2.43 $\oint E \cdot da = E \cdot 2\pi s L = \frac{1}{\epsilon_0} Q_{enc} \Rightarrow E = \frac{Q}{2\pi\epsilon_0 L}\frac{1}{s}\hat{s}$

$V(b) - V(a) = -\int_a^b E \cdot dl = -\frac{Q}{2\pi\epsilon_0 L}\int_a^b \frac{1}{s}\,ds = -\frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$

$V(a) - V(b) = \frac{Q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$ $\quad C = \frac{Q}{V} = \frac{2\pi\epsilon_0 L}{\ln\left(\frac{b}{a}\right)}$

2.50 $E = -\nabla V = -A\frac{\partial}{\partial r}\left(\frac{e^{-\lambda r}}{r}\right)\hat{r} = -A\left(\frac{r(-\lambda)e^{-\lambda r} - e^{-\lambda r}}{r^2}\right)\hat{r} = Ae^{-\lambda r}(1+\lambda r$

$\rho = \epsilon_0 \nabla \cdot E = \epsilon_0 A\left[e^{-\lambda r}(1+\lambda r)\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) + \frac{\hat{r}}{r^2}\cdot\nabla\left(e^{-\lambda r}(1+\lambda r)\right)\right]$

$\nabla\cdot\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(r)$ $\quad e^{-\lambda r}(1+\lambda r)\delta^3(r) = \delta^3(r)$

$\nabla(e^{-\lambda r}(1+\lambda r)) = \hat{r}\frac{\partial}{\partial r}(e^{-\lambda r}(1+\lambda r)) = \hat{r}(-\lambda e^{-\lambda r}(1+\lambda r) + e^{-\lambda r}\lambda) = \hat{r}(-\lambda^2 r e^{-\lambda r})$

$\frac{\hat{r}}{r^2}\cdot\nabla(e^{-\lambda r}(1+\lambda r)) = -\frac{\lambda^2}{r}e^{-\lambda r}$ $\quad \rho = \epsilon_0 A\left(4\pi\delta^3(r) - \frac{\lambda^2}{r}e^{-\lambda r}\right)$

$Q = \int \rho\, d\tau = \epsilon_0 A\left(4\pi\int\delta^3(r)\,d\tau - \lambda^2\int\frac{e^{-\lambda r}}{r}4\pi r^2\,dr\right)$

$= \epsilon_0 A\left(4\pi - \lambda^2 4\pi\int_0^\infty r e^{-\lambda r}\,dr\right)$

$Q = 4\pi\epsilon_0 A\left(1 - \frac{\lambda^2}{\lambda^2}\right) = 0$

$\int_0^\infty r e^{-\lambda r}\,dr = \frac{1}{\lambda^2}$

3.1 $z < R$, $\sqrt{z^2 + R^2 - 2zR} = (R - z)$ $\quad V_{ave} = \frac{q}{4\pi\epsilon_0}\frac{1}{2zR}((z+R) - (R-z)) = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$

$\frac{1}{4\pi\epsilon_0}\frac{Q_{enc}}{R}$ $\quad V_{ave} = V_{center} + \frac{Q_{enc}}{4\pi\epsilon_0 R}$

3.3 $\nabla^2 V = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dV}{dr}\right) = 0$ $\quad r^2\frac{dV}{dr} = c$ $\quad \frac{dV}{dr} = \frac{c}{r^2}$ $\quad V = -\frac{c}{r} + k$

$\nabla^2 V = \frac{1}{s}\frac{d}{ds}\left(s\frac{dV}{ds}\right) = 0$ $\quad s\frac{dV}{ds} = c$ $\quad \frac{dV}{ds} = \frac{c}{s}$ $\quad V = c\ln s + k$

3.13 $V(x,y) = \sum_{n=1} C_n e^{-n\pi x/a} \sin(n\pi y/a)$. $C_n = \frac{2}{a}\int_0^a V_0(y)\sin(n\pi y/a)\,dy$

$V_0(y) = \begin{cases} +V_0, & 0<y<a/2 \\ -V_0, & a/2<y<a\end{cases}$ $\quad C_n = \frac{2}{a}V_0\left[\int_0^{a/2}\sin(n\pi y/a)\,dy - \int_{a/2}^{a}\sin(n\pi y/a)\,dy\right]$

$= \frac{2V_0}{a}\left[-\frac{\cos(n\pi y/a)}{(n\pi/a)}\Big|_0^{a/2} + \frac{\cos(n\pi y/a)}{(n\pi/a)}\Big|_{a/2}^{a}\right]$

$= \frac{2V_0}{n\pi}\left(1+(-1)^n - 2\cos\left(\frac{n\pi}{2}\right)\right)$

$n=1,\ 0 \quad n=2,\ 4 \quad n=3,\ 0 \quad n=4,\ 0 \quad n=5,\ 0 \quad n=6,\ 4$

$C_n = \begin{cases} 8V_0/n\pi, & n=2,6\ldots \\ 0, & \text{other}\end{cases}$

$V(x,y) = \frac{8V_0}{\pi}\sum_{n=2,6\ldots}\frac{e^{-n\pi x/a}\sin(n\pi y/a)}{a}$

3.14 $V(x,y) = \frac{4V_0}{\pi}\sum_{n=1,3,5\ldots}\frac{1}{n}e^{-n\pi x/a}\sin(n\pi y/a)$ $\quad \sigma = -\epsilon_0\frac{\partial V}{\partial n}$

$\sigma(y) = -\epsilon_0\frac{\partial}{\partial x}\left[\frac{4V_0}{\pi}\sum\frac{1}{n}e^{-n\pi x/a}\sin(n\pi y/a)\right]\Big|_{x=0}$

$= -\epsilon_0\frac{4V_0}{\pi}\sum\frac{1}{n}\left(-\frac{n\pi}{a}\right)e^{-n\pi x/a}\sin(n\pi y/a)\Big|_{x=0}$

$= \frac{4\epsilon_0 V_0}{a}\sum_{n=1,3,5}\sin(n\pi y/a)$

3.15 a) $\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=0 \qquad V(b,y)=V_0(y)$

$V(x,y) = (Ae^{kx}+Be^{-kx})(C\sin ky + D\cos ky)$

$V(x,y) = AC(e^{n\pi x/a} - e^{-n\pi x/a})\sin(n\pi y/a) = (2AC)\sinh(n\pi x/a)\sin(n\pi y/a)$

$V(x,y) = \sum_{n=1}^{\infty} C_n\sinh(n\pi x)\sin(n\pi x/a)$

$\sum C_n\sinh(n\pi b/a)\sin(n\pi y/a) = V_0(y) \qquad C_n\sinh(n\pi b/a) = \frac{2}{a}\int_0 V_0(y)\sin(n\pi y/a)\,dy$

$C_n = \frac{2}{a\sinh(n\pi b/a)}\int_0^a V_0(y)\sin(n\pi y/a)\,dy$

b) $C_n = \frac{2}{a\sinh(n\pi b/a)}V_0\int_0^a\sin(n\pi y/a)\,dy$

$= \frac{2V_0}{a\sinh(n\pi b/a)}\times\begin{cases}0, & n\text{ even}\\ \frac{2a}{n\pi}, & n\text{ odd}\end{cases}$

$V(x,y) = \frac{4V_0}{\pi}\sum_{n=1,3,5}\frac{\sinh(n\pi x/a)\sin(n\pi y/a)}{n\sinh(n\pi b/a)}$