10/30/20

Reading Quiz 1 for PHYS 330

1 Vectors & Scalars

1. $a(\vec{B}+\vec{C}) = a\vec{B} + a\vec{C}$ $\qquad \vec{B} = b\hat{x} + c\hat{y} + d\hat{z}, \ \vec{C} = e\hat{x} + f\hat{y} + g\hat{z}$

$$a((b\hat{x}+c\hat{y}+d\hat{z})+(e\hat{x}+f\hat{y}+g\hat{z})) = a(b\hat{x}+c\hat{y}+d\hat{z}) + a(e\hat{x}+f\hat{y}+g\hat{z})$$

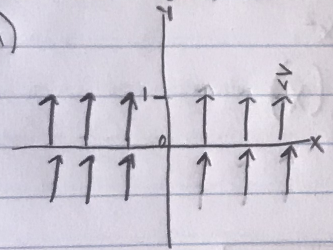
$$a((b+e)\hat{x}+(c+f)\hat{y}+(d+g)\hat{z}) = ab\hat{x} + ac\hat{y} + ad\hat{z} + ae\hat{x} + af\hat{y} + g\hat{z}$$

$$a(b+e)\hat{x} + a(c+f)\hat{y} + a(d+g)\hat{z} = a(b+e)\hat{x} + a(c+f)\hat{y} + a(d+g)\hat{z}$$

2. $\nabla(f(x,y) + \nabla g(x,y))$

- This combination of objects is wrong because its not clear on what the final product will be. On one hand, the gradient of f and laplacian of g could be calculated while the gradient of g could be calculated as a vecter and be added to f and then the gradient of $f + \nabla g$ could be calculated.

3. a)



b) $\vec{v} = \langle 0, 1 \rangle$ from pt $a = 0$ to $b = 1$

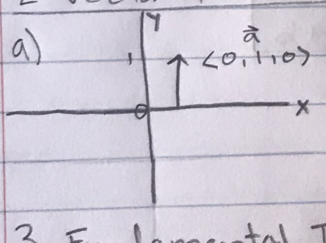
Verticle: $\int_0^1 1\, dy = y\Big|_0^1 = \boxed{1}$

Horizontal: $= \boxed{0}$

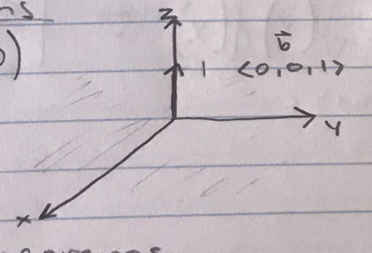
c) The divergence is 0 because as the vectors are the same in magnitude and direction.

2 Vector Rotations

1. a)



b)



c) $a = \sqrt{0^2 + 1^2 + 0^2} = \sqrt{1} = 1$

$b = \sqrt{0^2 + 0^2 + 1^2} = \sqrt{1} = 1$

$a = b$

3 Fundamental Theorems

1. $\vec{v} = s^{-1}\hat{\phi}$

$\int_S (\nabla \times \vec{v}) \cdot d\vec{a}$, if the surface is a closed sphere, would equal 0

4 Dirac Delta Functions

1. $f(x) \star g(x) = \left(f(x) - g(x) \big/ f(x) + g(x)\right), \quad \int_{-\infty}^{\infty} (f(x) \star g(x))\, \delta(x)\, dx$

- $f(x) = \cos(x),\ g(x) = \sin(x)$

$$\int_{-\infty}^{\infty} \left(\frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)}\right) \delta(x)\, dx = \int_{-\infty}^{\infty} \frac{\cos(x)}{\cos(x) + \sin(x)} \delta(x)\, dx - \cancel{\int_{-\infty}^{\infty} \frac{\sin(x)}{\cos(x) + \sin(x)} \delta(x)\, dx} \to 0$$

$$= f(0) = \cos(0) = \boxed{1}$$

• $f(x) = \cosh(x)$, $g(x) = \sinh(x)$

$$\int_{-\infty}^{\infty} \left(\frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)}\right) \delta(x)\, d(x)$$

$$= \int_{-\infty}^{\infty} \frac{\cosh(x)}{\cosh(x) + \sinh(x)} \delta(x)\, dx - \int_{-\infty}^{\infty} \frac{\cancel{\sinh(x)}}{\cosh(x) + \sinh(x)} \delta(x)\, dx \rightarrow 0$$

$$= f(0) = \cosh(0) = \boxed{1}$$

• $f(x) = a + ax + ax^2$, $g(x) = b + bx + bx^2$

$$\int_{-\infty}^{\infty} \frac{(a + ax + ax^2) - (b + bx + bx^2)}{(a + ax + ax^2) + (b + bx + bx^2)} \delta(x)\, d(x)$$

$$\int_{-\infty}^{\infty} \frac{a(1 + x + x^2) - b(1 + x + x^2)}{a(1 + x + x^2) + b(1 + x + x^2)} \delta(x)\, d(x)$$

$$\int_{-\infty}^{\infty} \frac{a-b}{a+b} \frac{\cancel{(1 + x + x^2)}}{\cancel{(1 + x + x^2)}} \delta(x)\, d(x) = \boxed{\frac{a-b}{a+b}}$$