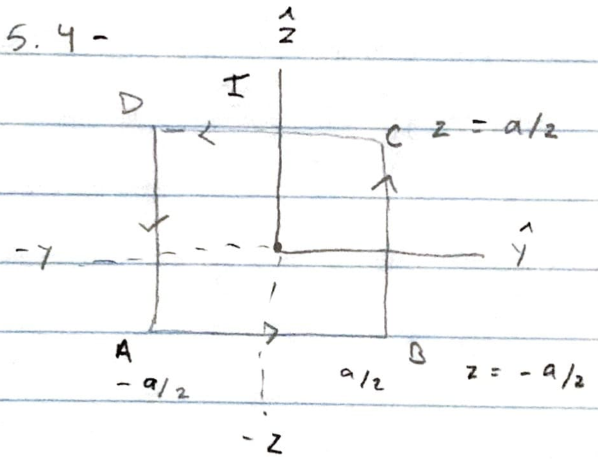
5.4, 5.7, 5.11, 5.12,
5.16, 5.19, 5.21,
ex. 5.12, 5.23, 5.27

Erandi Macias

Electro hw 5

5.4 -



$$\vec{F} = I\int d\vec{l} \times \vec{B}$$

$$F_{AB} = I\int dy\,\hat{y} \times k\left(-\frac{a}{2}\right)\hat{x}$$

$$= -Ik\frac{a}{2}\int_{a/2} (\hat{y}\times\hat{x})\,dy$$

$$= -Ik\frac{a}{2}\int_{-a/2}^{a/2} dy\,(-\hat{z})$$

$$= Ik\frac{a^2}{2}\hat{z}$$

$$\vec{F}_{BC} = I\int dz\,\hat{z} \times kz\hat{x}$$

$$= Ik\int_{-a/2}^{a/2} z\,dz\,(-\hat{y}) = 0$$

$$\vec{F}_{CD} = I\int dy(-\hat{y}) \times k\frac{a}{2}\hat{x}$$

$$= Ik\frac{a^2}{2}\hat{z}$$

$$F_{DA} = I\int dz(-\hat{z}) \times kz\hat{x}$$

$$= Ik\int_{-a/2}^{a/2} z\,dz\,(-\hat{z}\times\hat{x}) = 0$$

$$\vec{F} = Ik\frac{a^2}{2}\hat{z} + 0 + Ik\frac{a^2}{2}\hat{z} + 0 = Ika^2\hat{z}$$

5.7-

$$\nabla x = \hat{x}$$

$$\vec{p} = \int \vec{r}\,' \rho(\vec{r}\,')\, d\tau'$$

$$\frac{d\vec{p}}{dt} = \frac{d}{dt}\int \rho\, \vec{r}\, d\tau = \int \frac{\partial \rho}{\partial t}\, \vec{r}\, d\tau$$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \qquad \frac{d\vec{p}}{dt} = \int \frac{\partial \rho}{dt}\, \vec{r}\, d\tau$$

$$= -\int (\nabla \cdot \vec{J})\, \vec{r}\, d\tau$$

$$\nabla \cdot (x\vec{J}) = x(\vec{\nabla} \cdot \vec{J}) + \vec{J} \cdot (\nabla x)$$

$$\nabla \cdot (x\vec{J}) = x(\nabla \cdot \vec{J}) + J_x$$

$$\int (\nabla \cdot \vec{J})\, x\, d\tau = \int \nabla (x\vec{J})\, d\tau - \int \vec{J}_x\, d\tau$$

$$\int (\nabla \cdot \vec{J})\, x\, d\tau = \int \vec{\nabla} \cdot (x\vec{J})\, d\tau - \int \vec{J}_x\, d\tau$$

$$\int \vec{\nabla} (x\vec{J})\, d\tau = \int (x\vec{J}) \cdot d\vec{a}$$

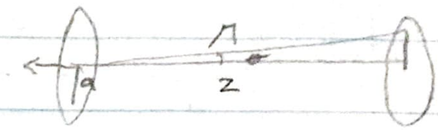
$$\int (\vec{\nabla} \cdot \vec{J})\, x\, d\tau = -\int J_x\, d\tau$$

$$\int (\vec{\nabla} \cdot \vec{J})\, \vec{r}\, d\tau = -\int \vec{J}\, d\tau$$

$$\frac{d\vec{p}}{dt} = \int \vec{J}\, d\tau$$

5.11 -

$$B = \frac{\mu_0 n I}{2} \int \frac{a^2}{(a^2+z^2)^{3/2}}\, dz$$



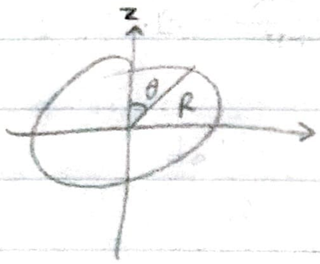
$Z = a \cot \theta$

$$dz = -\frac{a}{\sin^2\theta}\, d\theta \quad ; \quad \frac{1}{(a^2+z^2)^{3/2}} = \frac{\sin^3\theta}{a^3}$$

$$B = \frac{\mu_0 n I}{2} \int_{\theta_1}^{\theta_2} \frac{a^2 \sin^3\theta}{a^3 \sin^2\theta} (-a\, d\theta) = -\frac{\mu_0 n I}{2} \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta$$

$$= \frac{\mu_0 n I}{2} \Big[\cos\theta \Big]_{\theta_1}^{\theta_2} = \frac{\mu_0 n I}{2} (\cos\theta_2 - \cos\theta_1)$$

5.12 -



$$dB = \frac{\mu_0 \, dI}{2} \frac{(R\sin\theta)^2}{((R\sin\theta)^2 + (R\cos\theta)^2)^{3/2}}$$

$$= \frac{\mu_0}{2R} \sin^2\theta \, dI$$

$dI = K \, R \, d\theta$

$K = \sigma v$

$\sigma = \frac{Q}{4\pi R^2}$

$v = \omega R \sin\theta$

$$dI = \frac{Q}{4\pi \cancel{R^2}} \omega \cancel{R} \sin\theta \cancel{R} \, d\theta$$

$$= \frac{Q\omega}{4\pi} \sin\theta \, d\theta$$

$$B = \frac{\mu_0}{2R} \frac{Q\omega}{4\pi} \int_0^{\pi} \sin^3\theta \, d\theta = \frac{\mu_0 Q \omega}{8\pi R} \left(\frac{4}{3}\right)$$

$$= \frac{\mu_0 Q \omega}{6\pi R} \hat{z}$$

5.16 -

$$\vec{B} = \mu n I \hat{z}$$

(i) $\vec{B} = \mu_0 n_2 I \hat{z} + \mu_0 n_1 I (-\hat{z}) = \mu_0 I (n_2 - n_1) \hat{z}$

(ii) $\vec{B} = -\mu_0 n_2 I \hat{z}$

(iii) $\vec{B} = 0$

5.19 -

$$I = \int_{S_2} \vec{J} \cdot d\vec{a} - \int_{S_1} \vec{J} \cdot d\vec{a}$$

$$= \int_{S_1, S_2} \vec{J} \cdot \hat{n} \, d\bar{a}$$

$$= \int \cancel{\nabla \cdot \vec{J}}^{\,0} \, dv$$

$$= 0$$

5.21 –

Ampere's law: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

$$\nabla \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

$$\nabla \cdot (\nabla \times B) = \mu_0 \nabla \cdot \vec{J} = -\mu_0 \frac{\partial \rho}{\partial t} \quad \times$$

$$\vec{\nabla} \times \vec{E} = 0 \rightarrow \vec{\nabla} \cdot (\vec{\nabla} \times \vec{E}) = 0 \quad \checkmark$$

Ex. 5.12 -

$$\oint \vec{A} \cdot d\vec{l} = \int (\nabla \times \vec{A}) \cdot d\vec{a} = \int \vec{B} \cdot d\vec{a} = \Phi$$

$$\oint \vec{B} \cdot dl = \mu_0 I_{enc.}$$

$$\oint \vec{A} \cdot d\vec{l} = A(2\pi s) = \int \vec{B} \cdot d\vec{a} = \mu_0 n I (\pi s^2)$$

$$\vec{A} = \frac{\mu_0 n I}{2} s \hat{\phi} \quad \text{for } s \leq R$$

$$\int \vec{B} \cdot d\vec{a} = \mu_0 n I (\pi R^2)$$

$$\vec{A} = \frac{\mu_0 n I}{2} \frac{R^2}{s} \hat{\phi} \quad \text{for } s \geq R$$

* 5.23 -

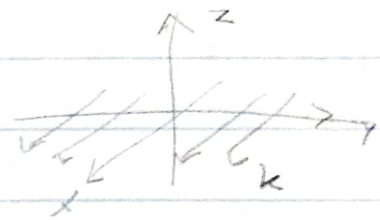
$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{I}{\mathscr{r}} \hat{z} \, dz = \frac{\mu_0 I}{4\pi} \hat{z} \int_{z_1}^{z_2} \frac{1}{\sqrt{z^2 + s^2}} dz$$

$$= \frac{\mu_0 I}{4\pi} \hat{z} \left[\ln \left(z + \sqrt{z^2 + s^2} \right) \right]_{z_1}^{z_2} \quad \text{using wolfram}$$

$$= \frac{\mu_0 I}{4\pi} \ln \left(\frac{z_2 + \sqrt{(z_2)^2 + s^2}}{z_1 + \sqrt{(z_1)^2 + s^2}} \right) \hat{z}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = -\frac{\partial A}{\partial s} \hat{\phi}$$

5.27 -



$$K = k\,\hat{x} = \frac{\mu_0 k}{2}\,\hat{y}$$

$$\vec{A} = A(z)\,\hat{x}$$

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ A(z) & 0 & 0 \end{vmatrix} = \frac{\partial A}{\partial z}\,\hat{y}$$

$$\vec{A} = -\frac{\mu_0 k}{2}\,\hat{x}$$