Tuesday Warm Up, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- **Homogeneous system:** Let k be a constant, and let $s_{\text{in}}(t)$ and $s_{\text{out}}(t)$ be the input and output signals to a system S, respectively. S is *homogeneous* if:

$$s_{\text{out}}(t) = S[s_{\text{in}}(t)] \tag{1}$$
$$k s_{\text{out}}(t) = S[k s_{\text{in}}(t)] \tag{2}$$

- **Additive system:** Let $s_1(t)$ and $s_2(t)$ be two input signals to a system S, with outputs $s_1'(t)$ and $s_2'(t)$. S is *additive* if:

$$s_1'(t) = S[s_1(t)] \tag{3}$$
$$s_2'(t) = S[s_2(t)] \tag{4}$$
$$s_1'(t) + s_2'(t) = S[s_1(t) + s_2(t)] \tag{5}$$

- **Shift-invariant system:** Let $s_{\text{in}}(t)$ and $s_{\text{out}}(t)$ be input and output signals to a system S, and let t_0 be a constant. S is *shift invariant* if:

$$s_{\text{out}}(t) = S[s_{\text{in}}(t)] \tag{6}$$
$$s_{\text{out}}(t - t_0) = S[s_{\text{in}}(t - t_0)] \tag{7}$$
$$\tag{8}$$

- **Synthesis:** combining input signal components together linearly to form an output signal.
- **Decomposition:** producing the output signal components linearly from an input signal.
- **Fundamental Concept of DSP:** Decomposing an input signal into components, passing them trough a linear system, and synthesizing the results produces the same output as passing the original signal through the system.
- **Impulse signal:** a single nonzero point in a string of zeros.
- **Impulse decomposition:** decomposing a digitized, sampled signal into a linear combination of impulse signals.
- **Even/Odd decomposition:** decomposing a digitized, sampled signal into even and odd signal components.

2 Linear Systems

1. Let a system S act on a signal $s(t)$ as follows: $S[s(t)] = s(t - T/2)$. (a) If $s(t) = 2\sin(2\pi f t)$, and $T = 1/f$, what is $S[s(t)]$? (b) Graph the input and output of S. (c) What is $s(t) + S[s(t)]$?

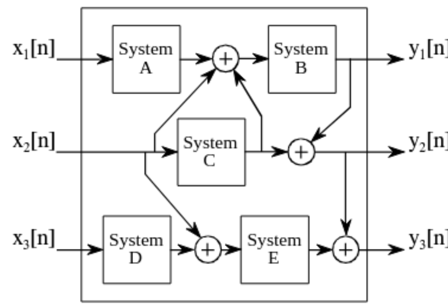

Figure 1: A DSP system with multiple inputs and outputs.

2. Let a system S act on a signal $s(t)$ as follows: $S[s(t)] = 2s - 1.5$. (a) If $s(t) = 1.5\sin(2\pi(60)t)$, what is $S[s(t)]$? (b) If $s(t) = -1.5\sin(2\pi(60)t)$, what is $S[s(t)]$? (c) Graph the outputs of (a) and (b). (d) Consider Fig. 1 (bottom). Develop expressions for $y_1[n]$, $y_2[n]$, and $y_3[n]$, and show that each is linear.

3. Suppose a signal component is the impulse $x[n] = [0\ 0\ 0\ 2\ 0\ 0...]$, with 100 total samples. (a) If $y[n] = S(x[n]) = -x[n-1]$, what is $y[n]$? (b) If $y[n] = S(x[n]) = (x[n])^2$, what is $y[n]$? (c) Are the systems S in parts (a) and (b) linear or non-linear?

4. Let $x[n]$ be a digitized, sampled signal with N samples. The *even* component is $x_{\text{E}} = (x[n] + x[N-n])/2$, and the *odd* component is $x_{\text{O}} = (x[n] - x[N-n])/2$. (a) Let $x[n] = [0\ 0\ 1\ 1\ 0\ 0]$. Is it even or odd? (b) Let $x[n] = [0\ -1\ 0\ 0\ 1\ 0]$. Is it even or odd?