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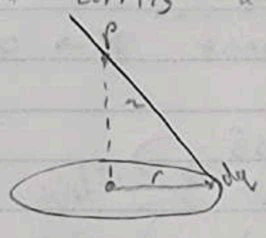
EMT HW #2

2.5) Find the electric field a distance z above the center of a circular loop of radius r (Fig 2.9) that carries a uniform line charge λ,



$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq'}{\eta^2}\hat{\eta}$$

$$dq = \lambda R\, d\phi$$

$$\eta = z\hat{z} - R\hat{s} \Rightarrow \eta^2 = z^2 + R^2$$

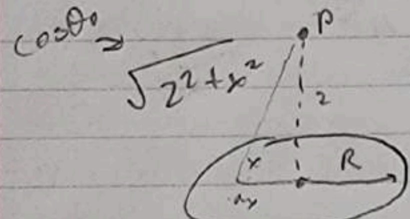
$$\hat{\eta} = \frac{(\hat{z} - \varepsilon\hat{s})}{(1+\varepsilon^2)^{1/2}}$$

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\, d\phi}{(z^2+R^2)}\frac{(\hat{z} - \varepsilon\hat{s})}{(1+\varepsilon^2)^{1/2}}$$

$$E = \frac{\lambda R}{4\pi\varepsilon_0 z^2(1+\varepsilon^2)^{3/2}}\int_0^{2\pi} d\phi\,(\hat{z} - \cancel{\varepsilon\hat{s}})$$

$$\boxed{E = \frac{2\pi R\lambda\,\hat{z}}{4\pi\varepsilon_0 z^2(1+\varepsilon^2)^{3/2}}}$$

2.6) Find the EF a distance z above the center of a flat circular disk of radius R (Fig. 2.10) that carries a uniform surface charge σ.
What does your formula give in the limit $R \to \infty$?
Check with $z \gg R$.



$$\vec{r} = z\hat{z} - s\hat{s}$$
$$r^2 = z^2 + s^2$$
$$\hat{r} = \frac{(z\hat{z} - s\hat{s})}{(z^2+s^2)^{1/2}}$$

Prob 2.5

$$dq' = s\,ds\,d\phi$$

$$s = z\tan\theta$$
$$ds = z\sec^2\theta\, d\theta \qquad \theta_0 = \tan^{-1}\left(\frac{R}{z}\right)$$

$$E = \frac{\sigma}{2\varepsilon_0} z\hat{z}\int_0^R \frac{s\,ds}{(s^2+z^2)^{3/2}} \Rightarrow \frac{\sigma}{2\varepsilon_0}\cos\theta\Big|_0^{\theta_0}\hat{z}$$

$$\Rightarrow \frac{\sigma}{2\varepsilon_0}\left(1-\cos\theta_0\right)\hat{z}$$

replace $\sqrt{z^2+x^2}$

$$= E = \frac{\sigma}{2\varepsilon_0}\hat{z}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) \Rightarrow \boxed{E \to \frac{\sigma\hat{z}}{2\varepsilon_0}}$$

check:

$$E = \frac{\sigma\hat{z}}{2\varepsilon_0} z\left(z^{-1} - (z^2+R^2)^{-\frac{1}{2}}\right)$$

$$= \frac{\sigma\hat{z}}{2\varepsilon_0} z\left(z^{-1}\left(1+\left(\frac{R}{z}\right)^2\right)^{-\frac{1}{2}}\right)$$

Cancel out 1/2 terms

$$E = \frac{\sigma \hat{z}}{2\varepsilon_0} z \left(\frac{1}{z^2} \cdot \left(\frac{R}{2}\right)^2 \right) \frac{* \, q}{* \, \pi}$$

$$\Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{\pi R^2 \sigma \hat{z}}{z^2}$$

$$\boxed{\Rightarrow E = \frac{1}{4\pi\varepsilon_0} \frac{Q \hat{z}}{z^2}}$$

2.11) $E = kr^3 \hat{r}$ k is constant

a) charge density ρ

$\rho = \varepsilon_0 (\nabla \cdot E)$ in spherical coords. $\nabla \cdot E = \frac{1}{r^2} \frac{d}{dr}(r^2 E_r)$

$E_r = kr^3 \hat{r}$ $\rho = \varepsilon_0 (\nabla \cdot E) = \varepsilon_0 \left\{ \frac{1}{r^2} \frac{d}{dr}\left(r^2 (kr^3)\right) \right\}$

$\rho = \boxed{5k\varepsilon_0 r^2}$

b) Gauss' Law: $\oint E \cdot da = \frac{q}{\varepsilon_0}$

$$q = \varepsilon_0 \int (kR^3 \hat{r}) \cdot (4\pi R^2) \hat{r} \Rightarrow \varepsilon_0 (4\pi k R^5) = \boxed{4\pi k \varepsilon_0 R^5}$$

$dq = \rho \, d\tau$

$= 5k\varepsilon_0 r^2 (4\pi r^2 dr)$ $q = \int dq$

$$= \int_0^R (5k\varepsilon_0 r^2)(4\pi r^2 dr) = \boxed{4\pi k \varepsilon_0 R^5}$$

2.12) Use Gauss's law to find the EF inside a uniformly charged solid sphere (charge density ρ).

$$\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$$

$$\oint E \cdot da = E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} Q_{enc} = \frac{1}{\varepsilon_0} \frac{4}{3} \pi r^3 \rho$$

$$\oint \vec{E} \cdot da = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0} \hat{r}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_{enc}}{r^2} \hat{r}$$

$q_{enc} = \frac{4}{3}\rho\pi r^3$

$q = \rho v$

$V = \frac{4}{3}\pi r^3$

$q_{enc} = \frac{Q_{enc}}{V} v$ $\quad \rho = \frac{Q_{in}}{V}$

$$\bar{E} = \frac{1}{\cancel{4}\pi\varepsilon_0} \frac{\frac{4}{3}\rho \pi r^{\cancel{3}}}{r^{\cancel{2}}} \hat{r} \Rightarrow \boxed{E = \frac{\rho r}{3\varepsilon_0} \hat{r}}$$

2.16) ρ cylinder radius a

Gauss' Law

$$\oint \vec{E}\cdot da = \frac{Q_{enc}}{\varepsilon_0} \qquad E\cdot 4\pi s^2 = \frac{1}{\varepsilon_0}\rho\pi s^2 l$$

$s<a$:

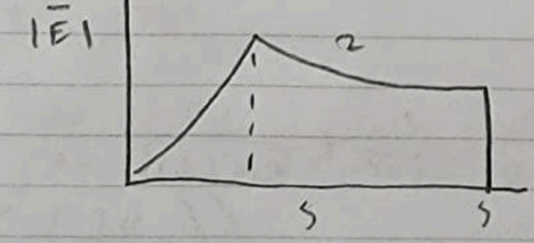
$$Q_{enc} = \rho\pi a^2 l \qquad \boxed{E_s = \frac{\rho s \hat{s}}{2\varepsilon_0}}$$

$a<s<b$:

$$E\cdot 4\pi s^2 = \frac{1}{\varepsilon_0 \rho\pi a^2 l} \Rightarrow \boxed{E_a = \frac{\rho a^2 \hat{s}}{2\varepsilon_0 s}}$$

$s>b$: $E\cdot 2\pi s l \Rightarrow \frac{1}{\varepsilon_0 Q_{enc}} = \boxed{0}$

Outer charge-density $-\rho$
Inner charge density $+\rho$

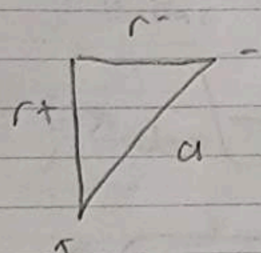


Since Efield will be 0 the further we move outside the cable

2.18) radius R $+\rho$ & $-\rho$

From 2.12;

$$E = \frac{\rho r}{3\varepsilon_0}\hat{r}$$



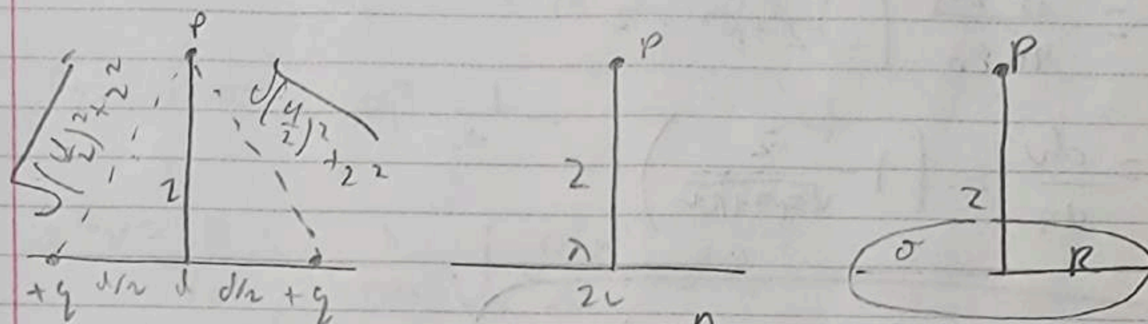
$$E(r) = \frac{\rho(\bar{r}-\bar{r}')}{3\varepsilon_0}$$

Partially overlapping,

$$E_{tot} = \frac{\rho\bar{r}}{3\varepsilon_0} - \frac{\rho(\bar{r}-\bar{d}')}{3\varepsilon_0}$$

$$\Rightarrow \boxed{E = \frac{\rho d}{3\varepsilon_0}}$$

2.25) $E = -\nabla V$ for each case



Eqn 2.27: $V(r) = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$

Eqn 2.30: $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\sqrt{(\frac{d}{2})^2+z^2}} + \frac{q}{\sqrt{(\frac{d}{2})^2+z^2}}\right)$

$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{\sqrt{(d/2)^2+z^2}}\right)$ = pot. @ point P

$E = -\nabla V$

$-\frac{dV}{dx} + \frac{dV}{dy} + \frac{dV}{dz}$

$= -\left(\frac{1}{4\pi\varepsilon_0} \frac{dV}{dx} \frac{2q}{\sqrt{(d/2)^2+z^2}} + \frac{1}{4\pi\varepsilon_0} \frac{dV}{dy} \frac{2q}{\sqrt{(d/2)^2+z^2}} + \frac{1}{4\pi\varepsilon_0} \frac{dV}{dz}\left(\frac{2q}{\sqrt{(d/2)^2+z^2}}\right)\right)$

$= -\left(\frac{1}{4\pi\varepsilon_0} \frac{dV}{dV} \frac{2q}{\sqrt{(d/2)^2+z^2}}\right)$

$$= \frac{\sigma\, 2\pi}{4\pi\, \varepsilon_0}\left(\sqrt{z^2+R^2} - z\right.$$

$$= \frac{dV}{dz} = \left(1 - \frac{-z}{\sqrt{z^2+R^2}}\right)$$

$$\boxed{\Rightarrow \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)\hat{z}}$$

2.29) Eqn 2.29 = $V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{\eta} d\tau$

Eqn 1.102 = $\nabla^2 \frac{1}{\mu} = -4\pi \delta^3(\mu)$

$\nabla^2 V = \frac{\rho}{\varepsilon_0} = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{\mu} d\tau'$

$\mu = (r - r')$

$\nabla^2 \frac{1}{\mu} = -4\pi \delta^3(\mu) = -4\pi \delta^3(r - r')$

Laplacian: $\nabla^2 V(r) = \frac{1}{4\pi\varepsilon_0} \int \nabla^2 \left(\frac{1}{\mu}\right) \rho(r') d\tau$

$\nabla^2 V(r) = \frac{1}{4\pi\varepsilon_0} \int -4\pi \delta^3(r - r') \rho(r') d\tau$

$\frac{-4\pi}{4\pi\varepsilon_0} \int \delta^3(r - r') \rho(r') d\tau$

$= \frac{1}{\varepsilon_0} \int \rho(r') \delta^3(r - r') d\tau = \rho(r)$

$= -\frac{1}{\varepsilon_0} (\rho(r)) \Rightarrow \nabla^2 V(r) = \frac{-\rho(r)}{\varepsilon_0}$