Tuesday Warm Up, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- $\sqrt{-1} = j$ ... The fundamental imaginary unit.
- $z = x + jy$ ... A complex number.
- $\Re\{z\} = x$, $\Im\{z\} = y$ ... Real and imaginary parts.
- $z^* = x - jy$ ... The complex conjugate of z.
- $|z| = \sqrt{zz^*} = \sqrt{x^2 + y^2}$ ... The magnitude of z.
- $\tan\phi = y/x$ ... The phase angle of z.
- $|z| = r$, so $x = r\cos\phi$, and $y = r\sin\phi$.
- **Complex response $R(f)$ of a high-pass filter with resistance r and capacitance C:** $R(f) = j\omega\tau/(1 + j\omega\tau)$, where $\omega = 2\pi f$, and $\tau = rC$.

2 Application of Complex Numbers: AC Circuit Filters

1. The response of a simple high-pass RC filter is

$$R(f) = j\omega\tau/(1 + j\omega\tau) \tag{1}$$

(See memory bank). (a) Find the magnitude[1] of Eq. 1. (b) Find the phase angle of Eq. 1. (c) Graph the magnitude and phase angle versus frequency, by hand. (d) Suppose a signal has a an amplitude of A at a frequency f: $A(f)$. The filtered amplitude is $R(f)A(f)$. If $A = 1$ at $f = 0.5$ kHz, $R = 1$ kΩ, and $C = 1$ μF, what is the filtered amplitude $A(f)R(f)$?[2]

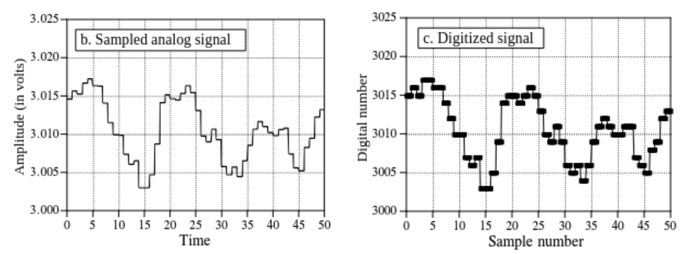

Figure 1: (Top) A sampled, analog signal. (Bottom) The data from (top), but digitized.

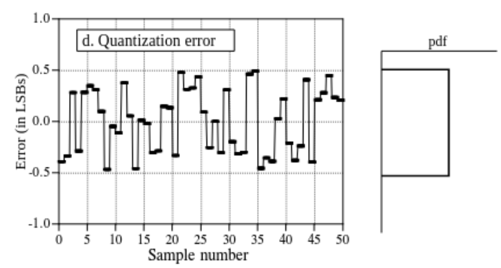

Figure 2: The quantization error for the signal in Fig. 1.

3 Statistics, Probability, Noise, and ADC/DAC

1. Consider the sampled, digitized signal in Fig. 1, with quantization error shown in Fig. 2. Note that when the signal is *digitized*, the error in voltage is equally likely to fall anywhere between two digital voltage levels: $[-1/2, 1/2]$ LSB (least significant bit). The error distribution is shown by the PDF on the right side of Fig. 2. (a) If the signal ranges between 0 and 3.3 Volts, and there are 10 bits available for digitization ($2^1 0 = 1024$ voltage levels), what is the voltage per level? (b) The voltage per level is called the LSB, or *least significant bit.* What is the LSB in millivolts? (c) What is the average quantization error in LSB? (d) What is the standard deviation of the quantization error in LSB?

[1]Hint: multiply the top and bottom by the complex conjugate of the denominator.
[2]This filtered amplitude is a result of the *convolution theorem*, which we will encounter in a later chapter.