Dylan.Z 3/17/25

Impulse Response

1.

a.

20 kHz = 20,000 samples per second

20,000 x 2 = 40,000 samples

b.

20,000 x 0.2 = 4,000

n = 0, 4000, 8000, 12000, 16000, 20000, 24000, 28000, 32000, 36000

Echo at 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8 seconds

c.

$$h[n] = \delta[n] + 0.5\delta[n-4000] + 0.25\delta[n-8000] + 0.125\delta[n-12000] + \dots$$

$$h[n] = \begin{cases} 1, & n = 0 \\ 0.5, & n = 4000 \\ 0.25, & n = 8000 \\ 0.125, & n = 12000 \\ \vdots \end{cases}$$

2.

$$y_{low}[n] = s[n] * l[n]$$
$$y_{high}[n] = s[n] * h[n]$$

$$y_{low}[n] + y_{high}[n] = (s[n] * l[n]) + (s[n] * h[n])$$

$$s[n] * (l[n] + h[n]) = s[n] * \delta[n] = s[n]$$

b.

$$h[n] = \delta[n] = s[n]$$

$$y_{low}[n] + y_{high}[n] = s[n]$$

$$s[n] * l[n] + s[n] * h[n] = s[n]$$

$$s[n] * (l[n] + h[n]) = s[n] * \delta[n]$$

$$l[n] + h[n] = \delta[n]$$

$$h[n] = \delta[n] - l[n]$$

c.

B: f_c of $h[n]$ is lower than that of $l[n]$

3 Discrete Fourier Transform

1.

a)

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi j (k/N) n}$$

$$X_k = \sum_{n=0}^{N-1} x_n e^{-2\pi j (k/N) n}$$

$$Y_k = \sum_{n=0}^{N-1} y_n e^{-2\pi j (k/N) n}$$

$$z_n = x_n + y_n$$

$$Z_k = \sum_{n=0}^{N-1} (x_n + y_n) e^{-2\pi j (k/N) n}$$

$$Z_k = \sum_{n=0}^{N-1} x_n e^{-2\pi j (k/N) n} + \sum_{n=0}^{N-1} y_n e^{-2\pi j (k/N) n}$$

$$Z_k = X_k + Y_k$$

$$W_k = \sum_{n=0}^{N-1} (a x_n) e^{-2\pi j (k/N) n}$$

$$W_k = a \sum_{n=0}^{N-1} x_n e^{-2\pi j (k/N) n} = a X_k$$

b)

$$x_n = \frac{1}{N} \sum_{k=0}^{N-1} X_k e^{2\pi j (k/N) n}$$

$$X_k = \delta[k] \qquad k = k_0$$

$$x_n = \frac{1}{N} e^{2\pi j (k/N) n}$$