HW#3: #43, 50, 1, 5, 3, 4, 2

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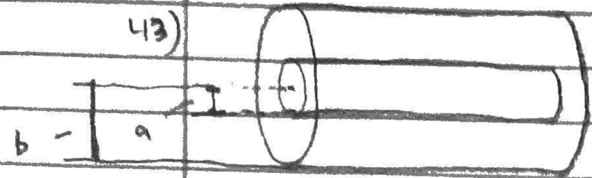
11-16-20

Ch. 2 -

43)



$C = Q/V$

$E = \frac{q}{2\pi s L \varepsilon_0}$; $\Delta V = \int_b^a E \cdot dl = \int_b^a \frac{q}{2\pi\varepsilon_0 L s} ds$ $\Big|_b^a$

$= \frac{q}{2\pi\varepsilon_0 L} \ln\left(b/a\right)$

$C = \frac{2\pi\varepsilon_0 L}{\ln(b/a)} \Rightarrow \boxed{\frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln(b/a)}}$

50) $V(r) = A \cdot \frac{e^{-\lambda r}}{r}$, Find $E(r)$, $\rho(r)$, Q

$\rho = \varepsilon_0 A \left(4\pi\delta^3(r) - \lambda^2 \frac{e^{-\lambda r}}{r}\right.$

a)

$E = -\nabla V$

$= -\partial/\partial r \left(\frac{Ae^{-\lambda r}}{r}\right)\hat{r}$

$= A\left[\frac{-r\lambda e^{-\lambda r} - e^{-\lambda r}}{r^2}\right]\hat{r} = Ae^{-\lambda r}$

$$\boxed{E(r) = \frac{Ae^{-\lambda r}(1 + r\lambda)}{r^2}\hat{r}}$$

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[Ch 3]

1) $V_{ave} = V_{center} + \frac{Q_{enc}}{4\pi\varepsilon_0 R}$, $V = \frac{kq}{r} = \frac{q}{4\pi\varepsilon_0 r}$

$\text{ᴧ}^2 = z^2 + R^2 - 2zR\cos\theta$

$\text{ᴧ} = \sqrt{z^2 + R^2 - 2zR\cos\theta}$

$V_{ave} = \frac{q}{4\pi\varepsilon_0\left(\sqrt{z^2 - R^2 - 2zR\cos\theta}\right)}$

$V_{ave} = \frac{1}{4\pi R^2}\frac{q}{4\pi\varepsilon_0}\int_0^{\pi}\left(z^2 + R^2 - 2zR\cos\theta\right)^{-1/2} R^2\sin\theta\, d\theta\, d\phi$

$= \frac{q}{4\pi\varepsilon_0}\frac{1}{2zR}\left[(z+R)-(z-R)\right] = \frac{1}{4\pi\varepsilon_0}\frac{q}{R} = \frac{q}{4\pi\varepsilon_0 R}$

$\boxed{V_{ave} = V_{center} + \frac{q}{4\pi\varepsilon_0 R}}$

3) $\nabla^2 V = \underbrace{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)}_{\text{only on } r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2 V}{\partial\phi^2}\right)$

$V = V(r)$

$\frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(r^2\cdot\frac{\partial V}{\partial r}\right) = 0 \Rightarrow \int\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right)dr = \int 0$

$r^2\left(\frac{\partial V}{\partial r}\right) = C_1 \Rightarrow \int\frac{dV}{dr}dr = \int\frac{C_1}{r^2}dr$

$$\boxed{V(r) = \frac{-C_1}{r} + C_2}$$

$$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s \cdot \frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\cancel{\left(\frac{\partial^2 V}{\partial \phi^2}\right)} + \cancel{\frac{\partial^2 V}{\partial z^2}}$$

$V = V(s)$

$$\int \frac{\partial}{\partial s}\left(s \cdot \frac{\partial V}{\partial s}\right) ds = \int 0 \, ds \Rightarrow r\frac{\partial V}{\partial s} = C_1$$

$$\int \frac{\partial V}{\partial s} ds = \int \frac{C_1}{s} ds \rightarrow \boxed{V(s) = C_1 \ln(s) + C_2}$$

13) $V = ?$

@ V_0 = constant

$x = 0$; $y = 0$ to $a/2$ @ $y = a/2$ to $y = a$ @ $-V_0$

$V(x,y) = X(x)\, Y(y)$

$$V(x,y) = \sum C_n e^{-n\pi x/a} \sin\left(\frac{n\pi y}{a}\right) \Rightarrow C_n = 2/a \int_0^a V_0(y) \sin\left(\frac{n\pi y}{a}\right) dy$$

$$V(x,y) = \begin{cases} V_0 & 0 < y < a/2 \\ -V_0 & a/2 < y < a \end{cases}$$

$$C_n = \frac{2}{a} \int_0^{a/2} V_0 \cdot \sin\left(\frac{n\pi y}{a}\right) dy - \int_{a/2}^{a} V_0 \sin\left(\frac{n\pi y}{a}\right) dy$$

$$= \frac{2V_0}{n\pi}\left(\left[-\cos\left(\frac{n\pi y}{a}\right)\right]\Bigg|_0^{a/2} - \left[\cos\left(\frac{n\pi y}{a}\right)\right]\Bigg|_{a/2}^{a}\right)$$

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$$\Rightarrow \frac{2V_0}{n\pi}\left(-\cos\left(\frac{n\pi}{2}\right) + \cos(0) + \cos(n\pi) - \cos\left(\frac{n\pi}{2}\right)\right)$$

$$= \frac{2V_0}{n\pi}\left(1 + (-1)^n - 2\cos\left(\frac{n\pi}{2}\right)\right)$$

$n = 1,\ 0$

$n = 2,\ 4$

$n = 3,\ 0$

$n = 4,\ 0$

$n = 6,\ 4$

$n = 2, 6, 10$

$n = 4n - 2$

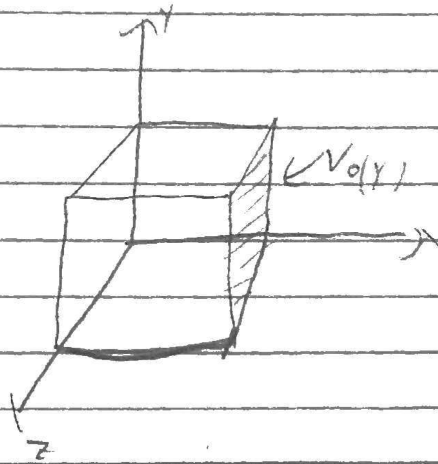
$$C_n = \frac{2V_0}{n\pi}(4) = \frac{8V_0}{n\pi}$$

$$\boxed{V(x,y) = \sum_{n=1}^{\infty} \frac{8V_0}{(4n-2)\pi} \cdot e^{-(4n-2)\pi x/a} \cdot \sin\left(\frac{(4n-2)\pi y}{a}\right)}$$

b) z from $(-\infty, \infty)$; $y=0$, $y=a$ & $x=0$

$x=b$, maintained @ $V_0(y)$

a) $V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$



1st $x=0$, $V=0$

$(A+B)(C\sin ky + D\cos ky) = 0$

$A = -B$

$A(e^{kx} - e^{-kx}) = \underline{\underline{A\sinh(kx)}}$

$= A$

$\boxed{\frac{e^{x} - e^{-x}}{2} = \sinh(x)}$

2nd

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$$\Rightarrow \frac{2V_0}{n\pi}\left(-\cos\left(\frac{n\pi}{2}\right) + \cos(0) + \cos(n\pi) - \cos\left(\frac{n\pi}{2}\right)\right)$$

$$= \frac{2V_0}{n\pi}\left(\underbrace{1 + (-1)^n - 2\cos\left(\frac{n\pi}{2}\right)}\right)$$

$n = 1,\ 0$
$n = 2,\ 4$
$n = 3,\ 0$
$n = 4,\ 0$
$n = 6,\ 4$

$n = 2, 6, 10$

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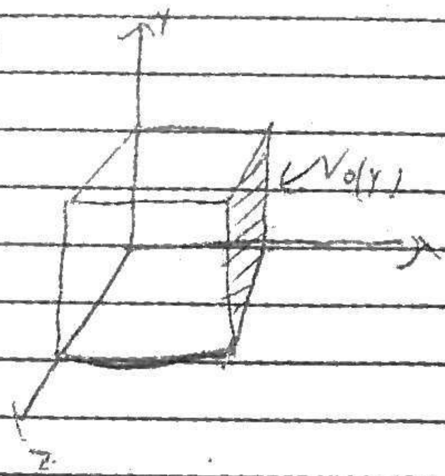
$$C_n = \frac{2V_0}{n\pi}(4) = \frac{8V_0}{n\pi}$$

$$\boxed{V(x,y) = \sum_{n=1}^{\infty} \frac{8V_0}{(4n-2)\pi} \cdot e^{-(4n-2)\pi x/a} \cdot \sin\left(\frac{(4n-2)\pi y}{a}\right)}$$

1b) z from $(-\infty, \infty)$; $y = 0,\ y = a,\ \&\ x = 0$

$x = b$, maintained @ $V_0(y)$

a) $V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$



1st: $x = 0,\ V = 0$

$(A + B)(C\sin ky + D\cos ky) = 0$

$A = -B$

$\frac{e^x - e^{-x}}{2} = \sinh x$

$A(e^{kx} - e^{-kx}) = \underline{A \sinh(kx)}$

$= A$

2nd

$(B = 0)$

$V(x,y) = \sinh(kx)\,(C\sin(ky) + D\cos(ky))$

1. $y = 0,\ V = 0$:

$V(x,y) = C\sinh(kx)\sin(ky)$

2. $y = a,\ V = 0$: $\sin(ka) = 0,\quad ka = n\pi$

$\boxed{k = \frac{n\pi}{a}}$

$V(x,y) = \sum_{n=1}^{\infty} C_n \sinh(n\pi x/a)\sin(n\pi y/a)$

$V(b,y) = \boxed{V_0(y) = \sum_{n=1}^{\infty} C_n \sinh(n\pi b/a)\sin(n\pi y/a)}$

$(x = b)$

$\int_0^a V_0(y)\sin(n\pi y/a)\,dy = C_n \sinh(n\pi b/a)\,(a/2)$

$\boxed{C_n = \frac{2}{a\sinh\left(\frac{n\pi b}{a}\right)}\int_0^a V_0(y)\left(\frac{n\pi y}{a}\right)dy}$

b) $= V_0\left[-\cos(n\pi y/a)\ a/n\pi\right]\Big|_0^a$

n being odd $\Rightarrow \frac{2V_0 a}{n\pi} = \frac{V_0 a}{n\pi}\left(-\cos(n\pi) + 1\right)$

even $n \Rightarrow 0$

$C_n = \frac{2}{a\sinh\left(\frac{n\pi b}{a}\right)} \Rightarrow \frac{4V_0}{n\pi\sinh\left(\frac{n\pi b}{a}\right)}$

$$\boxed{V(x,y) = \sum_{n=1,3,5}^{\infty} \frac{4V_0}{\pi}\,\frac{\sinh\left(\frac{n\pi x}{a}\right)\sin\left(\frac{n\pi y}{a}\right)}{n\sinh(n\pi b/a)}}$$