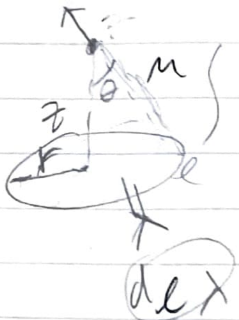
HW #2

2: 5, 6, 9, 12, 16, 18, 25, 21

2.5



only $\hat{z}$ left

$$\kappa^2 = z^2 + r^2$$

$$E = \left(\frac{1}{4\pi\epsilon_0}\int \frac{d\ell\lambda}{\kappa^2}\cos\theta\,\kappa\right)\hat{z}$$

$\kappa\cos\theta$ = vertical $\qquad \cos\theta = \frac{z}{\kappa}$

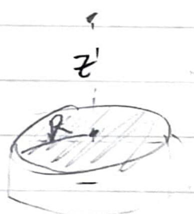
$$\tan\theta = \frac{r}{z}$$

$$\int \lambda\, d\ell = 2\pi\lambda$$

$$= k\,2\pi\left[\frac{\lambda}{(z^2+r^2)^2}\cdot\frac{z}{\kappa}\right]\hat{z}$$

$$= 2k\pi\lambda\left(\frac{z}{(z^2+r^2)^{3/2}}\right)\hat{z}$$

2.6



$\sigma \cdot 2\pi R \cdot dr = \Phi_{slice}$

$\lambda = \sigma \cdot 2\pi r$

$$E_{tot} = 2k\pi(\sigma\,2\pi.)\int_0^R \left[\frac{zr}{(z^2+r^2)^{3/2}}\right]\hat{z}$$

$R \to \infty$ $\quad r^2$ on the bottom is larger so the efield will go to zero in the integral.

9. $E = kr^3\hat{r}$

a) spherical!

$\nabla\cdot E = \frac{1}{r^2}\frac{d}{dr}(r^2E)$ $\nabla\cdot E = \frac{1}{\epsilon_0}\rho$

$\rho = \epsilon_0 \frac{1}{r^2}\left(\frac{1}{5} r^4 k\right) = 5\epsilon_0 k r^2$

b) $Q = \int \rho\, dV = \int_V 5\epsilon_0 k r^2 (r^2 \sin\phi\, dr\, d\phi\, d\theta)$

$\int_0^R r^2 dr \int_0^{2\pi} d\theta \int_0^{\pi} \sin\phi\, d\phi = 4\pi r^2$

2π 2

$\frac{r^5}{5}\Big|_0^R$

$= 20\epsilon_0 k\pi \int_0^R r^4 dr$

$= 20\epsilon_0 k\pi \left[\frac{R^5}{5}\right]$

12. $Q = \epsilon_0 \oint_S E\cdot da = \epsilon_0 (kr^3)(4\pi r^2)$

$= 4\epsilon_0 k r^5 \pi$

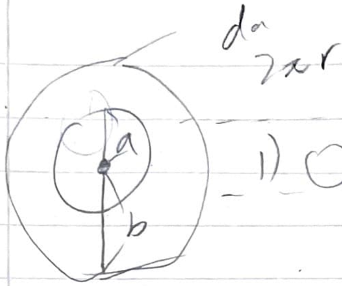
12. $\oint E\cdot da = \frac{1}{\epsilon_0} Q_{enc}$ $\frac{4}{3}\pi r^3$

$4\pi r^2$

$E = \frac{1}{\epsilon_0 4\pi r^2} Q_{enc}$

$\frac{1}{\epsilon_0 4\pi r^2}\rho\frac{4}{3}\pi r^3 = \frac{\rho}{3\epsilon_0} r$

16.



$r<a$

i) $\oint E \cdot da = E\, 2\pi r \cdot l = \frac{1}{\epsilon_0} Q$ $\qquad \rho \boxed{\cdot \pi r^2 l}$

$$E = \frac{\rho \pi r^2 l}{\epsilon_0 2\pi r l}$$

$$= \frac{\rho r}{2\epsilon_0}$$

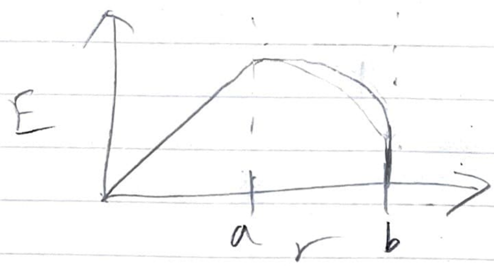
ii) $E \cdot 2\pi r l = \frac{1}{\epsilon_0}(\rho \pi a^2 l)$

$$E = \frac{\rho \pi a^2}{2\epsilon_0 r}$$

iii) $E \cdot 2\pi r l = \frac{1}{\epsilon_0} Q = 0$ neutral

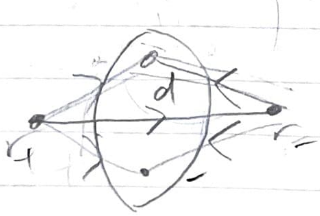
$E = 0$

17.



18.

2.12



$$E = \frac{\rho}{3\epsilon_0} r_+ + \frac{\rho(-r_-)}{3\epsilon_0} = \frac{\rho}{3\epsilon}(r_+ - r_-)$$

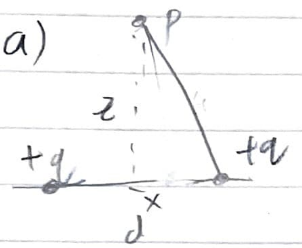
$$= \frac{\rho}{3\epsilon_0} d$$

$d = r_+ + (-r_-)$

25 $V(r) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$ $V = \frac{1}{4\pi\epsilon_0}\int\frac{\lambda(r')}{r}dl'$

$V = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma(r')}{r}da'$

a)

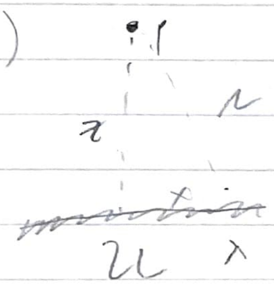


$E = -\nabla V$

$E = \frac{2}{4\pi\epsilon_0}\left[\frac{q+}{\sqrt{z^2+(\frac{d}{2})^2}}\right]$

$r^2 = z^2 + x^2$

b)



$V = \frac{1}{4\pi\epsilon_0}\int_0^{2L}\frac{\lambda\, dx}{(z^2+x^2)^{1/2}}$ wolfram

$= \frac{\lambda}{4\pi\epsilon_0}\left[\ln(x+(z^2+x^2)^{1/2}\Big|_0^{2L}\right]$

c) $V = \frac{1}{4\pi\epsilon_0}\int\frac{\sigma(r')}{(z^2+r^2)^{1/2}}dr$ $\sigma 2\pi R$

$V = \frac{2\pi\sigma}{4\pi\epsilon_0}\int\frac{r}{(z^2+r^2)^{1/2}}dr$

$u = z^2+r^2$

$du = 2r\,dr$

$dr = \frac{du}{2r}$

$= A\int_0^R\frac{1}{2}\frac{1}{\sqrt{u}}du$

$= \frac{\sigma}{4\epsilon_0}2\sqrt{z^2+r^2}\Big|_0^R$

$= \frac{\sigma}{2\epsilon_0}\left[\sqrt{z^2+R^2} - \overset{z}{\cancel{\sqrt{z^2}}}\right]$

29 2.29 1.102

$$V(r) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{\mathcal{r}} d\tau'$$

$$\nabla^2 V = -\frac{\rho}{\epsilon_0}$$

$$\nabla^2 \frac{1}{\mathcal{r}} = -4\pi\,\delta^3(\mathcal{r})$$

$\mathcal{r} = r - r'$

$$\frac{1}{4\pi\epsilon_0} \nabla^2 \int \frac{\rho(r')}{\mathcal{r}} d\tau = \frac{1}{4\pi\epsilon_0}$$

$$\frac{1}{4\pi\epsilon_0} \int \left(-4\pi\,\delta^3(\mathcal{r})\,\rho(r')\right) d\tau$$

$(r - r')$