Jackson Drummond

HW 4 4.10, 4.14, 4.15, 4.18, 4.26, 4.35

4.10) ~~A sphere of radius R carries a polarization~~

$$P(r) = kr \,, \quad \sigma_b \approx P \cdot \hat{n} \,, \quad \rho_b \approx -\nabla \cdot P$$

a) $P(R) = KR\hat{r}$, $\therefore \sigma_b = KR\cancel{\hat{r} \cdot \hat{n}} = \boxed{KR = \sigma_b}$

$$\rho_b = -\nabla \cdot KR\hat{r} \Rightarrow -\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr)\right)$$

$$\rho_b = -\frac{K}{r^2}\frac{\partial}{\partial r}(r^3) \Rightarrow \frac{-k}{\cancel{r^2}}(3\cancel{r^2}) \Rightarrow \boxed{-3K = \rho_b}$$

b) ~~Find the field inside & outside~~

$$\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0} \,, \quad q_{enc} = \rho_b \cdot \left(\frac{4}{3}\pi r^3\right)$$

$$E(4\pi \cancel{r^2}) = \frac{1}{\varepsilon_0}\left(-3K\frac{4}{3}\pi r^{\cancel{3}}\right) \Rightarrow \boxed{E_{in} = \frac{-kr}{\varepsilon_0}\hat{r}}$$

~~For outside~~ $q = \sigma_b(4\pi R^2) + \rho_b\left(\frac{4}{3}\pi R^3\right)$

$$q_{tot} \Rightarrow 4\pi KR^3 - 4\pi KR^3 = 0$$

$$\therefore \boxed{\vec{E}_{out} = \vec{0}}$$

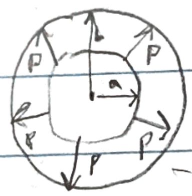
4.14) When a neutral dielectric is polarized, show that the total charge remains 0. Even if it'll wobble some.

Given: $\sigma_b \equiv P\cdot\hat{n}$ & $\rho_b \equiv -\nabla\cdot P$

$$Q_{tot} = \oint_S \sigma_b \, dS + \int_V \rho_b \, d\tau \Rightarrow \oint_S \vec{P}\cdot\hat{n}\, dS - \int_V \nabla\cdot\vec{P}\, d\tau$$

Based on the divergence therorm: $\oint_S \vec{P}\cdot d\vec{a} = \int_V \nabla\cdot\vec{P}\, d\tau$

$\boxed{Q_{tot} = 0}$, they cancel eachother out, ∴ no charge

4.15) 

$P(r) = \frac{K}{r}\hat{r}$ "frozen in" polarization

$\rho_b = -\nabla\cdot P$

$\oint \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$

a) $r < a$ $\quad -\nabla\cdot P(r) \Rightarrow 0$

$Q_{enc} = \int_V \rho_b \, d\tau \Rightarrow 0$ so $\oint \vec{E}\cdot d\vec{a} = 0$ & $\boxed{\vec{E} = 0}$

$r > b$

$Q_{tot} = \oint \sigma_b \, da + \int_V \rho_b \, d\tau$, $\quad P = 0$ ∴ $\rho_b = 0$

$Q_{tot} = 0$ ∴ $\boxed{\vec{E} = 0}$

$a < r < b$

$$\rho_b = -\nabla\cdot P \Rightarrow -\left(\frac{1}{r^2}\right)\left(\frac{\partial}{\partial r}\right)\left(r^2\frac{K}{r}\right) = -\frac{K}{r^2}$$

$$\sigma_b = \vec{P}\cdot\hat{n} \Rightarrow \frac{K}{r}\hat{r}\cdot\hat{n}, \quad \hat{n} = \hat{r} \Rightarrow \sigma_b = \frac{K}{r}$$

$$Q_{tot} = \left(-\frac{K}{r}\cdot 4\pi r^2\right)\Big|_a^r + \int_V \left(\frac{-K}{r^2}\right) d\tau$$

$\oint E\cdot da = \frac{Q_{enc}}{\epsilon_0}$

$$\Rightarrow \left(-4\pi r K\right)\Big|_a^r + -K\int_0^{2\pi}\int_0^{\pi}\int_a^r \frac{1}{r^2} r^2 \sin\theta \, dr\, d\theta\, d\phi$$

$\boxed{\vec{E} = \frac{-K}{\epsilon_0 r}\hat{r}}$

$$\Rightarrow \left(-4\pi r K - K\,2(2\pi)\, r\right)\Big|_a^r \Rightarrow -4\pi K r$$

4.15

b) $\oint D \cdot da = Q_{enc}$

$r<a$, $Q_{enc} = 0 \rightarrow D=0$ & $D = \epsilon_0 \vec{E} + \vec{P}$, $P=0$

$\therefore$ $\boxed{\vec{E} = 0}$

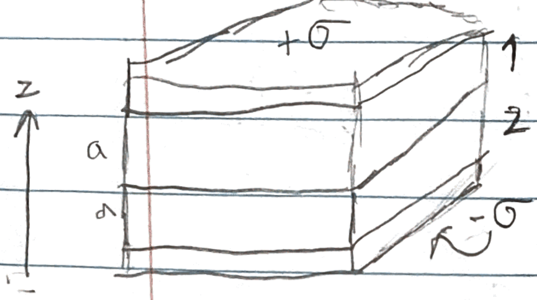
$r>b$, $Q_{enc} = 0$ $\therefore$ $P=0$ & $D=0$, so $\boxed{\vec{E}=0}$

$a<r<b$

Since $Q=0$ $\therefore$ $D=0$, $0 = \epsilon_0 \vec{E} + P$, $P = \frac{K}{r}\hat{r}$

$\therefore$ $\boxed{\vec{E} = -\frac{K}{\epsilon_0 r}\hat{r}}$

4.18



~~a) Find D~~ ~~b) find E_1 & E_2~~

c) Find P_1 & P_2 d) find potential difference between 1 & 2

e) find location & amt of bound charge

~~f) recalc field with bound charge~~

a) $\oint \vec{D} \cdot d\vec{a} = q_{free-enc}$, $\cancel{D(2a) = \sigma(a) \Rightarrow D = \frac{\sigma}{2}}$

Since we have two layers w/ charge moving down ↓, they add to cancel vector;

$$\boxed{\vec{D} = -\sigma \hat{z}}$$

b) $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, but $\vec{P} = \epsilon_0 (\chi_e) \vec{E}$ $D = \epsilon_0 (1 + \chi_e) E$

$\vec{D} = \epsilon \vec{E}$ so $\vec{E}_1 = \frac{\vec{D}}{\epsilon_1}$, $\vec{E}_2 = \frac{\vec{D}}{\epsilon_2}$, $\epsilon_1 = 2\epsilon_0$, $\epsilon_2 = \frac{3}{2}\epsilon_0$

$$\therefore \boxed{\vec{E}_1 = \frac{-\sigma}{2\epsilon_0}\hat{z}, \quad \vec{E}_2 = \frac{-2\sigma}{3\epsilon_0}\hat{z}}$$

c) $\vec{P} = D - \epsilon_0 E$

$$\vec{P}_1 = -\sigma\hat{z} - \frac{-\epsilon_0 \sigma}{2\epsilon_0}\hat{z} \Rightarrow -\sigma\left(1 - \frac{1}{2}\right)\hat{z} \Rightarrow \boxed{-\frac{\sigma}{2}\hat{z}}$$

$$\vec{P}_2 = -\sigma\hat{z} - \frac{-\epsilon_0 2\sigma}{3\epsilon_0} \Rightarrow -\sigma\left(1 - \frac{2}{3}\right)\hat{z} \Rightarrow \boxed{-\frac{\sigma}{3}\hat{z}}$$

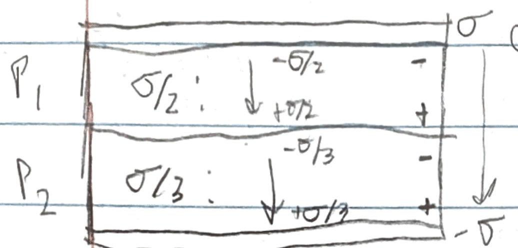
4.18)

d) $V = -\int \vec{E}\cdot d\vec{l}$, $E_1 = \frac{-\sigma}{2\epsilon_0}\hat{z}$, $E_2 = \frac{-2\sigma}{3\epsilon_0}\hat{z}$

$$\Delta V = \vec{E}_1 a + \vec{E}_2 a \Rightarrow \frac{a\sigma}{\epsilon_0}\left(\frac{1}{2}+\frac{2}{3}\right) \Rightarrow -\frac{7a\sigma}{6\epsilon_0}\hat{z}$$

the magnitude of difference is $\boxed{\frac{7a\sigma}{6\epsilon_0}}$

e) $\sigma_b = \vec{P}\cdot\hat{n}$, $\vec{P}_1 = \frac{-\sigma}{2}\hat{z}$, $\vec{P}_2 = \frac{-\sigma}{3}\hat{z}$



$\sigma\sum = (\sigma - \sigma/2) + (\sigma/2 - \sigma/3) + (\sigma/3 - \sigma) = 0$

$\sigma/2 \quad + \quad \sigma/6 \quad - \quad \frac{2}{3}\sigma$

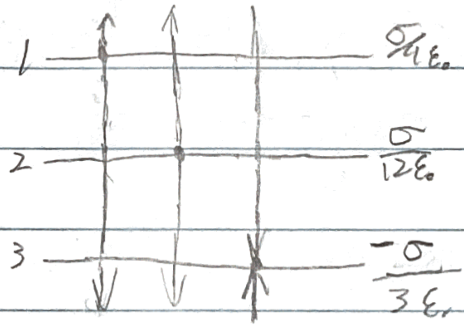
1) 2) 3)

$\oint \vec{E}\cdot da = \frac{q_{enc}}{\epsilon_0}$ $da = 2a$

1) $E(2a) = \frac{\sigma a}{2\epsilon_0}$, $E_1 = \frac{\sigma}{4\epsilon_0}$ 3) $E(2a) = \frac{-2\sigma a}{3\epsilon_0}$, $E_3 = \frac{-\sigma}{3\epsilon_0}$

2) $E(2a) = \frac{\sigma a}{6\epsilon_0}$, $E_2 = \frac{\sigma}{12\epsilon_0}$

E fields from free and bound charges



* factor $\left(\frac{\sigma}{\epsilon}\right)$

f)

$\frac{1}{4} + \frac{1}{12} - \frac{1}{3} = 0$ $-\frac{1}{4} + \frac{1}{12} - \frac{1}{3} = -\frac{1}{2}$

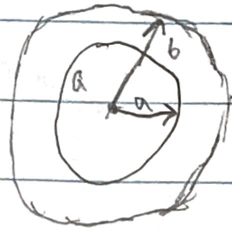
$-\frac{1}{4} - \frac{1}{12} - \frac{1}{3} = -\frac{2}{3}$ $-\frac{1}{4} - \frac{1}{12} + \frac{1}{3} = 0$

So there is no e-field outside box only inside

$\int_{2} \vec{E}$ or $\vec{E}_1 = \left(\frac{-1}{2}\right)\frac{\sigma}{\epsilon_0}$, since it's down & a vector $\vec{E}_1 = \frac{-\sigma}{2}\hat{z}$

$\int_{2}^{3} \vec{E}$ or $\vec{E}_2 = \left(\frac{-2}{3}\right)\frac{\sigma}{\epsilon_0}$, $\vec{E}_2 = \frac{-2\sigma}{3\epsilon_0}\hat{z}$

4.26



$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, $\chi_e \to \vec{D} = \epsilon \vec{E}$, $\vec{P} = \epsilon_0(\chi_e)\vec{E}$

$\oint D \cdot da = q$, $a = 4\pi r^2$, $q = Q$, $\epsilon = \epsilon_0(1+\chi_e)$

$r < a \to D = 0$, $r > b \to D = \frac{Q}{4\pi r^2}\hat{r}$

$$\therefore E = \begin{cases} r<a & = 0 \\ a<r<b & = \frac{Q}{4\pi\epsilon r^2}\hat{r} \\ r>b & = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \end{cases}, \quad U_{tot} = \frac{1}{2}\int \vec{D}\cdot\vec{E}\, d\tau$$

$$U = \frac{1}{2\epsilon}\left(\frac{Q}{4\pi}\right)^2 4\pi \left(\int_a^b \frac{1}{r^4} r^2\, dr + \int_b^\infty \frac{1}{r^4} r^2\, dr\right)$$

$$\Rightarrow \frac{Q^2}{8\pi}\left(\left(\frac{1}{\epsilon}\right)\left(\frac{-1}{r}\Big|_a^b\right) + \left(\frac{1}{\epsilon_0}\right)\left(\frac{-1}{r}\Big|_b^\infty\right)\right) = \frac{Q^2}{8\pi}\left(\frac{1}{\epsilon}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{\epsilon_0 b}\right)$$

$\epsilon = \epsilon_0(1+\chi_e)$, $U = \frac{Q^2}{8\pi}\left(\frac{1}{\epsilon_0(1+\chi_e)}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{\epsilon_0 b}\right)$

$$U = \frac{Q^2}{8\pi\epsilon_0(1+\chi_e)}\left(\left(\frac{1}{a} - \cancel{\frac{1}{b}}\right) + \frac{\cancel{\epsilon_0}(\cancel{1}+\chi_e)}{\cancel{\epsilon_0} b}\right)$$

$$\Rightarrow \boxed{U = \frac{Q^2}{8\pi\epsilon_0(1+\chi_e)}\left(\frac{1}{a} - \frac{\chi_e}{b}\right)}$$

4.35

χ_e, R $\qquad \oint \vec{D}\cdot d\vec{a} = q \Rightarrow \frac{q}{4\pi r^2}$

$\vec{D} = D_r \hat{r}$, $\vec{D} = \epsilon \vec{E}$, $\epsilon = \epsilon_0(1+\chi_e)$

since

$\vec{E} = \frac{D}{\epsilon} \Rightarrow \frac{q}{4\pi\epsilon_0(1+\chi_e)}\left(\frac{\hat{r}}{r^2}\right)$, inside $(r<R)$, outside permittivity disappears

$\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(r)$

Since $\vec{P} = \epsilon_0 \chi_e \vec{E}$, $\vec{P} = \frac{q\chi_e}{4\pi(1+\chi_e)}\left(\frac{\hat{r}}{r^2}\right)$, $\rho_b = -\nabla\cdot\vec{P}$

$\rho_b = \frac{-q\chi_e}{4\pi(1+\chi_e)}\left(\nabla\cdot\frac{\hat{r}}{r^2}\right) \Rightarrow \boxed{\frac{-q\chi_e}{1+\chi_e}\delta^3(r)}$, $\sigma_b = \vec{P}\cdot\hat{n}$, $r \to R$

$\sigma_b = \frac{q\chi_e}{4\pi(1+\chi_e)R^2}$ $\qquad Q_{tot(\sigma_b)} = 4\pi R^2 \sigma_b = \boxed{\frac{q\chi_e}{1+\chi_e}}$

$\therefore Q_{tot(\rho_b)} = \int \rho_b \, d\tau \Rightarrow \frac{-q\chi_e}{1+\chi_e}\int \delta^3(r)\, d\tau = \boxed{\frac{-q\chi_e}{1+\chi_e}}$