Phys 330 - Midterm

Andrew Householder

1.1.)

$$(\vec{A}\cdot\nabla)\vec{B} = \left(A_x \frac{\partial}{\partial x} + A_y \frac{\partial}{\partial y} + A_z \frac{\partial}{\partial z}\right)\vec{B}$$

a.)

$$(\vec{A}\cdot\nabla)\vec{B} = \left(A_x \frac{\partial}{\partial x}B_x + A_y \frac{\partial}{\partial y}B_x + A_z \frac{\partial}{\partial z}B_x\right)\hat{x}$$
$$+ \left(A_x \frac{\partial}{\partial x}B_y + A_y \frac{\partial}{\partial y}B_y + A_z \frac{\partial}{\partial z}B_y\right)\hat{y}$$
$$+ \left(A_x \frac{\partial}{\partial x}B_z + A_y \frac{\partial}{\partial y}B_z + A_z \frac{\partial}{\partial z}B_z\right)\hat{z}$$

b.) $(\hat{r}\cdot\nabla)\hat{r}$ $\qquad \hat{r} = \frac{\vec{r}}{r}$ $\qquad r = \sqrt{x^2+y^2+z^2}$

$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

So,

$$(\hat{r}\cdot\nabla)\hat{r} = \frac{1}{r}\left(\left(x\frac{\partial}{\partial x}\frac{x}{\sqrt{x^2+y^2+z^2}} + y\frac{\partial}{\partial y}\frac{x}{\sqrt{x^2+y^2+z^2}} + z\frac{\partial}{\partial z}\frac{x}{\sqrt{x^2+y^2+z^2}}\right)\hat{x}\right.$$
$$+(\ldots)\hat{y}$$
$$\left.+(\ldots)\hat{z}\right)$$

← similar only diff. numerators

$$= \frac{1}{r}\left(\left(x\left((x^2+y^2+z^2)^{-\frac{1}{2}} + x\left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}(2x)\right)\right.\right.$$
$$+ xy\left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}(2y)$$
$$\left. + xz\left(-\tfrac{1}{2}\right)(x^2+y^2+z^2)^{-3/2}(2z)\right)\hat{x}$$
$$+(\ldots)\hat{y}$$
$$\left.+(\ldots)\hat{z}\right)$$

$$= \frac{1}{r}\left(\left(\frac{x}{r} - \frac{x^3}{r^3} - \frac{xy^2}{r^3} - \frac{xz^2}{r^3}\right)\hat{x}\right.$$
$$+(\ldots)\hat{y}$$
$$\left.+(\ldots)\hat{z}\right)$$

$$= \frac{1}{r}\left(\left(\frac{x}{r} - \frac{x}{r^3}\underbrace{(x^2+y^2+z^2)}_{r^2}\right)\hat{x}\right.$$
$$+(\ldots)\hat{y}$$
$$\left.+(\ldots)\hat{z}\right)$$

$$= \frac{1}{r}\left(\left(\frac{x}{r} - \frac{x}{r}\right)\right)\hat{x} = 0$$

cancellation occurs in all components, since all that changes is which is in numerator. $\Rightarrow (\hat{r}\cdot\nabla)\hat{r} = 0$.

1.1.) cont.

c) $\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$ $\qquad$ $\vec{p} = q\vec{d} = qd\hat{x}$

$$\vec{E} = -\nabla V$$

$$V = V_0 r^2 + V_1 \qquad r^2 = x^2 + y^2 + z^2$$

$$V(x,y,z) = V_0 x^2 + V_0 y^2 + V_0 z^2 + V_1$$

$$\vec{E} = -\nabla V = -\left(2V_0 x\,\hat{x} + 2V_0 y\,\hat{y} + 2V_0 z\,\hat{z}\right)$$

$$(\vec{p}\cdot\nabla) = (qd)\hat{x}\cdot\left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right)$$

$$= qd\frac{\partial}{\partial x}$$

$$\Rightarrow (\vec{p}\cdot\nabla)\vec{E} = -\left(qd\frac{\partial}{\partial x}\right)\left(2V_0 x\,\hat{x} + 2V_0 y\,\hat{y} + 2V_0 z\,\hat{z}\right)$$

$$(\vec{p}\cdot\nabla)\vec{E} = -2qdV_0\hat{x} = \vec{F}$$

1.2) $J = \int_V e^{-r}\left(\nabla \cdot \frac{\hat{r}}{r^2}\right) d\tau$ note that $\left(\nabla \cdot \frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(r)$

a.) $J = \int_V e^{-r} 4\pi \delta^3(\vec{r})\, d\tau$ $\quad * \ \delta^3(\vec{r}) = \delta^3(\vec{r}-0) = \begin{cases} \infty & @\ 0 \\ 0 & \text{else} \end{cases}$

$$\boxed{J = 4\pi e^{-(0)} = 4\pi}$$

b.) transferring derivative:

$$J = -\int_V \frac{\hat{r}}{r^2} \cdot \nabla(e^{-r})\, d\tau + \oint_S e^{-r} \frac{\hat{r}}{r^2} \cdot d\vec{a}$$

$$\nabla e^{-r} = \frac{\partial}{\partial r} e^{-r} \hat{r} = -e^{-r}\hat{r}$$

$$\frac{\hat{r}}{r^2} \cdot -e^{-r}\hat{r} = \frac{-e^{-r}}{r^2}$$

$$= + \int_0^R \int_0^{2\pi} \int_0^{\pi} \frac{e^{-r}}{\cancel{r^2}} \cancel{r^2} \sin\theta\, d\theta\, d\phi\, dr + \int_0^{2\pi}\int_0^{\pi} e^{-r} \frac{\hat{r}}{r^2} \cdot r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}$$

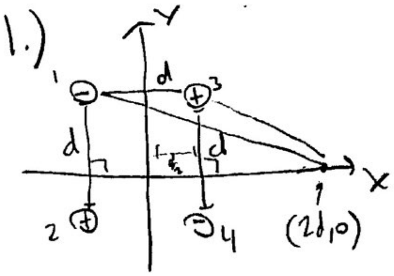
$$= 4\pi \int_0^R e^{-r}\, dr + 4\pi e^{-R}$$ ← taking surface integral to be on sphere at radius R.

$$= -4\pi \left[e^{-r}\right]_0^R + 4\pi e^{-R}$$

$$= -\cancel{4\pi e^{-R}} + 4\pi e^{-0} + \cancel{4\pi e^{-R}}$$

$$\boxed{J = 4\pi}$$

2.1.)



$$\vec{E} = K \sum_{i=1}^{4} \frac{q_i}{r_i^2} \hat{r}$$

a.) at $P=(0,0)$ the opposite corners cancel by symmetry, and

$$\underline{\vec{E} = 0 \quad \text{at origin}}$$

b.) $P=(2d,0)$

For charge 1.: $|\vec{r}| = \sqrt{\left(\frac{d}{2}\right)^2 + 3d^2} = \sqrt{\frac{d^2}{4} + 9d^2}$

$$\vec{E}_1 = K \frac{-q}{\frac{d^2}{4}+9d^2} \hat{r} \qquad \vec{r} = 3d\hat{x} - \frac{d}{2}\hat{y}$$

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|}$$

$$\vec{E}_1 = K \frac{(-q)}{\left(\frac{37d^2}{4}\right)^{3/2}} \left(3d\hat{x} - \frac{d}{2}\hat{y}\right)$$

Charge 3: $|\vec{r}|$ is same, $\vec{r} = 3d\hat{x} + \frac{d}{2}\hat{y}$

$$\vec{E}_2 = \frac{Kq}{\left(\frac{37d^2}{4}\right)^2} \left(3d\hat{x} + \frac{d}{2}\hat{y}\right)$$

Charge 3: $|\vec{r}| = \sqrt{\left(\frac{d}{2}\right)^2 + \left(\frac{3d}{2}\right)^2} = \sqrt{\frac{d^2}{4} + \frac{9d^2}{4}} = \sqrt{\frac{5d^2}{2}}$

$$\vec{E}_3 = \frac{Kq}{\frac{5d^2}{2}} \hat{r} \qquad \vec{r} = \frac{3d}{2}\hat{x} - \frac{d}{2}\hat{y}$$

$$= \frac{Kq}{\left(\frac{5d^2}{2}\right)^{3/2}} \left(\frac{3d}{2}\hat{x} - \frac{d}{2}\hat{y}\right)$$

Charge 4: $\vec{E}_4 = \frac{K(-q)}{\left(\frac{5d^2}{2}\right)^{3/2}} \left(\frac{3d}{2}\hat{x} + \frac{d}{2}\hat{y}\right)$

2.1.) cont.

$$\vec{E}_{total} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \vec{E}_4$$

x components all cancel..

$$\Rightarrow \quad \vec{E}_{total} = \frac{kq}{\left(\frac{37d^2}{4}\right)^{3/2}} d\hat{y} - \frac{kq}{\left(\frac{5d^2}{2}\right)^{3/2}} d\hat{y}$$

$$\Rightarrow \quad \vec{E}_{total} = kq\left(\left(\frac{4}{37d^2}\right)^{3/2} - \left(\frac{2}{5d^2}\right)^{3/2}\right) d\hat{y}$$

$P = (0, 2d)$

almost the same situation, except cancellation occurs in x direction & negatives flip...

$$\Rightarrow \quad \vec{E}_{total} = kq\left(\left(\frac{2}{5d^2}\right)^{3/2} - \left(\frac{4}{37d^2}\right)^{3/2}\right) d\hat{x}$$

2.2) $V(\vec{r}) = \dfrac{Ae^{-\lambda r}}{r} = Ae^{-\lambda r}\, r^{-1}$

$$\vec{E} = -\nabla V$$

$$-\nabla V = \frac{-\partial V}{\partial r}\hat{r} = -A\left[-\lambda e^{-\lambda r} r^{-1} + (-1)\, r^{-2} e^{-\lambda r}\right]\hat{r}$$

$$= +A\left[\frac{\lambda e^{-\lambda r}}{r} + \frac{e^{-\lambda r}}{r^2}\right]\hat{r}$$

$$= A\left[\frac{\lambda r e^{-\lambda r} + e^{-\lambda r}}{r^2}\right]\hat{r}$$

$$\boxed{\vec{E}(\vec{r}) = Ae^{-\lambda r}\left(\frac{1+\lambda r}{r^2}\right)\hat{r}}$$

Gauss' Law: $\nabla\cdot\vec{E} = \frac{1}{\epsilon_0}\rho \Rightarrow \rho = \epsilon_0(\nabla\cdot\vec{E})$

$$\nabla\cdot\vec{E} = \nabla\cdot Ae^{-\lambda r}(1+\lambda r)\left(\frac{\hat{r}}{r^2}\right)$$

$\Rightarrow$ by the product rule ...)

$$\nabla\cdot\vec{E} = Ae^{-\lambda r}(1+\lambda r)\underbrace{\left(\nabla\cdot\frac{\hat{r}}{r^2}\right)}_{=4\pi\delta^3(\vec{r})} + \frac{\hat{r}}{r^2}\cdot\nabla\left[Ae^{-\lambda r}(1+\lambda r)\right]$$

$$= \underbrace{Ae^{-\lambda r}(1+\lambda r)}\, 4\pi\delta^3(\vec{r}) + \left(\frac{\hat{r}}{r^2}\right)\cdot\left(A(-\lambda)e^{-\lambda r}(1+\lambda r) + Ae^{-\lambda r}\lambda r\right)$$

δ^3 takes over other functions at all points

$$= A4\pi\,\delta^3(\vec{r}) + \left(\frac{\hat{r}}{r^2}\right)\cdot\left(-A\lambda^2 e^{-\lambda r} r\right)$$

$$\nabla\cdot\vec{E} = A\left(4\pi\delta^3(\vec{r}) - \frac{\lambda^2 e^{-\lambda r}}{r}\right)$$

$$\epsilon_0(\nabla\cdot\vec{E}) = \epsilon_0 A\left(4\pi\delta^3(\vec{r}) - \frac{\lambda^2 e^{-\lambda r}}{r}\right) = \rho$$

but [illegible]?

2.2.) cont.

$$Q = \int_V \rho \, d\tau$$

$$= \epsilon_0 A \int_V \left(4\pi\delta^3(\vec{r}) - \frac{\lambda^2 e^{-\lambda r}}{r}\right) d\tau$$

$$= \epsilon_0 A \, 4\pi - \epsilon_0 A \int_0^\infty \int_0^{2\pi} \int_0^\pi \frac{\lambda^2 e^{-\lambda r}}{\cancel{r}} r^{\cancel{2}\,1} \sin\theta \, d\theta \, d\phi \, dr$$

$$= \epsilon_0 A 4\pi - \epsilon_0 A \int_0^\infty \int_0^{2\pi} \lambda^2 e^{-\lambda r} r \underbrace{\left[-\cos\theta\right]_0^\pi}_{2} d\phi \, dr$$

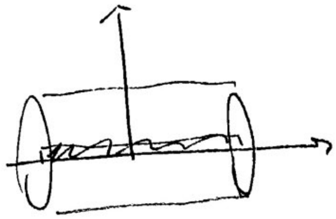
$$= \epsilon_0 A 4\pi - \epsilon_0 A (4\pi) \lambda^2 \underbrace{\int_0^\infty e^{-\lambda r} r \, dr}_{\frac{1}{\lambda^2}}$$

done in Sage

$$= \epsilon_0 A 4\pi - \epsilon_0 A 4\pi \cancel{\lambda^2} \frac{1}{\cancel{\lambda^2}}$$

$$\underline{\underline{Q = 0}}$$

2.3.)



Choose a gaussian cylinder of radius s and length l.

$$\oint \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc}$$

$$Q_{enc} = \int \lambda \, d\vec{l} = \lambda \int d\vec{l} = \lambda l$$

Choose $d\vec{a}$ in $\hat{s}$ direction...

$$\Rightarrow \oint \vec{E} \cdot d\vec{a} = \int E_{\hat{s}} \, da = E_{\hat{s}} \int da = E_{\hat{s}} A = E_{\hat{s}} (2\pi s l)$$

$$\Rightarrow \quad E_{\hat{s}} (2\pi s \cancel{l}) = \frac{\lambda \cancel{l}}{\epsilon_0}$$

$$E_{\hat{s}} = \frac{\lambda}{2\pi \epsilon_0 s}$$

a.) $\Rightarrow \boxed{\vec{E} = \frac{\lambda}{2\pi \epsilon_0 s} \hat{s}}$

b.) $\vec{F} = Q\vec{E} = \frac{q\lambda}{2\pi\epsilon_0 s} \hat{s} = m\vec{a}$

q, λ positive $\Rightarrow$ moves away from line of charge.

Suppose at $t=0$, $\vec{v} = v_0$

$$\vec{a} = \frac{q\lambda}{2\pi\epsilon_0 s} \hat{s} = \frac{d\vec{v}}{dt}$$

$$d\vec{v} = \frac{q\lambda}{2\pi\epsilon_0 s m} \hat{s} \, dt$$

$$\Rightarrow \quad \vec{v} = \left(\frac{q\lambda t}{2\pi\epsilon_0 s m} + v_0 \right) \hat{s} = \frac{d\vec{x}}{dt}$$

$$d\vec{x} = \left(\frac{q\lambda t}{2\pi\epsilon_0 s m} dt + v_0 dt \right) \hat{s}$$

$$\Rightarrow \boxed{\vec{x} = \left(\frac{q\lambda t^2}{4\pi\epsilon_0 s m} + v_0 t + s \right) \hat{s}}$$

3.1.) general solution via sep. of var.

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l \cos(\theta)$$

now at the spherical surface we have that

$$V = V_0(\theta)$$

- inside sphere we must have $(r \leq R)$

$$V(r,\theta) = \sum_l A_l r^l P_l \cos(\theta)$$

since $\frac{1}{r^{l+1}} \to \infty$ as $r \to 0$.

- Outside sphere $(r \geq R)$ we have

$$V(r,\theta) = \sum_l B_l r^{-(l+1)} P_l \cos(\theta)$$

since at $r = \infty$ $r^l \to \infty$

Enforcing continuity at surface $(r = R)$

$$V_0(\theta) = \sum_l A_l R^l P_l \cos\theta = \sum_l B_l R^{-(l+1)} P_l \cos\theta$$

$$\Rightarrow \quad A_l R^l = B_l R^{-(l+1)}$$

$$\Rightarrow \quad B_l = A_l R^{2l+1}.$$

the gradient of V is discontinuous at the boundary, however, and is given by

$$\left(\frac{\partial V_{out}}{\partial r} - \frac{\partial V_{in}}{\partial r}\right)\Bigg|_{r=R} = -\frac{1}{\epsilon_0}\sigma(\theta)$$

So, $\frac{\partial}{\partial r}\left(\sum B_l R^{-(l+1)} P_l \cos\theta\right) - \frac{\partial}{\partial r}\left(\sum A_l R^l P_l \cos\theta\right) = \frac{-\sigma}{\epsilon_0}$

$$\Rightarrow \quad \sum -(l+1) B_l R^{-(l+2)} P_l \cos\theta - \sum l A_l R^{l-1} P_l \cos\theta = \frac{-\sigma}{\epsilon_0}$$

but $B_l = A_l R^{2l+1}$

3.1.) cont.

$$\sum -(l+1) A_l R^{l-1} P_l \cos\theta - \sum l A_l R^{l-1} P_l \cos\theta = -\frac{\sigma}{\epsilon_0}$$

$$\Rightarrow \quad \sum (2l+1) A_l R^{l-1} P_l(\cos\theta) = \frac{\sigma(\theta)}{\epsilon_0}$$

So, $\sigma(\theta) = \epsilon_0 \sum (2l+1) A_l R^{l-1} P_l(\cos\theta)$

Now since P_l's orthogonal, fourier's trick shows

$$A_l = \frac{2l+1}{2R^l} \int_0^\pi V_0(\theta) P_l(\cos\theta) \sin\theta \, d\theta$$

Thus, $$\boxed{\begin{aligned} \sigma(\theta) &= \frac{\epsilon_0}{2R} \sum_{l=0}^{\infty} (2l+1)^2 C_l P_l(\cos\theta) \\ C_l &= \int_0^\pi V_0(\theta) P_l(\cos\theta) \sin\theta \, d\theta \end{aligned}}$$

b.) If $V_0(\theta) = P_2(\cos\theta)$, then C_2 only one that survives via orthogonality of P_ls.

$$\Rightarrow \quad C_2 = \int_0^\pi (P_2(\cos\theta))^2 \sin\theta \, d\theta = \frac{2}{5}$$ ↙ done in sage

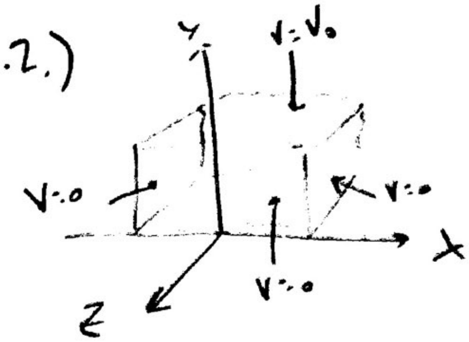
$$\Rightarrow \quad \sigma(\theta) = \frac{\epsilon_0}{2R} \left(\overbrace{2(2)+1}^{5^2}\right)^2 \left(\frac{2}{5}\right) P_2(\cos\theta)$$

$$= \frac{\epsilon_0}{\cancel{2}R} (2\cancel{5}^{5}) \left(\frac{\cancel{2}}{\cancel{5}}\right) P_2(\cos\theta)$$

$$\sigma(\theta) = \frac{5\epsilon_0}{R} P_2(\cos\theta)$$

$$\boxed{\sigma(\theta) = \frac{5\epsilon_0}{R}\left(\frac{3}{2}\cos(x)^2 - \frac{1}{2}\right)}$$

3.2.)



① $y=0 \rightarrow V=0$

② $x=b \rightarrow V=0$

③ $x=-b \rightarrow V=0$

④ $y=a \rightarrow V=V_0$

const.

$$\nabla^2 V = 0$$

$$\Rightarrow \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

Looking for seperable solutions $V = XY$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{C_1} + \underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{C_2} = 0 \qquad C_1 + C_2 = 0$$

let C_1 be negative...

$$\frac{d^2X}{dx^2} = -k^2 X \qquad \frac{d^2Y}{dy^2} = k^2 Y$$

$$\Rightarrow \quad Y = Ae^{ky} + Be^{-ky} \quad , \; X = C\sin(kx) + D\cos(kx)$$

following ① $0 = Ae^0 + Be^0 \Rightarrow B = -A.$

② $0 = C\sin(kb) + D\cos(kb)$

③ $0 = C\sin(-kb) + D\cos(-kb) \Rightarrow 0 = -C\sin(kb) + D\cos(kb)$

adding ②,③ $\quad 0 = 2D\cos(kb)$

$\therefore$ to always satisfy this $D=0.$ ← if works, soln. is unique!

Now $0 = C\sin(kb) \Rightarrow kb = n\pi$

$$\Rightarrow \underline{k = \frac{n\pi}{b}}$$

3.2.) Now $V = XY = A\left(e^{\frac{n\pi y}{b}} - e^{-\frac{n\pi y}{b}}\right) C \sin\left(\frac{n\pi}{b}x\right)$

$$\uparrow$$

$$2\sinh\left(\frac{n\pi}{b}y\right)$$

$$V = \underbrace{2AC}_{C_n} \sinh\left(\frac{n\pi}{b}y\right) \sin\left(\frac{n\pi}{b}x\right)$$

$$\Rightarrow V(x,y) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi}{b}y\right) \sin\left(\frac{n\pi}{b}x\right)$$

where C_n is given by Fourier's trick

$$C_n \sinh\left(\frac{n\pi}{b}a\right) = \frac{2}{b}\int_0^b V_0 \sin\left(\frac{n\pi}{b}x\right) dx$$

$$\Rightarrow C_n = \frac{2V_0}{\cancel{b}\sinh\left(\frac{n\pi}{b}a\right)} \left[\frac{\cancel{b}}{n\pi}\left(-\cos\left(\frac{n\pi}{b}x\right)\right)\right]_0^b$$

<u>if n even...</u>

$$-\cos(n\pi) - (-\cos(0))$$

$$= -1 - (-1) = 0$$

<u>if n odd</u>

$$-\cos(n\pi) - (-\cos(0))$$

$$= 1 + 1 = 2$$

$$\Rightarrow C_n = \begin{cases} \dfrac{4V_0}{n\pi \sinh\left(\frac{n\pi}{b}a\right)} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$$

So, $$\boxed{V(x,y) = \frac{4V_0}{\pi} \sum_{n \text{ odd}} \frac{\sinh\left(\frac{n\pi}{b}y\right)\sin\left(\frac{n\pi}{b}x\right)}{n \sinh\left(\frac{n\pi}{b}a\right)}}$$

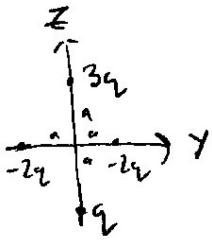
3.3.)

$$V_{mon}(\vec{r}) = K \frac{Q}{r}$$

$$V_{dip}(\vec{r}) = K \frac{\vec{p}\cdot\hat{r}}{r^2}$$

for point charges $\vec{p} = \sum_{i=1} q_i \vec{r}_i'$

i.)



$$\vec{p} = (3qa - qa)\hat{z} + (-2qa - (-2qa))\hat{y}$$

$$= 2qa\hat{z}$$

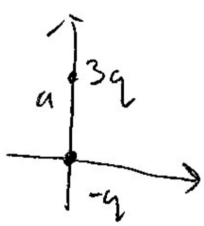
$$\Rightarrow V_{dip} = \frac{1}{4\pi\epsilon_0} \frac{2qa\,\hat{z}\cdot\hat{r}}{r^2} \qquad * \ \hat{z} = \cos\theta\hat{r}$$

$$\Rightarrow \boxed{V_{dip} = \frac{1}{4\pi\epsilon_0}\frac{2qa\cos\theta}{r^2}}$$

$V_{mon} = 0$ since $Q_{total} = 3q + q - 2q - 2q = 0.$

$$\Rightarrow V = V_{mon} + V_{dip} = \frac{1}{4\pi\epsilon_0}\frac{2qa\cos\theta}{r^2}$$

ii.)



$$\boxed{V_{mon} = \frac{1}{4\pi\epsilon_0}\frac{2q}{r}}$$

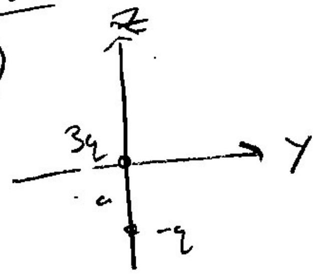
$$\vec{p} = (3qa + q(0))\hat{z} = 3qa\hat{z}$$

$$V_{dip} = \frac{1}{4\pi\epsilon_0}\frac{3qa\cos\theta}{r^2} \qquad \vec{p}\cdot\hat{r} = 3qa\cos\theta$$

$$\boxed{V = V_{mon} + V_{dip} = \frac{1}{4\pi\epsilon_0}\left(\frac{2q}{r} + \frac{3qa\cos\theta}{r^2}\right)}$$

3.3.) cont.

iii.)



$Q = 2q$

$\Rightarrow V_{mon} = \frac{1}{4\pi\epsilon_0}\frac{2q}{r}$

$\vec{p} = (3q(0) + -q(-a))\hat{z}$

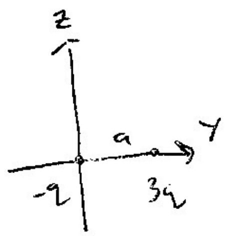
$\vec{p} = qa\,\hat{z}$

$\vec{p}\cdot\hat{r} = qa\cos\theta$

$\Rightarrow \quad V_{dip} = \frac{1}{4\pi\epsilon_0}\,\frac{qa\cos\theta}{r^2}$

$$V = V_{dip} + V_{mon} = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{r} + \frac{qa\cos\theta}{r^2}\right]$$

iv.)



$Q = 2q$

$\Rightarrow V_{mon} = \frac{1}{4\pi\epsilon_0}\frac{2q}{r}$

$\vec{p} = (3q(a) + -q(0))\hat{y}$

$\vec{p} = 3qa\,\hat{y}$ $\qquad$ * $\hat{y} = \sin\theta\sin\phi\,\hat{r}$

$\vec{p}\cdot\hat{r} = 3qa\sin\theta\sin\phi$

$V_{dip} = \frac{1}{4\pi\epsilon_0}\,\frac{3qa\sin\theta\sin\phi}{r^2}$

$$V = V_{mon} + V_{dip} = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{r} + \frac{3qa\sin\theta\sin\phi}{r^2}\right]$$