5.12 hint

HW #5 (Ch 5) # 5.14, 5.16, 5.17, 5.19, 5.20, 5.23, 5.26

5.14 a) $\oint B \cdot d\vec{l} = \mu_0 I_{enc}$

$B \cdot l = \mu_0 I$

$\boxed{\vec{B} = \frac{\mu_0 I}{l} \hat{s}}$

b) $\nabla \times \vec{B} = \mu_0 \vec{J}$

$\int (\nabla \times \vec{B}) \cdot d\vec{a} = \mu_0 \int \vec{J} \cdot d\vec{a}$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{J} \cdot d\vec{a}$

$B(l) = \mu_0 J(A)$

$B(2\cancel{\pi} a) = \mu_0 J(\cancel{\pi} a^2)$

$\boxed{\vec{B} = \frac{a \mu_0 J}{2} \hat{s}}$

5.16 two coaxial solenoids w/ current I

inner: radius a w/ n_1 turns per unit length

outer: radius b w/ n_2 turns per unit length

a) inside inner solenoid b) between solenoids c) outside both

$\vec{B} = \frac{\mu_0 N_1 I}{2\pi a} \hat{s}$ $\quad \vec{B} = \frac{\mu_0 N_2 I}{2\pi b} \hat{s}$ $\quad \vec{B} = 0$

5.17 a) between: $\vec{B} = \left(\frac{\mu_0(+\sigma)}{l} + \frac{\mu_0(-\sigma)}{l} \right) \hat{z} = 0$

above: $\vec{B} = \left(\frac{\mu_0(+\sigma)}{l} - \frac{\mu_0(-\sigma)}{l} \right) \hat{s} = \frac{2\mu_0 \sigma}{l} \hat{z}$

below: $\vec{B} = \left(\frac{\mu_0(-\sigma)}{l} - \frac{\mu_0(\sigma)}{l} \right) \hat{s} = \frac{-2\mu_0 \sigma}{l} \hat{z}$

b) $\vec{F}_{mag} = Q(\vec{v} \times \vec{B})$

$\vec{v} = v_x \hat{x} + v_y \hat{y} + v_z \hat{z}$, $\vec{B}_{above} = (2\mu_0 \sigma)/l \ \hat{z}$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ 0 & 0 & 2\mu_0\sigma/l \end{vmatrix} = ((2\mu_0\sigma/l) v_y)\hat{x} - ((2\mu_0\sigma/l) v_x)\hat{y} + \cancel{(0)\hat{z}}$$

$\boxed{\vec{F}_{mag} = Q\left(\frac{2\mu_0 \sigma v_y}{l} \hat{x} - \frac{2\mu_0 \sigma v_x}{l} \hat{y} \right)}$

c) $\vec{F} = Q\vec{E} + Q(\vec{v} \times \vec{B}) \rightarrow \vec{F} = 0$

$(\vec{v} \times \vec{B}) = -\vec{E} \rightarrow v \cdot B \sin\theta = -\vec{E}$

$\boxed{v = \frac{-\vec{E}}{B \sin\theta}}$

5.19 $I_{enc} = \int_S \vec{J} \cdot d\vec{a}$

We are suppose to use the surface where $\vec{J}$ & $\vec{A}$ originate.

5.20 a) $\rho = m/V$, $m_{copper} = 1.0552061 \times 10^{-25}$ kg

$r_{copper} = 1.4 \times 10^{-10}$ m

$$\rho = \frac{1.0552061 \times 10^{-25} \text{ kg}}{4/3\, \pi\, (1.4 \times 10^{-10} \text{ m})^3} = 9180.463 \frac{\text{kg}}{\text{m}^3}$$

b) $I = nqAV$

$1 C/s = I = 1\,A$, $A = \pi (10^{-3} m)^2$, $q = 1.60 \times 10^{-19}$ C

$n = \frac{1 \text{ electron}}{1 \text{ atom}} = 8.342 \times 10^{28}\, e^-/m^3$

$$V = \frac{I}{nqA} = \frac{1\ C/s}{(1.6 \times 10^{-19} C)(\pi\ 10^{-6}\ m)(8.342 \times 10^{28}\ m^{-3})} = 2.4 \times 10^{-5} \frac{m}{s}$$

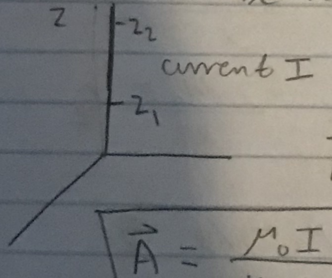
c) $$F = \frac{kq^2}{d^2} = \frac{\frac{1}{4\pi\varepsilon_0}(1.6 \times 10^{-19} C)^2}{(10^{-2} m)^2} = \frac{8.98 \times 10^9\ N\, m^2 C^{-2} (1.6 \times 10^{-19} C^2)}{(10^{-2} m)^2}$$

$F = 1.44 \times 10^{-5}$ N

d) • New force would be $.72 \times 10^{-5}$ N or 7.5×10^{-6} N

• it would be half of the original force

5.23



Eq 66: $\vec{A} = \frac{\mu_0 I}{4\pi} \int \left(\frac{1}{\imath}\right) d\vec{I}'$

$$\vec{A} = \frac{\mu_0 I}{4\pi} \int_{z_1}^{z_2} \frac{1}{\imath}\, d\vec{I}' = \frac{\mu_0 I}{4\pi} \left[\frac{I}{\imath} \right]_{z_1}^{z_2}$$

$$\boxed{\vec{A} = \frac{\mu_0 I}{4\pi \imath}(z_2 - z_1)\, \hat{z}}$$

5.26 a) $\vec{B} = \frac{I \mu_0}{2\pi s} \hat{\phi}$

$\vec{B} = \nabla \times \vec{A}$

$\int \vec{B} \cdot d\vec{a} = \int (\nabla \times \vec{A})\, d\vec{a}$

$\int \vec{B} \cdot d\vec{a} = \oint \vec{A}\, (d\vec{l}) = A$

$$\vec{A} = \frac{\mu_0 I}{2\pi} \int \frac{1}{s} ds = \boxed{\frac{\mu_0 I}{2\pi} \ln(s)\, \hat{z}}$$

b) $\oint B\, dl = \mu_0 I \rightarrow Bl = \mu_0 I \rightarrow B(2\pi s) = \mu_0 I \left(s^2/R^2\right)$

$$\vec{B} = \frac{\mu_0 I}{2\pi} \left(\frac{s}{R^2}\right) \longrightarrow \int \frac{\mu_0 I}{2\pi R^2}\, s\, ds$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi R^2} s^2\, \hat{z}}$$