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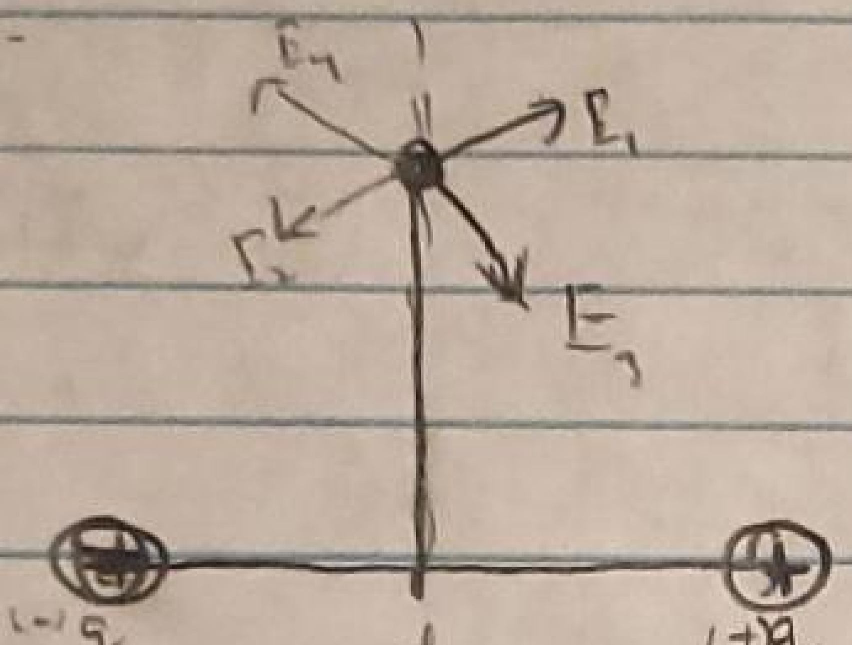
Quiz 2

1) a) $\vec{\tau} = \vec{p} \times \vec{E}$, $\vec{p} = q\vec{d}$

$(+)\vec{F} = q\vec{E}$
$(-)\vec{F} = -q\vec{E}$, $\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$

b) $\vec{p}_1$
$\vec{p}_2$ — opposite directions



$\bar{E}_1 = E_2$, $E_1 + E_2 = 0$
$E_3 = E_4$, $E_3 + E_4 = 0$

~~should be~~ , $\boxed{F_{net} = 0}$, because the charges are going in opposite of directions

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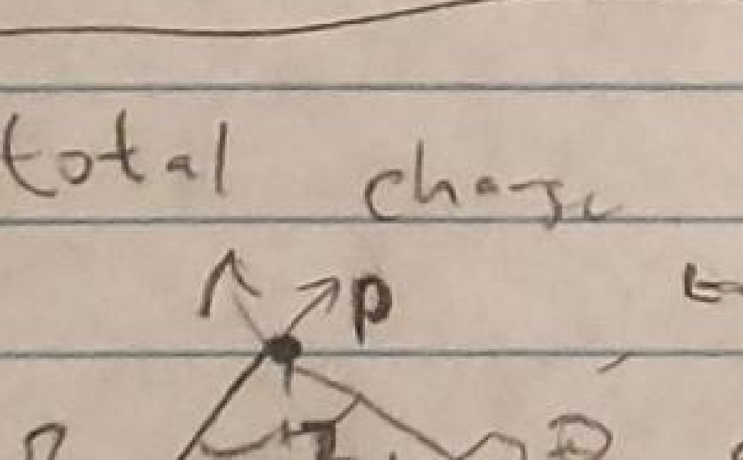
$\boxed{\text{Section II}}$ 2a) total charge $(Q = \lambda L)$, λ = charge density , $L \gg z$
L = total length

$\tan\theta = x/z$



$dq = \lambda dx$

$d\vec{E} = \dfrac{k\,dq\,\hat{\mathscr{r}}}{\mathscr{r}^2}$, $d\vec{E} = \dfrac{k\,dq\,\hat{\mathscr{r}}}{\mathscr{r}^2}$

$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$
$\vec{\mathscr{r}} = z\hat{z} - x\hat{x}$
$\mathscr{r}^2 = z^2 + x^2$
$\hat{\mathscr{r}} = \dfrac{\vec{\mathscr{r}}}{\mathscr{r}} = \dfrac{z\hat{z} - x\hat{x}}{(z^2 + x^2)^{1/2}}$

$= \dfrac{k\lambda dx(z\hat{z} - x\hat{x})}{(z^2+x^2)(z^2+x^2)^{1/2}}$

$= \dfrac{k\lambda dx(z\hat{z} - x\hat{x})}{(z^2+x^2)^{3/2}}\Bigg|^{L/2}_{-L/2}$

$$\int d\vec{E} = \int_{-L/2}^{L/2} \frac{k\lambda\, dx\,(z\vec{z} - x\vec{x})}{(z^2+x^2)^{3/2}}$$

$$\vec{E} = k\lambda \left\{ z\vec{z}\int_{-L/2}^{L/2} \frac{dx}{(z^2+x^2)^{3/2}} - \vec{x}\int_{-L/2}^{L/2} \frac{x\,dx}{(z^2+x^2)^{3/2}} \right\}$$

$x = z\tan(\theta)$

$dx = z\sec^2\theta\, d\theta$

$x\,dx = z^2\tan$

$$\vec{E} = k\lambda \left\{ z\vec{z}\int_{\theta_1}^{\theta_2} \frac{z\sec^2\theta\, d\theta}{(z^2+z^2\tan^2\theta)^{3/2}} - \int_{\theta_1}^{\theta_2} \frac{z\sec^2\theta\, d\theta \cdot z\tan\theta}{(z^2+(z^2\tan\theta)^2)^{3/2}} \right\}$$

$$= k\lambda \left\{ \frac{z^2\vec{z}}{z^3}\int_{\theta_1}^{\theta_2} \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)^{3/2}} - \int_{\theta}^{\theta} \frac{\sec^2\theta d\theta \cdot \tan\theta}{((1+\tan^2\theta))^{3/2}} \right\}$$

$$= k\lambda \left\{ \frac{\vec{z}}{z}\int_{\theta_1}^{\theta_2} \frac{\sec^2\theta\, d\theta}{\sec^3\theta} - \vec{x}\cancel{\int_{\theta}^{\theta_2} \frac{z^2\sec^2\theta \cdot \tan\theta}{z^3\sec^3\theta}} \right\} \to 0$$

$$= \frac{k\lambda\vec{z}}{z}(\sin\theta_2 - \sin\theta_1)$$

$$\vec{E} = \frac{k\lambda\vec{z}}{z}(\sin\theta_2 - \sin\theta_1) = \boxed{\frac{k\lambda L\vec{z}}{z}\left(1+\tfrac{1}{4}\left(\tfrac{L}{z}\right)^2\right)^{-1/2}}$$

for small zoom

$$\boxed{= \frac{k\lambda L\vec{z}}{z^2}\left(1+\frac{L^2}{4}\right)^{-1/2}}$$

$L >> z$

$$E \sim \frac{kQ}{z}\left(\frac{z}{zL^2}\right) = \boxed{\frac{2kQ}{zL}}$$

$\boxed{\text{Section 2}}$ 1b) Gauss's Law $\boxed{r = z}$

$Q_{encl} = \lambda L$, $\oint E \cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$E \int da = \frac{\lambda L}{\varepsilon_0}$, $\int da = 4\pi z^2 \cos\theta$, $\frac{z}{(z^2 + L^2/4)^{1/2}} = \cos\theta$

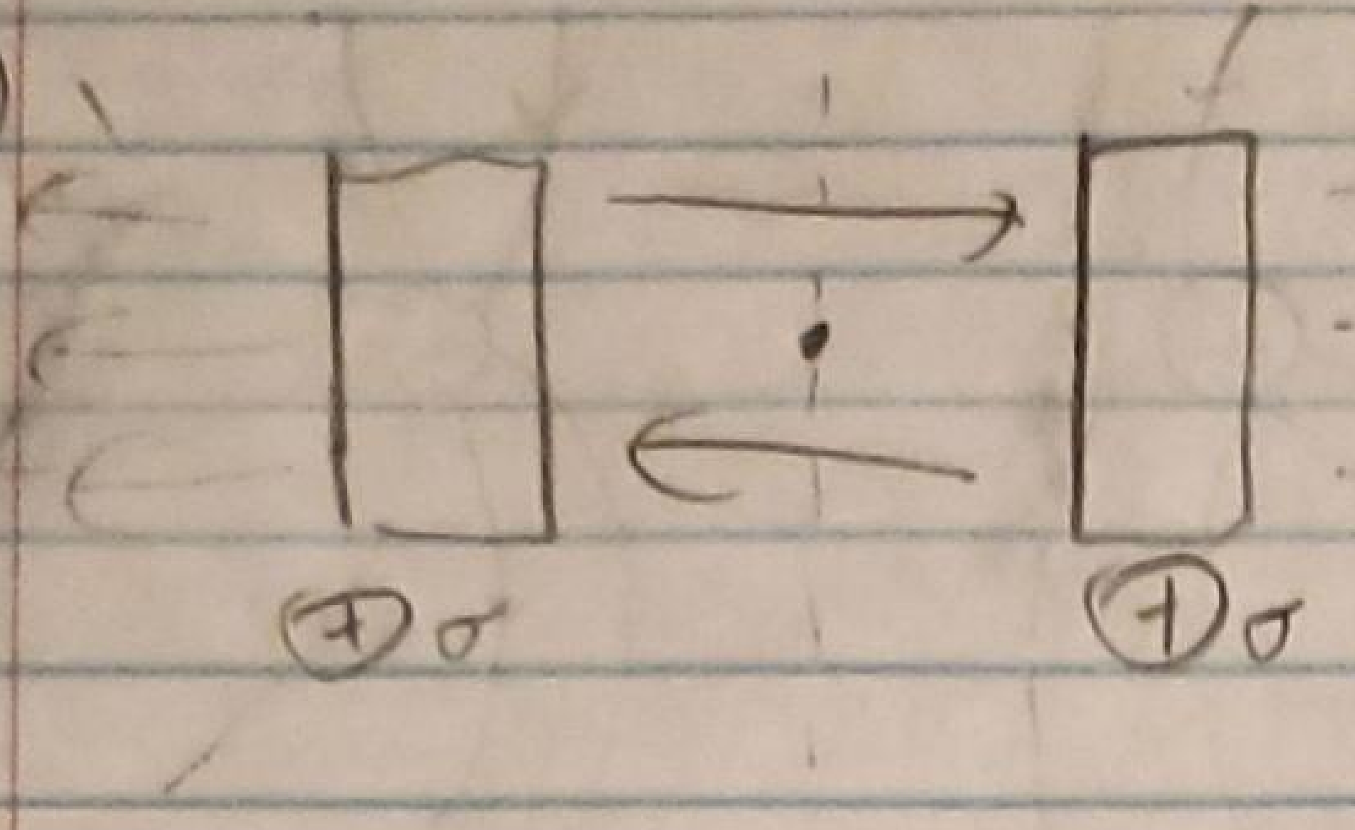
$E(4\pi \, z^2 \cos\theta \, \vec{z}) = \frac{\lambda L}{\varepsilon_0}$

$E = \frac{\lambda L}{4\pi\varepsilon_0 \, z \cos\theta} \vec{z} = \frac{k \lambda L}{(z^2 + L^2/4)^{1/2}} \frac{(z^2 + L^2/4)^{1/2}}{z}$

$$\boxed{E = \frac{k \lambda L \, \vec{z}}{z} (z^2 + L^2/4)^{-1/2}}$$

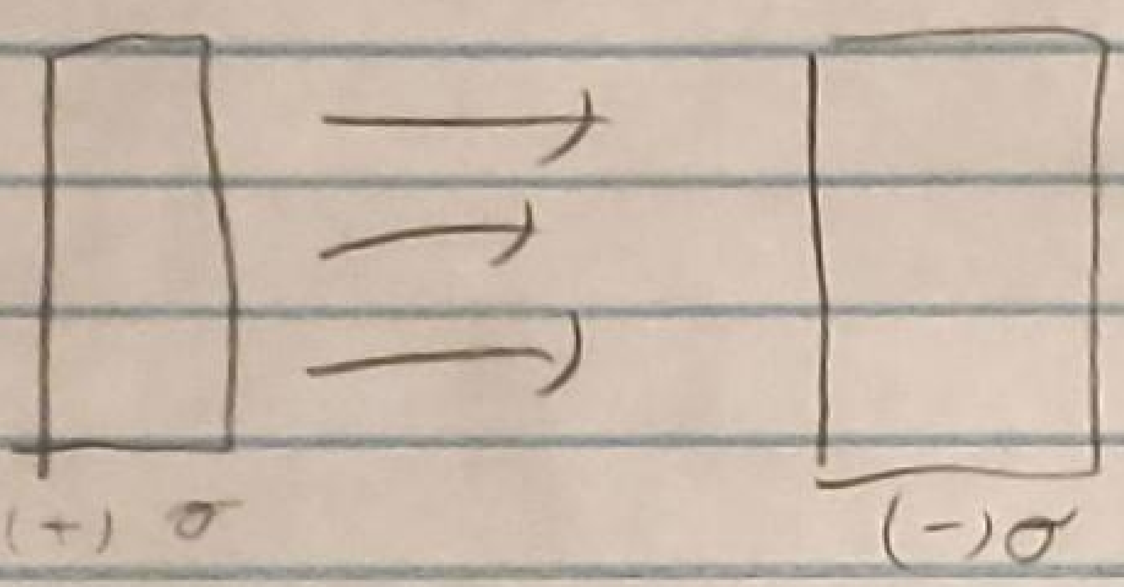
Section 21 | 2) ⊕⊕ (σ/A) σ $\vec{E} = \frac{\sigma}{2\varepsilon_0}$

a)



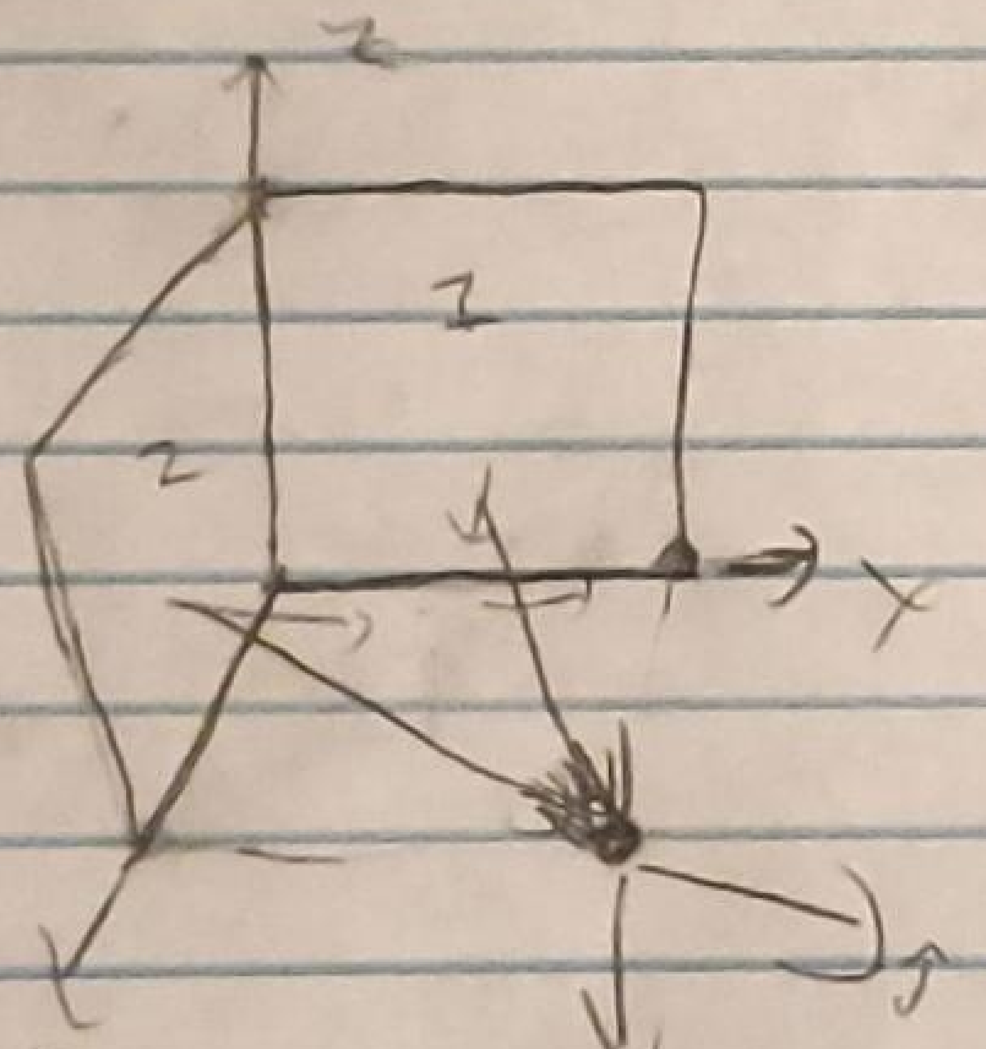
$$E_{TOT} = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = \boxed{0}$$

b)



$$E_{TOT} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{2\sigma}{2\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}}$$

c)



$E_{TOT} = E_x + E_y$ $\sqrt{2\sigma^2} =$

$$E_x = \frac{\sigma}{2\varepsilon_0}\hat{i} + \frac{\sigma}{2\varepsilon_0}\hat{j} = \sqrt{(\sigma/2\varepsilon_0)^2 + (\sigma/2\varepsilon_0)^2}$$

$$= \sqrt{\sigma^2/4\varepsilon_0^2 + (\sigma/4\varepsilon_0^2)}$$

$$= \sigma/2\varepsilon_0^2 + \sigma/2\varepsilon_0^2$$

$$E_y = \frac{\sigma}{2\varepsilon_0}\hat{i} + \frac{\sigma}{2\varepsilon_0}\hat{j}$$

$$E_{TOT} = \left(\frac{\sqrt{2\sigma^2}}{2\varepsilon_0} + \frac{\sqrt{2\sigma^2}}{2\varepsilon_0}\right) - \left(\frac{\sqrt{\sigma^2}}{2\varepsilon_0} \frac{\sqrt{\sigma^2}}{2\varepsilon_0}\right)$$

$$= \left(\frac{2\sqrt{2\sigma^2}}{2\varepsilon_0}\right) - \left(\frac{2\sqrt{2\sigma^2}}{2\varepsilon_0}\right)$$

$$\boxed{E_{TOT} = 0}$$

3) $\oint \vec{E} \cdot d\vec{l} = 0$, thus $\nabla \times \vec{E} = 0$

$$\vec{E} = -\nabla V$$

$$\vec{F} = \frac{-dV}{dx}\hat{x}$$

Show ! • $\int_a^B \vec{E} \cdot d\vec{l} = V(\vec{b}) - V(\vec{a})$, Arbitrary reform $V(\vec{r}) = -\int_\infty^r E(r')\,dr'$

Answer $\boxed{kq/r}$

$$V(b) - V(a) = -\int_o^b E \cdot dl - \int_o^a E \cdot dl = -\int_o^b E \cdot dl - \int_a^o E \cdot dl =$$

$$V(b) - V(a) = \boxed{-\int_a^b E \cdot dl}$$

$$V(r) = -\int_o^r E \cdot dl = \frac{-1}{4\pi\varepsilon_0}\int_\infty^r \frac{q}{r^2}\,dr' = \frac{1}{4\pi\varepsilon_0}\frac{q}{r'}\Big|_\infty^r = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\boxed{= \frac{kq}{r}}$$