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Homework 4 p 4.1, 4.7, Ex 4.2, 4.10, 4.15, 4.18

4.1) H atom situated between two metal plates 1mm apart which are connected to opp terminals of 500 V battery. What fraction of atomic radius does seperation distance d amount to?

$P = \alpha E$ and $p = qd$ so $qd = \alpha E$ (q: charge, α: given)

$d = \frac{\alpha E}{q}$ $\quad \frac{\alpha}{4\pi\epsilon_0} = .667 \times 10^{-30}$

$q = 1.6 \times 10^{-19} C$

$\alpha = (4\pi\epsilon_0)(.667 \times 10^{-30})$ $\quad E = \frac{V}{x}$ x (distance)

$d = \frac{(.667 \times 10^{-30} m^3)(4\pi\epsilon_0) V}{q x}$ $\quad x = 1 \times 10^{-3} m$

$\epsilon_0 = 8.85 \times 10^{-12} C^2/N \cdot m^2$

$d = \frac{(.667 \times 10^{-30})(4\pi)(8.85 \times 10^{-12})(500)}{(1.6 \times 10^{-19})(1 \times 10^{-3})}$

$d = 2.32 \times 10^{-16} m$

ratio of $\frac{d}{R} = \frac{(2.32 \times 10^{-16})}{(.5 \times 10^{-10})} = 4.69 \times 10^{-6}$

$R = .5 \times 10^{-10}$

Estimate voltage you would need w/ this apparatus to ionize atom

$d = R$

so $R = \frac{(.667 \times 10^{-30})(4\pi\epsilon_0) V}{q x}$

$$V = \frac{(.5\times10^{-19} ?)(1\times10^{-3})(1.6\times10^{-19})}{(1.67\times10^{-30})(4\pi)(8.85\times10^{-12})}$$

$V = 1.08\times10^{8}\ V$

4.7) Show that the energy of an ideal dipole p in electric field E is given by $U = -p\cdot E$.

We know $N(\text{Torque}) = pE\sin\theta = \vec{p}\times\vec{E}$

work done when rotating dipole thru $d\theta$ is
$dw = N d\theta$

$dw = pE\sin\theta\, d\theta$

$$W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = pE\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$$

$$= pE(-\cos(\theta_2) + \cos(\theta_1)) = -pE(\cos\theta_2 - \cos\theta_1)$$

we know that $W = U(\theta_2) - U(\theta_1)$

so $U = -pE\cos\theta = -p\cdot E$

4.10) A sphere of radius R carries polarization $P(\vec{r}) = k\vec{r}$, where k is constant, and r is vector from center

a) Calc bound charges σ_b and ρ_b

So $P(r) = kR\hat{r}$ and $\hat{r} = \hat{n}$

$\sigma_b = P \cdot \hat{n}$ $\quad \sigma_b = (kR\hat{r}) \cdot \hat{r} = kR$

Using spherical polar coords, electric polarization is $P(r) = r^2 k r$

we know $\rho_b = -\nabla \cdot P = -\left(\frac{1}{r^2}\frac{\partial (r^3 k)}{\partial r}\right)$

$= -\left(\frac{k}{\cancel{r^2}}\right)\left(3\cancel{r^2}\right) = -3k$

b) Find field inside and outside the sphere

For inside, we know $\rho = \frac{Q_{enc}}{V}$, so $Q_{enc} = \rho V$

$\rho = \rho_b$ and $V = \frac{4}{3}\pi r^3$

So $Q_{enc} = -\cancel{3}k\left(\frac{4}{\cancel{3}}\pi r^3\right) = -3\pi k r^3$

we also know that $\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$\oint E \cdot da = E(4\pi r^2)$

So $E = \frac{Q_{enc}}{4\pi\varepsilon_0 r^2} = -\frac{\cancel{4\pi} k r^{\cancel{3}}}{\cancel{4\pi}\varepsilon_0 \cancel{r^2}} = -\frac{kr}{\varepsilon_0}\hat{r}$

For outside, we know that $q_{tot} = q_{vol} + q_{surface}$ to find charge enclosed by gaussian surface

$q_{vol} = \rho_b\left(\frac{4\pi r^3}{3}\right) = -4\pi k r^3$ $r>R$

$q_{surf} = \sigma_b(4\pi r^2) = (kR)(4\pi r^2)$

$q_{tot} = -4\pi k r^3 + 4\pi k r^3 = 0$

So $\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$ $\bar{E} = 0$ since $Q_{enc} = 0$

4.15) thick spherical shell is made of dielectric material w/ a froze in polarization $P(r) = \frac{k}{r}\hat{r}$, k is constant and r is distance from center. Find electric in all 3 regions by two methods

a) locate all bound charge, use Gauss Law

$\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$P(\vec{r}) = \frac{k}{r}\hat{r}$, we know $\sigma_b = P \cdot \hat{n}$

$\hat{n} = -\hat{r}$

so $\sigma_b = -\frac{k}{r}$ so $\sigma_b(\text{at } r=a) = -\frac{k}{a}$

$\sigma_b(\text{at } r=b) = \frac{k}{b}$

we know $\rho_b = -\nabla \cdot P = -\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P)\right)$

$= -\frac{1}{r^2}\left(\frac{\partial}{\partial r}\left(r^2 \frac{k}{\cancel{r}}\right)\right) = -\frac{k}{r^2}$

For region of $r<a$, $Q_{enc}=0$
So $E=0$ for $r<a$
For $a<r<b$, $Q_{enc}=Q_{vol}+Q_{surface}$
$Q_{surface}=(\sigma_b)(4\pi r^2)=-\frac{k}{\cancel{r}}(4\pi r^{\cancel{2}})=4\pi rk$
$Q_{vol}=(\rho_b)(4\pi r^2)=-\frac{k}{\cancel{r^2}}(4\pi \cancel{r^2})$
$Q_{enc}=-\int_a^r 4\pi k\,dr+4\pi ak$
$=4\pi k(r-a)-4\pi ak=-4\pi rk+\cancel{4\pi ak}-\cancel{4\pi ak}$
$=-4\pi rk$
So $\oint E\cdot da=\frac{-4\pi rk}{\varepsilon_0}$
$E(\cancel{4\pi} r^{\cancel{2}})=\frac{-\cancel{4\pi} rk}{\varepsilon_0}$ $\quad E=-\frac{k}{r\varepsilon_0}\hat{r}$ for $a<r<b$
For $r>b$
$Q_{enc}=\int_a^b -4\pi k\,dr+\frac{k}{b}(4\pi b^2)-4\pi ak$
$=-4\pi k(b-a)+k4\pi b-4\pi ak$
$=\cancel{-4\pi kb}+\cancel{4\pi kb}+\cancel{k4\pi b}-\cancel{4\pi ak}=0$
So $E=0$ for $r>b$

b) No free charge
Use Eq 4.23, to find D, and then get E from Eq 4.21

4.23: $\oint D \cdot da = Q_{f_{enc}}$

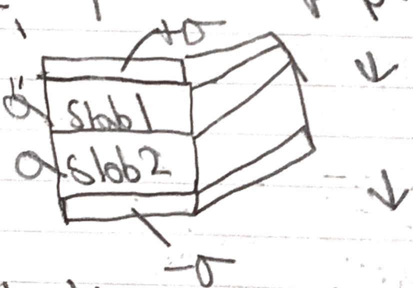
4.21: $D = \varepsilon_0 E + P$

We know that Q_{enc} is 0 for $r<a$ and $r>b$
so $\oint D \cdot da = 0$, $D = 0$

$0 = \varepsilon_0 E + P \Rightarrow E = -\frac{P}{\varepsilon_0}$

$P = \frac{k}{r}\hat{r}$ so $\boxed{E = -\frac{k}{r\varepsilon_0}\hat{r}}$

4.18) The space between the plates of a parallel plate capacitor is filled w/ two slabs of linear dielectric material. Each slab has thickness a, so total distance is 2a between plates. Slab 1 has dielectric constant 2, and Slab 2 has electric constant of 1.5. Free charge density on top plate is σ, and bottom plate is $-\sigma$.



a) Find the electric displacement D in each slab.

$\oint D \cdot da = Q_{f_{enc}}$

$DA = \pm \sigma A$ so $D = -\sigma \hat{z}$

Since we know the field pts down

b) Find the electric field E in each slab

$E = \frac{\sigma}{\varepsilon}$ we have two permittivity so

$\varepsilon = \varepsilon_0 \varepsilon_r$ ← permittivity of the medium

for slab 1 $\varepsilon_{r_1} = 2$

$E = \frac{\sigma}{\varepsilon_0 \varepsilon_{r_1}} = \frac{\sigma}{2\varepsilon_0} = -\frac{\sigma}{2\varepsilon_0}\hat{z}$

for slab 2 $E = \frac{\sigma}{1.5\,\varepsilon_0} = -\frac{2\sigma}{3\varepsilon_0}\hat{z}$

$1.5 = \frac{3}{2}$

c) Find the polarization P in each slab

$P = \varepsilon_0 \chi_e E$ $E = \frac{\sigma}{\varepsilon}$ so $P = \frac{\varepsilon_0 \chi_e \sigma}{\varepsilon}$

$P = \frac{\cancel{\varepsilon_0} \chi_e \sigma}{\cancel{\varepsilon_0} \varepsilon_r} = \frac{\chi_e \sigma}{\varepsilon_r}$ $\chi_e = \varepsilon_r - 1$

$P = \frac{(\varepsilon_r - 1)}{\varepsilon_r}\sigma = (1 - \varepsilon_r^{-1})\sigma$

for slab 1 $P = -(1 - 2^{-1})\sigma\hat{z} = -\frac{\sigma}{2}\hat{z}$

for slab 2 $P = -(1 - \frac{2}{3})\sigma\hat{z} = -\frac{\sigma}{3}\hat{z}$

d) Find the potential difference between

$\Delta V = -\int \vec{E}\cdot d\vec{l} = E_1 a + E_2 a$

$\Delta V = -\frac{\sigma a}{3\times 2\varepsilon_0} - \frac{2\sigma a}{3\varepsilon_0}\times 2 = -\frac{7\sigma a}{6\varepsilon_0}$

e) find the location and amount of all bound charge

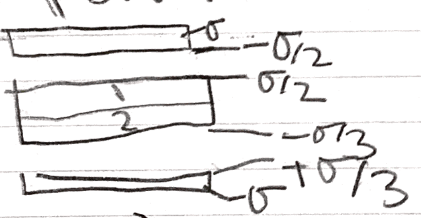
$\sigma_b = P \cdot \hat{n}$ $\quad \sigma_{1b}$ at top $= -\frac{\sigma}{2}$

σ_{1b} at bottom $= \frac{\sigma}{2}$

σ_{2b} at top $= -\frac{\sigma}{3}$ $\quad$ at bottom $\sigma_{2b} = \frac{\sigma}{3}$

So $\sigma_{b\,total} = -\frac{\sigma}{2} + \frac{\sigma}{2} - \frac{\sigma}{3} + \frac{\sigma}{3} = 0$

f) recalculate the field in each slab, confirm answer in part b).



for slab

$\sigma_{1\,top} = \frac{\sigma}{2}$ $\quad \sigma_{1\,bottom} = -\frac{\sigma}{2}$

so $\sigma_{1\,tot} = \frac{\sigma}{2} + +\frac{\sigma}{2} = \sigma$

thus $\vec{E}_1 = -\frac{\sigma}{2\varepsilon_0}\hat{z}$ since $E = \frac{\sigma}{2}$

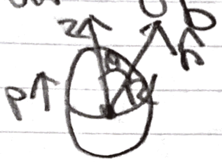
For slab 2 we know total surface for top is $\frac{2\sigma}{3}$ and bottom is $-\frac{2\sigma}{3}$

$\sigma_{2\,tot} = \frac{2\sigma}{3} - -\frac{2\sigma}{3} = \frac{4\sigma}{3}$

So $\vec{E} = \frac{\sigma}{2\varepsilon_0} = \left(\frac{\cancel{4}^{2}\sigma}{3}\right)\left(\frac{1}{\cancel{2}\varepsilon_0}\right) = -\frac{2\sigma\hat{z}}{3}$

Ex 4.2) Find the electric field produced by a uniformly polarized sphere of radius R

We know that $\rho_b = 0$ since $\vec{P}$ is uniform,
but $\sigma_b = \vec{P}\cdot\hat{n} = P\cos\theta$



We know from ex 3.9 that the potential for inside and outside the sphere will be

$$V_{inside}(r,\theta) = \frac{k}{3\varepsilon_0} r\cos\theta$$

$$V_{outside}(r,\theta) = \frac{k}{3\varepsilon_0}\frac{R^3}{r^2}\cos\theta$$

We know that $r\cos\theta = z$

So for electric field inside the sphere is

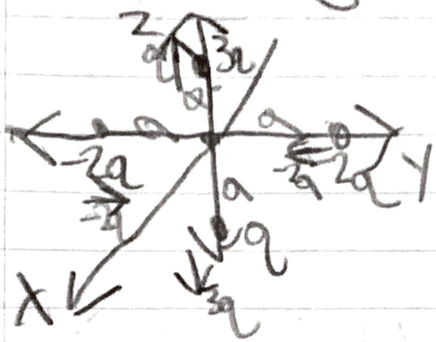
$$E = -\nabla V = -\frac{\partial}{\partial z}\left(\frac{k}{3\varepsilon_0} z\right)\hat{z} = -\frac{k}{3\varepsilon_0}\hat{z} = -\frac{\vec{P}}{3\varepsilon_0} \quad r<R$$

for electric field outside the sphere
let $r=R$

$$V_o(r,\theta) = \frac{k}{3\varepsilon_0}\frac{R^{\cancel{3}}}{\cancel{R^2}}\cos\theta = \frac{k\cos\theta}{3\varepsilon_0}$$

Chapter 3 bonus problem

3.29) 4 particles are placed, each distance a from origin. Find simple approx formula for potential, valid at pts from origin. (express answer in spherical coords)



We know dipole moment is $p = q\vec{d}$

$p = 3qa\hat{z} - qa\hat{z} + -2qa\hat{y} + 2qa(\hat{y})$

$p = (3qa\hat{z} - qa\hat{z}) + \cancel{(-2q)a\hat{y} + 2q(a\hat{y})}$

$p = 2qa\hat{z}$ Now we turn it into spherical coo

we know $V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \frac{p \cdot \hat{r}}{r^2}$

$\hat{z} = \hat{r}\cos\theta$ so $p \cdot \hat{r} = (2qa\hat{z}) \cdot \hat{r} = 2qa(\hat{z} \cdot \hat{r})$

$= 2qa(\hat{r}\cos\theta \cdot \hat{r}) = 2qa\cos\theta$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \frac{2qa\cos\theta}{r^2}}$$