Warm-up for Electromagnetic Theory (PHYS330)

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Abstract

Definition of the Fourier transform, and two interesting results. These tools may be useful for final projects.

1 Definition of the Fourier Transform

The *Fourier transform* is a way of representing a function of time (or space) as a function of frequency (or wavevector). Imagine a function of time: $E(t)$ having a partner function in the other space called $\widetilde{E}(\nu)$, where ν is the frequency. The Fourier transform turns $E(t)$ into $\widetilde{E}(\nu)$, and the inverse Fourier transform turns $\widetilde{E}(\nu)$ into $E(t)$. Let $j = \sqrt{-1}$. Here are the definitions:

$$\widetilde{E}(\nu) = \int_{-\infty}^{\infty} E(t) e^{-2\pi j \nu t} dt \tag{1}$$

$$E(t) = \int_{-\infty}^{\infty} \widetilde{E}(\nu) e^{2\pi j \nu t} d\nu \tag{2}$$

2 The Fourier transform of the Dirac Delta Function

Recall the main property of the Dirac delta function, $\delta(t - t_0)$:

$$f(t_0) = \int_{-\infty}^{\infty} f(t)\delta(t - t_0) dt \tag{3}$$

In this section, we aim to determine the Fourier transform of a sine wave. First, **compute the Fourier transform of the Dirac delta function** by inserting $\delta(t - t_0)$ for $E(t)$ in the definition of the Fourier transform.

[Answer: $e^{-2\pi j \nu t_0}$]

Now, write down the *inverse Fourier transform* of $e^{-2\pi j \nu t_0}$, and simplify the exponential under the integral sign.

Finally, in a separate place, write down the *Fourier transform* of $e^{2\pi j \nu_0 t}$, which is equivalent to computing the Fourier transform of a sine wave.

3 The Fourier transform of a Sine Wave

Finally, compare your expression for the Fourier transform of $e^{2\pi j \nu_0 t}$ to the inverse Fourier transform of $e^{-2\pi j \nu t_0}$, which was equal to the Dirac delta. Make the two integrals look as alike as possible. **To what is the Fourier transform of $e^{2\pi j \nu_0 t}$ equal**?

Because the solutions to boundary-value problems can be expressed as sums of sines and cosines, you now have the power to express them in *frequency space.* (There is a minor detail about changing the Fourier transform to relate position and k-vector: $f(x)$ goes with $\widetilde{f}(k)$).