677 hw Yifeng Luo

1. Bernoulli distri $f(x, p) = p^x (1-p)^{1-x}$

$$L(x,p) = f(x_1) f(x_2) \cdots f(x_{70}) = p^{\sum x_i} (1-p)^{n - \sum x_i}$$

$$l(x,p) = \sum x_i \log p + (70 - \sum x_i) \log(1-p)$$

$$\frac{\partial l}{\partial p} = \frac{\sum x_i}{p} - \frac{70 - \sum x_i}{1-p} = 0$$

$$\sum x_i - p \sum x_i - 70p + p \sum x_i = 0$$

$$\hat{p} = \frac{\sum x_i}{70}$$

$$\sum x_i = 12$$

$$\therefore \hat{p} = \frac{12}{70} = \frac{6}{35}$$

2. From the above problem, we have

$$L(x, \theta) = \theta^{\sum x_i} (1-\theta)^{n - \sum x_i}$$

when x are all 0

$$L(x,\theta) = (1-\theta)^n \qquad l(x,\theta) = n \log(1-\theta)$$

$$\frac{\partial l}{\partial \theta} = \frac{n}{1-\theta} = 0 \qquad \text{Can't solve for } \hat{\theta}$$

When x are all 1

$$L(x,\theta) = \theta^n \qquad l(x,\theta) = n \log(\theta)$$

$$\frac{\partial l}{\partial \theta} = \frac{n}{\theta} = 0 \qquad \text{Can't solve for } \hat{\theta}$$

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3. $x_1 — x_n \sim Po(\lambda)$ $f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$

$L(x,\lambda) = f(x_1) f(x_2) \cdots f(x_n)$

$= \frac{\lambda^{x_1} e^{-\lambda}}{x_1!} \cdot \frac{\lambda^{x_2} e^{-\lambda}}{x_2!} \cdots \frac{\lambda^{x_n} e^{-\lambda}}{x_n!}$

$= \frac{\lambda^{\sum x_i} e^{-\lambda n}}{\prod_{i=1}^{n} x_i!}$

$l(x,\lambda) = \sum x_i \log\lambda + (-n\lambda) - \sum \log x_i!$

$= \sum x_i \log\lambda - n\lambda - \sum \log x_i!$

$\frac{\partial l(x,\lambda)}{\partial\lambda} = \frac{\sum x_i}{\lambda} - n = 0 \Rightarrow \hat{\lambda} = \frac{\sum x_i}{n}$

when all observations are 0,

$L(x,\lambda) = \frac{e^{-\lambda n}}{n}$

$l(x,\lambda) = -\lambda n - \log n$

$\frac{\partial l(x,\lambda)}{\partial\lambda} = -n = 0$, no value for λ

4. $x_1 — x_n \sim N(\mu,\sigma^2)$ $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\{-\frac{1}{2\sigma^2}(x-\mu)^2\}$

$L(x,\mu,\sigma^2) = \prod \frac{1}{\sigma\sqrt{2\pi}} \exp\{-\frac{1}{2\sigma^2}(x_i-\mu)^2\}$

$= \frac{(2\pi)^{-\frac{n}{2}}}{\sigma^n} \exp\{-\frac{\sum(x_i-\mu)^2}{2\sigma^2}\}$

$l(x,\mu,\sigma^2) = -\frac{n}{2}\log(2\pi) - n\log\sigma + (-\frac{\sum(x_i-\mu)^2}{2\sigma^2})$

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$$\frac{\partial l(x,\mu,\sigma^2)}{\partial \sigma} = -\frac{n}{\sigma} + \frac{\sum(x_i-\mu)^2}{\sigma^3} = 0$$

$$\Rightarrow \hat{\sigma} = \sqrt{\frac{\sum(x_i-\mu)^2}{n}}$$

$$\hat{\sigma}^2 = \frac{\sum(x_i-\mu)^2}{n}.$$