Quiz 2

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PHYS 330

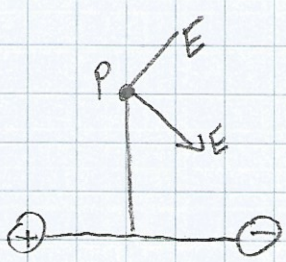
1) a) $\vec{\tau} = \vec{F} \times \vec{d}$ $\qquad \vec{p} = q\vec{d}$ $\qquad \vec{F}_+ = +q\vec{E}$

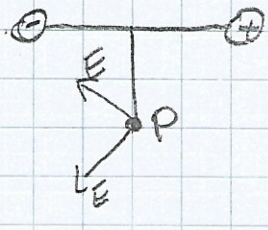
$\vec{F}_- = -q\vec{E}$

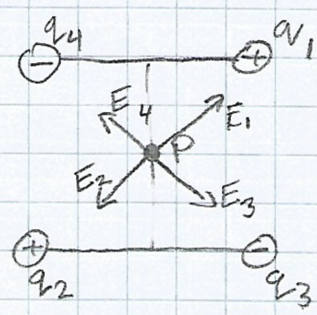
Torque = Force × Distance

$$\vec{\tau} = q\vec{E} \times \vec{d} \Rightarrow \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

b.)







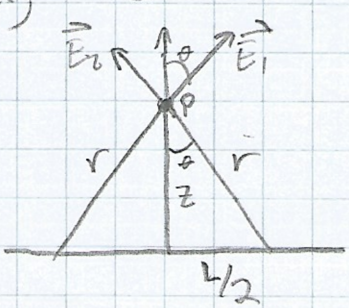
Should be zero

Dipole ⇒ $\vec{p} = q\vec{d}$ $\qquad E = k\frac{q}{r}$ $\qquad |q_1| = q_2$ $\qquad |r_1| = |r_2|$ $\qquad |E_1| = E_2$

Opposite directions: $\vec{E}_1 + \vec{E}_2 = 0$ $\qquad \vec{E}_3 + \vec{E}_4 = 0$

$$\boxed{\vec{E}_{net} = 0}$$

2) a.) $Q = \lambda L$



$$d\vec{E} = \frac{k\,dq\,\hat{\mathscr{r}}}{\mathscr{r}^2} = \frac{k\lambda\,dx\,(z\hat{z} - x\hat{x})}{(z^2+x^2)^{3/2}}$$

$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$

$\vec{\mathscr{r}} = z\hat{z} - x\hat{x}$

$\mathscr{r}^2 = z^2 + x^2$

$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}} = \frac{z\hat{z} - x\hat{x}}{(z^2+x^2)^{1/2}}$

$$\int d\vec{E} = \int_{-L/2}^{L/2} \frac{k\lambda\,dx\,(z\hat{z} - x\hat{x})}{(z^2+x^2)^{3/2}}$$

$$\vec{E} = k\lambda \left\{ z\hat{z} \int_{-L/2}^{L/2} \frac{dx}{(z^2+x^2)^{3/2}} - \hat{x} \int_{-L/2}^{L/2} \frac{x\,dx}{(z^2+x^2)^{3/2}} \right.$$

$x = z\tan(\theta)$

$dx = z\sec^2\theta\,d\theta$

$x\,dx = z^2\tan\theta\sec^2\theta\,d\theta$

$$\vec{E} = k\lambda \left\{ z\hat{z} \int_{\theta_1}^{\theta_2} \frac{z \sec^2\theta \, d\theta}{(z^2 + z^2 \tan^2\theta)^{3/2}} - \cdots \right\}$$

$$= k\lambda \left\{ \frac{z^2 \hat{z}}{z^3} \int_{\theta_1}^{\theta_2} \frac{\sec^2\theta \, d\theta}{(1 + \tan^2\theta)^{3/2}} - \cdots \right\}$$

$$= k\lambda \left\{ \frac{\hat{z}}{z} \int_{\theta_1}^{\theta_2} \frac{\sec^2\theta \, d\theta}{\sec^3\theta} - \cdots \right\}$$

$$= k\lambda \left\{ \frac{\hat{z}}{z} \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta - \cdots \right\}$$

$$= k\lambda \left\{ \frac{\hat{z}}{z} \sin\theta \Big|_{\theta_1}^{\theta_2} - \hat{x} \int_{\theta_1}^{\theta_2} \cancel{\frac{z^2 \tan\theta \sec^2\theta \, d\theta}{z^3 \sec^3\theta \, d\theta}} \right\} \quad \emptyset$$

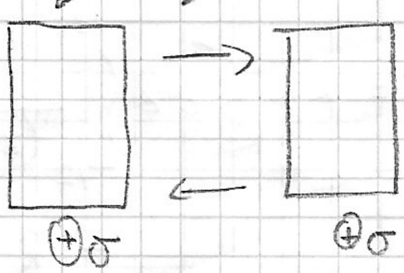
$$\vec{E} = \frac{k\lambda\hat{z}}{z} (\sin\theta_2 - \sin\theta_1) \qquad \sin\theta_1 = \frac{-L/2}{r}$$

$$= \frac{k\lambda L \hat{z}}{z} \left(z^2 + \frac{1}{4}L^2\right)^{-1/2}$$

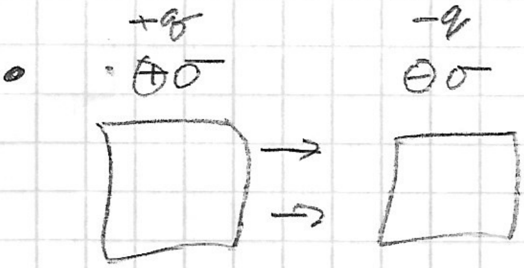
$$\boxed{\vec{E} = \frac{k\lambda L\hat{z}}{z^2} \left(1 + \frac{1}{4}\left(L/z\right)^2\right)^{-1/2}} \qquad E \sim \frac{kQ\hat{z}}{z^2} \quad Q = \lambda L \quad z \gg L$$

2) a) $\sigma/(2\varepsilon_0)$

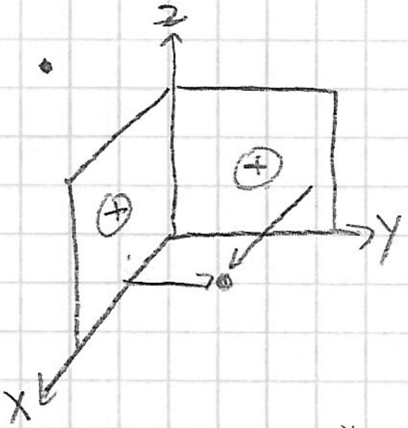
b) $+q \quad +q$



$$E_{Tot} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \boxed{0}$$



$$E_{TOT} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$$



$$E_{tot} = E_x + E_y$$

$$\frac{\sigma}{2\varepsilon_0}\hat{i} + \frac{\sigma}{2\varepsilon_0}\hat{j} = \sqrt{\left(\frac{\sigma}{2\varepsilon_0}\right)^2 + \left(\frac{\sigma}{2\varepsilon_0}\right)^2}$$

$$= \frac{2\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

3) $-\int_a^b \vec{E}\cdot d\vec{l} = V(\vec{b}) - V(\vec{a})$

$\hookrightarrow$ $V(r) = -\int_O^r E\cdot dl \quad (2.21)$

$$\underline{V(b) - V(a)} - \int_O^b E\cdot dl + \int_O^a E\cdot dl$$

$$= -\int_O^b E\cdot dl - \int_a^O E\cdot dl = \underline{-\int_a^b E\cdot dl}$$

$V(\vec{r}) = -\int_\infty^r E(r')dr'$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r} \quad = \frac{-1}{4\pi\varepsilon_0}\int_\infty^r \frac{q}{r'^2}dr'$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r'}\Big|_\infty^r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} = \boxed{\frac{kq}{r}}$$