PS 5 Solution

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x1

Generalized Roy Model: The union / non-union wage differential

$$\ln w_i^u = x_i'\beta^u + u_i^u \quad , \; u_i^u \sim \text{Normal}(0, \sigma_u^2)$$
$$\ln w_i^N = x_i'\beta^N + u_i^N \quad , \; u_i^N \sim \text{Normal}(0, \sigma_N^2)$$
$$U_i^* = \delta_0 + \delta_1(\ln w_i^u - \ln w_i^N) + x_i'\delta_2 + z_i'\delta_3 - v_i > 0$$
$$v_i \sim N(0, \sigma_v^2)$$
$$\text{Corr}(\sigma_u^2, \sigma_N^2) = \text{corr}(\sigma_u^2, \sigma_v^2) = \text{corr}(\sigma_N^2, \sigma_v^2) = 0$$

a) Randomly chosen individual:

$$\mathbb{E}[\ln w_i^u - \ln w_i^N \mid x_i] = x_i'\beta + \mathbb{E}[u_i^u \mid x_i] - x_i'\beta^N - \underbrace{\mathbb{E}[u_i^N \mid x_i]}_{\overset{!}{=}0}$$
$$= x_i'(\beta^u - \beta^N)$$

Exp. differential for union workers with x_i:

$$\mathbb{E}[\ln w_i^u - \ln w_i^N \mid x_i, U_i^* > 0]$$
$$= \mathbb{E}[\ln w_i^u - \ln w_i^N \mid \delta_0 + \delta_1(\ln w_i^u - \ln w_i^N) + x_i\delta_2 + z_i'\delta_3 - v_i > 0]$$
$$= x_i'(\beta^u - \beta^N) + \mathbb{E}[u_i^u - u_i^N \mid \delta_0 + \delta_1(x_i'(\beta^u - \beta^N)) + \delta_1(u_i^u - u_i^N) + x_i'\delta_2 + z_i'\delta_3 - v_i > 0]$$

Define $u_i \equiv u_i^u - u_i^N$, with $\text{Var}(u_i) = \text{Var}(u_i^u) + \text{Var}(u_i^N) - \underbrace{2\,\text{cov}(u_i^u, u_i^N)}_{\overset{!}{=}0}$

$$= \sigma_u^2 + \sigma_N^2$$

$$\mathbb{E}[\ln w_i^u - \ln w_i^N \mid U_i^* > 0]$$
$$= x_i'(\beta^u - \beta^N) + \mathbb{E}[u_i^u - u_i^N \mid \underbrace{v_i + \delta_1(u_i^N - u_i^u)}_{\text{error component}} < \delta_0 + \delta_1(x_i'(\beta^u - \beta^N)) + \underbrace{x_i'\delta_2 + z_i'\delta_3}_{\text{"known"}}]$$

Defining $D \equiv v_i + \delta_1 (u_i^N - u_i^u) = v_i - \delta_1 (u_i^u - u_i^N)$

(0) and $c \equiv \delta_0 + \delta_1 x_i'(\beta^u - \beta^N) + x_i'\delta_2 + z_i'\delta_3$, this can be rewritten

(1) $\mathbb{E}[\ln w_i^u - \ln w_i^N \mid u_i^* > 0] = x_i'(\beta^u - \beta^N) + \underbrace{\mathbb{E}[u_i^u - u_i^N \mid D < c]}_{(*)}$

Now we need to solve for (*)

$\Rightarrow$ regress $u_i^u - u_i^N$ on D, $(u_i^u - u_i^N) = a_1(D) + V$, $D \perp V$

Note that $Var(D) = \delta_1^2 \cdot (\sigma_u^2 + \sigma_N^2) + \sigma_v^2$

Then, $a_1 \equiv \frac{cov(u_i^u - u_i^N, D)}{Var(D)} = \frac{-\delta_1(\sigma_u^2 + \sigma_N^2)}{\delta_1^2 \cdot (\sigma_u^2 + \sigma_u^2) + \sigma_v^2}$ } normalized to 1

Further, $\mathbb{E}[D \mid D < c] = \sigma \cdot \mathbb{E}\left[\frac{D}{\sigma} \,\middle|\, \frac{D}{\sigma} < \frac{c}{\sigma}\right] = \underset{\underset{1}{\shortparallel}}{\sigma} \cdot \lambda(-c)$

Therefore, (1) becomes

$$x_i'(\beta^u - \beta^N) + \frac{-\delta_1(\sigma_u^2 + \sigma_N^2)}{\delta_1^2 \cdot (\sigma_u^2 + \sigma_N^2) + \sigma_v^2} \cdot \mathbb{E}[D \mid D < c]$$

$$= x_i'(\beta^u - \beta^N) + \frac{\delta_1(\sigma_u^2 + \sigma_N^2)}{\underbrace{\sqrt{\delta_1^2 \cdot (\sigma_u^2 + \sigma_N^2) + \sigma_v^2}}_{=1}} \lambda(-c)$$, with c defined in (0)

$$= x_i'(\beta^u - \beta^N) + \delta_1(\sigma_u^2 + \sigma_N^2)\lambda(-c)$$

b) This is not possible. As we have just seen in a), for a union worker ~~it holds~~ that $\mathbb{E}[u_i^u - u_i^N \mid u_i^* > 0] \neq 0$. More generally, for the sample of all union workers, it holds generally that $\mathbb{E}[u_i^u \mid u_i^* > 0] \neq 0$.

Similarly, for non-union, $\mathbb{E}[u_i^N \mid u_i^* \leq 0] \neq 0$, so that we have a selection problem (and in the context of plain OLS, endogeneity in some sense). OLS estimates for β^u, β^N are biased.

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c)

One could use a Tobit type 2. This allows the process of selection and the process of the outcome to be independent. (condit. on $\mathbf{x}$).

Or, what will be discussed in 5.2, could implement a biprobit. However, contrary to 5.2, the two eq. that are estimated with biprobit would be two wage equations (i.e. for w_i^u, and for w_i^N).

d) Heckman's two-step-method to control for the selection that occurs in sample. This is what the plain regression exercise b) would have ignored.

1st Step: Selection: - run probit on union dummy and estimate it's effect, denote by $\hat{\beta}$

- use this to find $\lambda(-\hat{c})$

2nd Step: Run OLS of $\ln w_i^u$ (and same for $\ln w_i^N$) on observables, controlling for $\lambda(-\hat{c})$.

$\Rightarrow$ Gives us $\hat{\beta}^u, \hat{\beta}^N$.

e)

Once we have good estimates for β^u, β^N, use these to create fitted values $\ln \hat{w}_i^u$, $\ln \hat{w}_i^N$.

Then, can include these in a probit of u_i^* on $(\ln \hat{w}_i^u - \ln \hat{w}_i^N), x_i, z_i$ (plus constant) to get the structural parameters δ_0 (for constant), δ_1 (for $(\ln \hat{w}_i^u - \ln \hat{w}_i^N)$), δ_2 (for x_i), δ_3 (for z_i).

f) Lee (1978)

- Rel. Return = just compare diff. groups
- relative return on education:

	union	non-union
	$\frac{ED_5 \text{ coeff.}}{ED_4 \text{ coeff.}} = \frac{0.139}{0.111} = 1.25$	$\frac{0.282}{0.157} = 1.796$

Relatively, the return on education (ED_5 vs. ED_4, i.e. "college premium") is much higher in the non-union sector, for non-union workers.

◦ Market experience: (same coeff. as squared term)

$\Rightarrow$ market experience has more of an effect in the unionized sector vs. ununionized, 0.016 vs. 0.012

◦ Female / Male: Gender effect ("male premium") larger in unionized sector, i.e. 0.317 vs. 0.267

◦ Race: effect larger for non-union sector, 0.095 vs. 0.186 (non-union)

Health impediments: in absolute values, larger for non-unionized (i.e. more negative) 0.055 vs. 0.088

)) Union coefficient -0.168, in paper defined as the coefficient $-\frac{f(\Psi_i)}{F(\Psi_i)}$

non-union 0.136, coeff. on $\frac{f(\Psi_i)}{1-F(\Psi_i)}$

For both sectors, observe positive selection. Individuals self-select into sector that gives them higher wages.

)) Judging from Table 6 in particular, it seems that the coefficient on $(\ln\hat{\omega}_u - \ln\hat{\omega}_n)$ is quite high in its magnitude, even when taking into account the standard error. Since these are the structual estimates, I'd say the wage diff. is very important in explaining the prob. of membership.

.) The reduced form "answers" a different question than the structual form. The reduced form yields the <u>net effects</u> of different factors / characteristics on membership. In particular, it results from personal preferences, firm considerations, cost-benefit-analysis, etc. These are underlying forces that do not affect the structual parameters, but do affect the reduced form estimates ("mutatis mutandis").

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Q2

a) See Stata do-file

b) In the present context (being a mother of a young child and LF participation) we suspect that there is dependence between these two decisions.

Biprobit takes this into account by allowing error dependence, measured by correlation ρ between error terms. Two standard, independent probits as in a) is like assuming a correlation of zero).

⇒ Here we should use biprobit, since there is evidence in favor of error correlation.

⇒ Efficiency gain relative to two sep. equation probits.

c)

Note: We assume that only 1 factor enters eq. of other, not both (which would not be possible), but of course could argue that unemployment might affect motherhood choice...

The estimated correlation is 0.29764 between the probit errors of being a mother an LF participation, even after controlling for unobservables.

This can be seen as evidence that there is positive correlation between unobservables, and that there is selection on unobservables.

d) From AET (2003)

→ bound treatment effect accounting for selection on unobservables.

Their method quantifies and bounds the selection bias due to selection on unobservables by varying ρ. We can then determine that ρ would have to be so that the coeff. on the kids dummy vanishes to zero.

- Regarding the bounds, the lower bound would be reached when the part of LF participation that is related to observables and the part related to unobservables have the same relationship with the kids dummy, i.e. motherhood.

- Upper bound: When motherhood is orthogonal to LF participation (i.e., the coeff. on kids dummy is zero in the first equation of the probit in (c)), $\varphi = 0$

$\Rightarrow$ In the present case, it seems plausible that there is more selection on observables than unobservables.

e) Imposing the constraint gives a $\hat{\varphi} = -0.50703$

$\Rightarrow$ Requires strong neg. correlation of errors to have kids dummy having no effect on LF.

f) $\varphi = -0.5$ seems not plausible. As argued (mentioned) before, unobservables hardly account for as much as observables in the selection. Then, such a negative rho <u>seems not</u> plausible.

g) Lower bound: "same" selection on unobservables than on observables.

Idea: probably can use from the paper that

$$0 \le \varphi \le \frac{cov(x'\beta, x'\gamma)}{var(x'\gamma)}$$ (in this case φ nonnegative)

Although, it seems not clear how to quantify this bound from the paper.

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*PS5 Question 2
use "/Users/Stefan/Desktop/FS19/Econometrics II - PhD/PS5/mroz.dta"

***Generating Dummy indicating of kids below 6 years of age

gen kidsbelow6=0
replace kidsbelow6=1 if kidslt6>0

***************************
            *a
***************************
*Probit inlf
probit inlf nwifeinc educ exper expersq age, robust

*Probit kidsbelow6
probit kidsbelow6 nwifeinc educ exper expersq age, robust


***************************
            *b
***************************
*Biprobit
biprobit inlf kidsbelow6 nwifeinc educ exper expersq age, robust


***************************
            *c
***************************
biprobit (inlf=kidsbelow6 nwifeinc educ exper expersq age) (kidsbelow6 nwifeinc educ exper expersq age), robust


***************************
            *e
***************************
constraint 1 kidsbelow6=0
biprobit (inlf=kidsbelow6 nwifeinc educ exper expersq age) (kidsbelow6 nwifeinc educ exper expersq age), constraint(1)


***************************
            *f
***************************
reg inlf kidsbelow6 nwifeinc educ exper expersq age
predict x_beta
replace x_beta=x_beta-kidsbelow6*_b[kidsbelow6]
reg kidsbelow6 nwifeinc educ exper expersq age
predict x_gamma
corr x_beta x_gamma
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