* t - distribution is used for testing comparison of means
* F - distribution is used for testing comparison of variances.

To test the significance of the difference between population variance

σ_1^2, σ_2^2 - Population variances

s_1^2, s_2^2 - sample variances

$H_0: \sigma_1^2 = \sigma_2^2, \quad H_1: \sigma_1^2 \neq \sigma_2^2$

Let $\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1}, \quad \sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1}, \quad v_1 = n_1 - 1, \; v_2 = n_2 - 1$

(i) If $\sigma_1^2 > \sigma_2^2$ then $F = \frac{\sigma_1^2}{\sigma_2^2}, \; (v_1 = n_1 - 1, \; v_2 = n_2 - 1)$

(ii) If $\sigma_1^2 < \sigma_2^2$ then $F = \frac{\sigma_2^2}{\sigma_1^2}, \; (v_1 = n_2 - 1, \; v_2 = n_1 - 1)$

1) A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variances?

Sol:

Given $n_1 = 13$, $\sigma_1^2 = 3.0$, $n_2 = 15$, $\sigma_2^2 = 2.5$

$H_0: \sigma_1^2 = \sigma_2^2$, $H_1: \sigma_1^2 \neq \sigma_2^2$ (Two tailed test)

Let LOS be 5%, If $\sigma_1^2 > \sigma_2^2$ then

$$F = \frac{\sigma_1^2}{\sigma_2^2} = \frac{3.0}{2.5} = 1.2$$

From F-table, $F_{5\%}(\nu_1 = 12, \nu_2 = 14) = 2.53$

$\therefore F < F_{5\%}$, H_0 is accepted.

$\therefore$ the two samples could have come from population with the same variance.

2) Two random variables gave the following results

$n_1 = 10$, $\Sigma(x_i - \bar{x})^2 = 90$, $n_2 = 12$, $\Sigma(y_i - \bar{y})^2 = 108$

Test whether the samples came from the populations with same variance.

Sol:

Given $n_1 = 10$, $n_2 = 12$, $\Sigma(x_i - \bar{x})^2 = 90$, $\Sigma(y_i - \bar{y})^2 = 108$

$H_0: \sigma_1^2 = \sigma_2^2$, $H_1: \sigma_1^2 \neq \sigma_2^2$

$$\therefore s_1^2 = \frac{1}{n_1}\Sigma(x_i - \bar{x})^2 = \frac{90}{10} = 9, \quad s_2^2 = \frac{1}{n_2}\Sigma(y_i - \bar{y})^2 = \frac{108}{12} = 9$$

$$\text{Now } \sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{10 \times 9}{9} = 10 \quad \& \quad \sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{12 \times 9}{11} = 9.82$$

If $\sigma_1^2 > \sigma_2^2$ then $F = \dfrac{\sigma_1^2}{\sigma_2^2} = \dfrac{10}{9.82} = 1.02$

Los = 5%

From F-table, $F_{5\%}(\nu_1 = 9, \nu_2 = 11) = 2.90$

$\therefore$ $F < F_{5\%}$, Ho is accepted.

$\therefore$ the two samples came from two populations with same variance.

3) From the following data test if the difference between variances is significant at 5% Los.

Sample	A	B
size	8	10
sum of the square of deviation from mean	84.4	102.6

Sol:

Given $n_1 = 8$, $n_2 = 10$, $\Sigma(x_i - \bar{x})^2 = 84.4$, $\Sigma(y_i - \bar{y})^2 = 102.6$

$H_0: \sigma_1^2 = \sigma_2^2$, $H_1: \sigma_1^2 \neq \sigma_2^2$

$$s_1^2 = \frac{\Sigma(x_i - \bar{x})^2}{n_1} = \frac{84.4}{8} = 10.55, \quad s_2^2 = \frac{\Sigma(y_i - \bar{y})^2}{n_2} = \frac{102.6}{10} = 10.26$$

Now $\sigma_1^2 = \dfrac{n_1 s_1^2}{n_1 - 1} = \dfrac{8 \times 10.55}{7} = 12.06$ and

$$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{10 \times 10.26}{9} = 11.4$$

$$\therefore F = \frac{\sigma_1^2}{\sigma_2^2} = \frac{12.06}{11.4} = 1.058$$

From F-table, $F_{5\%}(v_1=7, v_2=9) = 3.29$.

$\therefore F < F_{5\%}$, H_0 is accepted.

Hence Variances of the two populations are equal.

4) Two random samples drawn from normal populations are

Sample I	20	16	26	27	23	22	18	24	25	19		
Sample II	27	33	42	35	32	34	38	28	41	43	30	37

Sol: obtain estimates of the variances of the populations and test whether the two populations have the same variance.

Sol: Sample I

x_1	x_1^2
20	400
16	256
26	676
27	729
23	529
22	484
18	324
24	576
25	625
19	361
220	4960

$\bar{x}_1 = \frac{\Sigma x_1}{n_1} = \frac{220}{10}$

$= 22$

$s_1^2 = \frac{1}{n_1}\Sigma x_1^2 - \bar{x}_1^2$

$= \frac{4960}{10} - (22)^2$

$= 12$

Sample II

x_2	x_2^2
27	729
33	1089
42	1764
35	1225
32	1024
34	1156
38	1444
28	784
41	1681
43	1849
30	900
37	1369
420	15014

$\bar{x}_2 = \frac{\Sigma x_2}{n_2}$

$= \frac{420}{12} = 35$

$s_2^2 = \frac{1}{n_2}\Sigma x_2^2 - \bar{x}_2^2$

$= \frac{15014}{12} - (35)^2$

$= 26.17$

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Now $\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{10 \times 12}{9} = 13.33$ and

$$\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{12 \times 26.17}{11} = 28.55$$

If $\sigma_1^2 < \sigma_2^2$ then $F = \frac{\sigma_2^2}{\sigma_1^2} = \frac{28.55}{13.33} = 2.14$

From F-table, $F_{5\%}(v_1 = 11, v_2 = 9) = 3.28$

$\therefore F < F_{5\%}$, H_0 is accepted.

Hence the two populations have the same variance.

5. Two random samples gave the following data

	Size	mean	Variance
Sample I	8	9.6	1.2
Sample II	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population?

Sol:

Given $n_1 = 8$, $\bar{x}_1 = 9.6$, $s_1^2 = 1.2$, $n_2 = 11$, $\bar{x}_2 = 16.5$, $s_2^2 = 2.5$

$H_0: \bar{x}_1 = \bar{x}_2$, $H_1: \bar{x}_1 \neq \bar{x}_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{-6.9}{0.6864} = 10.05$$

$\nu = n_1 + n_2 - 2 = 8 + 11 - 2 = 17$

From t-table, $t_{5\%}(\nu = 17) = 2.11$

$\therefore |t| > t_{5\%}$, H_0 is rejected.

Hence the means of two samples differ significantly

$H_0 : \sigma_1^2 = \sigma_2^2$, $H_1 : \sigma_1^2 \neq \sigma_2^2$

Now $\sigma_1^2 = \dfrac{n_1 s_1^2}{n_1 - 1} = \dfrac{8 \times 1.2}{7} = 1.37$ and

$\sigma_2^2 = \dfrac{n_2 s_2^2}{n_2 - 1} = \dfrac{11 \times 2.5}{10} = 2.75$

If $\sigma_1^2 < \sigma_2^2$, then $F = \dfrac{\sigma_2^2}{\sigma_1^2} = \dfrac{2.75}{1.37} = 2.007$

From F-table, $F_{5\%}(\nu_1 = n_2 - 1 = 10, \nu_2 = n_1 - 1 = 7) = 3.64$

$\therefore F < F_{5\%}$, H_0 is accepted.

ie, the variances of the populations from which samples are drawn may be regarded as equal.

$\therefore$ The two samples could not have been drawn from the same normal population.

6 The nicotine contents in two random samples of tobacco are given below

Sample I	21	24	25	26	27	
Sample II	22	27	28	30	31	36

Can you say that the two samples came from the same population?

Sol:

Sample I

x_1	x_1^2
21	441
24	576
25	625
26	676
27	729
123	3047

$\bar{x}_1 = \frac{\Sigma x_1}{n} = \frac{123}{5} = 24.6$

$s_1^2 = \frac{\Sigma x_1^2}{n_1} - \bar{x}_1^2 = \frac{3047}{5} - (24.6)^2 = 4.24$

Sample II

x_2	x_2^2
22	484
27	729
28	784
30	900
31	961
36	1296
174	5154

$\bar{x}_2 = \frac{\Sigma x_2}{n_2} = \frac{174}{6} = 29$

$s_2^2 = \frac{\Sigma x_2^2}{n_2} - \bar{x}_2^2 = \frac{5154}{6} - (29)^2 = 18$

$H_0 : \bar{x}_1 = \bar{x}_2 \;,\; H_1 : \bar{x}_1 \neq \bar{x}_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = -1.92, \quad \nu = n_1 + n_2 - 2 = 5 + 6 - 2 = 9$$

From t-table, $t_{5\%} (\nu = 9) = 2.26$

$\therefore |t| < t_{5\%}$, H_0 is accepted.

$\therefore$ the means of two samples donot differ significantly.

$H_0 : \sigma_1^2 = \sigma_2^2 \;,\; H_1 : \sigma_1^2 \neq \sigma_2^2$

Now $\sigma_1^2 = \frac{n_1 s_1^2}{n_1 - 1} = \frac{5 \times 4.24}{4} = 5.3$, $\sigma_2^2 = \frac{n_2 s_2^2}{n_2 - 1} = \frac{6 \times 18}{5} = 21.6$

If $\sigma_1^2 < \sigma_2^2$ then $F = \frac{\sigma_2^2}{\sigma_1^2}$ $(\nu_1 = n_2 - 1, \nu_2 = n_1 - 1)$

$\therefore F = \frac{21.6}{5.3} = 4.07$

From F-table, $F_{5\%}(\nu_1 = 5, \nu_2 = 4) = 6.24$.

Since $F < F_{5\%}$, H_0 is accepted.

$\therefore$ The variance of the two populations can be regarded as equal.

Hence the two samples could have been drawn from the same normal population.