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Electro hw 4 4.10, 4.14, 4.15, 4.18, 4.26, 4.35

4.10 -

a) $\vec{P} = k\vec{r}$ $\qquad \sigma_b \equiv \vec{P}\cdot\hat{n}$ $\qquad \hat{n} = \hat{r}$

$$\sigma_b = kr\,\cancel{\hat{r}\cdot\hat{r}}^{\,1}$$

$$\sigma_b = kr$$

$$\rho_b = -\nabla\cdot\vec{P}$$

$$= -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr) = -\frac{k}{r^2}\frac{\partial}{\partial r}(r^3)$$

$$= -\frac{k}{\cancel{r^2}}(3\cancel{r^2})$$

$$= -3k$$

b)

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(\cancel{4\pi} r^{\cancel{2}}) = \frac{1}{\varepsilon_0}\left(-\cancel{3}k\,\frac{\cancel{4}}{\cancel{3}}\cancel{\pi}\, r^{\cancel{3}}\right)$$

$$\vec{E} = -\frac{kr}{\varepsilon_0}\hat{r} \qquad \text{inside}$$

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{1}{\varepsilon_0}\left(-3k\left(\frac{4}{3}\pi R^3\right) + kR(4\pi R^2)\right)$$

$$= \frac{1}{\varepsilon_0}\left(\cancel{-4\pi R^3 k + 4\pi R^3 k}^{\,0}\right)$$

$$\vec{E} = 0$$

4.14 -

$$Q_{tot} = \oint_S \sigma_b \, da + \oint_V \rho_b \, d\tau$$

$$= \oint_S \vec{P} \cdot d\vec{a} - \oint_V \vec{\nabla} \cdot \vec{P} \, d\tau$$

divergence theorem :

$$\oint_S \vec{P} \cdot d\vec{a} = \int_V \vec{\nabla} \cdot \vec{P} \, d\tau$$

$$Q_{enc} = 0$$

4.15 -

a)

$$\rho_b = -\vec{\nabla} \cdot \vec{P} = -\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{k}{r} \right)$$

$$= -\frac{k}{r^2}$$

$$\sigma_b = \vec{P} \cdot \hat{n} = \vec{P} \cdot \hat{r} = k/b \quad \text{at } r = b$$

$$= -\vec{P} \cdot \hat{r} = -k/a \quad \text{at } r = a$$

$$\vec{E} = \frac{1}{4\pi \varepsilon_0} \frac{q_{enc}}{r^2} \hat{r} \qquad r > b$$

$$\vec{E} = 0$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_{enc}}{r^2} \hat{r} \qquad r < a$$

$$q_{enc} = 0$$

$$\vec{E} = 0$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q_{enc}}{r^2} \hat{r} \qquad a < r < b$$

$$q_{enc} = -k/a\,(4\pi a^2) + \int_a^r \left(-k/r^2\right) 4\pi r^2\, dr$$

$$= -4\pi k a - 4\pi k (r - a)$$

$$= -4\pi k r$$

$$E = \frac{1}{\cancel{4\pi}\varepsilon_0} \left(\frac{-\cancel{4\pi} k \cancel{r}}{r^{\cancel{2}}} \right) \hat{r}$$

$$= -\frac{k}{\varepsilon_0 r} \hat{r}$$

b) $\oint \vec{D} \cdot d\vec{a} = q_{enc} = 0 \qquad \vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$$\vec{E} = 0$$

4.18 -

a)

$\int \vec{D} \cdot d\vec{a} = q_{f_{enc}}$

$D(2A) = \sigma A$

$D = \frac{\sigma}{2}$

$D = \sigma$

b) $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$= \varepsilon_0 \vec{E} + \varepsilon_0 (\chi_e) \vec{E}$

$= \varepsilon_0 (1 + \chi_e) \vec{E}$

$= \varepsilon_0 \varepsilon_r \vec{E}$

$\vec{D} = \varepsilon E$

$\vec{E}_1 = \frac{\vec{D}}{\varepsilon_1}$

$\vec{E}_1 = \frac{\sigma}{\varepsilon 2}$

$\vec{E}_2 = \frac{\vec{D}}{\varepsilon_2} = \frac{\sigma 2}{\varepsilon_0 3}$

c)

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$

$\vec{P} = \vec{D} - \varepsilon_0 \vec{E}$

$\vec{P}_1 = \sigma - \frac{\cancel{\varepsilon_0} \sigma}{2 \cancel{\varepsilon_0}}$

$= \sigma \left(1 - \frac{1}{2}\right) = \frac{\sigma}{2}$

$\vec{P}_2 = \sigma - \frac{\cancel{\varepsilon_0} 2 \sigma}{\cancel{\varepsilon_0} 3}$

$= \frac{\sigma}{3}$

d) $V = -\int_a^r \vec{E} \cdot d\vec{l}$

$V = E_1 a + E_2 a$

$= \frac{a\sigma}{\varepsilon_0}\left(\frac{1}{2} + \frac{2}{3}\right) = \frac{7}{6}\frac{a\sigma}{\varepsilon_0}$

e) $\sigma_b = \vec{P} \cdot \hat{n}$

4.26 -

$$W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\,d\tau = \frac{1}{2}\frac{Q^2}{(4\pi)^2}4\pi\left(\frac{1}{\varepsilon_0}\int_a^b \frac{1}{r^2}\frac{1}{r^2}r^2\,dr\right.$$

$$\left. + \frac{1}{\varepsilon_0}\int_b^\infty \frac{1}{r^2}\,dr\right)$$

$$= \frac{q^2}{8\pi}\left(\frac{1}{\varepsilon}\left(-\frac{1}{r}\right)\Big|_a^b + \frac{1}{\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_b^\infty\right)$$

$$= \frac{q^2}{8\pi\varepsilon_0}\left(\frac{1}{1+\chi_e}\left(\frac{1}{a}-\frac{1}{b}\right)+\frac{1}{b}\right)+\frac{1}{b}\right)$$

$$= \frac{q^2}{8\pi\varepsilon_0(1+\chi_e}\left(\frac{1}{a}+\frac{\chi_e}{b}\right)$$

4.35 -

$$\oint \vec{D}\cdot d\vec{a} = q_{f\,enc}$$

$$\vec{D} = \frac{q}{4\pi r^2}\hat{r} \qquad E = \frac{1}{\varepsilon}\vec{D} = \frac{q}{4\pi\varepsilon_0(1+\chi_e)r^2}\hat{r}$$

$$\vec{P} = \varepsilon_0\chi_e\vec{E} = \frac{q\chi_e}{4\pi(1+\chi_e)r^2}\hat{r}$$

$$\rho_b = -\nabla\cdot\vec{P} = -\frac{q\chi_e}{4\pi(1+\chi_e)}\left(\vec{\nabla}\cdot\frac{\hat{r}}{r^2}\right)$$

$$= -q\frac{\chi_e\,\delta^3(\vec{r})}{1+\chi_e}$$

$$\sigma_b = \vec{P} \cdot \hat{r} = \frac{q \chi_e}{4\pi (1+\chi_e) R^2}$$

$$q_{surf} = \sigma_b (4\pi R^2) = \frac{q \chi_e}{\cancel{4\pi} (1+\chi_e) \cancel{R^2}} (\cancel{4\pi R^2})$$

$$= \frac{q \chi_e}{1 + \chi_e}$$