HW7.

1. Snodgrass Problem:

① $\bar{x} = (0.225 + 0.262 + 0.217 + 0.24 + 0.23 + 0.229 + 0.235 + 0.217)/8$

$= 0.2319$

$\bar{y} = (0.209 + 0.205 + 0.196 + 0.21 + 0.202 + 0.207 + 0.224 + 0.223 + 0.22 + 0.201)/10 = 0.2097.$

$\sigma_x^2 = 2.7658 \times 10^{-5}$ $\sigma_y^2 = 0.7569 \times 10^{-5}$ $n=8$ $m=\cancel{9}\,10$

$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$

Let $\xi_i = x_i - \sqrt{\frac{n}{m}}\, y_i + \frac{1}{\sqrt{n \cdot m}} \sum y_i - \bar{y}$ $\bar{\xi} = \bar{x} - \bar{y}$

$S_\xi^2 = \frac{1}{n} \sum (\xi_i - \bar{\xi})^2 = 0.9461 \times 10^{-5}$

$U = \left| \frac{\bar{x} - \bar{y}}{S_\xi / \sqrt{n-1}} \right| = 19.074$ $\alpha = 5\%$ $T_7^{-1}(97.5\%) = 2.36$

$U = 19.074 > T_7^{-1}(97.5\%) \Rightarrow$ Hence, we reject H_0

② $T = |\bar{x} - \bar{y}| = 0.2319 - 0.2097, = 0.0222$

By creating a 1000 ~~th~~ times simulation, I got P-value of $\frac{6}{1000} = 0.006 < 0.05$, so we should reject the null hypothesis that these two essays were writen by the same person.

2. Hot Dog Problem

$X_1 - X_n \sim N(\mu, \sigma^2)$ $\bar{x} = \frac{\sum x_i}{n} = 156.66$ $S_x^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = 512.60$

$\frac{\sum x}{\sqrt{n}} = 5.06$ $\alpha = 10\%$ $T_{19}^{-1}(95\%) = 1.729.$

$CI: (\bar{x} - 1.729 \times 5.06, \bar{x} + 1.729 \times 5.06) = [148.1, 165.6]$

3) Reading score Prob.

$X_1 — x_n \sim N(\mu_1, \sigma_1^2)$ $y_1 — y_n \sim N(\mu_2, \sigma_2^2)$

$H_0 : \mu_1 \geq \mu_2$ $H_2 : \mu_1 < \mu_2$.

$\bar{\mu}_1 = \frac{\Sigma x_i}{n} = 1.5125$ $S_x^2 = 0.19$ $n = 8$

$\bar{\mu}_2 = 1.6683$ $S_y^2 = 0.17$ $m = 6$

$$U = \frac{(m+n-2)^{\frac{1}{2}}(\bar{x}_1 - \bar{x}_2)}{(\frac{1}{m}+\frac{1}{n})^{\frac{1}{2}}(S_{x_1}^2 + S_{x_2}^2)^{\frac{1}{2}}} = \frac{\sqrt{12}(1.5125 - 1.6683)}{(\frac{1}{6}+\frac{1}{8})^{\frac{1}{2}}(0.19+0.17)^{\frac{1}{2}}} = -1.6929 \sim t_{6+8-2} = t_{12}$$

$\alpha = 5\%$ $T_{12}^{-1}(97.5\%) = 2.179$.

$U = -1.6929 \geq -2.179$ Hence, We accept H_0 that $\mu_1 \geq \mu_2$.