HW 5
5.14, .16, .17, .19, .20, .23, .26

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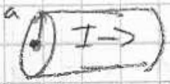
PHYS 330

5.14) Fig 40 

a.) eq. 5.57 $\oint B \cdot dl = \mu_0 I_{enc}$ enclosed current loop is $I_{enc} = 0$

$\oint B\,dl = \mu_0 (0)$

$\boxed{B = 0}$ magnetic field inside wire

$I_{enc} = I$ $B \oint dl = \mu_0 I$ ex 5.7 $\rightarrow$ $\boxed{B = \frac{\mu_0 I}{2\pi s}}$ outside the wire

b.) $I = \int_0^a J \cdot da$ $J \propto s$ $J = ks$ $da = (2\pi s)\, da$

$I = \int_0^a (ks)(2\pi s)\,ds = \frac{2}{3}\pi k a^3$ $k = \frac{3I}{2\pi a^3}$

$I = \int_0^s J \cdot da$ $I_{enc} = \int_0^s (ks)(2\pi s)\,ds = \frac{2\pi k s^3}{3}$

$I_{enc} = \frac{2}{3}\pi\left(\frac{3I}{2\pi a^3}\right) s^3 = I \frac{s^3}{a^3}$

$\oint B \cdot ds = \mu_0 I_{enc}$ $B(2\pi s) = \mu_0 \left(I \frac{s^3}{a^3}\right) = \boxed{\frac{\mu_0 I s^2}{2\pi a^3}}$ inside wire

$B(2\pi s) = \mu_0 (I) = \boxed{\frac{\mu_0 I}{2\pi s}}$ outside wire

5.16) magnetic field due to current carrying solenoid

$B = \mu_0 n I$ inside solenoid

$B = 0$ outside solenoid

magnetic field of outer solenoid $B_1 = \mu_0 n_2 I$

inner solenoid $B_2 = -\mu_0 n_1 I$

$B_i = \mu_0 (n_2 - n_1) I$ of inside the inner solenoid

$B_{ii} = \mu_0 n_2 I$ of outer solenoid

$B_{iii} = 0$ outside both

5.17) $B = \frac{\mu_0 K}{2}$ $K = \sigma v$

a) $B_{net} = \frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} = \mu_0 K = \boxed{\mu_0 \sigma v}$

net magnetic field

b) $F = \int (K \times B)\, da$ $f = (\sigma v \hat{x}) \times \left(\frac{\mu_0 K}{2} \hat{y}\right)$

Lorentz

$= \sigma v \frac{\mu_0 K}{2} \hat{z}$

$\boxed{f_m = \frac{\mu_0 (\sigma v)^2}{2}}$ magnetic force, upwards

c) $E = \frac{\sigma}{2\varepsilon_0}$ $f_e = \frac{\sigma^2}{2\varepsilon_0}$ $f_e = f_m$

$\frac{\sigma^2}{2\varepsilon_0} = \frac{\mu_0 (\sigma v)^2}{2}$ $v^2 = \frac{1}{\mu_0 \varepsilon_0}$ $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$v = \frac{1}{\sqrt{(4\pi \times 10^{-7})(8.85 \times 10^{-12})}} = \boxed{3 \times 10^8 \text{ m/s}}$ speed of upper plate

5.19.) $I_{enc} = \int_S J \cdot da$ Ampere's Law's $\oint B\, dl = \mu_0 \sum I_{enc}$

$\nabla \cdot J = 0$ $I_{enc} = \int_S J \cdot da$

Independent of the surface

Any surface can be used as the integral is independent of the surface.

5.20) a.) $\rho = \frac{\text{charge}}{\text{Volume}} = \left(\frac{\text{charge}}{\text{atom}}\right)\left(\frac{\text{atom}}{\text{mole}}\right)\left(\frac{\text{mole}}{\text{gram}}\right)\left(\frac{\text{gram}}{\text{Volume}}\right) = eN\left(\frac{1}{M}\right)d$

Copper: $e = 1.6 \times 10^{-19}$ C $N = 6 \times 10^{23}$ mol (Avogadro's #)

$M = 64$ gm/mol (atomic mass) $d = 9$ gm/cm^3 (density)

$\boxed{\rho = 1.4 \times 10^4 \text{ C/cm}^3}$ charge density b.) Current density $J = \frac{I}{A} = \frac{I}{\pi r^2} = \frac{1\,A}{\pi(5\times10^{-4}m)^2}$

$J = \rho v \qquad v = \frac{J}{\rho}$ $\qquad = 1.3\,A/m^2$

$= \frac{1.3\,A/m^2\left(\frac{1m}{10^4 cm^2}\right)}{1.4\,C/cm^3} = \boxed{9.1\times10^{-3}\,cm/s}$

c.) $\frac{F_{mag}}{length} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} = \frac{4\pi\times10^{-7}\,H/m}{2\pi}\left(\frac{(1A)(1A)}{1\,cm}\right)$

$\boxed{= 2\times10^{-7}\,N/cm}$

d.) Gauss' $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d} \qquad F_e = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_1\lambda_2}{d} \qquad \lambda = \frac{I}{v} \qquad \lambda_1 = \lambda_2$

$F_e = \frac{1}{v^2}\left(\frac{1}{2\pi\varepsilon_0}\right)\frac{I_1 I_2}{d} \qquad c^2 = \frac{1}{\mu_0\varepsilon_0} \qquad I_1 = I_2$

$F_e = \frac{1}{v^2}\left(\frac{\mu_0 c^2}{2\pi}\right)\left(\frac{I_1 I_2}{d}\right) \qquad \frac{1}{\varepsilon_0} = \mu_0 c^2$

$\frac{F_e}{F_{mag}} = \frac{\frac{1}{v^2}\left(\frac{\mu_0 c^2}{\cancel{2\pi}}\right)\left(\frac{I_1 I_2}{d}\right)}{\frac{\mu_0}{\cancel{2\pi}}\frac{I_1 I_2}{d}} = \frac{c^2}{v^2} = \left(\frac{3\times10^{10}\,cm/s}{9.1\times10^{-3}\,cm/s}\right)^2$

$= 1.1\times10^{25}$ ratio

$(1.1\times10^{25})(2\times10^{-7}\,N/cm) = \boxed{2\times10^{18}\,N/cm}$

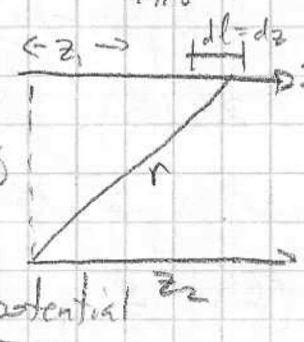
5.23) $A = \frac{\mu_0}{4\pi}\int\frac{I}{r}\,dl' = \frac{\mu_0 I}{4\pi}\int\frac{1}{r}\,dl' \qquad A = \frac{\mu_0}{4\pi}\int\frac{K}{r}\,da'$

$\frac{\mu_0 I}{4\pi s}\int_{\theta_1}^{\theta_2}\cos\theta\,d\theta = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$

magnetic vector potential $\vec{A} = \frac{\mu_0}{4\pi}\int\frac{I}{r}\,dz$

magnetic field due to straight current $\quad B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$

$\vec{A} = \frac{\mu_0}{4\pi}\int\frac{I\hat{z}}{r}\,dz \qquad r = \sqrt{s^2+z^2}$



$\vec{A} = \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2}\frac{dz\,\hat{z}}{\sqrt{s^2+z^2}}$

$= \boxed{\frac{\mu_0 I}{4\pi}\left[\frac{\ln\left(z_2+\sqrt{z_2^2+s^2}\right)}{\ln\left(z_1+\sqrt{z_1^2+s^2}\right)}\right]\hat{z}}$ magnetic potential

5.26 a) $A = A(s)\hat{z}$ $B = \nabla \times A$

vector potential, magnetic field

Cylindrical $\nabla \times A = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sA_\phi) - \frac{\partial A_s}{\partial \phi}\right]\hat{z}$

$\hat{s} = (\cos\phi)\hat{x} + (\sin\phi)\hat{y}$

$\hat{\phi} = -(\sin\phi)\hat{x} + (\cos\phi)\hat{y}$

$\hat{z} = z\hat{z}$

$dl_s = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$dl_s = ds\,\hat{s}$ $\quad dl_\phi = s\,d\phi\,\hat{\phi}$

$A_s = 0 \quad A_\phi = 0 \quad A(s) = A_z$

$\nabla \times A = -\frac{\partial A}{\partial s}\hat{\phi}$ $\quad B = \nabla \times A = -\frac{\partial A}{\partial s}\hat{\phi}$ magnetic field

long wire $B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$ $\quad \frac{-\partial A}{\partial s} = \frac{\mu_0 I}{2\pi s}$ $\quad \partial A = \frac{-\mu_0 I}{2\pi s}\partial s$

$A = -\int_a^s \frac{\mu_0 I}{2\pi s} ds = -\frac{\mu_0 I}{2\pi}\int_a^s \frac{1}{s} ds = \frac{-\mu_0 I}{2\pi}(\ln s)\Big|_a^s = \frac{-\mu_0 I}{2\pi}\ln\left(\frac{s}{a}\right)$

$\nabla \cdot A = \frac{\partial A_z}{\partial z} = 0$ $\quad$ curl of A

$\nabla \times A = -\frac{\partial A_z}{\partial s}\hat{\phi} = \frac{\mu_0 I}{2\pi s}\hat{\phi} = B$

$\boxed{\frac{-\mu_0 I}{2\pi}\ln\left(\frac{s}{a}\right)\hat{z}}$ vector potential

b) $\oint B \cdot dl = \mu_0 I$ $\quad J = \frac{I}{A} = \frac{I}{\pi s^2}$ $\quad I = J(\pi s^2)$

$B(2\pi s) = \mu_0 J \pi s^2$ $\quad J = \frac{I}{\pi R^2}\hat{\phi}$

$B(2\pi s) = \frac{\mu_0 I s}{2\pi R^2}\hat{\phi}$ $\quad B = \frac{-\partial A}{\partial s}\hat{\phi}$

$\partial A = \frac{-\mu_0 I s}{2\pi R^2}\partial s$ $\quad A' = \frac{-\mu_0 I}{2\pi R^2}\int_b^s s\,ds\,\hat{z} = \frac{-\mu_0 I}{2\pi R^2}\left(\frac{s^2}{2}\Big|_b^s\right)\hat{z}$

$= \frac{-\mu_0 I}{4\pi R^2}(s^2 - b^2)\hat{z}$

$\frac{-\mu_0 I}{2\pi}\ln\left(\frac{R}{a}\right) = \frac{-\mu_0 I}{4\pi R^2}(R^2 - b^2)$ $\quad A = \frac{-\mu_0 I}{2\pi}\left(\ln\frac{R}{b}\right)$

$2\ln\left(\frac{R}{a}\right) = \frac{R^2 - b^2}{R^2}$ $\quad A = \frac{-\mu_0 I}{2\pi}\frac{1}{2}\left(1 - \left(\frac{b}{R}\right)^2\right)$

$2\ln\left(\frac{R}{a}\right) = 1 - \frac{b^2}{R^2}$ $\quad A = \frac{-\mu_0 I}{4\pi R^2}(R^2 - b^2)$

$\ln\left(\frac{R}{b}\right) = \frac{1}{2}\left(1 - \left(\frac{b}{R}\right)^2\right)$

$A = \frac{-\mu_0 I}{4\pi R^2}(s^2 - R^2)\hat{z}$ for $s < R$

$A = \frac{-\mu_0 I}{2\pi}\ln\left(\frac{s}{R}\right)\hat{z}$ for $s \geq R$