HW#21 ~~5~~, ~~6~~, ~~9~~, ~~14~~, ~~16~~, ~~19~~, 25, 29

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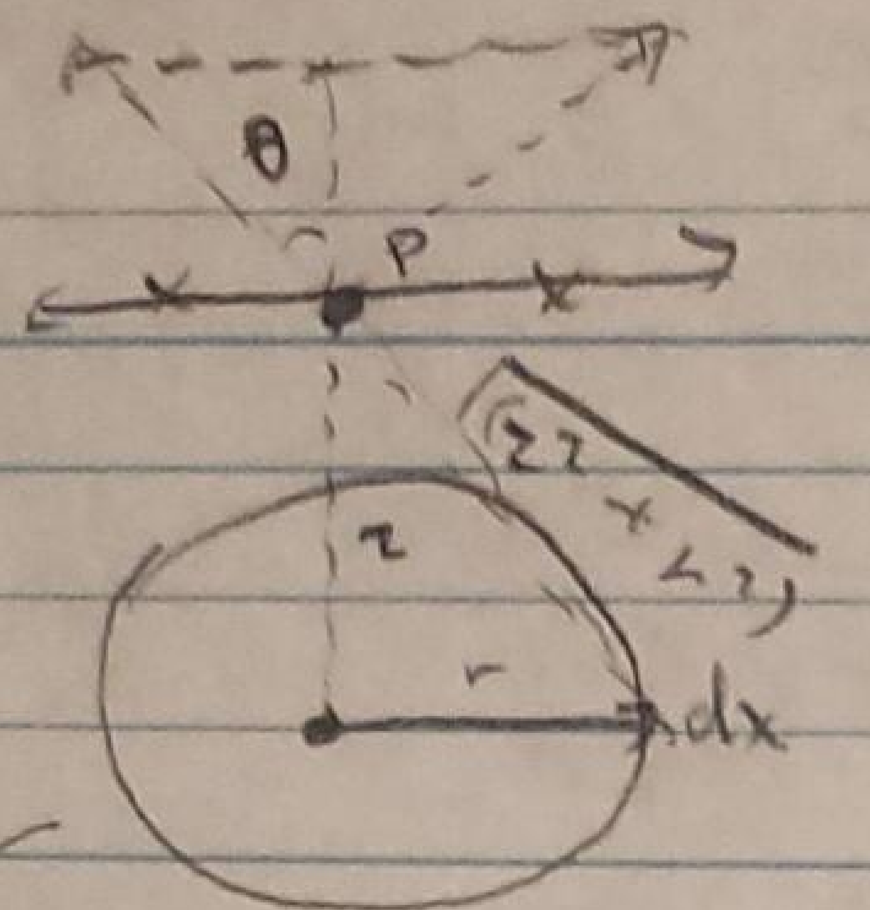
5)



$\vec{E} = \frac{kQ}{d} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$

goes from $0 \to 2\pi r$

λ = uniform line charge

$\cos\theta = \frac{z}{\sqrt{r^2+z^2}}$

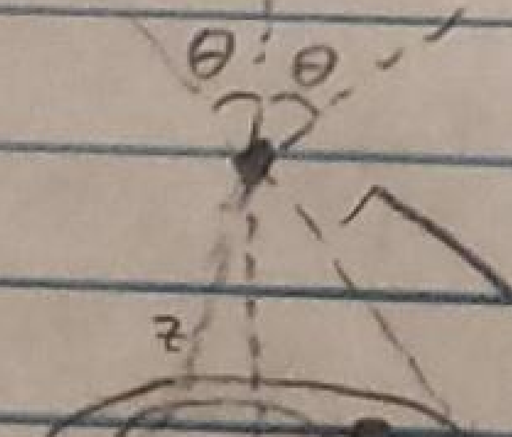
$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda dx}{(r^2+z^2)} \cdot \cos\theta$

$dE = \frac{1}{4\pi\varepsilon_0}\frac{\lambda z}{(z^2+r^2)^{3/2}}\,dx \Rightarrow$

$E = \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi r}\frac{\lambda z}{(z^2+r^2)^{3/2}}\,ds$

$$\boxed{E = \frac{\lambda\, z r}{2\varepsilon_0 (z^2+r^2)^{3/2}}}$$

6)



$\cos\theta = z/\sqrt{x^2+z^2}$

$\boxed{\sigma = Q = \text{uniform surface charge}}$

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} = 1/4\pi\varepsilon_0 \cdot Q/(x^2+z^2)$

$$\boxed{dE = k\left(\frac{dq}{(z^2+x^2)}\right)\cos\theta = k\left(\frac{dq \cdot z}{(z^2+x^2)^{3/2}}\right)}$$

$dq = \sigma(2\pi x)dx$

surface charge: $E(r) = k\int_0^{2\pi x} \frac{\sigma(r')}{\mathscr{r}^2}\hat{\mathscr{r}}\,da \Rightarrow k\cdot\sigma(2\pi x)dx$

$dE = k\,\sigma(2\pi x)\,dx \Rightarrow E = \int_s dE \Rightarrow \int_0^R \frac{1}{4\pi\varepsilon_0} (\sigma(2\pi x)dx)z / (z^2+x^2)^{3/2}$

$E = \frac{\sigma z}{2\varepsilon_0}\int_0^R \frac{x\,dx}{(z^2+x^2)^{3/2}} \Rightarrow \left[\frac{1}{\sqrt{x^2+z^2}}\right]\Bigg|_0^R$

$\boxed{E = \frac{\sigma z}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{z^2+R^2}}\right)}$, when $R \to \infty$

$\boxed{E_{R\to\infty} = \frac{\sigma}{2\varepsilon_0}}$

$\boxed{E_{z>>R} : \frac{\sigma\cdot(z^2+R^2)}{2\varepsilon_0}}$

q) $\vec{E} = kr^3\hat{r}$, (spherical coor.) $da = r^2\sin\theta\,d\theta\,d\phi\,\hat{r}$

a) $\rho = ?$ $\nabla\cdot E = \frac{\rho}{\varepsilon_0} \Rightarrow$

da $(\nabla\cdot kr^3\hat{r})\cdot\varepsilon_0 = \rho$

$(1/r^2 \cdot \partial/\partial r\,(r^2\cdot kr^3))\cdot\varepsilon_0 = \rho$

$1/r^2 \cdot \partial/\partial r(r^5)\,\varepsilon_0 = \rho$

$\boxed{5\varepsilon_0 k r^2 = \rho}$

b) $\vec{E} = kr^3 \hat{r}$, $\rho = 5\epsilon_0 k r^2$

Q=? , $\int \vec{E}\cdot da = \frac{Q}{\epsilon_0} \Rightarrow E\int \cdot da = \frac{Q}{\epsilon_0}$

$Q = \epsilon_0 \cdot (4\pi r^2)(kr^3)$

① $\boxed{Q = 4\pi r^5 k \epsilon_0}$

$\boxed{da = \rho d\tau} = 5\epsilon_0 k r^2 \cdot (4\pi r^2 dr)$

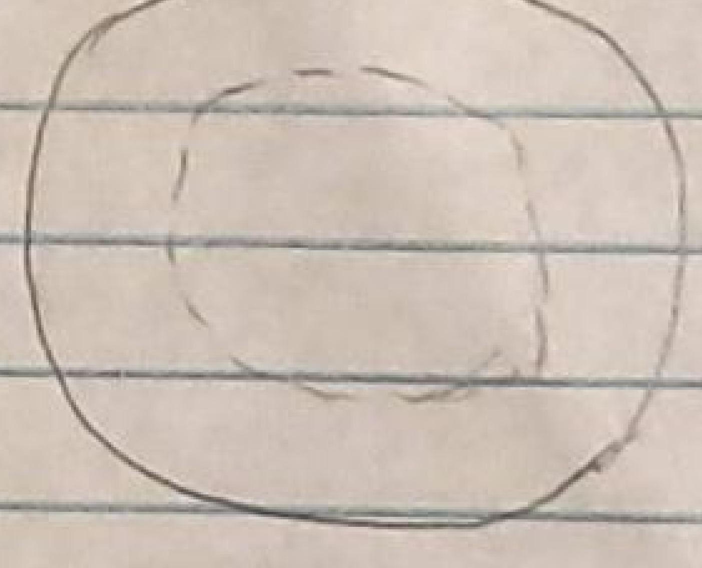
$Q = \int_0^R 5\epsilon_0 k r^4 4\pi dr \Rightarrow 4\pi \epsilon_0 k \int_0^R 5r^4 dr$

② $\boxed{Q = 4\pi r^5 \epsilon_0 k}$

12) $\vec{E} = ?$, Gauss's Law

uniformly charge sphere

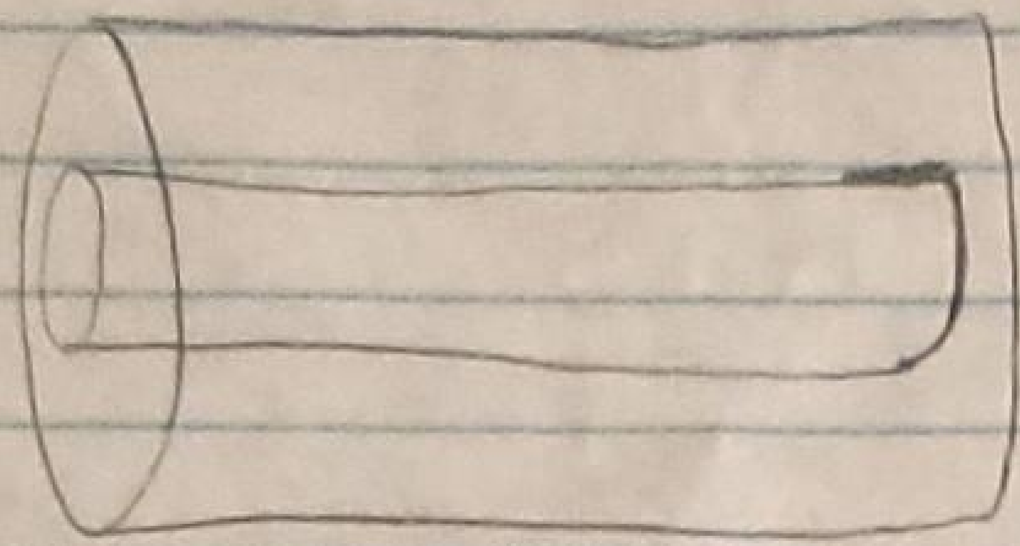
$\oint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$ $q_{enc} = \rho V = \rho(4/3 \pi r^3)$



$E = \frac{4\pi r^2 (q)}{\epsilon_0}$

$E = \frac{\cancel{4\pi r^2}\cdot(\rho \cdot 4/3 \pi r^3)}{\epsilon_0} \rightarrow \boxed{\vec{E} = \frac{\rho r}{3\epsilon_0}\hat{r}}$

1(c)



ρ = uniform volume density
a = uniform surface charge

a) $\vec{E} =$, $(s < a)$ inner cylinder

$$\oint E \cdot da = \frac{q_{enc}}{\varepsilon_0}$$

$$q_{enc} = \int \rho \, d\tau = 2\pi k l \int_0^s s'^2 \, ds' =$$

$$da = (2\pi s l)$$

$$= \frac{2}{3}\pi k l s^3$$

$$E = \frac{2\pi \rho l s^3}{3 \cdot 2 \varepsilon_0}$$

$$\boxed{E = \frac{\pi \rho s}{3\varepsilon_0}\hat{s}}$$

b) $(a < s < b)$

$$E = \frac{q_{enc}}{\varepsilon_0 (da)} = \frac{\rho(\pi a^2) l}{\varepsilon_0 \cdot (2\pi s l)}$$

$$q_{enc} = \int_V \rho \, dv = \int_0^l \int_0^{2\pi} \int_0^a \rho s \, ds \, d\phi \, dz =$$

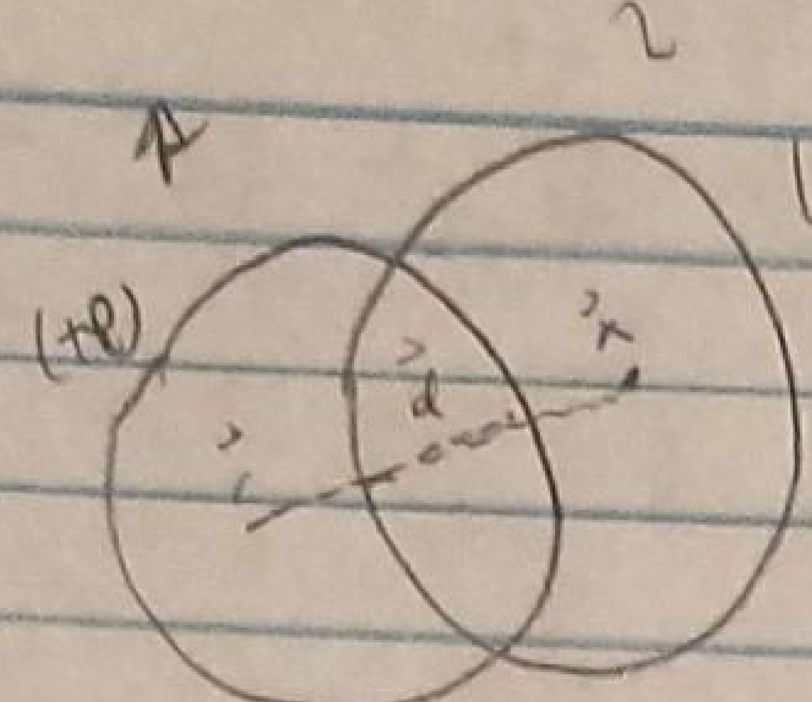
$$= \rho(\pi a^2) l$$

$$\boxed{E(s) = \frac{\rho a^2}{2 s \varepsilon_0}\hat{s}}$$

c) $(s < b)$

$q_{enc} = 0$, thus $\boxed{E = 0}$

18)



uniform volume charge densities

$\rho =$

(r=0) from (sphere 1)

From #2: $\vec{E}_1 = \frac{\rho r}{3\varepsilon_0}\hat{r}$

r>0

$\vec{E}_2 = -\frac{\rho(\vec{r}-\vec{d})}{3\varepsilon_0}$ (sphere 2)

$E_{TOT} = E_1 + E_2 = \frac{\rho}{3\varepsilon_0}(\vec{r}-\vec{r}+\vec{d}) = \boxed{\frac{\rho}{3\varepsilon_0}\vec{d}}$