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Electromagnetic detection of metallic particles

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Abstract: The effect of a conducting permeable spherical particle on the impedance of a solenoid is calculated, and consideration given to maximising the effect for detection purposes. For greatest sensitivity either the resistive change or the reactive change should be optimised, according to the method of detection; the frequency ranges needed to do so are indicated for both magnetic and nonmagnetic particles.

1 Introduction

The distortion of time-varying magnetic fields by the eddy currents which they produce has, for long, been used to locate or investigate metallic objects. Well developed applications include detectors for buried metal [1, 2] and nondestructive testing of the integrity and properties of metal products [3, 4]; a more recent application is sensing for automation [5]. The additional effect of permeability in the case of ferromagnetic materials can also be put to use; for example, in geophysical studies [6].

A further possible application is the detection of metallic particles to be used as tracers; for example, in the bulk flow of nonmetallic material. This differs from common applications in three respects: there is no need for absolute measurement, nor for separating effects due to different properties; there is a free choice of material; and the need for sensitivity is acute. The following Sections address the question of choosing the material and operating frequency to obtain the maximum sensitivity in detecting a relatively small metallic particle.

2 Basic detector

In electromagnetic detection or testing, a time-varying magnetic field produced by a coil is applied to the region containing the metal. The changes to the field, caused by the presence of metal, are measured by their effect on a circuit, which may or may not be that of the coil producing the field. The coil itself may be driven by sinusoidal alternating current or by a train of rectangular pulses; in the latter case, the response due to eddy currents which occurs after the end of each pulse is readily separable from that due to the original field [2]. In the case of AC excitation, the field distortion causes a change in the impedance of the driven coil, which may be detected, for example, by its effect on a tuned circuit of which the coil is part. In what follows, only the AC case is considered; the same principles could, if need arose, be extended to pulse excitation.

We consider, then, the effect of a small, isotropic and homogeneous spherical particle of radius a, conductivity σ and relative permeability μ_r on the impedance of an air-cored solenoid through which it passes; in the first instance, the field of the solenoid is supposed to be uniform. The effect of eddy currents in a conductor is frequently calculated from models with various simplifying assumptions: for example, when the skin depth is small, a conducting body becomes in effect a shell which may be regarded as a short-circuited turn of a transformer. In the case of a sphere, in a uniform alternating magnetic field, more general solutions are well known [7], but (unlike the more common case of a long cylinder [3]) they are not usually given explicitly in terms of the circuit producing the field. For the sake of completeness, a general solution in these terms is outlined in the following Section.

3 Theory

We suppose the sphere to be situated at the origin of co-ordinates, as shown in Fig. 1, in a region of otherwise uniform magnetic flux density $B_z e^{j\omega t}$. The resulting field follows from Maxwell's equations with the appropriate symmetry and boundary conditions. We assume, as is usual in eddy-current calculations, that displacement current is negligible: for ordinary metals this is easily satisfied over the frequency range met in conventional instrumentation, say up to 100 MHz.

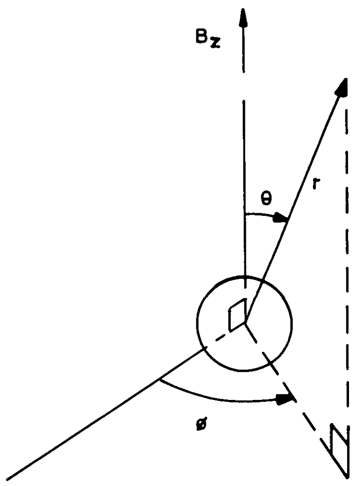

Fig. 1 *Co-ordinate system*

The vector potential outside the sphere is entirely azimuthal and may be written [7]

$$A_\phi = \tfrac{1}{2}B_z(r + Fa^3/r^2)\sin\theta\, e^{j\omega t} \quad (1)$$

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in which

$$F = \frac{(2\mu_r + 1)vI_{-1/2}(v) - (1 + v^2 + 2\mu_r)I_{1/2}(v)}{(\mu_r - 1)vI_{-1/2}(v) + (1 + v^2 - \mu_r)I_{1/2}(v)} \tag{2}$$

I_n signifies the modified Bessel function of order n and

$$v = a(j\omega\mu_r\mu_0\sigma)^{1/2} \tag{3}$$

As the skin depth δ for the material of the sphere is given by $(2/\omega\mu_r\mu_0\sigma)^{1/2}$, it follows that

$$v = (2j)^{1/2}a/\delta = u(1 + j) \tag{4}$$

if we write u for the ratio a/δ.

In eqn. 1, the term in r represents the vector potential of the uniform field undistorted by the sphere. We now suppose that the field is produced inside a solenoid of radius b by a current which is unaffected by the presence of the sphere. The impedance of the solenoid, including its inductive reactance and that part of its effective resistance which is attributable to dissipation in the sphere (and which appears in the circuit only by way of the distorted field), is then proportional to the flux linking it. The flux is most easily found by integrating A_ϕ around a path of radius b.

Choosing, for simplicity, the circular path $r = b$ and $\theta = \pi/2$, we have, from eqn. 1,

$$A_\phi = \tfrac{1}{2}B_z(b + Fa^3/b^2)e^{j\omega t}$$

and the flux enclosed is therefore

$$\Phi = 2\pi b A_\phi = B_z(\pi b^2 + \pi F a^3/b)e^{j\omega t}$$

the first term of which represents the flux in the absence of the sphere. Writing Z for the impedance of the solenoid, excluding the intrinsic resistance of its winding, and X_0 for its reactance without the sphere, we then have

$$\frac{Z}{X_0} = \frac{\pi b^2 + \pi F a^3/b}{\pi b^2} = 1 + Fa^3/b^3 \tag{5}$$

We may consider Z to be made up of the reactance X_0 together with increments, due to the sphere, of resistance and reactance ΔR and ΔX, respectively, so that eqn. 5 gives

$$\frac{\Delta R + j\Delta X}{jX_0} = F\frac{a^3}{b^3}$$

and, therefore,

$$\frac{\Delta R}{X_0} = -\mathrm{Im}\,(F)\frac{a^3}{b^3} \tag{6}$$

$$\frac{\Delta X}{X_0} = \mathrm{Re}\,(F)\frac{a^3}{b^3} \tag{7}$$

The quantity a^3/b^3 may be called the filling factor of the sphere in the solenoid, because it corresponds to the filling factor a^2/b^2 for a long solenoid containing a conducting cylinder; but here the factor a^3/b^3 does not signify an actual volume ratio.

Fig. 2 shows the complex dimensionless quantity F, computed for a range of values of u and μ_r, on an Argand diagram arranged to show resistance and reactance in a conventional orientation. The diagram shows expected features: for small u, corresponding to low frequency, eddy currents are weak and have little effect on inductance or resistance, but the inductance may be increased by a fraction up to twice the filling factor by virtue of the permeability; for high frequencies and hence large u, the skin depth and the resistance are small, but the eddy currents have their greatest effect in reducing inductance; between these extremes, the effective resistance reaches a maximum and the effects on inductance of

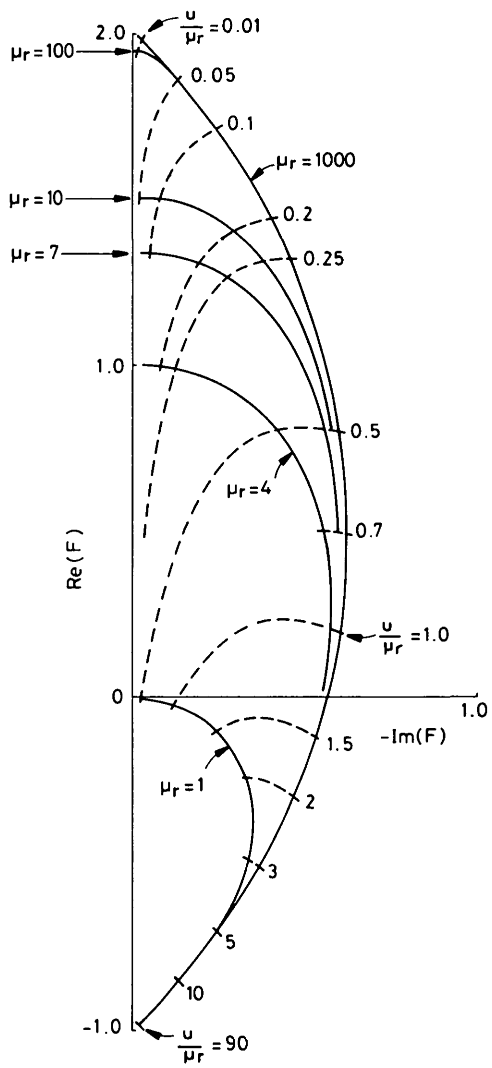

Fig. 2 *The function F*

permeability and of eddy currents tend to cancel. For $u \gg 1$, the value of F becomes quite closely a function of the ratio u/μ_r only: this can readily be deduced from eqn. 2 by noting that, for large u, we may set $I_{1/2}(v) = I_{-1/2}(v)$.

The foregoing derivation is based on the assumption of an initially uniform field B_z in the solenoid. If this is not so, the error in the vector potential due to the sphere itself will be negligible, provided that B_z is virtually uniform over the small region which the sphere will occupy; but, if the field is not uniform elsewhere, the total vector potential will not be correctly given by eqn. 1 and the calculated increments of impedance due to the sphere will be in error. Because it is the integrated effect of B_z over a cross-section of the solenoid which determines the flux through that section, and because, in practice, we may expect a particle to pass through every cross-section of the solenoid, we need consider only nonuniformity in the radial direction. The worst case will be that of a single loop of wire.

Excluding the contribution internal to the wire, a loop of mean radius b with wire of radius c has a self-inductance in air given by

$$L = \mu_0 b\{\ln(8b/c) - 2\}$$

If the loop carries current I, the flux density at its centre, which is the minimum value in its plane, is

$$B_z = \mu_0 I/2b$$

provided that $b \gg c$. If this value is applied uniformly over the plane bounded by the wire, the self-inductance is

$$L' = \frac{\mu_0}{2b} \pi(b - c)^2$$

Comparing L' with L for likely values of the ratio b/c, we find that L'/L is 0.53 for $b/c = 10$, and 0.33 for $b/c = 100$. The fractional increments predicted by Fig. 2 would, therefore, be too large by a factor between two and three, for a particle at the centre of a very compact coil

Nearer the wire of the loop, the flux density may be calculated from the standard expressions for the field due to a circular filament of current [7]: at radius $0.9b$ it is very nearly four times that at the centre, and over the entire plane would yield an inductance between $2L$ and $1.3L$ for b/c between 10 and 100; Fig. 2 would correspondingly underestimate the effect of a sphere in that position.

The worst discrepancy to be expected on account of a nonuniform field is therefore a factor of about three, for a particle on the axis, and this would be reduced for a solenoid of appreciable length. In practice the advantages of a relatively long solenoid would include, not only a more uniform field, but also a longer residence time for a moving particle.

4 Optimisation of sensitivity

Fig. 2 may be taken to represent the effect which can be achieved to a good approximation with a suitably designed detecting coil. For maximum response, the radius of the particle should be as large, and that of the coil as small, as practicably possible so as to maximise the filling factor a^3/b^3. (The choice of a also affects the response, because u is the ratio a/δ, but the filling factor has an overriding effect.) For given dimensions, the nature of the response is decided by the frequency used and the material of the particle.

For a moderately ferromagnetic particle, the magnitude of the impedance increment is roughly the same under all conditions, and under most is higher than that for a nonmagnetic material. The question, then, is whether to maximise $\Delta R/X_0$, on the one hand, or $\Delta X/X_0$ positively or negatively, on the other. The answer to this must depend on the actual detecting arrangement used.

If the coil forms a resonant circuit with a capacitor, detection may take the form of observing a change in the overall impedance around resonance, or, what is equivalent when the quality factor Q is high, a change in Q. We have

$$dQ = \frac{\partial Q}{\partial L} dL + \frac{\partial Q}{\partial R} dR$$

where R is the total series resistance of the coil and the capacitor is assumed perfect. As $Q = \omega L/R$,

$$dQ = \frac{\omega}{R} dL - \frac{\omega L}{R^2} dR$$

and

$$\frac{dQ}{Q} = \frac{dL}{L} - Q \frac{dR}{\omega L}$$

In the terms of eqns. 6 and 7, this may be written

$$\frac{\Delta Q}{Q} = \frac{\Delta X}{X_0} - Q \frac{\Delta R}{X_0}$$

and the effect of ΔR is magnified by the factor Q.

For any system which depends directly on a change in Q, for example a marginal oscillator or a system which detects the voltage change across a parallel-tuned circuit, there is evidently a good case for maximising $\Delta R/X_0$ while bearing in mind, if necessary, the variation of Q with frequency. Fig. 2 shows that $\Delta R/X_0$ is close to its maximum for $0.5 < u/\mu_r < 1$, provided that $\mu_r \geq 10$. We have

$$\frac{u}{\mu_r} = \frac{a}{\mu_r}\left(\frac{\omega \mu_r \mu_0 \sigma}{2}\right)^{1/2} = 2\pi a\left(\frac{10^{-7} f\sigma}{\mu_r}\right)^{1/2} \tag{8}$$

for cyclic frequency f. For a typical ferrous alloy, σ may be in the region of 8 $\mathrm{MSm^{-1}}$; if we take $a = 1$ mm then u/μ_r lies between 0.5 and 1.0, when

$$8 \times 10^3 \mu_r < f < 3.2 \times 10^4 \mu_r.$$

Frequencies in excess of 80 kHz are therefore required. The effective value of μ_r will, in practice, depend on the strength of the field used; too high a value will yield a frequency range in which Q, for a typical coil, is significantly below its maximum. There is, therefore, no strong case for using highly ferromagnetic alloys, which would raise the required frequency, not only because of their high permeability, but also because they normally have relatively low conductivity.

An alternative method of detection is to observe the phase shift in the impedance of the tuned circuit. Around resonance, the angle ϕ of the impedance is small, and it may be shown that

$$\Delta\phi = -Q \frac{\Delta X}{X_0} - \frac{\Delta R}{X_0}$$

so that here it is the change in reactance which is magnified. Such a method therefore requires the largest possible $\Delta X/X_0$ consistent with a high Q. Fig. 2 shows that 70% of the maximum attainable increment can be achieved for $u/\mu_r < 0.2$, provided that $\mu_r \geq 10$. If, as before, we take σ to be 8 $\mathrm{MSm^{-1}}$ and a to be 1 mm, this requires that

$$f < 1270\ \mu_r$$

which could allow frequencies up to about 1 MHz for readily attainable values of μ_r.

A third possibility, which is sometimes used, is to detect the change in frequency when the coil is part of an oscillator. Here, the effect of resistance is negligible, provided that oscillation continues, so again the reactance change should be maximised; the fractional change in frequency is given by $(\Delta X/X_0)/2$ and is, therefore, considerably less than the corresponding changes available for other methods of detection.

In practice, detection at the limits of sensitivity would pose problems not accounted for in the foregoing, such as the effect of noise and of drift due, for example, to temperature changes. For such reasons, circuits having two inductors arranged differentially, in a bridge or otherwise, may be preferable to the simple resonant form; the choice between optimising ΔR and ΔX would depend on the circuit used.

A nonmagnetic particle, while generally inferior in its available effect, can nevertheless produce resistive and inductive effects up to about half the maxima. The curve for $\mu_r = 1$, in Fig. 2, shows that $\Delta R/X_0$ attains half of its maximum value for values of u between two and three. For $a = 1$ mm as before, this requires that

$$1.01 \times 10^{12} < f\sigma < 2.28 \times 10^{12}$$

which, in the case of copper ($\sigma = 60$ MSm^{-1}), for example, means that f lies in the range 17 – 38 kHz. The same particle would produce more than 70% of the limiting (negative) reactance increment for $u > 5$, which corresponds to frequencies above about 100 kHz.

5 Conclusion

None of the cases considered in the preceding Section implies any great practical difficulty, so far as combinations of frequency and material properties are concerned. The choice between optimising resistive and reactive effects, and, hence, of the method of detection, could rest on the frequency required to maximise the natural Q-factor of the circuit, which is crucial in any resonance method, unless it is enhanced by active means. The same consideration could influence the choice between magnetic and nonmagnetic particles, as it could easily outweigh the weaker response of the latter. Nonmagnetic particles also have the advantage of virtually constant properties and, therefore, a more precisely predictable response.

The actual sensitivity attainable can be estimated only roughly, because of the strong dependence on the filling factor. The optimistic value of the ratio a/b can hardly exceed 0.1, and the filling factor is then 10^{-3}. The maximum value of $\Delta R/X_0$ is therefore about 0.6×10^{-3}, and, if we take 100 to be a readily attainable value of Q, then $\Delta Q/Q$ is 0.06; similarly, if the available $\Delta X/X_0$ is taken to be 1.4×10^{-3}, then $\Delta\phi$ would be 0.14 rad. These figures should be easily detectable, but they would, of course, fall rapidly with smaller a/b.

It is intended that these predictions should be put to the test; in the first instance, by direct measurement of impedance increments.

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