Warm-Up for March 7th, 2022

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1 Memory Bank

1. In two dimensions, solutions to the Laplacian at coordinates (x, y) are equal to the average value on a circle of radius R centered around (x, y):

$$V_{\text{ave}} = V(x, y) = \frac{1}{2\pi R} \oint_{\text{circle}} V dl \tag{1}$$

2. Obtaining voltage from field: $V(\mathbf{r}) = -\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{E} \cdot d\mathbf{l}$

2 Solutions to the Laplacian for Potential

1. Consider the following function of s in cylindrical coordinates:

$$V(s) = V_0 \left(\frac{\sin(s)}{s}\right) \tag{2}$$

Note that $V(0) = V_0$. Using Eq. 1, calculate the average value of $V(s)$ on a circle of radius $R = \pi$, centered on $s = 0$. Remark on the viability of Eq. 2 as a solution to Laplace's equation.

2. Recall the example of the neutral coaxial cable from the homework (Fig. 1). (a) Use Gauss' law to find the field $\mathbf{E}$ for $a < s < b$ in cylindrical coordinates. (b) Integrate the field to find the potential for $a < s < b$. (c) Following Eq. 1, compute the average potential over a circle of radius $s = a$. Does Eq. 1 hold? Why or why not?

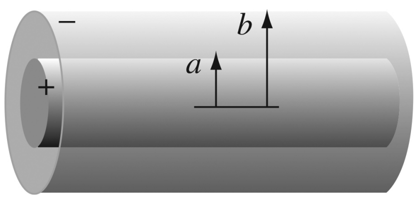

Figure 1: Figure 2.26 from the text, depicting a model for a neutral coaxial cable with static charge.