

HW3

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Exercise 4.5

		P_2		
		L	M	R
P_1	U	4, 3	2, 7	0, 4
	D	5, 5	5, -1	-4, -2

For this game, the pair of pure strategies (D, L) is a Nash equilibrium :

Let $\sigma_2^* = L$ and let $\sigma_1 = (p, 1-p)$, Then :

$$\pi_1(\sigma_1, L) = 4p + 5 - 5p = 5 - p \leq 5 = \pi_1(D, L)$$

Now let $\sigma_1^* = D$ and let $\sigma_2 = (1-p-q, p, q)$, Then :

$$\pi_2(D, \sigma_2) = 5 - 5p - 5q - p - 2q = 5 - 6p - 7q \leq 5 = \pi_2(D, L)$$

$\therefore (D, L)$ is a Nash equilibrium

Exercise 4.7

Consider the children's game "Rock-Scissors-Paper", where 2 children simultaneously make a hand sign corresponding to one of the three items. Playing "Rock" (R) beats "Scissors" (S), "Scissors" beats "Paper" (P), and "Paper" beats "Rock". When both children play the same action (both R, both S, or both P) the game is drawn.

- (a) Construct a payoff table for this game with a payoff of +1 for a win, -1 for losing, and 0 for a draw.
- (b) Solve this game.

(a) The payoff table :

		P_1		
		R	S	P
P_2	R	0, 0	-1, +1	+1, -1
	S	+1, -1	0, 0	-1, +1
	P	-1, +1	+1, -1	0, 0

(b) suppose P_2 plays (R, S, P) with $(p, q, 1-p-q)$

If P_1 is playing a completely mixed strategy at the Nash equilibrium, then:

$$\pi_1(R, \sigma_2^*) = \pi_1(S, \sigma_2^*) = \pi_1(P, \sigma_2^*)$$

$$\begin{aligned} \Leftrightarrow & p\pi_1(R, R) + q\pi_1(R, S) + (1-p-q)\pi_1(R, P) \\ &= p\pi_1(S, R) + q\pi_1(S, S) + (1-p-q)\pi_1(S, P) \\ &= p\pi_1(P, R) + q\pi_1(P, S) + (1-p-q)\pi_1(P, P) \end{aligned}$$

$$\Leftrightarrow q - 1 + p + q = -p + 1 - p - q = p - q$$

$$\Leftrightarrow \begin{cases} q = \frac{1}{3} \\ p = \frac{1}{3} \end{cases}$$

\therefore The same as P_2

\therefore the Nash equilibrium is (σ_1^*, σ_2^*) with $\sigma_1^* = \sigma_2^* = (\frac{1}{3}, \frac{1}{3})$