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Ethnomathematics and Numeral Systems

Ethnomathematics is the relationship between culture and mathematics, and how they are interconnected with each other. Ethnomathematics is a fairly new concept which was introduced by Brazilian mathematician, Ubiratan D'Ambrosio, in 1977 at the American Association for the Advancement of Science. Ethnomathematics allows numbers to be seen in a more abstract and conceptual way, rather than a “set/defined” way so to speak. You will notice that quantities can be expressed many different ways in different cultures and in our own culture today.

The oldest evidence of written mathematics dates as far back as 3000 BC, used by the ancient Sumerians in Mesopotamia. Although there was *written* evidence of mathematics in 3000 BC, it is worth mentioning that ancient people during the pre-historic time period used sticks, rocks, tally marks, and perhaps their fingers for basic counting. Representing small quantities in a physical way is practical, as you can use your ten fingers or ten rocks to represent a quantity of ten, but with larger quantities like 100, it starts to become impractical. Therefore, the need to develop a number system was important to represent larger numbers.

Throughout history, different civilizations used their own developed number system. Although quantities were represented differently in each of these number systems, they all utilized the same idea: the use of a base number and units to represent higher orders of the base number. The most widely

European (descended from the West Arabic)	0	1	2	3	4	5	6	7	8	9
Arabic-Indic	٠	١	٢	٣	٤	٥	٦	٧	٨	٩
Eastern Arabic-Indic (Persian and Urdu)	۰	۱	۲	۳	۴	۵	۶	۷	۸	۹
Devanagari (Hindi)	०	१	२	३	४	५	६	७	८	९

Figure 1 Arabic Numerals Table

used number system in the world today is the *decimal system* that originated in India around 1700 years ago, which uses the Arabic numerals "0, 1, 2, 3, 4, 5, 6, 7, 8, 9". The base number in this decimal system is 10, and the units themselves are the Arabic numerals. For example, to represent the quantity of 70 in decimal, you multiply the unit 7 to the base number 10, giving you the number 70. And to represent 700 in decimal, you multiply the unit 7 with the base number 10 twice (or 10^2), giving you the number 700. It is obviously clear how much more practical it is to represent a quantity of 700 with a developed number system rather than using 700 sticks or rocks. Let's take a look at some of the number systems developed in ancient civilizations.

The Egyptians during 3000-2001 BC used a base-ten number system that utilized hieroglyphics (as their writing system also used hieroglyphics). A line represents a quantity of 1, a loop represents the quantity of 10, a rope represents the quantity of 100, a flower represents the quantity of 1000, a finger represents the quantity of 10000, a tadpole represents the quantity of 100000, and god represent the quantity of 1000000. In figure 2, the quantity 12,427 is represented using Egyptian symbols with the most significant symbol (or digit) being on the left and the least significant symbols being on the right (reading from left to right like normal).

Figure 2 Egyptian Number System

In Ancient Rome, the Romans began using their developed Roman numeral system in 480 BC. This is a base-ten number system that utilizes capital letters, but it doesn't have a dedicated symbol to represent a quantity of 0. Figure 3 is a chart showing the quantity (or Arabic numeral) associated with each of the Roman numeral symbols. However, it is worth pointing out that something interesting happens when representing the quantities of 4 and 9. The quantity of 4 would not be represented as "IIII", and the quantity of 9 would not be represented as "VIIII". 4 is represented as "IV", and 9 is represented as "IX". In the case of 4, it can be thought as subtracting 1 (I) from 5 (V), which is why it is written as "IV". Similarly, in the case of 9, it is subtracting 1 (I) from 10 (X), which gives you "IX". As another example, the quantity of 49 is represented as "XLIX" instead of "IL" because "X" is the next symbol lower than "L" (so XL=40 + IX=9 gives 49). The Roman numeral system was very important for managing the Roman Empire, such as taxes, currency, and population.

Roman Numeral	Number
I	1
V	5
X	10
L	50
C	100
D	500
M	1000

Figure 3 Roman Numerals Table

The Mayans and Aztecs both used a base-twenty number system. Figure 4 depicts the Aztec numeral system with various symbols to represent the same quantity. For example, the quantity of 5 can be represented with either 5 dots or 1 bar, and the quantity of 10 can either be represented with 10 dots, 2 bars, or 1 diamond. The Aztec numeral system is similar to the Egyptian's in terms of using hieroglyphics to represent larger quantities.

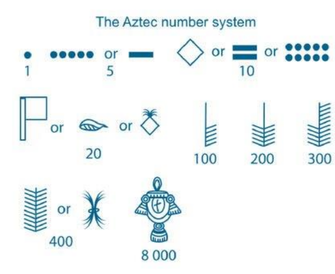

Figure 5 Aztec Numeral System

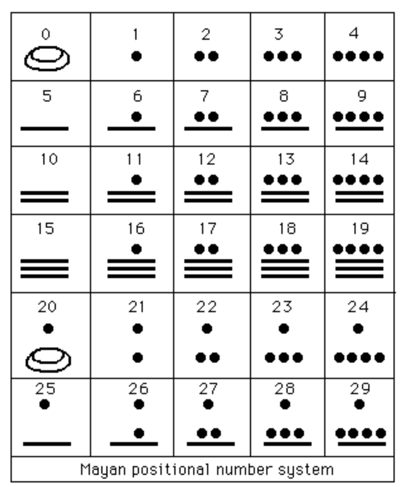

Figure 4 Mayan Numeral System

However, there is no special symbol to represent the quantity of 0. On the other hand, the Mayan numeral system uses a shell symbol to represent the quantity of 0. This was the only number system at the time (1500 BC) to represent the idea of having a quantity of 0, which is really interesting. To put this in perspective, the Arabic numeral system did not have a symbol to represent the quantity of 0 until 628 AD. The idea of 0 for the Mayan numeral system was crucial since it is a positional system, meaning the position of the symbol determines its power of the base number 20. In figure 5, these positions can be seen starting with the quantity of 20, so the most significant quantity (or digit) is at the top. The use of the Mayan and Aztec numeral systems was important for accounting purposes.

The most interesting out of all these numeral systems mentioned is the quipu knot system used by the Incans. The Incan quipu knot system uses the base-ten system to represent quantities, but on a string with different knots. An example of quipu knots can be seen in figure 6. Notice that the strings are tied on a main cord and are of different colors, allowing a variety of different ways to represent information such as food, land, number of animals, population, and taxes all on one cord! Figure 7 shows how the quantity 242 is represented as knots on a string. In the case of representing 0, a “figure-eight” knot would be tied in the corresponding digits place.

Figure 6 Incan Quipu Knots

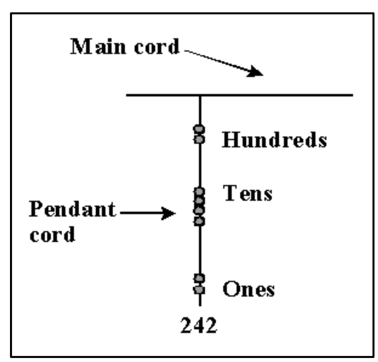

Figure 7 Quipu Knot Representation

After looking at some number systems used in ancient civilizations, it is worth learning about them to understand the culture and the importance behind number systems in general. In other words, it’s worth studying these number systems in an ethnomathematical way to understand the relationship between the math and culture, and how it all connects to the math we use today. Critics say that there should be a bigger focus on ethnomathematics in school to promote multiculturalism, which is very important in this day of age. Most people today do not know of these other numeral systems other than the widely-used Arabic numeral system. Now let’s take a look at the number systems we use today and how they relate to these ancient number systems.

The binary numeral system is heavily used today in the computer science field, but it was actually first presented by the ancient Hindu mathematician Pingala approximately in the 3rd or 2nd century BC. However, the binary number system we know today wasn’t studied until the 16th and 17th century in Europe. It is a very simple base-two number system that only uses 0s and 1s. The binary system is very useful in the computer science field because it allows for a “two-state system” for electrical signals: voltage (or ON) is represented as a 1 and no voltage (or OFF) is represented as a 0. It also allows for simple conversions and calculations using powers of 2. Figure 8 shows the place value of each “bit” as a power of 2, where the most significant bit is to the left (2^7) and the least significant bit is to the right (2^0). Figure 9 shows an example of converting from the binary number system to the normal decimal system. It’s a neat way to represent larger numbers with just 0s and 1s. The octal numeral system is also used in the computer science field today to represent binary numbers in a shorter way. It uses the

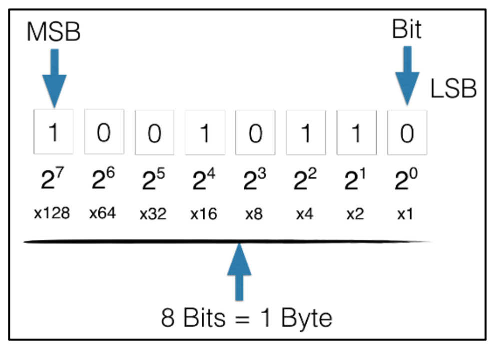

Figure 8 Binary System

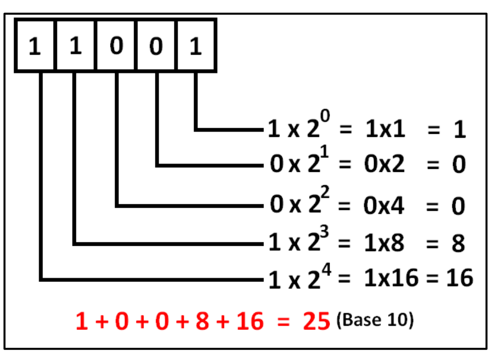

Figure 9 Binary to Decimal Conversion

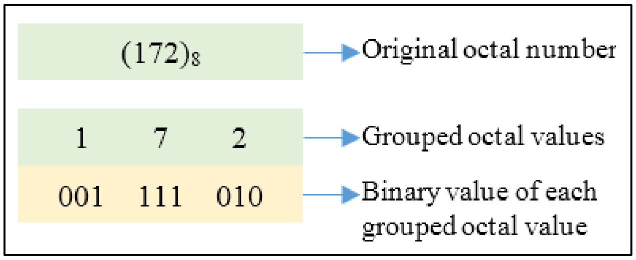

Figure 10 Binary to Octal Conversion

Arabic numerals from 0 to 7. It turns out the conversion between the octal and binary systems are very simple because they are multiples of each other. Figure 10 shows how simple this conversion is! The binary number being converted in the example is 001111010. Grouping every 3 bits together (001 111 010) will give you a digit for the octal number (because $2^3 = 8$). So, 001 represents 1 in decimal, 111 represents 7 in decimal, and 010 represents 2 in decimal. Putting it all together in order gives us the octal number 172. The hexadecimal numeral system yet another numeral system used in the computer science field, and it can represent both binary and octal numbers in a shorter way. It is a base-sixteen number system that uses the Arabic numerals from 0 to 9 and the letters A through F (see figure 11). Hexadecimal is a popular number system in computer science because it is used in the ASCII table (American Standard Code for Information Interchange)

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Figure 11 Number Systems

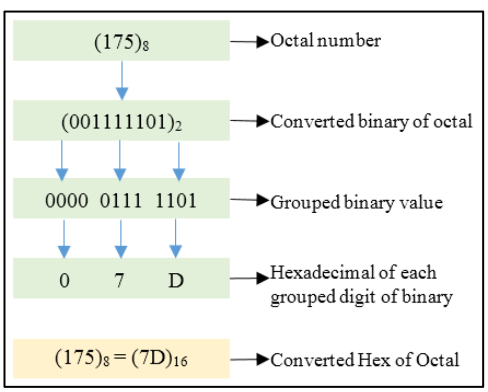

Figure 12 Octal to Hex Conversion using Binary

and used for color codes. Figure 12 shows a conversion going from octal to binary to hexadecimal. Converting from binary to hexadecimal is the same as octal, only you group 4 bits together instead of 3 (because $2^4 = 16$). These number systems convert nicely between each other.

Looking at these numeral systems ethnomathematically says a lot about today's culture. The binary, octal, and hexadecimal numeral systems are all used specifically for computer science, and today's world has a digital and computer-oriented culture. Learning these different number systems allows us to see that numbers are an abstract concept; physical quantities can be represented in so many different ways instead of the usual Arabic numeral way most people know. It also shows the idea of a number being a man-made concept. Throughout history, humans have developed number systems based off their needs and uses. For example, the Incan quipu system was developed to keep track of important information like population, food, and land plots since they had no way to express this information by writing. The binary and octal number systems were developed well before computers came to be, but it eventually became very useful for computer system applications. The hexadecimal number system was actually first introduced by International Business Machines (IBM) in 1963 so that 8 bits of a binary number can be represented as two digits in hexadecimal. While the binary and octal systems weren't necessarily developed with computers in mind, hexadecimal was specifically developed for the computer science field based off the binary and octal system. This is why the conversions between binary, octal, and hexadecimal are very nice, because their bases can be represented as a power of 2 ($16 = 2^4$, $8 = 2^3$, $2 = 2^1$). The hexadecimal number system is a great example of how a number system can be developed for a particular application.

In conclusion, although the Arabic numeral system is widely used throughout the world today, it is still important to teach and learn the various different numeral systems used throughout history by our ancestors, as well as the numeral systems being used today such as the Roman, binary, octal, and hexadecimal numeral systems. It allows people to see numbers in an abstract and flexible way instead of a "defined" and inflexible way. Math doesn't care about the number system used, as it is merely a "set of rules" like how to add and subtract numbers. You can still add, subtract, multiply, and divide in any of these number systems just like the decimal system. So, you can think of math as the "set of rules" and the number system as the "template" to apply the rules. This is why it is very useful and important to learn about the various number systems used in the world today and in the past.

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