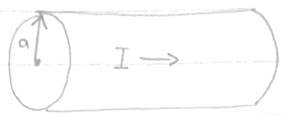
Chapter 5 Hw: 5.14, 5.16, 5.17, 5.19, 5.20, 5.23, 5.26

5.14



inside

a) $\oint B \cdot dl = \mu_0 I_{enc}$

$I_{enc} = 0$

$B(l) = \mu_0 (0)$ $\boxed{B = 0 \quad \text{magnetic field is zero inside}}$

outside

$\oint B \cdot dl = \mu_0 I_{enc}$

$B\,l = \mu_0 I_{enc}$

$B(2\pi s) = \mu_0 I_{enc}$

$$\boxed{B = \frac{\mu_0 I_{enc}}{2\pi s}}$$

b) inside

$I = \int_S J \cdot da$

$I = \int_0^a (Cs)(2\pi s)\,ds$

$= 2\pi C \int_0^a s^2\,ds = 2\pi C\left(\frac{a^3}{3}\right)$

$I = 2\pi C\left(\frac{a^3}{3}\right)$

$C = \frac{3I}{2\pi a^3}$

$I_{enc} = \int_0^s J \cdot da$

$= \int_0^s (Cs)(2\pi s)\,ds$

$= 2\pi C\left(\frac{s^3}{3}\right)$

$I_{enc} = 2\pi\left(\frac{3I}{2\pi a^3}\right)\left(\frac{s^3}{3}\right)$

$I_{enc} = I\frac{s^3}{a^3}$

$\oint B \cdot dl = \mu_0 I_{enc}$

$B(l) = \mu_0 I_{enc}$

$B(2\pi s) = \mu_0\left(I\frac{s^3}{a^3}\right)$

$$\boxed{B = \frac{\mu_0 I s^3}{2\pi a^3}}$$

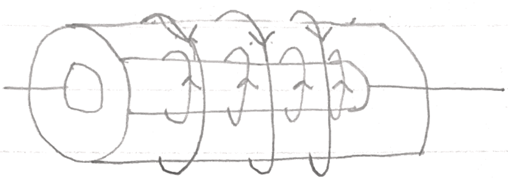
outside $\qquad I_{enc} = I$

$$\oint B \cdot dl = \mu_0 I_{enc}$$

$$B(2\pi s) = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi s}}$$

5.16



inner solenoid: radius a and n_1

outer one: radius b and n_2

$B = \mu_0 n I$ inside solenoid

$B = 0$ outside solenoid

i) inside the inner solenoid

$B_1 = \mu_0 n_2 I$ (outer) $\qquad B_2 = -\mu_0 n_1 I$ (inner)

$$B = \mu_0 I (n_2 - n_1)$$

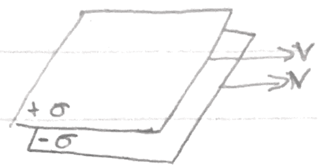
ii) between them

$$B = \mu_0 n_2 I$$

iii) outside both

$B = 0$ outside solenoid

5.17



$B = \frac{\mu_0 K}{2}$ $K = \sigma v$

a) between plates, above and below

above: $B = \frac{\mu_0 K}{2}$

below: $B = \frac{\mu_0 K}{2}$

between = above + below

$B = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} = \mu_0 K$

$\boxed{B = \mu_0 \sigma v}$

b) $F = \int (K \times B)\, da$ $B = \frac{\mu_0 K}{2} \hat{y}$ $K = \sigma v \hat{x}$

$f = K \times B$

$f = (\sigma v \hat{x}) \times \left(\frac{\mu_0 K}{2} \hat{y} \right)$

$= \sigma v \mu_0 \frac{K}{2} (\hat{x} \times \hat{y})$

$= \sigma v \mu_0 \frac{K}{2} \hat{z}$ $K = \sigma v$

$f = \sigma v \mu_0 \frac{\sigma v}{2} \hat{z}$

$\boxed{f = (\sigma v)^2 \frac{\mu_0}{2}}$

c) $f_e = f_m$ $f_e = \frac{\sigma^2}{2\varepsilon_0}$

$\frac{\cancel{\sigma^2}}{\cancel{2}\varepsilon_0} = (\cancel{\sigma} v)^2 \frac{\mu_0}{\cancel{2}}$

$\sqrt{v^2} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$ $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$ $\varepsilon_0 = 8.85 \times 10^{-12}$

$\mu_0 = 4\pi \times 10^{-7}$

$v = \frac{1}{\sqrt{(4\pi \times 10^{-7})(8.85 \times 10^{-12})}} = \boxed{3 \times 10^{8}\ m/s}$

5.19 $I_{enc} = \int_S J \cdot da$

If there are an infinite number of surfaces that have the same boundary line, you can choose or use any of the surfaces, as $\nabla \cdot J = 0$ and $\int J \cdot da$ is surface independent so you will get the same value no matter which surface you choose.

5.20

A) density ρ of mobile charges in a piece of copper

$\rho =$ charge/volume (C/cm³)

charge: 1.6×10^{-19} C

density of copper: $9.0 \frac{g}{cm^3} \times \frac{1}{64 g/mol} \times \frac{6.02 \times 10^{23} mol}{1}$

$= 8.47 \times 10^{22} \frac{1}{cm^3}$

$\rho = (1.6 \times 10^{-19} C)(8.47 \times 10^{22} \frac{1}{cm^3})$

$\boxed{\rho = 1.4 \times 10^4 \ C/cm^3}$

B) average velocity

$D = 1mm = 10^{-3}$ or $.001 m$

$I = 1A$

$J = \frac{I}{A}$ $J = \rho v$ $v = \frac{J}{\rho}$

$A = \pi r^2$

$= \pi \left(\frac{.001}{2}\right)^2 = 7.85 \times 10^{-7} m^2$

$J = \frac{I}{A} = \frac{1.0 A}{7.85 \times 10^{-7}} = 1.273 \times 10^6 \ A/m^2$

1 m = 100 cm

1 m² = 1000 cm²

127.38 A/cm²

$v = \frac{J}{\rho} = \frac{127.38 A/\cancel{cm^2}}{1.4 \times 10^4 C/\cancel{cm^3}}$

$\boxed{v = .0091 \ cm/s}$

c) force?? $d = 1cm$

$\left(\frac{N}{cm}\right)$ $F_{mag} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$ $I = 1.0A$

$= \frac{(4\pi \times 10^{-7} H/m)}{2\pi} \frac{(1.0A)^2}{1.0cm} = \boxed{1.97 \times 10^{-6} N/cm}$

D) $F_e = \frac{1}{2\pi\varepsilon_0} \frac{\lambda_1 \lambda_2}{d}$ $\lambda = \frac{1}{v}$

$= \frac{1}{2\pi\varepsilon_0} \frac{I_1 I_2}{v^2 d}$ $c^2 = \frac{1}{\mu_0 \varepsilon_0} \rightarrow \frac{1}{\varepsilon_0} = \mu_0 c^2$

$= \frac{\mu_0 c^2}{2\pi} \frac{I_1 I_2}{v^2 d}$

$3 \times 10^8 m/s = 3 \times 10^{10} cm/s$

$\frac{F_e}{F_{mag}} = \frac{\frac{\mu_0 c^2}{2\pi} \frac{I_1 I_2}{v^2 d}}{\frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}} = \frac{c^2}{v^2} = \frac{(3 \times 10^{10})^2}{(.0091?)^2} = \boxed{1.1 \times 10^{25}}$

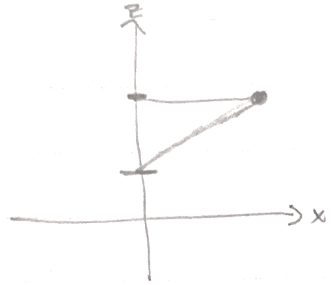
$\frac{F_e}{F_{mag}} = 1.1 \times 10^{25}$

$F_e = 1.1 \times 10^{25} (F_{mag})$

$F_e = 1.1 \times 10^{25} (1.97 \times 10^{-6}) (N/cm)$

$F_e = 2.17 \times 10^{19} N/cm$

5.23)



$y = 0$

$\vec{r} = x\hat{x} + z_0\hat{z}$

$\nabla \cdot \vec{A} = 0$

$\nabla^2 \vec{A} = -\mu_0 J(\vec{r}),$ 3 Poisson's

$\frac{\partial^2 A_x}{\partial x^2}\hat{x} + \frac{\partial^2 A_y}{\partial y^2}\hat{y} + \frac{\partial^2 A_z}{\partial z^2}\hat{z}$

$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,')}{\mathscr{r}}\, d\tau'$

$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int_{z_1}^{z_2} \frac{\vec{I}\, dz'}{\mathscr{r}}$

$\vec{A} \propto \hat{z}$

$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2} \frac{dz}{\sqrt{s^2+z^2}}\,\hat{z}$

$r = \sqrt{s^2+z^2}$

$= \frac{\mu_0 I}{4\pi}\left(\ln\left(z + \sqrt{s^2+z^2}\right)\right)\Big|_{z_1}^{z_2}$

$= \frac{\mu_0 I}{4\pi}\left(\ln\left(z_2 + \sqrt{z_2^2+s^2}\right) - \ln\left(z_1 + \sqrt{z_1^2+s^2}\right)\right)\hat{z}$

$$\boxed{\vec{A} = \frac{\mu_0 I}{4\pi}\left[\frac{\ln\left(z_2 + \sqrt{z_2^2+s^2}\right)}{\ln\left(z_1 + \sqrt{z_1^2+s^2}\right)}\right]\hat{z}}$$

$B = -\frac{\partial A}{\partial s}\hat{\phi}$

$= -\frac{\partial}{\partial s}\left[\frac{\mu_0 I}{4\pi}\ln\left(\frac{z_2 + \sqrt{z_2^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right)\right]\hat{\phi}$

$= -\frac{\mu_0 I}{4\pi}\left[\left(\frac{1}{z_2+\sqrt{z_2^2+s^2}}\right)\left(\frac{s}{\sqrt{z_2^2+s^2}}\right) - \left(\frac{1}{z_1+\sqrt{z_1^2+s^2}}\right)\left(\frac{s}{\sqrt{z_1^2+s^2}}\right)\right]\hat{\phi}$

$= -\frac{\mu_0 I s}{4\pi}\left[\left(\frac{1}{z_2+\sqrt{z_2^2+s^2}}\right)\left(\frac{z_2-\sqrt{z_2^2+s^2}}{z_2-\sqrt{z_2^2+s^2}}\right)\left(\frac{1}{\sqrt{z_2^2+s^2}}\right) - \left(\frac{1}{z_1+\sqrt{z_1^2+s^2}}\right)\left(\frac{z_1-\sqrt{z_1^2+s^2}}{z_1-\sqrt{z_1^2+s^2}}\right)\left(\frac{1}{\sqrt{z_1^2+s^2}}\right)\right]\hat{\phi}$

$= -\frac{\mu_0 I \cancel{s}}{4\pi}\left[\frac{z_2-\sqrt{z_2^2+s^2}}{(\cancel{z_2^2})-(\cancel{z_2^2}+s^2)}\left(\frac{1}{\sqrt{z_2^2+s^2}}\right) - \frac{z_1-\sqrt{z_1^2+s^2}}{(\cancel{z_1^2})-(\cancel{z_1^2}+s^2)}\left(\frac{1}{\sqrt{z_1^2+s^2}}\right)\right]\hat{\phi}$

$= +\frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{\sqrt{z_2^2+s^2}} - \cancel{1} - \frac{z_1}{\sqrt{z_1^2+s^2}} + \cancel{1}\right]\hat{\phi}$

5.23) continued

$$B = \frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{\sqrt{z_2^2+s^2}} - \frac{z_1}{\sqrt{z_1^2+s^2}}\right]\hat{\phi}$$

Equation 5.37 : $\frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right)$

$\sin\theta_2 = \frac{z_2}{\sqrt{z_2^2+s^2}}$

$\sin\theta_1 = \frac{z_1}{\sqrt{z_1^2+s^2}}$

$$\boxed{B = \frac{\mu_0 I}{4\pi s}\left[\sin\theta_2 - \sin\theta_1\right]}$$