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H.w#4

2. Impulse Response

1. Impulse response of audio echo system

(a) Sampling frequency is 20 kHz and duration is 2 seconds.
→ Number of samples = 20,000 × 2 = 40,000 samples.

(b) An echo every 0.2 seconds means 0.2 × 20,000 = 4,000 samples apart.
→ Non-zero $\delta[n]$ at positions: 0, 4000, 8000, ..., 36000.

(c) Each echo is half the amplitude of the previous one.
→ $h[n] = \delta[n] + (1/2)\delta[n-4000] + (1/4)\delta[n-8000] + \dots$

(d) For a 0.1 second sine-tone (10 Hz), the output would be the original signal followed by successively weaker echoes every 0.2 seconds.

2. Impulse response of a band-pass filter

(a) If the input signal s[n] is split and passed through l[n] and h[n] in parallel, and their outputs are summed, the result is the original signal.
→ So: $l[n] + h[n] = \delta[n]$

(b) Rearranging the above: $h[n] = \delta[n] - l[n]$

(c) To create a band-pass filter with different cutoff frequencies, we need:
→ A: the cutoff of l[n] is lower than that of h[n].

3. Discrete Fourier Transform, Filtering, and Noise

1. Discrete Fourier Transform properties

(a) Proving linearity:
Let $z[n] = a\cdot x[n] + b\cdot y[n]$
→ $\mathrm{DFT}\{z[n]\} = a\cdot\mathrm{DFT}\{x[n]\} + b\cdot\mathrm{DFT}\{y[n]\}$ → Linear (additive and homogeneous)

(b) Let $X[k] = \delta[k - k_0]$. Only one non-zero at k_0.
→ Inverse DFT gives $x[n] = (1/N)\cdot e^{(j2\pi k_0 n/N)}$, a complex sinusoid at k_0.

2. Spectrum of a Square Pulse

(a) Running the code shows that as the pulse width narrows, the Fourier spectrum widens. This is due to the time-frequency uncertainty principle.

(b) Measuring the width of time-domain and frequency-domain signals shows that their product is approximately constant.
→ This confirms the uncertainty principle.