Midterm for Electromagnetic Theory (PHYS330)

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Abstract

This exam may be completed at home, and covers chapters 1-4 of the course text and in-class examples. Class notes and the course text may be used (open book), but no internet sources are allowed. The daily warm-up exercises are good study materials for this exam.

1 Math Bootcamp

1. (a) If $\mathbf{A}$ and $\mathbf{B}$ are two vector functions, what does the expression $(\mathbf{A} \cdot \nabla)\mathbf{B}$ mean? That is, what are its x, y, and z components, in terms of the Cartesian components of $\mathbf{A}$, ∇, and $\mathbf{B}$? (b) Compute $(\hat{r} \cdot \nabla)\hat{r}$, where $\hat{r}$ is $\mathbf{r}/r$. (c) One can show that the *force* on a dipole induced by a non-uniform field is

$$\mathbf{F} = (\mathbf{p} \cdot \nabla)\mathbf{E} \tag{1}$$

Compute the force on a physical dipole located at the origin with $\mathbf{p} = q\mathbf{d} = qd\ \hat{\mathbf{x}}$ in a field with associated potential $V(\mathbf{r}) = V_0 r^2 + V_1$.

2. Evaluate the following integral using (a) the three-dimensional Dirac delta function, or (b) integration by parts. Solving both earns a bonus point.

$$J = \int_{\mathcal{V}} e^{-r} \left(\nabla \cdot \frac{\hat{\mathbf{r}}}{r^2} \right) \tag{2}$$

2 Electrostatics

1. Suppose two dipoles, each with dipole moment $\mathbf{p}$ pointed in opposite directions, form a square with alternating positive and negative charges and side length d. Calculate the field $\mathbf{E}_{\rm tot}$ at the following points P: (a) $P = (0, 0)$, (b) $P = (2d, 0)$, and $P = (0, 2d)$. Check units and take limits[1].

[1] This object is an electrostatic quadrupole.

2. The electric potential of some configuration is given by the expression

$$V(\mathbf{r}) = A\frac{e^{-\lambda r}}{r} \tag{3}$$