Quiz 3

Discussions about Vectors

3. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx)$

$a_n = \frac{1}{\pi}\int f(x)\cos(nx)\,dx$ $\quad b_n = \frac{1}{\pi}\int f(x)\sin(nx)\,dx$

$f(x) = \sin(3x)$ from $n=0$ to $n=\infty$

$a_n = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\cos(nx)\,dx$

$n=0$ $\quad a_0 = \frac{1}{\pi}\int_0^{2\pi}\sin(3x)\,dx$

$= \frac{1}{\pi}\left(-\frac{\cos(3x)}{3}\Big|_0^{2\pi}\right.$

$= \frac{1}{\pi}\left(-\frac{\cos(6\pi)}{3} + \frac{1}{3}\right) = \frac{1}{\pi}\left(-\frac{1}{3}+\frac{1}{3}\right) = 0$

$a_n = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\cos(nx)\,dx$

$\sin(3x)\cos(nx) = \frac{1}{2}(\sin(n+3) - \sin(n-3))$

$= \frac{1}{2\pi}\int_0^{2\pi}\sin((n+3)x)\,dx - \frac{1}{2\pi}\int_0^{2\pi}\sin((n-3)x)\,dx$

$= -\frac{1}{2\pi}\frac{\cos((n+3)x)}{n+3}\Big|_0^{2\pi} + \frac{1}{2\pi}\frac{\cos((n-3)x)}{n-3}\Big|_0^{2\pi}$

$= -\frac{1}{2\pi}\frac{\overset{1}{\cancel{\cos((n+3)(2\pi)}}}{n+3} - \frac{1}{n+3}\left(\frac{1}{2\pi}\right) + \frac{1}{2\pi}\frac{\overset{1}{\cancel{\cos((n-3)(2\pi))}}}{n-3} - \frac{1}{n-3}\left(\frac{1}{2\pi}\right)$

$= -\frac{1}{2\pi}\left(\frac{1}{n+3}\right)(\overset{0}{\cancel{1-1}}) + \frac{1}{2\pi}\left(\frac{1}{n+3}\right)(\overset{0}{\cancel{1-1}}) = 0$

$b_n = \frac{1}{\pi}\int_0^{2\pi}\sin(3x)\sin(nx)\,dx$

$\sin(3x)\sin(nx) = \frac{1}{2}(\cos(n-3) - \cos(n+3))$

$= \frac{1}{2\pi}\int_0^{2\pi}\cos((n-3)x)\,dx - \frac{1}{2\pi}\int_0^{2\pi}\cos((n+3)x)\,dx$

$= \frac{1}{2\pi}\left(\frac{\sin((n-3)x)}{n-3}\Big|_0^{2\pi} - \frac{1}{2\pi}\frac{\sin((n+3)x)}{n+3}\Big|_0^{2\pi}\right.$

$= \frac{1}{2\pi}\left(\frac{\overset{0}{\cancel{\sin(2\pi)}} - \overset{0}{\cancel{\sin(0)}}}{n-3}\right) - \frac{1}{2\pi}\left(\frac{\overset{0}{\cancel{\sin(2\pi)}} - \overset{0}{\cancel{\sin(0)}}}{n+3}\right)$

$= 0$

$n=3$

$$a_3 = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\cos(3x)\,dx \qquad u=\sin(3x) \qquad du = 3\cos(3x)\,dx$$

$$= \frac{1}{3\pi}\int_0^{2\pi} u\,du = \frac{1}{3\pi}\left(\frac{\sin^2(3x)}{2}\Big|_0^{2\pi}\right)$$

$$= \frac{1}{3\pi}\left(\frac{\overset{0}{\cancel{\sin^2(3(2\pi))}}}{2} - \frac{\overset{0}{\cancel{\sin^2(3(0))}}}{2}\right) = 0$$

$$b_3 = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\sin(3x)\,dx = \frac{1}{\pi}\int_0^{2\pi}\sin^2(3x)\,dx$$

$$\sin^2(3x) = \frac{1-\cos(6x)}{2}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} dx - \frac{1}{2\pi}\int_0^{2\pi}\cos(6x)\,dx$$

$$= \frac{1}{2\pi}\left(x\Big|_0^{2\pi}\right) - \frac{1}{12\pi}\sin(6x)\Big|_0^{2\pi}$$

$$= 1 - \frac{1}{12\pi}\overset{0}{\cancel{\sin(6(2\pi))}} + \frac{\overset{0}{\cancel{\sin(0)}}}{12\pi} = 1$$

All coefficients for $n=0$ to $n=\infty$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx)$$

$$= \cancel{\frac{0}{2}} + \sin(3x) + \cancel{\sum_{n=1}^{\infty}(0)}$$

$$\boxed{= \sin(3x)}$$

1. $\vec{V} = a\hat{x} + b\hat{y} + c\hat{z}$ equal to c?

B: $\vec{x}\cdot\hat{z}$

2. $\vec{x} = \sum_{i=1}^{n} c_i\hat{x}_i$ coefficient c_7??

D: $\hat{x}_7 \cdot \vec{x}$

Fourier's Trick and Boundary Value Problems

1. $V(x, y, z) \to 0 \qquad y \to \infty$

$$e^{-k(\infty)} = \frac{1}{e^{k(\infty)}} = 0 \qquad \frac{1}{(\infty)^2} = 0 \qquad e^{-k(\infty)^2} = \frac{1}{e^{-k(\infty)^2}} = 0$$

$y \to \infty$

$\sinh(x) = \sinh(x)$

B: $Y(y) = \sinh(x)$

2. $V_0(y, z) = V_0$

$$C_{n,m} = \frac{4V_0}{ab} \int_0^a \int_0^a \sin\left(\frac{n\pi y}{a}\right) \sin\left(\frac{n\pi z}{a}\right) dy\, dz$$

$$= \frac{4V_0}{ab} \int_0^a \sin\left(\frac{n\pi y}{a}\right) dy \int_0^a \sin\left(\frac{n\pi z}{a}\right) dz$$

$$= \frac{4V_0}{ab}\left[\left(-\frac{a}{n\pi}\cos\left(\frac{n\pi y}{a}\right)\Big|_0^a\right)\left(-\frac{a}{n\pi}\cos\left(\frac{n\pi z}{a}\right)\Big|_0^a\right)\right]$$

$$= \frac{4V_0}{ab}\left[\left[-\frac{a}{n\pi}(\cos(n\pi) - \cos(0))\right]\left[-\frac{a}{n\pi}\cos(n\pi) - \cos(0)\right]\right]$$

$$= \frac{4V_0}{ab}\left[\frac{a^2}{n\pi}(2)(2)\right]$$

$$= \frac{4V_0}{ab}\left[\frac{4a^2}{n^2\pi^2}\right] = \boxed{\frac{16 V_0 a}{\pi^2 n^2 b}}$$