Quiz 1

Eliott Bergerson
PHYS 330

1) 1.) $a(\vec{B}+\vec{C}) = a\vec{B}+a\vec{C}$

$B = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ $\quad C = C_x\hat{i} + C_y\hat{j} + C_z\hat{k}$

$a(B_x + B_y + B_z + C_x + C_y + C_z) = (B_x a + B_y a + B_z a) + (C_x a + C_y a + C_z a)$

$a(B_x + B_y + B_z) + a(C_x + C_y + C_z) = \boxed{a\vec{B}+a\vec{C}}$

2.) $\nabla(f(x,y) + \nabla g(x,y))$: You can not multiply gradients. Only cross or dot products

3.) $\vec{F}(x,y) = x\hat{i} + y\hat{j}$ $\quad \nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j}$

$\nabla \cdot \vec{F} = \frac{\partial}{\partial x} x + \frac{\partial}{\partial y}(y)$

$[\nabla \cdot \vec{F} = 1 + 1 = \boxed{2}$ Divergence

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & 0 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(0) - \frac{\partial}{\partial z}(y)\right) - \hat{j}\left(\frac{\partial}{\partial x}(0) - \frac{\partial}{\partial z}(x)\right) + \hat{k}\left(\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial y}(x)\right)$$

$$= \hat{i}(0) - \hat{j}(0) - \hat{k}(0) = \boxed{\emptyset}$$ no curling

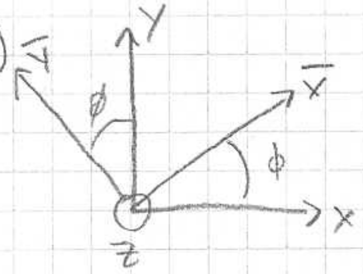
$\int_0^{2\pi} r\cos(t) + r\sin(t)\, dt$ $\quad x = r\cos(t)$, $y = r\sin(t)$ $\quad f(x,y) = x + y$, radius $= 1$

$\int_0^{2\pi} \cos(t) + \sin(t)\, dt$

$\sin(t)\Big|_0^{2\pi} + \cos(t)\Big|_0^{2\pi} = \boxed{0}$ Line Intgral

$1 - 1$ $\quad 0$

2.)



$$\begin{pmatrix} \bar{a}_x \\ \bar{a}_y \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} a_x \\ a_y \end{pmatrix}$$

$$\bar{\vec{a}}$$

$$\bar{a}_x = a_x \cos\phi + a_y \sin\phi$$

$$\bar{a}_y = -a_x \sin\phi + a_y \cos\phi$$

$$\bar{a}_x^2 + \bar{a}_y^2 = a_x^2 + a_y^2$$

$$\bar{a}_x^2 = a_x^2 \cos^2\phi + a_y^2 \sin^2\phi + \cancel{2a_x a_y \cos\phi \sin\phi}$$

$$\bar{a}_y^2 = a_x^2 \sin^2\phi + a_y^2 \cos^2\phi - \cancel{2a_x a_y \sin\phi \cos\phi}$$

$$\boxed{\bar{a}_x^2 + \bar{a}_y^2 = a_x^2 + a_y^2}$$

$$|\bar{\vec{a}}| = |\vec{a}|$$ Magnitude preserved

3.) $\int_S (\nabla \times \vec{v}) \cdot d\vec{a} = \emptyset$ When Stokes theorem is a closed surface, the surface integral is zero.

4) $f(x) \star g(x) = \left(\frac{f(x) - g(x)}{f(x) + g(x)} \right)$ $\int_{-\infty}^{\infty} (f(x) \star g(x))\, \delta(x)\, dx$

- $\int_{-\infty}^{\infty} (\cos(x) \star \sin(x))\, \delta(x)\, dx$

$\int_{-\infty}^{\infty} f(x)\, \delta(x)\, dx = f(0)$

$\frac{f(0) - g(0)}{f(0) + f(0)}$

$= \frac{\cos(0) - \sin(0)}{\cos(0) + \sin(0)} = \frac{1}{1} = \boxed{1}$

- $\int_{-\infty}^{\infty} (\cosh(x) \star \sinh(x))\, \delta(x)\, dx$

$= \frac{\cosh(0) - \sinh(0)}{\cosh(0) + \sinh(0)} = \boxed{1}$

- $f(x) = a + ax + ax^2 + \ldots$ $g(x) = b + bx + bx^2 + \ldots$

$\frac{f(x) - g(x)}{f(x) + g(x)} = \frac{f(0) - g(0)}{f(0) + g(0)}$

$= \boxed{\frac{a-b}{a+b}}$