HW #3

Linear Systems

①

a) system A – $A\{x[n]\} = 2x[n]-1$

$A\{kx[n]\} = 2(kx[n])-1$

$= 2kx[n]-1 \neq k(2x[n]-1)$

$= kA\{x[n]\}$

Fails homogeneity – not linear

System b: $B\{x[n]\} = 0.5x[n]$

$B\{kx[n]\} = 0.5kx[n]$

$= kB\{x[n]\}$

$B\{x_1[n]+x_2[n]\} = 0.5(x_1[n]+x[n])$

$= 0.5x_1[n]+0.5x_2[n]$

$= B\{x_1[n]\} + B\{x_2[n]\}$

System B passes both so linear

System A breaks homogeneity

① b) $A\{x[n]\} = 2x[n]-1$

$A'\{x[n]\} = 2x[n]$

$B\{x[n]\} = 0.5x[n]$

$A'\{B\{x[n]\}\} = A'\{0.5x[n]\} = 2\cdot 0.5x[n]$

$B\{A'\{x[n]\}\} = 0.5\cdot 2x[n] = x[n]$

② $f(t) = a_1\cos(2\pi f_1 t) + a_2\cos(2\pi f_2 t)$

$T_1 = \frac{1}{f_1}$

$T_2 = \frac{1}{f_2}$ $\quad f_2 = 2f_1$

1. $\int_{-\infty}^{\infty} f(t)\,\delta(t-T_1)\,dt$

2. $\int_{-\infty}^{\infty} f(t)\,\delta(t-T_1)\,dt$ $\quad f(t)\,\delta(t-T_2)\,dt$

$\int_{-\infty}^{\infty} f(t)\,\delta(t-T_1)\,dt = f(T_1)$

$f(T_1) = a_1\cos(2\pi f_1 T_1) + a_2\cos(2\pi f_2 T_1)$

$T_1 = \frac{1}{f_1} \rightarrow 2\pi f_1 T_1 = 2\pi$

$f_2 = 2f_1 \rightarrow f_2 T_1 = 2 \rightarrow f(T_1) = a_1\cos(2\pi) + a_2\cos(4\pi) = a_1(1) + a_2(1) = \boxed{a_1+a_2}$

$\int_{-\infty}^{\infty} f(t)\,\delta(t-T_2)\,dt = f(T_2)$

$f(T_2) = a_1\cos(2\pi f_1 T_2) + a_2\cos(2\pi f_2 T_2)$

$T_2 = \frac{1}{f_2} = \frac{1}{2f_1}$

$f_1 T_2 = \frac{1}{2} \rightarrow 2\pi f_1 T_2 = \pi$

$f_2 T_2 = 1 \rightarrow 2\pi f_2 T_2 = 2\pi$

$f(T_2) = a_1\cos(\pi) + a_2\cos(2\pi) = a_1(-1) + a_2(1)$

$\boxed{= -a_1 + a_2}$

② $T_g = \frac{d\phi(\omega)}{d\omega}$

low pass

$H(\omega) = \frac{1}{1+j\omega\tau}$

$\phi(\omega) = -\tan^{-1}(\omega\tau)$

$\tau = -\frac{d}{d\omega}(-\tan^{-1}(\omega\tau))$

$= \frac{\tau}{1+(\omega\tau)^2}$

high pass

$H(\omega) = \frac{j\omega\tau}{1+j\omega\tau}$

$\phi(\omega) = \frac{\pi}{2} - \tan^{-1}(\omega\tau)$

$T_g = -\frac{d}{d\omega}\left(\frac{\pi}{2} - \tan^{-1}(\omega\tau)\right) = \frac{\tau}{1+(\omega\tau)^2}$

③ $S(f) = \frac{a}{2}(\delta(f-f_0) + \delta(f+f_0))$

a)

$s(t) = \int_{-\infty}^{\infty} S(f) e^{j2\pi f t}\, df$

$s(t) = \frac{a}{2}\left(\int_{-\infty}^{\infty} \delta(f-f_0) e^{j2\pi f t} df + \int_{-\infty}^{\infty} \delta(f+f_0) e^{j2\pi f t} df\right)$

$= \frac{a}{2}(e^{j2\pi f_0 t} + e^{-j2\pi f_0 t})$

$= \boxed{a\cos(2\pi f_0 t)}$

b) $S(f) = \frac{a}{2j}(\delta(f-f_0) - \delta(f+f_0))$

$s(t) = \frac{a}{2j}(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t})$

$= a \cdot \frac{1}{2j}(e^{j2\pi f_0 t} - e^{-j2\pi f_0 t})$

$= a\sin(2\pi f_0 t)$

Convolution

① if $x[n] = \delta[n]$

$\hookrightarrow y[n] = \sum_{j=0}^{M-1} h[j]\cdot\delta[n-j]$

a)

$\delta[n-j] = 1$ when $j=n$

otherwise $0 \rightarrow y[n] = h[n]$

$x[n] = \delta[n] \rightarrow y[n] = h[n]$

b) $x[n] = \delta[n-n_0]$

$y[n] = \sum_{j=0}^{M-1} h[j]\cdot\delta[n-n_0-j]$

$y[n] = h[n-n_0]$

$x[n] = \delta[n-n_0] \rightarrow y[n] = h[n-n_0]$

③ $f(t) = a\delta(t-t_0)$

a) $F(f) = \int_{-\infty}^{\infty} a\delta(t-t_0)e^{-j2\pi f t}dt$

$\boxed{= ae^{-j2\pi f t_0}}$

$|F(f)| = |ae^{-j2\pi f t_0}|$

$= |a| \cdot |e^{-j\theta}| = a$

b) $F(f) = ae^{-j2\pi f t_0}$

$\phi = \arg(F(f)) = \arg(ae^{-j2\pi f t_0}) \boxed{= -2\pi f t_0}$

c) $\tau_g = -\frac{d\phi}{d\omega} \quad \omega = 2\pi f$

$\phi = -2\pi f t_0 \rightarrow \phi(\omega) = -\omega t_0$

$\tau_g = -\frac{d(-\omega t_0)}{d\omega} \boxed{= t_0}$

④ $\delta[n] = [1000000]^T$

$y[n] = S[x[n]] = 0.5x[n-2]$

$h[n] = S[\delta[n]]$

$S[x[n]] = 0.5x[n-2]$

$\hookrightarrow h[n] = 0.5\delta[n-2]$

a)

$\delta[n-2] = [0\ 0\ 1\ 0\ 0\ 0\ 0]$

$h[n] = 0.5 \cdot [0\ 0\ 1\ 0\ 0\ 0\ 0]$

$= [0\ 0\ 0.5\ 0\ 0\ 0\ 0]$

b) $s[n] = [0\ 1\ 1\ 1\ 1\ 1\ 1]$

$y[n] = 0.5x[n-2]$

$\hookrightarrow y[n] = 0.5s[n-2]$

$s[n-2] = [\ \ 0\ 1\ 1\ 1\ 1]$

$y[n] = 0.5 \cdot [0\ 0\ 0\ 1\ 1\ 1\ 1]$

$= [0\ 0\ 0\ 0.5\ 0.5\ 0.5\ 0.5]$

Fourier Transforms

①

a) $s(t) = a\delta(t-t_0)$

$S(f) = F\{s(t)\}$

$S(f) = \int_{-\infty}^{\infty} s(t) \cdot e^{-j2\pi f t}dt$

$= \int_{-\infty}^{\infty} a\delta(t-t_0) \cdot e^{-j2\pi f t}dt$

$= a\int_{-\infty}^{\infty} \delta(t-t_0) \cdot e^{-j2\pi f t}dt$

$\int_{-\infty}^{\infty} \delta(t-t_0) \cdot g(t)\,dt = g(t_0)$

$\boxed{S(f) = a \cdot e^{-j2\pi f t_0}}$

b) $H(f) = \frac{1}{1+j\omega\tau} = \frac{1}{1+j2\pi f\tau}$

$Y(f) = S(f) \cdot H(f) = ae^{-j2\pi f t_0} \cdot \frac{1}{1+j2\pi f\tau}$

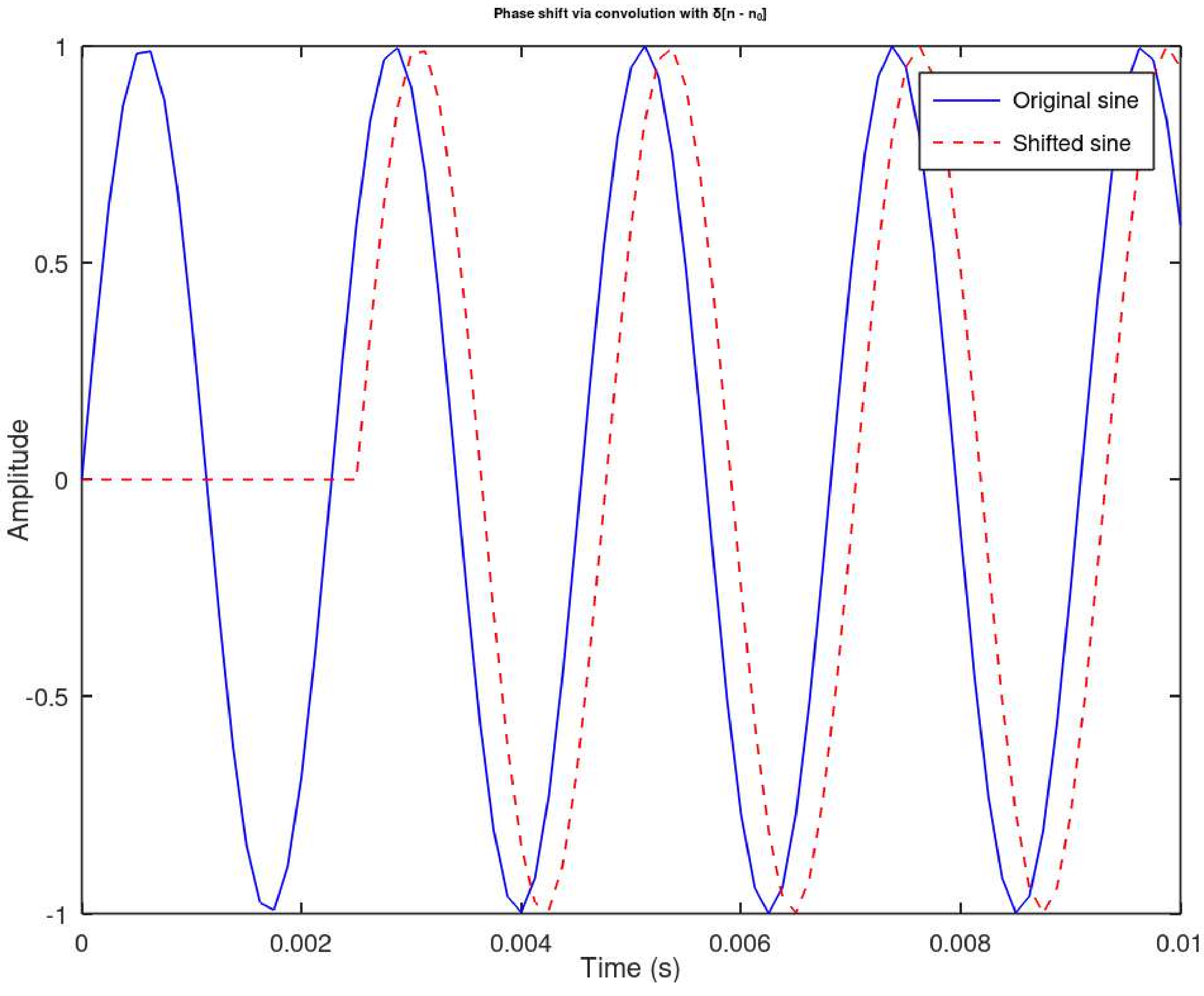
$|Y(f)| = |a| \cdot \left|\frac{1}{1+j2\pi f\tau}\right|$

$\boxed{= a \cdot \frac{1}{\sqrt{1+(2\pi f\tau)^2}}}$

c) $H(f) = \frac{j\omega\tau}{1+j\omega\tau} = \frac{j2\pi f\tau}{1+j2\pi f\tau}$

$Y(f) = ae^{-j2\pi f t_0} \cdot \frac{j2\pi f\tau}{1+j2\pi f\tau}$

$\boxed{= \frac{a \cdot 2\pi f\tau}{\sqrt{1+(2\pi f\tau)^2}}}$


Phase shift via convolution with δ[n - n0]
Original sine
Shifted sine
Amplitude
Time (s)
1
0.5
0
-0.5
-1
0
0.002
0.004
0.006
0.008
0.01