AI: Assignment 3

Symbolize the following proposition and discuss the truth.

1. Everyone has black hair.
2. Some people boarded the moon.
3. No one has boarded Jupiter.
4. Students studying in the US are not necessarily Asians.

Answer:

1. Let P(x) represent x is a person.

   Let Q(y) represent y has black hair.

   That *Everyone has black hair* means $\forall x\ P(x) \Rightarrow Q(x)$.

   Assume b is a girl with blonde hair, then P(b) is True but Q(b) is False.

   As a result, the proposition is **False**.

2. Let P(x) represent x is a person.

   Let Q(y) represent y boarded the moon.

   That *Some people boarded the moon* means $\exists x\ P(x) \wedge Q(x)$.

   Assume b is the man in Project Apollo, then P(b) is True and Q(b) is True.

   As a result, the proposition is **True**.

3. Let P(x) represent x is a person.

   Let Q(y) represent y has boarded Jupiter.

   That *No one has boarded Jupiter* means $\forall x\ P(x) \Rightarrow \neg Q(x)$.

4. Let P(x) represent x is a student studying in the US.

   Let Q(y) represent is Asian.

   That *Students studying in the US are not necessarily Asians* means $\exists x\ P(x) \wedge \neg Q(x)$.

Judge the following formula, which is tautology? What is the contradiction?

1. $\forall xF(x) \Rightarrow (\exists x \exists y G(x,y) \Rightarrow \forall x F(x))$
2. $\neg(\forall xF(x) \Rightarrow \exists yG(y)) \wedge \exists yG(y)$
3. $\forall x(F(x) \Rightarrow G(y))$

Answer:

1. Let P represent $\forall xF(x)$. Let Q represent $\exists x \exists y G(x,y)$.

   Then $\forall xF(x) \Rightarrow (\exists x \exists y G(x,y) \Rightarrow \forall x F(x))$ means $P \Rightarrow (Q \Rightarrow P)$

   $P \Rightarrow (Q \Rightarrow P)$ equals to $P \Rightarrow (P \vee \neg Q)$, equals to $(P \vee \neg Q) \vee \neg P$, equals to $P \vee \neg P \vee \neg Q$, which is always True.

   So the formula is a **tautology**.

2. Let P represent $\forall xF(x)$. Let Q represent $\exists yG(y)$.

   Then $\neg(\forall xF(x) \Rightarrow \exists yG(y)) \wedge \exists yG(y)$ means $\neg(P \Rightarrow Q) \wedge Q$

   $\neg(P \Rightarrow Q) \wedge Q$ equals to $\neg(Q \vee \neg P) \wedge Q$, equals to $(\neg Q \wedge P) \wedge Q$, equals to $\neg Q \wedge Q \wedge P$, which is always False.

   So the formula is a **contradiction**.

3. $\forall x(F(x) \Rightarrow G(x))$.

   Let F(x) be $x>2$, G(x) be $x >1$, then the formula is always True.

   Let F(x) be $x>2$, G(x) be $x<1$, then the formula is alway False.

   In all, it depends.

Which of the following are correct?

1. False $\models$ True.
2. $(A \wedge B) \models (A \Leftrightarrow B)$
3. $(A \wedge B) \Rightarrow C \models (A \Rightarrow C) \vee (B \Rightarrow C)$
4. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$
5. $(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$

Answer:

#1, 2, 3, 4 are correct, #5 is incorrect.

Conjunctive normal form.link:https://baike.baidu.com/item/%E5%90%88%E5%8F%96%E8%8C%83%E5%BC%8F/2459360

1. Obtaining conjunctive paradigm: $P \wedge (Q \Rightarrow R) \Rightarrow S$
2. Cut redundant connectives，Reserved {∨，∧，¬}
3. Move or remove the negation ~
4. distribution rates

Answer:

1. $P \wedge (Q \Rightarrow R) \Rightarrow S$

2. $= P \wedge (\neg Q \vee R) \Rightarrow S = \neg (P \wedge (\neg Q \vee R)) \vee S$

   $= \neg P \vee \neg (\neg Q \vee R) \vee S = \neg P \vee (\neg \neg Q \wedge \neg R) \vee S = \neg P \vee (Q \wedge \neg R) \vee S$

3. $= \neg P \vee S \vee (Q \wedge \neg R) = (\neg P \vee S \vee Q) \wedge (\neg P \vee S \vee \neg R)$

Arithmetic assertions can be written in first-order logic with the predicate symbol <, the function symbols + and ×, and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals.(Chapter 8.20)

1. Represent the property "x is an even number."
2. Represent the property "x is prime."
3. Goldbach's conjecture is the conjecture (unproven as yet) that every even number is equal to the sum of two primes. Represent this conjecture as a logical sentence.

Answer:

1. $\forall x\ Even(x) \Leftrightarrow \exists y\ x = y+y.$
2. $\forall x\ Prime(x) \Leftrightarrow \forall y,z\ (x=y\times z) \Rightarrow (y=1 \vee z=1).$
3. $\forall x\ Even(x) \Rightarrow \exists y,z\ Prime(y) \wedge Prime(z) \wedge (x = y+z).$