Thursday Warm Up, Unit 0: Foundations and Fundamentals

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1 Memory Bank

- $\bar{x} = \frac{1}{N}\sum_{i=0}^{N-1} x_i$ ... Sample mean.
- $\overline{x^2} = \frac{1}{N}\sum_{i=0}^{N-1} x_i^2$ ... Sample mean of the square.
- $s = \frac{1}{N-1}\sum_{i=0}^{N-1}(x_i - \bar{x})^2$ ... Sample std. deviation.
- $s^2 = \overline{x^2} - \overline{x}^2$ ... Formula for the variance.
- Let a **histogram** be defined by M bins i, with the data organized into M *frequencies* H_i.
- Total number of data points in a histogram: $N = \sum_{i=0}^{M-1} H_i$
- (1) Sample mean and (2) variance from histograms:
  1. $\bar{x} = \frac{1}{N}\sum_{i=0}^{M-1} iH_i$
  2. $s = \frac{1}{N-1}\sum_{i=0}^{M-1}(i - \bar{x})^2 H_i$
- For the following two formulas: $\omega = 2\pi f$, $\tau = RC$.
- **Low-pass filter response**, as a function of frequency:

$$R(f) = \frac{1}{1 + j\omega\tau} \tag{1}$$

- **High-pass filter response**, as a function of frequency:

$$R(f) = \frac{j\omega\tau}{1 + j\omega\tau} \tag{2}$$

2 Application of Complex Numbers: AC Circuit Filters

1. (a) Suppose a signal has an amplitude of A at a frequency f: $A(f)$. The filtered amplitude is $R(f)A(f)$. If $A = 3.3$ at $f = 50$ kHz, $R = 1$ kΩ, and $C = 1$ μF, what is the filtered amplitude $A(f)R(f)$, if the filter is *low-pass*? (b) Suppose a signal has an amplitude of A at a frequency f: $A(f)$. The filtered amplitude is $R(f)A(f)$. If $A = 3.3$ at $f = 5$ kHz, $R = 100$ kΩ, and $C = 1$ μF, what is the filtered amplitude $A(f)R(f)$, if the filter is *high-pass*?

3 ADC and DAC

1. Consider Fig. 1, which is adapted from Ch. 3 of the course text, when completing the following exercises:
   - If the sampling rate is 10 kHz, and the analog signal frequency is 2.5 kHz, what is the sampled frequency?
   - If the sampling rate is 10 kHz, and the analog signal frequency is 5 kHz, what is the sampled frequency?
   - If the sampling rate is 10 kHz, and the analog signal frequency is 15 kHz, what is the sampled frequency?
   - If the sampling rate is 10 kHz, and the analog signal frequency is 20 kHz, what is the sampled frequency?
2. If an analog signal is created by a DAC with range [0,5] V, and a max count of 4095, what is its amplitude if the DAC value is 2048 counts?
3. If a 4.0 V analog signal is created by a DAC with a max count of 4095, and its amplitude is 1024 in DAC counts, what is the maximum voltage range of the DAC if the minimum is 0 V?

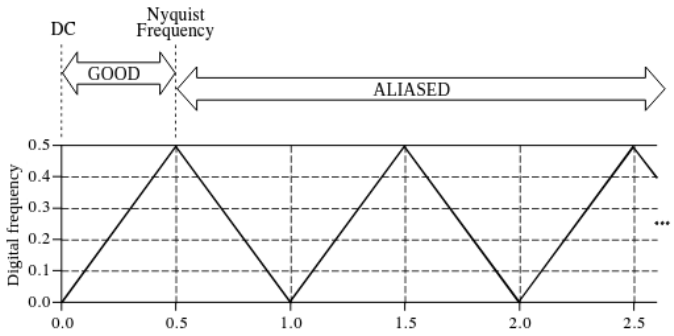

Figure 1: Digital versus analog frequency of sampled signals.