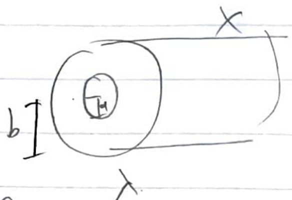
HW 3

2.43, 50 : 3.1, 3.13, 14, 15

2.43




$\frac{Q}{x} = \lambda$

$C = \frac{Q}{V}$

$V_t = V(a) + V(b)$

$\oint E \cdot da = E \cdot 2\pi r \cdot x = \frac{Q}{\epsilon_0}$

$E = \frac{Q}{2\pi r x \epsilon_0}$

$V(b) - V(a) = -\int_a^b E \cdot dx$

$= \frac{-Q}{2\pi x \epsilon_0} \int_a^b \frac{1}{r} dx$

$V = \frac{Q}{2\pi x \epsilon_0} [\ln(b) - \ln(a)]$

$$C = \frac{\frac{Q}{\frac{Q}{2\pi x \epsilon_0} \ln\left(\frac{b}{a}\right)}}{x} = \frac{\frac{2\pi \epsilon_0 x}{\ln\left(\frac{b}{a}\right)}}{x}$$

$$= \frac{2\pi \epsilon_0}{\ln\left(\frac{b}{a}\right)}$$

2.50 $V(r) = A \frac{e^{-\lambda r}}{r}$ $E(r)$ $\rho(r)$ $Q$

ans : $\rho = \epsilon_0 A \left(4\pi \delta^3(r) - \frac{\lambda^2 e^{-\lambda r}}{r}\right)$

$$E = -\nabla V = -A\left[\frac{r(-\lambda e^{-\lambda r}) - (e^{-\lambda r})1}{r^2}\right]\hat{r}$$

$$= +A\left[\frac{e^{-\lambda r}((1 + r\lambda) + 1)}{r^2}\right]\hat{r}$$

$\rho = \epsilon_0 \nabla \cdot E$

ch. 1

$$= A\epsilon_0\left[e^{-\lambda r}(\lambda r+1)\,\nabla\cdot\left(\frac{1}{r^2}\hat{r}\right) + \frac{1}{r^2}\hat{r}\cdot\nabla\left(e^{-\lambda r}(\lambda r+1)\right)\right]$$

$4\pi\,\delta^3(r)$ dirac so.

$\cancel{e^0(1)}\ 4\pi\,\delta^3(r)$

$$\frac{1}{r^2}\left[(-\lambda e^{-\lambda r})(\lambda r+1) + (e^{-\lambda r}\lambda)\right]$$

$$A\frac{1}{r^{\cancel{2}}}\left[-\lambda^2\cancel{r}e^{-\lambda r}\ \cancel{-\lambda e^{-\lambda r}}\ \cancel{+e^{-\lambda r}\lambda}\right]$$

$$\rho = A\epsilon_0\left(4\pi\delta^3(r) - \frac{\lambda^2}{r}e^{-\lambda r}\right)$$

$$Q = \int\rho\,d\tau = \int A\epsilon_0\left(4\pi\delta^3(r) - \frac{\lambda^2}{r}e^{-\lambda r}\right)d\tau$$

$\int_r\int_0^{\pi}\int_0^{2\pi} r^2\sin\theta\,d\theta\,d\phi\,dr = \int 4\pi r^2 dr$

$$= A\epsilon_0\left(\int 4\pi\,\delta^3(r)\,d\tau - \int\frac{\lambda^2}{r}e^{-\lambda r}4\pi r^2\,dr\right)$$

wolfram

$\lambda^2 4\pi\left(\frac{1}{\lambda^2}\right) = 4\pi$

$$= A\epsilon_0(4\pi - 4\pi) = 0$$

3.1 $V_{ave} = V_{cen} + \frac{Q_{enc}}{4\pi\epsilon_0 R'}$ $(z<R)$

$\sqrt{r^2} = \sqrt{z^2+R^2-2zR} = R-z$

3.14

$2z$

$V_{ave} = \frac{q}{4\pi\epsilon_0}\frac{1}{2zR}\left[(z+R)-(R-z)\right]$

$= \frac{q}{4\pi\epsilon_0 R}$

13 ex 3.3

a — V

a/2 — V, V

0 — V

x=0

V

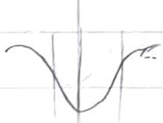
$V = \sum C_n e^{-n\pi x/a} \sin(n\pi y/a)$

$C_n = \frac{2}{a}\int_0^a V_0(y)\sin(n\pi y/a)\,dy$ $\qquad \frac{1}{(n\pi/a)}\cos(n\pi y/a)$

$C_n = \frac{2}{a}V_0\left(\frac{a}{n\pi/a}\right)\left[V_0\cos\left(\frac{n\pi y}{a}\right)\Big|_0^{a/2} - V_0\cos\left(\frac{n\pi y}{a}\right)\Big|_{a/2}^{a}\right]$

$= \frac{2V_0}{n\pi}\left[-\cos\left(\frac{n\pi}{2}\right)+1-\left(-\cos(n\pi)-\cos\left(\frac{n\pi}{2}\right)\right)\right]$

$= \frac{2V_0}{n\pi}\left[-2\cos\left(\frac{n\pi}{2}\right)+1+\cos(n\pi)\right]$



1: $-2(\cos\frac{\pi}{2})+1+\cos(\pi) = 0$

2: $-2(\cos(\pi))+1+1 = 4$

3: $-2(\cos\frac{3}{2}\pi)+1\pm1 = 0$

4: $-2\cos(2\pi)+1+1 = 0$

n has to be

$2+4\cdot n$

$= \frac{8V_0}{2\pi}$ for

$$V = \sum_{n=1}^{\infty} \frac{8V_0}{\pi(4n+2)} e^{-(4n+2)\pi x/a} \sin((4n+2)\pi y/a)_{4n}$$

14. $\sigma(y)$ 3.36 $x=0$

$$= -\epsilon_0 \frac{\partial V}{\partial n} \qquad V(x,y) = \frac{4V_0}{\pi} \sum_{1,3,5} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a)$$

$$+\epsilon_0 \frac{4V_0}{\cancel{\pi}} \sum \frac{1}{\cancel{n}} \left(+\frac{\cancel{n}\cancel{\pi}}{a}\right) \cancel{e^{-n\pi x/a}} \sin(n\pi y/a) \qquad x=0$$

$$= \frac{+4\epsilon_0 V_0}{a} \sum_{n=1,3,5} \sin\left(\frac{n\pi y}{a}\right)$$