HW2. Yifeng Luo

Y	Frequency	P(Y)
8	2	$\frac{2}{9}$
7	1	$\frac{1}{9}$
6	1	$\frac{1}{9}$
2	1	$\frac{1}{9}$
1	1	$\frac{1}{9}$
4	1	$\frac{1}{9}$
3	2	$\frac{2}{9}$

~~3~~ ~~8~~ ~~8~~ ~~7~~ ~~3~~ 6. 2 1 4.

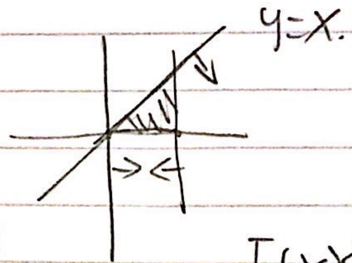
$$E(Y) = \frac{1}{9}(2\times 8+7+6+2+1+4+2\times 3)$$

$$= \frac{1}{9}(16+15+5+6)$$

$$= \frac{1}{9}\times 42$$

$$= \frac{14}{3}$$

2.



$0 \le x \le 1$

$0 \le y \le x$

$$E(XY) = \int_0^1 dx \int_0^x dy \, 12y^2 \cdot xy$$

$$= 12 \int_0^1 dx \int_0^x xy^3 \, dy = 12 \int_0^1 dx \left(\frac{1}{4} x y^4\right)\Big|_0^x$$

$$= 12 \cdot \frac{1}{4} \int_0^1 dx \; x^5 = 3\left(\frac{1}{6} x^6 \Big|_0^1\right) = \frac{1}{2}$$

3. $(x_1-2x_2+x_3)^2 = (x_1-2x_2)^2 + 2(x_1-2x_2)\cdot x_3 + x_3^2$

$= x_1^2 - 4x_1x_2 + 4x_2^2 + 2x_3x_1 - 4x_2x_3 + x_3^2$

$E[(x_1-2x_2+x_3)^2] = E(x_1^2) - 4E(x_1x_2) + 2E(x_3x_1) - 4E(x_2x_3) + E(x_3^2) + 4E(x_2^2)$

$\overset{\text{random} \Rightarrow \text{indep}}{=} E(x_1^2) - 4E(x_1)E(x_2) + 2E(x_3)E(x_1) - 4E(x_2)E(x_3) + 4E(x_2^2) + E(x_3^2)$

$x_i \sim Uni(0,1) \Rightarrow E(x_i) = \frac{0+1}{2} = \frac{1}{2}$

$Var(x_i) = \frac{(1-0)^2}{12} = \frac{1}{12}$

$E(x_i^2) = Var(x_i) + E(x_i)^2$

$= \frac{1}{12} + \frac{1}{4}$

$= \frac{4}{12} = \frac{1}{3}$

$E[(x_1-2x_2+x_3)^2] = \frac{1}{3} - 4\times\frac{1}{2}\times\frac{1}{2} + 2\times\frac{1}{2}\times\frac{1}{2} - 4\times\frac{1}{2}\times\frac{1}{2} + \frac{1}{3}\times 4 + \frac{1}{3}$

$= \frac{1}{3} - 1 + \frac{1}{2} - 1 + \frac{4}{3} + \frac{1}{3}$

$= \frac{1}{2}$

4. $P(X \le x) = \int_0^x e^{-t} dt$

$G(Y) = P(Y \le y) = P(e^{\frac{3x}{4}} \le y) = P(\frac{3}{4}x \le \ln y) = P(x \le \frac{4}{3}\ln y)$

$= \int_0^{\frac{4}{3}\ln y} e^{-t} dt = -e^{-t} \Big|_0^{\frac{4}{3}\ln y} = -(e^{-\frac{4}{3}\ln y} - 1)$

wrong

$= 1 - (e^{\ln y})^{-\frac{4}{3}} = 1 - y^{-\frac{4}{3}}$

$g(y) = G'(Y) = \frac{4}{3} y^{-\frac{7}{3}}, \quad y > 1$

$E(Y) = \int_1^\infty y \cdot \frac{4}{3} y^{-\frac{7}{3}} = \frac{4}{3}\int_1^\infty y^{-\frac{4}{3}} dy = \frac{4}{3}(-3) y^{-\frac{1}{3}} \Big|_1^\infty$

$E(Y) = \int_0^\infty e^{-x} \cdot e^{\frac{3}{4}x} dx = \int_0^\infty e^{-\frac{1}{4}x} dx = -4e^{-\frac{1}{4}x} \Big|_0^\infty = -4(0-1)$

$= 4.$

5. $X = 1, 2, \dots, 6.$

$Y = g(x) = 2x^2 + 1 \qquad P(x) = P(Y) = \frac{1}{6}$

$Y = \begin{cases} 2\times1+1 = 3 \\ 2\times4+1 = 9 \\ 2\times9+1 = 19 \\ 2\times16+1 = 33 \\ 2\times25+1 = 51 \\ 2\times36+1 = 73 \end{cases}$

$E(Y) = \frac{1}{6}(3+9+19+33+51+73) = \frac{188}{6} = \frac{94}{3}$

$E(X^2)=2\int_0^1 x^2-x^3\,dx = 2\left(\frac{1}{3}x^3-\frac{1}{4}x^4\right)\Big|_0^1=2\left(\frac{1}{3}-\frac{1}{4}\right)=\frac{1}{6}$

6. $E(Y^2)=E((2X+1)^2)=E(4X^2+4X+1)=4E(X^2)+4E(X)+1$.

$E(X)=\int_0^1 2(1-x)\cdot x\,dx=2\int_0^1 x-x^2\,dx=2\left(\frac{1}{2}x^2-\frac{1}{3}x^3\right)\Big|_0^1$

$=2\left(\frac{1}{2}-\frac{1}{3}-0\right)=\frac{1}{3}$

7. $E(Y^2)=4\times\frac{1}{6}+4\times\frac{1}{3}+1=3$

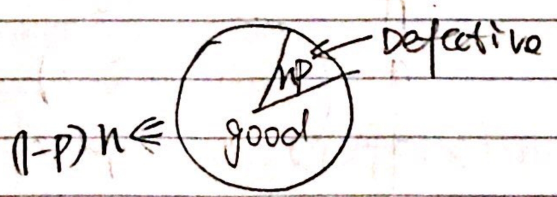
$(ax+b)^n=\sum_{i=0}^{n}\binom{n}{i}(ax)^{n-i}\cdot b^i$

$=\sum_{i=0}^{n}\binom{n}{i}a^{n-i}\cdot b^i\cdot x^{n-i}$

$E[(ax+b)]^n=E\left[\sum_{i=0}^{n}\binom{n}{i}a^{n-i}\cdot b^i\cdot x^{n-i}\right]$

$=\sum_{i=0}^{n}\binom{n}{i}a^{n-i}\cdot b^i\, E(x^{n-i})$

8.



X = # defective $\quad np-(1-p)n=np-n+np=2np-n$.

Y : # good

$n=20$. $p=5\%$. $X=1$ $Y=19$.

$E(X-Y)=E(X)-E(Y)=1-19=-18$.