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Quiz Ch. 1

1) a) $a(\vec{B} + \vec{C}) = a\vec{B} + a\vec{C}$

Proven by vector operation multiplication by scalar

$$a\left((B_x \vec{x} + B_y \vec{y} + B_z \vec{z}) + (C_x \vec{x} + C_y \vec{y} + C_z \vec{z})\right) =$$

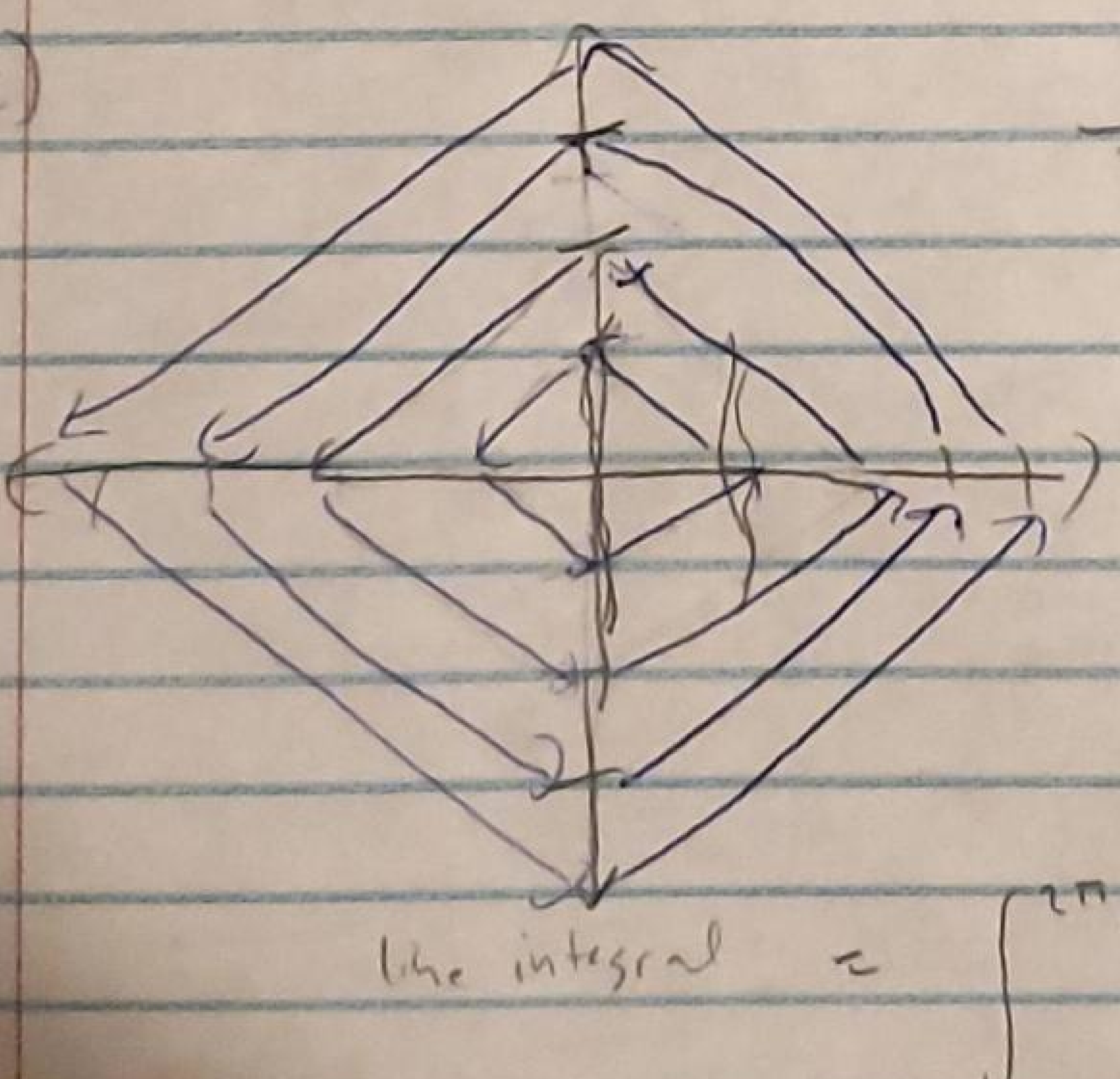
$$(aB_x)\vec{x} + (aB_y)\vec{y} + (aB_z)\vec{z} + (aC_x)\vec{x} + (aC_y)\vec{y} + (aC_z)\vec{z}$$

$$\boxed{a(\vec{B} + \vec{C}) = a\vec{B} + a\vec{C}}$$

b) $\nabla(f(x,y) + \nabla g(x,y)) \rightarrow$ $\boxed{\nabla f(x,y) + \nabla^2 g(x,y)}$

You can't add a first degree partial to a 2nd degree partial

c)



→ · Conservative vector field, curl = 0
if curl = 0, then the integral is then 0 because it is a close-loop

$x = r\cos(t)$ $\quad r = 1$
$y = r\sin(t)$ $\quad f(x,y) = 1$

line integral $= \int_0^{2\pi} r\cos(t) + r\sin(t)\, dt$

$\vec{F}(x,y) = x\hat{i} + y\hat{j}$

$= \sin(t) + \cos(t) \Big|_0^{2\pi}$

$\nabla = \partial/\partial x\, \hat{i} + \partial/\partial y\, \hat{j}$

$\sin(2\pi) - \cos(2\pi) = \boxed{0}$

$\nabla \cdot \vec{F} = \partial/\partial x(x) + \partial/\partial y(y) = \boxed{2}$, divergence closed sphere, $r = 1$ @ origin, 0

3) Let $\vec{v} = s^{-1} \phi$; (s, ϕ, z)

$$\boxed{\oint_S (\nabla \times \vec{v}) \cdot d\vec{a} = 0}$$

→ for any closed surface cause shrinks down to a point, corrollary 2

4) Let $f(x) * g(x) = \left(\frac{f(x) - g(x)}{f(x) + g(x)} \right)$

$\int_{-\infty}^{\infty} (f(x) * g(x)) \cdot \delta(x) \, dx$, if:

a) $f(x) = \cos(x)$ & $g(x) = \sin(x)$,

$$\int_{-\infty}^{\infty} \left(\frac{\cos(x) - \sin(x)}{\cos(x) + \sin(x)} \right) \delta(x) \, dx = \int_{-\infty}^{\infty} \left(\frac{\cos(0) - \sin(0)}{\cos(0) + \sin(0)} \right) \delta(x) \, dx$$

$$\boxed{= f(0)}$$

b) $f(x) = \cosh(x)$
$g(x) = \sinh(x)$

$$\int_{-\infty}^{\infty} \left(\frac{\cosh(x) - \sinh(x)}{\cosh(x) - \sinh(x)} \right) \delta(x) \, dx = \int_{-\infty}^{\infty} \left(\frac{\cosh(0) - \sinh(0)}{\cosh(0) - \sinh(0)} \right) \delta(x) \, dx$$

$$\boxed{= f(0)}$$

c) $f(x) = a + ax + ax^2 \ldots$
$g(x) = b + bx + bx^2 \ldots$

$$\int_{-\infty}^{\infty} \left(\frac{a + ax + ax^2 - b + bx + bx^2}{(a + ax + ax^2) + (b + bx + bx^2)} \right) \delta(x) \, dx = \int_{-\infty}^{\infty} \left(\frac{a + a(0) + a(0)^2 - b + b(0) + b(0)^2}{a + a(0) + a(0)^2 + b + b(0) + b(0)^2} \right) \delta(x) \, dx$$

$$\boxed{= f(0)}$$

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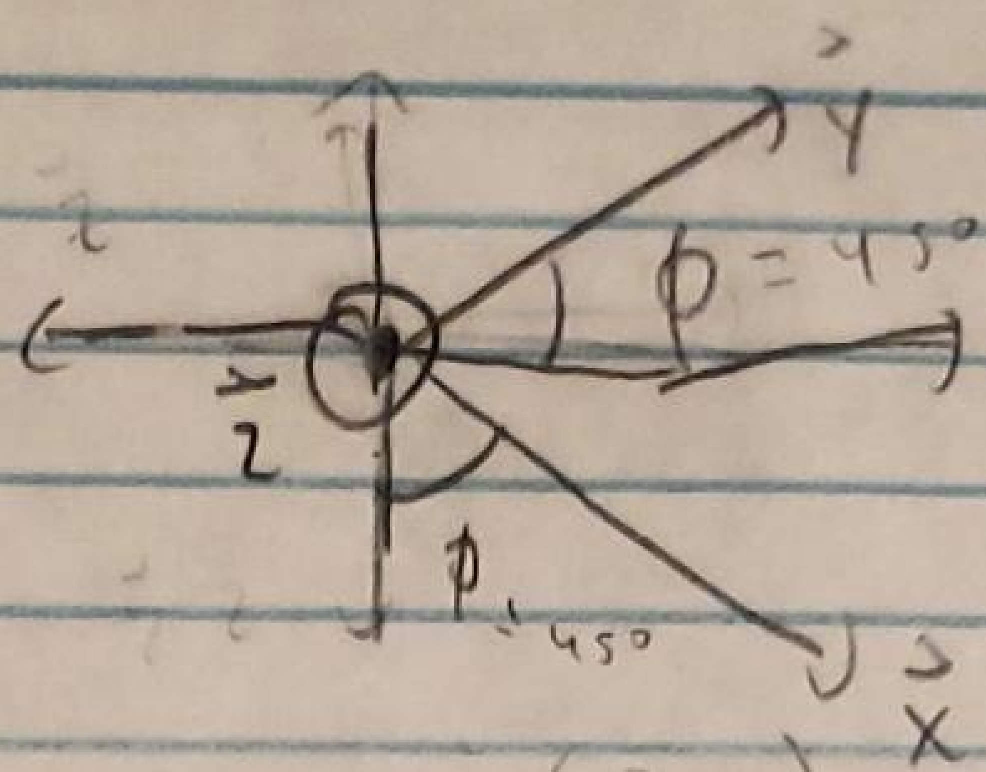
2) $a_{\vec{x}}$ - right

$a_{\vec{y}}$ - up

$a_{\vec{z}}$ - out



a) $\boxed{\vec{a} = \hat{i} + \hat{j}}$

b)
$$\boxed{\begin{aligned} a_{\vec{x}} &= -a_x\cos\phi + a_y\sin\phi \\ a_{\vec{y}} &= a_x\sin\phi + a_y\cos\phi \end{aligned}}$$

$$\underbrace{\begin{pmatrix} \vec{a}_x \\ \vec{a}_y \end{pmatrix}}_{\vec{a}} = \underbrace{\begin{pmatrix} \cos\phi & +\sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}}_{R} \underbrace{\begin{pmatrix} a_x \\ a_y \end{pmatrix}}_{a}$$

c)
$$a_{\vec{x}}^2 = a_x^2\cos^2\phi + a_y^2\sin^2\phi + 2a_xa_y\cos\phi\sin\phi$$
$$a_{\vec{y}}^2 = a_y^2\sin^2\phi + a_x^2\cos^2\phi - 2a_xa_y\sin\phi\cos\phi$$

$$\vec{a}_x^2 + \vec{a}_y^2 = a_x^2 + a_y^2$$

$$\boxed{|\vec{a}| = |a|}$$