# Electromagnetc Theory: PHYS330

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# Summary

# Week 4 Summary

1. Atoms, polarizations, and dipole moments
2. $\vec{P}$, dipole per unit volume, and bound charges
3. $\vec{D}$, the electric displacement
4. Linear dielectrics

# Atoms, polarizationsn, and dipole moments

# Atoms, polarizations, and dipole moments

Suppose an external field $\vec{E}$ induces a dipole moment $\vec{p}$ in an atomic charge distribution:

$$\boxed{\vec{p} = \alpha\vec{E}} \tag{1}$$

This statement is empirical, but it's true for "ordinary" field strengths: field isn't strong enough to ionize the atom.

# Atoms, polarizations, and dipole moments

**What is the electric field a distance $d$ from the center of a uniformly charged sphere?** *[Hint: use Gauss' law, and assume $\rho$ is constant in spherical coordinates].*

# Atoms, polarizations, and dipole moments

Result:

$$E = \frac{1}{4\pi\epsilon_0}\frac{qd}{a^3} \tag{2}$$

But then assume that $p = qd$, so

$$\alpha = 4\pi\epsilon_0 a^3 \tag{3}$$

| H | He | Li | Be | C | Ne | Na | Ar | K | Cs |
|---|---|---|---|---|---|---|---|---|---|
| 0.667 | 0.205 | 24.3 | 5.60 | 1.67 | 0.396 | 24.1 | 1.64 | 43.4 | 59.4 |

**Figure 1:** Do you understand the units of this table? The numbers are quoted as $\alpha/4\pi\epsilon_0$, *in units of* $10^{-30}$ $m^3$. *What would they be in namometers cubed?*

The trouble is that we cannot easily measure the volume of an atom (realm of quantum mechanics).

## Atoms, polarizations, and dipole moments

Molecules can also have a *permanent* dipole moment: polar molecules.

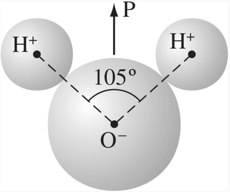


**Figure 2:** How would you calculate the dipole moment here?

Show that the torque on such a molecule in an external field is $\vec{\tau} = \vec{p} \times \vec{E}$ (Professor example).

# Atoms, polarizations, and dipole moments

If you have (approximately) aligned polar molecules with an external field $\vec{E} = E_0\hat{x}$, and then *reverse* the direction of the field, in what direction is the torque? Assume the dipole moments are in the xy-plane.

- A: $-\hat{z}$
- B: $\hat{y}$
- C: $\hat{z}$
- D: The torque is zero

## Atoms, polarizations, and dipole moments

In summary, there are two reasons there could be dipole moments within a material:

1. The atoms are *stretched* and you get an $\alpha = 4\pi\epsilon_0 a^3$
2. The atoms or molecules are *rotated* and you get a dipole moment $\vec{p}$ per atom/molecule.

Macroscopically, it is easier to demonstrate the polar molecule effect: `https://youtu.be/riMrg_kO__w`

# $\vec{P}$, dipole per unit volume, and bound charges

# $\vec{P}$, dipole per unit volume, and bound charges

We need to understand the field of a polarized material. Suppose we introduce the *dipole moment per unit volume*, $\vec{P}$.

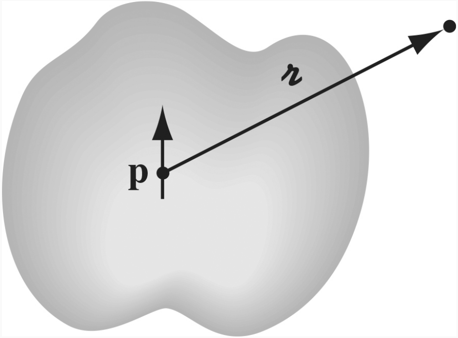


**Figure 3:** The definition of the dipole moment per unit volume, and geometry.

# $\vec{P}$, dipole per unit volume, and bound charges

This implies that

$$\vec{p} = \vec{P} d\tau' \tag{4}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^{\,2}} \tag{5}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\vec{P}(\vec{r'}) \cdot \hat{\boldsymbol{\mathscr{r}}}\ d\tau'}{\mathscr{r}^{\,2}} \tag{6}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \vec{P}(\vec{r'}) \cdot \nabla \left( \frac{1}{\mathscr{r}} \right) d\tau' \tag{7}$$

$$\nabla \cdot (f\vec{A}) = f(\nabla \cdot \vec{A}) + \vec{A} \cdot \nabla(f) \tag{8}$$

$$V(r) = \frac{1}{4\pi\epsilon_0} \left\{ \int_{\mathcal{V}} \nabla' \cdot \left( \frac{\vec{P}}{\mathscr{r}} \right) d\tau' - \int_{\mathcal{V}} \frac{1}{\mathscr{r}} \left( \nabla' \cdot \vec{P} \right) d\tau' \right\} \tag{9}$$

# $\vec{P}$, dipole per unit volume, and bound charges

$$V(r) = \frac{1}{4\pi\epsilon_0}\left\{\int_{\mathcal{V}} \nabla' \cdot \left(\frac{\vec{P}}{\mathcal{r}}\right) d\tau' - \int_{\mathcal{V}} \frac{1}{\mathcal{r}} \left(\nabla' \cdot \vec{P}\right) d\tau'\right\} \tag{10}$$

$$V(r) = \frac{1}{4\pi\epsilon_0}\left\{\oint_{\mathcal{S}} \left(\frac{\vec{P}}{\mathcal{r}}\right) \cdot d\vec{a}' - \int_{\mathcal{V}} \frac{1}{\mathcal{r}} \left(\nabla' \cdot \vec{P}\right) d\tau'\right\} \tag{11}$$

$$d\vec{a}' = da'\hat{n} \tag{12}$$

$$\sigma_b = \vec{P} \cdot \hat{n} \tag{13}$$

$$\rho_b = -\nabla \cdot \vec{P} \tag{14}$$

$$V(r) = \frac{1}{4\pi\epsilon_0}\left\{\oint_{\mathcal{S}} \frac{\sigma_b}{\mathcal{r}} da' - \int_{\mathcal{V}} \frac{\rho_b}{\mathcal{r}} d\tau'\right\} \tag{15}$$

The appearance of *bound charge.*

# $\vec{P}$, dipole per unit volume, and bound charges

Suppose we have a sphere with uniform polarization in the z-direction (and it is constant). The $\rho_b$ is zero because

- A: There is no bound charge inside a sphere.
- B: The divergence of a constant is zero.
- C: By symmetry.
- D: Otherwise the integral over $\rho_b$ would diverge.

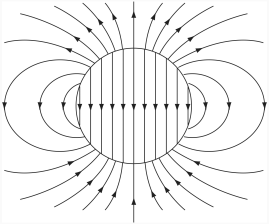

**Figure 4:** The uniformly polarized sphere. $\sigma_b = \vec{P} \cdot \hat{n} = P \cos\theta$.

# $\vec{P}$, dipole per unit volume, and bound charges

Think for a moment: in your own words, why do the field lines point in *opposite* directions just inside and just outside the surface of the sphere?

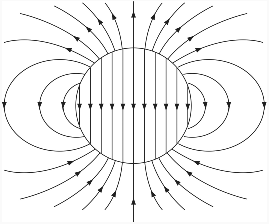

**Figure 5:** The uniformly polarized sphere. $\sigma_b = \vec{P} \cdot \hat{n} = P \cos\theta$.

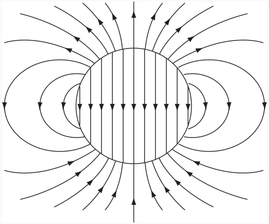

**Figure 6:** The uniformly polarized sphere. $\sigma_b = \vec{P} \cdot \hat{n} = P \cos\theta$.


**Figure 7:** Whenever you think of bound charge density versus surface charge density, think of this picture.

# $\vec{P}$, dipole per unit volume, and bound charges

**Conceptual question:** Given Eq. 15, what is the potential due a disk of surface bound charge $\sigma_b$ at a point slightly above the surface? *[Hint: if it helps, think of* $\sigma_b = \vec{P_0} \cdot \hat{z}$*, where* $P_0$ *is a constant.]*

# $\vec{D}$, the electric displacement

# Conclusion

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