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Quiz 1 Chapter 1 10/30

1.) Vectors and Scalors

1. $a(\vec{B}+\vec{C}) = a\vec{B} + a\vec{C}$ $\vec{B} = \langle B_x, B_y, B_z \rangle$, $\vec{C} = \langle C_x, C_y, C_z \rangle$

$\vec{B}+\vec{C} = (B_x+C_x)\hat{i} + (B_y+C_y)\hat{j} + (B_z+C_z)\hat{k}$

$a(\vec{B}+\vec{C}) = a(B_x+C_x)\hat{i} + a(B_y+C_y)\hat{j} + a(B_z+C_z)\hat{k}$

$= aB_x\hat{i} + aC_x\hat{i} + aB_y\hat{j} + aC_y\hat{j} + aB_z\hat{k} + aC_z\hat{k}$

$= aB_x\hat{i} + aB_y\hat{j} + aB_z\hat{k} + aC_x\hat{i} + aC_y\hat{j} + aC_z\hat{k}$

$= a\vec{B} + a\vec{C}$ ✓

2. What is wrong w/ combination of objects? $\nabla(f(x,y) + \nabla g(x,y))$

$\nabla(f(x,y) + \nabla g(x,y)) = \nabla f(x,y) + \nabla^2 g(x,y)$

The thing that is wrong w/ the combination is that you cannot add a gradient of a vector w/ the laplacian of the vector g(x,y).

3. Create a vector field that has a zero curl

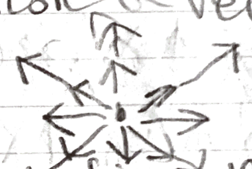
a)  has a zero curl

Fig 1.18a

b) Perform line integral w/ unit circle in xy plane as path & parametric eqn

$\vec{V} = x\hat{i} + y\hat{j} + z\hat{k}$ $x^2+y^2=1$ $\oint \vec{V}\cdot d\vec{l}$ $r=1$

$r(t) = \langle \cos(t), \sin(t) \rangle$

$0 < t < 2\pi$

$r'(t) = \langle -\sin(t), \cos(t) \rangle$ $x = r\cos t$ $y = r\sin t$

dx dy $x = \cos(t)$ $y = \sin(t)$

$\int V \cdot dl \quad dl = r'(t)\,dt \quad \int (x\hat{i} + y\hat{j}) \cdot r'(t)\,dt$

$= \int_0^{2\pi} \cos(t)\sin(t)\,dt + \sin(t)\cos(t)\,dt$

$= \int_0^{2\pi} (\cos(t)\sin(t) + \sin(t)\cos(t))\,dt = \int_0^{2\pi} 0\,dt = \boxed{0}$

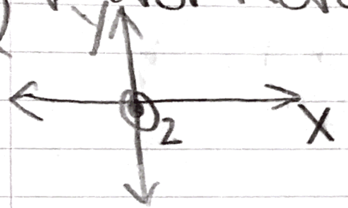
Yes this does make sense since we know that the curl is zero. So the integral must be zero as well.

c) divergence $\nabla \cdot \vec{v} = \frac{\partial(x)}{\partial x} + \frac{\partial(y)}{\partial y} + \frac{\partial(z)}{\partial z} = 1 + 1 + 1 = \boxed{3}$

$\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$

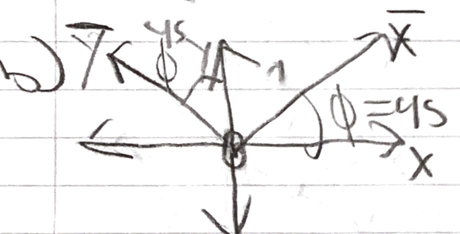
2) Vector Rotation

a)



$\vec{v} = \hat{i} + \hat{j} \qquad \phi = 45^\circ$

b)



$$\underbrace{\begin{pmatrix} \bar{V}_x \\ \bar{V}_y \end{pmatrix}}_{\bar{V}} = \underbrace{\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}}_{R} \underbrace{\begin{pmatrix} V_x \\ V_y \end{pmatrix}}_{\vec{V}}$$

$$\boxed{\begin{aligned} \bar{V}_x &= V_x\cos\phi + V_y\sin\phi \\ \bar{V}_y &= -V_x\sin\phi + V_y\cos\phi \end{aligned}}$$

c) $\bar{V}_x^2 = V_x^2\cos^2\phi + V_y^2\sin^2\phi + \cancel{2V_xV_y\cos\phi\sin\phi}$

$\bar{V}_y^2 = V_x^2\sin^2\phi + V_y^2\cos^2\phi - \cancel{2V_xV_y\sin\phi\cos\phi}$

$$|\vec{v}| = \bar{v}_x^2 + \bar{v}_y^2$$

$$\boxed{\begin{gathered}\bar{v}_x^2 + v_y^2 = v_x^2 + v_y^2 = |\vec{v}| \\ \text{The magnitude is the same.}\end{gathered}}$$

3) We know that for stokes theorem, corollary 2 states that $\int (\nabla \times \vec{v}) \cdot d\vec{a} = 0$ for only closed surface since the boundary shrink down to a point.

4) $\int_{-\infty}^{\infty} (f(x) \star g(x))\, \delta(x)\, dx$

$$f(x) \star g(x) = \left(\frac{f(x) - g(x)}{f(x) + g(x)}\right)$$

a) $f(x) = \cos(x)$ and $g(x) = \sin(x)$

$$\int_{-\infty}^{\infty} (f(x) \star g(x))\, \delta(x)\, dx = (f(0) \star g(0))$$

$$= \frac{f(0) - g(0)}{f(0) + g(0)} = \frac{\cos(0) - \sin(0)}{\cos(0) - \sin(0)} = \boxed{1}$$

b) $f(x) = \cosh(x)$ and $g(x) = \sinh(x)$

$$\int_{-\infty}^{\infty} (f(x) \star g(x))\, \delta(x)\, dx = \frac{f(0) - g(0)}{f(0) + g(0)} = \frac{\cosh(0) - \sinh(0)}{\cosh(0) - \sinh(0)} = \boxed{1}$$

c) $f(x) = a + ax + ax^2 \ldots$ and $g(x) = b + bx + bx^2 \ldots$

$$\int_{-\infty}^{\infty} (f(x) \star g(x))\, \delta(x)\, dx = \frac{f(0) - g(0)}{f(0) + g(0)} = \boxed{\frac{a - b}{a + b}}$$