Reading Quiz 1 for Electromagnetic Theory (PHYS330)

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Abstract

A summary of content covered in chapter 1 of Introduction to Electrodynamics.

1 Vectors and Scalars

1. Prove that scalar multiplication distributes over vector addition: $a(\vec{B} + \vec{C}) = a\vec{B} + a\vec{C}$. *Hint: break into components.*

2. What is wrong with this combination of objects? $\nabla(f(x,y) + \nabla g(x,y))$

3. (a) Create a vector field that has zero curl. (b) Perform the line integral with the unit circle in the xy-plane as the path. Does the result make sense? (c) What is the divergence of your field?

2 Vector Rotations

1. Imagine a Cartesian coordinate system (right-handed) with $\hat{x}$ right, $\hat{y}$ up, and $\hat{z}$ out of the page. (a) Define a vector in the xy-plane. (b) Rotate the coordinate system *about the z-axis* by 45 degrees, counter-clockwise. What are the new components of your vector? (c) Determine if it has the same magnitude (it should).

3 Fundamental Theorems

1. **Stokes' Theorem**. Let $\vec{v} = s^{-1}\hat{\phi}$ in cylindrical coordinates, where the three coordinates in the text are (s, ϕ, z). What is the result of $\int_S (\nabla \times \vec{v}) \cdot d\vec{a}$, if the surface is the *closed sphere* with radius 1, centered on the origin. *Hint: This problem is a trick...What happens when Stokes' Theorem gets a closed surface? Don't overthink it!*

4 Dirac Delta Functions

1. Let

$$f(x) \star g(x) = \left(\frac{f(x) - g(x)}{f(x) + g(x)} \right) \tag{1}$$

Evaluate $\int_{-\infty}^{\infty} (f(x) \star g(x)) \delta(x) dx$, if:

- $f(x) = \cos(x)$ and $g(x) = \sin(x)$.
- $f(x) = \cosh(x)$ and $g(x) = \sinh(x)$.
- $f(x) = a + ax + ax^2 + ...$, and $g(x) = b + bx + bx^2 + ...$