11/21/20

HW Set 3 # 2.43, 2.50, 3.1, 3.3, 3.13, 3.14, 3.15

2.43 $C = Q/V \qquad E = -\nabla V$

$$V = -\int_b^a \vec{E}\cdot d\vec{l} \qquad \vec{E} = \frac{kQ}{r^2}\hat{r}$$

$$V = -\int_b^a \frac{kQ}{r^2} = +kQ\left[+\frac{1}{r'}\right]_b^a = kQ\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$C = \frac{\cancel{Q}}{k\cancel{Q}\left(\frac{1}{a}-\frac{1}{b}\right)} = \boxed{\left(k\left(\frac{1}{a}-\frac{1}{b}\right)\right)^{-1}}$$

2.50
$$V(\vec{r}) = A\frac{e^{-\lambda r}}{r}$$

find $\vec{E}(\vec{r})$, $\rho(r)$, & Q_{total}

$$\vec{E} = -\nabla V$$

$$\vec{E} = -\left(\frac{\partial V}{\partial r} + \cancel{\frac{1}{r}\frac{\partial V}{\partial \theta}} + \cancel{\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}}\right) = -A\left(\frac{\partial}{\partial r}\left(e^{-\lambda r}\cdot r\right)\right)$$

$$= -A\left(e^{-\lambda r}(1) + r\left(-\lambda e^{-\lambda r}\right)\right) = -A\left(e^{-\lambda r}(1-\lambda r)\right)$$

$$\boxed{\vec{E} = -Ae^{-\lambda r}(1-\lambda r)}$$

$$|\vec{E}| = \left|\frac{kQ}{r^2}\hat{r}\right| \rightarrow E = \frac{kQ}{r^2} \rightarrow Q = \frac{E\cdot r^2}{k} = \frac{-Ae^{-\lambda r}(1-\lambda r)r^2}{k}$$

$$\boxed{Q = \frac{-Ar^2(1-\lambda r)}{ke^{-\lambda r}}}$$

$$\rho(r) = \frac{Q_{total}}{\text{Volume}} = \left(\frac{-Ar^2(1-\lambda r)}{ke^{-\lambda r}}\right)\Big/\text{Volume}$$

$$\boxed{\rho(r) = \frac{-Ar^2(1-\lambda r)}{ke^{-\lambda r}(\text{Volume})}}$$

3.1 $V = (kq)/\imath \qquad \imath^2 = z^2 + R^2 - 2zR\cos\theta \rightarrow \imath = (z^2+R^2-2zR\cos\theta)^{1/2}$

$$V(\vec{r}) = (4\pi R^2)^{-1}\oint V da \qquad da = R^2\sin\theta\, d\theta\, d\phi$$

$$V(\vec{r}) = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{4\pi \cancel{R^2}}\int (z^2+R^2-2zR\cos\theta)^{-1/2}\cancel{R^2}\sin\theta\, d\theta\, d\phi$$

$$= \frac{q\,(\cancel{2\pi})}{\cancel{16\pi^2}\,8\pi\varepsilon_0}\left[\frac{(z^2+R^2-2zR\cos\theta)^{-\frac{1}{2}+1\ \frac{1}{2}}}{\cancel{(-\frac{1}{2}+1)}\,1}\left(\frac{1}{2zR}\right)\right]_0^{\pi}$$

$$= \frac{+q}{8\pi z\varepsilon_0 R}\left[\sqrt{(z^2+R^2+2zR\cos(\pi))} - \sqrt{(z^2+R^2-2zR\cos(0))}\right]$$

$$= \frac{q}{8\pi z\varepsilon_0 R}\left[(\cancel{z}+R)-(\cancel{z}-R)\right] = \frac{(2R)q}{4\,8\pi\varepsilon_0 zR} = \frac{q}{4\pi\varepsilon_0 z} = \frac{kq}{z}$$

$V_{center} = (kq_1)/2$

$\oint E \cdot d\vec{a} = Q_{enc}/\varepsilon_0$

$|E|(4\pi r^2) = Q_{enc}/\varepsilon_0$

$E = \frac{k\,Q_{enc}}{r^2} \rightarrow V = \frac{k\,Q_{enc}}{r}$

$V_{average} = V_{center} + \frac{k\,Q_{enc}}{r}$

3.3 Laplace's Eq : $\nabla^2 V = 0$ $\qquad (\partial^2 V/\partial x^2) = 0$, $V(x) = mx + b$

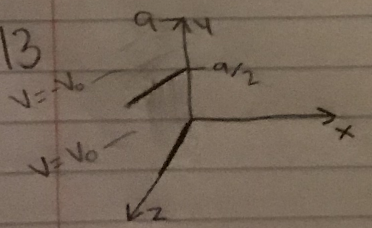
<u>Spherical</u> (depends on r)

$\frac{\partial^2 V}{\partial r^2} = 0$

<u>Cylindrical</u> (depends on s)

$\frac{\partial^2 V}{\partial s^2} = 0$

3.13



$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ (i) V=0 when y=0 (ii) V=0 when $y = \frac{a}{2}$

(iii) $V = V_0$ when x=0 (iv) $V \rightarrow 0$ as $x \rightarrow \infty$

$V(x,y) = X(x)Y(y) \rightarrow Y\frac{d^2V}{dx^2} + X\frac{d^2V}{dy^2} = 0$

$d^2V/dx^2 = k^2 X \longleftarrow X^{-1}(d^2V/dx^2) + Y^{-1}(d^2V/dy^2) = 0$

$d^2V/dy^2 = -k^2 Y \qquad C_1 + C_2 = 0$

$X(x) = Ae^{kx} + Be^{-kx} \qquad Y(y) = C\sin ky + D\cos ky$

$V(x,y) = (Ae^{kx} + Be^{-kx})(C\sin ky + D\cos ky)$