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12/9

Hw chapter 7 p 12, 15, 34

12.) A long solenoid, of radius a, driven by alternating current, so field inside is sinusoidal: $\vec{B}(t) = B_0 \cos(\omega t)\hat{z}$. A circular loop of wire, of radius a/2 and resistance R, is placed inside solenoid, and coaxial w/ it. Find current induced in loop as func of time.

we know $\phi = \underset{\text{mag}}{\vec{B}} \cdot \underset{\text{area}}{\vec{A}}$ $\quad \mathcal{E} = -\frac{d\phi}{dt}$

$\vec{A} = \pi\left(\frac{a}{2}\right)^2 \hat{z}$ $\quad \vec{B} = B_0 \cos(\omega t)\hat{z}$

$\phi = (B_0 \cos(\omega t))\hat{z} \cdot \pi\left(\frac{a}{2}\right)^2 \hat{z} = B_0 \frac{\pi a^2}{4} \cos(\omega t)$

So $\mathcal{E} = -B_0 \pi \frac{a^2}{4}(-\sin(\omega t)) \cdot \omega = \frac{B_0 \pi a^2}{4} \omega \sin(\omega t)$

from ohm law $I(t) = \frac{\mathcal{E}}{R}$

So $\boxed{I(t) = \frac{B_0 \pi a^2}{4R} \omega \sin(\omega t)}$

15.) A long solenoid w/ radius a and n turns per unit length carries $I(t)$ in the $\hat{\phi}$ dir. Find elec field (magnitude/dir) at dist s from the axis (both inside/outside the solenoid), in quasistatic approx

For inside solenoid $B(s) = \mu_0 n I(t) \hat{\phi}$

$A = \pi r^2$

$\phi = B \cdot A$

$I(t)\hat{\phi}$

$\Phi_B = \begin{cases} \mu_0 n I(\pi s^2) & \text{for } s < a \text{ (inside solenoid)} \\ \mu_0 n I(\pi a^2) & \text{for } s > a \text{ (outside solenoid)} \end{cases}$

For inside the solenoid, we know that

$-\frac{\partial \phi}{\partial t} = E \oint dl \qquad \phi = \mu_0 n I(\pi s^2)$

$-\mu_0 n \cancel{\pi} s^{\cancel{2}} \left(\frac{dI}{dt}\right) \hat{\phi} = E(2\cancel{\pi}\cancel{s})$

$E = \boxed{-\frac{\mu_0 n s}{2} \frac{dI}{dt} \hat{\phi}}$

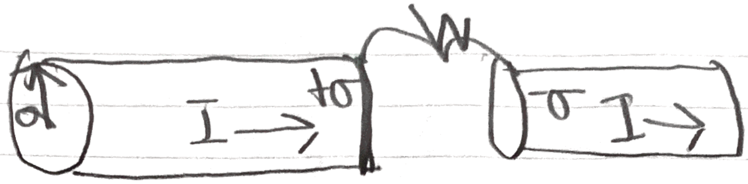
For outside the solenoid

$-\frac{\partial \phi}{\partial t} = E \oint dL \qquad \phi = \mu_0 n I(\pi a^2)$

$-(\mu_0 n \cancel{\pi} a^2) \frac{dI}{dt} \hat{\phi} = E(2\cancel{\pi} s)$

$E = \boxed{-\frac{\mu_0 n a^2}{2s} \frac{dI}{dt} \hat{\phi}}$

34) A fat wire, radius a, carries constant current I, uniformly distributed over its cross section. A narrow gap in the wire of width $w \ll a$, forms a parallel plate capacitor. Find the magnetic field in the gap, at dist $s < a$ from axis



$$J_d = \epsilon_0 \frac{dE}{dt} = \frac{I}{A} \qquad A = \pi a^2 \qquad \frac{dE}{dt} = \frac{1}{\epsilon_0 A} I$$

$$J_d = \frac{I}{\pi a^2}\hat{z} \qquad I = J_d \pi a^2$$

We know that $\oint B \cdot dL = \mu_0 I_{enc}$

$$B(2\pi s) = \mu_0 I_{enc}$$

$$I_{enc} = J_d \pi a^2$$

$$B(2\cancel{\pi} s) = \mu_0 (J_d \cancel{\pi} a^2)$$

$$\boxed{B = \frac{\mu_0 J_d a^2}{2s}\hat{\phi}}$$