Electromagnetc Theory: PHYS330

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Summary

Summary

1. Electromagnetism during the pandemic
    - Pace
    - Style
    - Need to review vectors, vector functions, and vector calculus
2. Challenge level: pre-requisites
    - Passed Calculus 1, 2, and 3
    - Passed Calculus-based physics 1, 2, and 3
    - Passed modern physics
3. Maxwell's equations live in 3D
4. Introduction to Electromagnetism by D. Griffiths (4th ed.)
5. First half of the text is recommended by publisher, retain for graduate school
6. Asynchronous content: www.youtube.com/918particle, and Moodle in folders

Homework

Homework

1. Reading: Chapter 1 by Friday/Saturday
2. Exercises: 1.54, 1.55, 1.56, 1.57, 1.59, 1.62, 1.63, 1.64

Today: the Dirac delta-function

The Dirac δ-function

Consider this function:

$$\vec{v} = \frac{1}{r^2}\hat{r} \tag{1}$$

with $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. What is the divergence?

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial\theta}(r\sin(\theta)v_\theta) + \frac{1}{r\sin(\theta)}\frac{\partial v_\phi}{\partial\phi} \tag{2}$$

The Dirac δ-function

So we find the divergence is zero. What is the result of a surface integral around the origin?

$$\oint \vec{v} \cdot d\vec{a} = \int_0^{2\pi} \int_0^{\pi} \left(\frac{\hat{r}}{R^2} \right) \cdot (R^2 \sin(\theta) d\theta d\phi \hat{r}) \tag{3}$$

The Dirac δ-function

(Let $d\tau$ be the volume element). Isn't the following *always* supposed to be true?

$$\int (\nabla \cdot \vec{v}) d\tau = \oint \vec{v} \cdot d\vec{a} \tag{4}$$

We must be dealing with a strange function...apparently all of the surface integral contribution comes from the origin, where the volume element is zero, but the function is infinite.

Think of a function that has an finite *integral* result, but is zero everywhere except one point. Nothing comes to mind.

The Dirac δ-function:

$$\delta(x) = 0 \quad \textit{if } x \neq 0 \tag{5}$$

$$\delta(x) = \infty \quad \textit{if } x = 0 \tag{6}$$

This function is called a *distribution*, not a real function. However, it has interesting properties:

$$f(x)\delta(x) = f(0)\delta(x) \tag{7}$$

$$\int_{-\infty}^{\infty} \delta(x)dx = 1 \tag{8}$$

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0) \tag{9}$$

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = f(a) \tag{10}$$

The Dirac δ-function

Show that

$$\delta(kx) = \frac{1}{|k|}\delta(x) \tag{11}$$

Try it here:

$$\int_{-\infty}^{\infty} \cos(2kx)\delta(kx)dx = \tag{12}$$

Another interesting thing

What is this integral?

$$\int_0^{2\pi} \sin(nx)\sin(mx)dx \tag{13}$$

The Dirac δ-function

Generalize to three dimensions:

$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z) \tag{14}$$

$$\int d\tau \delta^3(\vec{r}) = 1 \tag{15}$$

$$\int d\tau f(\vec{r})\delta^3(\vec{r} - \vec{a}) = f(\vec{a}) \tag{16}$$

Let $f(\vec{r}) = \cos^2(x) - \sin^2(y)$, and $\vec{a} = (0, \pi)$. Evaluate:

$$\int d\tau f(\vec{r})\delta^3(\vec{r} - \vec{a}) = \tag{17}$$

The Dirac δ-function

If the integral contains the origin:

$$\int \nabla \cdot \left(\frac{\hat{r}}{r^2} \right) d\tau = 4\pi \tag{18}$$

Thus we know

$$\nabla \cdot \left(\frac{\hat{r}}{r^2} \right) = 4\pi\delta^3(\vec{r}) \tag{19}$$

One of Maxwell's Equations: $\nabla \cdot \vec{E} = \rho/\epsilon_0$. This says the divergence of the E-field is charge density. If the E-field goes like $1/r^2$, then we know it's like a point charge. So the charge density of a point charge: $\delta^3(\vec{r})$.

Objects of Electromagnetism

Objects of Electromagnetism

What type of *object* is $\vec{f}(x, y, z) \cdot \vec{g}(x, y, z)$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

Objects of Electromagnetism

What type of *object* is $\vec{f}(x, y, z) \times \vec{g}(x, y, z)$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

Objects of Electromagnetism

What type of *object* is $\vec{h}(x, y, z) \cdot (\vec{f}(x, y, z) \times \vec{g}(x, y, z))$?

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Objects of Electromagnetism

What type of *object* is $\nabla f(x, y, z)$?

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Objects of Electromagnetism

What type of *object* is $\frac{\partial f(x,y,z)}{\partial x}$?

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Objects of Electromagnetism

What type of *object* is $\nabla \cdot \vec{f}(x, y, z)$?

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Objects of Electromagnetism

What type of *object* is $\nabla \times \vec{f}(x, y, z)$?

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Objects of Electromagnetism

What type of *object* is $\nabla \cdot (\nabla f(x, y, z))$?

- A: A scalar
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Objects of Electromagnetism

This object is the Laplacian of f:

$$\nabla \cdot (\nabla f(x, y, z)) = \nabla^2 f \tag{20}$$

Of all the possible *second derivatives* of the above objects this is the one we will encounter the most. The rest are zero or less important (grad of divergence). When you see a second derivative, think guilty until proven innocent, in EM.

Area Vectors and Surface Integrals

Area Vectors and Surface Integrals

Cartesian coordinates, six possibilities:

$$d\vec{a} = \pm dxdy\hat{z} \tag{21}$$

$$d\vec{a} = \pm dxdz\hat{y} \tag{22}$$

$$d\vec{a} = \pm dydz\hat{x} \tag{23}$$

You must always determine the vector $d\vec{a}$ before completing a surface integral.

Area Vectors and Surface Integrals

Let $\vec{v} = 2xz\hat{i} + (x+2)\hat{j} + y(z^2-3)\hat{k}$. Integrate $\vec{v}$ over the cube of side length 2 with one corner at the origin. (breakout rooms)

Area Vectors and Surface Integrals

Let $\vec{v} = s(2+\sin^2(\phi))\hat{s} + s\sin(\phi)\cos(\phi)\hat{\phi} + 3z\hat{z}$. (a) Find the divergence.

Area Vectors and Surface Integrals

Let $\vec{v} = s(2+\sin^2(\phi))\hat{s} + s\sin(\phi)\cos(\phi)\hat{\phi} + 3z\hat{z}$. (b) Test the divergence theorem using the quarter cylinder with radius 2 and height 5, the corner at the origin.

Conclusion

Summary

1. Electromagnetism and the module system
    - Pace
    - Style
    - Class decision
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