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Electromagnetic Theory Quiz #2

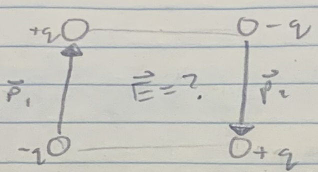
1. a) $\vec{\tau} = \vec{F} \times \vec{d}$

$\vec{p} = q\vec{d}$

$\vec{F}_+ = +q\vec{E}$

$\vec{F}_- = -q\vec{E}$

$\vec{\tau} = q\vec{E} \times \vec{d} \Rightarrow \boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

b) 

$\vec{E} = \Sigma pqd$

Should be zero because while one side has a certain electric field, the other dipole counters that because the charges are in opposite positions

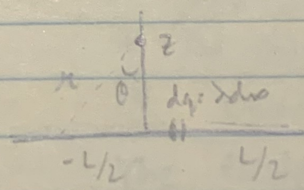
dipole $\Rightarrow \vec{p} = q\vec{d}$ $\quad E = k\frac{q}{r}$ $\quad q_1 = q_2$

$r_1 = r_2$ $\quad E_1 = E_2$

opposite directions : $\vec{E}_1 + \vec{E}_2 = 0$

$\vec{E}_3 + \vec{E}_4 = 0$

$\boxed{\vec{E}_{net} = 0}$



2. a) i) $Q = \lambda L$

$dq = \lambda dx$

$$\frac{k\lambda dx(z\hat{z} - x\hat{x})}{(z^2+x^2)(z^2+x^2)^{1/2}}$$

$$\int_{-L/2}^{L/2} \frac{k\lambda dx(z\hat{z} - x\hat{x})}{(z^2+x^2)^{3/2}}$$

$$E = k\lambda\left\{z\hat{z}\int_{-L/2}^{L/2} \frac{dx}{(z^2+x^2)^{3/2}} - \hat{x}\int_{-L/2}^{L/2} \frac{x\,dx}{(z^2+x^2)^{3/2}}\right.$$

...ric field $\vec{E}$. Perform the integral

$$V(\vec{r}) = -\int_{\infty}^{r} E(r')dr'$$

to find the potential formula for a point charge. [Answer: kq/r] (4) (5)

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2. a i cont)

$x = z\tan\theta$

$dx = z\sec^2\theta\, d\theta$

$x\,dx = z^2\tan\theta\sec^2\theta\, d\theta$

$$\vec{E} = k\lambda\left\{ z\hat{z}\int_{\theta_1}^{\theta_2} \frac{z\sec^2\theta\, d\theta}{(z^2+z^2\tan^2\theta)^{3/2}} = k\lambda\left(\frac{z^2\hat{z}}{z^3}\right)\int_{\theta_1}^{\theta_2} \frac{\sec^2\theta}{(1+\tan^2\theta)^{3/2}}\, d\theta\right.$$

$$= k\lambda\left(\frac{\hat{z}}{z}\right)\int_{\theta_1}^{\theta_2} \cos\theta\, d\theta$$

$$= \frac{k\lambda\hat{z}}{z}(\sin\theta_2 - \sin\theta_1) = \frac{k\lambda\hat{z}}{z}\left(\frac{1/2\,L}{(z^2+1/4\,L^2)^{1/2}} + \frac{1/2\,L}{(z^2+1/4\,L^2)^{1/2}}\right)$$

$$\boxed{\vec{E} = \frac{k\lambda\hat{z}L}{z}(z^2+1/4\,L^2)^{-1/2}}$$ if $L >> z$ the electric field is one

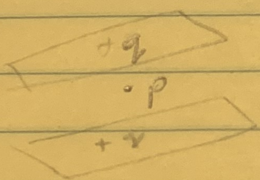
ii) Gauss' Law over line of charge $= \oint E\cdot da = \frac{1}{\varepsilon_0}Q_{enc}$

$Q = \lambda L$

$$E = \frac{1}{4\pi\varepsilon_0}Q\frac{\hat{z}}{z^2} = \frac{k\lambda L\hat{z}}{z^2}$$

$$\boxed{E = \frac{k\lambda L\hat{z}}{z^2}}$$

b) i)

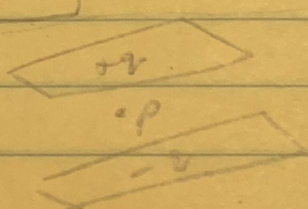


The field lines would be repelled from each other, creating a space of nothing in the center, meaning if the field point is there, there's no electric field

$(+q) - (+q) = 0$

$$\boxed{\frac{\sigma}{2\varepsilon_0} = 0}$$

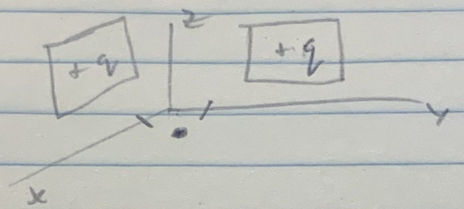
ii)



The electric field would go from the positive charged plate to the negative charged plate

$$\frac{2\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

2. b) iii) $\frac{\sigma}{2\varepsilon_0}\hat{i} + \frac{\sigma}{2\varepsilon_0}\hat{j} = \sqrt{\left(\frac{\sigma}{2\varepsilon_0}\right)^2 + \left(\frac{\sigma}{2\varepsilon_0}\right)^2} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$



3. a) $V(b) - V(a) = -\int_{O}^{b} E \cdot dl + \int_{O}^{a} E \cdot dl$

$\boxed{= -\int_{a}^{b} E \cdot dl}$

b) $V(\vec{r}) = \int_{\infty}^{r} E(r')dr' = -\frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r} \frac{q}{r'^2}dr' =$

$\frac{1}{4\pi\varepsilon_0}\frac{q}{r'}\Big|_{\infty}^{r} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \Rightarrow \boxed{\frac{kq}{r}}$

$k = \frac{1}{4\pi\varepsilon_0}$