Hw 4

4.1, 7, ex 4.2, p. 10, 18, 10

1. 1mm

$V = 500V$

$E = \frac{V}{x}$

$\frac{500V}{0.001m} = 5\times10^5 \, V/m$

H: $\alpha / 4\pi\epsilon_0$

$= 0.667 \quad 10^{-30} m^3$

$\alpha = 7.34\times10^{-41}$

$p = \alpha E = ed$

$d = 2\times10^{-16} m$

$R \approx 25pm$

wolfram

$25\times10^{-12} / 2\times10^{-16} = 125000$

$\boxed{125k : 1}$

13.6eV ?

7. $U = -p \cdot E$

$U = V_q - V_{q-}$

$-q \quad d \quad +q$

$\vec{d} = \vec{r}_+ - \vec{r}_-$

$V_q(r+d) - V_q(r)$

$p = qd \qquad p = \alpha E$

$= -q\int_r^{r+d} V\,dx = +q\int E \cdot de$

$= E\, d(q)$

p

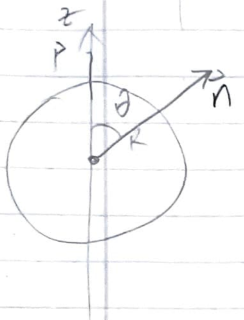
$\boxed{U = -pE}$

ex

Ex 4.2

$\sigma_b = P\cdot\hat{n} = P\cos\theta$

$V = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma_b}{\mathscr{r}}\,da$



$z = r\cos\theta$

$V(r,\theta) = \sum_{l=0}^{\infty} A_l r^l P(\cos\theta)$ inside

$\sum_{l=0}^{\infty} \frac{B}{r^{l+1}} P_l(\cos\theta)$ out

when $r = R$

$V_{in} = V_{out}$ $\therefore$

$A R^l = \frac{B}{R^{l+1}}$ $\qquad B = A R^{2l+1}$

$\frac{\partial}{\partial r}(V_{out} - V_{in}) = -\frac{1}{\epsilon_0}\sigma_b(\theta)$

$\frac{\partial}{\partial r} V_{out} = \sum -(l+1)\frac{B_l}{R^{l+2}} P\cos\theta$

$\frac{\partial}{\partial r} V_{in} = \sum l A_l R^{l-1} P(\cos\theta)$

$A = \frac{1}{2\epsilon_0 R^{l-1}}\int_0^{\pi} \sigma_0(\theta) P_l\cos\theta\sin\theta\,d\theta$

$\hookrightarrow kP\cos\theta$

$= \frac{k}{3\epsilon_0}$ $\quad\therefore\quad V = \frac{k}{3\epsilon_0} r\cos\theta$ inside

$$V_{out} = \frac{k}{3\epsilon_0}\frac{R^3}{r^2}\cos\theta$$

o-tile

$\uparrow\uparrow\uparrow E_0\hat{z}$

field + potential = 0. ∴ $V = E_0 \frac{R^3}{r^2}\cos\theta$.

for given E field.

10. $P(r) = kr$

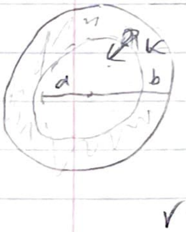
a) $\sigma_b = ?$ $\rho_b = ?$ $\sigma_b = kR$

$= P\hat{n}$ $\rho = -\nabla\cdot P$

$$\rho_b = -\frac{1}{r^2}\frac{d}{dr}\left(r^2(kr)\right) = -\frac{r^2}{r^2}3k = -3k$$

b). $E = \frac{1}{3\epsilon_0}(-3k)\cdot r\hat{r} = -\frac{k}{\epsilon_0} r\hat{r}$

15 $P(r) = \frac{k}{r}\hat{r}$ $\rho = -\frac{1}{r^2}(k) =$



$r<a$ and $r>b$ $E=0$ b/c $Q_{enc}=0$

$Q_{enc} = \sigma(4\pi a^2) + \int_a^r \rho\, d\tau$ $\hat{n} \parallel \hat{r}$

r = a sphere

$\sigma_b = \frac{k}{a}$

$$Q = 4\pi a k + 4\pi k \int_a^r -\frac{1}{r^2}$$

$= 4\pi a k - 4\pi k(r-a)$

$= -4\pi k r - 8\pi k a$

$\frac{Q_{enc}}{\epsilon} = EV$

$$\frac{(-4\pi k r - 8\pi k a)}{\epsilon(4\pi r^2)}$$

$$E = -\frac{(r+2a)}{\epsilon}$$

b) $\oint D\cdot da = Q_{free}$ $\qquad D \equiv \epsilon E + P$

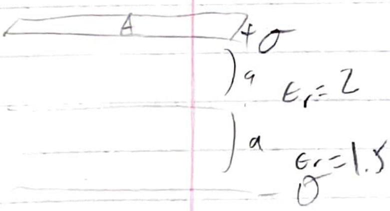
$= 0$ and $D = 0$

$\epsilon E = -P$

$\sigma_b = P\cdot\hat{n} = \frac{k}{a}, -\frac{k}{b}$ $\qquad E = \frac{k}{\epsilon_0 r}\hat{r}$ in shell

$E = 0$ else

18. a) D $\quad \int D\cdot da = Q = D\,\text{area}$



$\sigma A = D\,\text{area}$

$D = \sigma$

b) $E = \frac{D}{\epsilon}$ $\quad \epsilon = \epsilon_0\epsilon_r$ $\quad E_1 = \frac{\sigma}{2\epsilon_0}$ $\quad E_2 = \frac{\sigma}{(1.5)\epsilon_0}$

c) $P = \epsilon_0 \chi_e E$ $\quad P_1 = \frac{\sigma}{2}$ $\quad P_2 = \frac{3}{4}\sigma$

$\chi_e = \epsilon_r - 1$ $\quad \chi_1 = 1$ $\quad \chi_2 = 0.5$ $\quad \frac{3}{2}$

d) $V = E_1 a + E_2 a = \frac{a\sigma}{\epsilon_0}\left(\frac{1}{2} + \frac{2}{3}\right)$

$7/6$

e)