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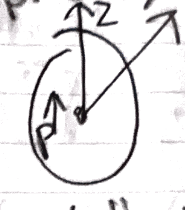
Quiz 4 11/20

1.1) • σ_b for uniformly polarized sphere w/ radius a

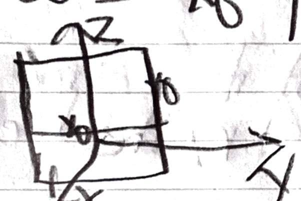
$\sigma_b = \vec{P}\cdot\hat{n} = \boxed{P\cos\theta}$



• σ_b for slab of dielectric material w/ thickness z_0 and lateral dimensions x_0 and y_0 w/ uniform $\vec{P}$ inside oriented along z axis.

we know $\sigma_b = \frac{q}{A} = \boxed{\frac{q}{x_0 \cdot y_0}}$

Area $= x_0 \cdot y_0$



1.2) Compute ρ_b for each polarization dist.

• $\vec{P} = P_0 \vec{r}$ (spherical coords) $\vec{r} = r\hat{r}$

$\rho_b = -\nabla\cdot\vec{P} = -\nabla\cdot(P_0 r\hat{r})$

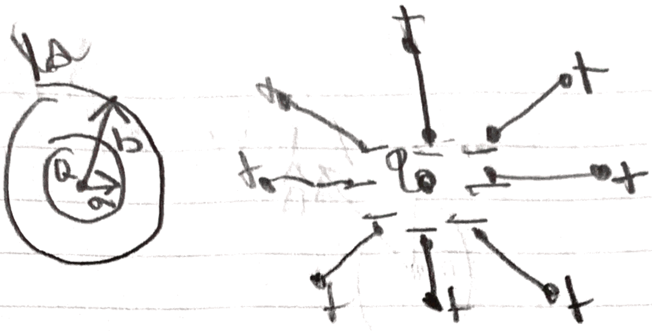
$= -\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 P_0 r) = -\frac{P_0}{r^2}\frac{\partial}{\partial r}(r^3) = -\frac{P_0 3\cancel{r^2}}{\cancel{r^2}} = \boxed{-3P_0}$

• $\vec{P} = P_0 P_2(\cos\theta)\hat{\theta}$ (spherical coords)

$\rho_b = -\nabla\cdot P) = -\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(P_0 P_2 \cos\theta\sin\theta)\right) = -\frac{P_0 P_2}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(\cos\theta\sin\theta)\right)$

$= -\frac{P_0 P_2}{r\sin\theta}(\cos^2\theta - \sin^2\theta) = \boxed{\frac{-P_0 P_2(\cos 2\theta)}{r\sin\theta}}$

2. Two diff systems are shown in fig 1 below. Show that E-field for a < r < b in Fig 1 (left) is equal to E field in fig 1 (right).



For the left fig. we know that
$D = \frac{Q}{4\pi r^2}\hat{r}$ for $a<r<b$

We also know that $D = \epsilon E$
so $E = \frac{Q}{4\pi \epsilon r^2}$ for $a<r<b$

For the right side, we know that if a free charge q is embedded in a large dielectric, the field it produce is
$$E = \frac{1}{4\pi\epsilon}\frac{q}{r^2}\hat{r}$$

Thus both sides have equal electric field