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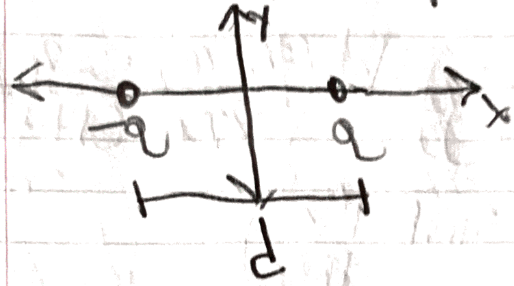
Chapter 2 Quiz

11/7

1.1 Pic physical dipole of 2 charges $+q$/$-q$ of equal mass, separated by distance d. $\vec{p} = q\vec{d}$ pointing from $-q$ to $+q$ in xy plane. Add external electric field $\vec{E} = E_0\hat{x}$, show that torque on dipole is $\tau = \vec{p} \times \vec{E}$



$\vec{p} = q\vec{d}$

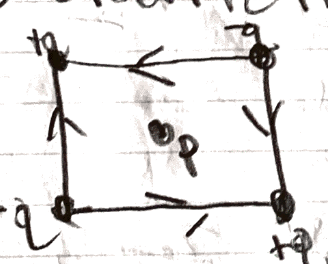
$\vec{F} = \pm q\vec{E}$

$\vec{E}$ could be $E_x\hat{x}$ or $E_y\hat{y}$

We also know that torque is equal to force times distance $\tau = Fd$

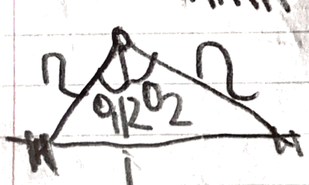
So $\tau = qEd = \vec{p} \times \vec{E}$

1.2 two dipole moments $\vec{p}_1$ and $\vec{p}_2$ pointed in opposite directions, forming square w/ alternating + and - charges. Calc electric field vector in center of square



Electric field vector would be zero as the vectors would cancel each other out

2.1) Compute elec field of continuous line of charge, $Q = \lambda L$. Field pt is distance z above center of line. Show limit that $L \gg z$.



$dq = \lambda dx$ $\quad \vec{\eta} = z\hat{z} - x\hat{x}$ $\quad \eta = (z^2 + x^2)^{1/2}$

$$d\vec{E} = k\frac{dq}{\eta^2}\hat{\eta} = k\frac{\lambda dx(z\hat{z} - x\hat{x})}{(z^2 + x^2)^{3/2}}$$

$$\int d\vec{E} = \int_{-L/2}^{L/2} \frac{k\lambda\, dx\,(z\hat{z} - \cancel{x\hat{x}})}{(z^2+x^2)^{3/2}} = k\lambda\left(z\hat{z}\int_{-L/2}^{L/2}\frac{dx}{(z^2+x^2)^{3/2}}\right)$$

$x = z\tan\theta \quad dx = z\sec^2\theta\, d\theta \quad x\,dx = z^2\tan\theta\sec^2\theta\, d\theta$

$$\vec{E} = k\lambda\left(z\hat{z}\int_{\theta_1}^{\theta_2}\frac{z\sec^2\theta\, d\theta}{(z^2+z^2\tan^2\theta)^{3/2}}\right) = k\lambda\left(\frac{\cancel{z}\hat{z}}{z^{\cancel{2}}}\int_{\theta_1}^{\theta_2}\frac{\sec^2\theta\, d\theta}{\underbrace{(1+\tan^2\theta)}_{\sec^2\theta}{}^{3/2}}\right)$$

$$= k\lambda\left(\frac{\hat{z}}{z}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta\right) = k\lambda\frac{\hat{z}}{z}\left(\sin\theta\Big|_{\theta_1}^{\theta_2}\right)$$

$\sin\theta_1 = \frac{-L/2}{r} \quad \sin\theta_2 = \frac{L/2}{r}$

$$= k\lambda\frac{\hat{z}}{z}\left(\frac{L/2}{(z^2+L^2/4)^{1/2}} + \frac{+L/2}{(z^2+L^2/4)^{1/2}}\right) = \frac{k\lambda L\hat{z}}{z}(z^2+L^2/4)^{-1/2}$$

limit of $L \gg z$ $\quad E \sim \frac{kQ}{z}\left(\frac{2}{L}\right) = \frac{2kQ}{zL}$

b) Gauss Law $\oint E\cdot da = \frac{Q_{enc}}{\varepsilon_0}$ $\quad Q = \lambda L$

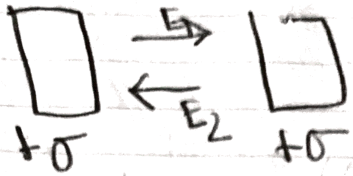
$E\int_S da = \frac{\lambda L}{\varepsilon_0}$ $\quad \int da = 4\pi r^2\cos\theta\hat{z}$ $\quad \frac{z}{(z^2+L^2/4)^{1/2}} = \cos\theta$

$E(4\pi r^2\cos\theta\,\hat{z}) = \frac{\lambda L}{\varepsilon_0}$

$E = \frac{\lambda L}{4\pi\varepsilon_0 r^2\cos\theta}\hat{z} = \frac{k\lambda L}{(z^2+L^2/4)^{\cancel{1/2}}}\times\frac{\cancel{(z^2+L^2/4)^{1/2}}}{z}$

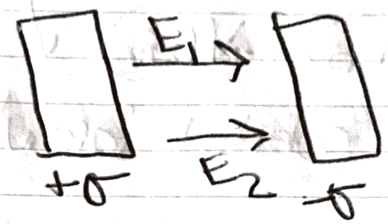
$\vec{E} = \frac{k\lambda L\hat{z}}{z}(z^2+L^2/4)^{-1/2}$

2.2 - two planes of + charge, field pt between plate



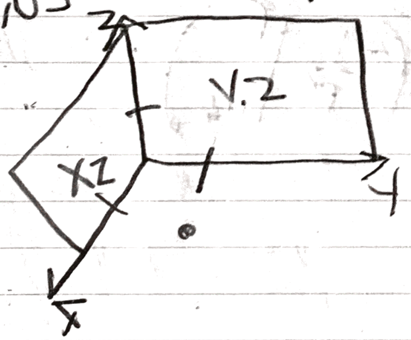
$E_{tot} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$ ✓

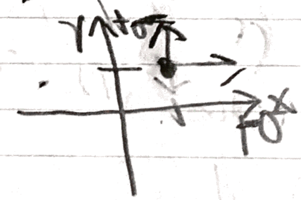
- two plates of charge, 1 + and 1 - charge, field pt in the middle



$E_{tot} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$ ✓

- two planes of + charge, one occupying yz plane, and the other occupying xz plane, and field pt is (1,1,0)





$\vec{E}_{tot} = \frac{\sigma}{2\varepsilon_0}\hat{i} + \frac{\sigma}{2\varepsilon_0}\hat{j}$

yz plane would have electric field going in the x axis
xz plane would have electric field going in y axis

for $|E| = \sqrt{\left(\frac{\sigma}{2\varepsilon_0}\right)^2 + \left(\frac{\sigma}{2\varepsilon_0}\right)^2} = \sqrt{\frac{\sigma^2}{4\varepsilon_0^2} + \frac{\sigma^2}{4\varepsilon_0^2}}$

$= \sqrt{\frac{\sigma^2}{2\varepsilon_0^2}} = \boxed{\frac{\sigma}{\sqrt{2}\varepsilon_0}}$ ✓

3.1) show that $-\int_a^b \vec{E}\cdot d\vec{L} = V(\vec{b}) - V(\vec{a})$

$$-\int_a^b E\cdot d\vec{L} = -\int_a^0 \vec{E}\cdot d\vec{L} - \int_0^b \vec{E}\cdot d\vec{L}$$

$$= \int_0^a \vec{E}\cdot d\vec{L} - \int_0^b E\cdot dL$$

We know $V(\vec{r}) = -\int_0^r \vec{E}\cdot d\vec{L}$

so $= -V(\vec{a}) + V(\vec{b}) = V(\vec{b}) - V(\vec{a})$ ✓

3.2) Peform Integrol to find the potentiol formula for pt charge.

$$V(\vec{r}) = -\int_\infty^r E(r')\,dr' = -\int_\infty^r \frac{1}{4\pi\varepsilon_0}\frac{q}{r'^2}\,dr'$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_\infty^r \frac{dr'}{r'^2} = \frac{-q}{4\pi\varepsilon_0}\left(-\frac{1}{r}\Big|_\infty^r\right)$$

$$= \frac{kq}{r}$$ ✓