Final Project Proposal

<u>Objective:</u>

The study of electrostatics has proven to be a very complex subject with many different applications such as vacuum tubes. For my theoretical project I would like to further study the emission of an electron charge inside a vacuum tube containing both a cathode and anode. I would like to gain a deeper understanding of the space charge cloud of electrons that builds up when a cathode is heated. I will explain the problem in detail but my objective is to develop a deeper understanding of the Child-Langmuir law and ultimately communicate it to everyone else.

Theoretical Problem:

Problem 2.53 In a vacuum diode, electrons are "boiled" off a hot **cathode**, at potential zero, and accelerated across a gap to the **anode**, which is held at positive potential V_0. The cloud of moving electrons within the gap (called **space charge**) quickly builds up to the point where it reduces the field at the surface of the cathode to zero. From then on, a steady current I flows between the plates.

Suppose the plates are large relative to the separation ($A \gg d^2$ in Fig. 2.55), so that edge effects can be neglected. Then V, ρ, and v (the speed of the electrons) are all functions of x alone.

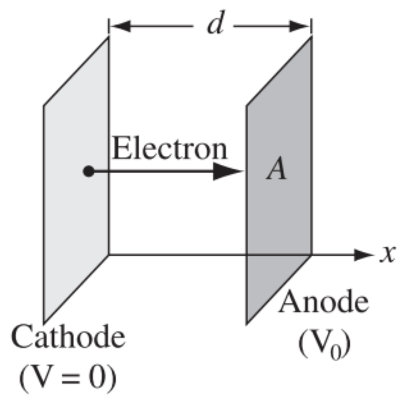

FIGURE 2.55

(a) Write Poisson's equation for the region between the plates.

(b) Assuming the electrons start from rest at the cathode, what is their speed at point x, where the potential is $V(x)$?

(c) In the steady state, I is independent of x. What, then, is the relation between ρ and v?

(d) Use these three results to obtain a differential equation for V, by eliminating ρ and v.

(e) Solve this equation for V as a function of x, V_0, and d. Plot $V(x)$, and compare it to the potential *without* space-charge. Also, find ρ and v as functions of x.

(f) Show that

$$I = K V_0^{3/2}, \tag{2.56}$$

and find the constant K. (Equation 2.56 is called the **Child-Langmuir law**. It holds for other geometries as well, whenever space-charge limits the current. Notice that the space-charge limited diode is *nonlinear*—it does not obey Ohm's law.)