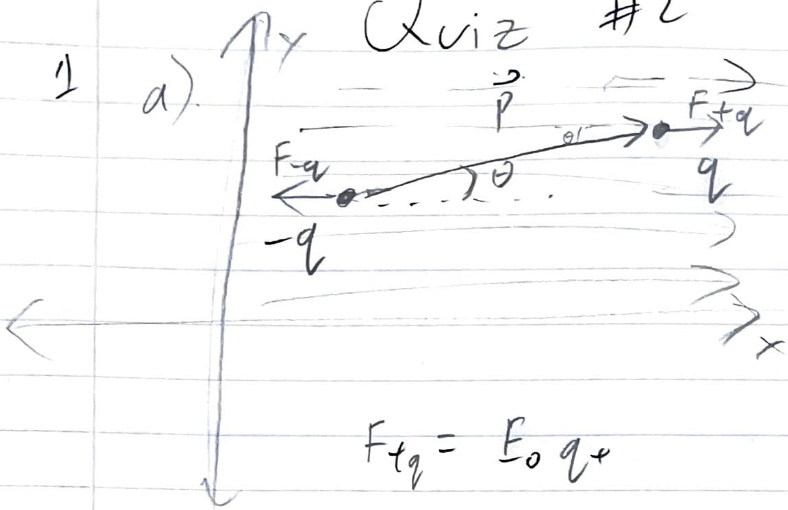
Quiz #2

1 a).



$\vec{p} = q\vec{d}$

$E = E_0 \hat{x}$ positive x direction

$\vec{E} = \frac{\vec{F}}{q}$

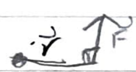
$\therefore \vec{F} = \vec{E}q$

$F_{+q} = E_0 q_+$

$F_{-q} = E_0 q_-$

$\vec{\tau} = \vec{r} \times \vec{F} = \vec{r} \sin\theta \vec{F}$

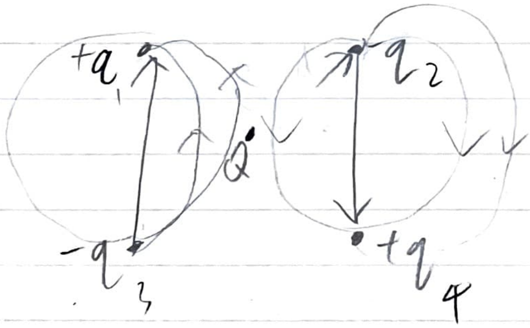
typically



$\vec{\tau} = \vec{p} \cdot E_0 \sin\theta$

$\vec{\tau} = \vec{p} \times \vec{E}$

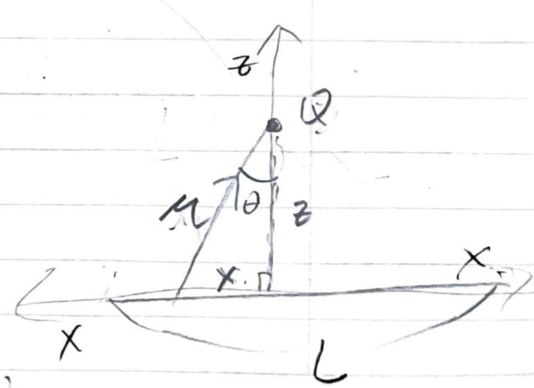
b))



$E = \frac{1}{4\pi\epsilon_0}$

0?

2. a $E(r) = \frac{1}{4\pi\epsilon_0} \int_0^L \frac{\lambda(r')}{r^2} \hat{r} \, dl'$



$r = z^2 + x^2$ $\quad \vec{r} = z\hat{z} - x\hat{x}$

$\hat{r} = \frac{\vec{r}}{r} = \frac{z\hat{z} - x\hat{x}}{(z^2+x^2)^{1/2}}$

$$\frac{k\,dq}{r^2}\hat{r} = k\lambda\,dx\,\frac{\hat{r}}{r^2}.$$

$$d\vec{E} = k\lambda\,dx\,\frac{(z\hat{z} - x\hat{x})}{(z^2+x^2)^{3/2}}$$

$$= k\lambda\left[z\hat{z}\int_0^L \frac{1}{(z^2+x^2)^{3/2}} - \hat{x}\int_0^L \frac{x\,dx}{(z^2+x^2)^{3/2}}\right]$$

wolfram $$k\lambda\left[z\left[\frac{x}{z^2\sqrt{z^2+x^2}}\right]_0^L\hat{z} - \left[-\frac{1}{\sqrt{x^2+z^2}}\right]_0^L\hat{x}\right]$$

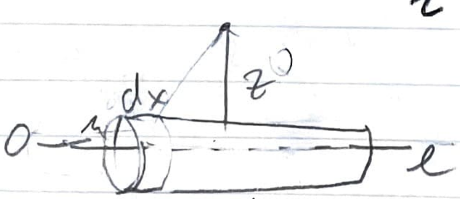
$$= k\lambda\left[\left(\frac{L}{z\sqrt{z^2+L^2}} - 0\right)\hat{z} + \left(\frac{1}{\sqrt{z^2}} - \frac{1}{\sqrt{L^2+z^2}}\right)\hat{x}\right]$$

$$= k\lambda\left(\frac{L}{z\sqrt{z^2+L^2}}\hat{z} + \left(\frac{1}{z} - \frac{1}{\sqrt{L^2+z^2}}\hat{x}\right)\right)$$

b) $L \gg z$! $$\frac{L}{zL}\hat{z} + \left(\frac{1}{z} - \frac{1}{\sqrt{L^2}}\right) \to 0$$

$$E \approx \frac{1}{z}\hat{z} + \frac{1}{z}\hat{x}$$

b)



$$\oint \vec{E}\,d\vec{A} = \frac{Q}{\varepsilon_0}$$

$$= E\oint dA$$

$$= E\,4\pi r^2 = \frac{Q_{inc}}{\varepsilon_0}$$

$$E_{\text{at-slice}} = \frac{Q_{inc}}{\epsilon\,4\pi z^2}$$

$$Q_{enc\,(slice)} = \lambda\, dx$$

$$\text{Total } E = \int_0^L \frac{\lambda\, dx}{4\pi \epsilon_0 r^2}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{L}{z\sqrt{z^2+L^2}}\hat{z} + \left(\frac{1}{z} - \frac{1}{\sqrt{L^2+z^2}}\right)\hat{x}\right]$$

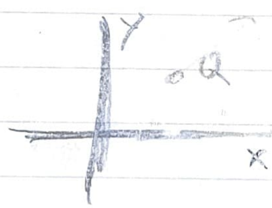
same set up a) part 1.

2. $E = \frac{\sigma}{2\epsilon_0}$ $\qquad E_1 = \frac{\sigma}{2\epsilon_0}$ $\qquad E_2 = \frac{\sigma}{2\epsilon_0}$

a) $E = \frac{\sigma_1 - \sigma_2}{2\epsilon}$

b) $E = \frac{\sigma_1 + \sigma_2}{2\epsilon_0}$

c)



$$E = \frac{\sigma_{xz}}{2\epsilon_0}\hat{x} + \frac{\sigma_{yz}}{2\epsilon_0}\hat{y}$$

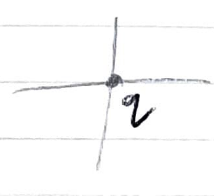
3. $\oint \vec{E} \cdot d\hat{l} = 0$ $\qquad \vec{E} = -\nabla V$

$$-\int_a^b \vec{E} \cdot d\vec{l} = V(b) - V(\vec{a})$$

$$-\int_a^b -\nabla V\, d\vec{l} = \int_a^b V'\, dl = V(b) - V(a)$$

FTC

$$V(\vec{r}) = -\int_\infty^r E(r')\, dr'$$



$$E = k\frac{q}{r^2}\hat{r}$$

$$V(r) = -\int_{\infty}^{r} \left(\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\right) d\vec{r} = +\frac{1}{4\pi\epsilon_0}\frac{q}{r}\Bigg|_{\infty}^{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} - 0$$

$$= k\frac{q}{r}$$