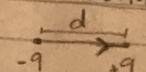
Reading Quiz 2

Distributions of Point Charges

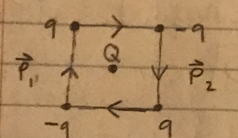
1. 

$\vec{p} = q\vec{d}$

$\vec{E} = E_0 \hat{x}$

$\vec{\tau} = \vec{p} \times \vec{E}$

$= (q\vec{d}) \times (E_0 \hat{x})$

2. 

$\vec{E} = kq/r^2 \, \hat{r}$

$\vec{E}_{tot} = \cancel{\frac{k(q)}{r^2}\hat{r}} + \cancel{\frac{k(-q)}{r^2}\hat{r}} + \cancel{\frac{k(q)}{r^2}\hat{r}} + \cancel{\frac{k(-q)}{r^2}\hat{r}}$

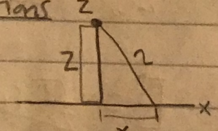
$\boxed{\vec{E} = 0}$

Continuous Charge Distributions

1 a) $Q = \lambda L$

total charge, charge density, total length

$k = \frac{1}{4\pi\varepsilon_0}$



$\vec{E} = k\int \frac{\lambda}{\imath^2}\hat{\imath}\, dl'$

$\vec{\imath} = \vec{r} - \vec{r}\,' = z\hat{z} - x\hat{x}$

$x: -\frac{L}{2}$ to $\frac{L}{2}$

$\vec{r} = z\hat{z}$, $\vec{r}\,' = x\hat{x}$, $dl' = dx$

$$\vec{E} = k\int \frac{\lambda}{\imath^2}\hat{\imath}\, dx$$

$\imath = \sqrt{z^2 + x^2}$

$\hat{\imath} = \vec{\imath}/\imath = \frac{(z\hat{z} - x\hat{x})}{\sqrt{z^2+x^2}}$

$$= k\int \frac{\lambda}{(z^2+x^2)}\left(\frac{z\hat{z} - x\hat{x}}{\sqrt{z^2+x^2}}\right)dx = \lambda k \int \frac{z\hat{z} - x\hat{x}}{(z^2+x^2)^{3/2}}\, dx$$

$$= \lambda k\left[\int_{-\frac{L}{2}}^{L/2} \frac{z\hat{z}}{(z^2+x^2)^{3/2}}dx - \int_{-\frac{L}{2}}^{L/2} \frac{x\hat{x}}{(z^2+x^2)^{3/2}}dx\right] = \lambda k \int_{-L/2}^{L/2} z\hat{z}\left(\frac{dx}{(z^2+x^2)^{3/2}}\right) - \hat{x}\left(\frac{x\,dx}{(z^2+x^2)^{3/2}}\right)$$

$$= \lambda k\left(z\hat{z}\left[\frac{x}{z^{\cancel{2}}(z^2+x^2)^{1/2}}\right]_{-L/2}^{L/2} + \hat{x}\cancel{\left[+\frac{1}{(z^2+x^2)^{1/2}}\right]_{-L/2}^{L/2}}\right)$$

$$= \frac{\lambda k \hat{z}}{z}\left[\frac{L/2}{\sqrt{z^2+\frac{L^2}{4}}} + \frac{+L/2}{\sqrt{z^2+L^2/4}}\right] = \frac{\lambda k \hat{z}}{z}\left(\frac{L}{\sqrt{z^2+L^2/4}}\right)$$

$$\boxed{\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda L}{(z^2+L^2/4)^{1/2}}\hat{z}}$$

when $L >> z$: $\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda L}{(z^2 + L^2/4)^{1/2}}\, \cancel{z} = \frac{1}{2\,\cancel{4}\pi\varepsilon_0} \frac{\lambda \cancel{L}}{\cancel{L}/2}$

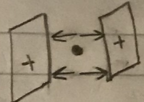
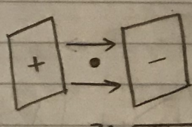
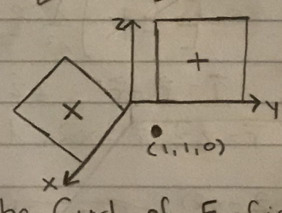
$$\boxed{\vec{E} = \frac{\lambda}{2\pi\varepsilon_0}}$$

b) $\phi_E = Q_{enc}/\varepsilon_0$ $\qquad \phi_E = \vec{E}\cdot\vec{A} = EA\cos\theta = (kq/r^2)(L)\,\cancel{\cos(0)}^{1}$

$Q_{enc} = \lambda L \qquad r^2 = z^2$

$$E = \frac{Q_{enc}}{A\cdot\varepsilon_0} = \frac{\lambda L}{z^2\varepsilon_0}$$

2. plane of charge w/ charge density σ has $\vec{E} = \sigma/2\varepsilon_0$

-  $\vec{E} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \boxed{0}$
-  $\vec{E} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$
- 

$\vec{E} = \frac{\sigma}{2\varepsilon_0}\cancel{(\cos 90°)} + \frac{\sigma}{2\varepsilon_0}(\sin 90°) = \frac{\sigma}{2\varepsilon_0}(1)$

$$\boxed{\vec{E} = \sigma/2\varepsilon_0}$$

The Curl of E-fields

1. $\oint \vec{E}\cdot d\vec{l} = 0, \quad \nabla\times\vec{E} = 0, \quad \vec{E} = -\nabla V$

- show $\quad -\int_a^b \vec{E}\cdot d\vec{l} = V(\vec{b}) - V(\vec{a})$

$V(\vec{r}) \equiv -\int_0^r \vec{E}\cdot d\vec{l} \qquad -\int_a^0 \vec{E}\cdot d\vec{l}$

$V(\vec{b}) - V(\vec{a}) = -\int_0^b \vec{E}\cdot d\vec{l} + (+\int_0^a \vec{E}\cdot d\vec{l}) = -\int_a^b \vec{E}\cdot d\vec{l}$ ✓

- $V(\vec{r}) = -\int_\infty^r E(r')dr' = -\int_\infty^r \frac{kq}{r'^2}dr' = -\int_\infty^r kq\,r'^{-2}\,dr^{-1}$

$= +kq\left[+r'^{-1}\right]_\infty^r = kq\left[\frac{1}{r} - \cancel{\frac{1}{\infty}}\right]$

$$\boxed{V(\vec{r}) = \frac{kq}{r}}$$