Warm-Up for February 28th, 2022

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1 Memory Bank

1. Integration by parts in 1D: $\int_a^b udv = uv|_a^b - \int vdu$
2. Integration by parts in 3D: $\int_{\mathcal{V}} (\nabla \cdot \mathbf{E}) V d\tau = \oint_{\mathcal{S}} V\mathbf{E} \cdot d\mathbf{A} - \int_{\mathcal{V}} \mathbf{E} \cdot (\nabla V) d\tau$
3. Work required to assemble a continuous charge distribution: $W = \frac{1}{2} \int \rho V(\mathbf{r}) d\tau$

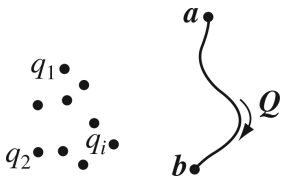

Figure 1: By bringing a new charge toward a charge distribution, we are performing work: $W = Q\Delta V = Q(V(\mathbf{r}) - V(\infty))$.

2 Product Rule

Using the product rule, show that

$$\int_0^\infty x e^{-x} dx = 1 \tag{1}$$

3 Work and Energy

Using items 2 and 3 in the memory bank and $\rho = \epsilon_0 \nabla \cdot \mathbf{E}$, show that the work to assemble a charge distribution is

$$W = \frac{\epsilon_0}{2} \left[\oint V\mathbf{E} \cdot d\mathbf{a} - \int \mathbf{E} \cdot (\nabla V) d\tau \right] \tag{2}$$

Substitute $\mathbf{E} = -\nabla V$ to find how $W \propto E^2$.