

1 Convergence of CBBA:

Theorem 1. *Provided that the scoring function is DMG (Diminishing Marginal Gain), the CBBA process with a conflict resolution phase over a static communication network with diameter D satisfies the following:*

1. *CBBA produces the same solution as SGA(sequential greedy algo) with the corresponding winning bid values and winning agent information being shared across the fleet*
2. *The convergence time T_c is bounded above by $N_{min}D$.*

Here $N_{min} = \min\{N_t, N_u L_t\}$ - means that assignment is said to be completed once N_{min} tasks have been assigned.

$D = \max_{i,k} d_{ik}$, where d_{ik} – shortest path length for any pair of UAVs

N_u Number of agents

N_t - Number of tasks

L_t - Maximum length of the bundle, i.e. each agent can be assigned a maximum L_t tasks

\mathcal{I} - Index set of agents where $\mathcal{I} \doteq \{1, \dots, N_u\}$

1.1 Static Network

The communication network of a fleet of unmanned vehicles can be modeled as an undirected graph with every edge length being unity. Suppose that this communication network is static and connected; then, there exists a (undirected) shortest path length $d_{ik} < \infty$ for every pair of agents i and k . The network diameter D is defined as the longest of all shortest path lengths $D = \max_{i,k} d_{ik}$

If the conflict resolution is assumed to be synchronized, i.e., every agents second phase in the t -th iteration takes place simultaneously, then the actual time τ can be equivalently represented by the iteration count t . In this case, the convergence time $T_c \in Z_+$ can be defined as the smallest iteration number at which a feasible assignment is found that will not change afterwards

$$T_c = \min t \in \mathcal{T}$$

where the set \mathcal{T} is defined as $\mathcal{T} = \{t \in Z_+ \mid \forall s \geq t :$

$$x_{ij}(s) = x_{ij}(t),$$

$$\sum_{i=1}^{N_u} x_{ij}(s) = 1,$$

$$\sum_{j=1}^{N_t} x_{ij}(s) \leq L_t,$$

$$\sum_{j=1}^{N_t} \sum_{i=1}^{N_u} x_{ij}(s) = N_{min}\}$$

with binary variable x_{ij}