5, 6, 9, 12, 16, 18, 25, 29

2.5

$$dE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{dq}{\mathscr{r}^2} \qquad \mathscr{r} = \sqrt{z^2+r^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda\cdot r\,d\phi}{\sqrt{z^2+r^2}}\cdot\frac{z}{(z^2+r^2)} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda r\cdot d\phi\cdot z}{(z^2+r^2)^{3/2}}$$

$$= \frac{\lambda r z}{4\pi\varepsilon_0(z^2+r^2)^{3/2}}\int_0^{2\pi} d\phi$$

$$\boxed{\vec{E} = \frac{2\pi r\lambda z}{4\pi\varepsilon_0(z^2+r^2)^{3/2}}\hat{z}}$$

2.6

$$\mathscr{r} = \sqrt{z^2+r^2} \qquad \lambda = \sigma\,dr \qquad E_{ring} = \frac{\cancel{2}\cancel{\pi} r(\sigma\,dr)z}{\cancel{4}\cancel{\pi}\varepsilon_0(z^2+r^2)^{3/2}}$$

$$E_{disk} = \frac{z\sigma}{2\cdot\varepsilon_0}\int_0^R\frac{2r\,dr}{(z^2+r^2)^{3/2}} \qquad \begin{aligned} u &= z^2+r^2 \\ du &= 2r\,dr\end{aligned}$$

$$= \frac{z\sigma}{2\cdot 2\varepsilon_0}\int_{z^2}^{z^2+R^2}\frac{du}{u^{3/2}} = \frac{z\sigma}{4\varepsilon_0}\left[-2u^{-1/2}\right]_{z^2}^{z^2+R^2}$$

$$= \frac{-2z\sigma}{4\varepsilon_0}\left[\frac{1}{\sqrt{u}}\right]_{z^2}^{z^2+R^2} = \frac{-2z\sigma}{4\varepsilon_0}\left[\frac{1}{\sqrt{z^2+R^2}}-\frac{1}{\sqrt{z^2}}\right]$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{z^2+R^2}}\right]\hat{z}}$$

2.9 a) $\rho = \varepsilon_0 \nabla \cdot E = \varepsilon_0 \left(\frac{1}{r^2}\right)\frac{\partial}{\partial r}\left(r^2 \cdot kr^3\right) = \frac{\varepsilon_0}{r^2}\left(5kr^4\right) = \boxed{5kr^2\varepsilon_0}$

b) $Q = \varepsilon_0 \oint E \cdot da = \varepsilon_0 (kR^3)(4\pi R^2) = \boxed{4\pi R^5 \varepsilon_0}$

$$Q = \int_0^R (5kr^2\varepsilon_0)(4\pi r^2 dr) = 20\pi\varepsilon_0 \int_0^R r^4 dr$$

$$= 20\pi\varepsilon_0 \left[\frac{1}{5}r^5\right]_0^R = \boxed{4\pi R^5 \varepsilon_0}$$

2.12 $\oint \bar{E} \cdot d\bar{a} = \frac{Q_{enc}}{\varepsilon_0}$

$E \cdot (4\pi r^2) = \frac{1}{\varepsilon_0}\left(\frac{4}{3}\pi r^3\right)\rho$

$$\boxed{E = \frac{r\rho}{3\varepsilon_0}\hat{r}}$$

$E \cancel{4\pi} r^2 = \frac{1}{\varepsilon_0}\left(\frac{4}{3}\cancel{\pi} R^3\right)\rho$

$$\boxed{E = \frac{\rho R^3}{r^2 \varepsilon_0}\hat{r}}$$

2.16 $\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$

i) $E(2\pi s l) = \frac{\rho(\pi s^2 l)}{\varepsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\rho s}{2\varepsilon_0}\hat{s}}$

ii) $E(2\pi s l) = \frac{\rho(\pi a^2 l)}{\varepsilon_0} \Rightarrow E = \frac{\rho a^2}{2\varepsilon_0 s}$

iii) $\vec{E} = 0$

2.18

$$E = \frac{r\rho}{3\varepsilon_0}\hat{r} = \frac{\rho}{3\varepsilon_0}\mathbf{r}$$

$$E_+ = \frac{+\rho}{3\varepsilon_0}\mathbf{r}$$

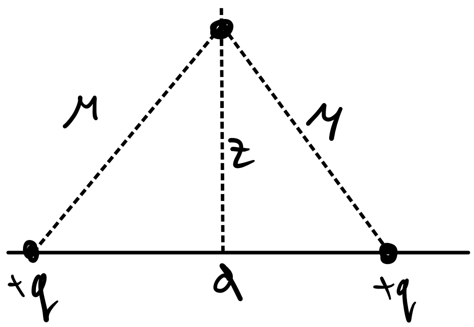
$$E_- = \frac{-\rho}{3\varepsilon_0}(r-d)$$

$$\Rightarrow E = \frac{\rho r}{3\varepsilon_0} - \frac{\rho r}{3\varepsilon_0} + \frac{\rho d}{3\varepsilon_0}$$

$$\boxed{E = \frac{\rho d}{3\varepsilon_0}}$$

2.25

a)



$$V(r) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$

$$\boxed{V(r) = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q}{\sqrt{z^2+(a/2)^2}}\right]}$$

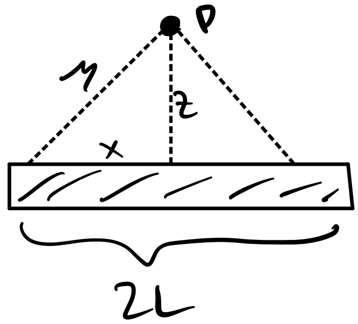
$$\vec{E} = -\vec{\nabla}V \qquad \vec{\nabla}V = \vec{\nabla}\frac{2q}{4\pi\varepsilon_0\sqrt{z^2+(a/2)^2}} = \frac{2q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}\left[\left(z^2+(a/2)^2\right)^{-1/2}\right]$$

$$= \frac{2q}{4\pi\varepsilon_0}\left[\frac{2z\cdot -\frac{1}{2}}{(z^2+(a/2)^2)^{3/2}}\right]$$

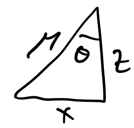
$$\boxed{E = \frac{-2qz}{4\pi\varepsilon_0\left(z^2+(a/2)^2\right)^{3/2}}\hat{z}}$$

The potential at point P would be 0 if the right charge is changed to $-q$.

b)



$$V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')\,d\tau'}{r}$$



$\sec\theta = \frac{r}{z}$

$\tan\theta = \frac{x}{z}$

$$V(r) = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{dx}{\sqrt{z^2+x^2}}$$

$x = z\tan\theta$

$dx = z\sec^2\theta\, d\theta$

$$= \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{L} \frac{z\sec^2\theta\, d\theta}{z\sec\theta} = \frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{L}\sec\theta\, d\theta$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\Big[\ln|\tan\theta+\sec\theta|\Big]_{-L}^{L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left|\frac{x}{z}+\frac{\sqrt{z^2+x^2}}{z}\right|\right]_{-L}^{L}$$

$$\boxed{V(r) = \frac{\lambda}{4\pi\varepsilon_0}\ln\left|\frac{L+\sqrt{z^2+L^2}}{-L+\sqrt{z^2+L^2}}\right|}$$

$$E = -\bar{\nabla} V = \frac{\lambda}{4\pi\varepsilon_0}\frac{\partial}{\partial z}\left[\ln\left|\frac{L+(z^2+L^2)^{1/2}}{-L+(z^2+L^2)^{1/2}}\right|\right]\hat{z}$$

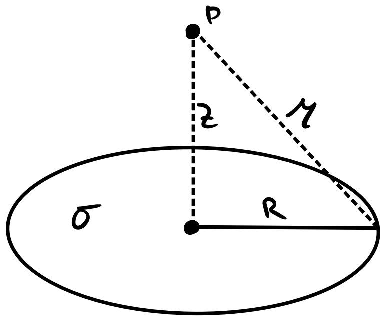
$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{(-L+(L^2+z^2)^{1/2})(2z)(\frac{1}{2})(z^2+L^2)^{-1/2} - (L+(z^2+L^2)^{1/2})(2z)(\frac{1}{2})(z^2+L^2)^{-1/2}}{(-L+(z^2+L^2)^{1/2})^2}\cdot\frac{(-L+\sqrt{L^2+z^2})}{(L+\sqrt{L^2+z^2})}\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{-L+\sqrt{L^2+z^2}}{L+\sqrt{L^2+z^2}}\left(\frac{\frac{(-L+\sqrt{L^2+z^2})z - z(L+\sqrt{z^2+L^2})}{\sqrt{L^2+z^2}}}{(-L+\sqrt{z^2+L^2})^2}\right)\right]$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{-L+\sqrt{L^2+z^2}}{L+\sqrt{L^2+z^2}}\left(\frac{-2Lz}{\sqrt{L^2+z^2}\,(-L+\sqrt{z^2+L^2})^2}\right)\right] = \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{-2Lz}{(L+\sqrt{L^2+z^2})(-L+\sqrt{L^2+z^2})(\sqrt{L^2+z^2})}\right]$$

$$= \frac{-2\lambda L z}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{z^2+L^2}\,(-L^2+z^2+L^2)}\right] = \boxed{\frac{-2\lambda L}{4\pi\varepsilon_0 z\sqrt{z^2+L^2}}\hat{z} = E}$$

c)



$s = r\theta$

$ds = r\,dr\,d\theta$

$P(r') = \sigma \cdot r\,dr \cdot d\theta$

$r = \sqrt{R^2+z^2}$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{P(r')\,d\tau}{r}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma \cdot dr \cdot r}{\sqrt{r^2+z^2}}\int_0^{2\pi} d\theta$$

$u = r^2+z^2$

$du = 2r\,dr$

$$= \frac{1}{2}\cdot\frac{\sigma}{2\varepsilon_0}\int_0^R \frac{2r \cdot dr}{\sqrt{r^2+z^2}}$$

$$= \frac{\sigma}{4\varepsilon_0}\left[2u^{1/2}\right]_{z^2}^{R^2+z^2}$$

$$\boxed{V(r) = \frac{\sigma}{2\varepsilon_0}\left[\sqrt{R^2+z^2} - z\right]}$$

$$\vec{E} = -\vec{\nabla} V$$

$$\nabla V = \frac{\sigma}{2\varepsilon_0}\frac{\partial}{\partial z}\left((R^2+z^2)^{1/2} - z\right)\hat{z} = \frac{\sigma}{2\varepsilon_0}\left[\frac{(\cancel{2}z)(\frac{1}{\cancel{2}})}{\sqrt{R^2+z^2}} - 1\right] = \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{\sqrt{R^2+z^2}} - 1\right)$$

$$E = -\nabla V = \boxed{\frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)\hat{z}}$$

2.2a) $v(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}')}{\eta} d\tau'$ $\qquad \nabla^2 v = \frac{-\rho}{\varepsilon_0}$

$$\nabla^2 V = \frac{1}{4\pi\varepsilon_0}\nabla^2\int \frac{\rho(r')}{\eta}d\tau' = \frac{1}{4\pi\varepsilon_0}(\rho(r'))\int \nabla^2\left(\frac{1}{\eta}\right)d\tau'$$

$$= \frac{\rho(r')}{4\pi\varepsilon_0}\int \vec{\nabla}\cdot\nabla\left(\frac{1}{\eta}\right)d\vec{\tau} = \frac{\rho(r')}{4\pi\varepsilon_0}\int \vec{\nabla}\cdot\left(-\frac{1}{\eta^2}\hat{\eta}\right)$$

$$= \frac{\rho(r')}{4\pi\varepsilon_0}\int_V -4\pi\,\delta^3(\vec{r})\,d\tau' = \boxed{\frac{-\rho(r')}{\varepsilon_0}}$$