Midterm for Electromagnetic Theory (PHYS330)

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Abstract

This exam may be completed at home, and covers chapters 1-3 of the course text and in-class examples. Class notes and the course text may be used (open book), but no internet sources are allowed. The daily warm-up exercises are good study materials for this exam.

1 Math Bootcamp

1. (a) If $\mathbf{A}$ and $\mathbf{B}$ are two vector functions, what does the expression $(\mathbf{A}\cdot\nabla)\mathbf{B}$ mean? That is, what are its x, y, and z components, in terms of the Cartesian components of $\mathbf{A}$, ∇, and $\mathbf{B}$? (b) Compute $(\hat{r}\cdot\nabla)\hat{r}$, where $\hat{r}$ is $\mathbf{r}/r$. (c) One can show that the *force* on a dipole induced by a non-uniform field is

$$\mathbf{F} = (\mathbf{p}\cdot\nabla)\mathbf{E} \tag{1}$$

Compute the force on a physical dipole located at the origin with $\mathbf{p} = q\mathbf{d} = qd\ \hat{\mathbf{x}}$ in a field with associated potential $V(\mathbf{r}) = V_0 r^2 + V_1$.

2. Evaluate the following integral using (a) the three-dimensional Dirac delta function, or (b) integration by parts. Solving both earns a bonus point.

$$J = \int_{\mathcal{V}} e^{-r}\left(\nabla\cdot\frac{\hat{\mathbf{r}}}{r^2}\right) \tag{2}$$

2 Electrostatics

1. Suppose two dipoles, each with dipole moment $\mathbf{p}$ pointed in opposite directions, form a square with alternating positive and negative charges and side length d. Calculate the field $\mathbf{E}_{\mathrm{tot}}$ at the following points P: (a) $P = (0, 0)$, (b) $P = (2d, 0)$, and $P = (0, 2d)$. Check units and take limits[1].

2. The electric potential of some configuration is given by the expression

$$V(\mathbf{r}) = A\frac{e^{-\lambda r}}{r} \tag{3}$$

In Eq. 3, A and λ are constants. Find the field $\mathbf{E}(\mathbf{r})$, the charge density ρ and the total charge Q in terms of A and λ. *Hint:* $\rho = \epsilon_0 A(4\pi\delta^3(\mathbf{r}) - \lambda^2 \exp(-\lambda r)/r)$. **Bonus:** compute the total energy stored in the field over all space.

3. (a) Use Gauss' Law to compute the field $\mathbf{E}$ as a function of the distance s from a long straight wire with positive charge density λ. (b) Calculate the position versus time of a positive point charge q with mass m if it is released a distance s from the wire.

[1]This object is an electrostatic quadrupole.

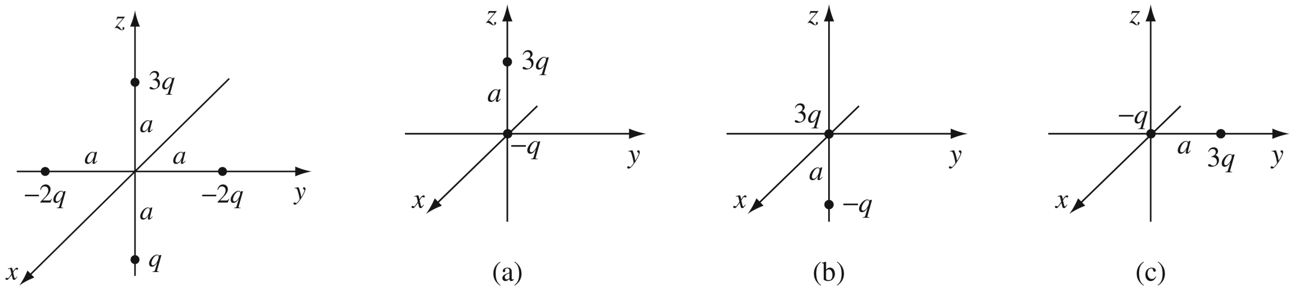

Figure 1: (Left) An arrangement of four charges near the origin. (Right, a-c) An arrangement of two charges near the origin, oriented three different ways.

3 Potentials

1. Suppose the potential $V_0(\theta)$ at the surface of a sphere of radius R is specified, and there is no charge inside or outside the sphere. (a) Show that the charge density on the sphere is given by

$$\sigma(\theta) = \frac{\epsilon_0}{2R} \sum_{l=0}^{\infty} (2l+1)^2 C_l P_l(\cos\theta) \tag{4}$$

$$C_l = \int_0^{\pi} V_0(\theta) P_l(\cos\theta) \sin\theta d\theta \tag{5}$$

(b) Produce the specific result for $\sigma(\theta)$ with $V_0(\theta) = P_2(\cos\theta)$.

2. For the infinite rectangular pipe in Example 3.4 from the text, suppose the constant potential V_0 is now only on one side. That is, at $y = 0$ and $x = \pm b$, the potential is zero. At $y = a$, the potential is V_0. Find the potential $V(x, y)$ inside the pipe. *Square pipes are examples of electromagnetic waveguides often used in microwave electronics.*

3. Consider Fig. 1. Using the monopole and dipole potentials in the multipole expansion, find the approximate potential in spherical coordinates for each charge arrangement, far from the origin. *Note: these arrangements may or may not have a monopole moment in addition to the dipole moment.*