$\check{4}, \check{7}, \check{11}, \check{12}, \check{16}, \check{19}, \check{21}, 23, \check{27}$, example 5.12

Ian Watanabe
ch.5 HW

$\boxed{4}$ Top: $F = IaB = Iak\left(\frac{a}{2}\right) = Ik\frac{a^2}{2}$ which is up

Bottom: $F = IaB = -Iak\left(\frac{a}{2}\right) = -Ik\frac{a^2}{2}$ which is up

$\therefore \quad F_{net} = 2\left(Ik\frac{a^2}{2}\right) = \boxed{Ika^2\,\hat{z}}$

$\boxed{7}$ $\frac{dp}{dt} = \frac{d}{dt}\int_V \rho r\,d\tau = \int \frac{\partial \rho}{\partial t} r\,d\tau = -\int (\nabla\cdot J) r\,d\tau = \int_V \nabla \cancel{(r J)}^{0}\,d\tau - \int_V J\,d\tau = -\int_V J\,d\tau$

$\nabla\cdot(xJ) = x(\nabla\cdot J) + J\cdot(\nabla x) \Rightarrow \nabla\cdot(xJ) = x(\nabla\cdot J) + J_x$

$\therefore \int_V (\nabla\cdot J) r\,d\tau = -\int_V J\,d\tau$

$\therefore \frac{dp}{dt} = \int J\,d\tau$

$\boxed{11}$ $I = nI\,dz \qquad z = a\cot\theta \Rightarrow dz = \frac{-a}{\sin^2\theta}d\theta$

$B = \frac{\mu_0 n I}{2}\int \frac{a^2}{(a^2+z^2)^{3/2}}dz$

$\frac{1}{(a^2+z^2)^{3/2}} = \frac{\sin^3\theta}{a^3}$

$\Rightarrow B = \frac{\mu_0 n I}{2}\int \frac{(-a)\,a^2\sin^3\theta}{a^3\sin^2\theta}d\theta = \frac{-\mu_0 n I}{2}\int \sin\theta\,d\theta$

$= \frac{\mu_0 n I}{2}\Big[\cos\theta\Big]_{\theta_1}^{\theta_2} = \boxed{\frac{\mu_0 n I}{2}(\cos\theta_2 - \cos\theta_1)}$

infinite solenoid

$\theta_2 = 0 \quad \theta_1 = \pi \quad \therefore \quad B = \frac{\mu_0 n I}{2}(\overset{1}{\cos(0)} - \overset{-1}{\cos(\pi)}) = B = \frac{\mu_0 n I}{2}(2) = \boxed{\mu_0 n I}$

$\boxed{12}$ $dB = \frac{\mu_0\,dI}{2}\underbrace{\left[\frac{(R\sin\theta)^2}{[(R\sin\theta)^2 + (R\cos\theta)^2]^{3/2}}\right]}_{\frac{R^2\sin^2\theta}{(R^2)^{3/2}} = \frac{\sin^2\theta}{R}} = \frac{\mu_0\,dI}{2R}\sin^2\theta$

$dI = KR\,d\theta$

$K = \sigma v$

$\sigma = \frac{Q}{4\pi R^2}$

$v = \omega R\sin\theta$

$dI = \omega R\sin\theta\left(\frac{Q}{4\pi R^2}\right)R\,d\theta = \frac{Q\omega\sin\theta}{4\pi}d\theta$

$u = \cos\theta$
$du = -\sin\theta\,d\theta$

$B = \frac{\mu_0 Q\omega}{2R\,4\pi}\int_0^\pi \sin^3\theta\,d\theta = \frac{\mu_0 Q\omega}{8\pi R}\left[\frac{1}{3}\cos^3\theta - \cos(\theta)\right]_0^\pi$

$\int \sin\theta(\sin^2\theta)$

$\int \sin\theta(1-\cos^2\theta)$

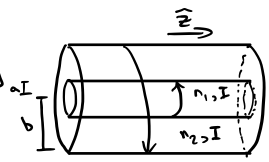
$\sin\theta - \sin\theta\cos^2\theta$

$\int \sin\theta + \int u^2\,du$

$-\cos\theta + \frac{1}{3}\cos^3\theta$

$= \frac{\mu_0 Q\omega}{8\pi R}\left[-\frac{1}{3} - 1\right] = \frac{-\mu_0 Q\omega}{2\,8\pi R}\cdot\frac{4}{3} = \boxed{\frac{-\mu_0 Q\omega}{6\pi R}}$

16



$\vec{B} = \mu_0 n I \hat{z}$

$\boxed{\vec{B} = 0}$ when outside both solenoids

$\boxed{\vec{B} = \mu_0 n_2 I \hat{z}}$ between both solenoids

$B = \mu_0 n_2 I \hat{z} + \mu_0 n_1 I(-\hat{z}) = \boxed{\mu_0 I (n_2 - n_1)\hat{z}}$ Inside both

19 $\nabla \times \vec{B} = \mu_0 \vec{J}$

$$I_{enc} = \int_s \vec{J} \cdot d\vec{a} = \frac{1}{\mu_0} \int_s (\nabla \times \vec{B}) \cdot d\vec{a} = \frac{1}{\mu_0} \oint \vec{B} \cdot d\vec{l}$$

It doesn't matter what the surface is because the integral is a line integral over a specified boundary on a surface and that surface is independent of this.

21 Amphere's Law: $\nabla \times B = \mu_0 J$

$\nabla \cdot (\nabla \times B) = \mu_0 \nabla \cdot J = -\mu_0 \frac{\partial \rho}{\partial t}$ Unless ρ is a constant then this would be inconsistent with the divergence of a curl is zero.

The other 2 maxwell equations are fine and there's no inconsistencies.

23 $A = \frac{\mu_0}{4\pi} \int \frac{I \hat{z}}{\mu} dz = \frac{\mu_0 I}{4\pi} \hat{z} \int_{z_1}^{z_2} \frac{dz}{\sqrt{z^2+s^2}} = \frac{\mu_0 I}{4\pi} \hat{z} \left[\ln(z + \sqrt{z^2+s^2}) \right]_{z_1}^{z_2} = \frac{\mu_0 I}{4\pi} \ln \left[\frac{z_2 + \sqrt{z_2^2+s^2}}{z_1 + \sqrt{z_1^2+s^2}} \right] \hat{z}$

$$B = \nabla \times A = \frac{-\partial A}{\partial s} \hat{\phi} = \frac{-\mu_0 I}{4\pi} \left[\frac{s}{\sqrt{z_2^2+s^2}} \cdot \frac{1}{z_2 + \sqrt{z_2^2+s^2}} - \frac{s}{\sqrt{z_1^2+s^2}} \cdot \frac{1}{z_2 + \sqrt{z_1^2+s^2}} \right] \hat{\phi}$$

$$= \frac{-\mu_0 I s}{4\pi} \left[\frac{z_2 - \sqrt{z_2^2+s^2}}{z_2^2 - (z_2^2+s^2)} \cdot \frac{1}{\sqrt{z_2^2+s^2}} - \frac{z_1 - \sqrt{z_1^2+s^2}}{z_1^2 - (z_1^2+s^2)} \cdot \frac{1}{\sqrt{z_1^2+s^2}} \right] \hat{\phi}$$

$$= \frac{-\mu_0 I s}{4\pi} \cdot \frac{-1}{s^2} \left(\frac{z_2}{\sqrt{z_2^2+s^2}} - 1 - \frac{z_1}{\sqrt{z_1^2+s^2}} + 1 \right) \hat{\phi} = \frac{\mu_0 I}{4\pi s} \left(\frac{z_2}{\sqrt{z_2^2+s^2}} - \frac{z_1}{\sqrt{z_1^2+s^2}} \right) \hat{\phi}$$

$\sin\theta_1 = \frac{z_1}{\sqrt{z_1^2+s^2}}$ $\sin\theta_2 = \frac{z_2}{\sqrt{z_2^2+s^2}}$

$$= \boxed{\frac{\mu_0 I}{4\pi s} (\sin\theta_2 - \sin\theta_1) \hat{\phi}}$$

$\boxed{27}$ $K = k\hat{x}$ $\quad \vec{B} = \begin{cases} \frac{\mu_0}{2} K\hat{y} & z<0 \\ -\frac{\mu_0}{2} K\hat{y} & z>0 \end{cases}$

$\nabla \times \vec{A} = \frac{\partial A_x}{\partial z}\hat{y} - \frac{\partial A_x}{\partial y}\hat{z} = \vec{B}$

Above: $\frac{\partial A_x}{\partial z} = \frac{-\mu_0 K}{2} \Rightarrow A_x = \frac{-\mu_0 K z}{2}$

Below: $\frac{\partial A_z}{\partial z} = \frac{\mu_0 K}{2} \Rightarrow A_x = \frac{\mu_0 K z}{2}$

$\boxed{\vec{A} = \frac{-\mu_0 K z}{2}\hat{x}}$

$\boxed{\text{Ex } 5.12}$

$\oint \vec{A}\cdot dl = \int (\nabla \times A)\cdot d\vec{a} = \int \vec{B}\cdot d\vec{a} = \Phi_B$

$\int \vec{B}\cdot dl = \mu_0 I_{enc}$ $\quad \vec{B} = \mu_0 N I$

$\oint \vec{A}\cdot dl = A\,2\pi r$

$\int \vec{B}\cdot d\vec{a} = \mu_0 N I(\pi r^2)$

$\boxed{A = \frac{\mu_0 N I r}{2}\hat{\phi} \text{ for } r<R}$

$A(2\pi r) = \mu_0 N I \pi R^2$

$\boxed{A = \frac{\mu_0 N I}{2}\frac{R^2}{r}\hat{\phi} \text{ for } r>R}$