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EMT HW #5

5.4) $\vec{B} = kz\hat{x}$

$k =$ constant

$\vec{F}_{I} = I(-a\hat{y}) \times (kz\hat{x})$

where $z = a/2$

$F_{I} = I(-a\hat{y}) \times (\frac{ka}{2}\hat{x})$

$\hat{y} \times \hat{x} = -\hat{z}$

$\vec{F}_{III} = -\frac{Ia^2k}{2}(\hat{z})^2$

So $\boxed{\vec{F}_{I} = \frac{-Ia^2k}{2}\hat{z}}$

$\vec{F}_{II} = \int I(dz\hat{z}) \times (kz\hat{x})$

$\Rightarrow \int I\,dz\,kz\hat{y} = Ik\int_{-a/2}^{a/2} z\,dz\,\hat{y} = \boxed{0}$

So

$F_{IV} = \int I(-dz\hat{z}) \times (kz\hat{x}) = -\int Ikz\,dz\,\hat{y}$

$\Rightarrow -Ik\int_{a/2}^{-a/2} z\,dz\,\hat{y} \Rightarrow \boxed{0}$

Then

$F_{net} = I_{II} + F_{III} = 2\left(\frac{Ia^2k}{2}\right)\hat{z} = \boxed{Ia^2k\hat{z}}$

5.7 $\int_V J\,d\tau = dp/dt$

$$\frac{dp}{dt} = \frac{d}{dt}\int_V \rho\, r\, d\tau = \int \left(\frac{d\rho}{dt}\right) r\, d\tau$$

$$\Rightarrow -\int (\nabla \cdot J)\, r\, d\tau$$

So $-\frac{d\rho}{dt} = \nabla \cdot J$

$$\nabla \cdot (xJ) = x(\nabla \cdot J) + J \cdot (\nabla x)$$

$$\Leftrightarrow x(\nabla \cdot J) + J \cdot \hat{x}$$

$$\Rightarrow x(\nabla \cdot J) + J_x$$

$$\Rightarrow \int_V \nabla \cdot (xJ) \Rightarrow \int_V x(\nabla \cdot J)\, d\tau + \int_V J_x\, d\tau$$

$\Rightarrow \nabla \cdot (xJ) = \int_S xJ \cdot da = 0$, J is an int current

$$\Rightarrow \int_V x(\nabla \cdot J)\, d\tau = -\int_V J_x\, d\tau \Rightarrow \int_V (\nabla \cdot J)\, r\, d\tau = -\int_V J\, d\tau$$

Then we get the beginning line

$$\int_V J\, d\tau = \frac{dp}{dt}$$

5.11) From Ch 5, using Eq 5.41

$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos(\theta)}{r^2}\right)2\pi R = \frac{\mu_0 I}{2} \frac{R^2}{(R^2+z^2)^{3/2}}$$

$$I \to nI$$

$$B = \frac{\mu_0 nI}{2}\int \frac{a^2}{(a^2+z^2)^{3/2}}\,dz$$

Where we have $z = a\cot(\theta)$

Given this,

$$dz = \frac{-a}{\sin^2(\theta)}\,d\theta$$

Also,

$$\frac{1}{(a^2+z^2)^{3/2}} = \frac{\sin^3(\theta)}{a^3}$$

$$B = \frac{\mu_0 nI}{2}\int \frac{a^2\sin^3(\theta)}{a^3\sin^2(\theta)}(-a\,d\theta)$$

$$\Rightarrow -\frac{\mu_0 nI}{2}\int \sin(\theta)\,d\theta = \frac{\mu_0 nI}{2}\cos(\theta)\Big|_{\theta_1}^{\theta_2} = \frac{\mu_0 nI}{2}(\cos(\theta_2) - \cos(\theta_1))$$

In an ∞ solenoid, $\theta_2 = 0$ & $\theta_1 = \pi$
Therefore

$B = \mu_0 nI$ since $(\cos(\theta_2) - \cos(\theta_1)) = 1-(-1) = 2$

$$B = \frac{\mu_0 nI}{\cancel{2}}\cancel{(2)} = \boxed{B = \mu_0 nI}$$

5.12 Using ex 5.6

Mag. Field about a loop, at the axis is:

$$B = \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{(z^2+a^2)^{3/2}} \quad \& \quad \sigma = \frac{Q}{4\pi R^2}$$

While looking at a loop we get

$dq = 2\pi(R\sin(\theta))Rd\theta\sigma$

↑ gives rad. of loop at an angle.

Current in the loop is given by

$$dI = dq/dt = 2\pi(R\sin\theta)Rd\theta\sigma \times \frac{\omega}{2\pi}$$

$a = R\sin(\theta)$, then we plug in a & dI

$$dB = \frac{\mu_0}{2} \frac{R^2\sin^2(\theta)dI}{R^3} = \frac{\mu_0}{2} \frac{\sin(\theta)dI}{R}$$

$$\Rightarrow dB = \frac{\mu_0 R\sigma\omega}{2}\sin^3(\theta)d\theta$$

Integrate from 0 to π

$$B_{total} = \int_0^{\pi} \frac{\mu_0 R\sigma\omega}{2}\sin^3(\theta)d\theta \Rightarrow \frac{\mu_0 R\sigma\omega}{2} \times \frac{4}{3}$$

Reduce & simplify

$$\boxed{B_{total} = \frac{2\mu_0 R\sigma\omega}{3}}$$

5.16 As shown in 5.11, $\underline{B = \mu_0 n I}$ (inside)

& Mag field outside is 0. $B_{outside} = 0$

1) The inner solenoid is solved by taking
Mag field due to the outer solenoid & the inner
solenoid.
Doing so we get $B_{outer} = \mu_0 n_2 I$ &
$B_{inner} = -\mu_0 n_1 I$
Therefore the magnetic field in the inner solenoid
point is $\boxed{B = \mu_0 (n_2 - n_1) I}$

2) Between the two, due to the inner solenoid
& since this one is between the two,
we get $\boxed{B = \mu_0 n_2 I}$

3) Since the 3rd point is outside both &
the mag field outside is 0,
the mag field outside both inner & outer solenoid
is $\boxed{0.}$

5.19)

$$I_{enc} = \int_S J \cdot da$$

$\nabla \times \vec{B} = \mu \vec{J}$, which is Ampere's Law

In Turn,

$$I_{encl} = \int_S J \cdot da = \frac{1}{\mu_0} \int_S (\nabla \times \vec{B}) \cdot d\vec{A}$$

Use Stokes theorem.

$$I_{enc} = \frac{1}{\mu_0} \oint_C \vec{B} \cdot d\vec{l}$$

We can choose any surface with the same boundary C, it is dependent on a line int,

Due to this it is independent of the surface used.

5.21)

$$\nabla \times B = \mu_0 J$$

$$\nabla (\nabla \times B) = \mu_0 (\nabla \times J) \Rightarrow \mu_0 \frac{d\rho}{dt}$$

- Currents are fixed due to it requiring ρ to be constant.

5.23) Find my vector potential.

∞ I (z_1, z_2)

$$A = \frac{\mu_0 I}{4\pi} \int dl' = \frac{\mu_0 I}{4\pi} \hat{z} \int_{z_1}^{z_2}$$

$$A = \frac{\mu_0 I}{4\pi} \ln \left(\frac{z_2 + \sqrt{z_2^2 + s^2}}{z_1 + \sqrt{z_1^2 + s^2}} \right) \hat{z}$$

$$\boxed{B = \nabla \times A = -\frac{dA}{ds} \hat{\phi}}$$

5.29) Vector potential above & below plane surface current in Ex 5.8

$$B = \frac{\mu_0 K}{2} \hat{y}$$

$$\vec{A} = A(z)\,\hat{x}$$

$$\frac{dA}{dz} = \frac{\mu_0 K}{2}$$

$$\boxed{A = \frac{\mu_0 K z}{2} \hat{x}}$$