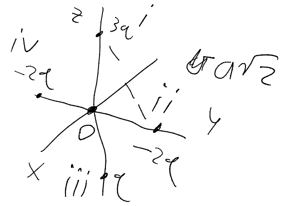
3.29

Thursday, December 10, 2020 12:07 AM

Find an expression for potential far from origin




$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_n}{r_n}\right)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{3q}{r_i} - \frac{2q}{r_{ii}} + \frac{q}{r_{iii}} - \frac{2q}{r_{iv}}\right)$$

$$r_i^2 = r^2 + \left(\frac{a\sqrt{2}}{2}\right)^2 - ra\sqrt{2}\cos\theta = r^2\left(1 - \frac{a\sqrt{2}}{r}\cos\theta + \frac{a^2}{2r^2}\right)$$

$r \gg d$

$$\frac{1}{r_i} = \frac{1}{r}\left(1 - \frac{a\sqrt{2}}{r}\cos\theta\right)^{-1/2} \approx \frac{1}{r}\left(1 + \frac{a\sqrt{2}}{2r}\cos\theta\right)$$

$$r_{ii}^2 = r^2 + \left(\frac{a\sqrt{2}}{2}\right)^2 + ra\sqrt{2}\cos\theta = r^2\left(1 + \frac{a\sqrt{2}}{r}\cos\theta + \frac{a^2}{2r^2}\right)$$

$$\frac{1}{r_{ii}} = \frac{1}{r}\left(1 + \frac{a\sqrt{2}}{r}\cos\theta\right)^{-1/2} \approx \frac{1}{r}\left(1 - \frac{a\sqrt{2}}{2r}\cos\theta\right)$$

$$\frac{1}{r_{iii}} \approx \frac{1}{r}\left(1 + \frac{a\sqrt{2}}{2r}\cos\theta\right), \quad \frac{1}{r_{iv}} \approx \frac{1}{r}\left(1 - \frac{a\sqrt{2}}{2r}\cos\theta\right)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{3q}{r}\left(1 + \frac{a\sqrt{2}}{2r}\cos\theta\right) + \frac{q}{r}\left(1 + \frac{a\sqrt{2}}{2r}\cos\theta\right) - \frac{2q}{r}\left(1 - \frac{a\sqrt{2}}{2r}\cos\theta\right) - \frac{2q}{r}\left(1 - \frac{a\sqrt{2}}{2r}\cos\theta\right)\right)$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\left(\frac{4q}{r}\left(1 + \frac{a\sqrt{2}}{2r}\cos\theta\right) - \frac{4q}{r}\left(1 - \frac{a\sqrt{2}}{2r}\cos\theta\right)\right)$$

$$= \frac{q}{\pi\epsilon_0 r}\left(1 + \frac{a\sqrt{2}}{2r}\cos\theta - 1 + \frac{a\sqrt{2}}{2r}\cos\theta\right) = \boxed{\frac{qa\sqrt{2}}{\pi\epsilon_0 r^2}\cos\theta}$$