2.50 $V(\vec{r}) = A \frac{e^{-\lambda r}}{r}$

$-\nabla V = E = -\left(A(-\lambda)\frac{e^{-\lambda r}}{r} + \frac{A e^{-\lambda r}}{r^2}\right)\hat{r}$

$= E = A\left(\frac{\lambda r e^{-\lambda r}}{r^2} + \frac{e^{-\lambda r}}{r^2}\right)\hat{r} = \boxed{A\left(\frac{\lambda e^{-\lambda r}}{r} + \frac{e^{-\lambda r}}{r^2}\right)\hat{r}}$

$\rho(r) = \epsilon_0 \nabla \cdot E = \epsilon_0 A\left(\frac{-\lambda^2 e^{-\lambda r}}{r} + \left(-\frac{\lambda e^{-\lambda r}}{r^2}\right) + \frac{\lambda e^{-\lambda r}}{r^2} + 4\pi\delta^3(r)\right)$

r^3 is too small to matter, so

$$\boxed{\rho(r) = \epsilon_0 A\left(4\pi\delta^3(r) - \frac{\lambda^2 e^{-\lambda r}}{r}\right)}$$

$$\int_{-\infty}^{\infty}\int_0^{\pi}\int_0^{2\pi} \epsilon_0 A\left(4\pi\delta^3(r) - \frac{\lambda^2 e^{-\lambda r}}{r}\right) r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \epsilon_0 A 2\pi(2)\int_{-\infty}^{\infty}\left(4\pi\delta^3(r)r^2 - r\lambda^2 e^{-\lambda r}\right)dr$$

$$= \epsilon_0 A(2\pi(2))\left[(\lambda r + 1)e^{-\lambda r} + 0\right]_{-\infty}^{\infty}$$

$$= \epsilon_0 A 4\pi\left[1 - 0 + 0\right] = \boxed{0}$$

3.3 Find general solution to Laplace of only radius for spherical and cylinder

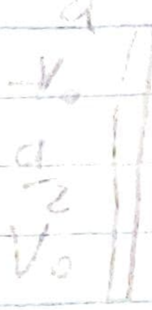
$$\nabla^2 V_{sphere} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \cancel{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right)}$$

$$+ \cancel{\frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}} = \boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right)}$$

$$\nabla^2 V_{cylinder} = \frac{1}{s}\frac{\partial}{\partial s}\left(s \frac{\partial V}{\partial s}\right) + \cancel{\frac{1}{s^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right)} + \frac{\partial^2 V}{\partial z^2}$$

$$= \boxed{\frac{1}{s}\frac{\partial}{\partial s}\left(s \frac{\partial V}{\partial s}\right)}$$

3.13 Find V From Ex 3.3,



(i) $V=V_0$ when $0<y<a/2$, $x=0$

(ii) $V=-V_0$ when $\frac{a}{2}<y<a$, $x=0$

(iii) $V=0$ when $y=0$

(iv) $V=0$ when $y=0$

(v) $V\to 0$ as $x\to\infty$

$V(x,y)=X(x)(Y(y))$ $\quad \frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0$

$$Y\frac{d^2X}{dx^2}+X\frac{d^2Y}{dy^2}=0$$

$$\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}=0$$

$$f(x)+g(Y)=0$$

$$\frac{1}{X}\frac{d^2}{dx^2}=C_1 \quad \frac{1}{Y}\frac{d^2}{dy^2}=C_2,\ C_1+C_2=0$$

$$d^2X/dx^2=k^2X \quad d^2Y/dy^2=-k^2Y$$

$X(x) = Ae^{kx} + Be^{-kx}$

$Y(y) = C\sin ky + D\cos ky$

$V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$

$x \to \infty,\ V \to 0 \therefore A = 0$

$= e^{-kx}(C\sin ky + D\cos ky)$

$V = 0$ when $y = 0 \therefore D = 0$

$= e^{-kx} C\sin ky$

$V = 0$ when $y = a,\ \therefore k = \frac{n\pi}{a}$

$e^{\left(-\frac{n\pi x}{a}\right)} \sin\left(\frac{n\pi y}{a}\right) C_n$

$V(x,y) = \sum_{n=1}^{\infty} C_n e^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)$

$V(0,y) = \sum_{n=1}^{\infty} C_n \cdot 1 \cdot \sin\left(\frac{n\pi y}{a}\right) = \begin{cases} -V_0, & \frac{a}{2} < y < a \\ V_0, & 0 < y < \frac{a}{2} \end{cases}$

$\sum_{n=1}^{\infty} C_n \left(\int_0^a \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{n'\pi y}{a}\right) dy\right) = \int_0^{a/2} V_0 \sin\left(\frac{n'\pi y}{a}\right) dy$

$+ \int_{a/2}^{a} -V_0 \sin\left(\frac{n'\pi y}{a}\right) dy$

$$\int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n'\pi y}{a}\right)dy = \begin{cases} 0, & n \neq n' \\ \frac{a}{2}, & n = n' \end{cases}$$

$$C_n = \frac{2}{a}\left(\int_0^{a/2} V_0 \sin\left(\frac{n\pi y}{a}\right) + \int_{a/2}^{a} -V_0 \sin\left(\frac{n\pi y}{a}\right) dy\right)$$

$$C_n = \frac{2V_0}{a}\left(\int_0^{a/2} \sin\left(\frac{n\pi y}{a}\right) dy - \int_{a/2}^{a} \sin\left(\frac{n\pi y}{a}\right) dy\right)$$

$$C_n = \frac{2V_0}{a}\left(\left[\frac{-a\cos\left(\frac{n\pi y}{a}\right)}{n\pi}\right]_0^{a/2} + \left[\frac{a\cos\left(\frac{n\pi y}{a}\right)}{n\pi}\right]_{a/2}^{a}\right)$$

$$= \frac{2V_0}{n\pi}\left[\cos(n\pi) - \cos\left(\frac{n\pi}{2}\right) - \cos\left(\frac{n\pi}{2}\right) + 1\right]$$

$$= \frac{2V_0}{n\pi}\left[\cos(n\pi) - 2\cos\left(\frac{n\pi}{2}\right) + 1\right]$$

$n=1$, $\frac{2V_0}{\pi}\left[-1-2(0)+1\right] = 0$

$n=2$, $\left[1-(-2)+1\right] \cdot \frac{2V_0}{n\pi} = \frac{4V_0}{\pi}$

$n=3$, $\left[-1-(0\cdot 2)+1\right] = 0$

$n=4$, $\left[1-2(1)+1\right] = 0$

$n=6$, $C_n = \frac{4V_0}{3\pi}$ $(C_8 = 0)$, $C_{10} = \frac{4V_0}{5\pi}$

$$C_n = \frac{2V_0}{a}\int_0^{a/2} \sin\left(\frac{n\pi y}{a}\right)dy - \frac{2V_0}{a}\int_{a/2}^{a} \sin\left(\frac{n\pi y}{a}\right)dy$$

$$= \frac{2V_0}{n\pi}\left[\cos(n\pi) \ldots - 2\cos\left(\frac{n\pi}{2}\right)\right]$$

$C_n = \frac{8V_0}{\pi}$, when n is not a factor of 4.

$$\boxed{V(x,y) = \frac{8V_0}{\pi}\sum_{n=0} \frac{1}{(n\pi)} \sin\left(\frac{2(n+2)\pi y}{a}\right)}$$

only at the [illegible]

all elements not factors of 4.

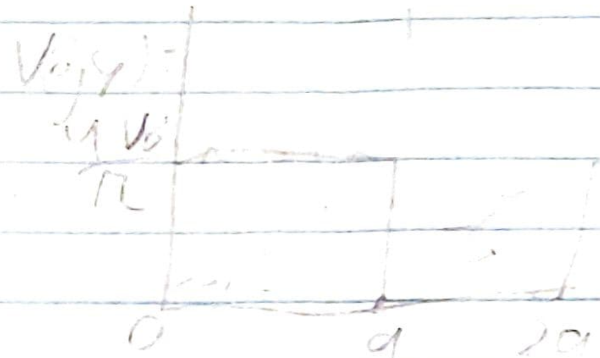
3.14 determine σ for the slot assuming it is constant V_0 at $x=0$

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=0} \frac{1}{(2n+1)} e^{-(2n+1)\pi x/a} \sin\left((2n+1)\pi y/a\right)$$

$$V(0,y) = \frac{4V_0}{\pi} \sum_{n} \frac{1}{2n+1} \sin\left(\frac{(2n+1)y\pi}{a}\right)$$

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial x}\Big|_{x=0}$$

We can approximate $\sin(y)$ component as simply equal to 1 for $0<y<1$ for our purposes because it is a square wave



$$\epsilon_0 \frac{-\partial V}{\partial x} = \frac{-4V_0\epsilon_0}{\pi} \sum_{n=0}^{\infty} \frac{-(2n+1)}{2n+1} e^{\frac{-(2n+1)x}{a}}$$

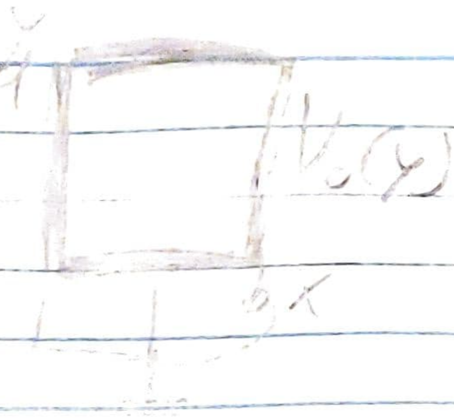
$$= \epsilon_0 \frac{4V_0}{\pi} \sum_{n=0}^{\infty} e^{\frac{-(2n+1)x}{a}}$$

$x=0$

$$\epsilon_0 \frac{4V_0}{\pi} \sum_{n=0}^{\infty} 1 = \boxed{\frac{4V_0\epsilon_0}{\pi}}$$

$\sigma \neq \epsilon_0$

3.15 (a) Develop general formula



(i) when $x=0$, $V=0$

(ii) when $y=0$, $V=0$

(iii) when $y=a$, $V=0$

(iiii) when $x=b$, $V=V_0(y)$

$$V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$$

$x=0$, $V=0$ ∴ $A+B=0$

$A=-B$

$$= (A(e^{kx} - e^{-kx}))(C\sin ky + D\cos ky)$$

$$\sinh kx = \frac{e^{kx} - e^{-kx}}{2}$$

$$=(C_1 \sinh kx)(C\sin ky + D\cos ky)$$

$y=0$, $V=0$, ∴ $D=0$

$y=a$, $V=0$, ∴ $k = \frac{n\pi}{a}$

$$\sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi}{a} x\right) C \sin\left(\frac{n\pi}{a} y\right)$$

$V(b,y)$ $V(x,y) = \sum_{n=1} C_n \sinh\left(\frac{n\pi b}{a}\right) \sin\left(\frac{n\pi}{a} y\right)$

$= V_0$

$$V(x,y) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi}{a} y\right)$$

(b) But like what if $V_0(y)$ were V_0 the whole time?

$$V(x,y) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi x}{a}\right) \sin\left(\frac{n\pi}{a} y\right)$$

$$V(b,y) = V_0 = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi b}{a}\right) \sin\left(\frac{n\pi}{a} y\right)$$

$$C_n \sinh\left(\frac{n\pi b}{a}\right) = \begin{cases} 0, & n \text{ is even} \\ \frac{4V_0}{n\pi}, & n \text{ is odd} \end{cases}$$

$$C_n = \frac{4V_0}{n\pi \sinh\left(\frac{n\pi b}{a}\right)}$$

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=0}^{\infty} \frac{1}{(2n+1)} \frac{\sinh\left(\frac{(2n+1)\pi x}{a}\right)}{\sinh\left(\frac{(2n+1)\pi b}{a}\right)} \sin\left(\frac{n\pi}{a} y\right)$$