Warm-Up for March 9th, 2022

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1 Memory Bank

1. Hyperbolic cosine: $2\cosh(kx) = \exp(kx) + \exp(-kx)$.
2. Single general solution to the Laplacian in Cartesian coordinates:

$$V(x,y) = (A\exp(kx) + B\exp(-kx))\,(C\sin(ky) + D\cos(ky)) \tag{1}$$

2 Solutions to the Laplacian for Potential

Supposed two infinitely long grounded metal plates are situated in the xz plane at $y = 0$ and $y = a$, and have widths $2b$. Suppose also that two infinitely long plates of width a are situated in the xy plane at $x = \pm b$, each with voltage V_0. See Fig. 1.

- In your own words, why does the single solution of the Laplacian reduce to Eq. 1? Determine the four boundary conditions of $V(x,y)$.

- In your own words, why must $A = B$ given these boundary conditions?

- Show that the general sum of solutions is

$$V(x,y) = \sum_i^{\infty} C_n \cosh(n\pi x/a)\sin(n\pi y/a) \tag{2}$$

- Use Fourier's Trick with $x = b$ to determine the C_n coefficients.

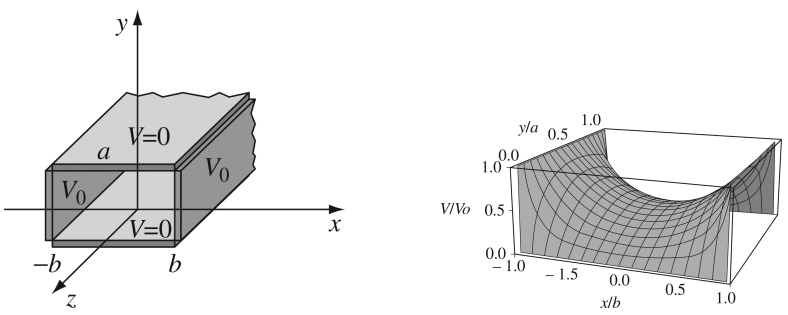

Figure 1: (Left) A rectangular pipe with two sides grounded and two sides at constant potential V_0. (Right) The shape of the solution (Eq. 2).