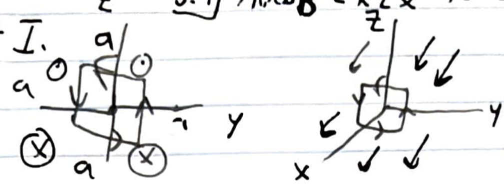
5.4 5.7 5.11 5.12 5.16 5.19 5.21 5.23 5.27 Raymond Hartig

5.12 repeat.

EM HW 5.

5.4| Applied $\vec{B} = kz\hat{x}$ Force on square loop in yz plane.

CC current I.



$F = \int (I \times B) d\ell$ left and right sides cancel by right hand rule.

$\vec{F}_{top} = \int (I \times B) d\ell \Rightarrow \vec{F}_{top} = I\int d\vec{\ell} \times \vec{B}$ I is constant. $= I\int\left(-\hat{y}dy \times \frac{ka}{2}\hat{x}\right)$

$= +I\int_{-a/2}^{a/2} \frac{ka}{2} dy\,\hat{z}$ $\vec{F}_{top} = \frac{Ika^2}{2}\hat{z}$. $\vec{F}_{bottom} = I\int d\vec{\ell} \times \vec{B} = I\int\left(\hat{y}dy \times \frac{-ka}{2}\hat{x}\right)$ $z<0$

$= \frac{Ika^2}{2}\hat{z}$ So $\boxed{\vec{F}_{total} = Ika^2\hat{z}}$

5.7| Show $\int_V \vec{J}\,d\tau = \frac{d\vec{p}}{dt}$. $\vec{p}$ is total dipole moment.

$\frac{d\vec{p}}{dt} = \frac{d}{dt}\int_V \rho\vec{r}\,d\tau = \int \frac{\partial\rho}{\partial t}\vec{r}\,d\tau = -\int (\nabla\cdot\vec{J})\vec{r}\,d\tau$. Product rule 5.

$\nabla\cdot(x\vec{J}) = x(\nabla\cdot\vec{J}) + \vec{J}\cdot(\nabla x)$. $\nabla x = \hat{x}$. so $\nabla\cdot(x\vec{J}) = x(\nabla\cdot\vec{J}) + J_x$

So $\int_V (\nabla\cdot\vec{J})x\,d\tau = \int_V \nabla\cdot(x\vec{J})\,d\tau - \int_V J_x\,d\tau$. $\int_V \nabla\cdot(x\vec{J})\,d\tau = 0$ so

$\int_V (\nabla\cdot\vec{J})x\,d\tau = -\int_V J_x\,d\tau \Rightarrow \int_V (\nabla\cdot\vec{J})\vec{r}\,d\tau = -\int_V \vec{J}\,d\tau$ so $\frac{d\vec{p}}{dt} = \int \vec{J}\,d\tau$.

5.11| $B = \frac{\mu_0 nI}{2}\int \frac{a^2}{(a^2+z^2)^{3/2}}dz$ from equation 5.41. $z = a\cot\theta$. Trig sub.

$dz = -\frac{a}{\sin^2\theta}d\theta$ so $\frac{1}{(a^2+z^2)^{3/2}} = \frac{\sin^3\theta}{a^3}$ So $B = \frac{\mu_0 nI}{2}\int \frac{a^2\sin^3\theta}{a^3\sin^2\theta}(-a\,d\theta)$

$\Rightarrow B = \frac{\mu_0 nI}{2}\int \frac{-a\sin\theta}{a}d\theta \Rightarrow B = \frac{-\mu_0 nI}{2}\int \sin\theta\,d\theta \Rightarrow \boxed{B = \frac{\mu_0 nI}{2}(\cos\theta_2 - \cos\theta_1)}$

Now infinite solenoid. $\theta_2 = 0$ and $\theta_1 = \pi$. $\boxed{B = \mu_0 nI}$

5.12| $dB = \frac{\mu_0 dI}{2}\frac{(R\sin\theta)^2}{[(R\sin\theta)^2+(R\cos\theta)^2]^{3/2}} = \frac{\mu_0}{2R}\sin^2\theta\,dI$ $dI = KR\,d\theta$ $K = \sigma v$ $v = \omega R\sin\theta$ $\sigma = \frac{Q}{4\pi R^2}$

$dI = \frac{Q}{4\pi R^2}\omega R\sin\theta R\,d\theta = \frac{Q\omega}{4\pi}\sin\theta\,d\theta$ $B = \frac{\mu_0}{2R}\frac{Q\omega}{4\pi}\int_0^\pi \sin^3\theta\,d\theta = \frac{\mu_0 Q\omega}{8\pi R}\left(\frac{4}{3}\right)$ $\boxed{\vec{B} = \frac{\mu_0 Q\omega}{6\pi R}\hat{z}}$

5.16 i) $\vec{B} = \mu_0 I(n_2 - n_1)\hat{z}$. ii) $\vec{B} = \mu_0 I n_2 \hat{z}$ iii) $\vec{B} = 0$.

5.19 This does not matter because $\int \vec{J}\cdot d\vec{a}$ works with any arbitrary surface because $\vec{J}$ has no divergence.

5.21 Arrrrmpere's Law consistency with electrodynamics.
Its not consistent because it says $\nabla \times \vec{B} = \mu_0 \vec{J}$ But if you use 5.29 you get $\nabla\cdot(\nabla\times B) = \mu_0 \nabla\cdot\vec{J}$ which is not consistent $\nabla\cdot\nabla\times f = 0$.
The other Maxwell Equations do not have this issue. So Ampere's Law needs some work done to it!

5.23 Get magnetic vector potential of a finite section of a thin straigh wire with current I. Check answer with 5.37.

$$A = \frac{\mu_0}{4\pi}\int \frac{I\hat{z}}{\mathscr{R}}\,dz = \frac{\mu_0 I}{4\pi}\hat{z}\int_{z_1}^{z_2}\frac{dz}{\sqrt{z^2+s^2}} = \frac{\mu_0 I}{4\pi}\hat{z}\left[\ln\left(z+\sqrt{z^2+s^2}\right)\right]\Bigg|_{z_1}^{z_2}$$

So $$\boxed{\vec{A} = \frac{\mu_0 I}{4\pi}\ln\left[\frac{z_2+\sqrt{z_2^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right]\hat{z}.}$$ Now for the B-field.

$B = \nabla\times A \Rightarrow \vec{B} = -\frac{\partial A}{\partial s}\hat{\phi}$. Done in sagemath. Lots of stuff going on in that derivative

$$\vec{B} = \frac{\mu_0 I}{4\pi s}\left[\frac{z_2}{\sqrt{z_2^2+s^2}} - \frac{z_1}{\sqrt{z_1^2+s^2}}\right]\hat{\phi}.$$

Let $\frac{z_2}{\sqrt{z_2^2+s^2}} = \sin\theta_2$ and $\frac{z_1}{\sqrt{z_1^2+s^2}} = \sin\theta_1$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)\hat{\phi}}$$

This holds with 5.37

5.27 Get vector potential below and above plane surface current in 5.8.

$\vec{K} = K\hat{x}$. $\vec{B} = \pm\frac{\mu_0 K}{2}\hat{y}$. + for $z<0$ and − for $z>0$!

$$\vec{B} = \nabla\times\vec{A} = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A(z) & 0 & 0\end{vmatrix} = \frac{\partial A}{\partial z}\hat{y} = \pm\frac{\mu_0 K}{2}\hat{y}$$

A is only going to be in $\hat{x}$ since A is parallel to $\vec{K}$.

$$\frac{\partial A}{\partial z}\cancel{\hat{y}} = \pm\frac{\mu_0 K}{2}\cancel{\hat{y}}$$

$$\frac{\partial A}{\partial z} = \pm\frac{\mu_0 K}{2} \Rightarrow \int \partial A = \pm\int\frac{\mu_0 K}{2}\partial z$$

$$\boxed{\vec{A} = \pm\frac{\mu_0 K}{2}z\hat{x}}$$ $\hat{x}$ because $\vec{K}\propto\hat{x}$!