Methodology, Modelling and Consulting Skills

Project Report

Modelling the Future Transportation of Dundee

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Executive Summary

Using mathematical models and linear programming to determine how the Dundee City Council should phase out it's Public EV charging stations to meet the expected demand of an all EV vehicle city within 2035. We have created an optimal solution based on the current potential locations and demand. The result that we found satisfied 80% of the demand, it recommended one large charging hub of fast charging stations outside the city centre, which makes sense because this would likely be used by taxis and council vehicles. For the rapid charging stations it recommended an even distribution throughout the city, avoiding the centre. To comply with the new Scottish government requirements we focus mainly on Fast and Rapid chargers. Additionally, we have created an additional model to future proof our recommended plans.

1 Introduction

Charging stations and charging points have distinct meanings. A charging station can contain multiple charging points, and there are three different types of charging points available (slow, fast and rapid).

We were given the task of finding the optimal layout of electric vehicle charging points for Dundee City Council as the UK government wants to phase out non-electric vehicles before 2035 [4]. The council had 4 requirements for our solution:

- Limit number charging point locations in each square. In particular in the city centre to limit the traffic.
- Limit the number of charging locations in the city centre to limit the increase in traffic in the city centre.
- Meet the demand for all types of car including hybrids, meaning that we need multiple types of charger.
- Not all of the demand needs to be fulfilled

We were supplied with 3 different future forecasts for the demand. And data relating to the current charging stations in Dundee, and the current interest points in the city. All of this data was given on a simple grid. Using this data we made a linear model which we used to find the optimal arrangement for the facilities.

Many different grants and government schemes are supporting this electric transformation. Scotland's choice is using the ChargePlace Scotland (CPS), which is Scotland's comprehensive electric vehicle charging network which has $\sim$ 2,402 publicly available charge points across the country. [1]

2 Literature Review

Using "New Thinking In GIScience"[2] we were able to find various models relevant to our problem.

One potential method to approach this problem would be to model it as a fixed-charge facility location problem, where we treat all grid squares as both demand sites and candidate locations for facilities (charging points).

Another potential approach is the set-covering problem, which takes the distance between supply and demand sites into account. This is something we should consider, as it would be inconvenient to have to drive across the city to charge your car.

Most facility-allocation models use a fixed-cost condition on the cost of implementing a facility. Our facilities can have different configurations and this means that we can't naively use the fixed-cost approach.

The Scottish government has released guidelines for Public chargers, requiring charging capacity to be above 7KwH [5]. Meaning slow chargers cannot meet the requirement. They are also ineffective as they cost as much as fast chargers. Additionally, another Scottish city namely the City of Edinburgh recently only implemented rapid and fast chargers [7]. Thus, we decided to not include slow chargers in our implementation. Fast and slow slow chargers also cover the same type of demand, so there are no vehicles going without their demand being met as a result of this decision. Some councils were also recommending fast and rapid chargers for public use, whereas slow chargers are recommended for personal use. 40% [5] of Dundee's housing has dedicated parking (not on-street), it follows that around half of that could be expected to adopt at home slow chargers slow personal chargers, so only 80% of our demand would need to be met.

The cost of installing a new charging point should be on average £750 for fast charging points and on average £1250 for a rapid charging point. [6]

3 Optimisation Model: Fixed Charge Facility Location

3.1 Assumptions

- There are no slow charging points to be implemented
- The cost per KWh always favours as many rapid as possible, so long as the ratio of the cost of fast to rapid is less than the ratio of power output. We would also recommend using as many rapid chargers regardless of the costs because it requires the fewest charging points and causes the least traffic. So we will ignore the cost in the objective function.
- Demand sites and supply sites are both the grid squares in the demand data. This is because it is a simple way to determine where cars can get to.
- Demand can be supplied by any grid square, however, doing so will be penalised in the objective function
- Travel distance is approximated by the distance between grid squares

- There are no operating/maintenance costs for the charging stations
- The charging stations produce no revenue
- The cost of building a charging point is fixed for each type of charger
- The council can build charging stations in any grid that isn't on the sea
- The proportion of cars that use only fast charging stations is around 42% [8]
- The proportion of fast charging stations is around 70%, this is the full proportion of fast-only compatible cars plus half the proportion of fast and rapid compatible cars
- The discount for potential charging points is fixed at some rate for each site.
- The council doesn't expect the full demand, we worked out that 80% is a good estimate in Section 2

3.2 Model Specification

3.2.1 Uncapacitated Facility Location Problem

For this part we introduce the fundamentals of a fixed-charge facility location problem. The decision variables are:

$$x_{i,j} = \begin{cases} 1 \text{ if demand in grid square } j \in \mathcal{J} \text{ is fulfilled by charging station in grid square } i \in \mathcal{I} \\ 0 \text{ otherwise} \end{cases}$$

$$z_i = \begin{cases} 1 \text{ if we open a charging station in grid square } i \in \mathcal{I} \\ 0 \text{ otherwise} \end{cases}$$

Note that indices $\mathcal{I}$ and $\mathcal{J}$ both represent all grid squares, however, we represent them separately to indicate demand sites and candidate charging station sites. To address the multiple types of charging points that are available, we must introduce a new index $k \in \mathcal{K}$, where $\mathcal{K} = \{1, 2, 3\}$, $k = 1$ represents slow charging points, $k = 2$ represents fast charging points, and $k = 3$ represents rapid charging points. We must also introduce a new variable:

The basic constraints included at this phase of the model are:

$$\begin{aligned} \sum_{i \in \mathcal{I}} x_{i,j} &= 1 \quad \forall j \in \mathcal{J} \\ x_{i,j} &\leq z_i + \sum_{k \in \mathcal{K}} C_{i,k} \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \\ x_{i,j} &\in \{0, 1\} \quad \forall i \in \mathcal{I}, j \in \mathcal{J} \\ z_i &\in \{0, 1\} \quad \forall i \in \mathcal{I} \end{aligned}$$

where $C_{i,k}$ is the number of current charging points of type k in grid square i. We include this in the constraint because we need a facility to be open in our model if it already is open in reality.

3.2.2 Distance Between Squares

It would be useful to have a variable that stores the distance between squares. We use this simple model in place of calculating the exact distance along the roads because we wanted to avoid scope creep in the project. This could be improved in future iterations of the model. $\mu_{i,j}$, which is defined as the distance between centre of grid square i and centre of grid square j measured in number of grid squares. To compute $\mu_{i,j}$, we must first represent each grid square in terms of two-dimensional spatial coordinates:

$$\begin{aligned} X &= \mathcal{I} \text{ or } \mathcal{J} \\ A &\in X \times X \\ B &\in X \times X \end{aligned}$$

Where A_1 is the x-coordinate, and A_2 is the y-coordinate. The Euclidean distance is then computed as:

$$\mu_{A,B} = \sqrt{(A_1 - B_1)^2 - (A_2 - B_2)^2}$$

3.2.3 Extending the Model

$$y_{i,k} = \text{ number of type } k \text{ charging points in grid square } i$$

We also want to limit the traffic in the city centre. To do this we assumed that the increase in traffic from the charging stations is directly proportional to the number of charging stations that we add. We also assumed that the number of charging stations in the city centre itself should be zero. This is because we want the increase in traffic in the city centre to be as small as possible. We calculated a value to be the limit of the number of charging stations in a grid square using the following equation:

$$\eta_i = \delta_i \sqrt{\min\{\ \delta_i\}^{-1}}$$

Where δ_i is the distance to the city centre as given in the data provided. $y_{i,k}$ is subject to the following constraints:

$$y_{i,k} \in \mathbb{Z}^+ \quad \forall i \in \mathcal{I}, k \in \mathcal{K}$$

$$\sum_{k \in \mathcal{K}} (y_{i,k} + C_{i,k}) \leq \eta_i z_i \quad \forall i \in \mathcal{I}$$

The second constraint here is included since we must have a charging station in grid square i for there to be any charging points.

The main difference between our problem and the standard fixed-cost model is that each of our facilities can have a different cost. We know the cost of each of the charging points in each facility, so by combining these costs and multiplying by $y_{i,k}$ we are able to calculate the cost of implementing each facility.

3.2.4 Additional Considerations

This constraint makes sure that if there is demand then we have a facility trying to meet it.

$$\sum_{i \in \mathcal{I}} \geq 1 \quad \forall j \in \mathcal{J}$$

There are also a number of additional considerations we must include to fully characterise our problem. We will first consider how much demand we wish to fulfill, introducing the constraint:

$$P \sum_{j \in \mathcal{J}} d_j \leq \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} (y_{i,k} + C_{i,k}) E_k$$

where d_j is the demand in grid square j and E_k is the power output, measured in KWh per year, of a type k charging point. P is a scalar data value that can be manually set, where $P \in [0, 1]$. P is included to indicate what proportion of the total demand we intend to fulfill, so we can consider the different solutions we obtain (e.g. charging point locations and cost of implementation) when we seek to satisfy different amounts of demand.

Next we want to make sure that the minimum demand is being met in each grid square.

$$\sum_{j \in \mathcal{J}} P x_{i,j} d_j \leq \sum_{k \in \mathcal{K}} (y_{i,k} + C_{i,k}) E_k \quad \forall i \in \mathcal{I}$$

where P is the proportion of the demand that we are requiring we meet.

We also want an easy way to decide if the grid square is in the water, or in the middle of a farm. To do this we set a site to be closed if it has no demand. This is not ideal, especially in the early demand case where there aren't as many grid squares with demand.

$$z_j <= 0 \quad \forall j \in \mathcal{J} : d_j = 0$$

For each grid square i the sum of charging points should be greater greater than z_i, this ensures that if a facility is open then we put charging points in the facility.

$$\sum_{k \in \mathcal{K}} y_{i,k} \geq z_i$$

3.2.5 Objective Function

Finally, we define the objective function, and therefore the optimisation problem, to be:

$$\min \quad \sum_{i \in \mathcal{I}} \sum_{k \in \mathcal{K}} y_{i,k} T_k + \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \mu_{i,j} x_{i,j} + Q \sum_{i \in \mathcal{I}} (z_i - \phi G_i)$$

where ϕ is the discount percentage for potential charging points, G_i is 1 if grid square i is a potential charging point and 0 otherwise, and Q is the expected average cost of building any facility, however, setting $Q = 1$ should have no effect on the final result.

The first term takes into account the cost of building each facility. The second term is the distance of each charging station to the demand that it fulfills. The third term is the cost of setting up all the facilities.

3.3 Potential Improvements

- Use road data to find the distances between demand sites and supply sites.
- Use road data to find whether a grid square is close to a road/not in the sea.
- Implement the Queuing Theory model to contrast and compare with our model. This would give us an idea for how convenient the model is for the customers.
- The main improvement to our model would be to have an estimate for the cost of implementing each charging station on a case by case basis.
- Use traffic and road data to calculate the demand in a form that is better for the queuing model.

4 Alternative Model: Queuing System

The Facility location problem that has been developed so far is a top down approach, where the objective function being minimised is the set up cost and demand fulfillment cost of the facilities. Although the model does consider the distance between sites of demand and facilities as the fulfillment cost, it does not explicitly take into account the waiting time and travelling time of customers. An alternative method of modelling the specified problem would be a bottom-up approach, where a model can be developed based on queuing systems to minimise the waiting time and travelling time of customers for a fixed budget. The mathematical theory and algorithms for a queuing system based model has been developed and presented in this section.

This model remains an exploratory analysis, and there are more aspects to consider before it can be fully implemented. For example, we may require more data, understand whether the model has an optimal solution, and develop a suitable algorithm to solve the model. Due to the time constraints and scope of this project, we are unable to implement these next steps, however, they serve as a road map to any further work.

Due to space constraints, the full background of mathematical theory that the model is derived from cannot all be included in this section. Interested readers are referred to Appendix A.

4.1 Notations

4.1.1 Constants, parameters and matrices

- $N = \{1, 2, ..., N\}$: index set of customers
- $M = \{1, 2, ..., M\}$: index set of potential facilities
- $K = \{1, 2\}$: index set of different kinds (fast and rapid) of charging station, or sometimes a sub-site will be called k
- $D \in \mathbf{R}^{N\times M}$: distance matrix from a customer to a facility
- C_k : cost for each category of charging station
- Λ : total demand rate of service request
- $\lambda := \lambda_i$: demand (arrival) rate (how many demands required in a unit time) of customer i, and here we assume each arrival customer has same demand (arrival) rate λ
- μ_k: service rate of charging point k
- γ_j: total demand rate of facility site j
- $W_{j,k}$: random waiting time of facility site j, charging station in category k
- $\mathbf{E}W_{j,k}$: expected waiting time of facility site j, charging station k
- $\bar{p}$: maximum number of facilities that can be opened

- K : maximal budget of Dundee council each year
- $\bar{W}_{j,k}$: upper limit (bound) of waiting time of each sub-site k at site j
- $N_{j,k}(i)$: the counting (arrival) process for customers in sub-site k of site j
- $\tau_k(i)$: the inter-arrival (service) time for each single facility of category k
- p_k : the probability that each customer would like to choose kind k of charging sub-site

4.1.2 Decision Variables

- $x_{i,j}$: if event A is defined as the customer i would like to go to site j, then

$$x_{i,j} := \mathbf{1}_{\{A\}}$$

- $y_{j,k}$: how many facilities in category K will be opened at site j
- z_j : if the site j is decided to be opened

4.2 Model Assumptions

- Any customer will go to the **closest** charging point(s) to receive service;
- Arrival of customers at each same category of charging stations in each site, is confined to a Poisson distribution but arrival process of each sub-site k will be based on the thinning property:

$$N_{j,k}(i) \sim Poi(p_k\gamma_j)$$

- The service time of category k (individual) charging point is also an exponential random variable but it only depends on the category of charging stations k:

$$\tau_k(i) \sim exp\,(\mu_k)$$

- There is an upper bound on the permissible expected waiting time of customers (charging time of each vehicle is finite) [3]:

$$\mathbf{E}W_{j,k} := \int_{w\in[0,\infty)} |W_{j,k}|\cdot f_{W_{j,k}}(w)dw = \bar{W}_{j,k} < \infty$$

- Furthermore, FIFO assumption is applied in our case (i.e. ideally, every customer will not cut in lines);
- Thus, this is a $(M/M/c_{j,k})$ Queueing system in each site in terms of same kind of charging points.

4.3 Essential Definitions

Some key definitions for the model are included below, however, this is not an exhaustive list of all definitions used in the derivation of the model. For remaining definitions, please refer to appendix A.3.

- Average number of traveling customers (**ANTC** for short) (With same λ and μ_k assumption in this model):

$$T := \sum_{i\in M}\sum_{j\in N} \lambda d_{ij}x_{ij}/v$$

- Idle rate:

$$\rho_{j,k} = \frac{\gamma_{j,k}}{y_{j,k}\mu_k} < 1$$

  should be well defined for each sub-queueing system, because the system will expand and never reach at stable state, otherwise;

- **(M)**The average number of waiting customers (**ALWL** for short) per unit time (By **Proposition 4.3.2.1**):

$$V = \sum_{i\in M}\sum_{j\in N}\sum_{k\in K} \lambda_i x_{i,j}\cdot \mathbf{E}W_{i,j,k}$$

  where we don't expand the expression because $W_{j,k}$ is too complicated; If each has the same arrival rate λ and service rate μ_k for each category of charging station, V can be rewritten as following :

$$V = \sum_{i\in M}\sum_{j\in N}\sum_{k\in K} \lambda x_{i,j}\cdot \frac{\mathbf{E}L_q^{j,k}}{\gamma_{j,k}} = \sum_{i\in M}\sum_{j\in N}\sum_{k\in K} \frac{1}{p_k}\cdot \mathbf{E}L_q^{j,k}$$

4.4 Algorithm Specified

4.4.1 Objective function: Minimizing The ANTC and The ALWL (See Appendix A.4.2)[3]

$$(\mathbf{Q}) \quad \min \left\{ \sum_{i \in M} \sum_{j \in N} \lambda x_{ij} \left(\frac{d_{i,j}}{v} + \sum_{k \in K} \mathbf{E} W_{j,k} \right) \right\}$$

4.4.2 Constraints

- Maximal Facilities constraint:

$$\sum_{j \in N} \sum_{k \in K} y_{j,k} \leq \bar{p}$$

- Maximal Capacity constraint:

$$\sum_{k \in K} y_{j,k} \leq \bar{p}_j, \forall j \in N$$

- Compulsory Arrival constraint:

$$\sum_{j \in N} x_{i,j} = 1, \forall i \in M$$

- Maximal Potential Demands constraint:

$$\sum_{i \in M} x_{i,j} \leq \sum_{k \in K} y_{j,k}, \forall j \in N$$

- Closest Facilities constraint [3]:

$$\sum_{j' \in N} d_{i,j'} x_{i,j'} \leq (d_{i,j} - \Delta) \cdot z_k + \Delta$$

- EITHER-OR constraint 1 between y and z:

$$1 - \sum_{k \in K} y_{j,k} \leq M \cdot (1 - z_j), \forall j \in N$$

- EITHER-OR constraint 2 between y and z:

$$\sum_{k \in K} y_{j,k} \leq M \cdot z_j, \forall j \in N$$

- Maximal Budget constraint:

$$\sum_{j \in N} \sum_{k \in K} C_k \cdot y_{j,k} \leq K$$

- Upper Bound for Waiting Time:

$$\sum_{i \in M} \sum_{k \in K} \lambda_i x_{i,j} \cdot \mathbf{E} W_{j,k} \leq \bar{W}, \forall j \in N$$

- Stable System Constraint:

$$\rho_{j,k} := \frac{\gamma_{j,k}}{y_{j,k} \cdot \mu_k} < 1$$

- Mixed-Integer Programming constraint:

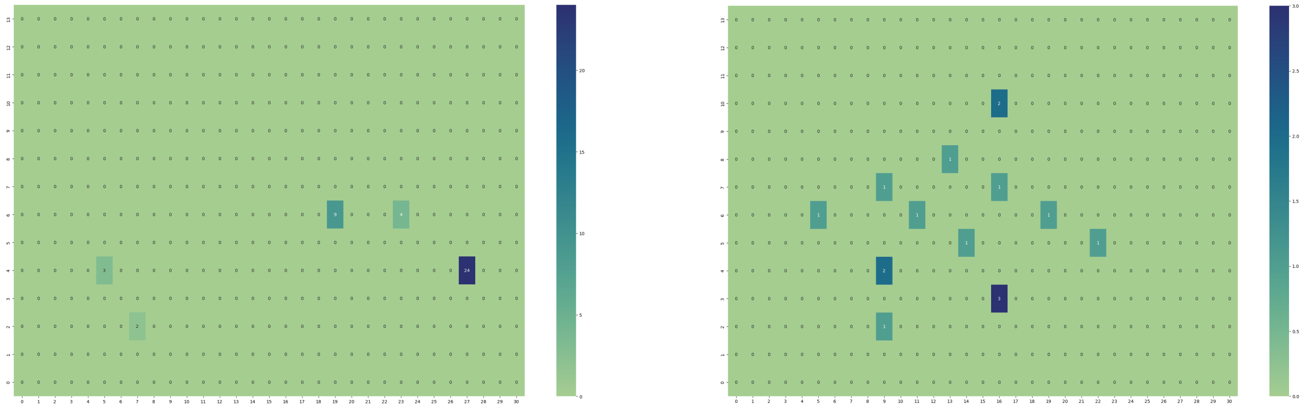
$$x_{i,j} \in \{0,1\},\ y_{j,k} \in \mathbf{Z}^+,\ z_j \in \{0,1\}$$

4.5 Potential Improvements

Not every linear programming problem is always solvable under every case. Therefore, the same is applied for the algorithm based on queuing system. From this perspective, an attempt to come up with an existence theorem for such an algorithm will be crucial for application in the real world. Specifically, in which scenarios it will generate finite optimal solutions, or prove to be an unbounded problem. It is still likely to be proven that the algorithm is solvable (i.e. has finite optimal solutions) all the time.

The other thing is grabbing the demand rate for customers and service rate for charging points, both of which have been specified above. One potential approach is by applying simulation.

Figure 1: Map view of recommended charging station locations. Blue points indicate fast charging points and red points indicate rapid charging points. The size of the points indicate the number of charging points at the location.



(a) Fast charging points

(b) Rapid charging points

Figure 2: Heat map displaying number of recommended number of charging points in each grid square: a) fast charging points, b) rapid charging points

5 Results

The data for our results can be found here:
https://drive.google.com/file/d/1Cd1fMA-2UN9lR-m7ypkj6m6EcoVgHfAS/view?usp=share˙link
The code we used to obtain and analyse the results can be found here
https://github.com/jackclennon/MMCScode

5.1 Recommendations

Using the assumptions outlined in section 3.1, we were able to obtain results using our models that can serve as an initial recommendation to the council for making construction plans for new charging points. Figure 1 displays the location and size of recommended charging stations on a map of the city of Dundee, where the size of the station refers to the number of charging points housed in the station. Figure 2 provides a more detailed view of the number of recommended fast and rapid charging points in each grid square.

The results of our model favoured placing chargers throughout the city, avoiding the city centre. With some stations only housing a couple of chargers, but some hubs that host a large amount of charging points further out from the city centre housing many chargers.

5.2 Sensitivity Analysis

Due to the complexity of the problem, we had to make many assumptions when developing a model that could yield actionable solutions, as outlined in section 3.1. We found it necessary to investigate and characterise the effect each of these assumptions have by altering the variables associated to the assumptions, individually, and comparing the solutions with our original solutions. This was conducted for the demand data, the potential discount in setup costs for charging stations, the lower bound on the number of fast charging points, and the range in possible power outputs, and different levels for the fulfillment. We ran the models for only a short time due to the limited time we had to produce the results. The relative gap was kept below 6%.

5.2.1 Methodology

We first altered the variables to change, then we simply checked how many charging stations (not points) stayed in the same place for each change in the variables. We used this to calculate the "agreeance" of the new data with the original data which gave us a metric for comparing results.

5.2.2 Findings

We were able to obtain some qualitative results for certain variables by inspecting the solutions. The model only responds to the presence of a discount in the set up cost of charging stations (see figure 6), different discounts yield the same solution. We also discovered that for lower lower bounds on the number of fast charging points, i.e. higher upper bounds for the number of rapid charging points, less charging stations and charging hubs (stations housing more than 10 charging points) are required.

After quantifying the agreeance, we found that that the model is very sensitive to changes in the demand data (see figure 4 in Appendix B), that there is a lot of variance between the different lower bounds on the number of fast charging points (see 7 is Appendix B), the model is sensitive but to a lesser extent for the various charger power levels (see figure 5 in Appendix B), the model is also sensitive to the fulfillment with the maximum agreeance between different fulfillment cases being only 30% (this was comparing 70% fulfillment to 80% fulfillment) (see figure 8 in Appendix B).

6 Conclusion

To analyse the final results we ran a similar analysis as we did for the sensitivity analysis across the different demand levels to determine the charging stations that exist in the model for the most time (see figure 3 in Appendix B). There was no charging station that appeared in all the demand cases. However, there was a single charging station that appears in demand cases 1, 2, and, 3. That was a charging station in grid square 229. The overlap between each demand case was low, meaning that the council would have to keep building new stations. This is more room for future work, we could find a way to propagate forward the results from one year to the next. One way to do this would be to find the solution for the final demand case, and use only those charging sites as the potential charging sites for the earlier demand cases.

To future proof potential demand and charging stations design infrastructure. We recommend to preform a bottom up analysis using the Stochastic Queuing theory [A]. This model allow an alternative view from the customers perspective and can. Additionally, to allow the council to cover all demands, socio-economic and energy capacity in each region should be accounted for in future.

We calculated the estimated cost for the charging stations, and we found that it was small (around £50,000), most of the cost will come from labour and land. We don't have this information so we didn't include an estimate for the cost. In future work we would like to find a reliable estimate for the total cost.

References

[1] CPS, *Accessing the network*, https://chargeplacescotland.org/accessing-network/, 2022, Accessed: 27/11/2022.

[2] Bin Li et al, *New thinking in giscience*, ch. 33.2, Springer, 2022.

[3] Qian Wang et al, *Algorithms for a facility location problem with stochastic customer demand and immobile servers*, Springer, 2002.

[4] HM Government, *Transitioning to zero emission cars and vans: 2035 delivery plan*, https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment˙data/file/1005301/transitioning-to-zero-emission-cars-vans-2035-delivery-plan.pdf, 2021, Accessed: 27/11/2022.

[5] GOV.SCOT, *About dundee 2021*, https://www.gov.scot/publications/scottish-building-regulations-proposed-changes-energy-standards-associated-topics/pages/7/, 2021, Accessed: 28/11/2022.

[6] HOMEHOW, *Electric car charger installation cost*, https://www.homehow.co.uk/costs/ev-charging-station-installation, 2021, Accessed: 28/11/2022.

[7] The City of Edinbrugh Council, *The city of edinbrugh council*, https://www.edinburgh.gov.uk/electric-vehicles-ev/find-electric-vehicle-charging-points-edinburgh/3, 2022, Accessed: 28/11/2022.

[8] SMMT, *October 2022 new car registrations*, https://media.smmt.co.uk/october-2022-new-car-registrations/?utm˙source=rssutm˙medium=rssutm˙campaign=october-2022-new-car-registrations, 2022, Accessed: 27/11/2022.

A Appendix A: $(M/M/c)$ Queueing Theory

A.1 PASTA (Convergence Theorem in CTMC)

If the Queueing system is a Poisson arrival $(M/\cdot/\cdot)$, then the following statement holds:

- This property is **true only for Poisson arrivals**.
- In long run, the fraction of customer finding the arrival of system in state A: $P_A^{(\infty)}$, is equivalent to the fraction of time when the system is in state A: P_A^*;
- $$P_A^* := \lim_{t\to\infty} \sum_{n=0}^{N} \frac{P_{j,A}^{(n)}}{N+1}, \forall j \in S$$
- $$P_A^{(\infty)} := \lim_{t\to\infty} P_{j,A}^{(n)}, \forall j \in S$$
- This is applicable under FIFO assumption.

A.2 Little's Formula

Notations:

- Counting process of customers: $\Lambda(t)$;
- Service time: B_n;
- Sojourn time: S_n;
- Waiting time: W_n;
- Arrival rate: $\Lambda := \lim_{T\to\infty} \frac{\int_0^T \Lambda(t)dt}{T}$;
- Average sojourn time : $\mathcal{S} := \lim_{T\to\infty} \frac{\sum_{n=0}^N S_n}{N}$;
- Average Service time: $\mathcal{B} := \lim_{T\to\infty} \frac{\sum_{n=0}^N B_n}{N}$;
- Average Number in system: $\mathcal{L} := \lim_{T\to\infty} \frac{\int_0^T L(t)dt}{T}$

Proof for Little's formula:

$$\mathcal{L} = \Lambda \mathcal{S}$$

Proof. Denote:

- $I_i(t) := \mathbf{1}_{\{N_i=1\}}$ and N_i is equal to 1 if individual i is in the system;
- $L(t)$ follows the definition above.

Then we have:

$$\int_0^T L(t)dt = \sum_{i=1}^{\Lambda(T)} \int_0^T I_i(t)dt$$

Consider at the end of time in such queueing system T:

- When $L(T) = 0$:
$$\int_0^T L(t)dt = \sum_{i=1}^{\Lambda(T)} \int_0^T I_i(t)dt = \sum_{i=1}^{\Lambda(T)} S_i$$
- When $L(T) > 0$:
$$\int_0^T L(t)dt = \sum_{i=1}^{\Lambda(T)} \int_0^T I_i(t)dt < \sum_{i=1}^{\Lambda(T)} S_i$$

Next:

$$\begin{aligned}\mathcal{L} &= \lim_{T\to\infty} \frac{\int_0^T L(t)dt}{T} \\ &= \lim_{T\to\infty} \frac{\sum_{i=1}^{\Lambda(T)} S_i}{T} + \lim_{T\to\infty} \frac{\epsilon_T}{T} \\ &= \lim_{T\to\infty} \frac{\sum_{i=1}^{\Lambda(T)} S_i}{\Lambda(T)} \cdot \frac{\Lambda(T)}{T} + \lim_{T\to\infty} \frac{\epsilon_T}{T} \\ &= \mathcal{S}\Lambda\end{aligned}$$

where $\lim_{T\to\infty} \frac{\epsilon_T}{T} = 0$ is the uncompleted sojourn time going to infinity because time is infinity but $\epsilon_T < \infty$.

A.3 Essential Ideas and Definitions

A.3.1 General Ideas: Mixed-Integer Programming

- Firstly, define all the random variables with an exponential or Poisson distribution (by the data-set provided);
- Next, set the objective function: minimize the waiting time/travelling time (here equivalently we minimize the Average number of traveling customers and Average waiting lines, however) of potential customers such that satisfy within an upper bound (facilities/cost/......);
- Finally, determine the constraints by assumptions.

A.3.2 Essential Definitions (Continued)

- Little's Formula (The most crucial results for basically everything below, PLEASE SEE APPENDIX A.2):

$$\mathcal{L} = \Lambda\mathcal{S}$$

- **Proposition 4.3.2.1:** Equivalently, Little's formula can be written as:

$$L_q := \mathcal{L} - L_s = \Lambda \cdot (\mathcal{S} - \mathcal{B}) := \Lambda \cdot \mathbf{E}W$$

- Aggregate (total) demand rate at open facility site j:

$$\gamma_j = \sum_{i\in M}\sum_{j\in N} \lambda x_{i,j}$$

- Furthermore, Aggregate (total) demand rate at sub-site k of site j:

$$\gamma_{j,k} = p_k \sum_{i\in M}\sum_{j\in N} \lambda x_{i,j}$$

- Average number of traveling customers (By **Little's formula**):

$$T := \sum_{i\in M}\sum_{j\in N} \lambda_i d_{ij} x_{ij}/v$$

- Expected waiting time in $M/M/c_{j,k}$:

$$\mathbf{E}W_{i,j,k} := \frac{\mathbf{E}L_q^{i,j,k}}{\gamma_{j,k}} = \Pi_W^{(i,j,k)} \frac{1}{(1-\rho_{i,j,k})y_{j,k}\mu_k}$$

  where

$$\Pi_W := \frac{(c\rho)^c}{c!} \cdot \left((1-\rho) \frac{1}{\sum_{n=0}^{c-1} \frac{(c\rho)}{n!} + \frac{(c\rho)^c}{c!}} \right), c := y_{j,k}$$

$$EL_q = \Pi_W \frac{\rho}{1-\rho}$$

$$\rho_{j,k} = \frac{\gamma_{j,k}}{y_{j,k}\mu_k} < 1$$

  If we assume same λ and μ_k in this model, then:

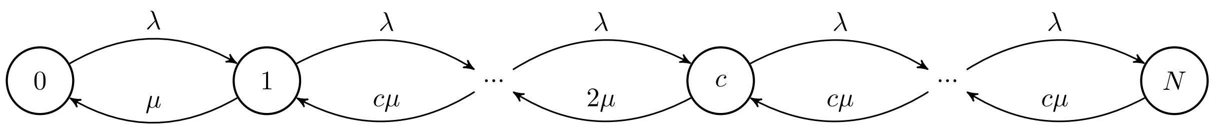
$$\mathbf{E}W_{j,k} := \Pi_W^{(j,k)} \frac{1}{(1-\rho_k)y_{j,k}\mu_k}$$

- Closest assumption [3]:
$$\sum_{j' \in N} d_{i,j'} x_{i,j'} \leq (d_{i,j} - \Delta) \cdot z_j + \Delta$$
where $\Delta := \max_{j \in N, i \in I}\{d_{i,j}\}$.

A.3.3 $M/M/c$ Queueing System Background

Notations and Basic Assumptions

- Markov Chain for $M/M/c$ Queueing system:



- $\rho_{i,j,k} := \frac{\lambda_i}{\mu_j c_{j,k}} < 1$ under this assumption, the steady state will exist (intuitively average service time will shorter than average demand rate)
- $p_0, p_1, ..., p_n, ...$: denote for steady state probability where $n \in 1, 2, 3, ...$ is the number of arrivals
- Global balance equation(GBL):
$$Q\Pi = 0$$
$$\sum_n \Pi_n = 1$$
- The long-run probability of zero arrival:
$$\pi_0 = p_0 = \frac{1}{\sum_{n=0}^{c-1} \frac{(c\rho)}{n!} + \frac{(c\rho)^c}{c!} \cdot \frac{1}{1-\rho}}$$
- The long-run probabilities of positive arrivals $n \leq c$:
$$p_n = \frac{\lambda}{n\mu} p_{n-1} = \frac{\lambda}{(n-1)\mu} p_{n-2} = \cdots = \frac{(c\rho)^n}{n!} p_0 = \frac{(\lambda/\mu)^n}{n!} p_0$$
- The long-run probabilities of positive arrivals $n \geq c$:
$$p_{c+n} = \rho^n p_c = \rho^n \frac{(\lambda/\mu)^c}{c!} p_0$$

Basic Properties: (The footnotes will be omitted)

- The (delay) probability Π_W:
$$\begin{aligned}
\Pi_W &:= \sum_{n=c}^{\infty} p_n \\
&= \sum_{n=0}^{\infty} p_{c+n} \\
&= \frac{p_c}{1-\rho} \\
&= \frac{(c\rho)^c}{c!} \cdot \left((1-\rho) \frac{1}{\sum_{n=0}^{c-1} \frac{(c\rho)}{n!} + \frac{(c\rho)^c}{c!}} \right)
\end{aligned}$$

- The expected length of queue $\mathbf{E}L_q$:

$$\begin{aligned}
\mathbf{E}L_q &= \sum_{n=0}^{\infty} n p_{c+n} \\
&= \sum_{n=0}^{\infty} n \rho^n p_c \\
&= p_c \sum_{n=0}^{\infty} n \rho^n \\
&= \frac{p_c}{1-\rho} \sum_{n=0}^{\infty} n(1-\rho)\rho^n \\
&= \Pi_W (1-\rho)\rho \frac{d}{d\rho} \sum_{n=0}^{\infty} \rho^n \\
&= \Pi_W \frac{\rho}{1-\rho}
\end{aligned}$$

- By Little's Formula where $\mathcal{L} = \Lambda \mathcal{S}$:

$$\mathbf{E}W := \mathbf{E}L_q/\gamma$$

$$T := \sum_{i \in M} \sum_{j \in N} \lambda_i x_{ij} \sum_{i \in M} \sum_{j \in N} d_{ij}/v = \sum_{i \in M} \sum_{j \in N} \lambda_i d_{ij} x_{ij}/v$$

B Appendix B: Sensitivity Analysis

Figure 3: Comparing the agreeance across demand cases

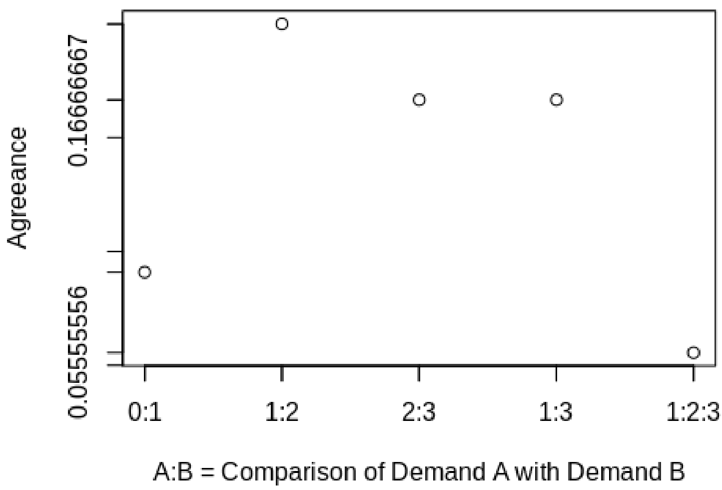


Figure 4: Comparing the agreeance for demand cases against the basic solution

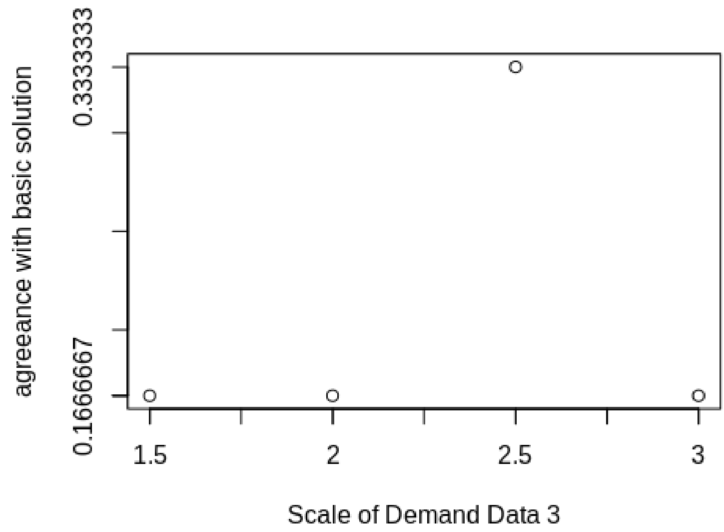


Figure 5: Comparing the agreeance for potential power levels against the basic solution

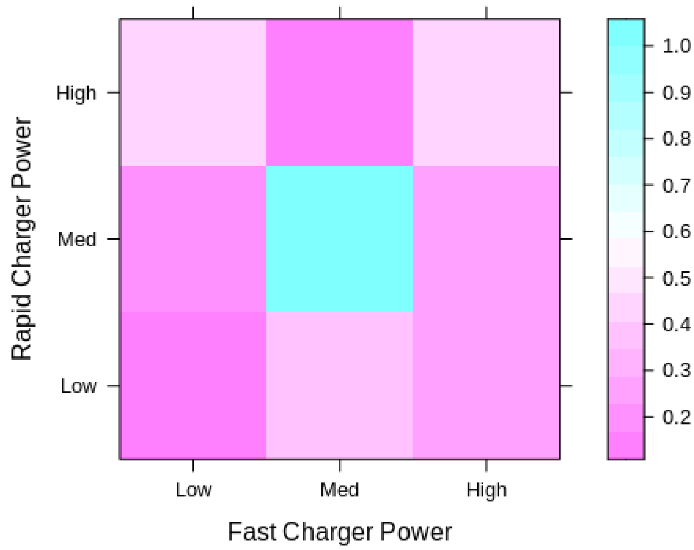


Figure 6: Comparing the agreeance for discount cases against basic solution

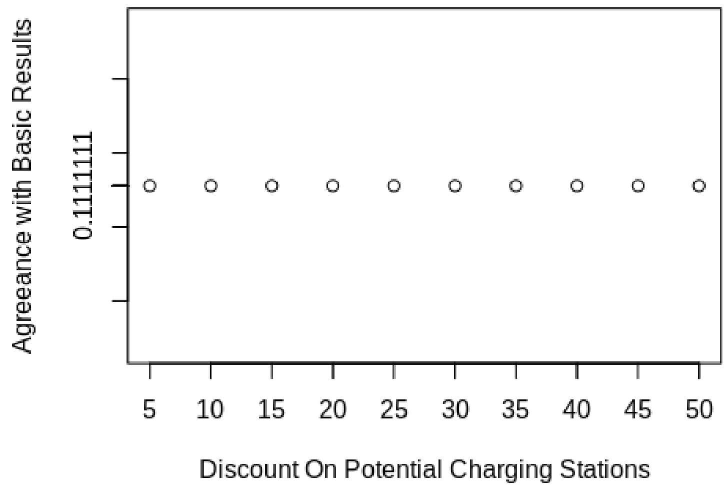


Figure 7: Comparing the agreeance across lower bounds for the percentage of fast charging points

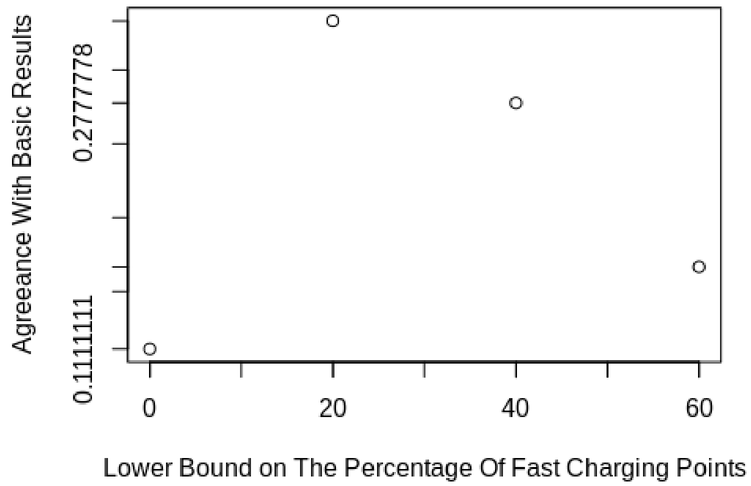


Figure 8: Comparing the agreeance across fulfillment for the percentage of fast charging points