Quiz 1

1

a) Show that $v(t) = \Re\{2.5e^{j\phi}\}$

where $v(t) = 2.5\cos(2\pi f t - \pi/4)$ $f = 1\ \text{KHz}$ $\phi = 2\pi f t - \pi/2$

use Euler's formula:

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{j\phi} = \cos\phi + j\sin\phi$$

Multiply by 2.5

$$2.5^{j\phi} = 2.5\cos\phi + j2.5\sin\phi$$

take the real part

$$\Re\{2.5^{j\phi}\} = 2.5\cos\phi = v(t)$$

b) Show that $v(t) = \Im\{2.5e^{j(\phi-\pi/2)}\}$

$$e^{j(\phi-\pi/2)} = \cos(\phi-\pi/2) + j\sin(\phi-\pi/2)$$

Multiply 2.5

$$2.5e^{j(\phi-\pi/2)} = 2.5\cos(\phi-\pi/2) + j2.5\sin(\phi-\pi/2)$$

take imaginary part

$$\Im\{2.5e^{j(\phi-\pi/2)}\} = 2.5\sin(\phi-\pi/2)$$

Using trig identity

$$\sin(x-\pi/2) = -\cos(x)$$

$$\sin(\phi-\pi/2) = -\cos\phi$$

Therefore

$$\Im\{2.5e^{j(\phi-\pi/2)}\} = -v(t)$$

2.

$$T = 1/f$$

a) $f = 1\,kHz$

$$T = \frac{1}{f} \Rightarrow \frac{1}{1\,kHz} = \frac{1\,Hz}{1000\,Hz} = \boxed{0.001 \text{ seconds}}$$

b) $T = \frac{1}{f} = f = \frac{1}{T}$

$$\frac{1}{0.5ns} = \frac{1}{5\cdot 10^{-9}} = \boxed{2.0\cdot 10^{8}\ Hz}$$

c) $\frac{f_s}{f} = \frac{50 kHz}{5 kHz} = \boxed{10 \text{ Samples per period}}$

d) $\Delta t = 1/f_s = 0.002 ms$

$f_s = \frac{1}{\Delta t} = \frac{1}{0.002 ms} = \frac{1}{2 \cdot 10^{-6} s} = 500{,}000 Hz = 500 kHz$

$\frac{500 kHz}{5 kHz} = \boxed{100 \text{ samples per period}}$

3) 0-2.56 Volts digitize to 256 steps

a) $\frac{2.56}{256} \approx 0.01 V = \boxed{10 mV}$

b) $\log_2(256) = 8$

c) $2^{16} = 65536$

$\frac{2.56}{65536} = 3.906 \cdot 10^{-5} V$ or $3.906 \cdot 10^{-2} mV$

5) $R(f) = j\omega T / (1 + j\omega T)$

a) $|R(f)| = RR^*$

$$= \sqrt{\left(\frac{-j\omega\tau}{1-j\omega\tau}\right)\left(\frac{j\omega\tau}{1+j\omega\tau}\right)} = \sqrt{\frac{(\omega\tau)^2}{1+(\omega\tau)^2}}$$

$$= \frac{\omega\tau}{\sqrt{1+(\omega\tau)^2}}$$

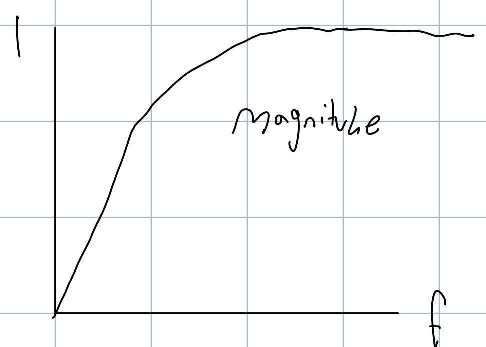
b)

$$\frac{\text{angle top}}{\text{angle bottom}} \quad \measuredangle H = \measuredangle_{top} - \measuredangle_{bottom}$$

$$\measuredangle top = 90^\circ = \pi/2$$
$$\measuredangle bottom = \tan^{-1}(\omega\tau)$$

$$\boxed{\phi = \frac{\pi}{2} - \tan^{-1}(\omega\tau)}$$

c)



d) $A(0.5\text{kHz}) = 1 \quad R = 1k\Omega = 1000\,\Omega \quad C = 1\,\mu F$

$f = 0.5\text{kHz} = 500\text{Hz}$

$\omega\tau = 2\pi(500)(1000)(1 \cdot 10^{-5}) = 3.14$

$$|R(0.5)| = \frac{3.14159}{\sqrt{1+(3.14159)^2}} = 0.9529$$

6)

a) $\frac{2.5 kHz}{10 kHz} = 0.25 = 2.5 kHz$

b) $\frac{5 kHz}{10 kHz} = 0.5 = 5 kHz$

c) $\frac{15 kHz}{10 kHz} = 1.5 = 5 kHz$

d) $\frac{20 kHz}{10 kHz} = 2 = 0 kHz$

7) $S[s(t)] = s(t - T/s)$

$s(t) = 2\sin(2\pi f t)$ $\quad T = 1/f$

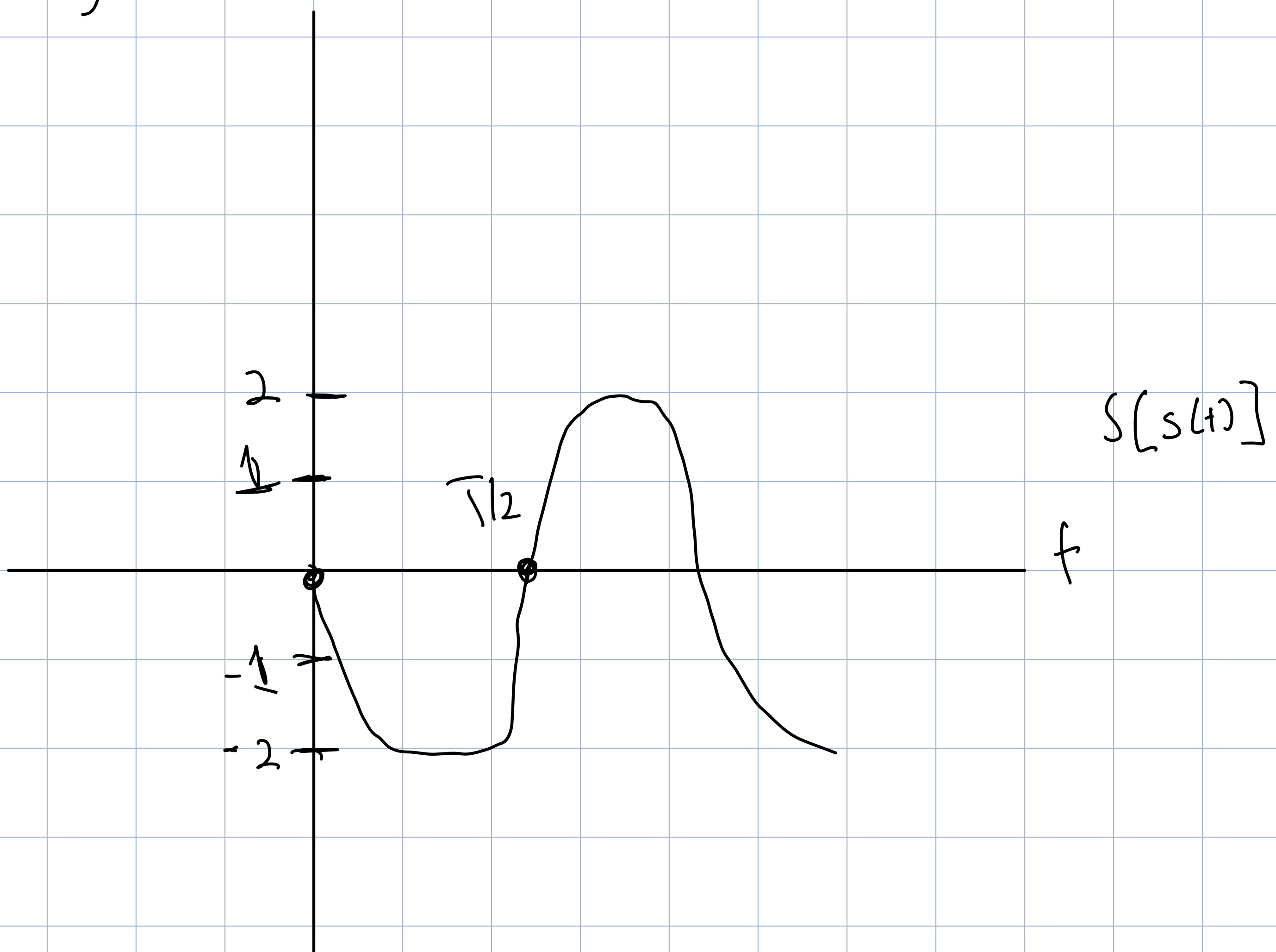
$S = (t - T/2) - s\left(t - \frac{1}{2f}\right)$

$2\sin\left(2\pi f t - 4\pi f \frac{1}{4f}\right) = 2\sin(2\pi f t - \pi)$

$\sin(x-\pi) = -\sin(x)$

$\boxed{S[s(t)] = -2\sin(2\pi f t)}$

b)



c) $s(t) + S[s(t)]$

$2\sin(2\pi f t) - \sin(2\pi f t)$

$s(t) + S[s(t)] = 0$

8) $X[n] = [0\,0\,0\,2\,0\,0\ldots]$ 100 samples

a) $Y[n] = S(x[n]) = -x[n-1]$

$y[n] = -x[n-1]$

Since $x[3] = 2$

$x[n-1] = 2$ when $n = 4$

$y[4] = -2$

$y[n] = [0\,0\,0\,-2, \ldots 0]$

b) $y[n] = S(x[n]) = (x[n])^2$

$X[3] = 2$

$Y[3] = (x[3])^2 = 4$

$Y[n] = [0\,0\,0\,4, \ldots, 0]$

c) The system in a) is linear

The system in b) is non-linear

9) even if $f(t) = f(-t)$ for all t

odd if $f(t) = -f(-t)$

- $\cos(2\pi f t)$
- $\exp^{-(t/\sigma)^2}$
- e^{-at}
- at^2+bt+c

- $\cos(2\pi f t) = \cos(-2\pi f t)$

$= \cos(2\pi f t)$

$\cos 2\pi f t$ ∴ is even

- $e^{-(t/\sigma)^2}$

$e^{(-(-t/\sigma)^2)}$

$= e^{(-(t^2/\sigma^2))}$

$= e^{(-(t/\sigma)^2)}$

$e^{(-(t/\sigma)^2)}$ ∴ is even

• e^{-at}

$$e^{-a(t)}$$

$$= e^{at}$$

Since $-f(t) \neq e^{at}$ undefined

• $at^2 + bt + c$

$$a(-t)^2 + b(-t) + c$$

$$= at^2 - bt + c$$

$\cdot f(-t) \neq at^2 - bt + c$

$f(t) \overset{\text{undefined}}{\neq} f(-t)$

$at^2 + bt + c$ is neither even or odd

1a) $F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$

a)

$$F\{ax(t)\} = aF\{x(t)\}$$

$$F\{ax(t)\} = a\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$$

$$F\{ax(t)\} = aX(f) \quad \text{homogenous}$$

b) $F\{x_1(t) + x_2(t)\} = F\{x_1(o)\} + F\{x_2(t)\}$

$$F\{x_1(t) + x_1(t)\} = \int_{-\infty}^{\infty} x_1(t)e^{-j2\pi ft}dt + \int_{-\infty}^{\infty} x_2(t)e^{-j2\pi ft}dt$$

$$\boxed{F(x_1(t) + x_2(t)) = X_1(f) + X_2(f)}$$

c) $F\{x(t-t_0)\} = \int_{-\infty}^{\infty} x(v)e^{-j\omega t}dt$

$$= \int_{-\infty}^{\infty} f(u)e^{-j\omega(u+t_0)}du$$

$$= e^{-j\omega t_0}\int_{-\infty}^{\infty} f(u)e^{-j\omega u}du$$

$$= e^{-j\omega t_0}F[f(t)]$$

11)

a) $F(\omega) = F[f(t)] = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$

$$F(w) = \int_{-\infty}^{\infty} a\delta(t-t_0)e^{-jwt}\,dt$$

$$\int_{-\infty}^{\infty} \delta(t-t_0)g(t)\,dt = g(t_0)$$

$$F(w) = ae^{-jwt_0}$$

$$F[a\delta(t-t_0)] = ae^{-jwt_0}$$

b) $|e^{-j2\pi f t_0}| = 1$

$$|F\{\delta(t-t_0)\}| = 1$$

c) Phase $e^{-j2\pi f t_0} = -2\pi f t_0$

$$\text{Phase}(F\{\delta(t-t_0)\}) = -2\pi f t_0$$

12) $F(f) = (a/2)(\delta(f-f_0) + a\delta(f+f_0))$

a) $\int_{-\infty}^{\infty} a\delta(t-t_0)e^{-2\pi j f_0 t}\,dt$

$$ae^{-2\pi j f_0 t_0}$$

$$F(f) = \frac{a}{2}\left[\delta(f-f_0) + \delta(f+f_0)\right]$$

$$F^{-1}\{F(f)\} = \frac{a}{2}\int_{-\infty}^{\infty} \delta(f-f_0)e^{2\pi j f t}\,df + \int_{-\infty}^{\infty} \delta(f+f_0)e^{2\pi j f t}\,df$$

$$\frac{a}{2}\left[e^{2\pi j f_0 t} + e^{-2\pi j f_0 t}\right]$$

b) $F(f) = \frac{a}{2j}\left(\delta(f-f_0) - \delta(f+f_0)\right)$

$$\frac{a}{2j}\int_{-\infty}^{\infty} \delta(f-f_0)e^{2\pi j f t}\,df - \frac{a^2}{2j}\int_{-\infty}^{\infty} \delta(f+f_0)e^{j2\pi f t}\,df$$

$$= \frac{a}{2j}e^{j2\pi f_0 t} - \frac{a^2}{2j}e^{-j2\pi f_0 t}$$

13)

a)

$A\cos(2\pi f_{L0} t) \; \vdots \; (m/A)\cos(2\pi f_m t)$

$$A\cos(2\pi f_{L0} t) = \frac{A}{2}\left(e^{j2\pi f_{L0} t} + e^{-j2\pi f_{L0} t}\right)$$

$$\frac{m}{A}\cos(2\pi f_A t) = \frac{m}{2A}\left(e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right)$$

b $$\left(A\cos(2\pi f_{Lo} t)\right) + \left(\frac{m}{A}\cos f_A t\right)$$

$$\left(\frac{A}{2}\left(e^{j2\pi f_{Lo} t} + e^{-j2\pi f_{Lo} t}\right)\right) \times \left(\frac{m}{2A}\left(e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right)\right)$$

$$\frac{Am}{4A}\left(e^{j2\pi(f_{Lo}+f_A)t} + e^{j2\pi(f_{Lo}-f_A)t} + e^{-j\pi(f_{Lo}+f_A)t}\right)$$

$$\frac{m}{4}\left(e^{j2\pi(f_{Lo}+f_A)t} + e^{j2\pi(f_{Lo}-f_A)t} + e^{-j2\pi(f_{Lo}-f_A)t}\right)$$

$$\frac{m}{4}\left(2\cos(2\pi(f_{Lo}+f_A)t) + 2\cos(2\pi(f_{Lo}-f_A)t)\right)$$

$$= \frac{m}{2}\cos(2\pi(f_{Lo}+f_A)t) + \frac{m}{2}\cos(2\pi(f_{Lo}-f_A)t)$$