01(1 + (2\pi fRC)^2) = (2\pi fRC)^2$

   $0.01 + 0.01(2\pi fRC)^2 = (2\pi fRC)^2$

   $0.01 = (1 - 0.01)(2\pi fRC)^2$

   $0.01 = .99(2\pi fRC)^2$

   $\frac{0.01}{0.99} = (2\pi fRC)^2$

   $0.0101 = (2\pi fRC)^2$

   $2\pi fRC = \sqrt{0.0101}$

   $C = \frac{\sqrt{0.0101}}{2\pi fRC}$

   $C = \frac{\sqrt{0.0101}}{2\pi(10 \times 10^6)(10 \times 10^3)}$

   $C = \frac{.1005}{6.28 \times 10^{11}}$

   $C \approx .1005(6.28^{-1})(10^{-11})$

   $C \approx 1.6 \times 10^{-13} F$

   $\boxed{C \approx 160 pF}$

5. **Bonus Point:** What is the phase shift introduced by the filters in the previous two exercises?