Homework 2

ADC

a) $\Delta V = \frac{2.55-0}{255} = 0.01V = 10mV$

b) $n=12 \to 2^{12}-1 = 4095$

$\Delta V = \frac{4.095-0}{4095} = 0.001V = 1mV$

c) $\Delta V = \frac{12}{2^n-1} < 0.001$

$\hookrightarrow 2^n - 1 > \frac{12}{0.001} = 12000$

$2^n > 12001$

$n > \log_2(12001) \approx 13.55$

$2^{14}-1 = 16383$

$\Delta V = \frac{12}{16383} \approx 0.000732V < 1mV$

so you need at least 14 bits

d) $2^n - 1 = 2047$

$\Delta V = \frac{5}{2047} \approx 0.002443 V/counts$

$Counts = \frac{2.52}{\Delta V} \Rightarrow 2.52 \cdot \frac{2047}{5} = 1031.7$

1032 counts

DAC

a) $V_{out} = 256 \cdot 0.0098V = 2.51V$

b) $\Delta V = \frac{5}{4095} \approx 0.001220 \, V/count$

$V_{out} = 2048 \cdot \frac{5}{4095} \approx 2.5006V$

c) $\Delta V = \frac{0.25}{128} = 0.01953125 \, V/count.$

$V_{max} = 511 \cdot \Delta V = 0.998V$

③ $f = 25 \cdot 10^6 Hz \quad R = 10^4 \Omega$

$\frac{V_{out}}{V_{in}} = \frac{0.33}{3.3} = 0.1$

$H(f) = \frac{1}{1+j\omega T} \quad T = RC$

$|H(f)| = \frac{1}{\sqrt{1+(\omega RC)^2}} = 0.1$

$\sqrt{1+(\omega RC)^2} = \frac{1}{0.1} = 10$

Prob & statistics, Noise

② $f_s = 500kHz$

a) $f_a = 50kHz$

$r = 50kHz$

$50 \leq 250$

$f_{digi} = 50kHz$

b) $f_a = 250kHz$

$r = 250kHz$

$r = 250 = f_s/2$

$f_{digi} = 250kHz$

c) $f_a = 750kHz$

$r = 750-500 = 250kHz$

$f_{digi} = 250kHz$

d) $f_a = 1000kHz$

$r = 1000 - 2 \cdot 500 = 0kHz$

$f_{digi} = 0$

$$1 + (\omega RC)^2 = 100$$

$$(\omega RC)^2 = 99$$

$$RC = \frac{\sqrt{99}}{\omega}$$

$$\omega = 2\pi f$$

$$\omega = 2\pi f = 2\pi \cdot 25 \cdot 10^6 = 1.57 \cdot 10^8 \text{ rads/s}$$

$$RC = \frac{\sqrt{99}}{2\pi f} \longrightarrow C = \frac{\sqrt{99}}{2\pi f R}$$

$$C = \frac{\sqrt{99}}{2\pi \cdot 25 \cdot 10^6 \cdot 10^4} = 6.33 \cdot 10^{-12} F$$

$$= \boxed{6.33 pF}$$

④ $$H(f) = \frac{j\omega T}{1 + j\omega \tau} \qquad \tau = RC$$

$V_{in} = 3.3V \quad V_{out} = 0.33v$

$R = 10k\Omega = 10^4 \Omega$

$f = 10MHz = 10 \cdot 10^6 Hz$

$$\frac{\omega RC}{\sqrt{1 + (\omega RC)^2}} = 0.1 \longrightarrow \frac{(\omega RC)^2}{1 + (\omega RC)^2} = 0.01$$

$$(\omega RC)^2 = 0.01(1 + (\omega RC)^2)$$

$$\hookrightarrow (\omega RC)^2 = 0.01 + 0.01(\omega RC)^2$$

$$\hookrightarrow (1 - 0.01)(\omega RC)^2 = 0.01$$

$$\frac{0.01}{0.99} = (\omega RC)^2$$

$$\omega RC = \sqrt{\frac{0.01}{0.99}}$$

$$C = \frac{1}{\omega R} \cdot \sqrt{\frac{0.01}{0.99}}$$

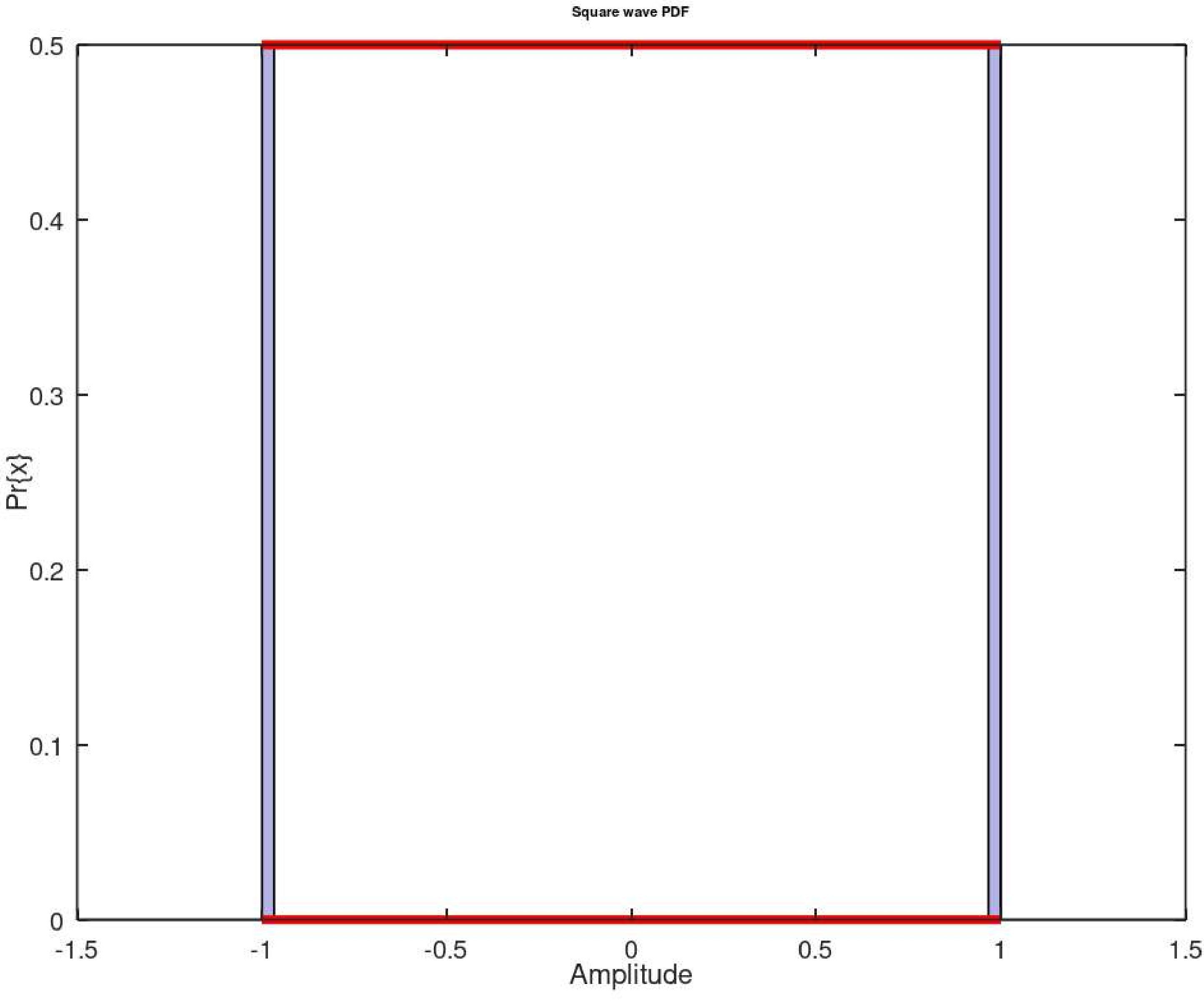
$$\omega = 2\pi f = 2\pi \cdot 10 \cdot 10^6$$
$$= 2\pi \cdot 10^7$$

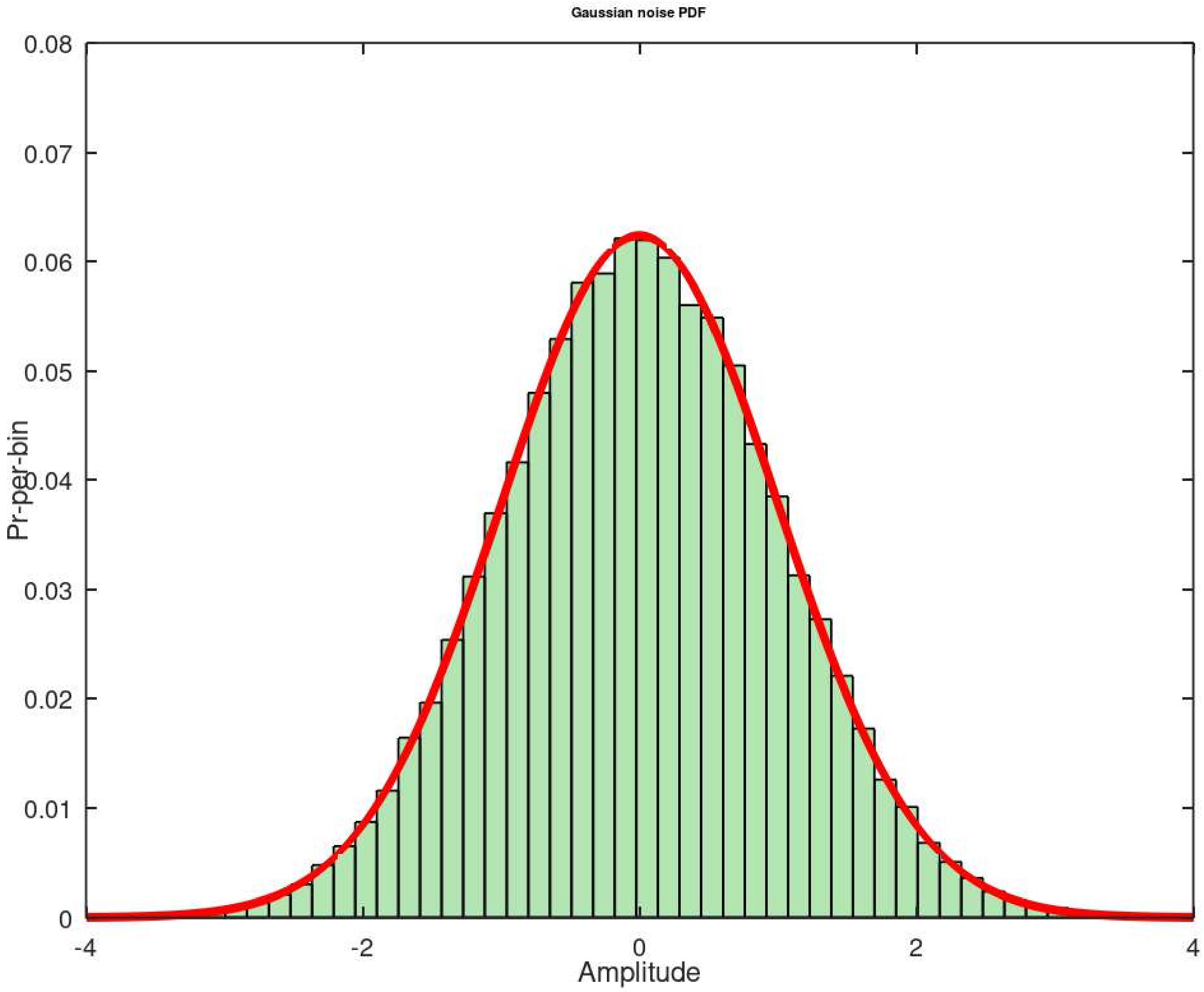
$$C = \frac{1}{2\pi \cdot 10^7 \cdot 10^4} \cdot \sqrt{\frac{0.01}{0.99}}$$
$$= \boxed{1.599 \cdot 10^{-13} F}$$

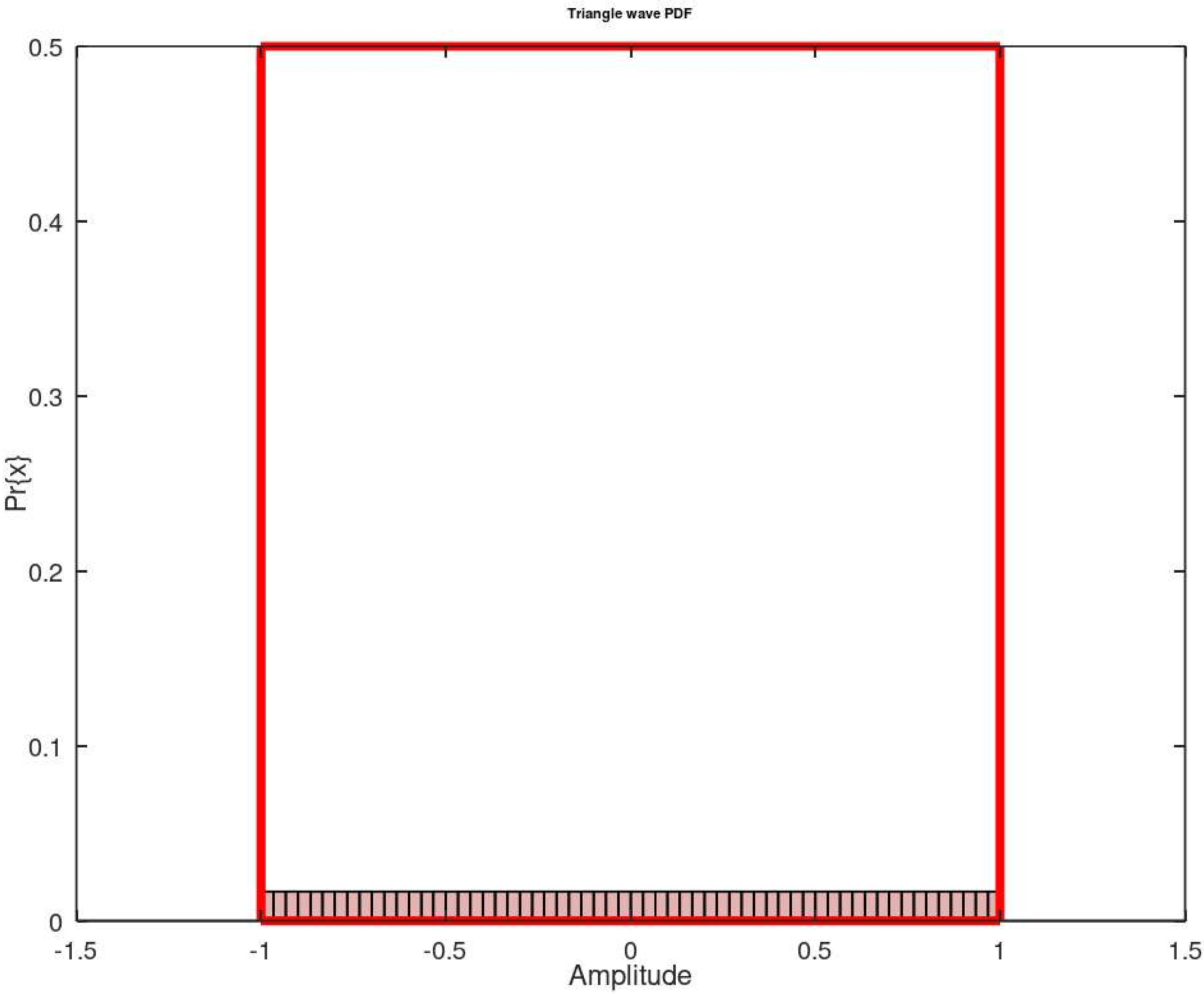
```
function y = square_wave(N, A)
  t = (0:N-1)/N;
  y = A * sign(sin(2*pi*t))';
endfunction

function y = triangle_wave(N, A)
  t = (0:N-1)/N;
  y = (2*abs(2*(t - floor(t + 0.5))) - 1)' * A;
endfunction

function y = gaussian_noise(N, mu, sigma)
  y = mu + sigma*randn(N,1);
endfunction
```


Square wave PDF
0.5
0.4
0.3
0.2
0.1
0
Pr{x}
-1.5
-1
-0.5
0
0.5
1
1.5
Amplitude


Gaussian noise PDF
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Pr-per-bin
-4
-2
0
2
4
Amplitude


Triangle wave PDF
Pr{x}
Amplitude
0.5
0.4
0.3
0.2
0.1
0
-1.5
-1
-0.5
0
0.5
1
1.5