Electromagnetic Theory Quiz 1
Dane Goodman 10/30/20

1. a) ① $\vec{B} = \langle x_1, y_1, z_1 \rangle$
$\vec{C} = \langle x_2, y_2, z_2 \rangle$
$\vec{B} + \vec{C} = \langle x_1 + x_2, y_1 + y_2, z_1 + z_2 \rangle$
$a(\vec{B} + \vec{C}) = a\langle x_1 + x_2, y_1 + y_2, z_1 + z_2 \rangle$
$= \langle a(x_1 + x_2), a(y_1 + y_2), a(z_1 + z_2) \rangle$
② $a\vec{B} + a\vec{C} = \langle ax_1, ay_1, az_1 \rangle + \langle ax_2, ay_2, az_2 \rangle$
$= \langle ax_1 + ax_2, ay_1 + ay_2, az_1 + az_2 \rangle$
$= \langle a(x_1 + x_2), a(y_1 + y_2), a(z_1 + z_2) \rangle$

$\boxed{① = ②, \text{ therefore } a(\vec{B} + \vec{C}) = a\vec{B} + a\vec{C}}$

b) What is wrong with this function is that you can't multiply a gradient to another gradient. Gradients only work with cross product and dot product, can't add a gradient and a function.

c) i) $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}$ $\vec{F}(x,y) = x\hat{i} + y\hat{j}$

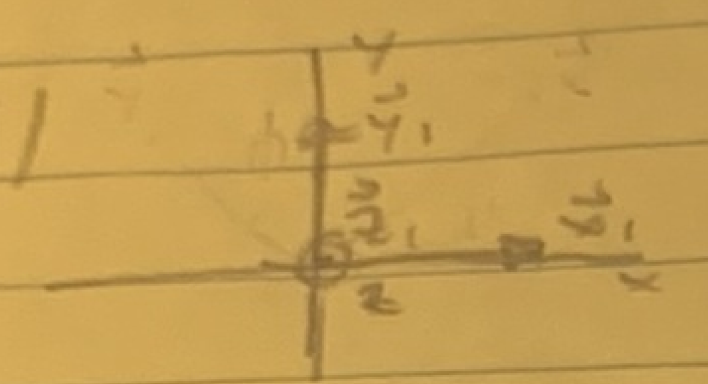
$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & 0 \end{vmatrix} = \left(\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(y)\right)\hat{i} - \left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}(x)\right) + \left(\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right)$$

$= 0\hat{i} - 0\hat{j} + 0\hat{k} = \boxed{0}$ $\boxed{\text{therefore no curl}}$

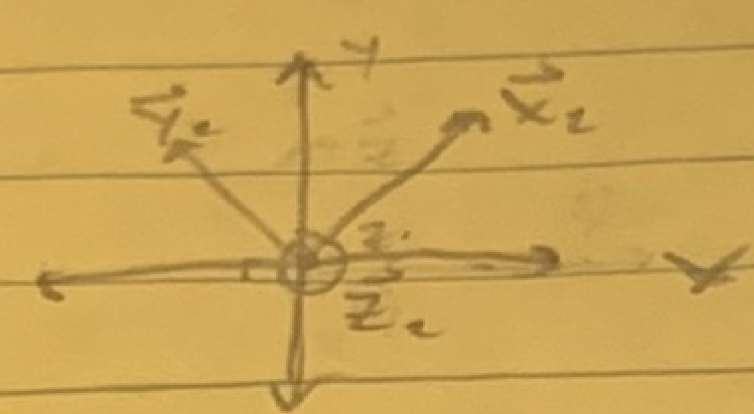
ii) $\int_0^{2\pi} \cos(x) + \sin(x)\, dx = \sin(x)\Big|_0^{2\pi} - \cos(x)\Big|_0^{2\pi}$
$= \boxed{0}$

iii) $\vec{F}(x,y) = x\hat{i} + y\hat{j}$ $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}$
$\nabla \cdot \vec{F} = \frac{\partial}{\partial x}(x) + \frac{\partial}{\partial y}(y) = 1 + 1 = \boxed{2}$

2. a) i)



ii)



iii) magnitude $= \sqrt{x^2+y^2+z^2}$ → use unit vector $(x=1,\ y=1,\ z=1)$

$\sqrt{\vec{x}_1 + \vec{y}_1 + \vec{z}_1} = \sqrt{1^2+1^2+1^2} = \sqrt{3}$ $\qquad \sqrt{3} = \sqrt{3}$ ✓

$\sqrt{\vec{x}_2 + \vec{y}_2 + \vec{z}_2} = \sqrt{1^2+1^2+1^2} = \sqrt{3}$

$\boxed{\text{It has same magnitude}}$

3. a) For a closed surface, $\oint (\nabla \times \vec{v}) \cdot d\vec{a} = 0$, which means it's a point, hence the coordinates are $(0,0,0)$ since it's centered at the origin.

4. a) i) $\int_{-\infty}^{\infty} (f(x) * g(x))\, \delta(x)\, dx$ if $f(x) = \cos(x) / g(x) = \sin(x)$

$\int \delta(x)\, dx = 0$

$$= \frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)}$$

$$= \frac{\cos(0) - \sin(0)}{\cos(0) + \sin(0)} = \frac{1-0}{1+0} = \frac{1}{1} = \boxed{1}$$

ii) if $f(x) = \cosh(x) / g(x) = \sinh(x)$

$$= \frac{\cosh(x) - \sinh(x)}{\cosh(x) + \sinh(x)} = \frac{\cosh(0) - \sinh(0)}{\cosh(0) + \sinh(0)} = \frac{1-0}{1+0} = \boxed{1}$$

iii) if $f(x) = a + ax + ax^2 + \dots$ and $g(x) = b + bx + bx^2 + \dots$

$$= \frac{(a+ax+ax^2+\dots) - (b+bx+bx^2+\dots)}{(a+ax+ax^2+\dots) + (b+bx+bx^2+\dots)} = \boxed{\frac{a-b}{a+b}}$$