Thursday Warm Up, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- $\sqrt{-1} = j$ ... The fundamental imaginary unit.
- $z = x + jy$ ... A complex number.
- $\Re\{z\} = x$, $\Im\{z\} = y$ ... Real and imaginary parts.
- $z^* = x - jy$ ... The complex conjugate of z.
- $|z| = \sqrt{zz^*} = \sqrt{x^2 + y^2}$ ... The magnitude of z.
- $\tan\phi = y/x$ ... The phase angle of z.
- $|z| = r$, so $x = r\cos\phi$, and $y = r\sin\phi$.
- **Complex response of a low-pass filter with resistance R and capacitance C:** $R(f) = 1/(1+j\omega\tau)$, where $\omega = 2\pi f$, and $\tau = RC$.

2 Application of Complex Numbers: AC Circuit Filters

1. Recall that the response of a simple low-pass RC filter is
$$R(f) = \frac{1}{1 + j\omega\tau} \quad (1)$$
(See memory bank). (a) Find the magnitude of Eq. 1.[1] (b) Find the phase angle of Eq. 1. (c) Graph the magnitude and phase angle versus frequency, by hand. (d) Suppose a signal has a an amplitude of A at a frequency f: $A(f)$. The filtered amplitude is $R(f)A(f)$. If $A = 1$ at $f = 1$ kHz, $R = 1$ kΩ, and $C = 1$ μF, what is the filtered amplitude $A(f)R(f)$?[2]

3 Statistics, Probability, and Noise

1. Consider a signal with 2.5 V amplitude, and a DC offset of 2.5 V: $s(t) = 2.5\sin(2\pi f t) + 2.5$. (a) Write a short code in `octave` that produces and plots this signal, with $f = 10$ Hz, and $\Delta t = 1$ ms. (b) Use the `randn` function to create a noise vector of the same size as $s(t)$, but with a mean of 0 and a standard deviation of 1.0: `n = randn(size(t))`. (c) Plot the signal plus noise on the same graph: `plot(t,z)`, where $z = s + n$. (d) What is the signal-to-noise ratio (SNR) of the sine wave plus noise? (e) Use the `hist` command to create a histogram of your noise values, and signal plus noise values.

[1] Hint: multiply the top and bottom by the complex conjugate of the denominator.

[2] This filtered amplitude is a result of the *convolution theorem*, which we will encounter in a later chapter.