Motion planning problem classification

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1 Problem characteristics

State representation w.r.t. the time derivative:

- (x) Kinematic (position)
- (x, \dot{x}) Kino-dynamic (position and velocity)
- $(x, \dot{x}, \ddot{x}, F, \tau)$ Dynamic (position, velocity, acceleration, forces, torques)

Planning space:

- q Joint space (joint angles)
- y Task space (base pose, end-effector pose, ...)

Output type:

- x^* position
- $x_{0:T}^*$ time indexed trajectory (with different time parametrizations)
- $x_{0:n}^*$ plain (non-timed) trajectory

Output type w.r.t. the control type:

- \bullet x position
- \dot{x} velocity
- \bullet \ddot{x} acceleration
- τ force/torque

Problem type:

- $\underset{x}{\operatorname{argmin}}(x^{\intercal}Qx)$ Unconstrined problem (e.g. Newton step minimization)
- $\bullet \ \operatorname{argmin}_{x}(Qx+b)$ Linear programming problem
- $\operatorname{argmin}(x^{\top}Qx + c^{\top}x), \text{s.t.} \leq b$ QP problem (with equality an inequality constraints)
- $\mathop{\rm argmin}_x(f(x)^{\!\top} Q f(x) + c^{\!\top} f(x), \text{s.t.} A g(x) \leq b$ Sequntial QP problem
- $\underset{x}{\operatorname{argmin}} \|f(x)\|^2$, s.t. $g(x) \leq 0$ NLP
- $\underset{x}{\operatorname{argmin}} \|f(x,i)\|^2$, s.t. $g(x,i) \leq 0$ Mixed integer NLP
- $\underset{x}{\operatorname{argmin}} f(x) = \text{True}$ Sampling problem (uni- or bi- directional)
- $\underset{x}{\operatorname{argmin}} ||f(x)||^2$, s.t. g(x) = True Sampling optimization problem

Problem type could also be Hierarchical, e.g. Hierarchical Unconstrained or Hierarchical QP.

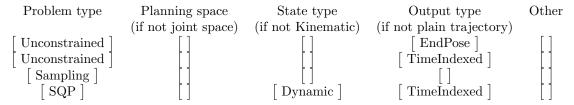
Constraint/cost term type:

- g(x) = c constant
- g(x) = Ax + b linear/affine
- $g(x) = x^{\mathsf{T}} A x$ squared/quadratic
- g(x) non-linear

Constraint/cost term differentiability:

- g(x) non-differentiable (e.g. collision check)
- $\frac{\partial g(x)}{\partial x}$ 1st order differentiable
- $\bullet \ \frac{\partial^2 g(x)}{\partial^2 x}$ 2nd order differentiable

2 Naming convention



The above problems will translate into:

- UnconstrainedEndPoseProblem (IK problem)
- UnconstrainedTimeIndexedProblem (AICO problem)
- SamplingProblem (OMPL problem)
- $\bullet \ \ SQPDynamic Time Indexed Torque Controlled Problem$