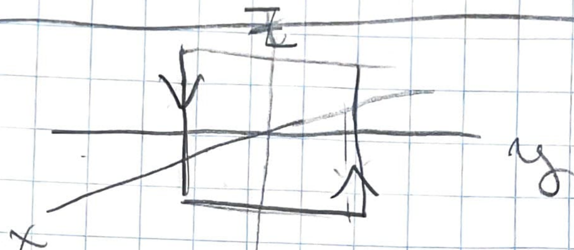
EM THEORY HW #5

5.4, 5.7, 5.11, 5.12, 5.16, 5.19, 5.21, 5.23, 5.27

$\boxed{4}$ $B = kz\hat{x}$



assuming Counter-Clockwise

Top $\rightarrow$ $IaB = Ia k\left(\frac{a}{2}\right) = Ik\frac{a^2}{2}$,

F Bottom $\rightarrow$ $IaB = -\frac{Ika^2}{2}$

$f_{net} = Ika^2\hat{z}$

$\boxed{7}$ $\frac{dp}{dt} = \frac{d}{dt}\int_V \rho r\, d\tau = \int\left(\frac{\partial\rho}{\partial t}\right) r\, d\tau = -\int(\nabla\cdot J) r\, d\tau$

$\nabla\cdot(xJ) = x(\nabla\cdot J) + J\cdot(\nabla x)$, $\nabla x = \hat{x}$, $\nabla\cdot(xJ) = x(\nabla\cdot J) + J$

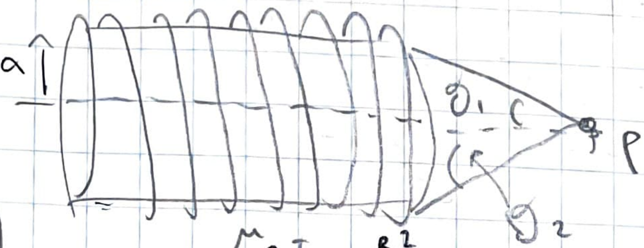
$\therefore \int_V(\nabla\cdot J)x\, d\tau = \int_V \nabla\cdot(xJ)\, d\tau - \int_V J_x\, d\tau$

$\int_S xJ\cdot da = 0$, as all inside, none on surface

$\therefore \int_V(\nabla\cdot J)x\, d\tau = -\int_V J_x\, d\tau$

$\int_V(\nabla\cdot J)r\, d\tau = -\int_V J\, d\tau$, $\frac{dp}{dt} = \int J\, d\tau$

$\boxed{\text{III}}$ find B @ P



5-41

$$B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{r^2}\right), \quad 2\pi R = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 n I}{2}\int \frac{a^2}{(a^2+z^2)^{\frac{3}{2}}}\,dz, \quad z = a\cot\theta$$

$$dz = -\frac{a}{\sin^2\theta}\,d\theta \quad \& \quad \frac{1}{(a^2+z^2)^{\frac{3}{2}}} = \frac{\sin^3\theta}{a^3}$$

$$B = \frac{\mu_0 n I}{2}\int \frac{a^2\sin^3\theta}{a^3\sin^2\theta}(-a\,d\theta) = -\frac{\mu_0 n I}{2}\int \sin\theta\,d\theta \rightarrow \frac{\mu_0 n I}{2}\cos\theta\Big]_{\theta_1}^{\theta_2}$$

$$= \frac{\mu_0 n I}{2}\left[\cos\theta_2 - \cos\theta_1\right]$$

for Sol. ∞, $\theta_1 = \pi$, $\theta_2 = 0$,

so $\cos(0) - \cos(\pi) = 2$, $\cancel{2}\cdot\frac{\mu_0 n I}{\cancel{2}} = \boxed{\mu_0 n I}$

$\boxed{\text{R}}$

Ex. 5.6 → $B(z) = \frac{\mu_0 I}{4\pi}\left(\frac{\cos\theta}{r^2}\right) 2\pi R = \frac{\mu_0 I}{2}\frac{R^2}{(R^2+z^2)^{\frac{3}{2}}}$

$B(r) = \frac{\mu_0}{4\pi}\int \frac{K(r')\times \hat{r}}{r^2}\,da'$

can use again as all horiz. components cancel, leaving the above statement still true

$$B(0) = \frac{\mu_0}{4\pi}\int \frac{K(r')\times(-R\hat{r})}{R^3}\,da' = \frac{\mu_0}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\pi} \frac{\sigma\omega R^2\cos\phi'\,\hat{\phi}}{R^3}R^2\cos\phi'\,d\theta'\,d\phi'$$

$$= \frac{\mu_0}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{2\pi} \frac{\sigma\omega R^2\cos\phi\,(\cos\phi'\,\hat{z} - \sin\phi\,\hat{s})R^2\cos\phi'\,d\theta'\,d\phi'}{R^3}$$

$$= \frac{\mu_0\sigma\omega R}{2}\left(\frac{4}{3}\hat{z} - 0\hat{s}\right) = \boxed{\frac{2}{3}\mu_0\sigma\omega R\hat{z}}$$

$\boxed{5.16}$

$B_{in} = \mu_0 n I$, B_{out} in $\hat{z}$ dir, $B_{in} = -\hat{z}$

(i) $B = \mu_0 I (n_2 - n_1)\hat{z}$, (ii) $B = \mu_0 I n_2 \hat{z}$, $B_{outside} = 0$

$\boxed{5.19}$ $I_{enc} = \int_S J \cdot da$

Should not matter at all which surface is used, $J \cdot da$ is surface independent.

$\nabla \cdot J = 0$

$\boxed{5.21}$ $1.46 \rightarrow \nabla \cdot (\nabla \times v) = 0$

Ampere's Law → $\nabla \times B = \mu_0 \bar{J}$, with continuity eq.

$\nabla \cdot (\nabla \times B) = \mu_0 \nabla \cdot J = -\mu_0 \frac{\partial \rho}{\partial t}$ $\times$ $\underbrace{\nabla \cdot (\nabla \times v) = 0}_{\text{inconsistent}}$

unless $\rho = \overset{\text{a constant}}{K}$, which is only ok in magnetostatics

All other Maxwell Eqtn's are ok, / Not problematic in this regard

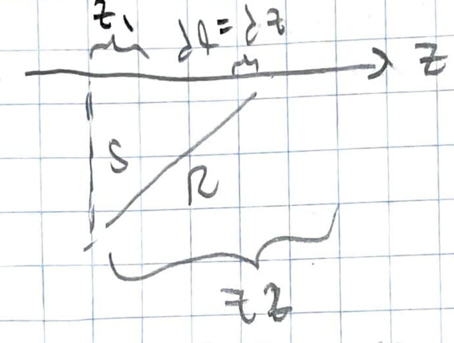
5.23

$5.66 \rightarrow \quad A = \frac{\mu_0}{4\pi}\int \frac{1}{R}\, dl' = \frac{\mu_0 I}{4\pi}\int \frac{1}{R}\, dl' \quad A = \frac{\mu_0}{4\pi}\int \frac{K}{R}\, da'$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\left(\frac{\cos^2\theta}{s^2}\right)\left(\frac{s}{\cos^2\theta}\right)\cos\theta\, d\theta = \frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2}\cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{2\pi s}\left(\sin\theta_2 - \sin\theta_1\right)$$

$$A = \frac{\mu_0}{4\pi}\int \frac{I\hat{z}}{R}\, dz = \frac{\mu_0 I}{4\pi}\hat{z}\int_{z_1}^{z_2}\frac{dz}{\sqrt{z^2+s^2}}$$



$$= \frac{\mu_0 I}{4\pi}\ln\left[\frac{z_2+\sqrt{(z_2)^2+s^2}}{z_1+\sqrt{(z_1)^2+s^2}}\right]\hat{z}$$

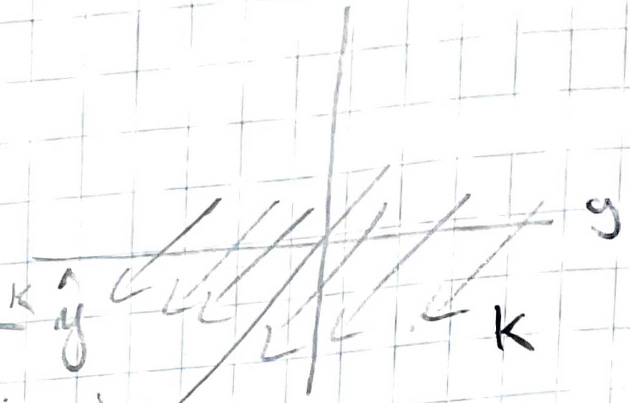
$$B = \nabla \times A = \frac{\partial A}{\partial s}\hat{\phi} = -\frac{\mu_0 I}{4\pi}\left[\frac{1}{z_2+\sqrt{(z_2)^2+s^2}}\,\frac{s}{\sqrt{(z_2)^2+s^2}} - \frac{1}{z_1+\sqrt{(z_1)^2+s^2}}\,\frac{s}{\sqrt{z_1^2+s^2}}\right]\hat{\phi}$$

$$= -\frac{\mu_0 I s}{4\pi}\left(-\frac{1}{s^2}\right)\left[\frac{z_2}{\sqrt{(z_2)^2+s^2}} - 1 - \frac{z_1}{\sqrt{(z_1)^2+s^2}} + 1\right]\hat{\phi} = \frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{\sqrt{z_2^2+s^2}} - \frac{z_1}{\sqrt{z_1^2+s^2}}\right]$$

$\sin\theta_1 = \frac{z_1}{\sqrt{z_1^2+s^2}}$ & $\sin\theta_2 = \frac{z_2}{\sqrt{z_2^2+s^2}}$)

$$\boxed{\frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right)\hat{\phi}}$$

$\boxed{5.27}$



$K = K\hat{x}$, $B = \pm \frac{\mu_0 K}{2}\hat{y}$

A // to K, only depends on x

z)

$A = A(z)\hat{x}$

$$B = \nabla \times A = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A(z) & 0 & 0 \end{vmatrix} = A(z)\frac{\partial}{\partial z}\hat{y}$$

or $\boxed{\frac{\partial A}{\partial z}\hat{y} = \pm \frac{\mu_0 K}{2}\hat{y}}$