Thursday Warm Up, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- $\cos(2\pi f_1 t) = (1/2)(\exp(2\pi j f_1 t) + \exp(-2\pi j f_1 t))$.
- $\sin(2\pi f_1 t) = (1/2j)(\exp(2\pi j f_1 t) - \exp(-2\pi j f_1 t))$.
- $F(f) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-2\pi j f t} dt$ ... The Fourier Transform.
- $\mathcal{F}^{-1}\{F(f)\} = \int_{-\infty}^{\infty} F(f) e^{2\pi j f t} df$ ... The Inverse Fourier Transform.
- **Convolution**: this is an operation that characterizes the response $h[n]$ of a linear system.

$$y[i] = h[n] * x[n] = \sum_{j=0}^{M-1} h[j] x[i-j] \tag{1}$$

In words, the output at sample i is equal to the produce of the system response h and the input signal x, summed over the proceeding M samples (from $j = 0$ to $j = M - 1$).

2 Interpreting Spectra: amplitude modulation (AM)

1. Consider Fig. 1 (top). In this exercise, we will develop an understanding of **amplitude modulation.** (a) Express the following functions as complex exponentials: $A\cos(2\pi f_1 t)$ and $(m/A)\cos(2\pi f_2 t)$. (b) Multiply the two functions, and show that the result is a pair of sinusoids at two new frequencies. What are the new frequencies? (c) Sketch what you think the Fourier spectrum would be, and compare it to Fig. 1 (bottom). Fig. 2 contains an LC resonator circuit, which retains the carrier frequency in the spectrum.

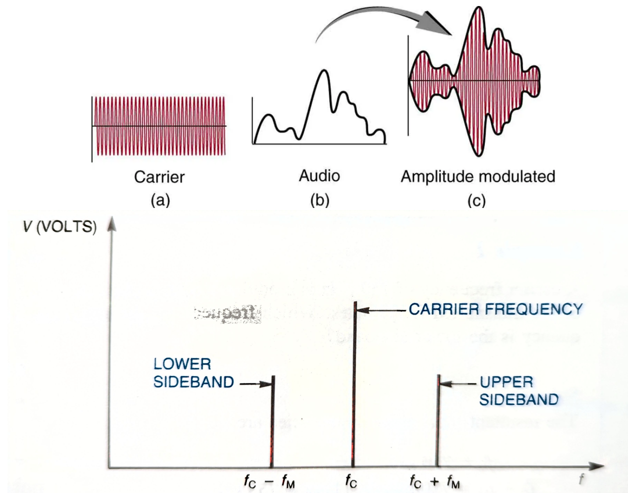

Figure 1: Amplitude modulation signal and spectrum.

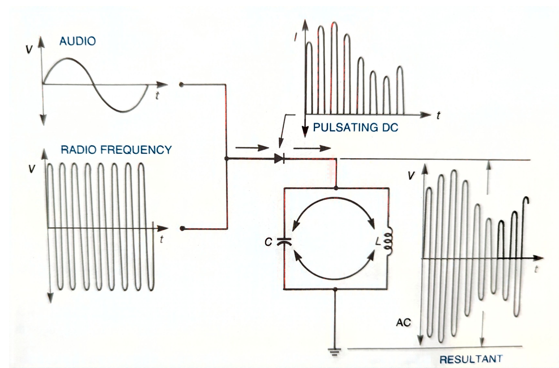

Figure 2: Amplitude modulation circuit.

3 Convolution and Impulse Response

1. Notice the definition of **convolution**, Eq. 1. Let $x[n]$ be the input signal, $h[n]$ be the response of a linear DSP system, and $y[n]$ be the output signal. (a) Show that $y[n] = h[n]$, if $x[n] = \delta[n]$. (b) What is $y[n]$, if $x[n] = [00001000]$, and $h[n] = [\frac{1}{2}\frac{1}{2}000000]$? (c) Using the `conv` function in `octave`, write a short code that produces the result of the convolution in (b).