Warm-Up for $\pi + 2$ Day, 2022

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March 16, 2022

1 Memory Bank

1. The dipole moment:
$$\mathbf{p} = \int \mathbf{r}'\rho(\mathbf{r}')d\tau' = \int \mathbf{r}'dq' \tag{1}$$

2. The potential from the dipole term of the multipole expansion:
$$V_{\text{dipole}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{r^2} \tag{2}$$

2 Multipole Expansions

1. Suppose there is a line charge along the $\hat{z}$-direction, with density $\pm\lambda$. For a length $L/2$ below the origin, the density is $-\lambda$, and for a length $L/2$ above the origin, the density is $+\lambda$. (a) Find the dipole moment, $\mathbf{p}$. (b) Insert $\mathbf{p}$ into Eq. 2 to find the potential. (c) Calculate the $\mathbf{E}$-field. Is the result resonable? Why doesn't the field limit to something like $\approx Q/r^2$?

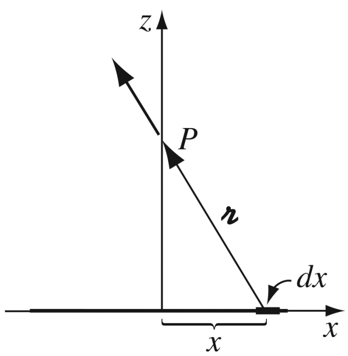

Figure 1: Suppose there is a positive line charge to the right, and a negative line charge to the left. *Assume the charge distribution is along the $\hat{z}$-direction, not the $\hat{x}$-direciton.*