Ch2 HW 5,6, 9,10, 18, 18,25,29

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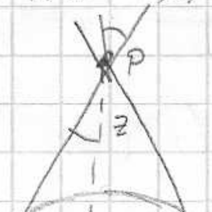
PHYS 330

2.5) Fig 2.9



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\cos\theta \qquad R^2 = r^2 + z^2$$

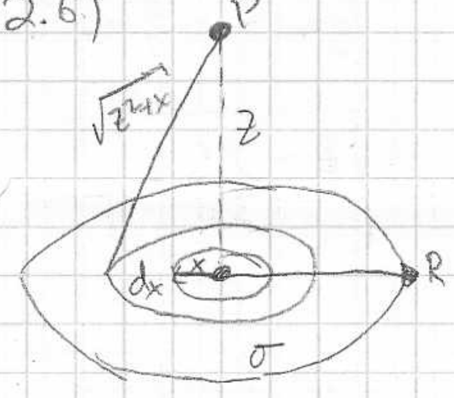
$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+z^2}\cos\theta$$

$$\cos\theta = \frac{z}{\sqrt{r^2+z^2}} \qquad dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}}$$

$$\vec{E} = \int_0^{2\pi} dE = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{z}{(r^2+z^2)^{3/2}}\,d\phi = \frac{\lambda\, z\, 2\pi r}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$\vec{E}_{net} = \frac{\lambda r z}{2\varepsilon_0 (r^2+z^2)^{3/2}}\hat{z}$$

2.6)



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\left(\sqrt{z^2+x^2}\right)^2}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{dq}{(z^2+x^2)}\left(\frac{z}{\sqrt{z^2+x^2}}\right) = \frac{1}{4\pi\varepsilon_0}\frac{(dq)z}{(z^2+x^2)^{3/2}}$$

$$\sigma = \frac{dq}{(2\pi x)dx} \qquad dq = \sigma(2\pi x)dx$$

$$dE = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi x)(dx)z}{(z^2+x^2)^{3/2}} = \frac{\sigma z}{2\varepsilon_0}\int_0^R\frac{x\,dx}{(z^2+x^2)^{3/2}}$$

$$= \frac{\sigma z}{2\varepsilon_0}\left(\frac{-1}{\sqrt{z^2+x^2}}\right)\Bigg|_0^R = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$

$$E_{R\to\infty} = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+\infty^2}}\right) = \boxed{\frac{\sigma}{2\varepsilon_0}}$$

$$E_{z\gg R} = \frac{\sigma}{2\varepsilon_0}\left(1-\left(1+\frac{R^2}{z^2}\right)^{-1/2}\right) = \boxed{\frac{\sigma R^2}{4\varepsilon_0 z^2}}$$

2.9) $E = kr^3 \hat{r}$

a) $\rho = \varepsilon_0 (\nabla \cdot E)$ $\qquad$ $\nabla \cdot E = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 E_r)$

$E_r = kr^3 \hat{r}$ $\qquad$ $\varepsilon_0 \left\{ \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 (kr^3) \right) \right\}$

$$\boxed{\rho = 5k\varepsilon_0 r^2}$$

b) $\oint E \cdot da = \frac{q}{\varepsilon_0}$ $\qquad$ $q_{enc} = \varepsilon_0 \int (kR^3 \hat{r}) \cdot (4\pi R^2) \hat{r} = \boxed{4\pi k \varepsilon_0 R^5}$

Gauss's Law

Spherical shell

$dq = \rho \, d\tau = 5k\varepsilon_0 r^2 (4\pi r^2 dr)$ $\qquad$ $q = \int dq$

$= \int_0^R (5k\varepsilon_0 r^2)(4\pi r^2 dr) = \boxed{4\pi k \varepsilon_0 R^5}$

2.12) $\oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$ $\qquad$ $E(a) = \frac{q_{enc}}{\varepsilon_0}$ $\qquad$ $q_{enc} = \rho \frac{4}{3} \pi r^3$

$E(4\pi r^2) = \frac{\rho \frac{4}{3} \pi r^3}{\varepsilon_0}$ $\qquad$ $E = \frac{\rho r}{3\varepsilon_0}$ $\qquad$ $\boxed{\vec{E} = \frac{\rho r}{3\varepsilon_0} \hat{r}}$

2.16) (i) $\int E \cdot da = \frac{q_{enc}}{\varepsilon_0}$ $\qquad$ $q_{enc} = \int \rho \, d\tau$

$q_{enc} = \int_0^l \int_0^{2\pi} \int_0^s \rho (s\,ds) \, d\phi \, dz = (2\pi \rho l)\left(\frac{s^2}{2}\right)$

$E(2\pi s l) = \frac{2\pi \rho l s^2}{2\varepsilon_0}$ $\qquad$ $\boxed{E(s) = \frac{\rho s}{2\varepsilon_0} \hat{s}}$

ii) $q_{enc} = \int_0^l \int_0^{2\pi} \int_0^a \rho s \, ds \, d\phi \, dz = \rho (\pi a^2) l$

$E(2\pi s l) = q_{enc}/\varepsilon_0 = \rho \pi a^2 l / \varepsilon_0$ $\qquad$ $\boxed{E(s) = \frac{\rho a^2}{2 s \varepsilon_0} \hat{s}}$

iii) $q_{enc} = 0$ $\qquad$ $\boxed{E(s) = 0}$

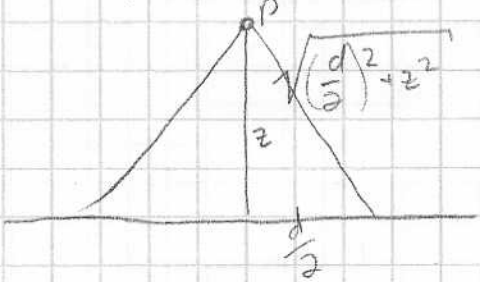
2.18) 2.15) $\vec{E}_+ = \frac{\rho r_+}{3\varepsilon_0}$ $E_- = \frac{-\rho}{3\varepsilon_0} r_-$ $E = E_+ + E_-$

$$E = \frac{\rho}{3\varepsilon_0}(r_+ - r_-) \qquad r_+ - r_- = d \qquad \boxed{E = \frac{\rho}{3\varepsilon_0} d}$$

2.25) a) 2.27) $V(r) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$

2.30) $V = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(r')}{r} dl'$

$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(r')}{r} da'$



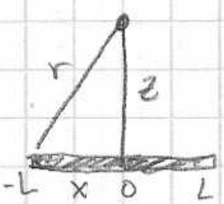
$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\sqrt{\left(\frac{d}{2}\right)^2 + z^2}} + \frac{q}{\sqrt{\left(\frac{d}{2}\right)^2 + z^2}}\right)$$

$$= \boxed{\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{\sqrt{\left(\frac{d}{2}\right)^2 + z^2}}\right)}$$

$$E = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$$

$$E = \frac{-2q}{4\pi\varepsilon_0} \frac{1}{\sqrt{\left(\frac{d}{2}\right)^2 + z^2}}\hat{z} = \boxed{\frac{1}{4\pi\varepsilon_0} \frac{2qz}{\left(\left(\frac{d}{2}\right)^2 + z^2\right)^{3/2}}\hat{z}}$$

b.)



$V = \frac{1}{4\pi\varepsilon_0} \int \frac{\lambda(r')}{r} dl'$ $\qquad V = \frac{1}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{\lambda\, dx}{\sqrt{x^2 + z^2}}$

$$= \frac{\lambda}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{dx}{\sqrt{x^2 + z^2}} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\sqrt{x^2 + z^2} + x\right)\Big|_{-L}^{L}$$

$$\boxed{= \frac{\lambda}{4\pi\varepsilon_0} \ln\left(\frac{\sqrt{L^2 + z^2} + L}{\sqrt{L^2 + z^2} - L}\right)}$$

$$E = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{2\lambda L}{z\sqrt{L^2 + z^2}}\hat{z}}$$

c.) $V = \frac{1}{4\pi\varepsilon_0} \int \frac{\sigma(r')}{r} da$

$$V = \frac{1}{4\pi\varepsilon_0} \int_0^R \frac{\sigma\, 2\pi r\, dr}{\sqrt{z^2+r^2}} = \frac{\sigma 2\pi}{4\pi\varepsilon_0} \int_0^R \frac{r\, dr}{\sqrt{z^2+r^2}}$$

$$= \frac{\sigma\, 2\pi}{4\pi\varepsilon_0} \left(r^2 + z^2 \right|_0^R = \boxed{\frac{\sigma}{2\varepsilon_0} \left(\sqrt{R^2+z^2} - z \right)}$$

$$E = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z} \right)$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)\hat{z}}$$

22a) 22a) $V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{\mathscr{r}} d\tau$ 1.102) $\nabla^2 \frac{1}{\mathscr{r}} = -4\pi\delta^3(\mathscr{r})$

Poisson's eqn $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ $V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{\mathscr{r}} d\tau'$

$$\nabla^2 \frac{1}{\mathscr{r}} = -4\pi\delta^3(\mathscr{r}) = -4\pi\delta^3(r-r')$$

Laplacian eqn: $\nabla^2 V(r) = \frac{1}{4\pi\varepsilon_0} \int \nabla^2\left(\frac{1}{\mathscr{r}}\right) \rho(r')\, d\tau'$

$$\nabla^2 V(r) = \frac{1}{4\pi\varepsilon_0} \int -4\pi\delta^3(r-r')\, \rho(r')\, d\tau'$$

$$= \frac{-1}{\varepsilon_0} \int \rho(r')\, \delta^3(r-r')\, d\tau'$$

$$\int_{\text{All Spaces}} \rho(r')\, \delta^3(r-r')\, d\tau' = \rho(r)$$

$$\nabla^2 V(r) = \frac{-1}{\varepsilon_0} \rho(r) =$$

$V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{r} d\tau'$ Satifies Poisson's equation