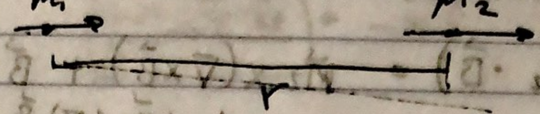
EM HW #6: 6.3, 6.7, 6.16

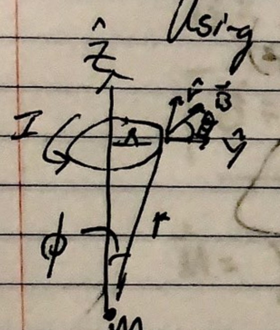
(6.3.)



$$\vec{B}_{dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}\right]$$

(6.2) $F = 2\pi I R B\cos\theta$

Using Ampére model of dipole



Thus $B\cos\theta = \vec{B}\cdot\hat{y}$

$$B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}_1\cdot\hat{r})(\hat{r}\cdot\hat{y}) - \vec{m}_1\cdot\hat{y}\right]$$

$m_1\cos\phi$ $\quad$ $\sin\phi$ $\quad$ $\cancel{0}$

$$B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}\,3m_1\cos\phi\sin\phi$$

$$\cos\phi = \frac{\sqrt{r^2-R^2}}{r} \qquad \sin\phi = \frac{R}{r}$$

$$B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}\,3m_1\,\frac{R\sqrt{r^2-R^2}}{r^2}$$

$$\Rightarrow F = 2\pi I R^2 \frac{\mu_0}{2\cdot 4\pi}\,3m_1\,\frac{\sqrt{r^2-R^2}}{r^5}$$

$I R^2\pi = m_2$

$$F = \frac{3\mu_0}{2\pi} m_1 m_2 \frac{\sqrt{r^2-R^2}}{r^5} \qquad \text{if } R \ll r$$

a.)

$$\boxed{F = \frac{3\mu_0}{2\pi} m_1 m_2 \frac{r}{r^4}}$$

6.3.) b.) $\vec{F} = \nabla(\vec{m}\cdot\vec{B})$ (6.3)

$$\vec{F} = \nabla(\vec{m}_2 \cdot \vec{B}) = \vec{m}_2 \times (\nabla \times \vec{B}) + \vec{B} \times \underbrace{(\nabla \times \vec{m}_2)}_{\emptyset} + (\vec{m}_2 \cdot \nabla)\vec{B} + \underbrace{(\vec{B}\cdot\nabla)\vec{m}_2}_{\emptyset}$$

$$\vec{F} = (\vec{m}_2 \cdot \nabla)\vec{B}$$

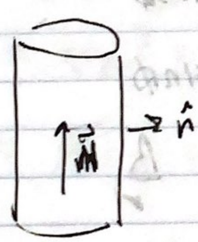
$$= \left(m_2 \frac{d}{dz}\right)\left[\frac{\mu_0}{4\pi}\frac{1}{z^3}\left(3(\vec{m}_1\cdot\hat{z})\hat{z} - \vec{m}_1\right)\right]$$

$3\vec{m}_1 - \vec{m}_1 = 2m_1$

$$= \frac{2\mu_0}{4\pi} m_1 m_2 \,\hat{z} \frac{d}{dz}\left(\frac{1}{z^3}\right)$$

$$\vec{F} = \frac{-3\mu_0}{2\pi}\frac{m_1 m_2}{z^4}\hat{z}$$

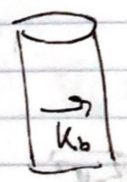
6.7.)



$\vec{J}_b = \nabla \times \vec{M} = 0$

↳ no curl

$\vec{K}_b = \vec{M} \times \hat{n} = M\hat{\phi}$



← solenoid

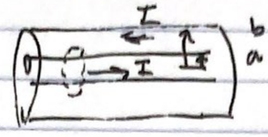
$\oint B \cdot dl = \mu_0 I_{enc}$

$\Rightarrow$ $\vec{B} = 0$ outside

inside $\vec{B} = \mu_0 K_b = \mu_0 \vec{M}$

$\vec{B} = \mu_0 \vec{M}$

6.16.)



$\rightarrow \hat{z}$

$$\oint \vec{H} \cdot d\vec{l} = I_{f\,enc}$$

$$H(2\pi s) = I_{f\,enc} = I$$

$$H = \frac{I}{2\pi s} \qquad \text{dir in } \hat{\phi}$$

$$\vec{H} = \frac{I}{2\pi s}\hat{\phi}$$

$$\vec{B} = \mu_0(1+\chi_m)\vec{H}$$

$$\boxed{\vec{B} = \mu_0(1+\chi_m)\frac{I}{2\pi s}\hat{\phi}}$$

$\leftarrow$ field btw. tubes

$$\vec{M} = \chi_m \vec{H} = \frac{\chi_m I}{2\pi s}\hat{\phi}$$

$$\vec{J}_b = \nabla \times \vec{M} = \frac{1}{s}\frac{\partial}{\partial s}\left(\cancel{s}\frac{\chi_m I}{2\pi \cancel{s}}\right)\hat{z} = 0$$

$$\vec{K}_b = \vec{M} \times \hat{n} = \frac{\chi_m I}{2\pi s}\hat{z}$$

at $s=a$, $s=b$

$$\vec{K}_b = \begin{cases} \dfrac{\chi_m I}{2\pi a} & s=a \\[2ex] -\dfrac{\chi_m I}{2\pi b} & s=b \end{cases}$$

Current enclosed in region btw. cylinders

$$I + \frac{\chi_m I}{\cancel{2\pi a}}\cancel{2\pi a} = (1+\chi_m)I$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} = \mu_0(1+\chi_m)I = B\,2\pi s$$

$$\Rightarrow \vec{B} = \frac{\mu_0(1+\chi_m)I}{2\pi s}\hat{\phi} \quad \checkmark$$