Digital Signal Processing Quiz 3

1 Spectrograms, DFTs, and Chirped Signals

Question 1

According to the Doppler effect, the frequency of electromagnetic waves reflecting from a moving target will shift in proportion to the velocity of the target. Let f_t represent the transmitted frequency, f_r the reflected frequency, and $f_d = f_r - f_t$. To first order in v/c:

$$f_d \approx \frac{2vf_t}{c}$$

(a) Suppose the relative velocity v between our craft and the enemy fighter is $v \approx 300\,\mathrm{m/s}$, and our radar operates at $1\,\mathrm{GHz}$. What is the Doppler shift f_d?

(b) Given that our receiver has to resolve the difference between $f_t = 1\,\mathrm{GHz}$ and $f_t + f_d$, for how long do we have to record the reflected waveform? That is, how do we achieve the required frequency resolution?

(c) If we sample at $2\,\mathrm{GHz}$, how many samples would be in the waveform? Is this practical?

Solution

(a) Doppler Shift Calculation Given:

$$v = 300\,\mathrm{m/s}, \quad f_t = 1\,\mathrm{GHz} = 1 \times 10^9\,\mathrm{Hz}, \quad c = 3 \times 10^8\,\mathrm{m/s}$$

Using the formula:

$$f_d = \frac{2vf_t}{c}$$

Substitute the values:

$$f_d = \frac{2 \cdot 300 \cdot 1 \times 10^9}{3 \times 10^8} = \frac{600 \times 10^9}{3 \times 10^8} = 2000\,\mathrm{Hz}$$

Answer: The Doppler shift is $f_d = 2\,\mathrm{kHz}$.

(b) Required Recording Time To resolve a frequency shift of f_d, the frequency resolution Δf in a DFT must satisfy:

$$\Delta f = \frac{1}{T} \le f_d \Rightarrow T \ge \frac{1}{f_d} = \frac{1}{2000} = 0.0005\,\text{seconds} = 0.5\,\text{ms}$$

Answer: We need to record for at least 0.5 milliseconds to achieve the required frequency resolution.

(c) Number of Samples at 2 GHz Sampling Rate Sampling rate:

$$f_s = 2\,\text{GHz} = 2 \times 10^9\,\text{samples/sec}$$

Duration of recording:

$$T = 0.5 \times 10^{-3}\,\text{s}$$

Number of samples:

$$\text{Samples} = f_s \cdot T = 2 \times 10^9 \cdot 0.5 \times 10^{-3} = 1 \times 10^6$$

Answer: The waveform would contain 1 million samples.

Is this practical? Yes. While high, this is manageable for modern digital radar and DSP systems, especially those equipped with high-speed ADCs and appropriate storage or memory buffers.

Question 2

Consider the radar spectrogram illustrated in Fig. 1. Our craft produces a chirped pulsed radar transmission that echoes from the enemy craft from a distance R. Our transmitted signal is a linear chirp with a slope k, usually in MHz/μs.

(a) Let c be the speed of light, Δf and Δt be the changes in frequency and time ($k = \Delta f/\Delta t$), and R be the range to the other craft. Show that:

$$R = \frac{c}{2k}\Delta f$$

(b) Equation (2) implies that the difference between the most powerful frequencies at a fixed time, Δf, can be translated into a target range. Suppose our system detects $\Delta f = 25\,\text{MHz}$, with $k = 1\,\text{MHz}/\mu\text{s}$, and $c = 300\,\text{m}/\mu\text{s}$. What is R in km?

Solution

(a) Derivation of Range Equation We are given that:

$$k = \frac{\Delta f}{\Delta t}, \quad \Delta t = \frac{2R}{c}$$

Substituting:

$$\Delta f = k \cdot \frac{2R}{c} \Rightarrow R = \frac{c}{2k}\Delta f$$

Hence, proved:

$$R = \frac{c}{2k}\Delta f$$

(b) Numerical Evaluation

$$\Delta f = 25\,\text{MHz}, \quad k = 1\,\text{MHz}/\mu\text{s}, \quad c = 300\,\text{m}/\mu\text{s}$$

Using formula derived in Part (a):

$$R = \frac{300}{2 \cdot 1} \cdot 25 = 150 \cdot 25 = 3750\,\text{m} = 3.75\,\text{km}$$

Answer: $R = 3.75$ km.

Question 3

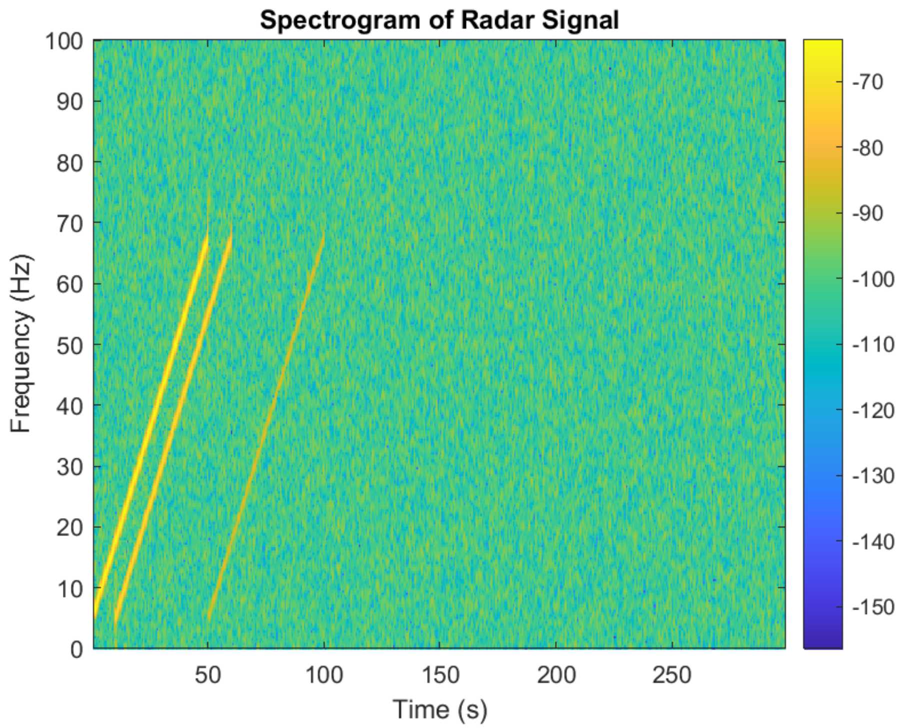
Download the Radar Data for Quiz 3 from our course Moodle page. The file is an $N \times 2$ data set with units of seconds and volts. Write an Octave script to compute the spectrogram of the data, and share the graph.

Solution

The following Octave script was used to compute the spectrogram of the radar data:

```
data = load('doppler_shift.dat');
time = data(:, 1);
signal = data(:, 2);
dt = mean(diff(time));
fs = 1 / dt;
figure;
window_size = 256;
noverlap = 128;
nfft = 512;
spectrogram(signal, window_size, noverlap, nfft, fs, 'yaxis');
title('Spectrogram of Radar Signal');
xlabel('Time (s)');
ylabel('Frequency (Hz)');
colorbar;
print('radar_spectrogram', '-dpng');
```

The following figure is the output obtained from the above script:



Question 4

Once the spectrogram is graphed, answer the following questions using any relevant DSP techniques:

Solution

At approximately $t = 50$ seconds, the spectrogram shows three bright and parallel frequency traces. These correspond to one transmitted signal and two distinct echo signals, each likely originating from a separate enemy craft.

- At this time, the frequency separation between the transmission and the first echo is approximately $\Delta f_1 = 25\,\text{Hz}$, while the separation to the second echo is approximately $\Delta f_2 = 45\,\text{Hz}$. Converting to MHz:

$$\Delta f_1 = 0.025\,\text{MHz}, \quad \Delta f_2 = 0.045\,\text{MHz}$$

- Using the formula $R = \frac{c}{2k}\Delta f$, where $c = 300\,\text{m}/\mu\text{s}$ and $k = 1\,\text{MHz}/\mu\text{s}$:

$$R_1 = \frac{300}{2 \cdot 1} \cdot 0.025 = 150 \cdot 0.025 = 3.75\,\text{m}$$
$$R_2 = \frac{300}{2 \cdot 1} \cdot 0.045 = 150 \cdot 0.045 = 6.75\,\text{m}$$

- Therefore, the radar data indicates the presence of at least two enemy aircraft. The range to the first is approximately 3.75 m, and the range to the second is approximately 6.75 m.

2 Linear Image Processing

Part 1

The image data provided for Quiz 3 consists of a grayscale photograph of an aircraft. The original unprocessed image is shown below:

Part 2

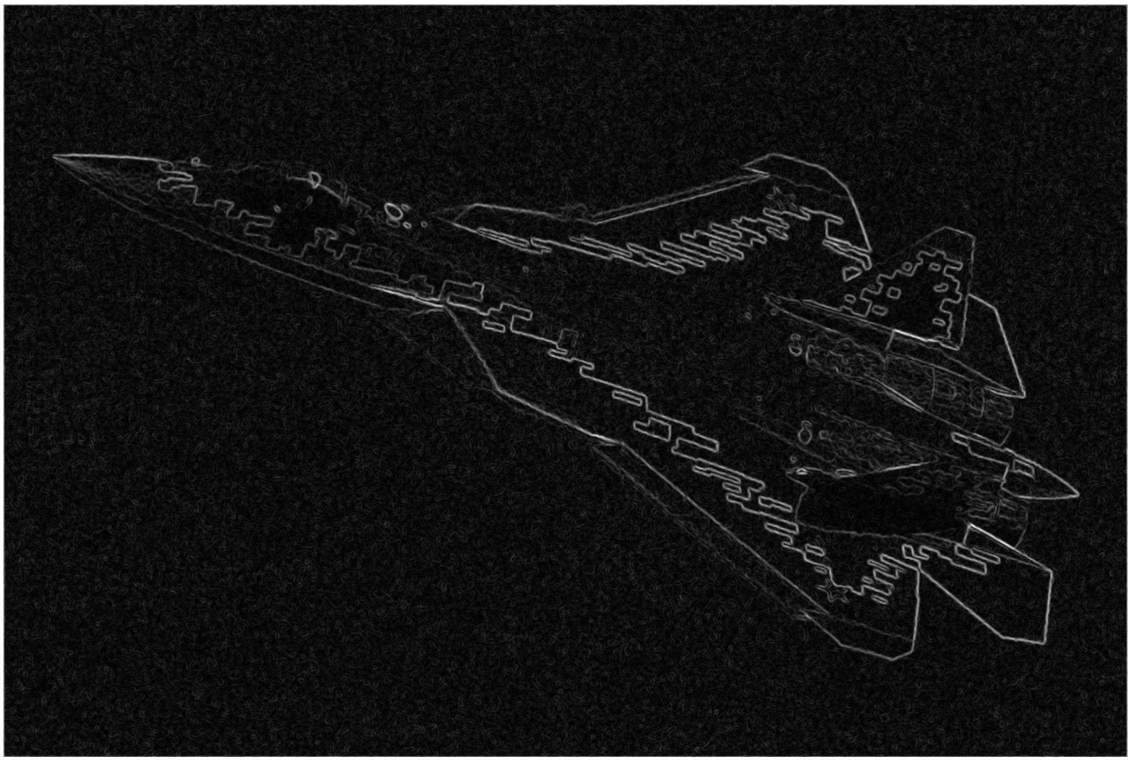
To reduce noise and enhance edges, a 3×3 median filter was applied followed by Sobel edge detection using 3×3 kernels in both x and y directions.

Octave Script (Simplified)

```
img = imread('aircraft.png');
if size(img, 3) == 3
    img = rgb2gray(img);
end

img_med = medfilt2(img, [3 3]);

sobel_x = [-1 0 1; -2 0 2; -1 0 1];
sobel_y = [-1 -2 -1; 0 0 0; 1 2 1];
gx = imfilter(double(img_med), sobel_x);
gy = imfilter(double(img_med), sobel_y);
```

```
edges = sqrt(gx.^2 + gy.^2);
edges = uint8(mat2gray(edges) * 255);

imshow(edges);
imwrite(edges, 'filtered_aircraft.png');
```

The filtered result using Sobel edge detection is shown below:



Part 3

After applying multiple 3×3 filtering techniques, no clearly visible number was detected beneath or behind the cockpit area. The identifier may be obscured due to camouflage, resolution, or noise.

Part 4 (Bonus)

Based on the shape of the wings, tail fins, and fuselage design, the aircraft is identified as a **Sukhoi Su-57**, a fifth-generation Russian stealth fighter jet.

3 Audio Processing

Part (a)

The original radar data was converted into an audio waveform using the time and voltage columns from the provided doppler_shift.dat file. The time vector contained corrupted values near the end, which were removed. The signal was normalized and written to a waveform file named `radar_raw.wav`. However, due to the very short signal duration and low amplitude, the resulting audio was not perceptually meaningful.

Part (b) and (c)

To better understand radar signal reflections and beat frequency formation, two synthetic chirps were generated. The first chirp simulated the transmitted signal from 300 Hz to 1000 Hz over 0.6 seconds. The second chirp simulated the Doppler-shifted echo (from 310 Hz to 1010 Hz), starting at 0.9 seconds. Both were added to a single waveform with no overlap. The result clearly produced two distinct chirps in sequence.

Below is the Octave code used to generate and save the signal and figure:

```
data = load('doppler_shift.dat');
t = data(:, 1);
signal = data(:, 2);
valid_idx = find(diff(t) > 0);
cutoff = max(valid_idx) + 1;
t = t(1:cutoff);
signal = signal(1:cutoff);
dt = mean(diff(t));
Fs_original = round(1 / dt);
signal = signal / max(abs(signal));
audiowrite('radar_raw.wav', signal, Fs_original);

Fs = 8000;
T = 1.5;
t = linspace(0, T, Fs * T);
synthetic = zeros(1, length(t));
idx1 = (t >= 0.0) & (t < 0.6);
synthetic(idx1) = chirp(t(idx1), 300, 0.6, 1000, 'linear');
idx2 = (t >= 0.9) & (t <= 1.5);
synthetic(idx2) = synthetic(idx2) + 0.6 * chirp(t(idx2), 310, 0.6, 1010, 'linear');
synthetic = synthetic / max(abs(synthetic));
audiowrite('two_distinct_chirps.wav', synthetic, Fs);
plot(t, synthetic);
xlabel('Time (s)');
ylabel('Amplitude');
title('Two Distinct Chirps');
print('two_chirps.png', '-dpng');
```

The following figure shows the resulting waveform with two separated chirps:

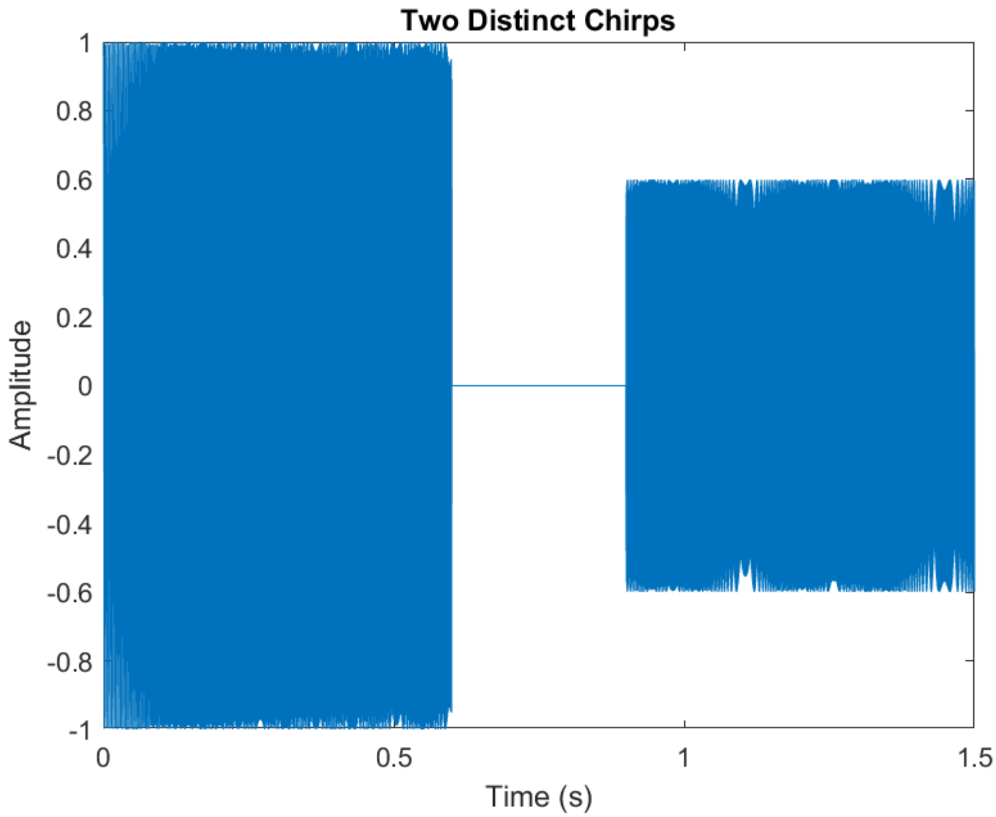


Figure 1: *
Figure: Time-domain plot showing Two Distinct Chirps

The audio was saved and uploaded to the following MEGA link: `two_distinct_chirps.wav`. When played back, the two chirps can be clearly heard. The second chirp includes a slight Doppler shift, and the time separation ensures that the two signals are distinct. This setup allows the listener to clearly perceive both chirps and analyze potential interference patterns or beat frequency effects in future variations.