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DSP Quiz 1.

1) $z = 4+4i$

$r = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$

$\boxed{z = 4\sqrt{2}\, e^{\frac{\pi}{4} i}}$

$\boxed{z = 4\sqrt{2} \exp\left(\frac{\pi}{4} i\right)}$

2) $z = 1$ $\boxed{z = \exp(i0) = 1.}$

$z = i$ $\boxed{z = \exp\left(\frac{\pi}{2} i\right)}$

$z = -1$ $\boxed{z = \exp(\pi i)}$

$z = -i$ $\boxed{z = \exp\left(\frac{3\pi}{2} i\right)}$

The phase angle is shifting $90°$ at a time because the numbers are as follows.

$z = 2\exp\left(\frac{\pi}{4} i\right) = 2\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) = 2\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

$\boxed{z = \sqrt{2} + i\sqrt{2}}$

$z = 5\exp(i\pi) = 5(\cos(\pi) + i\sin(\pi)) = 5(-1+0) = -5$

$\boxed{z = -5}$

$$v(t) = a_1 \exp(ix_1) + a_2 \exp(ix_2)$$

$$|v|^2 = v^* v = \ldots$$

<u>Case 1</u> $\phi_2 = \phi_1 + \pi$

$$v = a_1\cos(x_1) + ia_1\sin(x_1) + a_2\cos(x_2) + ia_2\sin(x_2)$$

$$\therefore \quad v = a_1\cos(x_1) + ia_1\sin(x_1) + a_2\cos(x_1+\pi) + ia_2\sin(x_1+\pi)$$

From trig properties we have that $\cos(x_1+\pi) = -\cos(x_1)$ and $\sin(x_1+\pi) = -\sin(x_1)$

So this $\Rightarrow$ $v = a_1\cos(x_1) + ia_1\sin(x_1) - a_2\cos(x_1) - ia_2\sin(x_1)$

$$= \cos(x_1)(a_1 - a_2) + \sin(x_1)(ia_1 - ia_2)$$

$$= \cos(x_1)(a_1 - a_2) + i\sin(x_1)(a_1 - a_2)$$

$$= (a_1 - a_2)(\cos(x_1) + i\sin(x_1))$$

$$v = (a_1 - a_2)e^{-ix_1}$$

Now $v^* v = [(a_1 - a_2)e^{-ix_1}][(a_1 - a_2)e^{ix_1}]$

$$\boxed{\text{So } |v|^2 = (a_1 - a_2)^2 \text{ for case 1.}}$$

Case 2 $\phi_2 = \phi_1$ so $x_1 = x_2$

$$v = a_1\cos(x_1) + ia_1\sin(x_1) + a_2\cos(x_1) + ia_2\sin(x_1)$$

$$= \cos(x_1)(a_1 + a_2) + i\sin(x_1)(a_1 + a_2)$$

$$= (a_1 + a_2)(\cos(x_1) + i\sin(x_1))$$

$$v = (a_1 + a_2)e^{-ix_1}$$

$$vv^* = |v|^2 = [(a_1 + a_2)e^{-ix_1}][(a_1 + a_2)e^{+ix_1}] \boxed{= (a_1 + a_2)^2}$$

This displays destructive (subtraction for out of phase) interference and constructive (addition for in phase) interference.

$$\phi_v = \arctan\left(\frac{Im\{v\}}{Re\{v\}}\right) \quad \text{and } V = a_1\exp(ix_1)+a_2\exp(ix_2)$$

<u>Case 1</u> $\quad \phi_2 = \phi_1 + \pi \rightarrow$

$$Re\{v\} = a_1\cos(x_1) - a_2\cos(x_1)$$

$$Im\{v\} = \sin(x_1)a_1 - \sin(x_1)a_2$$

$$\phi_v = \arctan\left(\frac{(a_1-a_2)\sin(x_1)}{(a_1-a_2)\cos(x_1)}\right) = \arctan(\tan(x_1)) = x_1$$

$$\boxed{\phi_v = x_1 \text{ for } \phi_2 = \phi_1+\pi}$$

<u>Case 2</u> $\quad \phi_2 = \phi_1 \Rightarrow x_1 = x_2$

$$Re\{v\} = a_1\cos(x_1) + a_2\cos(x_1) = \cos(x_1)(a_1+a_2)$$

$$Im\{v\} = a_1\sin(x_1) + a_2\sin(x_1) = \sin(x_1)(a_1+a_2)$$

$$\phi_v = \arctan\left(\frac{Im\{v\}}{Re\{v\}}\right) \Rightarrow \phi_v = \arctan\left(\frac{\sin(x_1)(a_1+a_2)}{\cos(x_1)(a_1+a_2)}\right)$$

$$\phi_v = \arctan(\tan(x_1)) = x_1$$

$$\boxed{\phi_v = x_1 \text{ for } \phi_1 = \phi_2}$$ case as well.

This makes sense because at these angles, the numbers have the same magnitude.

Recompute h w/ $L=0$ and $\tau = RC$. Graph $|h(\omega)|$

$$h = \frac{V_{out}}{V_{in}} \qquad V_{out} = \frac{1}{i\omega C} \qquad V_{in} = R + \frac{1}{i\omega C}$$

$$\Rightarrow \quad h = \frac{\frac{1}{i\omega C}}{R + \frac{1}{i\omega C}} \left(\frac{i\omega C}{i\omega C}\right) = \frac{1}{Ri\omega C + 1}\left(\frac{Ri\omega C - 1}{Ri\omega C - 1}\right)$$

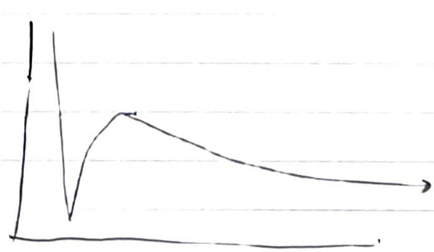
$$= \frac{Ri\omega C - 1}{-R^2\omega^2C^2 - 1} = \frac{1 - Ri\omega C}{R^2\omega^2C^2 + 1} = \frac{1}{R^2\omega^2C^2+1} - \frac{R\omega C}{R^2\omega^2C^2+1} i$$

$$\boxed{h(\omega) = \frac{1}{R^2\omega^2C^2+1} - \frac{R\omega C}{R^2\omega^2C^2+1} i}$$

$$\boxed{h(\omega) = \frac{1}{\tau^2\omega^2+1} - \frac{\tau\omega}{\tau^2\omega^2+1} i}$$

$$|h(\omega)| = \sqrt{\left(\frac{1}{\tau^2\omega^2+1} - \frac{\tau\omega i}{\tau^2\omega^2+1}\right)\left(\frac{1}{\tau^2\omega^2+1} + \frac{\tau\omega i}{\tau^2\omega^2+1}\right)}$$

$$\boxed{|h(\omega)| = \sqrt{\left(\frac{1}{\tau^2\omega^2+1}\right)^2 + \left(\frac{\tau\omega}{\tau^2\omega^2+1}\right)^2}}$$



code

```
τ = number
f = 0:0.01:2;  ⇒  ω = 2πf
h = abs(h(ω))
plot(f,h)
```