3.29, 3.3

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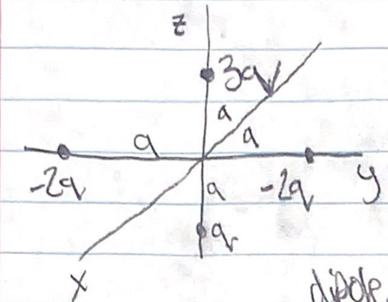
11/30/20
Phys 330

Homework 4 Extra Credit Questions

3.29).



approximate potential in spherical coordinates

dipole moment $p = qr$

$p = \sum q_i r_i$

total dipole moment =

$p = (3qaz - qaz) + (-2qay + (-2qa(-y)))$

$= -2qaz + (-2qay) + 2qay$

$= 2qaz$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$p = 2qa\hat{z}$

$pr = 2qazr$

$= 2qa(zr)$

$= 2qa(r\cos\theta r)$

$\underline{\underline{pr = 2qa r\cos\theta}}$

$V = \frac{1}{4\pi\varepsilon_0}\frac{pr}{r^2}$

$V = \frac{1}{4\pi\varepsilon_0}\frac{2qa\cos\theta}{r^2}$

$$\boxed{\frac{1}{4\pi\varepsilon_0}\frac{2qa\cos\theta}{r^2}}$$

3.3). radius R $\sigma = k\cos\theta$ surface charge

a) dipole moment of this charge distribution

$V(r) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_+} - \frac{q}{r_-}\right)$

$\frac{1}{\eta} = \frac{1}{r}\sum\left(\frac{r'}{r}\right)P_n(\cos\theta)$

$\frac{1}{\eta} = \frac{1}{r}\sum_{n=0}\left(\frac{r'}{r}\right)P_n(\cos\theta)$

$r' = d/2$

$$\frac{1}{r_+} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{d}{2r}\right)^n P_n(\cos\theta)$$

r_- take $\theta = 180 - \theta$ $\cos\theta \to -\cos\theta$

$$\frac{1}{r} = \frac{1}{r}\sum_{n=0}^{\infty}\left(\frac{d}{2r}\right)^n P_n(-\cos\theta)$$

$$P_n(-n) = (-1)^n P_n(x)$$

$$P_n(-\cos\theta) = (-1)^n P_n(\cos\theta)$$

$$V(r) = \frac{q}{4\pi\varepsilon_0 r}\sum_{n=0}\left(\frac{d}{2r}\right)^n P_n(\cos\theta) - \sum_{n=0}(-1)^n \frac{d}{2r} P_n \cos\theta$$

$n=1$ for dipole moment

$$V_{dipole} = \frac{q}{4\pi\varepsilon_0 r}(P_1\cos\theta) + P_1(\cos\theta)$$

$$= \frac{q}{4\pi\varepsilon_0}\ \frac{qd\cos\theta}{r^2}$$

$P_1(\cos\theta) = (\cos\theta)$ for $n=2$;

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{r}\left(\frac{d}{2r}\right)^2 (P_2(\cos\theta - P_2(\cos\theta))$$

$$= 0$$

for $n=3$;

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r}\left(\frac{d}{2r}\right)^3 P_3(\cos\theta + P_3(\cos\theta))$$

$$= \frac{2q}{4\pi\varepsilon_0 r}\left(\frac{d}{2r}\right)^3 P_3(\cos\theta)$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(\frac{d}{2r}\right)^3 (5\cos^3\theta - 3\cos\theta)$$

$$\boxed{= \frac{qd^3}{32\pi\varepsilon_0 r^4}(5\cos^3\theta - 3\cos\theta)}$$