4.1, 4.7, Example 4.2 , 4.10, 4.15, 4.18, EC 4.6 3.24 3.30
reproduce result
Using 3.9 flomckiss

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Phys 330

Homework #4 E&M

4.1). hydrogen atom $r = 1/2$ angstrom $\alpha = .667$ E-field
magnitude of the dipole moment $= p = \alpha E$
atomic polarity

$p = ed$ ← charge of hydrogen atom
separation distance

$\alpha E = ed \quad d = \frac{\alpha E}{e}$

Electric field and the potential is $E = \frac{V}{x}$
table 4.1

$\frac{\alpha}{4\pi\epsilon_0} = .667 \times 10^{-30} m^3 \qquad \alpha = (.667 \times 10^{-30}) 4\pi\epsilon_0$

$e = 1.6 \times 10^{-19}$

$\epsilon_0 = 8.85 \times 10^{-12} \qquad d = \frac{(.667 \times 10^{-30}) 4\pi\epsilon_0) \frac{V}{x}}{e} \qquad x = 1mm$

$$d = \frac{.667 \times 10^{-30} \, 4\pi (8.85 \times 10^{-12})(500)}{(1.6 \times 10^{-19})(1mm)} = \frac{2.31808 \times 10^{-19} m}{10^3 mm}$$

$$\frac{d}{R} = \frac{2.318 \times 10^{-16} m}{.5 \times 10^{-10} m} = 4.64 \times 10^{-6} \text{ unitless?}$$

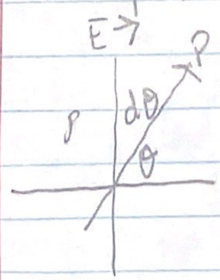
$d = R$

$d = \frac{\alpha 4\pi\epsilon_0 V}{ex}$

$R = \frac{\alpha 4\pi\epsilon_0 V}{ex} \qquad V = \frac{Rxe}{(.667 \times 10^{-30} m^3) 4\pi\epsilon_0}$

$$\frac{(.5 \times 10^{-10})(.001)(1.6 \times 10^{-19})}{(.667 \times 10^{-30}) 4\pi (8.85 \times 10^{-12})} = \boxed{1.078 \times 10^8 V}$$

4.7 $U = -p \cdot E \qquad$ Torque $= pE\sin\theta$



work $= dW = \tau d\theta$
$dW = pE\sin\theta d\theta \qquad W = \int_{\theta_1}^{\theta_2} pE\sin\theta d\theta$
$\cos\theta$

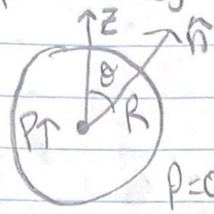
$W = pE(\cos\theta_2 - \cos\theta_1)$

$W = PEU$

$-pE(\cos\theta_2 - \cos\theta_1)$

$\boxed{U = PE\cos\theta}$

EX 4.2). Field produced by polarized sphere Radius R



$\rho = 0$

question about ex 3.9 and reproducing result from boundary conditions

phys 185 final project ideas

a & b coefficients inside and outside of sphere

legendre expansion polynomials r^l $r^{-(l+1)}$

voltage is continuous

σ

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$$V(r,\theta) = \begin{cases} \frac{P}{3\varepsilon_0} r\cos\theta & r \le R \\ \frac{P}{3\varepsilon_0} \frac{R^3}{r^2} \cos\theta, & r > R \end{cases}$$

$r\cos\theta = z$

$E = -\nabla V = -\frac{P}{3\varepsilon_0}\hat{z} = \frac{1}{3\varepsilon_0} P$ for $r < R$ inside *

condition 1 $V_i(R,\theta) = V_0(R,\theta)$

$r = R$

$\frac{\rho}{\varepsilon_0}$ $r2l+1$ $r^{(2l+1)}$

$q = \int \rho d\tau$

$q = \rho \times \frac{4}{3} \times R^3$ $4\pi R^3$

$\int \frac{3V}{4\pi R^3}$ Gauss's law

$\oint E ds = \frac{q\pi^3}{R^3}$

$E \cdot 4\pi r^2 = \frac{q r^2}{R^3}$

$\boxed{E_{inside} = \frac{P}{4\pi\varepsilon_0} \frac{q\pi}{R^3} r}$

$\int E \cdot da$

$\frac{1}{\varepsilon_0}\int \rho d\tau$

$\frac{\rho}{\varepsilon_0} \, 4/5 \times R^3$

4.10). A sphere of radius R carries a polarization $P(r) = Kr$

K is constant

r is vector from center

a). σ_b and ρ_b

b). field inside & outside

Radial $P(r) = KR\hat{r}$ $\hat{n} = \hat{r}$

$\sigma_b = P \cdot n$ $\sigma_b = KR\hat{r} \cdot \hat{r}$

$= KR$ = surface bound charge

volume electric Polarization $P(r) = r^2 Kr$

$\rho_b = -\nabla \cdot P = \frac{1}{r^2}\frac{\partial}{\partial r} P$

4.10). continued

$$\rho_b = -\left\{\frac{1}{r^2}\frac{d}{dr}(r^2 Kr)\right.$$

$$= -\frac{K}{r^2}\frac{d}{dr}(r^3)$$

$$= -\frac{K}{r^2}3r^2$$

$= -3K$ volume bound charge

$\rho_b = -3K$

$\rho = \frac{Q_{in}}{V}$ $\quad Q_{in} = \rho V$

$V = \frac{4}{3}\pi r^3$

$Q_{in} = \rho\left(\frac{4}{3}\pi r^3\right)$ $\quad q_{enc} = (\rho_b)\left(\frac{4}{3}\pi r^3\right)$

$q_{enc} = -(3K)\left(\frac{4}{3}\pi r^3\right)$

$\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$

$E \oint da = \frac{q_{enc}}{\varepsilon_0}$

$= E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0}$

$4\pi r^2 = SA$ $\quad E(\cancel{4\pi r^2}) = -\frac{3K\left(\cancel{4/3\pi r^3}\right)}{\varepsilon_0}$

$\boxed{E(r) = \frac{-Kr}{\varepsilon_0}}$ E field

$q_{tot} = q_{vol} + q_{surface}$

$q_{vol} = (\rho_b)\left(\frac{4}{3}\pi r^3\right)$

$q_{vol} = -(3K)\left(\frac{4}{3}\pi r^3\right)$

$q_{surface} = \sigma_b(4\pi r^2)$

$(KR)(4\pi r^2)$

$q_{tot} = (-3K)\frac{4}{3}\pi R^3 + (KR)(4\pi R^2)$

$= -(\cancel{K}R)(4\cancel{\pi}R^2) + (\cancel{K}R)(4\cancel{\pi}R^2)$

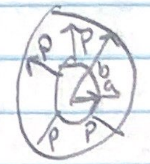
$= 0$

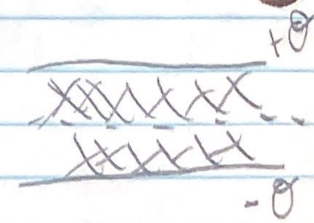
Symmetry

$\oint E \cdot da = \frac{q_{enc}}{E_0}$

$E(4\pi r^2) = \frac{0}{\varepsilon_0}$, constant $\quad \boxed{E = 0}$

4.15). $P(r) = \frac{k}{r}\hat{r}$





a). $\oint D \cdot da = Q_{f\,enc}$

$E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$

$x\frac{d}{dx}\left(\delta(x) = -\delta(x)\right) \quad \int u\,dv = uv - \int v\,du$

$u = x \quad du = \frac{d}{dx}$
$dv = \delta(x) \quad v =$

b). $P(r) = \frac{k}{r}\hat{r}$

$\rho_b = -\nabla \cdot P$

$\rho_b = -\left(\frac{1}{r^2}\frac{d}{dr}(r^2 P)\right)$

$\rho_b = -\left(\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{k}{r}\right)\right)$

$-\frac{d}{dr}\left(\frac{k}{r}\right) = \frac{-k}{r^2}$

$Q_{enc} = 0$

$\int E \cdot da = \frac{q_{enc}}{\varepsilon_0} \quad E(4\pi r^2) = \frac{q}{\varepsilon_0}$

$\boxed{E = 0}$ when $r < a$

$a < r < b$

$q_{enc} = q_{bound} + q_{surface}$

$q_{surf} = \rho_b(4\pi r^2)$

$\frac{k}{r}(4\pi r^2) = q_{surf}$

$q_{bound} = -\left(\frac{k}{r^2}\right) 4\pi r_0^2$

$q_{enc} = \frac{-k}{a}(4\pi a^2) + \int \left(\frac{-k}{r^2}\right) 4\pi r^2 dr$

$= -4\pi k a - k4\pi(r-a)$

$= -4\pi k r$

$\int E \cdot da = \frac{q_{enc}}{\varepsilon_0} = E(4\pi r^2) = \frac{-4\pi k r}{\varepsilon_0} \quad \boxed{E = \frac{-k}{r\varepsilon_0}\hat{r}}$

4.15). continued

when $r>b$

$$q_{enc} = \frac{-k}{a}(4\pi a^2) + \int_a^b \frac{-k}{r^2}(4\pi r^2 dr) + \frac{k}{b}(4\pi b^2)$$

$$= -4\pi k a - 4\pi k(b-a) + 4\pi k b$$

$$= 0$$

$$\int E \cdot da = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = 0$$

$$\boxed{E = 0 \text{ for } r>b}$$

c). $r<a$ $r>b$ $q_{enc} = 0$

Eqn 4.23 $\int D \cdot da = Q_{enc}$ $0 = Q_{enc}$ $\boxed{D=0}$

Eqn 4.21, $D = \varepsilon_0 E + P$

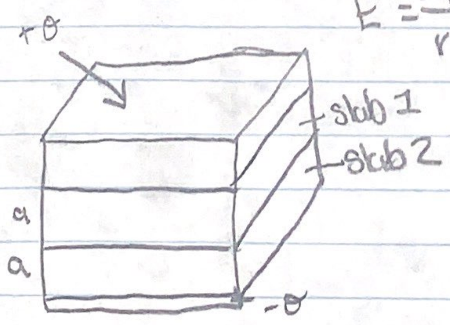
$D=0$ $0 = \varepsilon_0 E + P$

$-E$

$$E = -\left(\frac{1}{\varepsilon_0}\right)P$$

$$E = \frac{-k}{r\varepsilon_0} r \quad = \boxed{E = \left(\frac{-k}{(r\varepsilon_0)} r\right)}$$

4.18).



a). Top plate $\oint D \cdot da = q_{enc}$ $E = \frac{\sigma}{2\varepsilon_0}$ charge density → permitivity

$$E = \frac{\sigma}{2\varepsilon_1} + \frac{\sigma}{2\varepsilon_2}$$

slab 1 slab 2

$K_1\varepsilon_0$ & $K_2\varepsilon_0$

$$E = \frac{\sigma}{2\varepsilon_1} + \frac{\sigma}{2\varepsilon_2}$$

dieletric constants

$K_1 = 2 \quad K_2 = 1.5$

$$= \frac{\sigma}{2(K_1\varepsilon_0)} + \frac{\sigma}{2(K_2\varepsilon_0)}$$

$$E = \frac{\sigma}{2}\left(\frac{1}{2\varepsilon_0} + \frac{1}{1.5\varepsilon_0}\right)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2} + \frac{1}{1.5}\right) = \frac{7\sigma}{12\varepsilon_0}$$

$\boxed{D_{slab\,1} = K_1\varepsilon_0 E}$

$$D = 2\varepsilon_0\left(\frac{7\sigma}{12\varepsilon_0}\right) = \frac{7\sigma}{6}$$

$D_{slab\,2} = K_2\varepsilon_0 E$

$1.5 = K_2 \quad K_1 = 2$

$$D = (1.5)\,\varepsilon_0\left(\frac{7\sigma}{12\varepsilon_0}\right)$$

$$= \frac{3}{2}\varepsilon_0\left(\frac{7\sigma}{12\varepsilon_0}\right)$$

$$\boxed{= \frac{21\sigma}{24}}$$

$$\boxed{\begin{aligned} \text{Slab 1} &= \frac{7\sigma}{6} \\ \text{Slab 2} &= \frac{21\sigma}{24} \end{aligned}}$$

B). $E = \frac{\sigma}{\varepsilon} \quad \varepsilon = \varepsilon_0\varepsilon_r \quad \frac{\sigma}{\varepsilon_0\varepsilon_r} \qquad E_1 = \frac{\sigma}{\varepsilon_0\varepsilon_{r1}}$

$E_{field\ slab\,2} = \frac{\sigma}{\varepsilon_0\varepsilon_{r2}} \quad 1.5$

$$E_2 = \frac{\sigma}{\varepsilon_0(1.5)} \quad \boxed{= \frac{2\sigma}{3\varepsilon_0} = E_{field}}$$

C). Polarization: $P = \varepsilon_0 \chi_e E$ ← suceptibility

$$P = \frac{\varepsilon_0\chi_e\sigma}{\varepsilon}$$

$$P = \frac{\varepsilon_0\chi_e\sigma}{\varepsilon_0\varepsilon_r} \qquad \frac{\chi_e}{\varepsilon_r}\sigma \qquad \varepsilon_r - 1 = \chi_e$$

$$P = \left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)\sigma$$

$$(1 - \varepsilon_r^{-1})\sigma \qquad P_1 = (1 - 2^{-1})\sigma$$

4.18). continued

$P_1 = (1-2^{-1})\sigma$

$\boxed{= \tfrac{1}{2}\sigma \text{ polarization slab 1}}$

Slab 2 $= P = (1-\varepsilon_r^{-1})\sigma \quad P_2 = (1-(1.5)^{-1})\sigma$

$\boxed{= \sigma/3 \text{ slab 2 polarization}}$

D). $V = Ed$

$V = E_1 a + E_2 a$

$= (E_1 + E_2)a \quad \frac{\sigma}{2\varepsilon_0} = E_1 \quad \frac{2\sigma}{3\varepsilon_0} = E_2$

$V = \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{3\varepsilon_0}\right)a$

$\frac{\sigma}{\varepsilon_0}\left[\frac{1}{2} + \frac{2}{3}\right]a = \boxed{\frac{7\sigma}{6\varepsilon_0}a \text{ Potential difference}}$

E). $\sigma_b = P_1 \cdot \hat{n}$ from - to + $\quad \sigma_b = -P_1 = -\frac{\sigma}{2}$

$\sigma_b = P_1 \cdot \hat{n}$

$= P_1 = \frac{\sigma}{2}$

$\sigma_{b2} = P_2 \cdot \hat{n} = P_2 = \sigma/3$

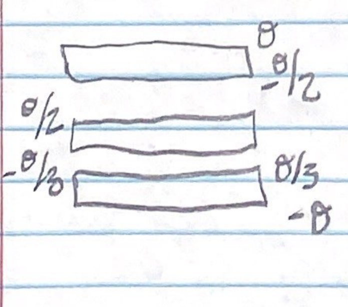
Top slab 2 $\sigma_{b2} = P_2 \cdot \hat{n} = -P_2$

$= -\sigma/3$

$\rho_b = -\frac{\sigma}{2} + \frac{\sigma}{2} + \sigma/3 - \sigma/3$

$\boxed{\rho_b = 0 \text{ Total bound charge}}$

f).



Slab 1 $= \sigma - \sigma/2 = \sigma/2$ above

Slab 1 $= \sigma/2 - \sigma/3 + \sigma/3 = -\sigma/2$ below

Slab 1 surface $= \sigma_1 = \sigma/2 + (\sigma/2) = \sigma$

$\boxed{E_1 = \frac{\sigma_1}{2\varepsilon_0}}$

$E = \frac{\sigma}{2\varepsilon_0}$ Efield for slab 1 = $\boxed{\frac{\sigma}{2\varepsilon_0}}$

$\sigma_{tot} = \sigma/2 + \sigma/2 - \sigma/3 = \frac{2\sigma}{3}$ above

slab 2 $= \left(\sigma/3 - \sigma\right) = -\frac{2\sigma}{3}$ below

$\sigma_2 = \frac{2\sigma}{3} + \frac{2\sigma}{3} = \frac{4\sigma}{3}$ @ surface

magnitude $= E_2 = \frac{\sigma_2}{2\varepsilon_0}$ $\sigma_2 = \frac{4\sigma}{3}$

$E_2 = \frac{\left(\frac{4\sigma}{3}\right)}{2\varepsilon_0}$ $\boxed{= \frac{2\sigma}{3\varepsilon_0} \text{ slab 2 Efield}}$