Did them out of order, sorry!.

HW #2 ✓ 2.5, ✓ 2.6, ✓ 2.9, ✓ 2.12, ✓ 2.16, ✓ 2.18, ✓ 2.25, ✓ 2.29

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2.12) Use Gauss's law to find the electric field inside a uniformly charged solid sphere (charge density ρ).

area of sphere $= 4\pi r^2$

$$\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{enc} \qquad da = 4\pi r^2$$

$$E\, 4\pi r^2 = \frac{1}{\varepsilon_0} Q_{enc} \qquad Q_{enc} = \text{volume} \cdot \rho = \frac{4}{3}\pi r^3 \rho$$

$$E\, 4\pi r^2 = \frac{1}{\varepsilon_0} \frac{4}{3}\pi r^3 \rho$$

$$\boxed{E = \frac{1}{3\varepsilon_0} r \rho \hat{r}}$$

2.16) Coaxial cable carries uniform volume charge density ρ on the inner cylinder ($r = a$) and a uniform surface charge density on outer cylinder shell ($r = b$). This surface charge is negative and is of just the right magnitude that the cable as a whole is electrically neutral. Find the electric field in each of the three regions (i) inside the inner cylinder ($s < a$), (ii) between the cylinders ($a < s < b$) (iii) outside the cable ($s > b$). Plot $|E|$ as a function of s.

(i) $\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$ $\qquad da = 2\pi s \cdot l$ cylinder minus ends

$$E\, 2\pi s l = \frac{1}{\varepsilon_0} \pi s^2 l \rho \qquad Q_{enc} = \text{volume} \cdot \rho = \pi s^2 l \cdot \rho$$

$$\boxed{E = \frac{1}{2\varepsilon_0} s \rho \hat{s}}$$

(ii) $\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$

$$E\, 2\pi s l = \frac{1}{\varepsilon_0} \pi a^2 l \rho$$

$$\boxed{E = \frac{1}{2\varepsilon_0 s} a^2 \rho \hat{s}}$$

(iii) outer cylinder is same as (i) but negative

$$\vec{E} = E_{\text{inside cyl}} + E_{\text{outside cyl}}$$

$$= \frac{1}{2\varepsilon_0} s\rho\hat{s} + -\frac{1}{2\varepsilon_0} s\rho\hat{s}$$

$$\boxed{\vec{E} = 0}$$

2.18) Two spheres, each of radius R and carrying uniform volume charge densities $+\rho$ and $-\rho$, are placed so that they partially overlap. Call the vector from the positive center to the negative center d. Show that the field in the region of overlap is constant and find its value.

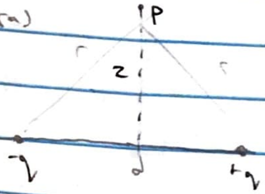
For one sphere $E = \frac{1}{3\varepsilon_0} \rho r \hat{r}$

$E_+ = \frac{1}{3\varepsilon_0} \rho r$ $\quad$ $E_{tot} = \frac{\rho}{3\varepsilon_0}(r_+ - r_-)$ $\quad$ $d = r_+ - r_-$

$E_- = \frac{-1}{3\varepsilon_0} \rho r$ $\quad$ $\boxed{E_{tot} = \frac{\rho}{3\varepsilon_0} d}$ all constants so E is constant

2.25) Find the potential at a distance z above the center of the charge distributions. In each case, compute $E = -\nabla V$. Suppose that we changed the right-hand charge in Fig. 2.34a to $-q$; what then is the potential at P? What field does that suggest?

(a)



$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{2q}{\sqrt{z^2+\left(\frac{d}{2}\right)^2}}}$$

$r = \sqrt{z^2+\left(\frac{d}{2}\right)^2}$

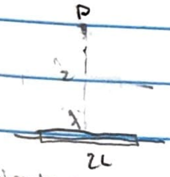
if $+q$ is $-q$ then $V = 0$

Only know V on x-axis so don't know E

$$E = -\nabla V$$

$$E = -\nabla \frac{1}{4\pi\varepsilon_0}\frac{2q}{\sqrt{z^2+\left(\frac{d}{2}\right)^2}}$$

(b)

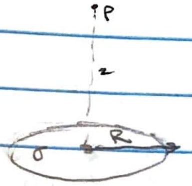


$dq = \lambda\, dx$

$$V = \frac{1}{4\pi\varepsilon_0}\int^{2L}\frac{\lambda\, dx}{\sqrt{z^2+\left(\frac{x}{2}\right)^2}}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{2L\lambda\, dx}{\sqrt{z^2+\left(\frac{x}{2}\right)^2}}}$$

(c)



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{R}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\pi R^2}{R}$$

$$\boxed{V = \frac{\sigma\pi R}{4\varepsilon_0}}$$

2.29). Check that Eq. 2.29 satisfies Poisson's equation, by applying the Laplacian and using Eq. 1.102.

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r')}{\mathscr{r}}\,d\tau'$$

$$\nabla^2\frac{1}{\mathscr{r}} = -4\pi\delta^3(\mathscr{r})$$

$$\nabla^2 V(r) = \nabla^2\left(\frac{1}{4\pi\varepsilon_0}\int\frac{\rho(r')}{\mathscr{r}}\,d\tau'\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\nabla^2\int\frac{\rho(r')}{\mathscr{r}}\,d\tau'$$

$$= \frac{1}{4\pi\varepsilon_0}\int\rho(r')\frac{\nabla^2}{\mathscr{r}}\,d\tau'$$

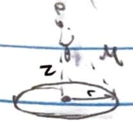
$$= \frac{1}{4\pi\varepsilon_0}\int\rho(r')\left(-4\pi\delta^3(\mathscr{r})\right)d\tau'$$ goes to ρ

$$= \frac{-4\pi}{4\pi\varepsilon_0}\int\rho(r')\,\delta^3(r-r')\,d\tau'$$

$$= -\frac{1}{\varepsilon_0}\tau$$

$$\boxed{= -\frac{1}{\varepsilon_0}\rho}$$ ← Poisson's equation

2.5) Find the electric field a distance z above the center of a circular loop of radius r that carries a uniform line charge λ.



$$E(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda(r')}{\mathscr{r}^2}\,\hat{\mathscr{r}}\cdot dl\,\hat{z} \qquad \int dl = \text{circumference} = 2\pi r$$

$$= \frac{1}{4\pi\epsilon_0}\,\frac{\lambda}{r^2+z^2}\left(\frac{z}{\sqrt{r^2+z^2}}\right)(2\pi r)\,\hat{z} \qquad \hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}}$$

$$\boxed{= \frac{\lambda z r}{2\epsilon(r^2+z^2)^{3/2}}\,\hat{z}} \qquad = \frac{z}{\sqrt{r^2+z^2}}$$

$\hat{\mathscr{r}} = \sqrt{r^2+z^2}$

$\mathscr{r}^2 = r^2+z^2$

2.6) Find the electric field at a distance z above the center of a flat circular disk of radius R that carries a uniform surface charge σ. What does it give in the limit $R\to\infty$? Check when $z >> R$.

disk is just collection of rings

ring (from 2.5) $\rightarrow \vec{E} = \frac{1}{4\pi\epsilon}\,\frac{2\lambda\pi r z}{(r^2+z^2)^{3/2}}\,\hat{z}$ $\qquad \lambda \rightarrow \sigma\, dr$

so for disk $\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^R \frac{2\pi r\sigma\, dr\, z}{(r^2+z^2)^{3/2}}$

$= \frac{2\pi\sigma z}{4\pi\epsilon}\int_0^R \frac{r\,dr}{(r^2+z^2)^{3/2}}$ ← I looked online how to integrate this

$$\boxed{\vec{E} = \frac{\sigma z}{2\epsilon}\left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right]\hat{z}}$$

When $R\to\infty$, $\frac{1}{\sqrt{R^2+z^2}} \rightarrow 0 \quad \therefore \vec{E} = \frac{\sigma z}{2\epsilon z}\hat{z} = \frac{\sigma}{2\epsilon}\hat{z}$

When $z >> R$, $\frac{1}{\sqrt{R^2+z^2}} \rightarrow \frac{1}{z} \quad \therefore \vec{E} = \frac{\sigma z}{2\epsilon}\left[\frac{1}{z} - \frac{1}{z}\right] = 0$

2.9) Suppose the electric field in some region is found to be $\vec{E} = kr^3\hat{r}$ in spherical coordinates (k is constant).

(a) Find the charge density ρ.

(b) Find the total charge contained in a sphere of radius R centered at the origin (in two different ways).

(a) $EA = \frac{q}{\epsilon_0}$ $\qquad$ A of sphere $= 4\pi r^2$

$(kr^3)(4\pi r^2) = \frac{q}{\epsilon_0}$ $\qquad$ volume $= \frac{4}{3}\pi r^3$

$q = 4\pi k r^5 \epsilon_0$

$\rho = \frac{q}{\text{volume}}$

$\rho = \frac{4\pi k r^5 \epsilon_0}{\frac{4}{3}\pi r^3}$

$$\boxed{\rho = 3kr^2\epsilon_0}$$

(b) 1st way: $E \cdot A = \frac{Q_{enc}}{\epsilon_0}$

$$(kR^3)(4\pi R^2) = \frac{Q_{enc}}{\epsilon_0}$$

$$\boxed{Q_{enc} = 4\pi k R^5 \epsilon_0}$$

2nd way:

$$Q_{enc} = \int \rho \, d\tau$$

$$= \int_0^R (3kr^2\epsilon_0)(4\pi r^2 \, dr)$$

$$= \int_0^R 12 k r^4 \epsilon_0 \, dr$$

$$= 12 k \epsilon_0 \pi \int_0^R r^4 \, dr$$

$$\boxed{= \frac{12}{5}\pi R^5 k \epsilon_0}$$