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PHYS 330

$\boxed{100\%}$ nice work

HW #2 2.5, 2.6, 2.9, 2.12, 2.16, 2.18, 2.25, 2.29

2.12) Use Gauss's Law to find the electric field inside a uniformly changed solid sphere (charge density ρ).

area of sphere $= 4\pi r^2$

$\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$ $\quad da = 4\pi r^2$ ✓

$E \cdot 4\pi r^2 = \frac{1}{\varepsilon_0} Q_{enc}$ $\quad Q_{enc} = \text{volume} \times \rho$

$E \cdot \cancel{4\pi r^2} = \frac{1}{\varepsilon_0} \frac{4}{3}\pi r^3 \rho$ $\quad = \frac{4}{3}\pi r^3 \rho$

$\boxed{E = \frac{1}{3\varepsilon_0} r\rho \hat{r}}$ ✓

2.16) Coaxial cable carries a uniform volume charge density ρ on the inner cylinder ($r = a$) and a uniform surface charge density on the right magnitude that the cable as a whole is electrically neutral. Find the electric field in each of the three regions: (i) inside the inner cylinder ($s < a$), (ii) between the cylinders ($a < s < b$), (iii) outside the able ($s > b$). plot $|E|$ as a function of s

(i) $\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$ $\quad da = 2\pi s \cdot \ell$
cylinder − ends

$E \cdot \cancel{2\pi} s \cdot \cancel{\ell} = \frac{1}{\varepsilon_0} \cancel{\pi} s^2 \cancel{\ell} \rho$ $\quad Q_{enc} = \text{volume} \times \rho$

$\boxed{E = \frac{1}{2\varepsilon_0} s\rho \hat{s}}$ ✓ $\quad = \pi s^2 \ell \cdot \rho$

$\propto s\hat{s}$

(ii) $\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$

$E \cdot \cancel{2\pi} s \cancel{\ell} = \frac{1}{\varepsilon_0} \cancel{\pi} a^2 \cancel{\ell} \rho$

$\boxed{E = \frac{1}{2\varepsilon_0 s} a^2 \rho \hat{s}}$ ✓ $\propto \hat{s}/s$

(iii) outer cyl same as (i) but negative

$\vec{E}_{tot} = E_{outer} + E_{inner}$

$= \frac{1}{2\varepsilon_0} s\rho \hat{s} + \frac{1}{-2\varepsilon_0} s\rho \hat{s}$

$\boxed{\vec{E} = 0}$ ✓ $q_{enc} = 0$

2.18) Two spheres, each of radius R and carrying uniform volume charge densities $+\rho$ & $-\rho$, respectively, are placed so that they partially overlap. Call the vector from the positive center to the negative center d. Show that the field in the region of overlap is constant, and its value.

For one sphere $E = \frac{1}{3\varepsilon_0}\rho r\hat{r}$

$$E_+ = \frac{1}{3\varepsilon_0}\rho r\hat{r} \qquad E_{tot} = \frac{\rho}{3\varepsilon_0}(r_+ - r_-)$$

$$E_- = \frac{-1}{3\varepsilon_0}\rho r\hat{r} \qquad \boxed{E_{tot} = \frac{\rho}{3\varepsilon_0}d}$$ all constants, so E is constant

→ I assume these are different...

2.25) (a) $V = \frac{1}{4\pi\varepsilon_0}\frac{2q}{(z^2+(\frac{d}{2})^2)^{1/2}}$

$$E = -\frac{1}{4\pi\varepsilon_0}2q\left(-\frac{1}{2}\right)\frac{z}{(z^2+(\frac{d}{2})^2)^{3/2}}\hat{z}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2qz}{(z^2+(\frac{d}{2})^2)^{3/2}}\hat{z}$$

(b) $V = \frac{1}{4\pi\varepsilon_0}\int_{-L}^{L}\frac{\lambda dx}{(x^2+z^2)^{1/2}}$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(x+\sqrt{z^2+x^2}\right)\Big|_{-L}^{L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{L+(z^2+L^2)^{1/2}}{z}\right)$$

$$E = -\frac{\lambda}{4\pi\varepsilon_0}\left\{\frac{1}{L+\sqrt{z^2+L^2}}\frac{1}{2}\frac{1}{\sqrt{z^2+L^2}}2z - \frac{1}{(-L+\sqrt{z^2+L^2})}\frac{1}{2}\frac{1}{\sqrt{z^2+L^2}}2z\right\}\hat{z}$$

$$= \frac{-\lambda}{4\pi\varepsilon_0}\frac{z}{\sqrt{z^2+L^2}}\left\{\frac{-L+\cancel{\sqrt{z^2+L^2}}-L-\cancel{\sqrt{z^2+L^2}}}{(z^2+L^2)-L^2}\right\}\hat{z}$$

$$= \frac{-2\lambda L}{4\pi\varepsilon_0}\frac{1}{z\sqrt{z^2+L^2}}\hat{z}$$

well done!

(c) $V = \frac{1}{4\pi\varepsilon_0}\int_0^R\frac{\sigma 2\pi r\,dr}{\sqrt{r^2+z^2}}$

$$= \frac{1}{2\cancel{4\pi}\varepsilon_0}\cancel{2\pi}\sigma\left(\sqrt{r^2+z^2}\right)\Big|_0^R$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2+z^2}-z\right)$$

$$E = -\frac{\sigma}{2\varepsilon_0}\left\{\frac{1}{2}\frac{1}{\sqrt{R^2+z^2}}2z-1\right\}\hat{z}$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{R^2+z^2}}\right]\hat{z}$$

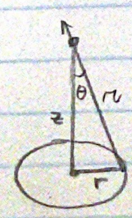
2.29) $\nabla^2 V = \frac{1}{4\pi\varepsilon_0}\nabla^2\int\left(\frac{\rho}{\mathscr{r}}\right)d\tau$

$$= \frac{1}{4\pi\varepsilon_0}\int\rho(r')\left(\nabla^2\frac{1}{\mathscr{r}}\right)d\tau$$

$$= \frac{1}{\cancel{4\pi}\varepsilon_0}\int\rho(r')\left[-\cancel{4\pi}\delta^3(r-r')\right]d\tau$$

$$= \frac{-1}{\varepsilon_0}\rho(r')$$

2.5)



$$E = \frac{1}{4\pi\varepsilon_0}\left\{\int\frac{\lambda dl}{\mathscr{r}^2}\cos\theta\right\}\hat{z}$$

$$\mathscr{r}^2 = r^2+z^2, \quad \cos\theta = \frac{z}{\mathscr{r}}, \quad \int dl = 2\pi r$$

$\Downarrow$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{\lambda(2\pi r)\,z}{(r^2+z^2)^{3/2}}$$ → in $\hat{z}$-direction

2.6) $E_{ring} = \frac{1}{4\pi\varepsilon_0} \frac{\overbrace{(\sigma dr)}^{\lambda} 2\pi r z}{(r^2+z^2)^{3/2}}$

Tot. charge of ring $= \sigma \cdot 2\pi r \cdot dr$

$= \lambda \cdot 2\pi r$

$\rightarrow \lambda = \sigma dr$

$$E_{disk} = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma z \int_0^R \frac{r}{(r^2+z^2)^{3/2}} dr$$
$$= \frac{1}{4\pi\varepsilon_0} 2\pi\sigma z \left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right]\hat{z}$$

$dq = \lambda dl = \lambda R d\phi$

$$E_{plane} = \frac{1}{\cancel{4\pi}\varepsilon_0} \cancel{2\pi}\sigma \hat{z}$$
$$= \frac{\sigma}{2\varepsilon_0}\hat{z}$$

$$\frac{1}{\sqrt{R^2+z^2}} = \frac{1}{z}\left(1+\frac{R^2}{z^2}\right)^{-1/2} \approx \frac{1}{z}\left(1-\frac{1}{2}\frac{R^2}{z^2}\right)$$
$$\approx \cancel{\frac{1}{z}} - \cancel{\frac{1}{z}} + \frac{1}{2}\frac{R^2}{z^3} = \frac{R^2}{2z^3}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{\cancel{2}\pi R^2\sigma}{\cancel{2}z^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{Q}{z^2} \quad Q = \pi R^2\sigma$$

2.9) (a) $\rho = \varepsilon_0 \nabla \cdot E$

$= \varepsilon_0 \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cdot kr^3)$

$= \varepsilon_0 \frac{1}{r^2} k(5r^4)$

$= 5\varepsilon_0 k r^2$ ✓

(b) $Q_{enc} = \varepsilon_0 \oint E \cdot da$

$= \varepsilon_0 (kR^3)(4\pi R^2)$

$= 4\pi\varepsilon_0 k R^5$ ✓

(c) $Q_{enc} = \int \rho \, d\tau$

$= \int_0^R (5\varepsilon_0 k r^2)(4\pi r^2 dr)$

$= 20\pi\varepsilon_0 k \int_0^R r^4 dr$

$= 4\pi\varepsilon_0 k R^5$ ✓