Tuesday Warm Up, Unit 0: Foundations and Fundamentals

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January 28, 2025

1 Memory Bank

- $\sqrt{-1} = j$ ... The fundamental imaginary unit.
- $z = x + jy$ ... A complex number.
- $\Re\{z\} = x$, $\Im\{z\} = y$ ... Real and imaginary parts.
- $z^* = x - jy$ ... The complex conjugate of z.
- $|z| = \sqrt{zz^*} = \sqrt{x^2 + y^2}$ ... The magnitude of z.
- $\tan\phi = y/x$ ... The phase angle of z.
- $|z| = r$, so $x = r\cos\phi$, and $y = r\sin\phi$.
- **Euler's Identity:** $e^{j\phi} = \cos\phi + j\sin\phi$
- $\cos\phi = (\exp(j\phi) + \exp(-j\phi))/2$
- $\sin\phi = (\exp(j\phi) - \exp(-j\phi))/(2j)$

2 Complex Numbers

1. Recall Euler's Identity: $\exp(j\phi) = \cos\phi + j\sin\phi$. Let z be a complex number, so that $z = x + y$, with $x = |z|\cos\phi$ and $y = |z|\sin\phi$ in the complex plane. Writing a complex number or signal like $z = |z|\exp(j\phi)$ is called putting the number or signal in *polar form.* (a) Put the following numbers or signals in polar form:
   - $z = 2 + 2j$
   - $z = 2 - 2j$
   - $z = -2 + 2j$
   - $z(t) = \cos(2\pi ft) + j\sin(2\pi ft)$
   - $z(t) = \cos(2\pi ft - \phi_0)$.

3 Statistics, Probability, and Noise

1. **Digitizing voltages**: Suppose we are dealing with an AC circuit that produces waveforms for audio systems. The output runs from -2.5 to 2.5 Volts. (a) What is the range if we add an offset of +2.5 V to the output signals? (b) If we can *digitize* the new voltage range into 256 steps, what is the voltage range between steps? (c) What power of 2 gives 256?

2. Consider the signal in the previous problem, with the signal of 2.5 V amplitude, and a DC offset of 2.5 V: $s(t) = 2.5\sin(2\pi ft) + 2.5$. (a) Write a short code in `octave` that produces and plots this signal, with $f = 100$ Hz, and $\Delta t = 1$ ms. (b) Use the `randn` function to create a noise vector of the same size as $s(t)$, but with a mean of 0 and a standard deviation of 1.0. Enter `help randn` for more information on the `randn` function. (c) Plot the signal and the signal plus noise on the same graph. To plot more than one curve on the same figure, use the `hold on` command. This will make the graph persist instead of disappearing when something new is plotted. (d) What is the signal-to-noise ratio (SNR) of the sine wave plus noise? (e) Use the `hist` command to create a histogram of your noise values, and signal plus noise values.