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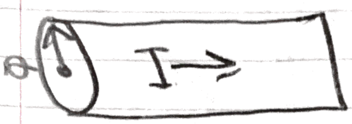
Homework 5 p 14, 16, 17, 19, 21, 23, 26

14) Steady current I flows down cylindrical wire of radius a. Find magnetic field inside/outside wire if

a) current is uniformly distributed over outside surface of wire

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$



we know the $I_{enc} = 0$

so $\oint \vec{B}\cdot d\vec{l} = \mu_0(0)$ $\quad \int d\vec{l} = 2\pi s$

$B\int dl = 0$

$B(2\pi s) = 0$

$B = 0$ magnetic field is 0 inside wire

for outside the wire, I_{enc} will just be I

so $B(2\pi s) = \mu_0 I$

$$\boxed{B = \frac{\mu_0 I}{2\pi s}}$$

b) current is distributed in such a way that J is proportional to s, the distance from the axis

For inside the wire we know $I = \int_0^a J\, da$

we know $J \propto s$ so $J = ms$

where m would be the proportionality constant

$da = 2\pi s\, ds$

$$I = \int_0^a ms(2\pi s)\, ds = 2\pi m \int_0^a s^2\, ds = 2\pi m \frac{a^3}{3}$$

$$m = \frac{3 I}{2\pi a^3}$$

for I_{enc}, $I_{enc} = \int_0^s \vec{J} \cdot da$

$I_{enc} = \int_0^s ms(2\pi s)ds = 2\pi m \frac{s^3}{3}$

$I_{enc} = 2\pi \frac{s^3}{3}\left(\frac{3I}{2\pi a^3}\right) = \frac{Is^3}{a^3}$

so $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$

$B(2\pi s) = \mu_0\left(\frac{Is^3}{a^3}\right)$

$B = \frac{\mu_0 I s^2}{2\pi a^3}$ for $s<a$

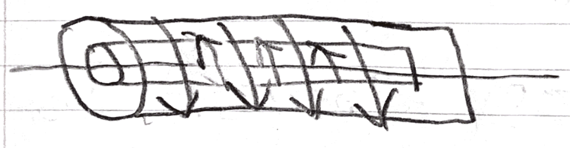
for outside the wire

$I_{enc} = I$ $\quad \oint \vec{B} \cdot d\vec{l} = \mu_0 I$

$B(2\pi s) = \mu_0 I$ $\quad B = \frac{\mu_0 I}{2\pi s}$

16) Two long coaxial solenoids each carry current I, but in oppisite direction. Inner Solenoid (radius a) has n_1 turns per unit length, outer one (radius b) has n_2.
Find $\vec{B}$ in each of the 3 regions.

1) Inside Inner solenoid



We know that $B = \mu_0 n I$ for pts inside solend and $B = 0$ for pts outside

So for inside inner solenoid

$B_1 = \mu_0 n_2 I$ (due to outer solenoid)

$B_2 = -\mu_0 n_1 I$ (due to inner solenoid)

right hand rule

So $\boxed{B_{tot} = \mu_0 I (n_2 - n_1)}$

ii) between them

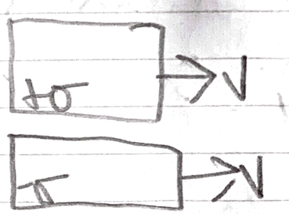
B_1 (due to inner solenoid) $= 0$

B_2 (outer solenoid) $= \boxed{\mu_0 n_2 I}$

iii) outside both

magnetic field would be 0 since B_1 and B_2 are both 0.

17) large parallel capacitor w/ uniform surface charge σ on upper plate and $-\sigma$ on lower is moving at constant speed v.



a) find magnetic field between the plates and also above/below them

$B = \frac{\mu_0 K}{2}$ from Ex 3.6, and $K = \sigma v$

we know that for the top plate
$B = \frac{\mu_0 k}{2}$

for the bottom plate
$B = \frac{\mu_0 k}{2}$

we know that the fields due to the plates cancel for pts above or below the plates

for in between
$B_{tot} = \frac{\mu_0 k}{2}\hat{y} + \frac{\mu_0 k}{2}\hat{y} = \mu_0 k = \boxed{\mu_0 \sigma v}$

b) Find magnetic force per unit area on upper plate

$F = \int (\vec{k} \times \vec{B}) da$ $\quad \vec{B} = \frac{\mu_0 k}{2}\hat{y}$

$k = \sigma v \hat{x}$

so $F = (\sigma v \hat{x}) \times \left(\frac{\mu_0 k}{2}\hat{y}\right)$

$= \sigma v \frac{\mu_0 k}{2}(\hat{x} \times \hat{y})$ $\quad \hat{x} \times \hat{y} = \hat{z}$

so force $= \sigma v \frac{\mu_0 k}{2}\hat{z} = \sigma v \frac{\mu_0 (\sigma v)}{2}\hat{z}$

$= \boxed{\frac{\mu_0 (\sigma v)^2}{2}}$ in the upwards direction

19) In calculating current enclosed by Amperian Loop one must evaluate integral of the form

$$I_{enc} = \int_S \vec{J} \cdot d\vec{a}$$

The problem is there are infinitely many surfaces that share the same boundary line. Which one are we supposed to use?

17cont) At what speed v would magnetic force balance electrical force

we know $E = \frac{\sigma}{2\varepsilon_0}$ and $f_e = \frac{\sigma^2}{2\varepsilon_0}$

so to balance forces, $f_e = f_m$

$$\frac{\sigma^2}{\cancel{2}\varepsilon_0} = \frac{\mu_0 (\sigma v)^2}{\cancel{2}} \qquad \frac{\cancel{\sigma^2}}{\varepsilon_0} = \mu_0 \cancel{\sigma^2} v^2$$

$$v^2 = \frac{1}{\varepsilon_0 \mu_0} \qquad \varepsilon_0 = 8.85 \times 10^{-12}\ C^2/N{\cdot}m^2$$

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

$$v^2 = \frac{1}{(8.85 \times 10^{-12})(4\pi \times 10^{-7})} = \frac{1}{1.112 \times 10^{-17}}$$

$$v = \frac{1}{\sqrt{1.112 \times 10^{-17}}} = \frac{1}{3.33 \times 10^{-9}} = \boxed{3 \times 10^8\ m/s}$$

19cont) we know thru Ampere's law that

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \Sigma I_{enc}$$

while the value of the current enclosed is

$$I_{enc} = \int_S \vec{J} \cdot da$$

We know that b/c of the thm of divergence less fields $\int \vec{J} \cdot da$ is independent of surface. The value we get from the integral will $= 0$ for closed surface

So $\nabla \cdot \vec{J} = 0$

Thus, for infinite # of surfaces that has same boundary line, the integral is independent of the surface.

20) a) find density ρ of mobile charges in pc of copper, assuming each atom contributes one free electron.

So $\rho = \text{charge}/\text{volume}$ Cu

we also know $\rho = \frac{\text{Charge}}{1 \text{ atom}} \times \frac{1 \text{ atom}}{1 \text{ mole}} \times \frac{1 \text{ mole}}{\text{gram}} \frac{\text{gram}}{\text{vol}}$

$$\rho = eN\left(\frac{1}{\text{mass of Cu}}\right)d$$

$e = 1.6 \times 10^{-19}$ $N = 6 \times 10^{23}$ mol mass $= 64$ g/mol

$d = 9$ gm/cm^3

$$\rho = (1.6 \times 10^{-19})(6 \times 10^{23})\left(\frac{1}{64}\right)(9)$$

$$\rho = 1.35 \times 10^{4} \, C/cm^3$$

b) Calc average electron velocity in copper wire 1mm in diameter, carrying current of 1A.

$d = 1mm = 1 \times 10^{-3} m \qquad r = \frac{d}{2} = \frac{(1 \times 10^{-3})}{2} = 5 \times 10^{-4} m$

$J = \frac{I}{A}$ and $J = \rho v$

$A(\text{area}) = \pi r^2 = \pi (5 \times 10^{-4})^2 = 7.85 \times 10^{-7}$

$J = \frac{1}{(7.85 \times 10^{-7})} = 1.27 \times 10^{6} \frac{A}{m^2} \cdot \frac{1m^2}{10^4 cm^2} = 127.32 \, A/cm^2$

$V = \frac{J}{\rho} = \frac{(127.32)}{(1.35 \times 10^4)} = .0094 \, cm/s$

c) What is the force of attraction between two such wires, 1cm apart?

$d = 1cm$

$F_{mag} = \frac{\mu_0 I_1 I_2}{2\pi d} \qquad \mu_0 = 4\pi \times 10^{-7} \, H/m \quad d = 1cm = .01m$

we know $I_1 = I_2 = 1A$

$F = \frac{(4\pi \times 10^{-7})(1A)(1A)}{2\pi (1cm)}$

$F = 2 \times 10^{-5} \, N/m$

d) If you could somehow remove stationary + charges, what would the electrical repulsion force be? How many times greater than magnetic force is it?

we know $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d}$ for cylindrical charge dist

so $F_e = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_1\lambda_2}{d}$ we also know $\lambda = \frac{I}{v}$

and $\lambda_1 = \lambda_2$, $I_1 = I_2$

So $F_e = \left(\frac{1}{2\pi\varepsilon_0}\right)\left(\frac{1}{v^2}\right)\left(\frac{I_1 I_2}{d}\right)$

we know $c^2 = \frac{1}{\mu_0\varepsilon_0}$ so $\frac{1}{\varepsilon_0} = \mu_0 c^2$

thus $F_e = \frac{\mu_0 c^2 I_1 I_2}{2\pi v^2 d}$

$\frac{F_e}{F_{mag}} = \frac{\cancel{\mu_0} c^2 \cancel{I_1 I_2}}{\cancel{2\pi} v^2 \cancel{d}} \cdot \left(\frac{\cancel{2\pi d}}{\cancel{\mu_0 I_1 I_2}}\right) = \frac{c^2}{v^2}$

we know $c = 3\times10^{10}$ cm/s and $v = .0094$ cm/s

$\frac{F_e}{F_{mag}} = \frac{(3\times10^{10})^2}{(.0094)^2} = 1.01\times10^{25}$

so $F_e = (1.01\times10^{25})F_{mag}$

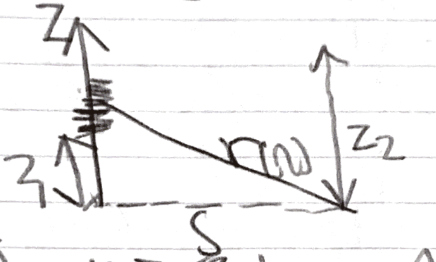
$F_e = (1.01\times10^{25})(2\times10^{-8}\text{ cm}) = 2.024\times10^{18}$ N/cm

23) Find magnetic vector potential of a finite segment of a straight wire carrying a current I.

23 cd) Eq 5.66 is $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I}{r}\, dl' = \frac{\mu_0 I}{4\pi}\int \frac{1}{r}\, dl'$; $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{K}}{r}\, da'$

Eq 5.37: $\frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2} \cos\theta \, d\theta = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$

$\vec{A}$ is mag vector potential, $B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$



$r = \sqrt{s^2+z^2}$

so $\vec{A} = \frac{\mu_0}{4\pi}\int \frac{I\hat{z}}{r}\, dz$

$\vec{A} = \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2} \frac{dz}{\sqrt{s^2+z^2}}\,\hat{z} = \frac{\mu_0 I}{4\pi}\ln\left(\frac{z+\sqrt{s^2+z^2}}{s}\right)\Big|_{z_1}^{z_2}$

$= \frac{\mu_0 I}{4\pi}\left[\ln\left(\frac{z_2+\sqrt{z_2^2+s^2}}{s}\right) - \ln\left(\frac{z_1+\sqrt{z_1^2+s^2}}{s}\right)\right]$

$= \boxed{\frac{\mu_0 I}{4\pi}\ln\left(\frac{z_2+\sqrt{z_2^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right)\hat{z}}$

$\hat{\phi}$ is direction of mag field perpendicular to $\hat{z}$

so $B = -\frac{\partial A}{\partial s}\hat{\phi}$

$B = \frac{\mu_0 I}{4\pi}\frac{\partial}{\partial s}\left(\ln\left(\frac{z_2+\sqrt{z_2^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right)\right)$

Use symbolab

$= \frac{-z_2^2 s - z_2 s\sqrt{z_2^2+s^2} + s z_1^2 + s z_1\sqrt{z_1^2+s^2}}{\sqrt{(z_2+\sqrt{z}}}$

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$$B = -\frac{\mu_0 I s}{4\pi}\left[\frac{z_2-\sqrt{z_2^2+s^2}}{(z_2)^2-[z_2^2+s^2]}\frac{1}{\sqrt{z_2^2+s^2}} - \frac{z_1-\sqrt{z_1^2+s^2}}{(z_1^2-(z_1^2+s^2))(\sqrt{z_1^2+s^2})}\right]$$

$$= -\frac{\mu_0 I s}{4\pi}\left(-\frac{1}{s^2}\right)\left[\frac{z_2}{\sqrt{z_2^2+s^2}} - \frac{z_1}{\sqrt{z_1^2+s^2}}\right]\hat{\phi}$$

$$B = \frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{\sqrt{z_2^2+s^2}} - \frac{z_1}{\sqrt{z_1^2+s^2}}\right]\hat{\phi}$$

$$\sin\theta_2 = \frac{z_2}{\sqrt{z_2^2+s^2}}$$

$$\sin\theta_1 = \frac{z_1}{\sqrt{z_1^2+s^2}}$$

thus this magnetic field is equivalent to eq 5.37.

2b) a) find the vector potential a distance s from an infinite straight wire carrying a current I. check that $\nabla\cdot A = 0$ and $\nabla\times A = B$ cylindrical coords

let $\vec{A} = A(s)\hat{z}$

$$B = \nabla\times A = \left(\frac{1}{s}\cancel{\frac{\partial A_z}{\partial\phi}}^{0} - \cancel{\frac{\partial A_\phi}{\partial z}}^{0}\right)\hat{s} + \left(\cancel{\frac{\partial A_s}{\partial z}}^{0} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left[\cancel{\frac{\partial(sA_\phi)}{\partial s}}^{0} - \cancel{\frac{\partial A_s}{\partial\phi}}^{0}\right]\hat{z}$$

$$\nabla\times A = -\frac{\partial A}{\partial s}\hat{\phi} = B$$

we also know that $\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$ for

Infinetly long wire

So $-\frac{\partial A}{\partial s} = \frac{\mu_0 I}{2\pi s}$ $\int \partial A = \int -\frac{\mu_0 I}{2\pi s} ds$

$A = -\int_a^s \frac{\mu_0 I}{2\pi s} ds = \boxed{-\frac{\mu_0 I}{2\pi} \ln\left(\frac{s}{a}\right)}$

$\nabla \cdot \vec{A} = \frac{\partial A_z}{\partial z} = 0$ b/c $\vec{A} = A(s)\hat{z}$ ✓

$\nabla \times A = -\frac{\partial A_z}{\partial s}\hat{\phi} = \frac{\mu_0 I}{2\pi s}\hat{\phi} = B$ ✓

b) Find magnetic potential inside the wire, if it has radius R and current is uniformly distributed.

$\oint \vec{B} \cdot d\vec{L} = \mu_0 I_{enc}$ $\oint \vec{B} \cdot d\vec{L} = \vec{B}(2\pi s)$

we know $J = \frac{I}{A}$ $A = \pi s^2$

So $\vec{J} = \frac{I}{\pi s^2}$ $I = \vec{J}(\pi s^2)$

So $\vec{B}(2\pi s) = \mu_0 \vec{J} \pi s^2$

we know that $\vec{J}$ is also equal to $\frac{I}{\pi r^2}\hat{\phi}$

so $\vec{B} = \frac{\mu_0 I s}{2\pi r^2}\hat{\phi}$

we know from the last part that $\vec{B} = -\frac{\partial A}{\partial s}\hat{\phi}$

$\frac{\partial A}{\partial s} = -\frac{\mu_0 I s}{2\pi r^2}$ $\int \partial A = \int -\frac{\mu_0 I s}{2\pi r^2}\partial s$

$A' = -\frac{\mu_0 I}{2\pi r^2}\int_b^s s\,ds\ \hat{z}$ for $s<r$

$= -\frac{\mu_0 I}{4\pi R^2}(s^2 - b^2)\hat{z}$

~~Now let~~ let $s=R$ $\quad -\frac{\mu_0 I}{4\pi R^2}(R^2 - b^2) = -\frac{\mu_0 I}{2\pi}\ln\frac{R}{a}$

$1 - \frac{b^2}{R^2} = 2\ln\frac{R}{a}$ $\quad$ let $a = b = R$

$0 = 0$ ✓

so $\vec{A} = -\frac{\mu_0 I}{4\pi R^2}(s^2 - R^2)\hat{z}$

3.30) Ex 3.9, derived exact potential for spherical sphere of radius R,
which comes surface charge $\sigma = k\cos\theta$
a) Calc dipole moment of this charge dist

$$\vec{p} = p\hat{z}$$

$$p = \int z\sigma\, da$$

We know $z = R\cos\theta$, $\sigma = k\cos\theta$, $da = R^2 \sin\theta\, d\theta\, d\phi$
$0<\theta<\pi$, $0<\phi<2\pi$

$$p = \int_0^{2\pi}\int_0^{\pi} (R\cos\theta)(k\cos\theta)(R^2\sin\theta)\, d\theta\, d\phi$$

$$= kR^3 2\pi \int \cos^2\theta \sin\theta\, d\theta$$

$u = \cos\theta$
$du = -\sin\theta\, d\theta$

$$= kR^3 2\pi \left(-\frac{\cos^3\theta}{3}\Big|_0^{\pi}\right) = kR^3 2\pi\left(\frac{1}{3} + \frac{1}{3}\right)$$

$$p = \frac{4\pi kR^3}{3}\hat{z}$$

b) Find the approx potential, at pts far from sphere,
compare exact answer. What can you conclude?
We know $V = \frac{1}{4\pi\varepsilon_0}\frac{p\cdot\hat{r}}{r^2}$

$$p\cdot\hat{r} = \frac{4\pi kR^3}{3}\hat{z}\cdot\hat{r} = \frac{4\pi kR^3}{3}\cos\theta$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi kR^3}{3r^2}\right)\cos\theta = \frac{kR^3}{3\varepsilon_0}\frac{\cos\theta}{r^2}$$

higher order multipole terms will have zero potential