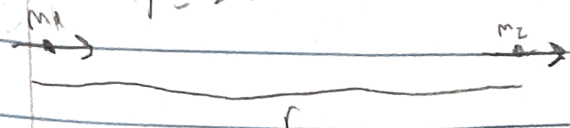
HW 6 Jackson Drums 6.3, 6.7, 6.16

6.3) $F = 2\pi I R B \cos\theta$, & $F = \nabla(m\cdot B)$




let $B\cos\theta = B\cdot\hat{y}$

& $B = \frac{\mu_0}{4\pi}\left(\frac{1}{r^3}\right)[3(m_1\cdot\hat{r})\hat{r} - m_1]$

a)

So then $B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left(3(m\cdot\hat{r})\hat{r}\cdot\hat{y} - \cancel{m_1\cdot\hat{y}}\right)$ goes to zero $\hat{r}\cdot\hat{y} = \sin\phi$, $m_1\cdot\hat{y} = m_1\cos\phi$

$\Rightarrow \frac{\mu_0}{4\pi}\frac{1}{r^3}(3m_1\cos\phi\sin\phi) = B\cos\theta$, $\sin\phi = \frac{R}{r}$ & $\cos\phi = \frac{\sqrt{r^2-R^2}}{r}$

So $F = \frac{\mu_0}{2}\left(\frac{3}{r^3}m_1\right)IR^2$ & if $m_2 = IR^2\pi$ then $F = \frac{3\mu_0}{2\pi r^3}m_1m_2\frac{\sqrt{r^2-R^2}}{r^5}$

~~Since $R \ll r$~~, for dipole $\boxed{F = \frac{3\mu_0}{2\pi}\frac{m_1 m_2}{r^4}}$

b) $r\to z$

$F = \nabla(m_2\cdot B) = (m_2\cdot\nabla)B = \left(m_2\frac{d}{dz}\right)B$

$F = \left(m_2\frac{d}{dz}\right)\left(\frac{\mu_0}{4\pi}\left(\frac{1}{z^3}\right)\underbrace{(3(m_1\cdot\hat{z})\hat{z} - m_1)}_{2m_1}\right)$, dipole constant in z direction, let $z\to r$

Such that $F = m_2\left(\frac{\mu_0}{2\pi}m_1\right)\hat{z}\cdot\frac{d}{dz}\left(\frac{1}{z^3}\right) = \boxed{\frac{-3\mu_0}{2\pi}\frac{m_1 m_2}{r^4}\hat{z}}$

6.7)

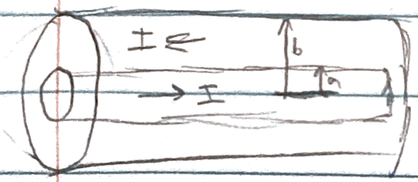

Well $\nabla\times M = 0$, since M is constant

there would be a bound surface currents s.t. $K_b = M\times\hat{n} = M\hat{\phi}$

bc it's a solenoid → the field outside is zero but inside is proportional to M. s.t. $B = \mu_0 K_b\hat{z} \Rightarrow \mu_0 M$

$\hat{z}\cdot\hat{\phi} = 1$

6.16)




Since $\oint H \cdot dl = I_{f\,enc}$, $H = \frac{I}{2\pi s}\hat{\phi}$

& $B = \mu_0(1+\chi_m)H$

So $\therefore$ $B = \frac{\mu_0(1+\chi_m)I}{2\pi s}\hat{\phi}$

$J_b = \nabla \times M \Rightarrow 0$, $K_b = M \times \hat{n} \Rightarrow \frac{\chi_m I}{2\pi s}\hat{z}$

$I + \frac{\chi_m I}{2\pi s} 2\pi s = (1+\chi_m) I$

So $\oint B \cdot dl = \mu_0 I_{enc} = \mu_0(1+\chi_m) I$ $\therefore$ B is the same