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Reading Quiz #1

1. $a(\vec{B}+\vec{C}) = a\vec{B} + a\vec{C}$

$\vec{B} = B_x\hat{i} + B_y\hat{j}$ $\quad \vec{C} = C_x\hat{i} + C_y\hat{j}$

$\vec{B}+\vec{C} = B_x\hat{i} + C_x\hat{i} + B_y\hat{j} + C_y\hat{j}$

$a(\vec{B}+\vec{C}) = a(B_x\hat{i} + C_x\hat{i} + B_x j + C_y\hat{j})$

$= aB_x\hat{i} + aB_y j + aC_x\hat{i} + aC_y\hat{j}$

$= a\vec{B} + a\vec{C}$

$= a(\vec{B}+\vec{C})$

2. $\nabla(f(x,y) + \nabla g(x,y))$

this makes a scalar

and you can't add a scalar to a vector.

3. $\nabla \times V = 0$

$\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} = 0$ $\quad \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} = 0$ $\quad \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} = 0$

a) $V = x\hat{i} + y\hat{j}$

b) $x^2 + y^2 = 1$

$x = r\cos(t) \qquad y = r\sin(t)$

$\vec{r}(t) = \langle \cos(t), \sin(t) \rangle \qquad 0 \le t \le 2\pi$

$v = \langle x, y \rangle$

$F(x(t), y(t)) = \langle \cos(t), \sin(t) \rangle$

$v' = \langle -\sin(t), \cos(t) \rangle$

$$\int_0^{2\pi} F(x(t), y(t)) \cdot v' \, dt$$

$$= \int_0^{2\pi} (\cos(t)(-\sin(t)) + (\sin(t)\cos(t)) \, dt$$

$$\cos(t)\sin(t) - \cos(t)\sin(t)$$

$$= \int_0^{2\pi} (0) \, dt = 0$$

yes, because for ever point it's opposite corresponding point in the $-x, -y$ will cancel the original point in opposite direction.

2. $\vec{V} = (2x+1)\hat{x} + (3y-x)\hat{y}$

$x' = x\cos\theta - y\sin\theta$

$y' = x\sin\theta + y\cos\theta$

$\cos(45^\circ) = \sin(45^\circ) = \frac{1}{\sqrt{2}}$

b) $v' = (2x+1)\frac{1}{\sqrt{2}} - (3y-x)\frac{1}{\sqrt{2}}\,\hat{x}$

$+ \frac{1}{\sqrt{2}}(2x+1) + (3y-x)\frac{1}{\sqrt{2}}\,\hat{y}$

c) $x=1$ $y=1$ $\vec{v} = 3\hat{x} + 2\hat{y}$

$|\vec{v}| = \sqrt{9+4} = \sqrt{13}$

$x=1$ $y=1$ $v' = \left(\frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}}\right)\hat{x} + \left(\frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}}\right)\hat{y}$

$|v'| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{5}{\sqrt{2}}\right)^2}$

$= \sqrt{\frac{1}{2} + \frac{25}{2}} = \sqrt{\frac{26}{2}} = \sqrt{13}$

3. $\vec{v} = s^{-1}\hat{\phi}$ (s, ϕ, z)

$\int_S (\nabla \times \vec{v}) \cdot da$ $s^2 + z^2 = 1$

$= \oint_P v \cdot dl$ $0 \leq \phi \leq 2\pi$

z

$v = \frac{1}{s}\hat{\phi}$

$\oint (\nabla \times v) \cdot da = 0$ for closed surface

4. $f(x) * g(x) = \left(\frac{f(x) - g(x)}{f(x) + g(x)}\right)$

$\int_{-\infty}^{\infty} (f(x) * g(x))\delta(x)\, dx$

$= f(0) * g(0)$

• $f(x) = \cos(x)$ $g(x) = \sin(x)$

$= 1 \cdot 0$

$= 0$

- $f(x) = \cosh(x) \quad g(x) = \sinh(x)$

  $= \cosh(0) \cdot \sinh(0)$

  $\;\; 1 \cdot 0$

  $= 0$

- $f(x) = a + ax + ax^2 + \dots \qquad g(x) = b + bx + bx^2 + \dots$

  $f(0) = a \qquad g(0) = b$

  $= ab$