# 2-spin

## Heisenberg Model

标准模型

$$H = J\sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j \qquad (1)$$

$$\text{展开：}\ H = J\sum (S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z)$$

Using a basis $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ for two spins the matrix representation of one 2-spin term is

$$\begin{pmatrix} J_{ij}/4 & 0 & 0 & 0 \\ 0 & -J_{ij}/4 & J_{ij}/2 & 0 \\ 0 & J_{ij}/2 & -J_{ij}/4 & 0 \\ 0 & 0 & 0 & J_{ij}/4 \end{pmatrix}. \qquad (4.5)$$

H

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.2500 | 0 | 0 | 0 |
| 2 | 0 | -0.2500 | 0.5000 | 0 |
| 3 | 0 | 0.5000 | -0.2500 | 0 |
| 4 | 0 | 0 | 0 | 0.2500 |

AFM

D

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.7500 | 0 | 0 | 0 |
| 2 | 0 | 0.2500 | 0 | 0 |
| 3 | 0 | 0 | 0.2500 | 0 |
| 4 | 0 | 0 | 0 | 0.2500 |

```
GS =

        0
   0.7071
  -0.7071
        0
```

FM

D

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.2500 | 0 | 0 | 0 |
| 2 | 0 | -0.2500 | 0 | 0 |
| 3 | 0 | 0 | -0.2500 | 0 |
| 4 | 0 | 0 | 0 | 0.7500 |

```
GS =

     1
     0
     0
     0
```

# Heisenberg Model

标准模型

$$H = J \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j \tag{1}$$

展开：$H = J \sum (S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z)$

# 2-spin

XXZ model

$$H = \left[ \frac{1}{2} (S_1^+ \otimes S_2^- + S_1^- \otimes S_2^+) + \Delta S_1^z \otimes S_2^z \right] \tag{27}$$

```
% XXZ 参数:
Delta = 1;
```

AFM

D — 4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.7500 | 0 | 0 | 0 |
| 2 | 0 | 0.2500 | 0 | 0 |
| 3 | 0 | 0 | 0.2500 | 0 |
| 4 | 0 | 0 | 0 | 0.2500 |

```
GS =

         0
    0.7071
   -0.7071
         0
```

```
% XXZ 参数:
Delta = 0.5;
```

D — 4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.6250 | 0 | 0 | 0 |
| | 0 | 0.1250 | 0 | 0 |
| | 0 | 0 | 0.1250 | 0 |
| | 0 | 0 | 0 | 0.3750 |

GS — 4x1 double

| | 1 |
|---|---|
| 1 | 0 |
| 2 | 0.7071 |
| 3 | -0.7071 |
| 4 | 0 |

```
% XXZ 参数:
Delta = 2.0;
```

D — 4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| | -1 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0.5000 | 0 |
| | 0 | 0 | 0 | 0.5000 |

GS — 4x1 double

| | 1 |
|---|---|
| 1 | 0 |
| 2 | 0.7071 |
| 3 | -0.7071 |
| 4 | 0 |

XXZ model

$$H = \left[\frac{1}{2}(S_1^+ \otimes S_2^- + S_1^- \otimes S_2^+) + \Delta S_1^z \otimes S_2^z\right] \tag{27}$$

# 2-spin

## AFM

```
% XXZ 参数:
Delta = 1;
```

D

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.7500 | 0 | 0 | 0 |
| 2 | 0 | 0.2500 | 0 | 0 |
| 3 | 0 | 0 | 0.2500 | 0 |
| 4 | 0 | 0 | 0 | 0.2500 |

```
GS =

     0
0.7071
-0.7071
     0
```

```
% XXZ 参数:
Delta = 2.0;
```

D

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| | -1 | 0 | 0 | 0 |
| | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0.5000 | 0 |
| | 0 | 0 | 0 | 0.5000 |

GS

4x1 double

| | 1 |
|---|---|
| 1 | 0 |
| 2 | 0.7071 |
| 3 | -0.7071 |
| 4 | 0 |

```
% XXZ 参数:
Delta = 0.5;
```

D

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| | -0.6250 | 0 | 0 | 0 |
| | 0 | 0.1250 | 0 | 0 |
| | 0 | 0 | 0.1250 | 0 |
| | 0 | 0 | 0 | 0.3750 |

D GS

4x1 double

| | 1 |
|---|---|
| 1 | 0 |
| 2 | 0.7071 |
| 3 | -0.7071 |
| 4 | 0 |

## FM

```
% XXZ 参数:
Delta = 1;
```

D

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.2500 | 0 | 0 | 0 |
| 2 | 0 | -0.2500 | 0 | 0 |
| 3 | 0 | 0 | -0.2500 | 0 |
| 4 | 0 | 0 | 0 | 0.7500 |

```
GS =

1
0
0
0
```

```
% XXZ 参数:
Delta = 2.0;
```

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | -0.5000 | 0 | 0 | 0 |
| 2 | 0 | -0.5000 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 |

```
GS =

1
0
0
0
```

```
% XXZ 参数:
Delta = 0.5;
```

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| | -0.3750 | 0 | 0 | 0 |
| | 0 | -0.1250 | 0 | 0 |
| | 0 | 0 | -0.1250 | 0 |
| | 0 | 0 | 0 | 0.6250 |

```
GS =

      0
-0.7071
-0.7071
      0
```

# Heisenberg Model

标准模型

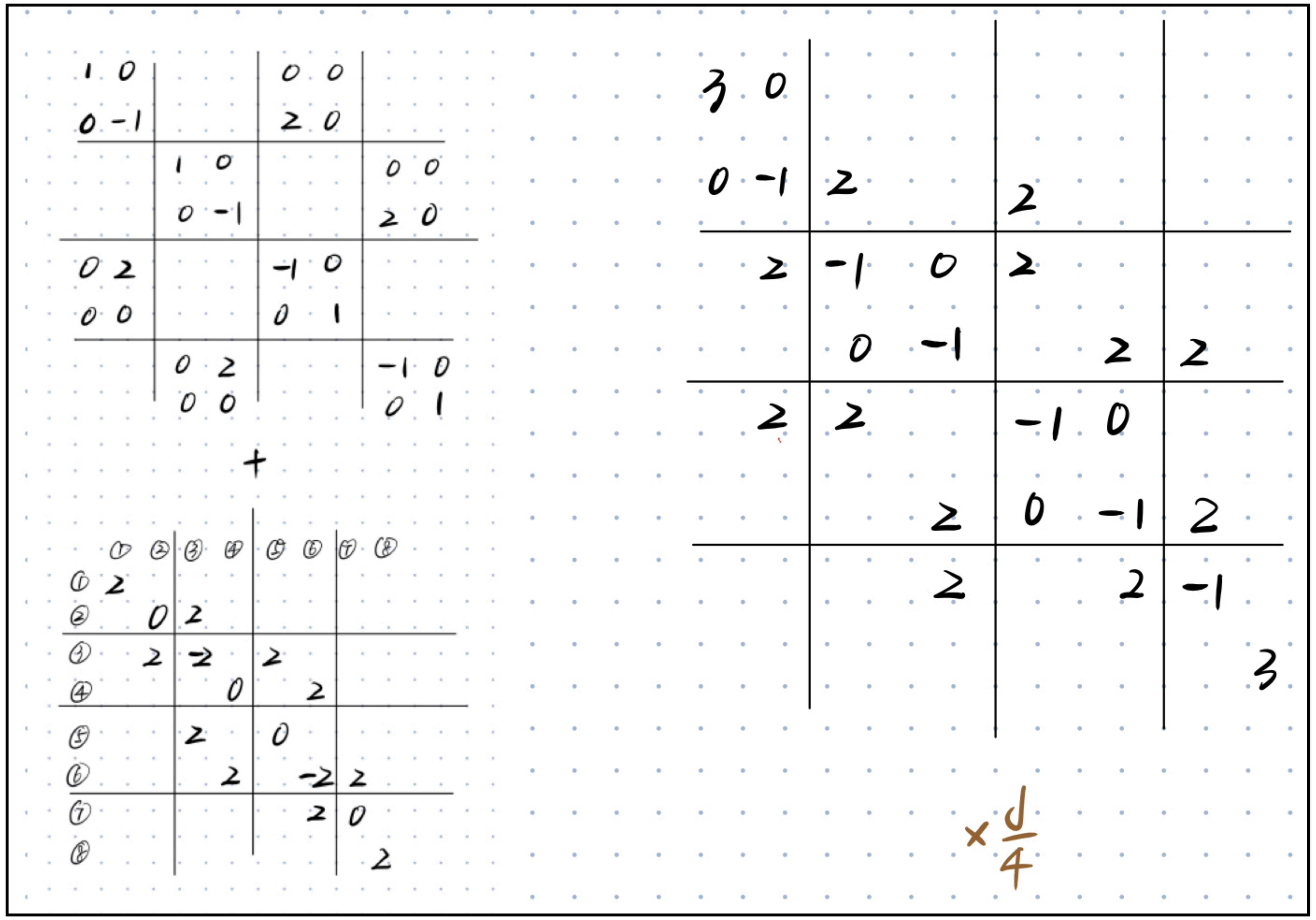
$$H = J \sum_{<i,j>} \mathbf{S}_i \cdot \mathbf{S}_j \tag{1}$$

展开： $H = J \sum (S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z)$

# 3-spin

H =

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | -1 | 2 | 0 | 2 | 0 | 0 | 0 |
| 0 | 2 | -1 | 0 | 2 | 0 | 0 | 0 |
| 0 | 0 | 0 | -1 | 0 | 2 | 2 | 0 |
| 0 | 2 | 2 | 0 | -1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 2 | 0 | -1 | 2 | 0 |
| 0 | 0 | 0 | 2 | 0 | 2 | -1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 |




XXZ model Delta=1时同样对得上

8×8 H矩阵:

$H_1 = H_{old} \otimes I$

$$\begin{bmatrix} 1 & & & \\ & -1 & 2 & \\ & 2 & -1 & \\ & & & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{array}{c|cc|cccc|cc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 1 & 0 & & & & & & \\ 2 & 0 & 1 & & & & & & \\ \hline 3 & & & -1 & 0 & 2 & 0 & & \\ 4 & & & 0 & -1 & 0 & 2 & & \\ \hline 5 & & & 2 & 0 & -1 & 0 & & \\ 6 & & & 0 & 2 & 0 & -1 & & \\ \hline 7 & & & & & & & 1 & 0 \\ 8 & & & & & & & 0 & 1 \end{array} \quad H_1$$

$H_2 = I \otimes H_{old}$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & & & \\ & -1 & 2 & \\ & 2 & -1 & \\ & & & 1 \end{bmatrix}$$

$$\begin{array}{c|cccc|cccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 1 & & & & & & & \\ 2 & & -1 & 2 & & & & & \\ 3 & & 2 & -1 & & & & & \\ 4 & & & & 1 & & & & \\ \hline 5 & & & & & 1 & & & \\ 6 & & & & & & -1 & 2 & \\ 7 & & & & & & 2 & -1 & \\ 8 & & & & & & & & 1 \end{array} \quad H_2$$


$$\begin{array}{c|cc|cc|cc|cc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 1 & 0 & & & & & & \\ 2 & 0 & 1 & & & & & & \\ \hline 3 & & & -1 & 0 & 2 & 0 & & \\ 4 & & & 0 & -1 & 0 & 2 & & \\ \hline 5 & & & 2 & 0 & -1 & 0 & & \\ 6 & & & 0 & 2 & 0 & -1 & & \\ \hline 7 & & & & & & & 1 & 0 \\ 8 & & & & & & & 0 & 1 \end{array} + \begin{array}{c|cc|cc|cc|cc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 1 & & & & & & & \\ 2 & & -1 & 2 & & & & & \\ \hline 3 & & 2 & -1 & & & & & \\ 4 & & & & 1 & & & & \\ \hline 5 & & & & & 1 & & & \\ 6 & & & & & & -1 & 2 & \\ \hline 7 & & & & & & 2 & -1 & \\ 8 & & & & & & & & 1 \end{array}$$


$$\begin{array}{c|cc|cc|cc|cc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 2 & & & & & & & \\ 2 & & 0 & 2 & & & & & \\ \hline 3 & & 2 & -2 & & 2 & & & \\ 4 & & & & 0 & & 2 & & \\ \hline 5 & & & 2 & & 0 & & & \\ 6 & & & & 2 & & -2 & 2 & \\ \hline 7 & & & & & & 2 & 0 & \\ 8 & & & & & & & & 2 \end{array}$$

$H_3$: $\quad I_2 \otimes S_3^{z,\pm} \qquad \begin{bmatrix} 1 & \\ & 1 \end{bmatrix} \otimes \cdots$

$S^- = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$S^+ = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\sigma^z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$$\begin{array}{cc|cc} 0/0/1 & 0/1/0 & & \\ 1/0/0 & 0/0/-1 & & \\ \hline & & 0/0/1 & 0/1/0 \\ & & 1/0/0 & 0/0/-1 \end{array}$$

$S^+ \otimes I \otimes S^-$ $\qquad$ $S^- \otimes I \otimes S^+$ $\qquad$ $S^z \otimes I \otimes S^z$

$$2\times \begin{array}{c|c} 0 & \begin{array}{cc|cc} 0 & 0 & & \\ 1 & 0 & & \\ \hline & & 0 & 0 \\ & & 1 & 0 \end{array} \\ \hline 0 & 0 \end{array} + 2\times \begin{array}{c|c} 0 & 0 \\ \hline \begin{array}{cc|cc} 0 & 1 & & \\ 0 & 0 & & \\ \hline & & 0 & 1 \\ & & 0 & 0 \end{array} & 0 \end{array} + \begin{array}{c|c} \begin{array}{cc|cc} 1 & 0 & & \\ 0 & -1 & & \\ \hline & & 1 & 0 \\ & & 0 & -1 \end{array} & 0 \\ \hline 0 & \begin{array}{cc|cc} -1 & 0 & & \\ 0 & +1 & & \\ \hline & & -1 & 0 \\ & & 0 & +1 \end{array} \end{array}$$

$$\begin{array}{cc|cc|cc|cc} 1 & 0 & & & 0 & 0 & & \\ 0 & -1 & & & 2 & 0 & & \\ \hline & & 1 & 0 & & & 0 & 0 \\ & & 0 & -1 & & & 2 & 0 \\ \hline 0 & 2 & & & -1 & 0 & & \\ 0 & 0 & & & 0 & 1 & & \\ \hline & & 0 & 2 & & & -1 & 0 \\ & & 0 & 0 & & & 0 & 1 \end{array} \quad H_3$$

# 手解


$$\begin{array}{cc|cc|cc|cc} 1 & 0 & & & 0 & 0 & & \\ 0 & -1 & & & 2 & 0 & & \\ \hline & & 1 & 0 & & & 0 & 0 \\ & & 0 & -1 & & & 2 & 0 \\ \hline 0 & 2 & & & -1 & 0 & & \\ 0 & 0 & & & 0 & 1 & & \\ \hline & & 0 & 2 & & & -1 & 0 \\ & & 0 & 0 & & & 0 & 1 \end{array}$$

$+$

$$\begin{array}{c|cc|cc|cc|cc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 2 & & & & & & & \\ 2 & & 0 & 2 & & & & & \\ \hline 3 & & 2 & -2 & & 2 & & & \\ 4 & & & & 0 & & 2 & & \\ \hline 5 & & & 2 & & 0 & & & \\ 6 & & & & 2 & & -2 & 2 & \\ \hline 7 & & & & & & 2 & 0 & \\ 8 & & & & & & & & 2 \end{array}$$


$$\begin{array}{cc|cc|cc|cc} 3 & 0 & & & & & & \\ 0 & -1 & 2 & & 2 & & & \\ \hline & 2 & -1 & 0 & 2 & & & \\ & & 0 & -1 & & 2 & 2 & \\ \hline & 2 & 2 & & -1 & 0 & & \\ & & & 2 & 0 & -1 & 2 & \\ \hline & & & 2 & & 2 & -1 & \\ & & & & & & & 3 \end{array}$$

$\times \frac{J}{4}$

加入纵场项：

$$H=\left[\frac{1}{2}(S_1^+\otimes S_2^-+S_1^-\otimes S_2^+)+\Delta S_1^z\otimes S_2^z\right]+h_b(S_1^z\otimes I_2)+h_b(I_1\otimes S_2^z)\quad(28)$$

# 2-spin

核对手解形式：

2-spin 加入纵场

$$h_b\cdot S_1^z\otimes I+h_b\cdot I\otimes S_2^z$$

$$\frac{1}{4}\times\begin{bmatrix}1&&&\\&-1&2&\\&2&-1&\\&&&1\end{bmatrix}+\begin{bmatrix}h_b&&&\\&h_b&&\\&&-h_b&\\&&&-h_b\end{bmatrix}+\begin{bmatrix}h_b&&&\\&-h_b&&\\&&h_b&\\&&&-h_b\end{bmatrix}$$

$$\frac{1}{4}\qquad\begin{bmatrix}2h_b&&&\\&0&&\\&&0&\\&&&-2h_b\end{bmatrix}\times\frac{1}{2}\ (\sigma\to S)\Rightarrow\begin{bmatrix}h_b&&&\\&0&&\\&&0&\\&&&-h_b\end{bmatrix}\ \checkmark$$

(用σ表示)

$$\frac{1}{4}-1=-\frac{3}{4}$$

H

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 0.5000 | 0 | 0 | 0 |
| 2 | 0 | -0.2500 | 0.5000 | 0 |
| 3 | 0 | 0.5000 | -0.2500 | 0 |
| 4 | 0 | 0 | 0 | 0 |

```
H = 4*H;
```

H

4x4 double

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 0 | 0 | 0 |
| 2 | 0 | -1 | 2 | 0 |
| 3 | 0 | 2 | -1 | 0 |
| 4 | 0 | 0 | 0 | 0 |

对得上

说明一下手解形式：最左边矩阵是无纵场且提出J/4的，

只有打勾的纵场矩阵是用S矩阵的正确表示（意味着比sigma矩阵表示差1/2倍）

然后令h=1/4,两个相加之后乘上4倍（把J/4这项消掉，矩阵元就是整数加减方便看）

# 纵场的检验： 3-spin

H

8x8 double

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 1.5000 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0.5000 | 0 | 0.5000 | 0 | 0 | 0 |
| 3 | 0 | 0.5000 | 0 | 0 | 0.5000 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | -0.5000 | 0 | 0.5000 | 0.5000 | 0 |
| 5 | 0 | 0.5000 | 0.5000 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 0.5000 | 0 | -0.5000 | 0.5000 | 0 |
| 7 | 0 | 0 | 0 | 0.5000 | 0 | 0.5000 | -0.5000 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

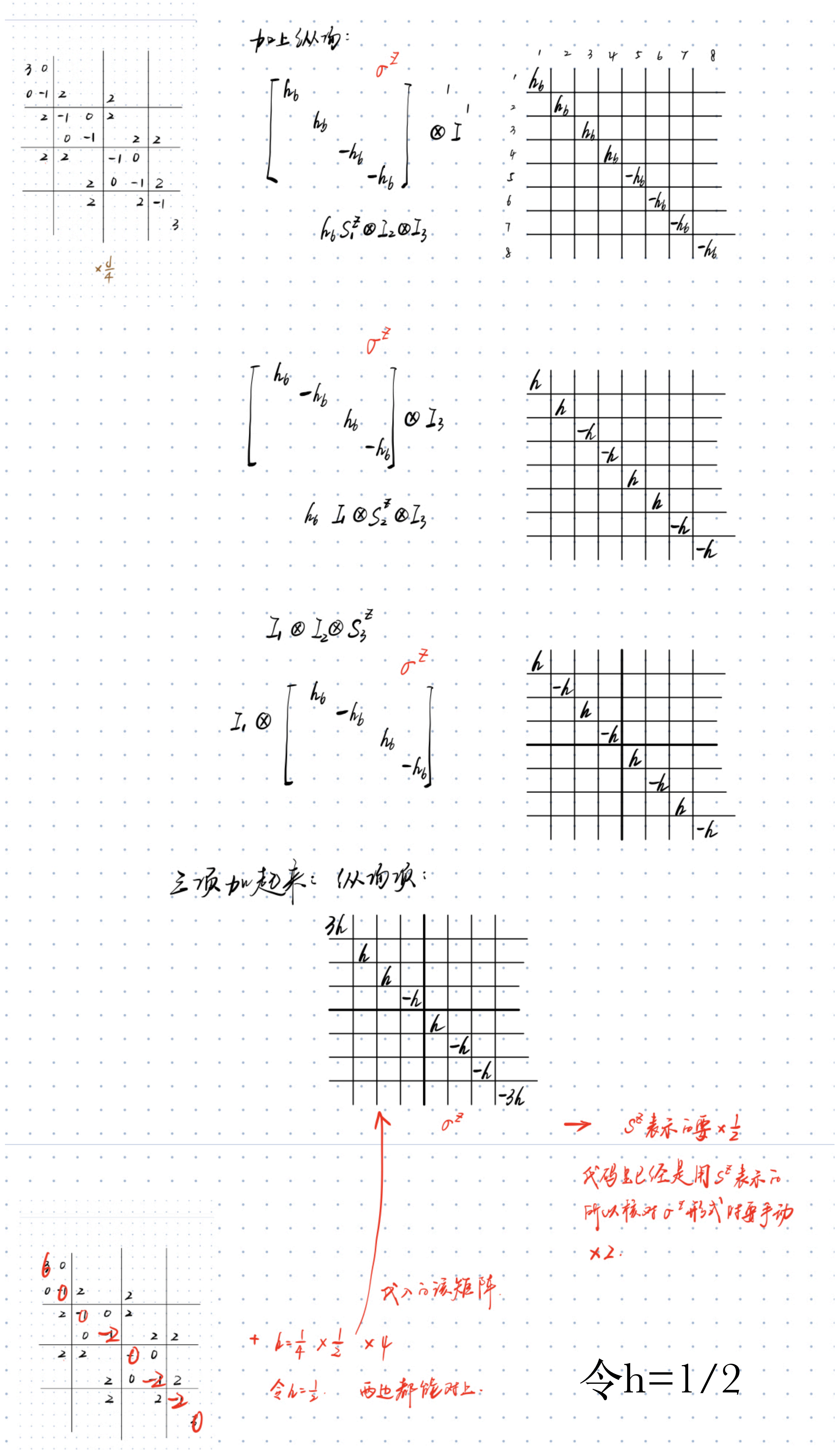
```
H = 4*H;
```

也能对上

H

8x8 double

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 6 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 2 | 0 | 2 | 0 | 0 | 0 |
| 3 | 0 | 2 | 0 | 0 | 2 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | -2 | 0 | 2 | 2 | 0 |
| 5 | 0 | 2 | 2 | 0 | 0 | 0 | 0 | 0 |
| 6 | 0 | 0 | 0 | 2 | 0 | -2 | 2 | 0 |
| 7 | 0 | 0 | 0 | 2 | 0 | 2 | -2 | 0 |
| 8 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |



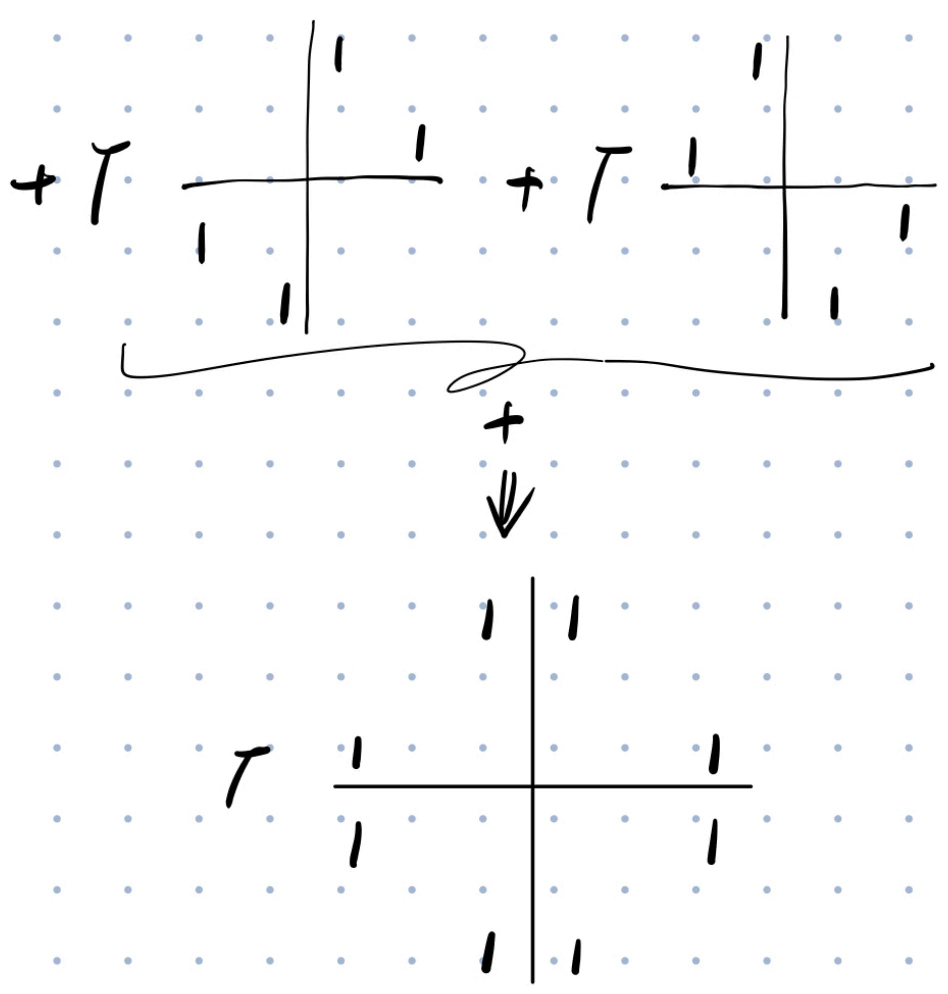
横场Ising:

$$H = \sum_{\langle i,j \rangle} J_{i,j}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x \tag{9}$$

# 横场的检验： 2/3-spin

```
>> sgamma

sgamma =

     0     1     1     0
     1     0     0     1
     1     0     0     1
     0     1     1     0
```

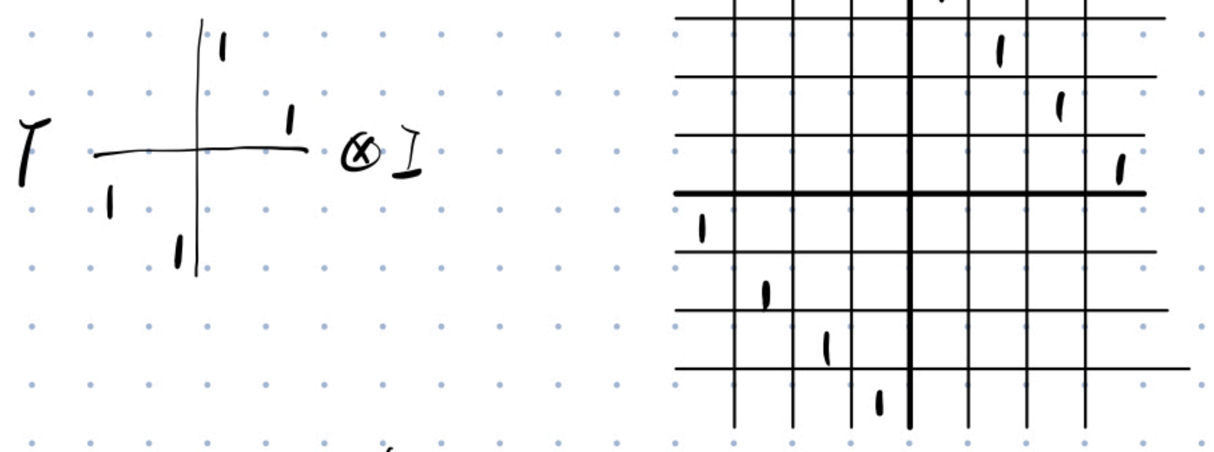
$\Gamma\sigma_1^x \otimes I_2 + \Gamma I_1 \otimes \sigma_2^x$



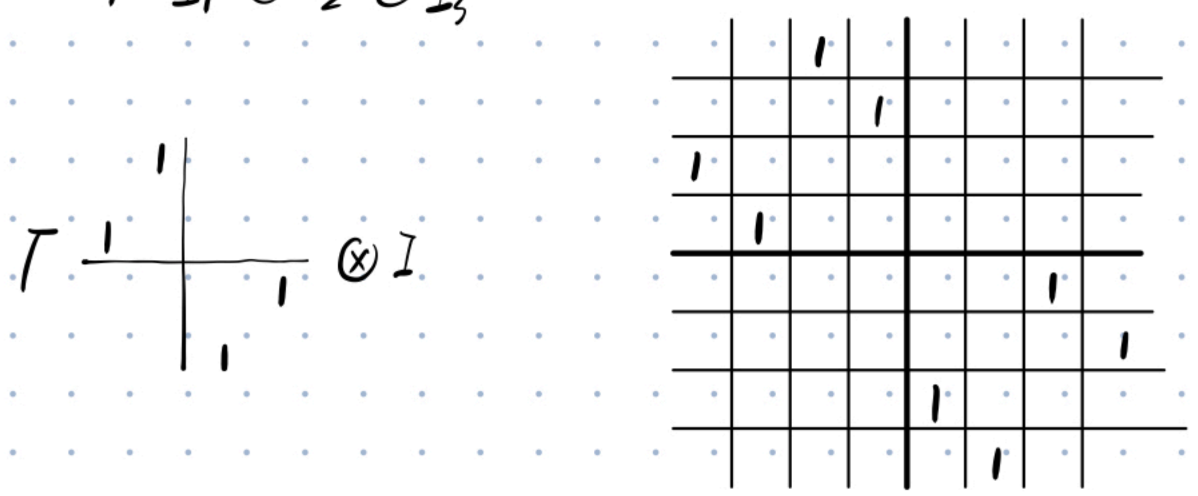
```
>> sgamma

sgamma =

     0     1     1     0     1     0     0     0
     1     0     0     1     0     1     0     0
     1     0     0     1     0     0     1     0
     0     1     1     0     0     0     0     1
     1     0     0     0     0     1     1     0
     0     1     0     0     1     0     0     1
     0     0     1     0     1     0     0     1
     0     0     0     1     0     1     1     0
```

8 × 8 横场矩阵:

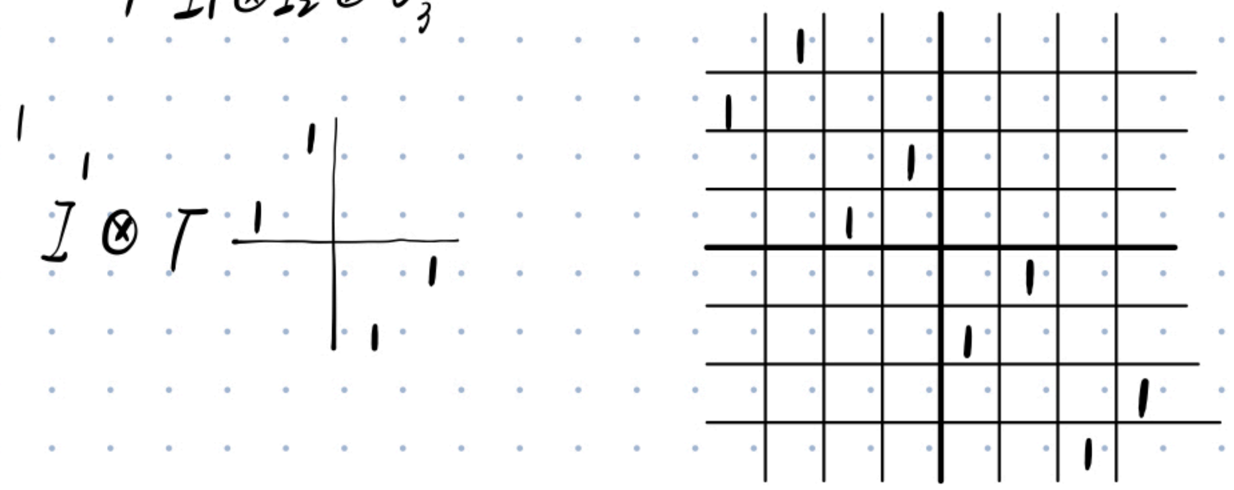
$\Gamma\sigma_1^x \otimes I_2 \otimes I_3$

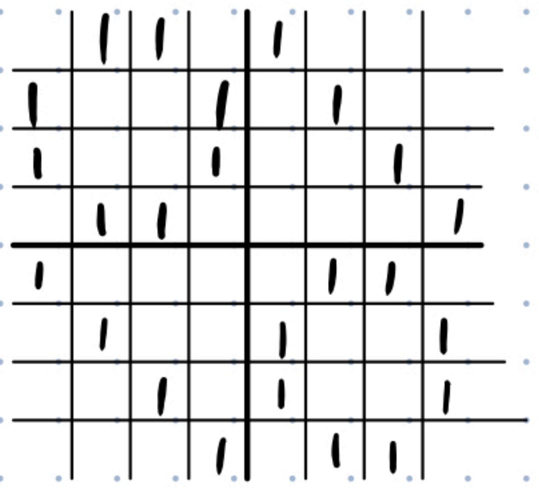


$\Gamma I_1 \otimes \sigma_2^x \otimes I_3$



$\Gamma I_1 \otimes I_2 \otimes \sigma_3^x$





横场Ising：

$$H = \sum_{\langle i,j \rangle} J_{i,j}\sigma_i^z\sigma_j^z - \Gamma \sum_i \sigma_i^x \tag{9}$$

# 2-spin

```
H =

     1     1     1     0
     1    -1     0     1
     1     0    -1     1
     0     1     1     1
```

Ising: $J\sigma_1^z \otimes \sigma_2^z$

Ising: 2-spin $J\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&1\end{pmatrix} + \Gamma\begin{pmatrix}0&1&1&0\\1&0&0&1\\1&0&0&1\\0&1&1&0\end{pmatrix}$

$J=\Gamma=1$ 时：

$$\begin{pmatrix}1&1&1&0\\1&-1&0&1\\1&0&-1&1\\0&1&1&1\end{pmatrix}$$

# 横场Ising model

$$H = \sum_{\langle i,j\rangle} J_{ij}\sigma_i^z\sigma_j^z - \Gamma\sum_i \sigma_i^x .$$

## 3-spin

```
H =

     3     1     1     0     1     0     0     0
     1    -1     0     1     0     1     0     0
     1     0    -1     1     0     0     1     0
     0     1     1    -1     0     0     0     1
     1     0     0     0    -1     1     1     0
     0     1     0     0     1    -1     0     1
     0     0     1     0     1     0    -1     1
     0     0     0     1     0     1     1     3
```

8x8矩阵的Ising只是Heisenberg矩阵的对角元（不乘上1/4系数），

此时再把横场加上，矩阵如左表示