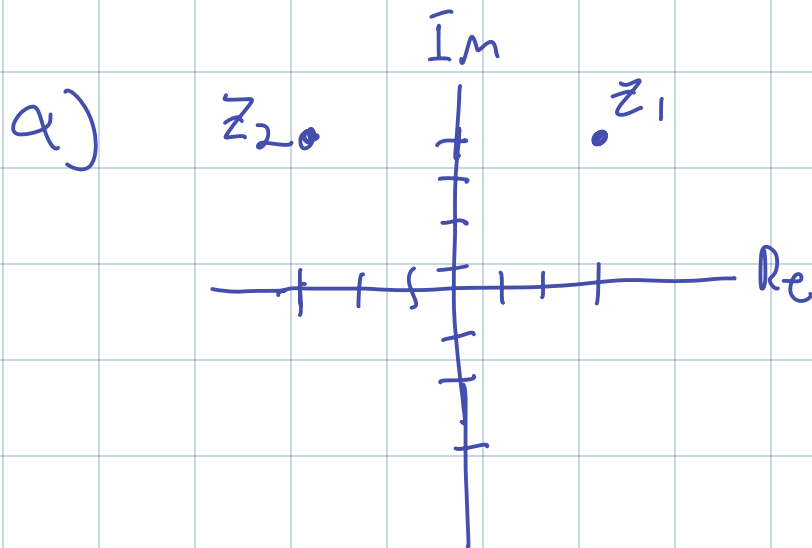


Problem Set 1 Due 1/31/2025

1. $Z_1 = 3 + 4j$ and $Z_2 = -3 + 4j$



b)

$$\begin{aligned} &3 + 4j \\ &-3 + 4j \\ &\textcircled{8j} \end{aligned}$$

c)

$$\begin{aligned} &3 + 4j \\ &-3 + 4j \\ &\textcircled{6} \end{aligned}$$

d) $(3 + 4j)(-3 + 4j)$

$$-9 + 12j - 12j + 16j^2$$

$$-9 - 16$$

$$= -27$$

e)

$$\frac{3+4j}{-3+4j} \left(\frac{-3-4j}{-3-4j} \right) = \frac{-9-12j-12j-16j^2}{9+12j-12j-16j^2} = \frac{7-24j}{27}$$

$$\frac{7}{27} - \frac{24j}{27}$$

f) $z_1 = 3+4j$

take the magnitude

$$\sqrt{zz^*}$$

$$= \sqrt{(3+4j)(3-4j)}$$

$$= \sqrt{9-12j+12j-16j^2}$$

$$= \sqrt{9+16}$$

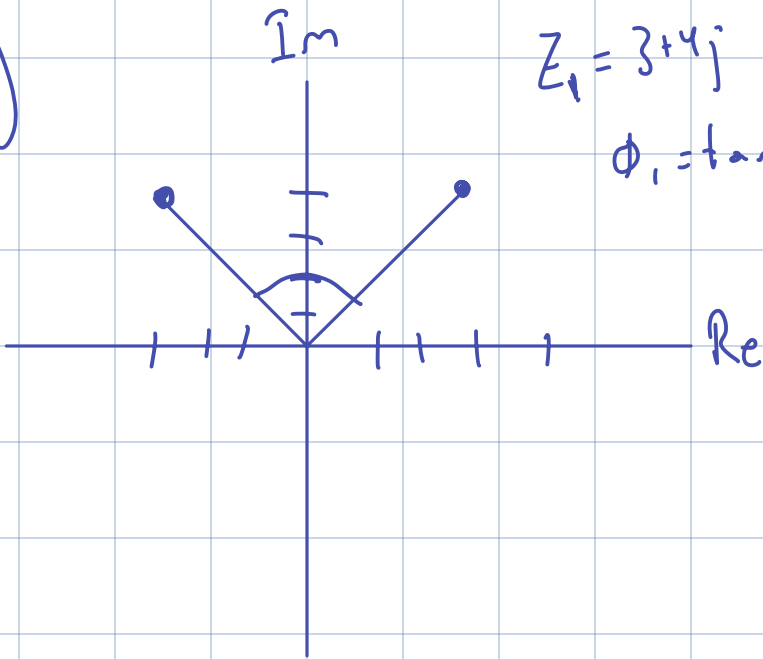
$$= \sqrt{25} = 5$$

g) $z_2 = 3-4j$

$$\sqrt{zz^*}$$

$$\begin{aligned}
 &= \sqrt{(-3-4j)(-3+4j)} \\
 &= \sqrt{9 - 12j + 12j - 16j^2} \\
 &= \sqrt{9 + 16} \\
 &= \sqrt{25} = 5
 \end{aligned}$$

h)



$$Z_1 = 3 + 4j$$

$$\phi_1 = \tan^{-1}\left(\frac{4}{3}\right)$$

$$Z_2 = -3 + 4j$$

$$\phi_2 = \tan^{-1}\left(\frac{4}{-3}\right)$$

j)

Polar form

$$Z_1 = 5e^{j \tan^{-1}\left(\frac{4}{3}\right)}$$

$$Z_2 = 5e^{j \tan^{-1}\left(\frac{4}{-3}\right)}$$

2) Euler's Identity: $e^{j\phi} = \cos\phi + j\sin\phi$

$$\cos\phi = \frac{e^{j\phi} + e^{-j\phi}}{2}$$

$$\cos(2\pi ft) = \frac{e^{2\pi jft} + e^{-2\pi jft}}{2}$$

$$\cos(2\pi ft) = \frac{(\cos(2\pi ft) + j\cancel{\sin(2\pi ft)}) + (\cos(-2\pi ft) - j\cancel{\sin(2\pi ft)})}{2}$$

$$= \frac{2\cos(2\pi ft)}{2}$$

$$\cos(2\pi ft) = \cos(2\pi ft) = \frac{e^{2\pi jft} + e^{-2\pi jft}}{2}$$

$$\sin(2\pi ft) = \frac{e^{2\pi jft} - e^{-2\pi jft}}{2}$$

$$\frac{e^{2\pi jft} - e^{-2\pi jft}}{2} = \frac{\cos(2\pi ft) + j\sin(2\pi ft) - (\cos(2\pi ft) - j\sin(2\pi ft))}{2}$$

$$\Sigma \frac{\cancel{\cos(2\pi ft)} + j\sin(2\pi ft) - \cancel{\cos(2\pi ft)} + j\sin(2\pi ft)}{2}$$

$$\frac{e^{2\pi jft} - e^{-2\pi jft}}{2} = \frac{2j\sin(2\pi ft)}{2} = j\sin(2\pi ft)$$

$$2. \quad V_1(t) = 4 \cos(2\pi f_1 t)$$

$$a) \quad V_2(t) = 4 \cos(2\pi f_2 t - \phi)$$

$$P = V_1(t) V_2(t)$$

$$P(t) = 4 \cos(2\pi f_1 t) \cdot 4 \cos(2\pi f_2 t - \phi)$$

$$= 16 \cos(2\pi f_1 t) \cos(2\pi f_2 t - \phi)$$

$$= 8 (\cos(2\pi f_1 - f_2) t + \phi) + \cos(2\pi (f_1 + f_2) t - \phi)$$

$$8 \cos(2\pi (f_1 - f_2) t + \phi) + \cos(2\pi (f_1 + f_2) t - \phi)$$

$$f_+ = f_1 + f_2 \quad f_- = f_1 - f_2$$

b)

$$P_{\max} = 8 [\cos(0) + \cos(4\pi f_1 t)]$$

$$= 8 [1 + \cos(4\pi f_1 t)]$$

$$\underbrace{\qquad\qquad\qquad}_{=1}$$

$$= 8 [1 + 1]$$

$$= 16$$

16 is the correct number because $\cos(2\pi f_1 t)$

and $\cos(2\pi f_L t - \phi)$ maximum value is one.

4.)

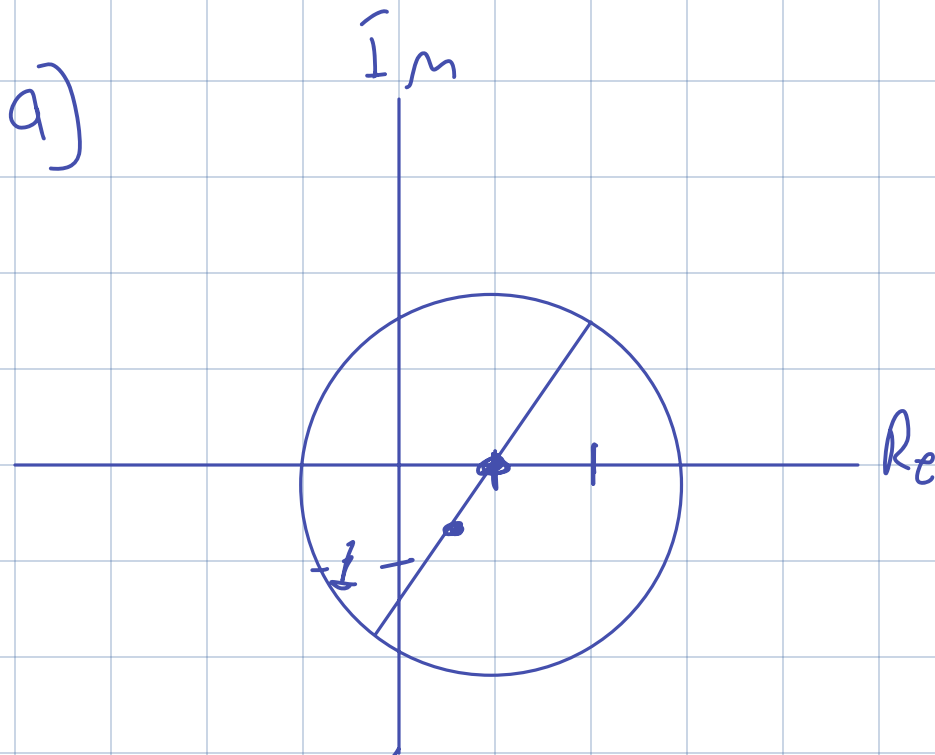
$$V_1(t) = \{ \exp[j(2\pi f t - \phi)] \}$$

$$V_2(t) = \{ \exp(2\pi j f t) \}$$

$$\exp(j(2\pi f t - \phi)) = \exp(j 2\pi f t) \cdot \exp(-j\phi)$$

$$\exp(2\pi j f t) = \exp(j 2\pi f t) \cdot 1$$

$$V_1(t) = \exp(-j\phi), \quad V_2(t) = 1$$



b)

$$Z = \exp(-j\phi) + 1$$

$$\exp(-j\phi) = \cos(\phi) - j\sin(\phi)$$

$$\begin{aligned} Z &= (\cos(\phi) - j\sin(\phi)) + 1 \\ &= (1 + \cos(\phi)) - j\sin(\phi) \end{aligned}$$

Magnitude of Z

$$|Z| = \sqrt{(1 + \cos(\phi))^2 + (-\sin(\phi))^2}$$

$$= \sqrt{1 + 2\cos(\phi) + \cos^2(\phi) + \sin^2(\phi)}$$

$$|Z| = \sqrt{2 + 2\cos(\phi)}$$

$$\theta = \tan^{-1}\left(\frac{-\sin(\phi)}{1 + \cos(\phi)}\right)$$

$$c) \quad |Z| = 2\left|\cos\left(\frac{\pi}{2}\right)\right| = 1.85$$

$$|Z| = 2 |\cos(\frac{\pi}{2})| = 2$$

$$|Z| = 2 |\cos(\frac{180^\circ}{2})| = 0$$

in phase (0°) the magnitude
doubles to \geq out of phase (180°)
the magnitude cancels to 0

