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EMT HW #6

3.6) a) using Eq 6.2

$$F = 2\pi IRB\cos\theta$$

$$B_1 = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(m_1\cdot\hat{r})\hat{r} - m_1\right]$$

$$B\cdot\hat{y} = |B||\hat{y}|\cos\theta = B\cos\theta$$

$$B\cos\theta = B\cdot\hat{y} = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3\underbrace{(m_1\cdot\hat{r})}_{m_1\cos\phi}\underbrace{\hat{r}\cdot\hat{y}}_{\sin\phi} - \underbrace{m_1\cdot\hat{y}}_{0}\right]$$

So

$$B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}3m_1\cos\phi\sin\phi$$

Plug into 6.2

$$F = 2\pi IR\left[\frac{\mu_0}{4\pi}\frac{1}{r^3}3m_1\cos\phi\sin\phi\right]$$

Replace $\sin\phi = R/r$

$\cos\phi = \frac{\sqrt{r^2-R^2}}{r}$ & Reduce Eq to:

$$F = 3\frac{\mu_0}{2\pi}I\pi R^2\frac{m_1\sqrt{r^2-R^2}}{r^5}, \quad \underline{I\pi R^2 = m_2}$$

Simplify to

$$\boxed{F = \frac{3\mu_0}{2\pi}\frac{m_1m_2}{r^4}}$$

b) using Eq 6.3
$F = \nabla(m \cdot B)$

$$\nabla(m_1 \cdot B) = (m_1 \cdot \nabla) B + (B \cdot \nabla) m_1 + m_1 \times (\nabla \times B) + B \times (\nabla \times m_1)$$
$$= (m_1 \cdot \nabla) B + 0 + 0 + 0$$

By this then $F = (m_2 \cdot \nabla) B$

$B = \frac{\mu_0}{4\pi} \frac{1}{z^2} [2m_1]$, solving for radial part of z axis

Sub $\frac{\mu_0}{4\pi} \frac{1}{z^2} [2m_1] = B$, $\nabla = \hat{x} \frac{d}{dx} + \hat{y} \frac{d}{dy} + \hat{z} \frac{d}{dz}$

into $F = (m_2 \cdot \nabla) B$

we then get

$\boxed{F = -\frac{3\mu_0}{2\pi} \frac{m_1 m_2}{z^4} \hat{z}}$ is the mag field between the two dipoles.

6.7)

For uniform mag bound current will be 0 so

$$\vec{J}_b = \vec{\nabla} \times \vec{m} = 0$$

El surface current density

$$\vec{K}_b = \vec{m} \times \hat{n} \Rightarrow m \sin\theta \, \hat{\phi}$$

$$\boxed{\vec{K}_b = m \hat{\phi}} \qquad (\sin(90) = 1)$$

Cylinder now acts like a solenoid, mag field outside solenoid is

1) $\boxed{\vec{B}_{out} = 0}$

Inside:

2) $\underline{\vec{B} = \mu_0 n I \hat{z}}$

n = # of turns per unit length

Surface current density

$$k_b = n \frac{dI}{dl} \Rightarrow n \, dI = k_b \, dl_\perp$$

$dl_\perp$ = perp. length to the current

For unit length: $nI = \int dI \Rightarrow \int k_b \, dl_\perp$

$$\Rightarrow \int m \, dl$$

$$\Rightarrow nI = m$$

From 2)

we get $\boxed{\vec{B} = \mu_0 m \hat{z}}$ to be the magnetic field inside the cylinder, 6.16) Ampere's Law

$\oint H \cdot dl = I \Rightarrow H = I/2\pi s$

$\Rightarrow B = \mu_0 (1+\chi_m) \frac{1}{2\pi s}$

$M = \chi_m H$

$\Rightarrow M = \frac{I \chi_m}{2\pi s} \phi$

Bound Current $\Rightarrow \vec{J} = \nabla \times \vec{m} = \frac{1}{s} \frac{d}{ds} \left(s \frac{\chi_m I}{2\pi s} \right) \hat{z} = 0$

$\vec{K} = M \times \hat{n} = \begin{cases} \frac{\chi_m I}{2\pi a} \hat{z}, & at\ s=a \\ -\frac{\chi_m I}{2\pi b} \hat{z}, & at\ s=b \end{cases}$

Amperian loop total enclosed current

$I + \frac{\chi_m I}{2\pi a} 2\pi a = (1+\chi_m) I$

$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \Rightarrow B 2\pi s - \mu_0 I (1+\chi_m)$

$\boxed{\Rightarrow B = \frac{\mu_0 I (1+\chi_m)}{2\pi s} \hat{\phi}}$