PS 5

$\ln w_i^U = x_i' \beta^U + u_i^U \qquad \rightsquigarrow u_i^U \sim N(0, \delta_u^2)$

$\ln w_i^N = x_i' \beta^N + u_i^N \qquad \rightsquigarrow u_i^N \sim N(0, \delta_v^2)$

$U_i^* = \delta_0 + \delta_1 (\ln w_i^U - \ln w_i^N) + x_i' \delta_2 + z_i' \delta_3 - v_i > 0$

a)

$$Pr(\text{being unionized} | x_i) = Pr(U_i > 0 | x_i) =$$

$$= Pr(\delta_0 + \delta_1 (\ln w_i^U - \ln w_i^N) + x_i \delta_2 + z_i' \delta_3 - v_i > 0 \,|\, x_i)$$

$$= Pr(\delta_0 + \delta_1 (x_i \beta^U + u_i^U - x_i \beta^N - u_i^N) + x_i' \delta_2 + z_i' \delta_3 - v_i > 0 \,|\, x_i)$$

$$= Pr(\delta_0 + \delta_1 x_i' (\beta^U - \beta^N) + x_i' \delta_2 + z_i' \delta_3 > v_i - \delta_1 (u_i^U - u_i^N) \,|\, x_i)$$

$$= Pr\left(\frac{\overbrace{\delta_0 + \delta_1 x_i' (\beta^U - \beta^N) + x_i' \delta_2 + z_i' \delta_3}^{A}}{\delta^*} > \frac{\overbrace{v_i - \delta_1 (u_i^U - u_i^N)}^{\varepsilon_i^*}}{\delta^*} \,\Big|\, x_i \right)$$

where $\delta^* = \sqrt{\delta_{v_i}^2 - \delta_1 (\delta_{u_i}^{U\,2} - \delta_u^{N\,2})}$

We need to assume $\delta^* \sim N$, because this doesn't follow directly from the assumptions stated in the PS.

$$= \Phi\left(\frac{\overbrace{\delta_0 + \delta_1 x_i' (\beta^U - \beta^N) + x_i' \delta_2 + z_i' \delta_2}^{A}}{\delta^*} \right) \text{ - probability of being in the union}$$

Expected union/non-union wage diff for a random person:

$$E(\ln w_i^U - \ln w_i^N) = E(\ln w_i^U) - E(\ln w_i^N) = \underline{(\beta^U - \beta^N) E(x_i)} + \underbrace{E(u_i^U) - E(u_i^N)}_{0}$$

For a person who is in the union

$$E(\ln W_i^U \mid d_i=1, X_i) =$$

$$= x_i' B^U + E(u_i^U \mid d_i=1, x_i)$$

$$= x_i' B^U + E(T\varepsilon_i^* + \varepsilon_i \mid A > \varepsilon^*)$$

$$= x_i' B^U + T_U E(\varepsilon_i^* \mid A > \varepsilon^*)$$

$$= x_i' B^U - T_U \lambda(A)$$

Note that:

$$E(\varepsilon_i^* \mid A > \varepsilon^*) = -\lambda(A) = \frac{\phi(A)}{\Phi(A)}$$

$$E(\varepsilon_i^* \mid A < \varepsilon^*) = \frac{\phi(A)}{1-\Phi(A)} = \lambda'(A)$$

Consider the regression

$$u_i^U = T_u \varepsilon_i^* + \varepsilon_i$$

where $\varepsilon_i^* = v_i - \delta_1(u_i^U - u_i^N)$

$$T_u = \frac{\text{cov}(u_i^u, \varepsilon_i^*)}{\text{Var}(\varepsilon_i^*)} = \frac{E(u_i^u, v_i - \delta_1(u_i^u - u_i^N))}{\text{Var}(\varepsilon_i^*)} = \frac{-\delta_1 \delta_u^2}{\delta^*}$$

$$E(\ln w_i^N \mid d_i=0, x_i) = E(\ln w_i^N \mid A < \varepsilon^*)$$

$$= x_i' B^N + T_n E(\varepsilon_i^* \mid A < \varepsilon^*)$$

$$= x_i' B^N + T_N \lambda'(A) =$$

$$= x_i' B^N + T_N \lambda'(A)$$

$$= x_i' B^N + T_N \lambda'(A)$$

$$T_n = \frac{\delta_1 \delta_N^2}{\delta^2}$$

$$\lambda'(A) = \frac{\phi(A)}{[1-\Phi(A)]}$$

$$E(\ln w_i^U - \ln w_i^N) = x_i'(B^U - B^W) + (T_U \lambda(A) + T_n \lambda'(A))$$

b) No, because we have a selection problem.
ie $E(u_i^U | d_i=1) \neq 0$ & $E(u_i^N | d_i=0) \neq 0$ → the exclusion restriction is violated for B^U & B^N, so the OLS estimate would be biased.

p 290

see how I defined A before on p.1

MLE

c) $L = Pr(d_i=1 | x_i, \theta_i, z_i) f(\ln(w_i) | x_i \theta_i, z_i) = \begin{cases} \ln w^U & \text{if } u_i^* \geq 0 \\ \ln w^N & \text{if } u_i^* < 0 \end{cases}$

$= \Phi(A) \cdot \phi(y - x_i' B^U) \cdot (1 - \Phi(A)) \cdot (y - x_i' B^N)$

We can either estimate the wage equations by MLE or by following the procedure described bellow.

Step 1 Reduced form probit / participation equation [union status equation]

d) substitute the equations for $\log w_{ui}$ & $\log w_{ni}$ into u_i^* and estimate the probit by MLE. } Reduced form probit

↳ see point a) for derivation

From this you get $\underbrace{\hat{\delta}_0}_{\delta^{**}} + \underbrace{\delta_1 (B^U - B^N) \cdot \delta_2}_{\delta^*}$, $\underbrace{\hat{\delta}_3}_{\delta^{*}}$

Step 2 estimating the wage equation

run the wage regression, using estimates from step (1).

$\ln w_i^U = x_i' B^U - J_u \lambda(A)$ → estimate from step 1 earlier

This gives you an estimate of B^U.

$\ln w_i^N = x_i' B^N + J_n \lambda(A)$ ⇒ get estimate of B^N

e) reestimating the "structural" probit to retrieve $\delta_0, \delta_1, \delta_2$ using estimates from step 2
→ plug the estimates from the wage regression pair into the probit from step 1, to get the structural ~~...~~ probit estimates

f) • Returns to education are higher in the non-unionised sector → for people with higher education is better not to be in the union
• HE has a higher return in the unionised sector
• males recieve higher wage then females, but the diff is smaller in the nonunionised sector
→ black-white wage diff smaller in unionised sector
- HLT has a less negative effect on unionised sector

g) in Table 1 it's $-\frac{\phi(.)}{\Phi(.)}$, in table 2 it's $\frac{\phi(.)}{1-\Phi(.)}$

So both selectivity variables / truncation effects are positive, meaning we only observe the upper section of the union wage distributions.
positive truncation effects stem for the fact that individuals self select into sectors that pays them better than the average person

h) It seems to be the most important factor explaining the union status

i)

1) table 6 → ~~the~~ biggest predictor of unionisation is the union non-union wage differential

Table 6 tells us that unions may prefer to hire higher educated workers, even though they themselves would prefer the non-unionized sector. The same results hold for the "race dummy"

Table 7 gives net effects on the union status.
There is difference b/w table 6 & 7 when it comes to education; ~~firms~~ unions prefer to choose more educated workers, but ~~some~~ educated workers prefer the non-unionised sector and the net effect is negative.
Unions tend to discriminate against non-whites, but the returns of non-whites are greater in the unions than non-unions & the net-effect is negative

↳ constructed by plugging back estimates of the wage equations into "reduced form probit"

(2)

b) This is essentially a SUR model.

We have two separate equations where the error terms are assumed to be correlated across equations. In the case of bivariate probit we have:

$$y_1 = \begin{cases} 1 \text{ if } y_1^* > 0 \\ 0 \text{ otherwise} \end{cases} \qquad y_2 = \begin{cases} 1 \text{ if } y_2^* > 0 \\ 0 \text{ otherwise} \end{cases}$$

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \Big| X \sim N\left[\begin{matrix} 0, \\ 0, \end{matrix} \begin{matrix} 1 & \rho \\ \rho & 1 \end{matrix}\right]$$

$$L(B_1, B_2) = \left(\prod P(y_1 = 1, y_2 = 1) \,|\, B_1, B_2\right)^{y_1 y_2} P(y_1 = 0, y_2 = 1 | B_1, B_1)^{(1-y_1)y_2}$$

$$P(y_1 = 1, y_2 = 0 | B_1 B_2)^{y_1(1-y_2)} P(y_1 = 0, y_2 = 0)^{(1-y_1)(1-y_2)}$$

↳ the advantage is that estimating the two models jointly we gain efficiency compared to if we estimated them separately by OLS

d) First they try different values of rho (correlation) between the error terms affecting C & errors affecting Y. to see at what value the negative affect of C on Y becomes 0 & afterwards positive.

Then they try to bound the effect from below by assuming that the correlation of observables with C is the same as the correlation of unobservables w C

ie part of mortality related to observable & part related to unobservables are assumed to have the same relationship with C.

$$\mathrm{Proj}(c^{*} | x'\gamma, \varepsilon) = \phi_0 + \phi_{x'\gamma}\, x'\gamma + \phi_\varepsilon \varepsilon$$ → Lower bound $\phi_{x'\gamma} = \phi_\varepsilon$
upper bound $\phi_\varepsilon = 0$

Upper bound would be the calculated effect assuming c is exogenous!

In the end they provide the framework on analyzing the relative strength of selection on unobservables & selection on observables i.e.

$$\frac{E(\varepsilon|c=1) - E(\varepsilon|c=0)}{\mathrm{var}(\varepsilon)} = \lambda \frac{E(x'\gamma|c=1) - E(x'\gamma|c=0)}{\mathrm{var}(x'\gamma)}$$

If $\lambda = 1$ - selection on observables is the same as selection on unobservables