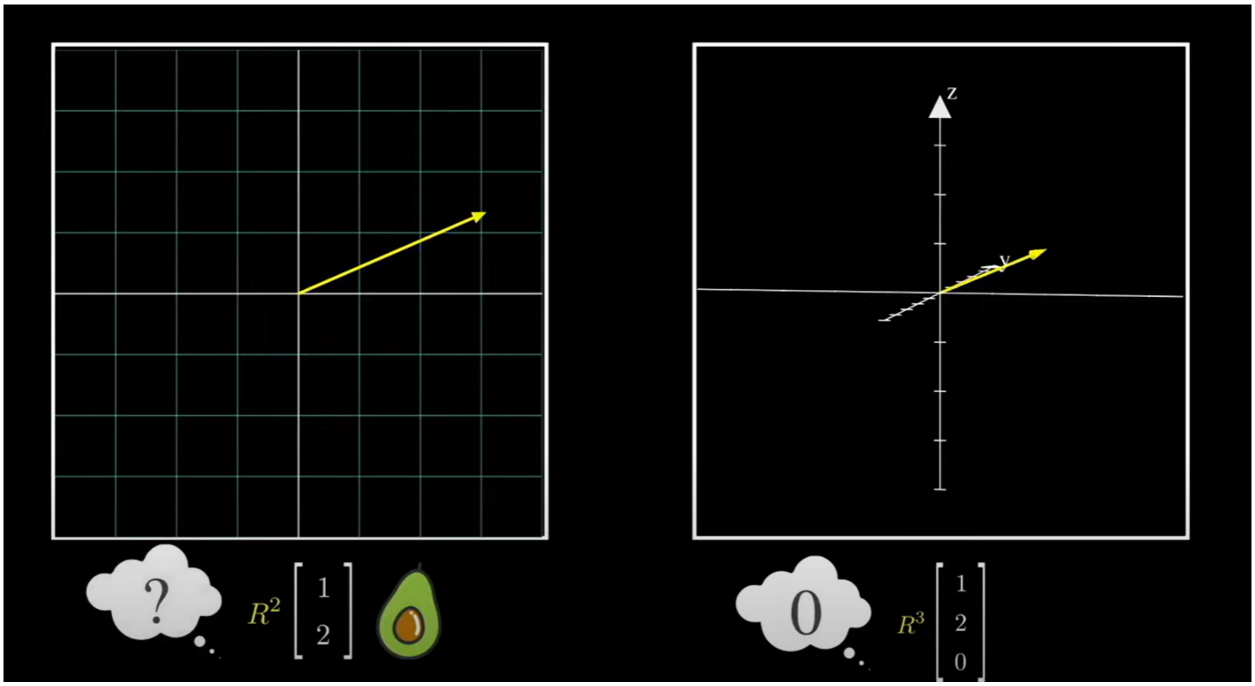
Single Value Decomposition Notes



Though these two vectors look similar, they are completely different.

If you move R2 around, you will never be able to change the value of Z, the third dimension. It's devoid of it.

How can we translate the two?

a 2x3 matrix can take R3 vector and translate it down into R2

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

This is intuitive. It just gets added up.

Matrixes apply transformation from R^N to R^M

Dimension Eraser

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Simplest form of R3 to R2

If multiplied with any vector, the X and Y are preserved. **the Z value is removed every time! irregardless of the Z**

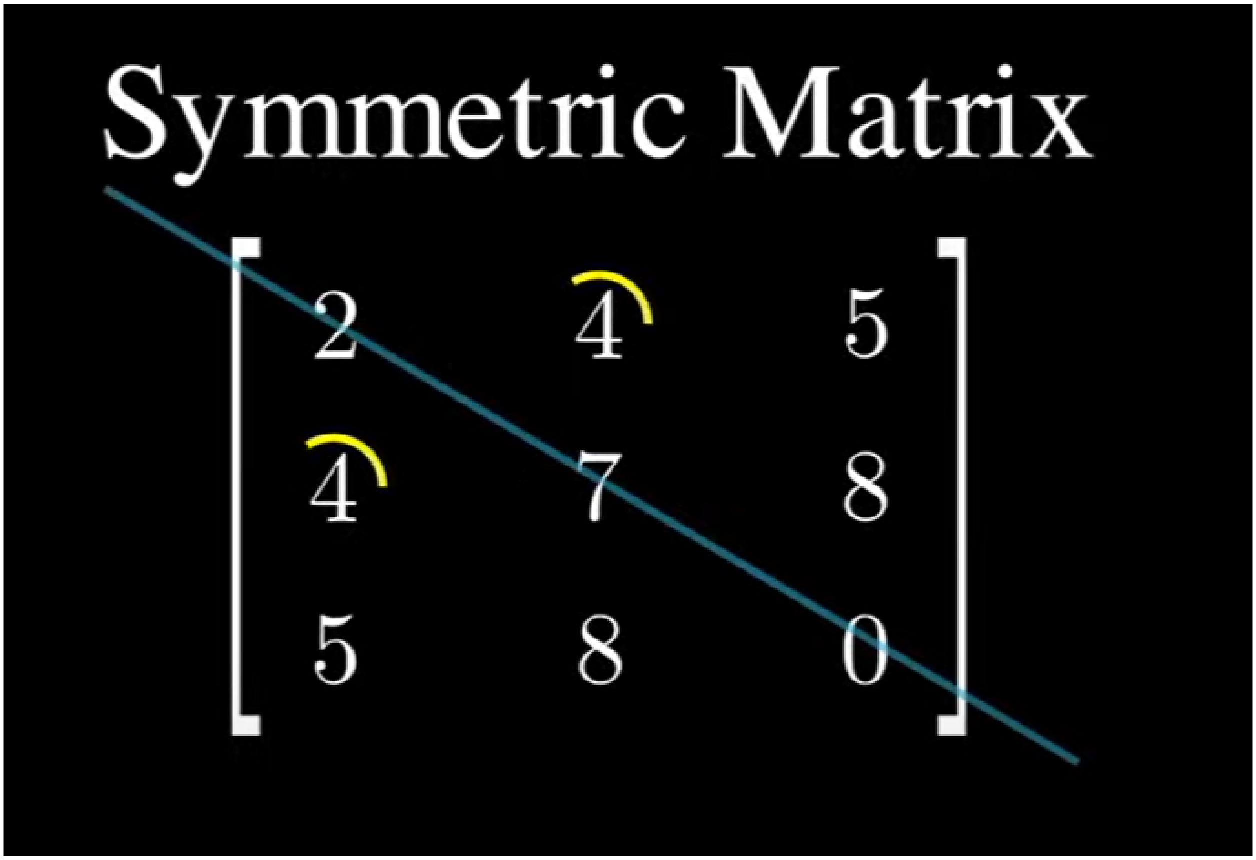
Dimension Adder

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

Appends 0 as a Z value for whatever R2 matrix it is multiplied with

R2 —> R3

Symmetric Matrix



Square matrix

On two sides of the diagonal line, the entries are identical

The eigenvectors of symmetric matrix are perpendicular to eachother

We can create symmetry out of nowhere **if we take the transpose and multiply it together**

$$\underset{A \quad 2\times 3}{\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}}\underset{A^T \quad 3\times 2}{\begin{bmatrix}1 & 4\\ 2 & 5\\ 3 & 6\end{bmatrix}} = \underset{AA^T \quad 2\times 2}{\begin{bmatrix}14 & 32\\ 32 & 77\end{bmatrix}}$$

- It’s also a square matrix

SVD

Any matrix A can be decomposed into 3 separate matrices

$$A = \underset{\text{Orthogonal}}{U}\ \underset{\substack{\text{Rectangular}\\ \text{Diagonal}}}{\Sigma}\ \underset{\text{Orthogonal}}{V^T}$$

Matrix U: left singular vectors of matrix A

Matrix V: normalized right singular vectors of matrix A

- One of the ways to break Matrix into singular pieces

Eigenvalues are restricted to square matrices, but SVD can work with rectangular matrices

- A way to find the important pieces of the matrix that add up into the whole matrix

First two pieces are the most important parts

Every matrix, rectangular, square, etc

- Can be written as product of 3 special matrices
- Rotation X stretch X Rotation