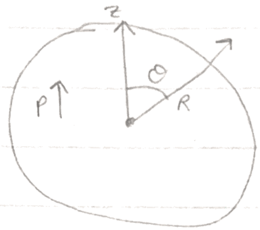
Quiz 4

Band Charges

1A) $\sigma_b = P\cdot\hat{n}$

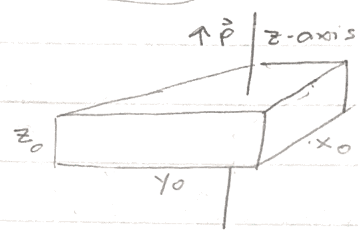
$n = (\cos\phi\sin\theta, \sin\phi\sin\theta, \cos\theta)$

$\sigma_b = P\cos\theta$



1B) $\sigma_b = P\cdot\hat{n}$

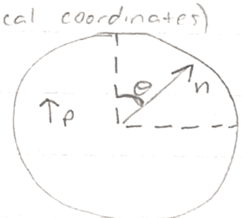
sliced $= \frac{q}{A_{end}} = \frac{q}{x_0 y_0}$



$\sigma_b = \frac{q}{x_0 y_0}$

$\vec{r} = r\cdot\hat{r}$

2A) $\vec{P} = P_0\vec{r}$ (spherical coordinates)



$\rho_b = -\nabla\cdot P$

$\nabla\cdot v = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r)$

$\nabla\cdot P = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2(P_0(r\cdot\hat{r})))$

$\nabla\cdot P = \frac{1}{r^2}\frac{\partial}{\partial r}(r^3 P_0)$

$\nabla\cdot P = \cancel{\frac{1}{r^2}}(3\cancel{r^2}P_0)$

$\boxed{-\nabla\cdot P = -3P_0}$

2b) $\vec{P} = P_0 P_2(\cos\theta)\hat{\theta}$

$\nabla\cdot v = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, v_\theta)$

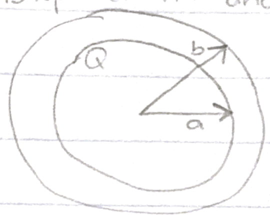
$\nabla\cdot p = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\, P_0 P_2\cos\theta)$

$\nabla\cdot p = \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(P_0 P_2\sin\theta\cos\theta)$

$= \frac{P_0P_2}{r\sin\theta}(\cos\theta\cos\theta - \sin\theta\sin\theta) = \frac{P_0P_2}{r\sin\theta}(\cos^2\theta - \sin^2\theta) = \frac{P_0P_2}{r\sin\theta}(\cos(2\theta))$

$\boxed{= -\frac{P_0P_2}{r\sin\theta}\cos(2\theta)}$

The Electric Displacement and Linear Dielectrics



$a < r < b$

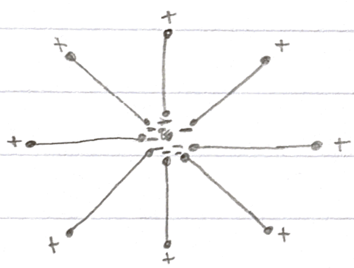
$$\oint D \cdot da = Q_{fenc}$$

$$DA = Q_{fenc}$$

$$D(4\pi r^2) = Q_{fenc}$$

$$D = \frac{Q}{4\pi r^2}\hat{r}$$

$$D = \varepsilon E \qquad E = \frac{D}{\varepsilon} \qquad E = \frac{\left(\frac{Q}{4\pi r^2}\right)}{\varepsilon} = \boxed{\frac{Q}{4\pi \varepsilon r^2}\hat{r}}$$



$$\bar{P} = \varepsilon_0 \chi_e \bar{E}$$

$$\bar{D} = \varepsilon_0 \bar{E} + \bar{P} = \varepsilon_0 (1 + \chi_e) E = \varepsilon \bar{E}$$

$$\bar{\nabla} \cdot \bar{D} = \rho_f \qquad \bar{\nabla} \times \bar{D} = 0$$

$$D = \varepsilon_0 \bar{E}_{vac} \qquad D = \varepsilon_0 (1 + \chi_e) E = \varepsilon \bar{E}$$

$$\bar{E} = \frac{1}{\varepsilon} D = \frac{1}{\varepsilon_r} E_{vac} = \frac{1}{K} \bar{E}_{vac}$$

$$E = \frac{1}{K} E_{vac} = \left(\frac{1}{4\pi\varepsilon}\right)\left(\frac{q}{r^2}\right)\hat{r} = \boxed{\frac{q}{4\pi \varepsilon r^2}\hat{r}}$$