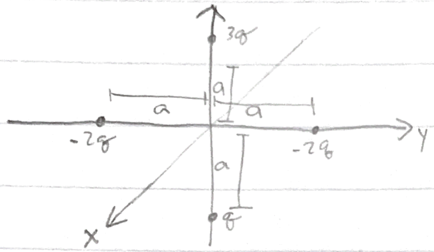
Bonus Points: 3.29, 3.30 and Resubmitted Work: 4.18

3.29)



spherical coords

$x = r\sin\theta\cos\phi$

$q = r\sin\theta\sin\phi$

$z = r\cos\theta$

$$p = \sum_{i=1}^{n} q_i r_i$$

$$p = (3qa\hat{z} - qa\hat{z}) + (-2qa\hat{y} + (-2qa(-\hat{y}))).$$

$$p = 2qa\hat{z} - \cancel{2qa}\hat{y} + \cancel{2qa}\hat{y}$$

$$= 2qa\hat{z}$$

$$p\cdot\hat{r} = (2qa\hat{z})\cdot(\hat{r})$$

$$= 2qa(\hat{z}\cdot\hat{r})$$

$$= 2qa(\hat{r}\cos\theta\cdot\hat{r})$$

$$= 2qa\cos\theta$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cdot\hat{r}}{r^2} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{2qa\cos\theta}{r^2}}$$

3.30) $\sigma = k\cos\theta$

a) $p = p\hat{z}$

$p = \int z\sigma\, da$

$= \int_0^{2\pi}\int_0^{\pi} (R\cos\theta)(k\cos\theta) R^2 \sin\theta \, d\theta\, d\phi$

$= 2\pi R^3 k \int_0^{\pi} \cos^2\theta \sin\theta \, d\theta$ $\quad$ $u = \cos\theta$, $du = -\sin\theta\, d\theta$ $\quad$ $\int -u^2 du$

$= 2\pi R^3 k \left(-\frac{\cos^3\theta}{3} \Big|_0^{\pi} \right.$

$= 2\pi R^3 k \left(-\frac{\cos^3(\pi)}{3} + \frac{\cos^3(0)}{3} \right)$

$= 2\pi R^3 k \left(\frac{1}{3} + \frac{1}{3} \right)$

$\boxed{= \frac{4\pi R^3 k}{3}}$

b) Eq 3.87: $V(r,\theta) = \frac{kR^3}{3\epsilon_0}\frac{1}{r^2}\cos\theta \quad (r \geq R)$

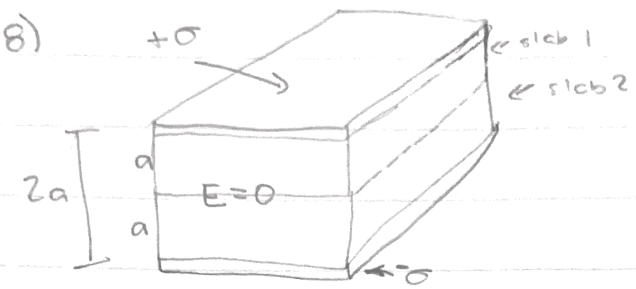
$V = \frac{1}{4\pi\epsilon_0}\frac{p\cdot\hat{r}}{r^2}$ $\quad$ $p = \frac{4}{3}\pi R^3 k\,\hat{z}$

$p\cdot\hat{r} = \frac{4}{3}\pi R^3 k (\hat{z}\cdot\hat{r})$

$V = \frac{1}{\cancel{4}\cancel{\pi}\epsilon_0}\frac{\frac{\cancel{4}}{3}\cancel{\pi}R^3 k\,(z\cdot\hat{r})}{r^2}$

$V = \frac{R^3 k}{3\epsilon_0 r^2}(\cos\theta)$ $\quad$ Higher multipoles will have a potential of zero.

4.18)



a) $\oint D \cdot da = Q_{enc} = 0$ slab 1 = $\varepsilon_1 = K_1 \varepsilon_0$ slab 2 = $\varepsilon_2 = K_2 \varepsilon_0$

$E = \frac{\sigma}{2\varepsilon_0}$ $E = \frac{\sigma}{2\varepsilon_1} + \frac{\sigma}{2\varepsilon_2} = \frac{\sigma}{2(K_1\varepsilon_0)} + \frac{\sigma}{2(K_2\varepsilon_0)}$

$E = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{K_1} + \frac{1}{K_2}\right) = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2} + \frac{1}{1.5}\right)$

$= \frac{\sigma}{2\varepsilon_0}\left(\frac{3}{6} + \frac{4}{6}\right) = \frac{\sigma}{2\varepsilon_0}\left(\frac{7}{6}\right)$

$= \frac{7\sigma}{12\varepsilon_0}$

slab 1: $D = K_1 \varepsilon_0 E$

$= 2\cancel{\varepsilon_0}\left(\frac{7\sigma}{12\cancel{\varepsilon_0}}\right) = \boxed{\frac{7\sigma}{6}}$

Slab 2: $D = K_2 \varepsilon_0 E$

$= (1.5)\cancel{\varepsilon_0}\left(\frac{7\sigma}{12\cancel{\varepsilon_0}}\right) = \boxed{\frac{21\sigma}{24}}$

b) $E = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$

Slab 1:

$E_1 = \frac{\sigma}{\varepsilon_0 \varepsilon_{r_1}} = \boxed{\frac{\sigma}{2\varepsilon_0}}$

Slab 2:

$E_2 = \frac{\sigma}{\varepsilon_0 \varepsilon_{r_2}} = \boxed{\frac{\sigma}{1.5\varepsilon_0}}$

c) $P = \varepsilon_0 \chi_e E$ $E = \frac{\sigma}{\varepsilon}$

$P = \frac{\varepsilon_0 \chi_e \sigma}{\varepsilon}$ $\varepsilon = \varepsilon_0 \varepsilon_r$

$P = \frac{\cancel{\varepsilon_0} \chi_e \sigma}{\cancel{\varepsilon_0} \varepsilon_r} = \frac{\chi_e}{\varepsilon_r}\sigma$ $\chi_e = \varepsilon_r - 1$

$P = \frac{\varepsilon_r - 1}{\varepsilon_r}\sigma = \left(1 - \frac{1}{\varepsilon_r}\right)\sigma$

Slab 1:

$P = \left(1 - \frac{1}{2}\right)\sigma = \boxed{\frac{1}{2}\sigma}$

Slab 2:

$P = \left(1 - \frac{1}{(1.5)}\right)\sigma$

$= \left(\frac{3}{3} - \frac{2}{3}\right)\sigma = \boxed{\frac{1}{3}\sigma}$

d) $V = Ed$ $\qquad E_1 = \frac{\sigma}{2\varepsilon_0} \quad E_2 = \frac{2\sigma}{3\varepsilon_0}$

$V = E_1 a + E_2 a$

$V = \frac{\sigma}{2\varepsilon_0} a + \frac{2\sigma}{3\varepsilon_0} a$

$V = \frac{\sigma}{\varepsilon_0} \left[\frac{1}{2} + \frac{2}{3} \right] a$

$V = \frac{7\sigma}{6\varepsilon_0} a$

e) $\sigma_b = P \cdot \hat{n}$

top slab 1: $\sigma = P_1 \cdot \hat{n}$

$= -P_1 = \frac{-\sigma}{2}$

bottom slab 1: $\sigma = P_1 \cdot \hat{n}$

$= +P_1 = \frac{\sigma}{2}$

top slab 2: $\sigma = P_2 \cdot \hat{n}$

$= -P_2 = \frac{-\sigma}{3}$

bottom slab 2: $\sigma = P_2 \cdot \hat{n}$

$= +P_2 = \frac{\sigma}{3}$

$P_{total} = -\frac{\sigma}{2} + \frac{\sigma}{2} - \frac{\sigma}{3} + \frac{\sigma}{3} = \boxed{0}$

f)