Warm-Up for April 18th, 2022

Dr. Jordan Hanson - Whittier College Dept. of Physics and Astronomy

April 18, 2022

1 Memory Bank

1. Recall that $\mathbf{a} = \int_{\mathcal{S}} d\mathbf{a} = \mathbf{a}$ is the vector area of a surface, and that if $\mathbf{c}$ is some constant vector:

$$\oint (\mathbf{c} \cdot \mathbf{r}) d\mathbf{l} = \mathbf{a} \times \mathbf{c} \tag{1}$$

2. Let $\mathbf{c} = \hat{\mathbf{r}}$, and reverse the order of the cross-product:

$$\oint (\hat{\mathbf{r}} \cdot \mathbf{r}') d\mathbf{l}' = -\hat{\mathbf{r}} \times \int d\mathbf{a}' \tag{2}$$

3. The *magnetic multipole expansion* for a line current I is

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \oint \frac{1}{\mathscr{r}} d\mathbf{l}' = \frac{\mu_0 I}{4\pi} \sum_{n=0} \frac{1}{r^{n+1}} \oint (r')^n P_n(\cos\alpha) d\mathbf{l}' \tag{3}$$

2 Magnetic Multipole Expansion

1. In the magnetic multipole expansion, set $n = 0$ with $P_1(\cos\alpha) = 1$ to calculate the monopole term. Why is it zero?

2. In the magnetic multipole expansion, set $n = 1$ with $P_1(\cos\alpha) = \cos\alpha$. Using Eq. 2, with $\cos\alpha = \hat{\mathbf{r}} \cdot \mathbf{r}'$ (Fig. 1), show that

$$\mathbf{A}_{dipole}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\mathbf{r}}}{r^2} \tag{4}$$

The vector $\mathbf{m}$ is a constant, the *magnetic dipole moment.* What is its definition?

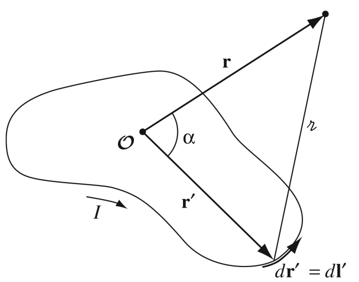

Figure 1: A line current of strength I and observed at displacement $\mathscr{r}$.