Quiz 1

1. $v(t) = 2.5\cos(2\pi f t - \pi/4)$

a) $v(t) = \Re[2.5 e^{j\phi} e^{j\omega t}]$

euler → $\cos x = \frac{e^{jx} + e^{-jx}}{2}$

$v(t) = 2.5\cos(2\pi f t - \pi/4)$

$= 2.5\, \Re[e^{j(2\pi f t - \pi/4)}]$

b) $\cos(2\pi f t - \pi/4) = \sin(2\pi f t - \pi/4 + \pi/2)$

$= \Re\{e^{j(2\pi f t - \pi/4 + \pi/2}\}$

$= 2.5 \cdot \Re\{e^{j(2\pi f t - \pi/4 + \pi/2}\}$

$= \Re\{2.5 e^{j(2\pi f t - \pi/4 + \pi/2}\}$

2. $f = 1\text{kHz}$

a) $T = \frac{1}{f} = \frac{1}{1000}s = \boxed{1ms}$

b) $f = \frac{1}{T} \rightarrow f = \frac{1}{5 \cdot 10^{-9}} = \boxed{200MHz}$

c) $f_s = 50\text{ kHz}$ $f = 5\text{kHz}$

Samples per $= \frac{50\text{k}\cancel{\text{Hz}}}{5\text{k}\cancel{\text{Hz}}} = \boxed{10}$

d) $\Delta t = \frac{1}{f_s} = 0.002ms$

Samples per $= \frac{T}{\Delta t} = \frac{0.002}{5 \cdot 10^{-3}} = \boxed{10}$

3. a) $\Delta V = \frac{\text{Range}}{\text{steps}}$

$= \frac{2.56V}{256} = \boxed{0.01V}$

b) $2^8 = \boxed{256}$

$\downarrow 2\times$

c) $2^{16} = 65536$ steps

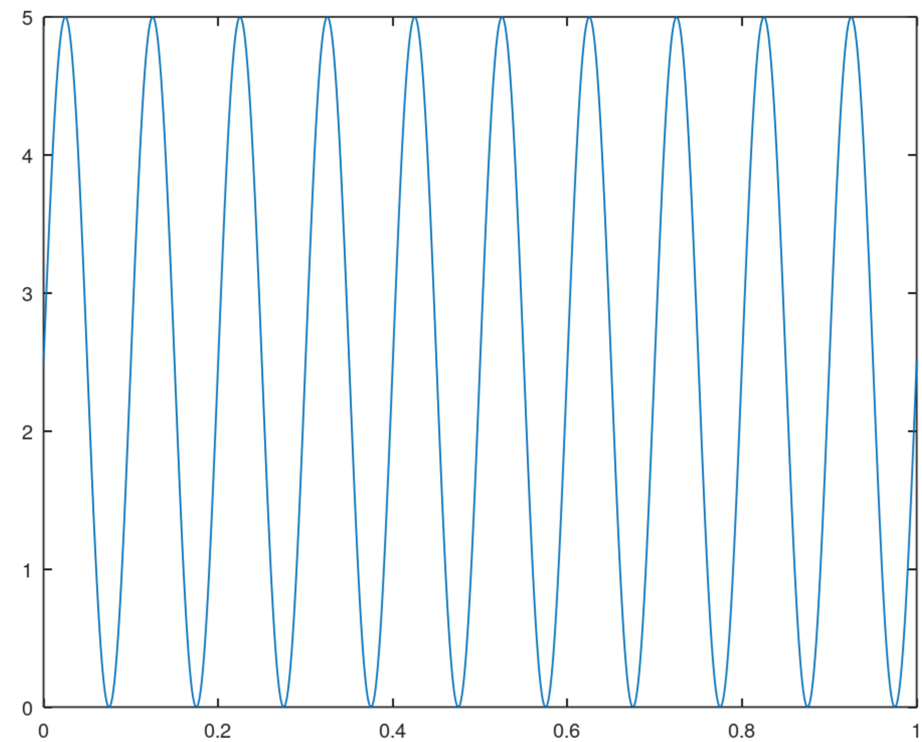
$\Delta V = \frac{2.56V}{65536} = 0.000039V$

$= \boxed{39\mu V}$

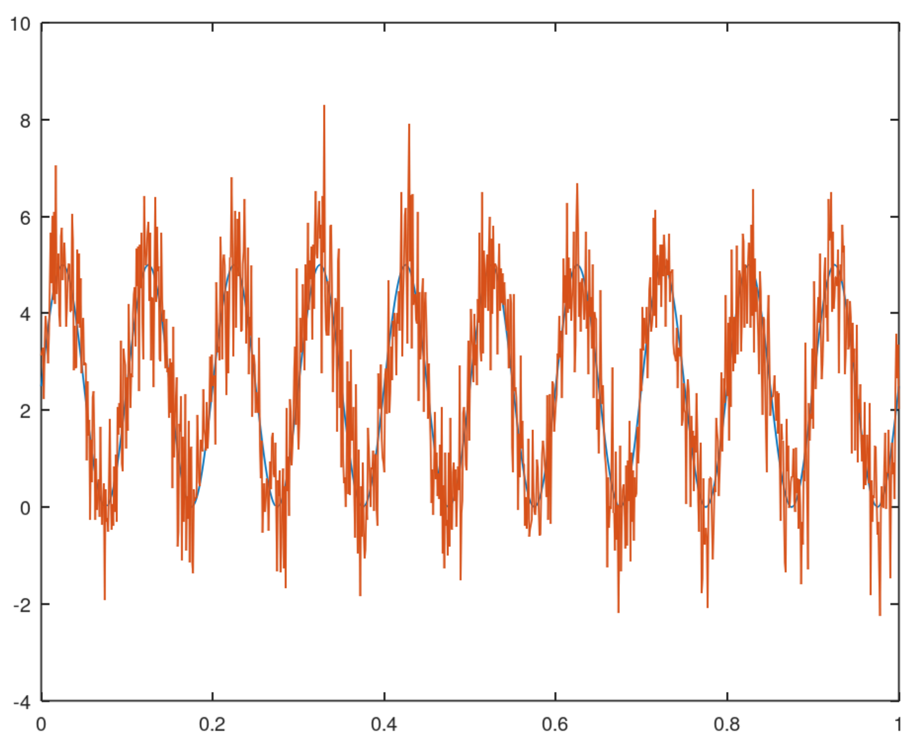
4

4.

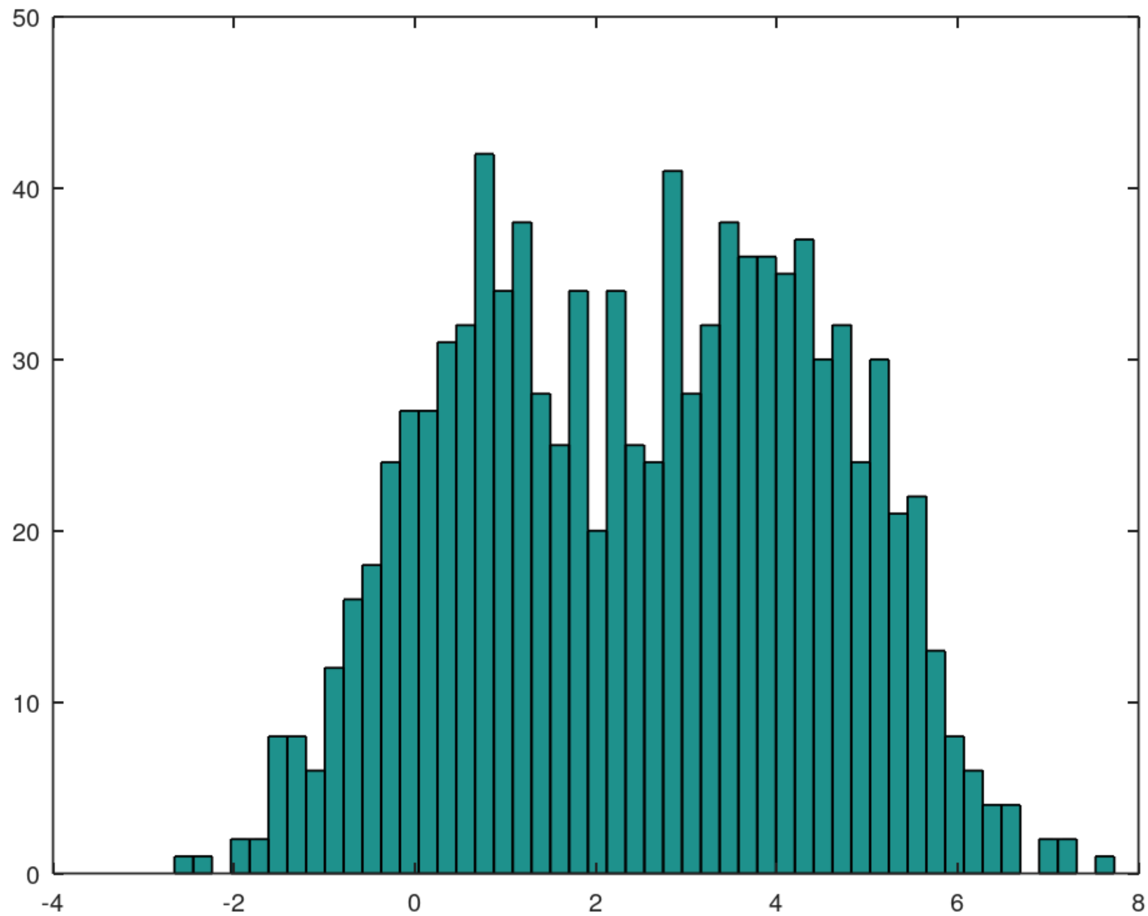
a)



b)



c)



5)

a) $h(f) = j\omega T/(1+j\omega T)$

$$|h(f)| = \left|\frac{j\omega T}{1+j\omega T}\right| = \frac{|\omega T|}{\sqrt{1+(\omega T)^2}} = \frac{\omega T}{\sqrt{1+(\omega T)^2}}$$

b) $\theta(f) = \tan^{-1}(\mathrm{Im}(h(f))/\mathrm{Re}(h(f)))$

$$\theta(f) = \tan^{-1}\left(\frac{\omega T}{1}\right) = \tan^{-1}(\omega \tau)$$

$$\frac{j\omega\tau}{1+j\omega\tau}$$

c) $A(f)h(f) = A\frac{\omega\tau}{\sqrt{1+\omega^2+\tau^2}}$

$A=1$ $f=0.5$

$h = 1\,k\Omega$

$C = 1\,\mu F$

$\omega = 2\pi(0.5\cdot 10^3)$

$\tau = (1\cdot 10^3)(1\cdot 10^{-6}) = 1\,ms$

$$= 1\cdot\frac{2\pi(500)(1)}{\sqrt{1+(2\pi(500)(1))^2}} = \boxed{0.99}$$

8) $x[n] = [0002000\ldots]$

a) $y[n] = S[x[n]] = -x[n-1]$

$n=2 \rightarrow y[n] = -x[n-1]$

The impulse shifts right by 1 so then the spike is at $n=3$ with val of -2

b) $y[n] = S[x[n]] = (x[n])^2$

$x[n] = 2$ at $n=2$

$y[n] = (2)^2 = \boxed{4}$ at $n=2$

c) equation a is linear * scaling & shifting

equation b is Nonlinear * squaring so no superposition

6) $f_s = |f - nf_s|$

a) $f = 2.5\,kHz$ $f_s/2 = 5\,kHz$

$f_s = 2.5\,kHz$

b) $f_s = 5\,kHz$

c) $f_s = \frac{15\,kHz}{10\,kHz} = 1.5\,kHz$

d) $f_s = \frac{20\,kHz}{10\,kHz} = 2\,kHz$

7) $S[s(t)] = s(t - T/2)$

a) $s(t) = 2\sin(2\pi f t)$ $T = 1/f$

$$S[s(t)] = 2\sin\left(2\pi f\left(t - \frac{1}{2f}\right)\right)$$

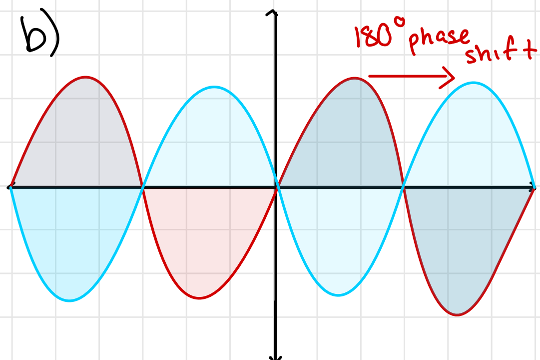
$$\downarrow$$

$$= 2[\sin(2\pi f t)\cos(\pi) - \cos(2\pi f t)\sin(\pi)]$$

$\cos(\pi) = -1$ $\sin(\pi) = 0$

$$S[s(t)] = \boxed{-2\sin(2\pi f t)}$$

b)



c) $s(t) + S[s(t)]$

$$s(t) + S[s(t)] = 2\sin(2\pi f t) - 2\sin(2\pi f t) = \boxed{0}$$

9)

- $\cos(2\pi f t)$
  - even
- $\exp(-(t/\sigma)^2)$
  - even
- $\exp(-\alpha t)$
  - neither?
- at^2+bt+c
  - even if $b=0$ or then neither

(10)

$$F(\omega)=F\{f(t)\}=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

a) $F\{af(t)\}=\int_{-\infty}^{\infty} a f(t)e^{-j\omega t}dt$

$$=a\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$$

$$=aF(\omega) \rightarrow F\{af(t)\}=aF(\omega)$$

b) $F\{f_1(t)+f_2(t)\}=F_1(\omega)+F_2(\omega)$

c) $F\{f(t-t_0)\}=e^{-j\omega t_0}F(\omega)$

(13) $\cos(2\pi f t)=\dfrac{e^{j2\pi f t}+e^{-j2\pi f t}}{2}$

$$A\cos(2\pi f_c t)=\frac{A}{2}\left(e^{j2\pi f_c t}+e^{-j2\pi f_c t}\right)$$

$$(m/A)\cos(2\pi f_A t)=\frac{m}{2A}\left(e^{j2\pi f_A t}+e^{-j2\pi f_A t}\right)$$

b) $A\cos(2\pi f_c t)\cdot(m/A)\cos(2\pi f_c t)$

$$=(A\cos(2\pi f_c t))\left(\frac{m}{A}\cos(2\pi f_A t)\right)=\frac{m}{2}\left(\cos(2\pi(f_c+f_A)t)+\cos(2\pi(f_c-f_A)t)\right)$$

$$=\boxed{f_c+f_A,\ f_c-f_A}$$

(11) $\delta(t-t_0)$

a) $G(\omega)=\int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt$

$$F\{\delta(t-t_0)\}=\int_{-\infty}^{\infty}\delta(t-t_0)e^{-j\omega t}dt$$

$$=\int_{-\infty}^{\infty} f(t)\delta(t-t_0)dt=f(t_0)$$

$$=F\{\delta(t-t_0)\}=\boxed{e^{-j\omega t_0}}$$

b) mag of $e^{-j\omega t_0}=1$

c) phase angle $=\boxed{-\omega t_0}$

(12) $F(f)=\frac{a}{2}\delta(f-f_0)+a\delta(f+f_0)$

$$f(t)=\int_{-\infty}^{\infty}F(f)e^{j2\pi f t}df$$

$$=\int_{-\infty}^{\infty}\left[\frac{a}{2}\delta(f-f_0)+a\delta(f+f_0)\right]e^{j2\pi f t}df$$

$$f(t)=\frac{a}{2}e^{j2\pi f_0 t}+ae^{-j2\pi f_0 t}$$

$$\Rightarrow e^{j\theta}+e^{-j\theta}=2\cos\theta$$

$$=\boxed{a\cos(2\pi f_0 t)}$$

b) $F(f)=\frac{a}{2j}\delta(f-f_0)-\frac{a}{2j}\delta(f+f_0)$

$$=\int_{-\infty}^{\infty}\left[\frac{a}{2j}\delta(f-f_0)-\frac{a}{2j}\delta(f+f_0)\right]e^{j2\pi f t}df$$

$$=\frac{a}{2j}e^{j2\pi f_0 t}-\frac{a}{2j}e^{-j2\pi f_0 t}$$

$$=e^{j\theta}-e^{-j\theta}=2j\sin\theta$$

$$=\boxed{a\sin(2\pi f_0 t)}$$

```
clear;
close all;

fs = 44100;
duration = 2.0;
num = fs * duration;

delta = zeros(num_samples, 1);

echo_spacing = 0.25;
echo_samples = round(echo_spacing * fs);

echo_response = zeros(num, 1);

amp = 2;
index = 1;
while index <= num
    echo_response(index) = amp;
    amp = amp / 2;
    index = index + echo_samples;
endwhile


f_tone = 329.63;
duration_tone = 0.25;
samples = round(duration_tone * fs);
sine_wave = [sin(2 * (pi^2) * f_tone * (0:samples-1)' / (3*fs));
zeros(num - samples, 1)];

output_signal = conv(sine_wave, echo_response,'full');

player1 = audioplayer(sine_wave, fs, 16);
playblocking(player1);

player2 = audioplayer(output_signal, fs, 16);
play(player2);
```