1.54)

$$\bar{V} = r^2\cos\theta\,\hat{r} + r^2\cos\phi\,\hat{\theta} - r^2\cos\theta\sin\phi\,\hat{\phi}$$

$$\oint\!\!\oint \bar{F}\cdot\overline{ds} = \iiint \mathrm{div}(\bar{F})\,dv$$

$$\nabla\cdot\bar{V} = \frac{\partial}{\partial r}(r^2\cos\theta) + \frac{\partial}{\partial\theta}(r^2\cos\phi) - \frac{\partial}{\partial\phi}(r^2\cos\theta\sin\phi)$$

$$= 2r\cos\theta + r^2\cos\phi - r^2\cos\theta\cos\phi$$

$$\underline{= r(2\cos\theta + r\cos\phi - r\cos\theta\cos\phi)}$$

$$\hat{r} = \frac{1}{r^2}$$

$$\hat{\theta} = \frac{1}{r\sin\theta}$$

$$\phi = \frac{1}{r\sin\theta}$$

$$V = r^2\cos\theta\,\hat{r} + \overset{\sin\theta}{\downarrow} r^2\cos\phi\,\hat{\theta} - r^2\cos\sin\phi\,\hat{\phi}$$

1.54

$$\nabla\cdot\bar{V} = \frac{1}{r^2}\frac{\partial}{\partial r}(V_1) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(V_2) - \frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}(V_3)$$

= [illegible]

$$= \frac{1}{\cancel{r^2}}(4r^{3}\cos\theta) + \frac{1}{\cancel{r}\sin\theta}(r^{\cancel{2}}\cos\theta\cos\phi) - \frac{1}{\cancel{r}\sin\theta}(r^{\cancel{2}}\cos\theta\cos\phi)$$

$$= 4r\underline{\cos\theta} + \frac{1}{\sin\theta}(r\underline{\cos\theta}\cos\phi) - \frac{1}{\sin\theta}(r\underline{\cos\theta}\cos\phi)$$

$$= \frac{r\cos\theta}{\cancel{\sin\theta}}\left(4\cancel{\sin\theta} + \cancel{\cos\phi - \cos\phi}\right)$$

$$\boxed{= 4r\cos\theta}$$

$\int \nabla\cdot\bar{v}\, d\bar{\tau}$ $[0, R]$

$= \iiint (4r\cos\theta) r^2 \sin\theta\, dr\, d\theta\, d\phi$ $[0, \pi/2]$

$= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^R (4r^3\cos\theta \sin\theta)\, dr\, d\theta\, d\phi$

$= \frac{\pi}{2}\int_0^{\pi/2}\int_0^R (4r^3\cos\theta \sin\theta\, dr\, d\theta$

~~$= \frac{\pi}{2}($~~

$= \frac{4r^3\pi}{2}\int_0^{\pi/2}\int_0^R \cos\theta \sin\theta\, dr\, d\theta$

$u = \sin\theta\, d\theta$

$du = \cos\theta\, d\theta$

$d\theta = \frac{du}{\cos\theta}$

$= 2r^3\pi\int_0^{\pi/2}\int_0^R u\cancel{\cos\theta}\,\frac{dr\,du}{\cancel{\cos\theta}}$

$u = \sin 0$

$u = 0$

$u = \sin(\pi/2)$

$u_2 = 0$

$= \frac{2r^3\pi}{}\int_0^R \frac{u^2}{2}\Big|_0^{\cancel{\pi/2}\;0} dr$

$= 2r^3\pi(0-0)\int_0^R dr$

~~$= 2r^3\pi\left(\frac{\pi}{4}\right)^2\left(\frac{1}{2}\right)$~~

~~$= \frac{2r^3\pi^3}{8} = \frac{r^3\pi^3}{4}\int_0^R dr$~~

~~$\frac{\pi^3}{4}\int_0^R r^3\, dr$~~

~~$= \frac{r^4}{4}\Big|_0^R$~~

$$=2r^3\pi\int_0^R dr$$

$$=2\pi\int_0^R r^3\,dr$$

$$=2\pi\frac{R^4}{4}$$

$$\boxed{=\frac{\pi R^4}{2}}$$

1.55 $V = ay\hat{x} + bx\hat{y}$ $[0, R]$

$$\iint_s (\bar{\nabla} \times \bar{A}) \cdot \bar{ds}$$
$$= \oint \bar{A} \cdot \bar{dr}$$

$$\nabla \times V = \begin{vmatrix} \hat{x} & \hat{y} & z \\ ay & bx & 0 \\ \partial_x & \partial_y & \partial_z \end{vmatrix}$$

$$(0-0)\hat{x}\,(0-0\hat{y}),\,(b-a)\hat{z}$$

$$= (b-a)\hat{z}$$

$$= \oint V \cdot \bar{dl}$$

$$= \oint ay\,dx + bx\,dy$$

$$x^2 + y^2 = R^2$$

$$y^2 = R^2 - x^2 \qquad 2x\,dx + 2y\,dy = 0$$
$$y = \sqrt{R^2 - x^2} \qquad 2x\,dx = -2y\,dy$$
$$\boxed{dy = -\left(\frac{x}{y}\right)}$$

$$= \int ay\,dx + bx\left(-\frac{x}{y}\right)dx$$

$$= \int \frac{1}{y}\left(ay^2 - bx^2\right)dx$$

$$= \int\int_0^{2\pi} \frac{a(R^2 - x^2) - bx^2}{\sqrt{R^2 - x^2}}\,dx$$

$$= \int \frac{a(R^2 - x^2)}{\sqrt{R^2 - x^2}} - \int \frac{bx^2}{\sqrt{R^2 - x^2}}\,dx$$

$$= a\int_0^R \frac{R^2 - x^2}{\sqrt{R^2 - x^2}}\,dx - b\int \frac{x^2}{\sqrt{R^2 - x^2}}\,dx$$

$$x = R\sin(t)$$
$$dx = R\cos(t)\,dt$$

$$= a\int_0^R \frac{R^2 - x^2}{\sqrt{R^2 - x^2}}dx - b\int \frac{x^2}{\sqrt{R^2 - x^2}} dx$$

$x = R\sin(t)$

$dx = R\cos(t)dt$

$uv - \int vdu$

e^{t^2}

$u = t^2$

$du = 2tdt$

$$= a\int_0^R \frac{R^2 - R^2\sin^2(t)}{\sqrt{R^2 - R^2\sin^2(t)}}(R\cos(t)dt)$$

$$= a\int_0^R \frac{R^2(1 - \sin^2(t))}{\cancel{R}\sqrt{1 - \sin^2(t)}}(R\cos(t)dt)$$

$$= a\int_0^R \frac{R\cos^2(t)}{\cancel{\cos(t)}} R\cancel{\cos(t)}dt$$

$$= aR^2\int_0^R \cos^2(t)dt$$

$$= aR^2\left[-2\cos(t)\sin(t)\right]_0^1$$

$$= -2aR^2\left[\overset{0}{\cos(1)}\overset{\pi/2}{\sin(1)} - \overset{1}{\cos(0)}\overset{0}{\sin(0)}\right]$$

$$= \int a(R^2 - x^2)^{1/2} - \frac{bx^2}{(R^2 - x^2)^{1/2}} dx$$

$x = r\cos\theta$

$y = r\sin\theta$

1.56) $\oint v \cdot dl = \iint (\nabla \times v) \cdot ds$

$\bar{V} = 6\hat{x} + yz^2\hat{y} + (3y+z)\hat{z}$

$\nabla \times \bar{V} = \begin{vmatrix} \partial_x & \partial_y & \partial_z \\ 6 & yz^2 & (3y+z) \end{vmatrix}$

$ds = dy\,dz\,\hat{x}$

$= (3 - 2yz)\hat{x} + 0 + 0$

$= \iint (3 - 2yz)\,dy\,dz$

$z = 2 - 2y$

$= \int_0^1 \int_0^{2-2y} (3 - 2yz)\,dy\,dz$

$z: [0, 2-2y]$

$y: [0, 1]$

$= \int_0^1 3z - yz^2 \Big|_{z=0}^{2-2y} dy$

$(2-2y)(2-2y)$

$4 - 4y + 4y^2$

$-4y$

$= \int_0^1 3(2-2y) - y(2-2y)^2\,dy$

$= 6 - 6y - y(4y^2 - 8y + 4)$

$= 6 - 6y - (4y^3 - 8y^2 + 4y) = 6 - 6y - 4y^3 + 8y^2 - 4y$

$= -(4y^3 - 8y^2 + 10y - 6)$ $\quad -4y^3 + 8y^2 - 10y + 6$

$= -\int_0^1 (4y^3 - 8y^2 + 10y - 6)\,dy$

$= -\left(y^4 - \frac{8}{3}y^3 + 5y^2 - 6y\right)\Big|_0^1$

$= -\left(\cancel{1} - \frac{8}{3} \cancel{+5} \cancel{-6}\right)$

$= -\left(-\frac{8}{3}\right)$

$= \boxed{8/3}$

1.57) $\int_c \vec{F} \cdot dr = \iint_s \nabla \times V \, ds$

$V = (r\cos^2\theta)\hat{r} - (r\cos\theta \sin\theta)\hat{\theta} + 3r\hat{\phi}$

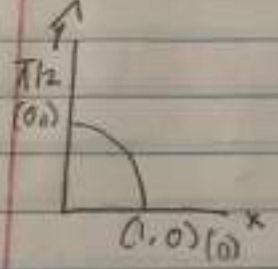
$\nabla \times \vec{v} = \quad \rho_r \qquad \rho_\theta \qquad \rho_\phi$

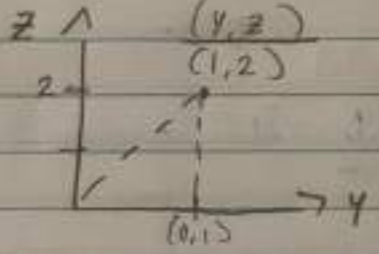
$(r\cos^2\theta) \qquad \cdot r\cos\theta \text{Sin}\theta \qquad 3r$

$\left(0-0,\ 3-0,\ \left(-\cos\theta \text{Sin}\theta - 2r\cos(\theta)\text{Sin}(\theta)\right)\right.$

$\left.-2r\left(\frac{\cos\theta \text{Sin}\theta}{2r} + \cos\theta \text{Sin}\theta\right)\right)$

$= 3\hat{\theta} - 2r\left(\frac{\cos\theta \text{Sin}\theta}{2r} + \cos\theta \text{Sin}\theta\right)\hat{\phi}$





need r, θ, z

$= \int$

1.5a)

I have no idea what
I am doing

$$\bar{V} = r^2 \sin\theta \,\hat{r} + 4r^2\cos\theta\,\hat{\theta} + r^2\tan\theta\,\hat{\phi}$$

$$\nabla\cdot\bar{V} = \frac{1}{r}(4r^3\sin\theta) + \frac{1}{r\sin\theta}(-4r^2\sin^2\theta)\hat{\theta} + \frac{1}{\sin\theta}(r^2\sin\theta\tan\theta)\hat{\phi}$$

$$\nabla\cdot\bar{V} = 2r\sin\theta - 4r^2\sin\theta + 0$$

$$= 2r(\sin\theta - 4r\sin\theta)$$

$$\frac{\sqrt{3}}{4} \cdot \frac{1}{2}$$

$$= \int_0^R\int_0^{\pi/6}\int_0^{2\pi}(2r\sin\theta - 4r^2\sin\theta)(r^2\sin\theta)\,dr\,d\theta\,d\phi$$

$$\frac{\pi}{6} \quad \frac{1}{2}$$

$$= \iint (2r^3\sin^2\theta(\phi) - 4r^4\sin^2\theta(\phi))\Big|_0^{2\pi} dr\,d\theta$$

$$= \iint_0^{\pi/6}(4\pi r^3\sin^2\theta - 8\pi r^4\sin^2\theta\,dr\,d\theta$$

$$= \int 4\pi r^3\left(\frac{\theta}{2} - \frac{\cos(\theta)\sin(\theta)}{2}\right) 8\pi r^4\left(\frac{\theta}{2} - \frac{\cos(\theta)\sin(\theta)}{2}\right)\Big|_0^{\pi/6}$$

$$= \int 4\pi r^3\left(\frac{\pi}{12} - \frac{\sqrt{3}/2(\frac{1}{2})}{2}\right) 8\pi r^4\left(\frac{\pi}{12} - \frac{\sqrt{3}/2(\frac{1}{2})}{2}\right)$$

$$= \int 4\pi r^3\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right.$$

$$\left(\frac{2\pi}{24} - \frac{3\sqrt{3}}{24}\right)$$

$$1$$

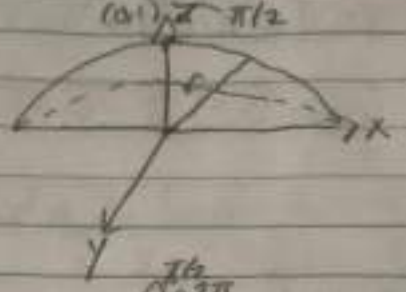
$$= \int \cancel{4}\pi r^3\left(\frac{2\pi - 3\sqrt{3}}{\cancel{24}\,6}\right) - \cancel{8}\pi r^4\left(\frac{2\pi - 3\sqrt{3}}{\cancel{24}\,3}\right)$$

$$= \int_0^R \frac{2\pi - 3\sqrt{3}}{6}(\pi)\,r^3 - \frac{2\pi - 3\sqrt{3}}{3}(\pi)\,r^4\,dr$$

$$\boxed{= \frac{2\pi - 3\sqrt{3}(\pi)}{6}\left(\frac{R^4}{4}\right) - \frac{2\pi - 3\sqrt{3}(\pi)}{3}\frac{R^5}{5}}$$

1.62) $a = \int_s da$ Sphere so $\theta : [0, \pi/2]$

a)



$\overline{da} = r^2 \sin\theta \, d\theta \, d\phi$

$\hat{r} = \cos\theta \hat{z}$

$$a = \int_0^{\pi/2}\int_0^{2\pi} \cos\theta \sin\theta r^2 \, d\theta \, d\phi$$

$$a = 2\pi r^2 \int_0^{\pi/2} \cos\theta \sin\theta \, d\theta$$

$$a = 2\pi r^2 \int_0^{\pi/2} \cancel{\cos\theta} \, u \left(\frac{du}{\cancel{\cos\theta}}\right)$$

$u = \sin\theta \, d\theta$

$du = \cos\theta \, d\theta$

$d\theta = \frac{du}{\cos\theta}$

$$a = 2\pi r^2 \int_0^{\pi/2} u \, du$$

$$a = 2\pi r^2 \frac{u^2}{2} \Big|_0^{\pi/2}$$

$$a = \pi r^2 \sin^2\theta \Big|_0^{\pi/2}$$

$$a = \pi r^2 1^2$$

$$\boxed{a = \pi r^2 \hat{z}}$$

b)

$a = \int_S da$

$\int \nabla T = \oint \bar{T} \cdot \overline{da}$

$T = 1$

$\nabla T = 0$

$\int 0 = \int \overline{da}$

$\boxed{\int 0 = \int da}$

c) $a_1 = a_2$

$\boxed{a_1 - a_2 = 0}$

d) $a = \frac{1}{2} \oint r \times dl$

e) $\oint (c \cdot r) \, dl = a \times c$

$T = \bar{c} \cdot \bar{r}$

$\nabla T = \nabla (\bar{c} \cdot \bar{r})$

1.63) $V = \frac{\hat{r}}{r}$ $\quad r = \sqrt{x^2+y^2+z^2}$

$\hat{r} = \frac{\bar{r}}{r}$ $\quad dl = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$$\nabla V = P_x\hat{x} + P_y\hat{y} + P_z\hat{z}$$

$$P_x = (x^2+y^2+z^2)^{1/2}$$

$$\frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2x)$$

$$P_x = \frac{x}{(x^2+y^2+z^2)}$$

$$\nabla V = \frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}$$

$$\nabla \cdot V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{1}{r^2}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(1) = 0$$

$$\oint V\,d\bar{a} = \int\left(\frac{1}{R^2}\hat{r}\right)\left(R^2\sin\theta\,d\theta\,d\phi\,\hat{r}\right)$$

$$= \left(\int_0^{\pi}\sin\theta\,d\theta\right)\left(\int_0^{2\pi}d\phi\right) = 4\pi$$

$$\nabla x(r^n\hat{r}) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 r^n) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^{n+2})$$

$$= \frac{1}{r^2}(n+2)r^{n-1} = (n+2)r^{n-1}$$

$$n=-2 \Rightarrow \nabla\cdot v = 4\pi\,\delta^3(r))$$

$$\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = 4\pi\,\delta^3(r)$$

b)

$$\nabla \times (r^n \hat{r}) = 0$$

$$\int (\nabla \times v)\, d\tau = 0 = -\oint v \times da$$

$$V = r^n \hat{r}$$

$$\hat{r} \times \hat{r} = 0$$

$$da = R^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$$

$$V \times da = (r^n \hat{r}) \times (R^2 \sin\theta\, d\theta\, d\phi\, \hat{r})$$

$$\boxed{V \times da = 0}$$