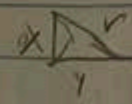
Midterm for electramagnetic theory

☆ 1) $(\bar{A}\cdot\nabla)\bar{B} -$

→ a)

$$\left(A_x + A_y + A_z\right)\cdot\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right)$$

$$A_x + A_y + A_z\hat{x}$$



b) Compute $(\hat{r}\cdot\nabla)\hat{r}$

$$= \frac{\bar{r}}{\|r\|} \quad \|r\| = r \qquad \|\bar{r}\| = \sqrt{x^2+y^2+z^2}$$

$$r^2 = x^2+y^2+z^2$$

$$\bar{r} = \langle x, y, z\rangle$$

$$\frac{g f' - f g'}{g^2}$$

$$= \left(\hat{x}\frac{\partial\hat{r}}{\partial x} + \hat{y}\frac{\partial\hat{r}}{\partial y} + \hat{z}\frac{\partial\hat{r}}{\partial z}\right)\frac{\langle x, y, z\rangle}{\sqrt{x^2+y^2+z^2}}$$

$$(x^2+y^2+z^2)^{1/2}$$

$$\frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2x)$$

$$\frac{\partial}{\partial x}\left(\frac{x}{(x^2+y^2+z^2)^{1/2}}\right) + \frac{\partial}{\partial y}\left(\frac{y}{(x^2+y^2+z^2)^{1/2}}\right) + \frac{\partial}{\partial z}\left(\frac{z}{(x^2+y^2+z^2)^{1/2}}\right)\cdot\hat{r}$$

$$= \frac{r(1) - \frac{x\left(\cancel{\frac{1}{2}}\right)(\cancel{2}x)}{r}}{x^2+y^2+z^2} + \frac{r - y^2/r}{x^2+y^2+z^2} + \frac{r - z^2/r}{x^2+y^2+z^2}$$

(finish →) $= r - x^2/r$

$$\left(\frac{r^2-x^2}{r} + \frac{r^2-y^2}{r} + \frac{r^2-z^2}{r}\right)\cdot\frac{1}{r^2}$$

$$\left(\frac{r^2-x^2}{r^{3/2}} + \frac{r^2-y^2}{r^{3/2}} + \frac{r^2-z^2}{r^{3/2}}\right)\cdot\hat{r}$$

$$\boxed{= \left(\hat{x}\frac{r^2-x^2}{r^{3/2}}\hat{r} + \hat{y}\frac{r^2-y^2}{r^{3/2}}\hat{r} + \hat{z}\frac{r^2-z^2}{r^{3/2}}\hat{r}\right)}$$

(b) $\cancel{(\hat{r}\cdot\nabla)\hat{r}}$

$\cancel{\left(\frac{\bar{r}}{|r|}\cdot\nabla\right)\cdot\hat{r}}$

$\bar{r} = \langle x+y+z \rangle$

$|r| = (x^2+y^2+z^2)^{1/2}$

$r^2 = x^2+y^2+z^2$

$r^2 - y^2 = x^2+z^2$

$r^2 - y^2 =$

$\cancel{\hat{x}\frac{x}{(x^2+y^2+z^2)^{1/2}} + \hat{y}\frac{y}{(x^2+y^2+z^2)^{1/2}} + \hat{z}\frac{z}{(x^2+y^2+z^2)^{1/2}}}$

$\bar{r} =$

$\cancel{= \hat{r}(\nabla\cdot\hat{r})}$

$\cancel{= \frac{1}{r^{3/2}}\left(\hat{x}\cdot\hat{r}(r^2-x^2) + \hat{y}\hat{r}(r^2-y^2) + \hat{z}\hat{r}(r^2-z^2)\right)}$

$(\hat{r}\cdot\nabla)\hat{r}$

$\left(\hat{r}_x\frac{\partial}{\partial x} + \hat{r}_y\frac{\partial}{\partial y} + \hat{r}_z\frac{\partial}{\partial z}\right)\cdot\hat{r}$

$\hat{r} = \frac{\bar{r}}{|r|}$

$\hat{r}^2 = \frac{\bar{r}^2}{|r^2|}$

$\bar{r} = \langle x+y+z \rangle$

$|r| = (x^2+y^2+z^2)^{1/2}$

$|r|^2 = x^2+y^2+z^2$

$= \left(\hat{r}_x^2\frac{\partial}{\partial x} + \hat{r}_y^2\frac{\partial}{\partial y} + \hat{r}_z^2\frac{\partial}{\partial z}\right)$

$= \bar{r}^2\left(\frac{1}{r^2}\frac{\partial}{\partial x} + \frac{1}{r^2}\frac{\partial}{\partial y} + \frac{1}{r^2}\frac{\partial}{\partial z}\right)$ – All go to zero b/c dirac delta

$f(x)$

$v = \frac{1}{r^2}\hat{r}$

$\boxed{\nabla\cdot v = 0}$

$\boxed{= 0}$

c)

$F = (p \cdot \nabla) E$ $\qquad$ $\bar{p} = \langle q d \hat{x} \rangle$

$\nabla V = -E$

$\nabla \cdot (V_0 r^2 + V_1) = -E$

$2V_0 r = -E$

$\boxed{E = -2V_0 r}$

$$F = \left(\hat{x} q d \frac{\partial}{\partial r}\right)\left(-2V_0 r\right)$$

$F = \hat{x} q d - 2V_0$

$\boxed{F = -2V_0 q d \hat{x}}$

2) $J = \int_V e^{-r} \left(\nabla \cdot \frac{\bar{r}}{r^2} \right) d\tau$ $\qquad \frac{\bar{r}}{r^2} = \bar{V}$

$\bar{J} = \int_V e^{-r} \left(\nabla \cdot \frac{\bar{r}}{r^2} \right) d\tau$ $\qquad \nabla \cdot \bar{V} = 4\pi \delta^3(r)$

$\bar{J} = \int_V e^{-r} 4\pi \delta^3(\hat{r}) d\tau$

$\bar{J} = \int_V e^{0}$

$\bar{J} = 0$

Chapter 2 electrostatics

* 1)

?

2) $$V(r) = \frac{Ae^{-\lambda r}}{r}$$

$$\nabla V = -E \qquad \nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot V = -E$$

$$\left(\hat{r}\,\frac{\partial}{\partial r}\right) \cdot \left(\frac{Ae^{-\lambda r}}{r}\right)$$

$$\hat{r} A \frac{\partial}{\partial r}\left(\frac{e^{-\lambda r}}{r}\right) \qquad \begin{aligned} u &= -\lambda r\, dr \\ du &= -\lambda\, dr \end{aligned}$$

$$\hat{r} A\left(\frac{-\lambda e^{-\lambda r}}{r} - \frac{e^{-\lambda r}}{r^2}\right) = -E$$

$$= -\frac{A(\lambda r + 1)e^{-\lambda r}}{r^2}\hat{r} = -E$$

$$\boxed{\bar{E} = \frac{A(\lambda r + 1)e^{-\lambda r}}{r^2}\hat{r}}$$

$$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$$

$$\rho = \varepsilon_0(\nabla \cdot E)$$

$$\rho = A\varepsilon_0\left(\nabla \cdot \left(\frac{\bar{r}}{r^2} \cdot \frac{(\lambda r + 1)e^{-\lambda r}}{r}\right)\right)$$

$$\rho = A\varepsilon_0\left(4\pi\delta^3(r) - \frac{\lambda^2 r^2 + r\lambda + 1)e^{-\lambda r}}{r^2}\right)$$

$$Q_{enc} = \int_V \rho \, d\tau$$

$$= \int \varepsilon_0 A \left(4\pi \delta^3(r) - \frac{\lambda^2 (e^{-\lambda r})}{r}\right) dr$$

(1)

$$= \varepsilon_0 A \left(4\pi \int \delta^3(r) d\tau - \int \frac{\lambda^2 (e^{-\lambda r})}{r} (4\pi r^2) dr\right)$$

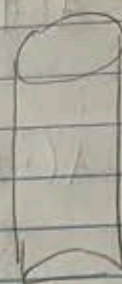
$$= \varepsilon_0 A \left(4\pi (1) - \lambda^2 4\pi \int e^{-\lambda r} r \, dr\right)$$

$$= \varepsilon_0 A \left(4\pi - \lambda^2 4\pi (e^{-\lambda x} - \lambda x e^{-\lambda x}\right)$$

$$\boxed{= \varepsilon_0 A \left(4\pi - \lambda^2 4\pi (\lambda x - 1) e^{-\lambda x}\right)}$$

3) $\oint E \cdot da = \frac{Q}{\varepsilon_0}$ $\lambda = \rho$ $\bar{A} = \bar{da} = 2\pi s h + 2\pi s^2$

$$\int \bar{E} \cdot da = \frac{Q}{\varepsilon_0}$$



field is constant so

$$\bar{E} \cdot \bar{A} = \frac{Q}{\varepsilon_0}$$

$Q_{enc} = \int_V \lambda \, d\tau$

$$E = \frac{Q}{\bar{A}\, \varepsilon_0}$$

$Q_{enc} = \pi s^2 h$

$$= \frac{\cancel{\pi} s^{\cancel{2}} h}{\varepsilon_0 (2\cancel{\pi}\cancel{s} h + 2\cancel{\pi} s^{\cancel{2}})}$$

$$\boxed{\bar{E} = \frac{s h}{\varepsilon_0 (2h + 2s)} \hat{s}}$$

* b) $\bar{F} = m\bar{a}$

$\bar{F} = Q\bar{E}$

$F = qE$

$$a = \frac{F}{m} = \frac{q\bar{E}}{m}$$

$\bar{a} = \frac{d^2(\bar{r})}{dt^2}$

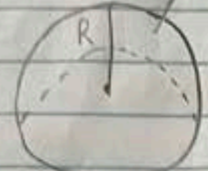
$$\boxed{\frac{d(\bar{s})^2}{dt^2} = \frac{q\bar{E}}{m}}$$

$$\frac{q\bar{E}}{m} = A\cos(ks) + B\sin(ks)$$

Ch 3. Potentials

$V_0(\theta)$ S.A

1)



$\sigma = -\varepsilon_0 \frac{\partial V}{\partial n}$ $\quad Q = \int \sigma \, da$

$0 =$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta \frac{\partial V}{\partial\theta}\right) = 0$$

$$V(r,\theta) = R(r)\,\Theta(\theta)$$

$$V(r,\theta) = \sum_{l=0}^{\infty}\left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$$

if $r<R = \sum_{l=0}^{\infty}(A_l r^l)(P_l \cos\theta)$

& $r>R = \sum_{l=0}^{\infty}\left(\frac{B_l}{r^{l+1}}\right)P_l(\cos(\theta))$

$$\sum_{l=0}^{\infty}(A_l r^l)(P_l(\cos\theta)) = \sum_{l=0}^{\infty}\left(\frac{B_l}{r^{l+1}}\right)P_l(\cos(\theta))$$

$$\sum_{l=0}^{\infty}(A_l r^l P_l(\cos\theta) - \left(\frac{B_l}{r^{l+1}}\right)P_l \cos\theta = 0$$

$$\sum_{l=0}^{\infty}P_l(\cos\theta)\left(A_l r^l - \frac{B_l}{r^{l+1}}\right) = 0$$

$$\cos(\theta)\sum_{l=0}^{\infty}P_l\left(A_l r^l - \frac{B_l}{r^{l+1}}\right) = 0$$

$\int_a^b \Rightarrow \phi = \int_a^a$

3.2) $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$

$V = 0$ when $y = 0$
$V = V_0$ when $y = a$
$V = 0$ when $x = b$
$V = 0$ when $x = -b$

$V(x,y) = X(x)Y(y)$

$\frac{1}{X}\frac{d^2X}{dx} + \frac{1}{Y}\frac{d^2Y}{dy^2} = 0$

$\frac{1}{X}\frac{d^2X}{dx^2} = \lambda$ & $\lambda + \frac{1}{Y}\frac{d^2Y}{dy^2} = 0$

$\frac{d^2X}{dx^2} = -\lambda X$ $\frac{d^2Y}{dy^2} = -\lambda Y$

Boundary

if $\lambda < 0$ then $\lambda = -k^2$

$\frac{d^2Y}{dy^2} = -k^2 Y \rightsquigarrow$

$X = 0$

$Y(y) = A\cos(ky) + B\sin(ky)$

$y = 0$ $V(x,0) = 0 = X(x)Y(0) = Y(0) = 0$

$y = a$ $V(x,a) = V_0 = X(x)Y(a) = Y(a) = V_0$

$Y(0) = 0 = A$

$Y(y) = B\sin(ky)$

for $X(a) = 0 \longrightarrow ka = n\pi$

$k = n\pi/a$

$\boxed{Y(y) = B\sin\left(\frac{n\pi y}{a}\right)}$

$\boxed{X(x) = Ce^{kx} + De^{-kx}}$

$V=0$ when $x=\pm b$

$V(x,y) = X(x)Y(y)$

$$V(x,y) = B\sin\left(\frac{n\pi y}{a}\right)\left[Ce^{(n\pi x/a)} + De^{-(n\pi x/a)}\right]$$

$$V(x,y) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi y}{a}\right)\left[C_n e^{(n\pi x/a)} + D_n e^{-(n\pi x/a)}\right]$$

$$V(-b,y) = 0 = \sum_{n=1}^{\infty} \sin(n\pi y/a)\left[C_n n^{\pi x - b/a} + D_n e^{(n\pi x/a}\right]$$

$$\boxed{V(x,y) = \sum_{n=1}^{\infty} \sin\left(\frac{n\pi y}{a}\right)\left[C_n \cosh n\pi x/a\right]}$$

$$V(b,y) = \sum C_n \cosh\left(\frac{n\pi b}{a}\right)\sin(n\pi y/a)$$

$$= -\frac{V_0 y}{a}$$

$$\sum C_n \cosh(n\pi b/a)\int_0^a \sin(n\pi y/a)\sin\left(\frac{n'\pi y}{a}\right)dy$$

$$= -\frac{V_0}{a}\int_0^a y\sin(n'\pi y/a)\,dy \rightarrow$$ looked up

$$-V_0\left(a\frac{2\sin(\pi n') - \pi n\cos(\pi n')}{n^2\pi^2}\right)$$

$$C_n\cosh(n\pi b/a) = \frac{\cancel{a}\sin(\pi n) - \pi n\cos(\pi n)}{\pi^2 n^2}\left(\frac{-V_0}{\cancel{a}}\right)$$

$$C_n = \frac{V_0 a\sin(\pi n) - \pi n\cos(\pi n)}{\pi^2 n^2}$$

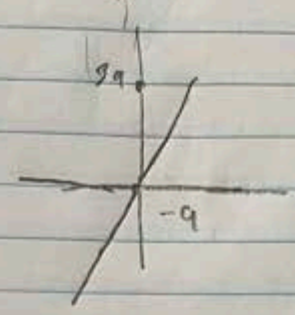
$$V(x,y) = \sum_{n=1}^{\infty} C_n\cosh(n\pi x/a)\sin(n\pi y/a)$$

where $C_n = \dfrac{V_0 a\sin(\pi n) - \pi n\cos(\pi n)}{\pi^2 n^2}$

3.3

Used page (154 - 156) for my equations

a)



$Q = 3q - q$

$\boxed{K = 4\pi\epsilon_0}$

mon = 2q

$P = [3q\, a\hat{z} + -q^0]$

$\bar{p} = 3qa\hat{z}$

$\hat{z} \cdot \hat{r} = \cos\theta$

$V(r) = V_m + V_d$

$= \frac{1}{K}\frac{Q}{r} + \frac{1}{K}\frac{p \cdot \hat{r}}{r^2}$

$$\boxed{= \frac{1}{K}\left(\frac{2q}{r} + \frac{3qa\cos\theta}{r^2}\right)}$$

b) $Q = 2q$

$\bar{p} = qa\hat{z}$

$V(r) = V_m + V_d$

$$\boxed{V(r) = \frac{1}{K}\left(\frac{2q}{r} + \frac{qa\cos\theta}{r^2}\right)}$$

c) $Q = 2q$

$p = 3qa\hat{y}$

$\hat{r} = \sin\theta\cos\phi\,\hat{x} + \sin\theta\sin\phi\,\hat{y} + \cos\theta\,\hat{z}$

$\hat{y} \cdot \hat{r} = \sin\theta\sin\phi$

$V(r) = V_m + V_d$

$$\boxed{V(r) = \frac{1}{K}\left(\frac{2q}{r} + \frac{3qa\sin\theta\sin\phi}{r^2}\right)}$$