HW2

Jack Cheng

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Problem 1

See following photo for solution.

Stochastic Control & Optimization

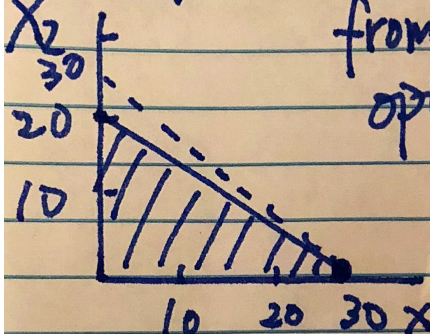
HW2. dc43342 Daxi Cheng.

Problem 1. Suppose we have x_1 forte, x_2 pi

Choose x_1, x_2

maximize $4x_1+5x_2$

$$\text{s.t} \begin{cases} 2x_1 + 3x_2 \leq 60 \\ x_1, x_2 \geq 0 \end{cases}$$

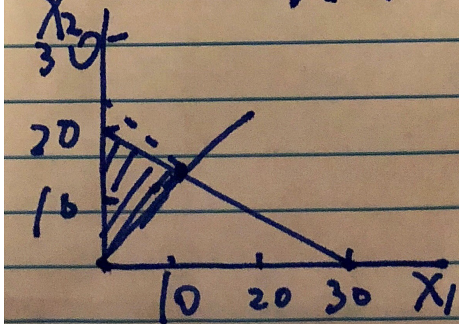


from the graph we get the optimal choice is $x_1 = 30$, $x_2 = 0$

where we have $4 \times 30 = 120$ points

if we add one constrain that:

$$x_1 - x_2 \leq 0$$



from the graph we have the solution is when

$$\begin{cases} x_1 - x_2 = 0 \\ 2x_1 + 3x_2 = 60 \end{cases} \Rightarrow x_1 = x_2 = 12$$

Where we get $9 \times 12 = 108$ points

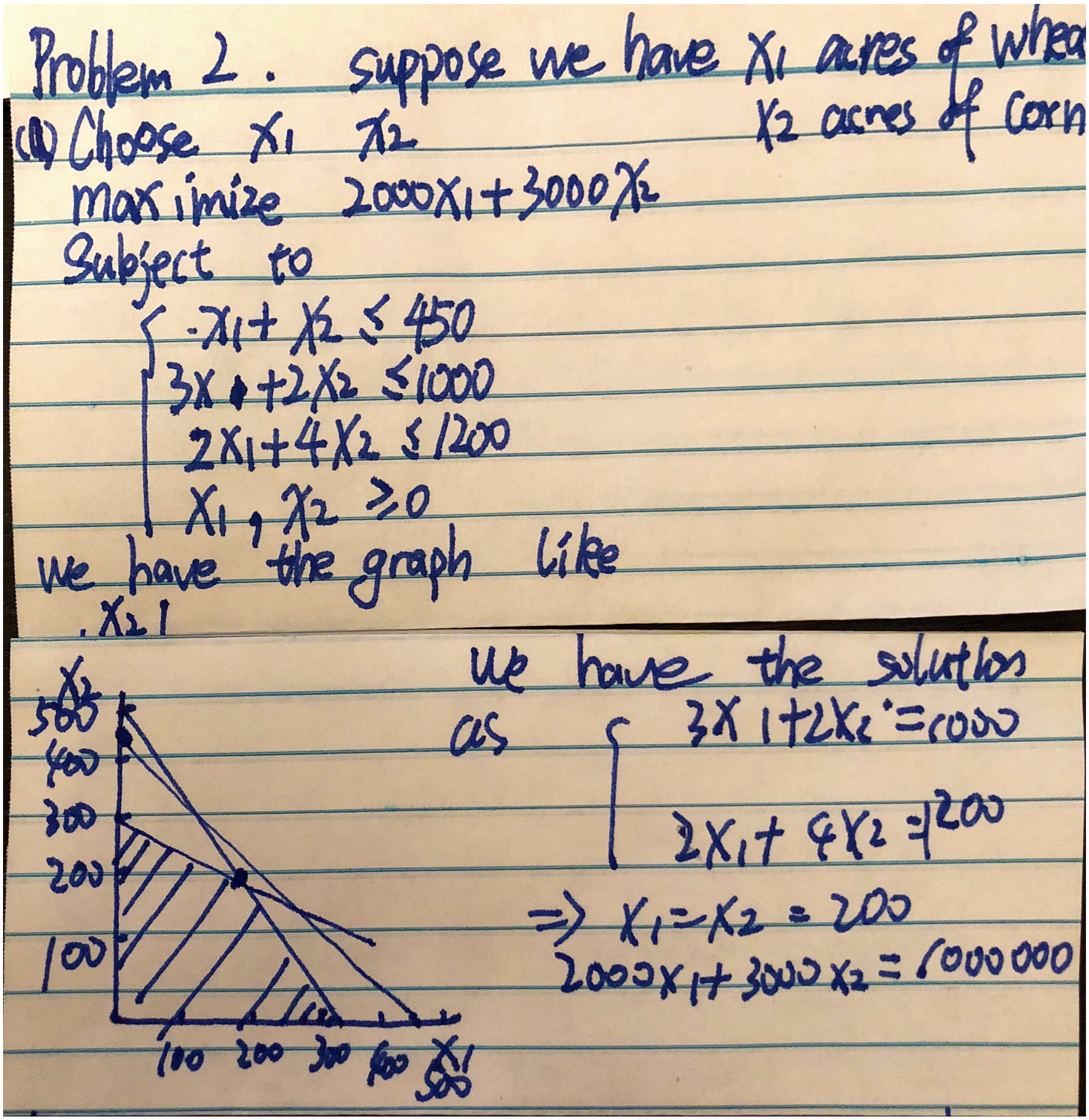
He would lose $120 - 108 = 12$ points.

Figure 1:

Problem 2

See the following picture for solution for question 1.

(1)



Problem 2. suppose we have x_1 acres of whea

(a) Choose x_1 x_2 $\quad$ x_2 acres of corn

maximize $2000x_1+3000x_2$

Subject to

$$\begin{cases} x_1+x_2 \le 450 \\ 3x_1+2x_2 \le 1000 \\ 2x_1+4x_2 \le 1200 \\ x_1, x_2 \ge 0 \end{cases}$$

we have the graph like

we have the solution as

$$\begin{cases} 3x_1+2x_2 = 1000 \\ 2x_1+4x_2 = 1200 \end{cases}$$

$\Rightarrow x_1 = x_2 = 200$

$2000x_1+3000x_2 = 1000000$

Figure 2:

(2)

```r
rm(list=ls())
library('lpSolve')
c=c(2000,3000)
A=matrix(c(1,3,2,1,2,4),3,2)
dir=c('<=','<=','<=')
b=c(450,1000,1200)
s=lp('max',c,A,dir,b)
s$solution
```

```
## [1] 200 200
```

```r
s$objval
```

```
## [1] 1e+06
```

```r
f=200
while (f<=2200) {
  b=c(450,1000,f)
  s=lp('max',c,A,dir,b)
  print(paste('When fertilizer is',f))
  print(s$solution)
  f=f+100
}
```

```
## [1] "When fertilizer is 200"
## [1] 100   0
## [1] "When fertilizer is 300"
## [1] 150   0
## [1] "When fertilizer is 400"
## [1] 200   0
## [1] "When fertilizer is 500"
## [1] 250   0
## [1] "When fertilizer is 600"
## [1] 300   0
## [1] "When fertilizer is 700"
## [1] 325.0  12.5
## [1] "When fertilizer is 800"
## [1] 300  50
## [1] "When fertilizer is 900"
## [1] 275.0  87.5
## [1] "When fertilizer is 1000"
## [1] 250 125
## [1] "When fertilizer is 1100"
## [1] 225.0 162.5
## [1] "When fertilizer is 1200"
## [1] 200 200
## [1] "When fertilizer is 1300"
## [1] 175.0 237.5
## [1] "When fertilizer is 1400"
## [1] 150 275
## [1] "When fertilizer is 1500"
## [1] 125.0 312.5
## [1] "When fertilizer is 1600"
```

```
## [1] 100 350
## [1] "When fertilizer is 1700"
## [1]  50 400
## [1] "When fertilizer is 1800"
## [1]   0 450
## [1] "When fertilizer is 1900"
## [1]   0 450
## [1] "When fertilizer is 2000"
## [1]   0 450
## [1] "When fertilizer is 2100"
## [1]   0 450
## [1] "When fertilizer is 2200"
## [1]   0 450
```

We can see that the farmer discontinues producing when fertilizer ≥ 1800, and stops producing corn when fertilizer ≤ 600.

Problem 3

We need to solve the vector of x1-x5 which stands for the number of purchasing investment 1-5 respectively.

Choose x_1, x_2, x_3, x_4, x_5 to Maximize $13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$ subject to:

$$11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 50$$

$$3x_1 + 6x_2 + 5x_3 + 1x_4 + 34x_5 \leq 20$$

$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

```r
c=c(13,16,16,14,39)
A=matrix(c(11,3,53,6,5,5,5,1,29,34),2,5)
dir=c('<=','<=')
b=c(50,20)
s=lp('max',c,A,dir,b)
s$solution
```

```
## [1] 0.0 0.0 2.5 7.5 0.0
```

```r
s$objval
```

```
## [1] 145
```

We should purchase 2.5 of investment3 and 7.5 of investment 4 to get an NPV of 145.

Problem 4

We need to solve the vector of x1-x3 which stands for the amount of corn, 2% milk and wheat bread respectively.

Choose x_1, x_2, x_3

to Minimize $0.18x_1 + 0.23x_2 + 0.05x_3$

subject to:

$$72x_1 + 121x_2 + 65x_3 \geq 2000$$

$$72x_1 + 121x_2 + 65x_3 \leq 2250$$

$$107x_1 + 500x_2 \geq 5000$$

$$107x_1 + 500x_2 \leq 50000$$

and $0 \leq x_1 \leq 10, 0 \leq x_2 \leq 10, 0 \leq x_3 \leq 10$

```
c=c(0.18,0.23,0.05)
A=matrix(c(72,72,107,107,1,0,0,121,121,500,500,0,1,0,65,65,0,0,0,0,1),7,3)
dir=c('>=','<=','>=','<=','<=','<=','<=')
b=c(2000,2250,5000,50000,10,10,10)
s=lp('min',c,A,dir,b)
s$solution
```

```
## [1]  1.944444 10.000000 10.000000
```

```
s$objval
```

```
## [1] 3.15
```

We should take about 1.94 units of corn, 10 units of 2% milk and 10 units of wheat bread at the cost of \$3.15.

Problem 5

Here are my assumptions for this question:

For the wood that we didn't cut down last year, we could cut them down this year or later. Each year the growth amount of a tree is fixed. However, for the trees that we have already cut down, there would be no growth.

We need to select $X_1 \rightarrow\ X_3,\ Y_1\ to\ Y_3$ which denotes the acre (not tons) we get from 1st unit and 2nd unit in year 1 to 3 respectively.

Choose $X_1 \rightarrow\ X_3,\ Y_1\ to\ Y_3$

to maximize $X_1 + 1.3X_2 + 1.4X_3 + Y_1 + 1.2Y_2 + 1.6Y_3$

subject to:

Total availability:

$$X_1 + X_2 + X_3 \leq 2$$

$$Y_1 + Y_2 + Y_3 \leq 3$$

And we have:

$$X_1 + Y_1 \geq 1.2$$

$$1.3X_2 + 1.2Y_2 \geq 1.5$$

$$1.4X_3 + 1.6Y_3 \geq 2$$

$$X_1 + Y_1 \leq 2$$

$$1.3X_2 + 1.2Y_2 \leq 2$$

$$1.4X_3 + 1.6Y_3 \leq 3$$

and $X_1, X_2, X_3, Y_1, Y_2, Y_3 \geq 0$

```r
c=c(1,1.3,1.4,1,1.2,1.6)
A=matrix(0,8,6)
A[1,1:3]=1
A[2,4:6]=1
A[3,1]=1
A[3,4]=1
A[4,1]=1
A[4,4]=1
A[5,2]=1.3
A[5,5]=1.4
A[6,2]=1.3
A[6,5]=1.4
A[7,3]=1.4
A[7,6]=1.6
A[8,3]=1.4
A[8,6]=1.6
dir=c('<=','<=','>=','<=','>=','<=','>=','<=')
b=c(2,3,1.2,2,1.5,2,2,3)
s=lp('max',c,A,dir,b)
s$solution
```

```
## [1] 0.4615385 1.5384615 0.0000000 1.1250000 0.0000000 1.8750000
```

```r
s$objval
```

```
## [1] 6.586538
```

That is that we get about 0.462, 1.538 and 0 acres from forest 1, 1.125,0 and 1.875 acres from forest 2 in year 1,2 and 3 respectively. In total, we would be able to get about 6.587 tons of wood.