

Electromagnetic Theory: PHYS330

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Summary

Summary

1. Electromagnetism during the pandemic
 - Pace
 - Style
 - Need to review vectors, vector functions, and vector calculus
2. Challenge level: pre-requisites
 - Passed Calculus 1, 2, and 3
 - Passed Calculus-based physics 1, 2, and 3
 - Passed modern physics
3. Maxwell's equations live in 3D
4. **Introduction to Electromagnetism by D. Griffiths (4th ed.)**
5. First half of the text is recommended by publisher, retain for graduate school
6. Asynchronous content: www.youtube.com/918particle, and Moodle in folders

Homework

Homework

1. Reading: Chapter 1 by Friday/Saturday
2. Exercises: 1.54, 1.55, 1.56, 1.57, 1.59, 1.62, 1.63, 1.64

Today: the Dirac delta-function

The Dirac δ -function

Consider this function:

$$\vec{v} = \frac{1}{r^2} \hat{r} \quad (1)$$

with $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. What is the divergence?

$$\nabla \cdot \vec{v} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v_r) + \frac{1}{r \sin(\theta)} \frac{\partial}{\partial \theta} (r \sin(\theta) v_\theta) + \frac{1}{r \sin(\theta)} \frac{\partial v_\phi}{\partial \phi} \quad (2)$$

The Dirac δ -function

So we find the divergence is zero. What is the result of a surface integral around the origin?

$$\oint \vec{v} \cdot d\vec{a} = \int_0^{2\pi} \int_0^\pi \left(\frac{\hat{r}}{R^2} \right) \cdot (R^2 \sin(\theta) d\theta d\phi \hat{r}) \quad (3)$$

The Dirac δ -function

(Let $d\tau$ be the volume element). Isn't the following *always* supposed to be true?

$$\int (\nabla \cdot \vec{v}) d\tau = \oint \vec{v} \cdot d\vec{a} \quad (4)$$

We must be dealing with a strange function...apparently all of the surface integral contribution comes from the origin, where the volume element is zero, but the function is infinite.

Think of a function that has a finite *integral* result, but is zero everywhere except one point. Nothing comes to mind.

The Dirac δ -function

The Dirac δ -function:

$$\delta(x) = 0 \quad \text{if } x \neq 0 \quad (5)$$

$$\delta(x) = \infty \quad \text{if } x = 0 \quad (6)$$

This function is called a *distribution*, not a real function. However, it has interesting properties:

$$f(x)\delta(x) = f(0)\delta(x) \quad (7)$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \quad (8)$$

$$\int_{-\infty}^{\infty} f(x)\delta(x) dx = f(0) \quad (9)$$

$$\int_{-\infty}^{\infty} f(x)\delta(x - a) dx = f(a) \quad (10)$$

The Dirac δ -function

Show that

$$\delta(kx) = \frac{1}{|k|} \delta(x) \quad (11)$$

Try it here:

$$\int_{-\infty}^{\infty} \cos(2kx) \delta(kx) dx = \quad (12)$$

Another interesting thing

What is this integral?

$$\int_0^{2\pi} \sin(nx) \sin(mx) dx \quad (13)$$

The Dirac δ -function

Generalize to three dimensions:

$$\delta^3(\vec{r}) = \delta(x)\delta(y)\delta(z) \quad (14)$$

$$\int d\tau \delta^3(\vec{r}) = 1 \quad (15)$$

$$\int d\tau f(\vec{r}) \delta^3(\vec{r} - \vec{a}) = f(\vec{a}) \quad (16)$$

Let $f(\vec{r}) = \cos^2(x) - \sin^2(y)$, and $\vec{a} = (0, 1)$. Evaluate:

$$\int d\tau f(\vec{r}) \delta^3(\vec{r} - \vec{a}) = \quad (17)$$

The Dirac δ -function

If the integral contains the origin:

$$\int \nabla \cdot \left(\frac{\hat{r}}{r^2} \right) d\tau = 4\pi \quad (18)$$

Thus we know

$$\nabla \cdot \left(\frac{\hat{r}}{r^2} \right) = 4\pi\delta^3(\vec{r}) \quad (19)$$

One of Maxwell's Equations: $\nabla \cdot \vec{E} = \rho/\epsilon_0$. This says the divergence of the E-field is charge density. If the E-field goes like $1/r^2$, then we know it's like a point charge. So the charge density of a point charge: $\delta^3(\vec{r})$.

Objects of Electromagnetism

What type of *object* is $\vec{f}(x, y, z) \cdot \vec{g}(x, y, z)$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

What type of *object* is $\vec{f}(x, y, z) \times \vec{g}(x, y, z)$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

What type of *object* is $\vec{h}(x, y, z) \cdot (\vec{f}(x, y, z) \times \vec{g}(x, y, z))$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

What type of *object* is $\nabla f(x, y, z)$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

What type of *object* is $\frac{\partial f(x,y,z)}{\partial x}$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

What type of *object* is $\nabla \cdot \vec{f}(x, y, z)$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

What type of *object* is $\nabla \times \vec{f}(x, y, z)$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

What type of *object* is $\nabla \cdot (\nabla f(x, y, z))$?

- A: A scalar
- B: A pseudoscalar
- C: A vector
- D: A pseudovector

This object is the Laplacian of f :

$$\nabla \cdot (\nabla f(x, y, z)) = \nabla^2 f \quad (20)$$

Of all the possible *second derivatives* of the above objects this is the one we will encounter the most. The rest are zero or less important (grad of divergence). When you see a second derivative, think guilty until proven innocent, in EM.

Area Vectors and Surface Integrals

Area Vectors and Surface Integrals

Cartesian coordinates, six possibilities:

$$d\vec{a} = \pm dx dy \hat{z} \quad (21)$$

$$d\vec{a} = \pm dx dz \hat{y} \quad (22)$$

$$d\vec{a} = \pm dy dz \hat{x} \quad (23)$$

You must always determine the vector $d\vec{a}$ before completing a surface integral.

Area Vectors and Surface Integrals

Let $\vec{v} = 2xz\hat{i} + (x+2)\hat{j} + y(z^2-3)\hat{k}$. Integrate \vec{v} over the cube of side length 2 with one corner at the origin. (breakout rooms)

Area Vectors and Surface Integrals

Let $\vec{v} = s(2 + \sin^2(\phi))\hat{s} + s \sin(\phi) \cos(\phi)\hat{\phi} + 3z\hat{z}$. (a) Find the divergence.

Area Vectors and Surface Integrals

Let $\vec{v} = s(2 + \sin^2(\phi))\hat{s} + s \sin(\phi) \cos(\phi)\hat{\phi} + 3z\hat{z}$. (b) Test the divergence theorem using the quarter cylinder with radius 2 and height 5, the corner at the origin.

Conclusion

Summary

1. Electromagnetism and the module system
 - Pace
 - Style
 - Class decision
2. Challenge level: pre-requisites
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