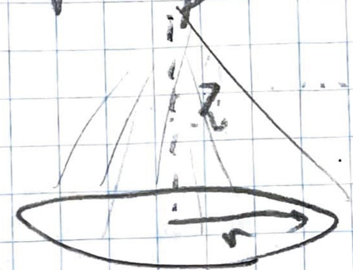
Prob 2.5

find the $\mathcal{E}$ field at dist. z above center in a circ. loop w/ radius "r"

unif. line charge "λ"

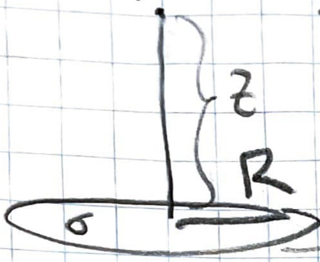


all Horiz. Cancel
due to symmetry

$R^2 = r^2 + z^2$ and $\cos\theta = \frac{z}{R}$, $\int dl = 2\pi r$

$$\boxed{\mathcal{E} = \frac{1}{4\pi\varepsilon_0} \frac{\lambda(2\pi r)}{(r^2+z^2)^{\frac{3}{2}}} \hat{z}}$$

2.6 find E field @ z above center of a disk, w/ radius R, surface chg σ



Horiz. cancel again,

again $R^2 = r^2 + z^2$, Ring $= \pi R^2$

$\sigma 2\pi z\, dR$

$= \lambda 2\pi R$,

$\lambda = \sigma\, dr$

line chg

$$\boxed{E_{ring} = \frac{1}{4\pi\varepsilon_0} \frac{(\sigma\, dr) 2\pi r z}{(r^2+z^2)^{\frac{3}{2}}}}$$

$$E_{disk} = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma z \left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\, \hat{z} \right],$$

when $R \gg z$, term 2 $\to 0$, $E_{plane} = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma \hat{z}$

when $z \gg R$, $\frac{1}{\sqrt{R^2+z^2}} = \left(1+\frac{R^2}{z^2}\right)^{-\frac{1}{2}} \approx \frac{1}{z}\left(1 - \frac{1}{2}\frac{R^2}{z^2}\right)$

$$E = \boxed{\frac{1}{4\pi\varepsilon_0} \frac{2\pi R^2 \sigma}{2z^2}}$$

$\boxed{2.9}$ $\mathcal{E} = kr^3\hat{r}$

a) $\rho = \varepsilon_0 \nabla \cdot \mathcal{E} = \varepsilon_0 \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \cdot kr^3) = \boxed{\varepsilon_0 \frac{1}{r^2} k(5r^4}$

$Q_{enc} = \varepsilon_0 \oint \vec{E} \cdot da = \varepsilon_0 (kR^3)(4\pi R^2) = \boxed{4\pi\varepsilon_0 kR^5}$

$Q_{enc} = \int \rho \, d\tau = \int_0^R (5\varepsilon_0 kr^2)(4\pi r^2 dr) =$

$20\pi\varepsilon_0 k \int_0^R r^4 dr = 4\pi 3_0 kR^5$

HW #2

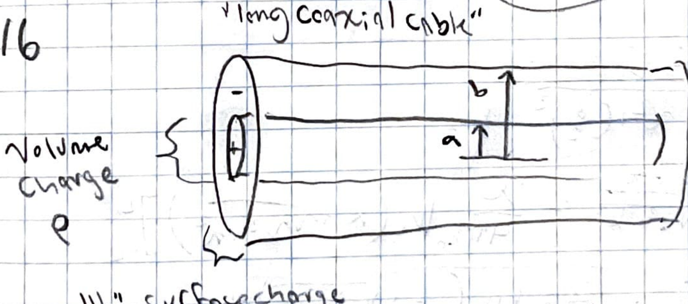
~~2.5~~, ~~2.6~~, 2.9, ~~2.12~~, ~~2.16~~, ~~2.18~~, ~~2.25~~, ~~2.29~~

2.12) use gauss's law to find the electric field inside a uniformly charged solid sphere (charge density ρ)

$$\oint E \cdot da = E(4\pi r^2) = \frac{1}{\varepsilon_0} Q$$

$$= \frac{1}{\varepsilon_0}\frac{4}{3}\pi r^3 \rho \;,\quad \boxed{E = \frac{1}{3\varepsilon_0}\rho r \hat{r}}$$

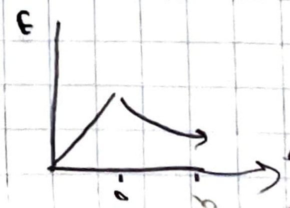
2.16



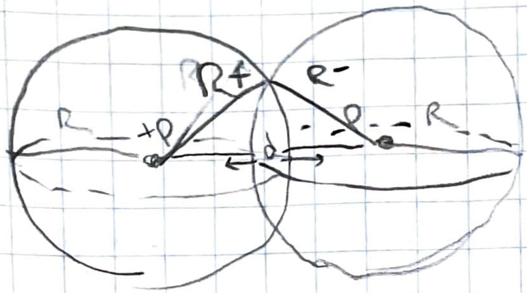
(i) $$\oint E \cdot da = \frac{E \cdot 2\pi s \cdot l}{2\pi s l} = \frac{\frac{1}{\varepsilon_0}\rho \pi s^2 l}{2\pi s l} = \boxed{\frac{\rho s}{2\varepsilon_0}\hat{s}}$$

(ii) $$\oint E \cdot da = E \cdot 2\pi s l = \frac{1}{\varepsilon_0}\rho \pi a^2 l \;,\quad = \boxed{\frac{\rho a^2}{2\varepsilon_0 s}\hat{s}}$$

(iii) $$\frac{\rho s}{2\varepsilon_0} - \frac{\rho a^2}{2\varepsilon_0 s} = 0 \;,\quad \boxed{Q_{enc} = 0}$$



2.18



$$E_1 = \frac{1}{3\varepsilon_0} \rho r \hat{r}$$

$$E_2 = -\frac{1}{3\varepsilon_0} \rho r \hat{r}$$

$$E_{tot} = \frac{\rho}{3\varepsilon_0}(R_+ - R_-)$$

$$R_+ - R_- = d \longrightarrow \boxed{E = \frac{\rho}{3\varepsilon_0} d}$$

2.25

for a) pt 1) $\boxed{V = \frac{1}{4\pi\varepsilon_0} \frac{2q}{\sqrt{x^2 + \left(\frac{d}{2}\right)^2}}}$

b pt 1) $V = \frac{1}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{\lambda\, dx}{\sqrt{z^2 + x^2}} = \frac{\lambda}{4\pi\varepsilon_0} \ln\left(x + \sqrt{z^2 + x^2}\right)\Big]_{-L}^{L}$

$= \frac{\lambda}{4\pi\varepsilon_0} \ln\left[\frac{L + \sqrt{z^2 + L^2}}{-L + \sqrt{z^2 + L^2}}\right] = \boxed{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{L + \sqrt{z^2 + L^2}}{z}\right)}$

c pt 1) $V = \frac{1}{4\pi\varepsilon_0} \int_0^R \frac{\sigma\, 2\pi r\, dr}{\sqrt{r^2 + z^2}} = \frac{1}{4\pi\varepsilon_0} 2\pi\sigma \left(\sqrt{r^2 + z^2}\right)\Big]_0^R$

$$\boxed{= \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + z^2} - z\right)}$$

2.25 pt2

a) $E = -\frac{1}{4\pi\varepsilon_0} 2q\left(-\frac{1}{2}\right)\frac{2x}{\left(x^2+\left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}} = \frac{1}{4\pi\varepsilon_0}\frac{2qz}{\left(z^2+\left(\frac{d}{2}\right)^2\right)^{\frac{3}{2}}}\hat{z}$

(checks out w/ 2.1)

b) $E = -\frac{\lambda}{4\pi\varepsilon_0}\left\{\frac{1}{\left(L+\sqrt{z^2+L^2}\right)}\frac{1}{2}\frac{1}{\sqrt{x^2+L^2}}2z - \frac{1}{\left(-L+\sqrt{z^2+L^2}\right)}\frac{1}{2}\frac{1}{\sqrt{z^2+L^2}}2z\right\}\hat{z}$

$= \frac{\lambda}{4\pi\varepsilon_0}\frac{z}{\sqrt{z^2+L^2}}\left\{\frac{-L+\sqrt{x^2+L^2}-L-\sqrt{z^2+L^2}}{(z^2+L^2)-L^2}\right\}\hat{z}$

$= \frac{2L\lambda}{4\pi\varepsilon_0}\frac{1}{z\sqrt{z^2+L^2}}\hat{z}$ (checks out w/ 2.2)

c) $E = -\frac{\sigma}{2\varepsilon_0}\left\{\frac{1}{2}\frac{1}{\sqrt{R^2+z^2}}2z-1\right\}\hat{z} = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{z}{\sqrt{R^2+z^2}}\right]\hat{z}$

(✓ w/ prob 2.6)

- If R-H charge in (a) is $-q$, $V=0$, suggests $E=-\nabla V=0$, this differs from (2.2)

We know V on z axis, cannot do $E_x = -\frac{\partial V}{\partial x}$ or $E_y = \frac{\partial V}{\partial y}$

This works as symmetry makes E_x, E_y zero so no worries, but now E is in x dir, and we have V on z axis only, cannot figure out E_x

2.29

Poisson's $\nabla^2 \phi = \frac{-\rho}{\varepsilon_0}$

2-29 → $V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{\mathscr{r}} \, d\tau'$

1.102 → $\nabla^2 \frac{1}{\mathscr{r}} = -4\pi \delta^3(\mathscr{r})$

$$\nabla^2 V = \frac{1}{4\pi\varepsilon_0} \nabla^2 \int \frac{\rho(r')}{\mathscr{r}} \, d\tau$$

$$\nabla^2 V = \frac{1}{4\pi\varepsilon_0} \int \rho(r') \left[\nabla^2 \frac{1}{\mathscr{r}}\right] d\tau$$

$$\nabla^2 V = \frac{1}{\cancel{4\pi}\varepsilon_0} \int \rho(r') [-\underbrace{\cancel{4\pi}}_{\text{comes out and cancels}} \delta^3(r-r')] \, d\tau$$

$$\boxed{\nabla^2 V = -\frac{\rho(r')}{\varepsilon_0}}$$