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ET Homework #4

4.1, 4.7, Ex: 4.2, 4.10, 4.15, 4.18

4.1) $p = \alpha E$ $\quad E = \frac{V}{x}$

$p = ed$

$ed = \alpha E$ $\quad \frac{\alpha}{4\pi\epsilon_0} = 0.667 \times 10^{-30}\ m^3$

$d = \frac{\alpha E}{e}$

$$d = \frac{(0.667 \times 10^{-30}\ m^3)4\pi\epsilon_0 V}{ex}$$

$d = 2.32 \times 10^{-16}\ m$

$\frac{d}{R} = \frac{2.32 \times 10^{-16}\ m}{0.5 \times 10^{-10}\ m} = \boxed{4.64 \times 10^{-6}}$

$d = R$

$$R = \frac{(0.667 \times 10^{-30}\ m^3)4\pi\epsilon_0 V}{ex}$$

$$V = \frac{Rxe}{(0.667 \times 10^{-30}\ m^3)4\pi\epsilon_0}$$

$V = 1.078 \times 10^8\ V = \boxed{1.1 \times 10^8\ V}$

4.7) $U = -p \cdot E$

$\tau = pE\sin\theta$

$dw = \tau d\theta$

$dw = pE\sin\theta\, d\theta$

$w = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2)$

$w = U(\theta_2) - U(\theta_1)$

$= -pE(\cos\theta_2 - \cos\theta_1)$

$\boxed{U = -pE\cos\theta = -p \cdot E}$

Example 4.2) Inside:

$q = \int \rho \, d\tau$

$q = \rho \times \frac{4}{3}\pi R^3$

$\rho = \frac{3q}{4\pi R^3}$

$\oint E \, ds = \frac{q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int \rho \, d\tau$

$= \frac{\rho}{\varepsilon_0} \frac{4}{3}\pi r^3$

$\oint E \, ds = \frac{q r^3}{R^3}$

$E \cdot 4\pi \cancel{r^2} = \frac{q r^3}{R^3}$

$\boxed{E_{inside} = \frac{1}{4\pi\varepsilon_0} \frac{q r}{R^3} \hat{r}}$ $\quad r < R$

Outside:

$\oint E \, ds = \frac{q_{enc}}{\varepsilon_0}$

$q_{enc} = \int \rho \, d\tau = \int_0^R \rho \, d\tau + \int_R^r \rho \, d\tau$

$q_{enc} = \int_0^R \frac{3q}{4\pi R^3} d\tau$

$q_{enc} = \frac{3q}{4\pi R^3} \int_0^R d\tau = \frac{3q}{4\pi R^3} \frac{4}{3}\pi R^3 = q$

$\boxed{\oint E \, ds = \frac{q}{4\pi\varepsilon_0 r^2} \hat{r} \quad r > R}$

4.10) a) $P(r) = kr$

$P(r) = kR\hat{r}$

$\hat{n} = \hat{r}$

$$\boxed{\sigma_b = P \cdot \hat{n} = kR\hat{r} \cdot \hat{r} = kR}$$

$P(r) = r^2 kr$

$\rho_b = -\nabla \cdot P = -\left\{\frac{1}{r^2}\frac{\partial}{\partial r}P\right\} = -\left\{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr)\right\}$

$= -\left\{\frac{k}{r^2}\frac{\partial}{\partial r}(r^3)\right\}$

$= -\left\{\frac{k}{r^2}3r^2\right\}$

$$\boxed{\rho_b = -3k}$$

b) $\rho = \frac{Q_{inside}}{V}$ $\quad Q_{inside} = \rho V$

$V = \frac{4}{3}\pi r^3$

$Q_{inside} = \rho\left(\frac{4}{3}\pi r^3\right)$

$q_{enc} = (\rho_b)\left(\frac{4}{3}\pi r^3\right)$

$q_{enc} = (-3k)\left(\frac{4}{3}\pi r^3\right)$

$\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$

$E\oint da = \frac{q_{enc}}{\varepsilon_0}$

$E(4\pi r^2) = -\frac{3k\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$

$$\boxed{E(r) = \frac{-kr}{\varepsilon_0}\hat{r} \quad \text{for inside}}$$

$q_{tot} = q_{vol} + q_{surface}$

$q_{vol} = (\rho_b)\left(\frac{4}{3}\pi r^3\right) = (-3k)\left(\frac{4}{3}\pi r^3\right)$

$q_{surface} = (\sigma_b)(4\pi r^2) = (kR)(4\pi r^2)$

$q_{tot} = -(kR)(4\pi R^2) + (kR)(4\pi r^2) = 0$

$\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$

$$\boxed{E = 0}$$

4.15) a) $P(r) = \frac{k}{r}\hat{r}$ $\quad \sigma_b = P \cdot \hat{n}$

$\hat{n} = -\hat{r}$ $\quad \sigma_b = \frac{k}{r}(\hat{r} \cdot \hat{r}) = \frac{k}{r}$

$r = a$

$\sigma_b = -\frac{k}{a}$

$r = b$

$\sigma_b = \frac{k}{b}$

$\rho_b = -\nabla \cdot P = -\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P)\right) = -\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{k}{r}\right)\right)$

$= -\frac{\partial}{\partial r}\left(\frac{k}{r}\right) = -\frac{k}{r^2}$

$q_{enc} = 0$ in $r < a$

$\oint E \cdot da = \frac{q_{enc}}{\epsilon_0} = 0$

$\boxed{E = 0 \text{ when } r < a}$

$q_{enc} = q_{bound} + q_{surface}$ in $a < r < b$

$q_{surface} = (\sigma_b)(4\pi r^2)$

$= \left(\frac{k}{r}\right)(4\pi r^2)$

$q_{bound} = (\rho_b)(4\pi r^2)$

$= -\left(\frac{k}{r^2}\right)(4\pi r^2)$

$q_{enc} = -\frac{k}{a}(4\pi a^2) + \int_a^r \left(-\frac{k}{r^2}\right)(4\pi r^2)\,dr$

$= -4\pi k a - k 4\pi (r - a)$

$= -4\pi k r$

$\oint E \cdot da = \frac{q_{enc}}{\epsilon_0}$

$E(4\pi r^2) = \frac{-4\pi k r}{\epsilon_0}$

$\boxed{E = -\frac{k}{r\epsilon_0}\hat{r} \text{ when } a < r < b}$

$q_{enc} = -\frac{k}{a}(4\pi a^2) + \int_a^b \left(-\frac{k}{r^2}\right)(4\pi r^2)\,dr + \frac{k}{b}(4\pi b^2)$

$= -4\pi k a - 4\pi k(b - a) + 4\pi k b$

$= 0$

$\oint E \cdot da = \frac{0}{\epsilon_0} = 0$

$\boxed{E = 0 \text{ when } r > b}$

4.15 cont) b) in $r<a$ and $r>b$:

$$Q_{fenc} = 0$$

$$\oint D \cdot da = Q_{fenc}$$

$$D = 0$$

$$D = \varepsilon_0 E + P$$

$$0 = \varepsilon_0 E + P$$

$$\boxed{E = 0 \text{ when } r<a \text{ and } r>b}$$

$$D = \varepsilon_0 E + P$$

$$E = \left(-\frac{1}{\varepsilon_0}\right) P$$

$$E = \left(-\frac{1}{\varepsilon_0}\right)\left(\frac{k}{r}\hat{r}\right)$$

$$\boxed{E = \left(-\frac{k}{r\varepsilon_0}\right)\hat{r} \text{ when } a<r<b}$$

4.18) a) $\oint D \cdot da = q_{enc}$

no charges inside slab, therefore $\oint D \cdot da = 0$

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_1} + \frac{\sigma}{2\varepsilon_2} = \frac{\sigma}{2(k_1\varepsilon_0)} + \frac{\sigma}{2(k_2\varepsilon_0)}$$

$$E = \frac{\sigma}{2}\left(\frac{1}{2\varepsilon_0} + \frac{1}{1.5\varepsilon_0}\right) = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2} + \frac{1}{1.5}\right) = \frac{7\sigma}{12\varepsilon_0}$$

$$D_{slab_1} = 2\varepsilon_0\left(\frac{7\sigma}{\underset{6}{12}\varepsilon_0}\right) = \frac{7\sigma}{6}$$

$$D_{slab_2} = 1.5\varepsilon_0\left(\frac{7\sigma}{12\varepsilon_0}\right) = \frac{21\sigma}{24}$$

Electric displacement $\boxed{\text{slab } 1 = \frac{7\sigma}{6} \qquad \text{slab } 2 = \frac{21\sigma}{24}}$

b) $E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$

$$E_1 = \frac{\sigma}{\varepsilon_0\varepsilon_{r_1}} = \frac{\sigma}{2\varepsilon_0} \qquad E_2 = \frac{\sigma}{1.5\varepsilon_0} = \frac{2\sigma}{3\varepsilon_0}$$

Electric field vector $\boxed{\text{slab } 1 = \frac{\sigma}{2\varepsilon_0} \qquad \text{slab } 2 = \frac{2\sigma}{3\varepsilon_0}}$

4.18 cont) c) $P = \varepsilon_0 \chi_e E = \frac{\varepsilon_0 \chi_e \sigma}{\varepsilon_0 \varepsilon_r}$

$$P = \left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)\sigma$$

$$P_1 = (1 - (2)^{-1})\sigma = \frac{\sigma}{2}$$

$$P_2 = (1 - (1.5)^{-1})\sigma = \frac{\sigma}{3}$$

polarization $\boxed{\text{slab } 1 = \frac{\sigma}{2} \qquad \text{slab } 2 = \frac{\sigma}{3}}$

d) $V = Ed = E_1 a + E_2 a = (E_1 + E_2)a$
$= \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{3\varepsilon_0}\right)a = \frac{7\sigma}{6\varepsilon_0}a$

$\boxed{\text{potential difference btwn slab 1 and 2} = \frac{7\sigma}{6\varepsilon_0}a}$

e) $\sigma_b = P \cdot \hat{n} = -P_1 = -\frac{\sigma}{2}$

$$\sigma_b' = P_1 \cdot \hat{n} = P_1 = \frac{\sigma}{2}$$

$$\sigma_{b2} = P_2 \cdot \hat{n} = P_2 = \frac{\sigma}{3}$$

$$\sigma_{b2}' = P_2 \cdot \hat{n} = -P_2 = -\frac{\sigma}{3}$$

$$\boxed{P_b = -\frac{\sigma}{2} + \frac{\sigma}{2} + \frac{\sigma}{3} - \frac{\sigma}{3} = 0}$$

f) Above slab 1: $\sigma - \frac{\sigma}{2} = \frac{\sigma}{2}$
below slab 1: $\frac{\sigma}{2} - \frac{\sigma}{3} + \frac{\sigma}{3} - \sigma = -\frac{\sigma}{2}$
$\sigma_1 = \frac{\sigma}{2} - \left(-\frac{\sigma}{2}\right) = \sigma$ $\qquad E_1 = \frac{\sigma_1}{2\varepsilon_0} = \boxed{\frac{\sigma}{2\varepsilon_0}}$

above slab 2: $\frac{\sigma}{2} + \frac{\sigma}{2} - \frac{\sigma}{3} = \frac{2\sigma}{3}$
below slab 2: $\frac{\sigma}{3} - \sigma = -\frac{2\sigma}{3}$
$\sigma_2 = \frac{2\sigma}{3} - \left(-\frac{2\sigma}{3}\right) = \frac{4\sigma}{3}$ $\qquad E_2 = \frac{\sigma_2}{2\varepsilon_0} = \frac{\frac{4\sigma}{3}}{2\varepsilon_0} = \boxed{\frac{2\sigma}{3\varepsilon_0}}$