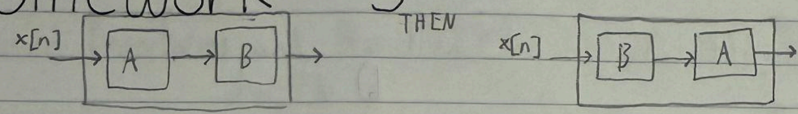
Homework # 3

1. IF



a) let $A\{x[n]\} = 2x[n] - 1$

$2(cx[n]) - 1 = 2cx[n] - 1$

$2cx[n] - 1 \neq cA\{x[n]\}$

$\boxed{\text{non linear : breaks rule of homogenity}}$

let $B\{x[n]\} = 0.5\,x[n]$

$0.5\,c\,x[n]) = cB\{x[n]\}$

$B\{x_1[n] + x_2[n]) = 0.5\,x_1[n] + 0.5\,x_2[n]$

$\boxed{\text{linear}}$

b) $2\{x[n]\} - 1 \rightarrow 2\{x[n]\}$

$A(B\{x[n]\}) = 2(0.5\{x[n]\}) = 1\{x[n]\} = x[n]$

$B(A\{x[n]\}) = 0.5(2\{x[n]\}) = x[n]$

$\boxed{A = 2\{x[n]\}}$

2. $f(t_0) = \int_{-\infty}^{\infty} f(t)\,\delta(t - t_0)\,dt$ $\quad f(t) = a_1\cos(2\pi f_1 t) + a_2\cos(2\pi f_2 t)$

$T_1 = \frac{1}{f_1}$ $\quad T_2 = \frac{1}{f_2}$ $\quad f_2 = 2f_1$

• $\int_{-\infty}^{\infty} f(t)\,\delta(t - T_1)\,dt$

$f(T_1) = a_1\cos(2\pi f_1 T_1) + a_2\cos(2\pi f_2 T_1)$

$T = \frac{1}{f_1}$ $\quad = a_1\cos(2\pi \cancel{f_1} \frac{1}{\cancel{f_1}}) + a_2\cos(2\pi 2\cancel{f_1} \frac{1}{\cancel{f_1}})$

$f_2 = 2f_1$

$= a_1\underbrace{\cos(2\pi)}_{=1} + a_2\underbrace{\cos(4\pi)}_{=1}$

$\boxed{f(T_1) = a_1 + a_2}$

• $\int_{-\infty}^{\infty} f(t)\,\delta(t - T_2)\,dt$

$f(T_2) = a_1\cos(2\pi f_2 T_2) + a_2\cos(2\pi f_2$

$T_2 = \frac{1}{f_2}$ $\quad = a_1\cos(2\pi 2f_1 \frac{1}{f_2}) + a_2\cos(2\pi 2f_1)$

$f_2 = 2f_1$

$= a_1\cos(\pi) + a_2(1)$

$\boxed{f(T_2) = -a_1 + a_2}$

3. $f(t) = a\,\delta(t-t_0)$

a)
$$F = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt$$
$$= \int_{-\infty}^{\infty} a\,\delta(t-t_0) e^{-i\omega t} dt$$
$$g(t_0) = \int_{-\infty}^{\infty} \delta(t-t_0)\, g(t)\, dt$$

$$F = ae^{-i\omega t_0} = |ae^{-i\omega t_0}|$$
$$= |a| \cdot |e^{-i\omega t_0}|$$
$$\boxed{|F| = a}$$

b) $\phi = -\omega t_0$

$f = \frac{\omega}{2\pi}$

$$\boxed{\phi = -2\pi f t_0}$$

c) $\phi' = (-\omega t_0)' \frac{d\phi}{d\omega}$

$$\boxed{T_g = t_0}$$

4.

a) $y[n] = S[x[n]] = 0.5x[n-2]$

$S[\delta[n]]$

$n = 1,\ \delta[n] = 1$

$\delta[n-2] = 1,\ n = 2$

$S[\delta[n]] = 0.5[\delta[n-2]] = 00001000$

$$\boxed{S[\delta[n]] = 0.5\delta[n-2]}$$

b) $s[n] = 01111111$

$y[n] = 0.5x[n-2]$

$s[n] = 01111111 \rightarrow s[n-2] = 00011111$

$0.5[00011111]$

Step response:
0, 0, 0, 0.5, 0.5, 0.5, 0.5, 0.5

3. Fourier Transforms and Basic Filters

1. $\omega = 2\pi f \quad \tau = RC \quad S(f)\ \frac{1}{1+j\omega\tau}$

a) $s(t) = a\delta(t-t_0)$

$= F(a\delta(t-t_0))$

$\boxed{S(f) = ae^{-j2\pi f t_0}}$

b) $S_{out} = \frac{S(f)}{1+j\omega\tau}$

$= \left|\frac{ae^{-j2\pi f t_0}}{1+j2\pi f\tau}\right| = |a| \cdot \left|\frac{1}{\sqrt{1+(2\pi f\tau)}}\right|$

$\boxed{|S_{out}| = \frac{|a|}{\sqrt{1+(2\pi f\tau)^2}}}$

c) $Y_{HP} = \frac{j\omega\tau}{1+j\omega\tau}$

$S_{out} = ae^{-j2\pi f t_0} \cdot \frac{j2\pi f\tau}{1+j2\pi f\tau}$

$\boxed{S_{out} = \frac{|a|\,2\pi f\tau}{\sqrt{1+(2\pi f\tau)^2}}}$

2. $Y_{LP} = \frac{1}{(1+j\omega\tau)} \qquad Y_{HP} = \frac{j\omega\tau}{1+j\omega\tau} \qquad \phi = \frac{\pi}{2} - \tan^{-1}(\omega\tau)$

$\boxed{T_g = \frac{2\pi\tau}{1+(2\pi f\tau)^2}}$

$\boxed{T_g = \frac{-2\pi\tau}{1+(2\pi f\tau)^2}}$

3.

a) $S(f) = \frac{a}{2}(\delta(f-f_0) + \delta[f+f_0)]$

$s(t) = \int S(f)e^{j2\pi f t}\,df$

$= \int \frac{a}{2}(\delta(f-f_0) + \delta(f+f_0))e^{j2\pi f t}\,df$

$= \frac{a}{2}(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t})$

$= \frac{a}{2}(2\cos(2\pi f_0 t))$

$= \boxed{a\cos(2\pi f_0 t)}$

b) $S(f) = \frac{a}{2j}(\delta(f-f_0) - \delta(f+f_0))$

$= \int \frac{a}{2j}[e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}]$

$= \frac{a}{2j}[2j\sin(2\pi f_0 t)]$

$= \boxed{a\sin(2\pi f_0 t)}$

4. Convolution and Octave Code

1. $y[i] = h[n] * x[n] = \sum_{j=0}^{M-1} h[j]\,x[i-j]$

a) $x[n] = \delta[n]$

$y[i] = h[i]\,\delta[i-i]$

$= h[i] \cdot 1$

$\boxed{y[i] = h[i]}$

b) $x[n] = \delta[n-n_0] \rightarrow y[n] = h[n-n_0]$

$y[i] = \sum_{j=0}^{M-1} h[j]\,x[i-j]$

$y[i] = \sum_{j=0}^{M-1} h[j]\,\delta[i-j-n_0]$

$\delta[i-j-n_0] = 1$

$\boxed{y[i] = h[i-n_0]}$

2.

```
fs = 8000;
t = 0:1/fs:0.02;
f = 440;
x = sin(2 * pi * f * t);
n0 = 20;
impulse = zeros(1, length(x));
impulse(n0 + 1) = 1;
y = conv(x, impulse, 'same');

subplot(2,1,1);
plot(t, y(1:length(t)));
title(['phase shift sine wave (n_0 = ', num2str(n0), ')']);
xlabel('Time (s)');
ylabel('Amplitude');
```