Electromagnetic Theory Quiz 1
Dane Goodman 10/30/20

1. a) ① $\vec{B} = \langle x_1, y_1, z_1 \rangle$
$\vec{C} = \langle x_2, y_2, z_2 \rangle$
$\vec{B} + \vec{C} = \langle x_1 + x_2, y_1 + y_2, z_1 + z_2 \rangle$
$a(\vec{B} + \vec{C}) = a\langle x_1 + x_2, y_1 + y_2, z_1 + z_2 \rangle$
$= \langle a(x_1 + x_2), a(y_1 + y_2), a(z_1 + z_2) \rangle$
② $a\vec{B} + a\vec{C} = \langle ax_1, ay_1, az_1 \rangle + \langle ax_2, ay_2, az_2 \rangle$
$= \langle ax_1 + ax_2, ay_1 + ay_2, az_1 + az_2 \rangle$
$= \langle a(x_1 + x_2), a(y_1 + y_2), a(z_1 + z_2) \rangle$

$\boxed{① = ②, \text{ therefore } a(\vec{B} + \vec{C}) = a\vec{B} + a\vec{C}}$

b) What is wrong with this function is that you can't multiply a gradient to another gradient. Gradients only work with cross product and dot product, can't add a gradient and a function.

c) i) $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}$ $\vec{F}(x,y) = x\hat{i} + y\hat{j}$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & 0 \end{vmatrix} = \left(\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(y)\right)\hat{i} - \left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}(x)\right) + \left(\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right)$$

$= 0\hat{i} - 0\hat{j} + 0\hat{k} = \boxed{0}$ therefore no curl

ii) $\int_0^{2\pi} \cos(x) + \sin(x)\, dx = \sin(x)\Big|_0^{2\pi} - \cos(x)\Big|_0^{2\pi}$
$= \boxed{0}$

iii) $\vec{F}(x,y) = x\hat{i} + y\hat{j}$ $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}$
$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) = 1 + 1 = \boxed{2}$