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ET Quiz #3

1. a) $\vec{V} = a\hat{x} + b\hat{y} + c\hat{z}$ $\qquad$ $\vec{V}\cdot\hat{z} = \hat{z}(a\hat{x} + b\hat{y} + c\hat{z})$

$\hat{x}_i \cdot \hat{x}_j = 0$ $\qquad$ $\vec{V}\cdot\hat{z} = c$

$\hat{x}_i \cdot \hat{x}_i = 1$

$\boxed{B}$

b) $\vec{x} = \sum_{i=1}^{n} C_i \hat{x}_i = C_7$

$\vec{V}\cdot\hat{x}_m = \sum_{i=1}^{n} C_n \hat{x}_i + \hat{x}_m = C_m$ $\qquad$ $\boxed{D}$

$\vec{V}\cdot\hat{x}_7 = \sum_{i=1}^{n} C_i \cdot \hat{x}_i \cdot \hat{x}_7 = C_7$

c) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx) + b_n \sin(nx)$ $\qquad$ $f(x) = \sin(3x)$

$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(nx)\,dx$

$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(nx)\,dx$

$$f(x) = \begin{cases} 1, & 0 \le x \le \pi \\ 0, & \pi \le x \le 2\pi \end{cases}$$

$a_n = \frac{1}{\pi}\int_0^{\pi} \sin(nx)\,dx$

$= \frac{1}{\pi}\int_0^{\pi} \sin(nx)\,dx = -\frac{1}{n\pi}\cos(nx)\Big|_0^{\pi}$

$= -\frac{1}{n\pi}(\cos(n\pi) - \cos(0))$

$= -\frac{1}{n\pi}(\cos(n\pi) - 1)$ $\qquad$ even n: 0

odd n: $\frac{2}{n\pi}$

$a_0 = 0$

$b_n = \frac{1}{\pi}\int_0^{\pi} \cos(nx)\,dx$

$b_n = \frac{1}{\pi n}\sin(nx)\Big|_0^{\pi} = \frac{1}{\pi n}(0 - 0) = 0$

$b_n = 0$

$f(x) = 0 + \sum_{n=\text{odd}}^{\infty} \frac{2}{n\pi}\cos(nx) = \frac{2}{\pi}\sum_{n=\text{odd}}^{\infty} \frac{\cos(nx)}{n}$

coefficients: all odd numbers from $n=0$ $n=\infty$

2. a) $V(x,y,z) \to 0 \quad y \to \infty$

$\sinh(\infty) = \infty$ $\quad \boxed{B}$

$\infty \neq 0$

b) $C_{n,m} = \frac{4V_0}{ab}\int_0^a \int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n\pi z}{a}\right) dy\,dz$

$= \frac{4}{ab} \cdot \int_0^a \sin\left(\frac{3\pi y}{a}\right) dy \cdot \int_0^b \sin\left(\frac{5\pi z}{b}\right) dz$

$\int_0^a \sin\left(\frac{3\pi y}{a}\right) dy = \frac{2}{3\pi} a$

$= \frac{4}{ab} \cdot \frac{2}{3\pi} a \cdot \int_0^b \sin\left(\frac{5\pi z}{b}\right) dz$

$\int_0^b \sin\left(\frac{5\pi z}{b}\right) dz = \frac{2}{5\pi} b$

$= \frac{4}{ab} \cdot \frac{2a}{3\pi} \cdot \frac{2b}{5\pi} = \boxed{\frac{16}{15\pi^2}}$

$= \frac{4}{ab} \cdot \int_0^a \sin\left(\frac{2\pi y}{a}\right) dy \cdot \int_0^b \sin\left(\frac{2\pi z}{b}\right) dz$

$\int_0^a \sin\left(\frac{2\pi y}{a}\right) dy = 0$

$= \frac{4}{ab} \cdot 0 \cdot \int_0^b \sin\left(\frac{2\pi z}{b}\right) dz$

$\int_0^b \sin\left(\frac{2\pi z}{b}\right) dz = 0$

$= \frac{4}{ab} \cdot 0 \cdot 0 = \boxed{0}$