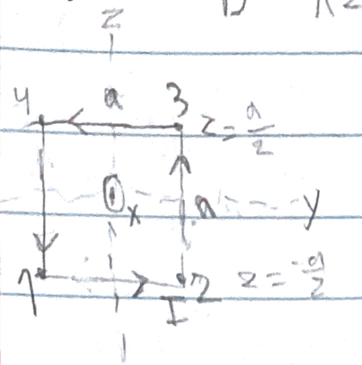
HW 5 ; 5.4, 5.7, 5.11, 5.12 | 5.16, 5.19, 5.21 | ex 5.12 | 5.23, 5.27

5.4) $B = kz\hat{x}$, $F = I\int d\vec{l} \times \vec{B}$

← Add my forces



$F_{12} = I\int dy\,\hat{y} \times k(\frac{-a}{2})\hat{x}$

$F_{12} = -Ik\frac{a}{2}\int dy\,(-\hat{z}) = Ik\frac{a^2}{2}\hat{z}$

Cancel each other

$F_{23} = I\int dz\,\cancel{\hat{z} \times kz\hat{x}} = 0$, $F_{41} = I\int \cancel{dz(-\hat{z}) \times kz\hat{x}} = 0$

$F_{34} = I\int dy(-\hat{y}) \times k\frac{a}{2}\hat{x} \Rightarrow Ik\frac{a^2}{2}\hat{z}$

$F_{net} = Ik\frac{a^2}{2}\hat{z} + Ik\frac{a^2}{2}\hat{z} = \boxed{Ika^2\hat{z}}$

5.7) $\int_V J\,d\tau = \frac{dp}{dt}$, note $\int_V \nabla\cdot(xJ)\,d\tau$, p is total dipole moment

$\frac{dp}{dt} = \frac{d}{dt}\int_V \rho r\,d\tau \Rightarrow -\int(\nabla\cdot J)r\,d\tau \Rightarrow \int_V \nabla\cdot(xJ)\,d\tau - \int_V J\hat{x}\,d\tau$

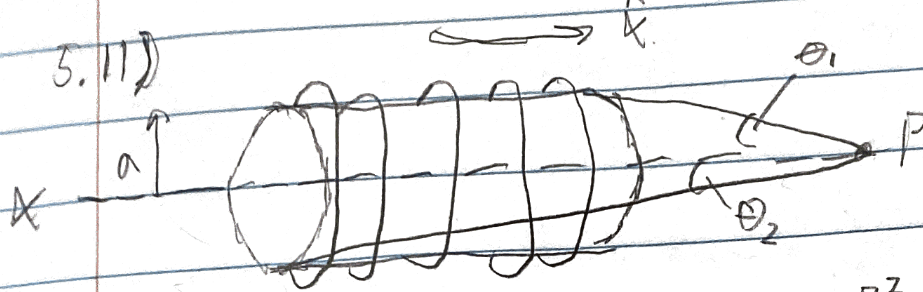
$\nabla\cdot(xJ) = x(\nabla\cdot J) + J\hat{x}$

J inside, surface is zero

$\Rightarrow \cancel{\oint xJ\cdot da} - \int_V J\hat{x}\,d\tau = -\int_V J\hat{x}\,d\tau \Rightarrow \int J\,d\tau$

$\boxed{\frac{dp}{dt} = \int_V J\,d\tau}$ q enclosed

5.11)



$$\vec{B} = \frac{\mu_0 I R^2}{2(R^2+z^2)^{3/2}}\hat{x}$$

$$B_{(z)} = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{r^2}\right)2\pi R = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}, \quad R = a$$

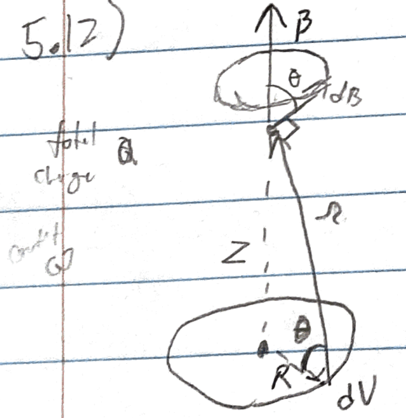
let $B = \frac{\mu_0 n I}{2}\int \frac{a^2}{(a^2+z^2)^{3/2}}dz$, $z = a\cot(\theta)$ so, $dz = -a\cot^2(\theta)$

$$\Rightarrow B = -\frac{\mu_0 n I}{2}\int_{\theta_1}^{\theta_2}\sin\theta\, d\theta$$

$$\Rightarrow B = \frac{\mu_0 n I}{2}(\cos(\theta_2) - \cos(\theta_1)), \quad \cos\theta_2 - \cos\theta_1 = 2$$

∂_a $B \Rightarrow \mu_0 n I$ - constant field

5.12)



$z = R\cos\theta$, $dI = K dl_\perp = KR\,d\theta$, $K = \sigma v$

$\sigma = \frac{Q}{4\pi R^2}$ then $v = \omega R\sin\theta$

$B(r) = \frac{\mu_0}{4\pi}\frac{qv\times\hat{r}}{r^2}$, $K = \frac{Q\omega}{4\pi R}\sin\theta$

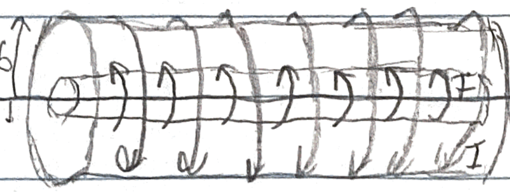
So $dI = \frac{Q\omega}{4\pi}\sin\theta\, d\theta$

So $dB = \frac{\mu_0}{2R}\sin^2\theta\, dI = \frac{\mu_0 Q\omega}{6\pi R}\sin^3\theta\, d\theta$, $\sin\theta = 1$

positive z direction

So $B = \frac{\mu_0 Q\omega}{6\pi R}\hat{z}$

5.16)



- inside inner solenoid
- between two
- outside

So outside solenoid would cancel & $B = 0$

Inside inner solenoid has radius a & & outter has radius b

So between the two must be $B = \mu_0 I (b - a) \hat{z}$

& B in inner solenoid would be: $B = \mu_0 I (a) \hat{z}$

5.19) $I = \int_S J \cdot da$, So Basically it doesn't matter

If $\nabla \cdot J = 0$ then the curl theorem can be used which converts the surface integral to a line integral. So using this process the choice of surface does not matter.

5.21) $\nabla \times B = \mu_0 J$, $\Rightarrow \nabla \cdot (\nabla \times B) = \mu_0 \nabla \cdot J$

which goes to $= -\mu_0 \frac{\partial \rho}{\partial t}$ if ρ is constant, $\nabla \cdot J = \frac{-\partial \rho}{\partial t}$

$\nabla \times E \Rightarrow 0$, so outside would be 0.

none of the seen doesn't have issues for maxwell's equation

$\nabla (\mathrm{curl}) = 0$

ex 5.12

Since $\oint A \cdot dl = \int (\nabla \times A) \cdot da = \int B \cdot da = \phi$

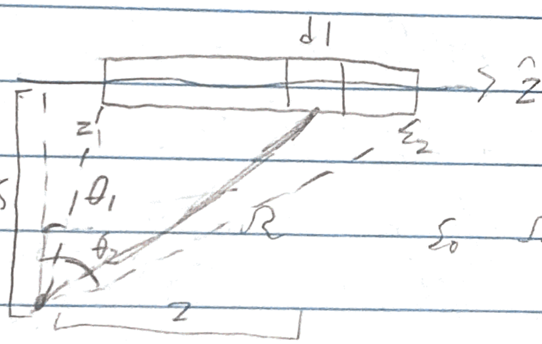
$\oint B \cdot dl = \mu_0 I_{enc}$ so $A(2\pi s) = \mu_0 n I (\pi s^2)$

so $A = \frac{\mu_0 n I}{2} s \hat{\phi}$ for $s \leq R$

$A = \frac{\mu_0 n I}{2} \frac{R^2}{s} \hat{\phi}$ for $s \geq R$ $\qquad \nabla \cdot A = 0$ ✓

So $\oint A \cdot dl = \int B \cdot da$

5.23



eqn 5.66 = $A = \frac{\mu_0}{4\pi} \int \frac{I}{\mathscr{R}} dl'$

so $\mathscr{R} = \sqrt{s^2 + z^2}$, so $A = \frac{\mu_0}{4\pi} \int \frac{I \hat{z}}{\mathscr{R}} dz$

$A \Rightarrow \frac{\mu_0}{4\pi} I \hat{z} \int_{z_1}^{z_2} \frac{dz}{\sqrt{s^2+z^2}} \Rightarrow \frac{\mu_0 I}{4\pi} \hat{z} \left(\ln(z + \sqrt{s^2+z^2}) \right|_{z_1}^{z_2}$

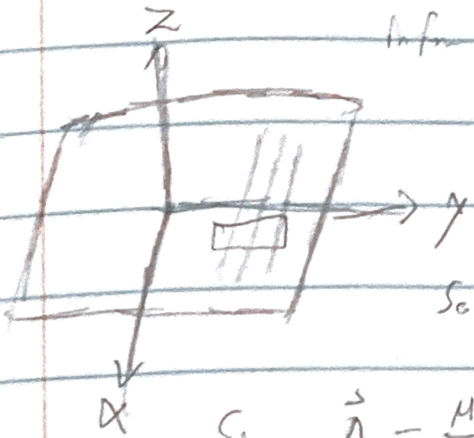
So $A = \frac{\mu_0 I}{4\pi} \left(\frac{z_2 + \sqrt{s^2 + z_2^2}}{z_1 + \sqrt{s^2 + z_1^2}} \right) \hat{z}$, eqn 5.37: $B = \frac{\mu_0 I}{4\pi s} (\sin\theta_2 - \sin\theta_1)$

$\vec{B} = \nabla \times A = -\frac{\partial A}{\partial s} \hat{\phi} \Rightarrow \frac{\mu_0 I}{4\pi s} \left(\frac{z_2}{\sqrt{s^2 + z_2^2}} - \frac{z_1}{\sqrt{s^2+z_1^2}} \right) \hat{\phi}$

$* = \sin\theta$

So $\vec{B} = \frac{\mu_0 I}{4\pi s} (\sin\theta_2 - \sin\theta_1) \hat{\phi}$

5.27



Infin uniform surface current $K = K\hat{x}$

$$B = \begin{cases} \frac{\mu_0}{2} K \hat{y} & z < 0 \\ -\frac{\mu_0}{2} K \hat{y} & z > 0 \end{cases}$$

So $\vec{B} = \nabla \times \vec{A} = B\hat{y}$

Since $\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\vec{K}}{\mathscr{r}} \, da \Rightarrow A(z)\hat{x}$

So $B = \frac{dA}{dz}$ $\therefore$ $A = \begin{cases} \frac{\mu_0 K}{2} z \hat{x} & z < 0 \\ -\frac{\mu_0 K}{2} z \hat{x} & z > 0 \end{cases}$