1

a) $\Re\{2.5e^{j\phi}\} = 2.5\cos\phi = 2.5\cos(2\pi ft - \frac{\pi}{4})$

$e^{ix} = \cos x + i\sin x \qquad \phi = 2\pi ft - \pi/4$

$2.5e^{j\phi} = 2.5(\cos\phi + i\sin\phi) = 2.5\cos\phi + 2.5i\sin\phi$

b.) $\Im\{2.5e^{j(\phi - \frac{\pi}{2})}\} = 2.5\sin(\phi \frac{\pi}{2}) = 2.5\cos(\phi)$

$= 2.5\cos(2\pi ft - \frac{\pi}{4})$

$\sin(x - \frac{\pi}{2}) = \cos(x)$

2

a) $Hz = \frac{cycle}{sec}$

$KHz = \frac{cycle \times 10^3}{sec}$

$KHz^{-1} = \frac{sec \times 10^{-3}}{cycle} = 1ms/cycle$

b) $\frac{1}{5\times 10^{-9}} = 2\times 10^8 Hz$

c) $\frac{1}{5}\times 10^{-3}s = period = 200s$

$200s \cdot 50000 Hz = 10^7$ samples

d) $\frac{1}{0.002ms} = \frac{1}{2\times 10^{-3}s} = 5\times 10^2 Hz = 500 Hz$

10^5 samples

3

a) $\Delta V = \frac{2.56}{256} = 0.01V$

b) $\log_2(256) = 8$

c) $256^2 = 65536 = 2^{16}$

$\frac{2.56}{65536} = 3.9 \times 10^{-5}$

```
%problem 4
clear;
close;
home;
f = 10;
fs = 1000;
dt = 1/fs;
T = 3
t = dt:dt:T;
s = 2.5*sin(2*pi*f.*t) + 2.5;
%plot(t,s);
n = randn(size(t));
z = s+n;
%plot(t,z);
mean(s-n);

hist(z)
```

5

a) $\frac{j\omega T}{(1+j\omega T)} = R(f)$

$$\frac{j\omega T}{1+j\omega T}\cdot\frac{1-j\omega T}{1-j\omega T} = \frac{j\omega T+(\omega T)^2}{1+(\omega T)^2} = \frac{j\omega T}{1+(\omega T)^2}+\frac{(\omega T)^2}{1+(\omega T)^2}$$

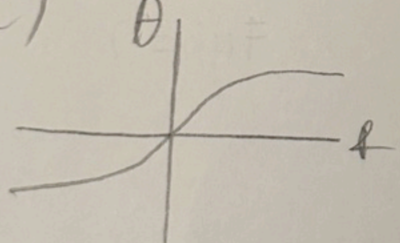
$$= \frac{(\omega T)^2}{1+(\omega T)^2}+\frac{j\omega T}{1+(\omega T)^2} = \frac{1}{1+(\omega T)^2}\left((\omega T)^2+j\,\omega T\right)$$

$$||R(f)|| = \frac{1}{1+(\omega T)^2}\sqrt{(\omega T)^4+(\omega T)^2} = \frac{1}{1+(\omega T)^2}\sqrt{\frac{1}{(\omega T)^2}\left((\omega T)^2+1\right)}$$

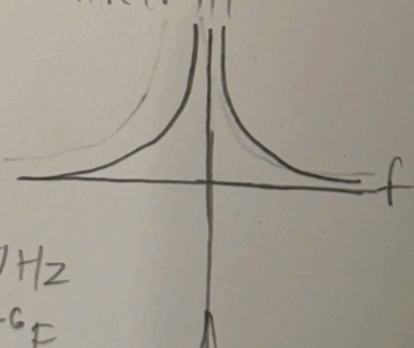
$$= \frac{1}{\omega T+(\omega T)^2}\sqrt{\omega T^2+1} = \frac{1}{2\pi f RC+(2\pi f RC)^2}\sqrt{(2\pi f RC)^2+1}$$

b) $$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\frac{(\omega T)^2}{1+(\omega T)^2}}{\frac{\omega T}{1+(\omega T)^2}}\right) = \tan^{-1}(\omega T)$$
$$= \tan^{-1}(2\pi f RC)$$

c)



$||R(f)||$



$A=1$ $f=0.5\,kHz = 500\,Hz$

$R=1000\Omega$ $C=1\times10^{-6}F$

$A(f)=1$

$$R(f)^2 = \left(\frac{j2\pi f RC}{1+j2\pi f RC}\right)^2 = \left(\frac{j\,2\pi\cdot500\cdot1000\cdot1\times10^{-6}}{1+j2\pi\cdot500\cdot1000\cdot1\times10^{-6}}\right)^2$$

$$= \left(\frac{j\pi}{1+j\pi}\right)^2 = \left(\frac{j\pi}{1+j\pi}\cdot\frac{1-j\pi}{1-j\pi}\right)^2$$

$$= \left(\frac{j\pi+\pi^2}{1+\pi^2}\right)^2 =$$

6

a) 10 KHz ≥ 5 KHz

2.5 KHz

b) 10 KHz ≥ 10 KHz

10 KHz

c) 7.5 KHz

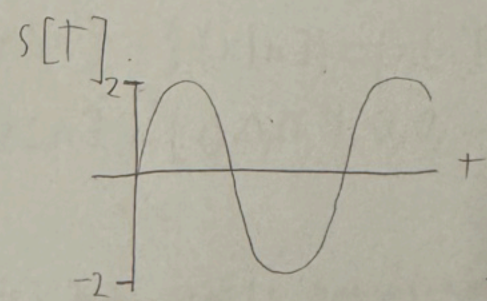
d) 0 Hz double period

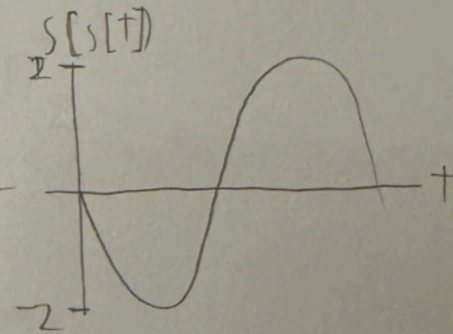
7 $s(s[t]) = s(t - \frac{T}{2})$

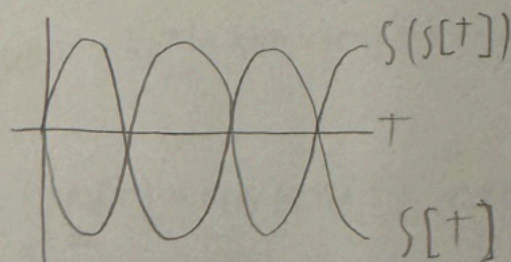
a) $s(t) = 2\sin(2\pi f t)$ $T = \frac{1}{f}$

$S(s[t]) = 2\sin(2\pi f(t - \frac{1}{2f})) = 2\sin(2\pi f t - \pi)$

b)







c) $s(t) + S(s[t]) = 0$

8 a) $y[n] = S(x[n]) = -x[n-1]$

$y[n] = [0, 0, -2, 0, 0, 0, \ldots]$

b) $y[n] = S(x[n]) = (x[n])^2$

$y[n] = [0, 0, 0, 4, 0, 0, \ldots]$

c)

a) no fails homogenous

b) yes both homogenous and additive

9 a) $\cos(2\pi f t) = \cos(-2\pi f t)$ even

b) $e^{-\left(\frac{x}{\sigma}\right)^2} = e^{-\left(\frac{-x}{\sigma}\right)^2}$ even

c) $-e^{-\alpha t} = e^{-\alpha t}$ odd

d) odd when $b \neq 0$ and $c \neq 0$ otherwise even

10 a) $\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-2\pi j f t} dt$

$\mathcal{F}\{kf(t)\} = k F(f)$

$= \int_{-\infty}^{\infty} kf(t) e^{-2\pi j f t} dt = k\int_{-\infty}^{\infty} f(t) e^{-2\pi j f t} dt$

$= k\ F(f)$

b) $\mathcal{F}(f_1(t)\} = F_1(f)$

$\mathcal{F}\{f_2(t\} = F_2(f)$

$F\{f_1(t) + f_2(t)\} = \int_{-\infty}^{\infty} (f_1(t) + f_2(T)) e^{-2\pi j f t} dt$

$= \int_{-\infty}^{\infty} f_1 e^{-2\pi j f T} dT + \int_{-\infty}^{\infty} f_2 e^{-2\pi j f t} dt$

$= F_1(f) + F_2(f)$

c) $\mathcal{F}\{f(t-n)\} = \int_{-\infty}^{\infty} f(t-n) e^{-2\pi j f t} dt$

$u = t - n \quad du = 1$

$t = u + n$

$= \int_{-\infty}^{\infty} f(u) e^{-2\pi j f (u+n)} du = F(f+n)$

so long as n is not complex ie contains a j component

11 a) $\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t) e^{-2\pi j f t} dt$

$$\mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} a\delta(t-t_0) e^{-2\pi j f t} dt$$

$$= a \int_{-\infty}^{\infty} \delta(t-T_0) e^{-2\pi j f t} dt$$

$$= a e^{-2\pi j f t_0}$$

b) $a = $ mag

c) $-2\pi f t_0 =$ phase angle

12 a) $F(f) = (a/2)(\delta(f-f_0) + a\delta(f+f_0))$

$$\mathcal{F}^{-1}\{F(f)\} = \int_{-\infty}^{\infty} F(f) e^{2\pi j f t} df$$

$$= \int_{-\infty}^{\infty} \left(\frac{a}{2}\right)(\delta(f-f_0) + a\delta(f+f_0)) e^{2\pi j f t} df$$

$$= \frac{a}{2}\int_{-\infty}^{\infty}\delta(f-f_0) e^{2\pi j f t} df + \frac{a^2}{2}\int_{-\infty}^{\infty}\delta(f+f_0) e^{2\pi j f t} df$$

$$= \frac{a}{2} e^{2\pi j f_0 t} + \frac{a^2}{2} e^{-2\pi j f_0 t} = F(t)$$

b) $F(f) = \frac{a}{2j}(\delta(f-f_0) - a\delta(f+f_0))$

$$\mathcal{F}^{-1}\{F(t)\} = \int_{-\infty}^{\infty} \frac{a}{2j}(\delta(f-f_0) - a\delta(f+f_0)) e^{2\pi j f t} df$$

$$= \frac{a}{2j}\int_{-\infty}^{\infty}\delta(f-f_0) e^{2\pi j f t} df - \frac{a^2}{2j}\int_{-\infty}^{\infty}\delta(f+f_0) e^{2\pi j f t} df$$

$$= \frac{a}{2j} e^{2\pi j f_0 t} - \frac{a^2}{2j} e^{-2\pi j f_0 t} = F(t)$$

13 a) $\cos(t) = \frac{e^{it} + e^{-it}}{2}$

$$A\cos(2\pi f_{LO} t) = A\frac{e^{j2\pi f_{LO} t} + e^{-j2\pi f_{LO} t}}{2}$$

$$\frac{m}{A}\cos(2\pi f_A t) = \frac{m}{A}\frac{e^{j2\pi f_A t} + e^{-j2\pi f_A t}}{2}$$

b) $A\frac{e^{j2\pi f_{LO} t} + e^{-j2\pi f_{LO} t}}{2} \frac{m}{A}\frac{e^{j2\pi f_A t} + e^{-j2\pi f_A t}}{2}$

$$= m\frac{\left(e^{j2\pi f_{LO} t} + e^{-j2\pi f_{LO} t}\right)\left(e^{j2\pi f_A t} + e^{-j2\pi f_A t}\right)}{4}$$

$$= m\frac{e^{j2\pi f_{LO} t + j2\pi f_A t} + e^{-j2\pi f_{LO} t + j2\pi f_A t} + e^{-j2\pi f_{LO} t - j2\pi f_A t} + e^{j2\pi f_{LO} t - j2\pi f_A t}}{4}$$

$$= m\frac{e^{j2\pi(f_{LO}+f_A)t} + e^{j2\pi(f_A - f_{LO})t} + e^{-j2\pi(f_{LO}+f_A)t} + e^{j2\pi(f_{LO}-f_A)t}}{4}$$

$$= \frac{m}{2}\frac{e^{j2\pi(f_{LO}+f_A)t} + e^{-j2\pi(f_{LO}+f_A)t}}{2} + \frac{m}{2}\frac{e^{j2\pi(f_{LO}-f_A)t} + e^{-j2\pi(f_{LO}-f_A)t}}{2}$$

$$= \frac{m}{2}\cos(2\pi(f_{LO}+f_A)t) + \frac{m}{2}\cos(2\pi(f_{LO}-f_A)t)$$

```
%Code Project #1
clear;
close;
home;

fs = 2000;
T = 2;
%d = zeros(fs*T);
%d(1) = 1;
%for i = 1:(T/0.25)
%	d(i) = 1/pow2(i);
%endfor;

d = zeros(1,T/0.25);

for i = 1:(T/0.25)
	d(i) = 1/pow2(i);
endfor;

f = 440;
sineT = 0.5;
dt = 1/fs;
t = dt:dt:sineT;
s = cat(2,sin(2*pi*f.*t) , zeros(1,fs * (T-sineT)));

outSound = conv(d,s);
plot(outSound);
player = audioplayer(outSound,fs,8);
play(player)
```