HW 4

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PHYS 330

4.1, 4.7, ex 4.2, 4.10, 4.15, 4.18

4.1) $p = \alpha E$ $\qquad p = ed$ — separation distance; charge of H atom

dipole momentum, atomic polarizability, Electric field

$$ed = \alpha E \qquad d = \frac{\alpha E}{e} \qquad E = \frac{V}{x}$$

Table 4.1 $\qquad \alpha = (0.667\times10^{-30}\,m^3)(4\pi\varepsilon_0)$

$$d = \frac{(0.667\times10^{-30})(4\pi\varepsilon_0)\left(\frac{V}{x}\right)}{e}$$

$\varepsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{N\cdot m^2}$

$V = 500\,V$

$x = 1\,mm$

$$d = \frac{(0.667\times10^{-30})\,4\pi\,(8.85\times10^{-12})(500)}{(1.6\times10^{-19})(1\times10^{-3})}$$

$$= 2.32\times10^{-16}\,m \qquad \frac{d}{R} = \frac{2.32\times10^{-16}\,m}{0.5\times10^{-10}\,m} = \boxed{4.64\times10^{-6}}$$

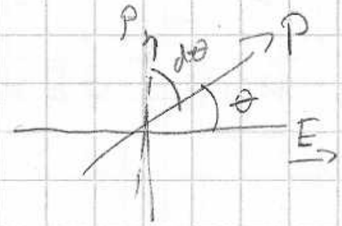
$$R = \frac{(0.667\times10^{-30})\,4\pi\varepsilon_0 V}{ex} \qquad V = \frac{Rxe}{(0.667\times10^{-30})\,4\pi\varepsilon_0}$$

$$V = \frac{(0.5\times10^{-10})(1\times10^{-3})(1.6\times10^{-19})}{(0.667\times10^{-30})\,4\pi\,(8.85\times10^{-12})} = \boxed{1.08\times10^{8}\,V}$$

4.7) $\underline{U = -p\cdot E}$

Torque $\tau = pE\sin\theta$

Work $dw = \tau\, d\theta$



$dw = pE\sin\theta\, d\theta$

$$w = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2) = U(\theta_2) - U(\theta_1)$$

$$\boxed{U = -pE\cos\theta = -p\cdot E}$$

4.10) $P(r) = kr$

a) $P(r) = kR\hat{r}$ $\quad \hat{n} = \hat{r}$ $\quad$ eq 4.11 $\sigma_b = P \cdot \hat{n}$

$$\sigma_b = kR\hat{r} \cdot \hat{r}$$

$$\boxed{\sigma_b = kR}$$

$P(r) = r^2 kr$ $\quad \rho_b = -\nabla \cdot P \overset{eq. 4.12}{=} -\left(\frac{1}{r^2}\frac{\partial}{\partial r} P\right)$

$$\rho_b = -\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 kr)\right) = -\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^3)\right)$$

$$= -\left(\frac{k}{r^2} 3r^2\right) = \boxed{-3k = \rho_b}$$

b) $\rho = \frac{Q_{inside}}{V}$ $\quad Q = \rho V$ $\quad V = \frac{4}{3}\pi r^3$

$$Q = \rho\left(\frac{4}{3}\pi r^3\right) \qquad q_{enc} = (\rho_b)\left(\frac{4}{3}\pi r^3\right)$$

$$q_{enc} = -(3k)\left(\frac{4}{3}\pi r^3\right)$$

Guass Law $\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$ $\quad E(4\pi r^2) = -\frac{3k\left(\frac{4}{3}\pi r^3\right)}{\varepsilon_0}$

$$\boxed{E(r) = \frac{-kr}{\varepsilon_0}\hat{r}} \quad \text{inside}$$

$q_{total} = q_{vol} + q_{surface}$ $\quad q_{vol} = -(3k)\left(\frac{4}{3}\pi r^3\right)$

$q_{tot} = (-3k)\frac{4}{3}\pi R^3 + (kR)(4\pi R^2) = 0$ $\quad q_{surface} = (kR)(4\pi r^2)$

Gauss $\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$ $\quad E(4\pi r^2) = \frac{0}{\varepsilon_0} = \boxed{0}$ outside

4.15) $P(r) = \frac{k}{r}\hat{r}$

a) Eq. 2.13 $\oint E \cdot da = \frac{1}{\varepsilon_0} Q_{enc}$ eq. 4.11 $\sigma_b = P \cdot n$

$\sigma_b = \frac{k}{r}(\hat{r} \cdot \hat{r}) = \frac{k}{r}$

$r = a$ $r = b$

$\sigma_b = -\frac{k}{a}$ $\sigma_b = \frac{k}{b}$ eq 4.12 $\rho_b = -\nabla \cdot P$

$= -\left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P)\right) = \frac{-k}{r^2}$

Gauss $\int E\, da = \frac{q_{enc}}{\varepsilon_0}$ $E = \frac{0}{4\pi\varepsilon_0 r^2}$

$\boxed{r < a \quad E = 0}$

$a < r < b$: $q_{enc} = q_{bound} + q_{surface}$ $q_{surface} = \frac{k}{b}(4\pi r^2)$

$q_{bound} = \left(-\frac{k}{r^2}\right)(4\pi r^2)$

$q_{enc\,bound} = -4\pi k r$

Gauss: $\boxed{E = \frac{-k}{r\varepsilon_0} r \quad a < r < b}$ $E(4\pi r^2) = \frac{-4\pi k r}{\varepsilon_0}$

$r > b$

$q_{enc} = \frac{-k}{a}(4\pi a^2) + \int_a^b \frac{-k}{r^2}(4\pi r^2 dr) + \frac{k}{b}(4\pi b^2) = 0$

Gauss: $\int E \cdot da = \frac{q_{enc}}{\varepsilon_0} = 0$ $\boxed{E = 0 \quad r > b}$

b) $r < a$ & $r > b$

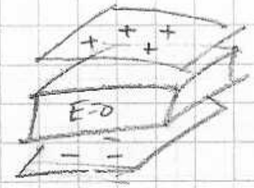
$Q_{f\,enc}$ eq 4.23 $\int D \cdot da = Q_{f\,enc}$ $D = 0$

$\boxed{\begin{array}{c} E = 0 \\ r < a \quad r > b \end{array}}$

eq 4.21: $D = \varepsilon_0 E + P$ $D = 0$

$E = -\left(\frac{1}{\varepsilon_0}\right) P$ $\boxed{\begin{array}{c} E = \frac{-k}{r\varepsilon_0} r \\ a < r < b \end{array}}$

4.18) a) $\oint D \cdot da = q_{enc} = 0$



$$E = \frac{\sigma}{2\varepsilon_0} \qquad E = \frac{\sigma}{2\varepsilon_1} + \frac{\sigma}{2\varepsilon_2}$$

$$E = \frac{\sigma}{2(k_1\varepsilon_0)} + \frac{\sigma}{2(k_2\varepsilon_0)} = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{2\varepsilon_0} + \frac{1}{1.5\varepsilon_0}\right) = \frac{7\sigma}{12\varepsilon_0} \qquad D_{slab1} = k_1\varepsilon_0 E$$

$$D = 2\varepsilon_0\left(\frac{7\sigma}{12\varepsilon_0}\right) = \boxed{\frac{7\sigma}{6}} \text{ slab1} \qquad D = 1.5\varepsilon_0\left(\frac{7\sigma}{12\varepsilon_0}\right) = \boxed{\frac{21\sigma}{24}} \text{ slab2}$$

b.) $$E = \frac{\sigma}{\varepsilon_0\varepsilon_r} \qquad E_1 = \frac{\sigma}{\varepsilon_0\varepsilon_{r_1}} = \boxed{\frac{\sigma}{2\varepsilon_0}} \text{ slap1} \qquad E_2 = \boxed{\frac{\sigma}{1.5\varepsilon_0}} \text{ slap2}$$

c.) $$P = \varepsilon_0 \chi_e E = \frac{\varepsilon_0\chi_e\sigma}{\varepsilon} = \frac{\varepsilon_0\chi_e\sigma}{\varepsilon_0\varepsilon_r} = \frac{\chi_e}{\varepsilon_r}\sigma$$

$$E = \frac{\sigma}{\varepsilon}$$

$$P = \left(\frac{\varepsilon_r - 1}{\varepsilon_r}\right)\sigma = (1-\varepsilon_r^{-1})\sigma$$

Slab 1: $P_1 = (1-2^{-1})\sigma = \boxed{\frac{1}{2}\sigma}$ Slab = $P_2 = (1-(1.5)^{-1})\sigma = \boxed{\frac{\sigma}{3}}$

d.) $$V = Ed = E_1 a + E_2 a = (E_1 + E_2)a \qquad E_1 = \frac{\sigma}{2\varepsilon_0} \quad E_2 = \frac{2\sigma}{3\varepsilon_0}$$

$$V = \left(\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{3\varepsilon_0}\right)a = \boxed{\frac{7\sigma}{6\varepsilon_0}a}$$ potential difference between 1 & 2

e.) $$\sigma_b = P_e \cdot \hat{n} \qquad \sigma_b = -P_1 = \frac{-\sigma}{2} \qquad \sigma_b' = P_1 \cdot \hat{n} = P_1 = \frac{\sigma}{2}$$

$$\sigma_{b2} = P_2 \cdot \hat{n} = P_2 = \frac{\sigma}{3} \qquad \sigma_{b2}' = P_2 \cdot \hat{n} = -P_2 = -\frac{\sigma}{3}$$

$$P_b = \frac{-\sigma}{2} + \frac{\sigma}{2} + \frac{\sigma}{3} - \frac{\sigma}{3} = \boxed{0}$$ bond charge

f.) total surface charge $\sigma - \frac{\sigma}{2} = \frac{\sigma}{2}$ slab 1 above

$\frac{\sigma}{2} - \frac{\sigma}{3} + \frac{\sigma}{3} - \sigma = \frac{-\sigma}{2}$ slab 1 below

Slabe 1 $\sigma_1 = \frac{\sigma}{2} - \left(-\frac{\sigma}{2}\right) = \sigma$ $\qquad E_1 = \frac{\sigma_1}{2\varepsilon_0} = \boxed{\frac{\sigma}{2\varepsilon_0}}$

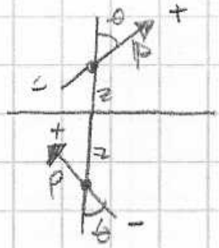
$\sigma = \frac{\sigma}{2} + \frac{\sigma}{2} - \frac{\sigma}{3} = \frac{2\sigma}{3}$ above slab2

$\left(\frac{\sigma}{3} - \sigma\right) = \frac{-2\sigma}{3}$ below slab2 $\qquad E_2 = \frac{\sigma_2}{2\varepsilon_0} = \frac{\frac{4\sigma}{3}}{2\varepsilon_0}$

surface charge: $\sigma_2 = \frac{2\sigma}{3} - \left(-\frac{2\sigma}{3}\right)$ $\qquad = \boxed{\frac{4\sigma}{2\varepsilon_0}}$

$= \frac{4\sigma}{3}$

Bonus 4.6) fig 4.7



$r = 2z$

$\underline{\vec{p}} = (p\cos\theta)\hat{r} + (p\sin\theta)\hat{\theta}$

eq. 3.103

$$\underline{E} = \frac{p}{4\pi\varepsilon_0 \underset{(2z)}{r^3}}\left((2\cos\theta)\hat{r} + (\sin\theta)\hat{\theta}\right)$$

$\vec{N} = \underline{\vec{p}} \times \underline{\hat{E}}$

$$\vec{N} = \left((p\cos\theta)\hat{r} + (p\sin\theta)\hat{\theta}\right) \times \left(\frac{p}{4\pi\varepsilon_0 (2z)^3}\left((2\cos\theta)\hat{r} + (\sin\theta)\hat{\theta}\right)\right)$$

$$\vec{N} = -\left(\frac{p^2 \sin\theta\cos\theta}{4\pi\varepsilon_0 (2z)^3}\right)\phi$$

$$|\vec{N}| = \left| -\left(\frac{p^2\left(\frac{\sin 2\theta}{2}\right)}{4\pi\varepsilon_0 (2z)^3}\right)\hat{\phi}\right| = \boxed{\frac{p^2 \sin 2\theta}{4\pi\varepsilon_0 (16z^3)} \quad \text{magnitude of torque on the dipole.}}$$

torque is positive when $0 < \theta < \frac{\pi}{2}$
↳ dipole rotates <u>counter clockwise</u>, stable @ $\underline{\theta = 0}$

torque is negative $\frac{\pi}{2} < \theta < \pi$
↳ dipole rotates <u>clockwise</u>, stable @ $\underline{\theta = \pi}$