due May 9th 2022

Roger Chen
5/4/2022
PHYS 330

Homework 6: #6.3, 6.7, 6.16

6.3) a) $B_{dip}(r) = \frac{\mu_0}{4\pi}\frac{m}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$

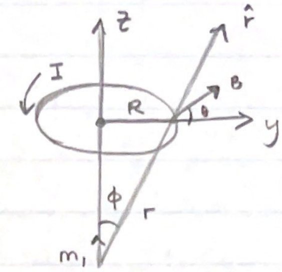
$m = (m\cdot\hat{r})\hat{r} + (m\cdot\hat{\theta})\hat{\theta}$

$= m\cos\theta\,\hat{r} - m\sin\theta\,\hat{\theta}$

$3(m\cdot\hat{r})\hat{r} - m = 3[m\cos\theta\,\hat{r}] - m\cos\theta\,\hat{r} + m\sin\theta\,\hat{\theta}$

$= 2m\cos\theta\,\hat{r} + m\sin\theta\,\hat{\theta}$

$= m[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}]$



$B_1 = \frac{\mu_0}{4\pi r^3}[3(m_1\cdot\hat{r})\hat{r} - m_1]$

$B\cdot\hat{y} = |B||\hat{y}|\cos\theta$

$= B\cos\theta$

$\vec{m}_1\cdot\hat{y} = 0$

$\hat{r}\cdot\hat{y} = \sin\phi$

$\vec{m}_1\cdot\hat{r} = m_1\cos\phi$

$\vec{B}\cdot\hat{y} = \frac{\mu_0}{4\pi}\frac{[3(m_1\cos\phi)\sin\phi - 0]}{r^3}$

$B\cos\theta = \frac{\mu_0}{4\pi r^3}3m_1\cos\phi\sin\phi$

$F = 2\pi I R B\cos\theta$

$\sin\phi = \frac{R}{r}$, $\cos\phi = \frac{\sqrt{r^2-R^2}}{r}$

$= 2\pi I R\left[\frac{\mu_0}{4\pi r^3}3m_1\cos\phi\sin\phi\right]$

$= 2\pi I R\left[\frac{\mu_0}{4\pi r^3}3m\frac{\sqrt{r^2-R^2}}{r}\frac{R}{r}\right]$

$= 2\pi I R^2\frac{\mu_0}{4\pi}\frac{3m_1\sqrt{r^2-R^2}}{r^5}$

$= 3\frac{\mu_0}{2\pi}I\pi R^2\frac{m_1\sqrt{r^2-R^2}}{r^5}$

$m_2 = I\pi R^2$

$= \frac{3}{2\pi}\mu_0 m_2 m_1\frac{\sqrt{r^2-R^2}}{5}$

$R \approx 0$

$= \frac{3}{2\pi}\mu_0 m_2 m_1\frac{\sqrt{r^2}}{r^5}$

$$\boxed{F = \frac{3\mu_0}{2\pi}\frac{m_2 m_1}{r^4}}$$

b) $F = \nabla(m_2\cdot B)$

$\nabla(m_1\cdot B) = (m_1\cdot\nabla)B + (B\cdot\nabla)m_1 + m_1\times(\nabla\times B) + B\times(\nabla\times m_1)$

$= (m_1\cdot\nabla)B + \cancel{0+0+0}$

$B = \frac{\mu_0}{4\pi}\frac{1}{z^3}[3(m_1\cdot\hat{z})\hat{z} - m_1]$

$= \frac{\mu_0}{4\pi}\frac{1}{z^3}[3m_1 - m_1]$

$= \frac{\mu_0}{4\pi}\frac{1}{z^3}[2m_1]$

$F = \left[m_2\cdot\left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\right]\left(\frac{\mu_0}{4\pi}\frac{1}{z^3}[2m_1]\right)$

$= \frac{\mu_0}{4\pi}(2m_1m_2)\left[\hat{x}\frac{\partial}{\partial x}\left(\frac{1}{z^3}\right) + \hat{y}\frac{\partial}{\partial y}\left(\frac{1}{z^3}\right) + \hat{z}\frac{\partial}{\partial z}\left(\frac{1}{z^3}\right)\right]$

$= \frac{\mu_0}{4\pi}(2m_1m_2)\left[0 + 0 + \hat{z}\frac{\partial}{\partial z}\left(\frac{1}{z^3}\right)\right]$

$= \frac{\mu_0}{4\pi}(2m_1m_2)\left[\frac{\partial}{\partial z}\left(\frac{1}{z^3}\right)\right]\hat{z}$

$= \frac{\mu_0}{4\pi}(2m_1m_2)[-3\cdot z^{-3-1}]\hat{z}$

$= \frac{3\mu_0}{4\pi}(2m_1m_2)[z^{-4}]\hat{z}$

$$= \boxed{-\frac{3\mu_0}{2\pi}\frac{m_1m_2}{z^4}\hat{z}}$$

→

6.7) uniform magnitisation of cyl. means

volume bond current $= 0$

surface bound current: $\vec{K}_B = \vec{m} \times \vec{n}$

$$= m \hat{z} \times \hat{r}$$

$$\vec{K}_B = m \hat{\phi}$$

This is the same for an infinite solenoid w/ surface current m

mag. field outside $= 0 \quad (r > R)$

mag. field inside $= \mu_0 m \hat{z} \quad (r < R)$

6.16) $\int H \cdot dl = I_{free} \Rightarrow H = \frac{I_{free}}{2\pi s}$

$B = \mu H = \mu_0 (1 + \chi_m) H$

$B = \mu_0 (1 + \chi_m) \frac{I}{2\pi s}$

$M = \chi_m H$

$\Rightarrow \quad M = \frac{I \chi_m}{2\pi s}$

$\bar{J} = \bar{\nabla} \times \bar{M} = \frac{1}{s} \frac{\partial (s M_\phi)}{\partial s} \hat{z} = 0$

$\bar{K}_a = \bar{M} \times \hat{n} \big|_{s=a} = \frac{I \chi_m}{2\pi s} \hat{\phi} \times \hat{s} = \frac{I \chi_m}{2\pi a} \hat{z}$

$\bar{K}_b = \bar{M} \times \hat{n} \big|_{s=b} = \frac{I \chi_m}{2\pi s} \hat{\phi} \times \hat{s} = \frac{-I \chi_m}{2\pi b} \hat{z}$

$I_{tot} = I + \int K dl$

$= I + \int \frac{I \chi_m}{2\pi a} \hat{z} \cdot dl \hat{z}$

$= I + \frac{I \chi_m}{2\pi a} 2\pi a$

$= I(1 + \chi_m)$

$\int \bar{B} \cdot d\bar{l} = \mu_0 I_{enc} \Rightarrow \bar{B} 2\pi s = \mu_0 I (1 + \chi_m)$

$$\Rightarrow \boxed{\bar{B} = \frac{\mu_0 I (1 + \chi_m)}{2\pi s} \hat{\phi}}$$