HW #6 #6.3, 6.7, 6.16 & Fourier Transform Worksheet

6.3) Find the force of attraction between two magnetic dipoles $\vec{m}_1$ & $\vec{m}_2$, oriented as shown in 6.7, a distance r apart (a) using Eq. 6.2 and (b) using Eq. 6.3

$6.2: \; F = 2\pi I R B \cos\theta \qquad 6.3: \; \vec{F} = \nabla(\vec{m}\cdot\vec{B})$

$B\cos\theta = \vec{B}\cdot\hat{j} \qquad \vec{B}_{\vec{m}} = \frac{\mu_0}{4\pi}\frac{3(\vec{m}_1\cdot\hat{r})(\hat{r}-\vec{m}_1)}{r^3}$

$B\cos\theta = \frac{\mu_0}{4\pi}\frac{3(\vec{m}_1\cdot\hat{r})(\hat{r}\cdot\hat{j})-(\vec{m}_1\cdot\hat{j})}{r^3}$

$B\cos\theta = \frac{\mu_0}{4\pi}\frac{3m_1\sin\phi\cos\phi}{r^3}$

$m_2 = \pi I R^2 \qquad \sin\phi = \frac{R}{r} \qquad \cos\phi = \frac{\sqrt{r^2-R^2}}{r}$

$F = 2\pi I R \frac{\mu_0}{4\pi}\frac{3m_1\cos\phi\sin\phi}{r^3}$

$F = 2\pi I R \frac{\mu_0}{4\pi}\frac{3m_1}{r^3}\frac{\sqrt{r^2-R^2}}{r}\frac{R}{r}$

$F = \frac{3m_1}{2}\pi I R^2 \mu_0 \frac{\sqrt{r^2-R^2}}{r^5}$

$F = \frac{3}{2}\mu_0 m_1 m_2 \frac{\sqrt{r^2-R^2}}{r^5}$

$R \ll r \rightarrow \boxed{F = \frac{3}{2}\mu_0 m_1 m_2 \frac{1}{r^4}}$

(b) $\vec{F} = \nabla(m_2\cdot B)$

$= m_2(\nabla\cdot B)$

$= m_2 \frac{d}{dr}\left[\frac{\mu_0}{4\pi}\frac{1}{r^3}3(\vec{m}_1\cdot\hat{r})(\hat{r}-\vec{m}_1)\right]$

$= m_2 \frac{d}{dz}\left[\frac{\mu_0}{4\pi}\frac{1}{z^3}3(\vec{m}_1\cdot\hat{z})(\hat{z}-\vec{m}_1)\right]$

$\boxed{\vec{F} = -\frac{3}{2}\frac{\mu_0}{\pi}\frac{m_1 m_2}{r^4}\hat{z}}$

6.7) An infinitely long circular cylinder carries a uniform magnetization $\vec{M}$ parallel to its axis. Find the magnetic field inside & outside

$K_b = \nabla\times\hat{n} = M\hat{\phi}$

$\boxed{B_{out} = 0}$

$B_{in} = \mu_0 K_b \hat{z}$

$\boxed{B_{in} = \mu_0\vec{M}}$

6.16) A coaxial cable consists of two very long cylindrical tubes separated by linear insulating material of magnetic susceptibility, χ_m. A current I flows down the inner conductor & returns along the outer one; in each case the current distributes itself uniformly over the surface. Find the magnetic field in the region between tubes

$$\oint \vec{H} \cdot d\vec{l} = I_{free} \qquad \vec{H} = \frac{I}{2\pi s}\hat{\phi} \qquad \vec{B} = \mu_0(1+\chi_m)\vec{H}$$

$$\vec{B} = \frac{\mu_0(1+\chi_m)I}{2\pi s}\hat{\phi} \qquad M = \chi_m H = \frac{\chi_m I}{2\pi s}\hat{\phi}$$

$$J_b = \nabla \times M = 0$$

$$K_b = M \times \hat{n} = \begin{cases} \frac{\chi_m I}{2\pi a}\hat{z} & \text{when } s=a \\ -\frac{\chi_m I}{2\pi b}\hat{z} & \text{when } s=b \end{cases}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$B(2\pi s) = \mu_0(1+\chi_m)I$$

$$\boxed{B = \frac{\mu_0(1+\chi_m)I}{2\pi s}\hat{\phi}}$$

FOURIER TRANSFORM WORKSHEET

Compute the Fourier transform of the Dirac delta function:

$$\tilde{E}(\nu) = \int_{-\infty}^{\infty} E(t)\, e^{-2\pi j \nu t}\, dt$$

$$\tilde{E}(t_0) = \int_{-\infty}^{\infty} \delta(t-t_0)\, e^{-2\pi j \nu t_0}\, dt$$

$$\tilde{E}(t_0) = e^{-2\pi j \nu t_0} \cancel{\int_{-\infty}^{\infty} \delta(t-t_0)\, dt}$$

$$\boxed{= e^{-2\pi j \nu t_0}}$$

Inverse Fourier transform

$$E(t) = \int_{-\infty}^{\infty} \tilde{E}(\nu)\, e^{2\pi j \nu t}\, d\nu$$

$$= \int_{-\infty}^{\infty} \left(e^{-2\pi j \nu t_0}\right) e^{2\pi j \nu t}\, d\nu$$

$$\boxed{= \delta(t-t_0)}$$

Write down the Fourier transform of $e^{2\pi j \nu_0 t}$ which is equivalent to computing the Fourier transform of a sine wave

$$E(t) = \sin(2\pi \nu_0 t) = \left[\frac{e^{2\pi j \nu_0 t} - e^{-2\pi j \nu_0 t}}{2j}\right]$$

$$E(t) = \int_{-\infty}^{\infty} \left[\frac{e^{2\pi j \nu_0 t} - e^{-2\pi j \nu_0 t}}{2j}\right] e^{-2\pi j \nu t}\, dt$$

$$E(t) = \frac{1}{2j}\left[\int_{-\infty}^{\infty} e^{2\pi j \nu_0 t}\, e^{-2\pi j \nu t}\, dt - \int_{-\infty}^{\infty} e^{-2\pi j \nu_0 t}\, e^{-2\pi j \nu t}\, dt\right]$$