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Ch2 HW

5) $E = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(r')}{|r-r'|^3} (r-r') \, dl'$ $\Rightarrow$

$\frac{1}{4\pi\varepsilon_0} \int \frac{\lambda}{(\sqrt{(0-x')^2+(0-y')^2+(z-0)^2})^3} \left((0,0,z)-(x',y',0)\right) ds' \Rightarrow$

$r' = r\langle \cos\theta', \sin\theta', 0 \rangle$ $\Rightarrow$

$E = \frac{\lambda}{4\pi\varepsilon_0} \int_0^{2\pi} \frac{1}{(\sqrt{(0-r\cos\theta')^2+(0-\sin\theta')^2+(z-0)^2})^3} \left(\langle 0,0,z\rangle - \langle r\cos\theta, r\sin\theta, 0\rangle\right)$

$\Rightarrow$

$E = \frac{\lambda}{4\pi\varepsilon} \int_0^{2\pi} \frac{1}{(r^2+z^2)^{3/2}} (-r\cos\theta, r\sin\theta, z) \, r\, d\theta'$ $\Rightarrow$

$E = \frac{\lambda}{4\pi\varepsilon_0} \frac{1}{(r^2+z^2)^{3/2}} \left\langle \int_0^{2\pi} -r\cos\theta, \int_0^{2\pi} r\sin\theta, z\int_0^{2\pi} \right\rangle$ $\Rightarrow$

$E = \frac{\lambda}{4\pi\varepsilon_0} \frac{r}{(r^2+z^2)^{3/2}} \langle \cancel{-r(\theta)}^{0}, \cancel{-r(\theta)}^{0}, z\cancel{(2\pi)}_{1} \rangle$ $\Rightarrow$

$E = \frac{\lambda}{4\pi\varepsilon_0} \frac{r}{(r^2+z^2)^{3/2}} \langle 0, 0, 1 \rangle$ $\Rightarrow$

$E = \frac{\lambda}{4\pi\varepsilon_0} \frac{2\pi r z}{(r^2+z^2)^{3/2}} \langle \hat{z} \rangle$

6) $E = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(r')}{\eta^2} \hat{\eta}\, da' = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(r')}{|r-r'|^3} \cancel{\left(\frac{r-r'}{|r-r'|}\right)} da'$

$\Rightarrow$ similar to problem 5 $\Rightarrow$

$$E = \frac{\sigma}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{R} \frac{1}{\left(\sqrt{(0-r\cos\theta)^2 + (0-r\sin\theta)^2 + (z-0)^2}\right)^3} \left(\langle 0,0,z\rangle - \langle x',y',0\rangle\right)$$

$\Rightarrow$

$$\left(\langle 0, 0, z\rangle - r'\langle \cos\theta', \sin\theta', 0\rangle\right) \Rightarrow$$

$$E = \frac{\sigma}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{R} \frac{r}{(r^2+z^2)^{3/2}} \langle -r\cos\theta, -r\sin\theta, z\rangle \Rightarrow$$

$$\cancel{E = \frac{\sigma}{4\pi\varepsilon_0} \frac{r}{(r^2+z^2)^{3/2}} \int_0^{2\pi}\int_0^{R}}$$

$$E = \frac{\sigma}{4\pi\varepsilon_0} \left\langle \int_0^{2\pi}\int_0^{R} \frac{-r^2\cos\theta}{(r^2+z^2)^{3/2}}, \int_0^{2\pi}\int_0^{R} \frac{-r^2\sin\theta}{(r^2+z^2)^{3/2}}, \int_0^{2\pi}\int_0^{R} \frac{zr}{(r^2+z^2)^{3/2}} \right\rangle$$

$\Rightarrow$

$$\left\langle \int_0^{R} \frac{-r^2}{(r^2+z^2)^{3/2}} \left(\int_0^{2\pi} \cos\theta\right), \int_0^{R} \frac{-r^2}{(r^2+z^2)^{3/2}} \left(\int_0^{2\pi} \sin\theta\right), \int_0^{R} \frac{zr^2}{(r^2+z^2)^{3/2}} \left(\int_0^{2\pi}\right) \right\rangle$$

$0 \qquad \Rightarrow \qquad 0 \qquad 2\pi$

$$E = \frac{\sigma}{4\pi\varepsilon_0} \left\langle 0, 0, 2\pi z \int_0^{R} \frac{r'}{(r'^2+z^2)^{3/2}} \right\rangle \Rightarrow$$

$$E = \frac{\sigma z}{2\varepsilon_0} \langle 0,0,1\rangle \int_0^{R} \frac{r'}{(r'^2+z^2)^{3/2}}\, dr'$$

$u = r'^2 + z^2$, $du = 2r'dr' \rightarrow \frac{du}{2} = r'dr'$

$$\int \frac{1}{u^{3/2}} \frac{du}{2} \Rightarrow$$

$$\frac{\sigma z \hat{z}}{4\varepsilon_0} \int_{z^2}^{R^2+z^2} u^{-3/2} \Rightarrow$$

$$-2u^{-1/2} \Big|_{z^2}^{R^2+z^2} \Rightarrow$$

$$E = \frac{\sigma z \hat{z}}{2\varepsilon_0} \left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right)$$

9) a) $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

$$\rho = \varepsilon_0 (\nabla \cdot E) \Rightarrow$$

$$\rho = \varepsilon_0 \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 E_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(E_\theta \sin\theta) + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial\phi} \right) \Rightarrow$$

$$\rho = \varepsilon_0 \left(\frac{1}{\cancel{r^2}}\frac{\partial}{\cancel{\partial r}}\cancel{r^2}(kr^{\cancel{3}}\hat{r}) + \cancel{\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(E_\theta \sin\theta)}_{0} + \cancel{\frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial\phi}}_{0} \right) \Rightarrow$$

$$\rho = \varepsilon_0 \frac{1}{r^2}\frac{\partial}{\partial r}(kr^5)$$

$$\rho = \varepsilon_0 \cancel{\frac{1}{r^2}}(5kr^4\hat{r}) \Rightarrow$$

$$\rho = \varepsilon_0 5kr^2\hat{r}$$

b) $Q_1 = \iiint \rho \, dV$

$$\int_0^\pi \int_0^{2\pi} \int_0^R (5\varepsilon_0 kr^2)(r^2 \sin\theta \, dr \, d\phi \, d\theta) \Rightarrow$$

$$\underbrace{\int_0^R 5\varepsilon_0 kr^4}_{5\varepsilon_0 k(R^5/5)} \underbrace{\int_0^{2\pi} d\phi}_{2\pi} \underbrace{\int_0^\pi \sin\theta \, d\theta}_{2}$$

$$Q = 4\pi \cancel{5}\varepsilon_0 k\left(\frac{R^5}{\cancel{5}}\right)$$

$$Q = k4\pi R^5$$

$$Q_2 = \iiint \rho \, dV \Rightarrow$$

$$\iiint \varepsilon_0 (\nabla \cdot E) \, dV \Rightarrow$$

$$\varepsilon_0 \iiint (\nabla \cdot E) \, dV \Rightarrow$$

$$\varepsilon_0 \oiint E \cdot dS$$

$$E_{surface} = kR^3\hat{r} \Rightarrow$$

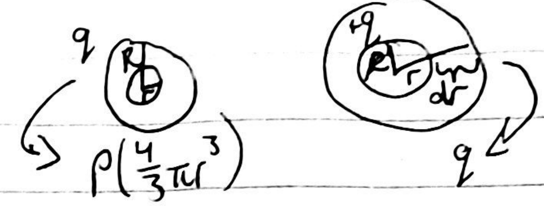
$$\varepsilon_0 \oiint kR^3\hat{r} \, dS \Rightarrow$$

$$\varepsilon_0 kR^3\hat{r} \underbrace{\oiint dS}_{0} \Rightarrow$$

$$\varepsilon_0 kR^3\hat{r}(R^2) \Rightarrow$$

$$Q_2 = \varepsilon_0 kR^5\hat{r}$$

12) $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$



$$\iiint_0^2 \nabla \cdot E\, dV \Rightarrow \iiint \frac{\rho}{\varepsilon_0}\, dV \Rightarrow$$

$$\frac{1}{\varepsilon_0}\iiint \rho\, dV \Rightarrow$$

$$\underbrace{\frac{1}{\varepsilon_0}\rho\frac{4}{3}\pi r^3}_{r<R}, \quad \underbrace{\frac{q}{\varepsilon_0}}_{r>R}$$

$$\Rightarrow$$

$$\oiint E \cdot dS = \oiint [E r(\hat{r}) \cdot \hat{r}\, dS] \Rightarrow$$

$$\oiint E r \hat{r} \cdot \hat{r}\, dS = \begin{cases} \frac{1}{\varepsilon_0}\rho\frac{4}{3}\pi r^3 \\ q/\varepsilon_0 \end{cases} \Rightarrow$$

$$\begin{cases} \frac{q}{\cancel{4/3\pi} R^3}\left(\cancel{\tfrac{4}{3}\pi} r^3\right)\frac{1}{\varepsilon_0} \\ q/\varepsilon_0 \end{cases} \Rightarrow$$

$$E(r)\oiint dS = \begin{cases} \frac{q r^3}{\varepsilon_0 R^3} \\ q/\varepsilon_0 \end{cases} \Rightarrow$$

$$E(r)(4\pi r) = \begin{cases} \frac{q r^3}{\varepsilon_0 R^3} \\ q/\varepsilon_0 \end{cases} \Rightarrow$$

$$E_r = \begin{cases} \frac{q r^{\cancel{3}2}}{\varepsilon_0 R^3} \cdot \frac{1}{4\pi \cancel{r}} \\ \frac{q}{\varepsilon_0} \cdot \frac{1}{4\pi r} \end{cases}$$

16) $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$

i) ~~s > a~~ $s < a$

ii) $a < s < b$

iii) $s < b$ — 0 since wire is ~~[illegible]~~ neutral.

$$\int_0^L \int_0^{2\pi} \int_0^s \nabla \cdot E \, dV = \int_0^L \int_0^{2\pi} \int_0^s \frac{\rho}{\varepsilon_0} dV = \begin{cases} \iiint s\,ds\,d\phi\,dz & s< \\ \iiint s\,ds\,d\phi\,dz & a< \end{cases}$$

$\Rightarrow$

$$\frac{\rho}{\varepsilon_0} \int_0^L \int_0^{2\pi} (E \cdot dS) = \begin{cases} \int_0^L dz \int_0^{2\pi} d\phi \int_0^s s\,ds \\ (\cdots) \end{cases} \Rightarrow$$

$$\begin{cases} L(2\pi)\left(\frac{s^2}{2}\right) \Rightarrow \\ L(2\pi)(s^2/2) \end{cases}$$

$$\underbrace{\int_0^L \int_0^{2\pi} E(s)\hat{s} \cdot (\hat{s}\, s\,d\phi\,dz)} = \begin{cases} \frac{\rho}{\varepsilon_0} L(2\pi)\frac{s^2}{2} \\ \frac{\rho}{\varepsilon_0} L(2\pi)\frac{s^2}{2} \end{cases} \Rightarrow$$

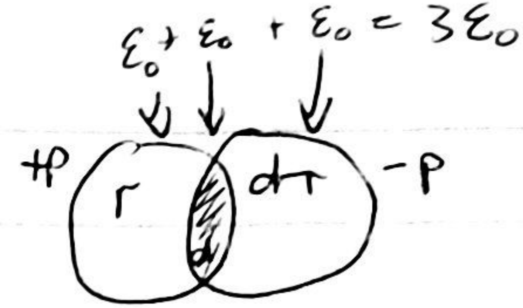
$E(s)\hat{s} \int_0^L \int_0^{2\pi} d\phi\,dz \Rightarrow$

$\int_0^L dz \int_0^{2\pi} d\phi \Rightarrow$

$$\frac{\cancel{\hat{s}} E(s) \cancel{(L)} \cancel{(2\pi)}}{\cancel{2\pi L \hat{s}}} = \begin{cases} \frac{\rho}{\varepsilon_0} \cancel{L(2\pi)} \frac{s^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{s 2\pi L}} \Rightarrow \\ \rho/\varepsilon_0 \cancel{L(2\pi)} \frac{s^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{2\pi L s}} \end{cases}$$

$$E(s) = \begin{cases} \frac{\rho s}{\varepsilon_0 2} \\ \frac{\rho s}{\varepsilon_0 2} \end{cases}$$

18) $E = \begin{cases} \frac{q r^2}{\varepsilon_0 R^3} \frac{1}{4\pi} \\ \frac{q}{\varepsilon_0} \frac{1}{4\pi r} \end{cases}$



$E_+ = \frac{+\rho}{\varepsilon_0} r \cdot 3$ & $E_- = \frac{-\rho}{3\varepsilon_0}(r-d)$

$E = E_+ + E_- \Rightarrow$

$E = \frac{+\cancel{\rho}}{3\varepsilon_0} r + \frac{-\cancel{\rho}}{3\varepsilon_0}(r-d) \Rightarrow$

$E = \frac{\rho}{3\varepsilon_0} d$

29)

$$V(r) = \frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(r')}{\eta}\, d\tau' \Rightarrow$$

$$\nabla^2 V(r) = \nabla^2\left[\frac{1}{4\pi\varepsilon_0}\iiint \frac{\rho(r')}{\eta}\, d\tau'\right] \Rightarrow$$

$$= \frac{1}{4\pi\varepsilon_0}\nabla^2 \iiint \frac{\rho(r')}{\eta}\, d\tau' \Rightarrow$$

$$\nabla^2 V(r) = \frac{1}{4\pi\varepsilon_0}\iiint \rho(r')\nabla^2\left(\frac{1}{\eta}\right) d\tau' \Rightarrow$$

$$= \frac{1}{\cancel{4\pi}\varepsilon_0}\iiint \rho(r')\, \cancel{4\pi}\,\delta^3(\eta)\, d\tau' \Rightarrow$$

$$= \frac{-1}{\varepsilon_0}\iiint \rho(r')\,\delta^3(\eta)\, d\tau' \Rightarrow$$

$$\iiint \rho(r')\delta^3(r-r')\, d\tau' \Rightarrow$$

$$= -\frac{1}{\varepsilon_0}\rho(r)\iiint \cancel{\delta^3(r-r')\, d\tau'}^{\,0}$$

$$= -\frac{\rho(r)}{\varepsilon_0}\hat{r}$$