PHYS 330 HW # 6

Riley Sullivan

6.3, 6.7, 6.16

6.3

$\boxed{6.2}$ $F = 2\pi IRB\cos\theta$

$\boxed{6.3}$ $F = \nabla(m \cdot B)$

(a) $F = 2\pi IRB\cos\theta$, $\frac{\mu_0}{4\pi}\left[\frac{3(m_1\cdot\hat{r})\hat{r} - m_1}{r^3}\right]$, $B\cos\theta = B\cdot\hat{y}$

$\therefore B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(m_1\cdot\hat{r})(\hat{r}\cdot\hat{y}) - (m_1\cdot\hat{y})\right]$, $m_1\cdot\hat{y} = 0$, $\hat{r}\cdot\hat{y} = \sin\phi$

$B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}3m_1\sin\phi\cos\phi$

$F = 2\pi IR\frac{\mu_0}{4\pi}\frac{1}{r^3}3m_1\sin\phi\cos\phi$, $\sin\phi = \frac{R}{r}$, $\cos\phi \frac{\sqrt{r^2-R^2}}{r}$

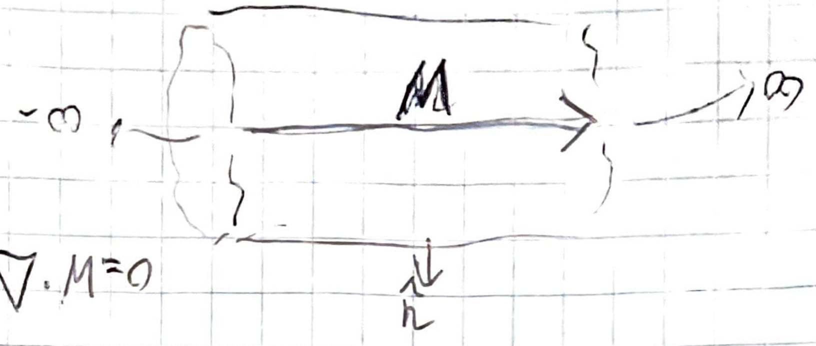
$F = 3\frac{\mu_0}{2}m_1 IR^2\frac{\sqrt{r^2-R^2}}{r^5}$, $IR^2\pi = m_2$,

$F = \frac{3\mu_0}{2\pi}m_1 m_2\frac{\sqrt{r^2-R^2}}{r^5}$, f dip, $R \ll r$ so $\boxed{F = \frac{3\mu_0}{2\pi}\frac{m_1 m_2}{r^4}}$

(b) $F = \nabla(m_2\cdot B) = (m_2\cdot\nabla)B = m_2\frac{\partial}{\partial z}\left[\frac{\mu_0}{4\pi}\frac{1}{z^3}(3(m_1\cdot\hat{z})\hat{z} - m_1)\right]$

$F = \frac{\mu_0}{2\pi}m_1 m_2\hat{z}\frac{\partial}{\partial z}\left(\frac{1}{z^3}\right) = \boxed{\frac{3\mu_0}{2\pi}\frac{m_1 m_2}{z^4}\hat{z}}$

6.7

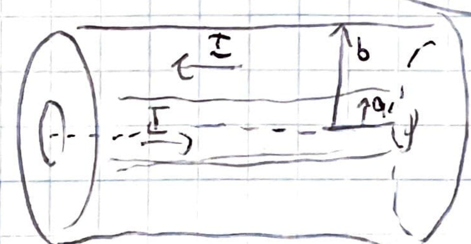


$J_b = \nabla \cdot M = 0$

$K_b = M \times \hat{n} = M\hat{\phi}$

$B = \mu_0 K_b = \mu_0 M$, $\boxed{B = \mu_0 M}$

6.16



$\oint H \cdot dl = I_{enc} = I$, $H = \frac{I}{2\pi s}\hat{\phi}$, $B = \mu_0(1+\chi_m)H = \mu_0(1+\chi_m)\frac{I}{2\pi s}\hat{\phi}$

$M = \chi_m H = \frac{\chi_m I}{2\pi s}\hat{\phi}$

$J_b = \nabla \times M = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\chi_m I}{2\pi s}\right)\hat{z} = 0$, $K_b = M \times \hat{n} = \begin{cases} \frac{\chi_m I}{2\pi a}\hat{z} & @\ s=a \\ -\frac{\chi_m I}{2\pi b}\hat{z} & @\ s=b \end{cases}$

$I + \frac{\chi_m I}{2\pi a} = (1+\chi_m) I$,

$\oint B \cdot dl = \mu_0 I_{enc} = \mu_0(1+\chi_m)I$

$\boxed{B = \frac{\mu_0(1+\chi_m)I}{2\pi s}\hat{\phi}}$