Hw5:

1. $X_1 — X_n \sim B(p)$ $T=\sum_{i=1}^{n} X_i$ $f(x,p)=p^x(1-p)^{1-x}$

$L(p;x)=p^{\sum x_i}(1-p)^{n-\sum x_i} = 1\cdot p^T(1-p)^{n-T}$

$m(x)=1$ $v(t(x),p)=p^{t(x)}(1-p)^{n-t(x)}$

2. $X_1 — X_n \overset{iid}{\sim} Geo(p)$ $T=\sum_{i=1}^{n} X_i$ $f(x,p)=(1-p)^{x-1}\cdot p$

$L(p,x)=p^n(1-p)^{\sum x_i - n} = 1\cdot p^n(1-p)^{\sum x_i - n}$

$m(x)=1$ $V(t(x),p)=p^n(1-p)^{t(x)-n}$

3. $X_1 — X_n \sim NegBinom(r,p)$ $T=\sum_{i=1}^{n} X_i$ $f(x,r,p)=\binom{x+r-1}{x-1}p^x(1-p)^r$

$L(r,p,x)=p^{\sum x_i}(1-p)^{nr}\prod_{i=1}^{n}\binom{x+r-1}{x-1}$

$m(x)=\prod\binom{x+r-1}{x-1}$ $V(t(x),r,p)=p^{t(x)}(1-p)^{nr}$

4. $X_1 — X_n \sim Gamma(\alpha,\beta)$ $T=\sum_{i=1}^{n} X_i$ $f(x,\alpha,\beta)=\frac{\beta^\alpha}{\Gamma(\alpha)}x^{\alpha-1}e^{-\beta x}$

$L(x,\alpha,\beta)=\left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^n\left(\prod x_i\right)^{\alpha-1}e^{-\beta\sum x_i}$

$m(x)=\Gamma(\alpha)^{-n}\left(\prod x_i\right)^{\alpha-1}$

$V(t(x),\beta)=\beta^{n\alpha}e^{-\beta t(x)}$

5. $T=\prod x_i$ $m(x)=e^{-\beta\sum x_i}$ $V(t(x),\alpha)=\left(\frac{\beta^\alpha}{\Gamma(\alpha)}\right)^n(t(x))^{\alpha-1}$