

Prob 2.5

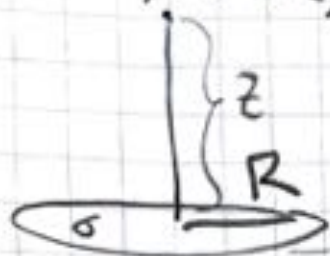
find the \vec{E} field at dist. z above center
in a circ. loop of radius " r "
with line charge " λ "



all Horiz. Cancel
due to Symmetry

$$R^2 = r^2 + z^2, \text{ and } \cos \theta = \frac{z}{R}, \int dl = 2\pi r$$
$$E = 4\pi\epsilon_0 \frac{\lambda(2\pi r)}{(r^2 + z^2)^{\frac{3}{2}}} z$$

2.6) find E field @ z above center of
 a disk, w/ radius R , surface chg σ



Horiz. cancel again,
 again $R^2 = r^2 + z^2$, $R_{ring} = R$

$$E_{ring} = \frac{1}{4\pi\epsilon_0} \frac{(\sigma 2\pi r) z}{(r^2 + z^2)^{3/2}}$$

$$\begin{aligned} \sigma 2\pi R_{ring} &= 2\pi\sigma R \\ \lambda &= \sigma 2\pi r \\ \text{line chg} \end{aligned}$$

$$E_{disk} = \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \left[\frac{1}{z} - \frac{1}{\sqrt{R^2 + z^2}} \right] \hat{z}$$

When $R \gg z$, term 2 \nearrow , $E_{plane} = \frac{1}{4\pi\epsilon_0} 2\pi\sigma \hat{z}$

When $z \gg R$, $\frac{1}{\sqrt{R^2 + z^2}} = \left(1 + \frac{R^2}{z^2}\right)^{-1/2} \approx \frac{1}{z} \left(1 - \frac{1}{2} \frac{R^2}{z^2}\right)$

$$E = \frac{1}{4\pi\epsilon_0} \frac{2\pi R^2 \sigma}{z^2}$$

1 (2.9) | $\epsilon = k r^3 \hat{r}$

a) $\rho = \epsilon_0 \nabla \cdot \epsilon = \epsilon_0 \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \cdot k r^3) = \epsilon_0 \frac{1}{r^2} k (5 r^4)$

$Q_{enc} = \epsilon_0 \oint \epsilon \cdot d\mathbf{a} = \epsilon_0 (k R^3) (4\pi R^2) = 4\pi \epsilon_0 k R^5$

$Q_{enc} = \int \rho d\tau = \int_0^R (5\epsilon_0 k r^2) (4\pi r^2 dr) =$

$20\pi \epsilon_0 k \int_0^R r^4 dr = 4\pi 3.0 k R^5$

HW #2

~~2.5, 2.6, 2.9, 2.12, 2.16, 2.18, 2.25, 2.29~~

2.12) use gauss's law to find the electric field inside a uniformly charged solid sphere (charge density ρ)

$$\oint \mathbf{E} \cdot d\mathbf{a} = E (4\pi r^2) = \frac{1}{\epsilon_0} Q_{enc}$$

$$= \frac{1}{\epsilon_0} \frac{4\pi r^3}{3} \rho \quad \Rightarrow \quad \boxed{E = \frac{1}{3\epsilon_0} \rho r \hat{r}}$$

2.16

"long coaxial cable"

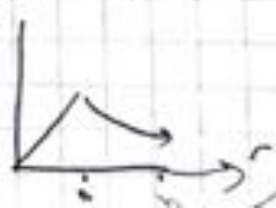


no surface charge

$$(i) \oint \mathbf{E} \cdot d\mathbf{a} = \frac{E \cdot 2\pi s \cdot l}{2\pi s l} = \frac{1}{\epsilon_0} \frac{\rho \pi s^2 l}{2\pi s l} = \boxed{\frac{\rho s}{2\epsilon_0} \hat{s}}$$

$$(ii) \oint \mathbf{E} \cdot d\mathbf{a} = E \cdot 2\pi s l = \frac{1}{\epsilon_0} \rho \pi a^2 l \quad \Rightarrow \quad \boxed{\frac{\rho a^2}{2\epsilon_0 s} \hat{s}}$$

$$(iii) \frac{\rho s}{2\epsilon_0} - \frac{\rho a^2}{2\epsilon_0 s} = 0, \quad \boxed{Q_{enc} = 0} \quad \left| \quad \begin{matrix} \uparrow \\ \text{E} \end{matrix} \right.$$



2.18



$$E_1 = \frac{1}{3}\epsilon_0 \rho r \hat{r}$$

$$E_2 = -\frac{1}{3}\epsilon_0 \rho r \hat{r}$$

$$E_{\text{tot}} = \frac{\rho}{3\epsilon_0} (R_+ - R_-)$$

$$E = \frac{\rho}{3\epsilon_0} d$$

$$R_+ - R_- = d$$

2.25

for a) $V = \frac{1}{4\pi\epsilon_0} \frac{2q}{\sqrt{x^2 + (\frac{L}{2})^2}}$

b) $V = \frac{1}{4\pi\epsilon_0} \int_{-L}^L \frac{\lambda dx}{\sqrt{x^2 + z^2}} = \frac{\lambda}{4\pi\epsilon_0} \ln(x + \sqrt{x^2 + z^2}) \Big|_{-L}^L$
 $= \frac{\lambda}{4\pi\epsilon_0} \ln \left[\frac{L + \sqrt{L^2 + z^2}}{-L + \sqrt{L^2 + z^2}} \right] = \left[\frac{2\lambda}{2\pi\epsilon_0} \ln \left(\frac{L + \sqrt{L^2 + z^2}}{z} \right) \right]$

c) $V = \frac{1}{4\pi\epsilon_0} \int_0^R \frac{\sigma 2\pi r dr}{\sqrt{r^2 + z^2}} = \frac{1}{4\pi\epsilon_0} 2\pi\sigma (\sqrt{r^2 + z^2}) \Big|_0^R \Rightarrow$
 $= \frac{\sigma}{2\epsilon_0} (\sqrt{R^2 + z^2} - z)$

L-25 pt 2

$$a) E = -\frac{1}{4\pi\epsilon_0} 2q \left(-\frac{1}{z}\right) \frac{2x}{\left(x^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{3}{2}}} = \frac{1}{4\pi\epsilon_0} \frac{2qz}{\left(x^2 + \left(\frac{L}{2}\right)^2\right)^{\frac{3}{2}}} \hat{z}$$

(checks out w/ 2.1)

$$b) E = -\frac{2}{4\pi\epsilon_0} \left\{ \frac{1}{(L + \sqrt{x^2 + L^2})^{\frac{1}{2}}} - \frac{1}{\sqrt{x^2 + L^2}} \right\} 2z - \frac{1}{(L - \sqrt{x^2 + L^2})^{\frac{1}{2}}} + \frac{1}{\sqrt{x^2 + L^2}} \hat{z}$$

$$= \frac{2}{4\pi\epsilon_0} \frac{z}{\sqrt{x^2 + L^2}} \left\{ \frac{-L + \sqrt{x^2 + L^2} - L - \sqrt{x^2 + L^2}}{(x^2 + L^2) - L^2} \right\} \hat{z}$$

$$= \frac{2Lz}{4\pi\epsilon_0} \frac{1}{\pm \sqrt{x^2 + L^2}} \hat{z} \quad (\text{check out w/ 2.2})$$

$$c) E = -\frac{0}{2\epsilon_0} \left\{ \frac{1}{2\sqrt{x^2 + z^2}} 2z - 1 \right\} \hat{z} = \frac{0}{2\epsilon_0} \left[1 - \frac{z}{\sqrt{x^2 + z^2}} \right] \hat{z}$$

(✓ w/ prob 2.6) ☺

- If R-H charge in (a) is $-q$, $V=0$, suggests $E = -\nabla V = 0$,
this differs from (2.1)

We know V on \hat{x} axis, cannot do $E_x = -\frac{\partial V}{\partial x}$ or $E_y = -\frac{\partial V}{\partial y}$
This works as symmetry makes E_x, E_y zero so no worries,
but now E is in x dir, and we have V on \hat{x} axis only,
cannot figure out E_x

2.29
Poisson's $\nabla^2 \phi = \frac{-\rho}{\epsilon_0}$

$$2.29 \rightarrow V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')}{R} d\tau'$$

$$1.102 \rightarrow \nabla^2 \frac{1}{R} = -4\pi \delta^3(R)$$

$$\nabla^2 V = \frac{1}{4\pi\epsilon_0} \nabla^2 \int \frac{\rho(r')}{R} d\tau'$$

$$\nabla^2 V = \frac{1}{4\pi\epsilon_0} \int \rho(r') \left[\nabla^2 \frac{1}{R} \right] d\tau'$$

$$\nabla^2 V = \frac{1}{4\pi\epsilon_0} \int \rho(r') \left[-4\pi \delta^3(r-r') \right] d\tau'$$

cancel out
in & cancels

$$\boxed{\nabla^2 V = -\frac{\rho(r')}{\epsilon_0}}$$