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Reading Quiz 3 for Electromagnetic Theory (PHYS330)

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Abstract

A summary of content covered in chapter 3 (so far) of Introduction to Electrodynamics.

1 Discussions about Vectors (Prelude to Fourier's Trick)

✓ 1. Let $\vec{v} = a\hat{x} + b\hat{y} + c\hat{z}$. Which of the following is equal to c?

- A: $\vec{v}\cdot\vec{v} - |\vec{v}|$
- B: $\vec{v}\cdot\hat{z}$
- C: $\hat{x}\cdot\vec{v}$
- D: $\sqrt{\vec{v}^2}$

$a+b+c - \sqrt{a^2-b^2-c^2}$

$a\hat{x} + b\hat{y} + c\hat{z} \cdot \hat{z} = (0+0+c) = c$

1 2. Let $\vec{x} = \sum_{i=1}^{n} c_i\hat{x}_i$ be an n-dimensional vector and the set of $\hat{x}_i$ represent orthonormal basis vectors. How do you obtain the coefficient c_7?

- A: $\hat{x}_i\cdot\vec{x}$
- B: $n = 7$
- C: $\hat{x}\cdot\vec{x}$
- D: $\hat{x}_7\cdot\vec{x}$

$\sum_{i=1}^{7} = c_1\hat{x}_1 + c_2\hat{x}_2 + \dots c_7\hat{x}_7$

$\hat{x}_7\cdot\vec{x} = \hat{x}_7 \cdot c_7\hat{x}_7 = c_7$

✓ 3. Suppose we are trying to develop the Fourier series for a function $f(x)$. Recall the definition of a Fourier series:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx) \quad (1)$$

However, the function we are trying to model is $f(x) = \sin(3x)$. Write down all coefficients in the Fourier series from $n = 0$ to $n = \infty$.

2 Fourier's Trick and Boundary Value Problems

✓ 1. If $V(x, y, z) \to 0$ as $y \to \infty$, which of the following cannot be part of the solution for $V(x, y, z)$?

- A: $Y(y) = e^{-ky}$ ✓
- B: $Y(y) = \sinh(x)$
- C: $Y(y) = 1/y^2$ 0
- D: $Y(y) = e^{-ky^2}$ 0

✓ 2. Below is Eq. 3.50 from section 3.3 of the text, with $V_0(y, z) = V_0$:

$$C_{n,m} = \frac{4V_0}{ab}\int_0^a\int_0^a \sin(n\pi y/a)\sin(n\pi z/a)dydz \quad (2)$$

Reproduce the result in Eq. 3.51 for $C_{n,m}$.

$1.3)\ f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \sin(nx) + b_n \cos(nx)$

$f(x) = \sin(3x)$

$a_n = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\sin(nx)\,dx$

$a_0 = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\,0\,dx = 0$

trig identity $\sin(3x)\sin(nx) = \frac{1}{2}(\cos(n-3) - \cos(n+3))$

$a_n = \frac{1}{2\pi}\int_0^{2\pi} \cos((n-3)x)\,dx - \frac{1}{2\pi}\int_0^{2\pi} \cos((n+3)x)\,dx$

$= \frac{1}{2\pi}\frac{\sin((n-3)x)}{n-3}\Big|_0^{2\pi} - \frac{1}{2\pi}\frac{\sin((n+3)x)}{n+3}\Big|_0^{2\pi}$

$= \frac{1}{2\pi}\frac{(0-0)}{n-3} - \frac{1}{2\pi}\frac{(0-0)}{n+3} = 0$ $\quad n-3 \neq 0$, $n+3 \neq 0$

$b_n = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\cos(nx)\,dx$

trig identity $\sin(3x)\cos(nx) = \frac{1}{2}(\sin(n+3) - \sin(n-3))$

$= \frac{1}{2\pi}\int_0^{2\pi} \sin((n+3)x)\,dx - \frac{1}{2\pi}\int_0^{2\pi} \sin((n-3)x)$

$= -\frac{1}{2\pi}\frac{\cos((n+3)x)}{n+3}\Big|_0^{2\pi} + \frac{1}{2\pi}\frac{\cos((n-3)x)}{n-3}\Big|_0^{2\pi}$

$$= -\frac{1}{2\pi}\frac{\cos(2(n+3)\pi)-1}{n+3} + \frac{1}{2\pi}\frac{\cos(2(n-3)\pi)-1}{n-3}$$

$$= \frac{-1}{2\pi(n+3)}(1-1) + \frac{1}{2\pi(n-3)}(1-1) = 0$$

So let $n=3$

$$a_3 = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\sin(3x) = \frac{1}{\pi}\int_0^{2\pi} \sin^2(3x)$$

trig identity $\sin^2(3x) = \frac{1}{2}(1-\cos(6x))dx$

$$\frac{1}{2\pi}\int_0^{2\pi} dx - \frac{1}{2\pi}\int_0^{2\pi}\cos(6x)dx = 1 - \frac{1}{12\pi}\sin(6x)\Big|_0^{2\pi} = 1$$

$u=6x$

$\frac{du}{6}=\frac{6}{6}$

$$b_3 = \frac{1}{\pi}\int_0^{2\pi} \sin(3x)\cos(3x) = \frac{1}{3\pi}\,\frac{\sin^2(3x)}{2}\Big|_0^{2\pi} = 0$$

$u = \sin(3x)$

$du = \cos(3x)\cdot 3$

So $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\sin(nx) + b_n\cos(nx) = 0 + \sin(3x)$ ✓

2.2) $V_0(y,z) = V_0$ $\quad C_{n,m} = \frac{4V_0}{ab}\int_0^a\int_0^a \sin\left(\frac{n\pi y}{a}\right)\sin\left(\frac{n\pi z}{a}\right)dy\,dz$

$$= \frac{4V_0}{ab}\int_0^a \sin\left(\frac{n\pi y}{a}\right)dy \int_0^a \sin\left(\frac{n\pi z}{a}\right)dz$$

$$\frac{4V_0}{ab}\left(-\cos\left(\frac{n\pi y}{a}\right)\cdot\frac{a}{n\pi}\Big|_0^a\right)\left(-\cos\left(\frac{n\pi z}{a}\right)\cdot\frac{a}{n\pi}\Big|_0^a\right)$$

$$\frac{4V_0}{ab}\left(\frac{a}{n\pi}(-\cos(n\pi)+1)\right)\left(\frac{a}{n\pi}(-\cos(n\pi)+1)\right)$$

$$\frac{4V_0 a}{bn^2\pi^2}\left((-\cos(n\pi)+1)\right)\left(-\cos(n\pi)+1\right)$$

n is even, $C_{n,m}=0$

n is odd, $C_{nm}=\frac{4V_0 a}{n^2\pi^2}(2)(2)=\boxed{\frac{16V_0 a}{bn^2\pi^2}}$