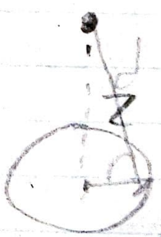
Adam Wildanger the First

2.5 distance z above loop radius r



$\imath = \sqrt{z^2 + r^2}$ $\quad \vec{\imath} = z\hat{z} + r\hat{r}$

$r\hat{r} = (x\hat{x} + y\hat{y})$

$\vec{\imath} \approx z\hat{z}$

$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{1}{\imath^2} \hat{\imath}\, dq$

$d\vec{E}_z = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{\imath^2} \cos\theta$

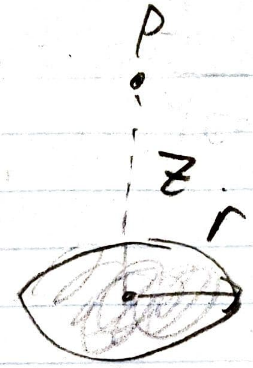
$= \frac{1}{4\pi\epsilon_0} \frac{dq}{\sqrt{R^2+z^2}} \cos\theta = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{\sqrt{R^2+z^2}} \cdot \frac{z}{\sqrt{R^2+x^2}}$

$= \frac{1}{4\pi\epsilon_0} \cdot \frac{z\,dq}{(R^2+z^2)^{1/2}} \Rightarrow \frac{z}{4\pi\epsilon_0 (R^2+z^2)^{3/2}} \int_0^{2\pi R} \lambda\, dq$

$= \frac{2\pi z}{4\pi\epsilon_0 (R^2+z^2)} = \boxed{\frac{z\lambda}{2\epsilon_0 (R^2+z^2)}}$

(2.6) disk

$R \to \infty$? $z >> R$?



$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{\sigma z\, da}{(r^2+z^2)^{3/2}}$$

$$\frac{\sigma}{4\pi\epsilon_0}\int_0^{2\pi}\int_0^{R} \frac{z\, r\,dr\,d\phi}{(r^2+z^2)^{3/2}} = \frac{\sigma z}{2\epsilon_0}\int_0^{R} \frac{r}{(r^2+z^2)^{3/2}}\,dr$$

$r^2+z^2 = u$

$du = 2r\,dr$

$$= \frac{\sigma z}{2\epsilon_0}\left[\frac{1}{z} - \frac{1}{\sqrt{z^2+R^2}}\right]$$

$\frac{du}{2} = r\,dr$

$$= \boxed{\frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]}$$

$u(R) = R^2+z^2$

$$\lim_{R \to \infty} = \boxed{\frac{\sigma}{2\epsilon_0}}$$

$$\lim_{z >> R} = \frac{\sigma}{4\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2}}\right] = \frac{\sigma}{4\epsilon_0}\left[1-1\right] = \boxed{0}$$

2.9 $\vec{E} = kr^3\hat{r}$ in spherical.

(a) find ρ

$$\nabla\cdot E = \frac{1}{\epsilon_0}\rho$$

$$\nabla\cdot\vec{E} = \frac{1}{r^2}\frac{\partial}{\partial r}(kr^5) = \frac{5kr^4}{r^2} = 5kr^2$$

$$5kr^2 = \frac{1}{\epsilon_0}\rho$$

$$\boxed{5\epsilon_0 k r^2 = \rho}$$

(b) Find total charge in sphere $r=R$
do it 2 ways

$$\int_V \nabla\cdot\vec{E}\,d\tau = \int_0^R\int_0^\pi\int_0^{2\pi} 5\epsilon_0 k r^2\cdot r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\epsilon_0 5k\cdot 2\pi\int_0^R\int_0^\pi r^4\sin\theta\,dr\,d\theta = \epsilon_0 5k\cdot 2\pi\left(-\cos\theta\Big|_0^\pi\right)\int_0^R r^4\,dr$$

$$= 5k\cdot 2\pi(-(-2))\frac{r^5}{5} = \boxed{4\pi\epsilon_0 k r^5}$$

$$\oint_S \vec{V}\cdot d\vec{a} = \oint_{sphere} \vec{E}\cdot d\vec{a} = \frac{Q_{enc}}{\epsilon_0}$$

$r = R$

$d\vec{a} = dl_\theta\, dl_\phi = r^2 \sin\theta\, d\theta\, d\phi$

$$\int_0^{2\pi}\int_0^{\pi} k r^5 \sin\theta\, d\theta\, d\phi = 2\pi \int_0^{\pi} k r^5 \sin\theta\, d\theta$$

$$= 2\pi k r^5 \cdot [2] \Rightarrow \boxed{4\pi k r^5 \cdot \epsilon_0} = Q_{enc}$$

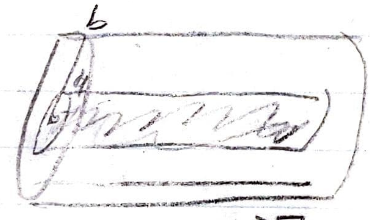
2.12 Use Gauss's law for $\vec{E}$ in sphere charge density ρ

$r=R$

$$\oint_S \vec{E}\cdot d\vec{a} = \oint_S |\vec{E}|\, da = \int_0^{\pi}\int_0^{2\pi} \rho\, R^2 \sin\theta\, d\theta\, d\phi$$

$$= 2\pi\rho R^2 \int_0^{\pi} \sin\theta\, d\theta = 2\pi\rho R^2 \cdot 2 = \boxed{4\pi\rho R^2}$$

2.16 Find $\vec{E}$ inside cylinder between the two, outside



(i)

$$\int_V \rho\, d\tau = \int_0^{a}\int_0^{2\pi}\int_0^{z} \rho\, s\, ds\, d\phi\, dz = z\rho \int_0^{a}\int_0^{2\pi} s\, ds\, d\phi$$

$$= 2z\rho\pi \int_0^{a} s\, ds = \boxed{z\rho\pi a^2}$$

(ii)

$$0 = \int_V^{b} \rho\, d\tau + \int_{s=b} \sigma\, da = \int_0^{b}\int_0^{2\pi}\int_0^{z} \rho\, s\, ds\, d\phi\, dz$$

$$+ \int_0^{2\pi}\int_0^{z} \sigma\, s\, d\phi\, dz = z\rho\pi b^2 + \sigma z b \int_0^{2\pi} d\phi$$

$$= z\rho\pi b^2 + \sigma z b 2\pi = 0 \qquad \sigma = \frac{-\rho b}{2}$$

$$\sigma \cancel{z}\cancel{b} 2\pi = -\cancel{z}\rho\cancel{\pi} b^{\cancel{2}}$$

$$\int_V \rho\, d\tau - \int_a \frac{\rho b}{2}\, da = z\rho\pi(b-a)^2 - \frac{\rho b}{2}\int_0^{z}\int_0^{2\pi} s\, d\phi\, dz\, da$$

$$= z\rho\pi(b-a)^2 - \pi\rho b(b-a)\int_0^{z} dz$$

$$= (z\rho\pi)(b-a))((b-a)-b) = (z\rho\pi)(b-a)(-a)$$

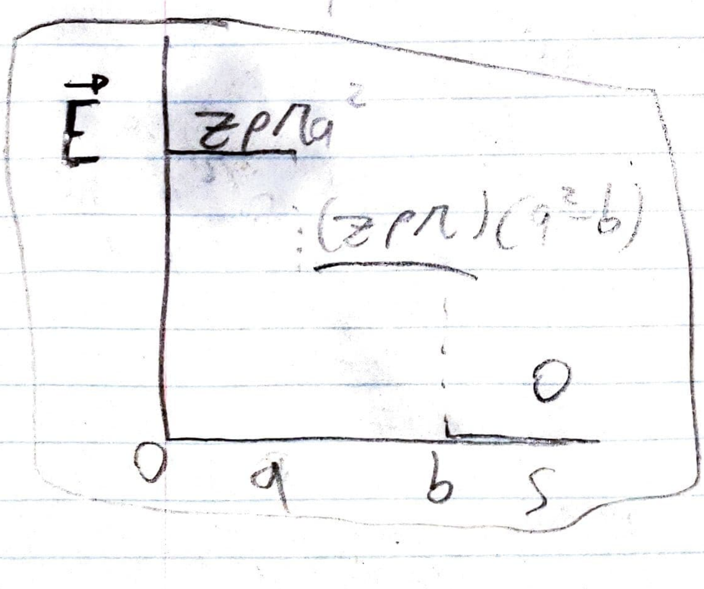
$$= \boxed{(z\rho\pi)(a^2-b)}$$

iii) $\int_V \rho\, d\tau - \frac{\rho b}{2}\int_a da$

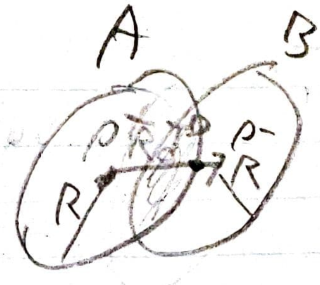
$s = 2b$

$$= \int_b^{2b}\int_0^{2\pi}\int_0^{z} \rho s\, ds\, d\phi\, dz - \frac{\rho b}{2}\int_0^{2\pi}\int_0^{z} s\, d\phi\, dz$$

$$= z\rho\pi(b^2) - \rho b^2 \cdot z\pi = \boxed{0}$$



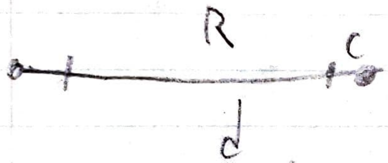
2.18 Find charge in overlap



Charge in sphere is $4\pi\rho R^2$

sphere A: $4\pi\rho r^2$ sphere B: $-4\pi\rho r^2$

$r=$

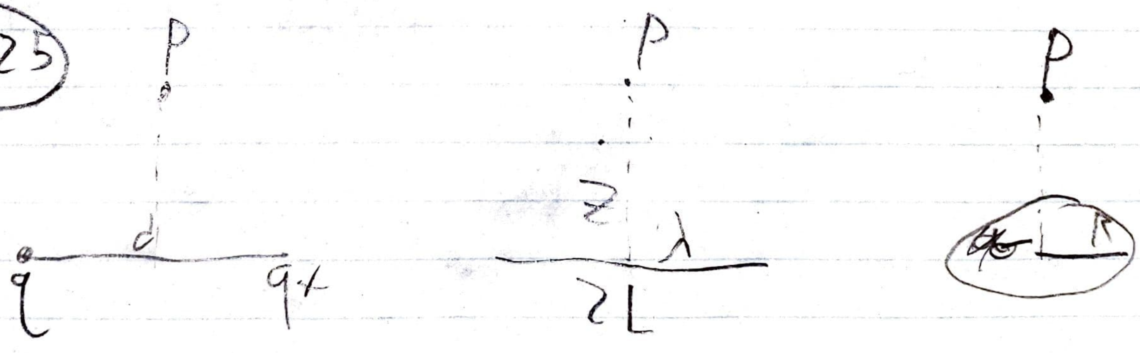


$|d|-|R|=c$

$|d|=|R|+|c|$

$4\pi\rho(R-c)^2-4\pi\rho(R-c)^2=0$ □

2.25



Find potential

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i}$$

$$V = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}\,')}{r} \, dl' \qquad V = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}\,')}{r} \, da'$$

potential of point charges

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{q+q}{\sqrt{z^2 + \left(\frac{d}{2}\right)^2}} = \boxed{\frac{2q}{4\pi\epsilon_0 \sqrt{z^2 + \frac{d^2}{4}}}}$$

If right q is $-q$

$$\frac{1}{4\pi\epsilon_0} \cdot \frac{q(-q)}{\sqrt{z^2 + \left(\frac{d}{2}\right)^2}} = \boxed{0}$$

This discrepancy is because the field flows from $+q$ to $-q$, making no potential

line

$du = 2\ell \, d\ell$

$u = z^2 + \ell^2$

$$V = \frac{1}{4\pi\epsilon_0} \int_0^{2L} \frac{\lambda \ell}{\sqrt{z^2 + \ell^2}} \, d\ell = \frac{1}{8\pi\epsilon_0} \int_{z^2}^{z^2+4L^2} \frac{1}{\sqrt{u}} \, du$$

$$= \frac{1}{8\pi\epsilon_0} \cdot \lambda u^{1/2} \Big|_{z^2}^{z^2+4L^2} = \boxed{\frac{\lambda \sqrt{z^2 + 4L^2}}{8\pi\epsilon_0} + \frac{\lambda z}{8\pi\epsilon_0}}$$

$$\boxed{\nabla V = \frac{-\lambda L}{2\pi\epsilon_0}}$$ Looks good

$\theta = \frac{\pi}{2}$

$\hat{r} = \sqrt{z^2+r^2}$

disc

$$V = \frac{1}{4\pi\epsilon_0}\int_0^R\int_0^{2\pi} \frac{\sigma r \sin\theta}{\sqrt{z^2+r^2}} \hat{r}\, dr\, d\theta$$

$$= \frac{\sigma \cdot 1 \cdot 2\pi}{4\pi\epsilon_0}\int_0^R \frac{r\,dr}{\sqrt{z^2+r^2}} = \frac{2\pi\sigma}{4\pi\epsilon_0}\cdot\left[\sqrt{z^2+r^2} - |z|\right]_0^R$$

$$= \frac{2\pi\sigma}{4\pi\epsilon_0}\left[\sqrt{z^2+R^2} - |z| - |z| + |z|\right]$$

$$= \boxed{\frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right)}$$

$$E = -\nabla V \Rightarrow \boxed{\frac{\sigma}{2\epsilon_0}\cdot\frac{R}{\sqrt{R^2+z^2}}}$$

Seems off. Will ask in class

(2.29) Check eq 2.29 satisfies Poisson.
Use Laplacian using Eq 1.102

eq 2.29: $V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{\eta} d\tau'$

$\nabla^2 \frac{1}{\eta} = -4\pi\delta^3(\vec{\eta})$

$\nabla^2 V = -\frac{\rho}{\epsilon_0}$

$\nabla \cdot \nabla \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}')}{\eta} d\tau'$

$= \frac{1}{4\pi\epsilon_0} \frac{-4\pi\delta^3(\vec{r})}{\eta} \int \rho(\vec{r}')d\tau'$

$= -\frac{\delta^3(\vec{r})}{\epsilon_0} \cdot \nabla^2 \int \rho(\vec{r}')d\tau'$

$= -\frac{1}{\epsilon_0} \cdot \nabla\rho(\vec{r}) = \boxed{\frac{\rho}{\epsilon_0}}$ ☐