Warm-Up for March 28th, 2022

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1 Memory Bank

1. Definition of voltage:
$$V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l} \tag{1}$$
2. Work to place a point charge Q with reference point at infinity:
$$U = QV(\mathbf{r}) \tag{2}$$
3. Field of a dipole (coordinate-free):
$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[3(\mathbf{p}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}\right] \tag{3}$$

2 Dipoles, Polarization, and Energy

1. Show that the energy of an ideal dipole $\mathbf{p}$ in an electric field $\mathbf{E}$ is $U = -\mathbf{p}\cdot\mathbf{E}$. *[Hint: use Eq. 1].*

2. (a) Show that the interaction energy between two dipoles separated by a displacement $\mathbf{r}$ is
$$U = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[\mathbf{p}_1\cdot\mathbf{p}_2 - 3(\mathbf{p}_1\cdot\hat{\mathbf{r}})(\mathbf{p}_2\cdot\hat{\mathbf{r}})\right] \tag{4}$$
(b) Consider Fig. 1. If the dipoles are at right angles, show that Eq. 4 reduces to $U = -(3k/r^3)p_1 p_2$, with $k = 1/4\pi\epsilon_0$.

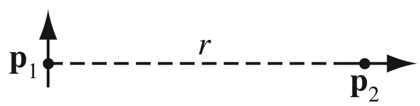

Figure 1: Two orthogonal dipoles separated by a distance r.