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HW 6.

6.3| Attraction between m_1 and m_2.

$F = 2\pi I R B\cos\theta$. $\vec{B} = \frac{\mu_0}{4\pi} \frac{3(m_1\cdot\hat{r})\hat{r} - m_1}{r^3}$ $B\cos\theta = \vec{B}\cdot\hat{y}$ here.

$B\cos\theta = \frac{\mu_0}{4\pi}\frac{1}{r^3}\left[3(\vec{m}_1\cdot\hat{r})(\hat{r}\cdot\hat{y}) - (\vec{m}_1\cdot\hat{y})\right]$. $\vec{m}_1\cdot\hat{y} = 0$ and $\hat{r}\cdot\hat{y} = \sin\phi$

$\vec{m}_1\cdot\hat{r} = m_1\cos\theta$

$B\cos\theta = \frac{\mu_0}{4\pi}\frac{3m_1\sin\phi\cos\phi}{r^3}$ So $\vec{F} = 2\pi I R \frac{\mu_0}{4\pi}\frac{1}{r^3} 3m_1\sin\phi\cos\phi$ $\sin\phi = \frac{R}{r}$

$\cos\phi = \frac{\sqrt{r^2-R^2}}{r}$

$\vec{F} = \frac{3\mu_0}{2} m_1 I R^2 \frac{\sqrt{r^2-R^2}}{r^5}$ $m_2 = I R^2 \pi$ so $\vec{F} = \frac{3\mu_0}{2\pi} m_1 m_2 \frac{\sqrt{r^2-R^2}}{r^5}$.

$$\boxed{\vec{F} = \frac{3\mu_0}{2\pi}\frac{m_1 m_2}{r^4}}$$

$F = \nabla(\vec{m}_2\cdot\vec{B}) = (\vec{m}_2\cdot\nabla)\vec{B} = (m_2\frac{d}{dz})\left[3(\vec{m}_1\cdot\hat{z})\hat{z} - \vec{m}_1)\right]$

$= \frac{\mu_0}{2\pi} m_1 m_2 \hat{z}\frac{d}{dz}\left(\frac{1}{z^3}\right)$, let $z = r$,

$$\boxed{\vec{F} = -\frac{3\mu_0}{2\pi}\frac{m_1 m_2}{r^4}\hat{z}}$$

6.7| $\nabla\times\vec{M} = \vec{J}_b = \frac{1}{s}\frac{\partial}{\partial s}(sks^2)\hat{z} = 3ks\hat{z}$ $\vec{K}_b = \vec{M}\times\vec{n} = ks^2(\hat{\phi}\times\hat{s}) = -kR^2\hat{z}$,

$B\cdot 2\pi s = \mu_0 I_{enc} = \mu_0\int_0^s J_b\,da = 2\pi k\mu_0 s^3 \Rightarrow \boxed{\vec{B} = \mu_0 k s^2\hat{\phi}}$

Outside though, $I_{enc} = 0$ so $\boxed{\vec{B} = 0.}$

6.16| $\oint \vec{H}\cdot d\vec{l} = I_{enc}$ $\vec{H} = \frac{I}{2\pi s}\hat{\phi}$. $\vec{B} = \mu_0(1+\chi_m)\vec{H}$

$\boxed{\vec{B} = \mu_0(1+\chi_m)\frac{I}{2\pi s}\hat{\phi}}$ $\vec{M} = \chi_m\vec{H}$ so $\boxed{\vec{M} = \frac{\chi_m I}{2\pi s}\hat{\phi}}$

$\vec{J}_b = \nabla\times\vec{M} = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\chi_m I}{2\pi s}\right)\hat{z} = 0$. So $\boxed{\vec{J}_b = 0}$

$$\boxed{\vec{K}_b = \vec{M}\times\vec{n} = \begin{cases} \frac{\chi_m I}{2\pi a}\hat{z} & s = a \\ -\frac{\chi_m I}{2\pi b}\hat{z} & s = b \end{cases}}$$

Now use a loop between the cylinders.

$I_{enc} + \frac{\chi_m I_{enc}}{2\pi a} = (1+\chi_m)I$

$\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{enc} \Rightarrow \mu_0(1+\chi_m)I \Rightarrow B 2\pi s = \mu_0(1+\chi_m)I$

$$\boxed{\vec{B} = \frac{\mu_0(1+\chi_m)I}{2\pi s}\hat{\phi}}$$