2.5)

$R = \sqrt{z^2 + r^2}$

$\cos\theta = \frac{Z}{R} = \frac{Z}{\sqrt{z^2+r^2}}$

$dq = \lambda dl = \lambda r d\theta$

$K = 4\pi\varepsilon_0$

$$E = \frac{1}{K}\int_0^{2\pi} \frac{dq}{R^2}$$

$$= \frac{1}{K}\int_0^{2\pi} \frac{\lambda r d\theta}{z^2+r^2} \cos\theta \, \hat{z}$$

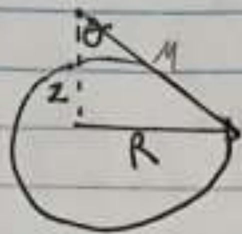
$$= \frac{1}{K}\int_0^{2\pi} \frac{\lambda r}{(z^2+r^2)}\left(\frac{Z}{(z^2+r^2)^{1/2}}\right) d\theta \, \hat{z}$$

$$= \frac{\lambda r Z}{K(z^2+r^2)^{3/2}}\int_0^{2\pi} d\theta \, \hat{z}$$

$$= \frac{\lambda r Z(\cancel{2\pi})}{2\cancel{4\pi}\varepsilon_0(z^2+r^2)^{3/2}} \hat{z}$$

$$\boxed{\bar{E} = \frac{\lambda r Z}{2\varepsilon_0(z^2+r^2)^{3/2}}}$$

2.6)



$$E = \frac{1}{K}\int \frac{dq}{r^2}\hat{r}$$

$$\vec{r} = z\hat{z} - R\hat{R}$$

$$r = \sqrt{z^2+R^2} \qquad \|\vec{r}\| = (z^2+R^2)^{1/2}$$

$$\cos\theta = \frac{z}{r} = \frac{z}{(z^2+R^2)^{1/2}} \qquad \hat{r} = \frac{z\hat{z} - R\hat{R}}{(z^2+R^2)^{1/2}}$$

$$dq = \sigma\, da = \sigma R\, dR\, d\phi$$

$$\tan\theta = \frac{R}{z}$$

$$z\tan\theta = R$$

$$E = \frac{1}{K}\int_0 \frac{\sigma R\, dR\, z}{(z^2+R^2)(z^2+R^2)^{1/2}}\hat{r}$$

$$E = \frac{1}{K}\left(\frac{\sigma}{z^2+R^2}\right)\int_0 R\,dR\left(\frac{z\hat{z}}{(z^2+R^2)^{1/2}} - \cancel{\frac{R\hat{R}}{(z^2+R^2)^{1/2}}}\right)$$

$$E = \frac{\sigma z}{K}\hat{z}\int_0 \frac{R\,dR}{(z^2+R^2)^{3/2}}$$

$$E = \frac{\sigma z \hat{z} \cos R}{K}$$

2.a) $\epsilon_0\left(\nabla\cdot E = \frac{1}{\epsilon_0}\rho\right)$

$\epsilon_0\langle \partial_r, \partial_\theta, \partial_\phi\rangle \cdot \langle Kr^3, 0, 0\rangle = \rho$

a) $\epsilon_0 \langle 3Kr^2 = \rho$

$\rho = 3\epsilon_0 K r^2$

$d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$

b) b₁) $Q_{enc} = \int_V \rho \, d\tau$

$Q_{enc} = \iiint (3\epsilon_0 K r^2) r^2 \sin\theta \, dr \, d\theta \, d\phi$

$-\cos(\theta)$

$= 3\epsilon_0 K \int_0^R r^4 dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi$

$= 3\epsilon_0 K \frac{R^5}{5} (2\pi)(2) \quad 1-(-1) = 2$

Q_{enc} $\boxed{= \frac{12\pi\epsilon_0 K R^5}{5}}$

b₂) $da = r^2 \sin\theta \, d\theta \, d\phi$

$Q_{enc} = \epsilon_0 \oint E \cdot da$

$Q_{enc} = \epsilon_0 \int_0^{\pi}\int_0^{2\pi} K r^3 \hat{r} \cdot r^2 \sin\theta \, d\theta \, d\phi$

$= \epsilon_0 \quad K r^5 \, 2\pi (1 - (-1))$

$\boxed{= 4\pi\epsilon_0 K r^5}$

2.12)

$$\oint_S E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$$

$$Q_{enc} = \int_0^R \int_0^{\pi} \int_0^{2\pi} \rho \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\rho \frac{4\pi R^3}{3}$$

$$\oint E \cdot da \qquad da = 4\pi R^2$$

$$|E| \int_0^{2\pi} \int_0^{\pi} da \rightarrow r^2 \sin\theta \, d\theta$$

$$|E| \, 4\pi R^2 = \frac{\rho}{\varepsilon_0} \frac{4\pi R^3}{3} \hat{r}$$

$$\boxed{E = \frac{\rho R}{3\varepsilon_0} \hat{r}}$$

2.16) $\frac{1}{\epsilon_0}\oint \bar{E}\cdot da = Q_{enc} \rightarrow$

Cylinder

$$E \cdot 4\pi s^2 = \frac{1}{\epsilon_0}\rho \pi s^2 l$$

$Q_e = \rho \pi a^2 l$

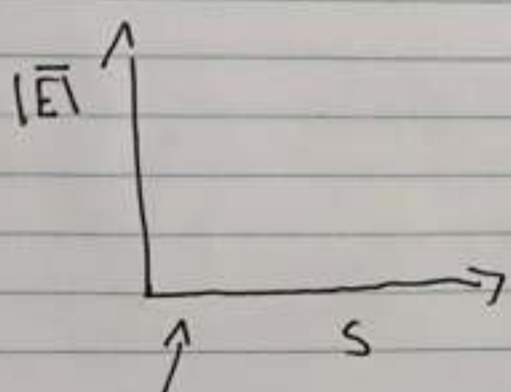
$$\boxed{E_1 = \frac{\rho s}{2\epsilon_0}}$$

<u>$a < s < b$</u>

$$E(4\pi s^2) = \frac{1}{\epsilon_0 \rho \pi a^2 l} = E = \frac{\rho a^2}{2\epsilon_0 s}$$

<u>$s > b$</u>

$$E(2\pi s l) = \frac{1}{\epsilon_0 Q_e} = 0$$



No clue?

2.18) $d = R_+ - R_-$

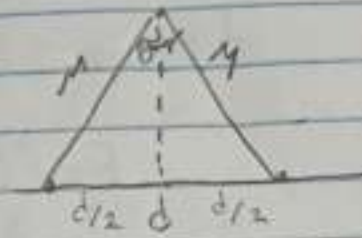
$$E = \frac{\rho R_+}{3\epsilon_0} - \frac{\rho R_-}{3\epsilon_0}$$

$$\boxed{|E| = \frac{\rho d}{3\epsilon_0}}$$

2.25)

$2q$

$E = -\nabla V$



$$V(r) = \frac{1}{k}\frac{2q}{r}$$

$$V(r) =$$

$$r^2 = \left(\frac{d}{2}\right)^2 + z^2$$

$$r = \sqrt{(d/2)^2 + z^2}$$

$$V(r) = \frac{2kq}{r} = \frac{2kq}{(z^2 + (d/2)^2)^{1/2}}$$

?

$$\frac{dV}{dz} = \frac{2kqz}{(z^2 + (d/2)^2)^{3/2}}$$

2L $\quad dq' = \lambda dx'$ $\qquad E = -\nabla V$

$$dV = \frac{k\lambda dx'}{(x'^2 + z^2)^{1/2}}$$

$$\boxed{\bar{E} = 2kL\lambda \frac{1}{z\sqrt{z^2 + L^2}}}$$

B. 6

$$dv = \frac{k\,dq'}{r} = \frac{k\sigma\,da'}{\sqrt{s'^2 + z^2}} = \frac{k\sigma s'\,ds'\,d\phi'}{\sqrt{s'^2 + z^2}}$$

$$V = \frac{\sigma}{2\varepsilon_0}\left((R^2 + z^2)^{1/2} - z\right)$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{(z^2 + R^2)^{1/2}}\right)}$$