5.14 a) $\oint B \cdot dl = B\, 2\pi s = \mu_0 I_{enc}$ $B = \begin{cases} 0 & s<a \\ \frac{\mu_0 I}{2\pi s}\hat{\phi} & s>a \end{cases}$

b) $J = ks$ $I = \int_0^a J\, da = \int_0^a ks(2\pi s)\,ds = \frac{2\pi k a^3}{3}$ $k = \frac{3I}{2\pi a^3}$

$I_{enc} = \int_0^s k\bar{s}(2\pi \bar{s})\,d\bar{s} = \frac{2\pi k s^3}{3} = I\frac{s^3}{a^3}$ $I\frac{s^3}{a^3}$ $s<a$ $I_{enc} = I$ $s>a$

$B = \begin{cases} \frac{\mu_0 I s^2}{2\pi a^3}\hat{\phi} & s<a \\ \frac{\mu_0 I}{2\pi s}\hat{\phi} & s>a \end{cases}$

5.16 inside field $\mu_0 n I$ points $-\hat{z}$ outside field 0 points $\hat{z}$

i) $B = \mu_0 I(n_2 - n_1)\hat{z}$ ii) $B = \mu_0 I n_1 \hat{z}$ iii) $B=0$

5.17 a) $B = \mu_0 \sigma v$ between plates $B=0$ everywhere else

b) $F = \int (K \times B)\,da \Rightarrow f = K \times B$ $K = \sigma v$ $B = \mu_0 \sigma v/2$

$f_m = \mu_0 \sigma^2 v^2/2$ $\uparrow$

c) lower plate $\sigma/2\varepsilon_0$ upper plate $f_e = \sigma^2/2\varepsilon_0$

balance if $\mu_0 v^2 = 1/\varepsilon_0$ or $v = 1/\sqrt{\varepsilon_0 \mu_0} = c$

5.19 This is a steady current and $J$ is divergenceless so $\int J \cdot da$ is independent of surface for the bounding line

5.20 a) $\rho = (1.6\times10^{-19})(6.0\times10^{23})\left(\frac{9.0}{64}\right) = 1.4\times10^4\ C/cm^3$

b) $J = \frac{I}{\pi s^2} = \rho v$ $v = \frac{I}{\pi s^2 \rho} = \frac{1}{\pi(2.5\times10^{-3})(1.4\times10^4)} = 9.1\times10^{-3}\ cm/s$

c) $f_m = \frac{\mu_0}{2\pi}\left(\frac{I_1 I_2}{d}\right) = \frac{4\pi\times10^{-7}}{2\pi} = 2\times10^{-7}\ N/cm$

d) $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d}$ $f_e = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_1\lambda_2}{d} = \frac{1}{v^2}\frac{1}{2\pi\varepsilon_0}\frac{I_1 I_2}{d} = \frac{c^2}{v^2}\frac{\mu_0}{2\pi}\left(\frac{I_1 I_2}{d}\right) = \frac{c^2}{v^2} f_m$

$\frac{f_e}{f_m} = \frac{c^2}{v^2} = \left(\frac{3.0\times10^{10}}{9.1\times10^{-3}}\right)^2 = 1.1\times10^{25}$ $f_e = 1.1\times10^{25}(2\times10^{-7}) = 2\times10^{18}\ N/cm$

5.23 $A = \frac{\mu_0}{4\pi}\int \frac{I\hat{z}}{r}\,dz = \frac{\mu_0 I}{4\pi}\hat{z}\int_{z_1}^{z_2}\frac{dz}{\sqrt{z^2+s^2}} = \frac{\mu_0 I}{4\pi}\ln\left(\frac{z_2+\sqrt{(z_2)^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right)\hat{z}$

$B = \nabla\times A = -\frac{\partial A}{\partial s}\hat{\phi} = -\frac{\mu_0 I}{4\pi}\left(\frac{1}{z_2+\sqrt{z_2^2+s^2}}\frac{s}{\sqrt{z_2^2+s^2}} - \frac{1}{z_1+\sqrt{z_1^2+s^2}}\frac{s}{\sqrt{z_1^2+s^2}}\right)\hat{\phi}$

$= -\frac{\mu_0 I s}{4\pi}\left(-\frac{1}{s^2}\right)\left(\frac{z_2}{\sqrt{z_2^2+s^2}} - 1 - \frac{z_1}{\sqrt{z_1^2+s^2}} + 1\right)\hat{\phi} = \frac{\mu_0 I}{4\pi s}\left(\underbrace{\frac{z_2}{\sqrt{z_2^2+s^2}}}_{\sin\theta_2} - \underbrace{\frac{z_1}{\sqrt{z_1^2+s^2}}}_{\sin\theta_1}\right)\hat{\phi}$

$= \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)\hat{\phi}$

5.26 a) $A = A(s)\hat{z}$ $\quad B = \nabla\times A = -\frac{\partial A}{\partial s}\hat{\phi} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$ $\quad \nabla\cdot A = \frac{\partial A_z}{\partial z} = 0$

$\frac{\partial A}{\partial s} = -\frac{\mu_0 I}{2\pi s}$ $\quad A(s) = -\frac{\mu_0 I}{2\pi}\ln(s/a)\hat{z}$

b) $\oint B\cdot dl = B\,2\pi s = \mu_0 I_{enc} = \mu_0 J\pi s^2 = \mu_0\frac{I}{\pi R^2}\pi s^2 = \frac{\mu_0 I s^2}{R^2}$

$B = \frac{\mu_0}{2\pi}\frac{Is}{R^2}\hat{\phi}$ $\quad \frac{\partial A}{\partial s} = -\frac{\mu_0 I}{2\pi}\frac{s}{R^2}$ $\quad A = -\frac{\mu_0 I}{4\pi R^2}(s^2-b^2)\hat{z}$

$-\frac{\mu_0 I}{2\pi}\ln(R/a) = -\frac{\mu_0 I}{4\pi R^2}(R^2-b^2)$

$$A = \begin{cases} -\frac{\mu_0 I}{4\pi R^2}(s^2-R^2)\hat{z} & s \leq R \\ -\frac{\mu_0 I}{2\pi}\ln(s/R)\hat{z} & s \geq R \end{cases}$$