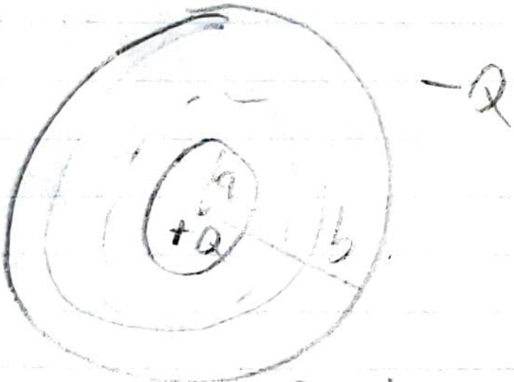
Adam

2.43 $\frac{C}{L} = \frac{Q}{VL}$



$\frac{\lambda_{enc}}{\epsilon_0} = \frac{Q}{\epsilon_0}$

$\oint \vec{E} \cdot d\vec{A} = \oint (E r \hat{r}) \cdot (r\, d\phi\, dz\, \hat{r}) = \int_0^{2\pi}\int_0^L r E_r\, d\phi\, dz$

$= 2\pi L E_r r$

$\frac{Q}{\epsilon_0} = 2\pi L E_r r \rightarrow E_r = \frac{Q}{\epsilon_0 2\pi L r}$

$V = -\int_b^a E_r\, dr = -\int_b^a \frac{Q}{\epsilon_0 2\pi L r}\, dr = -\frac{Q}{\epsilon_0 2\pi L}\int_b^a \frac{1}{r}\, dr$

$= \frac{-Q}{\epsilon_0 2\pi L} \ln(r)\Big|_b^a = \frac{-Q}{\epsilon_0 2\pi L}(\ln(a) - \ln(b))$

$= \frac{Q}{\epsilon_0 2\pi L}\left(\ln\left(\frac{b}{a}\right)\right)$

$\frac{C}{L} = \frac{Q}{VL} \rightarrow \frac{Q}{\frac{Q}{\epsilon_0 2\pi L}\ln\left(\frac{b}{a}\right) L} = \boxed{\frac{2\pi\epsilon_0}{\ln\left(\frac{b}{a}\right)}}$

(2.50) $V(\vec{r}) = A\frac{e^{-\lambda r}}{r}$

Find $\vec{E}(\vec{r})$

$\rho(r)$

Q

$E = -\nabla V = -A\frac{-\lambda r e^{-\lambda r} - e^{-\lambda r}}{r^2} = Ae^{-\lambda r}\frac{(\lambda r+1)}{r^2}\hat{r}$

$\boxed{\vec{E}(\vec{r}) = Ae^{-\lambda r}\frac{(\lambda r+1)}{r^2}\hat{r}}$

$\rho(r)$

$\frac{\rho}{\epsilon_0} = \nabla\cdot\vec{E} = \frac{\hat{r}}{r^2}A\nabla e^{-\lambda r}(\lambda r+1) + Ae^{-\lambda r}(\lambda r+1)\nabla\cdot\frac{\hat{r}}{r^2}$

this becomes 1 as $r \to 1$

$= A\frac{-\lambda^2 r e^{-\lambda r}}{r^2} + Ae^{-\lambda r}(1+\lambda\ldots)4\pi\delta(\vec{r})$

$= A\frac{-\lambda^2 e^{-\lambda r}}{r} + A\,4\pi\delta(\vec{r}) = \frac{\rho}{\epsilon_0}$

$\boxed{\rho = A\epsilon_0\left(4\pi\delta(\vec{r}) - \frac{\lambda^2 e^{-\lambda r}}{r}\right)}$

$$Q = \int_V \rho(\vec{r})\, d\vec{r} = A\epsilon_0 \int_0^R \int_0^\pi \int_0^{2\pi} \left(4\pi\,\delta(\vec{r}) - \frac{\lambda^2 e^{-\lambda r}}{r}\right)$$

$$\cdot r^2 \sin\theta\, dr\, d\theta\, d\phi$$

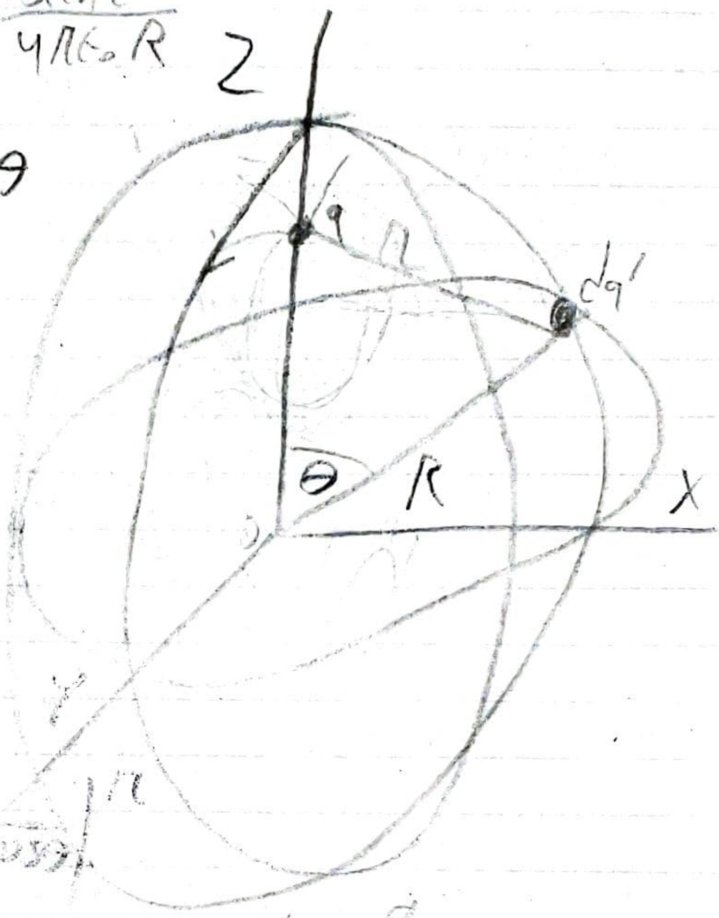
$$= A\epsilon_0 \cdot 2\pi \int_0^R \int_0^\pi \left(4\pi\,\delta(r) r^2 - \lambda^2 r\, e^{-\lambda r}\right) \sin\theta\, dr\, d\theta$$

$$= A\epsilon_0 \cdot 4\pi \left(\int_0^R 4\pi\,\delta(r)\, r^2\, dr - \int_0^R \lambda^2 r e^{-\lambda r}\, dr\right)$$

$$= \boxed{A\epsilon_0 \cdot 4\pi \left(\frac{4\pi R^3}{3} - \frac{e^{\lambda R} - \lambda R - 1}{e^{\lambda R}}\right)}$$

As I cannot differentiate π and π, script r will be $\mathfrak{r}$.

3.1 $V_{ave} = V_{center} + \frac{Q_{enc}}{4\pi\epsilon_0 R}$



$\pi^2 = z^2 + R^2 - 2zR\cos\theta$

$V_{center} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{\mathfrak{r}}$

$\mathfrak{r} = \sqrt{z^2 + R^2 - 2zR\cos\theta}$

$V = \frac{1}{4\pi R^2} \frac{q}{4\pi\epsilon_0}$

$\int \frac{R^2 \sin\theta \, d\theta \, d\phi}{z^2 + R^2 - 2zR\cos\theta}$

$= \frac{q}{4\pi\epsilon_0} \frac{1}{2zR} \sqrt{z^2 + R^2 - 2zR\cos\theta} \Big|^{\pi}$

$= \frac{q}{4\pi\epsilon_0} \frac{1}{2zR} \left[\ldots - 2zR \right] = \frac{1}{4\pi\epsilon_0} \frac{q}{z} = V_{center}$

external

$V_{ave} = V_{center} + V_{int}$

$V_{int} = \frac{q}{4\pi\epsilon_0 R}$

but if more than 1 q, then

$V_{int} = \frac{Q_{enc}}{4\pi\epsilon_0 R}$ by superposition

$$\boxed{V_{int} = V_{center} + \frac{Q_{enc}}{4\pi\epsilon_0 R}}$$

3.3 Find general solution of Laplace for spherical and cylindrical.
V depends on r and s respectively

Spherical

$$\nabla \nabla V = \frac{\partial^2 V}{\partial r^2} + 0 + 0 = \boxed{\frac{\partial^2 V}{\partial r^2}}$$

Cylinder

$$\nabla \nabla V = \frac{\partial^2 V}{\partial s^2} + 0 + 0 = \boxed{\frac{\partial^2 V}{\partial s^2}}$$

3.13 1

$V(x,y) = X(x)Y(y)$

$Y\left(\frac{d^2X}{dx^2}\right) + X\left(\frac{d^2Y}{dy^2}\right) = 0$

$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dy^2} = 0$

$f(x) + g(y) = 0$

$V = V_0$ from $0 \to \frac{a}{2}$
$V = -V_0$ from $\frac{a}{2} \to a$

$V(x,y) = e^{-kx}(C\sin ky + D\cos ky)$

$V(0,0) = V_0 = 1\cdot(0 + D\cdot 1) = D = V_0$

$V(0,a) = -V_0 = 1\cdot(C\sin ky + V_0\cos ky)$
if $k = \frac{(2n+1)\pi}{a}$, the sin is always 0
and cos is always -1 when y is a

$$V(x,y) = e^{-kx}\left(C\sin\left(\frac{(2n+1)\pi y}{a}\right) + V_0\cos\left(\frac{(2n+1)\pi y}{a}\right)\right)$$

Since sine is just not doing anything worthwhile and I need a square wave, I declare extermination on the coefficient of C. Now mathematical justice accounts in all balance. The C

$$V(x,y) = e^{-kx}\left(V_0\cos\left(\frac{(2n+1)\pi y}{a}\right)\right)$$

$$\boxed{V(x,y) = \sum_{n=0}^{\infty} V_0 e^{-\frac{(2n+1)\pi x}{a}}\left(\cos\left(\frac{(2n+1)\pi y}{a}\right)\right)}$$

$$\lim_{x\to\infty} = 0$$

$$V(0,y) = \sum_{n=0}^{\infty} V_0 \ldots$$

(3.14) Find $\sigma(y)$ for 3.3 assuming it is conductor at constant potential

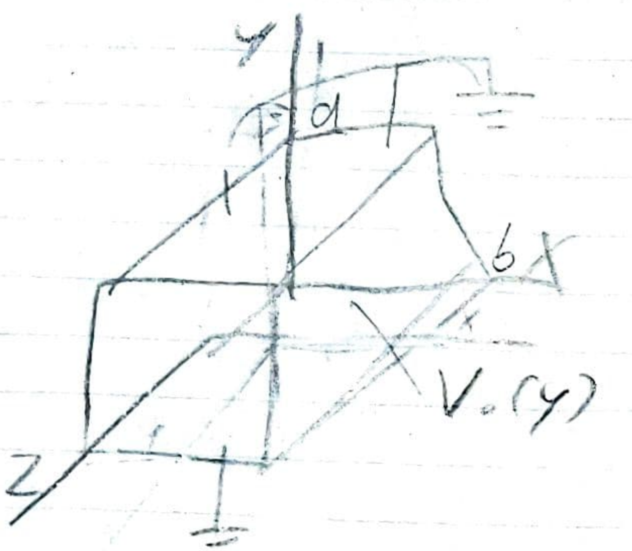
$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n} = -\epsilon_0 \frac{\partial V}{\partial x}\bigg|_{x=0}$$

$$V(x,y) = \frac{2V_0}{\pi} \tan^{-1}\left(\frac{\sin(\pi y/a)}{\sinh(\pi x/a)}\right)$$

$$-\epsilon_0 \frac{\partial V}{\partial x} \text{ @ } x=0 = -\epsilon_0 \frac{-\pi}{a \sin(\frac{\pi y}{a})} \frac{2V_0}{\pi}$$

potential is constant, so $\sin\left(\frac{\pi y}{a}\right) = 1$

$$\boxed{\sigma = \frac{2\epsilon_0 V_0}{a}}$$



(3.15)

(a) V_0 inside?

$$V(x,y) = e^{-kx}(C\sin ky + D\cos ky)$$

$$V(0,0) = 0 = 1(D) = D = 0$$

$$V(0,a) = 0 = 1 \cdot (C \sin ka)$$

$$0 = C\sin ka$$

$$0 = \sin ka$$

$$k = \frac{\pi}{a}$$

(i) $V = 0$ when $x = 0$

(ii) $V = 0$ when $y = 0$

(iii) $V = 0$ when $y = a$

(iiii) $V = V_0(y)$ when $x = b$

$V(x,y) = \ldots e^{-\frac{\ldots}{}} \ldots ky$

$$V(x,y) = \sum_{n=1}^{\infty} C e^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)$$

$$V(0,y) = \sum_{n=1}^{\infty} C \sin\left(\frac{n\pi y}{a}\right) = V_0(y)$$

$$= \sum_{n=1}^{\infty} C \int_0^a \sin\left(\frac{n\pi y}{a}\right)\left(\sin\frac{n'\pi y}{a}\right)dy = \int_0^a V_0(y)\sin\left(\frac{n'\pi y}{a}\right)dy$$

$$\int \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n'\pi y}{a}\right) = \begin{cases} 0 & \text{if } n' \neq n \\ \frac{a}{2} & \text{if } n' = n \end{cases}$$

$$C = \frac{2}{a}\int_0^a V_0(y)\sin\left(\frac{n'\pi y}{a}\right)dy$$

$$\boxed{V(x,y) = \sum_{n=1}^{\infty} \frac{2}{a}\int_0^a V_0(y)\sin\left(\frac{n\pi y}{a}\right)dy \cdot e^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)}$$