THE PRIMER, EXAMPLE CHAPTER: APPLING AN INTEGRAL

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CONTENTS

1. Confronting the Puzzle - Sum up a signal over time
   - *Classification*: A problem involving an **integral**
   - *Abstraction*: Trade the **objects** for **symbols** and act
   - *Hypothetical*: Changing **properties** of **symbols**
2. Create an algorithm - one path
   - *Riemann sums* and *pseudo-code*
   - **Build** and **Test**
3. Design a circuit - another path
   - *Problem objects to circuit elements*
   - **Build** and **Test**
4. Study the mechanics result

Symbolic Content: Depict idea through visual design

Question Content: Interactivity and machine learning

CONFRONTING THE PUZZLE

We encounter a signal. *A bug walks across a magic pad (a sensor). The pad gives only the current velocity of the bug. We'd like to know how far the bug has traveled...*

Symbolic content: Visual scene $\rightarrow$ *Classification*

Symbolic content: Animate bug in scene $\rightarrow$ *Classification*

Define **average velocity** and **displacement**:

$$\vec{x} = \vec{x}_{\mathrm{f}} - \vec{x}_{\mathrm{i}} \tag{1}$$

$$\vec{\vec{v}} = \frac{\Delta \vec{x}}{\Delta t} \tag{2}$$

CONFRONTING THE PUZZLE

Question Content: What is the average velocity → *Classification*

- A Conceptually wrong
- B Right
- C Numerically wrong
- D Unrelated

Question Content: Apply the average velocity → *Hypothetical*

- A Conceptually wrong
- B Right
- C Numerically wrong
- D Unrelated

Symbolic content: Observe that shrinking the time-window Δt makes $\vec{v}$ more like the *slope* of $\vec{x}(t)$, or the instantaneous velocity.

Taking the limit:

$$\lim_{\Delta t \to 0} \vec{\vec{v}} = \vec{v}(t) = \frac{d\vec{x}}{dt} \tag{3}$$

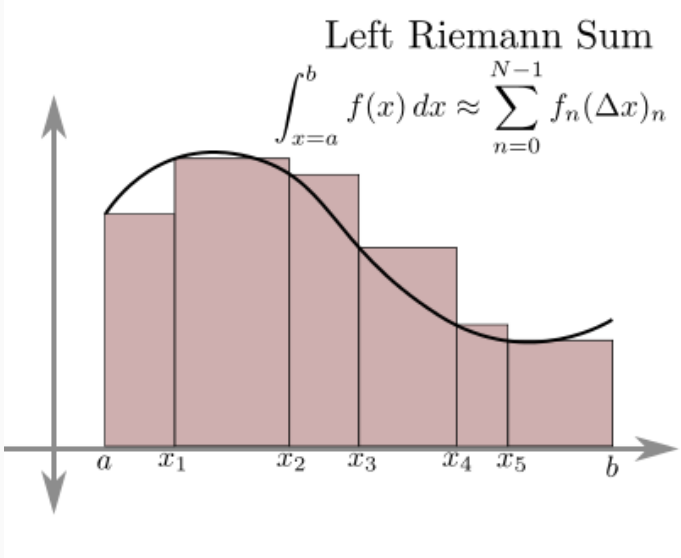
$$\int_0^{x_0} \frac{dx}{dt} dt = x_0 \tag{4}$$

Symbolic content: Represent Eq. 4 visually $\rightarrow$ *Abstraction*

ONE PATH - DEVELOP AN ALGORITHM

Symbolic content:

Breaking an integral into a: *a Riemann sum* $\rightarrow$ *Abstraction*



$$x_0 = \sum_{n=0}^{N-1} f_n(\Delta x)_n \tag{5}$$

DEVELOP AN ALGORITHM

Question Content: Adding to approximate integral → *Classification*

- A Conceptually wrong
- B Right
- C Numerically wrong
- D Unrelated

Question Content: Choose correct algorithm → *Hypothetical*

- A Conceptually wrong
- B Right
- C Numerically wrong
- D Unrelated

DEVELOP AN ALGORITHM

```
define x_0(t_data,v_data):
     delta_t = t_data[2]-t_data[1]
     x_0 = 0
     for i in v_data:
          x_0 += i*delta_t
     return x_0
```

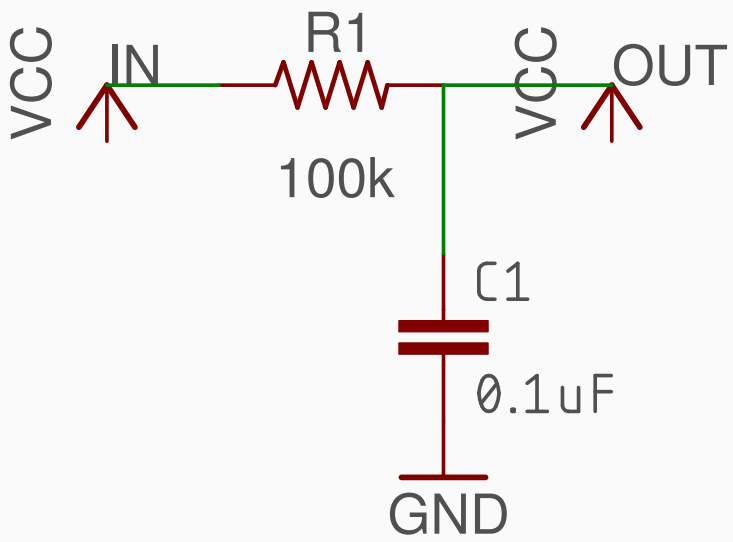
Study outputs given different data $\rightarrow$ *hypothetical*

ANOTHER PATH - DESIGN A CIRCUIT

Taking the limit:

$$\int_0^{x_0} \frac{dx}{dt} dt = x_0 \tag{6}$$

Symbolic content: Represent Eq. 4 visually → *Classification*



DESIGN A CIRCUIT

Question Content: Classifying resisters, capacitors → *Classification*

- A Conceptually wrong
- B Right
- C Numerically wrong
- D Unrelated

Question Content: Simple circuit mathematics → *Hypothetical*

- A Conceptually wrong
- B Right
- C Numerically wrong
- D Unrelated

Show in steps that this circuit obeys → *Abstraction*

$$v_{\text{out}}(t) = \frac{1}{RC}\int_{t_1}^{t_2} v_{\text{in}}(t)dt \tag{7}$$

- See $v_{\text{out}}(t)$ is like x_0, and $v_{\text{in}}(t)$ is like $\vec{v}(t)$ → *Classification*
- *Build the circuit* (hypothetically in the book) → *Abstraction*
- *Test the circuit* for different outputs → *Hypothetical*

Symbolic content: Show circuit and graph inputs and outputs

STUDY THE MECHANICS RESULT

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Using both or either paths (**Scalar**), solve the problem of x_0 using the content $\rightarrow$ *Hypothetical*

- Solution for the case of no motion
- Solution for the case of constant velocity
- Solution for the case of constant acceleration

Symbolic content: Show these solutions graphically

→ *Abstraction* (**Symbolic content**): Deduce equations for displacement, given each situation

$$x_0 = x_\mathrm{i} \tag{8}$$

$$x_0 = vt + x_\mathrm{i} \tag{9}$$

$$x_0 = \frac{1}{2}at^2 + vt + x_\mathrm{i} \tag{10}$$

CONCLUSION

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