Electromagnetc Theory: PHYS330

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Summary

Week 6 Summary

1. Current and Newton's Law
2. Flux rule from Lorentz force
3. Faraday's Law: Inspired by Symmetry
   - Induced E-fields
   - Quasi-static behavior
4. Inductors and analog filtering (special topic)

Current and Newton's Law

Current and Newton's Law

Why does current move at a constant velocity, if it is driven by an electric field?

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) = \sigma\vec{E} \tag{1}$$

This formula governs the current density as a function of force per unit charge, and it assumes small drift velocities so that the magnetic contributions are zero.

What is the acceleration of an electron in the middle of a parallel plate capacitor with empty space in the middle? Let the charge density be 1 μ C per cm^2. The mass of an electron is 9.1×10^{-31} kg, and $\epsilon_0 = 8.85 \times 10^{-12}$ F/m.

Why does current move at a constant velocity, if it is driven by an electric field?

$$\vec{J} = \sigma(\vec{E} + \vec{v} \times \vec{B}) = \sigma \vec{E} \tag{2}$$

This formula governs the current density as a function of force per unit charge, and it assumes small drift velocities so that the magnetic contributions are zero.

What is the acceleration of a charge flowing in a conductor that connects the parallel plates of the capacitor? Let the charge density be 1 μ C per cm^2. The mass of an electron is 9.1×10^{-31} kg, and $\epsilon_0 = 8.85 \times 10^{-12}$ F/m. Let the total effective resitance be 1 Ω.

Current and Newton's Law

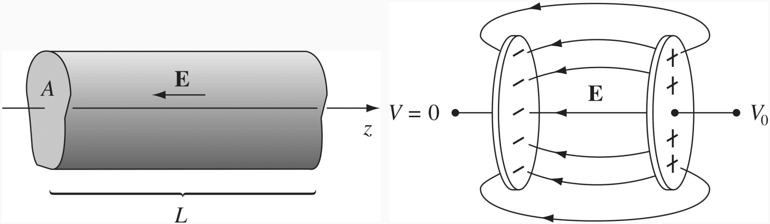

Figure 1: (Left) A conductor. (Right) A capacitor with the same geometry.

- Convince yourself that if the potential difference between the left and right sides of the conductor (left) is $V_0 - 0 = V_0$, that $V(z) = (V_0/L)z$.
- The E-field is therefore $\vec{E} = -V_0/L\hat{z}$, but the current does not accelerate.

Motional EMF Problems: Flux Rule from the Lorentz Force

Flux Rule from Lorentz Force

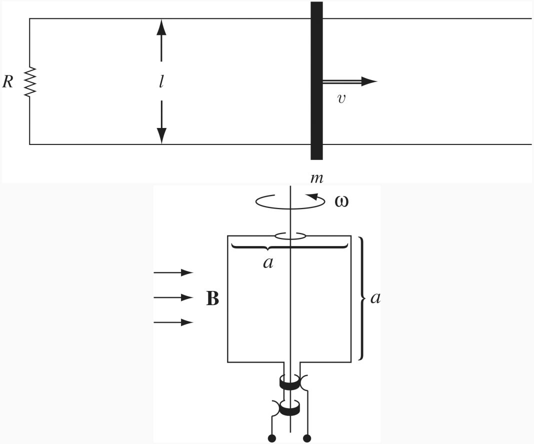

Figure 2: Motional emf problems resulting from the Lorentz force, solvable by the flux rule. (Top) Frictionless rails problem ... rail guns. (Bottom): the AC generator.

Faraday's Law: Inspired by Symmetry

In words: E-fields (via currents) generate B-fields. Changing B-fields induce E-fields (in loops). In differential form:

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}} \tag{3}$$

In integral form (via Stoke's Theorem):

$$\boxed{\oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \int \vec{B} \cdot d\vec{a}} \tag{4}$$

Faraday's Law

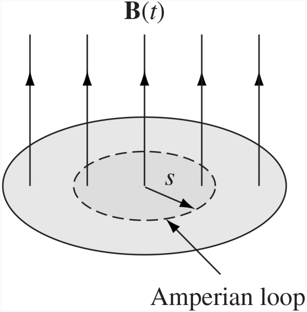

Figure 3: Cylindrical symmetry and the use of Faraday's Law to obtain emf.

- What is the magnitude and shape of the $\vec{E}$-field a distance s from the origin?
- What is the current I in a loop of wire at the same radius?

Faraday's Law

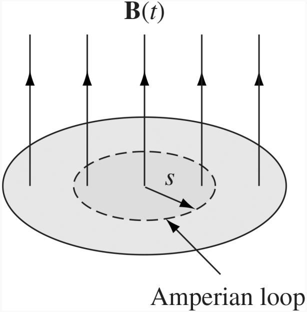

Figure 4: Cylindrical symmetry and the use of Faraday's Law to obtain emf.

- What is the acceleration of a point charge located a distance s from the origin?

Exercise 7.13: A square loop of wire, with sides of length a, lies in the first quadrant of the xy-plane, with one corner at the origin. In this region, there is a nonuniform time-dependent magnetic field $\vec{B}(y,t) = ky^3t^2\hat{z}$ (where k is constant). Find the emf induced in the loop.

Quasi-static behavior: $I(t)$ varies slowly enough to use the techniques of *magnetostatics* to predict $\vec{B}(t)$[1].

An infinitely long wire caries a current $I(t)$. Determine the induced E-field as a function of distance s from the wire. (Find the direction first).

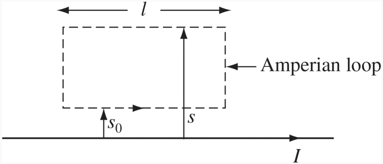

Figure 5: Obtaining the E-field generated by a long wire.

[1] Imagine the changing B-field changes so fast that the speed of light comes into play. For example, when the observer is so far away that the field's "wave" takes time to reach the observer.

Faraday's Law

A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R.

- (a) If the current in the solenoid is increasing at a constant rate $dI/dt = k$, what current flows and in what direction?
- (b) If the current is constant but the solenoid is removed to the left, what total charge passes through the resistor?

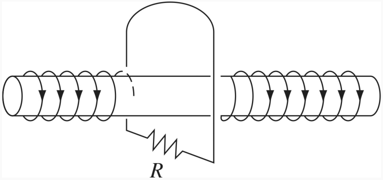

Figure 6: Obtaining the E-field generated by a long solenoid.

Special Topic: Inductors and Analog Filtering of Voltage Signals

Inductors and Analog Filtering of Voltage Signals

There is so much more here, especially in electrical engineering!

Remember the AC-circuit elements from before? We can now understand the origin of the impedences! Recall that Ohm's law is $v = iR$ in *both the time-domain and the frequency domain.*

- (The resistor is trivial). if R is a constant, then the Fourier transform of a constant is a constant. Thus, $Z_R = R$.
- Definition of capacitance: $Q = CV$. Take the derivative of both sides and then take the Fourier transform of both sides, compare to Ohm's law.
- Definition of inductance: $L = \phi_B / i$, the ratio of magnetic flux to current. Solve for the flux, then take the negative derivative of both sides. Use Lenz's law $v = -d\phi_B/dt$. Now take the Fourier transform of both sides and rearrange. Compare to Ohm's law.

$$\mathcal{F}(f(t)) = \tilde{F}(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt \qquad (5)$$

In Eq. 5, ω is the angular frequency, measured in radians per unit time. Let $f(t) = g'(t)$. Substituting into Eq. 5, and integrating by parts, we have

$$\tilde{F}(\omega) = g(t)e^{-j\omega t}|_{-\infty}^{\infty} + j\omega \int_{\infty}^{-\infty} g(t)e^{-j\omega t}dt \qquad (6)$$

For physical signals that represent finite energy, $\lim_{|t|\to\infty} g(t) = 0$. This requirement simplifies Eq. 6 by making the first term on the right-hand side vanish.

Inductors and Analog Filtering of Voltage Signals

$$\tilde{F}(\omega) = j\omega \int_{\infty}^{-\infty} g(t)e^{-j\omega t}dt = j\omega\mathcal{F}(g(t)) \tag{7}$$

The result may be summarized:

$$\boxed{\mathcal{F}(g'(t)) = j\omega\mathcal{F}(g(t))} \tag{8}$$

Example: The response of a circuit with inductors, a capacitor and a resistor.

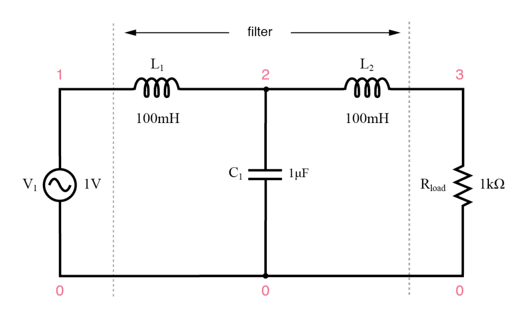

Figure 7: What is the response of the circuit? That is, amplitude as a function of frequency?

Conclusion

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