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11/6/20 PHYS

Reading Quiz 2 for Electromagnetic Theory (PHYS330)

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November 6, 2020

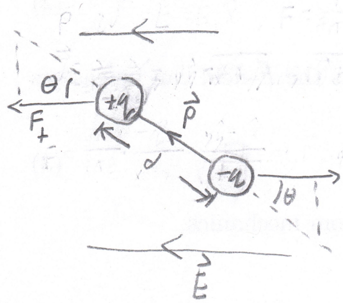
Abstract

A summary of content covered in chapter 2 of Introduction to Electrodynamics.

1 Distributions of Point Charges

1. Picture a *physical dipole* of two charges $+q$ and $-q$ of equal magnitude, separated by a distance d. Define the dipole moment as $\vec{p} = q\vec{d}$ pointing from $-q$ to q somewhere in the xy-plane. Now add an external electric field $\vec{E} = E_0\hat{x}$. Show that the *torque* on the dipole is $\vec p = q\vec d$, $\vec E = E_0\hat x$, $F = q\vec E$, $\tau = F\cdot d$

$$\vec{\tau} = \vec{p} \times \vec{E} = \vec p \vec E \sin\theta \quad (1)$$

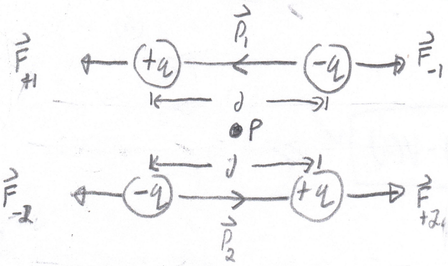


$\vec F_+ = +q\vec E\sin\theta$

$\vec F_- = -q\vec E\sin\theta \Rightarrow \vec\tau = dq\vec E\sin\theta \Rightarrow \vec\tau = p\vec E\sin\theta$

$\boxed{\vec\tau = p\vec E\sin\theta = \vec p\times\vec E}$

2. Imagine two dipoles, each with dipole moments $\vec{p}_1$ and $\vec{p}_2$ pointed in opposite directions, forming a square with alternating positive and negative charges. Calculate the electric field vector in the center of the square.



$\vec p_1 = q\vec d$, $\vec p_2 = q\vec d$, $\vec F = qE$

$\vec E = \frac{1}{4\pi\epsilon_0}\left(\frac{+2q}{d^2} + \frac{-2q}{d^2}\right)\hat d \Rightarrow \boxed{\vec E = 0}$

// Also symmetry shows the forces will cancel, so $\vec E$ should equal 0. //

2 Continuous Charge Distributions

$dQ = \lambda dL$

1. (a) Compute the electric field of a continuous line of charge, with total charge $Q = \lambda L$, where λ is the charge density and L is the total length. Take the field point to be a distance z above the center of the line of charge. Show what happens in the limit that $L \gg z$. (b) Obtain the same result as (a) using Gauss' Law.

(a) $dq = \lambda dx$, $\vec{\mathscr{r}} = \vec r - \vec r\,' \Rightarrow \vec{\mathscr{r}} = z\hat z - x\hat x$, $\mathscr{r}^2 = z^2 + x^2$, $dl' = dx$

$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}} = \frac{z\hat z - x\hat x}{(z^2+x^2)^{1/2}}$ $\quad \vec E = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda}{\mathscr{r}^2}\hat{\mathscr{r}}\, dl'$

$\vec E = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(z\hat z - x\hat x)}{(z^2+x^2)^{3/2}}dx \Rightarrow \vec E = \frac{\lambda}{4\pi\epsilon_0}\left[z\hat z\int_{-L/2}^{L/2}\frac{1}{(z^2+x^2)^{3/2}}dx - \hat x\int_{-L/2}^{L/2}\frac{1}{(z^2+x^2)^{3/2}}dx\right]$

$\vec E = \frac{\lambda}{4\pi\epsilon_0}\left[z\hat z\frac{x}{z^2\sqrt{z^2+x^2}}\right]_{-L/2}^{L/2} - \hat x\left[\cancel{\frac{1}{\sqrt{z^2+x^2}}}\right]_{-L/2}^{L/2} \Rightarrow \vec E = \frac{\lambda}{4\pi\epsilon_0}\left[\frac{(\frac{L}{2})\hat z}{z\sqrt{z^2+\frac{1}{4}L^2}} + \frac{(+\frac{L}{2})\hat z}{z\sqrt{z^2+\frac{1}{4}L^2}}\right]$

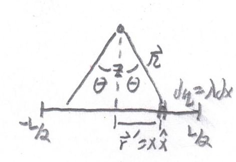
$\boxed{\vec E = \frac{1}{4\pi\epsilon_0}\frac{\lambda L}{z\sqrt{z^2+\frac{1}{4}L^2}}\hat z}$

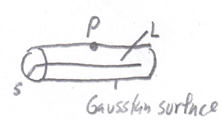
(b) $\oint_S \vec E\cdot d\vec a = \frac{1}{\epsilon_0}Q$

$\int \vec E\cdot d\vec a = \int|\vec E|da = |\vec E|\int_0^L\int_0^{2\pi} s\, dl\, d\phi$

$|\vec E|L2\pi s = \frac{Q}{\epsilon_0}$ $\quad$ // $Q = \lambda L$, $s = z$ //

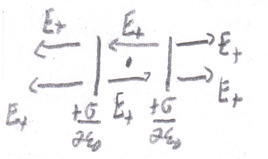
$|\vec E|L2\pi z = \frac{\lambda L}{\epsilon_0} \Rightarrow \boxed{\vec E = \frac{\lambda}{2\pi\epsilon_0 z}}$





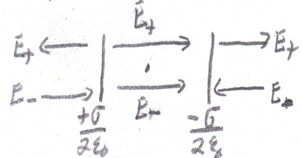
2. Assuming a plane of charge with charge density (Coulombs per unit area) σ has an electric field $\sigma/(2\epsilon_0)$, what electric fields would occur in each of the following situations:

   - Two planes of positive charge, and the field point is somewhere between the plates.
   - Two planes of charge, one positive and one negative, and the field point is somewhere between the plates.
   - Two planes of positive charge, one occupying the yz-plane, and the other occupying the xz-plane, and the field point is $(1, 1, 0)$.

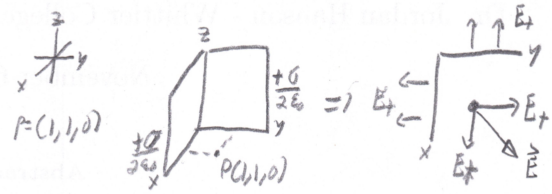


$\boxed{E=0}$

Positives repel



$\boxed{\vec{E}=\frac{\sigma}{\epsilon_0}}$



$\boxed{\vec{E}=\frac{\sigma}{2\epsilon_0}\hat{x}+\frac{\sigma}{2\epsilon_0}\hat{y}}$

3 The Curl of $\vec{E}$-fields

1. According to Eq. 2.19 in the text, the close loop line integral for the E-field of a point charge is

$$\oint \vec{E} \cdot d\vec{l} = 0 \Rightarrow \nabla \times \vec{E} = 0 \tag{2}$$

This implies $\nabla \times \vec{E} = 0$. According to the Helmholtz theorem in Ch. 1, this means the $\vec{E}$-field can be cast as the gradient of a scalar function known as *the potential*, V:

$$\vec{E} = -\nabla V \tag{3}$$

The minus sign is a convention that is analogous to the minus sign in $\vec{F} = -\frac{dU}{dx}\hat{x}$ from mechanics.

   - Show that

$$-\int_a^B \vec{E} \cdot d\vec{l} = V(\vec{b}) - V(\vec{a}) \tag{4}$$

   - Assume a point charge at the origin, and label its electric field $\vec{E}$. Perform the integral

$$V(\vec{r}) = -\int_{\infty}^{r} E(r')dr' \tag{5}$$

   to find the potential formula for a point charge. *[Answer: kq/r]*

$$-\int_a^B \vec{E}\cdot d\vec{l} = -\int_a^{\mathcal{O}} \vec{E}\cdot d\vec{l} + \int_{\mathcal{O}}^{B} \vec{E}\cdot d\vec{l} \Rightarrow -\int_{\mathcal{O}}^{B} \vec{E}\cdot d\vec{l} - \int_a^{\mathcal{O}} \vec{E}\cdot d\vec{l} \Rightarrow -\int_{\mathcal{O}}^{B} \vec{E}\cdot d\vec{l} - \left(-\int_{\mathcal{O}}^{a} \vec{E}\cdot d\vec{l}\right) \Rightarrow \boxed{V(b)-V(a)}$$

$// V(B) = -\int_{\mathcal{O}}^{B} \vec{E}\cdot d\vec{l},\ V(a) = -\int_{\mathcal{O}}^{a} //$

$$V(\vec{r}) = -\int_{\infty}^{r} E(r')dr' \Rightarrow -\int_{\infty}^{r}\left(k\frac{q}{r'^2}\right)dr' \Rightarrow +k\left[\frac{+q}{r'}\right]_{\infty}^{r} \Rightarrow k\left[\frac{q}{r}-\cancel{\frac{q}{\infty}}^{0}\right] \Rightarrow \boxed{V(\vec{r}) = k\frac{q}{r}}$$