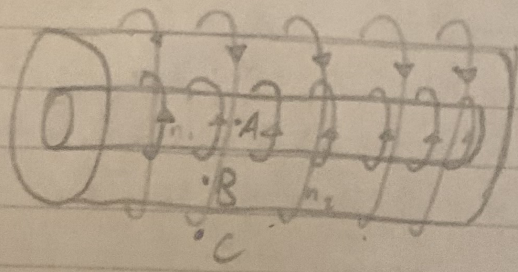
5.16) $B = \mu_0 n I$



$B_1 = \mu_0 n_2 I \qquad B_2 = -\mu_0 n_1 I$

$$\boxed{\begin{aligned} B_A &= \mu_0 (n_2 - n_1) I \\ B_B &= \mu_0 n_2 I \\ B_C &= 0 \end{aligned}}$$

5.17) a) $B = \frac{\mu_0 K}{2} \qquad K = \sigma v$

$B_{net} = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} = \mu_0 K = \boxed{\mu_0 \sigma v}$

b) $F = \int (K \times B) da \qquad f = K \times B$

$B = \frac{\mu_0 K}{2} \hat{y} \qquad K = \sigma v \hat{x}$

$f_m = (\sigma v \hat{x}) \times \left(\frac{\mu_0 K}{2} \hat{y}\right) = \sigma v \frac{\mu_0 K}{2} (\hat{x} \times \hat{y})$

$= \sigma v \frac{\mu_0 K}{2} \hat{z}$

$\boxed{f_m = \frac{\mu_0 (\sigma v)^2}{2}}$

c) $E = \frac{\sigma}{2\varepsilon_0}$

$f_e = \frac{\sigma^2}{2\varepsilon_0} \qquad f_e = f_m$

$\frac{\cancel{\sigma^2}}{\cancel{2}\varepsilon_0} = \frac{\mu_0 \cancel{\sigma^2} v^2}{\cancel{2}}$

$\boxed{v = 3 \times 10^8 \text{ m/s}} \qquad \sqrt{v^2} = \sqrt{\frac{1}{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

5.19) $I_{enc} = \int_s J \cdot da$

$\oint B\,dl = \mu_0 \sum I_{enc}$

$I_{enc} = \int_s J \cdot da$

$\nabla \cdot J = 0$

$\therefore$ for ~~no~~ surfaces with same boundary line, any surface can be taken as the integral is independent of the surface

5.20) a) $\rho = eN\left(\frac{1}{m}\right)d$

$\boxed{\rho = 1.4 \times 10^4 \ C/cm^3}$

b) $d = 1 \text{ mm} = 10^{-3} \ m^3$

$r = 5 \times 10^{-4} \ m^3$

$A = \pi r^2 = 7.85 \times 10^{-7} \ m^2$

$J = \frac{I}{A} = 1.2738 \times 10^6 \ A/m^2$

$J = \rho v$

$v = \frac{J}{\rho} = \boxed{9.1 \times 10^{-3} \ cm/s}$

c) $d = 0.01 \ m$

$\frac{F_{mag}}{\text{length}} = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} = \boxed{2 \times 10^{-7} \ N/cm}$

d) $E = \frac{1}{2\pi\varepsilon_0} \frac{\lambda}{d}$

$F_e = \frac{1}{2\pi\varepsilon_0} \frac{\lambda_1 \lambda_2}{d}$

$\lambda = \frac{I}{v}$

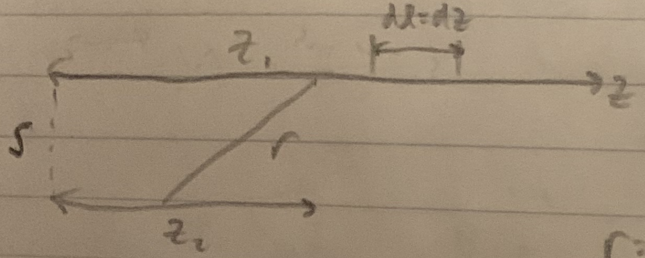
$F_e = \frac{1}{v^2}\left(\frac{1}{2\pi\varepsilon_0}\right)\frac{I_1 I_2}{d}$

$c^2 = \frac{1}{\mu_0 \varepsilon_0}$ $\qquad \frac{1}{\varepsilon_0} = c^2 \mu_0$

$F_e = \frac{1}{v^2}\left(\frac{\mu_0 c^2}{2\pi}\right)\frac{I_1 I_2}{d}$ $\qquad \frac{F_e}{F_{mag}} = \frac{\frac{1}{v^2}\left(\frac{\cancel{\mu_0} c^2}{\cancel{2\pi}}\right)\cancel{\frac{I_1 I_2}{d}}}{\cancel{\frac{\mu_0}{2\pi}}\cancel{\frac{I_1 I_2}{d}}}$

$\frac{F_e}{F_{mag}} = \frac{c^2}{v^2}$ $\qquad \boxed{F_e = 2 \times 10^{18} \ N/cm}$

5.23) $A = \frac{\mu_0}{4\pi}\int \frac{I}{r}\, dz$ $\quad$ $B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$



$r = \sqrt{s^2+z^2}$

$$A = \frac{\mu_0}{4\pi}\int \frac{I\hat{z}}{r}\, dz = \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2} \frac{\hat{z}\, dz}{\sqrt{z^2+s^2}}$$

$$= \frac{\mu_0 I \hat{z}}{4\pi}\int_{z_1}^{z_2} \frac{dz}{\sqrt{z^2+s^2}} = \frac{\mu_0 I \hat{z}}{4\pi} = \left[\ln\left(z+\sqrt{z^2+s^2}\right)\right]_{z_1}^{z_2}$$

$$A = \frac{\mu_0 I}{4\pi}\left(\frac{\ln(z_2+\sqrt{z_2^2+s^2})}{\ln(z_1+\sqrt{z_1^2+s^2})}\right)\hat{z}$$

$$B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$$

$$B = -\frac{\partial A}{\partial s}\hat{\phi}$$

$$B = -\frac{\partial}{\partial s}\left(\frac{\mu_0 I}{4\pi}\ln\left(\frac{z_2+\sqrt{z_2^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right)\right)\hat{\phi}$$

$$= -\frac{\mu_0 I}{4\pi}\left(\left(\frac{1}{z_2+\sqrt{z_2^2+s^2}}\right)\left(\frac{s}{\sqrt{z_2^2+s^2}}\right) - \left(\frac{1}{z_1+\sqrt{z_1^2+s^2}}\right)\left(\frac{s}{\sqrt{z_1^2+s^2}}\right)\right)\hat{\phi}$$

$$= -\frac{\mu_0 I}{4\pi}\left(-\frac{1}{s}\right)\left(\frac{z_2}{\sqrt{z_2^2+s^2}} - 1 - \frac{z_1}{\sqrt{z_1^2+s^2}} + 1\right)\hat{\phi}$$

$$B = \frac{\mu_0 I}{4\pi s}\left(\frac{z_2}{\sqrt{z_2^2+s^2}} - \frac{z_1}{\sqrt{z_1^2+s^2}}\right)\hat{\phi}$$

$\sin\theta_2 = \frac{z_2}{\sqrt{z_2^2+s^2}}$ $\quad$ $\sin\theta_1 = \frac{z_1}{\sqrt{z_1^2+s^2}}$

$$B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$$

Dane
Goodman

ET Homework #5

5.14, 5.16, 5.17, 5.19, 5.20, 5.23, 5.26

5.14) a) i) $\oint B \cdot dl = \mu_0 I_{enc}$

$I_{enc} = 0$

$B \oint dl = \mu_0 (0)$

$B(2\pi s) = 0$

$\boxed{B = 0 \quad \text{when } s < a \text{ inside the wire}}$

ii) $\oint B \cdot dl = \mu_0 I_{enc}$

$I_{enc} = I$

$B \oint dl = \mu_0 (I)$

$\boxed{B = \frac{\mu_0 (I)}{2\pi s} \quad \text{when } s > a \text{ outside the wire}}$

b) when $s < a$:

$I = \int_0^a J \cdot da$

$J \propto s$

$J = ks$

$I = \int_0^a (ks)(2\pi s) ds$

$= 2\pi k \frac{a^3}{3}$

$k = \frac{3I}{2\pi a^3}$

$I_{enc} = \int_0^s J \cdot da$

$I_{enc} = \int_0^s (ks)(2\pi s) ds$

$= 2\pi k \frac{s^3}{3} \qquad = 2\pi \left(\frac{3I}{2\pi a^3}\right) \frac{s^3}{3} = \frac{I s^3}{a^3}$

$B \oint ds = \mu_0 \left(I \frac{s^3}{a^3}\right) \qquad \boxed{B = \frac{\mu_0 I s^2}{2\pi a^3}}$

when $s > a$: $I_{enc} = I$

$\boxed{B = \frac{\mu_0 I}{2\pi s}}$

5.26) a) $A = A(s)\hat{z}$

$B = \nabla \times A$

$\nabla \times A = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_s}{\partial \phi}\right)\hat{z}$

$\hat{s} = (\cos\phi)\hat{x} + (\sin\phi)\hat{y}$

$\hat{\phi} = -(\sin\phi)\hat{x} + (\cos\phi)\hat{y}$

$\hat{z} = z\hat{z}$

$dl_z = dx\hat{x} + dy\hat{y} + dz\hat{z}$

$dl_s = ds\hat{s}$

$dl_\phi = s\,d\phi\,\hat{\phi}$

$\nabla \times A = -\frac{dA}{ds}\hat{\phi}$

$B = -\frac{dA}{ds}\hat{\phi}$

$B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$

$-\frac{dA}{ds} = \frac{\mu_0 I}{2\pi s}$ $\qquad dA = -\frac{\mu_0 I}{2\pi s}ds$

$A = -\int_a^s \frac{\mu_0 I}{2\pi s}ds$

$= -\frac{\mu_0 I}{2\pi}(\ln(s)\Big|_a^s = -\frac{\mu_0 I}{2\pi}\ln\left(\frac{s}{a}\right)$

$\nabla \cdot A = \frac{\partial A_z}{\partial z} = 0$ $\qquad \nabla \times A = -\frac{dA}{ds}\hat{\phi} = \frac{\mu_0 I}{2\pi s}\hat{\phi} = B$

$\boxed{= -\frac{\mu_0 I}{2\pi}\ln\left(\frac{s}{a}\right)\hat{z}}$

b) $\oint B \cdot dl = B(2\pi s) = \mu_0 I$

$J = \frac{I}{A} = \frac{I}{\pi s^2}$

$I = J(\pi s^2)$

$B(2\pi s) = \mu_0 J \pi s^2$

$J = \frac{I}{\pi R^2}\hat{\phi}$

$B = -\frac{dA}{ds}\hat{\phi}$ $\qquad B(2\pi s) = \frac{\mu_0 I}{\pi R^2}\pi s^2$ $\qquad B = \frac{\mu_0 I s}{2\pi R^2}\hat{\phi}$

5.26 cont) $\frac{dA}{ds} = -\frac{\mu_0 I s}{2\pi R^2}$

$$dA = \frac{-\mu_0 I s}{2\pi R^2} ds$$

$$A' = -\frac{\mu_0 I}{2\pi R^2} \int_b^s s\,ds\,\hat{z}$$

$$= -\frac{\mu_0 I}{2\pi R^2} \left(\frac{s^2}{2} \right|_b^s \hat{z}$$

$$= -\frac{\mu_0 I}{4\pi R^2} (s^2 - b^2) \hat{z}$$

$$-\frac{\mu_0 I}{2\pi} \ln\left(\frac{R}{a}\right) = -\frac{\mu_0 I}{4\pi R^2} (R^2 - b^2)$$

$$2 \ln\left(\frac{R}{a}\right) = \left(\frac{R^2 - b^2}{R^2}\right) = 1 - \frac{b^2}{R^2}$$

$$\ln\left(\frac{R}{b}\right) = \frac{1}{2}\left(1 - \left(\frac{b}{R}\right)^2\right)$$

$$A = -\frac{\mu_0 I}{2\pi} \left(\ln\left(\frac{R}{b}\right)\right)$$

$$= -\frac{\mu_0 I}{4\pi} \left(1 - \left(\frac{b}{R}\right)^2\right)$$

$$= -\frac{\mu_0 I}{4\pi R^2} (R^2 - b^2)$$

$$\boxed{A = \begin{cases} -\frac{\mu_0 I}{4\pi R^2} (s^2 - R^2) \hat{z} & \text{for } s < R \\ -\frac{\mu_0 I}{2\pi} \ln\left(\frac{s}{R}\right) \hat{z} & \text{for } s \geq R \end{cases}}$$