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Reading Quiz 3 for Electromagnetic Theory (PHYS330)

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Abstract

A summary of content covered in chapter 3 (so far) of Introduction to Electrodynamics.

1 Discussions about Vectors (Prelude to Fourier's Trick)

1. Let $\vec{v} = a\hat{x} + b\hat{y} + c\hat{z}$. Which of the following is equal to c?

- A: $\vec{v}\cdot\vec{v} - |\vec{v}|$
- $\boxed{\text{B: } \vec{v}\cdot\hat{z}}$
- C: $\hat{x}\cdot\vec{v}$
- D: $\sqrt{\vec{v}^2}$

z Y X

2. Let $\vec{x} = \sum_{i=1}^{n} c_i\hat{x}_i$ be an n-dimensional vector and the set of $\hat{x}_i$ represent orthonormal basis vectors. How do you obtain the coefficient c_7?

- A: $\hat{x}_i\cdot\vec{x}$
- B: $n = 7$
- C: $\hat{x}\cdot\vec{x}$
- $\boxed{\text{D: } \hat{x}_7\cdot\vec{x}}$

3. Suppose we are trying to develop the Fourier series for a function $f(x)$. Recall the definition of a Fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx) \quad = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + \sum_{n=1}^{\infty} b_n^{(1)} \cos(nx)$$

However, the function we are trying to model is $f(x) = \sin(3x)$. Write down all coefficients in the Fourier series from $n = 0$ to $n = \infty$.

$a_0 = \int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{\pi} \sin(3x)\,dx = -\frac{\cos(3x)}{3}\Big|_{-\pi}^{\pi} = \frac{(-1 - -1)}{3} = 0$ $\quad a_n = \int_{-\pi}^{\pi} \sin(3x)\sin(nx)\,dx = \left(\frac{1}{n+3} - \frac{1}{n-3}\right)\cdot\sin(n\pi)$

$\boxed{a_0 = 0,\ a_n = 0,\ b_n = 0}$ see work page b_n

2 Fourier's Trick and Boundary Value Problems

1. If $V(x,y,z) \to 0$ as $y \to \infty$, which of the following cannot be part of the solution for $V(x,y,z)$?

- A: $Y(y) = e^{-ky}$ $\quad \frac{1}{e^{ky}}$ $\quad \lim_{y\to\infty} A = 0$ $\quad \lim_{y\to\infty} B = 0$ $\quad \lim_{y\to\infty} B = \sinh(x) \neq 0$
- $\boxed{\text{B: } Y(y) = \sinh(x)}$ $\quad 1$
- C: $Y(y) = 1/y^2$ $\quad \frac{1}{y^2}$
- D: $Y(y) = e^{-ky^2}$ $\quad \frac{1}{e^{ky^2}}$ $\quad \lim_{y\to\infty} D = 0$

2. Below is Eq. 3.50 from section 3.3 of the text, with $V_0(y,z) = V_0$:

$$C_{n,m} = \frac{4V_0}{ab}\int_0^a\int_0^a \sin(n\pi y/a)\sin(n\pi z/a)dydz \tag{2}$$

Reproduce the result in Eq. 3.51 for $C_{n,m}$.

① $L=\pi$

③ $a_0=\frac{1}{2L}\int_{-\pi}^{\pi} f(x)dx=\frac{1}{2\pi}\int_{-\pi}^{\pi}\sin(3x)dx=\frac{1}{2\pi}\cdot\frac{-\cos(3x)}{3}\Big|_{-\pi}^{\pi}$

$=\frac{1}{2\pi}\left(\frac{-1--1}{3}\right)=0$

$a_n=\frac{1}{L}\int_{-\pi}^{\pi}\sin(3x)\sin(nx)dx=\left(\frac{-6\sin(\pi n)}{n^2-9}, \frac{1}{\pi}\right)$

$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}\sin(3x)\cos(nx)dx=0$

$b_n=\left(\frac{1}{n+3}-\frac{1}{n-3}\right)\cdot\sin(n\pi)$

$\frac{1}{}$

Because n is always an integer, and sine of an integer multiple of π is always zero, therefore, the coefficients of $a_0=0$, $a_n=0$, and $b_n=0$.

ok odd

2. (2) $C_{n,m} = \frac{4V_0}{ab} \int_0^a \sin\left(\frac{n\pi x}{a}\right) dx \cdot \int_0^b \sin\left(\frac{m\pi z}{b}\right) dz$

$$C_{n,m} = \frac{4V_0}{ab} \left[\frac{-\cos\left(\frac{n\pi x}{a}\right) \cdot a}{n\pi} \right]_0^a \cdot \left[\frac{-\cos\left(\frac{m\pi z}{a}\right)}{m\pi} \right]_0^b$$

$$= \frac{4V_0}{ab} \left[\frac{a\cos\left(\frac{n\pi a}{a}\right)}{n\pi} - \frac{(-\cos(0))}{n\pi} \right] \cdot \left[\frac{b\cos(m\pi)}{m\pi} - \frac{\cos}{m\pi} \right.$$

$$= \frac{4V_0}{ab} \left[\frac{1}{n\pi} - \frac{\cos(n\pi)}{n\pi} \right] \cdot \left[\frac{1}{m\pi} - \frac{\cos(m\pi)}{m\pi} \right]$$

if $n = 2$ or ... even int.

$$= \frac{4V_0}{ab} \left[\frac{1}{n\pi} - \frac{1}{n\pi} \right] \cdot \left[\frac{1}{m\pi} - \frac{\cos(m\pi)}{m\pi} \right]$$

$$\frac{4V_0}{ab} [0] \cdots = \boxed{0} \square$$

if m, is even

$$\left[\frac{1}{m\pi} - \frac{\cos 0}{m\pi} \right] = 0, \quad \frac{4V_0}{ab} \cdots 0 = \boxed{0} \square$$

if both are odd

$$\frac{4V_0}{ab} \left[\frac{1}{n\pi} - \frac{-1}{n\pi} \right] \left[\frac{1}{m\pi} - \frac{-1}{m\pi} \right] = \frac{4V_0}{ab} \left[\frac{2}{n\pi} \right] \left[\frac{2}{m\pi} \right]$$

$$= \boxed{\frac{16V_0}{abnm\pi^2}} \square$$