4) $v_1(t) = 3 \cdot \exp(j(2\pi f t - \phi))$

$V_2(t) = 3 \cdot \exp(j2\pi f t)$

$\downarrow$

$V_1(t) = 3e^{j2\pi f t}e^{-j\phi}$

$V_2(t) = 3e^{j2\pi f t}$

$V_1(t) + V_2(t) = e^{j2\pi f t} \cdot (3e^{-j\phi} + 3)$

b) $z = 3e^{-j\phi} + 3$

$e^{-j\phi} = \cos(\phi) - j\sin(\phi)$

$\hookrightarrow z = 3(\cos\phi - j\sin\phi) + 3$

$= 3\cos\phi - 3j\sin\phi + 3$

$z = (3\cos\phi + 3) - j3\sin\phi$

$|z| = \sqrt{(3\cos\phi + 3)^2 + (3\sin\phi)^2}$

$\theta = \arg(z) = \tan^{-1}\left(\frac{-3\sin\phi}{3\cos\phi + 3}\right)$

$= \tan^{-1}\left(\frac{-\sin\phi}{\cos\phi + 1}\right)$

c) $\phi = 45^\circ = \pi/4$

$\cos(\phi) = \sin(\phi) = \frac{\sqrt{2}}{2}$

$3\cos\phi + 3 = 3 \cdot \frac{\sqrt{2}}{2} + 3$

$= \frac{3\sqrt{2}}{2} + 3$

≈ 5.12

$-3\sin\phi = -3\frac{\sqrt{2}}{2} \approx -2.12$

$|z| \approx \sqrt{(5.12)^2 + (-2.12)^2}$

$\approx \sqrt{26.2 + 4.5} = \sqrt{30.7} \approx 5.54$

$\theta = \tan^{-1}\left(\frac{-2.12}{5.12}\right) \approx -22.6^\circ$

$V(t) = 5.54 \cdot e^{j(2\pi f t - 22.6^\circ)} \rightarrow$ Amp $= 5.54$, Phase $= -22.6^\circ$

d) $\phi = 0^\circ$ in phase, tot amp $= 6$

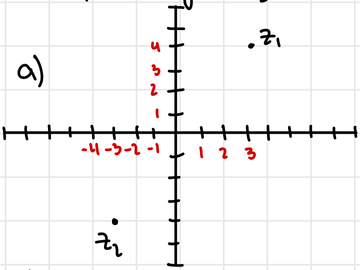
$\theta = 180^\circ$ out of phase, tot amp $= 0$

$\phi = 90^\circ$ phase difference, amp $= \sqrt{3^2 + 3^2} = 4.24$

Homework 1

① $z_1 = 3+4j \quad z_2 = -3+4j$

a)



b) $z_1 + z_2 = (3-3)+(4+4)j = 0+8j$

c) $z_1 - z_2 = (3+3)+(4-4)j = 6+0j$

d) $z_1 \times z_2 = (3+4j)(-3+4j)$
$= -9+12j-12j+16j^2$
$= -9-16 = -25$

e) z_1/z_2

$$\frac{z_1}{z_2} = \frac{3+4j}{-3+4j} \cdot \frac{-3-4j}{-3-4j} = \frac{(3+4j)(-3-4j)}{(-3+4j)(-3-4j)}$$

$$= \frac{-9-12j-16j^2}{9+16} = \frac{7-24j}{9+16} \longrightarrow \frac{7}{25} - \frac{24}{25}j$$

f) $|z_1| = \sqrt{3^2+4^2} = \sqrt{9+16} = 5$

g) $|z_2| = \sqrt{(-3)^2+4^2} = \sqrt{9+16} = 5$

h) $\phi_1 = \tan^{-1}(4/3) \approx 53.13°$

i) $\phi_2 = \tan^{-1}(4/-3) = 126.87°$

j) $z_1 = 5(\cos(0.927) + j\sin(0.927))$
$z_2 = 5(\cos(2.214) + j\sin(2.214))$

② $\cos(2\pi ft) = \dfrac{e^{2\pi jft} + e^{-2\pi jft}}{2}$

$\sin(2\pi ft) = \dfrac{e^{2\pi jft} - e^{-2\pi jft}}{2j}$

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$
$e^{-j\theta} = \cos(\theta) - j\sin(\theta)$

$\theta = 2\pi ft$

$e^{j2\pi ft} = \cos(2\pi ft) + j\sin(2\pi ft)$
$e^{-j2\pi ft} = \cos(2\pi ft) - j\sin(2\pi ft)$

$e^{j2\pi ft} + e^{-j2\pi ft} = [\cos(2\pi ft) - j\sin(2\pi ft)] + [\cos(2\pi ft) + j\sin(2\pi ft)]$

$\downarrow$

$$\cos(2\pi ft) = \frac{e^{j2\pi ft} + e^{-j2\pi ft}}{2}$$

$e^{j2\pi ft} - e^{-j2\pi ft} = [\cos(2\pi ft) - j\sin(2\pi ft)] - [\cos(2\pi ft) + j\sin(2\pi ft)]$

$\downarrow$

$$\sin(2\pi ft) = \frac{e^{j2\pi ft} - e^{-j2\pi ft}}{2j}$$

③ $v_1(t) = 4\cos(2\pi f_1 t)$
$v_2(t) = 4\cos(2\pi f_2 t - \phi)$

a)

$v_1(t)v_2(t) = 4\cos(2\pi f_1 t) \cdot 4\cos(2\pi f_2 t - \phi)$
$= 16 \cdot \cos(2\pi f_1 t) \cdot \cos(2\pi f_2 t - \phi)$
$= 8 \cdot [\cos(2\pi(f_1 - f_2)t + \phi) + \cos(2\pi(f_1+f_2)t - \phi)]$

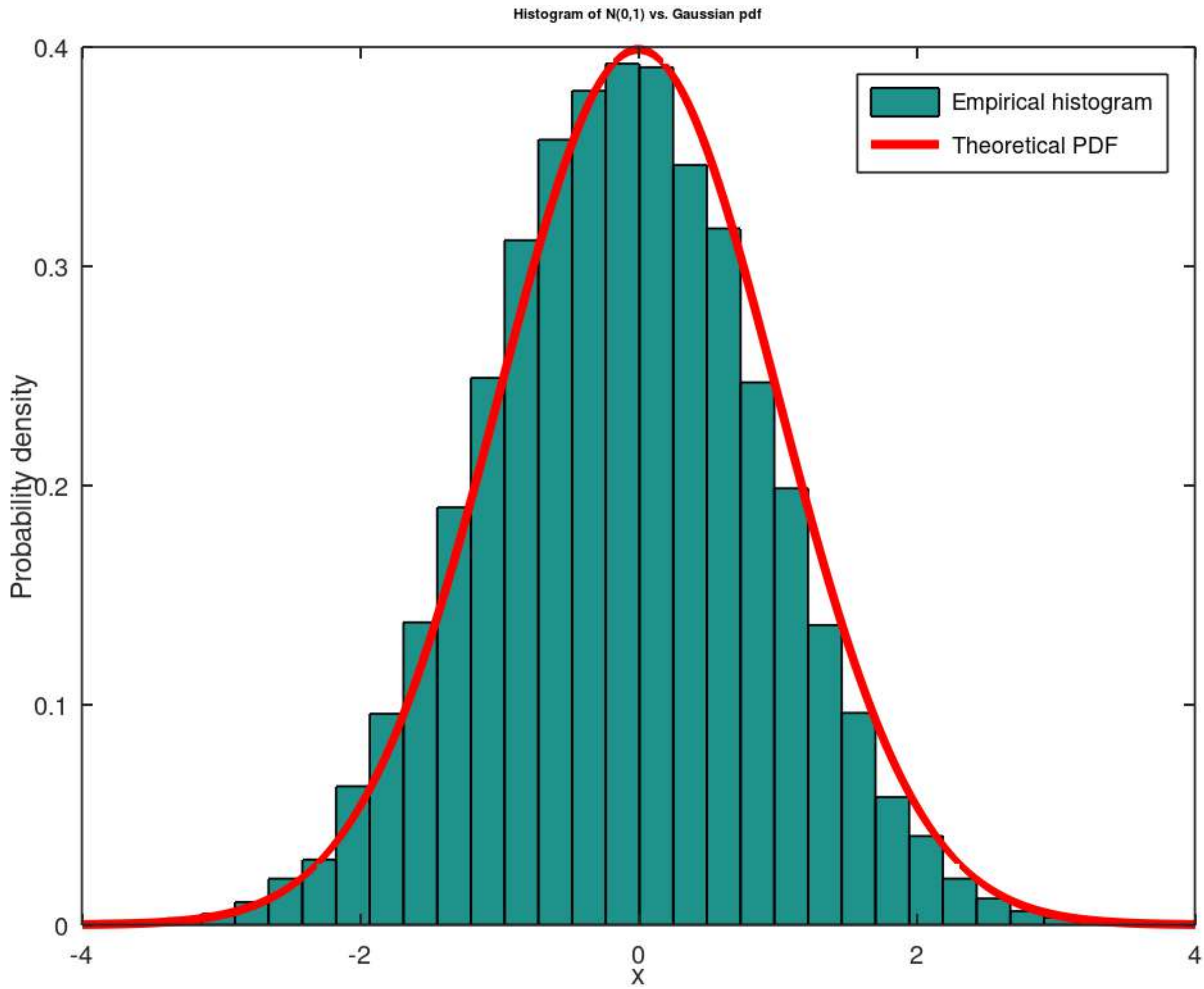
b) $P_{max} = 16 \quad f_1 = f_2 = f \quad \phi = 0$

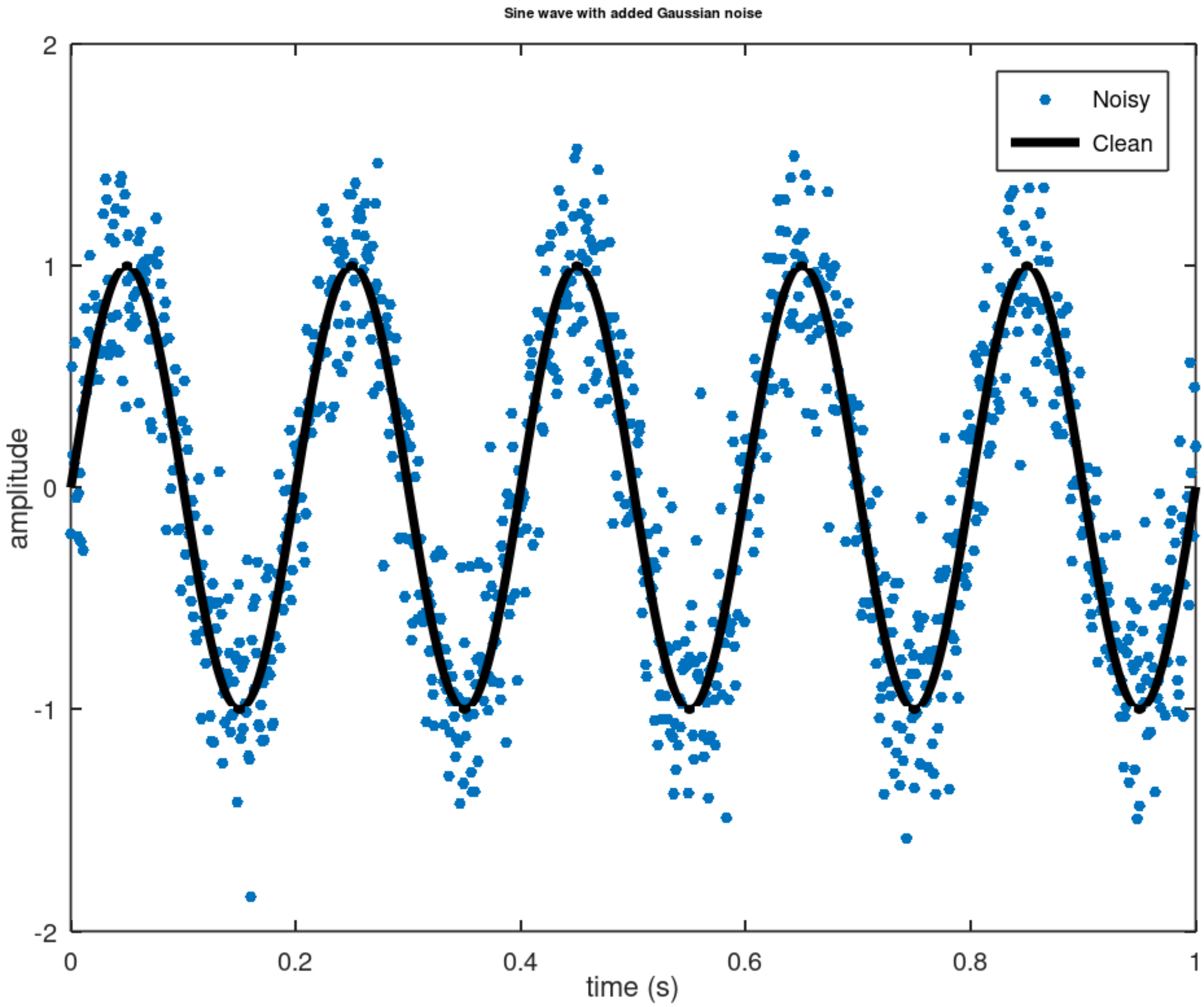
$v_1(t) = 4\cos(2\pi ft)$

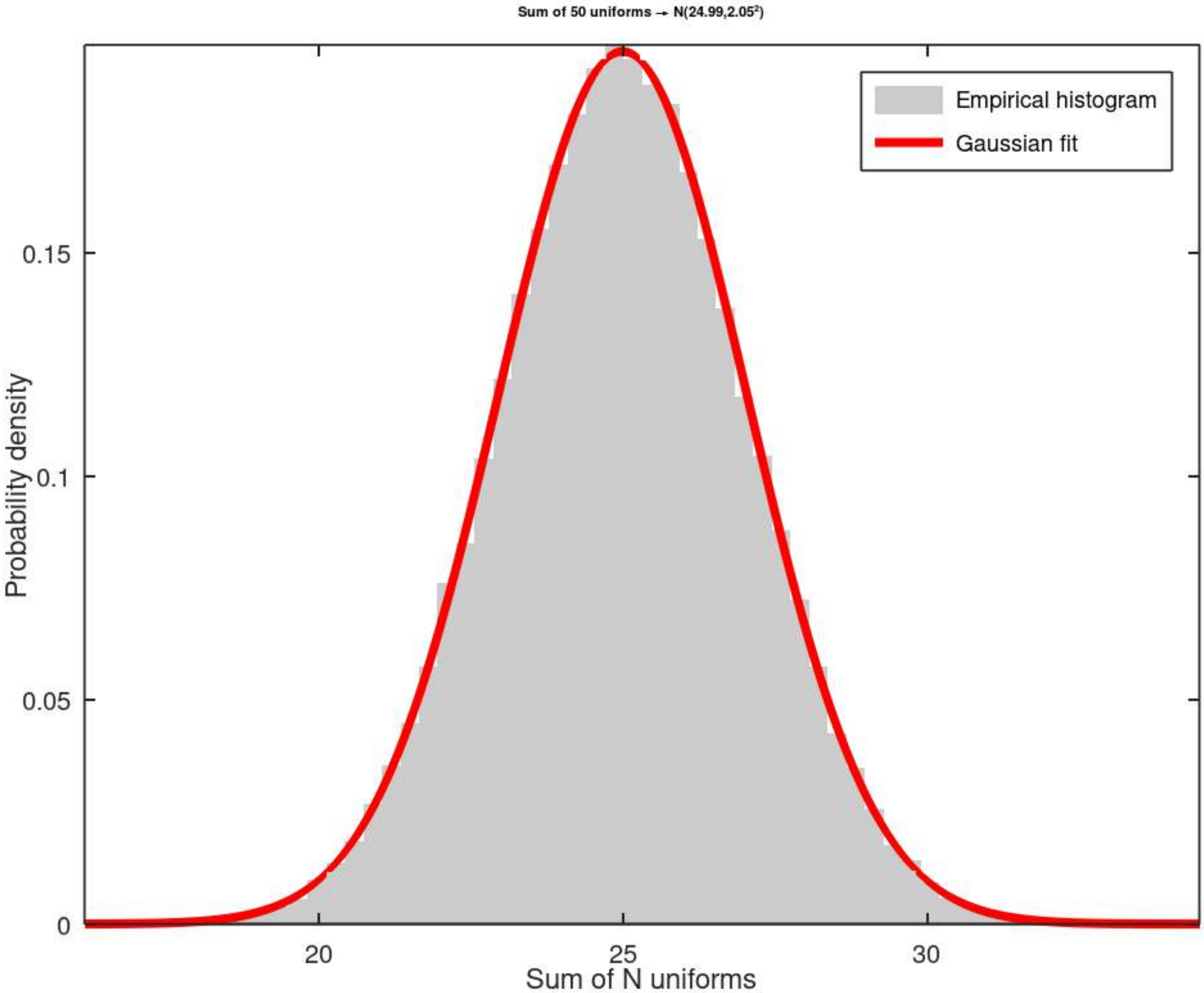
$v_2(t) = 4\cos(2\pi ft)$

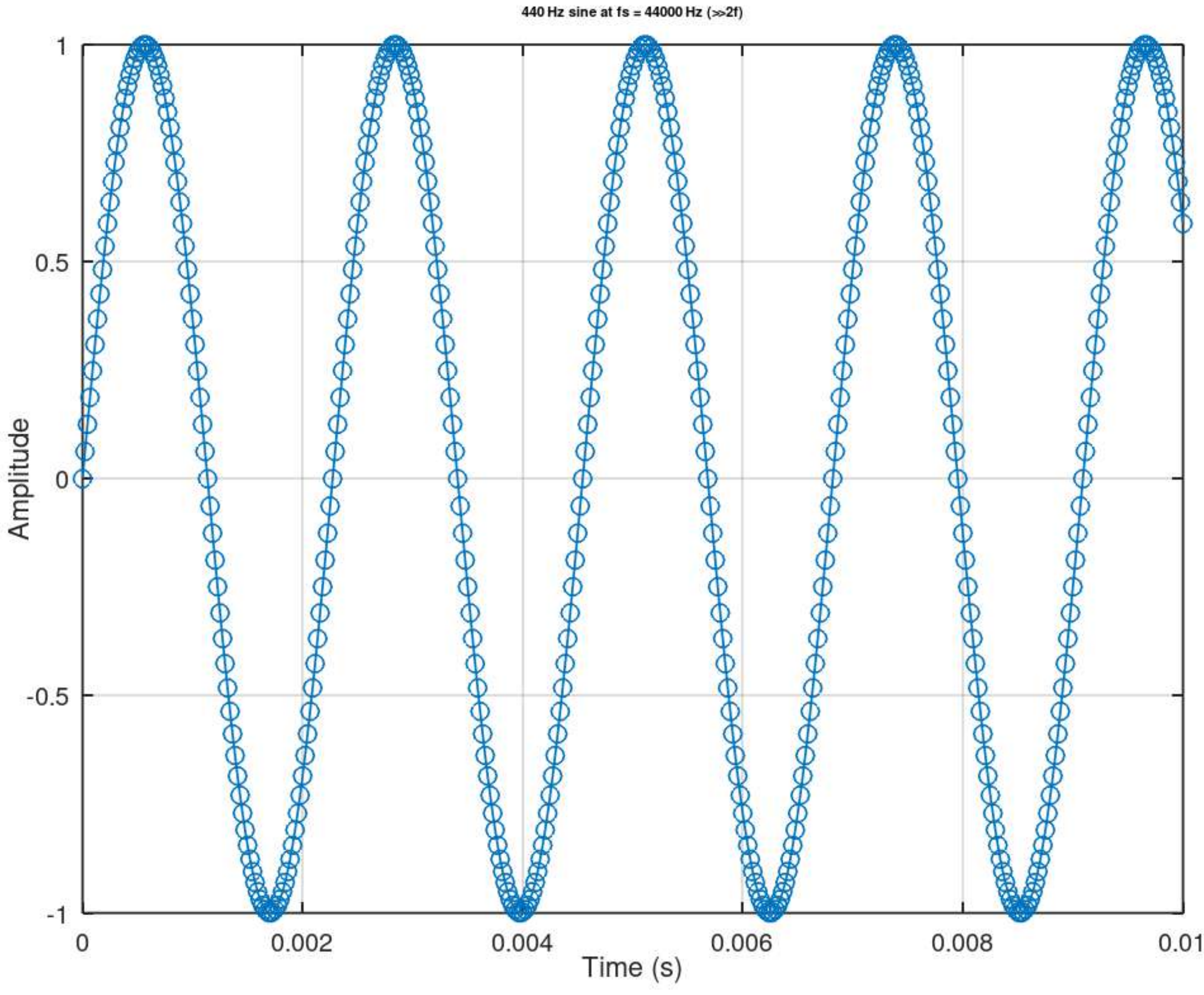
$\hookrightarrow v_1(t)v_2(t) = 4\cos(2\pi ft) \cdot 4\cos(2\pi ft)$
$= 16\cos^2\cancel{(2\pi ft)}^{1}$ $\quad \cos^2(x) \in [0,1] \rightarrow 1$
$= 16$

*both sinusoids have max amplitude 4 & when in phase
$(4 \cdot \cos(t))^2 = 16\cos^2(t) \longrightarrow max = 16$


Histogram of N(0,1) vs. Gaussian pdf
Empirical histogram
Theoretical PDF
Probability density
x
0.4
0.3
0.2
0.1
0
-4
-2
0
2
4


Sine wave with added Gaussian noise
Noisy
Clean
amplitude
time (s)
2
1
0
-1
-2
0
0.2
0.4
0.6
0.8
1


Sum of 50 uniforms → N(24.99,2.05²)
Empirical histogram
Gaussian fit
Probability density
0
0.05
0.1
0.15
20
25
30
Sum of N uniforms


440 Hz sine at fs = 44000 Hz (>>2f)
Amplitude
Time (s)
1
0.5
0
-0.5
-1
0
0.002
0.004
0.006
0.008
0.01