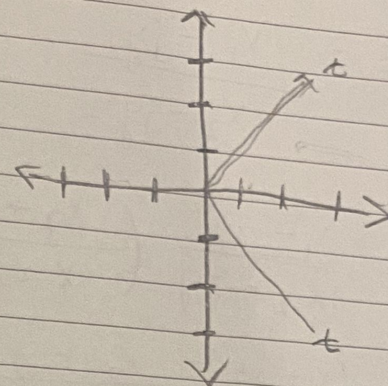
Homework 1

Eli Bedford

$$z^* = \frac{x_1 - jy_1}{x_2 - jy_2}$$



$$\left(z = \frac{x_1 + jy_1}{x_2 + jy_2}\right)$$

$$z = \frac{x_1 + jy}{x_2 + jy}\left(\frac{x_2 - jy_2}{x_2 - jy_2}\right)$$

$$z = \frac{(x_1 + jy)(x_2 - jy_2)}{x_2^2 - y_2^2}$$

$$z = \frac{x_1 x_2 + y_1 + y_2 - x_1 y_2 j + x_2 y_1 j}{x_2^2 + y_2^2}$$

$$z = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + j\,\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$$

$x_1 = 1$ $x_2 = 2$ $y_1 = 3$ $y_2 = 4$

$$\frac{(1)(2) + (3)(4)}{2^2 + 4^2} + j\,\frac{(2)(3) - (1)(4)}{2^2 + 4^2}$$

$$\frac{14}{20} + \frac{2}{20}j$$

$$\boxed{\frac{7}{10} + \frac{1}{10}j}$$

$$z = \frac{x_1 - jy_1}{x_2 - jy_2}$$

$$z = \frac{x_1 - jy}{x_2 - jy}\left(\frac{x_2 + jy_2}{x_2 + jy_2}\right)$$

$$z = \frac{(x_1 - jy)(x_2 + jy_2)}{(x_2^2 - y_2^2)}$$

$$z = \frac{x_1 x_2 + y_1 y_2 - x_1 - x_1 y_2 j + x_2 y_1 j}{x_2^2 + y_2^2}$$

$$z = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} - j\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}$$

$z = x_1 = 1 \quad x_2 = 2 \quad y_1 = 3 \quad y_2 = 4$

$$\frac{(1)(2) + (3)(4)}{(2)^2 + (4)^2} - \frac{j(2)(4) - (1)(3)}{(2)^2 + (4)^2}$$

2 (Bonus)

Prove $e^{jx} = \cos(x) + j\sin(x)$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} \cdots$$

$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \cdots$$

$$e^{jx} = 1 + (jx) + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} + \cdots$$

$$= (1 + (jx) + \frac{x^2}{2!} + j\frac{x^3}{3!} + \frac{x^4}{4!}$$

sin

$$\underbrace{\underbrace{\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!}\right)}_{\cos} + j\left(x + \frac{x^3}{3!} + \frac{x^5}{5!}\right)}_{e^{jx} = \cos(x) + \sin(x)}$$

3. prove $\cos(x) = \frac{1}{2}(e^{jx} + e^{-jx})$

$$e^{jx} = \underline{\cos x} + j\sin x \qquad j = \sqrt{-1}$$

$$\cos x = e^{jx} - j\sin x$$

$$= \frac{(\cos x + j\sin x) + (\cos - j\sin x)}{2}$$

$$= \frac{2\cos x}{2} = \cos x$$

$$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$$