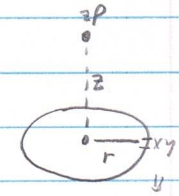
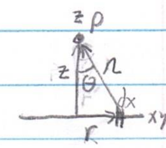
Jesse Chi-Su-G
11/9/20 PHYS

# HW2



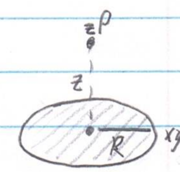



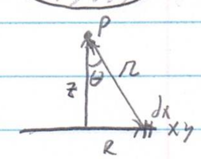

(2.5) $\vec{E} = \oint d\vec{E}\cos\theta\,\hat{z}$ // $\cos\theta = \frac{z}{\mathcal{R}} = \frac{z}{\sqrt{z^2+r^2}}$ , $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{dq}{\mathcal{R}^2} = \frac{k\,dq}{z^2+r^2}$

$\vec{E} = \oint \frac{k z\,dq}{(z^2+r^2)^{3/2}}\hat{z}$ // $dq = r\lambda\,d\phi$

$\vec{E} = \int_0^{2\pi} \frac{k z r \lambda}{(z^2+r^2)^{3/2}} d\phi\,\hat{z} = \frac{k z r \lambda\,\hat{z}}{(z^2+r^2)^{3/2}} \int_0^{2\pi} d\phi$

$\vec{E} = \frac{k z r \lambda\,\hat{z}}{(z^2+r^2)^{3/2}}(2\pi) \Rightarrow \vec{E} = \frac{1}{2 4\pi\varepsilon_0}\cdot\frac{z r \lambda 2\pi\,\hat{z}}{(z^2+r^2)^{3/2}} \Rightarrow \boxed{\vec{E} = \frac{z r \lambda}{2\varepsilon_0 (z^2+r^2)^{3/2}}\hat{z}}$




$u = z^2 + r^2 \Rightarrow du = 2r\,dr$
$r = \frac{du}{2r}$

(2.6) $\vec{E}_{loop} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi z r \lambda}{(z^2+r^2)^{3/2}}\hat{z}$ // $\vec{E} = d\vec{E}$ , $\lambda \to \sigma\,dr$

$d\vec{E}_{disk} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\pi z r \sigma\,dr}{(z^2+r^2)^{3/2}}\hat{z} \Rightarrow \vec{E} = \frac{1}{4\pi\varepsilon_0}\int_0^R \frac{2\pi z r \sigma\,dr}{(z^2+r^2)^{3/2}}\hat{z}$

$\vec{E} = \frac{\cancel{2}\cancel{\pi} z\sigma}{\cancel{2}4\cancel{\pi}\varepsilon_0}\hat{z}\int_0^R \frac{r}{(z^2+r^2)^{3/2}}dr \Rightarrow \vec{E} = \frac{z\sigma}{2\varepsilon_0}\hat{z}\int_0^R \frac{r\,dr}{(z^2+r^2)^{3/2}}$ , // $u = z^2+r^2$, $du = 2r\,dr$
$dr = \frac{du}{2r}$

$\vec{E} = \frac{z\sigma}{2\varepsilon_0}\hat{z}\int_0^R \frac{du}{2(u)^{3/2}} \Rightarrow \vec{E} = \frac{z\sigma}{4\varepsilon_0}\hat{z}\int_0^R u^{-3/2}du \Rightarrow \vec{E} = \frac{z\sigma}{4\varepsilon_0}\hat{z}\left[-2u^{-1/2}\right]_0^R \Rightarrow \vec{E} = \frac{z\sigma}{4\varepsilon_0}\hat{z}\left[\frac{-2}{\sqrt{z^2+r^2}}\right]_0^R$

$\vec{E} = \frac{z\sigma}{4\varepsilon_0}\hat{z}\left[\frac{-2}{\sqrt{z^2+R^2}} + \frac{2}{z}\right] \Rightarrow \vec{E} = -\frac{z\sigma}{2\varepsilon_0}\hat{z}\left[\frac{1}{\sqrt{z^2+R^2}} - \frac{1}{z}\right] \Rightarrow \boxed{\vec{E} = \frac{z\sigma}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right)\hat{z}}$

When $R \to \infty$, $\vec{E} = \frac{z\sigma}{2\varepsilon_0}\left(\frac{1}{z} - \cancel{\frac{1}{\infty}}^{0}\right)\hat{z} \Rightarrow \boxed{\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z}}$ When $R \to \infty$

When $z \gg R$, $\vec{E} = \frac{z\sigma}{2\varepsilon_0}\left(\frac{1}{z} - \frac{1}{z\sqrt{1+(R/z)^2}}\right)\hat{z} = \frac{z\sigma}{2\varepsilon_0}\cdot\frac{1}{z}\left(1 - \frac{1}{\sqrt{1+(R/z)^2}}\right)\hat{z}$ , // $z \gg R$

$\vec{E} = \frac{\sigma}{2\varepsilon_0}$