Reading Quiz 2 for Electromagnetic Theory (PHYS330)

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Abstract

A summary of content covered in chapter 2 of Introduction to Electrodynamics.

1 Distributions of Point Charges

1. Picture a *physical dipole* of two charges $+q$ and $-q$ of equal magnitude, separated by a distance d. Define the dipole moment as $\vec{p} = q\vec{d}$ pointing from $-q$ to q somewhere in the xy-plane. Now add an external electric field $\vec{E} = E_0\hat{x}$. Show that the *torque* on the dipole is

$$\vec{\tau} = \vec{p} \times \vec{E} \tag{1}$$

2. Imagine two dipoles, each with dipole moments $\vec{p}_1$ and $\vec{p}_2$ pointed in opposite directions, forming a square with alternating positive and negative charges. Calculate the electric field vector in the center of the square.

2 Continuous Charge Distributions

1. (a) Compute the electric field of a continuous line of charge, with total charge $Q = \lambda L$, where λ is the charge density and L is the total length. Take the field point to be a distance z above the center of the line of charge. Show what happens in the limit that $L \gg z$. (b) Obtain the same result as (a) using Gauss' Law.

2. Assuming a plane of charge with charge density (Coulombs per unit area) σ has an electric field $\sigma/(2\epsilon_0)$, what electric fields would occur in each of the following situations:

   - Two planes of positive charge, and the field point is somewhere between the plates.
   - Two planes of charge, one positive and one negative, and the field point is somewhere between the plates.
   - Two planes of positive charge, one occupying the yz-plane, and the other occupying the xz-plane, and the field point is $(1, 1, 0)$.

3 The Curl of $\vec{E}$-fields

1. According to Eq. 2.19 in the text, the close loop line integral for the E-field of a point charge is

$$\oint \vec{E} \cdot d\vec{l} = 0 \tag{2}$$

   This implies $\nabla \times \vec{E} = 0$. According to the Helmholtz theorem in Ch. 1, this means the $\vec{E}$-field can be cast as the gradient of a scalar function known as *the potential,* V:

$$\vec{E} = -\nabla V \tag{3}$$

   The minus sign is a convention that is analogous to the minus sign in $\vec{F} = -\frac{dU}{dx}\hat{x}$ from mechanics.

   - Show that

$$-\int_a^B \vec{E} \cdot d\vec{l} = V(\vec{b}) - V(\vec{a}) \tag{4}$$

   - Assume a point charge at the origin, and label its electric field $\vec{E}$. Perform the integral

$$V(\vec{r}) = -\int_\infty^r E(r')dr' \tag{5}$$

     to find the potential formula for a point charge. *[Answer: kq/r]*