HW3

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Problem 1

1)

See following photo for solution.

Choose x_1, x_2.

obj:

1 (1) maximize $Z = -x_1 + 4x_2$.

s.t. $-10x_1 + 20x_2 \le 22$

$5x_1 + 10x_2 \le 49$

≤ 5

$x_i \ge 0$

x_i are all integers

$(3.8, 3.0) \to 8.2$

+ $x_1 \le 3$

$(3.0, 2.6) \to 7.4$

+ $x_1 \ge 4$

$(4.0, 2.9) \to 7.6$

+ $x_2 \le 2$

$(1.8, 2) \to 6.2$

+ $x_2 \ge 3$

infeasible

+ $x_2 \le 2$

$(4, 2) \to 4$

+ $x_2 \ge 3$

infeasible

+ $x_1 \le 1$

$(1, 1.6) \to 5.4$

+ $x_1 \ge 2$

$(2, 2) \to 6$

So the solution would be

$x_1 = x_2 = 2$ objective $= 6$.

2)

```
library(lpSolve)
c = c(-1, 4)
A = matrix(c(-10, 5, 1, 20, 10, 0), 3, 2)
dir = rep("<=", 3)
b = c(22, 49, 5)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 3.8 3.0
```

```
s$objval
```

```
## [1] 8.2
```

Bound on $x_1 \leq 3$

```
A = matrix(c(-10, 5, 1, 1, 20, 10, 0, 0), 4, 2)
dir = rep("<=", 4)
b = c(22, 49, 5, 3)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 3.0 2.6
```

```
s$objval
```

```
## [1] 7.4
```

Bound on $x_1 \leq 3$ & $x_2 \leq 2$

```
A = matrix(c(-10, 5, 1, 1, 0, 20, 10, 0, 0, 1), 5, 2)
dir = rep("<=", 5)
b = c(22, 49, 5, 3, 2)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 1.8 2.0
```

```
s$objval
```

```
## [1] 6.2
```

Bound on $x_1 \leq 3$ & $x_2 \leq 2$ & $x_1 \leq 1$

```
A = matrix(c(-10, 5, 1, 1, 0, 1, 20, 10, 0, 0, 1, 0), 6, 2)
dir = rep("<=", 6)
b = c(22, 49, 5, 3, 2, 1)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 1.0 1.6
```

```
s$objval
```

```
## [1] 5.4
```

Bound on$ x_1 3$ & $x_2 \leq 2$ & $x_1 \geq 2$

```
A = matrix(c(-10, 5, 1, 1, 0, 1, 20, 10, 0, 0, 1, 0), 6, 2)
dir = c(rep("<=", 5), ">=")
b = c(22, 49, 5, 3, 2, 2)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 2 2
```

```
s$objval
```

```
## [1] 6
```

Bound on$ x_1 3$ & $x_2 \geq 3$

```
A = matrix(c(-10, 5, 1, 1, 0, 20, 10, 0, 0, 1), 5, 2)
dir = c(rep("<=", 4), ">=")
b = c(22, 49, 5, 3, 3)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 2
```

It is infeasible.

Bound on $x_1 \geq 4$

```
A = matrix(c(-10, 5, 1, 1, 20, 10, 0, 0), 4, 2)
dir = c(rep("<=", 3), ">=")
b = c(22, 49, 5, 4)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 4.0 2.9
```

```
s$objval
```

```
## [1] 7.6
```

Bound on $x_1 \geq 4$ & $x_2 \leq 2$

```
A = matrix(c(-10, 5, 1, 1, 0, 20, 10, 0, 0, 1), 5, 2)
dir = c(rep("<=", 3), ">=", "<=")
b = c(22, 49, 5, 4, 2)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 4 2
```

```
s$objval
```

```
## [1] 4
```

Bound on $x_1 \geq 4$ & $x_2 \geq 3$

```
A = matrix(c(-10, 5, 1, 1, 0, 20, 10, 0, 0, 1), 5, 2)
dir = c(rep("<=", 3), ">=", ">=")
b = c(22, 49, 5, 4, 3)
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 2
```

This is infeasible.

Now let’s check with the integet programming method:

```
c = c(-1, 4)
A = matrix(c(-10, 5, 1, 20, 10, 0), 3, 2)
dir = rep("<=", 3)
b = c(22, 49, 5)
s = lp("max", c, A, dir, b, all.int=TRUE)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 2 2
```

```
s$objval
```

```
## [1] 6
```

The optimal solution would be $x_1 = x_2 = 2$, and the objective value is 6.

There are in total 6 feasible solutions as above.

3)

There are 6 feasible solution and 8 branches, the difference is 2.

Problem 2

Choose x_1 to x_4 which stands for how many factory to build th factory from the first one to the forth one, respectively.

To maximize the total profit of $9x_1 + 5x_2 + 6x_3 + 4x_4$

Subject to the constrains that total cost:$6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 11$

At most one of the warehouse in Austin and the warehouse in Dallas can be built: $x_3 + x_4 \leq 1$

And at least one of the factory in Austin and the factory in Dallas should be built: $x_1 + x_2 \geq 1$

and all $x_i \geq 0$ and $x_i \leq 1$ and are all integers.

```
c <- c(9, 5, 6, 4)
A <- matrix(c(6, 1, 0, 3, 1, 0, 5, 0,1, 2, 0, 1), 3, 4)
A = rbind(A, diag(4))
dir = c("<=", ">=", rep("<=", 5))
b = c(11, rep(1, 6))
s = lp("max", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 1 1 0 1
```

```
s$objval
```

```
## [1] 18
```

So, the optimal investment strategy is a factory in Austin, a factory in Dallas, and a warehouse in Dallas. Total profit is 18.

Problem 3

1)

We need to choose x_1 to x_{12} which stand for whether to build up an airport for a given city respectively as ordered.

To minimize the total number of airports which is the sum of x_1 to x_{12}.

Subject to that all x_is are binary. And see picture for constraints:

(Just fill all the Xs with one and sumed up greater or equal to 1)

1 ATL	2 BOS	3 CHI	4 DEN	5 HOU	6 LAX	7 NO	8 NY	9 PIT	10 SLC	11 SF	12 SEA	
X_1+		X_3+		X_5+		X_7+	X_8+	X_9				≥ 1
	X_2+						X_8+	X_9				≥ 1
X_1+		X_3+				X_7+	X_8+	X_9				≥ 1
			X_4+						X_{10}			≥ 1
X_1+				X_5+		X_7+						≥ 1
					X_6+				$X_{10}+$	X_{11}		≥ 1
X_1+		X_3+		X_5+		X_7						≥ 1
X_1+	X_2+	X_3+					X_8+	X_9				≥ 1
X_1+	X_2+	X_3+					X_8+	X_9				≥ 1
			X_4+		X_6+				$X_{10}+$	$X_{11}+$	X_{12}	≥ 1
					X_6+				$X_{10}+$	$X_{11}+$	X_{12}	≥ 1
									$X_{10}+$	$X_{11}+$	X_{12}	≥ 1

2)

```
c = rep(1, 12)
atl = c(1,0,1,0,1,0,1,1,1,0,0,0)
bos = c(0,1,0,0,0,0,0,1,1,0,0,0)
chi = c(1,0,1,0,0,0,1,1,1,0,0,0)
den = c(0,0,0,1,0,0,0,0,0,1,0,0)
hou = c(1,0,0,0,1,0,1,0,0,0,0,0)
lax = c(0,0,0,0,0,1,0,0,0,1,1,0)
no = c(1,0,1,0,1,0,1,0,0,0,0,0)
ny = c(1,1,1,0,0,0,0,1,1,0,0,0)
pit = c(1,1,1,0,0,0,0,1,1,0,0,0)
slc = c(0,0,0,1,0,1,0,0,0,1,1,1)
sf = c(0,0,0,0,0,1,0,0,0,1,1,1)
sea = c(0,0,0,0,0,0,0,0,0,1,1,1)
A = matrix(c(atl, bos, chi, den, hou, lax, no, ny, pit, slc, sf, sea), 12, 12)
dir = rep(">=", 12)
b = rep(1, 12)
s = lp("min", c, A, dir, b)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
##  [1] 1 0 0 0 0 0 0 1 0 1 0 0
```

```
s$objval
```

```
## [1] 3
```

So, the optimal choice is ATL, NY and SLC, we build 3 airports in total.

Problem 4

1)

First by breaking down the problem, there are in total 17 ways of cutting the paper.

Number	25	37	54	Waste
1	1	0	0	95
2	1	1	0	58
3	1	1	1	4
4	1	2	0	21
5	2	0	0	70

Number	**25**	**37**	**54**	**Waste**
6	2	0	1	16
7	2	1	0	33
8	3	0	0	45
9	3	1	0	8
10	4	0	0	20
11	0	1	0	83
12	0	1	1	29
13	0	2	0	46
14	0	3	0	9
15	0	0	1	66
16	0	0	2	12
17	1	0	1	41

We need to choose x_i for i=1 to 17 as number of cut we make for each of these 17 ways.

To minimize the total cost which is
$95x_1 + 58x_2 + 4x_3 + 21x_4 + 70x_5 + 16x_6 + 33x_7 + 45x_8 + 8x_9 + 20x_{10} + 83x_{11} + 29x_{12} + 46x_{13} + 9x_{14} + 66x_{15} + 12x_{16} + 41x_{17}$

So that we can exactly meet the customer's need:

$x_1 + x_2 + x_3 + x_4 + 2x_5 + 2x_6 + 2x_7 + 3x_8 + 3x_9 + 4x_{10} + x_{17} = 233$

$x_2 + x_3 + 2x_4 + x_7 + x_9 + x_{11} + x_{12} + 2x_{13} + 3x_{14} = 148$

$x_3 + x_6 + x_{12} + x_{15} + 2x_{16} + x17 = 106$

And all x_is ≥ 0 and are all integers.

2)

```
c = c(95,58,4,21,70,16,33,45,8,20,83,29,46,9,66,12,41)
A1=c(1,1,1,1,2,2,2,3,3,4,0,0,0,0,0,0,1)
A2=c(0,1,1,2,0,0,1,0,1,0,1,1,2,3,0,0,0)
A3=c(0,0,1,0,0,1,0,0,0,0,0,1,0,0,1,2,1)
A = rbind(A1,A2,A3)
dir = rep("=", 3)
b = c(233, 148, 106)
s = lp("min", c, A, dir, b, all.int=TRUE)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
##  [1]   0   0 106   0   0   0   0   0  33   7   0   0   0   3   0   0   0
```

```
s$objval
```

```
## [1] 855
```

The solution would be 106 of (25+37+54) cutting and 33 of (3x25+37) cutting, 7 of (4x25)cutting and 4 of (3x37) cutting. Total waste is 855.

Problem 5

1)

We need to choose x_1 to x_7 which denotes the number of workers who start working from Sunday to Saturday.

To minimize the total cost which is:(Calculate each cost as example given by the question)

$330x_1 + 300x_2 + 330x_3 + 360x_4 + 360x_5 + 360x_6 + 360x_7$

Subject to: $x_1 + x_4 + x_5 + x_6 + x_7 \geq 5$

$x_1 + x_2 + x_5 + x_6 + x_7 \geq 13$

$x_1 + x_2 + x_3 + x_6 + x_7 \geq 12$

$x_1 + x_2 + x_3 + x_4 + x_7 \geq 10$

$x_1 + x_2 + x_3 + x_4 + x_5 \geq 14$

$x_2 + x_3 + x_4 + x_5 + x_6 \geq 8$

$x_3 + x_4 + x_5 + x_6 + x_7 \geq 6$

All x_is are positive intigers.

2)

```
c = c(330, 300, 330, 360, 360, 360, 360)
A = matrix(c(1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1,
1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 1, 1, 1, 1, 0, 0, 1), 7, 7)
dir = rep(">=", 7)
b = c(5, 13, 12, 10, 14, 8, 6)
s = lp("min", c, A, dir, b, all.int=TRUE)
s$status
```

```
## [1] 0
```

```
s$solution
```

```
## [1] 1 8 2 0 3 0 1
```

```
s$objval
```

```
## [1] 4830
```

So the optimal solution would be to have:

1 person will work Sun to Thu

8 people will work Mon to Fri

2 people will work Tue to Sat

3 people will work Thu to Mon

1 person will work Sat to Wed.

And the total cost would be 4830.

3)

From Monday to Friday is the most frequent working pattern.