Solutions for Homework 6

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1 Problem 6.3

Find the force of attraction between two magnetic dipoles, $\mathbf{m}_1$ and $\mathbf{m}_2$, oriented as shown in Fig. 6.7, a distance r apart, (a) using Equation 6.2, and (b) using Equation 6.3.

The dipole moments are parallel to each other, and separated by a distance r.

- (a) Equation 6.2 says that $F = 2\pi IRB\cos\theta$. Let $B\cos\theta = \mathbf{B}\cdot\hat{\mathbf{y}}$ (Fig. 1), and The $\mathbf{B}$-field of $\mathbf{m}_1$ is

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{3(\mathbf{m}_1\cdot\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{m}_1}{r^3} \tag{1}$$

So multiplying both sides by $\hat{\mathbf{y}}$ $(\mathbf{B}\cdot\hat{\mathbf{y}})$ gives

$$B\cos\theta = \frac{\mu_0}{4\pi}\frac{3(\mathbf{m}_1\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}\cdot\hat{\mathbf{y}} - \mathbf{m}_1\cdot\hat{\mathbf{y}}}{r^3} = \frac{\mu_0}{4\pi}\frac{3m_1\cos\phi\sin\phi}{r^3} \tag{2}$$

Using $m_2 = \pi IR^2$, and Fig. 1, we can show that

$$F = \frac{3\mu_0}{2\pi}m_1m_2\frac{\sqrt{R^2-r^2}}{r^5} \rightarrow \frac{3\mu_0}{2\pi}m_1m_2\frac{1}{r^4} \tag{3}$$

In the last step, we have applied $R \ll r$ for microscopic dipoles.

- (b) Using Equation 6.3:

$$\mathbf{F} = \nabla(\mathbf{m}_2\cdot\mathbf{B}) = (\mathbf{m}_2\cdot\nabla)\mathbf{B} = m_2\frac{d}{dz}\left(\frac{\mu_0}{4\pi}\frac{1}{z^3}(3(\mathbf{m}_1\cdot\hat{\mathbf{z}})\hat{\mathbf{z}} - \mathbf{m_1})\right) \tag{4}$$

The dipole moments are constant vectors in the z-direction. Taking the derivative with respect to z, we find

$$\mathbf{F} = -\frac{3\mu_0}{2\pi}\frac{m_1m_2}{r^4}\hat{\mathbf{z}} \tag{5}$$

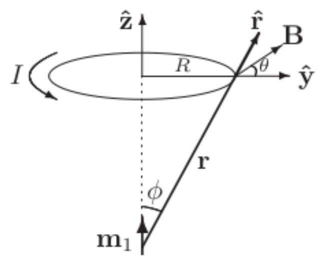

Figure 1: Diagram for Problem 6.3 (a).

2 Problem 6.7

An infinitely long circular cylinder carries a uniform magnetization $\mathbf{M}$ parallel to its axis. Find the magnetic field (due to $\mathbf{M}$) inside and outside the cylinder.

The curl of a constant magnetization vector is zero, but there is a bound surface current:

$$\mathbf{K}_b = \nabla\times\hat{\mathbf{n}} = M\hat{\phi} \tag{6}$$

But if this surface current is strictly circumferential, then that's a solenoid. The field outside the solenoid is zero, and the field inside should be proportional to M. Using the standard solenoid formula:

$$\mathbf{B} = \mu_0K_b\hat{\mathbf{z}} = \mu_0\mathbf{M} \tag{7}$$

3 Problem 6.16

A coaxial cable consists of two very long cylindrical tubes, separated by linear insulating material of magnetic susceptibility, χ_m. A current I flows down the inner conductor and returns along the outer one; in each case, the current distributes itself uniformly over the surface. Find the magnetic field in the region between the tubes.

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{\text{f,enc}} \tag{8}$$

$$\mathbf{H} = \frac{I}{2\pi s}\hat{\phi} \tag{9}$$

$$\mathbf{B} = \mu_0(1 + \chi_m)\mathbf{H} \tag{10}$$

$$\mathbf{B} = \frac{\mu_0(1 + \chi_m)I}{2\pi s}\hat{\phi} \tag{11}$$

Using the standard formulas for bound current density and surface currents, we find that there is no bound **J**, but there are bound surface currents. Using Ampère's Law with these currents gives the same field.