Definition 0.1 (Complex of R-modules [1]1.10)**.** A **complex of R-Modules is a sequence of modules F_i and maps $F_i to F_{i-1}$ such that the compositions $F_{i+1} \to F_i \to F_{i-1}$ are all zero. The homology of this complex at F_i is the module**

$$\ker(F_i \to F_{i-1}) \operatorname{im}(F_{i+1} \to F_i)$$

A free resolution of an R-module M is a complex

$$\mathcal{F} : \ldots \to F_n \vec{\phi_n} \ldots \to F_1 \vec{\phi_1} F_0$$

of free R-Modules such that $\operatorname{coker} \phi_1 = M$ and $\mathcal{F}$ is exact (sometimes we add " $\to 0$ " to the right of $\mathcal{F}$ and then insist that $\mathcal{F}$ be exact except at F_0). We shall sometimes abuse this notation and say that an exact sequence

$$\mathcal{F} : \ldots \to F_n \vec{\phi_n} \ldots \to F_1 \vec{\phi_1} F_0 \to M \to 0$$

is a resolution of M. The image of the map ϕ_i is called the ith syzygy module of M. A resolution $\mathcal{F}$ is a graded free resolution if R is a graded ring, the F_i are graded free modules, and the maps are homogeneous maps of degree 0. Of course only graded modules can have graded free resolutions. If for some $n < \inf$ we have $F_{n+1} = 0$, but $F_i \neq 0 \forall 0 \leq i \leq n$, then we shall say that $\mathcal{F}$ is a finite resolution of length n.

Literatur

[1] David Eisenbud. *Commutative Algebra*, volume 150 of *Graduate Texts in Mathematics*. Springer-Verlag, 1995.