Ch. 5 # ~~14, 16, 17, 18, 20, 22, 34~~

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14)

a) · Uniformly distributed

$\oint B \cdot dl = \mu_0 I_{enc} \Rightarrow B = \frac{\mu_0 I_{enc}}{2\pi s}$

If inside wire: $\boxed{B_{in} = 0 \text{ , since } I_{enc} = 0}$

If outside wire: $\boxed{B_{out} = \frac{\mu_0 I}{2\pi s}}$

b) $J \sim s$

$J = ks \qquad I = \int_0^a J \cdot da = \int_0^a ks \cdot (2\pi s \cdot ds)$

$B = \frac{\mu_0 I_{enc}}{2\pi s}$

$= \int_0^a ks^2 \cdot 2\pi \, ds \Rightarrow 2\pi k \left[\frac{s^3}{3}\right]_0^a$

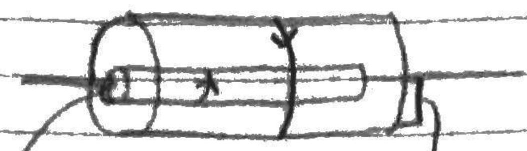
$I_{enc} = \frac{2\pi k a^3}{3} \Rightarrow k = \frac{3I}{2\pi a^3}$

$I_{enc} = \frac{I s^3}{a^3}$

$\boxed{B_{in} = \frac{\mu_0 I s^2}{2\pi a^3}}$ $\boxed{B_{out} = \frac{\mu_0 I}{2\pi s}}$

✱ solenoids

Fig. 5.42

16) 

$a = n_1$ $b = n_2$

i) inner solenoid; $\rho = a = n_1$

$I_{enc} = nIL$

$\oint B \cdot dl = \mu_0 I_{enc}$

$BL = \mu_0 n I L$

$B = \mu_0 n_1 I$

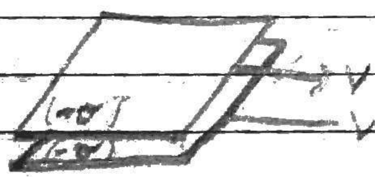
$\boxed{B_a = \mu_0 a I}$

ii) btwn solenoids; $\rho = b - n_2$

$\boxed{B_{ab} = \mu_0 (n_2 - n_1) I}$

iii) outside both?

$\boxed{I_{enc} = 0, \text{ thus } B = 0}$

17) (- Fig. 5.43) 

a) $B = \frac{\mu_0 K}{2}$, $K = \sigma v$

$B_{TOT} = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2}$

$\boxed{B = \mu \sigma v}$

b) $F_B = \int (K \times B) da$

$f_B = K \times B$

$f_B = \sigma v \cdot \frac{\mu_0 K}{2} \Rightarrow \boxed{f_B = \frac{\mu_0 (\sigma v)^2}{2}}$

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c) $V = ?$

$$E = \frac{\sigma}{2\varepsilon_0}, \quad F_B = \frac{\mu_0 (\sigma v)^2}{2}$$

$$F_B = F_E$$

$$\frac{\mu_0 \sigma^2 v^2}{2} = \frac{\sigma^2}{2\varepsilon_0} \Rightarrow v^2 = \frac{2\sigma^2}{2\varepsilon_0 \mu_0 \sigma^2} \Rightarrow \frac{1}{\varepsilon_0 \mu_0}$$

$$V = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} = \sqrt{\frac{1}{(4\pi\cdot 10^{-7})(8.85\cdot 10^{-12})}}$$

$$\boxed{= 3\cdot 10^{8}\ m/s}$$

19) $I_{enc} = \int_S J \cdot da$

$\oint B \cdot dl = \mu_0 I_{enc}$, doesn't matter

$I_{enc} = \oint J \cdot da$, $\boxed{\text{show } J \cdot \nabla = 0}$

$\boxed{\text{because it is independent of surface}}$

$\boxed{I_{enc} = 0}$

20) $\rho = \frac{q N d}{M} = \frac{(1.6\cdot 10^{-19} C)(6.0\cdot 10^{23})(9.0\ g/cm^3)}{64\ g/mol}$

(Charge/volume)

$$\boxed{= 1.4\cdot 10^{4}\ C/cm^3}$$

b) $d = 1mm \rightarrow 0.001 m$

$I = 1A$

$V_{avg} = ?$

$r = \frac{d}{2} = \frac{10^{-3}}{2} = 5\cdot10^{-4} m$

$J = \rho v$

$A = \pi r^2 = \pi(5\cdot10^{-4} m)^2 = 7.85\cdot10^{-7} m^2$

$J = I/A = 1/7.85\cdot10^{-7} m^2 = 1.3\cdot10^{6} A/m^2$

$V = \frac{J}{\rho} = \frac{1.3\cdot10^{16} A/m^2}{1.4\cdot10 \ C/cm^3} = \boxed{9.1\cdot10^{-7} cm/s}$

c) $d = 1cm = 0.01m$

$I_{net} = I_1 \cdot I_2$

$\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi s} = \frac{(4\pi\cdot10^{-7} H/m)(1.0A)}{2\pi(0.01m)}$

$\boxed{F_B = 2\cdot10^{-7} N/cm}$

2)

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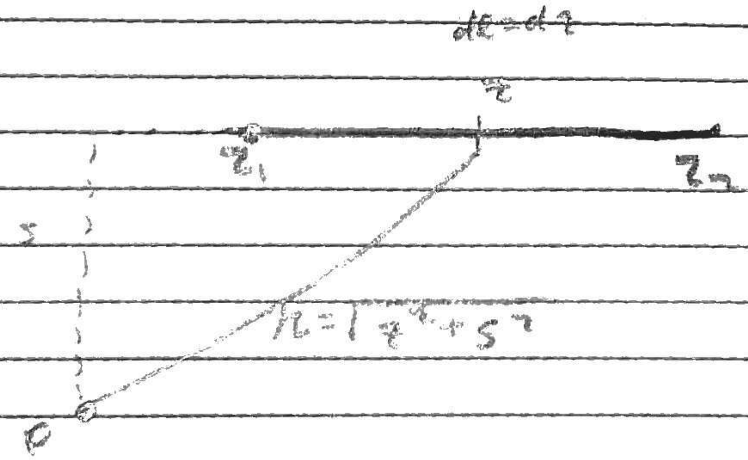
23) V_B

(5.66) - $A(r) = \frac{\mu_0}{4\pi}\int \frac{K(r')}{r}\, da'$, $A = \frac{\mu_0 I}{4\pi}\int 1/r \cdot dl$

(5.37) - $\boxed{B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_2)}$

(5.37) $A = \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2} \frac{dz}{\sqrt{s^2+z^2}}\hat{z} \Rightarrow \boxed{\frac{\mu_0 I}{4\pi}\ln\left[\frac{z_2+\sqrt{z_2^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right]\hat{z}}$



$\vec{B} = \nabla \cdot \vec{A} = -\frac{\partial A}{\partial s}\hat{\phi} = \frac{-\mu_0 I}{4\pi}\,\partial/\partial s\left[\ln\left(z_2+\sqrt{s^2+z_2^2}\right) - \ln\left(z_1+\sqrt{s^2+z_1^2}\right)\right]\hat{\phi}$

(Chain Rule) $= \frac{-\mu_0 I}{4\pi}\left[\frac{1}{z_2+\sqrt{s^2+z^2}}\cdot\frac{s}{\sqrt{s^2+z^2}} - \frac{1}{z_1+\sqrt{s^2+z_1^2}}\,\frac{s}{\sqrt{s^2+z_1^2}}\right]\hat{\phi}$

$= \frac{-\mu_0 I s}{4\pi}\left[\frac{z_2 - \sqrt{z^2+s^2}}{\sqrt{z_2^2+s^2}\cdot(z^2-(z_2^2+\cancel{s^2}))} - \frac{z_1-\sqrt{z_1^2+s^2}}{(z_1^2-(z^2+s^2))\sqrt{z_1^2+s^2}}\right]\hat{\phi}$

$\vec{B} = \frac{\mu_0 I}{4\pi s}\left(\frac{z_2}{\sqrt{z^2+s^2}} - \frac{z_1}{\sqrt{z^2+s^2}}\right)\vec{\phi} \Rightarrow \frac{\mu_0}{4\pi s}(\sin\theta_2 - \sin\theta_1)$

26) a) Check $\boxed{\nabla \cdot A = 0}$ & $\boxed{\nabla \times A = B}$

infinite straight line

$B = \frac{\mu_0 I}{2\pi s}\vec{\phi}$, $\frac{-\partial A}{\partial s} = \frac{\mu_0 I}{2\pi s}$

$B = \frac{-\partial A}{\partial s}$ $A = -\int_a^s \frac{\mu_0 I}{2\pi s} ds \Rightarrow \frac{-\mu_0 I}{2\pi s} \ln\left(s/a\right)$

$\nabla \times A = B$

$\nabla \times \left(\frac{-\mu_0 I}{2\pi s} \ln(s/a)\right) = B$

$\nabla \cdot A = 0$ because $\frac{\partial A}{\partial s} = 0$ when taking divergence

b) Radius ... uniformly distributed $I_{enc} = AJ$

$\Rightarrow (\pi s^2) J$

A

$\oint B \cdot dl = \mu_0 I \Rightarrow$ $B(2\pi s) = \mu_0 I_{enc}$

$B(2\pi s) = \mu_0 J \pi s^2 \Rightarrow B = \frac{\mu_0 I s}{2\pi R^2}\vec{\phi}$

$\frac{\partial A}{\partial s} = \frac{\mu_0 I s}{2\pi R^2}\vec{\phi}$

$A = \int^s \frac{\mu_0 I s}{2\pi R^2} ds\, \vec{z} \Rightarrow \frac{-\mu_0 I}{4\pi R^2}(s^2 - b^2)\vec{z}$

$$\boxed{A = \frac{-\mu_0 I (s^2 - R^2)}{4\pi R^2}\vec{z}}$$