Striker Hard
11/5/20
Phys 330

2.5, 2.6, 2.9, 2.12, 2.16, 2.18, 2.25, 2.29

Chapter 2 HW: Electrodynamics

2.12). Gausses law E-field in a solid sphere ρ = charge density

$\oint_S E \cdot da = \frac{1}{\epsilon_0} Q_{enc}$ $\quad \oint E \cdot da = E \cdot 4\pi r^2 = \frac{1}{\epsilon_0} Q_{enc} = \frac{1}{\epsilon_0} \frac{4}{3}\pi r^3 \rho$

$\oint \bar{E} \cdot dA = \frac{q_{enc}}{\epsilon_0}$

$E(4\pi r^2) = \frac{q_{enc}}{\epsilon_0} \hat{r}$

$\bar{E} = \frac{1}{4\pi\epsilon_0} \frac{q_{enc}}{r^2} \hat{r}$ $\quad Q_{enc} \to V$

$q_{enc} = \frac{Q_{enc}}{V} V$ $\quad \rho = \frac{Q_{in}}{V}$ $\quad q_{enc} = \rho V$ $\quad V = \frac{4}{3}\pi r^3$

$q_{enc} = \frac{4}{3}\rho\pi r^3$

$\frac{4/3}{4} = 1/3$

$\bar{E} = \frac{1}{\cancel{4\pi}\epsilon_0} \frac{4/3 \rho \cancel{\pi} r^{\cancel{3}}}{r^2} \hat{r}$

$\frac{1/3 \rho r}{\epsilon_0} \hat{r}$ $\quad \boxed{E = \frac{\rho r}{3\epsilon_0} \hat{r}}$

2.16). ρ cylinder radius a

using gausses law $\oint \bar{E} \cdot da = \frac{Q_{enc}}{\epsilon_0}$ $\quad E \cdot 4\pi s^2 = \frac{1}{\epsilon_0} \rho\pi s^2 l$

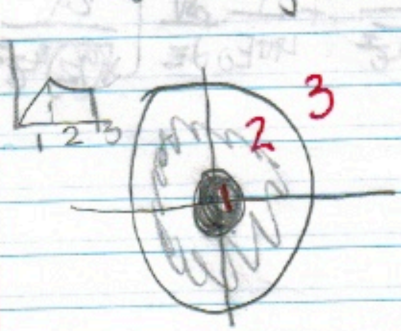
$s < a$ $\quad \boxed{E_s = \frac{\rho s}{2\epsilon_0} \hat{s}}$

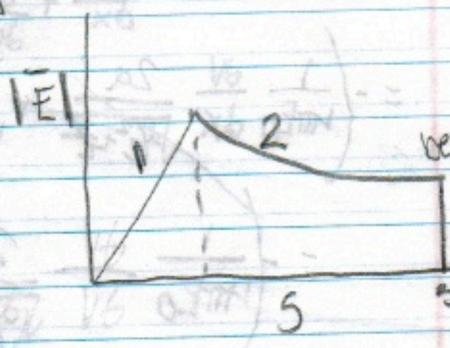
$Q_{enc} = \rho\pi a^2 l$

$a < s < b$ $\quad E \cdot 4\pi s^2 = 1/\epsilon_0 \, \rho\pi a^2 l$ $\quad \boxed{E_a = \frac{\rho a^2}{2\epsilon_0 s} \hat{s}}$

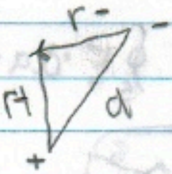
$s > b$ $\quad E \cdot 2\pi s \cdot l = \boxed{\frac{1}{\epsilon_0} Q_{enc} = 0}$

outter charge-density $-\rho$
inner charge density $+\rho$





2.18). radius R $+\rho$ & $-\rho$ from 2.12 $E=\frac{\rho r}{3\epsilon_0}\hat{r}$

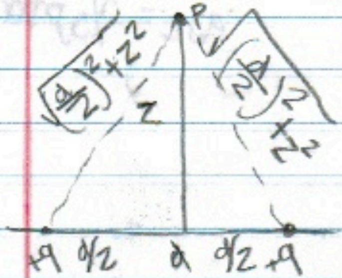


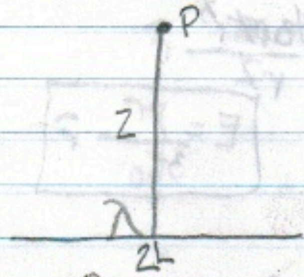
$$E(r)=\frac{\rho(\bar{r}-\bar{r}')}{3\epsilon_0}$$

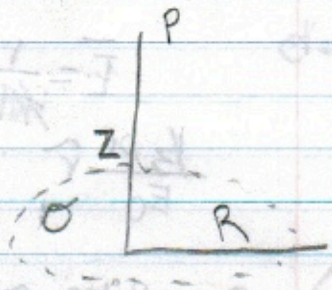
Spheres are partially overlapping therefore

$$E_{tot}=\frac{\rho\bar{r}}{3\epsilon_0}-\frac{\rho(\bar{r}-\bar{d}')}{3\epsilon_0}=\boxed{\bar{E}=\frac{\rho\bar{d}}{3\epsilon_0}}$$

2.25). $E=-\nabla V$ for each case







Eqn 2.27 = $V(r)=\frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$, Eqn 3.30

$$V=\frac{1}{4\pi\epsilon_0}\left(\frac{q}{\sqrt{(\frac{d}{2})^2+z^2}}+\frac{q}{\sqrt{(d/2)^2+z^2}}\right)$$ common denominator

$$\boxed{V=\frac{1}{4\pi\epsilon_0}\left(\frac{2q}{\sqrt{(d/2)^2+z^2}}\right)=\text{potential @ point P}}$$

$E=-\nabla V \qquad -\frac{dV}{dx}+\frac{dV}{dy}+\frac{dV}{dz}$

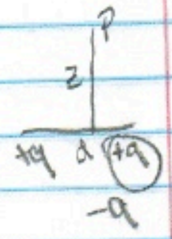
$$=-\left(\cancel{\frac{1}{4\pi\epsilon_0}\frac{dV}{dx}\frac{2q}{\sqrt{d/2^2+z^2}}}+\cancel{\frac{1}{4\pi\epsilon_0}\frac{dV}{dy}\frac{2q}{\sqrt{d/2^2+z^2}}}+\frac{1}{4\pi\epsilon_0}\frac{dV}{dz}\left(\frac{2q}{\sqrt{(d/2)^2+z^2}}\right)\right)$$

$$=-\left(\frac{1}{4\pi\epsilon_0}\frac{dV}{dV}\frac{2q}{\sqrt{(d/2)^2+z^2}}\right)$$

2.25 continued

$$= \frac{\sigma 2\pi}{4\pi E_0}\left(\sqrt{z^2+R^2}-z\right)$$

$$= \frac{dv}{dz} = \left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \quad \boxed{= \frac{\sigma}{2E_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z}}$$



If we were to change the right hand charge to a "-q" we would end up with a net charge of zero since the two charges would offset.

2.29). Poissons equation Eqn 2.29 Eqn 1.102

Eqn 2.29 $= V(r) = \frac{1}{4\pi E_0}\int \frac{p(r')}{\mathscr{r}} d\tau$

Eqn 1.102 $= \nabla^2 \frac{1}{\mathscr{r}} = -4\pi \delta^3(\mathscr{r})$

$$\nabla^2 V = \frac{p}{E_0} = \frac{1}{4\pi E_0}\int \frac{p(r')}{\mathscr{r}} d\tau'$$

$$\nabla^2 \frac{1}{\mathscr{r}} = -4\pi\delta^3(\mathscr{r}) = -4\pi\delta^3(r-r')$$

Laplacian: $\nabla^2 V(r) = \frac{1}{4\pi E_0}\int \nabla^2\left(\frac{1}{\mathscr{r}}\right) p(r') d\tau$

$$\nabla^2 V(r) = \frac{1}{4\pi E_0}\int -4\pi\delta^3(r-r') p(r') d\tau$$

$$\frac{-4\pi}{4\pi E_0}\int \delta^3(r-r') p(r') d\tau$$

$$= \frac{-1}{E_0}\int p(r')\delta^3(r-r') d\tau$$

$$\int p(r')\delta^3(r-r') d\tau = p(r)$$

$$= -\frac{1}{E_0}(p(r)) \quad \boxed{= \frac{-p(r)}{E_0} = \nabla^2 V}$$

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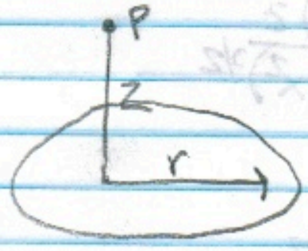
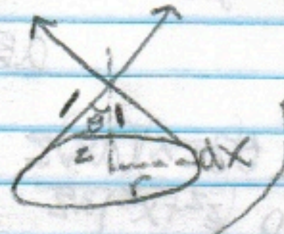
Electrodynamics: 2.5, 2.6, 2.9

2.5). z circular loop radius r   SOH CAH TOA



circular loop



$R^2 = r^2 + z^2$

$\cos\theta = \frac{z}{\sqrt{r^2+z^2}}$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\cos\theta$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+z^2}\cos\theta$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(r^2+z^2)}\left(\frac{z}{\sqrt{r^2+z^2}}\right)^{1/2}$$

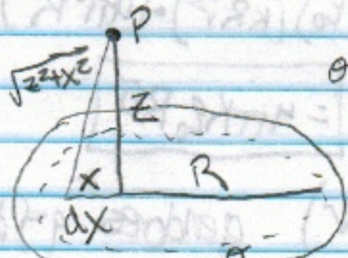
wolfram $E = \int dE$

$$E = \int \frac{\lambda z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}\, ds$$

$$\frac{\lambda}{4\pi\varepsilon_0}\frac{z}{(r^2+z^2)^{3/2}}(2\pi r)$$

$$\boxed{E = \frac{\lambda z r}{2\varepsilon_0 (r^2+z^2)^{3/2}}}$$

2.6).



σ = charge density of disk

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{(dq)}{\sqrt{z^2+x^2}}\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{dq}{(z^2+x^2)}\left(\frac{z}{\sqrt{z^2+x^2}}\right)$$

$$\frac{1}{4\pi\varepsilon_0}\frac{dq}{(z^2+x^2)}\left(\frac{z}{\sqrt{z^2+x^2}}\right)$$

$$\frac{1}{4\pi\varepsilon_0}\frac{dq}{z^2+x^2}\frac{z}{z^2+x^{2\,1/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{(dq)z}{(z^2+x^2)^{3/2}}$$

Charge density:

$$\sigma = \frac{dq}{(2\pi x)dx} \qquad dq = \sigma(2\pi x)dx$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{(dq)z}{(z^2+x^2)^{3/2}}$$

$$E = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{\sigma(2\pi x)(dx)z}{(z^2+x^2)^{3/2}} \qquad \int_0^R \frac{x\,dx}{(x^2+z^2)^{3/2}}$$

$$= \frac{-1}{\sqrt{\cancel{x}^2+z^2}^{3/2}}$$

$$E = \frac{\sigma z}{2\varepsilon_0}\left\{\frac{-1}{\sqrt{z^2+x^2}}\right\}_0^R$$

2.9). $E = Kr^3\hat{r}$ K is constant

a). charge density ρ

$\rho = \varepsilon_0(\nabla\cdot E)$ in spherical cordinates $\nabla\cdot E = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2E_r)$

$$E_r = Kr^3\hat{r}$$

$$\varepsilon_0\left\{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2(Kr^3))\right\} \quad \boxed{\rho = 5K\varepsilon_0 r^2}$$

Gauses law: $\oint E\cdot da = \frac{q}{\varepsilon_0}$ $\quad q_{enc} = \varepsilon_0\int(KR^3\hat{r})\cdot(4\pi R^2)\hat{r}$

$$= \varepsilon_0(4\pi KR^5) \quad \boxed{= 4\pi K\varepsilon_0 R^5}$$

$dq = \rho d\tau$

$$= 5K\varepsilon_0 r^2(4\pi r^2 dr) \quad q_{enclores}\ q = \int dq$$

$$= \int_0^R (5K\varepsilon_0 r^2)(4\pi r^2 dr)$$

$$\boxed{= 4\pi K\varepsilon_0 R^5}$$