HW#4: 1, ~~2~~, ~~3~~, ~~4~~, ~~5~~, ~~6~~, Ex 4.2

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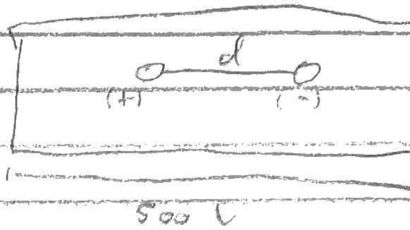
1) • 500 V battery $H = 0.667$

• $d = 1\,mm \Rightarrow d$

$V = qd$ or $V = \alpha E = \alpha \frac{qd}{4\pi\varepsilon_0 a^3}$

or p

$qd = \alpha E$, $E = \frac{qd}{4\pi\varepsilon_0 a^3}$



$\boxed{P_{H} = D \cdot 500}$

$D = \frac{\alpha E}{e}$

$\alpha = (0.667 \cdot 10^{-30} m^3)\, 4\pi\varepsilon_0$

$\alpha = 0.667 \cdot 10^{-30} m^3 / 8.99 \cdot 10^9$

$D = \frac{\alpha\,(500\,V)}{(1.6 \cdot 10^{-19})\,(0.001\,m)} = 2.32 \cdot 10^{-16}\,m$

$\boxed{D = R}$

Fraction of separation $= D/R = \boxed{4.64 \cdot 10^{-6}}$

$V = \frac{R\,q\,d}{4\pi\varepsilon_0 a^3} \rightarrow \frac{(0.5 \cdot 10^{-10})\,(1.6 \cdot 10^{-19})\,(0.001\,m)}{4\pi\,(8.85 \cdot 10^{-12})\,(0.667 \cdot 10^{-30})}$

$\boxed{= 1.07 \cdot 10^{8}\,V}$

7) $\boxed{U = -p \cdot E}$ $\tau = \vec{p} \times \vec{E}$

$\boxed{U = -pE\cos\theta}$ $\tau = pE\sin\theta$

$W = \Delta U = U_f - U_a$

$W = \oint \vec{F} \cdot dr = 0 \Rightarrow$ Work in spherical coordinate! $W = \int \vec{\tau} \cdot d\vec{\phi}$

$s = r\phi$

$\vec{r} \times \vec{\theta} \Rightarrow \vec{\phi}$

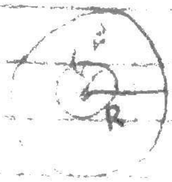
$\vec{\tau} \Rightarrow$ xy-plane

↳ thus! $\int_{\theta_1}^{\theta_2} \tau \cdot d\theta\, \vec{\phi}$

$= \int_{\theta_1}^{\theta_2} (\vec{p} \times \vec{E}) \cdot \vec{\phi}\, d\theta = \int_{\theta_1}^{\theta_2} (pE\cos\theta)\, d\theta\, \vec{\phi} \Rightarrow pE[\cos\theta_2 - \cos\theta_1]$

$W = U_f - U_i$

$W = -pE\cos\theta_2 \ldots$

10)  $P(r) = k\vec{r}$

$\vec{r} = r\cdot\hat{r}$

a) $\sigma_b = P\cdot\hat{n} = P r\hat{r}\hat{n} = k\vec{r}\hat{n} = \boxed{kR}$

$\rho_b = -\nabla\cdot P = -1/r^2 \, \partial/\partial r\left(r^2(k\vec{r})\right)$

$= -1/r^2 \cdot \partial/\partial r\left(kr^3\right) \Rightarrow -1/r^2\left(3r^2\right) = -3$

$\boxed{\rho_b = -3k}$

b) E_{in} & E_{out}

$Q_{enc} = \rho V = \underbrace{\rho\left(4/3\pi r^3\right)}_{(-3k)}$ sphere

$E(\vec{r}) = \int E\cdot da = \frac{Q_{enc}}{\varepsilon_0} =$

$E\int da = \frac{\rho V}{\varepsilon_0} \Rightarrow E = \frac{\rho V}{a\cdot\varepsilon_0} = \frac{(-3k)(4/3\pi r^3)}{(4\pi r^2)\varepsilon_0}$

$E = \frac{-kr}{\varepsilon_0} \Rightarrow \boxed{E_{in}(r) = \frac{-kr}{\varepsilon_0}}$

$\boxed{E_{out} = 0}$ – no charge outside of the sphere

b) $E = ?$

$E = \oint \vec{E} \cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0}$ slab 1

$E = \frac{\sigma A}{2A\varepsilon_0} = \frac{\sigma}{2\varepsilon_0} \Rightarrow \boxed{E_1 = \frac{-\sigma}{2\varepsilon_0}\vec{z}}$

$D = \varepsilon_0 \vec{E} + P$ $\quad P = \varepsilon_0 \chi_{e} \vec{E}$

$D = (\varepsilon_0 E_2 + \varepsilon_0 \chi_{e_2} \vec{z})$

$\boxed{D = \varepsilon_0 \varepsilon_{r_2} E}$ $\quad \varepsilon_{r_2} = 1.5 = 3/2$

$\boxed{E_2 = \frac{\sigma}{\varepsilon_0 \varepsilon_r}\vec{z}} \rightarrow \boxed{\frac{2\sigma}{\varepsilon_0 3}(\vec{z})}$

c) $P = ?$

$\boxed{\varepsilon_r = 1 + \chi_e}$

$P = D - \varepsilon_0 \vec{E}$

$\boxed{E = \frac{D}{\varepsilon} = \frac{D}{\varepsilon_r \varepsilon_0}}$

$P_1 = (\cdot \sigma \vec{z}) - \varepsilon_0 E_1$

$P_1 = (\cdot \sigma \vec{z}) - \varepsilon_0 \left(\frac{-\sigma}{2\varepsilon_0}\vec{z}\right)$

$\boxed{P_1 = -\frac{\sigma}{2}\vec{z}}$ $\quad P_2 = (\cdot \sigma \vec{z}) - \varepsilon_0 \left(\frac{2\sigma}{3\varepsilon_0}\right)\vec{z}$

$\boxed{P_2 = -\frac{2\sigma}{3}\vec{z}}$

area

d) $\Delta V = -\int \vec{E} \cdot d\vec{l} = E_1 a + E_2 a = -\frac{\sigma a}{2\epsilon_0} \quad \frac{2\sigma a}{3\epsilon_0}$

$$\boxed{V = -\frac{7a\sigma}{6\epsilon_0}}$$

e) $\sigma = P \cdot \vec{n}$

$\sigma_{b1} = P_1 \cdot \vec{n}$

$$\sigma_{b1} = \frac{-\sigma}{2} \qquad \sigma_{b2} = \frac{\sigma}{2}$$

$$\sigma_{b3} = \frac{-\sigma}{3} \qquad \sigma_{b4} = \frac{-\sigma}{3}$$

$\rho = \boxed{-\nabla \cdot P = 0}$ in dialetric materials

f) $E_1 = \frac{\sigma}{2\epsilon_0}, \quad E_2 = \frac{2\sigma}{3\epsilon_0}$

either $\frac{-\sigma}{2}$ or $\frac{\sigma}{2} \Rightarrow$ $\boxed{E_1 = \pm \frac{\sigma}{2\epsilon_0}}$

or $\frac{-2\sigma}{3}$ or $\frac{2\sigma}{3} \Rightarrow$ $\boxed{E_2 \pm \frac{2\sigma}{3\epsilon_0}}$

15) $P(r) = \frac{k}{r}\vec{r}$

$\underline{r=a}$

a) $\sigma_b = \vec{P}\cdot\hat{n} = \frac{k\vec{r}\cdot\hat{n}}{r} = \boxed{\frac{k}{r}} \to \boxed{\frac{k}{a}}$

$\rho_b = -\nabla\cdot P = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\left(\frac{k}{r}\right)\right) = \frac{1}{r^2}\frac{\partial}{\partial r}(r)$

$\boxed{= -k/r^2}$

$\vec{E} = ?$: $\oint E\cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$\underline{\text{if } r > a}$: $\boxed{E = 0}$

$\underline{\text{if } r > b}$: $\boxed{\vec{E} = 0}$

if $\underline{a<r<b}$:

$q = \rho v$

$q_b = -\frac{k}{r^2}(4\pi r^2)$ $\qquad$ $q_s = \left(\frac{k}{a}\right)(4\pi a^2)$

$= -4\pi k$ $\qquad$ $= 4\pi k r$

$q_{tot} = q_b + q_s$ $\qquad$ from ($a \to r$)

$= -4\pi k a + 4\pi k(r-a)$

$= -4\pi k r$

$E(\cancel{4\pi r^2}) = \frac{-4\pi k r}{\varepsilon_0} \to \boxed{E_{tot} = \frac{-k}{r\varepsilon_0}\vec{r}}$

b) $\vec{D} = ?$

(4.23) $\oint D \cdot da = Q_{f_{enc}}$ → Equals "0" outside region

$\boxed{D = 0 \quad \text{thus} \quad E = 0}$

(4.21) $D = \epsilon_0 \vec{E} + \vec{P}$

$0 = \epsilon_0 \vec{E} + P \rightarrow E = P/\epsilon_0 \Rightarrow \frac{-k}{r\epsilon_0}$

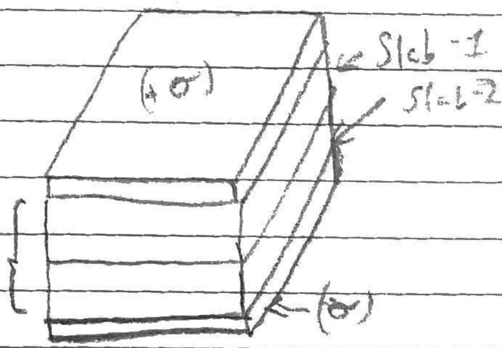
$\boxed{\vec{E} = \frac{-k}{r\epsilon_0}\hat{r}}$

18) thickness - a

$2a = d$

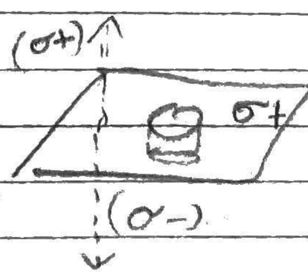
Top plate: $\epsilon_{r1} = 2$, $(+\sigma)$

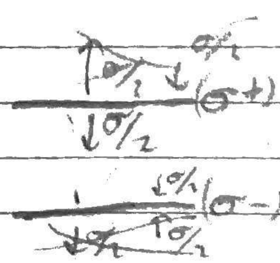
Bottom plate: $\epsilon_{r2} = 1.5$, $(-\sigma)$



a) $\vec{D} = ?$

$\oint \vec{D} \cdot d\vec{A} = q_{enc}$





$D(2A) = q_{enc} \Rightarrow \frac{\sigma A}{2A}$

$D = \frac{\sigma}{2}$, thus $D_1 = \sigma/2 + \sigma/2 = \sigma(-\hat{z})$

$\boxed{D = 0}$ $\boxed{D = -\sigma \hat{z}}$

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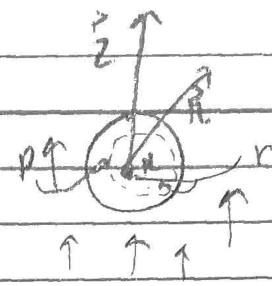
Ex. 4.2: $E = ?$



$\pi = \sqrt{z^2 + a^2 - 2za\cos\theta}$

$\sigma_b = P \cdot \hat{n} = P\hat{z} \cdot \cos\theta = \boxed{P\cos\theta}$

$\rho_B = -\nabla \cdot P = \boxed{0}$

polarized sphere equals 0.

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma_b \, da}{\pi} = \frac{P}{4\pi\varepsilon_0}\int \frac{\cos\theta \, R^2 \sin\theta \, d\theta \, d\phi}{\sqrt{z^2 + a^2 - 2zr\cos\theta}}$$

$$V(z) = \frac{PR^3}{3\varepsilon_0 r^2} \qquad \underline{r > R}$$

if $\underline{r < R}$: $\frac{Pr}{3\varepsilon_0}$

if $\underline{r < R}$: $E_z = -\nabla V = \frac{-P}{3\varepsilon_0}$

if $\underline{r > R}$: $E_z = \frac{-2Pr^3}{3\varepsilon_0 z^3}$