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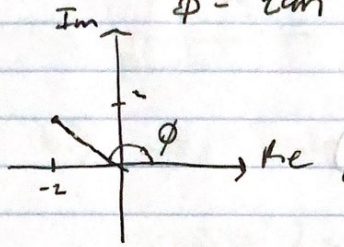
DSP - Quiz 2

Review Material

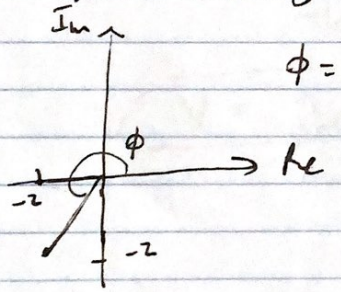
1.) a) $z = -2 + 2j$

$\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{2}{-2}\right) = \tan^{-1}(-1) = \frac{3\pi}{4}, \frac{7\pi}{4}$



$\boxed{\phi = \frac{3\pi}{4}}$

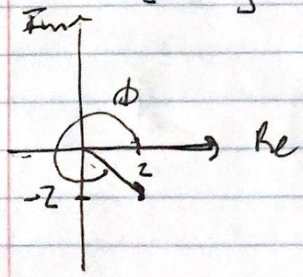
b.) $z = -2 - 2j$



$\phi = \tan^{-1}\left(\frac{-2}{-2}\right) = \tan^{-1}(1) = \frac{\pi}{4}, \frac{5\pi}{4}$

$\boxed{\phi = \frac{5\pi}{4}}$

c.) $z = 2 - 2j$



$\phi = \tan(-1) = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\boxed{\phi = \frac{7\pi}{4}}$

2) a) $v(t) = 4\cos(2\pi(10)t + 30°)$

$v(0) = 4\cos(30°)$

$v(0) = \text{Re}\{4e^{j(30°)}\}$

$\boxed{V = 4e^{j(30°)} = 4e^{j\frac{\pi}{6}}}$

b.) $v(t) = 2\sin(2\pi(10)t - 60°)$

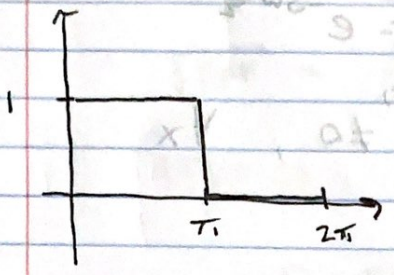
$v(0) = 2\sin(-60°) = \text{Im}\{2e^{j(-60°)}\}$

$-60° = -\frac{\pi}{3} = \frac{5\pi}{3}$

$\boxed{V = 2e^{j(-60°)} = 2e^{j(\frac{5\pi}{3})}}$

Fourier Analysis

$$f(x) = \begin{cases} 1, & 0 \le x < \pi \\ 0, & \pi < x \le 2\pi \end{cases}$$



$$F(w) = \int_{-\infty}^{\infty} f(x)\, e^{-jwx}\, dx$$

$$= \int_0^{\pi} (1)\, e^{-jwx}\, dx$$

$u = jwx$

$du = jw\,dx \Rightarrow dx = (jw)^{-1}\, du$

$$= \int_0^{\pi} (jw)^{-1} e^{-u}\, du$$

$$= (jw)^{-1} \left[-1\, e^{-u}\right]_0^{\pi}$$

$$= -(jw)^{-1} \left[e^{-jwx}\right]_0^{\pi}$$

$$= -(jw)^{-1} \left[e^{-jw\pi} - e^{-jw(0)}\right]$$

$$= -(jw)^{-1} \left(e^{-jw\pi/2}\right) \left[e^{-jw\pi/2} - e^{jw\pi/2}\right]$$

$$= \frac{1}{jw} \left[e^{jw\pi/2} - e^{-jw\pi/2}\right] \left(e^{-jw\pi/2}\right)$$

$$= \frac{1}{jw} \left[2j \sin\left(\frac{w\pi}{2}\right)\right] \left(e^{-jw\frac{\pi}{2}}\right)$$

$$F(w) = \frac{2}{w} \left(\sin\left(\frac{w\pi}{2}\right)\right) \left(e^{-jw\frac{\pi}{2}}\right)$$

phase shift ↗

1.) phase term: $e^{-jw\pi/2} \Rightarrow \phi(w) = -w\frac{\pi}{2}$

2) $|F| = \sqrt{FF^*}$ $\quad F = \frac{2}{\omega}\sin\left(\frac{\omega\pi}{2}\right)e^{-j\omega\frac{\pi}{2}}$

$F^* = \frac{2}{\omega}\sin\left(\frac{\omega\pi}{2}\right)e^{j\omega\frac{\pi}{2}}$

$FF^* = \frac{4}{\omega^2}\sin^2\left(\frac{\omega\pi}{2}\right)e^{j\omega\frac{\pi}{2}}e^{-j\omega\frac{\pi}{2}}$

$\sqrt{FF^*} = \frac{2}{\omega}\sin\left(\frac{\omega\pi}{2}\right)$

$|F| = 0 = \frac{2}{\omega}\sin\left(\frac{\omega\pi}{2}\right)$

$0 = \sin\left(\frac{\omega\pi}{2}\right)$

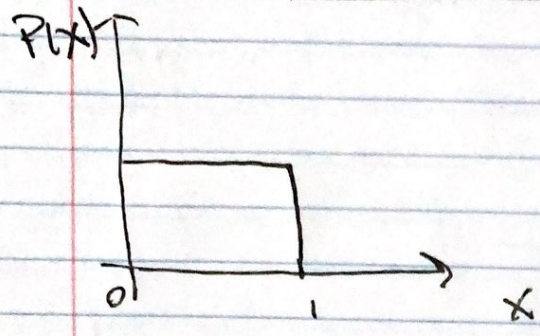
$\sin^{-1}(0) = \frac{\omega\pi}{2}$

$\omega = \frac{2}{\pi}\sin^{-1}(0) = \frac{2}{\pi}(n\pi)$, $n \in \mathbb{Z}$

$\omega = 2n$, $n \in \mathbb{Z}$

↑ frequencies when $|F(\omega)| = 0$

Probability and Statistics



$$P(x) = \frac{1}{1-0} = 1$$

1.) $$\int_0^1 P(x)\,dx = \int_0^1 1\,dx = [x]_0^1 = 1-0 = 1$$

2.) $$\langle x \rangle = \int_0^1 x P(x)\,dx$$

$$= \int_0^1 x(1)\,dx = \left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2} - 0 = \boxed{\frac{1}{2} = \mu}$$

$$\langle x^2 \rangle = \int_0^1 x^2 P(x)\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3}$$

3.) $$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$= \frac{1}{3} - \left(\frac{1}{2}\right)^2 = \frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{1}{12}$$

So, $$\boxed{\sigma = \frac{1}{\sqrt{12}}}$$