HW3 2.43, 2.50
3.1, 3, 13, 14, 15

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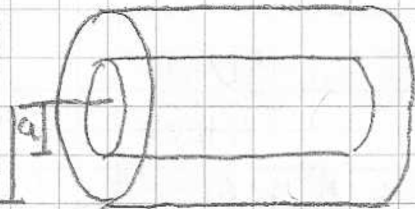
PHYS 330
11/15/2020

2.43.)



$$C = \frac{q}{V} \qquad E = \frac{q}{2\pi s l \varepsilon_0}\hat{s}$$

$$V(a) - V(b) = V = -\int_b^a \frac{q}{2\pi s l \varepsilon_0}\,ds$$

$$= -\frac{q}{2\pi\varepsilon_0 l}\left(\log s\right|_b^a = \frac{q}{2\pi\varepsilon_0 l}(\log a - \log b) = \frac{q}{2\pi\varepsilon_0 l}\ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}}$$

2.50) $V(r) = A\dfrac{e^{-\lambda r}}{r}$ $\qquad E = -\nabla V$

$$= -\frac{\partial}{\partial r}\left(A\frac{e^{-\lambda r}}{r}\right) = -A\frac{\partial}{\partial r}\left(\frac{e^{-\lambda r}}{r}\right)$$

$$= -A\left(\frac{re^{-\lambda r}(-\lambda) - e^{-\lambda r}}{r^2}\right)\hat{r} = \boxed{\frac{Ae^{-\lambda r}(1+r\lambda)}{r^2}\hat{r}}$$

$\rho(r)$: $\rho = \varepsilon_0 \nabla E$ $\quad = \varepsilon_0 \dfrac{\partial}{\partial r}\left(\dfrac{Ae^{-\lambda r}(1+r\lambda)}{r^2}\right)\hat{r}$ $\qquad \nabla\dfrac{\hat{r}}{r^2} = 4\pi\delta^3(r)$

$$= A(-\lambda^2 r e^{-\lambda r})\frac{1}{r^2} = -\frac{A\lambda^2}{r}e^{-\lambda r}$$

$$\boxed{\rho = A\varepsilon_0\left(4\pi\delta^3(r) - \frac{\lambda}{r}e^{-\lambda r}\right)}$$

$$Q = \int \rho\, d\tau = \int A\varepsilon_0\left(4\pi\delta^3(r) - \frac{\lambda}{r}e^{-\lambda r}\right)d\tau$$

$$Q = 0 = \int A\varepsilon_0 4\pi\delta^3(r)\,d\tau - \int A\varepsilon_0\frac{\lambda^2}{r}e^{-\lambda r}\,d\tau$$

$$\boxed{Q = 0}$$

3.1) $V_{ave} = V_{center} + \frac{Q_{enc}}{4\pi\varepsilon_0 R}$ $\qquad V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

$r^2 = z^2 + R^2 - 2Rz\cos\theta \qquad r = \sqrt{z^2+R^2-2zR\cos\theta}$

$V = \frac{q}{4\pi\varepsilon_0\left(z^2+R^2-2zR\cos\theta\right)^{1/2}}$ $\qquad V_{ave} = \frac{\int V\cdot da}{4\pi R^2}$

$da = R^2\sin\theta\, d\theta\, d\phi$

$V_{ave} = \frac{1}{4\pi R^2}\cdot\frac{q}{4\pi\varepsilon_0}\int\left(z^2+R^2-2zR\cos\theta\right)^{1/2} R^2\sin\theta\, d\theta\, d\phi$

$V_{ave} = \frac{1}{4\pi R^2}\frac{qR^2}{4\pi\varepsilon_0}\int\frac{1}{R}\sin\theta\, d\theta\, d\phi = \frac{q}{4\pi\varepsilon_0 R}$

$V_{ave} = V_{center}$ $\qquad \boxed{V_{ave} = V_{center} + \frac{1}{4\pi\varepsilon_0}\frac{Q_{enc}}{R}}$

3.3) $\frac{1}{r^2}\frac{\partial}{\partial r}\underbrace{\left(r^2\frac{\partial V}{\partial r}\right)}_{\text{Constant}} = 0$ $\qquad \partial V = \frac{C}{r^2}\partial r$ $\qquad V = \frac{-C}{r} + \underbrace{B}_{\text{Constant}}$

$\underline{V \text{ depends on } r}$

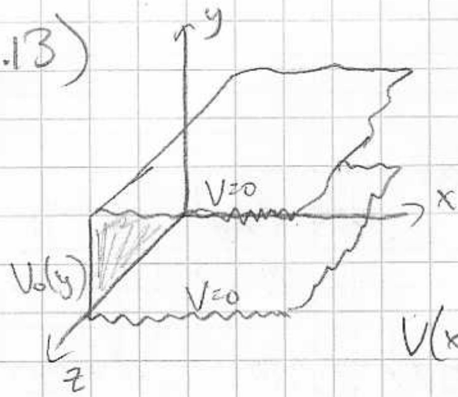
$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial\phi^2} + \frac{\partial^2 V}{\partial z^2} = 0$

$s\frac{\partial V}{\partial s} = \text{Constant} \qquad \frac{\partial V}{\partial s} = \frac{C}{s} \qquad \partial V = \frac{C}{s}\partial s$

$\int\frac{a}{x}dx = a\ln x + C \qquad V = \underbrace{C\ln s + B}_{\text{Constant}}$

$\underline{V \text{ depends on } s}$

3.13)



$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

$$V(x,y) = X(x)\,Y(y)$$

$$V(x,y) = \sum_{n=1}^{\infty} C_n e^{-n\pi x/a} \sin\frac{n\pi y}{a}$$

$$C_n = \frac{2}{a}\int_0^a V_0(y)\sin\left(\frac{n\pi y}{a}\right)dy$$

$$V(0,y) = \begin{cases} +V_0 & 0 < y < a/2 \\ -V_0 & a/2 < y < a \end{cases}$$

$$C_n = \frac{2}{a}\left(\int_0^{a/2} V_0 \sin\frac{n\pi y}{a}\,dy - \int_{a/2}^{a} V_0 \sin\frac{n\pi y}{a}\,dy\right)$$

$$C_n = \frac{2V_0}{n\pi}\left(1 + (-1)^n - 2\cos\frac{n\pi}{2}\right)$$

$n =$ odds $= 0$
$n = 4, 8, 12, \ldots = 0$
$n = 2, 6, 10, \ldots = \frac{8V_0}{n\pi}$

$$V(x,y) = \sum_{n=} C_n e^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)$$

$$= \boxed{\frac{8V_0}{\pi}\sum_{n} \frac{e^{-\frac{\pi n x}{a}}\sin\left(\frac{n\pi y}{a}\right)}{n}}$$

3.14 $$V(x,y) = \frac{4V_0}{\pi}\sum_{n=\text{odds}} \frac{1}{n} e^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)$$

$$\sigma = -\epsilon_0 \frac{\partial V}{\partial n}$$

$$\sigma(y) = -\epsilon_0 \left(\frac{\partial V}{\partial x}\right|_{x=0}$$

$$\sigma(y) = -\epsilon_0 \frac{\partial}{\partial x}\left(\frac{4V_0}{\pi}\sum \frac{1}{n} e^{-\frac{n\pi x}{a}} \sin\left(\frac{n\pi y}{a}\right)\right|_{x=0}$$

$$\boxed{\sigma(y) = \frac{4\epsilon_0 V_0}{a}\sum_{n=\text{odds}} \sin\left(\frac{n\pi y}{a}\right)}$$

3.15) $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$

$V(x,0) = 0$
$V(x,a) = 0$
$V(0,y) = 0$
$V(b,y) = V_0(y)$

$V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$ $\qquad 0 = (Ae^{kx} + Be^{-kx})D$

$D = 0$

$0 = (A+B)C\sin ky \qquad A = -B$

$V(x,y) = AC\left(e^{\frac{n\pi x}{a}} - e^{\frac{-n\pi x}{a}}\right)\sin\frac{n\pi y}{a} = 2AC\sinh\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$

$V(x,y) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right) \qquad V_0(y) = \sum_{n=1}^{\infty} C_n \sinh\left(\frac{n\pi b}{a}\right)\sin\left(\frac{n\pi y}{a}\right)$

Fourier's theorem: $\quad C_n \sinh\left(\frac{n\pi b}{a}\right) = \frac{2}{a}\int_0^a V_0(y)\sin\left(\frac{n\pi y}{a}\right)dy$

$C_n = \frac{2}{a\sinh\left(\frac{n\pi b}{a}\right)}\int_0^a V_0(y)\sin\left(\frac{n\pi y}{a}\right)dy \qquad V_0(y) = V_0$

$V_0(y) = \frac{2V_0}{a\sinh\left(\frac{n\pi b}{a}\right)}\frac{a}{n\pi}\left(-\cos\left(\frac{n\pi y}{a}\right)\right)\Big|_0^a = \frac{2V_0}{a\sinh\left(\frac{n\pi b}{a}\right)}\frac{a}{n\pi}\left(-\cos(n\pi)+1\right)$

$n = \text{even} \quad 1-\cos n\pi = 0$
$n = \text{odd} \quad 1-\cos n\pi = 2$

$C_n = \frac{4V_0}{n\pi\sinh\left(\frac{n\pi b}{a}\right)}$

$$V(x,y) = \frac{4V_0}{\pi}\sum_{n=\text{odd}}\frac{\sinh\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)}{n\sinh\left(\frac{n\pi b}{a}\right)}$$