RF Field Engineer Course: A Practical Introduction

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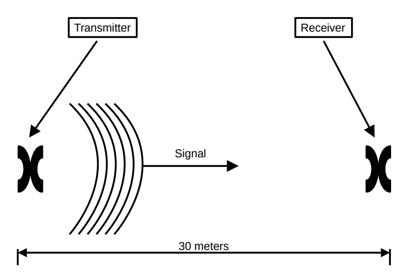
Review of Introduction Material

Reading: *Stimson3 ch. 1-6*

- Week 0: Units and estimation. **Key skills**: mental math, decibels, graphical analysis
    - Electromagnetic units, wavelength, frequency, wave speed, and period
    - Estimation and approximation
    - Decibels
    - Analysis of spectrograms
- Week 1: Wave concepts
    - Reflection
    - Refraction
    - Diffraction
    - Phase and amplitude
    - Polarization

Warm-up

A test facility is set up to characterize RF antennas. A transmitter sends a 200 MHz *sine tone that lasts for* 50 ns *to a receiver* 30 m *away. How many wavelengths is this distance? Has the sine tone ended before the first part of the signal reaches the receiver?*



- How many wavelengths?
- How long is the sine tone?

Warm-up

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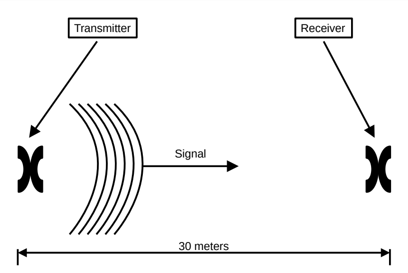

- The wavelength is 1.5 m, so $30\,\mathrm{m}/1.5\,\mathrm{m} = 20.0$ wavelengths.
- How long is the sine tone?

Warm-up

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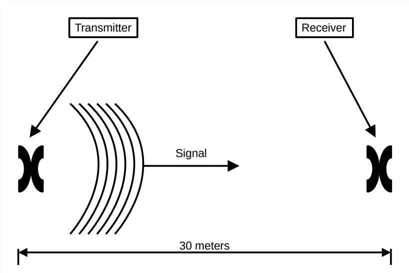

- $30\,\text{m}/1.5\,\text{m} = 20.0$ wavelengths.
- $\Delta x = c\Delta t$, so $\Delta x = 0.3\,\text{m GHz}\ \ 50\,\text{ns} = 15\,\text{m}$. Other way: the wavelength is 1.5 m, and the period is 5 ns, therefore the tone is on for 10 periods, or 10 wavelengths (15 m).

Warm-up

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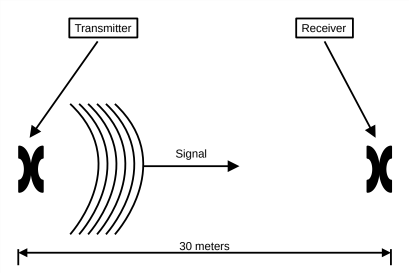

- If the transmitter tone resumes the same pattern as soon as the signal is no longer in the receiver, what is the duty cycle?

Warm-up

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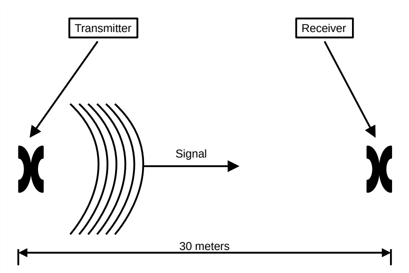


- 50 percent. The signal covers half of the space between the transmitter and receiver. Alternatively, the signal spends 100 ns arriving at the receiver, but it is only on for 50 ns.

Week 1 Summary

Reading: *Stimson3 ch. 1-6*

- Week 1: Units and estimation. **Key skills**: mental math, **wave concepts**
    - Electromagnetic units, estimation, and decibels
    - Waves and the wave equation
    - Reflections, refraction, and diffraction
    - Phase, amplitude, frequency, polarization
- Week 2: Basic Training in Mathematics. **Key skills**: estimate pulse bandwidth, pulse trains and uncertainty principle
    - **Complex numbers**: applications to phasors and radio waves, complex imdedance of filters and antennas
    - **Fourier series and transforms**, filters and attenuation, properties of waveforms, power spectra, and spectrograms, cross-correlation and convolution
    - **Statistics and probability**: applications to noise, signal-to-noise ratio

Waves and the Wave Equation

Waves and the Wave Equation

The 1D wave equation, involving displacement x and time t:

$$\boxed{\frac{\partial^2 E}{\partial t^2} - c^2 \frac{\partial^2 E}{\partial x^2} = 0} \tag{1}$$

- Solutions come in the form $F(x - ct) + G(x + ct)$.
- Note the units.
- Concentrate on sinusoids, e.g.:
  $E(t, x) = E_0 \cos(2\pi ft - 2\pi x/\lambda)$

$$E(t, x) = E_0 \cos(2\pi/\lambda(\lambda ft - x)) \tag{2}$$

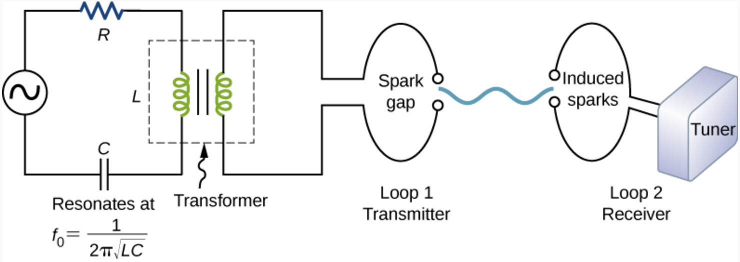
$$k = 2\pi/\lambda \tag{3}$$

$$E(t, x) = E_0 \cos(k(ct - x)) \tag{4}$$

$$E(t, x) = E_0 \cos(\omega t - kx) \tag{5}$$

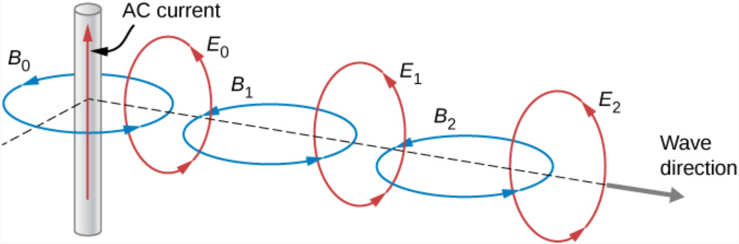
The 1D wave equation, involving displacement x and time t:

$$\boxed{\frac{\partial^2 E}{\partial t^2} - c^2 \frac{\partial^2 E}{\partial x^2} = 0} \tag{6}$$



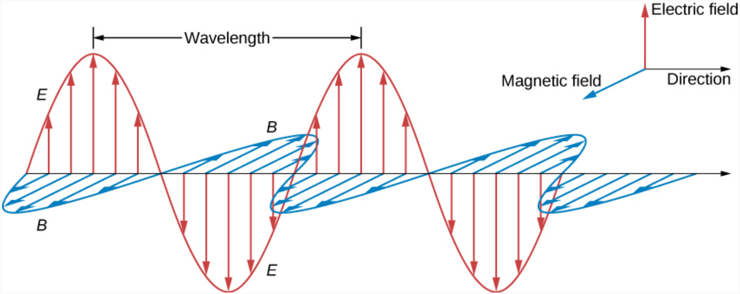
The 1D wave equation, involving displacement x and time t:

$$\boxed{\frac{\partial^2 E}{\partial t^2} - c^2 \frac{\partial^2 E}{\partial x^2} = 0} \tag{7}$$



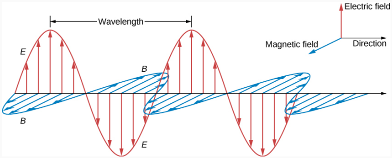
The 1D wave equation, involving displacement x and time t:

$$\boxed{\frac{\partial^2 E}{\partial t^2} - c^2 \frac{\partial^2 E}{\partial x^2} = 0} \tag{8}$$



Waves and the Wave Equation

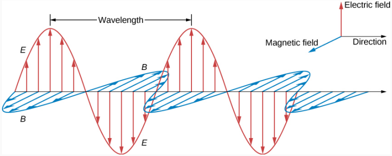
Think about the plane wave $E(t,x) = E_0 \cos(2\pi f t - kx)$. Suppose we freeze time, and vary the position x only. What value of x, in terms of k, represents one wavelength? If $k = 5\,\mathrm{cm}^{-1}$, what is the wavelength?



- Find the relationship between λ and k.
- Compute λ.

Waves and the Wave Equation

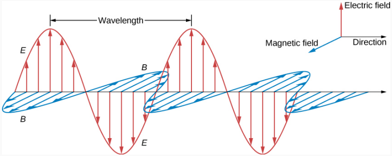
Think about the plane wave $E(t, x) = E_0 \cos(2\pi f t - kx)$. Suppose we freeze time at $t = 0$, and vary the position x only. If $x = \lambda$, the wavelength, what is the value of k? If $k = 5\,\mathrm{cm}^{-1}$, what is the wavelength?



- It must be the case that $kx = 2\pi$. This means we have traversed one wavelength: $x = \lambda$. Thus, $k = 2\pi/\lambda$.
- Compute λ.

Waves and the Wave Equation

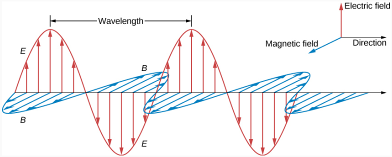
Think about the plane wave $E(t, x) = E_0 \cos(2\pi f t - kx)$. *Suppose we freeze time at* $t = 0$, *and vary the position* x *only. If* $x = \lambda$, *the wavelength, what is the value of* k? *If* $k = 5\,\mathrm{cm}^{-1}$, *what is the wavelength?*



- It must be the case that $kx = 2\pi$. This means we have traversed one wavelength: $x = \lambda$. Thus, $k = 2\pi/\lambda$.
- That means that $\lambda = 2\pi/k \approx 1.25\,\mathrm{cm}$.

Waves and the Wave Equation

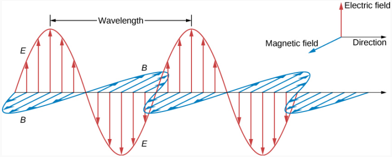
This wave (left) is linearly polarized, in that the E-field is projected onto one axis of the coordinate system. Suppose the E-field is entirely along the x-direction, and the B-field is entirely along the y-direction, and the wave is moving in the z-direction.



- If we have a dipole antenna aligned with the x-direction, measured signal amplitude is maximized.
- If we have a dipole antenna aligned with the y-direction, measured signal amplitude is minimized.
- How do we quantify this?

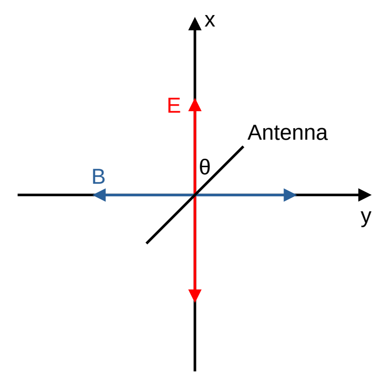
Waves and the Wave Equation

This wave (left) is linearly polarized, in that the E-field is projected onto one axis of the coordinate system. Suppose the E-field is entirely along the x-direction, and the B-field is entirely along the y-direction, and the wave is moving in the z-direction.



- How do we quantify this? The measured amplitude is proportional to the amount of *projected* E-field onto the linearly polarized antenna.
- $|\vec{E}_{obs}| = |\vec{E}|\cos(\theta)$, where θ is the angle between the antenna and x-direction.

Why is it proportional to cosine? Answer: that's the way vectors work. We can think of the polarization of a plane wave as a 2D vector. The projection of a 2D vector is explained below.



- Let $\hat{x}$, $\hat{y}$, and $\hat{z}$ represent vectors of length 1.0 in the x, y, and z directions, respectively.
- A vector of length 4 in the x-direction is $\vec{v} = 4\hat{x}$. If the 4 is $4\,\mathrm{V\,m^{-1}}$, then the vector has those same units.
- A 2D vector can have two *components:* $\vec{E} = E_x\hat{x} + E_y\hat{y}$.

Waves and the Wave Equation

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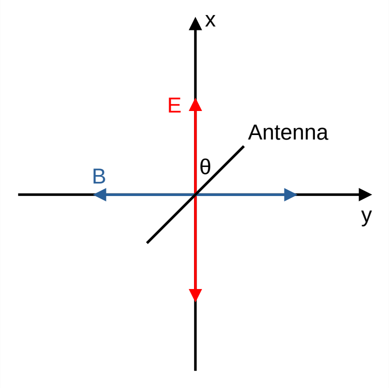

- Like componens add: $\vec{E}_1 = E_{1x}\hat{x} + E_{1y}\hat{y}$, $\vec{E}_2 = E_{2x}\hat{x} + E_{2y}\hat{y}$, $\vec{E}_1 + \vec{E}_2 = (E_{1x} + E_{2x})\hat{x} + (E_{1y} + E_{2y})\hat{y}$
- The magnitude: $|\vec{E}| = \sqrt{E_x^2 + E_y^2}$
- The direction: $\theta = \arctan(E_y/E_x)$ (in this case).
- Components: $E_x = |E|\cos(\theta)$, $E_y = |E|\sin(\theta)$

Waves and the Wave Equation

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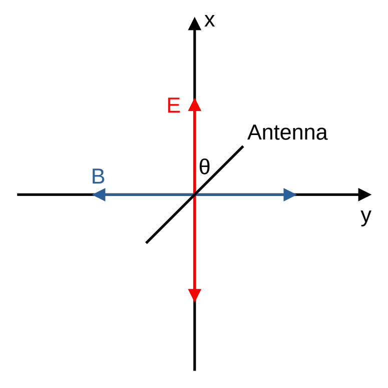
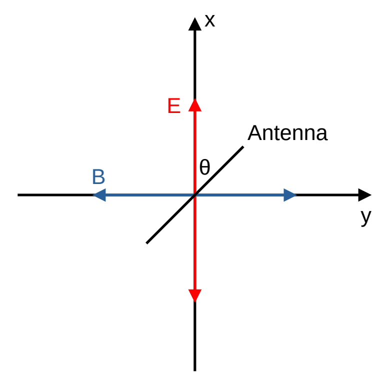


- Vectors can multiply via the *dot-product*: $\vec{E}_1 = E_{1x}\hat{x} + E_{1y}\hat{y}$, $\vec{E}_2 = E_{2x}\hat{x} + E_{2y}\hat{y}$. $\vec{E}_1 \cdot \vec{E}_2 = E_{1x}E_{2x} + E_{1y}E_{2y}$.
- How much of $\vec{E}_1$ is along the x-direction? Let one of the vectors in the dot product be $\hat{x}$: $\vec{E}_1 \cdot \hat{x} = E_{1x} \cdot 1 + E_{1y} \cdot 0$. (Remember, $\hat{x}$ is just along the x-direction and has a length of 1). $E_{1x} = |\vec{E}_1| \cos(\theta) = \vec{E}_1 \cdot \hat{x}$.
-

Waves and the Wave Equation

Suppose a radar system on a small boat is not receiving a signal well from another craft. The linearly-polarized antenna is meant to be oriented vertically, but was tilted 45 degrees by storm winds. By how much will the signal-to-noise ratio (SNR) improve after putting it right?



- By what factor will the E-field increase after righting the antenna?
- How to convert to SNR?

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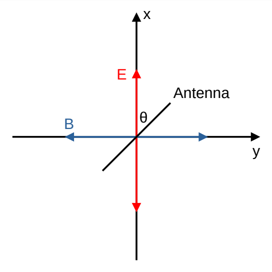
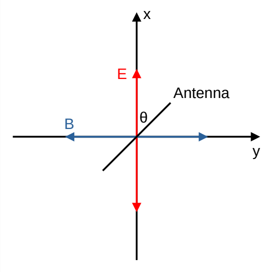


- Currently[a], the E-field amplitude is down by a factor of $\cos(\pi/4)$. Putting the antenna to the normal orientation will increase the signal amplitude by $1/\cos(\pi/4) = \sqrt{2}$, or a factor of 1.414.
- How to convert to SNR?

[a] Do you remember how to convert degrees to radians?

Waves and the Wave Equation

Suppose a radar system on a small boat is not receiving a signal well from another craft. The linearly-polarized antenna is meant to be oriented vertically, but was tilted 45 degrees by storm winds. If received power is proportional to the square of the E-field, by how much will the signal-to-noise ratio (SNR) improve after putting it right?



- Putting the antenna to the normal orientation will increase the signal amplitude by $1/\cos(\pi/4) = \sqrt{2}$, or a factor of 1.414.
- SNR is $10\log_{10}(P_2/P_1)$. Power is proportional to the square of the E-field: $SNR_{dB} = 10\log_{10}(E_2^2/E_1^2) = 20\log_{10}(E_2/E_1) = 20\log_{10}(\sqrt{2}) = 3\ dB$

Waves and the Wave Equation

Why is received power proportional to E-field squared? It turns out that the energy per unit area per unit time flowing in an EM plane wave is called the Poynting vector

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B} \tag{9}$$

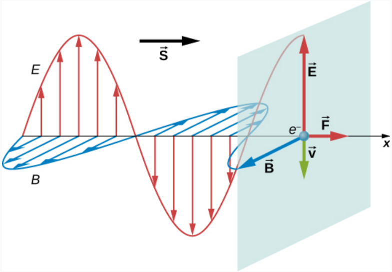
Note also that, for a plane wave,

$$|\vec{E}| = c|\vec{B}| \tag{10}$$

So the Poynting vector is proportional to E^2. The Poynting vector is the power per unit area flowing in an EM plane wave.

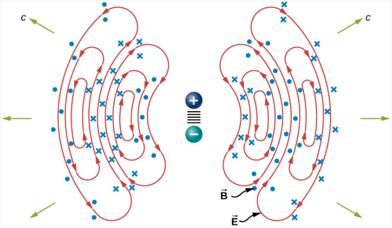
What's that symbol between the E-field and the B-field?

The Poynting vector is an example of a ***cross-product*** *between two vectors. The RHR governs the direction, and the magnitude is proportional to the sine of the angle between the vectors. In this case, the angle is 90 degrees.*



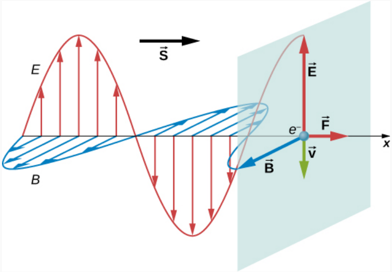
- $|\vec{S}| = \frac{1}{\mu_0}|\vec{E}||\vec{B}| \sin\theta$
- $\hat{x} \times \hat{y} = \hat{z}, \hat{z} \times \hat{x} = \hat{y}, \hat{y} \times \hat{z} = \hat{x}$ ...
- The right hand rule (RHR) tells you directions of E-field, B-field and S. S shares the same direction with the effective force on electrons in a conductor imparted by the wave.

The polarization state of the radiation is governed by the original motion of the charges that created it.



- Imagine positive and negative charges moving linearly (at left), and how this creates linear (dipole) radiation.
- There's no reason a system *has* to have just one linear motion.
- Suppose there are *two*, but one is 90 degrees out of phase with the other: $\vec{E}(z,t) = E_0(\cos(\omega t - kz), \sin(\omega t - kz), 0)$

Suppose you are listening for a transmitter that we know transmits a signal linearly polarized in the vertical direction. However, our only choices for the receiver antenna are a linear horizontally polarized mount, and an antenna designed to receive circular polarization. Which antenna will receive more of the signal?



- First, compute the dot product of the circularly polarized E-field with the linear antenna.
- Second, compute the dot product with the circular polarization.

Waves and the Wave Equation

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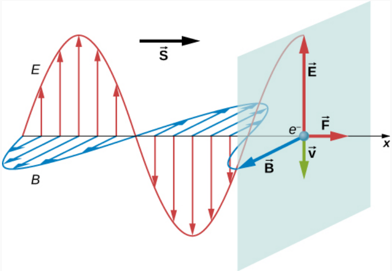
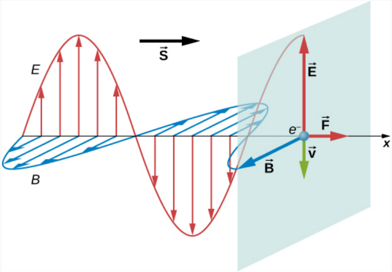


- Conceptualize the polarization vector of the signal like $\vec{E}(z,t) = (0, E(z,t), 0)$, where $E(z,t) = E_0 \sin(\omega t - kz)$. Then, conceptualize the horizontal polarization of the antenna as $\vec{H} = (H, 0, 0)$.
- Second, compute the dot product with the circular polarization.

Suppose you are listening for a transmitter that we know transmits a signal linearly polarized in the vertical direction. However, our only choices for the receiver antenna are a linear horizontally polarized mount, and an antenna designed to receive circular polarization. Which antenna will receive more of the signal?



- The received signal voltage will be like the dot product: $\vec{E} \cdot \vec{H}$, but this is zero.
- Second, compute the dot product with the circular polarization.

Waves and the Wave Equation

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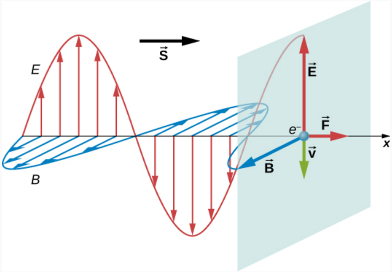

- The received signal voltage will be like the dot product: $\vec{E} \cdot \vec{H}$, but this is zero.
- Think just in terms of polarization vectors, and imagine the time and space variables become invisible.

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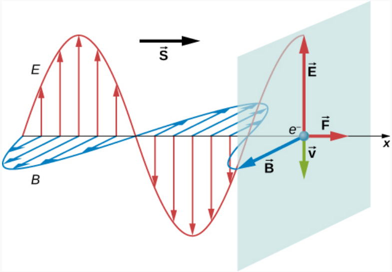
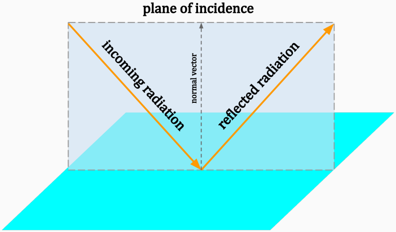


- The received signal voltage will be like the dot product: $\vec{E} \cdot \vec{H}$, but this is zero.
- $\vec{P}_1 = (0, 1, 0)$ and $P_2 = (\sin(\theta), \cos(\theta), 0)$. Average $\vec{P}_1 \cdot \vec{P}_2$ to find $4/\pi$. This is about -4 dB in power.

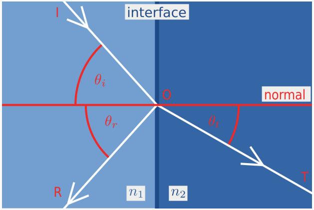
Waves and the Wave Equation

Reflections occur at interfaces between different media in which the speed of light changes.



Consider linearly polarized waves. The s-polarization is perpendicular to the plane of incidence, and the p-polarization is parallel to the plane of incidence.

The s and p polarizations have different reflection coefficients for reflected power. This reflectance leads to things like surface clutter.



$$R_s = \left| \frac{n_1 \cos(\theta_i) - n_2 \cos(\theta_t)}{n_1 \cos(\theta_i) + n_2 \cos(\theta_t)} \right|^2 \tag{11}$$

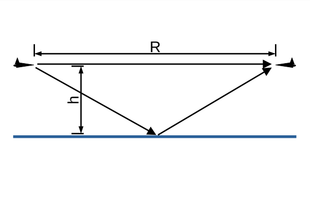
$$R_p = \left| \frac{n_1 \cos(\theta_t) - n_2 \cos(\theta_i)}{n_1 \cos(\theta_t) + n_2 \cos(\theta_i)} \right|^2 \tag{12}$$

The fraction of transmitted power is given by $T = 1 - R$. How to relate θ_i with θ_t? Snell's Law:

$$n_1 \sin(\theta_i) = n_2 \sin(\theta_t) \tag{13}$$

Waves and the Wave Equation

Suppose one aircraft sends a radio signal to another, but the transmission beam is broad enough to admit two paths: one direct path and one reflected path from the ocean below. Both craft have an altitude of $h = 1\,\text{km}$, and they are at a range of $R = 10\,\text{km}$. What is the time duration between the direct and reflected pulses? Assuming the radiated power goes as inverse distance squared, and that 10 percent of power is reflected, how much less powerful is the reflected pulse?



- Work out the difference in distance.
- Convert to time.
- Convert distance ratio to radiated power ratio. Estimate the effect of reflection.

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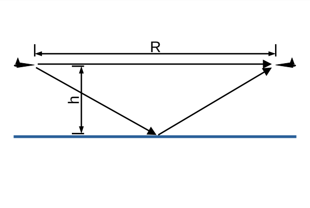
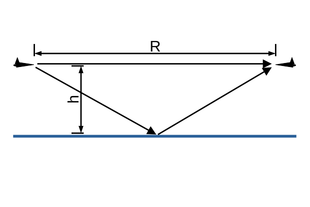


- Difference in distance is $0.2\,\mathrm{km}$.
- Convert to time.
- Convert distance ratio to radiated power ratio. Estimate the effect of reflection.

Waves and the Wave Equation

Suppose one aircraft sends a radio signal to another, but the transmission beam is broad enough to admit two paths: one direct path and one reflected path from the ocean below. Both craft have an altitude of $h = 1\,\text{km}$, and they are at a range of $R = 10\,\text{km}$. What is the time duration between the direct and reflected pulses? Assuming the radiated power goes as inverse distance squared, and that 10 percent of power is reflected, how much less powerful is the reflected pulse?



- Difference in distance is $0.2\,\text{km}$.
- Speed of light: $0.3\,\text{km}\,\mu\text{s}^{-1}$. $0.2\,\text{km}/0.3\,\text{km}\,\mu\text{s}^{-1} = 0.67\,\mu\text{s}$.
- Convert distance ratio to radiated power ratio. Estimate the effect of reflection.

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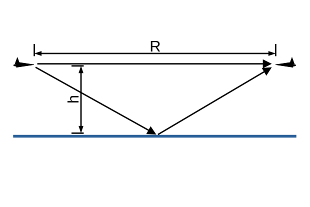

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- Distance ratio: $(10/10.2)$, so power ratio is $(10/10.2)^2$. In dB: $20\log_{10}(10/10.2) \approx -0.2$ dB. Reflection coefficient: -10 dB.

Conclusion

Reading: *Stimson3 ch. 1-6*

- Week 1: Units and estimation. **Key skills**: mental math, wave concepts
  - Electromagnetic units, estimation, and decibels
  - Waves and the wave equation
  - Reflections, refraction, and diffraction
  - Phase, amplitude, frequency, polarization
- Week 2: Basic Training in Mathematics. **Key skills**: estimate pulse bandwidth, pulse trains and uncertainty principle
  - **Complex numbers**: applications to phasors and radio waves, complex imdedance of filters and antennas
  - **Fourier series and transforms**, filters and attenuation, properties of waveforms, power spectra, and spectrograms, cross-correlation and convolution
  - **Statistics and probability**: applications to noise, signal-to-noise ratio