Index of Refraction Studies

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Introduction

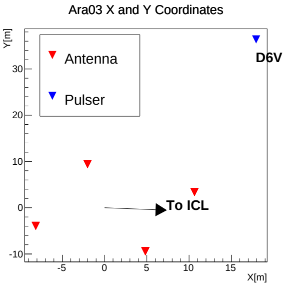
- An accurate refractive index (RI) model is important for reduction of systematic errors in ARA measurements
- This presentation will summarize all RI measurements and studies (of the past year and ongoing) made by UNL using ARA station data
- For past presentations on this topic see (in presented order):
  - 1) `http://ara.physics.wisc.edu/docs/0011/001165/001/ICL_Rooftop_Pulser_Analysis_2013data.pdf`
  - 2) `http://ara.physics.wisc.edu/docs/0011/001191/001/IoR_Measurement.pdf`
  - 3) `http://ara.physics.wisc.edu/docs/0012/001242/001/RefractiveIndexMeasurement2011And2013Data.pdf`
  - 4) `http://ara.physics.wisc.edu/docs/0012/001273/001/SurfacePulserStudy.pdf`
  - 5) `http://ara.physics.wisc.edu/docs/0013/001339/002/SurfacePulserStudyFull.pdf`

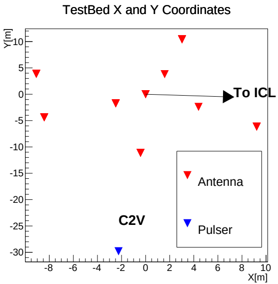
History

- Past RI measurements have used data from:
  - Testbed local pulser C2V (2011)
  - ICL roof top pulser (2013) (testbed and Ara03)
  - Ara03 local pulser D6V (2013)
- Recent progress has been made towards a RI measurement using 2015 surface pulser data (Ara02 and 03)

Reconstruction Notes

- Analytic Sphere Method (ASM) used for preliminary reconstruction
- Sergei Avdeev's calibration was used for the testbed and Thomas Meures's for Ara03
- Time differences found using correlations
- This geometry facilitates an extra check that the analysis results make sense

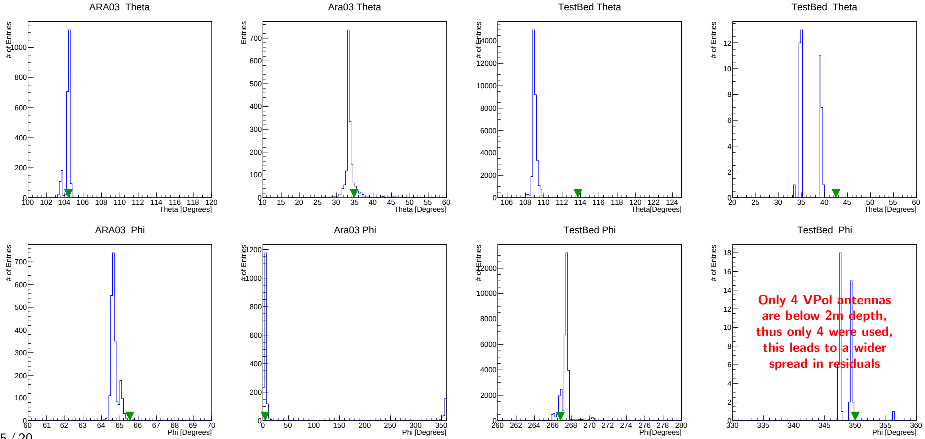




Reconstruction Results

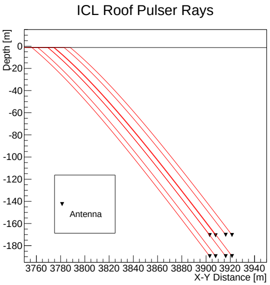
- Good directional reconstruction signifies sensitivity to the pulsers and ability to measure RI
- Ara02 ICL and testbed pulser C1H reconstructions had large directional residuals and were not used for past RI measurements at UNL
- No ray tracing used for theta markers

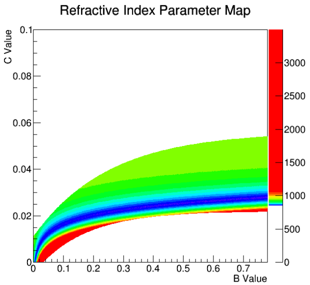
↓ D6V ↓ ICL ↓ C2V ↓ ICL



Sensitivity

- Default model: $n = A - B * e^{-C*depth}$
- Past studies revealed parameters B and C are correlated in the case of nearly identical ray paths (ICL roof and local pulsers)
    - Sensitivity to RI comes from differences in ray paths (where time difference occurs), for ICL roof and local pulsers this occurs at depth of the station, thus sensitivity is limited to a small range of RI, leading to many possible values of B and C, this forms the blue band in χ^2 at the bottom right
- Due to these limitations we measure RI at the array center depth

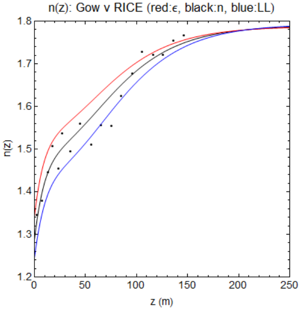




$$\chi^2 = \frac{\left(\mathrm{dt}_{\mathrm{meas}} - \mathrm{dt}_{\mathrm{pred}}\right)^2}{\sigma^2}$$

Alternate Models

- The plot below shows RICE measurements of RI (black dots)
- Different functional models were considered, the best fit to RICE data includes 8 parameters and three exponential decays
- With given data, we are sensitive to the index of refraction value at the locality of the three stations: testbed, Ara02(not used) and Ara03
    - Effectively we can measure $\mathrm{n}(\mathrm{Z}_{\mathrm{testbed}})$ and $\mathrm{n}(\mathrm{Z}_{\mathrm{Ara03}})$, two datapoints
    - The 8 parameter model cannot be constrained with this limitation



- $\rho(z) = \mathsf{A} - \mathsf{B} * e^{-\mathsf{C}*\mathsf{depth}} - \mathsf{D} * e^{-\mathsf{E}*\mathsf{depth}} - \mathsf{F} * e^{-\mathsf{G}*\mathsf{depth}}$
- Black: $n(\rho) = 1 + a_1 * \rho(z)$
- Red: $n(\rho) = \sqrt{1 + a_2 * \rho(z)}$
- Blue: $n(\rho) = a_3 * \rho(z)$

Measurement Procedure

Local in-ice pulser:

- We measured RI at the array center using the formula: $n = \frac{\Delta T * c}{\Delta D}$
  - ΔT is found from waveform correlation
  - ΔD is distance difference of (Pulser to antenna 1) - (Pulser to antenna 2), from ray tracing

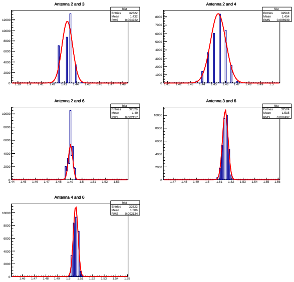
ICL:

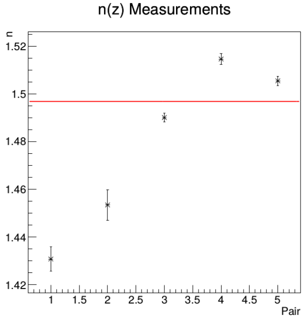
- We measured RI at the array center using the formula: $n = \frac{(\Delta T - \Delta T_S) * c}{\Delta D}$
  - ΔT_S is ray time difference above the surface, estimated using ray tracing
  - ΔD is distance difference of (Surface entry 1 to antenna 1) - (Surface entry 2 to antenna 2), from ray tracing

For Both:

- For best sensitivity only antenna pairs of one top and one bottom will be used

- Measurement error was found from the width of a Gaussian fit to the distribution of n values (left plot, example: testbed, C2V pulser)
- The mean of the Gaussian is taken as the measured n
- Systematic error was determined from the RMS of the distribution of antenna pair measurements (right plot)





Results

	Measurement	Measurement Error	Systematic Error
TestBed Local Pulser	1.49692	0.00114	0.03188
TestBed ICL Pulser	1.52494	0.00482	0.08260
Ara03 Local Pulser	1.73299	0.00081	0.04814
Ara03 ICL Pulser	1.77852	0.00052	0.01299

- Measurements involving the same station were combined using errors as weights

TestBed Weighted Average: $n_1 = 1.49840 \pm 0.00111 \pm 0.02974$

Ara03 Weighted Average: $n_2 = 1.76523 \pm 0.00044 \pm 0.01254$

Propagation to Model Parameters

- Solving for parameters B and C of the default model we obtain:

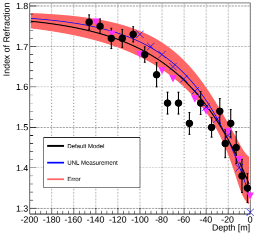
$$B = (A - n_1) * \left(\frac{A - n_2}{A - n_1}\right)^{-\frac{z_1}{z_2 - z_1}}$$

$$C = -\frac{ln\left|\frac{A - n_2}{A - n_1}\right|}{z_2 - z_1}$$

* where z_1 is the average depth of the TestBed and z_2 is the average depth of Ara03

- Error propagation derivation by Rami Kamalieddin, details in extras section

$$\mathrm{B} = 0.45396 \pm 0.00301 \pm 0.08368$$
$$\mathrm{C} = 0.01886 \pm 0.00019 \pm 0.00547$$

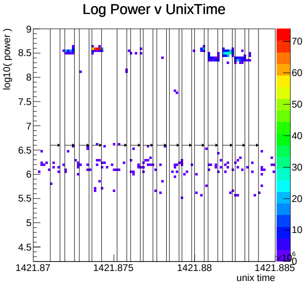


* Parameters B and C are correlated which has been accounted for in the error band (red)

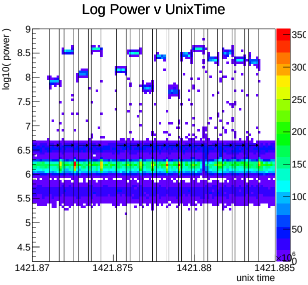
Moving Forward

- During the 2014-2015 pole season a surface pulser was placed on the ice and left running at 1 Hz for 10 min at 15 locations, at both Ara02 and 03
- Missing data in the filtered set is filled in by the unbiased set, this is visible in the total waveform power vs unixtime plots below (due to very high power pulses)

↓ Filtered Set



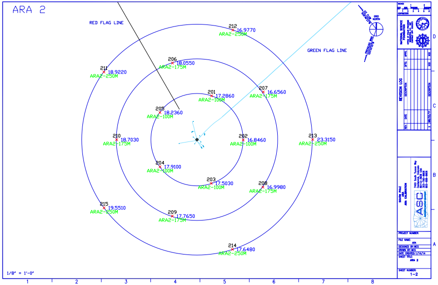
↓ Unbiased Set

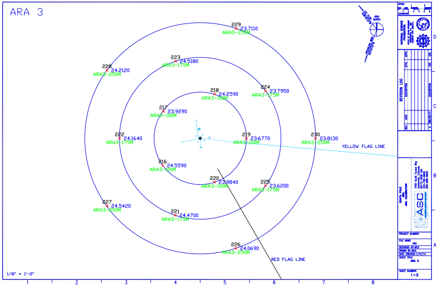


- We plan to use this data to obtain more RI measurements with the goal to reduce the width of the error band

Surface Pulser Reconstruction Notes

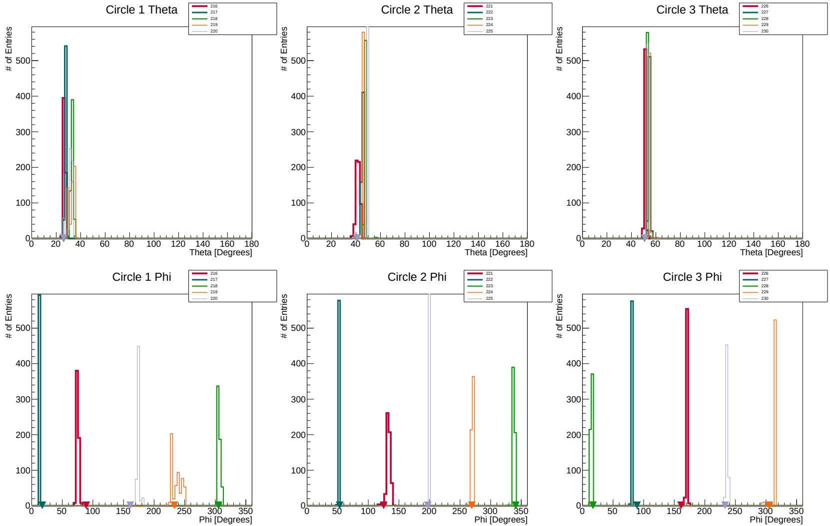
- Preliminary reconstruction using ASM with high voltage threshold hit time finding
- A layout of surface pulser locations is plotted below for Ara02 and 03
- As with past RI measurements, a good reconstruction would indicate an ability to make RI measurements





Surface Pulser Reconstruction

- VPol reconstruction shown
- Good results for all locations and polarizations (all shown in extras section) has demonstrated that this data can be used for a RI measurement
- Currently working on the RI measurement using this data

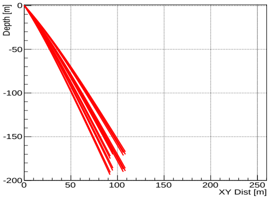


Surface Pulser Sensitivity

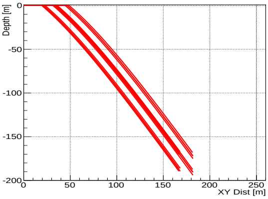
- Rays from the 2 outer most pulser circles are similar to our previous measurements (majority of ray path difference occurs at depth of the station)
  - We can use this to further constrain our RI measurement

- We believe the inner most circle could provide a sensitivity to the ice from the surface down to the depth of the station

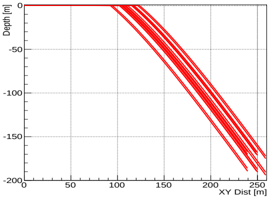
Inner Circle ↓



Middle Circle ↓



Outer Circle ↓



Conclusions

- Preliminary reconstructions were performed to persuade ourselves that a RI measurement could be made with the considered data
- Using data from 2011 and 2013 for the testbed and Ara03 we obtained numerical RI model parameters of:
  - $A = 1.78 \pm 0.005$ (Not measured, inherited)
  - $B = 0.45396 \, {}^{+}_{-} \, 0.00301 \, {}^{+}_{-} \, 0.08368$
  - $C = 0.01886 \, {}^{+}_{-} \, 0.00019 \, {}^{+}_{-} \, 0.00547$
- Surface pulser data from 2015 has recently been reconstructed well, we plan to use this data to reduce uncertainty in the fit to the default RI model
- More thought will be given towards fitting alternative models using the inner most surface pulser locations

Extras

Extra - Error Propagation

$$n_1 = A - B^{-cz_1} \qquad n_2 = A - B^{-cz_2}$$

z_1, n_1, σ_1: TestBed average depth, n at that depth, and error on n (Respectively)
z_2, n_2, σ_2: Ara03 average depth, n at that depth, and error on n (Respectively)
Note: Depths are treated as positive numbers

$$\begin{pmatrix} \sigma_B^2 & \sigma_B\sigma_C \\ \sigma_B\sigma_C & \sigma_C^2 \end{pmatrix} = \begin{pmatrix} \frac{\delta B}{\delta n_1} & \frac{\delta B}{\delta n_2} \\ \frac{\delta C}{\delta n_1} & \frac{\delta C}{\delta n_2} \end{pmatrix} \begin{pmatrix} \sigma_{n_1}^2 & \sigma_{n_1}\sigma_{n_2} \\ \sigma_{n_1}\sigma_{n_2} & \sigma_{n_2}^2 \end{pmatrix} \begin{pmatrix} \frac{\delta B}{\delta n_1} & \frac{\delta C}{\delta n_1} \\ \frac{\delta B}{\delta n_2} & \frac{\delta C}{\delta n_2} \end{pmatrix}$$

*Thanks to Rami Kamalieddin for derivation of covariance matrix propagation
*Solving for the needed formulae:

$$B = (A - n_1)\left(\frac{A-n_2}{A-n_1}\right)^{\frac{-z_1}{z_2-z_1}}$$

$$\frac{\delta B}{\delta n_1} = \frac{z_2\left(\frac{A-n_2}{A-n_1}\right)^{\frac{z_1}{z_1-z_2}}}{z_1-z_2}$$

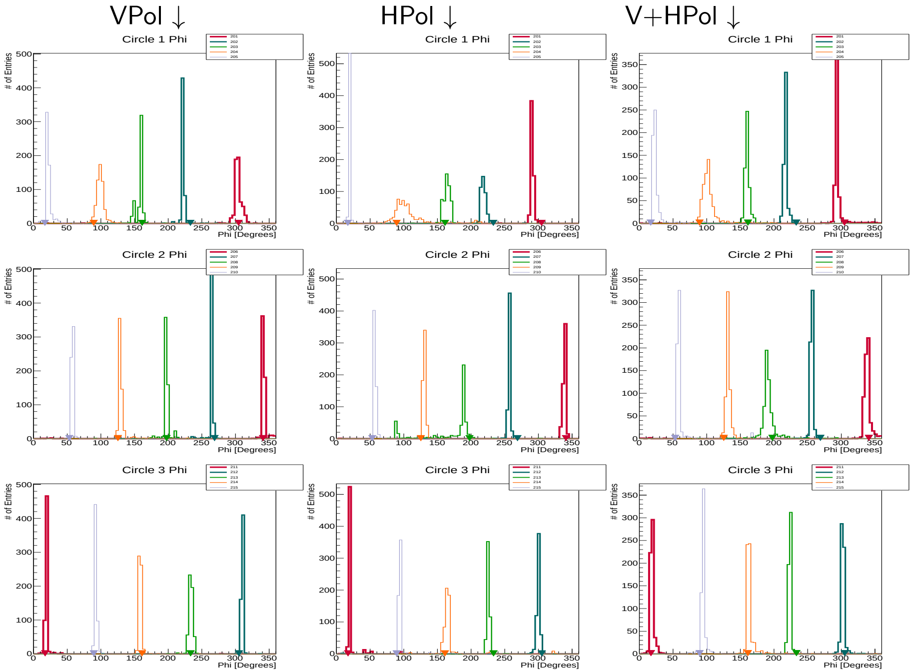
$$\frac{\delta B}{\delta n_2} = \frac{z_1\left(\frac{A-n_2}{A-n_1}\right)^{\frac{z_2}{z_1-z_2}}}{z_2-z_1}$$

$$C = \frac{-ln\left|\frac{A-n_2}{A-n_1}\right|}{z_2-z_1}$$

$$\frac{\delta C}{\delta n_1} = \frac{1}{(A-n_1)(z_1-z_2)}$$

$$\frac{\delta C}{\delta n_2} = \frac{1}{(A-n_2)(z_2-z_1)}$$

Ara02 - Surface Pulser Reconstruction



Ara03 - Surface Pulser Reconstruction

VPol ↓ HPol ↓ V+HPol ↓