Homework 4: 4.1, 4.7, Example 4.2, 4.10, 4.15, 4.18

4.1 separation distance d: ?? voltage !?

$p = ed$ $\quad p = \alpha E$

$d = \frac{p}{e}$

$d = \frac{\alpha E}{e}$ $\quad \frac{\alpha}{4\pi\varepsilon_0} = 0.667\times10^{-30}\ m^3$ $\quad e = (1.6\times10^{-19}C)$

$\alpha = (0.667\times10^{-30})(4\pi\varepsilon_0)$

$E = \frac{V}{x} = \frac{500V}{1mm} = \frac{500}{.001m}$

$$d = \frac{(0.667\times10^{-30})(4\pi\varepsilon_0)\left(\frac{500}{.001}\right)}{(1.6\times10^{-19})}$$

$$= \frac{(0.667\times10^{-30})(8.85\times10^{-12})(500000)(4\pi)}{(1.6\times10^{-19})}$$

$$= 2.32\times10^{-16} m$$

$$\frac{d}{R} = \frac{2.32\times10^{-16}}{0.5\times10^{-10}} = \boxed{4.64\times10^{-6}}$$

Voltage ??

$$d = \frac{(0.667\times10^{-30})(8.85\times10^{-12})\left(\frac{V}{x}\right)(4\pi)}{e}$$

$$V = \frac{d\times e}{(0.667\times10^{-30})(8.85\times10^{-12})(4\pi)}$$ $\quad d = R = 0.5\times10^{-10} m$

$$V = \frac{(0.5\times10^{-10})(.001)(1.6\times10^{-19})}{(0.667\times10^{-30})(8.85\times10^{-12})(4\pi)}$$

$$= \boxed{1.08\times10^{8} V}$$

4.7 Show that the energy of an ideal dipole p is an electric field E is given by $U = -p \cdot E$

$\vec{\tau} = pE\sin\theta$

$dw = \tau \, d\theta$

$\int dw = \int \tau \, d\theta$

$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta$

$W = \int_{\theta_1}^{\theta_2} pE\sin\theta \, d\theta$

$W = pE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$

$= pE \left(-\cos\theta \Big|_{\theta_1}^{\theta_2} \right.$

$= pE(-\cos(\theta_2) + \cos(\theta_1))$

$W = \Delta U = U_f - U_0$

$U_f \quad U_0$

$= -pE(\cos(\theta_2) - \cos(\theta_1))$

$= -pE\cos\theta$ dot product rule ; $A \cdot B = AB\cos\theta$

$= -p \cdot E$

4.10 $P(r) = kr$ Radius R

a) $\sigma_b = ??$ $\rho_b = ??$

$\sigma_b = P \cdot \hat{n}$ $P(r) = KR\hat{r}$

$\hat{r} = \hat{n}$

$\sigma_b = (KR\hat{r}) \cdot (\hat{r})$

$\boxed{\sigma_b = KR}$

$\rho_b = ??$ spherical polar coords.

$\rho_b = -\nabla \cdot P$ $P = r^2 Kr$

$= -\left(\frac{1}{r^2}\left(\frac{\partial}{\partial r} P\right)\right)$

$= -\left(\frac{1}{r^2}\left(\frac{\partial}{\partial r} r^2 Kr\right)\right)$

$= -\left(\frac{1}{\cancel{r^2}}\left(3\cancel{r^2}K\right)\right)$

$\boxed{\rho_b = -3K}$

b) Find the field inside and outside the sphere

field inside ??

$\rho = \frac{Q_{in}}{V}$

$Q_{in} = V\rho$ $V = \frac{4}{3}\pi r^3$ $\rho = -3K$

$Q_{in} = \left(\frac{4}{\cancel{3}}\pi r^3\right)(-\cancel{3}K)$

$Q_{in} = -4\pi r^3 K$ $Q_{in} = q_{enc}$

$\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$

$EA = q_{enc}/\varepsilon_0$

$E\,(\cancel{4\pi r^2}) = \frac{-\cancel{4\pi} r^{\cancel{3}}K}{\varepsilon_0}$

$$\boxed{E = \frac{-Kr}{\varepsilon_0}\hat{r}}$$

field outside:??

$$q_{total} = q_{vol} + q_{surface}$$

$$q_{vol} = (\rho_b) V \qquad q_{surface} = (\sigma_b) A$$

$$= -(\cancel{3}k)\left(\frac{4}{\cancel{3}}\pi r^3\right) \qquad = (kR)(4\pi r^2)$$

$$q_{total} = -4\pi r^3 k + 4\pi r^2 R k$$

$$= -4\pi R^3 k + 4\pi R^2 k = 0$$

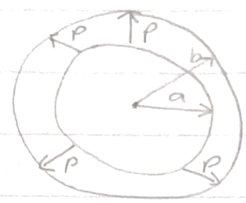
$$\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$$

$$\boxed{E = 0}$$

4.15 $P(r) = \frac{k}{r}\hat{r}$

$\rho_f = 0$ $\sigma_b = \vec{P}\cdot\hat{r}$

$\rho_b = -\nabla\cdot\vec{P}$



$\oint \vec{p}\cdot d\vec{a} = Q_f$

a) Use Gauss's law $\oint E\cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$\sigma_b = \vec{p}\cdot\hat{r}$

$= \frac{k}{r}(\hat{r}\cdot\hat{r}) = \frac{k}{r}$

$r=a$ $\sigma_b = -\frac{k}{a}$ $r=b$ $\sigma_b = \frac{k}{b}$

$\rho_b = -\nabla\cdot\vec{p}$

$= -\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P)\right)$

$= -\left(\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{k}{r}\right)\right)$

$= -\left(\frac{1}{r^2}k\right)$

$r<a$

$\oint E\cdot da = \frac{Q_{enc}}{\varepsilon_0}$ $Q_{enc} = 0$

$E(A) = 0$

$\boxed{E = 0}$

$r>b$

$Q_{enc} = -\frac{k}{a}(4\pi a^2) + \int_a^b \frac{-k}{r^2}(4\pi r^2 dr) + \frac{k}{b}(4\pi b^2)$

$= -4\pi ka - 4\pi k(b-a) + 4\pi kb$

$= 0$

$\oint E\cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$\boxed{E = 0}$

$a<r<b$

$Q_{enc} = Q_{bound} + Q_{surface}$

$Q_{bound} = (\rho_b)(4\pi r^2)$

$= \left(-\frac{k}{\cancel{r^2}}\right)(4\pi \cancel{r^2}) = -4\pi k$

$Q_{enc} = -4\pi ka + \int_a^r -4\pi k\, dr$

$= -\cancel{4\pi ka} - 4\pi kr + \cancel{4\pi ka}$

$= -4\pi kr$

$Q_{surface} = (\rho_b)(4\pi r^2)$

$= \left(-\frac{k}{\cancel{r}}\right)(4\pi r^{\cancel{2}}) = 4\pi kr$

$\oint E\cdot da = \frac{Q_{enc}}{\varepsilon_0}$

$E(\cancel{4\pi r^2}) = \frac{-\cancel{4\pi} k\cancel{r}}{\varepsilon_0}$

$\boxed{E = \frac{-k}{r\,\varepsilon_0}\hat{r}}$

4.15 b) $\oint D \cdot da = Q_f$ $\quad Q_f = 0 \quad r < a \quad r > b$

$Q_{enc} = 0 \quad E = 0$

$D = 0$

$D = \varepsilon_0 E + P$

$0 = \varepsilon_0 E + P$

$E = -\dfrac{P}{\varepsilon_0}$ $\quad P = \frac{k}{r}\hat{r}$

$$\boxed{E = \frac{-k}{r\varepsilon_0}\hat{r}}$$