HW CH. 1 [~~1.54~~, ~~1.55~~, ~~1.56~~, ~~1.57~~, ~~1.59~~, ~~1.62~~, ~~1.63~~, 1.64

1.54) $\vec{v} = r^2\cos\theta\,\hat{r} + r^2\cos\phi\,\hat{\theta} - r^2\cos\theta\sin\phi\,\hat{\phi}$

$$\nabla\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial v_\phi}{\partial\phi}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2(r^2\cos\theta)) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r^2\cos\phi\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(-r^2\cos\theta\sin\phi)$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^4\cos\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(r^2\cos\phi\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(-r^2\cos\theta\sin\phi)$$

$$= \frac{1}{r^2}(4r^3\cos\theta) + \frac{1}{r\sin\theta}(r^2\cos\phi\cos\theta) + \frac{1}{r\sin\theta}(-r^2\cos\theta\cos\phi)$$

$$= \frac{1}{r^2}(4r^3\cos\theta) = 4r\cos\theta$$

$$\int_V \nabla\cdot\vec{v}\,d\tau = \int_V 4r\cos\theta\,d\tau = \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^R (4r\cos\theta)(r^2\sin\theta\,dr\,d\phi\,d\theta)$$

$$= \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\int_0^R 4r^3\cos\theta\sin\theta\,dr\,d\phi\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} R^4\,\phi\,\cos\theta\sin\theta\Big|_0^{\frac{\pi}{2}}d\theta$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} R^4\cos\theta\sin\theta\,d\theta \quad [u\text{ sub}, u=\sin\theta, du=\cos\theta\,d\theta \quad d\theta=\frac{du}{\cos\theta}]$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} R^4 u\,du = \frac{\pi}{2}\frac{1}{2}u^2R^4\Big]_0^{\frac{\pi}{2}} = \frac{\pi}{2}\cdot\frac{1}{2}(\sin\theta)R^4\Big]_0^{\frac{\pi}{2}} = \boxed{\frac{\pi}{4}R^4}$$

I → $d\vec{a} = -r\,dr\,d\theta\,\hat{\phi}$ $\quad \phi = 0 \rightarrow \vec{v}\cdot d\vec{a} = 0$

II → $d\vec{a} = r\,dr\,d\theta\,\hat{\phi}$ $\quad \phi = \frac{\pi}{2}$, $\vec{v}\cdot d\vec{a} = (-r^2\cos\theta\sin\phi)(r\,dr\,d\theta\,\hat{\phi})$

$$\int_0^{\frac{\pi}{2}}\int_0^R -r^3\cos\theta\,dr\,d\theta = -\frac{1}{4}\int_0^{\frac{\pi}{2}} R^4\cos\theta\,d\theta = -\frac{1}{4}R^4$$

III → $da = r\,dr\,d\phi\,\hat{\theta}$, $\theta = \frac{\pi}{2} \rightarrow \vec{v}\cdot d\vec{a} = (r^2\cos\phi)(r\,dr\,d\phi)$

$$\int \vec{v}\cdot d\vec{a} = \int_0^{\frac{\pi}{2}}\int_0^R r^3\cos\phi\,dr\,d\theta = \frac{1}{4}\int_0^{\frac{\pi}{2}} R^4\cos\phi\,d\phi = \frac{1}{4}R^4$$

IV → $d\vec{a} = r^2\sin\theta\,d\theta\,d\phi\,\hat{r}$, $r = R$, $\vec{v}\cdot d\vec{a} = (R^2\cos\theta)(R^2\sin\theta\,d\theta\,d\phi)$

$$\int \vec{v}\cdot d\vec{a} = \int_0^{\frac{\pi}{2}} R^4\cos\theta\sin\theta\,d\theta\,d\phi = \frac{\pi}{4}R^4$$

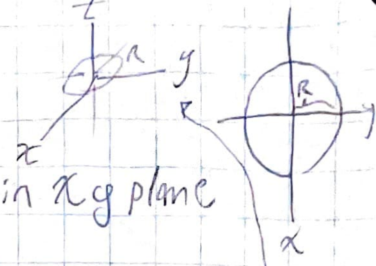
$$\oint \vec{v}\cdot d\vec{a} = \cancel{-\frac{1}{4}R^4} + \cancel{\frac{1}{4}R^4} + \boxed{\frac{\pi}{4}R^4}$$

Results Match!!!

1.55

Stoke's THRM

$\vec{v} = ay\hat{x} + bx\hat{y}$, circular path R @ origin in xy plane



$$\int_S (\nabla\times\vec{v})\cdot d\vec{a} = \oint_P \vec{v}\cdot d\vec{l}$$

$$\int (\nabla\times\vec{v})\, d\vec{a} \qquad d\vec{a} = dx\,dy\,\hat{z} \qquad \nabla\times\vec{V} \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ V_x & V_y & 0 \end{vmatrix} = \hat{k}\left(\frac{\partial}{\partial x}bx - \frac{\partial}{\partial y}ay\right) = \hat{k}(b-a)$$

$$\int_S (b-a)\hat{n}\cdot dx\,dy\,\hat{k} = \int (b-a)\,dx\,dy = (b-a)\int_S dx\,dy$$

full circle enclosed,

$A = \int da = \boxed{\pi R^2}$

$$\int_S (\nabla\times\vec{v})\cdot d\vec{a} = \pi R^2(b-a)$$

$$\oint_P \vec{v}\cdot d\vec{l}$$

$$\vec{v}\cdot d\vec{l} = (ay\hat{x} + bx\hat{y})(dx\hat{x} + dy\hat{y}) = ay\,dx + bx\,dy$$

$x = R\cos\theta$, $y = R\sin\theta$

$dx = -R\sin\theta\, d\theta \quad dy = R\cos\theta\, d\theta$

$$\vec{v}\cdot d\vec{l} = -ar^2\sin^2\theta\, d\theta + br^2\cos^2\theta\, d\theta = R^2(b\cos^2\theta - a\sin^2\theta)\,d\theta$$

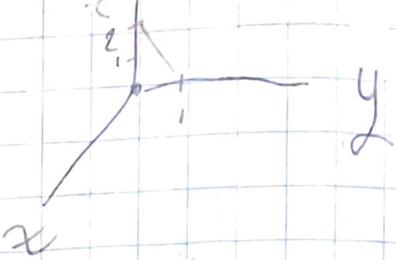
$$\rightarrow R^2\int_0^{2\pi}(b\cos^2\theta - a\sin^2\theta)\,d\theta = R^2\left[b\int_0^{2\pi}\cos^2\theta\, d\theta - a\int_0^{2\pi}\sin^2\theta\, d\theta\right]$$

$$= R^2[b\pi - a\pi] = \pi R^2(b-a)$$

$$\rightarrow \pi R^2(b-a) = \pi R^2(b-a)$$

stoke's thrm ✓

1.56)



line integral

$v = 6\hat{x} + yz^2\hat{y} + (3y+z)\hat{z}$

Path 1 $(0,0,0) \to (0,1,0)$

$\vec{u}\cdot d\vec{l} = yz^2\,dy$, $z=0$, $\vec{u}\cdot d\vec{l} = 0$: (,

Path 2 $(y = -2x+2)$, $(0,1,0) \to (0,0,-2y+2)$

so $z = -2y+2$

$\int \vec{m}\cdot d\vec{l} = \int yz^2\,dy + (3y+z)\,dz$

$z = (-2y+2)$; $dz = -2\,dy$

$\int [y(-2y+2)^2 dy + (y+2)(-2\,dy)] = 4y^2 - 8y + 4$

$\int 4y^3 - 8y + 4y - 2y - 4 = \int_0^1 (4y^3 - 8y^2 + 2y - 4)\,dy = \left[y^4 - \frac{8}{3}y^3 + y^2 - 4y\right]_1^0$

$= \frac{8}{3}(+4-2 = \boxed{\frac{8}{3}+2} = 4_2$

Path 3 $(0,0,2) \to (0,0,0)$

$\int_2^0 3y + z\,dz = \int_2^0 z\,dz = \frac{z^2}{2}\Big]_2^0 = -2$, $\frac{8}{3} + 2 - 2$

$\boxed{= \frac{8}{3}}$

Stoke's thrm

$\int (\nabla \times \vec{v})\cdot d\vec{a}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6 & yz^2 & 3y+z \end{vmatrix} = \hat{x}\left(\frac{\partial}{\partial y}(3y+z) - \frac{\partial}{\partial z}(yz^2)\right) - \hat{y}\left(\frac{\partial}{\partial x}(3y+z) - \frac{\partial}{\partial z}(6)\right) + \hat{z}\left(\frac{\partial}{\partial x}(yz^2) - \frac{\partial}{\partial y}(6)\right)$$

$= \hat{x}(3-2yz)$

$\downarrow$

$(\nabla \times \vec{v})\cdot da$ $(3-2yz)(dy\,dz)$

$$\int_0^1 \int_0^{-2y+2} (3-2yz)\,dy\,dz$$

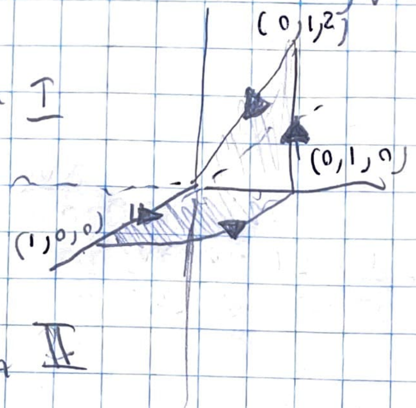
$$= \int_0^1 (3z - yz^2)\Big|_0^{-2y+2} = \int_0^1 3(-2y+2) - y(-2y+2)^2\,dy$$

$$\int_0^1 [-6y + 6 - 4y^3 + 8y^2 - 4y]\,dy = [y^4 + \frac{8}{3}y^3 - 5y^2 + 6y$$

$= 8/3$

1.57) $v = (r\cos^2\theta)\,\hat{r} - r\cos\theta\sin\theta\,\hat{\theta} + 3r\,\hat{\phi}$

Path I



$\theta = \frac{\pi}{2}$ $\phi = 0$ $r = 0 \to 1$

$\vec{v} \cdot d\vec{l} = r\cos^2\theta\, dr = 0$

So path 1 = 0

Path II

$r = 1$ $\phi = 0 \to \frac{\pi}{2}$ $\theta = \frac{\pi}{2}$ $\vec{v} \cdot d\vec{l} = (3r)(r\sin\theta)\,d\phi = 3r^2\sin\theta\, d\phi$

$= 3\int_0^{\frac{\pi}{2}} d\phi = \frac{3\pi}{2}$ Path $\boxed{II = \frac{3\pi}{2}}$

Path III $r = 1 \to \sqrt{5}$ $\theta = \frac{\pi}{2} \to \tan^{-1}(\frac{1}{2})$

$y = r\sin\theta$; $y = 1$, $r = \frac{1}{\sin\theta}$; $dr = \frac{-\cos\theta}{\sin^2\theta}\, d\theta$

$-\frac{\cos^2\theta}{\sin\theta}\left(\frac{\cos\theta}{\sin^2\theta}\right) d\theta - \frac{\cos\theta\sin\theta}{\sin^2\theta}\, d\theta = -\frac{\cos^3\theta}{\sin^3\theta}\, d\theta - \frac{\cos\theta}{\sin\theta}\, d\theta$

$-\left(\frac{\cos^3\theta + \cos\theta\sin^2\theta}{\sin^3\theta}\right) d\theta = \frac{-\cos\theta}{\sin^3\theta}\, d\theta$

$\int_{\frac{\pi}{2}}^{\tan^{-1}(\frac{1}{2})} \frac{\cos\theta}{\sin^3\theta}\, d\theta$ { I had to look on photomath for answer } $\boxed{III = 2}$

Path IV $\phi = \frac{\pi}{2}$, $r = \sqrt{5}$, $\theta = \tan^{-1}(\frac{1}{2})$

$r\cos^2\theta\, dr = \frac{4}{5} r\, dr$

$\frac{4}{5}\int_{\sqrt{5}}^{0} r\, dr = -\frac{2}{5} r^2 \Big]_0^{\sqrt{5}} = -2$; $0 + 2 - 2 + \boxed{\frac{3\pi}{2}} = IV$

Cont on next pg.

1.57 Cont.

$$\int (\vec{\nabla} \cdot \vec{v}) \cdot d\vec{a} = \oint \vec{v} \cdot d\vec{l}$$

$$\nabla \times V = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r} r v_\phi\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(r v_\theta) \frac{\partial v_r}{\partial\theta}\right]\hat{\phi}$$

$$= \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(3r\sin\theta) + \frac{\partial}{\partial\phi}(r\cos\theta\sin\theta)\right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}(r\cos^2\theta) - \frac{\partial}{\partial r}(3r^2)\right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(r^2\cos\theta\sin\theta) - \frac{\partial}{\partial\theta}(r\cos^2\theta)\right]\hat{\phi}$$

$$= \frac{1}{r\sin\theta}[3r\cos\theta]\hat{r} + \frac{1}{r}[-6r]\hat{\theta} + \frac{1}{r}[2r\cos\theta\sin\theta + 2r\cos\theta\sin\theta]\hat{\phi}$$

$$\vec{\nabla} \times \vec{v} = 3\cot\theta\,\hat{r} - 6\hat{\theta}$$

$$dI_1 = -r\,dr\,d\theta\,\hat{\phi} \qquad dI_I = -r\sin\theta\,dr\,d\phi\,\hat{\theta},\ \theta = \frac{\pi}{2}$$

$$(\vec{\nabla} \times \vec{v}) \cdot dI = 0 \qquad = 6\,r\,dr\,d\phi$$

$$\int_0^1 \int_0^{\frac{\pi}{2}} 6\,r\,dr\,d\phi = 3\pi \int_0^1 r\,dr = \frac{3\pi}{2} r^2 \Big|_0^1 = \boxed{\frac{3\pi}{2}}$$

Verified result! ✓

$$1.59)\ \vec{v} = r^2\sin\theta\,\hat{r} + 4r^2\cos\theta\,\hat{\theta} + r^2\tan\theta\,\hat{\phi}$$

$\rightarrow$ Checking div thrm

$$\int_V (\nabla\cdot\vec{v})\,d\tau = \oint \vec{v}\cdot d\vec{a}$$

$$\vec{\nabla}\cdot\vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi} v_\phi$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}(r^4\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(4r^2\cos\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(r^2\tan\theta)$$

$$= 4r\sin\theta + \frac{4r(-\sin^2\theta + \cos^2\theta)}{\sin\theta} = 4r\sin\theta - 4r\sin\theta + \frac{4r\cos^2\theta}{\sin\theta}$$

$$= 4r\frac{\cos^2\theta}{\sin\theta}$$

$$\int_V\left(4r\frac{\cos^2\theta}{\sin\theta}\right) r^2\sin\theta\,dr\,d\theta\,d\phi = 4\int_V r^3\cos^2\theta\,dr\,d\theta\,d\phi$$

$$= \int_0^R\int_0^{\pi/6}\int_0^{2\pi} r^3\cos^2\theta\,dr\,d\theta\,d\phi = R^4\int_0^{\pi/6}\int_0^{2\pi}\cos^2\theta\,d\theta\,d\phi = 2\pi R^4\int_0^{\pi/6}\cos^2\theta\,d\theta$$

$$= 2\pi R^4\left(\frac{1}{24}\,3\sqrt{3} + 2\pi\right)$$

$$= \boxed{\frac{\pi R^4}{12}(2\pi + 3\sqrt{3})}$$

$$\oint_S \vec{v}\cdot d\vec{a} \qquad \theta \quad \phi: 0\to 2\pi \quad r: 0\to R \qquad d\vec{a} = r\sin\theta\,d\phi\,dr\,\hat{\theta} = \frac{\sqrt{3}}{2}r\,d\phi\,dr\,\hat{\theta}$$

$$\vec{v}\cdot d\vec{a} = (4r\cos\theta)\left(\frac{\sqrt{3}}{2}r\,d\phi\,dr\right) = 2\sqrt{3}r^3\cos\theta\,d\phi\,dr = \sqrt{3}r^3\,d\phi\,dr$$

$$\sqrt{3}\int_S r^3\,d\phi\,dr = \sqrt{3}\int_0^R\int_0^{2\pi} r^3\,d\phi\,dr = \frac{\sqrt{3}}{2}\pi R^4$$

$$r = R \quad \phi: 0\to 2\pi,\ \theta = 0\to\frac{\pi}{6} \qquad d\vec{a} = (r^2\sin\theta)(r^2\sin\theta\,d\theta\,d\phi) = r^4\sin^2\theta\,d\theta\,d\phi = R^4\sin^2\theta\,d\theta\,d\phi$$

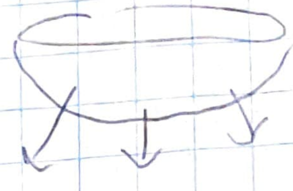
$$R^4\int_0^{\pi/6}\int_0^{2\pi}\sin^2\theta\,d\theta\,d\phi = 2\pi R^4\int_0^{\pi/6}\sin^2\theta\,d\theta = \pi R^4\left(\frac{\sqrt{3}}{2} + \frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= \frac{\pi R^4}{12}(2\pi + 3\sqrt{3}) \checkmark$$

verified! ✓

1.62 $a = \int_S da$ vector area of bowl with rad. "R"

$d\vec{a} = R^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$



in $\hat{r}$

a) $R^2 \int_0^{\frac{\pi}{2}} \int^{\frac{\pi}{2}} \sin\theta \, d\theta \, d\phi = 2\pi R^2 \int_0^{\pi/2} \sin\theta \cos\theta \, d\theta$

$= 2\pi R^2 \left(\frac{\sin^2\theta}{2} \right) \Big|_0^{\frac{\pi}{2}} \Rightarrow = \boxed{\pi R^2}$

b) the vector area in any closed surface is zero as when you assign vector area to the boundary of a surface, but since there is no boundary, the vector area is subsequently zero

c) again, if you assign vector area to the boundary of a surface, all surfaces sharing that boundary will share that same vector area

d) $a = \frac{1}{2} \oint r \times dl$, for one of the triangles, $a = \frac{1}{2}(\vec{r} \times d\vec{l})$, $d\vec{a}$ would add to total area $\vec{a}$

e) $\oint (c \cdot r) \, dl = a \times c$ for any constant $\vec{v}$ "C"

$T = \vec{c} \cdot \vec{r}$, $\nabla T = \nabla(\vec{c} \cdot \vec{r})$

$\nabla \times r = 0$, $\vec{r}$ has/causes no curl, $\nabla T = \nabla(c \cdot v) = (C \cdot \nabla)$

$= (c_x \frac{\partial}{\partial x} + c_y \frac{\partial}{\partial y} + c_z \frac{\partial}{\partial z})(x\hat{x} + y\hat{y} + z\hat{z}) = c_x\hat{x} + c_y\hat{y} + c_z\hat{z}$,

$= C$ as this is constant vector, $\nabla T = \vec{C}$

so $-\int \nabla T \times d\vec{a} = -\int \vec{c} \times \vec{a} = \boxed{\vec{a} \times \vec{c}}$

1.63

$\text{div}\left(\frac{\hat{r}}{r}\right)$

$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{r^2}{r}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(r) = \frac{1}{r^2}$

div. thrm $\int_V (\nabla \cdot \vec{v})\, d\tau = \int_S \vec{v} \cdot d\vec{a}$

Using sphere w rad "R"

$\int_V (\nabla \cdot \vec{v})\, d\tau = \int_V \left(\frac{1}{r^2}\right)(r^2 \sin\theta\, dr\, d\theta\, d\phi) = \int_V \sin\theta\, dr\, d\theta\, d\phi$

$= \int_0^R \int_0^\pi \int_0^{2\pi} \sin\theta\, dr\, d\theta\, d\phi = 2\pi R \int_0^\pi \sin\theta\, d\theta = 2\pi R[-\cos\theta]\Big|_0^\pi = 2\pi R[1+1] = 4\pi R$

$\int_S \vec{v} \cdot d\vec{a} = \int_S \left(\frac{1}{r}\right)(r^2 \sin\theta\, d\theta\, d\phi) = R\int \sin\theta\, d\theta\, d\phi = R\int_0^\pi \int_0^{2\pi} \sin\theta\, d\theta\, d\phi$

again, $= \boxed{4\pi R}$

No δ function needed

$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 r^n) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^{n+2}) = \frac{1}{r^2}(n+2)r^{n+1} = \frac{r^{n+1}}{r^2}(n+2)$

$\frac{r^n}{r}(n+2) = r^{n-1}(n+2) = (n+2), n=1 \Rightarrow \nabla \cdot r^n \hat{r} = (n+2)r^{n-1}$

$\nabla \times r^n \hat{r} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\, v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right] = 0$

$\boxed{\nabla \times r^n \hat{r} = 0}$

$\boxed{1.64}$

let $-\frac{1}{4\pi}\nabla^2 \frac{1}{\sqrt{r^2+\epsilon^2}}$ $\quad r \to \sqrt{r^2+\epsilon^2}$, $\epsilon \to 0$

$D(r,\epsilon) = -\frac{1}{4\pi}\nabla^2 \frac{1}{\sqrt{r^2+\epsilon^2}}$, as $\epsilon \to 0$, $\delta^3(\vec{r})$

a) show that $D(r,\epsilon) = \left(\frac{3\epsilon^2}{4\pi}\right)(r^2+\epsilon^2)^{-\frac{5}{2}}$

$$\to -\frac{1}{4\pi}\nabla^2\left((r^2+\epsilon^2)^{-\frac{1}{2}}\right) = -\frac{1}{4\pi}\frac{1}{r^2}\frac{\partial}{\partial r}\left[-r^3(r^2+\epsilon^2)^{-\frac{3}{2}}\right]$$

$$= \frac{1}{4\pi}\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^3(r^2+\epsilon^2)^{-\frac{3}{2}}\right) = \frac{1}{4\pi r^2}\left[3r^2(r^2+\epsilon^2)^{-\frac{3}{2}} - 3(r^2+\epsilon^2)r^4\right]$$

$$= \frac{3}{4\pi}\left[\frac{1}{(r^2+\epsilon^2)^{\frac{3}{2}}} - \frac{r^2}{(r^2+\epsilon^2)^{5/2}}\right] = \frac{3\epsilon^2}{4\pi(r^2+\epsilon^2)^{\frac{5}{2}}}, \quad r \to 0 \quad \frac{3}{4\pi\epsilon^3},$$

as can be seen, when ϵ approaches zero, this goes to inf.

when ϵ approaches zero,

$r \to 0$

$$\to \int_0^\infty \frac{3\epsilon\, 4\pi r^2}{4\pi(r^2+\epsilon^2)^{\frac{5}{2}}}\,dr = \frac{3\epsilon^2}{4\pi}\int_0^\infty \frac{4\pi r^2}{(r^2+\epsilon^2)^{\frac{5}{2}}} = 3\epsilon^2\int_0^\infty \frac{r^2}{(r^2+\epsilon^2)^{\frac{5}{2}}}\,dr$$

$$3\epsilon^2\left(\frac{1}{3\epsilon^2}\right) = 1$$

$\delta^3(\vec{r})$ has all same properties that this does ☺

these all show [illegible]