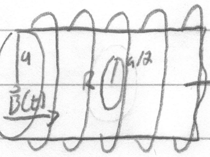
Jesse Chi-Su
12/4/20 PHYS

HW 7

7.12, 15, 34



7.12 Solenoid, radius a, w/ inside B-field: $\vec{B}(t) = B_0\cos(\omega t)\hat{z}$.

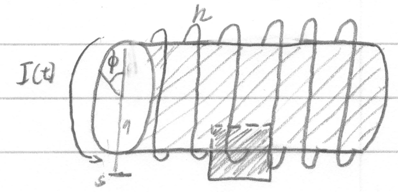
Loop in solenoid, radius $\frac{a}{2}$, resistance R. Find current $I(t)$. // $V = IR \Rightarrow I = \frac{V}{R}$, $V = \mathcal{E}$, $I = \frac{\mathcal{E}}{R}$ (1)

$\Phi = \int B\, d\vec{a} = B(\pi r^2)$ // $r = \frac{a}{2} \Rightarrow \Phi = B\pi\frac{a^2}{4}$, // $B = \vec{B}(t)$ // $\mathcal{E} = \frac{-\partial\Phi}{\partial t}$ (2) $\rightarrow \Phi = \int B\cdot d\vec{a}$ (3)

$\Phi = \frac{\pi a^2}{4}\cdot B_0\cos(\omega t) \Rightarrow \mathcal{E} = \frac{-\partial\Phi}{\partial t} = \frac{-\pi a^2}{4}B_0\frac{\partial}{\partial t}(\cos(\omega t)) = \frac{\pi a^2}{4}B_0\sin(\omega t)(\omega) \Rightarrow \mathcal{E} = \frac{\pi a^2}{4}B_0\omega\sin(\omega t)$

$I = \frac{\mathcal{E}}{R} \Rightarrow \boxed{I(t) = \frac{\pi a^2}{4R}B_0\omega\sin(\omega t)}$

7.15 W/ Quasistatic approx., find $\vec{E}$-field in & out solenoid.



Quasistatic: (Ex. 5.9) $\vec{B}$-field in solenoid: $\vec{B}\begin{cases}\mu_0 n I\hat{z}, & s<a\\ 0, & s>a\end{cases}$

// Amperian loop

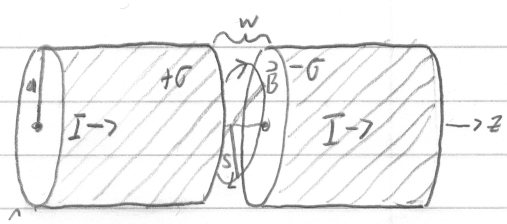
$\oint \vec{E}_{in}\cdot d\vec{l} = \frac{-\partial\Phi_1}{\partial t}$, $\Phi_1 = \int B\cdot d\vec{a}_1 \Rightarrow \Phi_1 = B\int_0^a 2\pi s'\,ds' \Rightarrow 2\pi B\left[\frac{1}{2}s'^2\right]_0^a \Rightarrow \Phi_1 = B\pi a^2 \Rightarrow \Phi_1 = \mu_0 n I\pi a^2$

$\oint \vec{E}_{out}\cdot d\vec{l} = -\frac{\partial\Phi_2}{\partial t}$, $\Phi_2 = \int B\cdot d\vec{a}_2 \Rightarrow \Phi_2 = B\int_0^s 2\pi s'\,ds' \Rightarrow 2\pi B\left[\frac{1}{2}s'^2\right]_0^s \Rightarrow \Phi_2 = B\pi s^2 \Rightarrow \Phi_2 = \mu_0 n I\pi s^2$

$\oint \vec{E}_{in}\cdot d\vec{l} = \frac{-\partial\Phi_1}{\partial t} \Rightarrow \vec{E}\cdot(2\pi s) = -\pi\mu_0 n a^2\frac{\partial I}{\partial t} \Rightarrow \boxed{\vec{E} = \frac{-\mu_0 n a^2}{2}\frac{\partial I}{\partial t}\hat{\phi}}$

$\oint \vec{E}_{out}\cdot d\vec{l} = -\frac{\partial\Phi_2}{\partial t} \Rightarrow \vec{E}\cdot(2\pi s) = -\mu_0 n\pi s^2\frac{\partial I}{\partial t} \Rightarrow \boxed{\vec{E} = \frac{-\mu_0 n s}{2}\frac{\partial I}{\partial t}\hat{\phi}}$

7.34 Find $\vec{B}$-field in gap @ distance ($s<a$) from axis.



$\nabla\times\vec{B} = \mu_0\vec{J} \Rightarrow \int_S(\nabla\times\vec{B})\cdot d\vec{S} = \mu_0\int\vec{J}\cdot d\vec{S}$

$\oint\vec{B}\cdot d\vec{l} = \mu_0\int\vec{J}\cdot d\vec{a}$

$\hookrightarrow \vec{J}_d = \epsilon_0\frac{\partial\vec{E}}{\partial t}$ // $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{z} \Rightarrow \vec{E} = \frac{1}{\epsilon_0}\frac{Q}{A} \Rightarrow \frac{\partial\vec{E}}{\partial t} = \frac{1}{\epsilon_0 A}\frac{\partial Q}{\partial t} \Rightarrow \frac{\partial\vec{E}}{\partial t} = \frac{I}{\epsilon_0 A}\hat{z}$

$\vec{J}_d = \epsilon_0\left(\frac{I}{\epsilon_0 A}\right)\hat{z} \Rightarrow \vec{J}_d = \frac{I}{\pi a^2}\hat{z}$

$\vec{B}(2\pi s) = \mu_0\frac{I}{\pi a^2}\int^s d\vec{a} \Rightarrow \vec{B}(2\pi s) = \frac{\mu_0 I}{\pi a^2}(\pi s^2) \Rightarrow \boxed{\vec{B} = \frac{\mu_0 I s}{2\pi a^2}\hat{\phi}}$