Solutions for Homework 4

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1 Problem 4.10

A sphere of radius R carries a polarization

$$\mathbf{P}(\mathbf{r}) = k\mathbf{r} \tag{1}$$

In Eq. 1, k is a constant and and $\mathbf{r}$ is the vector from the center.

- *(a) Calculate the bound charges σ_b and ρ_b.*
- *(b) Find the field inside and outside the sphere.*

(a) The surface bound charge is at radius R, so $\sigma_b = \mathbf{P}\cdot\hat{\mathbf{n}} = kR$. The volumetric bound charge is $\rho_b = -\nabla\cdot\mathbf{P} = -3k$. Note the total charge should add up to zero: $(4\pi R^2)\sigma_b + (4/3)\pi R^3\rho_b = 4\pi R^3 k - 4\pi R^3 k = 0$. (b) The field of any constant volumetric charge density $-3k$ should be calculable via Gauss' law. We find, after integrating over a Gaussian surface of radius $r < R$:

$$\oint \mathbf{E}\cdot d\mathbf{a} = \mathbf{E}\cdot\mathbf{A} = \frac{1}{\epsilon_0}\rho = \frac{-4k\pi r^3}{\epsilon_0} \tag{2}$$

$$\mathbf{E} = -\frac{3kr}{\epsilon_0}\hat{\mathbf{r}} = -\mathbf{P}/\epsilon_0 \tag{3}$$

(b) Note that, because the net charge is zero, the field outside the sphere is zero.

2 Problem 4.14

When you polarize a neutral dielectric, the charge moves a bit, but the total remains zero. This fact should be reflected in the bound charges σ_b and ρ_b. Prove from Eqs. 4.11 and 4.12 that the toal bound charge vanishes.

First, let's integrate the total volumetric bound charge:

$$-q = \int_{\mathcal{V}} \rho_b d\tau = -\int_{\mathcal{V}} \nabla\cdot\mathbf{P} d\tau = -\oint \mathbf{P}\cdot\hat{\mathbf{n}} da = -\oint \sigma_b da \tag{4}$$

Next, the total surface bound charge is

$$q = \oint_S \sigma_b da \tag{5}$$

Now we see that $Q = -q + q = 0$.

3 Problem 4.15

A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a frozen-in polarization

$$\mathbf{P}(\mathbf{r}) = \frac{k}{r}\hat{\mathbf{r}} \tag{6}$$

In Eq. 6, k is a constant and r is the distance from the center. There is no free charge in the problem. Find the electric field in all three regions by two different methods:

- (a) Locate all the bound charge, and use Gauss' Law to calculate the field it produces.
- (b) Use Eq. 4.23 to find $\mathbf{D}$, and then get $\mathbf{E}$ from Eq. 4.21. [Notice the second method is faster, and it avoids any explicit reference to the bound charges].