Phys 330 Midterm

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1) (a) If $\vec{A}$ and $\vec{B}$ are two vector functions, what does the expression $(A \cdot \nabla)B$ mean? That is, what are its x, y, and z components, in terms of the Cartesian components of A ∇ and B?

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z} \qquad \nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

$$\vec{A}\cdot\nabla = A_x\frac{\partial}{\partial x} + A_y\frac{\partial}{\partial y} + A_z\frac{\partial}{\partial z}$$

$$\vec{B} = B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

$$(\vec{A}\cdot\nabla)\vec{B} = \left(A_x\frac{\partial}{\partial x} + A_y\frac{\partial}{\partial y} + A_z\frac{\partial}{\partial z}\right)\left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right)$$

$$= A_x\frac{\partial}{\partial x}\left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right) + A_y\frac{\partial}{\partial y}\left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right) +$$

$$A_z\frac{\partial}{\partial z}\left(B_x\hat{x} + B_y\hat{y} + B_z\hat{z}\right)$$

$$\boxed{= A_x\frac{\partial}{\partial x}B_x\hat{x} + A_y\frac{\partial}{\partial y}B_y\hat{y} + A_z\frac{\partial}{\partial z}B_z\hat{z}}$$

(b) Compute $(\hat{r}\cdot\nabla)\hat{r}$, where $\hat{r}$ is $\vec{r}/|r|$.

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z} \qquad |r| = \sqrt{x^2+y^2+z^2} \qquad \frac{\vec{r}}{|r|} = \frac{x\hat{x} + y\hat{y} + z\hat{z}}{\sqrt{x^2+y^2+z^2}}$$

$$\hat{r}\cdot\nabla = \left(\frac{x}{\sqrt{x^2+y^2+z^2}}\hat{x} + \frac{y}{\sqrt{x^2+y^2+z^2}}\hat{y} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{z}\right)\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)$$

$$= \frac{x}{\sqrt{x^2+y^2+z^2}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\hat{x} + \frac{y}{\sqrt{x^2+y^2+z^2}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\hat{y} + \frac{z}{\sqrt{x^2+y^2+z^2}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\hat{z}$$

$$(\hat{r}\cdot\nabla)\hat{r} = \left[\frac{x}{\sqrt{x^2+y^2+z^2}}\frac{\partial}{\partial x}\left(\frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}\right) + \frac{x}{\sqrt{x^2+y^2+z^2}}\frac{\partial}{\partial y}\left(\frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}\right) + \frac{x}{\sqrt{x^2+y^2+z^2}}\frac{\partial}{\partial z}\left(\frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}\right)\right]\hat{x}$$

$$+ \left[\frac{y}{\sqrt{x^2+y^2+z^2}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(\frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}\right)\right]\hat{y} + \left[\frac{z}{\sqrt{x^2+y^2+z^2}}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\left(\frac{x\hat{x}+y\hat{y}+z\hat{z}}{\sqrt{x^2+y^2+z^2}}\right)\right]\hat{z}$$

$$= \left(\frac{x}{\sqrt{x^2+y^2+z^2}}\left[\frac{y^2+z^2}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{y}{\sqrt{x^2+y^2+z^2}}\left[\frac{-xy}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{z}{\sqrt{x^2+y^2+z^2}}\left[\frac{-xz}{(x^2+y^2+z^2)^{3/2}}\right]\right)\hat{x}$$

$$+ \left(\frac{x}{\sqrt{x^2+y^2+z^2}}\left[\frac{-xy}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{y}{\sqrt{x^2+y^2+z^2}}\left[\frac{x^2+z^2}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{z}{\sqrt{x^2+y^2+z^2}}\left[\frac{[illegible]}{(x^2+y^2+z^2)^{3/2}}\right]\right)\hat{y}$$

$$+ \left(\frac{x}{\sqrt{x^2+y^2+z^2}}\left[\frac{-xz}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{y}{\sqrt{x^2+y^2+z^2}}\left[\frac{-yz}{(x^2+y^2+z^2)^{3/2}}\right] + \frac{z}{\sqrt{x^2+y^2+z^2}}\left[\frac{x^2+y^2}{(x^2+y^2+z^2)^{3/2}}\right]\right)\hat{z}$$

$$= \left[\frac{xy^2+xz^2}{(x^2+y^2+z^2)^2} - \frac{xy^2}{(x^2+y^2+z^2)} - \frac{xz^2}{(x^2+y^2+z^2)^2}\right]\hat{x} + \left[\frac{[illegible]}{(x^2+y^2+z^2)^2} + \frac{[illegible]}{(x^2+y^2+z^2)^2} - \frac{[illegible]}{(x^2+y^2+z^2)^2}\right]\hat{y}$$

$$+ \left[\frac{-zx^2}{(x^2+y^2+z^2)^2} - \frac{zy^2}{(x^2+y^2+z^2)^2} + \frac{zx^2+zy^2}{(x^2+y^2+z^2)^2}\right]\hat{z} = 0\hat{x} + 0\hat{y} + 0\hat{z} \; \boxed{= 0}$$

(c) One can show that the force on a dipole induced by a non-uniform field is $\vec{F} = (\vec{p}\cdot\nabla)\vec{E}$. Compute the force on a physical dipole located at the origin with $\vec{p} = q\vec{d} = qd\hat{x}$ in a field with associated potential $V(\vec{r}) = V_0 r^2 + V$

$$\vec{p}\cdot\nabla = qd\frac{\partial}{\partial x} \qquad \vec{F} = qd\frac{\partial}{\partial x}\vec{E} \qquad \vec{E} = -\nabla V = -\frac{\partial}{\partial r}(V_0 r^2 + V) = -2V_0\vec{r}$$

$$\vec{r} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z} \qquad \vec{F} = qd\frac{\partial}{\partial x}(-2V_0\vec{r}) = -2qdV_0\frac{\partial}{\partial x}(r_x\hat{x} + r_y\hat{y} + r_z\hat{z})$$

$$\boxed{\vec{F} = -2V_0 qd\frac{\partial}{\partial x}r_x\hat{x}}$$

2) Evaluate the following integral using 3D Dirac delta function.

$$J = \int_V e^{-r} \left(\nabla \cdot \frac{\hat{r}}{r^2} \right)$$

from textbook: $\nabla \cdot \frac{\hat{r}}{r^2} = 4\pi \delta^3 \vec{r}$

$$J = \int_V e^{-r} (4\pi \delta^3 \vec{r})$$

$$= 4\pi \int_V e^{-r} \delta^3 \vec{r}$$

$$= 4\pi \int_V e^0$$

$$= 4\pi (1) = \boxed{4\pi}$$

2 1) Suppose two dipoles, each with dipole moment $\vec{p}$ pointed in opposite, form a square with alternating positive and negative charges and side length d. Calculate the field $\vec{E}_{total}$ at the following points P: (a) $P = (0,0)$, (b) $P = (2d, 0)$, and $P = (0, 2d)$.

$-q$ q d d $(0,0)$ q $-q$

$$\vec{p}_1 = [q(0)] + [-q(d)] = -qd\hat{y} \qquad \vec{p}_2 = [qd(\hat{x} + \hat{y})] + [-q(d\hat{x})]$$

$$= qd\hat{x} + qd\hat{y} - qd\hat{x} = qd\hat{y}$$

$$\vec{p} = \vec{p}_1 + \vec{p}_2$$

$$\vec{p} = -qd\hat{y} + qd\hat{y} = 0$$

$$E_{dip}(r, \theta) = \frac{p}{4\pi \varepsilon_0 r^3}(2\cos\theta \hat{r} + \sin\theta \hat{\theta})$$

$$E_{dip_1}(r, \theta) = \frac{-qd\hat{y}}{4\pi \varepsilon_0 r^3} 2\cos\theta \hat{r} + \frac{-qd\hat{y}}{4\pi \varepsilon_0 r^3} \sin\theta \hat{\theta}$$

$$= \frac{-2\cos\theta \, qd \sin\theta \sin\phi}{4\pi \varepsilon_0 r^3} - \frac{\sin\theta \, qd \cos\theta \sin\phi}{4\pi \varepsilon_0 r^3}$$

$$= \frac{-3qd \cos\theta \sin\theta \sin\phi}{4\pi \varepsilon_0 r^3} =$$

$$E_{dip_2}(r, \theta) = \frac{3qd \cos\theta \sin\theta \sin\phi}{4\pi \varepsilon_0 r^3}$$

(a) $E_{dip_1}(0,0) = \frac{-3qd \cos(0)\sin(0)\sin(0)}{4\pi \varepsilon_0 r^3} = 0$

$E_{dip_2}(0,0) = \frac{3qd \cos(0)\sin(0)\sin(0)}{4\pi \varepsilon_0 r^3} = 0$

$\boxed{E_{dip}(0,0) = 0 + 0 = 0}$

(b) $E_{dip_1}(2d, 0) = \frac{-3qd \cos(2d)\sin(2d)\sin(0)}{4\pi \varepsilon_0 r^3} = 0$

$E_{dip_1}(2d, 0) = \frac{3qd(\cos 2d)\sin(2d)\sin(0)}{4\pi \varepsilon_0 r^3} = 0$

$\boxed{E_{dip}(0, 2d) = 0 + 0 = 0}$

(c) $E_{dip_1}(0, 2d) = 0$

$E_{dip_2}(0, 2d) = 0$

$\boxed{E_{dip}(2d, 0) = 0 + 0 = 0}$

2) The electric potential of some configuration is given by the expression $V(\vec{r}) = A\frac{e^{-\lambda r}}{r}$. A and λ are constants. Find the field $\vec{E}(\vec{r})$, the charge density ρ and the total charge Q in terms of A and λ.

$\rho = \epsilon_0 A\left(4\pi\delta^3(\vec{r}) - \lambda^2 e^{-\lambda r}/r\right)$ ← Hint

$$E = -\nabla V$$
$$= -\frac{\partial}{\partial r} A\frac{e^{-\lambda r}}{r}$$
$$= -A\frac{r(-\lambda e^{-\lambda r}) - e^{-\lambda r}}{r^2}$$
$$= A\frac{\lambda r e^{-\lambda r} + e^{-\lambda r}}{r^2}$$
$$\boxed{\vec{E} = A e^{-\lambda r}\frac{\lambda r + 1}{r^2}\hat{r}}$$

$$\nabla \cdot E = \frac{1}{\varepsilon_0}\rho$$
$$\nabla\cdot\left(Ae^{-\lambda r}\left(\frac{\lambda r+1}{r^2}\right)\hat{r}\right) = \frac{1}{\varepsilon_0}\rho$$
$$\nabla\cdot\left(Ae^{-\lambda r}(\lambda r+1)\left(\frac{\hat{r}}{r^2}\right)\right) = \frac{1}{\varepsilon_0}\rho$$
$$4\pi\delta^3\vec{r}$$
$$Ae^{-\lambda r}(\lambda r+1)(4\pi\delta^3\vec{r}) = \frac{1}{\varepsilon_0}\rho$$
$$\boxed{\rho = \varepsilon_0 A\left(4\pi\delta^3\vec{r} - \lambda^2 e^{-\lambda r}/r\right)}$$

$$dq = \rho\, d\tau'$$
$$Q = \int \rho\, d\tau$$
$$Q = \int \varepsilon_0 A\left(4\pi\delta^3\vec{r} - \lambda^2 e^{-\lambda r}/r\right) d\tau$$
$$Q = \varepsilon_0 A\left[4\pi\int\delta^3(\vec{r})\,d\tau - \lambda^2\int\frac{e^{-\lambda r}}{r}(4\pi r^2)\,dr\right]$$
$$= \varepsilon_0 A\left[4\pi(1) - \lambda^2 4\pi\int e^{-\lambda r} r\, dr\right]$$
$$= \varepsilon_0 A\left[4\pi - \lambda^2 4\pi\frac{1}{\lambda^2}\right]$$
$$= \varepsilon_0 A[4\pi - 4\pi]$$
$$\boxed{Q = 0}$$

3) (a) Use Gauss' Law to compute the field $\vec{E}$ as a function of the distance s from a long straight wire with positive charge density λ.

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \qquad Q_{enc} = L\lambda \qquad L = \text{length of wire}$$
$$\vec{E}A = \frac{Q_{enc}}{\varepsilon_0} \qquad A = 2\pi s L$$
$$\vec{E}(2\pi s L) = \frac{L\lambda}{\varepsilon_0}$$
$$\vec{E} = \frac{L\lambda}{2\pi s L\varepsilon_0}$$
$$\boxed{\vec{E} = \frac{\lambda}{2\pi s\varepsilon_0}\hat{s}}$$

(b) Calculate the position versus time of a positive point charge q with mass m if it is released a distance s from the wire.

$$\vec{F} = Q\vec{E}$$
$$ma = Q\left(\frac{\lambda}{2\pi s\varepsilon_0}\right)$$
$$\frac{dv}{dt} = \frac{Q\lambda}{2\pi s m\varepsilon_0}$$
$$\frac{d^2 s}{dt^2} = \frac{Q\lambda}{2\pi m s\varepsilon_0}$$
$$\frac{ds}{dt} = \int\frac{Q\lambda}{2\pi m s\varepsilon_0}\,dt$$

$$\frac{ds}{dt} = \frac{Q\lambda}{2\pi m\varepsilon_0}\int\frac{1}{s}\,dt$$
$$\frac{ds}{dt} = \frac{Q\lambda}{2\pi m\varepsilon_0}\frac{t}{s}$$
$$s(t) = \frac{Q\lambda}{2\pi m s\varepsilon_0}\int t\,dt$$
$$s(t) = \frac{Q\lambda}{2\pi m s\varepsilon_0}\frac{1}{2}t^2 + C$$
$$\boxed{s(t) = \frac{Q\lambda t^2}{4\pi m s\varepsilon_0} + C}$$ C is a constant

3 1) Suppose the potential $V_0(\theta)$ at the surface of a sphere of radius R is specified, and there is no charge inside or outside the sphere.

(a) Show that the charge density on the sphere is given by

$$\sigma(\theta) = \frac{\varepsilon_0}{2R} \sum_{l=0}^{\infty} (2l+1)^2 C_l P_l(\cos\theta) \qquad C_l = \int_0^{\pi} V_0(\theta) P_l(\cos\theta) \sin\theta \, d\theta$$

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}}\right) P_l \cos\theta$$

when $r<R$ $\quad V(r,\theta) = \sum_{l=0}^{\infty} (A_l r^l) P_l \cos\theta$

$r>R$ $\quad V(r,\theta) = \sum_{l=0}^{\infty} \left(\frac{B_l}{r^{l+1}}\right) P_l \cos\theta$

no charge inside or outside so $\sum_{l=0}^{\infty} (A_l r^l) P_l \cos\theta = \sum_{l=0}^{\infty} \left(\frac{B_l}{r^{l+1}}\right) P_l \cos\theta$

$$V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{A_l r^{2l+1}}{r^{l+1}}\right) P_l \cos\theta \qquad A_l r^l = \frac{B_l}{r^{l+1}}$$

$$= \sum_{l=0}^{\infty} \left(A_l r^l \left(1 + \frac{r^{l+1}}{r^{l+1}}\right)\right) P_l \cos\theta \qquad B_l = A_l r^l (r^{l+1})$$

$$= \sum_{l=0}^{\infty} (A_l r^l (1+1)) P_l \cos\theta \qquad B_l = A_l r^{2l+1}$$

$$= \sum_{l=0}^{\infty} (2A_l r^l) P_l \cos\theta$$

$$\sigma(\theta) = -\varepsilon_0 \frac{\partial V}{\partial r}\Big|_{r=R}$$

$$\frac{-\sigma(\theta)}{\varepsilon_0} = \frac{\partial V}{\partial r}\Big|_{r=R} \qquad A_l = \frac{2l+1}{2R^l} \int_0^{\pi} V_0(\theta) P_l(\cos\theta) \sin\theta \, d\theta$$

$$= \frac{\partial V}{\partial r}\left(\sum_{l=0}^{\infty} (2A_l r^l) P_l \cos\theta\right)\Big|_{r=R}$$

$$= \sum_{l=0}^{\infty} (2l A_l R^{l-1})(P_l \cos\theta)$$

$$\sigma(\theta) = \varepsilon_0 \sum_{l=0}^{\infty} 2l A_l R^{l-1} P_l \cos\theta$$

$$= \varepsilon_0 \sum 2l \left(\frac{2l+1}{2R^2}\right) \int_0^{\pi} V_0(\theta) P_l(\cos\theta) \sin\theta \, d\theta \, (P_l \cos\theta)$$

$$= \varepsilon_0 \sum_{l=0}^{\infty} \frac{2l^2+1}{R^2} C_l P_l \cos\theta$$

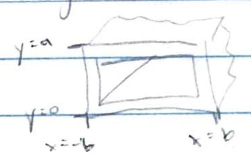
$$\boxed{\sigma(\theta) = \frac{\varepsilon_0}{R^2} \sum_{l=0}^{\infty} (2l^2+1) C_l P_l \cos\theta}$$

(b) Produce the specific result for $\sigma(\theta)$ with $V_0(\theta) = P_2(\cos\theta)$

$$\sigma(\theta) = \frac{\varepsilon_0}{2R} \sum_{l=0}^{\infty} (2l+1)^2 P_l(\cos\theta) \int_0^{\pi} V_0(\theta) P_l(\cos\theta) \sin\theta \, d\theta$$

$$\boxed{\sigma(\theta) = \frac{\varepsilon_0}{2R} \sum_{l=0}^{\infty} (2l+1)^2 P_l(\cos\theta) \int_0^{\pi} P_2(\cos\theta) P_l(\cos\theta) \sin\theta \, d\theta}$$

2) For the infinite rectangular pipe in Ex. 3.4 from the text, suppose the constant potential V_0 is now only on one side. That is, at $y=0$ and $x = \pm b$, the potential is zero. At $y=a$, the potential is V_0. Find the potential $V(x,y)$ inside the pipe. Square pipes are examples of electromagnetic waveguides often used in microwave electronics



(i) $V=0$ when $y=0$

(ii) $V=0$ when $x=-b$

(iii) $V=0$ when $x=b$

(iv) $V=V_0$ when $y=a$

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$$

$$V(x,y) = X(x)Y(y)$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} + \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = 0$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = C_1 \quad \frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = C_2$$

$$\frac{d^2X}{dx^2} = -k^2 X \qquad \frac{d^2Y}{dy^2} = k^2 Y$$

$$X(x) = A\cos(kx) + B\sin(kx)$$

when $x=-b$ $V=0 \rightarrow X(x)Y(y)=0 \rightarrow X(-b)=0$

$x=b$ $V=0 \rightarrow X(b)=0$

$$0 = A\cos(kb) + B\sin(kb)$$

$$A\cos(kb) = -B\sin(kb)$$

$$A = -B\frac{\sin(kb)}{\cos(kb)} \qquad k = \frac{n\pi}{b} \text{ so } \frac{0}{1} \text{ not } \frac{1}{0} \leftarrow \text{illegal}$$

$$A = -B(0)$$

$$A = 0 \rightarrow X(x) = (0)\cos(kx) + B\sin(kx) = B\sin\left(\frac{n\pi}{b}x\right)$$

$$Y(y) = Ce^{ky} + De^{-ky} = Ce^{\frac{n\pi y}{x}} + De^{-\frac{n\pi y}{x}}$$

$$V(x,y) = X(x)Y(y) = B\sin\left(\frac{n\pi}{b}x\right)\left[Ce^{\frac{n\pi y}{b}} + De^{-\frac{n\pi y}{b}}\right]$$

$$= \sin\left(\frac{n\pi}{b}x\right)\left[BCe^{\frac{n\pi y}{b}} + BDe^{-\frac{n\pi y}{b}}\right]$$

BC & BD are still constants can still be written as C & D

$$= \sin\left(\frac{n\pi}{b}x\right)\left[Ce^{\frac{n\pi}{b}y} + De^{-\frac{n\pi}{b}y}\right]$$

$$\boxed{V(x,y) = \sum_{n=0}^{\infty} \sin\left(\frac{n\pi}{b}x\right)\left[Ce^{\frac{n\pi}{b}y} + De^{-\frac{n\pi}{b}y}\right]}$$

3) Using the monopole and dipole potentials in the multipole expansion, find the approximate potential in spherical coordinates for each charge arrangement, far from the origin.

(a) Monopole moment : $Q_{total} = 3q + (-q)$

$= 2q$

Dipole moment : $\vec{p} = \sum_{i=1}^{2} q_i r_i$

$\vec{p} = [3q(a)] + [-q(0)]$

$\vec{p} = 3qa\,\hat{z}$

$\hat{z}\cdot\hat{r} = \cos\theta$

$$V_{mon} = \frac{Q_{total}}{4\pi\varepsilon_0 r} = \frac{2q}{4\pi\varepsilon_0 r}$$

$$V_{dip} = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2} = \frac{3qa\,\hat{z}\cdot\hat{r}}{4\pi\varepsilon_0 r^2} = \frac{3qa\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\boxed{V(r) = V_{mon} + V_{dip} = \frac{2q}{4\pi\varepsilon_0 r} + \frac{3qa\cos\theta}{4\pi\varepsilon_0 r^2}}$$

(b) Monopole moment : $Q_{total} = 3q + (-q) = 2q$

Dipole moment : $\vec{p} = [3q(0)] + [-q(a)]$

$\vec{p} = -qa(-\hat{z})$

$-\hat{z}\cdot\hat{r} = -1$

$$V_{mon} = \frac{2q}{4\pi\varepsilon_0 r} \qquad V_{dip} = \frac{-qa(\hat{z})\cdot\hat{r}}{4\pi\varepsilon_0 r^2} = \frac{qa\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\boxed{V(r) = \frac{2q}{4\pi\varepsilon_0 r} + \frac{qa\cos\theta}{4\pi\varepsilon_0 r^2}}$$

(c) Monopole moment : $Q_{total} = -q + 3q = 2q$

Dipole moment : $\vec{p} = [-q(0) + 3q(a)]$

$\vec{p} = 3qa\,\hat{y}$

$\hat{y}\cdot\hat{r} = \sin\theta\sin\phi$

$$V_{mon} = \frac{2q}{4\pi\varepsilon_0 r} \qquad V_{dip} = \frac{3qa\sin\theta\sin\phi}{4\pi\varepsilon_0 r^2}$$

$$\boxed{V(r) = \frac{2q}{4\pi\varepsilon_0 r} + \frac{3qa\sin\theta\sin\phi}{4\pi\varepsilon_0 r^2}}$$