Warm-Up for April 8th, 2022

Dr. Jordan Hanson - Whittier College Dept. of Physics and Astronomy

April 11, 2022

1 Memory Bank

1. Lorentz Force ... $\mathbf{F} = I\mathbf{L} \times \mathbf{B}$
2. Torque ... $\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
3. Continuity Equation ... $\nabla \cdot \mathbf{J} = -\frac{\partial \rho}{\partial t}$

2 Torque on a Current Loop, Current Density

1. Suppose there is a square loop of current in the xy-plane, with side length a, centered on the origin. There is a constant, uniform magnetic field $\mathbf{B} = B\hat{\mathbf{x}}$. Let $\boldsymbol{\mu} = I\mathbf{A}$, where $\mathbf{A}$ is an area vector for the loop. Show that there is a torque on the loop, equal to $\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B}$.

2. Suppose the outflow of charged particles from some exploding astrophysical event has spherical symmetry and decays exponentially:

$$\mathbf{J} = I_0 \left(\frac{e^{-\lambda t}}{r^2} \right) \hat{\mathbf{r}} \tag{1}$$

Using the continuity equation, show that the charge lost by the exploding object after a time λ^{-1} is

$$Q(t) = \frac{4\pi I_0}{\lambda} \left(e^{-1} - 1 \right) \tag{2}$$