chapter 6 HW
3, 7, 16

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Phys 330

3) $F = 2\pi I R B\cos\theta$

$F = \nabla(m \cdot B)$

$B\cos\theta = \frac{\mu_0}{4\pi}\left[\frac{3(m_1\cdot\hat{r})(\hat{r}\cdot\hat{y})-(m_1\cdot\hat{y})}{r^3}\right]$

$m_1\cdot\hat{y} = 0 \qquad m_1\cdot\hat{r} = m_1\cos\theta$

$\hat{r}\cdot\hat{y} = \sin\phi$

$B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3m_1\sin\phi\cos\phi\right]$

$\cos\phi = \frac{\sqrt{r^2-R^2}}{r}$

$\sin\phi = \frac{R}{r}$

$F = 2\pi I R\left[\frac{\mu_0}{4\pi}\frac{1}{r^3}(3m_1\sin\phi\cos\phi)\right]$

$F = \cancel{2}\pi I R^2\left[\frac{\mu_0}{{}_2\cancel{4}\pi}\cdot 3m_1\frac{\sqrt{r^2-R^2}}{r^5}\right]$

$m_2 = IR^2\pi$

$F = (m_2 m_1)\frac{3\mu_0}{2\pi}\frac{\sqrt{r^2-R^2}}{r^5}$

when $r > R$: $F = \frac{3\mu_0}{2\pi}\frac{\sqrt{r^2}}{r^5}m_1m_2 = \boxed{\frac{3\mu_0}{2\pi}\frac{m_1m_2}{r^4} = F}$

b) $F = \nabla(m_2\cdot B)$

$3(m_1\cdot\hat{r})\hat{r} - m_1 = 2m_1$

$= (m_2\cdot\nabla)B$

$= m_2\frac{d}{dr}\left[\frac{\mu_0}{4\pi}\frac{1}{r^3}(3(m_1\cdot\hat{r})\hat{r} - m_1)\right]$

$= m_2 m_1\cdot\frac{2\mu_0}{4\pi}\cdot\frac{d}{dr}\left(\frac{1}{r^3}\right) = m_1m_2\cdot\frac{\mu_0}{2\pi}\cdot 3\left(\frac{-1}{z^4}\right)$

$F = \frac{-3\mu_0}{2\pi}\frac{m_1m_2}{r^4}\hat{z}$

7 $J_b = \nabla \times M = 0$
$K_b = M \times \hat{n} = M\hat{\phi}$

Since $K_b = M\hat{\phi}$, it means that the field is just a surface current around an angle. This means that the field outside is zero since the example is just a solenoid.

$$B = \mu_0 K_b = \mu_0 \cdot M\hat{\phi} \quad \therefore \quad \boxed{B = \mu_0 \mathbf{M}}$$

16 $\oint H \cdot dl = I_{enclosed}$

$$H = \frac{1}{2\pi s}\hat{\phi} \qquad B = \mu_0(1+\chi_m)H = \mu_0(1+\chi_m)\frac{1}{2\pi s}\hat{\phi}$$

$$M = \chi_m H = \frac{\chi_m I}{2\pi s}\hat{\phi}$$

$$J_b = \nabla \times M = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\chi_m I}{2\pi s}\right)\hat{z} = 0 \qquad K_b = M \times \hat{n} = \frac{\chi_m I}{2\pi a}\hat{z} \text{ when } s=a$$

$$-\frac{\chi_m I}{2\pi b}\hat{z} \text{ when } s=b$$

$$I_{TOT} = I + \frac{\chi_m I}{2\pi a}(2\pi a)$$
$$= I(1+\chi_m)$$

$$\oint B\, dl = \mu_0 I_{enc} = \mu_0(1+\chi_m)I$$
$$\Rightarrow \boxed{B = \frac{\mu_0(1+\chi_m)I}{2\pi s}\hat{\phi}}$$