5.14; 5.16; 5.17, 5.19 Amperes Law
5.20; 5.23; 5.26

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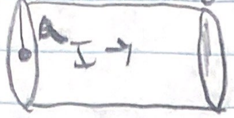
Phys 330

Homework #5 Electrodynamics

5.14). Steady current "I" cylindrical wire radius a
magnetic field inside and outside of wire

a). Outside of the surface of the wire



$s<a$

Amperes law $\oint B \cdot dl = \mu_0 I_{enc}$

$\mu_0 I_{enc} = 0$

$I_{enc} = 0$

$\boxed{B=0}$

$s>a$ $\oint B \cdot dl = \mu_0 I$ $I_{enc} = I$

$\frac{B(2\pi s)}{2\pi s} = \frac{\mu_0 I}{2\pi s}$ $\boxed{B = \frac{\mu_0 I}{2\pi s} \text{ outside the wire}}$

b). $I = \int_0^a J \cdot da$ $s>a$

$J \propto s$ $J = ks$

$I = \int_0^a (ks)(2\pi s)\,ds$

$I = 2\pi k \frac{a^3}{3}$

$\boxed{k = \frac{3I}{2\pi a^3}}$

$s<a$

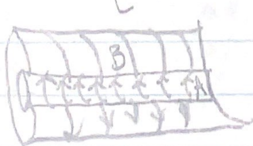
$I = \int_0^a J \cdot da$ $I_{enc} = \int_0^s (ks)(2\pi s)\,ds$

$I = 2\pi k \frac{s^3}{3}$

$I_{enc} = 2\pi\left(\frac{3I}{2\pi a^3}\right)\frac{s^3}{3}$ $\boxed{= I\frac{s^3}{a^3}}$

c

5.16).



$B = \mu_0 n I$ inside

n_2 = rotations of inner solenoid. Zero current outside

n_1 = rotations of inner solenoid

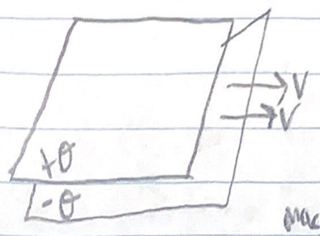
$B_1 = \mu_0 n_2 I$

$B_2 = -\mu_0 n_1 I$

inner solenoid magnetic field $= B_A = \mu_0(n_2-n_1)I$
inner solenoid $= \mu_0(n_2-n_1)I$

outside inner solenoid but inside outter $=0$
Magnetic field @ point $B = \mu_0 n_2 I$
*sandwiched layer
outside of outter $=0$ since <u>no charge encased</u>

5.17).



magnetic field above, below & between

uniform surface current K, $B = \frac{\mu_0 K}{2}$

charge density

Surface charge density $K = \sigma v$ - velocity

a). Top plate; $B = \frac{\mu_0 K}{2}$
Bottom Plate; $B = \frac{\mu_0 K}{2}$

$$\frac{\mu_0 K}{2} + \frac{\mu_0 K}{2} = \mu_0 K = B$$
$$\boxed{B = \mu_0 \sigma v}$$

B). Lorenz force $F = \int (K \times B)\, da$
$$f = K \times B$$
$$B = \frac{\mu_0 K}{2}\hat{y} \quad K = \sigma v \hat{x}$$

$$f = (\sigma v \hat{x}) \times \left(\frac{\mu_0 K}{2}\hat{y}\right)$$
$$= \sigma v \frac{\mu_0 K}{2}(\hat{x} \times \hat{y})$$
$$= \sigma v \frac{\mu_0 K}{2}\hat{z}$$

$\boxed{f = \frac{\mu_0(\sigma v)^2}{2}}$ magnitude force on upper plate direction is upwards.

5.17). continued

c). E field on lower plate

$E = \frac{\sigma}{2\varepsilon_0}$ force on upper plate

$f = \frac{\sigma^2}{2\varepsilon_0}$

$$\frac{\sigma^2}{2\varepsilon_0} = \frac{\mu_0 \sigma^2 v^2}{2}$$

$$v^2 = \frac{1}{\mu_0 E_0} \qquad v = \sqrt{\frac{1}{\mu_0 \varepsilon_0}}$$

$$v = \sqrt{\frac{1}{(4\pi\times10^{-7})(8.85\times10^{-12})}}$$

$$\boxed{v = 3\times10^{8} m/s}$$

5.14). $I_{enc} = \int_S J \cdot da$

amperes law

$$\oint B\cdot dl = \mu_0 \Sigma I_{enc}$$

$I_{enc} = \int_S J\cdot da$ J·da is independent from the surface

$$\nabla \cdot J = 0$$

Because the integral diverges, I will always be zero for a closed surface

5.20). a). density of ρ one free electron

$\rho = \frac{\text{charge}}{\text{volume}} = eN\left(\frac{1}{M}\right)d$

avogadros #, density of copper, mass of copper

charge of electron 1.6×10^{-19}

6.0×10^{23} = avogadros #

64 gm/mol = mass of copper

9 gm/cm^3 = density of copper

$$\rho = (1.6\times10^{-19})(6\times10^{23})\left(\frac{1}{64 g/mol}\right)(9 g/cm^3)$$

$\boxed{= 1.4\times10^{4} C/cm^3 \text{ charge density}}$

b). diameter of wire = 1mm $\quad$ radius $= \frac{d}{2}$

$= .001m$

$\frac{.001}{2} = 5\times10^{-4}m$

$A = \pi r^2$

$= \pi(5\times10^{-4})^2$

$= 7.85\times10^{-7}m^2$

current density $= J = \frac{I}{A}$ $\quad I = 1amp$

$J = \frac{1}{7.85\times10^{-7}m^2} = 1.2738\times10^{6}$

$J = \rho V$

$V = \frac{J}{\rho}$

$\frac{1.2738\times10^{6}(.0001)}{1.4\times10^{4}} = 9.1\times10^{-3}$

c). $d = 1.0cm$

$= 1.0cm(.01)$, $\frac{F_{mag}}{lenght} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$

$4\pi\times10^{-7}$ for μ_0 $\quad I_1 = I_2$

$\frac{F_{mag}}{lenght} = \frac{(4\pi\times10^{-7})(1A)(1)}{2\pi \quad 1cm}$

magnetic field $\boxed{2\times10^{-7}}$

D). $E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{d}$

$F_e = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_1\lambda_2}{d}$

$\lambda = \frac{1}{V}$

$\lambda_1 = \lambda_2 = \frac{1}{V}$ $\quad I_1 = I_2$

$F_e = \frac{1}{2\pi\varepsilon_0}\frac{\lambda_1\lambda_2}{d}$

$F_e = \frac{1}{V^2}\left(\frac{1}{2\pi\varepsilon_0}\right)\frac{I_1 I_2}{d}$

$c^2 = \frac{1}{\mu_0\varepsilon_0}$ $\quad \frac{1}{\varepsilon_0} = \mu_0 c^2$

$F_e = \frac{1}{V^2}\left(\frac{1}{2\pi\varepsilon_0}\right)\frac{I_1 I_2}{d}$

5.20). $F_e = \frac{1}{v^2}\left(\frac{\mu_0 c^2}{2\pi}\right)\frac{I_1 I_2}{d}$ $\quad F_{mag} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$

$$\frac{F_e}{F_{mag}} = \frac{\frac{1}{v^2}\left(\frac{\mu_0 c^2}{2\pi}\right)\frac{I_1 I_2}{d}}{\frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}}$$

$$\frac{f_{ele}}{f_{mag}} = \frac{c^2}{v^2} \quad c = 3\times10^{10} \quad 9.1\times10^{-3} = v$$

$$\left(\frac{3\times10^{10}}{9.1\times10^{-3}}\right)^2 = 1.1\times10^{25}$$

$$f_{ele} = 1.1\times10^{25}$$

$$(1.1\times10^{25}) f_{mag}$$

$$(1.1\times10^{25})(2\times10^{-7})$$

$$2\times10^{18}$$

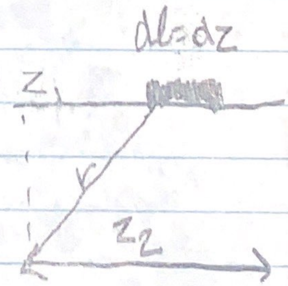
$$\boxed{2\times10^{18}\ N/cm}$$

5.23). $A = \frac{\mu_0}{4\pi}\int \frac{1}{r} dl' = \frac{\mu_0 I}{4\pi}\int \frac{1}{r} dl \quad A = \frac{\mu_0}{4\pi}\int \frac{K}{r} da'$

eqn 5.37 $\frac{\mu_0 I}{4\pi}\int_0^{\theta_2} \cos\theta d\theta = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$

Potential $A = \frac{\mu_0}{4\pi}\int \frac{1}{r} dz$

M field $B = \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$



$r = \sqrt{s^2+z^2}$

$$A = \frac{\mu_0}{4\pi}\int \frac{I\hat{z}}{r}\,dz$$

$$A = \frac{\mu_0 I}{4\pi}\int_{z_1}^{z_2} \frac{dz\,\hat{z}}{\sqrt{z^2+s^2}}$$

$$= \frac{\mu_0 I \hat{z}}{4\pi}\int_{z_1}^{z_2} \frac{dz}{\sqrt{z^2+s^2}}$$

$$= \frac{\mu_0 I \hat{z}}{4\pi} \ln\left(z+\sqrt{z^2+s^2}\right)\Big|_{z_1}^{z_2}$$

$$A = \frac{\mu_0 I}{4\pi}\left[\ln\left(z_2+\sqrt{z_2^2+s^2}\right) - \ln\left(z_1+\sqrt{z_1^2+s^2}\right)\right]\hat{z}$$

$$\boxed{= \frac{\mu_0 I}{4\pi}\left(\frac{\ln\left(z_2+\sqrt{z_2^2+s^2}\right)}{\ln\left(z_1+\sqrt{z_1^2+s^2}\right)}\right)\hat{z}}$$ magnetic potential

$$B = \frac{\mu_0 I}{4\pi s}\left(\sin\theta_2 - \sin\theta_1\right)$$

$$B = -\frac{dA}{ds}\hat{\phi}$$

$$\frac{\mu_0 I}{4\pi}\,\frac{\ln\left(z_2+\sqrt{z_2^2+s^2}\right)}{\ln\left(z_1+\sqrt{z_1^2+s^2}\right)} = A$$

$$B = -\frac{dA}{ds}\left(\frac{\mu_0 I}{4\pi}\ln\left(\frac{z_2+\sqrt{z_2^2+s^2}}{z_1+\sqrt{z_1^2+s^2}}\right)\right.$$

$$= -\frac{\mu_0 I}{4\pi}\left(\frac{1}{z_2+\sqrt{z_2^2+s^2}}\right)\left(\frac{s}{\sqrt{z_2^2+s^2}}\right) - \left(\frac{1}{z_1+\sqrt{z_1^2+s^2}}\right)\left(\frac{s}{\sqrt{z_1^2+s^2}}\right)$$

$$\boxed{= \frac{\mu_0 I s}{4\pi}\left(\frac{1}{z_2+\sqrt{z_2^2+s^2}}\right)\left(\frac{z_2-\sqrt{z_2^2+s^2}}{z_2-\sqrt{z_2^2+s^2}}\right)\frac{1}{z_2^2+s^2}\hat{\phi}}$$

5.26). vector potential s infinite straight wire

$\nabla\cdot A = 0 \quad \nabla\times A = B$

5.26). a). $A = A(s)\hat{z}$
continued

$B = \nabla \times A$

$$\nabla \times A = \left(\frac{1}{s}\frac{\partial A}{\partial \phi} - \frac{\partial A}{\partial z}\right)\hat{s} + \left(\frac{\partial A}{\partial z} - \frac{\partial A}{\partial s}\right) + \frac{1}{s}\left(\frac{\partial}{\partial s}(sA) - \frac{\partial A}{\partial \phi}\right)\hat{z}$$

where $\hat{s} = (\cos\phi)\hat{x} + (\sin\phi)\hat{y}$

$\hat{\phi} = -(\sin\phi)\hat{x} + (\cos\phi)\hat{y}$

$\hat{z} = z\hat{z}$

$dl = dx\hat{x} + dy\hat{y} + dz\hat{z}$

$dl = dx\hat{x} + dy\hat{y} + dz\hat{z}$

curl of the vector

$\nabla \times A = -\frac{\partial A}{\partial s}\phi$

magnetic field;

$B = \nabla \times A$

$= -\frac{\partial A}{\partial s}\hat{\phi}$

$B = \frac{\mu_0 I}{2\pi s}\phi \qquad B = -\frac{\partial A}{\partial s}$

$-\frac{\partial A}{\partial s} = \frac{\mu_0 I}{2\pi s}\phi \qquad \partial A = -\frac{\mu_0 I}{2\pi s}ds$

$A = -\int \frac{\mu_0 I}{2\pi s}ds$

$= \frac{\mu_0 I}{2\pi}\int \frac{1}{s}ds = \ln(s)$

$= \frac{\mu_0 I}{2\pi}\ln(s)\Big|_a^s$

$\boxed{= -\frac{\mu_0 I}{2\pi}\ln\left(\frac{s}{a}\right) = \text{vector Potential}}$

B). amperes law $\int B \cdot dl = B(2\pi s)$

$J = \frac{I}{A}$ current density

$J = \frac{I}{A}$

$J = \frac{I}{\pi s^2}$
area

$I = J(\pi s^2)$

$B(2\pi s) = \mu_0 \frac{I}{\pi R^2} \pi s^2$

$B = -\frac{\partial A}{\partial s} \phi$

$\frac{\partial A}{\partial s} = -\frac{\mu_0 I s}{2\pi R^2}$

$\boxed{\partial A = -\frac{\mu_0 I s}{2\pi R^2} \partial s}$ vector potential