

Equations of motion look like:

$$\frac{d}{dt}(\vec{x}) = f(t, \vec{x})$$

Example equations of motion:

$$\frac{d}{dt}(x_1) = x_2$$

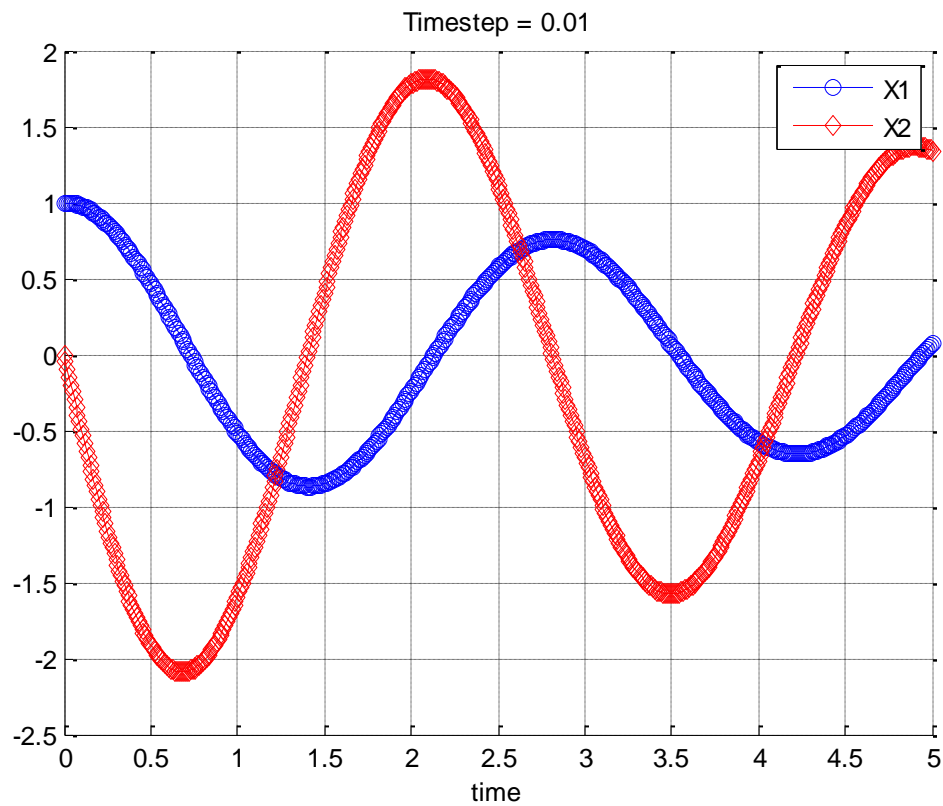
$$\frac{d}{dt}(x_2) = -kx_1 - cx_2$$

$$k = 5, \quad c = 1/4$$

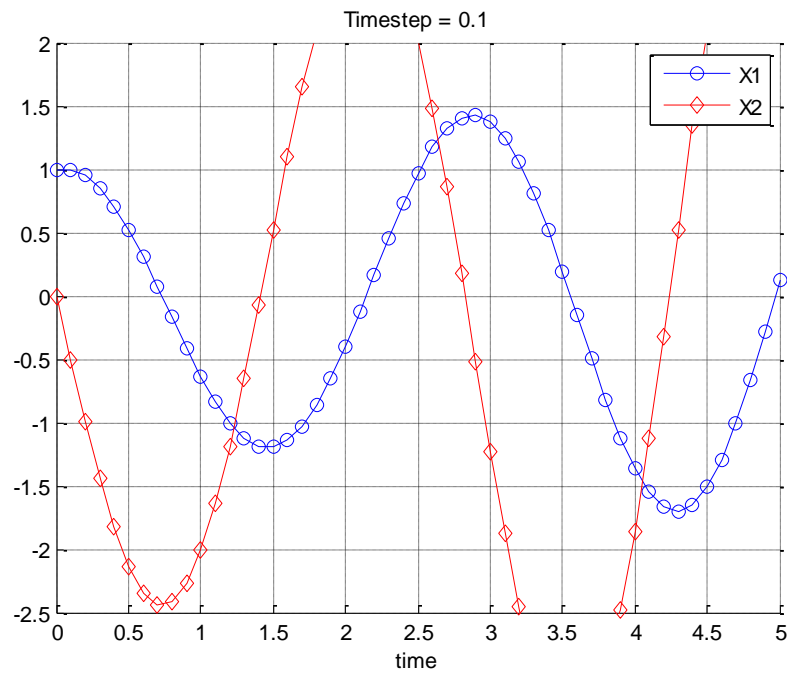
Example integrator: First Order Forward Euler

$$\vec{x}(t + dt) = \vec{x}(t) + dt * f(t, \vec{x}(t))$$

Starting with an initial state of $x_1(0) = 1$ and $x_2(0) = 0$, we can march the state forward in time using the above equation with our choice of timestep. Let's begin with $dt = 0.01$:



If we increase the size of the timestep by a factor of ten to only 0.1 seconds, the integrator performs poorly (even goes unstable!). This solution is inaccurate due to simulation error. ☹️



Going the other direction, a factor of ten decrease to 0.001 seconds gives no change from the original $dt=0.01$ case. To achieve better accuracy with larger timesteps, a more sophisticated integrator is needed.

