

Electromagnetic Theory: PHYS330

Jordan Hanson

November 5, 2020

Whittier College Department of Physics and Astronomy

Summary

Week 2 Summary

1. Homework discussions
 - Proofs! Glorious proofs.
 - Exercises with *checking* fundamental theorems
2. Electrostatics and Coulomb forces
 - Charge distributions, superposition, and the Coulomb force
 - A note about the *far-field*
 - Setting up integrals, taking limits, checking units
 - The divergence of electric fields
 - The curl of electric fields
3. Electric Potential
 - Definitions, fundamental theorem for gradients
 - Reference points
 - Laplace equation ...
4. Work, energy, and conductors

Homework

Homework, Week 2

Unlike last week, these exercises come from *within* the chapter. Ideally, you should look at all of the problems within the chapter as you study.

- Exercise 2.5
- Exercise 2.6
- Exercise 2.9
- Exercise 2.12
- Exercise 2.16
- Exercise 2.18
- Exercise 2.25
- Exercise 2.29

Charge distributions, Superposition, and the Coulomb Force

Charge distributions, Superposition, and the Coulomb Force

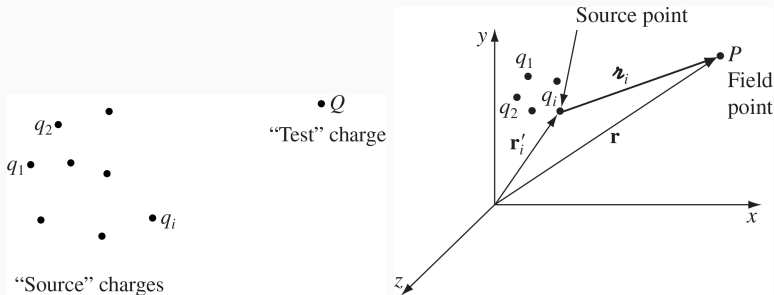


Figure 1: The basic problem of electrostatics. Note the definition of the separation vector, and the vectors to the field point and to all the source charges.

Charge distributions, Superposition, and the Coulomb Force

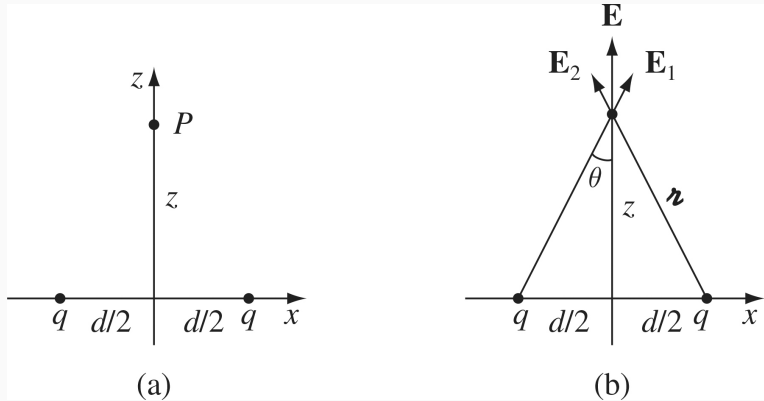


Figure 2: Begin with a dipole, and then a *physical* dipole.

Charge distributions, Superposition, and the Coulomb Force

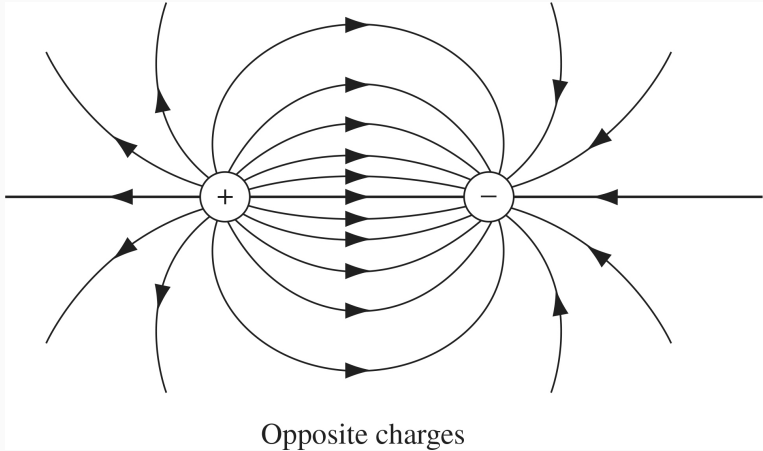


Figure 3: Field of a *physical* dipole.

Charge distributions, Superposition, and the Coulomb Force

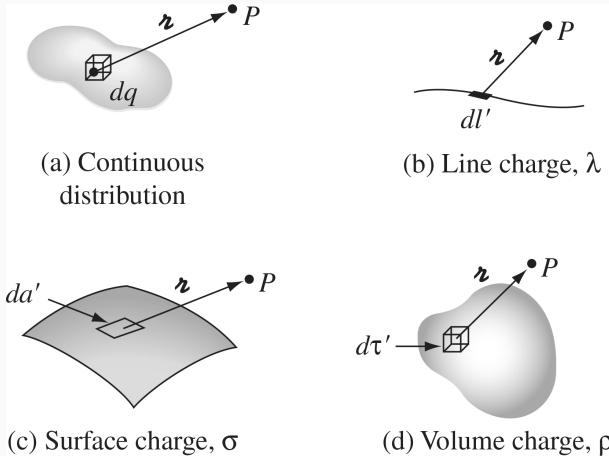


Figure 4: The continuous limit implies a variety of symmetries and geometries over which we integrate, rather than sum.

Charge distributions, Superposition, and the Coulomb Force

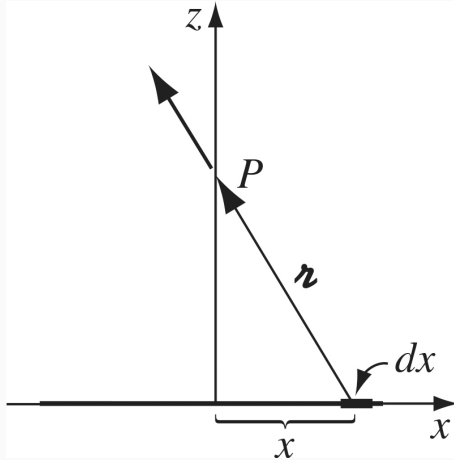


Figure 5: A continuous line density of charge. Integration yields the electric field.

Charge distributions, Superposition, and the Coulomb Force

Useful calculations:

1. Continuous line charge, length L .
2. Continuous plane of charge, radius R .
3. Loop of charge, radius R , a distance z above the center.

Why are these interesting? One example is that these shapes are used as *antennas*. Give some alternating current at the right voltage and impedance to a shape of metal, then you've got your antenna that radiates a certain way.

Professor do these examples.¹

¹Remember from PHYS180? Remember? Yeah...good times.

A Note about the Far-Field

The Far-Field

One way to express the **far-field** approximation (compare to Fraunhofer and Fresnel limits in diffraction):

$$\vec{r} = \vec{r'} + \vec{z} \quad (1)$$

$$\vec{z} = \vec{r} - \vec{r'} \quad (2)$$

$$z = \sqrt{r^2 - 2\vec{r} \cdot \vec{r'} + r'^2} \quad (3)$$

$$z = r\sqrt{1 - 2\vec{r} \cdot \vec{r'} r^{-2} + r'^2 r^{-2}} \quad (4)$$

$$z \approx r\sqrt{1 - 2\vec{r} \cdot \vec{r'} r^{-2}} \quad (5)$$

$$z \approx r\left(1 - \vec{r} \cdot \vec{r'} r^{-2}\right) \quad (6)$$

$$z \approx r\left(1 - \hat{r} \cdot \vec{r'} r^{-1}\right) \quad (7)$$

$$z \approx r - \hat{r} \cdot \vec{r'} \quad (8)$$

The Divergence of \vec{E} -fields

The Divergence of \vec{E} -fields

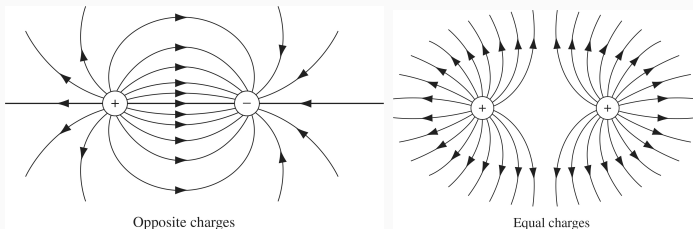


Figure 6: Field lines are an important theoretical concept.

The Divergence of \vec{E} -fields

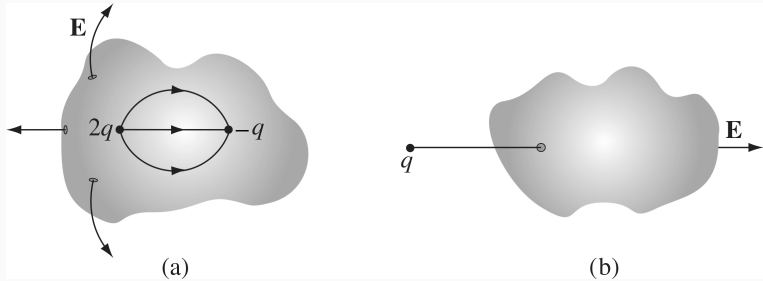


Figure 7: The concept of a closed Gaussian surface.

The Divergence of \vec{E} -fields

$$\oint \vec{E}_i \cdot d\vec{a} = \frac{1}{4\pi\epsilon_0} \int_0^\pi \int_0^{2\pi} \frac{q_i \hat{r}}{r^2} \cdot r^2 \sin\theta d\theta d\phi \hat{r} = \frac{q_i}{\epsilon_0} \quad (9)$$

$$\vec{E} = \sum_{i=1}^n \vec{E}_i \quad (10)$$

$$\oint \vec{E} \cdot d\vec{a} = \sum_{i=1}^n \left(\oint \vec{E}_i \cdot d\vec{a} \right) \quad (11)$$

$$\oint \vec{E} \cdot d\vec{a} = \sum_{i=1}^n \left(\frac{q_i}{\epsilon_0} \right) \quad (12)$$

$$\oint \vec{E} \cdot d\vec{a} = \frac{Q_{tot}}{\epsilon_0} \quad (13)$$

Gauss' Law: the total flux is proportional to the contained charge.

The Divergence of \vec{E} -fields

The divergence theorem:

$$\oint_S \vec{E} \cdot d\vec{a} = \int_V (\nabla \cdot \vec{E}) d\tau \quad (14)$$

Remark that the total charge is the integral over the 3D charge density:

$$\frac{Q_{tot}}{\epsilon_0} = \frac{1}{\epsilon_0} \int_V \rho d\tau \quad (15)$$

This implies

$$\oint_S \vec{E} \cdot d\vec{a} = \int_V (\nabla \cdot \vec{E}) d\tau = \frac{1}{\epsilon_0} \int_V \rho d\tau \quad (16)$$

Looking at the last two expressions:

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}} \quad (17)$$

The Divergence of \vec{E} -fields

Differential form of Gauss' Law:

$$\boxed{\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}} \quad (18)$$

Consider a different argument:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{\mathbf{r}}}{r^2} \rho(\vec{r}') d\tau' \quad (19)$$

$$\nabla \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} \int \nabla \cdot \left(\frac{\hat{\mathbf{r}}}{r^2} \right) \rho(\vec{r}') d\tau' \quad (20)$$

$$\nabla \cdot \vec{E} = \frac{1}{4\pi\epsilon_0} \int 4\pi\delta^3(\mathbf{r}) \rho(\vec{r}') d\tau' \quad (21)$$

$$\nabla \cdot \vec{E} = \frac{4\pi}{4\pi\epsilon_0} \int \delta^3(\vec{r} - \vec{r}') \rho(\vec{r}') d\tau' \quad (22)$$

$$\nabla \cdot \vec{E} = \rho(\vec{r})/\epsilon_0 \quad (23)$$

The Divergence of \vec{E} -fields

(Refresh with delta-functions): Use this charge distribution and Eq. 19 to find the \vec{E} -field.

$$\rho(\vec{r}') = q\delta^3(\vec{r}' - x\hat{x}) - q\delta^3(\vec{r}' + x\hat{x}) \quad (24)$$