Electromagnetc Theory: PHYS330

Jordan Hanson

October 26, 2020

Whittier College Department of Physics and Astronomy

Summary

Summary

1. Electromagnetism and the module system
    - Pace
    - Style
    - Class decision
2. Challenge level: pre-requisites
    - Passed Calculus 1, 2, and 3
    - Passed Calculus-based physics 1, 2, and 3
    - Passed modern physics
3. Maxwell's equations live in 3D
4. Introduction to Electromagnetism by D. Griffiths (4th ed.)
5. First half of the text is recommended by publisher, retain for graduate school
6. Asynchronous content: www.youtube.com/918particle, and Moodle in folders

Homework

Homework

1. Reading: Chapter 1 by Friday/Saturday
2. Exercises: 1.54, 1.55, 1.56, 1.57, 1.59, 1.62, 1.63, 1.64

Today: the Dirac delta-function

The Dirac δ-function

Consider this function:

$$\vec{v} = \frac{1}{r^2}\hat{r} \tag{1}$$

with $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. What is the divergence?

$$\nabla \cdot \vec{v} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin(\theta)}\frac{\partial}{\partial \theta}(r\sin(\theta)v_\theta) + \frac{1}{r\sin(\theta)}\frac{\partial v_\phi}{\partial \phi} \tag{2}$$

The Dirac δ-function

So we find the divergence is zero. What is the result of a surface integral around the origin?

$$\oint \vec{v} \cdot d\vec{a} = \int_0^{2\pi} \int_0^{\pi} \left(\frac{\hat{r}}{R^2} \right) \cdot (R^2 \sin(\theta) d\theta d\phi \hat{r}) \tag{3}$$

The Dirac δ-function

(Let $d\tau$ be the volume element). Isn't the following *always* supposed to be true?

$$\int (\nabla \cdot \vec{v}) d\tau = \oint \vec{v} \cdot d\vec{a} \tag{4}$$

We must be dealing with a strange function...apparently all of the surface integral contribution comes from the origin, where the volume element is zero, but the function is infinite.

Think of a function that has an finite *integral* result, but is zero everywhere except one point. Nothing comes to mind.

The Dirac δ-function

The Diract δ-function:

$$\delta(x) = 0 \quad if\, x \neq 0 \tag{5}$$

$$\delta(x) = \infty \quad if\, x = 0 \tag{6}$$

This function is called a *distribution*, not a real function. However, it has interesting properties:

$$f(x)\delta(x) = f(0)\delta(x) \tag{7}$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \tag{8}$$

$$\int_{-\infty}^{\infty} f(x)\delta(x) dx = f(0) \tag{9}$$

$$\int_{-\infty}^{\infty} f(x)\delta(x-a) dx = f(a) \tag{10}$$

The Dirac δ-function

Show that

$$\delta(kx) = \frac{1}{|k|}\delta(x) \tag{11}$$

One more interesting thing

What is this integral?

$$\int_0^{2\pi} \sin(nx)\sin(mx)dx \tag{12}$$

Conclusion

Summary

1. Electromagnetism and the module system
    - Pace
    - Style
    - Class decision
2. Challenge level: pre-requisites
    - Passed Calculus 1, 2, and 3
    - Passed Calculus-based physics 1, 2, and 3
    - Passed modern physics
3. Maxwell's equations live in 3D
4. Introduction to Electromagnetism by D. Griffiths (4th ed.)
5. First half of the text is recommended by publisher, retain for graduate school