HW Ch. 7 #7.12, 7.15, 7.34

7.12 $\vec{B}(t) = B_0 \cos(\omega t)\hat{z}$ radius : a

$\mathcal{E} = -d\phi/dt$

$$\phi = \int \vec{B}_{solenoid} \cdot d\vec{a} = \int \mu_0 \left(\frac{N}{L}\right) I(t)\,\hat{z} \cdot da\,\hat{z}, \quad \frac{N}{L} = n$$

$$\phi = \mu_0 n I(t) \int da = \mu_0 n I(t) A, \quad A = \pi r^2 = \pi a^2$$

$$\phi = \pi a^2 \mu_0 n I(t)$$

$$-\frac{d\phi}{dt} = -\pi a^2 \mu_0 n \frac{dI}{dt} = \mathcal{E}mf = f$$

$$\vec{J} = \sigma \vec{f}$$

$$\boxed{\vec{J} = -\pi a^2 \sigma \mu_0 n \dot{I}\,\hat{\phi}}$$

7.15 radius a, n turns per unit length (N/L)

find $\vec{E}$ inside & outside of solenoid

$\vec{B} = \mu_0 n I \hat{\phi}$, $\nabla \times \vec{E} = -\partial \vec{B}/\partial t$

$\int (\nabla \times \vec{E}) \cdot d\vec{a} = -\partial/\partial t \int \vec{B} \cdot d\vec{a}$

$\oint \vec{E} \cdot d\vec{l} = -\partial/\partial t\,(\phi_B)$

$E(\text{circumference}) = -\partial/\partial t\,(\phi_B)$, circumference $= 2\pi a$

$E(2\pi a) = -\partial/\partial t\,(\phi_B)$

$\phi = \int \mu_0 n I \hat{\phi} \cdot da\,\hat{\phi} = \mu_0 n I \int da = \mu_0 n I(t)(\pi a^2)$

$$-\frac{d\phi_B}{dt} = -\left(\pi a^2 \mu_0 n \frac{dI}{dt}\right)$$

$$E = \frac{-d\phi_B/dt}{2\pi a} = \frac{-\cancel{\pi} a^2 \mu_0 n (dI/dt)}{2\cancel{\pi a}} = \frac{-\mu_0 n \dot{I}}{2}$$

$$\boxed{\begin{aligned} \vec{E} &= \frac{-\mu_0 n \dot{I}}{2}\hat{\phi} \quad \text{(inside)} \\ \vec{E} &= 0\,\hat{\phi} \quad \text{(outside)} \end{aligned}}$$

7.34 wire w/ radius: a, constant current: I w/ width: w

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\int (\nabla \times \vec{B}) \cdot d\vec{l} = \int \mu_0 \vec{J} + \cancel{\int \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}}^{\,0}$$

$$B(\text{circumference}) = \mu_0 I$$

$$B(2\pi a) = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi a}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi a} \hat{\phi}}$$