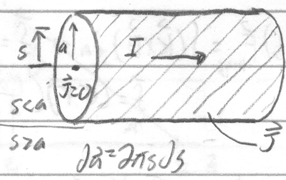
Jesse Chf-5v-6.
11/29/20 PHYS

HW5 S.14, 16, 17, 19, 20, 23, 26

S.14.



(a) $\vec{B}_{in}$ & $\vec{B}_{out}$ w/ I uniformly distributed

$\oint \vec{B}\cdot d\vec{l} = \mu_0 I \Rightarrow B\oint dl = \mu_0 I \Rightarrow B(2\pi s) = \mu_0 I \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$

For s<a, since $\vec{J}$ is outside surface, $\vec{J}_{in}=0$. So, $\boxed{\vec{B}_{in}=0}$

For s>a, $\vec{J}$ exists. So, $\boxed{\vec{B}_{out} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$

(b) $J = ks \Rightarrow I = \int J\, da \Rightarrow I = \int_0^a (ks)(2\pi s)\,ds \Rightarrow I = 2\pi k\int_0^a s^2\, ds \Rightarrow I = 2\pi k\left[\frac{1}{3}s^3\right]_0^a$,

$I = \frac{2}{3}\pi a^3 k \Rightarrow k = \frac{3I}{2\pi a^3}$

$J = \frac{3Is}{2\pi a^3}$

For s<a,

$\oint \vec{B}\cdot d\vec{a} = \mu_0 I_{enc}$, $I_{enc} = \int \vec{J}\cdot d\vec{a} \Rightarrow B(2\pi s) = \mu_0\int \frac{3Is'}{2\pi a^3}(2\pi s')\,ds' \Rightarrow B(2\pi s) = \frac{3\mu_0 I}{a^3}\int_0^s s'^2\, ds$,

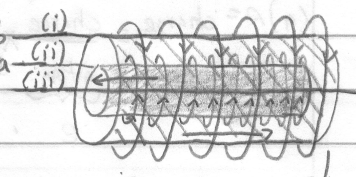
$B(2\pi s) = \frac{3\mu_0 I}{a^3}\left[\frac{1}{3}s'^3\right]_0^s \Rightarrow B(2\pi s) = \frac{\mu_0 I s^{3}}{a^3} \Rightarrow B = \frac{\mu_0 I s^2}{2\pi a^3} \Rightarrow \boxed{\vec{B}_{in} = \frac{\mu_0 I s^2}{2\pi a^3}\hat{\phi}}$

For s>a, $I_{enc} = I$. $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$,

$\boxed{\vec{B}_{out} = \frac{\mu_0 I}{2\pi s}\hat{\phi}}$

S.16

(Ex. 5.9)

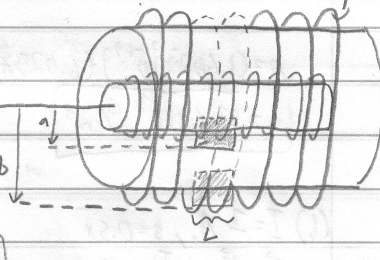


(iii) $\boxed{\vec{B}=0}$

(ii) $\boxed{\vec{B} = \mu_0 n_2 I\hat{z}}$

(i) $\oint \vec{B}\cdot d\vec{l} = (B(a)-B(b))L \Rightarrow B(a)L = \mu_0 I_{enc} = \mu_0 n_1 I L \Rightarrow B(a) = \mu_0 n_2 I$

$\Rightarrow B(b)L = \mu_0 I_{enc} = \mu_0 n_2 I L \Rightarrow B(b) = \mu_0 n_1 I$,



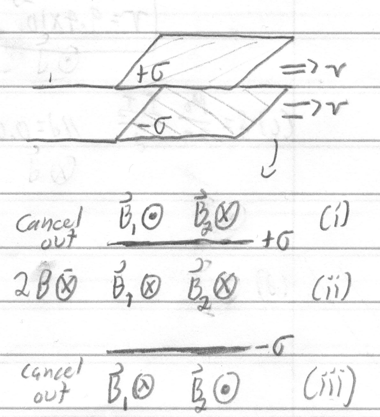
$\oint \vec{B}\cdot d\vec{l} \Rightarrow B = \mu_0 n_2 I - \mu_0 n_1 I \Rightarrow B = \mu_0 I(n_2 - n_1)$,

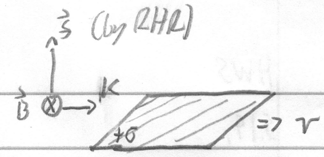
$\boxed{\vec{B} = \mu_0 I(n_2-n_1)\hat{z}}$

S.17 (Ex. 5.8)

(a) $\vec{B} = \frac{\mu_0}{2}K$ // $K = \sigma v \Rightarrow \vec{B} = \frac{\mu_0}{2}\sigma v$

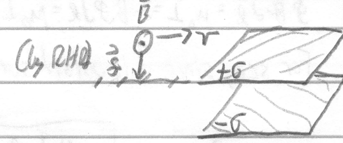
So, { i) $B=0$; ii) $B = \mu_0\sigma v$; iii) $B=0$ }





(b) $\vec{F}_{mag} = \int (\vec{K}\times\vec{B})\,da \Rightarrow \vec{f} = \vec{K}\times\vec{B}$ // $K=\sigma v,\ B=\frac{\mu_0}{2}\sigma v$

$\vec{f}_m = (\sigma v)(\frac{\mu_0}{2}\sigma v) \Rightarrow \boxed{\vec{f}_m = \frac{\mu_0}{2}\sigma^2 v^2 \text{ (up)}}$



(c) // $\vec{E}_{plate} = \frac{\sigma}{2\epsilon_0} \Rightarrow \vec{f}_e = (\sigma)(\frac{1}{2\epsilon_0}\sigma) \Rightarrow \underline{\vec{f}_e = \frac{1}{2\epsilon_0}\sigma^2}$ ↓

$f_m = f_e \Rightarrow \frac{\mu_0}{\cancel{2}}\cancel{\sigma^2} v^2 = \frac{1}{\cancel{2}\epsilon_0}\cancel{\sigma^2} \Rightarrow v^2 = \frac{1}{\epsilon_0\mu_0} \Rightarrow v = \sqrt{\frac{1}{\epsilon_0\mu_0}}$ // $\epsilon_0 = \frac{1}{\mu_0 c^2}$

$v = \sqrt{\frac{\mu_0}{\mu_0}c^2} \Rightarrow v = c \Rightarrow \boxed{v = 3\times10^8 \frac{m}{s}}$

S.19 $I_{enc} = \int_S \vec{J}\cdot d\vec{a}$. W/ ∞-many surfaces that share same boundary line, which to use?
// Recall: Ch.1 - Theorem 2, "Divergence-less fields",
where $\int \vec{J}\cdot d\vec{a}$ independent of surface, for any given boundary line.

$\boxed{\text{Since } \int \vec{J}\cdot d\vec{a} \text{ is independent of surface, one is free to use whichever they like.}}$

S.20

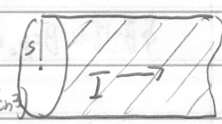
(a) $\rho = \frac{\text{charge}}{\text{volume}} = \underbrace{\frac{\text{charge}}{\text{atom}}}_{e} \times \underbrace{\frac{\text{atom}}{\text{mole}}}_{N_A} \times \underbrace{\frac{\text{mole}}{\text{gram}}}_{M_{copper}} \times \underbrace{\frac{\text{gram}}{\text{volume}}}_{d_{copper}} \Rightarrow \rho = eN_A\left(\frac{1}{M_{copper}}\right)d_{copper}$

// e - electron charge
// N_A - Avogadro's #
// M_{copper} - molar mass of Cu
// d_{copper} - density of Cu

$\rho = (1.602\times10^{-19})(6.023\times10^{23})(\frac{1}{63.55})(8.96)\left(C\cdot\frac{1}{\cancel{mol}}\cdot\frac{\cancel{mol}}{\cancel{g}}\cdot\frac{\cancel{g}}{cm^3}\right)$ ↓

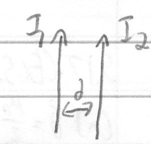
$\boxed{\rho = 13.6\times10^{3}\ C/cm^3}$

(b) $I = 1$ A., $s = 0.5$ x



$\vec{J} = \frac{dI}{da_\perp} = \frac{I}{\pi s^2},\ \vec{J} = \rho v \Rightarrow \frac{I}{\pi s^2} = \rho v \Rightarrow v = \frac{I}{\pi s^2 \rho} = \frac{1}{\pi(0.05\,cm)^2(13.6\times10^3\ C/cm^3)}$

$\boxed{v = 9.4\times10^{-3}\ cm/s}$



(c) $f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$ // $d = 0.01$ m. // $I_1 = I_2 = 1$ A. $\Rightarrow f = \frac{(4\pi\times10^{-7}\frac{N}{A^2})(1A^2)}{2\pi\ (0.01m)}$ ↓

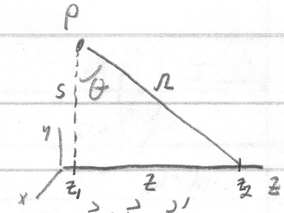
$\boxed{f = 2\times10^{-5}\ N/m}$

(d)

-Su-G.
PHYS

,19,

5.23 $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}}{\eta}\,dl' = \frac{\mu_0 I}{4\pi}\int \frac{1}{\eta}\,dl'\,\hat{z} = \frac{\mu_0 I \hat{z}}{4\pi}\int_{z_1}^{z_2} \frac{1}{\sqrt{z^2+s^2}}\,dz$



$\vec{\eta} = \vec{r} - \vec{r}'$

$\vec{\eta} = z\hat{z} - s\hat{s}$

$\eta = \sqrt{z^2+s^2}$

U-sub: $\tan\theta = \frac{z}{s} \Rightarrow z = s\tan\theta$, $\cos\theta = \frac{s}{\eta}$, $\sec\theta = \frac{\eta}{s}$

$dz = s\sec^2\theta\,d\theta$

$$\vec{A} = \frac{\mu_0 I \hat{z}}{4\pi}\int_{z_1}^{z_2} \frac{s\sec^2\theta}{\sqrt{s^2\tan^2\theta + s^2}}\,d\theta = \frac{\mu_0 I \hat{z}}{4\pi}\int_{z_1}^{z_2}\frac{s\sec^2\theta}{s(\sec\theta)}\,d\theta$$

$$\vec{A} = \frac{\mu_0 I \hat{z}}{4\pi}\int_{z_1}^{z_2}\frac{1}{\cos\theta}\,d\theta = \frac{\mu_0 I \hat{z}}{4\pi}\Big[\ln(\tan\theta + \sec\theta)\Big]_{z_1}^{z_2}$$

$$= (\dots)\left[\ln\left(\frac{z}{s} + \frac{\sqrt{z^2+s^2}}{s}\right)\right]_{z_1}^{z_2} \Rightarrow \frac{\mu_0 I \hat{z}}{4\pi}\left[\ln\left(z + \sqrt{z^2+s^2}\right)\right]_{z_1}^{z_2}$$

$$\boxed{\vec{A} = \frac{\mu_0 I}{4\pi}\left[\ln\left(\frac{z_2 + \sqrt{z_2^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right)\right]\hat{z}}$$

$$\vec{B} = \nabla\times\vec{A} = -\frac{\partial A_z}{\partial s}\hat{\phi} = \frac{-\mu_0 I}{4\pi}\frac{\partial}{\partial s}\left(\ln\left(z_2+\sqrt{z_2^2+s^2}\right) - \ln\left(z_1 + \sqrt{z_1^2+s^2}\right)\right)\hat{\phi}$$

$$= (\dots)(s)\left(\frac{1}{z_2+\sqrt{z_2^2+s^2}}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{z_2^2+s^2}}\cancel{2}s - \frac{1}{z_1+\sqrt{z_1^2+s^2}}\cdot\frac{1}{\cancel{2}}\cdot\frac{1}{\sqrt{z_1^2+s^2}}\cancel{2}s\right)\hat{\phi}$$

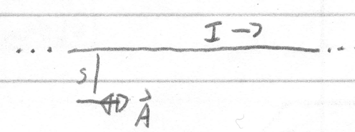
$$= \frac{-\mu_0 I s}{4\pi}\left(\frac{z_2 - \sqrt{z_2^2+s^2}}{z_2 - z_2^2 - s^2}\cdot\frac{1}{\sqrt{z_2^2+s^2}} - \frac{z_1 - \sqrt{z_1^2+s^2}}{z_1 - z_1^2 - s^2}\cdot\frac{1}{\sqrt{z_1^2+s^2}}\right)\left(\frac{-1}{s^2}\right)\hat{\phi}$$

$$= \frac{\mu_0 I}{4\pi s}\left(\frac{z_2}{\sqrt{z_2^2+s^2}} - \frac{z_1}{\sqrt{z_1^2+s^2}}\right)\hat{\phi}$$

// $\sin\theta_1 = \frac{z_1}{\sqrt{z_1^2+s^2}}$, $\sin\theta_2 = \frac{z_2}{\sqrt{z_2^2+s^2}}$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right)}$$

// 5.26



(a)