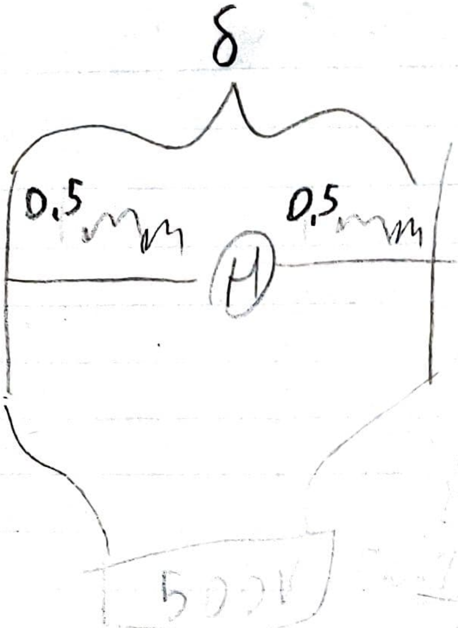
Adam

4.1



$\vec{p} = q\vec{d} = \alpha\vec{E} = \alpha\frac{V}{\delta}$

$\frac{\alpha}{4\pi\epsilon_0} = 0{,}667 \cdot 10^{-30}\ m^3$

$d = \frac{\alpha}{4\pi\epsilon_0} \cdot 4\pi\epsilon_0 \frac{V}{q\delta}$

$= 0{,}667 \cdot 10^{-30} \cdot 4\pi\epsilon_0 \frac{500}{0{,}001}$

$= \cdot \frac{5\times10^5}{e}$

$= 2{,}32 \times 10^{-16}\ m$

alpha

$\alpha = 4\pi\epsilon_0 a^3 = 3\epsilon_0 v$

$\frac{2{,}32 \times 10^{-16}}{\text{angstrom}/2} = \frac{2{,}32 \times 10^{-16}}{2 \cdot 10^{-}}$

$= \boxed{4{,}64 \times 10^{-6}}$

$a = \frac{\alpha}{4\pi\epsilon_0} \cdot 4\pi\epsilon_0 \frac{V}{q\delta}$

$\frac{q a \delta}{\alpha 4\pi\epsilon_0} \cdot 4\pi\epsilon_0 = V$

$V = \frac{q_e \cdot 10^{-10} \cdot 10^{-3}}{4\pi\epsilon_0 \cdot 2} \frac{1}{0{,}667 \cdot 10^{-30}} = \boxed{1{,}08 \cdot 10^{8}\ V}$

(4.7) Show $U = -\vec{p} \cdot \vec{E}$

$$W = Fd$$

$$\vec{p} = q \cdot \vec{d}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \hat{r} \qquad \vec{F}_E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_1 q_0}{r^2} \hat{r}$$

$$\vec{E} \cdot \vec{p} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_0}{r^2} \hat{r} \cdot q_1 \vec{d} = \vec{F}_E \cdot \vec{d} = V$$

But this is [illegible] a dipole, therefore we reverse [illegible] for the energy in [illegible] dipole

$$-\vec{F}_E \cdot \vec{d} = U = -\vec{p} \cdot \vec{E}$$

Example 4.3

Ex 4.2 Find $\vec{E}$ produced by uniformly polarized sphere of radius R.

$$V(r,\theta)=\sum_{l=0}^{\infty} A_l r^l P_l(\cos\theta) \quad r\le R$$

$$V(r,\theta)=\sum_{l=0}^{\infty} \frac{B_l}{r^{l+1}} P_l(\cos\theta) \quad r\ge R$$

$$\sum_{l=0}^{\infty} A_l R^l P_l(\cos\theta)=\sum_{l=0}^{\infty}\frac{B_l}{R^{l+1}} P_l(\cos\theta)$$

$$B_l = A_l R^{2l+1}$$

$$\left.\frac{\partial V_{out}}{\partial r}-\frac{\partial V_{in}}{\partial r}\right|_{r=R} = -\frac{1}{\epsilon_0}\sigma_0(\theta)$$

$$-\sum_{l=0}^{\infty}(l+1)\frac{B_l}{R^{l+2}}P_l(\cos\theta) - \sum_{l=0}^{\infty} l A_l R^{l-1} P_l(\cos\theta) = -\frac{1}{\epsilon_0}\sigma_0(\theta)$$

$$\sum_{l=0}^{\infty}(2l+1)A_l R^{l-1} P_l(\cos\theta) = \frac{1}{\epsilon_0}\sigma_0(\theta)$$

$$A_l=\frac{1}{2\epsilon_0 R^{l-1}}\int_0^\pi \sigma_0(\theta) P_l(\cos\theta)\sin\theta\, d\theta$$

$$\sigma_0(\theta)=k\cos\theta = k P_1(\cos\theta)$$

$$A_1 = \frac{k}{2\epsilon_0}\int_0^\pi (P_1(\cos\theta))^2 \sin\theta\, d\theta = \frac{k}{3\epsilon_0}$$

$$V(r,\theta)=\frac{k}{3\epsilon_0} r\cos\theta \quad r\le R$$

$V(r,\theta) = \frac{kR^3}{3\epsilon_0}\frac{1}{r^2}\cos\theta \quad (r \geq R)$

If $\sigma_0(\theta)$ is from our field on our sphere then $k = 3\epsilon_0 E_0$, inside is $E_0 r\cos\theta = E_0 z$ and field is $-E_0\hat{z}$ inside to cancel field

outside, $V = E_0\frac{R^3}{r^2}\cos\theta$

$E = -\nabla V$

$E = -(\frac{1}{r^2}\frac{\partial}{\partial r}($[illegible]$) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(E_0\frac{R^3}{r^2}\cos\theta\sin\theta)$

$= -(0 + \frac{1}{r\sin\theta}(\frac{E_0R^3}{r^2}(\cos^2\theta - \sin^2\theta))$

$= -(0 + \frac{1}{r\sin\theta}(\frac{E_0R^3}{r^2}(\cos^2\theta - (1-\cos^2\theta))$

$= -\frac{E_0R^3(1-2\cos^2\theta)}{r^3\sin\theta}$ $\boxed{\frac{E_0R^3 \text{[illegible]}}{r^3} - \frac{2E_0R^3\cos\theta\cot\theta}{r^3}}$

(4.10) shere radius R has polarization $\vec{P}(\vec{r}) = k\vec{r}$

(a) find σ_b and ρ_b

$\sigma_b \equiv \vec{P} \cdot \hat{n}$

$\rho_b \equiv -\nabla \cdot \vec{P}$

$\sigma_b = \vec{P} \cdot \hat{n}$

$\hat{n} = \hat{r}$ $\therefore$ $\sigma_b = \boxed{kR}$

$\rho_b = -\nabla \cdot \vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cdot kr)$

$= -\frac{1}{r^2}\frac{\partial}{\partial r}(kr^3) = \boxed{-3k}$

(b) Find E_{in} and E_{out}

$\vec{E}_{in} = \frac{\rho \vec{r}}{3\epsilon_0} = \boxed{-\frac{k}{\epsilon_0}\hat{r}}$

$Q_{tot} = 4\pi R^2 \sigma + \frac{4}{3}\pi R^3 \rho = 4\pi k R^3 - 4\pi k R^3 = 0$

$\vec{E}_{out} = \boxed{0}$

4.15 $\sigma_b = \vec{P} \cdot \hat{n}$

$\vec{P}(\vec{r}) = \frac{k}{r}\hat{r}$

$\rho_b = -\nabla \cdot \vec{P}$

$\sigma_a = \frac{k}{a}$

$\rho_b = -\nabla \cdot \vec{P} = -\frac{1}{r^2}\frac{\partial}{\partial r}(rk) = \frac{-k}{r^2}$

$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int$

$r = r$ $\quad \hat{r} = \hat{r}$

$E_a = \frac{1}{4\pi\epsilon_0}\int$

$= \frac{k}{2\epsilon_0}\int_0^{\pi} \sin\theta \, d\theta = \boxed{\frac{k}{a\epsilon_0}}$

$\hat{r} = \hat{r}$

$E_b = \boxed{\frac{k}{b\epsilon_0}}$

$E_{ab} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r})}{r^2}\, d\tau = \frac{1}{4\pi\epsilon_0}\int_b^a\int_0^{\pi}\int_0^{2\pi} \frac{-k}{r^4} r^2 \sin\theta \, dr\, d\theta\, d\phi$

$= \frac{k}{2\epsilon_0}\int_b^a\int_0^{\pi} \frac{k}{r^2} \sin\theta\, dr\, d\theta = \frac{-k}{\epsilon_0}\int_b^a \frac{1}{r^2}\, dr$

$= \boxed{\frac{1}{2\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)}$

$\oint \vec{D} \cdot d\vec{a} = Q_{fenc}$ $\vec{P}(\vec{r}) = \frac{k}{r}\hat{r}$

$Q_{fenc} = 0$

$\vec{D} = 0$

$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

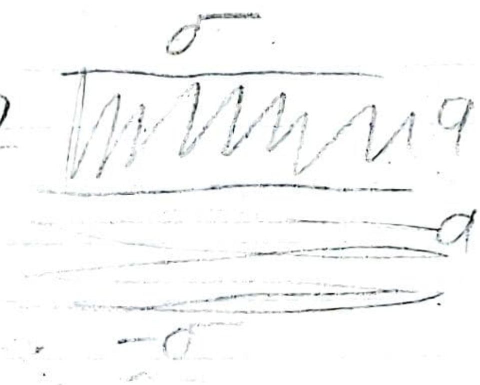
$0 = \epsilon_0 \vec{E} + \vec{P}$

$-\vec{P} = \epsilon_0 \vec{E}$

$\frac{k}{r\epsilon_0}\hat{r} = \vec{E}$

$$\boxed{\begin{aligned} \vec{E}_{in} &= \frac{k}{a\epsilon_0}\hat{r} \\ E_{sphere} &= \frac{k}{\epsilon_0}\left(\frac{1}{b} - \frac{1}{a}\right)\hat{r} \\ E_{out} &= \frac{k}{\epsilon_0 b}\hat{r} \end{aligned}}$$

4.18 (a) Find $\vec{D}$ in each slab.



$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$

$\oint \vec{D} \cdot d\vec{a} = Q_f$

$A D = Q$

$Q = \sigma A$

$D = \frac{Q}{A} = \frac{\sigma A}{A}$

$\boxed{\vec{D}_{top} = -\sigma \hat{z}}$

$\vec{E}_{top} = \frac{\sigma}{2\epsilon_0}\hat{z}$

$E_{bot} = \frac{\sigma}{2\epsilon_0}$

(b) Find $\vec{E}$

$\vec{E}_{top} = \frac{1}{\epsilon_r \epsilon_0}\vec{D} = \boxed{-\frac{\sigma}{2\epsilon_0}\hat{z}}$ $\quad$ $\vec{E}_{bot} = \frac{1}{\epsilon_r \epsilon_0}\vec{D} = \boxed{-\frac{2\sigma}{3\epsilon_0}\hat{z}}$

(c) Find $\vec{P}$ for each $\quad \vec{P} = \epsilon_0 \chi_e \vec{E}$

$\vec{P}_{top} = \vec{D}_{top} - \epsilon_0 \vec{E}_{top} = -\sigma \hat{z} + \frac{\sigma}{2}\hat{z} = \boxed{-\frac{\sigma}{2}\hat{z}}$

$\vec{P}_{bot} = \vec{D}_{bot} - \epsilon_0 \vec{E}_{bot} = -\sigma \hat{z} + \frac{2\sigma}{3}\hat{z} = \boxed{\frac{-1\sigma}{3}\hat{z}}$

$\Delta V \quad V_{bot} = -\int_{bot}^{top} \vec{E} \cdot d\vec{l} = (E_{top} + E_{bottom})a$

$= \left(\frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{3\epsilon_0}\right)a$

$V = E$

$V = \vec{E}_{top}$ $\quad \vec{E}_{bottom}$ $\quad \frac{\sigma}{\epsilon_0} + \frac{\sigma}{3\epsilon_0}$

(e) $\boxed{\rho_b = -\nabla \cdot \vec{P} = 0}$ $\quad \sigma_b = \vec{P} \cdot \hat{n} = \{$

$$\sigma_b = \vec{P} \cdot \hat{n} = \begin{cases} -\frac{\sigma}{2} & \text{top} \\ \frac{\sigma}{2} & \text{top mid} \\ \frac{\sigma}{3} & \text{bottom mid} \\ -\frac{\sigma}{3} & \text{bottom} \end{cases}$$

For mid, $\vec{P}$ is opposed to $\hat{n}$

(f) Find $\vec{E}$ again

$\vec{Q}_{top} = \sigma - \frac{\sigma}{2} = \frac{\sigma}{2}$ $\quad E_{above\ top} = \frac{-\sigma}{2 \cdot 2\epsilon_0} \hat{z} = \frac{-\sigma}{4\epsilon_0} \hat{z}$

$Q_{top\,mid} = \frac{\sigma}{2} - \frac{\sigma}{2} + \frac{\sigma}{2} - \sigma = -\frac{\sigma}{2}$

$E_{top\,mid} = \frac{-\sigma}{4\epsilon_0} \hat{z}$

$\boxed{E_{top} = \frac{-\sigma}{2\epsilon_0} \hat{z}}$

$Q_{very\ bottom} = \frac{\sigma}{3} - \sigma = -\frac{2}{3}\sigma$, $E_{vb} = \frac{-\sigma}{3\epsilon_0} \hat{z}$

$Q_{mb} = \sigma - \sigma/2 + \frac{\sigma}{3} - \sigma/3 = \frac{2\sigma}{3}$, $E_{mb} = \frac{-\sigma}{3\epsilon_0} \hat{z}$

$\boxed{E_{bottom} = \frac{-2\sigma}{3\epsilon_0} \hat{z}}$