11/20/20

Reading Quiz 3

Discussions about Vectors (Prelude to Fourier's Trick)

1. $\vec{V} = a\hat{x} + b\hat{y} + c\hat{z} \longrightarrow c\hat{z} = \vec{V} - a\hat{x} - b\hat{y}$

$\vec{V} \cdot \hat{z} = (a\hat{x} + b\hat{y} + c\hat{z}) \cdot (0\hat{x} + 0\hat{y} + 1\hat{z}) = c(\hat{z} \cdot \hat{z}) = c$

B: $\vec{V} \cdot \hat{z}$

2. $\vec{x} = \sum_{i=1}^{n} C_i \hat{x}_i$

- Can obtain the coefficient C_7 with **D: $\hat{x}_7 \cdot \vec{x}$**

3. Fourier series: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx)$

$f(x) = \sin(3x)$ *odd function

$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$, $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\sin(nx)\,dx$

$= b_0 \sin(0)x + b_1 \sin(1x)$ $= \frac{2}{\pi}\int_0^{\pi} \sin(\overset{A}{3}x)\sin(\overset{B}{n}x)\,dx$

$+ b_2 \sin(2x) + b_3 \sin(3x)$ $\sin A \sin B = \frac{1}{2}(\cos(A-B) - \cos(A+B))$

$= (2/\pi)\int_0^{\pi} \frac{1}{2}(\cos(3x-nx) - \cos(3x+nx))\,dx$

$+ b_4 \sin(4x) + \ldots\ldots$ $= \frac{1}{\pi}\int_0^{\pi} \cos((3-n)x) - \cos((3+n)x)\,dx$

$= \frac{1}{\pi}\left[\frac{\sin((3-n)x)}{3-n} - \frac{\sin((3+n)x)}{3+n}\right]_0^{\pi}$

$= \frac{1}{\pi}\left(\frac{\sin((3-n)\pi)}{3-n} - \frac{\sin((3+n)\pi)}{3+n}\right)$

$= 0$ for all n

Fourier's Trick & Boundary Value Problems

1. $V(x,y,z) \to 0$ as $y \to \infty$

B: $Y(y) = \sinh(x)$ can't be part of the solution

2. Equation 3.50: $C_{n,m} = \frac{4V_0}{ab}\int_0^a \int_0^b \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{m\pi z}{b}\right)dy\,dz$

$V_0(y,z) = V_0$

$C_{n,m} = \frac{4V_0}{ab}\left[\int_0^a \sin\left(\frac{n\pi y}{a}\right)dy \int_0^b \sin\left(\frac{m\pi z}{b}\right)dz\right]$

$= \begin{cases} 0, & \text{if } n \text{ or } m \text{ is even} \\ (16V_0/\pi^2 nm), & \text{if } n \text{ \& } m \text{ are odd} \end{cases}$