SAFE RETURN DOUBTFUL: WEEK 2 PART I

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SUMMARY

1. A few words about the reading, the next reading quiz
2. Warm-up: work with friction, mass, and distance.
    - $W = \mu mgd$
    - **Now in 2D! Because navigation takes place in two-dimensions**
    - In the future, we'll put this together with calories
    - **3D: terrain (different frictions) and *elevation***
3. **Lecture: My expeditions to Moore's Bay, Antarctica**

WARM-UP

Moving a load of food and equipment against friction:

$$W = \mu m g d \tag{1}$$

- Suppose we are pulling a load of gear with a tractor across sand. The gear has mass $m = 300$ kg, the distance is $d = 12$ km, and $g = 9.81$ m/s^2. If the friction coefficient is 0.2, how much work (in Watts) does the tractor do?
- Did you know that there are 7600 kcal of energy in one liter of gasoline? Convert this to megajoules.
- How many liters of gas is required to move the load?
- Assuming that the tractor also burns 0.4 liters each kilometer just to move itself. How many liters is this, if we go 12 km?
- How many liters of gasoline are required, in total?

Motion (Navigation) in 2D

(10%) Problem 9: After being deserted by his crew on an island in the Caribbean, Captain Blackbeard builds a raft to escape and set out to sea on it. The wind seems quite steady, at first blowing him due east for *15* km and then *7* km in a direction *10* degrees north of east. Confident that he will eventually reach safety, he falls asleep. When he wakes up, he notices the wind is now blowing him gently *11* degrees south of east, and after traveling for *29* km in that direction, he finds himself back on the island!

50% Part (a) How far, as the crow flies, in kilometers, did the wind blow him while he was sleeping?

50% Part (b) Assuming the wind blew in the same direction all night, while he was sleeping, what direction in degrees north of west, did the wind blow Jack during the night?

Figure 1: This is an example problem from introductory physics course. To solve the problem, we need to understand a piece of math called a *vector*.

Navigation in the film The Hunt for Red October:
`https://youtu.be/4unk6siO-tI`

Physics requires mathematical objects to build equations that capture the behavior of nature. Two examples of such objects are scalar and vector quantities. Each type of object obeys similar but different rules.

1. Scalar quantities
    - mass: $m_1 + (m_2 + m_3) = (m_1 + m_2) + m_3$
    - speed: $v_1(v_2 + v_3) = v_1 v_2 + v_1 v_3$
    - charge: $q_1 \left(\frac{1}{q_1}\right) = 1$, $q_1(0) = 0$
2. Vector quantities
    - displacement: $\Delta x = \vec{x}_f - \vec{x}_i$
    - velocity: $\vec{v}_1 + (\vec{v}_2 + \vec{v}_3) = (\vec{v}_1 + \vec{v}_2) + \vec{v}_3$

Professor: show how to break into components, connection to trigonometry.

A vector may be expressed as *a list of scalars*: $\vec{v} = (4, 2)$ (a vector with two *components*), $\vec{u} = (3, 4, 5)$ (three *components*). Now, we know how to add and subtract scalars. How do we add and subtract vectors?

What is
$(1, 3, 8)+$
$(0, 2, 1)$?
Answer: $(1, 5, 9)$

In other words, when adding vectors, we add them component by component. **Professor: work several examples.**

How do we subtract vectors? In the same fashion:

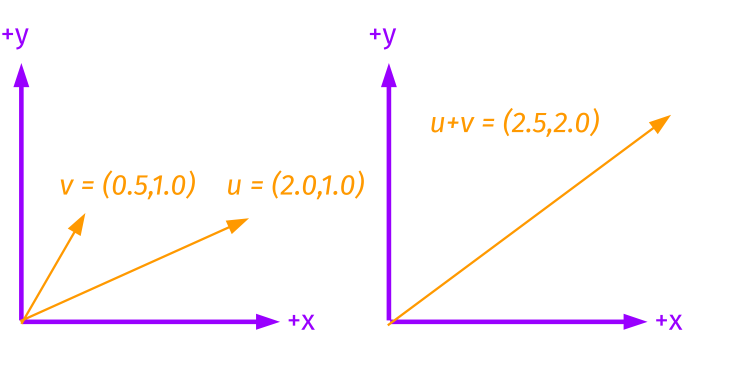
What is
$(1, 3, 8) -$
$(0, 2, 1)$?
Answer: $(1, 1, 7)$

In other words, when subtracting vectors, we subtract them component by component. **Professor: work several examples.**

The components of a vector may describe quantities in a coordinate system, such as *Cartesian coordinates* - after René Descartes. Vectors in the 3D Cartesian coordinate system (x,y,z) may be written in the following notation:

$$\boxed{\vec{v} = a\hat{i} + b\hat{j} + c\hat{k}}$$

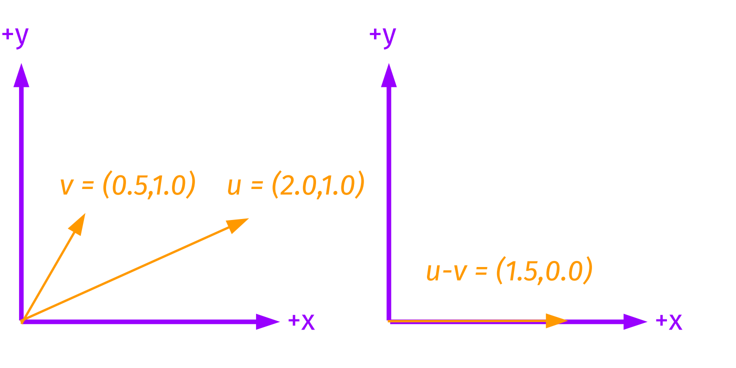
- a: The amount in the +x-direction, $\hat{i}$: a vector of length 1, in the +x-direction
- b: The amount in the +y-direction, $\hat{j}$: a vector of length 1, in the +y-direction
- c: The amount in the +z-direction, $\hat{k}$: a vector of length 1, in the +z-direction



(a)

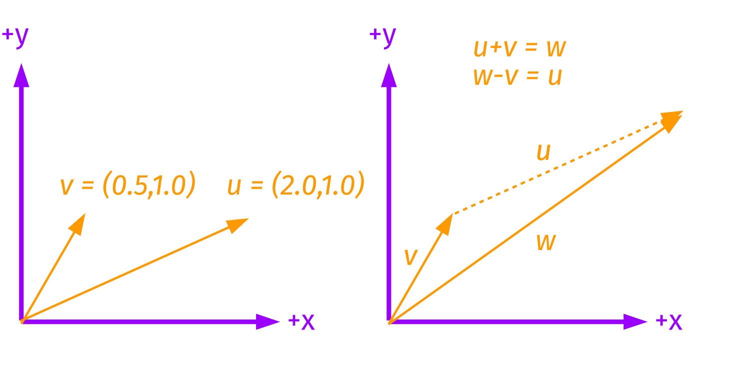
(b)

Figure 2: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} + \vec{v}$? Adding components: $\vec{u} + \vec{v} = 2.5\hat{i} + 2.0\hat{j}$.



(a) (b)

Figure 3: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) What is $\vec{u} - \vec{v}$? Subtracting components: $\vec{u} - \vec{v} = 1.5\hat{i} + 0.0\hat{j}$.



(a) (b)

Figure 4: (a) Two vectors in a two-dimensional Cartesian coordinate system: $\vec{u} = 0.5\hat{i} + 1.0\hat{j}$ and $\vec{v} = 2.0\hat{i} + 1.0\hat{j}$. (b) To compute $\vec{w} - \vec{v}$, arrange the vectors to get a sense of the result, $\vec{u}$.

The *length* or *norm* of a vector $\vec{v} = a\hat{i} + b\hat{j}$ is $|\vec{v}| = \sqrt{a^2 + b^2}$.

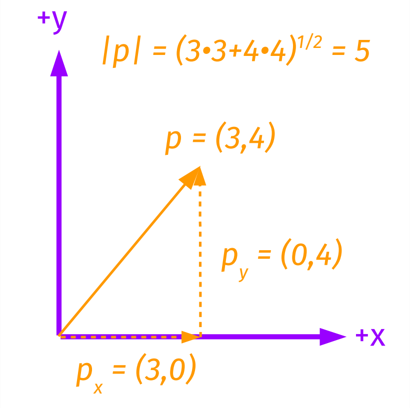


Figure 5: Computing the norm of a vector $\vec{p}$.

An object moves 10 km at $\theta = 60°$ North of East. How far East did it go, and how far North did it go?

- Break the *displacement* vector into North part and East part by drawing a triangle. One leg is the East part and one leg is the North part.
- Use sine and cosine, plus the angle, to find the length of each triangle leg.

We define the *position* of an object as a vector locating it in a given coordinate system. The scalar *distance* is the norm of the position vector, that is, the distance to to the origin.

Now we can introduce the concept of dislacement: a vector describing a movement of an object.

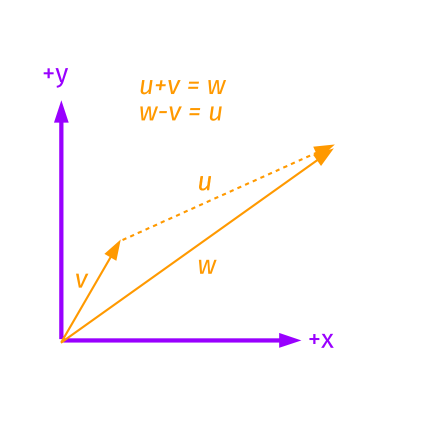


Figure 6: Suppose an object moves from position $\vec{v}$ to $\vec{w}$. In this case, the displacement is $\vec{u}$. **Thus, the final position is the initial position, plus the displacement.**

Download the Java applet (you may need to update Java):
`https://phet.colorado.edu/en/simulation/vector-addition`

Average velocity is the ratio of the displacement to the elapsed time.

$$\boxed{\vec{v}_{\mathrm{avg}} = \frac{\Delta \vec{x}}{\Delta t}} \tag{2}$$

The *average speed* is the norm of the average velocity:

$$\boxed{v_{\mathrm{avg}} = \frac{|\Delta \vec{x}|}{\Delta t}} \tag{3}$$

If the motion is in one dimension, then the average speed is

$$v_{\mathrm{avg}} = \frac{x_{\mathrm{f}} - x_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} \tag{4}$$

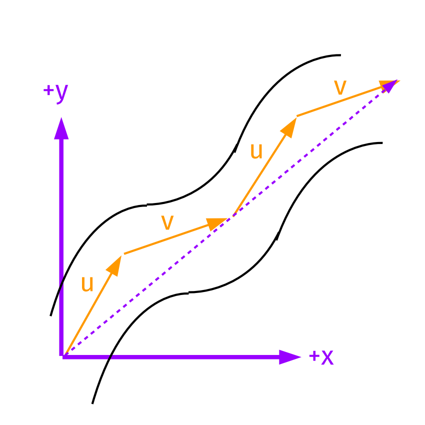


Figure 7: A Formula-1 driver keeps his car on the track by following a path approximated by the position vectors *u*, *v*, *u*, and *v*. The dashed arrow represents the total displacement.

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THE STORY OF ARIANNA, PART I