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Quiz 3: Digital Signal Processing

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1 Spectrograms, DFTs, and Chirped Signals

1. According to the **Doppler effect**, the frequency of electromagnetic waves reflecting from a moving target will shift in proportion to the velocity of the target. Let f_t represent the transmitted frequency, f_r represent the reflected frequency, and $f_d = f_r - f_t$. To first order in v/c,

$$f_d \approx 2v\frac{f_t}{c} \tag{1}$$

(a) Suppose the relative velocity v between our craft and the enemy fighter is $v \approx 300$ m s^{-1}, and our radar operates at 1 GHz. What is the Doppler shift, f_d? (b) Given that our receiver has to resolve the difference between $f_t = 1$ GHz and $f_t + f_d$, for how long do we have to record the reflected waveform? That is, how do we achieve the required frequency resolution? (c) If we sample at 2 GHz, how many samples would be in the waveform? Is this practical?

2. Consider the radar spectrogram illustrated in Fig. 1. Our craft produces a chirped pulsed radar transmission that echoes from the enemy craft from a distance R. Our transmitted signal is a linear chirp with a slope k, usually in MHz/μs. (a) Let c be the speed of light, Δf and Δt be the changes in frequency and time ($k = \Delta f/\Delta t$), and R be the *range* to the other craft. Show that

$$R = \frac{c}{2k}\Delta f \tag{2}$$

(b) Equation 2 implies that the difference between the most powerful frequencies at a fixed time, Δf, can be translated into a target *range*. Suppose our system detects $\Delta f = 25$ MHz, with $k = 1$ MHz/μs, and $c = 300$ m MHz. What is R, in km?

3. **Download the Radar Data for Quiz 3** from our course Moodle page. The file is a $N \times 2$ data set with units of seconds and volts. Write an `octave` script to compute the spectrogram of the data, and share the graph.

4. Once the spectrogram is graphed, answer the following questions, using any and all relevant DSP techniques:
   - Select a time at which the transmission and echo signals overlap. One way to measure Δf is the `findpeaks()` in the `octave` package `signal`. Calculate Δf in MHz.
   - What is the range to the enemy craft, in km?
   - *Wait ... how many enemy craft are there?* What is the distance to the **second** aircraft, in km?

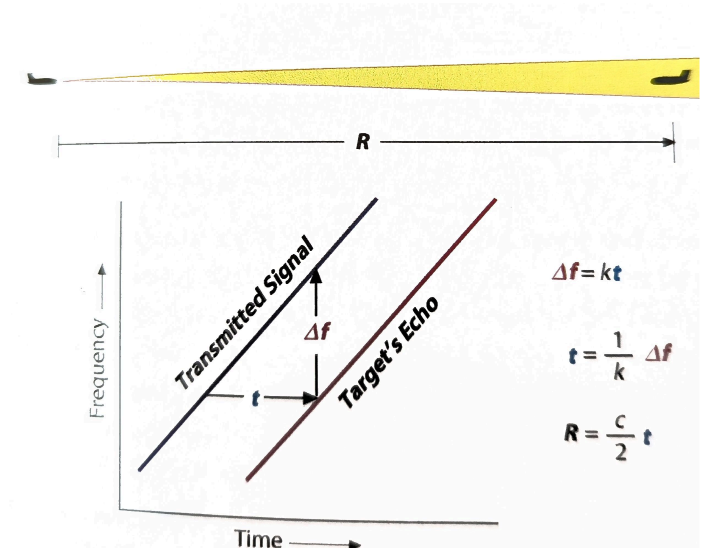


Figure 1: In a basic chirping RF signal scheme used to detect enemy aircraft, a transmitted RF signal is linearly chirped. Information in the radar echo can be used to find the range, R.

2 Linear Image Processing

1. **Download the Image Data for Quiz 3** from our course Moodle page. The file is a $N \times M$ image with greyscale pixels.
2. Create an appropriate 3×3 filter kernel to filter the image. Filter the image to reduce noise and enhance the edges.
3. What is the number written below and behind the cockpit?
4. **Bonus:** What type of aircraft is this?

3 Audio Processing

1. **Bonus:** (a) Convert the data used to produce the radar spectrogram into audio data. (b) Filter the noise as appropriate, using the filter scheme of your choice. (c) Play the audio and listen for the two chirps. Do the interfere and create a beat frequency? Sometimes, f_d in Doppler schemes is called the *beat frequency.*

1).

a) $f_d = \frac{2 v f_t}{c}$

$v = 300 m/s$

$f_t = 1 GHz = 10^9 Hz$

$c = 3\times10^8 m/s$

$$f_d = \frac{2 \cdot \cancel{3}00 m/s \cdot 10^9 Hz}{\cancel{3}\times10^8 m/s} = 2000 Hz = \underline{2kHz}$$

b). we can calculate the period of changing frequency, which is 2 kHz

$$T = \frac{1}{\Delta f} = \frac{1}{2kHz} = \underline{0.5 ms}$$

c). $N = f_s \cdot T = 2\times10^9 \text{sampl/s} \times 0.5\times10^{-3} s = \underline{10^6 \text{ samples} = 10M \text{ samples}}$

It is practical for DSP. However.

$f_s \geq 2 f_{max}$ for preventing aliasing

where in this case, $2 f_{max} = 2\times(\overbrace{GHz + 2kHz}^{f_{max}})$

But $f_s = 2GHz$

Thus. $f_s < 2 f_{max}$, which could cause aliasing

2. Consider the radar spectrogram illustrated in Fig. 1. Our craft produces a chirped pulsed radar transmission that echoes from the enemy craft from a distance R. Our transmitted signal is a linear chirp with a slope k, usually in MHz/μs. (a) Let c be the speed of light, Δf and Δt be the changes in frequency and time ($k = \Delta f / \Delta t$), and R be the *range* to the other craft. Show that

$$R = \frac{c}{2k} \Delta f \tag{2}$$

(b) Equation 2 implies that the difference between the most powerful frequencies at a fixed time, Δf, can be translated into a target *range*. Suppose our system detects $\Delta f = 25$ MHz, with $k = 1$ MHz/μs, and $c = 300$ m MHz. What is R, in km?

2). a)

$$k = \frac{\Delta f}{\Delta t}$$

$$\left.\begin{array}{l} \Delta t = \dfrac{2R}{c} \Rightarrow R = \dfrac{c\Delta t}{2} \\[2ex] \Delta f = k \Delta t \Rightarrow \Delta t = \dfrac{\Delta f}{k} \end{array}\right\} R = \frac{c}{2} \cdot \frac{\Delta f}{k} = \frac{c}{2k} \Delta f$$

b). $R = \dfrac{300}{2 \cdot 1} \times 25 = 3750\text{m} = \underline{3{,}75\text{km}}$

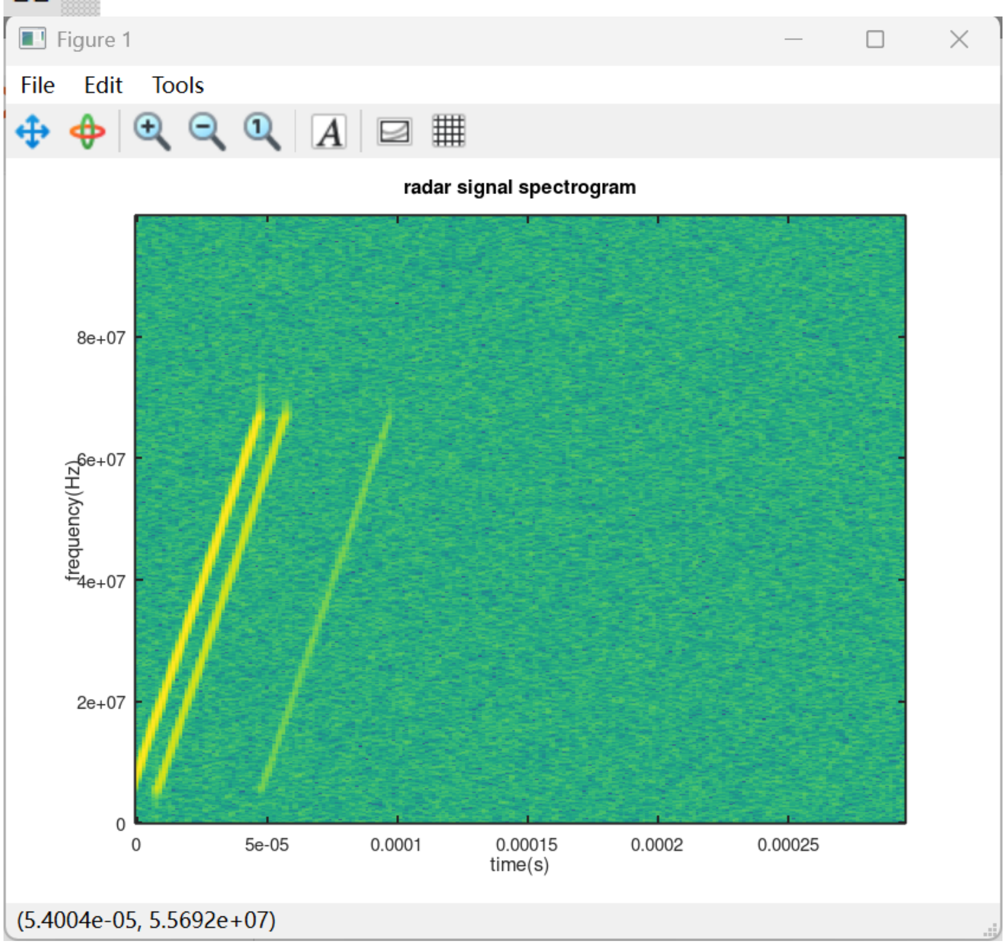
3. **Download the Radar Data for Quiz 3** from our course Moodle page. The file is a $N \times 2$ data set with units of seconds and volts. Write an `octave` script to compute the spectrogram of the data, and share the graph.

```
clear;
close;
home;
pkg load signal;
pkg list;


fname = 'C:/Users/49902/Desktop/doppler_shift.dat.dat';
data = dlmread(fname);


t = data(:,1);
v = data(:,2);
dt = t(2) - t(1);
fs = 1 / dt;

figure 1;
specgram(v, 2048, fs, 1024, 768);;
title('radar signal spectrogram');
xlabel('time(s)');
ylabel('frequency(Hz)');
```



4. Once the spectrogram is graphed, answer the following questions, using any and all relevant DSP techniques:

- Select a time at which the transmission and echo signals overlap. One way to measure Δf is the `findpeaks()` in the `octave` package `signal`. Calculate Δf in MHz.
- What is the range to the enemy craft, in km?
- *Wait ... how many enemy craft are there?* What is the distance to the **second** aircraft, in km?

a).

```
%========================================================= Problem 4 ==============================
[S, F, T] = specgram(v, 2048, fs, 1024, 768);
target_time = 3e-5;
[~, idx] = min(abs(T - target_time));%find closest time frame
slice = abs(S(:, idx));%spectrum slice @ t = 3e-5


[pks, locs] = findpeaks(slice, 'MinPeakHeight', max(slice) * 0.2);%find frequency of the peak
%Problem 4.1 find Δf
if length(locs) >= 3
    f1 = F(locs(1));
    f2 = F(locs(2));
    f3 = F(locs(3));

    delta1 = abs(f2 - f1) / 1e6;
    delta2 = abs(f3 - f1) / 1e6;

    printf('\nOverlap at t = %.5f s\n', T(idx));
    printf('peak frequencies: %.2f MHz, %.2f MHz, %.2f MHz\n', f1/1e6, f2/1e6, f3/1e6);
    printf('Δf1 = %.2f MHz\n', delta1);
    printf('Δf2 = %.2f MHz\n', delta2);
else
    disp('not detected');
end
```

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命令窗口

```
Overlap at t = 0.00003 s
peak frequencies: 32.81 MHz, 34.18 MHz, 45.12 MHz
Δf1 = 1.37 MHz
Δf2 = 12.30 MHz
>>
```

$$\left.\begin{array}{l}\Delta f_1 = 1.37\text{ MHz}\\ \Delta f_2 = 12.3\text{ MHz}\end{array}\right\}\ 2 \text{ enemy aircraft.}$$

b). $R_1 = \frac{c}{2k} \cdot \Delta f_1 = \frac{300}{2} \cdot 1.37 = \underline{0.21 \text{ km}}$

$R_2 = \frac{c}{2k} \cdot \Delta f_2 = \frac{300}{2} \cdot 12.3 = \underline{1.85 \text{ km}}$

c). Two enemy aircraft detected. closest one is 210 meters away, further one is 1.85 km away.

2 Linear Image Processing

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2. Create an appropriate 3×3 filter kernel to filter the image. Filter the image to reduce noise and enhance the edges.
3. What is the number written below and behind the cockpit?
4. **Bonus:** What type of aircraft is this?

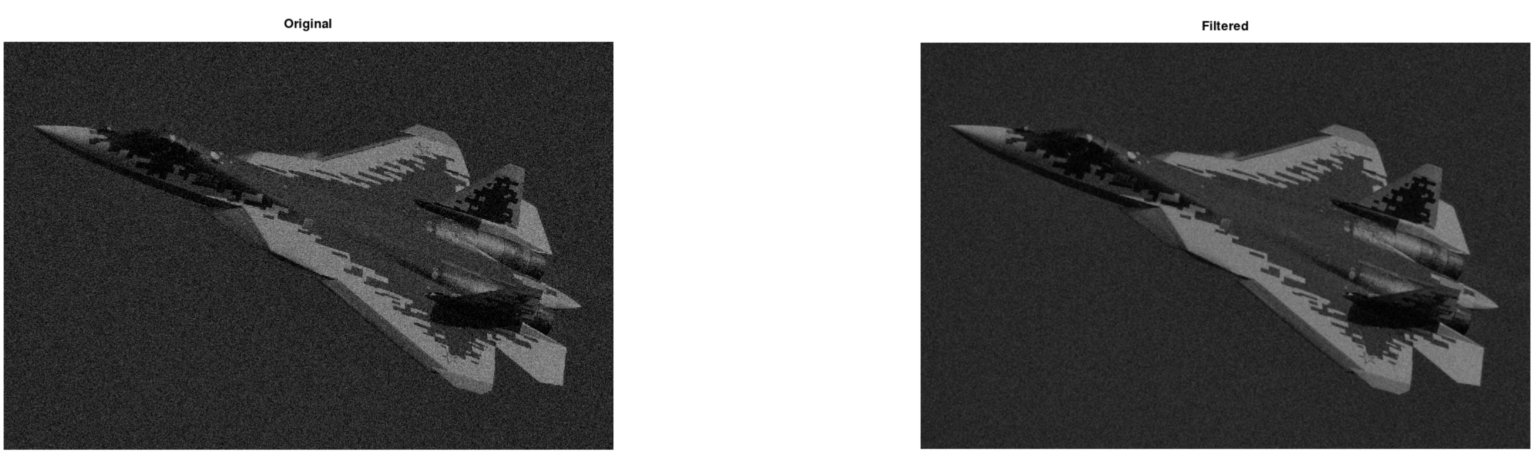
1), 2).

```
clear;
close;
home;
pkg load signal;

I = imread('C:/Users/49902/Desktop/aircraft.jpg');


% 3*3 average filter
K = ones(3, 3) / 9;
I_filtered = conv2(I, K, 'valid'); %2d convolution

subplot(1,2,1); imshow(uint8(I)); title('Original');
subplot(1,2,2); imshow(uint8(I_filtered)); title('Filtered');
title('original vs filtered img');
```



3). The number is 054

4? I have 0 knowedge about aircraft, But I looked up on Wiki and find the original picture :D

The **Sukhoi Su-57** (Russian: Сухой Су-57; NATO reporting name: **Felon**)[5][6] is a twin-engine stealth multirole fighter aircraft developed by Sukhoi.[7] It is the product of the PAK FA (Russian: ПАК ФА, prospective aeronautical complex of front-line aviation) programme, which was initiated in 1999 as a more modern and affordable alternative to the MFI (Mikoyan Project 1.44/1.42). Sukhoi's internal designation for the aircraft is **T-50**. The Su-57 is the first aircraft in Russian military service designed with stealth technology and is intended to be the basis for a family of stealth combat aircraft.

Su-57