Tuesday Warm Up, Unit 0: Foundations and Fundamentals

Prof. Jordan C. Hanson

January 21, 2025

1 Memory Bank

- $\sqrt{-1} = j$ ... The fundamental imaginary unit.
- $z = x + jy$ ... A complex number.
- $\Re\{z\} = x$, $\Im\{z\} = y$ ... Real and imaginary parts.
- $z^* = x - jy$ ... The complex conjugate of z.
- $|z| = \sqrt{zz^*} = \sqrt{x^2 + y^2}$ ... The magnitude of z.
- $\tan\phi = y/x$ ... The phase angle of z.
- $|z| = r$, so $x = r\cos\phi$, and $y = r\sin\phi$.
- **Taylor Series:** Let $f(t)$ be a continuous, differentiable function. Let $f^n(t)$ be the n-th derivative of $f(t)$, with $f^0(t) = f(t)$. The Taylor series is an infinite series, equivalent to $f(t)$, given by

$$f(t) = \sum_{n=0}^{\infty} \frac{f^n(t_0)}{n!}(t - t_0)^n \tag{1}$$

- **Euler's Identity:** $e^{j\phi} = cos\phi + j\sin\phi$

2 Complex Numbers and Signals

1. Let $z_1 = 2 - 2j$, and $z_2 = -2 + 2j$. Evaluate: (a) $z_1 + z_2$ (b) $z_1 - z_2$ (c) $z_1 * z_2$ (d) $|z_1|$ (e) $|z_2|$.

2. Let $z_1 = 2 - 2j$, and $z_2 = -2 + 2j$. Evaluate: (a) $\tan\phi_1$ (b) $\tan\phi_2$.

3. Using the Taylor series with $t_0 = 0$, show that

$$\cos(t) \approx 1 - \frac{t^2}{2} + \frac{t^4}{24} - ... \tag{2}$$

and

$$\sin(t) \approx t - \frac{t^3}{6} + \frac{t^5}{120} - ... \tag{3}$$

4. Using the Taylor series with $t_0 = 0$, show that

$$e^t = \sum_{n=0}^{\infty} \frac{t^n}{n!} \tag{4}$$

5. Using the results of the previous two exercises, show that

$$e^{j\phi} = \cos\phi + j\sin\phi \tag{5}$$

3 Application: AC Waveforms and Circuits

1. Suppose we have a voltage signal as a function of time: $v(t) = 2.5\cos(2\pi ft - \pi/4)$. The siganl has an amplitude of 2.5 Volts, a frequency $f = 1kHz$, and a phase shift of $\pi/4$ (45 degrees). Let $\phi = 2\pi ft - \pi/4$. (a) Show that

$$v(t) = \Re\{2.5e^{j\phi}\} \tag{6}$$

(b) Show that

$$v(t) = \Im\{2.5e^{j(\phi - \pi/2)}\} \tag{7}$$