

INTD290: Number Systems in pre-Columbian Context

Dr. Jordan Hanson - Whittier College Dept. of Physics and Astronomy

January 8, 2021

1 How to Submit this Assignment

Once you answer the questions, take a picture of your work and convert it to a PDF. Submit the PDF to the assignment link on Moodle.

2 Introduction to Digits and Bases

[Asynchronous Lesson 0.1: corresponding video] In pre-Columbian scientific communities, we do not encounter the same systems of numbers as those used within the European scientific revolution. Based on the video 0.1, answer the following questions.

1. Imagine seeing four people standing under a tree. which of the following symbols describes the number of people under the tree?

- A: 4
- B:
- C: - - - -
- ☒ D: all of the above

2. How many digits are there in the decimal system?

- A: 8
- ☒ B: 10
- C: 16
- D: 20

3. How many digits would there be in a base-8 system?

- ☒ A: 8
- B: 10
- C: 16
- D: 20

4. Write the number 255 as the sum of digits times powers of 10, as in video 0.1.

$$255 = 2 \times 10^2 + 5 \times 10^1 + 5 \times 10^0$$

$\downarrow \quad \quad \downarrow \quad \quad \downarrow$
 $200 + 50 + 5$

3 Base-2, or Binary

[Asynchronous Lesson 0.2: corresponding video] We move forward with base-2 or binary number systems. Watch the video 0.2 and answer the following questions.

1. Convert the following binary numbers to decimal numbers:

$$\begin{aligned} \bullet 1000 &= 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0 = 8 + 0 + 0 + 0 = 8 \\ \bullet 1001 &= 1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 0 + 0 + 1 = 9 \end{aligned}$$

$$\bullet 1111 = 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 8 + 4 + 2 + 1 = 15$$

2. Convert the following decimal numbers to binary numbers:

$$\begin{aligned} \bullet 32_{10} &= 16 \text{ r } 0 \rightarrow 16_{10} = 8 \text{ r } 0 \rightarrow 8_{10} = 4 \text{ r } 0 \rightarrow 4_{10} = 2 \text{ r } 0 \rightarrow 2_{10} = 1 \text{ r } 0 \rightarrow 1_{10} = 0 \text{ r } 1 \} 100000 \\ \bullet 42_{10} &= 21 \text{ r } 0 \rightarrow 21_{10} = 10 \text{ r } 1 \rightarrow 10_{10} = 5 \text{ r } 0 \rightarrow 5_{10} = 2 \text{ r } 1 \rightarrow 2_{10} = 1 \text{ r } 0 \rightarrow 1_{10} = 0 \text{ r } 1 \} 101010 \\ \bullet 11_{10} &= 5 \text{ r } 1 \rightarrow 5_{10} = 2 \text{ r } 1 \rightarrow 2_{10} = 1 \text{ r } 0 \rightarrow 1_{10} = 0 \text{ r } 1 \} 1011 \\ \bullet 17_{10} &= 8 \text{ r } 1 \rightarrow 8_{10} = 4 \text{ r } 0 \rightarrow 4_{10} = 2 \text{ r } 0 \rightarrow 2_{10} = 1 \text{ r } 0 \rightarrow 1_{10} = 0 \text{ r } 1 \} 10001 \end{aligned}$$

4 Base-16, or Hexadecimals

[Asynchronous Lesson 0.3: corresponding video] We move forward with base-16 or hexadecimal number systems. Watch the video 0.3 and answer the following questions.

1. How do you write 12 in hexadecimal?

- A: 12
- ☒ B: C
- C: D
- D: 1A

2. Let's convert the number 255 to hexadecimal, a base-16 number system. (a) First, divide 255 by 16 and start a column of the remainders. (b) Divide the result of (a) by 16 again, and record the remainder. (c) Repeat this process until the result is less than 16. This is your final remainder, because you can't divide by 16 again. (d) Line up the remainders to get the hexadecimal expression for 255.

$$\begin{array}{r} 16 \overline{) 255} \\ \underline{-16} \\ 95 \\ \underline{-80} \\ 15 \end{array}$$

$$r15 \quad r15 = FF$$

5 Base-20 Systems

[Asynchronous Lesson 0.3: corresponding video] Finally, we've built up to understanding the basic Mayan numerical patterns. Answer the following questions.

1. Suppose we introduce a base-20 number system. We need 20 digits, including 0-19. Use the Arabic numerals 0-9, plus letters from the alphabet A-K as digits representing the numbers 10-19. (a) What are the first three powers of 20: 20^0 , 20^1 , 20^2 ? (b) So how would you represent the decimal number 400 in your base-20 system? (c) How would you represent 401?

$$20^0 = 1 \quad 20^1 = 20 \quad 20^2 = 400 \quad 400 = 20 \times 20$$

$$401 = 20 \times 20 + 1$$

2. Convert the following numbers to your base-20 system:

$$\begin{aligned} \bullet 25 &= 15 \\ \bullet 45 &= 25 \\ \bullet 425 &= 115 \\ \bullet 625 &= 185 \end{aligned} \quad 1 \times 20^2 + 12 \times 20^1 + 5 \times 20^0$$

3. You've converted the following numbers to base-20:

$$\begin{aligned} \bullet 25 - 20 &= 5 \\ \bullet 45 - 20 &= 25 \\ \bullet 425 - 20 &= 405 \\ \bullet 625 - 20 &= 605 \end{aligned}$$

Now write these numbers as the Mayans wrote them using the digits in Fig. 1. Subtract 20 from each of them