Homework 3
Chaotic Dynamics - CSCI 4446

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0 Problem 0

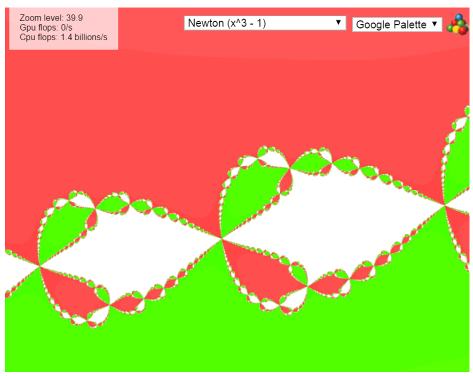

Figure 1: $x^3 - 1 = 0$ zoomed in at 40x.

It's amazing how the fractal behavior continues infinitely.

1 Problem 1

$$\lim_{x\to\infty} \frac{\log 2^n}{(\log \frac{1}{\frac{5}{12}})^n} = \lim_{x\to\infty} \frac{\log 2}{\log 12/5} \approx 0.791744069$$

2 Problem 2

2.1 a)

$$u_1 = x$$

$$u_2 = \dot{u_1} = \dot{x}$$

$$u_3 = \dot{u_2} = \ddot{x}$$

$$\dot{u_3} = \dddot{x} = \frac{1}{2}(3\tan\frac{u_3}{2} - 16\log u_2 + u_1)$$

Therefore, our 3 ODE's are:

$$\dot{u_1} = \dot{x}$$

$$\dot{u_2} = \ddot{x}$$

$$\dot{u_3} = \dddot{x} = \frac{1}{2}(3\tan\frac{u_3}{2} - 16\log u_2 + u_1)$$

2.2 b)

$$\dot{z} = yz + \log y = y\dot{y} + \log y = \dot{x}\ddot{x} + \log\dot{x} = \dddot{x}$$

Therefore, our single ODE is:

$$\dddot{x} - \dot{x}\ddot{x} - \log\dot{x} = 0$$

2.3 c)

Both a)and b) ODE systems are nonlinear, because they both have a logarithmic function and tangent function in one.

3 3

3.1 a)

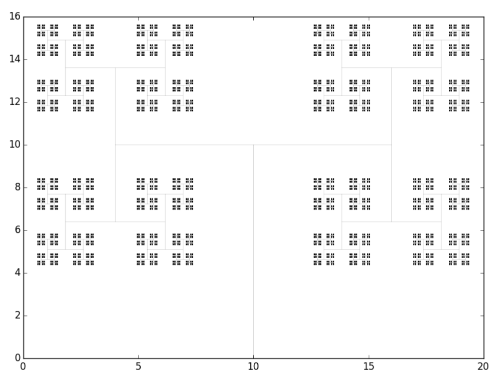

Figure 2: fractal tree with 90 degress angle

3.2 b)

If it's half as long, then we have a tree that has all of its "children" branches being shorter than parents. Therefore, the tree "converges" to a similar shape as in part a).

If it's $2^{(\frac{1}{2})}$, then each children branch is longer than its parent. Therefore, the tree "diverges".

If we make ratio less than 0.5, then the tree would converge even faster.

3.3 c)

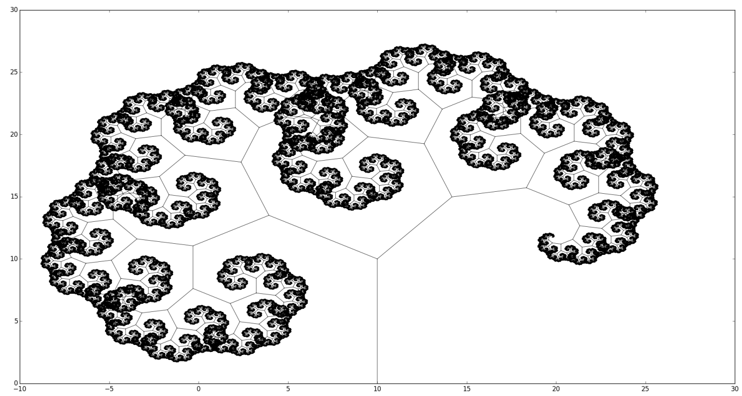

Figure 3: LeftRatio = 0.7, RightRatio=0.6, LeftAngle = 60, RightAngle = 40

3.4 d)

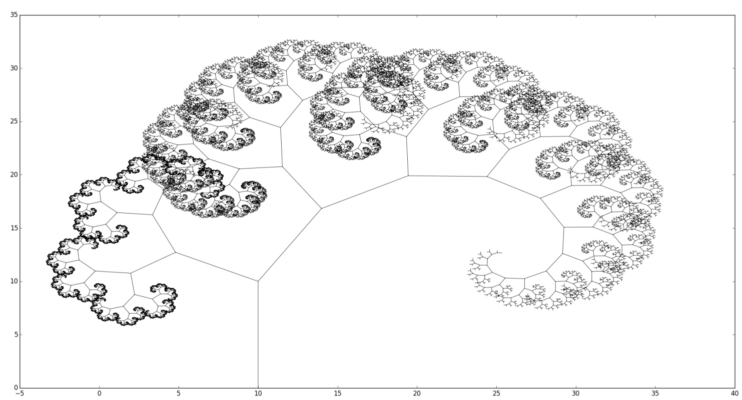

Figure 4: Randomized lengths, angles. Rewrite them at every recursive step.

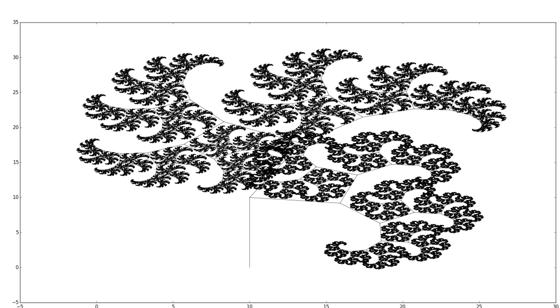

Figure 5: Randomized lengths, angles. Don't rewrite them at every recursive step.