Ray Kaw DSP Problem set #3

2. $f(t) = a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t)$

$$\int_{-\infty}^{\infty} (a_1 \cos(2\pi f_1 T_1) + a_2 \cos(2\pi f_2 t))\, \delta(t - T_1)\, dt$$

$$f(T_1) = a_1 \cos(2\pi f_1 \cdot \tfrac{1}{f_1}) + a_2 \cos(2\pi f_2 \cdot \tfrac{1}{f_1})$$
$$f(T_1) = a_1 \cdot 1 + a_2 \cdot 1$$
$$= a_1 + a_2$$

$$\int_{-\infty}^{\infty} (a_1 \cos(2\pi f_1 t) + a_2 \cos(2\pi f_2 t))\, \delta(t - T_2)\, dt$$

$$f(T_2) = a_1 \cos(2\pi f_1 T_2) + a_2 \cos(2\pi f_2 T_2)$$
$$f(T_2) = a_1 \cos(\pi) + a_2 \cos(2\pi)$$

$$f(T_2) = a_1 \cdot (-1) + a_2 \cdot 1$$
$$= -a_1 + a_2$$

3. $F(t) = \int_{-\infty}^{\infty} a\,\delta(t - t_0)\, e^{-j2\pi f t}\, dt$

$F(f) = a e^{-j2\pi f t_0}$

$|F(f)| = a$

b. $\phi(f) = -2\pi f t_0$

c. $T_g = -\dfrac{d\phi}{d\omega}$

$$\frac{d\phi}{d\omega} = -\frac{t_0}{\pi}$$
$$= \frac{t_0}{\cancel{\pi}} \cdot \cancel{\pi}$$

$$= T_g = t_0$$

3. Fourier Transforms and Basic Filters

$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t} dt$

$= a\delta(t - t_0) e^{-j2\pi f t} dt$

$S(f) = a e^{-j2\pi f t_0}$

b. $Y(f) = \frac{S(f)}{1 + j\omega\tau}$

$= \frac{a e^{-j2\pi f t_0}}{1 + j(2\pi f)\tau}$

$|Y(f)| = \frac{|a|}{|1 + j(2\pi f)\tau|}$

$|Y(f)| = \frac{|a|}{\sqrt{1 + (2\pi f \tau)^2}}$

c. $H(f) = \frac{j\omega\tau}{1 + j\omega\tau}$

$S(f) \cdot H(f) = \frac{a e^{-j2\pi f t_0}}{1 + j(2\pi f)\tau} \cdot j(2\pi f)\tau$

$\omega = 2\pi f$

$|Y(f)| = \frac{|a| \cdot (2\pi f \tau)}{\sqrt{1 + (2\pi f \tau)^2}}$

$= \frac{|a|(2\pi f \tau)}{\sqrt{1 + (2\pi f \tau)^2}}$

Roy nau DSP Problem set #3

2. $H(f) = \frac{1}{1+j2\pi f\tau}$

$T_g(f) = -\frac{d}{df}\left[-\tan^{-1}(2\pi f\tau)\right]$

$= \frac{2\pi T}{1+(2\pi f T)^2}$

low pass $= \frac{2\pi T}{1+(2\pi f\tau)^2}$

$\frac{j\omega T}{1+j\omega T} = \frac{j2\pi f T}{1+j2\pi f\tau}$

$\frac{\pi}{2} - \tan^{-1}(2\pi f T)$

$= \frac{-d}{df}\left[\frac{\pi}{2} - \tan^{-1}(2\pi f\tau)\right]$

high pass filter $= \frac{2\pi\tau}{1+(2\pi f\tau)^2}$

3.(a) $f(t) = \int_{-\infty}^{\infty} \frac{a}{2}(\delta(f-f_0) + \delta(f+f_0))e^{j2\pi ft}\,df$

$\frac{a}{2}\int_{-\infty}^{\infty} \delta(f+f_0)e^{j2\pi ft}\,df = \frac{a}{2}e^{-j2\pi f_0 t}$

$\frac{a}{2}(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t})$

$f(t) = a\cos(2\pi f_0 t)$

$= a\cos(2\pi f_0 t)$

b. $S(f) = \frac{a}{2j}(\delta(f-f_0) - \delta(f+f_0))$

$= \frac{a}{2j}\int_{-\infty}^{\infty} \delta(f-f_0)e^{j2\pi ft}\,df - \frac{a}{2j}\int_{-\infty}^{\infty} \delta(f+f_0)e^{j2\pi ft}\,df$

$f(t) = \frac{a}{2j}(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t})$

$= e^{ja} - e^{-ja} = 2j\sin(a)$

$f(t) = a\sin(2\pi f_0 t)$

Ray Rau DSP Problem set #3

1. $A\{x[n]\} = 2x[n] - 1$

$A\{x_1[n] + x_2[n]\} = 2(x_1[n] + x_2[n]) - 1$

$= 2x_1[n] + 2x_2[n] - 1)$

$= 2x_1[n] + 2x_2[n] - 2$

System A is not linear due to it not following the additivity property. $2x_1[n] + 2x_2[n] - 1 \neq 2x_1[n] + 2x_2[n] - 2$

System B

$B\{x[n]\} = 0.5x[n]$

$B\{x_1[n]\} + B\{x_2[n]\}$

$= 0.5x_1[n] + 0.5x_2[n]$

System B is linear.

B. $A'\{x[n]\} = 2x[n]$

$A'\{x_1[n]\} + A'\{x_2[n]\}$

$= 2x_1[n] + 2x_2[n]$

$B\{x[n]\} = 0.5x[n]$

$A'\{B\{x[n]\}\} = A'\{0.5x[n]\}$

$= x[n]$

$B\{A'\{x[n]\}\} = B\{2x[n]\} = 0.5(2x[n]) = x[n]$

System A is not linear and system B is linear.