due April 22, 2022

repeat the arguments & focus on how the closed line integral of magnetic vector potential is equal to the magnetic flux

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PHYS 330

HW 5: #5.4, 5.7, 5.11, 5.12, 5.16, 5.19, 5.21, 5.12, 5.23, 5.27

5.4) $F = I\int dl \times B$

left & right side cancel eachother b/c the forces exist in the top & bottom

$F_{top} = I\int dl \times B$

$= I\int_0^a dz\hat{y} \times kz\hat{x}$

$= IK\int_0^a z\,dz\,\hat{z}$

$= IK\frac{z^2}{2}\hat{z}$

$= \frac{1}{2}IKa^2\hat{z}$

$F_{bottom} = \frac{1}{2}IKa^2\hat{z}$

$F_{net} = IKa^2\hat{z}$

5.7) $\iiint_V(\nabla\cdot F)\,dV = \oiint_S(F\cdot n)\,dS$

$\bar{p} = \rho\bar{r}$ where ρ is volume charge density

$\frac{d\bar{p}}{dt} = \frac{d}{dt}\int_V \rho\bar{r}\,d\tau$

$= \int\left(\frac{\partial\rho}{\partial t}\right)r\,d\tau$

$-\nabla\cdot\bar{J} = \frac{\partial\rho}{\partial t}$ where J is volume current density

$\frac{d\bar{p}}{dt} = -\int(\nabla\cdot\bar{J})\bar{r}\,d\tau$

$\nabla\cdot(x\bar{J}) = x(\nabla\cdot\bar{J}) + J\cdot(\nabla x)$, $\nabla x = \hat{x}$

$= x(\nabla\cdot\bar{J}) + J\cdot\hat{x}$

$= x(\nabla\cdot\bar{J}) + J_x$

$\int_V(\nabla\cdot\bar{J})x\,d\tau = \underbrace{\int_V\nabla\cdot(x\bar{J})\,d\tau}_{\Downarrow} - \int_V\bar{J}_x\,d\tau$

$\int_S x\bar{J}\cdot d\bar{a} \Rightarrow 0$ b/c J is volume and it's ∅ on surface

$\int_V(\nabla\cdot\bar{J})x\,d\tau = -\int_V\bar{J}_x\,d\tau$

Thus $\boxed{\frac{d\bar{p}}{dt} = \int_V J_x\,d\tau}$

5.11) Suppose n is large ⇒ # of loops in length dz is ndz and

$dB = \frac{2\pi Indz}{c}\hat{z}\frac{a^2}{(z^2+a^2)^{3/2}}$

$\tan\theta = \frac{a}{z} \Rightarrow (1+\tan^2\theta)\,d\theta = \frac{d\theta}{\cos^2\theta} = -\frac{a}{z^2}dz = -\frac{\tan^2\theta}{a}dz$

$dz = \frac{a\,d\theta}{\sin^2\theta}$

$dB = \frac{2\pi Indz}{c}\hat{z}\frac{a^2}{(z^2+a^2)^{3/2}} = \frac{2\pi In}{c}\hat{z}\left(-\frac{a\,d\theta}{\sin^2\theta}\right)\frac{\sin^3\theta}{a}$

$= -\hat{z}\frac{2\pi nI}{c}\sin\theta\,d\theta$

$B = -\hat{z}\frac{2\pi In}{c}\int_{\theta_1}^{\theta_2}\sin\theta\,d\theta$

$\boxed{B = \frac{2\pi In}{c}(\cos\theta_2 - \cos\theta_1)}$

For infinite solenoid $\theta_2 = 0$ & $\theta_1 = \pi$

$\Rightarrow B = \hat{z}\frac{2\pi In}{c}(\cos\theta_2 - \cos\pi)$

$\boxed{B = \frac{4\pi In}{c}\hat{z}}$

→

5.12) Charge on elementary ring $= 2\pi a \sin\theta \cdot a d\theta \cdot \sigma$

current on ring $\Rightarrow dI = \cancel{2\pi} a^2 \sigma \sin\theta d\theta \cdot \frac{w}{\cancel{2\pi}} = w\sigma a^2 \sin\theta d\theta$

mag. field due to current rings $\Rightarrow d\bar{B} = \hat{z} \frac{\mu_0}{4\pi} \frac{2\pi dI \cdot AP^2}{(AP^2 + AO^2)^{3/2}}$

$= \hat{z} \frac{\mu_0}{2} \frac{w\sigma \cancel{a^2} \sin\theta d\theta \, a^2 \sin^2\theta}{\cancel{a^3}}$

$= \hat{z} \frac{\mu_0}{2} w\sigma a \sin^3\theta d\theta$

Total mag. field at O

$\bar{B} = \hat{z} \frac{\mu_0}{2} w\sigma a \int_0^\pi \sin^3\theta d\theta$

$\boxed{\bar{B} = \frac{2\mu_0 w a \sigma}{3} \hat{z}}$

5.16) i) $B = \mu_0 n_1 - \mu_0 n_2 I$ ii) $B = \mu_0 n_2 I$ (right)

$B = \mu_0 (n_1 - n_2) I$ iii) $B = 0$

5.19) Ampere's Law $\Rightarrow$ steady current

$\nabla \times \vec{B} = \mu_0 \vec{J}$

$I_{enc} = \int_S \vec{J} \cdot d\vec{a}$

$= \frac{1}{\mu_0} \oint_S (\nabla \times \vec{B}) \cdot d\vec{A}$

Stoke's Theorem

$I_{enc} = \frac{1}{\mu_0} \oint_C \vec{B} \cdot d\vec{l}$ This form implies we can choose any surface w/ boundary C $\Rightarrow$ indepedent of surface

5.21) Ampere's Law

$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$= \mu_0 \int_S \vec{J} \cdot d\vec{s}$

$\int (\nabla \times \vec{B}) \cdot d\vec{s} = \mu_0 \int_S \vec{J} \cdot d\vec{s}$

$\boxed{\nabla \times B = \mu_0 \vec{J}}$

$\mu_0 \nabla \cdot \vec{J} = -\mu_0 \frac{\partial \rho}{\partial t}$

$\nabla \times \vec{E} = 0$

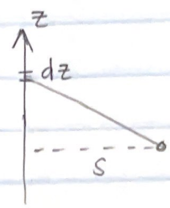
$\Rightarrow$ Ampere's Law is not valid outside of magneto statics, other equations have no defect

5.23) $\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I\hat{z}}{r} dz$

$= \frac{\mu_0}{4\pi} \int_{z_1}^{z_2} \frac{dz}{(z^2+s^2)^{1/2}} \hat{z}$

$= \frac{\mu_0 I}{4\pi} z \left[\ln(z + \sqrt{z^2+s^2}) \right] \Big|_{z_1}^{z_2}$

$$\boxed{\vec{A} = \frac{\mu_0 I}{4\pi} \ln \left[\frac{z_2 + \sqrt{z_2^2+s^2}}{z_1 + \sqrt{z_1^2+s^2}} \right] \hat{z}}$$



5.27) surface current density $\Rightarrow \bar{K} = K\hat{x}$

mag. field $\Rightarrow \vec{B} = \pm \frac{\mu_0 K}{2} \hat{y}$ where $+$ is $z<0$

$-$ is $z>0$

$\vec{B} = \vec{\nabla} \times \vec{A}$

$\hookrightarrow \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A(z) & 0 & 0 \end{vmatrix} = \pm \frac{\mu_0 K}{2} \hat{y}$

$\Rightarrow \frac{\partial A(z)}{\partial z} \hat{y} = \pm \frac{\mu_0 K}{2} \hat{y}$

$A(z) = \pm \frac{\mu_0 K}{2} z$

Therefore $\vec{A} = \pm \frac{\mu_0 K}{2} z \hat{x}$