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H.w#1

1. Complex Numbers and Signals

(a) Graph z1 and z2 in the complex plane:

z1 = 3 + 4j → Located in the first quadrant.

z2 = -3 + 4j → Located in the second quadrant.

(b) z1 + z2 = (3+4j) + (-3+4j) = 8j

(c) z1 - z2 = (3+4j) - ((-3+4j)) = (6+0j)

(d) z1 * z2 = (3+4j) * (-3+4j) = (-25+0j)

(e) z1 / z2 = (3+4j) / (-3+4j) = (0.28-0.96j)

(f) |z1| = 5.0

(g) |z2| = 5.0

(h) ϕ1 = angle of z1 = 53.13°

(i) ϕ2 = angle of z2 = 126.87°

(j) Polar form of z1: 5.0∠53.13°

Polar form of z2: 5.0∠126.87°

2. Use Euler's Identity

To prove the identities using Euler's formula:

Euler's identity:
e^{jθ} = cos(θ) + j sin(θ)
e^{-jθ} = cos(θ) - j sin(θ)

Add and divide by 2:
cos(2πft) = (e^{j2πft} + e^{-j2πft}) / 2

Subtract and divide by 2j:
sin(2πft) = (e^{j2πft} - e^{-j2πft}) / 2j

3. Signal Multiplication

Given:
$v1(t) = 4 \cos(2\pi f1t)$
$v2(t) = 4 \cos(2\pi f2t - \phi)$

(a) $P = v1(t)v2(t)$
$= 16 \cos(2\pi f1t) \cos(2\pi f2t - \phi)$
Using identity:
$\cos(A)\cos(B) = 0.5[\cos(A + B) + \cos(A - B)]$

$P = 8 [\cos(2\pi(f1+f2)t - \phi) + \cos(2\pi(f1-f2)t + \phi)]$

=> $f+ = f1 + f2$ and $f- = f1 - f2$

(b) P_max = 16 if $\phi = 0$ and $f1 = f2$. Because:
$\cos(0)\cos(0) = 1*1 = 1 \rightarrow 16*1 = 16$

4. Phase Offset Complex Signals

Let:
$v1(t) = \Im\{\exp(j(2\pi ft - \phi))\} = \sin(2\pi ft - \phi)$
$v2(t) = \Im\{\exp(j(2\pi ft))\} = \sin(2\pi ft)$

(a) Represent using complex numbers: $\exp(-j\phi)$, 1

(b) Add: $\exp(-j\phi) + 1 = r\angle\theta$ (use polar conversion)

(c) For $\phi = 45°$: $\exp(-j\pi/4) + 1$
Convert to rectangular: $\cos(-\pi/4) + j \sin(-\pi/4) = \sqrt{2}/2 - j\sqrt{2}/2$
Then add 1: $(1 + \sqrt{2}/2) - j\sqrt{2}/2$

Amplitude = $\sqrt{[(1 + \sqrt{2}/2)^2 + (\sqrt{2}/2)^2]}$

When $\phi = 0$: signals add constructively (max amplitude)

When $\phi = 180°$: signals cancel out (zero amplitude)

5. Uniform Random Noise in Octave

The PDF is:
$p(x) = 1 / (b - a), a \le x \le b$

Default: $a = 0$, $b = 1$

Octave Code:

```
x = rand(10000, 1);
hist(x, 30);
```

This shows a nearly flat histogram (uniform).

Summing many rand() values leads to Gaussian distribution (Central Limit Theorem)

6. Gaussian Noise and SNR

PDF of Gaussian:

$p(x) = (1 / \sqrt{(2\pi\sigma^2)}) * \exp(-(x - \mu)^2 / (2\sigma^2))$

(a) Histogram resembles Gaussian curve

(b) Plot in Octave appears like random fluctuations = noise

(c) Add noise: y = sin(x) + 0.1*randn(size(x))

(d) As signal amplitude drops, it gets buried in noise

SNR = Signal Power / Noise Power = A^2 / σ^2

7. ADC and DAC

Octave Code:

```
f = 10; fs = 2*f; t = 0:1/fs:1;
x = sin(2*pi*f*t);
plot(t, x);
```

When fs = 2*f (Nyquist Rate): signal is accurately captured

If fs < 2*f: aliasing occurs, distorted signal appears