THE PRIMER, EXAMPLE CHAPTER: APPLING AN INTEGRAL

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CONFRONTING THE PUZZLE

We encounter a signal. *A bug walks across a magic pad (a sensor). The pad gives only the current velocity of the bug. We'd like to know how far the bug has traveled...*

Symbolic content: Visual scene → *Classification*

Symbolic content: Animate bug in scene → *Classification*

Define **average velocity** and **displacement**:

$$\vec{x} = \vec{x}_{\mathrm{f}} - \vec{x}_{\mathrm{i}} \tag{1}$$

$$\bar{\vec{v}} = \frac{\Delta\vec{x}}{\Delta t} \tag{2}$$

Symbolic content: Observe that shrinking the time-window Δt makes $\vec{\bar{v}}$ more like the *slope* of $\vec{x}(t)$, or the instantaneous velocity.

Taking the limit:

$$\lim_{\Delta t \to 0} \vec{\bar{v}} = \vec{v}(t) = \frac{d\vec{x}}{dt} \tag{3}$$

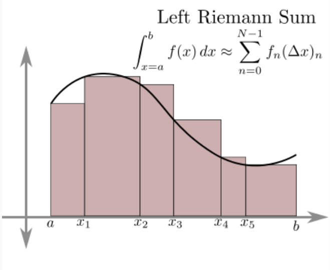
$$\int_0^{x_0} \frac{dx}{dt} dt = x_0 \tag{4}$$

Symbolic content: Represent Eq. 4 visually $\to$ *Abstraction*

ONE PATH - DEVELOP AN ALGORITHM

Symbolic content:

Breaking an integral into a: *a Riemann sum* → *Abstraction*



$$x_0 = \sum_{n=0}^{N-1} f_n(\Delta x)_n \tag{5}$$

```
define x_0(t_data,v_data):
    delta_t = t_data[2]-t_data[1]
    x_0 = 0
    for i in v_data:
        x_0 += i*delta_t
    return x_0
```

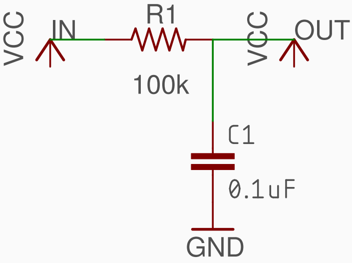
Study outputs given different data → *hypothetical*

ANOTHER PATH - DESIGN A CIRCUIT

Taking the limit:

$$\int_0^{x_0} \frac{dx}{dt} dt = x_0 \tag{6}$$

Symbolic content: Represent Eq. 4 visually → *Classification*



Show in steps that this circuit obeys $\rightarrow$ *Abstraction*

$$v_{\mathrm{out}}(t) = \frac{1}{RC}\int_{t_1}^{t_2} v_{\mathrm{in}}(t)dt \tag{7}$$

- See $v_{\mathrm{out}}(t)$ is like x_0, and $v_{\mathrm{in}}(t)$ is like $\vec{v}(t)$ $\rightarrow$ *Classification*
- *Build the circuit* (hypothetically in the book) $\rightarrow$ *Abstraction*
- *Test the circuit* for different outputs $\rightarrow$ *Hypothetical*

Symbolic content: Show circuit and graph inputs and outputs

STUDY THE MECHANICS RESULT

Using both or either paths (**Scalar**), solve the problem of x_0 using the content $\rightarrow$ *Hypothetical*

- Solution for the case of no motion
- Solution for the case of constant velocity
- Solution for the case of constant acceleration

Symbolic content: Show these solutions graphically
$\rightarrow$ *Abstraction* (**Symbolic content**): Deduce equations for displacement, given each situation

STUDY THE MECHANICS RESULT

Arrive at the answers:

$$x_0 = x_\mathrm{i} \tag{8}$$

$$x_0 = vt + x_\mathrm{i} \tag{9}$$

$$x_0 = \frac{1}{2}at^2 + vt + x_\mathrm{i} \tag{10}$$

CONCLUSION

1. Confronting the Puzzle - Sum up a signal over time
2. Create an algorithm - one path
3. Design a circuit - another path
4. Study the mechanics result