Cole Goodman

ET HW #2

2.5)
$$dE = \frac{k\,dq}{R^2}\cos\theta$$

$$R^2 = r^2 + z^2$$

$$dE = \frac{k\,dq}{r^2+z^2}\cos\theta$$

$$\cos\theta = \frac{z}{\sqrt{r^2+z^2}}$$

$$dE = \frac{k\,dq}{r^2+z^2}\left(\frac{z}{\sqrt{r^2+z^2}}\right)$$

$$\int dE = \int \frac{k\,dq}{r^2+z^2}\left(\frac{z}{\sqrt{r^2+z^2}}\right)$$

$$E = \int_0^{2\pi r} \frac{\lambda z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}\,ds$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{z}{(r^2+z^2)^{3/2}}(2\pi r)$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\frac{zr}{(r^2+z^2)^{3/2}} = \boxed{\frac{\lambda r z\hat{z}}{2\varepsilon_0 (r^2+z^2)^{3/2}}}$$

2.6)
$$dE = \frac{k\,dq\,z}{(z^2+x^2)^{3/2}} \qquad \sigma = \frac{dq}{(2\pi x)\,dx}$$

$$dq = \sigma(2\pi x)\,dx$$

$$E = \int_0^R \frac{k\sigma(2\pi x)(dx)z}{(z^2+x^2)^{3/2}}$$

$$= \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{x\,dx}{(x^2+z^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\left[\frac{-1}{\sqrt{z^2+x^2}}\right]_0^R$$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \qquad \lim_{R\to\infty} E = \frac{\sigma}{2\varepsilon_0}(1-0) = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

2.9) a) $\vec{E} = kr^3\hat{r}$

$\rho = \varepsilon_0(\nabla\cdot E)$

$\nabla\cdot E = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2E_r)$

$E_r = kr^3\hat{r}$

$\rho = \varepsilon_0\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2(kr^3))\right)$

$\boxed{\rho = 5k\varepsilon_0 r^2}$

b) i) $\oint E\cdot da = \frac{q}{\varepsilon_0}$

$q_{enc} = \varepsilon_0\int(kR^3\hat{r})\cdot(4\pi R^2)\hat{r}$

$= \varepsilon_0(4\pi kR^5)$

$\boxed{= 4\pi k\varepsilon_0 R^5}$

ii) $dq = \rho d\tau$

$= 5k\varepsilon_0 r^2(4\pi r^2 dr)$

$q = \int 5k\varepsilon_0 r^2(4\pi r^2 dr)$

$\boxed{= 4\pi k\varepsilon_0 R^5}$

2.12) $\oint E\cdot da = \frac{q_{enc}}{\varepsilon_0}$ $\qquad q_{enc} = \rho\times\frac{4}{3}\pi r^3$

$E(a) = \frac{q_{enc}}{\varepsilon_0}$

$E(4\pi r^2) = \frac{\rho\times\frac{4}{3}\pi r^3}{\varepsilon_0}$

$E = \frac{\rho r}{3\varepsilon_0}$

$\boxed{\vec{E} = \frac{\rho r}{3\varepsilon_0}\hat{r}}$

2.16) $\int \mathbf{E} \cdot da = \frac{q_{enc}}{\epsilon_0}$

i) $q_{enc} = \int \rho \, d\tau = \int_0^{l} \int_0^{2\pi} \int_0^{s} \rho (s\,ds) \, d\phi \, dz$

$q_{enc} = (2\pi \rho l)(\frac{1}{2} s^2)$

$E(2\pi s l) = \frac{2\pi \rho l s^2}{2\epsilon_0}$

$\boxed{E(s) = \frac{\rho s}{2\epsilon_0} \hat{s}}$

ii) $q_{enc} = \int_0^{l} \int_0^{2\pi} \int_0^{a} \rho s \, ds \, d\phi \, dz$

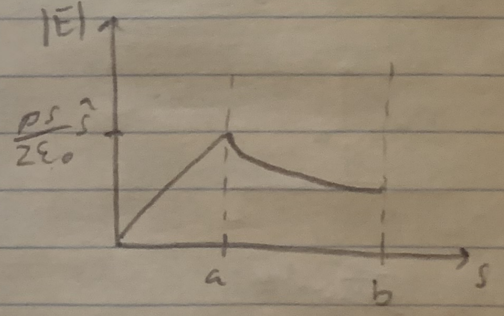
$= \rho (\pi a^2 / l$

$E(2\pi s l) = \frac{q_{enc}}{\epsilon_0} = \frac{\rho \pi a^2 l}{\epsilon_0}$

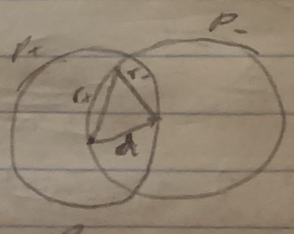
$\boxed{E(s) = \frac{\rho a^2}{2s\epsilon_0} \hat{s}}$

iii) $q_{enc} = 0$

$\boxed{E(s) = 0}$



2.18)



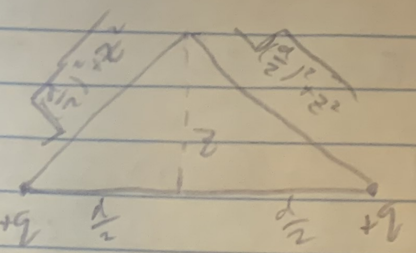
$E_+ = \frac{\rho}{3\epsilon_0} r_+$ $\quad$ $E_- = \frac{-\rho}{3\epsilon_0} r_-$

$E = E_+ + E_-$

$E = \frac{\rho}{3\epsilon_0} (r_+ - r_-)$ $\quad$ $r_+ - r_- = d$

$\boxed{E = \frac{\rho}{3\epsilon_0} d}$

2.25) i) $V(r) = k\sum_{i=1}^{n} \frac{q_i}{r_i}$



$$V = k\left(\frac{2q}{\sqrt{\left(\frac{d}{2}\right)^2+z^2}}\right)$$

$$E = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$$

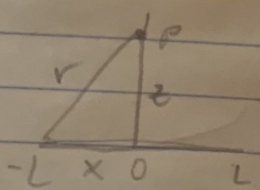
$$E = -\left(\frac{\partial}{\partial x}\left(\frac{2kq}{\sqrt{\left(\frac{d}{2}\right)^2+z^2}}\right)\hat{x} + \frac{\partial}{\partial y}\left(\frac{2kq}{\sqrt{\left(\frac{d}{2}\right)^2+z^2}}\right)\hat{y} + \frac{\partial}{\partial z}\left(\frac{2kq}{\sqrt{\left(\frac{d}{2}\right)^2+z^2}}\right)\hat{z}\right)$$

$$= -\frac{2q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}\left[\frac{1}{\sqrt{\left(\frac{d}{2}\right)^2+z^2}}\right)\hat{z}$$

$$= -\frac{2q}{4\pi\varepsilon_0}\left(\left(-\frac{1}{2}\right)\frac{2z}{\left(\left(\frac{d}{2}\right)^2+z^2\right)^{3/2}}\right)\hat{z}$$

$$E(2.34(a)) = k\left(\frac{2qz}{\left(\left(\frac{d}{2}\right)^2+z^2\right)^{3/2}}\right)\hat{z}$$

ii) $V = k\int \frac{\lambda(r')}{r} dl'$



$$V = k\int_{-L}^{L} \frac{\lambda dx}{\sqrt{z^2+x^2}} = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{1}{\sqrt{z^2+x^2}}dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\sqrt{x^2+z^2}+x\right)\Big|_{-L}^{L} = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\sqrt{L^2+z^2}+L}{\sqrt{L^2+z^2}-L}\right)$$

$$E = \left[-\frac{\partial}{\partial x}\left(\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\sqrt{L^2+z^2}+L}{\sqrt{L^2+z^2}-L}\right)\right)\hat{x} - \frac{\partial}{\partial y}\left(\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\sqrt{L^2+z^2}+L}{\sqrt{L^2+z^2}-L}\right)\right)\hat{y}\right.$$

$$\left.-\frac{\partial}{\partial z}\left(\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{\sqrt{L^2+z^2}+L}{\sqrt{L^2+z^2}-L}\right)\right)\hat{z}\right]$$

2.25 cont) $E = -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{\sqrt{L^2+z^2}-L}{\sqrt{L^2+z^2}+L}\right)\left(\frac{(\sqrt{L^2+z^2}-L)\left(\frac{2z}{2\sqrt{L^2+z^2}}\right)-(\sqrt{L^2+z^2}+L)\left(\frac{2z}{2\sqrt{L^2+z^2}}\right)}{(\sqrt{L^2+z^2}-L)^2}\right)\hat{z}$

$$= -\frac{\lambda}{4\pi\varepsilon_0}\left(\frac{1}{(\sqrt{L^2+z^2})^2-L^2}\right)\left(\frac{2Lz}{\sqrt{L^2+z^2}}\right)\hat{z}$$

$$\boxed{E(2.34(d)) = \frac{2k\lambda L}{z\sqrt{L^2+z^2}}\hat{z}}$$

iii) $V = k\int \frac{\sigma(r')}{r} da'$

$$V = k\int_0^R \frac{\sigma 2\pi r}{\sqrt{z^2+r^2}} dr$$

$$= \frac{\sigma 2\pi}{4\pi\varepsilon_0}\int_0^R \frac{r}{\sqrt{z^2+r^2}} dr$$

$$V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{r^2+z^2}\right)\Big|_0^R = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)$$

$$E = -\frac{\partial}{\partial x}\left(\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)\right)\hat{x} - \frac{\partial}{\partial y}\left(\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)\right)\hat{y}$$
$$- \frac{\partial}{\partial z}\left(\frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)\right)\hat{z}$$

$$= -\frac{\sigma}{2\varepsilon_0}\frac{\partial}{\partial z}\left(\sqrt{R^2+z^2}-z\right)\hat{z}$$

$$\boxed{E(2.34(c)) = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{R^2+z^2}}\right)\hat{z}}$$

If we change one of the charges to $-q$, the electric potential would be zero, therefore the electric field would be zero. But this doesn't match 2.2, because the potential on the z-axis isn't enough to calculate the total electric field

2.29) $V(r) = k \int \frac{\rho(r')}{\mathscr{r}} d\tau'$

$\nabla^2 \frac{1}{\mathscr{r}} = -4\pi \delta^3(\mathscr{r})$

$\nabla^2 V = -\frac{\rho}{\varepsilon_0}$

$V(r) = k \int \frac{\rho(r')}{\mathscr{r}} d\tau'$

$\nabla^2 \frac{1}{\mathscr{r}} = -4\pi \delta^3(r - r')$

$\nabla^2 V(r) = k \int \nabla^2 \left(\frac{1}{\mathscr{r}}\right) \rho(r') d\tau'$

$\nabla V(r) = k \int \left(-4\pi \delta^3(r - r')\right) \rho(r') d\tau'$

$= -\frac{1}{\varepsilon_0} \int \rho(r') \delta^3(r - r') d\tau'$

$\int \rho(r') \delta^3(r - r') d\tau' = \rho(r)$

$$\boxed{\nabla^2 V(r) = -\frac{1}{\varepsilon_0} \rho(r) = -\frac{\rho(r)}{\varepsilon_0}}$$

Therefore, the equation $V(r) = k \int \frac{\rho(r')}{r} d\tau'$ satisfies Poisson's equation