2.5 $E = \frac{1}{4\pi\epsilon_0}\left[\int \frac{\lambda\, dl}{\mathscr{r}^2}\cos\theta\right]\hat{z}$ $\mathscr{r}^2 = r^2 + z^2$ $\cos\theta = \frac{z}{\mathscr{r}}$

$\int dl = 2\pi r$

$$E = \frac{1}{4\pi\epsilon_0}\frac{\lambda(2\pi r)z}{(r^2+z^2)^{3/2}}\hat{z}$$

2.6 $E_r = \frac{1}{4\pi\epsilon_0}\frac{(\sigma\, dr)2\pi r z}{(r^2+z^2)^{3/2}}$

$$E_d = \frac{1}{4\pi\epsilon_0}2\pi\sigma z\int_0^R \frac{r}{(r^2+z^2)^{3/2}}dr$$
$$= \frac{1}{4\pi\epsilon_0}2\pi\sigma z\left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right)\hat{z}$$

$R \gg z$ $E_p = \frac{1}{4\pi\epsilon_0}2\pi\sigma\hat{z} = \frac{\sigma}{2\epsilon_0}\hat{z}$

$R \ll z$ $\frac{1}{\sqrt{R^2+z^2}} = \frac{1}{z}\left(1+\frac{R^2}{z^2}\right)^{-1/2} \approx \frac{1}{z}\left(1-\frac{1}{2}\frac{R^2}{z^2}\right) \approx \frac{1}{z} - \frac{1}{z} + \frac{1}{2}\frac{R^2}{z^3} = \frac{R^2}{2z^3}$

$E = \frac{1}{4\pi\epsilon_0}\frac{2\pi R^2\sigma}{2z^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{z^2}$ $Q = \pi R^2\sigma$

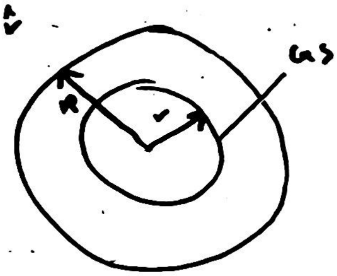
2.9 a) $\rho = \epsilon_0\nabla\cdot E = \epsilon_0\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\cdot kr^3) = \epsilon_0\frac{1}{r^2}k(5r^4) = 5\epsilon_0 k r^2$

b) $Q_{enc} = \epsilon_0\oint E\cdot da = \epsilon_0(kR^3)(4\pi R^2) = 4\pi\epsilon_0 k R^5$

$Q_{enc} = \int\rho\, d\tau = \int_0^R(5\epsilon_0 k r^2)(4\pi r^2 dr) = 20\pi\epsilon_0 k\int_0^R r^4 dr = 4\pi\epsilon_0 k R^5$

2.12 $\oint E\cdot da = E\cdot 4\pi r^2 = \frac{1}{\epsilon_0}Q_{enc} = \frac{1}{\epsilon_0}\frac{4}{3}\pi r^3\rho$ $E = \frac{1}{3\epsilon_0}\rho r\hat{r}$

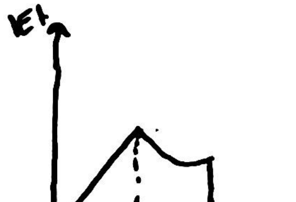
$Q_{tot} = \frac{4}{3}\pi R^3\rho$ $E = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}\hat{r}$

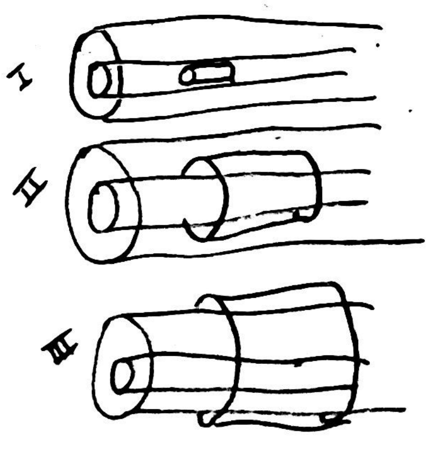


2.16 I. $\oint E\cdot da = E\cdot 2\pi s\cdot l = \frac{1}{\epsilon_0}Q_{enc} = \frac{1}{\epsilon_0}\rho\pi s^2 l$ $E = \frac{\rho s}{2\epsilon_0}\hat{s}$

II. $\oint E\cdot da = E\cdot 2\pi s\cdot l = \frac{1}{\epsilon_0}Q_{enc} = \frac{1}{\epsilon_0}\rho\pi a^2 l$ $E = \frac{\rho a^2}{2\epsilon_0 s}\hat{s}$

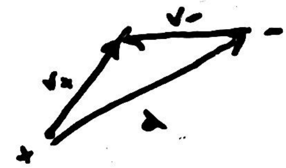
III. $\oint E\cdot da = E\cdot 2\pi s\cdot l = \frac{1}{\epsilon_0}Q_{enc} = 0$ $E = 0$





2.18 $E_+ = \frac{\rho}{3\epsilon_0} r_+ \qquad E_- = -\frac{\rho}{3\epsilon_0} r_- \qquad E = \frac{\rho}{3\epsilon_0}(r_+ - r_-)$

$r_+ - r_- = d \qquad E = \frac{\rho}{3\epsilon_0} d$



2.25 a) $V = \frac{1}{4\pi\epsilon_0} \frac{2q}{\sqrt{z^2 + (d/2)^2}}$

b) $V = \frac{1}{4\pi\epsilon_0} \int_{-L}^{L} \frac{\lambda dx}{\sqrt{z^2+x^2}} = \frac{\lambda}{4\pi\epsilon_0} \ln(x + \sqrt{z^2+x^2})\Big|_{-L}^{L} = \frac{\lambda}{4\pi\epsilon_0} \ln\left(\frac{L+\sqrt{z^2+L^2}}{z}\right)$

c) $V = \frac{1}{4\pi\epsilon_0} \int_0^R \frac{\sigma 2\pi r dr}{\sqrt{r^2+z^2}} = \frac{1}{4\pi\epsilon_0} 2\pi\sigma \left[\sqrt{r^2+z^2}\right]\Big|_0^R = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2} - z\right)$

$E = -\frac{\partial V}{\partial z}\hat{z}$

a) $E = -\frac{1}{4\pi\epsilon_0} 2q\left(-\frac{1}{2}\right) \frac{2z}{(z^2+(d/2)^2)^{3/2}}\hat{z} = \frac{1}{4\pi\epsilon_0} \frac{2qz}{(z^2+(d/2)^2)^{3/2}}\hat{z}$

b) $E = -\frac{\lambda}{4\pi\epsilon_0} \frac{z}{\sqrt{z^2+L^2}} \left(\frac{-L+\sqrt{z^2+L^2} - L - \sqrt{z^2+L^2}}{(z^2+L^2) - L^2} \right) = \frac{2L\lambda}{4\pi\epsilon_0} \frac{1}{z\sqrt{z^2+L^2}}\hat{z}$

c) $E = -\frac{\sigma}{2\epsilon_0}\left(\frac{1}{2}\frac{1}{\sqrt{R^2+z^2}} 2z - 1\right)\hat{z} = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)\hat{z}$

2.29 $\nabla^2 V = \frac{1}{4\pi\epsilon_0} \nabla^2 \int \left(\frac{\rho}{\mathscr{r}}\right) d\tau = \frac{1}{4\pi\epsilon_0} \int \rho(r')\left(\nabla^2 \frac{1}{\mathscr{r}}\right) d\tau$

$= \frac{1}{4\pi\epsilon_0} \int \rho(r')\left(-4\pi\,\delta^3(r-r')\right) d\tau$

$= -\frac{1}{\epsilon_0}\rho(r)$