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Reading Quiz 3 for Electromagnetic Theory (PHYS330)

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Abstract

A summary of content covered in chapter 3 (so far) of Introduction to Electrodynamics.

1 Discussions about Vectors (Prelude to Fourier's Trick)

1. Let $\vec{v} = a\hat{x} + b\hat{y} + c\hat{z}$. Which of the following is equal to c?
   - ~~A: $\vec{v}\cdot\vec{v} - |\vec{v}|$~~ $\Rightarrow \vec{v}\cdot\vec{v} = a^2+b^2+c^2 \Rightarrow |\vec{v}| = \sqrt{a^2+b^2+c^2} \Rightarrow \vec{v}\cdot\vec{v} - |\vec{v}| = a^2+b^2+c^2-\sqrt{a^2+b^2+c^2} \neq c$
   - $\boxed{\text{B: } \vec{v}\cdot\hat{z}}$ $(a\hat{x}+b\hat{y}+c\hat{z})\cdot(\hat{z}) = a(0)+b(0)+c(1) = \boxed{c}$
   - C: $\hat{x}\cdot\vec{v}$ = $(\hat{x})\cdot(a\hat{x}+b\hat{y}+c\hat{z}) = a(1)+b(0)+c(0) = (a) \neq c$
   - ~~D: $\sqrt{\vec{v}^2}$~~ $= \vec{v}$

2. Let $\vec{x} = \sum_{i=1}^{n} c_i\hat{x}_i$ be an n-dimensional vector and the set of $\hat{x}_i$ represent orthonormal basis vectors. How do you obtain the coefficient c_7? Orthogonal - $\int_a^b f_n(y) f_{n'}(y)\,dy = 0$ if $n' \neq n$.
   - ~~A: $\hat{x}_i\cdot\vec{x}$~~ $= 0 \neq c_7$
   - $\boxed{\text{B: } n = 7}$ $\Rightarrow \vec{x} = \sum_{i=1}^{7} c_1\hat{x}_1 + c_2\hat{x}_2 + \dots + \boxed{c_7\hat{x}_7}$
   - ~~C: $\hat{x}\cdot\vec{x}$~~ $= 0 \neq c_7$
   - D: $\hat{x}_7\cdot\vec{x}$ $\Rightarrow \hat{x}_7\cdot\vec{x} = c_1(\hat{x}_1\cdot\hat{x}_7) + c_2(\hat{x}_2\cdot\hat{x}_7) + \dots + \overset{\text{0, since } n = n'}{\cancel{c_7(\hat{x}_7\cdot\hat{x}_7)}} + \dots + c_n(\hat{x}_n\cdot\hat{x}_7)$

3. Suppose we are trying to develop the Fourier series for a function $f(x)$. Recall the definition of a Fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx) \tag{1}$$

However, the function we are trying to model is $f(x) = \sin(3x)$. Write down all coefficients in the Fourier series from $n = 0$ to $n = \infty$.

$a_n = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\cos(nx)\,dx \Rightarrow a_0 = \frac{1}{\pi}\int_0^{2\pi}\sin(3x)\,dx \Rightarrow a_0 = \frac{1}{\pi}\left[\frac{1}{3}\cos(3x)\right]_{2\pi}^{0} \Rightarrow a_0 = \frac{1}{3\pi}[1-1] \Rightarrow a_0 = 0$

$a_n = 0$ & $b_n = 0$

$\left[f(x) = \frac{a_0}{2} + a_1\sin(x) + b_1\cos(x) + a_2\sin(2x) + b_2\cos(2x) + a_3\sin(3x) + b_3\cos(3x) + \dots + a_n\sin(nx) + b_n\cos(nx) \right]$

2 Fourier's Trick and Boundary Value Problems

1. If $V(x,y,z) \to 0$ as $y \to \infty$, which of the following cannot be part of the solution for $V(x,y,z)$?
   $Y(y) = C\sinh(ky) + D\cosh(ky)$ or $Y(y) = Ae^{ky} + Be^{-ky}$
   - ~~A:~~ $Y(y) = e^{-ky}$
   - ~~B:~~ $Y(y) = \sinh(x)$
   - $\boxed{\text{C: } Y(y) = 1/y^2}$ → cannot be expressed in differentiation of Laplace's eqn.
   - ~~D:~~ $Y(y) = e^{-ky^2}$

2. Below is Eq. 3.50 from section 3.3 of the text, with $V_0(y,z) = V_0$:

$$C_{n,m} = \frac{4V_0}{ab}\int_0^a\int_0^a \sin(n\pi y/a)\sin(\overset{m}{\cancel{n}}\pi z/a)\,dy\,dz \tag{2}$$

Reproduce the result in Eq. 3.51 for $C_{n,m}$.

$\boxed{2.2}$

$$C_{n,m} = \frac{4V_0}{ab}\underbrace{\int_0^a \sin\left(\frac{n\pi y}{a}\right)dy}_{①} \underbrace{\int_0^b \sin\left(\frac{m\pi z}{b}\right)dz}_{②}$$

① $\int_0^a \sin\left(\frac{n\pi y}{a}\right)dy$ // $u=\frac{n\pi y}{a} \Rightarrow du = \frac{n\pi}{a}dy \Rightarrow dy = \frac{a}{n\pi}du \to y=0, u=0 \Rightarrow y=a, u=n\pi$ //

$$\int_0^{n\pi} \sin(u)\frac{a}{n\pi}du \Rightarrow \frac{a}{n\pi}\int_0^{n\pi}\sin(u)du = \frac{a}{n\pi}\cos(u)\Big|_{n\pi}^{0} = \frac{a}{n\pi}\left(1-\cos(n\pi)\right) \begin{cases} \text{if } n \text{ even, } ① = 0 \\ \text{if } n \text{ odd, } ① = \frac{2a}{n\pi} \end{cases}$$

② $\int_0^b \sin\left(\frac{m\pi z}{b}\right)dz$ // $u=\frac{m\pi z}{b} \Rightarrow du = \frac{m\pi}{b}dz \Rightarrow dz = \frac{b}{m\pi}du \to z=0, u=0;\ z=b, u=m\pi$ //

$$\int_0^{m\pi} \sin(u)\frac{b}{m\pi}du \Rightarrow \frac{b}{m\pi}\cos(u)\Big|_{m\pi}^{0} \Rightarrow \frac{b}{m\pi}\left(1-\cos(m\pi)\right) \begin{cases} \text{if } n \text{ even, } ② = 0 \\ \text{if } n \text{ odd, } ① = \frac{2b}{m\pi} \end{cases}$$

So, when:

n even, m even $\to C_{n,m} = \frac{4V_0}{ab}(0)(0) = 0$

n even, m odd $\to C_{n,m} = \frac{4V_0}{ab}(0)\left(\frac{2b}{m\pi}\right) = 0$

n odd, m even $\to C_{n,m} = \frac{4V_0}{ab}\left(\frac{2a}{n\pi}\right)(0) = 0$

n odd, m odd $\to C_{n,m} = \frac{4V_0}{ab}\left(\frac{2a}{n\pi}\right)\left(\frac{2b}{m\pi}\right) = \frac{16V_0}{\pi^2 nm}$.

So,

$$\boxed{C_{n,m} = \frac{4V_0}{ab}\int_0^a \sin\left(\frac{n\pi y}{a}\right)dy \int_0^b \sin\left(\frac{m\pi z}{b}\right)dz = \begin{cases} 0, \text{ if } n \text{ or } m \text{ even,} \\ \frac{16V_0}{\pi^2 nm}, \text{ if } n \text{ and } m \text{ odd.} \end{cases}}$$

✓ equal to 3.51.

$\boxed{1.3}$ $a_n = \frac{1}{\pi}\int_0^{2\pi}\sin(3x)\cos(nx)\,dx \Rightarrow a_0 = \frac{1}{\pi}\int_0^{2\pi}\sin(3x)(1)dx \Rightarrow a_0 = \frac{1}{\pi}\left(\frac{1}{3}\cos(3x)\right)\Big|_{2\pi}^{0} \Rightarrow a_0 = \frac{1}{3\pi}(1-1) \Rightarrow \underline{a_0 = 0}$

$a_1 = \frac{1}{\pi}\int_0^{2\pi}\sin(3x)\cos(2x)dx \Rightarrow a_1 = \frac{1}{\pi}\int_0^{2\pi}$