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PHYS 330 HW 3   3.3, 3.5, 3.6, 3.13, 3.14, 3.15, 3.16, 3.19, 3.22, 3.24, 3.26

3.3) General solution to Laplace's equation in spherical coordinates where V only depends on r. Gen. solution in cylindrical when V depends only on s.

Laplacian in spherical: $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right)$

only depends on 1 variable so $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$

multiply by r^2 both sides $\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) = 0$

∴ must be constant

now $r^2 \frac{\partial V}{\partial r} = k$

$\frac{\partial V}{\partial r} = k r^{-2}$

$V(r) = -k r^{-1} + C$

$\boxed{V(r) = \frac{-k}{r} + C}$ for spherical

Laplacian in cylindrical: $\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s \frac{\partial V}{\partial s}\right)$

only depends on 1 variable so $\frac{1}{s}\frac{\partial}{\partial s}\left(s \frac{\partial V}{\partial s}\right) = 0$

multiply both sides by s $\frac{\partial}{\partial s}\left(s \frac{\partial V}{\partial s}\right) = 0$

must be constant

now $s \frac{\partial V}{\partial s} = k$

$\frac{\partial V}{\partial s} = k s^{-1}$

$\boxed{V(s) = k\ln(s) + C}$ for cylindrical

3.5) Prove field is uniquely determined when ρ is given & either V or $\frac{\partial V}{\partial n}$ is specified on each boundary surface.

$\nabla \cdot E_1 = \frac{\rho}{\epsilon_0}$ $\nabla \cdot E_2 = \frac{\rho}{\epsilon_0}$ $E_3 = E_1 - E_2$ $\nabla \cdot E_3 = 0$

$\oint E_3 \cdot d\vec{a} = 0$ product rule: $\nabla \cdot (V_3 E_3) = V_3(\nabla \cdot E_3) + E_3 \cdot (\nabla V_3) = -(E_3)^2$

$\int_V \nabla \cdot (V_3 E_3) = -\int_V (E_3)^2 \, d\tau$

$\oint_S V_3 E_3 \cdot d\vec{a} = -\int_V (E_3)^2 \, d\tau$

$\int (E_3)^2 \, d\tau = 0$ since $E_3 = E_1 - E_2$ and $E_3 = 0$

then $\boxed{E_1 = E_2}$

3.6) Second uniqueness theorem but w/ Green's identity with $T = V = V_3$

Green's identity:

$$\int_V [T\nabla^2 V + (\nabla T)\cdot(\nabla V)]\, d\tau = \oint_S (T\nabla V)\, d\vec{a}$$

$$\int_V [V_3\nabla^2 V_3 + (\nabla V_3)\cdot(\nabla V_3)]\, d\tau = \oint_S (V_3 \nabla V_3)\cdot d\vec{a}$$

$$\nabla^2 V_3 = \nabla^2 V_2 - \nabla^2 V_1 = \frac{\rho}{\epsilon_0} - \frac{\rho}{\epsilon_0} = 0$$

$$\nabla V_3 = -E_3$$

$$\int_V [0 + -E_3 \cdot -E_3]\, d\tau = \oint_S V_3(-E_3)\cdot d\vec{a}$$

$$\boxed{\int_V E_3^2\, d\tau = -\oint V_3 E_3 \cdot d\vec{a}}$$

3.13) Potential in infinite slot of Ex. 3.3 w/ boundary @ $x=0$ w/ two metal strips, one from $y=0$ to $y=\frac{a}{2}$ w/ constant potential V_0 and other $y=\frac{a}{2}$ to $y=a$ at potential $-V_0$

Gen. solution from Ex. 3.3:

$$V(x,y) = \sum_{n=1}^{\infty} C_n e^{-n\pi x/a} \sin(n\pi y/a)$$

$$C_n = \frac{2}{a}\int_0^a V_0(y)\sin(n\pi y/a)\, dy$$

$$C_n = \frac{2}{a}\left[\int_0^{a/2} V_0(y)\sin(n\pi y/a)\, dy + \int_{a/2}^{a} -V_0(y)\sin(n\pi y/a)\, dy\right]$$

$$= \frac{2}{a}V_0\left[\int_0^{a/2}\sin(n\pi y/a)\, dy - \int_{a/2}^{a}\sin(n\pi y/a)\, dy\right]$$

$$u = n\pi y/a \quad du = \frac{n\pi}{a}dy \quad dy = \frac{a}{n\pi}du$$

$$= \frac{2}{a}V_0\left[\int_0^{a/2}\sin(u)\frac{a}{n\pi}\, du - \int_{a/2}^{a}\sin(u)\frac{a}{n\pi}\, du\right]$$

$$= \frac{2}{a}\frac{a}{n\pi}V_0\left[\cos(n\pi y/a)\Big|_0^{a/2} - \cos(n\pi y/a)\Big|_{a/2}^{a}\right]$$

$$= \frac{2V_0}{n\pi}\left[\cos\left(\frac{n\pi}{2}\right) - \cos(0) - \cos(n\pi) + \cos\left(\frac{n\pi}{2}\right)\right]$$

$$C_n = \frac{2V_0}{n\pi}\underbrace{\left(2\cos\left(\frac{n\pi}{2}\right) - 1 + (-1)^n\right)}$$

$=0$ when n is odd or mult of 4 else is 4 so $C_n = \frac{8V_0}{n\pi}$

$n = 4j+2$

$$V(x,y) = \sum_{j=0}^{\infty} \frac{8V_0}{(4j+2)\pi} e^{-(4j+2)\pi x/a} \sin((4j+2)\pi y/a)$$

$$\boxed{= \frac{8V_0}{\pi}\sum_{j=0}^{\infty} \frac{e^{-(4j+2)\pi x/a}\sin((4j+2)\pi y/a)}{4j+2}}$$

3.14) For Ex. 3.3 infinite slot determine charge density $\sigma(y)$ on strip

@ $x=0$ assuming it's conductor w/ constant potential $-V_0$

For conductors: $\frac{\partial V}{\partial n} = -\frac{\sigma}{\varepsilon_0} \rightarrow \sigma = -\varepsilon_0 \frac{\partial V}{\partial n}\Big|_{x=0}$

$$V(x,y) = \frac{4V_0}{\pi} \sum_{n=1,3,5} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a)$$

$$\frac{\partial V}{\partial n} = \frac{\partial}{\partial x}\left[\frac{4V_0}{\pi} \sum_{n=1,3,5} \frac{1}{n} e^{-n\pi x/a} \sin(n\pi y/a)\right]\Bigg|_{x=0}$$

$$\sigma = -\varepsilon_0 \left(\frac{4V_0}{\pi} \sum \frac{1}{n}\left(\frac{-n\pi}{a}\right) e^{-n\pi x/a} \sin(n\pi y/a)\right)\Bigg|_{x=0}$$

$$\sigma = \frac{4V_0\varepsilon_0}{\pi} \sum \frac{\pi}{a} e^{0} \sin(n\pi y/a)$$

$$\boxed{\sigma = \frac{4V_0\varepsilon_0}{a} \sum_{n=1,3,5\ldots} \sin(n\pi y/a)}$$

3.15) rectangular pipe parallel to z-axis, infinite, has 3 grounded metal sides @ $y=0$, $y=a$, $x=0$ 4^{th} side @ $x=b$ at potential $V_0(y)$

(a) Gen. formula for potential inside

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

boundary conditions
(i) $V=0$ when $y=0$
(ii) $V=0$ when $y=a$
(iii) $V=0$ when $x=0$
(iv) $V=V_0$ when $x=b$

$$V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$$

$B=-A$ for condition (iii)

$D=0$ for condition (i)

$k=\frac{n\pi}{a}$ for condition (ii)

$$V(x,y) = \left(Ae^{\frac{n\pi x}{a}} - Ae^{\frac{-n\pi x}{a}}\right)\left(C\sin\frac{n\pi}{a}y\right)$$

$$= AC\left(e^{\frac{n\pi x}{a}} - e^{\frac{-n\pi x}{a}}\right)\sin\left(\frac{n\pi y}{a}\right)$$

$$= \underbrace{2(AC)}_{\text{constant}} \sinh(n\pi x/a)\sin(n\pi y/a)$$

$$V(x,y) = \sum_{n=0}^{\infty} C_n \sinh(n\pi x/a)\sin(n\pi y/a)$$

(a) $C_n = \frac{2}{a\sinh(n\pi b/a)} \int_0^a V_0(y)\sin(n\pi y/a)\,dy$

(b) when $V_0(y) = V_0$

when n is odd $C_n = \frac{4V_0}{n\pi\sinh(n\pi)}$

$$V(x,y) = \sum_{n=0}^{\infty} \left(\frac{4V_0}{n\pi\sinh(n\pi)}\right)\sinh(n\pi x/a)\sin(n\pi y/a)$$

$$\boxed{= \frac{4V_0}{\pi}\sum_{n=0}^{\infty} \frac{\sinh(n\pi x/a)\sin(n\pi y/a)}{n\sinh(n\pi)}}$$

3.(6) Cubical box side lengths a, made of 5 metal plates grounded, separate insulated top w/ potential V_0.

(i) $V=0$ when $x=0$

(ii) $V=0$ when $x=a$

(iii) $V=0$ when $y=0$

(iv) $V=0$ when $y=a$

(v) $V=0$ when $z=0$

(vi) $V=V_0$ when $z=a$

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}+\frac{\partial^2 V}{\partial z^2}=0$$

$$V(x,y,z)=X(x)\,Y(y)\,Z(z)$$

$$\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}+\frac{1}{Z}\frac{d^2Z}{dz^2}=0$$

$$\frac{1}{X}\frac{d^2X}{dx^2}=C_1 \qquad \frac{1}{Y}\frac{d^2Y}{dy^2}=C_2 \qquad \frac{1}{Z}\frac{d^2Z}{dz^2}=C_3$$

$$\underbrace{C_1+C_2}_{\text{negative}}+\underbrace{C_3}_{\text{positive}}=0$$

$$C_1=-k^2 \quad C_2=-l^2 \quad C_1+C_2=-C_3=k^2+l^2$$

$$X(x)=A\cos(kx)+B\sin(kx)$$

$$Y(y)=C\cos(ly)+D\sin(ly)$$

$$Z(z)=Ee^{-z\sqrt{k^2+l^2}}+Fe^{z\sqrt{k^2+l^2}}$$

(i) $\rightarrow A=0$ (iii) $\rightarrow C=0$ (ii) $\rightarrow k=\frac{n\pi}{a}$ (iv) $\rightarrow l=\frac{m\pi}{a}$ (v) $\rightarrow E=-F$

$$V(x,y,z)=\sum_n\sum_m C_{nm}\sin(n\pi x/a)\sin(m\pi y/a)\sinh\left(\pi\sqrt{n^2+m^2}\,\underbrace{\tfrac{z}{a}}_{\substack{\text{becomes } 1 \\ \text{b/c } z=a \\ \text{when } V=V_0}}\right)$$

$$V_0=\sum_n\sum_m C_{n,m}\sin(n\pi x/a)\sin(m\pi y/a)\sinh(\pi\sqrt{n^2+m^2})$$

$$C_{n,m}\sinh(\pi\sqrt{n^2+m^2})=\left(\frac{2}{a}\right)^2 V\int_0^a\int_0^a\sin(n\pi x/a)\sin(m\pi y/a)\,dx\,dy$$

$$C_{n,m}=\frac{4V_0}{a^2\sinh(\pi\sqrt{n^2+m^2})}\left(\frac{4a^2\sin^2(\pi m/2)\sin^2(\pi n/2)}{\pi^2 nm}\right) \rightarrow \text{from wolfram alpha}$$

when n & m are odd $\rightarrow C_{n,m}=\frac{16V_0}{\pi^2 nm}$

$$V(x,y,z)=\sum_n\sum_m\left(\frac{16V_0}{\pi^2 nm}\right)\sin(n\pi x/a)\sin(m\pi y/a)\sinh(\pi\sqrt{n^2+m^2}\,z/a)$$

$$\boxed{V(x,y,z)=\frac{16V_0}{\pi^2}\sum_n\sum_m\frac{1}{nm}\sin(n\pi x/a)\sin(m\pi y/a)\sinh(\pi\sqrt{n^2+m^2}\,z/a)}$$

when $x=y=z=\frac{a}{2}$

$$=\frac{16V_0}{\pi^2}\sum_n\sum_m\frac{1}{nm}\sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{m\pi}{2}\right)\sinh\left(\frac{\pi}{2}\sqrt{n^2+m^2}\right)$$

$$=\frac{16V_0}{\pi^2}\sum_n\sum_m\frac{1}{nm}(-1)^{n+1}(-1)^{m+1}\sinh\left(\frac{\pi}{2}\sqrt{n^2+m^2}\right)$$

3.19) Potential at surface of sphere w/ radius R is $V_0 = k\cos 3\theta$ where k is a constant. Potential inside & outside of sphere & $\sigma(\theta)$ on sphere?

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta \rightarrow V_0 = k[4\cos^3\theta - 3\cos\theta]$$

$$V_0 = k[\alpha P_3 \cos\theta - \beta P_1 \cos\theta]$$

$$4\cos^3\theta - 3\cos\theta = \alpha \tfrac{1}{2}(5\cos^3\theta - 3\cos\theta) - \beta\cos\theta$$

$$= \tfrac{5}{2}\alpha\cos^3\theta - \tfrac{3}{2}\alpha\cos\theta - \beta\cos\theta$$

$$4\cos^3\theta - 3\cos\theta = \tfrac{5}{2}\alpha\cos^3\theta - \left(\tfrac{3}{2}\alpha + \beta\right)\cos\theta$$

$4 = \frac{5}{2}\alpha$ $\qquad 3 = \frac{3}{2}\alpha + \beta$ $\qquad V_0 = k\left[\left(\frac{8}{5}\right)P_3\cos\theta - \left(\frac{3}{5}\right)P_1\cos\theta\right]$

$\frac{8}{5} = \alpha$ $\qquad 3 = \frac{3}{2}\left(\frac{8}{5}\right) + \beta$ $\qquad V_0 = \frac{k}{5}\left[8P_3\cos\theta - 3P_1\cos\theta\right]$

$3 = \frac{24}{10} + \beta$

$\frac{15}{5} - \frac{12}{5} = \beta = \frac{3}{5}$

$V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l \cos\theta \quad r \le R \qquad V(r,\theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) \quad r \ge R$

$$A_l = \frac{2l+1}{2R^l}\int_0^\pi V_0(\theta)P_l(\cos\theta)\sin\theta\, d\theta$$

$$= \frac{2l+1}{2R^l}\int_0^\pi \left[\frac{k}{5}(8P_3\cos\theta - 3P_1\cos\theta)\right]P_l(\cos\theta)\sin\theta\, d\theta$$

$$= \frac{k}{5}\frac{2l+1}{2R^l}\left[8\int_0^\pi P_3\cos\theta P_l\cos\theta\sin\theta\, d\theta - 3\int_0^\pi P_1\cos\theta P_l\cos\theta\sin\theta\, d\theta\right.$$

3.22) $V(r,\theta) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{r^2+R^2} - r\right)$ (a) use this w/ $P_l(1)=1$ to evaluate first three terms in expansion of $V(r,\theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta)$ for potential of disk at points off axis, $r>R$. (b) Find potential for $r<R$ with $V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta)$

(a) $V(r,\theta) = \sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}}(1) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{r^2+R^2} - r\right)$

$\sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{r^2+R^2} - r\right)$

$\sqrt{r^2+R^2}$ when $r>R$ $= r\left[1 + \frac{1}{2}\left(\frac{R}{r}\right)^2 - \frac{1}{8}\left(\frac{R}{r}\right)^4 + \dots\right]$

$\sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} = \frac{\sigma}{2\varepsilon_0}\left[r\left(1 + \frac{1}{2}\frac{R^2}{r^2} - \frac{1}{8}\frac{R^4}{r^4}\right) - r\right]$

$= \frac{\sigma R^2}{2\varepsilon_0 r} - \frac{\sigma R^4}{8\varepsilon_0 r^3}$

when $l=0$, r^1 1st term has r^1 so $B_0 = $ 1st term $= \frac{\sigma R^2}{2\varepsilon_0}$

when $l=1$, r^2, no term with r^2 so $B_1 = 0$

when $l=2$, r^3, 2nd term has r^3 so $B_2 = $ 2nd term $= -\frac{\sigma R^4}{8\varepsilon_0}$

$V(r,\theta) = \left(\frac{\sigma R^2}{2\varepsilon_0}\right)\left(\frac{1}{r}\right) + (0)\left(\frac{1}{r^2}\right) - \left(\frac{\sigma R^4}{8\varepsilon_0}\right)\left(\frac{1}{r^3}\right)P_2\cos\theta$

this back in b/c the P_l part from initial

(a) $\boxed{V(r,\theta) = \frac{\sigma R^2}{2\varepsilon_0}\left(\frac{1}{R} - \frac{R^2}{4r^3}P_2\cos\theta\right)}$

(b) $\sum_{l=0}^{\infty} A_l r^l (1) = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{r^2+R^2} - r\right)$ for top region

$\sqrt{r^2+R^2}$ when $r<R$ $= R\left[1 + \frac{1}{2}\left(\frac{r}{R}\right)^2 - \frac{1}{8}\left(\frac{r}{R}\right)^4 + \dots\right]$

$\sum_{l=0}^{\infty} A_l r^l = \frac{\sigma}{2\varepsilon_0}\left[R\left(1 + \frac{r^2}{2R^2} - \frac{r^4}{8R^4}\right) - r\right]$

$\sum_{l=0}^{\infty} A_l r^l = \frac{\sigma}{2\varepsilon_0}\left[R + \frac{r^2}{2R} - \frac{r^4}{8R^3} - r\right]$

term w/ r^0 is 1st $\to A_0 = \frac{\sigma R}{2\varepsilon_0}$

term w/ r^1 is 4th $\to A_1 = \frac{-\sigma}{2\varepsilon_0}$

term w/ r^2 is 2nd $\to A_2 = \frac{\sigma}{4\varepsilon_0 R}$

$V(r,\theta) = \frac{\sigma R}{2\varepsilon}(r^0) - \frac{\sigma}{2\varepsilon_0}(r^1)P_1\cos\theta + \frac{\sigma}{4\varepsilon_0 R}(r^2)P_2\cos\theta$

$\boxed{V(r,\theta) = \frac{\sigma}{2\varepsilon_0}\left[R - rP_1\cos\theta + \frac{r^2}{2R}P_2\cos\theta\right]}$ for top region

bottom region is same but $P(1) = -1$ so

$\boxed{V(r,\theta) = \frac{\sigma}{2\varepsilon_0}\left[R + rP_1\cos\theta + \frac{r^2}{2R}P_2\cos\theta\right]}$ for bottom region

3.24) Solve Laplace's equation by separation of variables in cylindrical assuming no z dependence

$$\frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} = 0$$

$$V(s, \phi) = S(s)\,\Phi(\phi)$$

$$\frac{1}{s}\Phi\frac{d}{ds}\left(s\frac{dS}{ds}\right) + \frac{1}{s^2}S\frac{d^2\Phi}{d\phi^2} = 0 \quad \text{mult by } s^2$$

$$s\Phi\frac{d}{ds}\left(s\frac{dS}{ds}\right) + S\frac{d^2\Phi}{d\phi^2} = 0 \quad \text{divide by } S\Phi$$

$$\frac{s}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right) + \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = 0$$

$$\frac{s}{S}\frac{d}{ds}\left(s\frac{dS}{ds}\right) = C \qquad \frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2} = C_2 \qquad C_1 + C_2 = 0$$

$$s\frac{d}{ds}\left(s\frac{dS}{ds}\right) = k^2 S \qquad \frac{d^2\Phi}{d\phi^2} = -k^2\Phi \qquad C_1 = k^2 \quad C_2 = -k^2$$

when $S = s^n \rightarrow s\frac{d}{ds}\left(s\frac{d}{ds}(s^n)\right) = k^2 s^n \rightarrow s\frac{d}{ds}(sns^{n-1}) = k^2 s^n \rightarrow sn\frac{d}{ds}(s^n) = k^2 s^n$

$$sn^2 s^{n-1} = k^2 s^n \rightarrow s^n n^2 = k^2 s^n \rightarrow n^2 = k^2 \rightarrow n = \pm k$$

$S(s) = Cs^k + Ds^{-k}$ ← confused how we got this step (from solutions)

$s\frac{d}{ds}\left(s\frac{dS}{ds}\right) = (0)S \rightarrow s\frac{dS}{ds}$ must be a constant $\rightarrow s\frac{dS}{ds} = C$

$\frac{dS}{ds} = \frac{C}{s} \rightarrow dS = ds\left(\frac{C}{s}\right) \rightarrow S = C\ln s + D$ ← 2nd constant from indefinite integration

$$V(s, \phi) = (Cs^k + Ds^{-k})(C\ln s + D)$$

3.26) Charge density $\sigma(\phi) = a\sin 5\phi$, where a is constant, glued over surface of ∞ cylinder of rad R. Potential inside & out

Answer from 3.24 was

$$V(s,\phi) = a_0 + b_0 \ln(s) + \sum_{k=1}^{\infty} \left[s^k (a_k \cos(k\phi) + b_k \sin(k\phi)) + s^{-k}(c_k \cos(k\phi) + d_k \sin(k\phi))\right]$$

can remove parts not valid when $s=0$ because s must be able to $=0$ inside cylinder

$$V_{inside}(s,\phi) = a_0 + \sum_{k=1}^{\infty} s^k (a_k \cos(k\phi) + b_k \sin(k\phi))$$

can remove parts not valid when $s \to \infty$ b/c s must be able to $\to \infty$ outside

$$V_{outside}(s,\phi) = a_0 + \sum_{k=1}^{\infty} s^{-k}(c_k \cos(k\phi) + d_k \sin(k\phi))$$

$$\sigma = -\varepsilon_0 \left(\frac{\partial V_{out}}{\partial s} - \frac{\partial V_{in}}{\partial s}\right)\Big|_{s=R}$$

$$a\sin 5\phi = -\varepsilon_0 \left[\frac{\partial}{\partial s}\left(a_0 + \sum_{k=1}^{\infty} s^{-k}(c_k \cos(k\phi) + d_k \sin(k\phi))\right) - \frac{\partial}{\partial s}\left(a_0 + \sum_{k=1}^{\infty} s^k (a_k \cos(k\phi) + b_k \sin(k\phi))\right)\right]$$

$$a\sin 5\phi = -\varepsilon_0 \sum_{k=1}^{\infty} \left[-kR^{-k-1}(c_k \cos(k\phi) + d_k \sin(k\phi)) - kR^{k-1}(a_k \cos(k\phi) + b_k \sin(k\phi))\right]$$

$$= -\varepsilon_0 \sum_{k=1}^{\infty} \left[-k(\cos(k\phi) + \sin(k\phi))\left[\frac{1}{R^{k+1}}(c_k + d_k) + R^{k-1}(a_k + b_k)\right]\right]$$

Don't know what to do next!