Warm-Up for April 13th, 2022

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1 Memory Bank

1. Ampère's Law with **B**-fields, **A**-fields.

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{1}$$

$$\nabla^2 \mathbf{A} = -\mu_0 \mathbf{J} \tag{2}$$

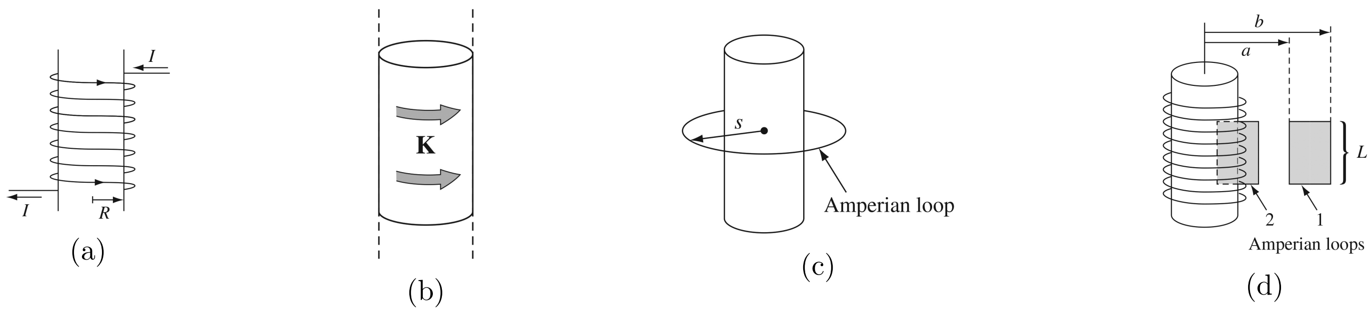

Figure 1: (a) A current wound around a solid cylinder is called a solenoid. (b) This leads to a surface current **K**. (c) An Amperian loop aids in finding the **B**-field. (d) Loops inside and outside determine where $\mathbf{B} \neq 0$.

2 B-fields and A-fields with Solenoids

1. Consider Fig. 1. A cylindrical current is called a *solenoid*. (a) Using geometric arguments, prove that the field inside the solenoid is strictly in the $\hat{\mathbf{z}}$ direction (parallel to the cylinder). Use Fig. 1 (c) to show that $B_\phi = 0$. (b) The surface current density is $\mathbf{K} = nI\hat{\boldsymbol{\phi}}$, where $n = N/L$, the turns per unit length. Use the integral form of Ampère's Law to show that $\mathbf{B} = \mu_0 n I \hat{\mathbf{z}}$ inside the cylinder.

2. The definition of the *vector potential* is $\mathbf{B} = \nabla \times \mathbf{A}$, because $\nabla \cdot \mathbf{B} = 0$. (a) Perform a surface integral on both sides of the definition of **A** and use the curl-theorem to show that $\oint \mathbf{A} \cdot d\mathbf{l} = \int \mathbf{B} \cdot d\mathbf{a} = \Phi_B$. Obtain **A** for the solenoid inside and out, using **B**. [1]

[1] You may assume **A** is parallel to the current. See Eq. 5.65 in the text.