

Reading Quiz #1 Electro Magnetic Theory

1. $a(\vec{B} + \vec{C}) = a\vec{B} + a\vec{C}$
 $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$$\begin{aligned} \vec{C} &= C_x\hat{i} + C_y\hat{j} + C_z\hat{k} \\ a(\vec{B} + \vec{C}) &= a(B_x\hat{i} + B_y\hat{j} + B_z\hat{k} + C_x\hat{i} + C_y\hat{j} + C_z\hat{k}) \\ &= (B_xa + B_ya + B_za) + (C_xa + C_ya + C_za) \\ &= a(B_x + B_y + B_z) + a(C_x + C_y + C_z) \\ &= a\vec{B} + a\vec{C} \quad \checkmark \end{aligned}$$

2. $\nabla(f(x,y) + g(x,y))$

Unable to distribute gradient over parenthesis. Can't add gradients if Im not mistaken only take dot and crossproduct of them.

3. $f(x,y) = x\hat{i} + y\hat{j}$ has zero curl x & y plane

$$\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \quad \nabla \cdot \vec{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) + 0$$

$$\nabla \cdot \vec{F} = 1 + 1 = 2$$

$$\nabla \cdot \vec{F} = 2 \quad \boxed{\text{divergence} = 2}$$

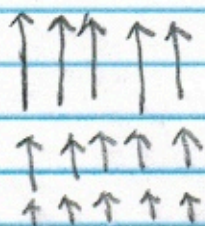
Curl $\nabla \times \vec{F} = \det$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & 0 \end{vmatrix}$$

$$\hat{i}\left(y\frac{\partial}{\partial z} - (0)\frac{\partial}{\partial y}\right) - \hat{j}\left(0\frac{\partial}{\partial x} - x\frac{\partial}{\partial z}\right) + \hat{k}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right)$$

$$\hat{i}(0) - \hat{j}(0) + \hat{k}(0) = 0 \quad \boxed{\text{no curl}}$$

figure 1.18

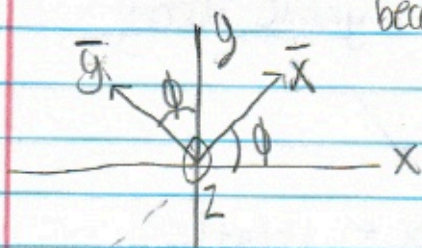


line integral $\int_0^{2\pi} \cos(t) + \sin(t)$

$$\begin{aligned} &\int_0^{2\pi} (\cos(t) + \sin(t)) \\ &= \sin(t) - \cos(t) \Big|_0^{2\pi} \\ &= \sin(2\pi) - \cos(2\pi) - (\sin(0) - \cos(0)) \\ &= 0 - 1 - (0 - 1) = 0 \end{aligned}$$

$$\begin{aligned} f(x,y) &= x^2 + y^2 \\ x &= r\cos(t) \\ y &= r\sin(t) \\ \text{radius} &= 1 \end{aligned}$$

2).



because we're rotating along z axis, a_x & a_y are changing

$$\vec{a}_x = \cos\phi \quad \sin\phi$$

$$\vec{a}_y = \sin\phi \quad \cos\phi$$

$$a_x = a_x \cos\phi + a_y \sin\phi$$

$$a_y = -a_x \sin\phi + a_y \cos\phi$$

$$\vec{a}_x^2 + \vec{a}_y^2 = a_x^2 + a_y^2$$

$$\vec{a}_x^2 = a_x^2 \cos^2\phi + a_y^2 \sin^2\phi + 2a_x a_y \cos\phi \sin\phi$$

$$a_y^2 = a_x^2 \sin^2\phi + a_y^2 \cos^2\phi - 2a_x a_y \sin\phi \cos\phi$$

$$\boxed{\vec{a}_x^2 + \vec{a}_y^2 = a_x^2 + a_y^2} \quad \text{magnitude is preserved}$$

$$|\vec{a}| = |\vec{a}| \quad \checkmark$$

3).

$$\int_P (\nabla \times \vec{v}) \cdot d\vec{a} = \oint \vec{v} \cdot d\vec{l}$$

Because there is no a radius of one and because it is closed, per the corollary 2 from the textbook for any closed surface, since the boundary line shrinks down to a point, the right side of the equation vanishes hence the result of $\oint \vec{v} \cdot d\vec{l} = 0$ therefore the entire integral $= 0$

$$4). f(x) * g(x) = \frac{f(x) - g(x)}{f(x) + g(x)}$$

$$\int_{-\infty}^{\infty} f(x) \delta(x) dx = f(0)$$

$$\frac{\cos(0) - \sin(0)}{\cos(0) + \sin(0)} = \boxed{1} \quad f(0) * g(0)$$

$$\frac{\cosh(0) - \sinh(0)}{\cosh(0) + \sinh(0)}$$

$$\frac{1-0}{1+0} = \boxed{1}$$

$$\frac{\cosh(0) - \sinh(0)}{\cosh(0) + \sinh(0)}$$

$$\frac{1-0}{1+0} = \boxed{1}$$

$$\boxed{a+ax+ax^2+\dots} \quad \boxed{b+bx+bx^2+\dots}$$

$$\frac{a+ax+ax^2-b-bx-bx^2}{a+ax+ax^2+b+bx+bx^2}$$

$$\boxed{\frac{a-b}{a+b}}$$

$$\frac{a+a(0)+a(0)^2-b+b(0)+b(0)^2}{a+a(0)+a(0)^2-b+b(0)+b(0)^2} = \boxed{1}$$