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Reading Quiz 2 for Electromagnetic Theory (PHYS330)

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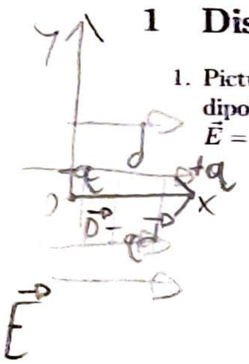
Abstract

A summary of content covered in chapter 2 of Introduction to Electrodynamics.

1 Distributions of Point Charges



1. Picture a *physical dipole* of two charges $+q$ and $-q$ of equal magnitude, separated by a distance d. Define the dipole moment as $\vec{p} = q\vec{d}$ pointing from $-q$ to q somewhere in the xy-plane. Now add an external electric field $\vec{E} = E_0\hat{x}$. Show that the *torque* on the dipole is

$$\vec{\tau} = \vec{p} \times \vec{E} \quad (1)$$

$\tau = rF\sin\theta \Rightarrow \boxed{\vec\tau = \vec p \times \vec E}$

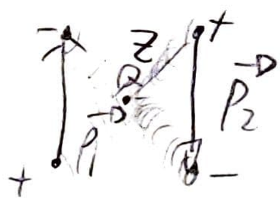
$r = \vec p$

$F = \vec E$

sin θ applied to 2 forces is cross product

2. Imagine two dipoles, each with dipole moments $\vec{p}_1$ and $\vec{p}_2$ pointed in opposite directions, forming a square with alternating positive and negative charges. Calculate the electric field vector in the center of the square.

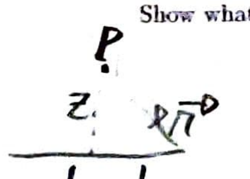
$\vec E(z) = \frac{1}{4\pi\epsilon_0}\frac{\vec p}{z^3}$



$E(z) = \frac{1}{4\pi\epsilon_0}\frac{\vec P_1}{z^3} + \frac{1}{4\pi\epsilon_0}\frac{\vec P_2}{z^3}$ if $\vec p_1 = -\vec p_2$ then $E(z) = \frac{0}{4\pi\epsilon_0} = \boxed{0}$ □

2 Continuous Charge Distributions

1. (a) Compute the electric field of a continuous line of charge, with total charge $Q = \lambda L$, where λ is the charge density and L is the total length. Take the field point to be a distance z above the center of the line of charge. Show what happens in the limit that $L \gg z$. (b) Obtain the same result as (a) using Gauss' Law.



$E(\vec r) = \frac{1}{4\pi\epsilon_0}\int_0^L \frac{\lambda(\vec r')}{\eta^2}\hat\eta \, dl'$

$\eta = \sqrt{\left(\frac{l}{2}\right)^2 + z^2}$

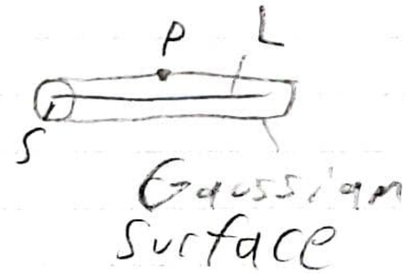
$\hat\eta = \frac{z\hat z - l\hat l}{\sqrt{z^2+l^2}}$

$= \frac{\lambda}{4\pi\epsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}}\left(\frac{z\hat z}{\sqrt{(z^2+l^2)^3}} - \frac{l\hat l}{\sqrt{(z^2+l^2)^3}}\right)dl = \frac{2\lambda L}{4\pi\epsilon_0 z\sqrt{4z^2+l^2}} = \boxed{\frac{Q}{2\pi\epsilon_0 z\sqrt{4z^2+l^2}}}$

$\lim_{L \gg z} \frac{\lambda L}{2\pi\epsilon_0 z\sqrt{L^2}} = \boxed{\frac{\lambda}{2\pi\epsilon_0 z}}$

b) $\oint_S \vec E \cdot d\vec a = \frac{1}{\epsilon_0} Q$

(2b) $\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} Q$



$\int \vec{E} \cdot d\vec{a} = \int |\vec{E}| da = |\vec{E}| \int_0^L \int_0^{2\pi} s \, d\ell \, d\phi$

$= |\vec{E}| L 2\pi s = \frac{Q}{\epsilon_0} \Rightarrow \vec{E} L 2\pi s = \frac{\lambda L}{\epsilon_0}$

$\Rightarrow \vec{E} = \frac{\lambda}{2\pi s \epsilon_0}$

$\Rightarrow$

$\vec{E} = \frac{\lambda}{2\pi s \epsilon_0} \hat{s}$

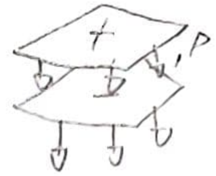
2. Assuming a plane of charge with charge density (Coulombs per unit area) σ has an electric field $\sigma/(2\epsilon_0)$, what electric fields would occur in each of the following situations:

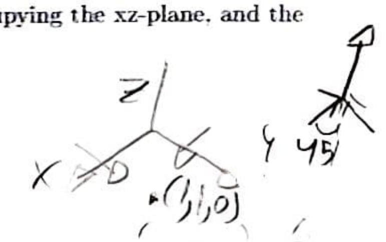
a • Two planes of positive charge, and the field point is somewhere between the plates.

b • Two planes of charge, one positive and one negative, and the field point is somewhere between the plates.

c • Two planes of positive charge, one occupying the yz-plane, and the other occupying the xz-plane, and the field point is (1, 1, 0).

(a) $\frac{\sigma}{2\epsilon_0}\hat{p} + \frac{-\sigma}{2\epsilon_0}\hat{p} = \boxed{0}$





(b) $\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$

(c) $\frac{\sigma}{\epsilon_0}\hat{x} + \frac{\sigma}{\epsilon_0}\hat{y} = \frac{\sqrt{2}\sigma}{2 \cdot 2\epsilon_0} + \frac{\sqrt{2}\sigma}{2 \cdot 2\epsilon_0} = \boxed{\frac{\sqrt{2}\sigma}{2\epsilon_0} \text{ in xy direction}}$

3 The Curl of $\vec{E}$-fields

1. According to Eq. 2.19 in the text, the close loop line integral for the E-field of a point charge is

$$\oint \vec{E} \cdot d\vec{l} = 0 \tag{2}$$

This implies $\nabla \times \vec{E} = 0$. According to the Helmholtz theorem in Ch. 1, this means the $\vec{E}$-field can be cast as the gradient of a scalar function known as *the potential,* V:

$$\vec{E} = -\nabla V \tag{3}$$

The minus sign is a convention that is analogous to the minus sign in $\vec{F} = -\frac{dU}{dx}\hat{x}$ from mechanics.

a) • Show that

$$-\int_a^B \vec{E} \cdot d\vec{l} = V(\vec{b}) - V(\vec{a}) \tag{4}$$

b) • Assume a point charge at the origin, and label its electric field $\vec{E}$. Perform the integral

$$V(\vec{r}) = -\int_\infty^r E(r')dr' \tag{5}$$

to find the potential formula for a point charge. *[Answer: kq/r]*

(a) $\oint \vec{E} \cdot d\vec{l} = 0, \ \vec{E} = 0$

$\vec{E} = -\nabla \vec{V} \Rightarrow 0 = -\nabla \vec{V} = 0$

$-\int_a^B \vec{E} \cdot d\vec{l} = V(\vec{b}) - V(\vec{a}) = \int_a^B 0 \, d\vec{l} = 0 = V(\vec{b}) - V(\vec{a})$. □

$\Rightarrow \boxed{V(\vec{a}) = V(\vec{b})}$ □

(b) $V(\vec{r}) = -\int_\infty^r E(r')dr' = -\int_\infty^r \frac{kq}{4\pi\epsilon_0 r'^2} dr' = \frac{kq}{4\pi\epsilon_0 r'}\Big|_\infty^r$

$\vec{E} = \frac{kq}{4\pi\epsilon_0 r^2}$

$= \frac{kq}{4\pi\epsilon_0 r} \Rightarrow \boxed{\frac{kq}{r}}$ (k is arbitrary constant, absorbs other coefficients)