

HW1

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Problem 1

By the definition of the product of two matrices, we know that if $C=AB$ then:

$$C_{ij} = \sum_{k=1}^P A_{ik} B_{kj}$$

$$C_{ij}^T = C_{ji} = \sum_{k=1}^P A_{jk} B_{ki} = \sum_{k=1}^P B_{ik}^T A_{kj}^T$$

(Since A_{jk} and B_{ki} here are two numbers so we can change the order) That is the proof of

$$(AB)^T = B^T A^T$$

Then since by definition of the transpose of product, we have.

$$I = I^T = (A^{-1}A)^T = A^T(A^{-1})^T$$

So by definition we have:

$$(A^T)^{-1} = (A^{-1})^T$$

Problem 2

According to the question, we can convert all the conditions into the following problem:

$$Ax = b$$

so we have

$$x = A^{-1}b$$

```
A=matrix(c(1,1,0,0.14,1,0,1,0.2,1,0,0,0.2,1,0,0,0.1),4,4)
b=c(250,(250*0.25*0.55)/(1-0.55),250*0.25,250*0.15)
solve(A)%*%b
```

```
##           [,1]
## [1,] 76.38889
## [2,] 62.50000
## [3,] 31.94444
## [4,] 79.16667
```

This is the amount of money we invested in these four assets respectively.

Problem 3

```
A=matrix(c(2,3,2,4,2,3,3,3,2,7,4,5),3,4)
A
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    2    4    3    7
## [2,]    3    2    3    4
## [3,]    2    3    2    5
```

```
b=c(100000,50000,60000)
b
```

```
## [1] 1e+05 5e+04 6e+04
```

So the condition would be:

$$Ax \leq b$$

With A and b as above. x is the vector that marks Variant 1 to 4 respectively.

Problem 4

We can convert the question into the math expressions below:

```
A=matrix(c(1,1,1,1,0,0,0,0,0,0,1,
           -1,0,0,0,1,1,1,0,0,0,1,
            0,-1,0,0,-1,0,0,1,1,0,1,
            0,0,-1,0,0,-1,0,-1,0,1,1,
            0,0,0,-1,0,0,-1,0,-1,-1,1),11,5)
A
```

```
##      [,1] [,2] [,3] [,4] [,5]
## [1,]    1   -1    0    0    0
## [2,]    1    0   -1    0    0
## [3,]    1    0    0   -1    0
## [4,]    1    0    0    0   -1
## [5,]    0    1   -1    0    0
## [6,]    0    1    0   -1    0
## [7,]    0    1    0    0   -1
## [8,]    0    0    1   -1    0
## [9,]    0    0    1    0   -1
## [10,]   0    0    0    1   -1
## [11,]    1    1    1    1    1
```

```
b=c(7-52,21-24,7-38,0-45,34-16,25-17,27-7,7-5,3-30,14-52,0)
b
```

```
## [1] -45 -3 -31 -45 18 8 20 2 -27 -38 0
```

Now, we just need to solve the equation:

$$A^T A r = A^T b$$

Which is:

$$r = (A^T A)^{-1} A^T b$$

```
r=solve(t(A)**A)**t(A)**b
r
```

```
##      [,1]
## [1,] -24.8
## [2,]  18.2
## [3,]  -8.0
## [4,]  -3.4
## [5,]  18.0
```

This is the scores we got for each team respectively.