Warm-Up for April 11th, 2022

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1 Memory Bank

1. Divergence and Curl of **B**-fields, and current density:

$$\nabla \cdot \mathbf{B} = 0 \tag{1}$$
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{2}$$
$$I = \int \mathbf{J} \cdot d\mathbf{a} \tag{3}$$

2 Calculating B-fields

1. Take Equation 2 as a given, and perform a surface integral of both sides. Let $I_{\rm enc}$ be the sum of currents that penetrate the surface. (a) Use the curl theorem to show

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I} \tag{4}$$

(b) Using Equation 1, prove that **B**-fields can have no monopole moment. That is, there are no **B**-fields like $\mathbf{B} = (B_0/r^2)\hat{\mathbf{r}}$.

2. Consider the surface current density in Fig. 1 (left). From dimensional analysis, we find that a surface current density times a distance equals a current (Fig. 1, right). Show that the corresponding **B**-field is constant and in the $\hat{\mathbf{y}}$-direction, depending on the sign of z.

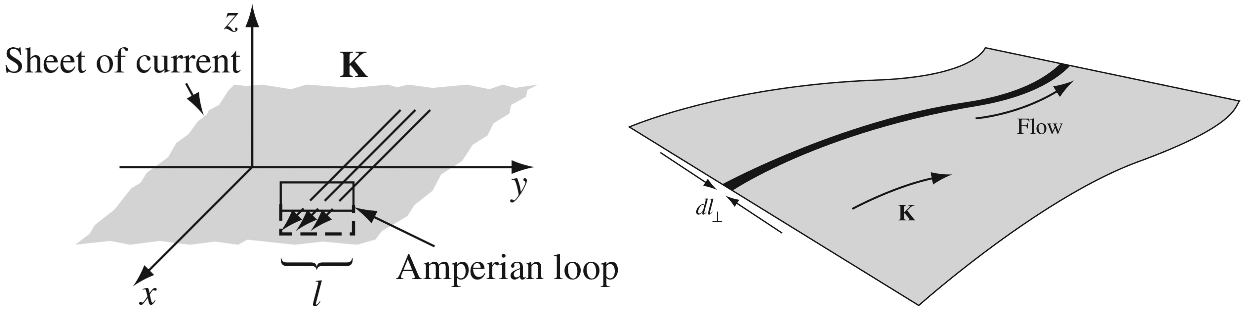

Figure 1: (Left) An infinite uniform surface current density: $\mathbf{K} = K\hat{\mathbf{x}}$. (Right) A uniform surface current density times a distance $dl_\perp$ gives a current dI.