INTD290: Number Systems in pre-Columbian Context

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January 8, 2021

1 How to Submit this Assignment

Once you answer the questions, take a picture of your work and convert it to a PDF. Submit the PDF to the assignment link on Moodle.

2 Introduction to Digits and Bases

[**Asynchronous Lesson 0.1: corresponding video**] In pre-Columbian scientific communities, we do not encounter the same systems of numbers as those used within the European scientific revolution. Based on the video 0.1, answer the following questions.

1. Imagine seeing four people standing under a tree. which of the following symbols describes the number of people under the tree?

   - A: 4
   - B: ....
   - C: - - - -
   - D: *all of the above*

2. How many digits are there in the decimal system?

   - A: 8
   - B: 10
   - C: 16
   - D: 20

3. How many digits would there be in a base-8 system?

   - A: 8
   - B: 10
   - C: 16
   - D: 20

4. Write the number 255 as the sum of digits times powers of 10, as in video 0.1.

$$2 \times 10^2 + 5 \times 10^1 + 5 \times 10^0$$

$$200 + 50 + 5 = 255 \checkmark$$

3 Base-2, or Binary

[**Asynchronous Lesson 0.2: corresponding video**] We move forward with base-2 or binary number systems. Watch the video 0.2 and answer the following questions.

1. Convert the following binary numbers to decimal numbers:

   - 1000 = 8 $1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$
   - 1001 = 9 $1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

- 1101 = 13   *(handwritten notes)*
- 1111 = 15

2. Convert the following decimal numbers to binary numbers:
   - 32 = 100000   *(handwritten work)*
   - 42 = 101010
   - 11 = 1011
   - 17 = 10001

4 Base-16, or Hexadecimals

[Asynchronous Lesson 0.3: corresponding video] We move forward with base-16 or hexadecimal number systems. Watch the video 0.3 and answer the following questions.

0 - 9, A, B, C, D, E, F

1. How do you write 12 in hexadecimal?
   - A: 12
   - (B) C
   - C: D
   - D: 1A

2. Let's convert the number 255 to hexadecimal, a base-16 number system. (a) First, divide 255 by 16 and start a column of the remainders. (b) Divide the result of (a) by 16 again, and record the remainder. (c) Repeat this process until the result is less than 16. This is your final remainder, because you can't divide by 16 again. (d) Line up the remainders to get the hexadecimal expression for 255.

   255/16   15 r 15
   15/16   0 r 15

   255 = FF

5 Base-20 Systems

[Asynchronous Lesson 0.3: corresponding video] Finally, we've built up to understanding the basic Mayan numerical patterns. Answer the following questions.

1. Suppose we introduce a base-20 number system. We need 20 digits, including 0-19. Use the Arabic numerals 0-9, plus letters from the alphabet A-K as digits representing the numbers 10-19. (a) What are the first three powers of 20: 20^0, 20^1, 20^2? (b) So how would you represent the decimal number 400 in your base-20 system? (c) How would you represent 401?

   a) $20^0 = 1$, $20^1 = 20$, $20^2 = 400$

   b) $1 \times 20^2 + 0 \times 20^1 + 0 \times 20^0 = 400$

   c) $1 \times 20^2 + 0 \times 20^1 + 1 \times 20^0 = 401$

2. Convert the following numbers to your base-20 system:
   - 25 = $1 \times 20^1 + 5 \times 20^0$
   - 45 = $2 \times 20^1 + 5 \times 20^0$
   - 425 = $1 \times 20^2 + 1 \times 20^1 + 5 \times 20^0$
   - 625 = $1 \times 20^2 + A \times 20^1 + 5 \times 20^0$
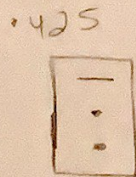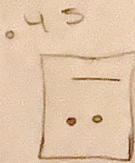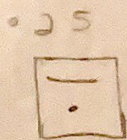
3. You've converted the following numbers to base-20:
   - 25
   - 45
   - 425
   - 625

Now write these numbers as the Mayans wrote them, using the digits in Fig. 1. Subtract 20 from each of them, and write the results using Mayan digits. (You can put your work on a separate page)

.25

.45

.425

.625

0 1 2 3 4

5 6 7 8 9

10 11 12 13 14

15 16 17 18 19

Figure 1:   The 20 digits of the Mayan system. The digit for 0 resembles an empty shell. The dots are worth 1 and the bars are worth 5.