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11/9

Hw chapter 2 prob 5, 6, 9, 12, 16, 18, 25, 29

5) Find electric field a distance z above center of circular loop of radius r that carries uniform line charge λ

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{R^2}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(r^2+z^2)}\cos\theta$$

$R = \sqrt{r^2+z^2}$ $R^2 = r^2+z^2$

$\cos\theta = \frac{z}{R} = \frac{z}{\sqrt{r^2+z^2}}$ $dq = \lambda dx$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(r^2+z^2)^{3/2}} \quad E = \int dE = \int_0^{2\pi r}\frac{\lambda z}{4\pi\varepsilon_0(r^2+z^2)^{3/2}}dx$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\frac{z}{(r^2+z^2)^{3/2}}(2\pi r) = \boxed{\frac{\lambda z r}{2\varepsilon_0(r^2+z^2)^{3/2}}}$$

6) Find electric field distance z above center of flat circular disk of radius R that carries σ. What does your formula give in limit $R \to \infty$? also check case $z \gg R$. $dq = \sigma da$ $da = s\,ds\,d\phi$

we know that $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{(z^2+r^2)}\cos\theta = \frac{1}{4\pi\varepsilon_0}\frac{dq\,z}{(z^2+r^2)^{3/2}}$

$dq = \sigma(2\pi r)dr$

$$E = \int_0^R \frac{1}{4\pi\varepsilon_0}\frac{(dq)z}{(z^2+r^2)^{3/2}} = 2\pi\sigma k z\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$

$$E = \frac{2\pi\sigma z}{4\pi\varepsilon_0}\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$$ use Symbolab

$$= \frac{2\sigma}{2\varepsilon_0}\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}} = \frac{2\sigma}{2\varepsilon_0}\left(\frac{-1}{(z^2+r^2)^{1/2}}\right)\Bigg|_0^R$$

$$= \frac{2\sigma}{2\varepsilon_0}\left(\frac{-1}{(z^2+R^2)^{1/2}} + \frac{1}{z}\right) = \boxed{\frac{\sigma}{2\varepsilon_0}\left(\frac{-z}{(z^2+R^2)^{1/2}} + 1\right)}$$

Now for $R \to \infty$ $\quad E = \frac{\sigma}{2\varepsilon_0}\left(0 + 1\right) = \boxed{\frac{\sigma}{2\varepsilon_0}}$

For $z \gg R$

$$E \sim \frac{\sigma}{2\varepsilon_0}\left(\frac{-\cancel{z}}{\cancel{z}} + 1\right) = \boxed{0}$$

9) Suppose electric field in some region is found to be $E = kr^3\hat{r}$ in spherical coords (k is constant)

a) find charge density ρ

$$\rho = \varepsilon_0(\nabla \cdot E)$$

Since we're in spherical coords

$$\nabla \cdot E = \frac{1}{r^2}\left(\frac{\partial}{\partial r}(r^2 E_r)\right) = \frac{1}{r^2}\left(\frac{\partial}{\partial r}(r^2(kr^3))\right) -$$

$= \frac{1}{r^2}(5r^4k) = 5r^2k$ so $\boxed{\rho = \varepsilon_0(5r^2k)}$

b) find total charge contained in sphere of radius R, centered at origin. (2 different ways)
1st way is using gauss law

$\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$Q_{enc} = \varepsilon_0 \cdot \oint E \cdot da = \varepsilon_0 \cdot (5(kR^3\hat{r}) \cdot (4\pi R^2)\hat{r}$

$= \varepsilon_0 \cdot 5\, 4\pi R^5 k = \boxed{\varepsilon_0(4\pi R^5 k)}$

2nd way

$dq = \rho\, d\tau$

$= (5r^2k\varepsilon_0)\, d\tau = (5r^2k\varepsilon_0)(4\pi r^2 dr$

$d\tau = 4\pi r^2 dr$

$q = \int dq = \int_0^R (5k\varepsilon_0 r^2) 4\pi r^2 dr$

$= 20k\varepsilon_0\pi \int_0^R r^4 dr = 20k\varepsilon_0\pi \left(\frac{r^5}{5}\right|_0^R =$

$= \boxed{4\pi k\varepsilon_0 R^5}$

2) Use Gauss Law to find electric field inside a uniformly charged solid sphere (charge density ρ).

Gauss law $\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$E(a)$

$E(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}$

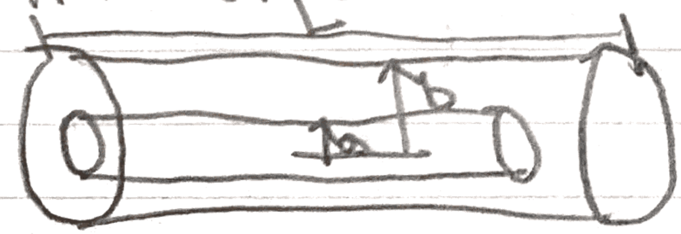
$\vec{E}(4\pi r^2) = \frac{Q_{enc}}{\varepsilon_0}\hat{r}$ $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{r^2}\hat{r}$

$Q_{enc} = \int_V \rho \, d\tau = \rho \frac{4}{3}\pi r^3$

$\vec{E} = \frac{1}{\cancel{4\pi}\varepsilon_0 r^2}\left(\frac{\rho \cancel{4\pi} r^3}{3}\right)\hat{r}$

$= \boxed{\frac{\rho r}{3\varepsilon_0}\hat{r}}$

16) long coaxial cable (fig 2.26) carries uniform volume charge density ρ on inner cylinder (radius a), and uniform surface charge density on outer cylindrical shell (radius b). This surface charge is $-$, and is of just the right magnitude that the cable as a whole is electrically neutral. Find electric field in each of the three region



SV.

1) inside the inner cylinder ($s<a$) radius

Gauss law $\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$Q_{enc} = \int \rho \, d\tau \qquad d\tau = s\,ds\,d\phi\,dz$

$= \int_0^L \int_0^{2\pi} \int_0^s \rho \, (s\,ds\,d\phi\,dz) = \rho 2\pi L \left(\frac{s^2}{2}\right)$

$\oint E \cdot da = E(2\pi s L) = \frac{Q_{enc}}{\varepsilon_0}$

$|E|(2\pi s l) = \frac{2\pi \rho l s^2}{\varepsilon_0 2}$

$$|E| = \boxed{\frac{\rho s}{2\varepsilon_0}\hat{s}}$$

ii) between the cylinders ($a<s<b$)

$$Q_{enc} = \int_0^L \int_0^{2\pi} \int_0^a \rho s\, ds\, d\phi\, dz = \rho \cancel{2}\pi L \frac{a^2}{\cancel{2}}$$

$$|E|(2\pi s L) = \frac{\rho \pi L a^2}{\varepsilon_0}$$

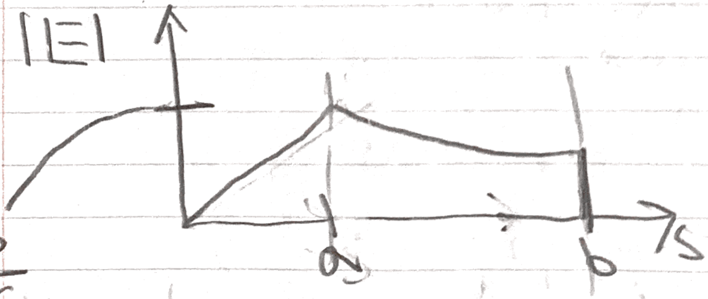
$$|E| = \frac{\rho \cancel{\pi} \cancel{L} a^2}{2 \cancel{\pi} \cancel{L} \varepsilon_0}\hat{s} = \boxed{\frac{\rho a^2}{2 s \varepsilon_0}\hat{s}}$$

iii) outside the cable ($s>b$)

$Q_{enc} = 0$

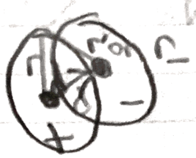
so $\oint E \cdot da = \frac{0}{\varepsilon_0} = \boxed{0}$

thus $E(s) = \underline{0}$



$\frac{\rho s}{2\varepsilon_0}$

13) Two spheres each of radius R and carrying +ρ and −ρ, respectively, placed so they partially overlap. Call vector from positive center to neg. center d. Show that the field in region is constant, and find its value.



From prob 2.12 we know

$E = \frac{\rho r^+}{3\varepsilon_0}\hat{r}$ for a uniformly charged solid sphere

this will be for + sphere

$E = \frac{\rho r^-}{3\varepsilon_0}$ for the negative sphere

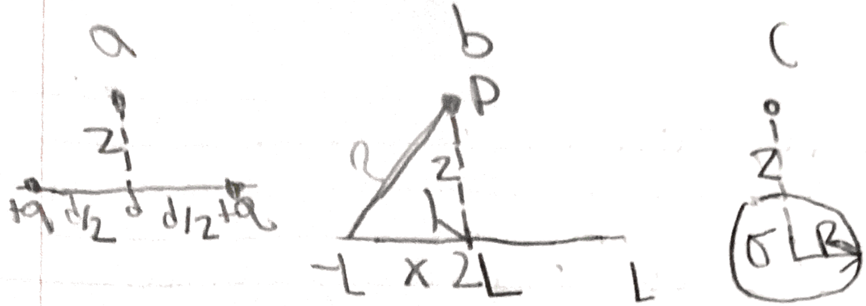
$E_{tot} = E^+ + E^-$

$E = \frac{\rho r^+}{3\varepsilon_0} + \frac{\rho r^-}{3\varepsilon_0} = \frac{\rho}{3\varepsilon_0}(r^+ + r^-)$

we know that $r^+ + r^- = d$

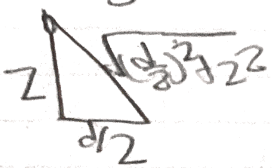
So $E = \boxed{\frac{\rho}{3\varepsilon_0} d}$

25) Use eqn 2.27 / 2.3, find potential at distance z above center of charge. Each case, compute $E = -\nabla V$, compare answer to Ex 1, 2, prob 6. Suppose we change right hand charged in Fig 2.34a to −q, what is potential at P? What field does that suggest?



$$V(r)=\frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \qquad V=\frac{1}{4\pi\varepsilon_0}\int\frac{\lambda(r')}{r}\,dl \qquad V=\frac{1}{4\pi\varepsilon_0}\int\frac{\sigma(r')}{r}\,da$$

For fig a



$$V=\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\sqrt{\frac{d^2}{4}+z^2}}+\frac{q}{\sqrt{\frac{d^2}{4}+z^2}}\right)=\frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{\sqrt{\frac{d^2}{4}+z^2}}\right)$$

$$E=-\nabla V=-\left(\frac{\partial V}{\partial x}\hat{i}+\frac{\partial V}{\partial y}\hat{j}+\frac{\partial V}{\partial z}\hat{k}\right)=-\frac{2q}{4\pi\varepsilon_0}\frac{\partial}{\partial z}\left(\frac{1}{\sqrt{\frac{d^2}{4}+z^2}}\right)\hat{k}$$

$$=-\frac{2q}{4\pi\varepsilon_0}\left(-\frac{1}{2}\cdot\frac{2z}{(\frac{d^2}{4}+z^2)}\right)=\boxed{\frac{2qz}{4\pi\varepsilon_0(\frac{d^2}{4}+z^2)}}\hat{k}$$

For fig b

$$V=\frac{1}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{\lambda\,dx}{\sqrt{x^2+z^2}}=\frac{\lambda}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{dx}{\sqrt{x^2+z^2}}$$ use Symbolab

$$=\frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{L+\sqrt{L^2+z^2}}{-L+\sqrt{L^2+z^2}}\right)$$

Use wolfframe

$$E=-\nabla V=-\frac{\lambda}{4\pi\varepsilon_0}\frac{\partial}{\partial z}\left(\ln\left(\frac{L+\sqrt{L^2+z^2}}{-L+\sqrt{L^2+z^2}}\right)\right)$$

$$\boxed{=\frac{-\lambda}{4\pi\varepsilon_0}\left(\frac{-2}{z^2+L^2-L(z^2+L^2)}\right)^{1/2}\hat{k}}$$

25 con't) For fig C

$$V = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{\sigma 2\pi r\,dr}{\sqrt{z^2+r^2}} = \frac{2\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{r\,dr}{\sqrt{z^2+r^2}} \qquad \begin{matrix} u = z^2+r^2 \\ \underline{du = 2r} \end{matrix}$$

$$= \frac{\pi\sigma}{4\pi\varepsilon_0}\int_0^R \frac{du}{\sqrt{u}} = \frac{\cancel{\pi}\sigma}{\cancel{4}\cancel{\pi}\varepsilon_0}\, \cancel{4}\sqrt{z^2+r^2} = \frac{\sigma}{2\varepsilon_0}\sqrt{z^2+r^2}\Big|_0^R$$

$$\frac{u^{1/2+1}}{1/2}$$

$$\frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)^{1/2} \qquad E = -\nabla V = -\frac{\sigma}{2\varepsilon_0}\frac{\partial}{\partial z}\left(\sqrt{z^2+R^2} - z\right)^{1/2}$$

$$= \boxed{\frac{\sigma}{2\varepsilon_0}\left(-\frac{1}{2}\frac{\left(-z+(z^2+R^2)^{1/2}\right)^{1/2}}{(x^2+R^2)^{1/2}}\right)\hat{z}}$$

If one of the charges is change to $-q$ then the electric potential at pt P is 0 and thus the electric field will be 0

29) Check that Eq 2.29 satisfies Poisson eqn, by applying Laplacian and using Eq 1.102

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(r')}{\imath}\,d\tau' \quad 2.29$$

$$\nabla^2 \frac{1}{\imath} = -4\pi\delta^3(\vec{\imath}) \quad 1.102$$

so $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ poisson eqn

$\vec{\eta} = \vec{r} - \vec{r}'$ $\qquad \nabla^2 \frac{1}{\eta} = -4\pi\,\delta(\vec{r} - \vec{r}')$

apply Laplacian to 2.29

$$\nabla^2 V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \nabla^2 \frac{\rho(\vec{r}')}{\eta}\, d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\int \left(-4\pi\,\delta^3(\vec{r} - \vec{r}')\right)\rho(\vec{r}')\, d\tau' = -\frac{1}{\varepsilon_0}\int \rho(\vec{r}')\,\delta^3(\vec{r} - \vec{r}')\, d\tau'$$

for dirac delta func in 3 dim

$$\int_{\text{allspace}} \rho(\vec{r}')\,\delta^3(\vec{r} - \vec{r}') = \rho(\vec{r})$$

so $\nabla^2 V(\vec{r}) = -\frac{\rho(\vec{r})}{\varepsilon_0}$ ✓