Hypothesis Testing I

1. $X \sim Exp(\lambda)$ $f(x) = \lambda e^{-\lambda x}$ $F(x) = 1 - e^{-\lambda x}$

$H_0: \lambda \geq 1$ $H_1: \lambda < 1$ Reject if $x \geq 1$

(a) Power function: $\beta(\lambda) = P(X \geq 1 | \lambda) = 1 - P(X < 1)$

$\therefore \beta(\lambda) = 1 - (1 - e^{-\lambda}) = e^{-\lambda}$

(b) $\alpha = \sup \beta(\lambda) = e^{-1} = 0.3678$.

2. It is a binomial distribution, two tail problem

$f(y) = \binom{n}{y} p^y (1-p)^{\cancel{20}\, n - y}$

$\Rightarrow f(y) = \binom{20}{y} p^y (1-p)^{20-y}$

$H_0: p = 0.2$ $H_1: p \neq 0.2$

Rej: $y \geq 7$ or $y \leq 1$

(a) $\beta(p) = P(Y \geq 7 | p) + P(Y \leq 1 | p)$

$= 1 - P(Y \leq 7 | p) + P(Y \leq 1 | p)$

$= 1 - \sum_{y=0}^{6} \binom{20}{y} p^y (1-p)^{20-y} + \sum_{0}^{1} \binom{20}{y} p^y (1-p)^{20-y}$

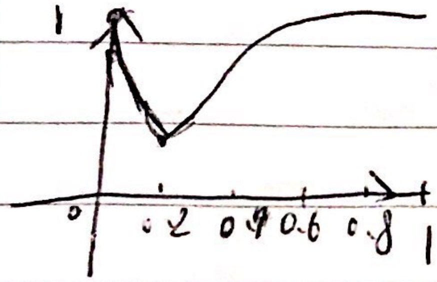
$= \sum_{0,1,7,\dots,20} \binom{20}{y} p^y (1-p)^{20-y}$

$\beta(p=0) = 1$ $\beta(p=0.1) = 0.394$ $\beta(p=0.2) = 0.1558$

$\beta(p=0.3) = 0.399$ $\beta(p=0.4) = 0.7505$ $\beta(p=0.5) = 0.9423$

$\beta(p=0.6) = 0.993$ $\beta(p=0.7) = 0.999$ $\beta(p=0.8) = 0.999$

$\beta(p=0.9) = 1$ $\beta(p=1) = 1$



(b) $\alpha = \sup B(p) = 0.1558$.

3. $f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$

$H_0 : \mu = \mu_0 \quad H_1 : \mu \neq \mu_0$

Rej: $T(x) > c \quad |\bar{x} - \mu_0| > c \quad \alpha = 0.05$.

$B(\mu) = P(T(x) > c) = P(|\bar{x} - \mu_0| > c) = P\left(\frac{\sqrt{n}\,|\bar{x} - \mu_0|}{\sigma} > \frac{\sqrt{n}\,c}{\sigma}\right)$

$= P\left(|z| > \frac{\sqrt{n}\,c}{\sigma}\right) = 1 - \Phi\left(\frac{\sqrt{n}\,c}{\sigma}\right)$

$\alpha = B(\mu) = 0.05 \Rightarrow c = \frac{\Phi^{-1}(0.995)}{\sqrt{n}} = \frac{1.96}{\sqrt{25}} = 0.392$.

4. $f(p, x) = \binom{9}{x} p^x (1-p)^{9-x}$

$H_0 : p = 0.4 \quad H_1 : p \neq 0.4$

(a) $P(Y \leq c_1 | p = 0.4) + P(Y \geq c_2 | p = 0.4)$

$= P(Y \leq c_1 | p = 0.4) + 1 - P(Y \leq c_2 | p = 0.4) < 0.1$

$\therefore 0.9 < P(Y < c_2 | p = 0.4) - P(Y \leq c_1 | p = 0.4)$

when $c_1 \geq 2 \quad P(Y \leq c_1 | p = 0.4) > 0.23$

$c_2 \leq 5 \quad P(Y \geq c_2 | p = 0.4) \geq 0.26$

$\therefore c_1 \leq 1 \quad c_2 \geq 6$

c_1	c_2	P
1	6	0.169
1	7	0.0956
0	6	0.109
-1	6	0.993

$\therefore$ We choose $c_1 = 1 \quad c_2 = 7$

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(b) $B(p) = P(Y \geq 7 \mid p) + P(Y \leq 1 \mid p)$

$$= \sum_{0,1,7,\dots,9} \binom{9}{Y} p^{Y} (1-p)^{9-Y}$$

$$\alpha = \sup_{p=0.4} B(p) = 0.0956.$$