4.14)

$$\sigma_b = P \cdot \hat{n} \quad (4.11)$$
$$\rho_b = -\nabla \cdot P \quad (4.12)$$

Gaus's theorerm of Divs.

$$\oint_s P \cdot da = \int_V \nabla \cdot p \, d\tau$$

$$Q_{tot} = \oint_s \sigma_b \, da + \int_V \rho_b \, d\tau$$

$$Q_{tot} = \oint_s P \cdot da - \int_V \nabla \cdot P \, d\tau$$

$$Q_{tot} = \int_V \nabla \cdot P \, d\tau - \int_V \nabla \cdot P \, d\tau$$

$$\boxed{= 0}$$

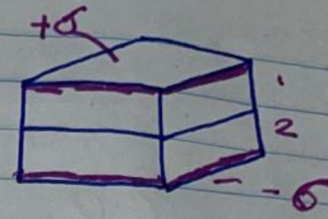
4.15)

4.18)

a) $\oint D \cdot da = q_{enc}$

$D\cancel{A} = \sigma\cancel{A}$

$\boxed{D = \sigma \quad \& \quad D = 0}$



b) $D = \varepsilon E \Rightarrow E = D/\varepsilon$

$E = \sigma/\varepsilon$

$E_1 = \sigma/2\varepsilon_0$

$E_2 = \sigma/1.5\varepsilon_0$

c) $P = \varepsilon_0 \chi_e E =$

$= \frac{\cancel{\varepsilon_0} \chi_e \times \sigma}{\varepsilon_r \cancel{\varepsilon_0}} = (1 - 1/\varepsilon_r)\sigma$

$P_1 = (1 - 1/2)\sigma = \boxed{\frac{\sigma}{2}}$

$\boxed{P_2 = \frac{\sigma}{3}}$

d) $V = E_1 a + E_2 a = a(\sigma/2\varepsilon_0 + \sigma/1.5\varepsilon_0) = 7\sigma a/6\varepsilon_0$

$V = \frac{7\sigma a}{6\varepsilon_0}$

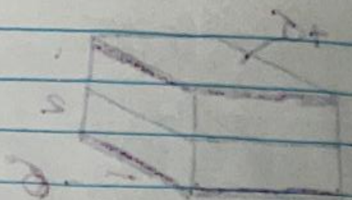
e) $\sigma_b = -P_1 = -\frac{\sigma}{2}$ (top slab 1)

$\sigma_b = P_1 = \frac{\sigma}{2}$ (bottom)

$\sigma_b = -P_2 = -\sigma/3$ (top S. 2

$\sigma_b = P_2 = \sigma/3$ (bottom S. 3)

4.26)



$$\bar{E} = \frac{\bar{D}}{\epsilon_0(1+\chi_e)} \begin{cases} 0 & r<a \\ \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} & r>b \end{cases}$$

$$\bar{D} = \frac{Q}{4\pi r^2} = \begin{cases} 0 & r<a \\ \frac{Q}{4\pi r^2} & r>a \end{cases}$$

$$w = \frac{1}{2}\int \bar{D}\cdot\bar{E}\, d\tau$$

$$= \frac{1}{2}\int_a^b D\cdot E\, d\tau + \frac{1}{2}\int_b^\infty D\cdot E\, d\tau$$

$$= \frac{1}{2}\int_a^b \frac{Q^2}{8\pi^2 r^2 \epsilon}(4\pi r^2 dr) + \frac{1}{2}\int_b^\infty \frac{Q^2}{8\pi^2 r^4 \epsilon_0}(4\pi r^2 dr)$$

$$= \frac{Q^2}{8\pi}\left[\frac{1}{\epsilon}\int_a^b \frac{1}{r^2}dr + \frac{1}{\epsilon_0}\int_b^\infty \frac{1}{r^2}dr\right]$$

$$= \frac{Q^2}{8\pi}\left(\frac{1}{\epsilon}\left(\frac{-1}{r}\right|_a^b + \frac{1}{\epsilon_0}\left(\frac{-1}{r}\right)\Big|_b^\infty\right.$$

$$= \frac{Q^2}{8\pi}\left(\frac{1}{\epsilon}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{\epsilon_0}\left(\frac{1}{b}-\cancel{\frac{1}{\infty}}\right)\right)$$

$$= \frac{Q^2}{8\pi}\left[\frac{1}{\epsilon_0(1+\chi_e)}\left[\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{b\epsilon_0}\right.$$

$$= \frac{Q^2}{8\pi\epsilon_0(1+\chi_e)}\left(\frac{1}{a}+\frac{\chi_e}{b}\right)$$

4.35)

$$\oint \bar{D}\cdot d\bar{a} = Q_{enc}$$

$$\bar{D} = \frac{q}{4\pi r^2}\hat{r}$$

$$\bar{E} = \frac{1}{\epsilon}\bar{D}$$

$$= \frac{1}{\epsilon}\frac{q}{4\pi r^2}\hat{r} =$$

$$\bar{E} = \frac{q}{4\pi\epsilon_0(1+\chi_e)}\frac{\hat{r}}{r^2}$$

$$\bar{P} = \epsilon_0 \chi_e \bar{E}$$

$$= \frac{q\chi_e}{4\pi(1+\chi_e)r^2}\hat{r}$$

$4\pi\delta^3(r)$

function!

$$\rho_b = -\nabla\cdot\bar{P}$$

$$= \frac{-q\chi_e}{4\pi(1+\chi_e)}\left(\nabla\cdot\frac{\hat{r}}{r^2}\right)$$

$$= \frac{-q\chi_e}{\cancel{4\pi}(1+\chi_e)}\cancel{4\pi}\,\delta^3$$

$$\rho_b \boxed{= \frac{-q\chi_e\,\delta^3}{(1+\chi_e)}}$$

$\sigma_b = \vec{P} \cdot \hat{n}$

$= -\dfrac{q\chi_e}{4\pi(1+\chi_e)R^2}$

$Q_{surf} = \sigma_b(4\pi R^2)$

$\boxed{= -\dfrac{q\chi_e}{(1+\chi_e)}}$

$\int \rho_b \, d\tau = \dfrac{-q\chi_e}{1+\chi_e} \cancel{\int \delta^3(s)\,ds}$

$\boxed{= \dfrac{-q\chi_e}{1+\chi_e}}$

So center?

Homework 4

4.10) $\overline{P(r)} = k\bar{r}$

a) $\sigma_b = ?$ $\quad \rho_b = ?$

$\hat{n} = \hat{r}$

$\sigma_b = \bar{P}\cdot\hat{n}$

$\boxed{\sigma_b = kr\,\hat{r}\cdot\hat{r}}$

$\boxed{= kR}$

$\rho_b = -\nabla\cdot\bar{P} \quad = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 k r\right)$

$= \frac{-k}{r^2}\frac{\partial}{\partial r}\left(r^3\right)$

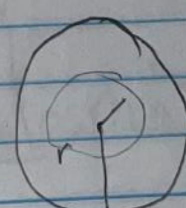
$= \frac{-k}{\cancel{r^2}}\, 3\cancel{r^2}$

$\boxed{\rho_b = -3k}$

b)

inside

$\oint \bar{E}\cdot da = \frac{q_{enc}}{\varepsilon_0} = \int_V \rho\, d\tau$



$E(4\pi r^2) = \frac{1}{\varepsilon_0}\left(-\cancel{3}k\left(\frac{4}{\cancel{3}}\pi r^3\right)\right)$

$E = -\frac{\cancel{4}k r^{\cancel{3}}\cancel{\pi}}{\cancel{4\pi r^2}\varepsilon_0}$

$\boxed{\bar{E}_{in} = -\frac{k r\hat{r}}{\varepsilon_0}}$

b) Out

$$\oint \bar{E} \cdot d\bar{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{1}{\varepsilon_0}\left(-\cancel{3}K \frac{4}{\cancel{3}} \pi R^3 + KR(4\pi R^2)\right)$$

$$= \frac{1}{\varepsilon_0}\left(-4K\pi R^3 + 4KR^3\pi\right)$$

$$E(4\pi r^2) = 0$$

$$\boxed{E = 0}$$

$$\boxed{E_{out} = 0}$$