Distributed tasking problem for track and search

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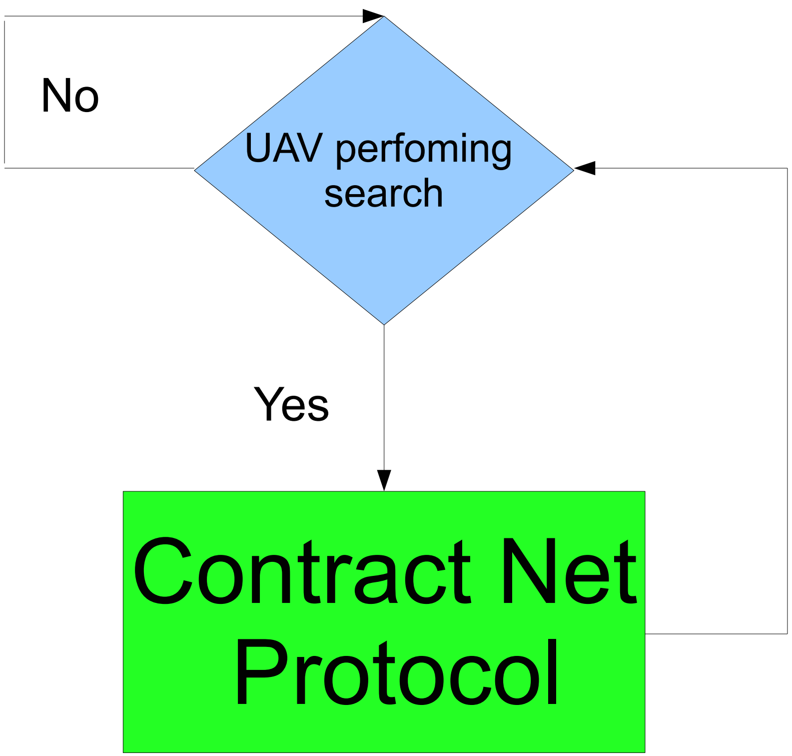
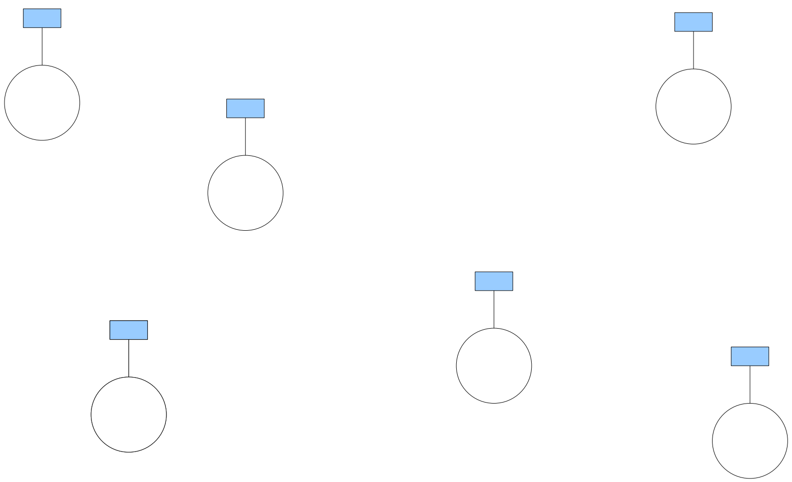
Introduction



- Recognition;
- Announcement ;
- Bidding;
- Awarding;
- Expediting.

Contract Net Stages



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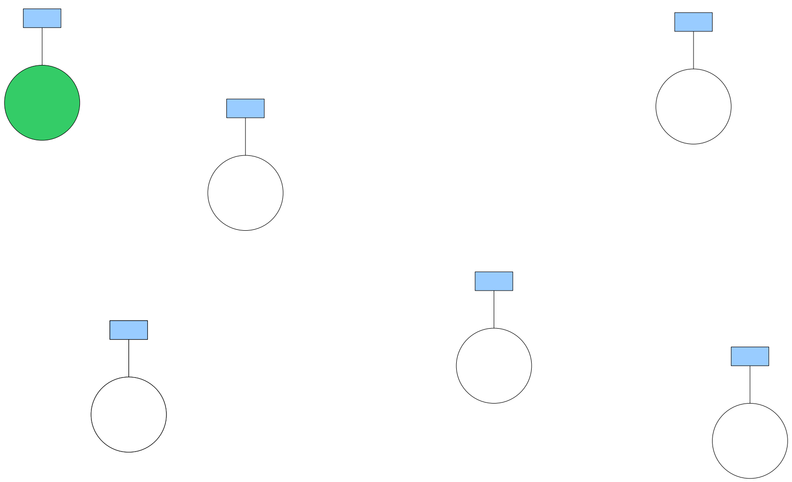


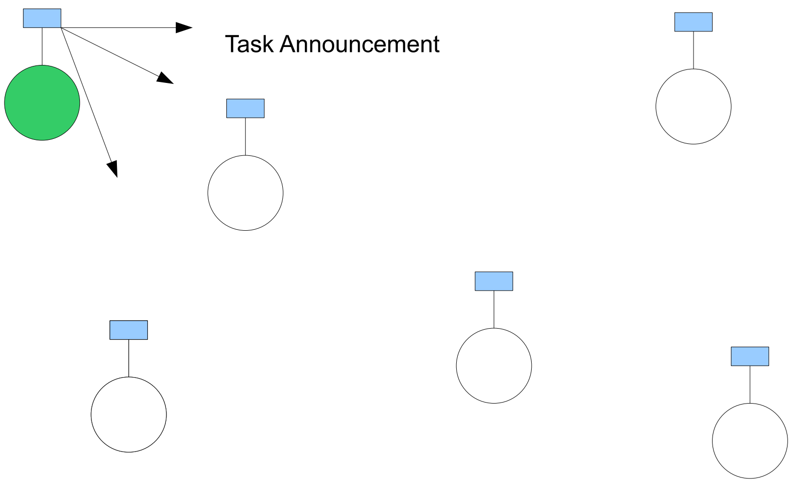
- Recognition;
- Announcement ;
- Bidding;
- Awarding;
- Expediting.

In this stage, an agent recognises it has a problem it wants help with.
Agent has a goal, and either

- realises it cannot achieve the goal in isolation - does not have capability;
- realises it would prefer not to achieve the goal in isolation (typically because of solution quality, deadline, etc)

As a result, it needs to involve other agents.

Contract Net Stages



- Recognition;
- Announcement ;
- Bidding;
- Awarding;
- Expediting.

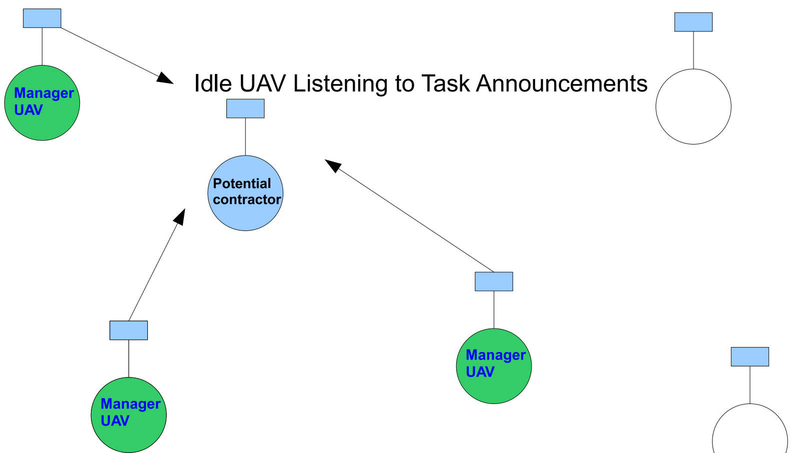
In this stage, the agent with the task sends out an announcement of the task which includes a specification of the task to be achieved.

Specification must encode:

- description of task itself (maybe executable);
- any constraints (e.g., deadlines, quality constraints).
- meta-task information (e.g.,bids must be submitted by...)

The announcement is then broadcast.

Contract Net Stages



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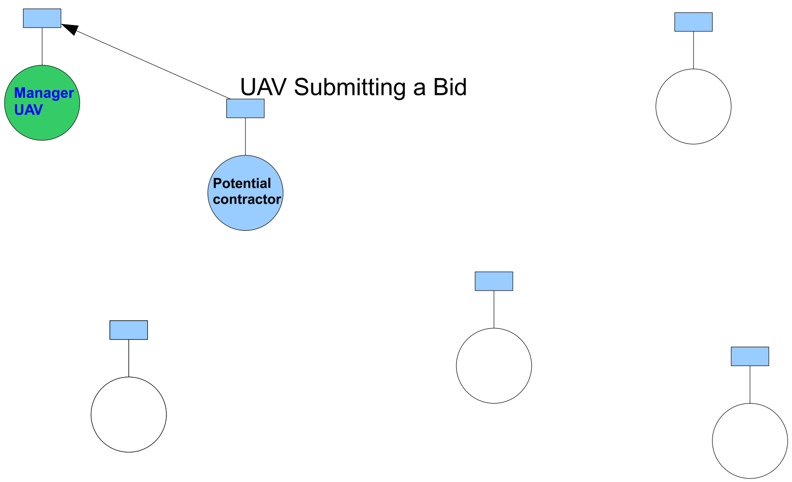


- Recognition;
- Announcement ;
- Bidding;
- Awarding;
- Expediting.

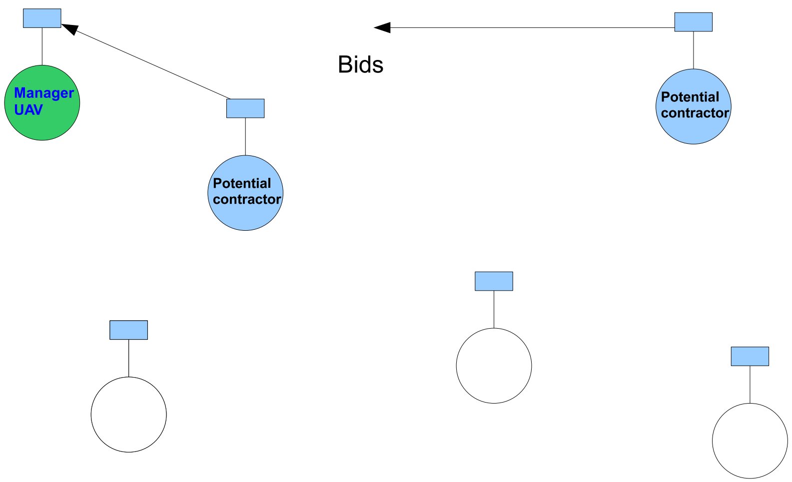
UAVs that receive the announcement decide for themselves whether they wish to bid for the task.

Factors:

- agent must decide whether it is capable of expediting task;
- agent must determine quality constraints and price information (if relevant).

If they do choose to bid, then they submit a tender.

Contract Net Stages



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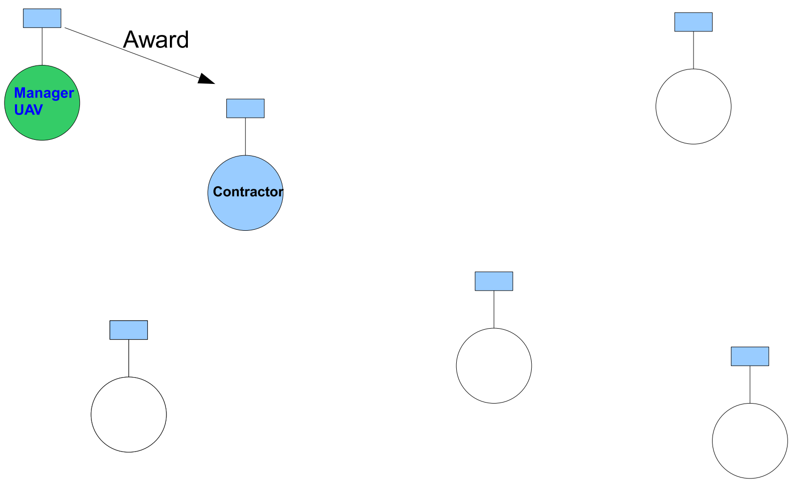

- Recognition;
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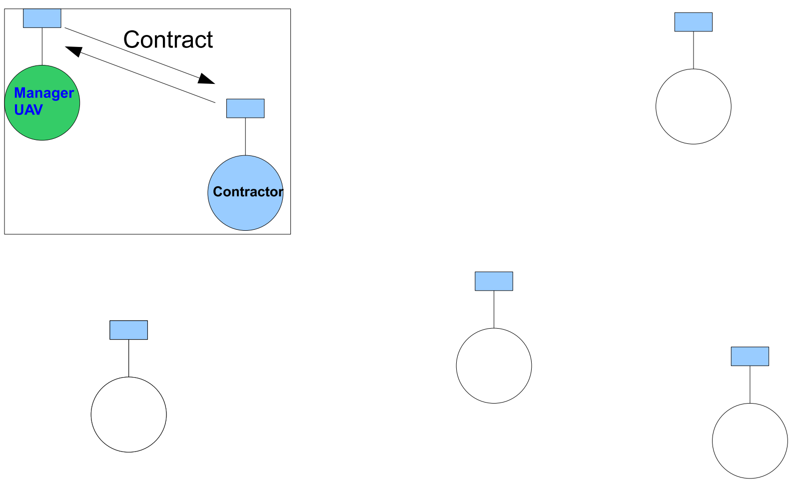
- Expediting.

Agent that sent task announcement must choose between bids and decide who to "award the contract" to.

The result of this process is communicated to agents that submitted a bid.

Contract Net Stages



- Recognition;
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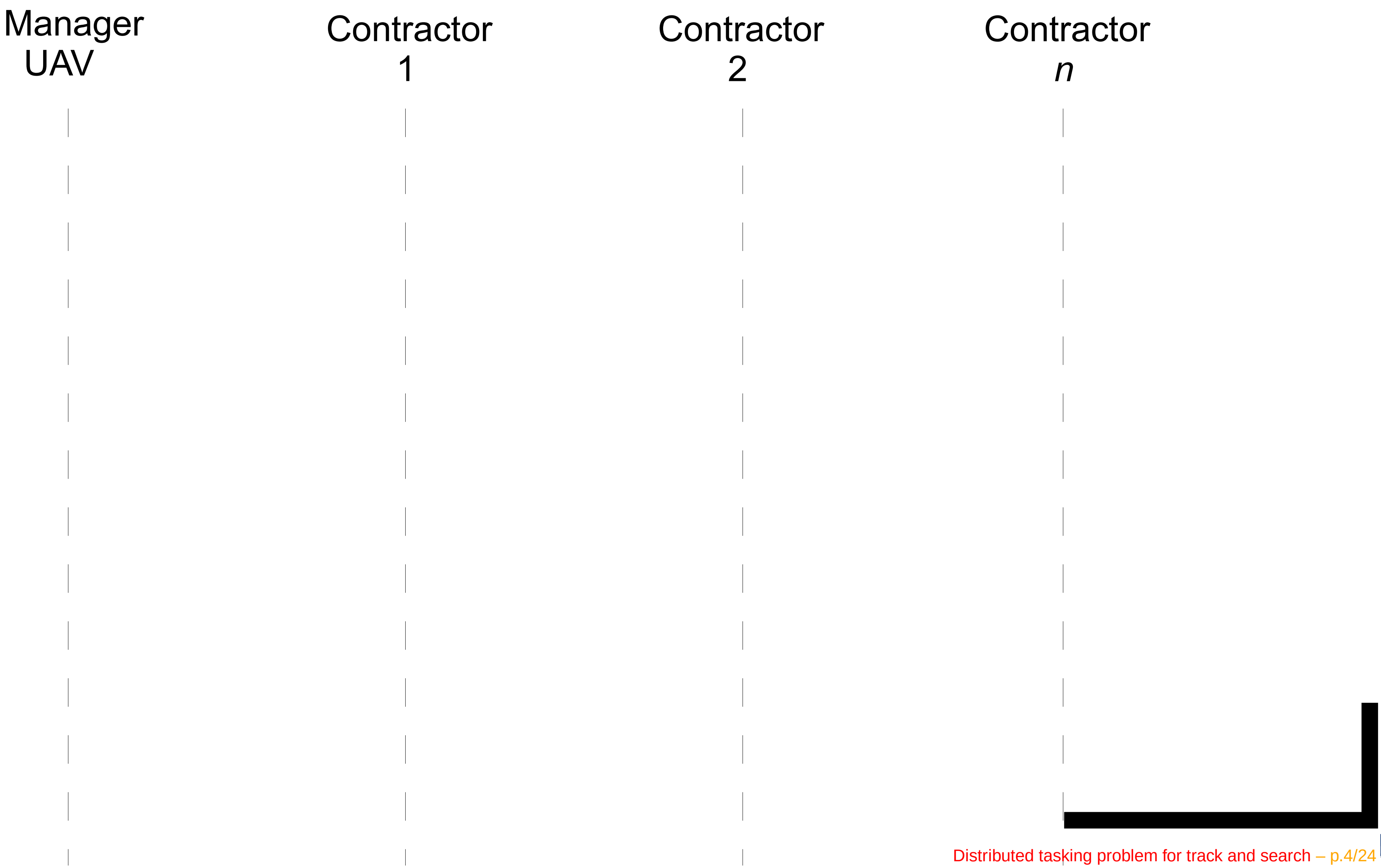
- Awarding;
- Expediting.

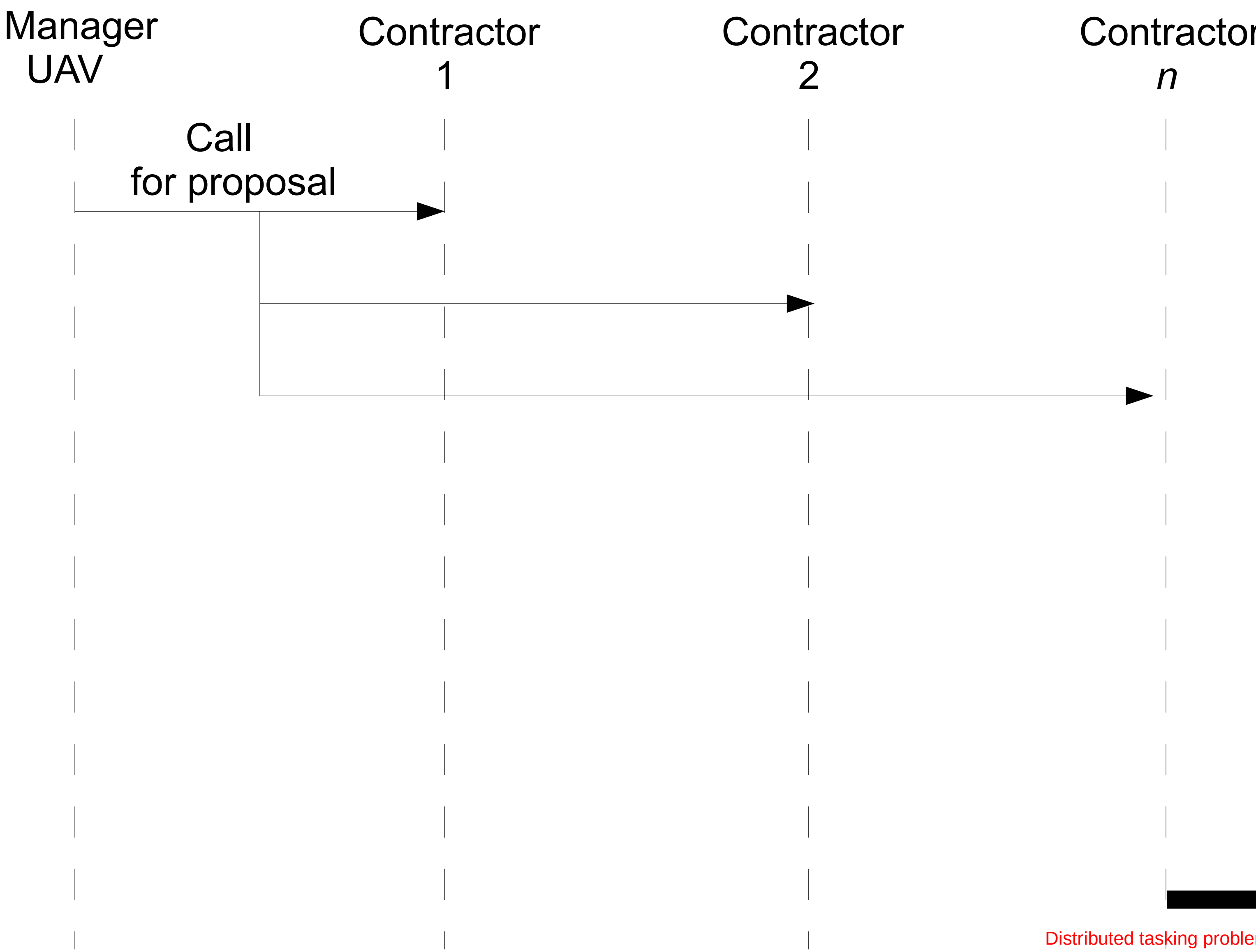
Agent that sent task announcement must choose between bids and decide who to "award the contract" to.
The result of this process is communicated to agents that submitted a bid.
The successful contractor then expedites the task.
May involve generating further manager-contractor relationships: sub-contracting.

- May involve another contract net.

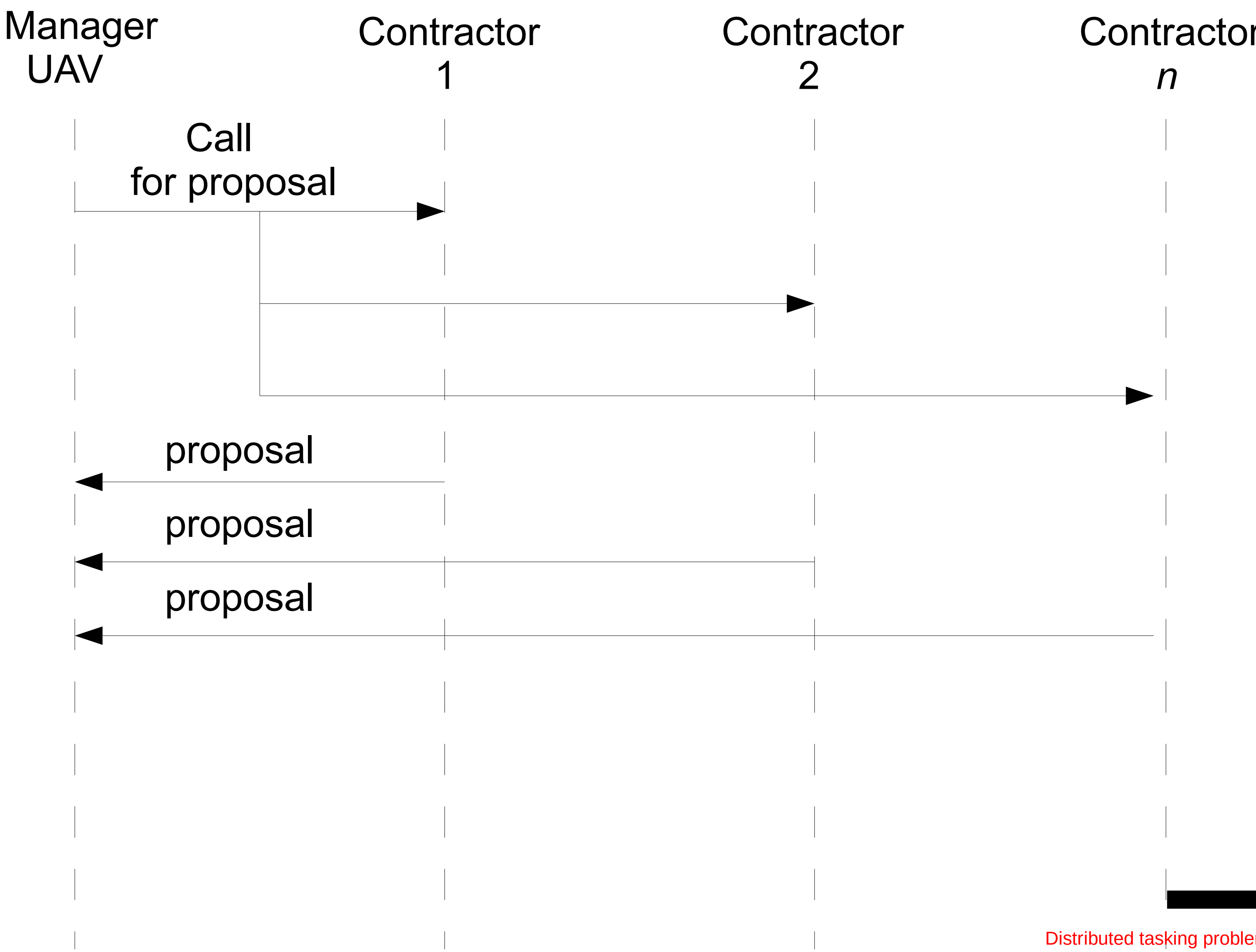
Procedure Diagram



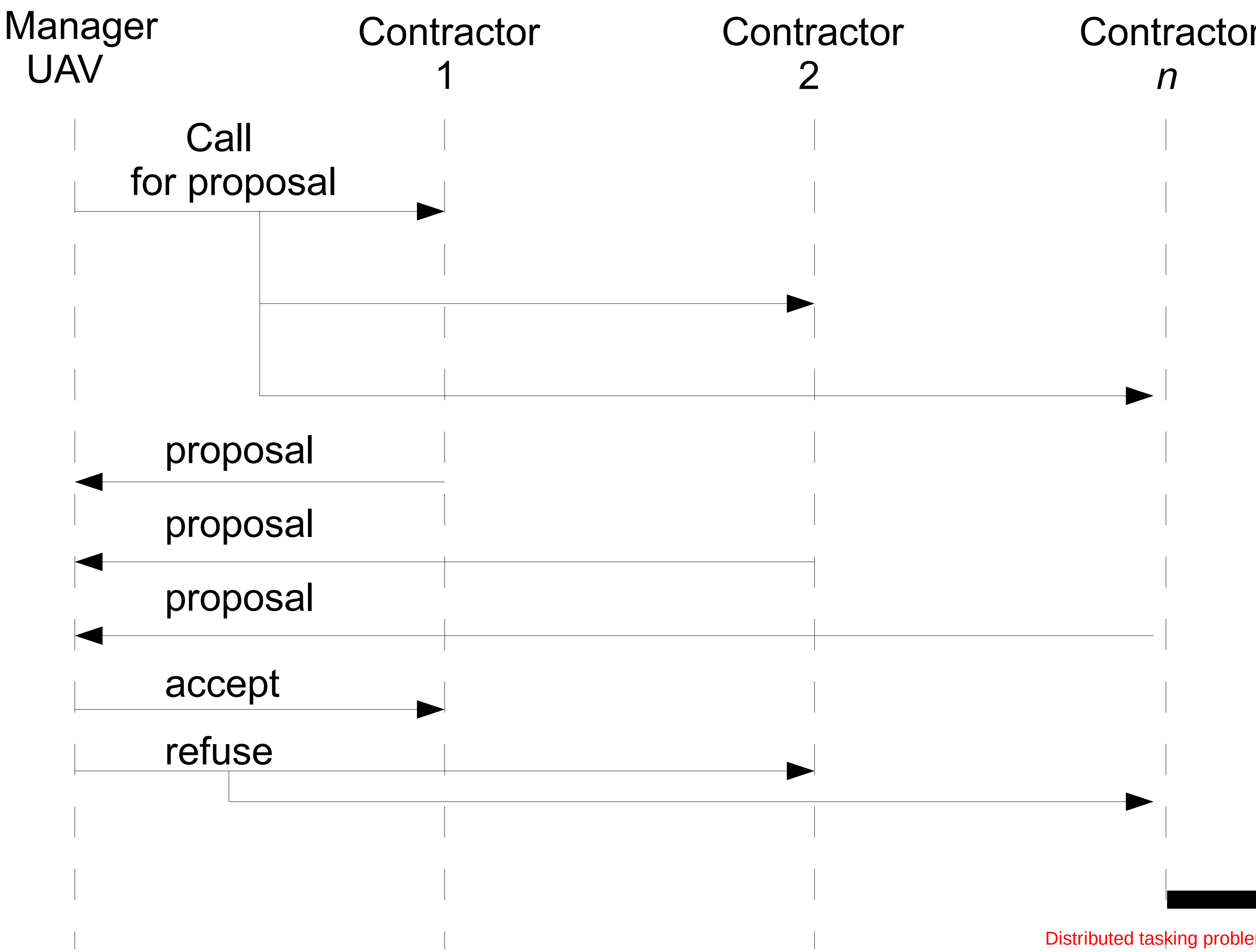
Procedure Diagram



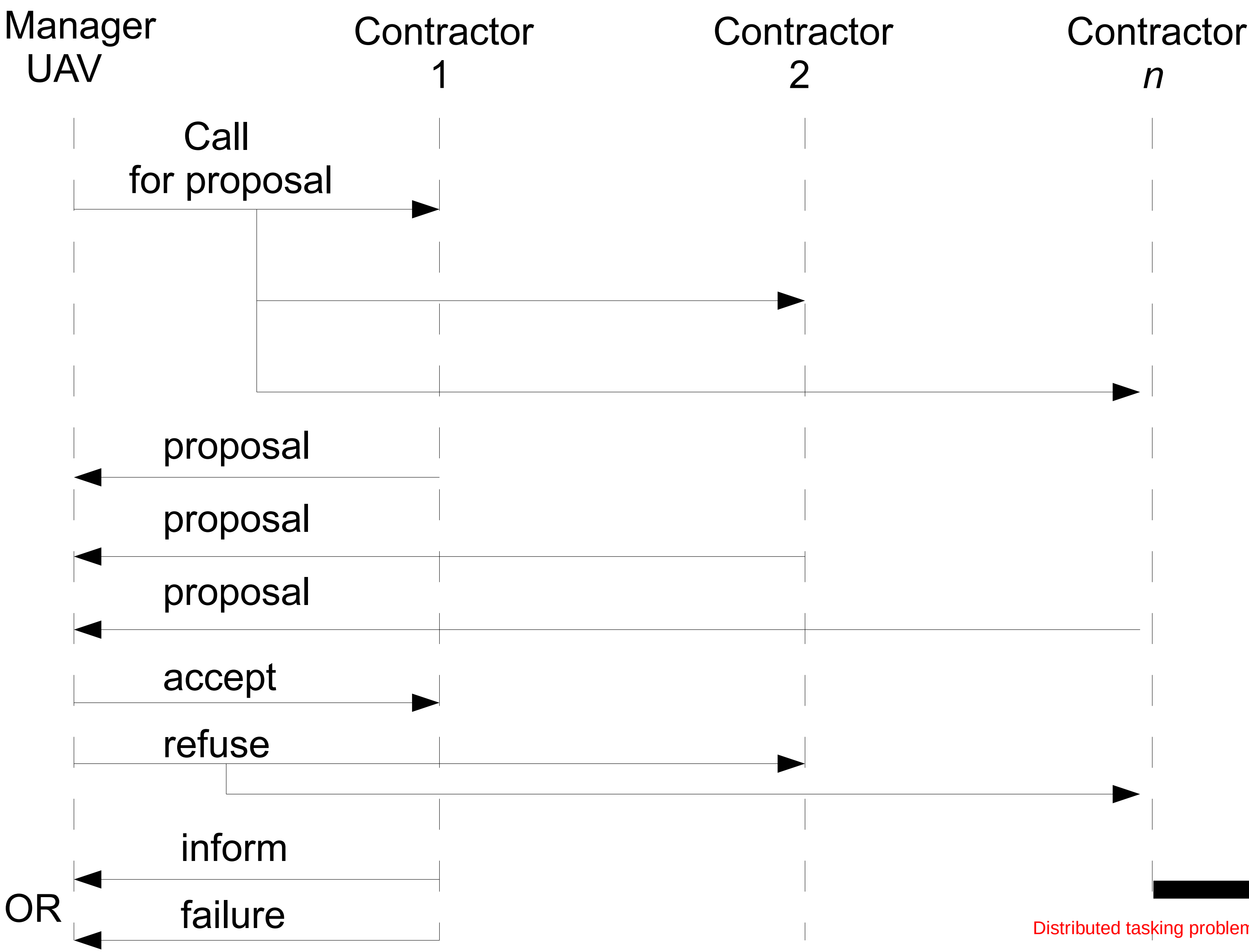
Procedure Diagram



Procedure Diagram



Procedure Diagram

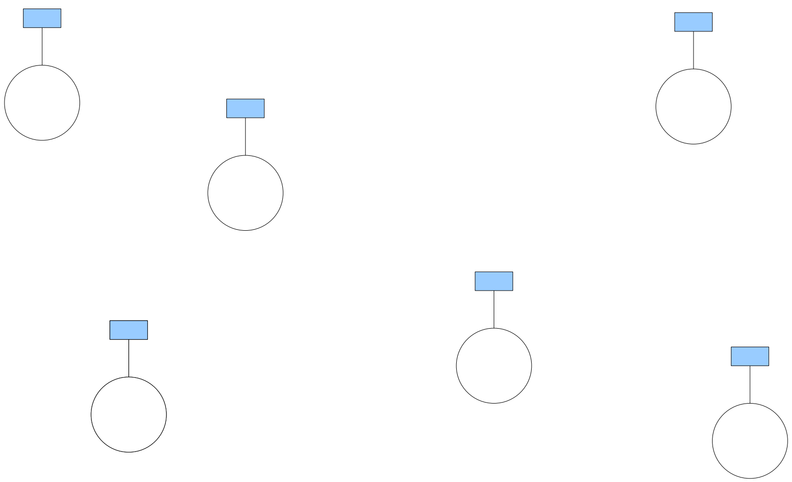


Issues for Implementing Contract Net

How to. . .

- ... specify tasks?
- ... specify quality of service?
- ... decide how to bid?
- ... select between competing offers?
- ... differentiate between offers based on multiple criteria?

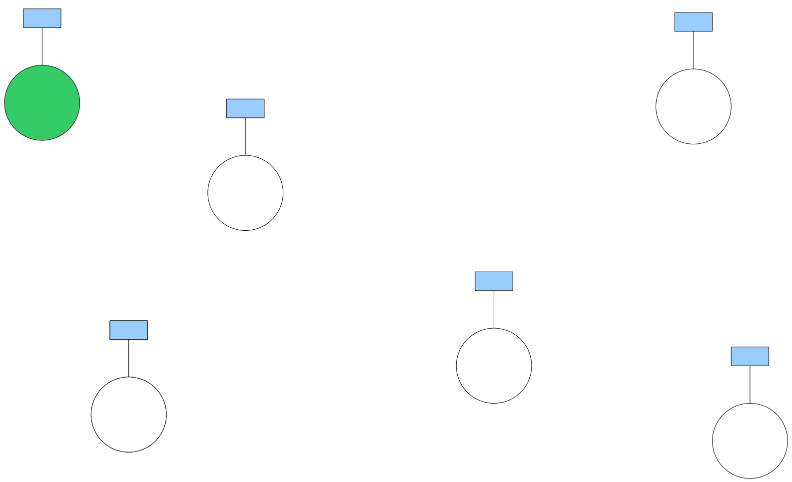
Agent Information



- A bundle of targets, $\mathbf{b}_i \doteq \{b_{i1}, ..., b_{i|\mathbf{b}_i|}\}$
- A corresponding path, $\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$
- A vector of times $\tau_i \doteq \{\tau_{i1}, ..., \tau_{i|\tau_i|}\}$

i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$					
$Path$					
$Time$					

Agent Information



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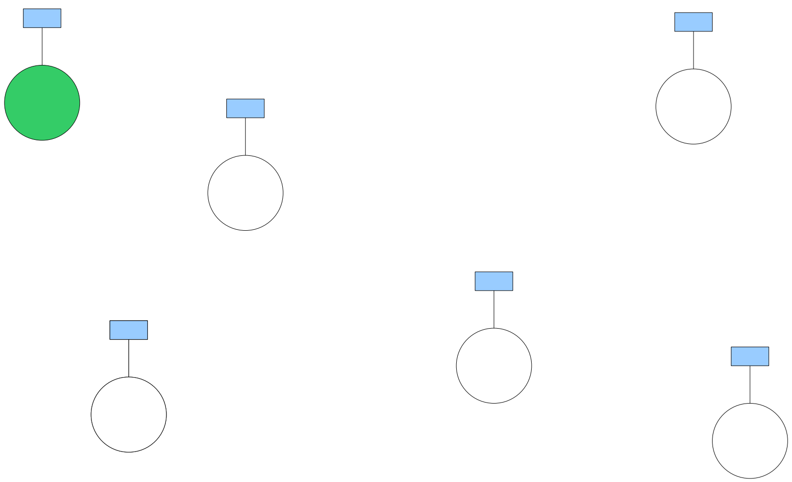
- A corresponding path, $\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$
- A vector of times $\tau_i \doteq \{\tau_{i1}, ..., \tau_{i|\tau_i|}\}$

i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$				$b_i = [b_{i1}]$
$Path$					
$Time$					

Agent Information

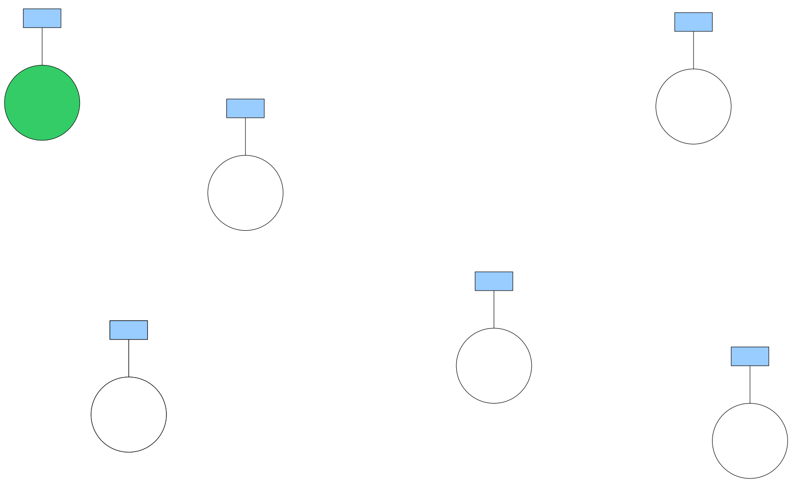


Recognition phase

Type of target (static or moving);

i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$				$b_i = [b_{i1}]$
$Path$					
$Time$					

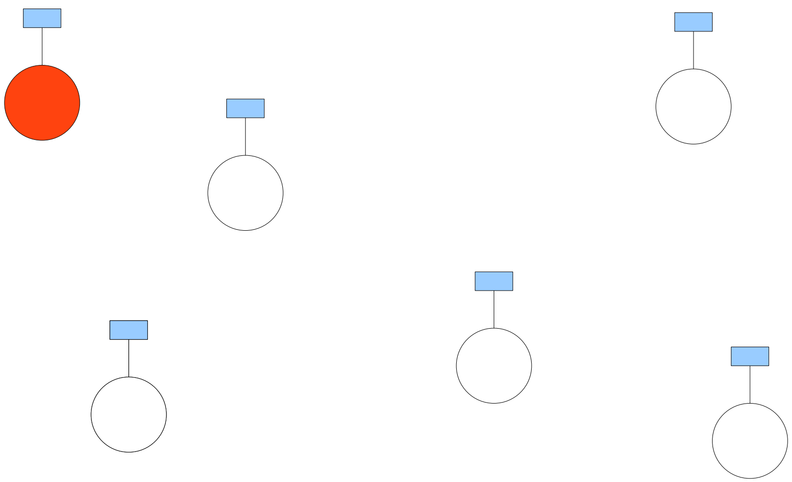
Agent Information



Recognition phase
Type of target (static or moving);
Switch to track and update own vectors of information, and disseminate to other UAVs.

i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$				$b_i = [b_{i1}]$
$Path$					
$Time$					

Agent Information



Recognition phase

Type of target (static or moving);

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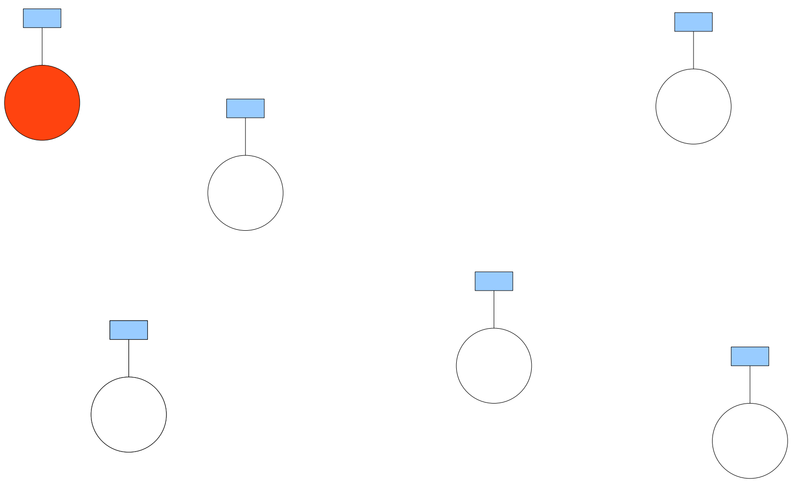
i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$				$b_i = [b_{i1}, b_{i2}]$
$Path$	p_{i1}				$p_i = [p_{i1}]$
$Time$					

Agent Information



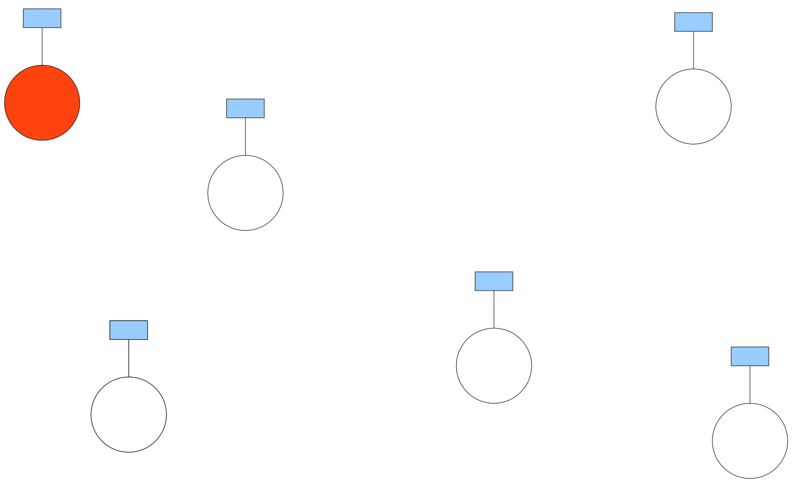
Recognition phase

Type of target (static or moving);

Switch to track and update own vectors of information, and disseminate to other UAVs.

i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$				$b_i = [b_{i1}]$
$Path$	p_{i1}				$p_i = [p_{i1}]$
$Time$	τ_{i1}				$\tau_i = [\tau_{i1}]$

Agent Information



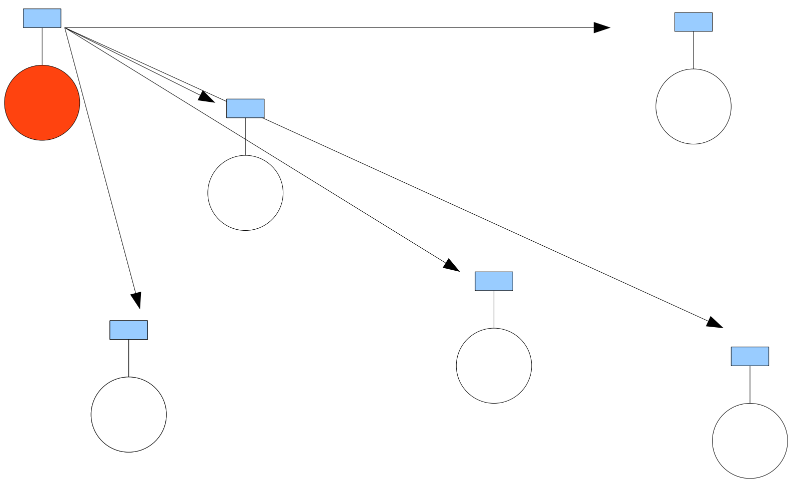
Recognition phase

Type of target (static or moving);

Switch to track and update own vectors of information, and disseminate to other UAVs.

i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Winning\ \ Agent$	i				$z_i = [z_{i1}]$
$WinningBids$	y_{i1}				$y_i = [y_{i1}]$

Agent Information



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i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$				$b_i = [b_{i1}]$
$Winning \ \ Agent$	i				$z_i = [z_{i1}]$
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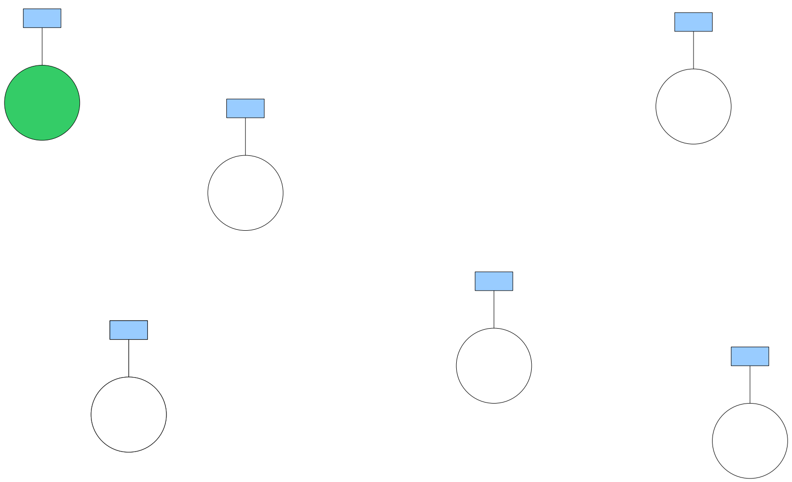
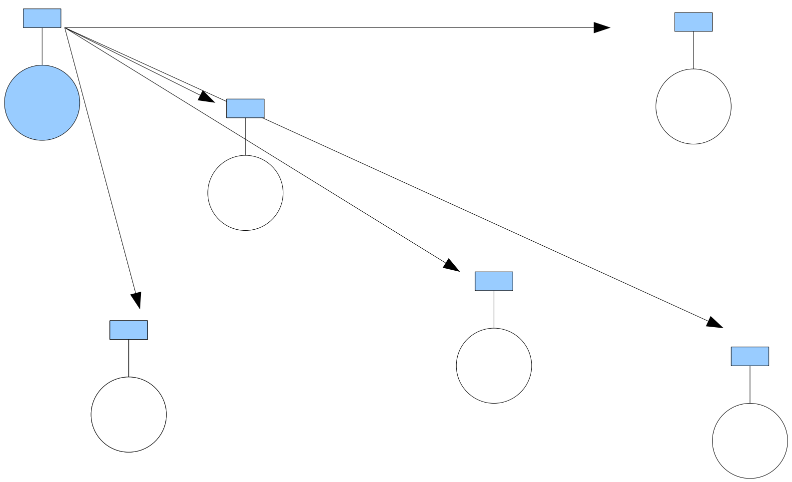


Recognition phase

Type of target (static or moving); Do the same until number of static target $nst \leq 2$

i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$				$b_i = [b_{i1}]$
$Path$					
$Time$					

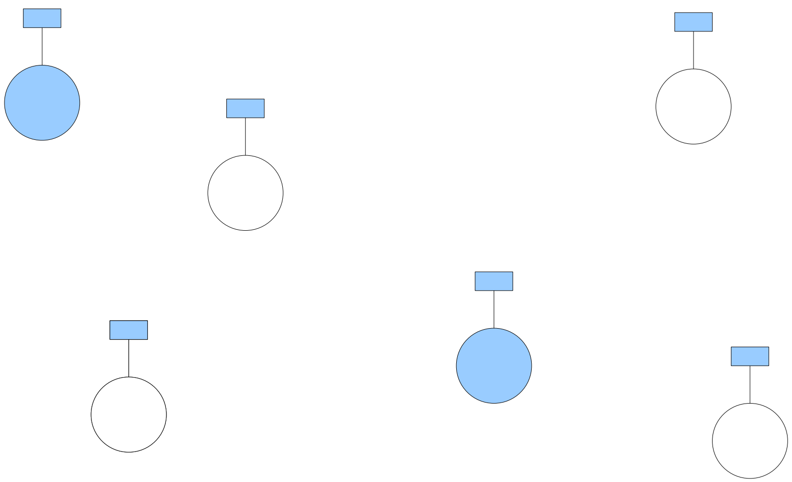
Agent Information



Recognition phase
Type of target (static or moving);
Switch to track and update own vectors of information, and disseminate to other UAVs.

i	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$				$b_i = [b_{i1}]$
$Winning\ \ Agent$	i				$z_i = [z_{i1}]$
$WinningBids$	y_{i1}				$y_i = [y_{i1}]$

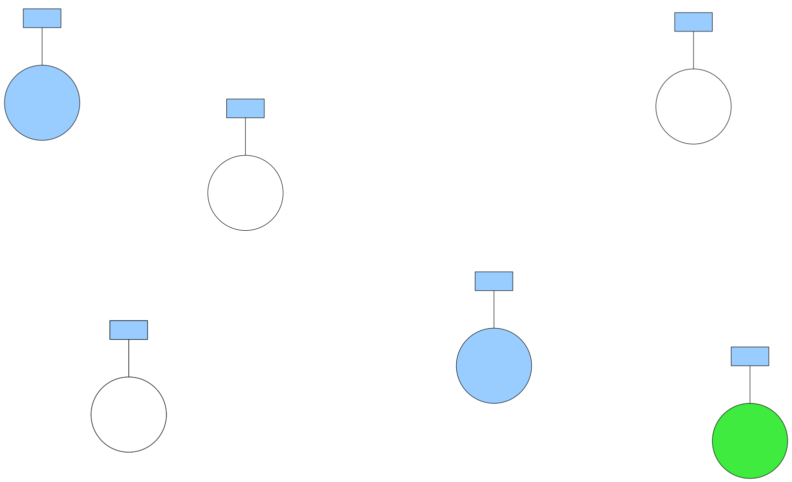
Agent Information



For example two static targets a found then each UAVs have the following information

k	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$	$\checkmark$			$b_k = [b_{k1}, b_{k2}]$
$Winning\ \ Agent$	i	j			$z_k = [z_{k1}, z_{k2}]$
$WinningBids$	y_{k1}	y_{k2}			$y_k = [y_{k1}, y_{k2}]$

Agent Information

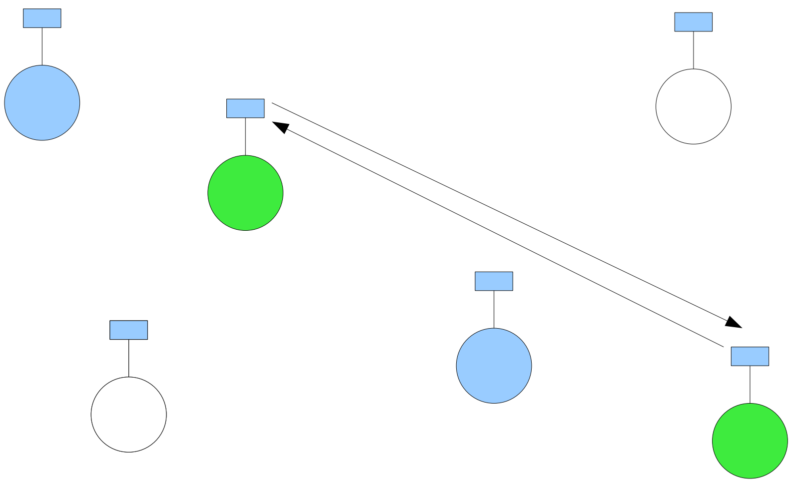


Recognition phase

Type of target (static or moving);

k	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$	$\checkmark$	$\maltese$		$b_k = [b_{k1}, b_{k2}, b_{kk}]$
$Winning\ \ Agent$	i	j			$z_k = [z_{k1}, z_{k2}]$
$WinningBids$	y_{k1}	y_{k2}			$y_k = [y_{k1}, y_{k2}]$

Agent Information



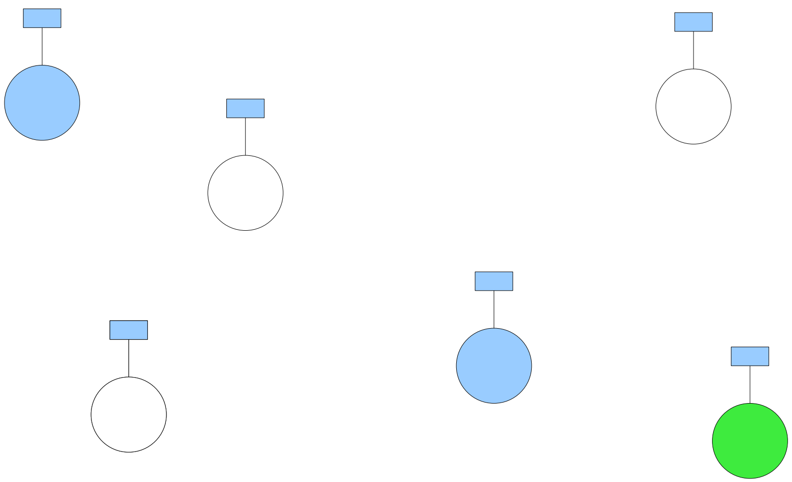
Recognition phase

Type of target (static or moving);

Check of existence of another new static target, if exist more then 1, select a manager UAV.

k	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$	$\checkmark$	$\maltese$	$\maltese$	$[b_{k1}, b_{k2}, b_{kk}, b_{kN_t}]$
$Winning\ \ Agent$	i	j			$z_k = [z_{k1}, z_{k2}]$
$WinningBids$	y_{k1}	y_{k2}			$y_k = [y_{k1}, y_{k2}]$

Agent Information



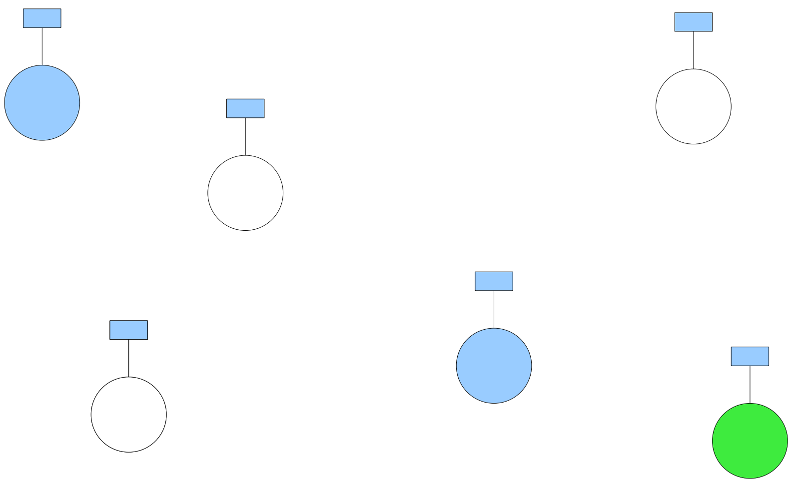
Recognition phase

Then do "Separation procedure": where input are: locations of static targets and

Number of subgroups =Total static target - number of new static targets)

k	$Target_1$	$Target_2$	$Target_k$	$Target_{N_t}$	$Values$
$Bandle$	$\checkmark$	$\checkmark$	✠	✠	$[b_{k1}, b_{k2}, b_{kk}, b_{kN_t}]$
$Winning\ \ Agent$	i	j			$z_k = [z_{k1}, z_{k2}]$
$WinningBids$	y_{k1}	y_{k2}			$y_k = [y_{k1}, y_{k2}]$

Agent Information



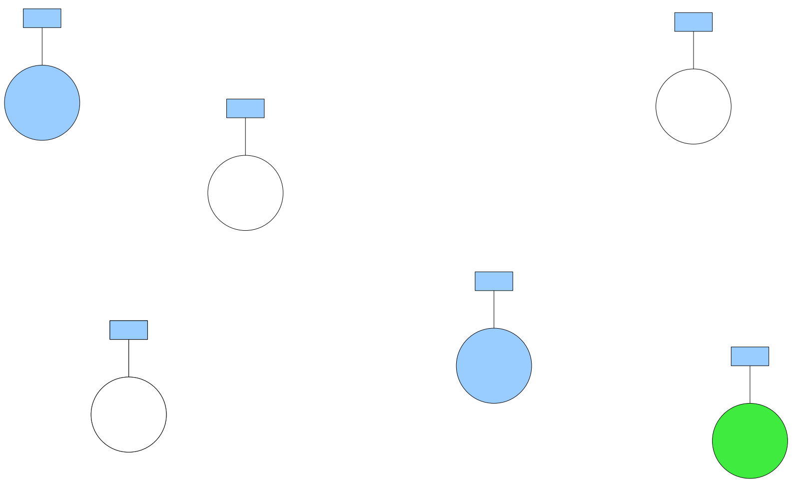
Recognition phase

Then do "Separation procedure": where input are: locations of static targets and

Number of subgroups = current length of bundle |b| - number of new static targets)

k	$Task_1$	$Task_2$			$Values$
$Bandle$	✓	✓	✠	✠	$[b_{k1}, b_{k2}, b_{kk}, b_{kN_t}]$
$Winning\ Agent$					$z_k = [z_{kt1}, z_{kt2}]$
$WinningBids$	y_{kt1}	y_{kt2}			$y_k = [y_{kt1}, y_{kt2}]$

Agent Information



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- Expediting.

k	$Task_1$	$Task_2$			$Values$
$Bandle$	$\checkmark$	$\checkmark$	✠	✠	$[b_{k1}, b_{k2}, b_{kk}, b_{kN_t}$
$Winning\ \ Agent$					$z_k = [z_{kt1}, z_{kt2}]$
$WinningBids$	y_{kt1}	y_{kt2}			$y_k = [y_{kt1}, y_{kt2}]$

Definition of UAVs and targets

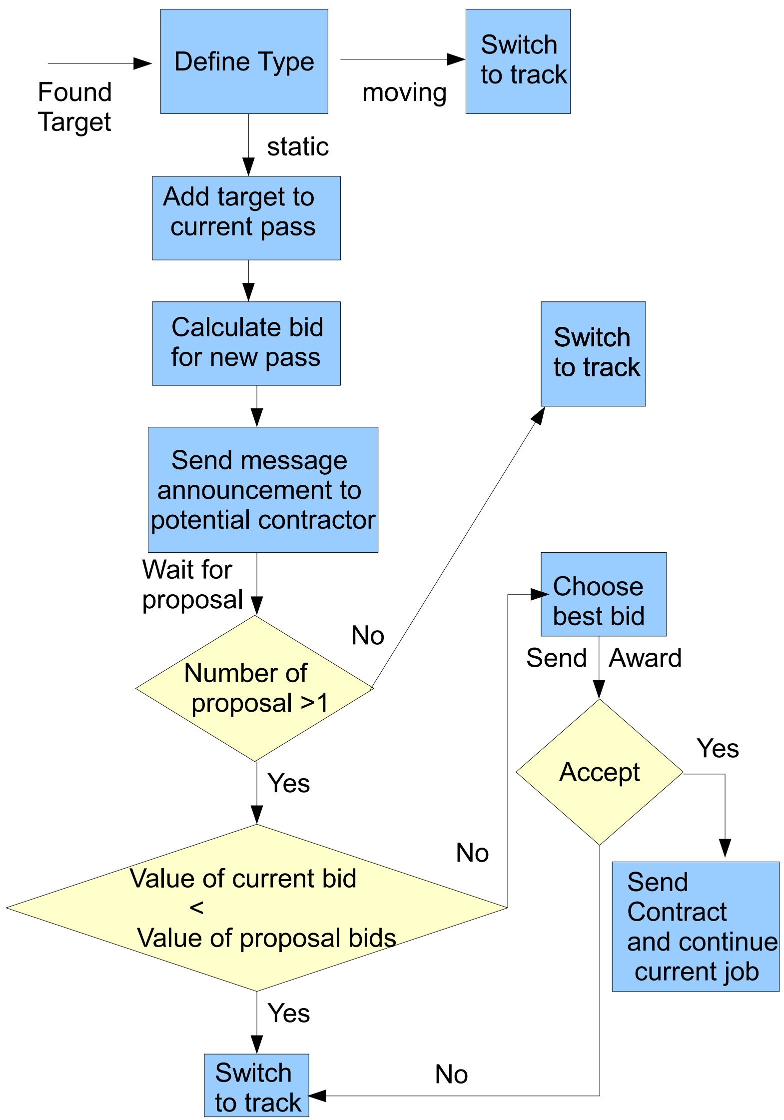
Possible target (task) fields:

- id - task id;
- type -task type;
- value -task reward;
- start-task start time (sec);
- end - task expiry time (sec);
- duration -task default duration (sec);
- x- task position (meters);
- y-task position (meters);
- z-task position (meters).

Possible UAVs fields:

- id- agent id;
- type- agent type;
- avail- agent availability (expected time in sec);
- x- agent position (meters);
- y- agent position (meters);
- z- agent position (meters);
- velocity - agent cruise velocity (m/s));
- fuel-(agent fuel per meter)).

Manager statecharts



Manager UAV (Case 1)

Manager UAV_i					$Values$
$Bundle$					$b_i = []$
$Path$					$p_i = []$
$Time$					$\tau_i = []$

$Performing\ \ Search$

Manager UAV_i					$Values$
$Winning\ \ Agent$					$z_i = []$
$WinningBids$					$y_i = []$

Manager UAV (Case 1)

Manager UAV_i	$Target_1$				$Values$
$Bundle$					$b_i = []$
$Path$					$p_i = []$
$Time$					$\tau_i = []$

$Found \;\; target$

Manager UAV_i					$Values$
$Winning \;\; Agent$					$z_i = []$
$WinningBids$					$y_i = []$

Manager UAV (Case 1)

Manager UAV_i	$Target_1$				$Values$
$Bundle$					$b_i = []$
$Path$					$p_i = []$
$Time$					$\tau_i = []$

$moving$

Manager UAV_i					$Values$
$Winning\ \ Agent$					$z_i = []$
$WinningBids$					$y_i = []$

Manager UAV (Case 1)

Manager UAV_i	$Target_1$				$Values$
$Bundle$	$\checkmark$				$b_i = []$
$Path$					$p_i = []$
$Time$					$\tau_i = []$

$$\Downarrow$$

$Switch\ to\ track$

Manager UAV (Case 1)

Manager UAV_i	$Target_1$				$Values$
$Bundle$	✠				$b_i = []$
$Path$					$p_i = []$
$Time$					$\tau_i = []$

$static$

Manager UAV_i					$Values$
$Winning\ \ Agent$					$z_i = []$
$WinningBids$					$y_i = []$

Manager UAV (Case 1)

Manager UAV_i	$Target_1$				$Values$
$Bundle$	✠				$b_i = [b_{i1}]$
$Path$	p_{i1}				$p_i = [p_{i1}]$
$Time$	τ_{i1}				$\tau_i = [\tau_{i1}]$

$Calculate\ arrival\ time\ \tau_{i1}(p)\ and\ corresponding\ bid\ \ y_{i1}$

Manager UAV_i					$Values$
$Winning\ \ Agent$					$z_i = []$
$WinningBids$					$y_i = []$

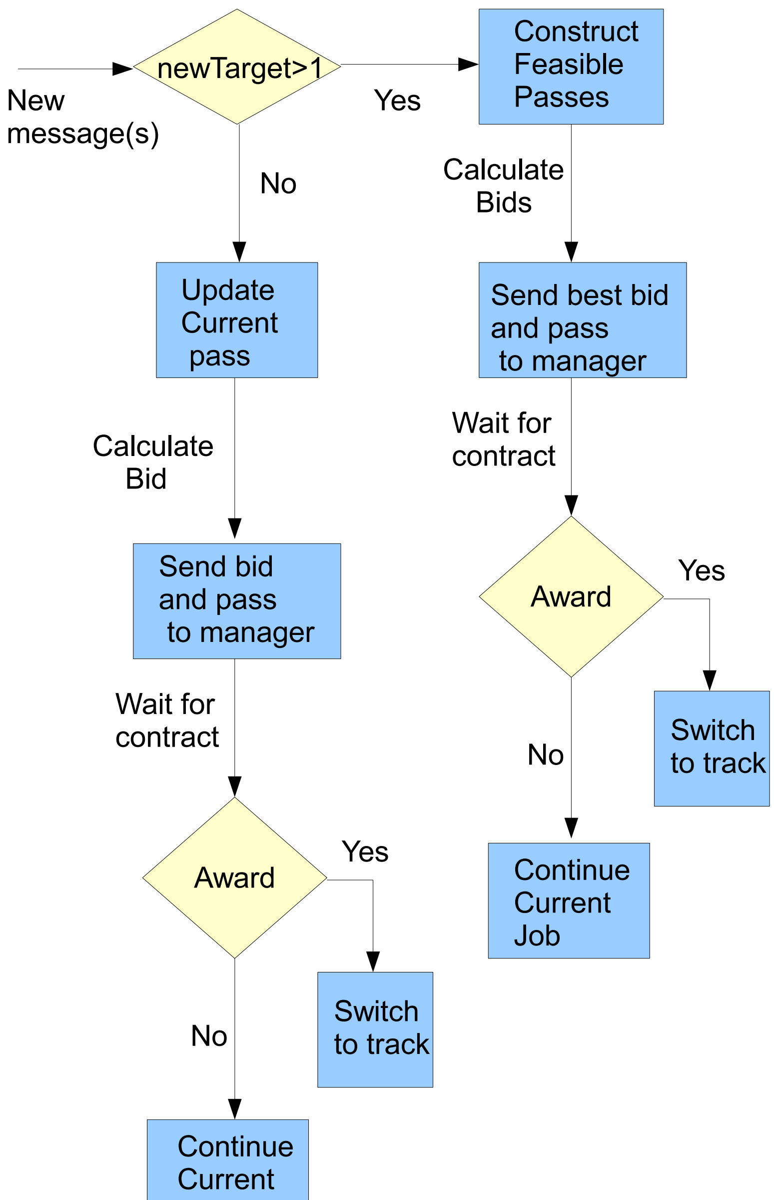
Manager UAV (Case 1)

Manager UAV_i	$Target_1$				$Values$
$Bundle$	✠				$b_i = [b_{i1}]$
$Path$	p_{i1}				$p_i = [p_{i1}]$
$Time$	τ_{i1}				$\tau_i = [\tau_{i1}]$

$Calculate\ arrival\ time\ \tau_{i1}(p)\ and\ corresponding\ bid\ \ y_{i1}$

Manager UAV_i	$Target_1$				$Values$
$Winning\ \ Agent$	i				$z_i = [z_{i1}]$
$WinningBids$	y_{i1}				$y_i = [y_{i1}]$

Potential contractors statecharts



Potential Contractors

Potential Contractor UAV_j					$Values$
$Bundle$					$b_j = []$
$Path$					$p_j = []$
$Time$					$\tau_j = []$

$Performing\ \ Search$

Potential Contractors

Potential Contractor UAV_j	$Target_1$				$Values$
$Bundle$	✠				$b_i = []$
$Path$					$p_i = []$
$Time$					$\tau_i = []$

$Recieve\ message$

Potential Contractors

Potential Contractor UAV_j	$Target_1$				$Values$
$Bundle$	✠				$b_j = [b_{j1}]$
$Path$	p_{j1}				$p_j = [p_{j1}]$
$Time$	τ_{j1}				$\tau_j = [\tau_{j1}]$

$Calculate\ arrival\ time\ \tau_{j1}(p)\ and\ corresponding\ bid\ y_{j1}$

Potential Contractor UAV_j	$Target_1$				$Values$
$Winning\ \ Agent$	j				$z_j = [z_{j1}]$
$WinningBids$	y_{j1}				$y_j = [y_{j1}]$

Potential Contractors

UAV_k		$Target_2$	$Target_k$		$Values$
$Bundle$		✓	✓		$b_k = [b_{k2}, b_{kk}]$
$Path$		p_{k2}	p_{kk}		$p_k = [p_{kk}, p_{k2}]$
$Time$		τ_{k2}	τ_{kk}		$\tau_k = [\tau_{kk}, \tau_{k2}]$

$Performing\ Tracking$

Potential Contractors

UAV_k	$Target_1$	$Target_2$	$Target_k$		$Values$
$Bundle$	✠	✓	✓		$b_k = [b_{k2}, b_{kk}]$
$Path$		p_{k2}	p_{kk}		$p_k = [p_{kk}, p_{k2}]$
$Time$		τ_{k2}	τ_{kk}		$\tau_k = [\tau_{kk}, \tau_{k2}]$

$Recieve\ \ message$

Potential Contractors

UAV_k	$Target_1$	$Target_2$	$Target_k$		$Values$
$Bundle$	$\checkmark$	$\checkmark$	$\checkmark$		$\mathbf{b}_i \leftarrow (\mathbf{b}_i \oplus_{end} 1)$
$Path$		p_{k2}	p_{kk}		$p_k = [p_{kk}, p_{k2}]$
$Time$		τ_{k2}	τ_{kk}		$\tau_k = [\tau_{kk}, \tau_{k2}]$

$Update\ current\ bundle\ of\ targets$

Potential Contractors

UAV_k	$Target_1$	$Target_2$	$Target_k$		$Values$
$Bundle$	$\checkmark$	$\checkmark$	$\checkmark$		$\mathbf{b}_k \leftarrow (\mathbf{b}_k \oplus_{end} 1)$
$Path$	p_{k1}	p_{k2}	p_{kk}		$\mathbf{p}_k \leftarrow (\mathbf{p}_k \oplus_{n_1} 1)$
$Time$		τ_{k2}	τ_{kk}		$\tau_k = [\tau_{kk}, \tau_{k2}]$

Update current pass

Potential Contractors

UAV_k	$Target_1$	$Target_2$	$Target_k$		$Values$
$Bundle$	✓	✓	✓		$\mathbf{b}_k \leftarrow (\mathbf{b}_k \oplus_{end} 1)$
$Path$	p_{k1}	p_{k2}	p_{kk}		$\mathbf{p}_k \leftarrow (\mathbf{p}_k \oplus_{n_1^*} 1)$
$Time$		τ_{k2}	τ_{kk}		$\tau_k = [\tau_{kk}, \tau_{k2}]$

Update current pass

$\Longrightarrow$ And optimal location n_1^* is then given by $n_1^* = \max_{n_1} c_1(\tau_{k1}^*(\mathbf{p}_k \oplus_{n_1} 1))$

Potential Contractors

UAV_k	$Target_1$	$Target_2$	$Target_k$		$Values$
$Bundle$	$\checkmark$	$\checkmark$	$\checkmark$		$\mathbf{b}_k \leftarrow (\mathbf{b}_k \oplus_{end} 1)$
$Path$	p_{k1}	p_{k2}	p_{kk}		$\mathbf{p}_k \leftarrow (\mathbf{p}_k \oplus_{n_1^*} 1)$
$Time$	τ_{k1}	τ_{k2}	τ_{kk}		$\tau_k \leftarrow (\tau_k \oplus_{n_{1^*}} \tau_{k1}(\mathbf{p}_k \oplus_{n_{1^*}}$

$Update\ current\ pass$

$\Longrightarrow$ And optimal location n_1^* is then given by $n_1^* = \max_{n_1} c_1(\tau_{k1}^*(\mathbf{p}_k \oplus_{n_1} 1))$

Potential Contractors

UAV_k	$Target_1$	$Target_2$	$Target_k$		$Values$
$Bundle$	✓	✓	✓		$\mathbf{b}_k \leftarrow (\mathbf{b}_k \oplus_{end} 1)$
$Path$	p_{k1}	p_{k2}	p_{kk}		$\mathbf{p}_k \leftarrow (\mathbf{p}_k \oplus_{n_1^*} 1)$
$Time$	τ_{k1}	τ_{k2}	τ_{kk}		$\tau_k \leftarrow (\tau_k \oplus_{n_{1*}} \tau_{k1}(\mathbf{p}_k \oplus_{n_{1*}}$

$Update\ current\ pass$

$\Longrightarrow$ And optimal location n_1^* is then given by $n_1^* = \max_{n_1} c_1(\tau_{k1}^*(\mathbf{p}_k \oplus_{n_1} 1))$

$\Longrightarrow$ Then the final score for new task j(which is include $|b_k|$ targets) is $c_{kj}(\mathbf{p}_i) = c_j(\tau_{kj}^*(\mathbf{p}_k \oplus_{n_j^*} j))$

Compare bids

For case, when bundle of manager UAV3 was not empty $|b_3| \neq \emptyset$

	$Proposal1$	$Proposal2$	$Proposal3$
$UAV1$	c_{11}	-	-
$UAV2$	-	c_{22}	-
$UAV3$	-	-	c_{33}

Compare bids

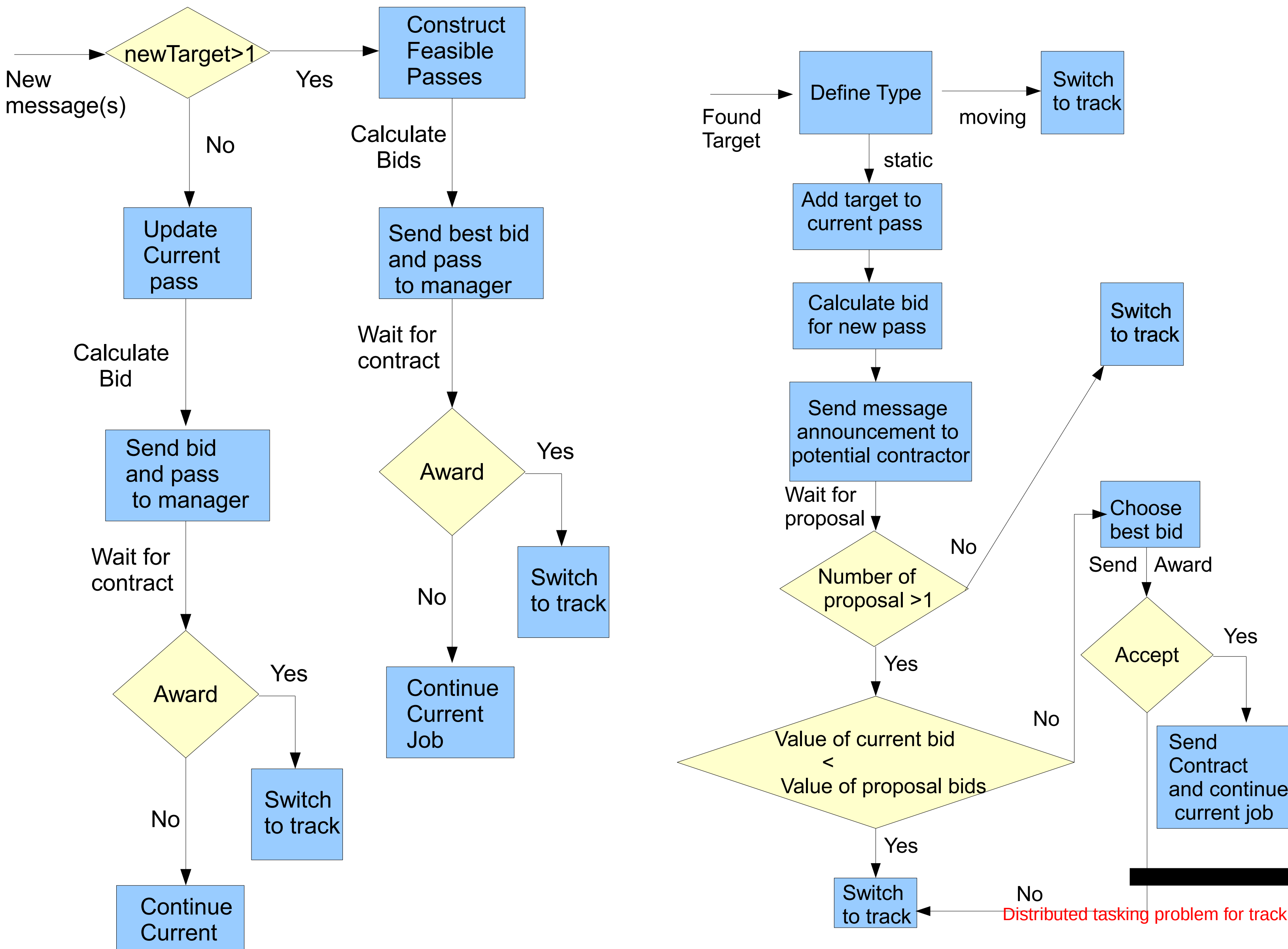
For case, when bundle of manager UAV3 was not empty $|b_3| \neq \emptyset$

	$Proposal1$	$Proposal2$	$Proposal3$
$UAV1$	c_{11}	-	-
$UAV2$	-	c_{22}	-
$UAV3$	-	-	c_{33}

For case, when bundle of manager UAV3 was empty $|b_3| = \emptyset$

	$Proposal1$	$Proposal2$	$Proposal3$
$UAV1$	c_{11}	-	-
$UAV2$	-	c_{22}	-
$UAV3$	c_{31}	c_{32}	c_{33}

Potential contractors and Manager



Problem statement

$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \rightarrow \max$$

$$subject\ \ to:$$

$$\sum_{j=1}^{N_t} x_{ij} \leq L_t, \ \ \forall i \in \mathcal{I}$$

$$\sum_{i=1}^{N_a} x_{ij} \leq 1, \ \ \forall j \in \mathcal{J}$$

$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

where $x_{ij} = 1$ if agent i is assigned to task j, and $\mathbf{x}_i \doteq \{x_{i1}, ..., x_{iN_t}\}$ is a vector of assignments for agent i, whose j-th element is x_{ij}.

The summation term in brackets in the objective function represents the local reward for agent i.

N_a Number of agents

N_t- Number of tasks

L_t- Maximum length of the bundle, i.e. each agent can be assigned a maximum L_t tasks

$\mathcal{I}$- Index set of agents where $\mathcal{I} \doteq \{1, ..., N_a\}$

$\mathcal{J}$- Index set of tasks where $\mathcal{J} \doteq \{1, ..., N_t\}$

Problem statement

$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \rightarrow \max$$

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The summation term in brackets in the objective function represents the local reward for agent i.

$\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$ - The variable length vector represent the path for agent i,an ordered sequence of tasks where the elements are the task indices, $p_{in} \in \mathcal{J}$ for $n = 1, ..., |\mathbf{p}_i|$, i.e. its n-th element is $j \in \mathcal{J}$ if agent i conducts task j at the n-th point along the path. The current length of the path is denoted by $|\mathbf{p}_i| \leq L_t$.

Problem statement

$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \rightarrow \max$$

$$subject\ \ to:$$

$$\sum_{j=1}^{N_t} x_{ij} \leq L_t, \quad \forall i \in \mathcal{I}$$

$$\sum_{i=1}^{N_a} x_{ij} \leq 1, \quad \forall j \in \mathcal{J}$$

$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

where $x_{ij} = 1$ if agent i is assigned to task j, and $\mathbf{x}_i \doteq \{x_{i1}, ..., x_{iN_t}\}$ is a vector of assignments for agent i, whose j-th element is x_{ij}.

The summation term in brackets in the objective function represents the local reward for agent i.

An assignment is said to be free of conflicts if each task is assigned to no more than one agent.

Key assumptions

$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \to \max$$

$$subject\ \ to:$$

$$\sum_{j=1}^{N_t} x_{ij} \leq L_t, \ \ \forall i \in \mathcal{I}$$

$$\sum_{i=1}^{N_a} x_{ij} \leq 1, \ \ \forall j \in \mathcal{J}$$

$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

- The score c_{ij} that agent i obtains by performing task j is defined as a function of the arrival time τ_{ij} at which the agent executes the task (or possibly the expected arrival time in a probabilistic setting).
- The arrival time τ_{ij} is uniquely defined as a function of the path $\mathbf{p}_i$ that agent i takes.
- The path $\mathbf{p}_i$ is uniquely defined by the assignment vector of agent i, $\mathbf{x}_i$.

Key assumptions

$$\sum_{i=1}^{N_a} \left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij} \right) \rightarrow \max$$

$$subject \;\; to:$$

$$\sum_{j=1}^{N_t} x_{ij} \leq L_t, \quad \forall i \in \mathcal{I}$$

$$\sum_{i=1}^{N_a} x_{ij} \leq 1, \quad \forall j \in \mathcal{J}$$

$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

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- The arrival time τ_{ij} is uniquely defined as a function of the path $\mathbf{p}_i$ that agent i takes.
- The path $\mathbf{p}_i$ is uniquely defined by the assignment vector of agent i, $\mathbf{x}_i$.

An example is the problem involving time-discounted values of targets, in which the sooner an agent arrives at the target, the higher the reward it obtains. Or for scenario involves re-visit tasks, where previously observed targets must be revisited at some scheduled time. In this case the score function would have its maximum at the desired re-visiting time and lower values at other re-visit times.

Six vectors of information for agent

- A bundle, $\mathbf{b}_i \doteq \{b_{i1}, ..., b_{i|\mathbf{b}_i|}\}$
  of variable length whose elements are defined by $b_{in} \in \mathcal{J}$ for $n = 1, ..., |\mathbf{b}_i|$. The current length of the bundle is denoted by b_i, which cannot exceed the maximum length L_t, and an empty bundle is represented by $b_i = \emptyset$ and $|\mathbf{b}_i| = 0$. The bundle represents the tasks that agent i has selected to do, and is ordered chronologically with respect to when the tasks were added (i.e. task b_{in} was added before task $b_{i(n+1)}$).
- A corresponding path, $\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$
- A vector of times $\tau_i \doteq \{\tau_{i1}, ..., \tau_{i|\tau_i|}\}$
- A winning agent list $\mathbf{z}_i \doteq \{z_{i1}, ..., z_{iN_t}\}$ of size N_t
- A winning bid list $\mathbf{y}_i \doteq \{y_{i1}, ..., y_{iN_t}\}$ of size N_t
- Vector of timestamps $\mathbf{s}_i \doteq \{s_{i1}, ..., s_{iN_a}\}$, of size N_a

Six vectors of information for agent

- A bundle, $\mathbf{b}_i \doteq \{b_{i1}, ..., b_{i|\mathbf{b}_i|}\}$
- A corresponding path, $\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$
  whose elements are defined by $p_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$ for $n = 1, ..., |\mathbf{b}_i|$. The path contains the same tasks as the bundle, and is used to represent the order in which agent i will execute the tasks in its bundle. The path is therefore the same length as the bundle, and is not permitted to be longer than L_t; $|\mathbf{p}_i| = |\mathbf{b}_i| \leq L_t$.
- A vector of times $\tau_i \doteq \{\tau_{i1}, ..., \tau_{i|\tau_i|}\}$
- A winning agent list $\mathbf{z}_i \doteq \{z_{i1}, ..., z_{iN_t}\}$ of size N_t
- A winning bid list $\mathbf{y}_i \doteq \{y_{i1}, ..., y_{iN_t}\}$ of size N_t
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- A vector of times $\tau_i \doteq \{\tau_{i1}, ..., \tau_{i|\tau_i|}\}$

  whose elements are defined by τ_{in} for $n = 1, ..., |\tau_i|$. The times vector represents the corresponding times at which agent i will execute the tasks in its path, and is necessarily the same length as the path.
- A winning agent list $\mathbf{z}_i \doteq \{z_{i1}, ..., z_{iN_t}\}$ of size N_t
- A winning bid list $\mathbf{y}_i \doteq \{y_{i1}, ..., y_{iN_t}\}$ of size N_t
- Vector of timestamps $\mathbf{s}_i \doteq \{s_{i1}, ..., s_{iN_a}\}$, of size N_a

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- A vector of times $\tau_i \doteq \{\tau_{i1}, ..., \tau_{i|\tau_i|}\}$
- A winning agent list $\mathbf{z}_i \doteq \{z_{i1}, ..., z_{iN_t}\}$ of size N_t

  where each element $z_{ij} \in \{\mathcal{I} \cup \emptyset\}$ for $j = 1, ..., N_t$ indicates who agent i believes is the current winner for task j. Specifically, the value in element z_{ij} is the index of the agent who is currently winning task j according to agent i, and is $z_{ij} = \emptyset$; if agent i believes that there is no current winner.
- A winning bid list $\mathbf{y}_i \doteq \{y_{i1}, ..., y_{iN_t}\}$ of size N_t
- Vector of timestamps $\mathbf{s}_i \doteq \{s_{i1}, ..., s_{iN_a}\}$, of size N_a

Six vectors of information for agent

- A bundle, $\mathbf{b}_i \doteq \{b_{i1}, ..., b_{i|\mathbf{b}_i|}\}$
- A corresponding path, $\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$
- A vector of times $\tau_i \doteq \{\tau_{i1}, ..., \tau_{i|\tau_i|}\}$
- A winning agent list $\mathbf{z}_i \doteq \{z_{i1}, ..., z_{iN_t}\}$ of size N_t
- A winning bid list $\mathbf{y}_i \doteq \{y_{i1}, ..., y_{iN_t}\}$ of size N_t

  where the elements $y_{ij} \in [0, \infty)$ represent the corresponding winners bids and take the value of 0 if there is no winner for the task.
- Vector of timestamps $\mathbf{s}_i \doteq \{s_{i1}, ..., s_{iN_a}\}$, of size N_a

Six vectors of information for agent

- A bundle, $\mathbf{b}_i \doteq \{b_{i1}, ..., b_{i|\mathbf{b}_i|}\}$
- A corresponding path, $\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$
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- A winning bid list $\mathbf{y}_i \doteq \{y_{i1}, ..., y_{iN_t}\}$ of size N_t
- Vector of timestamps $\mathbf{s}_i \doteq \{s_{i1}, ..., s_{iN_a}\}$, of size N_a

  where each element $s_{ik} \in [0, \infty)$ for $k = 1, ..., N_a$ represents the timestamp of the last information update agent i received about agent k, either directly or through a neighboring agent.

Six vectors of information for agent

- A bundle, $\mathbf{b}_i \doteq \{b_{i1}, ..., b_{i|\mathbf{b}_i|}\}$
- A corresponding path, $\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$
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- A winning bid list $\mathbf{y}_i \doteq \{y_{i1}, ..., y_{iN_t}\}$ of size N_t
- Vector of timestamps $\mathbf{s}_i \doteq \{s_{i1}, ..., s_{iN_a}\}$, of size N_a

$$\Downarrow$$

Each agent must carry these vectors of information in order to be able to perform decentralized algorithm which consists of iterations between two phases:
a bundle building phase where each vehicle greedily generates an ordered bundle of tasks, and a
consensus phase where conflicting assignments are identified and resolved through local communication between neighboring agents

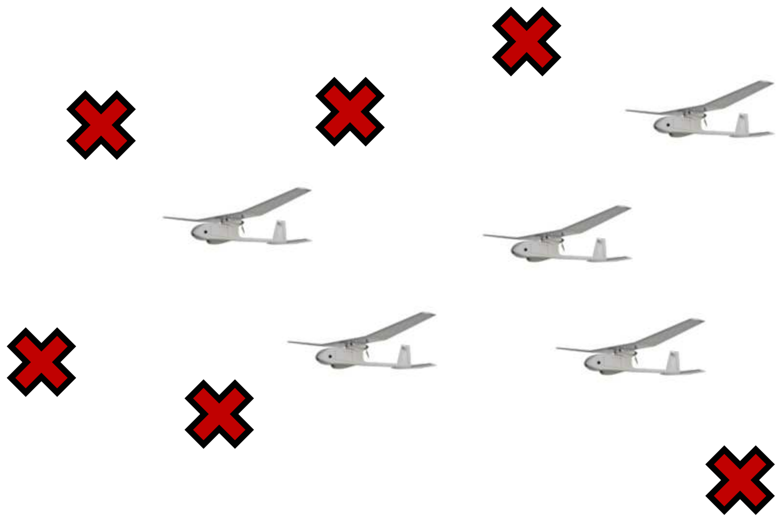
Six vectors of information for agent

- A bundle, $\mathbf{b}_i \doteq \{b_{i1}, ..., b_{i|\mathbf{b}_i|}\}$
- A corresponding path, $\mathbf{p}_i \doteq \{p_{i1}, ..., p_{i|\mathbf{p}_i|}\}$
- A vector of times $\tau_i \doteq \{\tau_{i1}, ..., \tau_{i|\tau_i|}\}$
- A winning agent list $\mathbf{z}_i \doteq \{z_{i1}, ..., z_{iN_t}\}$ of size N_t
- A winning bid list $\mathbf{y}_i \doteq \{y_{i1}, ..., y_{iN_t}\}$ of size N_t
- Vector of timestamps $\mathbf{s}_i \doteq \{s_{i1}, ..., s_{iN_a}\}$, of size N_a

$$\Downarrow$$

Algorithm will iterates between these two phases until no changes to the information vectors occur anymore.

Agent Information



$$\sum_{i=1}^{N_a} \left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij} \right) \rightarrow \max$$

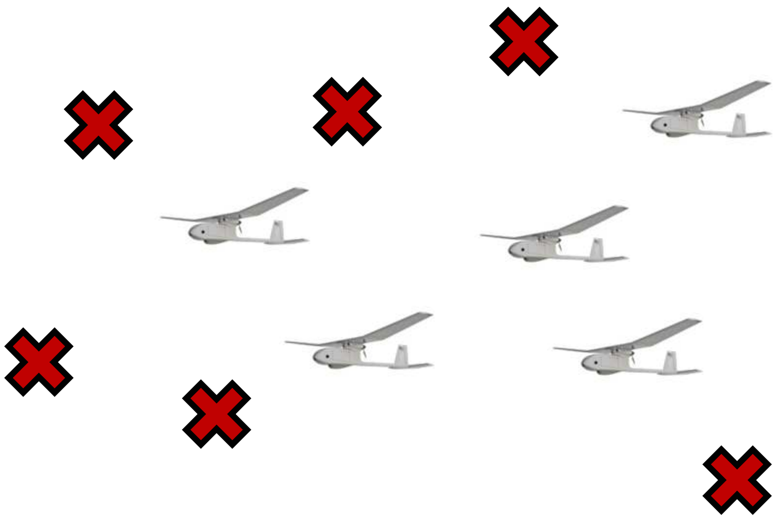
$$\sum_{j=1}^{N_t} x_{ij} \leq L_t, \quad \forall i \in \mathcal{I}$$

$$\sum_{i=1}^{N_a} x_{ij} \leq 1, \quad \forall j \in \mathcal{J}$$

$$x_{ij} \in \{0, 1\}, \forall (i, j) \in \mathcal{I} \times \mathcal{J}$$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	✓		✓		
$Path$					
$Time$					

Agent Information



$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \to \max$$

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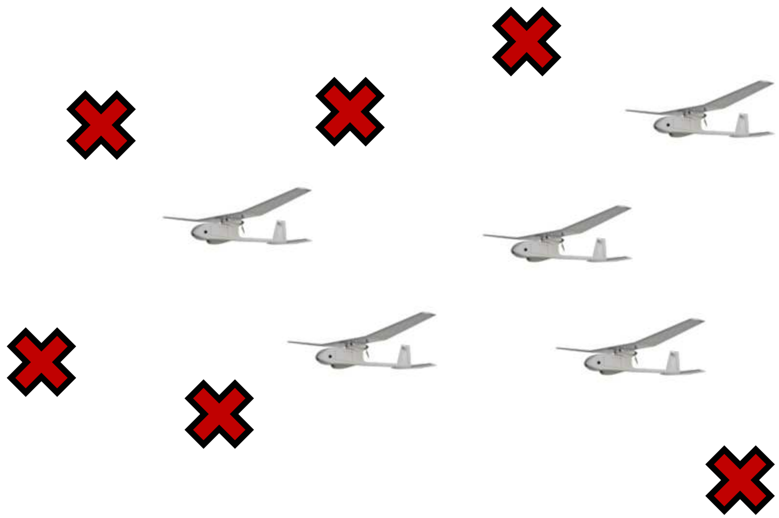
$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓		2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$					
$Time$					

Agent Information



$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \to \max$$

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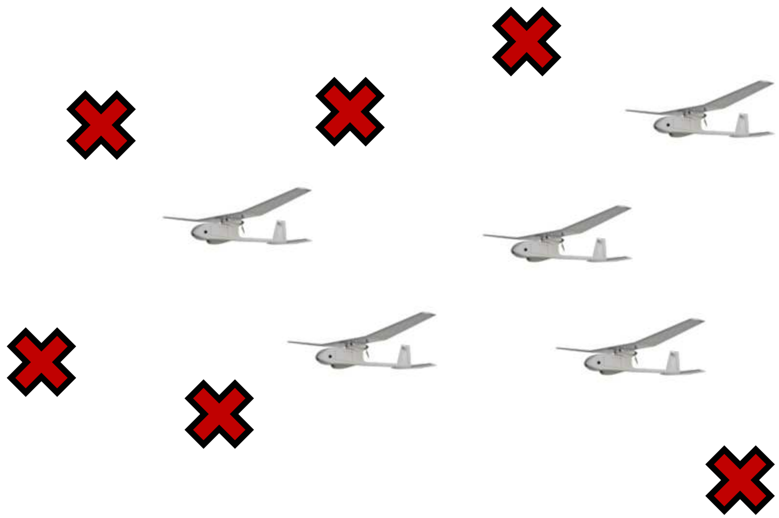
i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓		2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$					

Agent Information



$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \to \max$$

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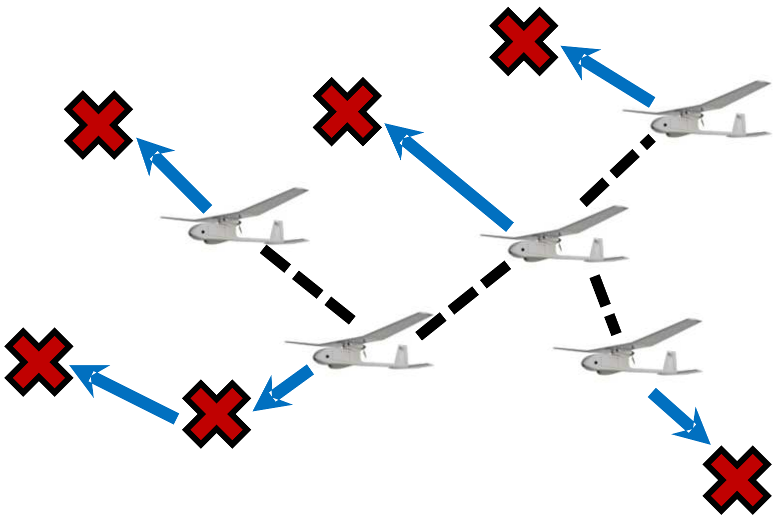
$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓		2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

Agent Information



$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \rightarrow \max$$

$$\sum_{j=1}^{N_t} x_{ij} \le L_t, \quad \forall i \in \mathcal{I}$$

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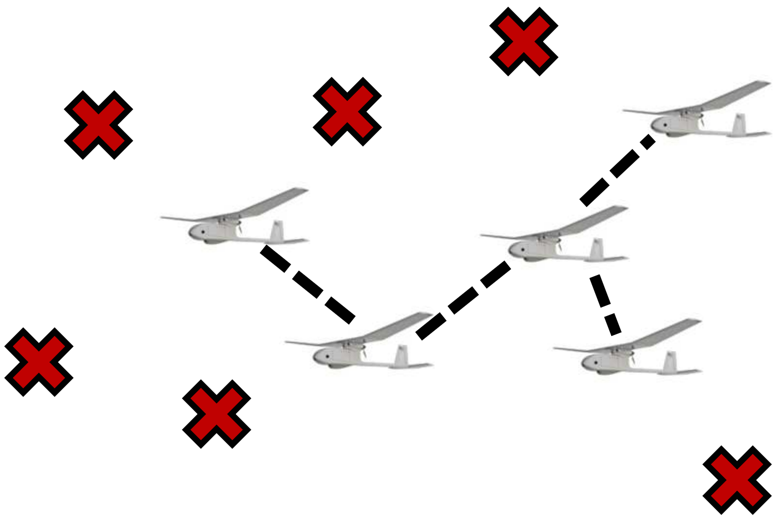
$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓		2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

Agent Information



$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \rightarrow \max$$

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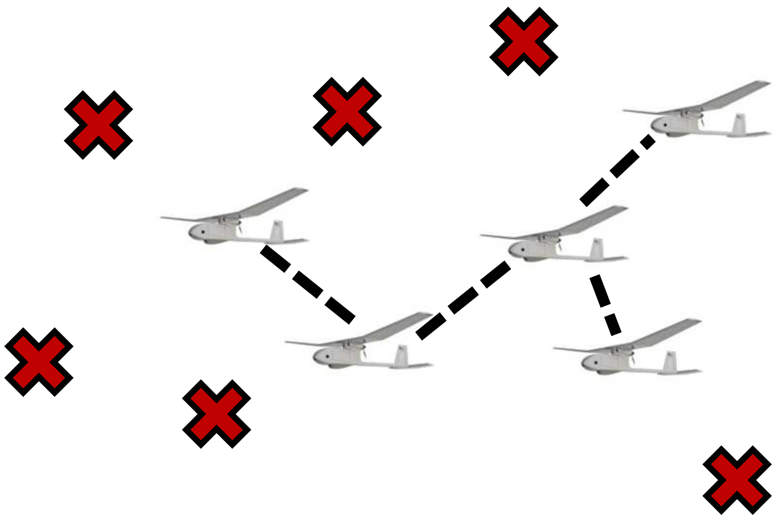
$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	2	4	i	k	$z_i = [z_{21}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$					

Agent Information



$$\sum_{i=1}^{N_a}\left(\sum_{j=1}^{N_t} c_{ij}(\tau_{ij}(\mathbf{p}_i(\mathbf{x_i})))x_{ij}\right) \to \max$$

$$\sum_{j=1}^{N_t} x_{ij} \leq L_t, \quad \forall i \in \mathcal{I}$$

$$\sum_{i=1}^{N_a} x_{ij} \leq 1, \quad \forall j \in \mathcal{J}$$

$$x_{ij} \in \{0,1\}, \forall (i,j) \in \mathcal{I} \times \mathcal{J}$$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 $\checkmark$		2 $\checkmark$		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$Calculate\ a\ score\ \ c_{ij} = c_{i2}\ \ and\ compare\ with\ current$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$Calculate\ a\ score\ \ c_{ij} = c_{i2}\ \ and\ compare\ with\ current$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$Calculate\ a\ score\ \ c_{ij} = c_{i2}\ \ and\ compare\ with\ current$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 $\checkmark$	✠	2 $\checkmark$		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$To\ calculate\ best\ score\ for\ task\ j,\ we\ "insert"\ \ the\ task\ in\ some\ location\ n_j$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$To\ calculate\ best\ score\ for\ task\ j,\ we\ first\text{"}insert\text{"}\ the\ task\ in\ some\ location\ n_j$

And new path becomes $(\mathbf{p}_i \oplus_{n_j} j)$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$To\ calculate\ best\ score\ for\ task\ j,\ we\ first\text{"}insert\text{"}\ \ the\ task\ in\ some\ location\ n_j$

And new path becomes $(\mathbf{p}_i \oplus_{n_j} j)$ and second calculate the optimal execution time for this new path:

$$\tau^*_{ij}(\mathbf{p}_i \oplus_{n_j} j) = \max_{\tau_{ij} \in [0,\infty)} c_j(\tau_{ij})$$

$$subject\ \ to:$$

$$\tau^*_{ik}(\mathbf{p}_i \oplus_{n_j} j) = \tau^*_{ik}, \forall k \in \mathbf{p}_i$$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$\Longrightarrow$ optimal score for the task at location n_j is $c_j(\tau^*_{ij}(\mathbf{p}_i \oplus_{n_j} j))$.

$$\tau^*_{ij}(\mathbf{p}_i \oplus_{n_j} j) = \max_{\tau_{ij} \in [0,\infty)} c_j(\tau_{ij})$$

$$subject\ \ to:$$

$$\tau^*_{ik}(\mathbf{p}_i \oplus_{n_j} j) = \tau^*_{ik}, \forall k \in \mathbf{p}_i$$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$\Longrightarrow$ optimal score for the task at location n_j is $c_j(\tau^*_{ij}(\mathbf{p}_i \oplus_{n_j} j))$.

$\Longrightarrow$ And optimal location n^*_j is then given by $n^*_j = \max_{n_j} c_j(\tau^*_{ik}(\mathbf{p}_i \oplus_{n_j} j))$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$\Longrightarrow$ optimal score for the task at location n_j is $c_j(\tau^*_{ij}(\mathbf{p}_i \oplus_{n_j} j))$.

$\Longrightarrow$ And optimal location n^*_j is then given by $n^*_j = \max_{n_j} c_j(\tau^*_{ik}(\mathbf{p}_i \oplus_{n_j} j))$

$\Longrightarrow$ Final score for task j is $c_{ij}(\mathbf{p}_i) = c_j(\tau^*_{ij}(\mathbf{p}_i \oplus_{n^*_j} j))$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓	✠	$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

$\Longrightarrow$ optimal score for the task at location n_j is $c_j(\tau_{ij}^*(\mathbf{p}_i \oplus_{n_j} j))$.

$\Longrightarrow$ And optimal location n_j^* is then given by $n_j^* = \max_{n_j} c_j(\tau_{ik}^*(\mathbf{p}_i \oplus_{n_j} j))$

$\Longrightarrow$ Final score for task j is $c_{ij}(\mathbf{p}_i) = c_j(\tau_{ij}^*(\mathbf{p}_i \oplus_{n_j^*} j))$

$\Longrightarrow$ Final step is to select the highest scoring task to add to the bundle

$j^* = \max_{j \notin \mathbf{p}_i} c_{ij}(\mathbf{p}_i) h_{ij}$, where $h_{ij} = \mathbf{I}(c_{ij}(\mathbf{p}_i) > y_{ij})$ the indicator function

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✠	2 ✓		$b_i = [b_{i1}, b_{i2}]$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✓	2 ✓		$\mathbf{b}_i \leftarrow (\mathbf{b}_i \oplus_{end} j^*)$
$Path$	2		1		$p_i = [p_{i1}, p_{i2}]$
$Time$	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
Bandle	1 ✓	✓	2 ✓		$\mathbf{b}_i \leftarrow (\mathbf{b}_i \oplus_{end} j^*)$
Path	2		1		$\mathbf{p}_i \leftarrow (\mathbf{p}_i \oplus_{n_{j*}} j^*)$
Time	20		10		$\tau_i = [\tau_{i1}, \tau_{i2}]$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
Winning Agent	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
WinningBids	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Bandle$	1 ✓	✓	2 ✓		$\mathbf{b}_i \leftarrow (\mathbf{b}_i \oplus_{end} j^*)$
$Path$	2		1		$\mathbf{p}_i \leftarrow (\mathbf{p}_i \oplus_{n_{j*}} j^*)$
$Time$	20		10		$\tau_i \leftarrow (\tau_i \oplus_{n_{j*}} \tau^*_{ij*}(\mathbf{p}_i \oplus_{n_{j*}} j^*$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$	i	4	i	k	$z_i = [z_{i1}, z_{42}, z_{ik}, z_{kN_t}]$
$WinningBids$	9	5	8	7	$y_i = [y_{i1}, y_{42}, y_{ik}, y_{kN_t}]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
Bandle	1 ✓	✓	2 ✓		$\mathbf{b}_i \leftarrow (\mathbf{b}_i \oplus_{end} j^*)$
Path	2		1		$\mathbf{p}_i \leftarrow (\mathbf{p}_i \oplus_{n_{j*}} j^*)$
Time	20		10		$\tau_i \leftarrow (\tau_i \oplus_{n_{j*}} \tau^*_{ij*}(\mathbf{p}_i \oplus_{n_{j*}} j^*$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
Winning Agent	i	i	i	k	$z_i = [z_{i1}, z_{i2}, \ldots]$
WinningBids	9	$c_{ij*}(\mathbf{p}_i)$	8	7	$y_i = [y_{i1}, y_{i2}, \ldots]$

Bundle construction(Task selection)

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
Bandle	1 ✓	✓	2 ✓		$\mathbf{b}_i \leftarrow (\mathbf{b}_i \oplus_{end} j^*)$
Path	2		1		$\mathbf{p}_i \leftarrow (\mathbf{p}_i \oplus_{n_{j*}} j^*)$
Time	20		10		$\tau_i \leftarrow (\tau_i \oplus_{n_{j*}} \tau^*_{ij*}(\mathbf{p}_i \oplus_{n_{j*}} j^*$

$Bundle\ recursion\ continues\ until\ |\mathbf{b}_i| = L_t\ or\ h_{ij} = 0\ for\ all\ j \notin \mathbf{p}_i$

i	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
Winning Agent	i	i	i	k	$z_i = [z_{i1}, z_{i2}, \ldots]$
WinningBids	9	$c_{ij*}(\mathbf{p}_i)$	8	7	$y_i = [y_{i1}, y_{i2}, \ldots]$

Consensus

i, (receiver)	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$					$z_i = [z_{i1}, z_{i2}, ...]$
$WinningBids$					$y_i = [y_{i1}, y_{i2}, ...]$

$$Update: \quad z_{ij} = z_{kj}, \quad y_{ij} = y_{kj}$$

$$Reset: \quad z_{ij} = \emptyset, \quad y_{ij} = 0$$

$$Leave: \quad z_{ij} = z_{ij}, \quad y_{ij} = y_{ij}$$

k, (sender)	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$					$z_k = [z_{k1}, z_{k2}, ...]$
$WinningBids$					$y_k = [y_{k1}, y_{k2}, ...]$

Decision Rules

Agent k thinks z_{kj} is	Agent i thinks z_{ij} is	Receiver Action
k	i	if $y_{kj} > y_{ij} \rightarrow update$
k	k	$update$
k	$m \notin \{i, k\}$	$if\ s_{km} > s_{im}$ or $y_{kj} > y_{ij} \rightarrow update$
k	$none$	$update$

$$s_{ik} = \begin{cases} \tau_r(i.e.\ message\ reception\ time), & if\ g_{ik} = 1; \\ \max\{s_{mk} \mid m \in \mathcal{I}, g_{im} = 1\}, & otherwise \end{cases}$$

Agent k thinks z_{kj} is	Agent i thinks z_{ij} is	Receiver Action
i	i	$leave$
i	k	$reset$
i	$m \notin \{i, k\}$	$if\ s_{km} > s_{im} \rightarrow reset$
i	$none$	$leave$

Decision Rules

Agent k thinks z_{kj} is	Agent i thinks z_{ij} is	Receiver Action
$m \notin \{i,k\}$	i	$if\ s_{km} > s_{im}\ and\ y_{kj} > y_{ij} \rightarrow update$
$m \notin \{i,k\}$	k	$if\ s_{km} > s_{im} \rightarrow update$ $else \rightarrow reset$
$m \notin \{i,k\}$	m	$s_{km} > s_{im} \rightarrow update$
$m \notin \{i,k\}$	$n \notin \{i,k,m\}$	$if\ s_{km} > s_{im}\ and\ s_{kn} > s_{in} \rightarrow update$ $if\ s_{km} > s_{im}\ and\ y_{kj} > y_{ij} \rightarrow update$ $if\ s_{kn} > s_{in}\ and\ s_{im} > s_{km} \rightarrow reset$
$m \notin \{i,k\}$	$none$	$if\ s_{km} > s_{im} \rightarrow update$

Agent k thinks z_{kj} is	Agent i thinks z_{ij} is	Receiver Action
$none$	i	$leave$
$none$	k	$update$
$none$	$m \notin \{i,k\}$	$if\ s_{km} > s_{im} \rightarrow update$
$none$	$none$	$leave$

Decision Rules

i, (receiver)	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$					$z_i = [z_{i1}, z_{i2}, ...]$
$WinningBids$					$y_i = [y_{i1}, y_{i2}, ...]$

$$Update: \quad z_{ij} = z_{kj}, \quad y_{ij} = y_{kj}$$

$$Reset: \quad z_{ij} = \emptyset, \quad y_{ij} = 0$$

$$Leave: \quad z_{ij} = z_{ij}, \quad y_{ij} = y_{ij}$$

k, (sender)	$Task_1$	$Task_2$	$Task_k$	$Task_{N_t}$	$Values$
$Winning\ \ Agent$					$z_k = [z_{k1}, z_{k2}, ...]$
$WinningBids$					$y_k = [y_{k1}, y_{k2}, ...]$

Algorithm summary

- Calculate marginal score for all tasks

$$c_{ij}(\mathbf{p}_i) = \begin{cases} 0, & if\ j \in \mathbf{p}_i; \\ \max\limits_{n \le l_b} S_{path}(\mathbf{p}_i \oplus_n j) - S_{path}(\mathbf{p}_i), & otherwise \end{cases}$$

- Determine which tasks are winnable
- Select the index of the best eligible task, j^*, and select best location in the plan to insert the task, n_j^*
- If $c_{ij^*} \le 0$, then return. otherwise, continue
- Update agent information
- Update shared information vectors
- if $l_b = L_t$, then return, otherwise, go to 1.

Algorithm summary

- Calculate marginal score for all tasks
- Determine which tasks are winnable

  $$h_{ij} = \mathbf{I}(c_{ij}(\mathbf{p}_i) > y_{ij}), \forall j \in \mathcal{J}$$

- Select the index of the best eligible task, j^*, and select best location in the plan to insert the task, n_j^*
- If $c_{ij^*} \leq 0$, then return. otherwise, continue
- Update agent information
- Update shared information vectors
- if $l_b = L_t$, then return, otherwise, go to 1.

Algorithm summary

- Calculate marginal score for all tasks
- Determine which tasks are winnable
- Select the index of the best eligible task, j^*, and select best location in the plan to insert the task, n_j^*

$$j^* = \max_{j \in \mathcal{J}} c_{ij} h_{ij}$$

$$n_j^* = \max_{n \in \{0,\ldots,l_b\}} S_{path}(\mathbf{p}_i \oplus_n j^*)$$

- If $c_{ij^*} \leq 0$, then return. otherwise, continue
- Update agent information
- Update shared information vectors
- if $l_b = L_t$, then return, otherwise, go to 1.

Algorithm summary

- Calculate marginal score for all tasks
- Determine which tasks are winnable
- Select the index of the best eligible task, j^*, and select best location in the plan to insert the task, n_j^*
- If $c_{ij^*} \leq 0$, then return. otherwise, continue

- Update agent information
- Update shared information vectors
- if $l_b = L_t$, then return, otherwise, go to 1.

Algorithm summary

- Calculate marginal score for all tasks
- Determine which tasks are winnable
- Select the index of the best eligible task, j^*, and select best location in the plan to insert the task, n_j^*
- If $c_{ij^*} \leq 0$, then return. otherwise, continue
- Update agent information

$$\mathbf{b}_i \leftarrow (\mathbf{b}_i \oplus_{l_b} j^*)$$

$$\mathbf{p}_i \leftarrow (\mathbf{p}_i \oplus_{n_j^*} j^*)$$

- Update shared information vectors
- if $l_b = L_t$, then return, otherwise, go to 1.

Algorithm summary

- Calculate marginal score for all tasks
- Determine which tasks are winnable
- Select the index of the best eligible task, j^*, and select best location in the plan to insert the task, n_j^*
- If $c_{ij^*} \leq 0$, then return. otherwise, continue
- Update agent information
- Update shared information vectors

$$y_{i(j^*)} = c_{i(j^*)}$$

$$z_{i(j^*)} = i$$

- if $l_b = L_t$, then return, otherwise, go to 1.

Algorithm summary

- Calculate marginal score for all tasks
- Determine which tasks are winnable
- Select the index of the best eligible task, j^*, and select best location in the plan to insert the task, n_j^*
- If $c_{ij^*} \leq 0$, then return. otherwise, continue
- Update agent information
- Update shared information vectors
- if $l_b = L_t$, then return, otherwise, go to 1.

Simulation

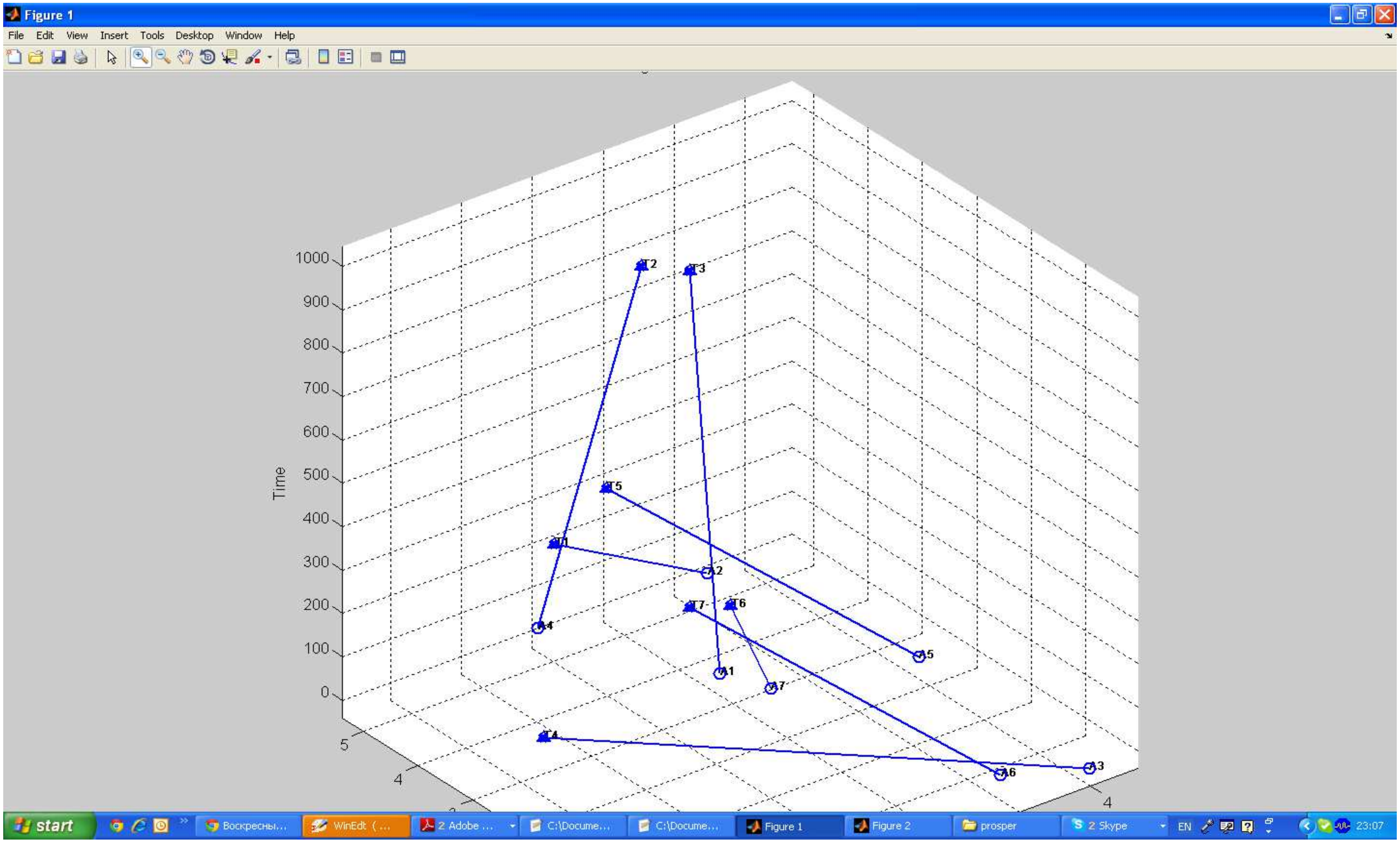


Figure 1
File Edit View Insert Tools Desktop Window Help
Time
1000
900
800
700
600
500
400
300
200
100
0
5
4
4
T2
T3
T5
T1
A2
T7
T6
A4
A5
A1
A7
T4
A6
A3
start
WinEdt
Figure 1
Figure 2
prosper
2 Skype
EN
23:07

The end

Thank you!