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Quiz #3

Section 2

2) a) $V(x,y,z) \Rightarrow 0$ as $y \to \infty$, which can't be part of the sol'n?

$$\boxed{B: \; Y(y) = \sinh(x)}$$

$$\frac{\sinh(\infty) = \infty}{\infty \neq 0}$$

b) Eq. 3.51:

$$C_{n,m} = \frac{4V_0}{ab} \int_0^a \sin(n\pi y/a)\, dy \cdot \int_0^b \sin(m\pi z/b)\, dz$$

$$① = \begin{cases} 0, & \text{if } n \text{ or } m \text{ is even} \\ \frac{16V_0}{\pi nm}, & \text{if } n \text{ and } m \text{ are odd} \end{cases} ②$$

if $n=3, m=2$:

$$= \frac{4V_0}{ab} \int_0^a \sin\left(\frac{(3)\pi y}{a}\right) \cdot \int_0^b \sin\left(\frac{(2)\pi z}{b}\right) dz$$

①

$$= \frac{4V_0}{ab} \left(\frac{2}{3\pi} a\right) \cdot 0 \Rightarrow \boxed{0}$$

if $n=3$, $m=3$:

$$= \frac{4V_0}{ab} \int_0^a \sin\left(\frac{(3)\pi y}{a}\right) \cdot \int_0^b \sin\left(\frac{(3)\pi z}{b}\right) dz$$

②

$$= \frac{4V_0}{ab} \cdot \left(\frac{2}{3\pi} a\right) \cdot \left(\frac{2}{3\pi} b\right) = \boxed{\frac{16V_0}{9\pi}}$$

Section 1

1) $\vec{v} = a\vec{x} + b\vec{y} + c\vec{z}$, $c = ?$

Fourier's Trick: $\vec{v} \cdot \vec{z} = \vec{z}(\underbrace{a\vec{x}}_{=0} + \underbrace{b\vec{y}}_{=0} + \underbrace{c\vec{z}}_{=1})$

B	$\vec{v} \cdot \vec{z} = c$

2) $\vec{x} = \sum_{i=1}^{n} c_i x_i$, n-dimensional vector , $c_7 = ?$

$\boxed{D: \vec{x}_7 \cdot \vec{x}}$

Fourier's Trick: $\vec{v} \cdot \vec{x}_m = \sum_{i=1}^{n} c_i x_i \cdot \vec{x}_m = c_m$

$\vec{v} \cdot x_7 = \sum_{i=1}^{n} c_i x_i \cdot x_7 = c_7$

3) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx)$

modeling: $f(x) = \sin(3x)$, from $n=0$ to $n=\infty$

$a_n = 1/\pi \int_0^{2\pi} \sin(3x)\cos(nx)\,dx$

if $n=0$; $a_0 = 1/\pi \int_0^{2\pi} \sin(3x)\,dx \Rightarrow 1/\pi \left(-\cos(3x)/3\right)\Big|_0^{2\pi}$

$= 1/\pi \left(-\cos(6\pi)/3 + 1/3\right) = 0$, thus $a_0 = 0$

$a_n = 1/\pi \int_0^{2\pi} \sin(3x)\cos(nx)\,dx$

trig. functions

$= 1/\pi \int_0^{2\pi} 1/2(\sin(n+3) - \sin(n-3))$

$$= \frac{1}{2\pi}\int_0^{2\pi} \sin((n+3)x)\,dx - \frac{1}{2\pi}\int_0^{2\pi} \sin((n-3)x)\,dx$$

$$= -\frac{1}{2\pi}\left.\frac{\cos((n+3)x)}{n+3}\right|_0^{2\pi} + \frac{1}{2\pi}\left.\frac{\cos((n-3)x)}{n-3}\right|_0^{2\pi}$$

$$= -\frac{1}{2\pi}\left(\frac{1}{n+3}\right)\cdot(0) + \frac{1}{2\pi}\left(\frac{1}{n-3}\right)\cdot(0) = 0$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\cdot\sin(nx)\,dx$$

trig. Function

$$\boxed{= \frac{1}{2}\left(\cos(n-3) - \cos(n+3)\right)}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \cos((n-3)x)\,dx - \frac{1}{2\pi}\int_0^{2\pi} \cos((n+3)x)\,dx$$

$$= \frac{1}{2\pi}\left.\cancel{\left(\frac{\sin((n-3)x)}{n-3}\right)}\right|_0^{2\pi} - \frac{1}{2\pi}\left.\cancel{\left(\frac{\sin(n+3)x}{n+3}\right)}\right|_0^{2\pi}$$

$$\boxed{b_n = 0}$$

$$b_0 = \frac{1}{\pi}\int_0^{2\pi} \cos(0\cdot\pi)\,dx \Rightarrow$$

$$\boxed{b_0 = 1}$$

There are no coefficients for the Fourier series from $\sin(3x)$.