EM THEORY HW # 4

Riley Sullivan

4.10, 4.14, 4.15, 4.18, 4.26, 4.35

4.10)

$P(r) = kr$

$\boxed{4.12}$

ⓐ $\sigma_b = P \cdot \hat{n} = kR$, $\rho_b = -\nabla \cdot P = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr)$

$= \frac{-3kr^2}{r^2} = \boxed{-3k} = \rho_b$

ⓑ when $r < R$, $E = \frac{1}{3\varepsilon_0}\rho r \hat{r}$, $E = \frac{(-3k)}{3\varepsilon_0} = -\left(\frac{k}{\varepsilon_0}\right) r$

When $r > R$, $Q_{tot} = kR(4\pi R^2) + (-3k)(\frac{4}{3}\pi R^3)$

$= 4\pi kR^3 + (-4\pi kR^3)$

$\boxed{Q_{tot}\{r>R\} = 0}$

$\boxed{4.14}$ $4.11 \rightarrow \boxed{\sigma_b = P \cdot \hat{n}}$, $4.12 \rightarrow \boxed{\rho_b = -\nabla \cdot P}$

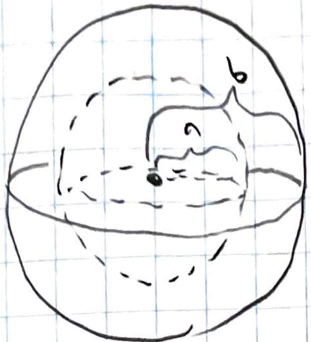
$Q_{tot} = \oint_S \sigma_b \, da + \int_V \rho_b \, d\tau = \oint_S P \cdot da - \int_V \nabla \cdot P \, d\tau$

$\therefore \oint_S P \cdot da = \int_V \nabla \cdot P \, d\tau$

$Q_{tot} = \left[\cancel{\int_V \nabla \cdot P \, d\tau} - \cancel{\int_V \nabla \cdot P \, d\tau}\right] = 0$

$Q_{tot} = 0$

$\boxed{4.15}$



$P(r) = \frac{k}{r}\hat{r}$

$\boxed{a}$ $\rho_b = -\nabla \cdot P = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{k}{r^2}\right)$

$\sigma_b = P \cdot \hat{n}, \quad \left[@ r=b \rightarrow P\cdot\hat{r} = \frac{k}{b} \quad @ r=a \rightarrow -P\cdot\hat{r} = -\frac{k}{a} \right]$

$\frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}$ → when $r<a$, $Q_{enc}=0$, and when $r>b$, $Q_{enc}=0$ [4.14 – QED]

So $E = 0$

When $a<r<b$ $\quad Q_{enc} = -\frac{k}{a}(4\pi a^2) + \int_a^r \left(\frac{-k}{r^2}\right) 4\pi r^2\, dr = -4\pi k a - 4\pi k r$

$E = -\left(\frac{k}{\varepsilon_0 r}\right)\hat{r}$

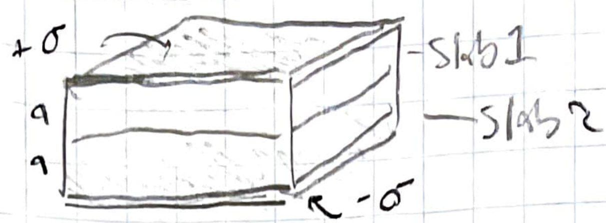
$\oint D\cdot da = Q_{f\,enc} = 0, \quad D = 0$ at all pts

$D = \varepsilon_0 E + P = 0,$

$E = \left(\frac{-1}{\varepsilon_0}\right)P$

$\therefore\ E = 0\ [r<a\ \&\ r>b], \quad E = \left[\frac{k}{\varepsilon_0 r}\right]\hat{r}\ @\ a<r<b$

4.18



(a)

$\int D \cdot da = Q_{f\,enc}$, $DA = \sigma A$, $D = \sigma$

(b) $D = \varepsilon E$, $E = \frac{\sigma}{\varepsilon_1}$ in Slab 1 ; $\frac{\sigma}{\varepsilon_2}$ for Slab 2

$\varepsilon = \varepsilon_0 \varepsilon_r \Rightarrow \varepsilon_1 = 2\varepsilon_0$, $\varepsilon_2 = \frac{3}{2}\varepsilon_0$

So $\boxed{E_1 = \frac{\sigma}{2\varepsilon_0}}$, $\boxed{E_2 = \frac{2\sigma}{3\varepsilon_0}}$

(c) $P = \varepsilon_0 \chi_e E$,

$P = \frac{\varepsilon_0 \chi_e \sigma}{\varepsilon_0 \varepsilon_r} = \frac{\chi_e}{\varepsilon_r} \sigma$; $\chi_e = \varepsilon_r - 1$

$P = 1 - \varepsilon_r^{-1} \sigma$

$P_1 = \frac{\sigma}{2}$, $P_2 = \frac{\sigma}{3}$

(d) $V = E_1 a + E_2 a$, $\frac{\sigma a}{2\varepsilon_0} + \frac{2\sigma a}{3\varepsilon_0} = \boxed{\frac{7\sigma a}{6\varepsilon_0}}$

(e) $\rho_b = 0$, $\sigma_b = P_1$ @ bottom of slab 1 $= \frac{\sigma}{2}$

σ_b is P_2 @ bottom of 2 $= \frac{\sigma}{3}$

σ_b @ top slab 1 $= -\frac{\sigma}{2}$; σ_b @ top slab 2 $= -\frac{\sigma}{3}$

(f) Slab 1 $\left\{ \begin{array}{l} \sigma \text{ above} = \frac{\sigma}{2} \\ \sigma \text{ below} = -\frac{\sigma}{2} \end{array} \right\}$ $E_1 = \frac{\sigma}{2\varepsilon_0}$

Slab 2 $\left\{ \begin{array}{l} \text{above} = \frac{2\sigma}{3} \\ \text{below} = -\frac{2\sigma}{3} \end{array} \right\}$ $E_2 = \frac{2\sigma}{3\varepsilon_0}$

$\boxed{4.26}$

Eq 4.58

$$W = \frac{1}{2}\int D\cdot E \, d\tau$$

Ex 4.5

$$D = \begin{bmatrix} 0, & (r<a) \\ \frac{Q}{4\pi r^2}\hat{r} & (r>a) \end{bmatrix}, \quad E = \begin{cases} 0 & (r<a) \\ \frac{Q}{4\pi\varepsilon r^2}\hat{r} & (a<r<b) \\ \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r} & (r>b) \end{cases}$$

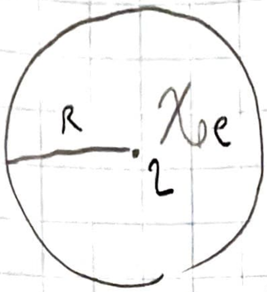
$$W = \frac{1}{2}\int D\cdot E \, d\tau = \frac{1}{2}\frac{Q^2}{(4\pi)^2}4\pi\left[\frac{1}{\varepsilon}\int_a^b \frac{1}{r^2}\cdot\frac{1}{r^2}r^2\,dr + \frac{1}{\varepsilon_0}\int_b^\infty \frac{1}{r^2}\,dr\right]$$

$$W = \frac{Q^2}{8\pi}\left[\frac{1}{\varepsilon}\left(\frac{-1}{r}\right)\Big]_a^b + \frac{1}{\varepsilon_0}\left(\frac{-1}{r}\right)\Big]_b^\infty\right]$$

$$W = \frac{Q^2}{8\pi\varepsilon_0}\left[\frac{1}{(1+\chi_e)}\left(\frac{1}{a}-\frac{1}{b}\right)+\frac{1}{b}\right]$$

$$\boxed{W = \frac{Q^2}{8\pi\varepsilon_0(1+\chi_e)}\left(\frac{1}{a}+\frac{\chi_e}{b}\right)}$$

4.35



$$\oint D \cdot da = Q_{fenc}, \quad D = \frac{q}{4\pi r^2}\hat{r}, \quad E = \frac{1}{\varepsilon} D = \frac{q}{4\pi\varepsilon_0 (1+\chi_e)} \frac{\hat{r}}{r^2}$$

$$P = \varepsilon_0 \chi_e E$$

$$P = \frac{q\chi_e}{4\pi(1+\chi_e)} \frac{\hat{r}}{r^2}$$

$$\rho_b = -\nabla \cdot P = -\frac{q\chi_e}{4\pi(1+\chi_e)} \left(\nabla \cdot \frac{\hat{r}}{r^2}\right)$$

$$\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi \delta^3(r)$$

$$\rho_b = -\frac{q\chi_e}{1+\chi_e} \delta^3(r)$$

$$\sigma_b = P \cdot \hat{r} = \frac{q\chi_e}{4\pi(1+\chi_e)R^2}$$

$$Q_{surface} = \sigma_b (4\pi R^2) = \boxed{q\frac{\chi_e}{1+\chi_e}}$$

$$\int \rho_b \, d\tau = -\frac{q\chi_e}{1+\chi_e} \int \delta^3(r) \, d\tau = \boxed{-q\frac{\chi_e}{1+\chi_e}}$$