Electromagnetc Theory: PHYS330

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Summary

Week 6 Summary

1. Current and Newton's Law
2. Flux rule from Lorentz force
3. Faraday's Law: Inspired by Symmetry
    - Induced E-fields
    - Quasi-static behavior
4. Inductors and analog filtering (special topic)

Current and Newton’s Law

Current and Newton's Law

Why does current move at a constant velocity, if it is driven by an electric field?

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) = \sigma\vec{E} \tag{1}$$

This formula governs the current density as a function of force per unit charge, and it assumes small drift velocities so that the magnetic contributions are zero.

What is the acceleration of an electron in the middle of a parallel plate capacitor with empty space in the middle? Let the charge density be 1 μ C per cm^2. The mass of an electron is 9.1×10^{-31} kg, and $\epsilon_0 = 8.85 \times 10^{-12}$ F/m.

Current and Newton's Law

Why does current move at a constant velocity, if it is driven by an electric field?

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) = \sigma \vec{E} \tag{2}$$

This formula governs the current density as a function of force per unit charge, and it assumes small drift velocities so that the magnetic contributions are zero.

What is the acceleration of a charge flowing in a conductor that connects the parallel plates of the capacitor? Let the charge density be 1 μ C per cm^2. The mass of an electron is 9.1×10^{-31} kg, and $\epsilon_0 = 8.85 \times 10^{-12}$ F/m. Let the total effective resitance be 1 Ω.

Current and Newton's Law

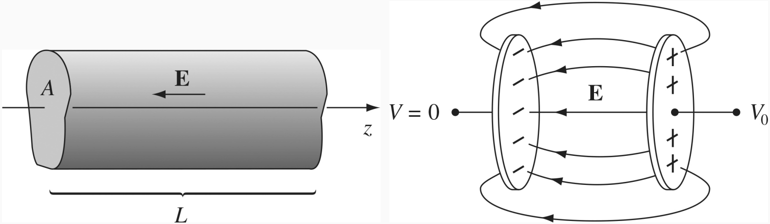
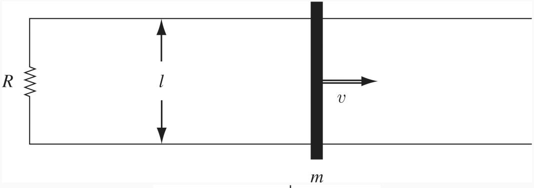


Figure 1: (Left) A conductor. (Right) A capacitor with the same geometry.

- Convince yourself that if the potential difference between the left and right sides of the conductor (left) is $V_0 - 0 = V_0$, that $V(z) = (V_0/L)z$.
- The E-field is therefore $\vec{E} = -V_0/L\hat{z}$, but the current does not accelerate.

Motional EMF Problems: Flux Rule from the Lorentz Force

Flux Rule from Lorentz Force



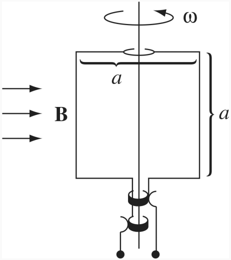


Figure 2: Motional emf problems resulting from the Lorentz force, solvable by the flux rule. (Top) Frictionless rails problem ... rail guns. (Bottom): the AC generator.

Faraday's Law: Inspired by Symmetry

Faraday's Law

In words: E-fields (via currents) generate B-fields. Changing B-fields induce E-fields (in loops). In differential form:

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \tag{3}$$

In integral form (via Stoke's Theorem):

$$\boxed{\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}} \tag{4}$$

Faraday's Law

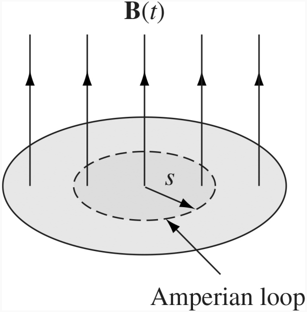


Figure 3: Cylindrical symmetry and the use of Faraday's Law to obtain emf.

- What is the magnitude and shape of the $\vec{E}$-field a distance s from the origin?
- What is the current I in a loop of wire at the same radius?

Faraday's Law

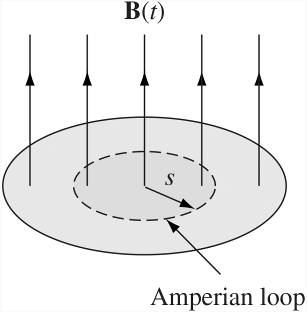


Figure 4: Cylindrical symmetry and the use of Faraday's Law to obtain emf.

- What is the acceleration of a point charge located a distance s from the origin?

Exercise 7.13: A square loop of wire, with sides of length a, lies in the first quadrant of the xy-plane, with one corner at the origin. In this region, there is a nonuniform time-dependent magnetic field $\vec{B}(y,t) = ky^3t^2\hat{z}$ (where k is constant). Find the emf induced in the loop.

Special Topic: Inductors and Analog Filtering of Voltage Signals

Conclusion

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