ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

path to success

PRE-MEDICAL

PHYSICS

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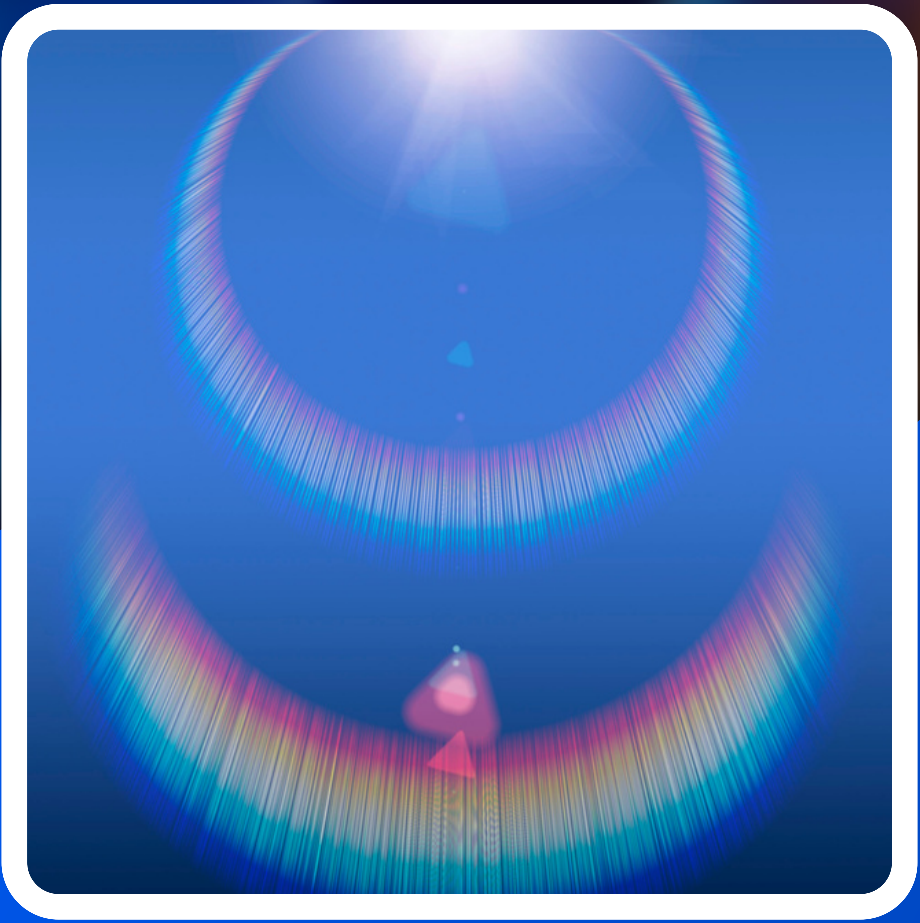

STUDY MATERIAL

Ray optics and optical Instruments

ENGLISH MEDIUM

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RAY OPTICS AND OPTICAL INSTRUMENTS

INTRODUCTION

Optics is the branch of physics which deals with the behavior of light waves. Under many circumstances, the wavelength of light is negligible compared with the dimensions of the device as in the case of ordinary mirrors and lenses. A light beam can then be treated as a ray whose propagation is governed by simple geometric rules. The part of optics that deals with such a phenomena is known as geometrical optics.

PROPAGATION OF LIGHT

Light travels along straight line path in a certain medium or in vacuum. The path of light changes only where the medium changes. We call this rectilinear (straight–line) propagation of light. A bundle of light rays is called a beam of light.

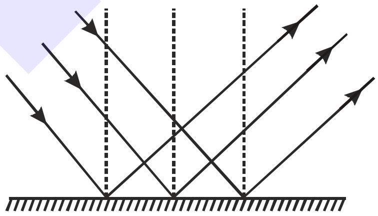
- Apart from vacuum and gases, light can travel through some liquids and solids as well. A medium in which light can travel without attenuation over large distances is called a transparent medium. Water, glycerine, glass and clear plastics are transparent. A medium in which light cannot travel is called opaque. Wood, metals, bricks, etc., are opaque. In materials like oil, light can travel some distance, but its intensity reduces rapidly. Such materials are called translucent.

1. REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light bounces back into the same medium. This phenomenon of light is called Reflection of light.

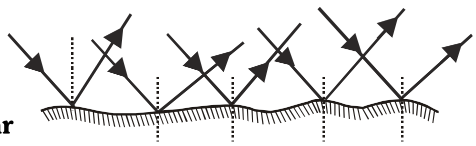
(i) Regular / Specular reflection :

When reflection takes place from a perfect plane surface then rays remain parallel after reflection . It is called **Regular reflection**.



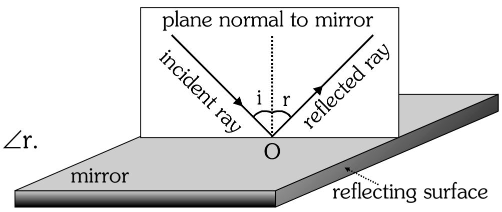
(ii) Irregular / Diffused reflection

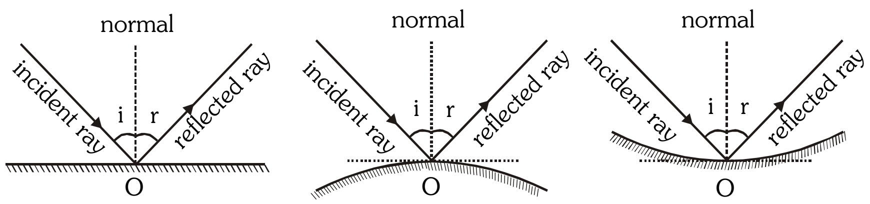
When the surface is rough, light is reflected from the bits of its plane surfaces in different directions. This is called **Irregular reflection**. This process enables us to see an object from any position.



LAWS OF REFLECTION

- Incident ray, reflected ray and normal at the point of incidence all lie in the same plane.
- The angle of reflection is equal to the angle of incident i.e. $\angle i = \angle r$.



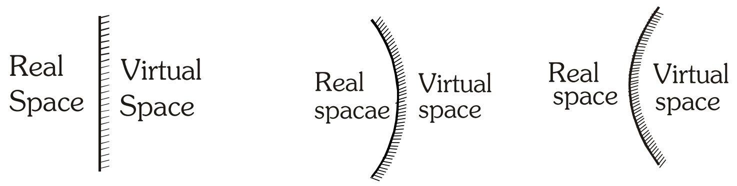


REAL AND VIRTUAL SPACES

A mirror, plane or spherical divides the space into two regions ;

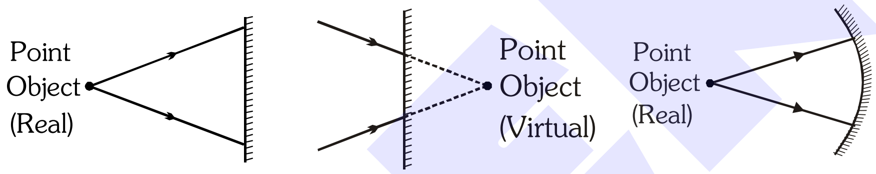
(a) Real space, the side where the reflected rays exist.

(b) Virtual space is on the other side where the reflected rays do not exist.



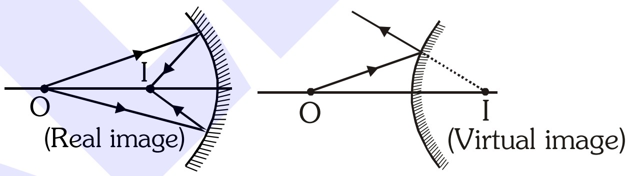
OBJECT

Object is decided by incident rays only. A point object is that point from which the incident rays actually diverge (real object) or towards which the incident rays appear to converge (virtual object).



IMAGE

Image is decided by reflected or refracted rays only. A point image is that point at which the refracted / reflected rays actually converge (real image) or from which the refracted /reflected rays appear to diverge (virtual image).

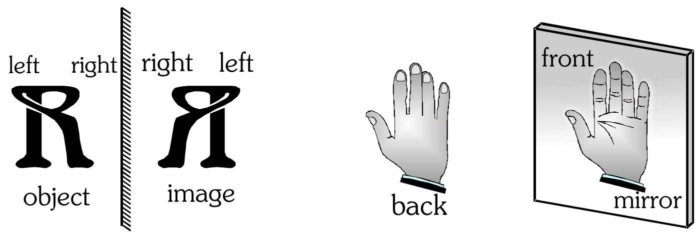


2. REFLECTION FROM PLANE MIRROR

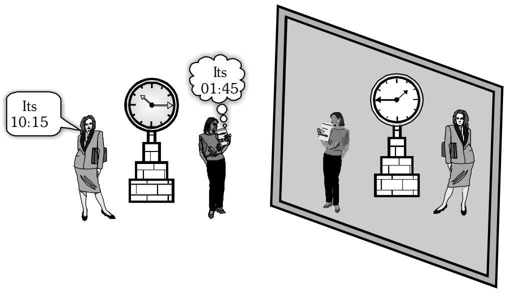
A plane mirror is a mirror with perfectly plane reflecting surface.

Plane mirror is the perpendicular bisector of the line joining object and image.

- The image formed by a plane mirror suffers **lateral–inversion,** i.e., left is turned into right and vice–versa with respect to object in the image formed by a plane mirror.



When a wall clock is placed in front of a plane mirror then the clock is the object and its time is object time and the image of the clock is observed by a person standing in front of a plane mirror then time seen by him is as follows.



(i) Object Time = A^H

Image Time = $12 - A^H$

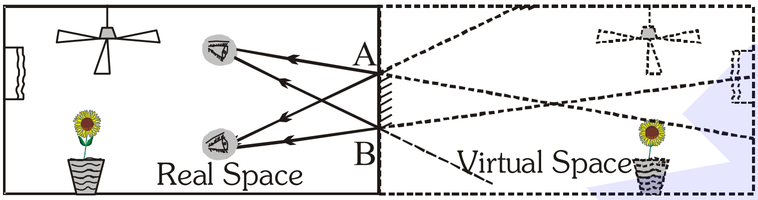
(ii) Object Time = $A^H\,B^M$

Image Time = $11 - 60' - A^H\,B^M$

(iii) Object Time = $A^H\,B^M C^S$

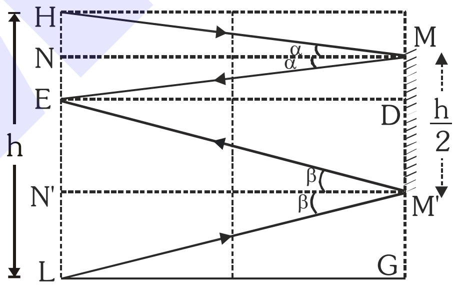
Image Time = $11 - 59' - 60'' - A^H B^M C^S$

- A plane mirror behaves like a window to a virtual world.



AB $\rightarrow$ Plane mirror

- To see the complete image in a plane mirror the minimum length of plane mirror should be half the height of a person.



From figure. ΔHNM and ΔENM are congruent

$\therefore$ EN = HN $\qquad \therefore$ MD = EN = $\dfrac{1}{2}$HE

Similarly ΔEN' M' and ΔLN' M' are congruent

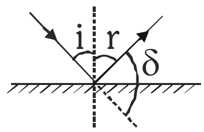
$\therefore$ EN' = N'L $\qquad \therefore$ M'D = EN' = $\dfrac{1}{2}$EL

Length of the mirror MM' = MD + M'D = $\dfrac{1}{2}$HE + $\dfrac{1}{2}$EL

$$= \frac{1}{2}\,(HE + EL) = \frac{1}{2}HL$$

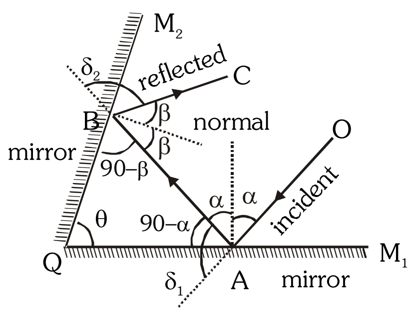
$\therefore$ Minimum length of mirror is just half the height of the person.

- This result does not depend on position of eye (height of the eye from ground).
- This result is independent of the distance of person from the mirror.



- Deviation for a single mirror $\delta = 180 - (i + r)$; $\angle i = \angle r$; $\delta = 180 - 2i$

- Total deviation produced by the combination of two plane mirrors which are inclined at an angle θ from each other.

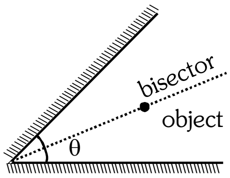


$\delta = \delta_1 + \delta_2 \;= 180 - 2\alpha + 180 - 2\beta \;= 360 - 2\,(\alpha + \beta)$ ...(i)

From ΔQAB, $\qquad \theta + 90 - \alpha + 90 - \beta = 180 \Rightarrow \theta = \alpha + \beta$ ...(ii)

Putting the value of θ in (i) from (ii), $\delta = 360 - 2\theta$

- If there are two plane mirrors inclined to each other at an angle θ the number of images (n) of a point object formed are determined as follows.



(a) If $\frac{360°}{\theta} = m$ is even then number of images n = m – 1

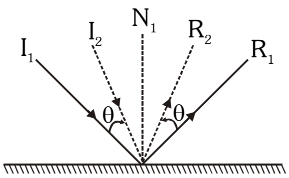
(b) If $\frac{360°}{\theta} = m$ is odd. There will be two cases:

(i) When object is not on the bisector, then number of images n = m

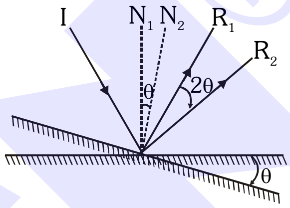
(ii) When object is on the bisector, then number of images n = m – 1

- If the object is placed between two plane mirrors then multiple images are formed due to successive reflections. At each reflection, a part of light energy is absorbed. Therefore, distant images get fainter.
- Keeping the mirror fixed if the incident ray is rotated by some angle, the reflected ray also gets rotated by the same angle but in opposite sense. (See Fig. 1)

(Fig. 1)



(Fig. 2)



- Keeping the incident ray fixed, if the mirror is rotated by some angle, then the reflected ray rotates by double the angle in the same sense. (See Fig. 2)
- $\vec{v}_{on} = -\vec{v}_{in}$, $\vec{v}_{op} = \vec{v}_{ip}$

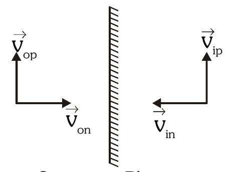
though speed of object and image are the same

v_{op} = component of velocity of object parallel to the mirror.

v_{on} = component of velocity of object normal to the mirror.

v_{ip} = component of velocity of image parallel to the mirror.

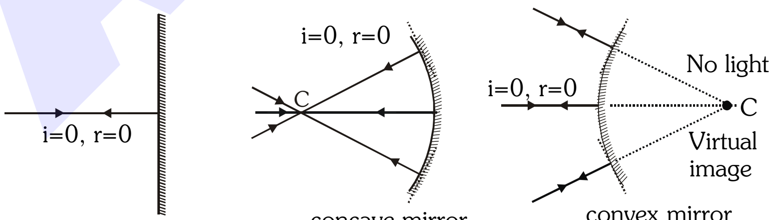
v_{in} = component of velocity of image normal to the mirror.



Stationary Plane Mirror

GOLDEN KEY POINTS

- **Rectilinear propagation of light :** In a homogeneous transparent medium light travels along straight line.
- When a ray is incident normally on a boundary after reflection it retraces its path.



plane mirror concave mirror convex mirror

- The frequency, wavelength and speed of light do not change after reflection.
- Eye is most sensitive for yellow green colour (555 nm.) and least sensitive for violet and red colours.

Due to this reason :

(i) Commercial vehicles are painted with yellow colour.

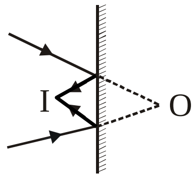
(ii) Sodium lamps [yellow colour (5890 Å & 5996 Å)] are used in street lamps.

Illustrations

Illustration 1.

A plane mirror often forms a virtual image. Can it even form a real image ? Explain your answer.

Solution

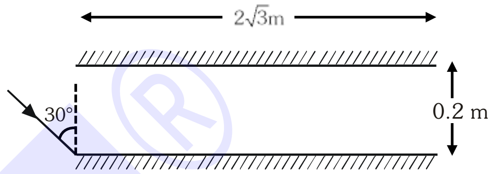


Yes, a plane mirror can form a real image if the object is virtual

i.e. if a convergent beam of light is incident on it.

Illustration 2.

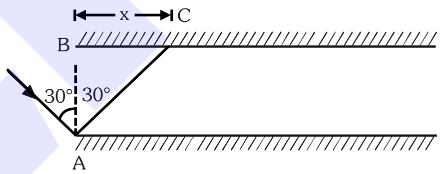
Two plane mirrors each of length $2\sqrt{3}$m are placed parallel to each other at a distance of 0.2m. A light ray is incident on one mirror at an angle of 30° as shown. Find the maximum number of reflection (including the first one) before it emerges out.



(1) 28 (2) 32 (3) 30 (2) 34

Solution

Let horizontal distance covered along mirrors before next reflection be x



$\therefore$ In ΔABC,

$$\tan 30^\circ = \frac{x}{0.2} \Rightarrow x = \frac{0.2}{\sqrt{3}}$$

So, number of reflections = $\dfrac{\text{length of mirror}}{x} = \dfrac{2\sqrt{3}}{(0.2/\sqrt{3})} = 30$

Illustration 3.

An object is placed between two plane mirrors inclined at 30° to each other. How many images will be formed?

Solution

$$n = \frac{360^\circ}{\theta} - 1 = \frac{360^\circ}{\theta} - 1 = 11$$

Illustration 4.

A boy 1.50 m tall with his eye-level at 1.38 m from the ground stands before a mirror fixed on a wall. Indicate by means of a ray diagram how the mirror should be positioned so that he can view himself fully. What should be the minimum length of the mirror? Does the answer depend on the eye level?

Solution

Minimum length of mirror = $\dfrac{H}{2} = \dfrac{1.5}{2} = 0.75$ m

No, the answer does not depend on the eye level or distance between person and mirror.

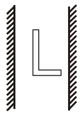
BEGINNER'S BOX-1

1. A boy is 1.8 m tall and can see his image in a plane mirror fixed on a wall. His eyes are 1.6 m from the floor level. The minimum length of the mirror to see his full image is :-

(1) 0.9 m (2) 0.85 m

(3) 0.8 m (4) can't be determined

2. An object shaped as 'L' is placed between two parallel plane mirrors as shown.



In the first seven closest images, how many laterally inverted images are formed ?

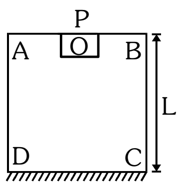
(1) 1 (2) 2 (3) 3 (4) 4

3. A plane mirror is placed horizontally on level ground at a distance of 60 m from the foot of a tower. Light rays from the top of tower falling just on the edge of the mirror suffers a deviation of 90°. The height of the tower is :-

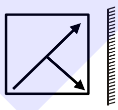
(1) 30 m (2) 60 m (3) 90 m (4) 120 m

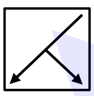
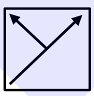
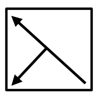
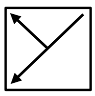
4. A plane mirror makes an angle of 30° with the horizontal. If a vertical ray strikes the mirror, find the angle between the mirror and the reflected ray ?

5. Figure shows the plane (top view) of a cubical room, with the wall CD as a plane mirror; each side of the room is 3 metres in length. A camera P is placed at the mid point of the wall AB. At what distance should the camera be focused to photograph the image of an object placed at A ?



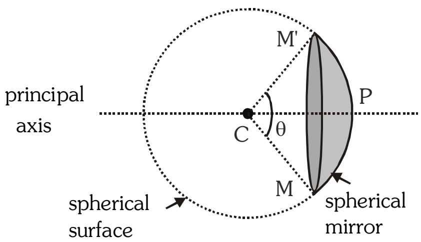
6. The correct mirror–image of the figure given is :-

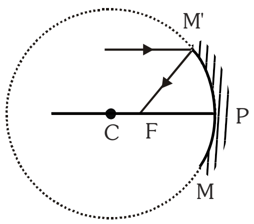


(1)  (2)  (3)  (4) 

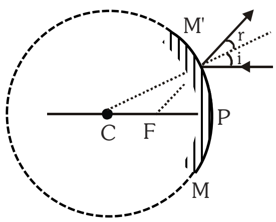
3. SPHERICAL MIRROR

Curved mirror is a part of a hollow sphere. If reflection takes place from the inner surface then the mirror is called concave and if its outer surface acts as reflector it is convex.





concave mirror



convex mirror

DEFINITIONS FOR THIN SPHERICAL MIRRORS

(i) **Pole** is any point on the reflecting surface of the mirror. For convenience we take it as the central P of the mirror (as shown).

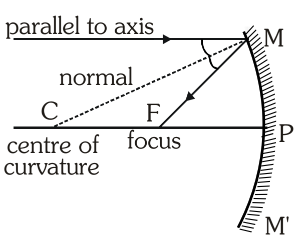
(ii) **Principal–section** is any section of the mirror such as MM' passing through the pole.

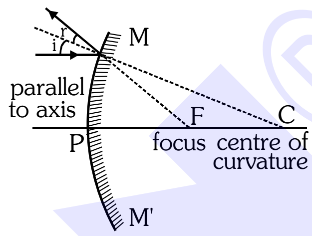
(iii) **Centre of curvature** is the centre C of the sphere of which the mirror is a part.

(iv) **Radius of curvature** is the radius R of the sphere of which the mirror is a part.

(v) **Principal–axis** is the line CP, joining the pole and centre of curvature of the mirror.

(vi) **Principal– focus** is an image point F on the principal axis for which object is at infinity.

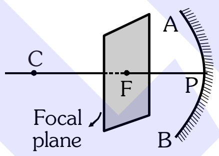


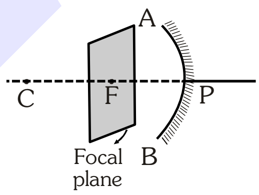


(vii) **Focal–length** is the distance PF between pole P and focus F along the principal axis.

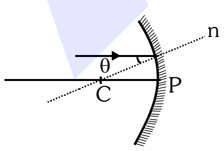
(viii) **Linear Aperture,** in reference to a mirror, is the effective diameter of the light reflecting area of the mirror.

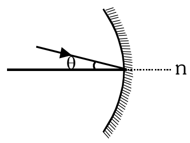
(ix) **Focal Plane** is the plane passing through focus and perpendicular to the principal axis.

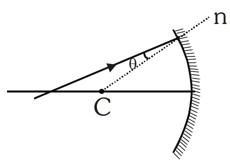




(x) **Paraxial Rays** Those rays which make small angle with normal at point of incidence and hence are close to principal axis.



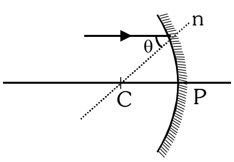




(θ is very small)

(xi) **Marginal rays :**

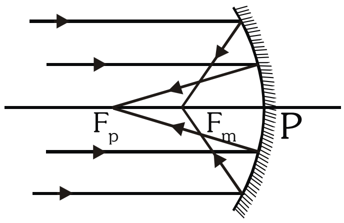
Rays having a large angle of incidence



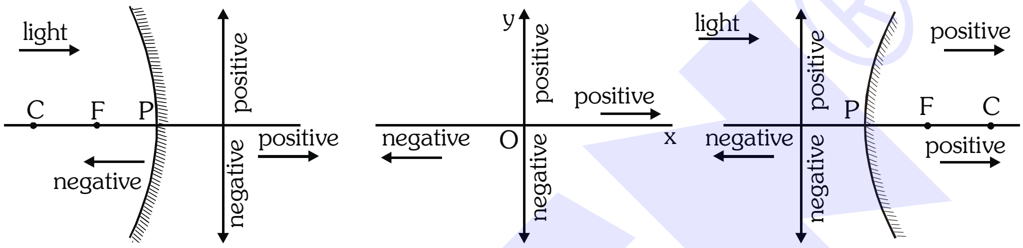
(θ is large)

SPHERICAL ABERRATION

When all rays are incident on a spherical mirror in a direction parallel to the axis, the marginal rays (i.e. the rays incident on the mirror just close to the edge) come to focus at a point nearest to the mirror called the marginal focus (F_m). The paraxial (or central) rays come to focus further away at 'F_p'. Thus, the image of the distant object is not formed at one point but is spread along the axis between 'F_m' and 'F_p' this defect is called spherical aberration.



SIGN – CONVENTION

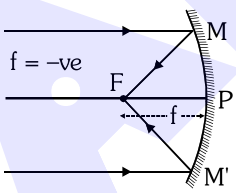


- Along the principal axis, distances are measured from the pole (pole is taken as the origin).
- Distances in the direction of incident light are taken positive while those along opposite direction negative.
- The distances above the principal axis are taken positive while below it negative.
- Whenever and wherever possible incident light is taken to travel from left to right.

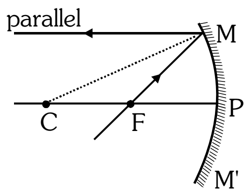
RULES FOR IMAGE FORMATION (FOR PARAXIAL RAYS ONLY)

(These rules are based on the laws of reflection $\angle i = \angle r$)

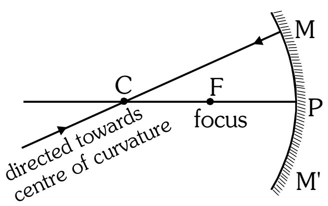
- A light ray parallel to the principal axis after reflection from the mirror passes or appears to pass through its focus (by definition of focus).



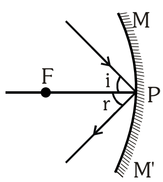
- A light ray passing through or directed towards focus, becomes parallel to the principal axis after reflection from the mirror.



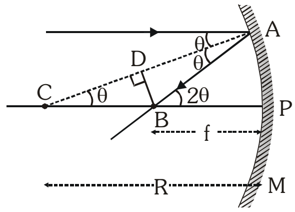
- A ray passing through or directed towards centre of curvature, retraces its path (as for it $\angle i = 0$ and so $\angle r = 0$) after reflection from the mirror.



- Incident and reflected rays at the pole of a mirror are symmetrical about the principal axis $\angle i = \angle r$.



Relation between f and R for a spherical mirror



1. For marginal rays, In ΔABC, AB = BC and

AC = CD + DA = 2BC $\cos\theta$

$\Rightarrow$ R = 2BC$\cos\theta$

$BC = \dfrac{R}{2\cos\theta}$ and

$BP = PC - BC = R - \dfrac{R}{2\cos\theta}$

2. For paraxial rays

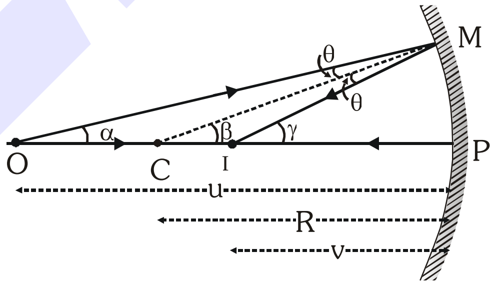
(θ is small $\therefore \cos\theta \approx 1$)

Hence $BC = \dfrac{R}{2}$ and $BP = \dfrac{R}{2}$. Thus, point B is the midpoint of PC and is defined as focus so

$BP = f = \dfrac{R}{2}$

Relation between u,v and f for a curved mirror

An object is placed at a distance u from the pole of a mirror for small angles and its image is formed at a distance v (from the pole).



If angle is very small : $\alpha = \dfrac{MP}{u}$, $\beta = \dfrac{MP}{R}$, $\gamma = \dfrac{MP}{v}$

from Δ CMO, $\beta = \alpha + \theta \Rightarrow \theta = \beta - \alpha$

from Δ CMI, $\gamma = \beta + \theta \Rightarrow \theta = \gamma - \beta$

so we can write $\beta - \alpha = \gamma - \beta \Rightarrow 2\beta = \gamma + \alpha$

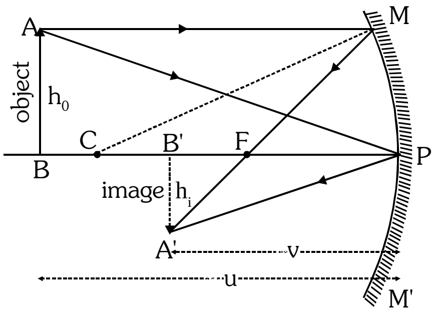
$$\therefore \quad \frac{2}{R} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

As per sign convention for object/image for spherical mirrors

Real object	u – ve	Real image	v – ve
Virtual object	u + ve	Virtual image	v + ve

MAGNIFICATION

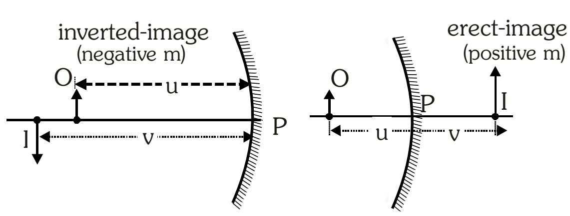
Transverse or lateral or linear magnification



Linear magnification $m_T = \dfrac{\text{height of image}}{\text{height of object}} = \dfrac{h_i}{h_o}$

ΔABP and $\Delta A'B'P$ are similar so $\dfrac{-h_i}{h_o} = \dfrac{-v}{-u} \Rightarrow \dfrac{-h_i}{h_o} = -\dfrac{v}{u}$

Magnification $m_T = -\dfrac{v}{u}$; $m_T = -\dfrac{v}{u} = \dfrac{f}{f-u} = \dfrac{f-v}{f} = \dfrac{h_i}{h_o}$



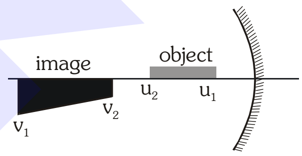
If one dimensional object is placed perpendicular to the principal axis then linear magnification is called transverse or lateral magnification. $\boxed{m_T = \frac{h_i}{h_o} = -\frac{v}{u}}$

Magnification	Image	Magnification	Image
$\lvert m \rvert > 1$	enlarged	$\lvert m \rvert < 1$	diminished
$m < 0$	inverted	$m > 0$	erect

- **Longitudinal magnification**

  If a rod is placed along the principal axis then linear magnification is called longitudinal or axial magnification.

  Longitudinal magnification : $m_L = \frac{\text{length of image}}{\text{length of object}} = \frac{|v_2 - v_1|}{|u_2 - u_1|}$



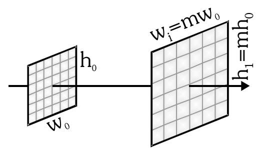
  For small objects only : $m_L = \frac{dv}{du}$

  differentiation of $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ yields $-\frac{dv}{v^2} - \frac{du}{u^2} = 0 \Rightarrow -\frac{dv}{du} = \left[\frac{v}{u}\right]^2$ so $-m_L = m_T^2 \Rightarrow m_L = -m_T^2$

- **Superficial magnification**

  If two dimensional object is placed with its plane perpendicular to the principal axis then its magnification is known as superficial magnification

  Linear magnification $m_T = \frac{h_i}{h_o} = \frac{w_i}{w_o}$



  $h_i = m_T h_o$, $w_i = m_T w_o$ Also $A_{obj} = h_o \times w_o$

  $A_{image} = h_i \times w_i = m_T h_o \times m_T w_o = m_T^2 \; A_{obj}$

  Superficial magnification $m_s = \frac{\text{area of image}}{\text{area of object}} = \frac{A_{image}}{A_{object}} = m_T^2$

IMAGE FORMATION BY SPHERICAL MIRRORS

Concave mirror

(i) **Object :** Placed at infinity

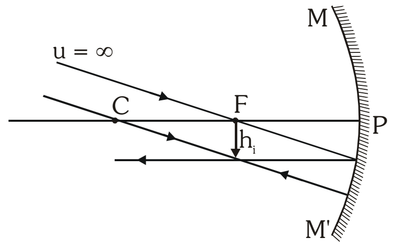
Image : real, inverted, highly diminished, at F

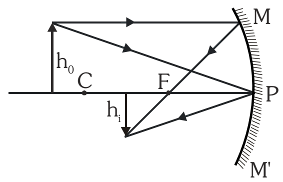
$|m| << 1$ & $m<0$

(ii) **Object :** Placed in between infinity and C

Image : real, inverted, diminished, in between C and F

$|m| < 1$ & $m<0$

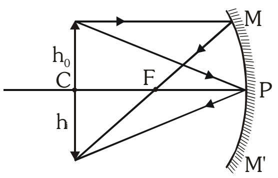




(iii) **Object :** Placed at C

Image : real, inverted, equal, at C

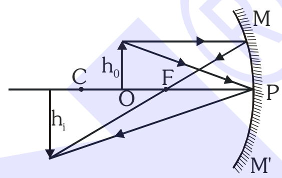
($m = -1$)



(iv) **Object :** Placed in between F and C

Image : real, inverted, enlarged beyond C

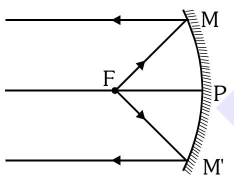
$|m| > 1$ & $m<0$



(v) **Object :** Placed at F

Image : real, inverted, very large

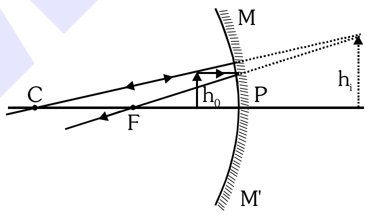
(assumed) at infinity ($m<<-1$)



(vi) **Object :** Placed between F and P

Image : virtual, erect, enlarged and

behind the mirror ($m > +1$)

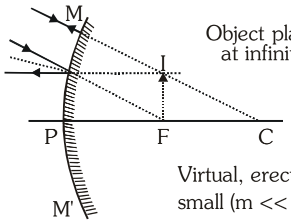


For concave mirror

Position of Object	Position of Image	Magnification
$-\infty$	F	$|m| << 1$ & $m<0$
$-\infty - C$	C – F	$|m| < 1$ & $m<0$
C	C	$m = -1$
C – F	$-\infty - C$	$|m| > 1$ & $m<0$
Between C and F, near F	$-\infty$	$m <<-1$
Between F and P, near F	$+\infty$	$m >>1$

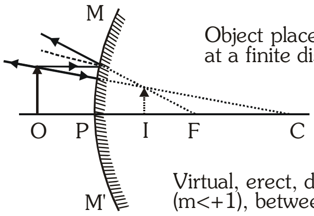
Convex mirror

Image is always virtual and erect, whatever be the position of the object and m is always positive.



Object placed at infinity

Virtual, erect and very small (m << +1) at F



Object placed at a finite distance

Virtual, erect, diminished (m<+1), between P and F

POWER OF A MIRROR

The power of a mirror is defined as $P = -\dfrac{1}{f(m)} = -\dfrac{100}{f(cm)}$

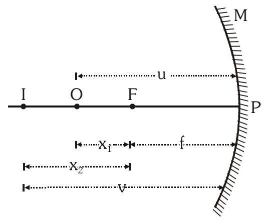
VELOCITY OF THE IMAGE OF A MOVING OBJECT

When the object is approching the focus of a concave mirror from infinity with speed v_{obj}.

$$v = \frac{uf}{u-f} \Rightarrow \frac{dv}{dt} = \frac{(u-f)\frac{du}{dt}f - uf\left(\frac{du}{dt}-0\right)}{(u-f)^2} \Rightarrow v_{image} = \frac{dv}{dt} = -\frac{f^2}{(u-f)^2}\frac{du}{dt} = -m^2 v_{obj}$$

velocity of image = $-m^2$ × velocity of object

NEWTON'S FORMULA



In case of spherical mirrors if object distance (x_1) and image distance (x_2) are measured from the focus instead of pole, then $u = -(f+x_1)$ and $v = -(f+x_2)$,

by $\dfrac{1}{v}+\dfrac{1}{u}=\dfrac{1}{f} \Rightarrow -\dfrac{1}{(f+x_2)} - \dfrac{1}{(f+x_1)} = -\dfrac{1}{f}$

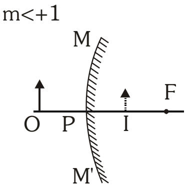
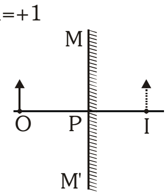
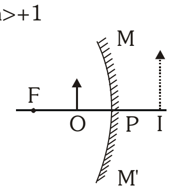
on solving $\boxed{x_1 x_2 = f^2}$ This is Newton's formula.

GOLDEN KEY POINTS

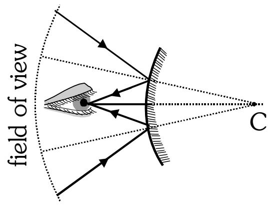
- **Difference between real and virtual image for spherical mirror**

Real Image	Virtual Image
(i) Inverted w.r.t. object	(i) Erect w.r.t. object
(ii) Can be obtained on screen	(ii) Cannot be obtained on screen
(iii) Its magnification is negative	(iii) Its magnification is positive
(iv) It is formed in front of the mirror	(iv) It is formed behind the mirror

- For a real extended object, if the image formed due to a single mirror is erect it is always virtual (i.e.,m is + ve) and in this situation the size of image is as follows :

Image is smaller than object the mirror is convex	Image is equal to object size the mirror is plane	Image is larger than object the mirror is concave
m<+1 	m=+1 	m>+1 

- **Convex mirrors** forms erect, virtual and diminished images. In a convex mirror the field of view is more wider as compared to a plane mirror. It is used as a rear–view mirror in vehicles.



- **Concave mirrors** form erect, virtual and enlarged images, so these are used by dentists for examining teeth. Due to their converging property concave mirrors are also used as reflectors in automobile head lights and search lights.
- As focal length of a spherical mirror (f=R/2) depends only on its radius and is independent of the wavelength of light and refractive index of medium. Hence it follows that the focal length of a spherical mirror in air or water and for red or blue light is the same.

Illustrations

Illustration 5.

The focal length of a concave mirror is 30 cm. Find the position of the object in front of the mirror, so that the image is three times the size of the object.

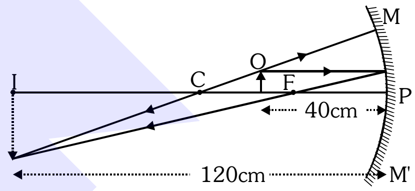
Solution

As the object is in front of the mirror it is real and for real object the magnified image formed by concave mirror can be inverted (i.e.,real) or erect (i.e.,virtual). So there are two possibilities.

(a) If the image is inverted (i.e., real)

$$m = \frac{f}{f-u} \Rightarrow -3 = \frac{-30}{-30-u} \Rightarrow u = -40 \text{ cm}$$

Object must be 40 cm away from the mirror (in between C and F).



(b) If the image is erect (i.e., virtual)

$$m = \frac{f}{f-u} \Rightarrow 3 = \frac{-30}{-30-u} \Rightarrow u = -20 \text{ cm}$$

Object must be 20 cm away from the mirror (in between F and P).

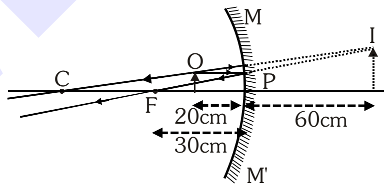


Illustration 6.

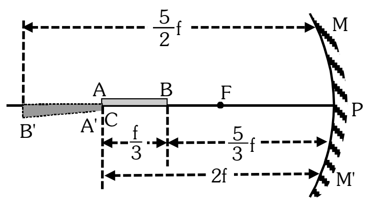
A thin rod of length $\frac{f}{3}$ is placed along the principal axis of a concave mirror of focal length f such that its image which is real and elongated, just touches one end of the rod. What is the magnification ?

Solution

Image is real and enlarged, the object must be between C and F.

One end A' of the image coincides with the end A of rod itself.

So $v_A = u_A$ $\frac{1}{v_A} + \frac{1}{v_A} = \frac{1}{-f}$, i.e., $v_A = u_A = -2f$



so it clear that the end A is at C. $\because$ the length of the rod is $\frac{f}{3}$

$\therefore$ Distance of the other end B from P is $u_B = 2f - \frac{f}{3} = \frac{5}{3}f$

If the distance of image of end B from P is v_B then $\frac{1}{v_B} + \frac{1}{-\frac{5}{3}f} = \frac{1}{-f} \Rightarrow v_B = -\frac{5}{2}f$

$\therefore$ the length of the image $|v_B| - |v_A| = \frac{5}{2}f - 2f = \frac{1}{2}f$ and magnification $m = \frac{|v_B| - |v_A|}{|u_B| - |u_A|} = \frac{\frac{1}{2}f}{-\frac{1}{3}f} = -\frac{3}{2}$

Negative sign implies that image is inverted with respect to object and so it is real.

Illustration 7.

The sun subtends an angle θ radians at the pole of a concave mirror of focal length f. What is the diameter of the image of the sun formed by the mirror ?

Solution

Angle subtend by object equal to angle subtend by image at the pole of mirror.

Diameter of image = fθ

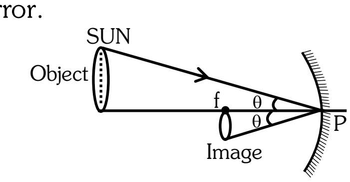


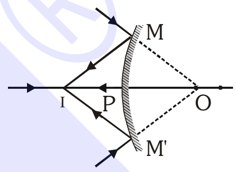
Illustration 8.

A beam of light converges towards a point O, behind a convex mirror of focal length 20 cm. Find the nature and position of the image if the point O is (a) 10 cm behind the mirror (b) 30 cm behind the mirror.

Solution

(a) For this situation object will be virtual as shown in figure.

Here u =+10 cm and f = +20 cm.



$\therefore \quad \frac{1}{v}+\frac{1}{+10}=\frac{1}{+20}$ i.e., $v = -20$ cm

i.e., the image will be at a distance of 20 cm in front of the mirror and will be real, erect and enlarged with

$m=-\left[-\frac{20}{10}\right]=+2$

(b) For this situation also object will be virtual as shown in Figure.

Here, u = + 30 cm and f = + 20 cm

$\therefore \quad \frac{1}{v}+\frac{1}{+30}=\frac{1}{+20}$ i.e., $v = +60$ cm

i.e.,the image will be at a distance of 60 cm behind the mirror and will be virtual, inverted and enlarged with $m=-\left[+\frac{60}{30}\right]=-2$

Illustration 9.

An object is placed in front of a convex mirror at a distance of 50 cm. A plane mirror is introduced covering the lower half of the convex mirror. If the distance between the object and the plane mirror is 30 cm, it is found that there is no parallax between the images formed by the two mirrors. What is the radius of curvature of the convex mirror ? Also calculate magnification produced by the convex mirror.

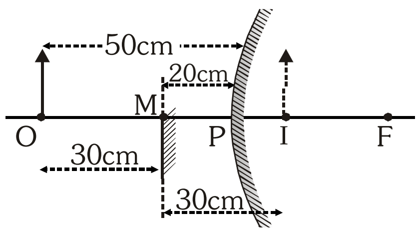
Solution

It is clear that virtual image in plane mirror is 30 cm behind it and there is no parallax so images formed by two mirrors will coincide and u = – 50 cm the distance of image formed by plane mirror from convex mirror v = PI = MI – MP = MO – MP = 30 – 20 = 10 cm [MI = MO]

Since this image coincides with image formed by convex mirror, so for

convex mirror $\frac{1}{+10}+\frac{1}{-50}=\frac{1}{f}$

$\Rightarrow \quad f=\frac{50}{4}=12.5$ cm so $R = 2f = 25$ cm

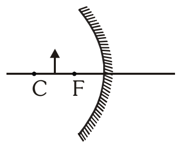
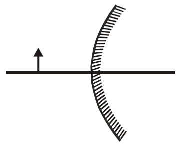
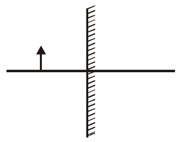
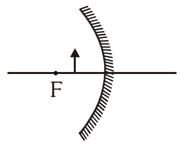


The image formed by convex mirror is erect, virtual and diminished.

magnification $m=-\left[\frac{v}{u}\right]=-\left[\frac{+10}{-50}\right]=+\left[\frac{1}{5}\right]$

BEGINNER'S BOX-2

1. Column-I contains a list of mirrors along with the position of object. Match this with Column-II describing the nature of the image.

Column I	**Column II**
(A) 	(P) real, inverted, enlarged
(B) 	(Q) virtual, erect, enlarged
(C) 	(R) virtual, erect, diminished
(D) 	(S) virtual, erect

2. A man has a shaving mirror of focal length 0.2 m. How far should the mirror be held from his face in order to give an image of two fold magnification ?

3. A convex mirror has a focal length f. A real object is placed at a distance of $\frac{f}{2}$ from the pole. Find out the position, magnification and nature of the image.

4. A motor car is fitted with a rear view mirror of focal length 20 cm. A second motor car 2 m broad and 2.16 m high is 6 m away from first car. Find the position of second car as seen in the mirror of the first car.

5. A virtual image three times the size of the object is obtained with a concave mirror of radius of curvature 36 cm. Find the distance of the object from the mirror.

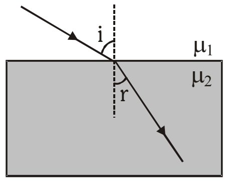
6. The focal length of a concave mirror is 30 cm. Where should an object be placed so that its image is three times magnified, real and inverted ?

7. A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to receive a sharp image ? Describe the nature and size the image. If the candle is moved closer to the mirror, how should the screen have to be moved ?

4. REFRACTION OF LIGHT

Refraction is the phenomenon in which direction of propagation of light changes at the boundary when it passes from one medium to the other. During refraction frequency does not change.

- **Laws of Refraction**



(i) Incident ray, refracted ray and normal always lie in the same plane.

(ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1 \sin i = \mu_2 \sin r$ (Snell's law)

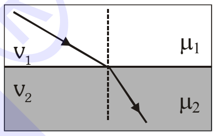
Absolute refractive index (n or μ)

It is defined as the ratio of speed of light in free space 'c' to that in a given medium v. Hence μ or $n = \dfrac{c}{v}$

Denser is the medium, lesser will be the speed of light and so greater will be the refractive index,

$v_{glass} < v_{water}$, $\quad \therefore \mu_G > \mu_W$

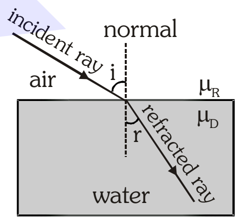
Relative refractive index



When light passes from one medium to other, then refractive index of medium 2 relative to 1 is written as ${}_1\mu_2$ and is defined as

$${}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{(c/v_2)}{(c/v_1)} = \frac{v_1}{v_2}$$

- **Bending of light ray**

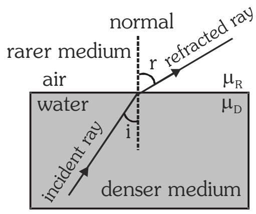


According to Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

(i) If light passes from rarer to denser medium $\mu_1 = \mu_R$ and $\mu_2 = \mu_D$

so that $\dfrac{\sin i}{\sin r} = \dfrac{\mu_D}{\mu_R} > 1 \Rightarrow \angle i > \angle r$

In passing from rarer to denser medium, the ray bends towards the normal.



(ii) If light passes from denser to rarer medium $\mu_1 = \mu_D$ and $\mu_2 = \mu_R$

$$\frac{\sin i}{\sin r} = \frac{\mu_R}{\mu_D} < 1 \Rightarrow \angle i < \angle r$$

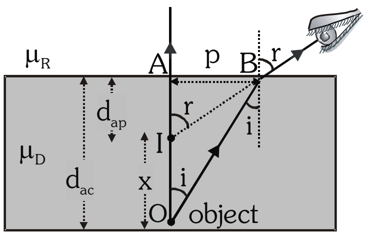
In passing from denser to rarer medium, the ray bends away from the normal.

APPARENT DEPTH AND NORMAL SHIFT

If a point object in denser medium is observed from rarer medium and boundary is plane, then from Snell's law we have $\quad \mu_D \sin i = \mu_R \sin r$...(i)

If the rays OA and OB are close enough then p ≈ small

$\sin i \simeq \tan i = \dfrac{p}{d_{ac}}$ and $\sin r \simeq \tan r = \dfrac{p}{d_{ap}}$



here d_{ac} = actual depth, d_{ap} = apparent depth

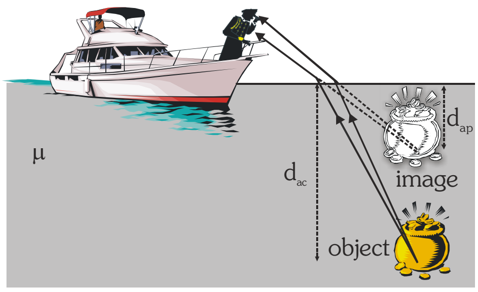
So that equation (i) becomes $\mu_D \dfrac{p}{d_{ac}} = \mu_R \dfrac{p}{d_{ap}} \Rightarrow \dfrac{d_{ac}}{d_{ap}} = \dfrac{\mu_D}{\mu_R} = \dfrac{\mu_1}{\mu_2}$

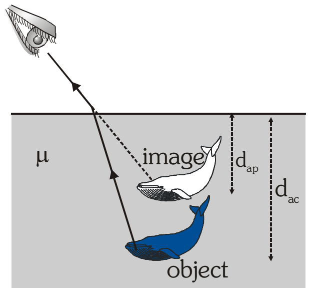
(If $\mu_R = 1$, $\mu_D = \mu$) then $d_{ap} = \dfrac{d_{ac}}{\mu}$ so $d_{ap} < d_{ac}$ ...(ii)

The distance between object and its image, is called normal shift (x)

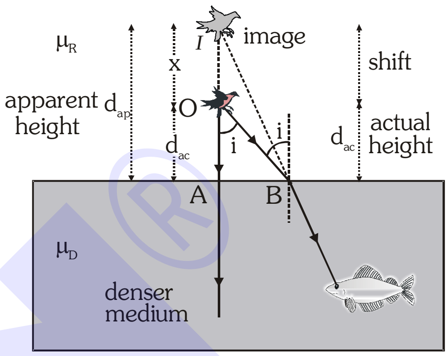
$x = d_{ac} - d_{ap} \qquad \left[\because d_{ap} = \dfrac{d_{ac}}{\mu}\right]; x = d_{ac} - \dfrac{d_{ac}}{\mu} = d_{ac}\left[1 - \dfrac{1}{\mu}\right]$ ...(iii) If $d_{ac} = d$ then $\boxed{x = d\left[1 - \dfrac{1}{\mu}\right]}$

This result is valid only when observer is at the top.





Object in a rarer medium as seen from a denser medium



$$\frac{d_{ac}}{d_{ap}} = \frac{\mu_1}{\mu_2} = \frac{\mu_R}{\mu_D} = \frac{1}{\mu}(<1)$$

$d_{ap} = \mu\, d_{ac}$ i.e., $d_{ap} > d_{ac}$

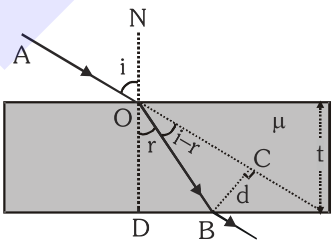
A flying object appears to be higher than in reality.

$x = d_{ap} - d_{ac} \Rightarrow x = [\mu - 1]\, d_{ac}$

LATERAL SHIFT

The perpendicular distance between incident and emergent ray is known as lateral shift.

Lateral shift d = BC and t = thickness of slab

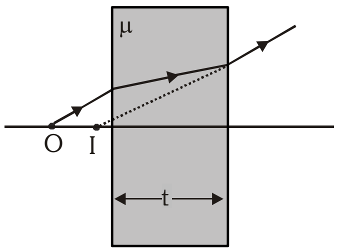


In ΔBOC : $\sin(i-r) = \dfrac{BC}{OB} = \dfrac{d}{OB} \Rightarrow d = OB\sin(i-r)$ ...(i)

In ΔOBD : $\cos r = \dfrac{OD}{OB} = \dfrac{t}{OB} \Rightarrow OB = \dfrac{t}{\cos r}$ ...(ii)

From (i) and (ii) $d = \dfrac{t}{\cos r}\sin(i-r)$

TRANSPARENT GLASS SLAB (Normal shift)



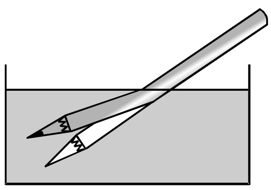
When an object is placed in front of a glass slab, it shifts the image in the direction of incident light and forms an image at a distance x given by :

$$x = t\left[1 - \frac{1}{\mu}\right]$$

SOME ILLUSTRATIONS OF REFRACTION

- **Bending of an object**

  When pencil in a denser medium is seen from a rarer medium it appears to be bent.



- **Twinkling of stars**

  Due to fluctuations in the refractive index of different layers of atmosphere, the refraction becomes irregular so that the light sometimes reaches the eye and sometimes it does not. This gives the effect of twinkling of stars.

GOLDEN KEY POINTS

- μ is a scalar and has no units and dimensions.
- If ε_0 and μ_0 are electric permittivity and magnetic permeability of free space respectively while ε and μ are those of a given medium, then according to electromagnetic theory,

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} \text{ and } v_m = \frac{1}{\sqrt{\varepsilon\mu}} \Rightarrow n_m = \frac{c}{v_m} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r\mu_r}$$

- As in vacuum or free space, speed of light of all wavelengths is maximum and equal to c, so for all wavelengths the refractive index of free space is minimum and is $\mu = \frac{c}{v_m} = \frac{c}{c} = 1$.

Illustrations

Illustration 10.

A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 upto the same height then what will be the apparent depth ?

Solution

Here, real depth = 12.5 cm; apparent depth = 9.4 cm; μ = ?

$$\because \quad \mu = \frac{\text{real depth}}{\text{apparent depth}} \qquad \therefore \qquad \mu = \frac{12.5}{9.4} = 1.33.$$

Now, in the second case, μ = 1.63, real depth = 12.5 cm; apparent depth d_{ap} = ?

$$\therefore \quad 1.63 = \frac{12.5}{d_{ap}} \qquad \Rightarrow \qquad d_{ap} = \frac{12.5}{1.63} = 7.67 \text{ cm.}$$

Illustration 11.

The bottom of a container is made of glass 4 cm thick (μ=1.5). The container contains two immiscible liquids A and B upto depths of 6 cm and 8 cm respectively. What is the shift of the a scratch on outer surface of the bottom of the glass slab when viewed through the container? Refractive indices of A and B are 1.4 and 1.3 respectively.

Solution

$$x = d_1\left[1-\frac{1}{\mu_1}\right] + d_2\left[1-\frac{1}{\mu_2}\right] + d_3\left[1-\frac{1}{\mu_3}\right] = 4\left[1-\frac{1}{1.5}\right] + 6\left[1-\frac{1}{1.4}\right] + 8\left[1-\frac{1}{1.3}\right]$$

$$= 4\times\frac{0.5}{1.5} + 6\times\frac{0.4}{1.4} + 8\times\frac{0.3}{1.3} = 4.88\,\text{cm}$$

Illustration 12.

A mark at the bottom of liquid appears to rise by 0.2 m. The depth of the liquid is 2 m. Find out the refractive index of the liquid.

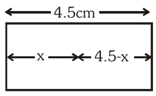
Solution

$$\text{Shift } \; x = d_{ac}\left(1-\frac{1}{\mu}\right) \quad \Rightarrow \quad 0.2 = 2\left(1-\frac{1}{\mu}\right) \Rightarrow 1-\frac{1}{\mu} = 0.1 \Rightarrow \mu = 1.11.$$

Illustration 13.

An air bubble inside a cubical block of glass of side 4.5 cm seems to be at 2 cm from one face and 1 cm from the other face opposite to the first when viewed normally. What is the real distance of bubble from first face ?

Solution



Refractive index $\mu = \dfrac{\text{Real depth}}{\text{App. depth}} = \dfrac{x}{2}$ .... (i) ; $\mu = \dfrac{4.5 - x}{1}$ .... (ii)

From (i) and (ii) $x = 3$ cm.

Illustration 14.

A fish is at the depth of 30 cm below the surface of water. If refractive index of water is $\frac{4}{3}$ then find out the distance between the fish and its image ?

Solution

Shift $x = d\left(1 - \dfrac{1}{\mu}\right)$ where $d = 30$ cm, $\mu = \dfrac{4}{3}$ So $x = 30\left(1 - \dfrac{1}{4/3}\right) \Rightarrow x = \dfrac{30}{4} = 7{\cdot}5$ cm

So distance between the fish and its image is 7.5 cm.

BEGINNER'S BOX-3

1. Width of a slab is 6 cm whose $\mu = \dfrac{3}{2}$. If its rear surface is silvered and object is placed at a distance 28 cm from the front face. Calculate the final position of the image from the silvered surface.

2. A light ray is moving from denser (refractive index=μ) to air. If the angle of incidence is half the angle of refraction, find out the angle of refraction.

3. Light of wavelength 8000 Å enters from air into water ($\mu_{water} = 4/3$). What is the change in the frequency of light in water ?

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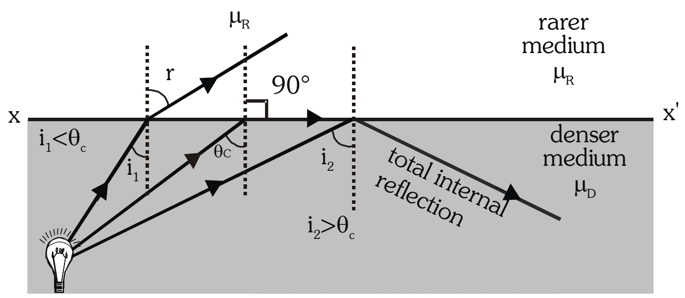
5. TOTAL INTERNAL REFLECTION

When light ray travels from denser to rarer medium, it bends away from the normal. If the angle of incidence is increased, the angle of refraction also increases. At a particular value of angle, the refracted ray subtends 90^0 angle with the normal, this angle of incidence is known as critical angle (θ_C). If angle of incidence increases further, the ray comes back to the same medium. This phenomenon is known as total internal reflection.



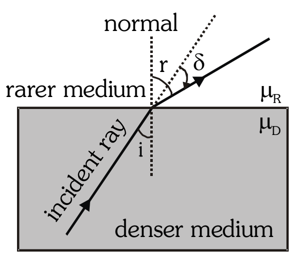
CONDITIONS

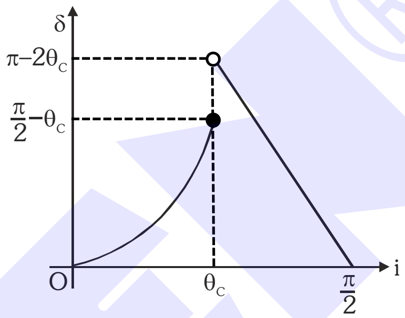
- Angle of incidence > critical angle $[i > \theta_c]$
- Light should travel from denser to rarer medium for example Glass to air, water to air, Glass to water

Applying Snell's Law at boundary xx' yields $\mu_D \sin\theta_C = \mu_R \sin 90° \Rightarrow \sin\theta_C = \frac{\mu_R}{\mu_D}$

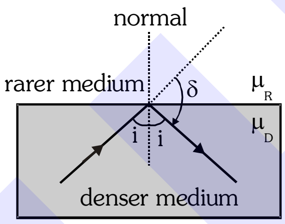
Graph between angle of deviation (δ) and angle of incidence (i) as ray goes from denser to rarer medium

- If $i < \theta_c$; $\mu_D \sin i = \mu_R \sin r$; $r = \sin^{-1}\left(\frac{\mu_D}{\mu_R}\sin i\right)$ so $\delta = r - i = \sin^{-1}\left(\frac{\mu_D}{\mu_R}\sin i\right) - i$

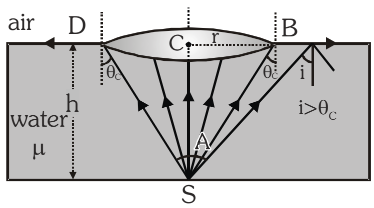




- If $i > \theta_c$; $\delta = \pi - 2i$



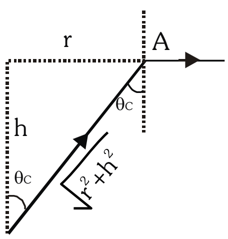
- A point source is situated at the bottom of a tank filled with a liquid of refractive index μ upto h height . It is found that light comes out of liquid surface through a circular portion above the source.



$$\sin\theta_C = \frac{r}{\sqrt{r^2+h^2}} \text{ and } \sin\theta_C = \frac{1}{\mu} \Rightarrow \frac{1}{\mu} = \frac{r}{\sqrt{r^2+h^2}} \Rightarrow \frac{1}{\mu^2} = \frac{r^2}{r^2+h^2}$$

$$\Rightarrow \mu^2 r^2 = r^2 + h^2 \Rightarrow (\mu^2 - 1)r^2 = h^2$$

radius of circular portion $r = \frac{h}{\sqrt{\mu^2-1}}$ and area $= \pi r^2$



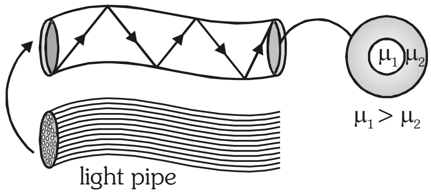
Vertex angle $A = 2\theta_C$

For pure water, $A = 2 \times 49° = 98°$

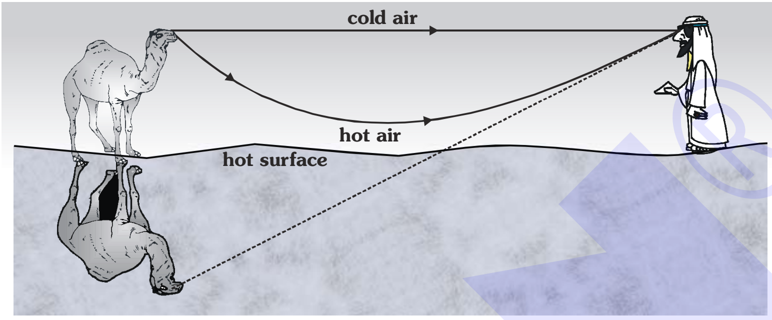
APPLICATIONS OF TOTAL INTERNAL REFLECTION

- **Sparkling of diamond :** The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so $\theta_c = 24°$. Diamond is cut in such a manner that, once the light enters into it, when it tends come out then $i > \theta_C$. So TIR will take place repeatedly inside it. The light which beams out from a few places entering into the eyes of the observer makes it sparkle.

- **Optical Fibre :** In optical fibre light propogates through multiple total internal reflections along the axis of a glass fibre of few microns radius in which index of refraction of core is greater than that of surroundings (cladding).



- **Mirage and optical looming** : Mirage is caused due to total internal reflection in deserts and other hot regions where , refractive index of air near the surface of earth becomes lesser than that above it due to heating of the earth. Light from distant objects approach the surface of earth with successively increasing i, till i > θ_C so that TIR takes place so that inverted images appear along with the objects as shown in figure.



Similar to 'mirage' in deserts, 'optical looming' takes place in polar regions due to TIR. Here μ of different air layers decrease with height and so an inverted image of an object is formed in the sky which appears to be suspended in air.

GOLDEN KEY POINTS

- A diver in water at a depth d sees the world outside through a horizontal circle of radius. $r = d \tan \theta_c$.
- In case of total internal reflection, as all (i.e. 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirror or refraction from lenses there is some loss of intensity as the entire light cannot be reflected or refracted. Due to this reason, images formed by TIR are much brighter those than formed by mirrors or lenses.

Illustrations

Illustration 15.

A ray of light from a denser medium strikes a rarer medium at an angle of incidence i. If the reflected and refracted rays are mutually perpendicular to each other then what is the value of critical angle ?

Solution

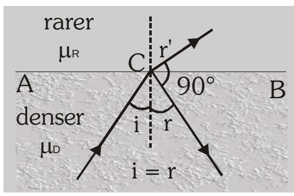
The situation in accordance with the given problem is shown in figure.

Applying Snell's law at the boundary at C,

$\mu_D \sin i = \mu_R \sin r'$ $\Rightarrow$ $\mu = \dfrac{\mu_D}{\mu_R} = \dfrac{\sin r'}{\sin i}$ ... (i)

But according to given problem, $r' + 90° + r = 180°$

$r' + r = 90°$ i.e., $r' = 90° - r$ or $r' = 90° - i$ [as $\angle r = \angle i$]



So equation (i) reduces to $\mu = \dfrac{\sin(90° - i)}{\sin i} = \dfrac{\cos i}{\sin i} = \cot i$ ... (ii)

But by definition, $\theta_C = \sin^{-1}\dfrac{1}{\mu}$ and from equation (ii) $\mu = \cot i$

So $\theta_C = \sin^{-1}\left[\dfrac{1}{\cot i}\right] = \sin^{-1}(\tan i)$.

Illustration 16.

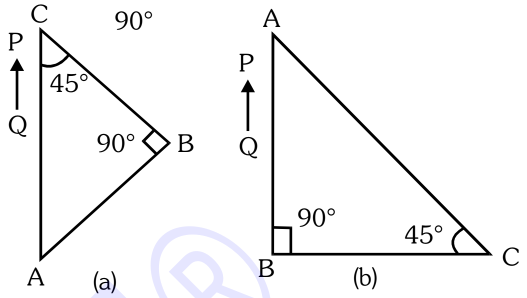
If the critical angle for a certain medium and vacuum is 30° find the velocity of light in the medium.

Solution

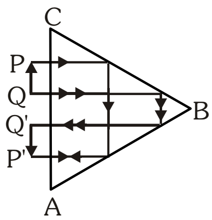
$$\mu = \frac{1}{\sin\theta_c} = \frac{1}{\sin 30^\circ} = 2 \Rightarrow \text{ velocity of light in the medium is } v_{\text{medium}} = \frac{c}{\mu} = \frac{3\times10^8}{2} = 1.5\times10^8 \text{ ms}^{-1}.$$

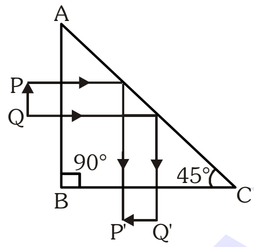
Illustration 17.

An object is placed in front of a right angled prism ABC in two positions (a) and (b) as shown. The prism is made of crown glass with critical angle 41°. Trace the path of rays starting form P and Q as shown in the figures (a) and (b) entering normal to the prism



Solution

(a) 

(b) 

For the refraction on the inclined surfaces of the prism i=45° (>C=41°). So the rays undergoes total internal reflection.

BEGINNER'S BOX-4

1. If light travels a distance x in time t_1 sec in air and 10x distance in time t_2 in a certain medium, then find the critical angle of the medium.

2. Calculate the critical angle for glass-air interface if a ray of light incident from air on a glass surface is deviated through 15° when angle of incidence is 45°.

3. A ray of light travels from denser medium having refractive index $\sqrt{2}$ to air, What should be the angle of incidence for the ray to emerge out ?

6. REFRACTION AT CURVED SURFACE

μ_1 = refractive index of the medium in which the incident ray lies.

μ_2 = refractive index of the medium in which refracted ray lies.

O = Object

P= pole

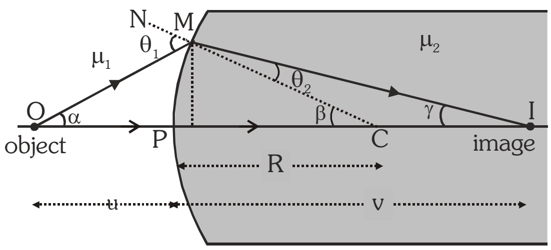
C= centre of curvature

R = PC = radius of curvature

Refraction from curved surface

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

if angles are very small then : $\mu_1\theta_1 = \mu_2\theta_2$ ...(i)



But $\theta_1 = \alpha + \beta$ ...(ii)

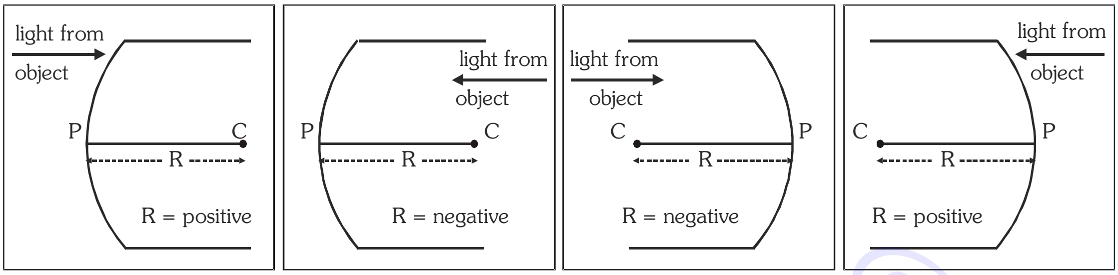
$\beta = \theta_2 + \gamma$ ...(iii)

from (i), (ii) and (iii) $\mu_1(\alpha+\beta) = \mu_2(\beta-\gamma)$

$\Rightarrow \mu_1\alpha + \mu_1\beta = \mu_2\beta - \mu_2\gamma \quad \Rightarrow \mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta$

$$\Rightarrow \frac{\mu_1 PM}{-u} + \frac{\mu_2 PM}{v} = \frac{(\mu_2-\mu_1)PM}{R} \Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2-\mu_1)}{R}$$

SIGN CONVENTION FOR RADIUS OF CURVATURE

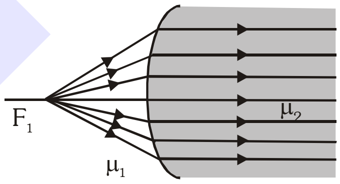


These are valid for all types of refracting surfaces – convex, concave or plane. In case of plane refracting surface $R \to \infty$, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R} \Rightarrow \frac{\mu_2}{v} - \frac{\mu_1}{u} = 0$ i.e. $\frac{u}{v} = \frac{\mu_1}{\mu_2}$ or $\frac{d_{Ac}}{d_{Ap}} = \frac{\mu_1}{\mu_2}$

FOCAL LENGTH OF A SINGLE SPHERICAL SURFACE

A single spherical surface as two principal focal points which are as follows :-

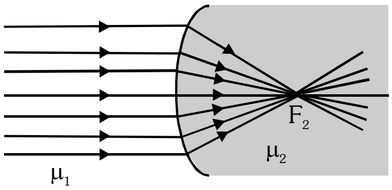
(i) First focus: The first principal focus is the point on the axis where an object should be placed so that the image is formed at infinity. That is when



$u = f_1$, $v = \infty$, then from $-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \left(\frac{\mu_2-\mu_1}{R}\right)$

We get $-\frac{\mu_1}{f_1} = \frac{\mu_2-\mu_1}{R} \Rightarrow f_1 = \frac{-\mu_1 R}{(\mu_2-\mu_1)}$

(ii) Second focus: Similarly, the second principal focus is the point where parallel rays get focussed. That is $u_1 = -\infty$, $v_1 = f_2$, then



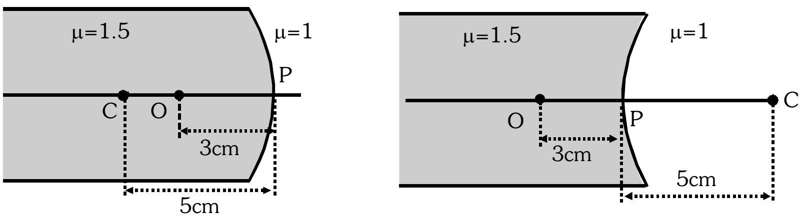
$\frac{\mu_2}{f_2} = \frac{\mu_2-\mu_1}{R}$; $f_2 = \frac{\mu_2 R}{(\mu_2-\mu_1)}$

(iii) Ratio of Focal lengths: $\boxed{\frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}}$ or $\boxed{\frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} = 0}$

Illustrations

Illustration 18.

An air bubble in glass (μ = 1.5) is situated at a distance of 3 cm from a spherical surface of diameter 10 cm as shown in figure. At what distance from the surface will the bubble appear if the surface is (a) convex (b) concave ?



Solution

For the refraction at curved surface $\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{(\mu_2-\mu_1)}{R}$

(a) $\mu_1 = 1.5,\quad \mu_2 = 1,\ R = -5$ cm and $u = -3$ cm $\Rightarrow \frac{1}{v}-\frac{(1.5)}{(-3)}=\frac{1-1.5}{(-5)} \Rightarrow v = -2.5$ cm

the bubble will appear at a distance of 2.5 cm from the convex surface within the glass.

(b) $\mu_1 = 1.5,\ \mu_2 = 1,\ R = 5$ cm and $u = -3$ cm $\Rightarrow \frac{1}{v}-\frac{(1.5)}{(-3)}=\frac{1-1.5}{(5)} \Rightarrow v = -1.66$ cm

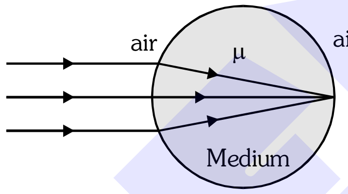
the bubble will appear at a distance of 1.66 cm from the concave surface within the glass.

Note : If the surface is plane then $R \to \infty$

cases (a) or (b) would yield $\frac{1}{v}-\frac{(1.5)}{(-3)}=\frac{(1-1.5)}{\infty} \Rightarrow v = -2$ cm

Illustration 19.

Calculate the value of refractive index (μ) for the given situation.



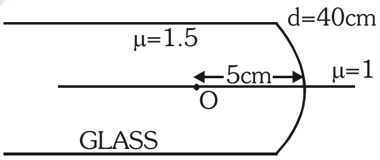
Solution

$$\because \quad \frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R} \Rightarrow \frac{\mu}{2R}-\frac{1}{-\infty}=\frac{\mu-1}{R}$$

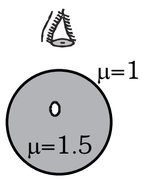
$$\Rightarrow \quad \frac{\mu}{2R}=\frac{\mu-1}{R} = \Rightarrow \mu = 2\mu - 2 \Rightarrow \mu = 2.$$

BEGINNER'S BOX-5

1. An object O in glass (μ = 1.5) is situated at a distance of 5 cm from a spherical surface of diameter 40 cm as shown in the figure. Find the distance of the image from the surface.

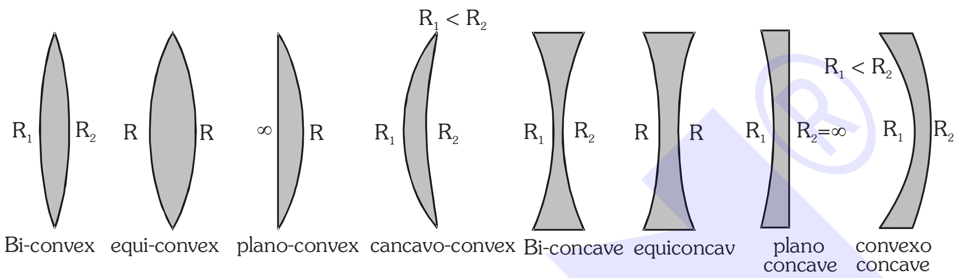


2. There is a small air bubble inside a glass sphere (μ = 1.5) of radius 10 cm. The bubble is 4 cm below the surface and is viewed normally from the outside as shown in figure. Find the apparent depth of the bubble.

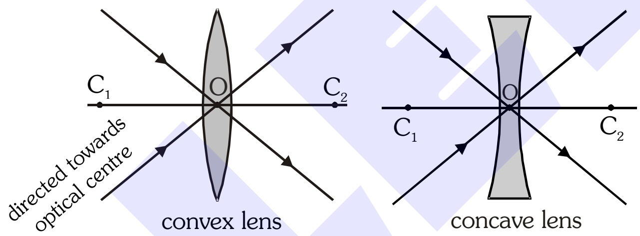


7. LENS

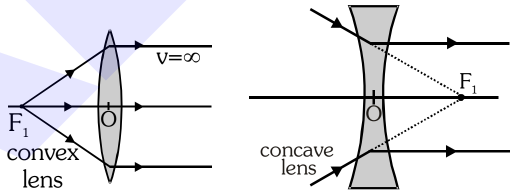
- A lens is a portion of a transparent material with two refracting surfaces such that at least one is curved with refractive index of its material being different from that of the surroundings.
- A thin spherical lens with refractive index greater than that of surroundings behaves as a convergent or convex lens, i.e., converges parallel rays if its central (i.e. paraxial) portion is thicker than marginal one.
- However if the central portion of a lens is thinner than marginal one, it diverges parallel rays passing through it and behaves as divergent or concave lens. This is how we classify convergent and divergent lenses.



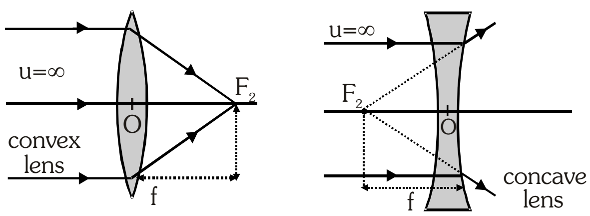
- **Optical Centre :** It is a point O for a given thin lens through which any ray passes undeviated



- **Principal Axis : C_1C_2** is a line passing through optical centre and perpendicular to the lens.
- **Principal Focus :** A lens has two focal points. First focal point is an object point on the principal axis corresponding to which the image is formed at infinity.



Whereas second focal point is an image point on the principal axis corresponding to which object lies at infinity



- **Focal Length f** is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appears to converge.
- **Aperture :** In reference to a lens, aperture means the effective diameter of the circular area through which light enters the lens. Intensity of image formed by a lens depends on the light passing through the lens will equivalently depend on the square of aperture, i.e.,

$$\text{Intensity} \propto (\text{Aperture})^2$$

LENS–MAKER'S FORMULA

In case of image formation by a lens

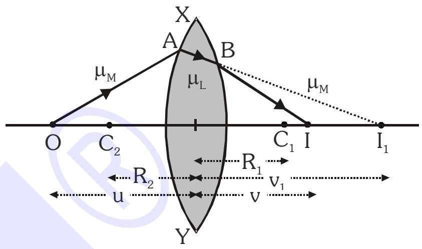


Image formed by first surface acts as object for the second surface.

So, from the formula of refraction at curved surface $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$

For first surface A, $\dfrac{\mu_L}{v_1}-\dfrac{\mu_M}{u}=\dfrac{\mu_L-\mu_M}{R_1}$ ...(i) $[\because \mu_2=\mu_L, \mu_1=\mu_M]$

For second surface B, $\dfrac{\mu_M}{v}-\dfrac{\mu_L}{v_1}=\dfrac{\mu_M-\mu_L}{R_2}=-\dfrac{\mu_L-\mu_M}{R_2}$ ...(ii) $[\because \mu_2=\mu_M, \mu_1=\mu_L, \mu_1=\mu_2, u\to v_1]$

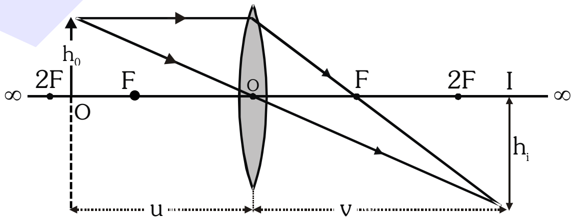
Adding (i) and (ii)

$$\mu_M\left[\frac{1}{v}-\frac{1}{u}\right]=(\mu_L-\mu_M)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\Rightarrow\frac{1}{v}-\frac{1}{u}=\frac{\mu_L-\mu_M}{\mu_M}\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]\text{...(iii)}\left(\because\ \mu=\frac{\mu_L}{\mu_M}\right)$$

Now if object is at infinity, Image will be formed at the focus, i.e, u=–∞, v=f, So $\boxed{\dfrac{1}{f}=(\mu-1)\left[\dfrac{1}{R_1}-\dfrac{1}{R_2}\right]}$ ...(iv)

This is known as lens makers formula. By equating (iii) and (iv), $\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f}$ this is known as lens formula or Gaussian form of lens equation.

Magnification $: m=\dfrac{\text{height of image}}{\text{height of object}}=\dfrac{h_i}{h_o}=\dfrac{v}{u}=\dfrac{f}{f+u}=\dfrac{f-v}{f}$



RULES FOR IMAGE FORMATION

- A ray passing through optical centre proceeds undeviated through the lens.
- A ray passing through first focus or directed towards it, becomes parallel to the principal axis after refraction from the lens.
- A ray passing parallel to the principal axis passes or appears to pass through F_2 after refraction through the lens.

For Convergent or convex lens

Position of Object	Position of Image	Magnification
$-\infty$	F	$\|m\| << 1$ & $m<0$
Between ∞ & 2F	F–2F	$\|m\| < 1$ & $m<0$
2F	2F	$m = -1$
F – 2F	∞–2F	$\|m\| > 1$ & $m<0$
Between 2F & F, near F	$+\infty$	$m <<-1$
Between F & O, near F	$-\infty$	$m >>1$
F – O	In front of lens	$m >1$

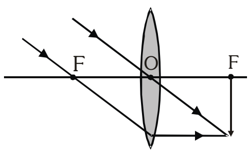
IMAGE FORMATION FOR CONVEX LENS (CONVERGENT LENS)

(i) Object is placed at infinity

Image :

at F, real, inverted and very small in size

$|m| << 1$ & $m<0$

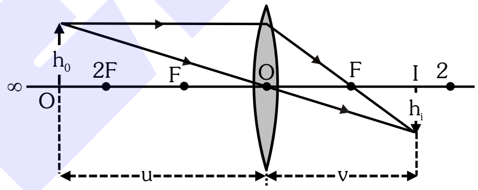


(ii) Object is placed in between ∞ – 2F

Image :

between (F – 2F), inverted and real small in size (diminished)

$|m| < 1$ & $m<0$

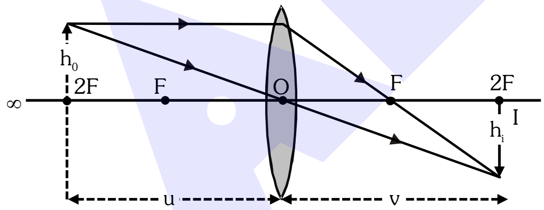


(iii) Object is placed at 2F

Image :

(at 2F), real inverted and equal (in size)

($m = -1$)

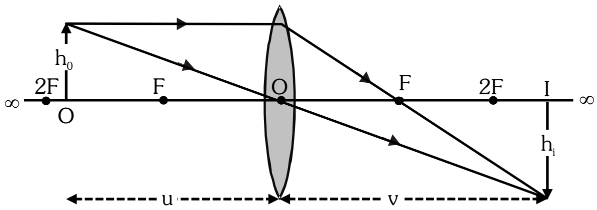


(iv) Object is placed in between 2F – F

Image :

between (2F – ∞) real, inverted and enlarged

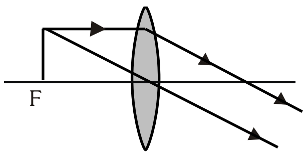
$|m| > 1$ & $m<0$



(v) Object is placed at F

Image :

at infinity, real, inverted and highly enlarged



(vi) Object is placed in between F – O

Image :

virtual (in front of lens) erect and enlarged ($m > +1$)

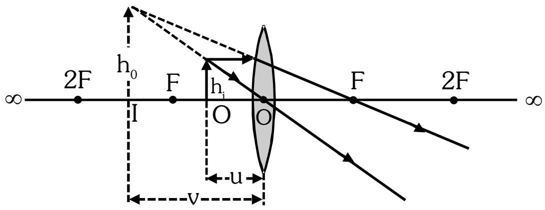


IMAGE FORMATION FOR CONCAVE LENS (DIVERGENT LENS)

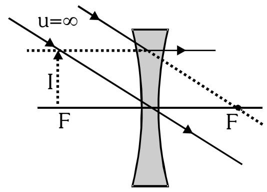
Image is virtual, diminished, erect and between lens and object when object is real.

(i) Object is placed at infinity

Image :

At F, virtual, erect

and diminished ($m << +1$)

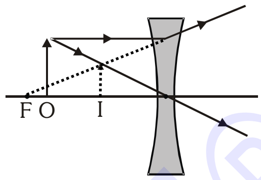


(ii) Object is placed in front of lens

Image :

between F and optical centre

virtual, erect and diminished ($m < +1$)



Sign convention for object/image in case of lens

Real object	$u = -$ ve
Real image	$v = +$ ve
Virtual object	$u = +$ ve
Virtual image	$v = -$ ve

POWER OF LENS

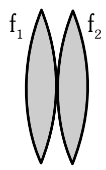
Reciprocal of focal length in metres is known as power of lens in dioptres.

SI UNIT : dioptre (D) **Power of lens :** $P = \dfrac{1}{f(m)} = \dfrac{100}{f(cm)}$ dioptres [in air]

$$P = \frac{\mu_{outer}}{f(m)} = \frac{\mu_{outer}100}{f(cm)} \text{ dioptres [in medium]}$$

COMBINATION OF LENSES

Two thin lenses are placed in contact with each other

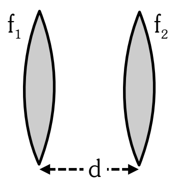


Power of combination $P = P_1 + P_2 \Rightarrow \dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

Use sign convention while solving numericals

Two thin coaxial lenses are placed at a small separation d

(provided incident rays are parallel to the principal axis).



$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \Rightarrow P = P_1 + P_2 - P_1 P_2 d$$

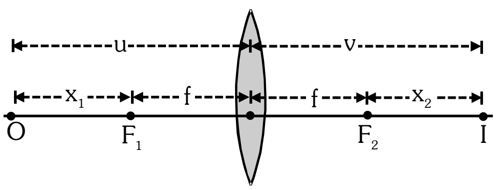
Use proper sign convention when solving numericals.

- **Newton's Formula**

$$f = \sqrt{x_1 x_2}$$

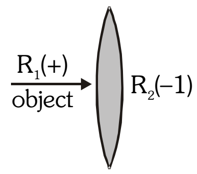
x_1 = distance of object from the I[st] focus.

x_2 = distance of image from the II[nd] focus.

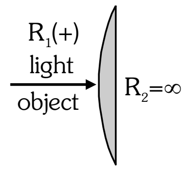


GOLDEN KEY POINTS

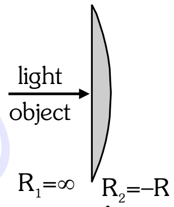
- Focal length of equiconvex lens placed in air :
  refractive index of lens $\mu_L = \mu$, refractive index of surrounding medium $(\mu_M) = 1$
  $R_1 = +R$, $R_2 = -R$



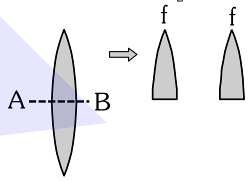
$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \left(-\frac{1}{R}\right)\right] \Rightarrow \text{Focal length } f = \frac{R}{2(\mu - 1)}$$



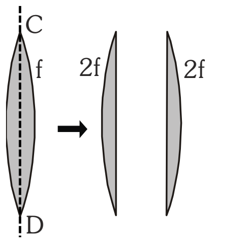
- Focal length of planoconvex lens placed in air :
  $$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right] \Rightarrow \text{Focal length } f = \frac{R}{(\mu - 1)}$$
  If the object is placed towards the plane surface, then
  $$\frac{1}{f} = (\mu - 1)\left[\frac{1}{\infty} - \left(-\frac{1}{R}\right)\right] \Rightarrow \text{Focal length } f = \frac{R}{(\mu - 1)}$$



- If an equiconvex lens of focal length f is cut into two identical parts by a horizontal plane AB then the focal length of each part will be equal to that of the initial lens; because μ, R_1 and R_2 will remain unchanged. Only intensity of image will be reduced.
  $\because$ intensity $I \propto (\text{aperture})^2$



- If the same lens is cut into equal parts by a vertical plane CD, the focal length of each part will be double the initial value but intensity of image will remain unchanged.



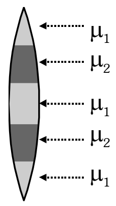
focal length of each part becomes 2f

For equiconvex lens $\frac{1}{f} = \frac{(\mu - 1)2}{R}$ For plano convex lens $\frac{1}{f_1} = \frac{\mu - 1}{R}$

So $\frac{1}{f} = \frac{2}{f_1} \Rightarrow f_1 = 2f$

(f=focal length of original lens)

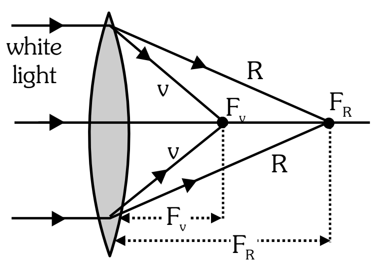
If a lens is made of number of layers of different refractive indices for a given wavelength of light, the no. of images corresponding to an object is equal to types of refractive indices,

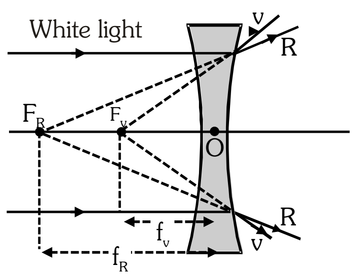


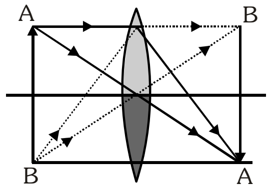
as $\frac{1}{f} \propto (\mu - 1)$

For the figure shown number of images = 2

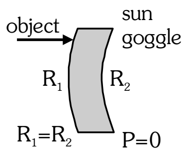
- Focal length of a lens depends on wavelength. $\because \frac{1}{f} \propto (\mu - 1) \propto \frac{1}{\lambda} [f \propto \lambda] \therefore f_R > f_V$







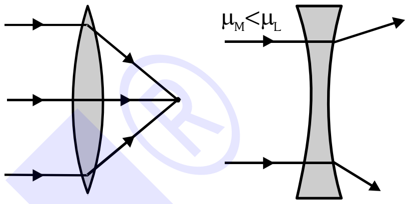
- If half portion of a lens is covered by black paper then intensity of image will be reduced but complete image will be formed.
- Sun–glasses or goggles :
  radii of curvature of two surfaces are equal with centres of curvatures on the same side of the lens



$R_1 = R_2 = +R$ so $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{R}\right]$

$\Rightarrow \frac{1}{f} = 0 \Rightarrow f = \infty$ and $P = 0 \Rightarrow$ sun glasses or goggles have no power.

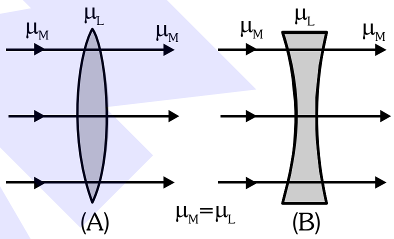
- If refractive index of medium < refractive index of lens



If $\mu_M < \mu_L$ then $\frac{\mu_L}{\mu_m} > 1$ or $\left(\frac{\mu_L}{\mu_m} - 1\right) > 0$

Convex lens behave as convergent lens.
While concave lens behave as divergent lens.

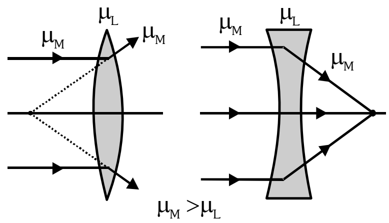
- Refractive index of medium = Refractive index of lens ($\mu_M = \mu_L$)



$\frac{1}{f} = \left(\frac{\mu_L}{\mu_M} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$; $\frac{1}{f} = 0 \Rightarrow f = \infty$ & $P = 0$

Lens will behave as plane transparent plate

- Refractive index of surrounding medium > Refractive index of lens



$\mu_M > \mu_L \Rightarrow \frac{\mu_L}{\mu_M} < 1$ and $\left(\frac{\mu_L}{\mu_m} - 1\right) < 0$

convex lens will behave as divergent lens and concave lens will behave as convergent lens. An air bubble in water behaves as a concave lens.

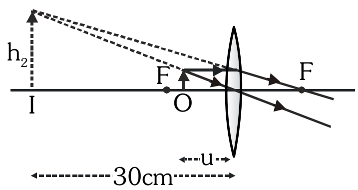
Illustrations

Illustration 20.

A person is looking at an object using magnifying lens has a focal length of 10 cm. (a) Where should an object be placed if the image is to be 30 cm away from the lens ? (b) What will be the magnification ?

Solution

(a) In case of magnifying lens, it is convergent in nature and the image is erect, enlarged, virtual, between infinity and object and on the same side of the lens.



f = 10 cm and v = – 30 cm

and hence from lens-formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

we have $\frac{1}{-30} - \frac{1}{u} = \frac{1}{10}$ i.e., $u = -7.5$ cm

So the object must be placed at a distance of 7.5 cm (which is < f) in front of the lens.

(b) $m = \left[\frac{h_2}{h_1}\right] = \frac{v}{u} = \frac{-30}{-7.5} = 4$ i.e., image is erect, virtual and four times the size of object.

Illustration 21.

An object 25 cm high is placed in front of a convex lens of focal length 30 cm. If the height of the image formed is 50 cm, find the distance between the object and the image ?

Solution

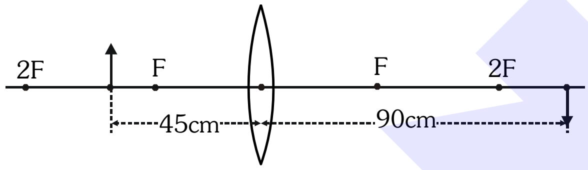
As the object is in front of the lens, it is real. If the image is inverted and real then

$h_1 = 25$ cm, $f = 30$ cm, $h_2 = -50$ cm

$$m = \frac{h_2}{h_1} = \frac{-50}{25} = -2 \Rightarrow m = \frac{f}{f+u} \quad \Rightarrow -2 = \frac{30}{30+u}$$

$$u = -45 \text{ cm} \Rightarrow \quad m = \frac{v}{u} \Rightarrow \quad -2 = \frac{v}{-45} \quad \Rightarrow \quad v = 90 \text{ cm}$$

As in this situation, the object and image are on the opposite sides of the lens, the distance between object and image is $d_1 = u + v = 45 + 90 = 135$ cm



If the image is erect (i.e., virtual)

$$m = \frac{f}{f+u} \quad \Rightarrow 2 = \frac{30}{30+u} \quad \Rightarrow \quad u = -15 \text{ cm} \quad \Rightarrow \quad m = \frac{v}{u} \quad \Rightarrow 2 = \frac{v}{-15} \quad \Rightarrow v = -30 \text{ cm.}$$

As in this situation both image and object are in front of the lens, the distance between object and image is $d_2 = v - u = 30 - 15 = 15$ cm.

Illustration 22.

A needle placed 45 cm away from a lens forms an image on a screen placed 90 cm away on the other side of the lens. Identify the type of lens and determine its focal length, What is the size of the image, if the size of the needle is 5 cm?

Solution

Here, $u = -45$ cm, $v = 90$ cm, $f = ?$, $h_2 = ?$, $h_1 = 5$ cm,

$$\because \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \therefore \quad \frac{1}{90} + \frac{1}{45} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1+2}{90} = \frac{1}{f} \quad \Rightarrow \quad \text{or} \quad f = 30 \text{ cm}$$

As f is positive, the lens is converging

$$\because \quad \frac{h_2}{h_1} = \frac{v}{u} \qquad \therefore \frac{h_2}{5} = \frac{90}{-45} = -2$$

$\Rightarrow \quad h_2 = -10$ cm.

Minus sign indicates that image is real and inverted

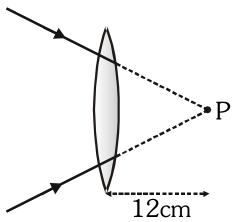
Illustration 23.

A beam of light converges to a point P. A lens is placed in the path of the convergent beam 12 cm from P. At what point does the beam converge if the lens is (a) a convex lens of focal length 20 cm. (b) a concave lens of focal length 16 cm.

Solution

Here, the point P on the right of the lens acts as a virtual object,

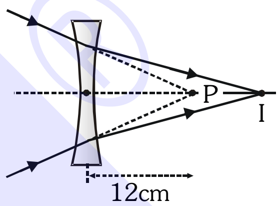
$\therefore$ u = 12 cm, v = ?



(a) f = 20 cm $\because \frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ $\therefore$

$$\frac{1}{v}-\frac{1}{12}=\frac{1}{20}$$

$$\Rightarrow \frac{1}{v}=\frac{1}{20}+\frac{1}{12}=\frac{3+5}{60}=\frac{8}{60} \Rightarrow v = \frac{60}{8} = 7.5 \text{ cm}$$



(b) f = –16 cm, u = 12 cm,

$$\therefore \frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{-16}+\frac{1}{12}=\frac{3+4}{48}=\frac{1}{48}$$

$\Rightarrow$ v = 48 cm

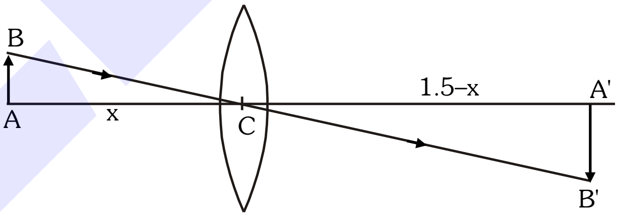
Hence image is formed 48 cm to the right of the lens, where the beam would converge.

Illustration 24.

An object is placed at a distant of 1.50 m from a screen and a convex lens placed in between produces an image magnified 4 times on the screen. What is the focal length and the position of the lens ?

Solution

$$m = \frac{h_2}{h_1} = -4$$



Let the lens be placed at a distance x from the object.

Then u = – x, and v = (1.5 – x)

using $m = \frac{v}{u}$, we get $-4 = \frac{1.5-x}{-x}$ $\Rightarrow x = 0.3$ m

The lens is placed at a distance of 0.3 m from the object (or 1.20 m from the screen)

For focal length, we may use

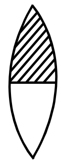
$$m = \frac{f}{f+u} \quad \text{or} \quad -4 = \frac{f}{f+(-0.3)} \Rightarrow f = \frac{1.2}{5} = 0.24 \text{ m}$$

BEGINNER'S BOX-6

1. An object is placed at the distance of 30 cm in front of a convex lens of focal length 10 cm. Find the position of the image, its nature and magnification.
2. A planoconvex lens has a focal length of 30 cm and an index of refraction 1.5. Find the radius of the convex surface.
3. A biconvex lens (μ = 1.5) of focal length 0.2 m acts as a divergent lens of power 1D when immersed in a liquid. Find the refractive index of the liquid.
4. Two thin converging lenses of focal lengths 20 cm and 40 cm are placed in contact. Find the effective power of the combination.

5. An object placed 20 cm in front of a convex lens has its image 40 cm behind the lens. Find the power of the lens.

6. A lens shown in figure is made of two different materials. A point object is placed on its axis. How many images will be formed.



7. A convex lens of focal length f produces a real image of size is $\frac{1}{n}$ times the size of the object. Find the position of the object.

8. COMBINATION OF LENSES AND MIRRORS

When several lenses or mirrors are used, the image formation is considered one after another in sequences of steps, The image formed by the lens facing the object serves as an object for the next lens or mirror, the image formed by the second lens acts as an object for the third, and so on, The total magnification in such situations will be given by

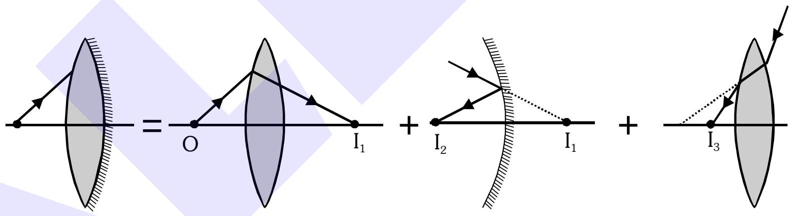
$$m = \frac{I}{O} = \frac{I_1}{O} \times \frac{I_2}{I_1} \times \ldots \qquad \Rightarrow m = m_1 \times m_2 \times \ \ldots$$

Power of Lens [in air] $P_L = \frac{1}{f_L}$ Converging lens $P_L = +ve$ Diverging lens $P_L = -ve$

Power of mirror $P_m = -\frac{1}{f_m}$ Convex mirror $P_M = -ve$ Concave mirror $P_M = +ve$

SILVERING OF LENS

Calculate the focal length of the equivalent mirror of a equiconvex lens silvered at one side.

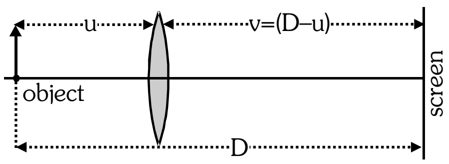


$P = P_L + P_M + P_L = 2P_L + P_M$

$$= \frac{2(\mu-1)\times 2}{R} + \frac{2}{R} = \frac{4\mu - 4 + 2}{R} \Rightarrow P = \frac{4\mu - 2}{R} = -\frac{1}{F}$$

DISPLACEMENT METHOD

It is used for determination of focal length of convex lens in laboratory. A thin convex lens of focal length f is placed between an object and a screen fixed at a distance D apart.If D > 4f there are two positions of lens corresponding to which a sharp image of the object is formed on the screen.



By lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{D-u}-\frac{1}{-u}=\frac{1}{f}\Rightarrow u^2-Du+Df=0\Rightarrow u=\frac{D\pm\sqrt{D(D-4f)}}{2}$ there are three possibilities :

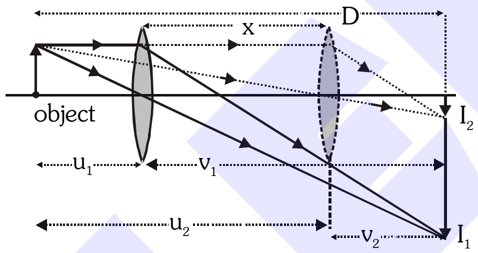
(i) for $D < 4f$ u will be imaginary hence physically no position of lens is possible

(ii) for $D = 4f$ $u=\frac{D}{2}=2f$ so only one position of lens is possible

and since $v = D - u = 4f - 2f$, $v = 2f$

(iii) for $D > 4f$ $u_1=\frac{D-\sqrt{D(D-4f)}}{2}$ and $u_2=\frac{D+\sqrt{D(D-4f)}}{2}$

So there are two positions of lens for which real image will be formed on the screen.(for two distances u_1 and u_2 of the object from lens)



If the distance between two positions of lens is x then

$$x=u_2-u_1=\frac{D+\sqrt{D(D-4f)}}{2}-\frac{D-\sqrt{D(D-4f)}}{2}=\sqrt{D(D-4f)}\Rightarrow x^2=D^2-4\,Df\Rightarrow f=\frac{D^2-x^2}{4D}$$

Distance of image corresponds to two positions of the lens :

$$v_1=D-u_1=D-\frac{1}{2}[D-\sqrt{D(D-4f)}]=\frac{1}{2}[D+\sqrt{D(D-4f)}]=u_2\Rightarrow v_1=u_2$$

$$v_2=D-u_2=D-\frac{1}{2}[D+\sqrt{D(D-4f)}]=\frac{1}{2}[D-\sqrt{D(D-4f)}]=u_1\Rightarrow v_2=u_1$$

Distances of object and image are interchangeable. for the two positions of the lens

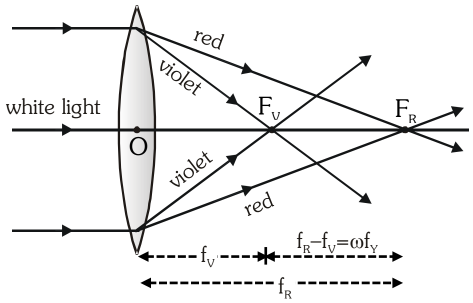
Now $x = u_2 - u_1$ and $D = v_1 + u_1 = u_2 + u_1\ [\because\ v_1 = u_2]$

so $u_1=v_2=\frac{D-x}{2}$ and $u_2=v_1=\frac{D+x}{2}$; $m_1=\frac{I_1}{O}=\frac{v_1}{u_1}=\frac{D+x}{D-x}$ and $m_2=\frac{I_2}{O}=\frac{v_2}{u_2}=\frac{D-x}{D+x}$

Now $m_1\times m_2=\frac{D+x}{D-x}\times\frac{D-x}{D+x}\Rightarrow\frac{I_1I_2}{O^2}=1\Rightarrow O=\sqrt{I_1I_2}$

9. CHROMATIC ABERRATION

The image of an object due to white light formed by a lens is usually coloured and blurred. This defect of image is called chromatic aberration which arises due to the fact that the focal length of a lens is different for different colours.



For a single lens $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ and as μ of lens is maximum for violet while minimum for red so, violet gets focussed nearest to the lens while red is farthest from it.

Longitudinal or Axial Chromatic Aberration

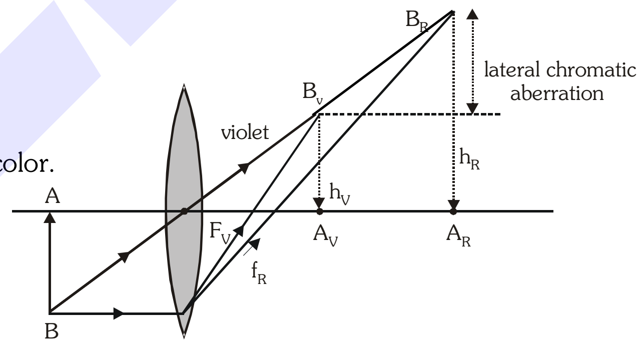
When an object O situated on the axis of a lens is illuminated by white light, then images of different colors are formed at different points along the axis. The formation of images of different colors at different positions is called 'axial' or longitudinal chromatic aberration. The axial distance between the red and the violet images $I_R - I_V$ is a measure of longitudinal aberration. When white light is incident on lens, image is obtained at different point on the axis because focal length of lens depends on wavelength. $f \propto \lambda \Rightarrow f_R > f_V$

$f_R - f_V = \omega f_y \Rightarrow$ Axial or longitudinal chromatic aberration

If the object is at infinity, then the longitudinal chromatic aberration is equal to the difference in focal–lengths $(f_R - f_V)$ for the red and the violet rays.

LATERAL CHROMATIC ABERRATION

As the focal–length of the lens varies from colour to colour, the magnification $m = \left[\frac{f}{u+f}\right]$ produced by the lens also varies from colour to color. Therefore, for a finite–sized object AB, the images due to different colors formed by the lens are of different sizes.



The formation of images of different colours in different sizes is called lateral chromatic aberration. The difference in the heights of the red image $B_R A_R$ and the violet image $B_V A_V$ is a measure of as lateral chromatic aberration. $\quad LCA = h_R - h_V$

ACHROMATISM

If two or more lenses are combined together in such a way that this combination produces images due to all colours at the same point then this combination is known as achromatic combination of lenses.

Condition for achromatism, [when two lenses are in contact].

$$\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2}$$

and equivalent focal length $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$ (Apply sign convention while solving numerical)

When lens are not contact, we must keep then at separation $d = \frac{\omega_1 f_2 + \omega_2 f_1}{\omega_1 + \omega_2}$

Illustration 25.

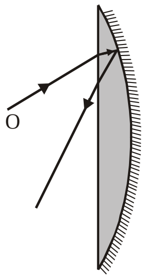
Radius of curved surface of a plano convex lens is 20 cm and refractive index of lens material is 1.5. Calculate equivalent focal length of lens if :–

(i) curved surface is silvered.

(ii) plane surface is silvered.

Solution

(i)



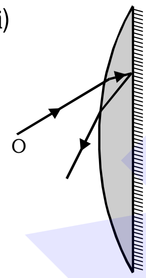
$$P = 2P_L + P_M$$

$$P = \frac{2(\mu-1)}{R} + \frac{2}{R}$$

$$P = \frac{2\mu}{R} \Rightarrow P = \frac{2\times1.5}{20} = -\frac{1}{f}$$

$$\Rightarrow f = -\frac{20}{2\times1.5} = -\frac{20}{3}\text{cm}$$

(ii)



$$P = 2P_L + P_M$$

$$P = \frac{2(\mu-1)}{R} + \frac{1}{\infty}$$

$$P = \frac{2(\mu-1)}{R} \Rightarrow f = -\frac{1}{P}$$

$$\Rightarrow f = -\frac{R}{2(\mu-1)} = \frac{-20}{2(1.5-1)} = -20\text{cm}$$

BEGINNER'S BOX-7

1. In the displacement method the distance between the object and the screen is 70 cm and the focal length of the lens is 16 cm, find the separation between the magnified and diminished image position of the lens.

2. An achromatic doublet of focal length 90 cm is to be made of two lens. The material of one of the lenses has 1.5 times the dispersive power of the other. The doublet is converging type. Find the focal length of each lens.

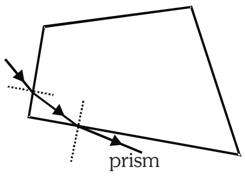
3. The dispersive power of material of a lens of focal length 20 cm is 0.08. Find the longitudinal chromatic aberration of the lens ?

4. The dispersive powers of the materials of the two lenses are in the ratio of 4/3. If the focal length of this achromatic combination is 60 cm, find the focal length of the lenses.

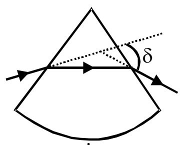
5. What is the axial chromatic aberration in case of a lens which focusses violet ray 20.1 cm and red ray 20.3 cm away from it.

10. PRISM

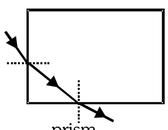
A prism is a portion of a homogeneous, transparent medium (such as glass) enclosed by two plane surfaces inclined at an angle. These surfaces are called the 'refracting surfaces' and the angle between them is called the 'refracting angle' or the 'angle of prism'.



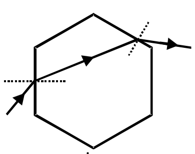
prism



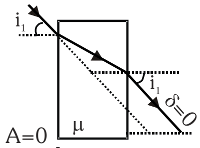
prism



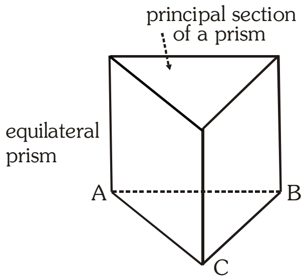
prism

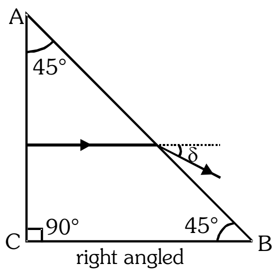


prism

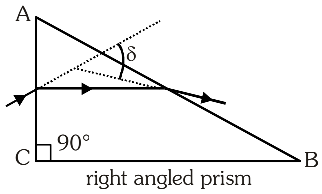


It is not a prism



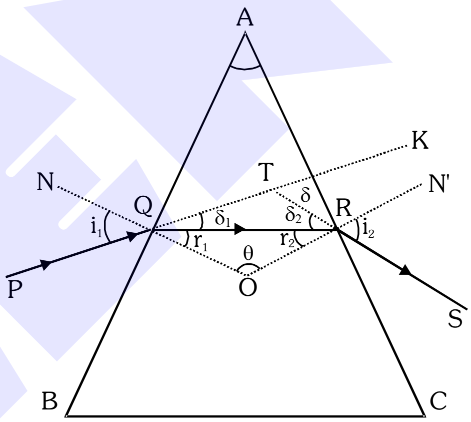


right angled
isosceles prism



right angled prism

DEVIATION



PQ = incident ray

QR = refracted ray

RS = emergent ray

A = prism angle

i_1 = incident angle on face AB

i_2 = emergent angle on face AC

r_1 = refracted angle on face AB

r_2 = incident angle on face AC

Angle of deviation on face AB is $\delta_1 = i_1 - r_1$

Angle of deviation on face AC is $\delta_2 = i_2 - r_2$

Total angle of deviation

$\delta = \delta_1 + \delta_2 \Rightarrow \delta = (i_1 - r_1) + (i_2 - r_2) = i_1 + i_2 - (r_1 + r_2)$ ...(i)

In ΔQOR $r_1 + r_2 + \theta = 180°$ ...(ii)

In the quadrilateral AQOR

$A + 90° + \theta + 90° = 360° \Rightarrow A + \theta = 180°$ ...(iii)

from (ii) and (iii) $r_1 + r_2 = A$ ...(iv)

from (i) and (iv) Total angle of deviation $\delta = i_1 + i_2 - A$

from Snell's law for the refraction at surface AB $\mu_1 \sin i_1 = \mu_2 \sin r_1$

and at surface AC $\mu_2 \sin r_2 = \mu_1 \sin i_2$

CONDITION OF MINIMUM DEVIATION

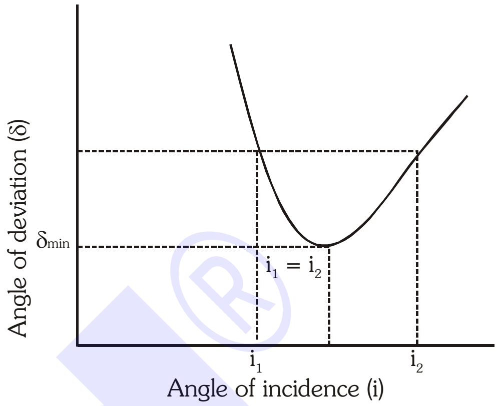
For minimum deviation

In this condition $i_1 = i_2 = i \Rightarrow r_1 = r_2 = r$ and since $r_1 + r_2 = A \therefore r + r = A \Rightarrow 2r = A \Rightarrow r = \frac{A}{2}$

Minimum deviation $\delta_{min} = 2i - A;\ i = \frac{A+\delta_{min}}{2},\ r = \frac{A}{2}$

If prism is placed in air $\mu_1 = 1; 1 \times \sin i = \mu \sin r$

$$\sin\left[\frac{A+\delta_{min}}{2}\right] = \mu \sin\frac{A}{2} \Rightarrow \mu = \frac{\sin\left[\frac{A+\delta_{min}}{2}\right]}{\sin\frac{A}{2}}$$



if angle of prism is small $A < 10°$ then $\sin\theta \approx \theta$

$$\mu = \frac{\frac{(A+\delta_{min})}{2}}{\frac{A}{2}} = \frac{A+\delta_{min}}{A}$$

$\Rightarrow A + \delta_{min} = \mu A \Rightarrow \delta_{min} = (\mu - 1)A$

NO EMERGENCE CONDITION

Let maximum incident angle on the face AB is $i_{max} = 90°$

$1 \times \sin 90° = \mu \sin r_1; \sin r_1 = \frac{1}{\mu} = \sin\theta_C\ ;\ r_1 = \theta_C$ ...(i)

if TIR occurs at face AC then $r_2 > \theta_C$ ...(ii)

$r_1 + r_2 = A$ ...(iii)

from (i) and (ii) $r_1 + r_2 > \theta_C + \theta_C \Rightarrow r_1 + r_2 > 2\theta_C$ ...(iv)

from (iii) and (iv) $A > 2\theta_C \Rightarrow \frac{A}{2} > \theta_C \Rightarrow \sin\frac{A}{2} > \sin\theta_C \Rightarrow \sin\frac{A}{2} > \frac{1}{\mu} \Rightarrow \frac{1}{\sin\frac{A}{2}} < \mu$

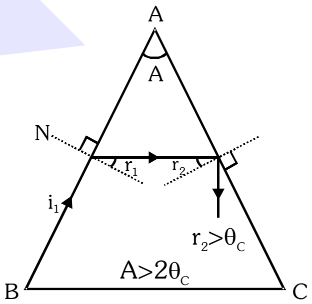


Illustration 26.

Angle of incidence is 45° on a prism having refractive index $\sqrt{2}$ and prism angle 75°. Find angle of deviation.

Solution

For 1st face,

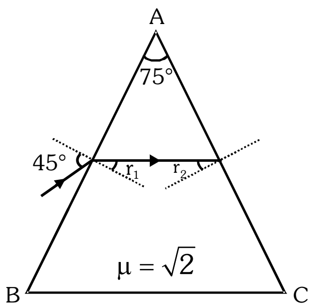
$\sin 45° \times 1 = \sin r_1 \times \sqrt{2}$

$\Rightarrow \sin r_1 = \frac{1}{2} \Rightarrow r_1 = 30°$

as we know, $r_1 + r_2 = A$

$30° + r_2 = 75°$

$\Rightarrow\ r_2 = 45°$



Now for 2nd face,

$\sin 45° \times \sqrt{2} = \sin e \times 1$

$\Rightarrow\ \sin e = 1$

$\Rightarrow\ e = 90°$

So $\delta = i + e - A$

$= 45° + 90° - 75°$

$= 60°$

Note : Deviation angle is maximum when light ray has grazing emergence.

GOLDEN KEY POINTS

- Angle of prism or refracting angle of prism is the angle between the faces on which light is incident and from which it emerges.
- If the faces of a prism on which light is incident and from which it emerges are parallel then the angle of prism will be zero and as incident ray will emerge parallel to itself, deviation will also be zero, i.e., the prism will act as a transparent plate.
- If μ of the material of the prism is equal to that of surrounding, no refraction will take place at its faces and light will pass through it undeviated, i.e., $\delta = 0$.

Illustrations

Illustration 27.

A ray of light passes through an equilateral prism such that the angle of incidence is equal to the angle of emergence and the either is equal to 3/4th of the angle of prism. Calculate the angle of deviation. Refractive index of prism is 1.5.

Solution

$A = 60°$, $\mu = 1.5$; $i_1 = i_2 = \frac{3}{4}A = 45°$, $\delta = ?$

$\because A + \delta = i_1 + i_2$ $\therefore 60° + \delta = 45° + 45° \Rightarrow \delta = 90° - 60° = 30°$

Illustration 28.

A prism of refractive index 1.53 is placed in water of refractive index 1.33. If the angle of prism is 60°, calculate the angle of minimum deviation in water. (sin 35.1° = 0.575)

Solution

Here, ${}^a\mu_g = 1.33$, ${}^a\mu_w = 1.53$, $A = 60°$, ${}^w\mu_g = \frac{{}^a\mu_g}{{}^a\mu_w} = \frac{1.53}{1.33} = 1.15 \because {}^w\mu_g = \frac{\sin\frac{A+\delta_m}{2}}{\sin\frac{A}{2}}$

$\therefore \frac{\sin(A+\delta_m)}{2} = {}^w\mu_g \times \sin\frac{A}{2} = 1.15 \sin\frac{60°}{2} = 0.575 \Rightarrow \frac{A+\delta_m}{2} = \sin^{-1}(0.575) = 35.1°$

$\therefore \delta_m = 35.1 \times 2 - 60 = 10.2°$

Illustration 29.

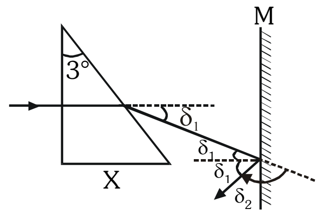
A thin prism of 5° angle gives a deviation of 3·2°. Find the refractive index of the material.

Solution

Angle of deviation $\delta = A(\mu - 1) \Rightarrow 3{\cdot}2° = 5°(\mu - 1) \Rightarrow \mu = 1{\cdot}64$

Illustration 30.

A small angled prism of angle 3° is made of a material of $\mu = 1{\cdot}5$. A ray of light is made incident as shown in the figure M is a plane mirror. Find the angle of deviation for the ray reflected from the mirror M with respect to the incident ray.



Solution

For small angled prism deviation $\delta_1 = (\mu - 1)A$

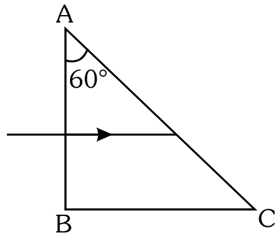
$\delta_1 = (1{\cdot}5 - 1)3 = 1{\cdot}5°$ (at emergence from prism) ; $\delta_2 = (180° - 2\delta_1) = (180 - 2 \times 1{\cdot}5) = 177°$

total deviation $\delta = \delta_1 + \delta_2 = 1{\cdot}5° + 177° = 178{\cdot}50°$.

BEGINNER'S BOX-8

1. Angle of incidence is 45° in the condition of minimum deviation for a prism of refracting angle 60°. Find the angle of deviation.

2. A light ray is incident normally on the surface AB of a prism of refracting angle 60°. If the light ray does not emerge from AC, then find the refractive index of the prism.



3. Calculate the refractive index of the material of an equilateral prism for which the angle of minimum deviation is $\frac{\pi}{3}$ radian.

4. A ray of light passing through a prism having $\mu = \sqrt{2}$ suffers minimum deviation. It is found that angle of incidence is double the angle of refraction within the prism. Find angle of the prism.

5. The angle of minimum deviation measured with a prism is 30° and the angle of prism is 60°. Find the refractive index of the material of the prism.

6. A ray incident at 15° on a refracting surface of a prism of angle 30° suffers a deviation of 55°. Find the angle of emergence.

11. DISPERSION OF LIGHT

When white light is incident on a prism then it is split into seven colours. This phenomenon is known as dispersion. Prism introduces different refractive indices with different wavelengths

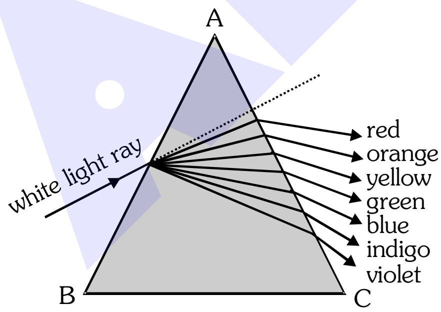
As $\delta_{min} = (\mu - 1)A \because \lambda_R > \lambda_V$ So $\mu_V > \mu_R \Rightarrow \delta_{m(violet)} > \delta_{m(red)}$

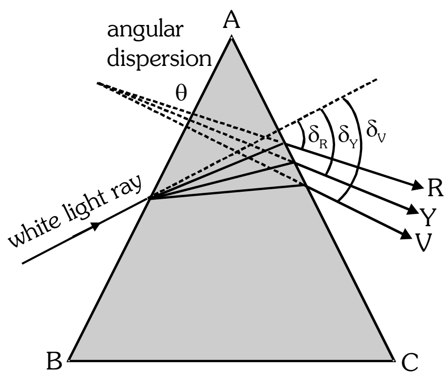
ANGULAR DISPERSION

It is the difference between the angles of deviation for violet colour and red colour

Angular dispersion $\theta = \delta_V - \delta_R = (\mu_V - 1)A - (\mu_R - 1)A = (\mu_V - \mu_R)A$

It depends on prism material and on the angle of prism $\theta = (\mu_V - \mu_R)A$





DISPERSIVE POWER (ω)

It is ratio of angular dispersion (θ) to mean colour deviation (δ_y)

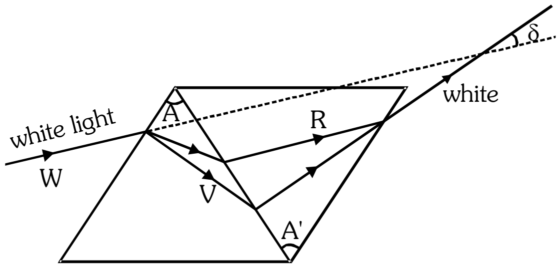
Dispersive power $\omega = \frac{\theta}{\delta_Y} \Rightarrow \omega = \frac{(\mu_V - \mu_R)A}{(\mu_Y - 1)A} = \frac{\mu_V - \mu_R}{\mu_Y - 1} \Rightarrow \omega = \frac{\mu_V - \mu_R}{\mu_Y - 1}$

Refractive index of mean colour $\mu_Y = \frac{\mu_V + \mu_R}{2}$. Dispersive power depends only on the material of the prism.

COMBINATION OF PRISMS

Deviation without dispersion ($\theta = 0°$)

Two or more thin prisms are combined in such a way that deviation occurs i.e. emergent light ray makes certain angle with incident light ray but dispersion does not occur i.e., white light is not split into different colours.



Total dispersion = $\theta = \theta_1 + \theta_2$

$= (\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A'$

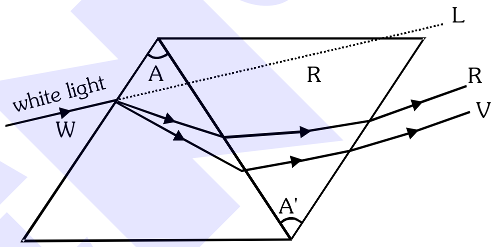
For no dispersion $\theta = 0$; $(\mu_V - \mu_R)A + (\mu'_V - \mu'_R)A' = 0$

Therefore, $A' = -\dfrac{(\mu_V - \mu_R)A}{\mu'_V - \mu'_R}$

–ve sign indicates that prism angles are arranged in opposite manner.

Dispersion without deviation ($\delta = 0°$)

Two or more thin prisms combine in such a way that dispersion occurs i.e., white light is splitted into different colours but deviation does not occur i.e., emergent light ray remains parallel to incident light ray.



Total deviation is $\delta = \delta_1 + \delta_2$

$\Rightarrow \delta = 0; \quad (\mu - 1)A + (\mu' - 1)A' = 0 \quad \Rightarrow \quad A' = -\dfrac{(\mu - 1)A}{\mu' - 1}$

–ve sign indicates that prism angles are arranged in opposite manner.

GOLDEN KEY POINTS

- Alike refractive index, dispersive power has no units and dimensions which depends on the material of the prism and is always positive.
- As for a given prism dispersive power is constant, i.e., dispersion of different wavelengths will be different and will be maximum for violet and minimum for red (as deviation is maximum for violet and minimum for red).
- As for a given prism $\theta \propto \delta$ so a single prism produces both deviation and dispersion of light simultaneously, i.e., a single prism cannot give deviation without dispersion or dispersion without deviation.

Illustrations

Illustration 31.

White light is passed through a prism of angle 5°. If the refractive indices for red and blue colours are 1.641 and 1.659 respectively, calculate the angle of dispersion between them.

Solution

As for small angled prism $\delta = (\mu - 1)A$,

$\delta_B = (1.659 - 1) \times 5° = 3.295°$ and $\delta_R = (1.641 - 1) \times 5° = 3.205°$

so $\theta = \delta_B - \delta_R = 3.295° - 3.205° = 0.090°$.

Illustration 32.

Prism angle of a prism is 10°. Their refractive index for red and violet colours is 1.51 and 1.52 respectively. Then find the dispersive power.

Solution

Dispersive power of prism $\omega = \left(\dfrac{\mu_v - \mu_r}{\mu_y - 1}\right)$ but $\mu_y = \dfrac{\mu_v + \mu_r}{2} = \dfrac{1.52 + 1.51}{2} = 1.515$

Therefore $\omega = \dfrac{1.52 - 1.51}{1.515 - 1} = \dfrac{0.01}{1.515} = 0.019$.

Illustration 33.

The refractive indices of flint glass for red and violet colours are 1.644 and 1.664 respectively. Calculate its dispersive power.

Solution

Here, μ_r = 1.644, μ_v = 1.664, ω = ?

Now $\mu_y = \dfrac{\mu_v + \mu_r}{2} = \dfrac{1.664 + 1.644}{2} = 1.654 \quad \because \ \omega = \dfrac{\mu_v - \mu_r}{\mu_y - 1} = \dfrac{1.664 - 1.644}{1.654 - 1} = 0.0305$.

Illustration 34.

In a certain spectrum produced by a glass prism of dispersive power 0.0305, it was found that μ_r = 1.645 and μ_v = 1.665. What is the refractive index for yellow colour ?

Solution

Here, ω = 0.031, μ_r = 1.645 μ_v = 1.665, μ_y = ?

$$\because \quad \omega = \frac{\mu_v - \mu_r}{\mu_y - 1}$$

$$\therefore \quad \mu_y - 1 = \frac{\mu_v - \mu_r}{\omega} = \frac{1.665 - 1.645}{0.0305} = \frac{0.020}{0.305} = 0.655$$

$\therefore$ μ_y = 0.655 + 1 = 1.655.

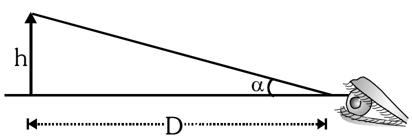
BEGINNER'S BOX-9

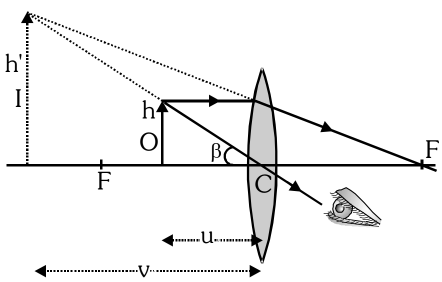
1. White light is passed through a prism of angle 5°. If the refractive index for red and blue colours are 1·641 and 1·659 respectively, then find the angle of dispersion between them.
2. White light is passed through a prism of angle 10°. If the refractive index for red and violet colours are 1.641 and 1.659 respectively, then find the-
   (a) angles of deviation for violet and red colours.
   (b) angular dispersion
   (c) dispersive power
3. For a certain material the refractive indices for red, violet and yellow colour lights are 1.52, 1.64 and 1.60 respectively. Find the dispersive power of the material.

12. OPTICAL INSTRUMENTS

Simple microscope : It is a convergent lens.

When the object is placed between the focus and the optical centre a virtual, magnified and erect image is formed.





Magnifying power (MP) = $\dfrac{\text{visual angle with instrument } (\beta)}{\text{maximum visual angle for unaided eye } (\alpha)} \Rightarrow MP = \dfrac{\frac{h}{-u}}{\frac{h}{-D}} = \dfrac{D}{u}$

(i) When the image is formed at infinity :

From lens equation $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{-\infty} - \dfrac{1}{-u} = \dfrac{1}{f} \Rightarrow u = f$. So $MP = \dfrac{D}{u} = \dfrac{D}{f}$

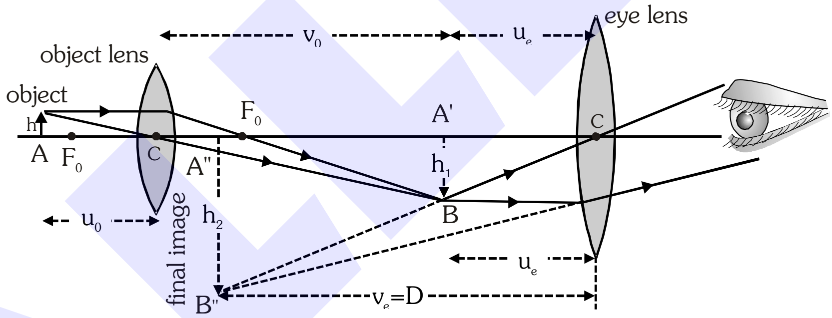
(ii) If the image is at minimum distance of clear vision D :

$\dfrac{1}{-D} - \dfrac{1}{-u} = \dfrac{1}{f} \Rightarrow \dfrac{1}{u} = \dfrac{1}{D} + \dfrac{1}{f}$ [v = –D and u = –ve]

Multiplying both the sides by D $\dfrac{D}{4} = 1 + \dfrac{D}{f} \Rightarrow \dfrac{D}{4} = 1 + \dfrac{D}{f}$

Compound Microscope

Compound microscope is used to get more magnified image as compared to a simple microscope. Object is placed in front of the objective lens and the image is seen through the eye piece. The aperture of objective lens is less as compared to eye piece because object is very near so collection of more light is not required. Generally object is placed between F – 2F due to this a real, inverted and magnified image is formed between 2F – ∞. It is known as intermediate image A'B'. The intermediate image acts as an object for the eye piece. Now the distance between both the lens are adjusted in such a way that intermediate image falls between the optical centre of eye piece and its focus. In this condition, the final image is virtual, inverted and magnified.



Total magnifying power =

Linear magnification of objective lens × angular magnification MP of eye lens = $m_0 m_e = \dfrac{v_0}{u_0}\dfrac{D}{u_e}$

(i) When final image is formed at least distance of distinct vision.

$$MP = \frac{v_0}{u_0}\left[1 + \frac{D}{f_e}\right] = \frac{f_0}{(f_0 + u_0)}\left[1 + \frac{D}{f_e}\right] = \frac{f_0 - v_0}{f_0}\left[1 + \frac{D}{f_e}\right] = \frac{h_1}{h}\left(1 + \frac{D}{f_e}\right)$$

Length of the tube $L = v_0 + |u_e|$

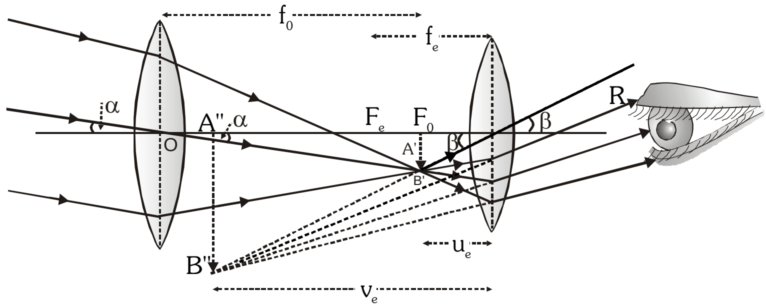
(ii) When final image is formed at infinity. $\dfrac{1}{v_e} - \dfrac{1}{u_e} = \dfrac{1}{f_e} \Rightarrow \dfrac{1}{\infty} + \dfrac{1}{u_e} = \dfrac{1}{f_e} \Rightarrow u_e = f_e$

$MP = \dfrac{v_0}{u_0}\left[\dfrac{D}{f_e}\right] = \dfrac{f_0}{(f_0 + u_0)}\left[\dfrac{D}{f_e}\right] = \dfrac{(f_0 - v_0)}{f_0}\left[\dfrac{D}{f_e}\right] = \dfrac{h_1}{h}\left[\dfrac{D}{f_e}\right]$. Separation between the lens = $v_0 + f_e$

Sign convention for solving numericals u_0 = –ve, v_0 = +ve, f_0 = +ve,

u_e = –ve, v_e = –ve, f_e = +ve, m_0 = –ve, m_e = +ve, M = –ve

Astronomical Telescope



A telescope is used to see distant objects. The objective forms the image A'B' at its focus. This image A'B' acts as an object for eyepiece and it forms the final image A"B".

$$\text{MP} = \frac{\text{visual angle with instrument } (\beta)}{\text{visual angle for unaided eye } (\alpha)} \Rightarrow \text{MP} = \frac{\frac{h'}{-u_e}}{\frac{h'}{f_0}} = -\frac{f_0}{u_e} \quad [A'B' = h']$$

(i) If the final image is formed at infinity then, $v_e = -\infty$, $u_e = -ve$

$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow u_e = f_e. \text{ So } \text{MP} = -\frac{f_0}{f_e} \text{ and length of the tube } L = f_0 + f_e$$

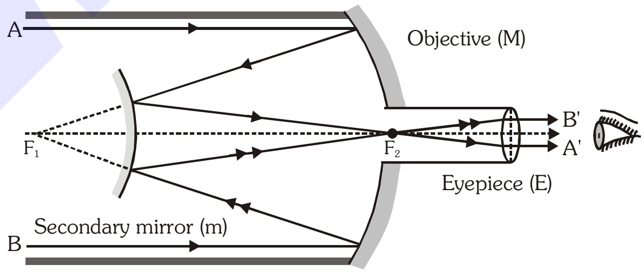
(ii) If the final image is at least distance of distinct vision then : $v_e = -D$, $u_e = -ve$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right] \text{ So } \text{MP} = -\frac{f_0}{u_e} = -\frac{f_0}{f_e}\left[1 + \frac{f_e}{D}\right]$$

Length of the tube is $L = f_0 + |u_e|$

Cassegrain's telescope

This telescope consists of a paraboloidal mirror M as the objective, and a convex elliptical mirror m called the secondary mirror. F_1 and F_2 are the two conjugate foci of the mirror m.

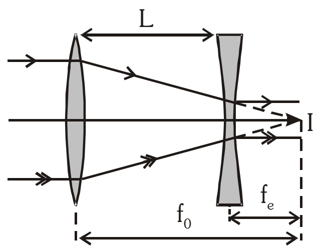


It is easy to see that the angular magnification of the telescope, i.e.,

$$\text{MP} = \frac{\text{focal length of objective } (-f_o)}{\text{focal length of eyepiece } (f_e)} \Rightarrow M = -\frac{f_o}{f_e}$$

Galilean telescope

If in an astronomical telescope, the convergent eye–piece is replaced by a divergent lens which is placed in such a way that rays from objective are directed towards its focus (Figure), final image will be erect, enlarged and virtual. This telescope is also used to see distant terrestrial objects and is called **Galilean telescope** and for it



$$\text{MP} = \frac{f_0}{f_e} \text{ with } L = 'f_0' + 'f_e'$$

The intermediate image in this telescope is outside the tube. Hence the telescope cannot be used for making measurements.

S.No.	Compound – Microscope	S.No.	Astronomical – Telescope
1.	It is used to increase the visual angle of near tiny object.	**1.**	It is used to increase the visual angle of distant large objects.
2.	The objective and eye lens both are convergent, with short focal lengths and apertures.	**2.**	The objective is of large focal length and aperture while eye lens is of short focal length and aperture and both are convergent.
3.	Final image is inverted, virtual and enlarged and formed somewhere between D to ∞ from the eye.	**3.**	Final image is inverted, virtual and enlarged and formed somewhere between D to ∞ from the eye.
4.	MP does not change appreciably if objective and eye lens are interchanged as [MP ~ (LD / f_0 f_e)]	**4.**	MP becomes ($1/m^2$) times of its initial value if objective and eyelens are interchanged as MP ~ [f_0 / f_e]
5.	MP is increased by decreasing the focal length of both the lenses.	**5.**	MP is increased by increasing the focal length of objective and by decreasing the focal length of the eyepiece

Illustrations

Illustration 35.

A man with normal near point 25 cm away reads a book with small print using a magnifying glass, which is a thin convex lens of focal length 5 cm.

(a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass ?

(b) What is the maximum and minimum MP possible using the above simple microscope ?

Solution

(a) As for normal eye far and near points are ∞ and 25 cm away respectively, so for magnifier $v_{max} = -\infty$ and $v_{min} = -25$ cm. However, for a lens as $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow u=\frac{f}{(f/v)-1}$

So u will be minimum when v = minimum = –25 cm i.e. $(u)_{min} = \frac{5}{-(5/25)-1} = -\frac{25}{6} = -4.17\,cm$

Ans u will be maximum when v = maximum = ∞ i.e., $u_{max} = \frac{5}{\left(\frac{5}{\infty}-1\right)} = -5$ cm

So the nearest and farthest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) As in case of simple magnifier MP = (D/u). So MP will be minimum when u is max = 5 cm

$$\Rightarrow (MP)_{min}\left[=\frac{D}{f}\right] = \frac{-25}{-5} = 5$$ and MP will be maximum when u is min = (25/6) cm

$$\Rightarrow (MP)_{max}\left[=1+\frac{D}{f}\right] = \frac{-25}{-(25/6)} = 6$$

Illustration 36.

A thin convex lens of focal length 5 cm is used as a simple microscope by a person with normal near point located (25 cm) away. What is the magnifying power of the microscope ?

Solution Here, f = 5 cm; D = 25 cm, MP = ?, $MP = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 6$

Illustration 37.

A compound microscope consists of an objective of focal length 2.0 cm and an eye piece of focal length 6.25 cm, separated by a distance of 15 cm. How far should an object be placed from the objective in order to obtain the final image at (a) the least distance of distinct vision (25 cm) (b) infinity ?

Solution

Here, f_0 = 2.0 cm; f_e = 6.25 cm, u_0 = ?

(a) $v_e = -25$ cm $\because \frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$ $\therefore \frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{6.25} = \frac{-1-4}{25} = \frac{-5}{25} \Rightarrow u_e = -5$ cm

As distance between objective and eye piece = 15 cm; $v_0 = 15 - 5 = 10$ cm

$$\because \frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \therefore \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{10} - \frac{1}{2} = \frac{1-5}{10} \Rightarrow u_0 = \frac{-10}{4} = -2.5 \text{ cm}$$

(b) $\because v_e = \infty$, $u_e = f_e = 6.25$ cm $\therefore v_0 = 15 - 6.25 = 8.75$ cm.

$$\because \frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \Rightarrow \frac{1}{u_0} = \frac{1}{v_0} - \frac{1}{f_0} = \frac{1}{8.75} - \frac{1}{20} = \frac{2-8.75}{17.5} \Rightarrow u_0 = \frac{-17.5}{6.75} = -2.59 \text{cm}$$

Illustration 38.

A small telescope has an objective of focal length 144 cm and an eyepiece of focal length 6.0 cm. What is the magnifying power of the telescope ? What is the separation between the objective and the eyepiece? The final image is formed at infinity.

Solution Here, f_0 = 144 cm; f_e = 6.0 cm, MP = ?, L = ?

$MP = \frac{-f_0}{f_e} = \frac{-144}{6.0} = -24$ and $L = f_0 + f_e = 144 + 6.0 = 150.0$ cm.

Illustration 39.

Diameter of the moon is 3.5×10^3 km and its distance from earth is 3.8×10^5 km. It is seen by a telescope whose objective and eyepiece have focal lengths 4 m and 10 cm respectively. What will the angular diameter of the image of the moon ?

Solution

$MP = -\frac{f_0}{f_e} = -\frac{400}{10} = -40$. Angle subtended by the moon at the objective $= \frac{3.5 \times 10^3}{3.8 \times 10^5} = 0.009$ radians.

Thus angular diameter of the image = MP × visual angle of moon = 40 × 0.009 = 0.36 radians $= \frac{0.36 \times 180}{3.14} \simeq 21°$

BEGINNER'S BOX-10

1. Magnification of a compound microscope is 30. Focal length of eye piece is 5 cm and the image is formed at least distance of distinct vision. Find the magnification of objective.

2. The powers of the lenses of a telescope are 0.5 and 20 dioptres. If the final image is formed at the minimum distance of distinct vision (25 cm) then what will be length of the tube ?

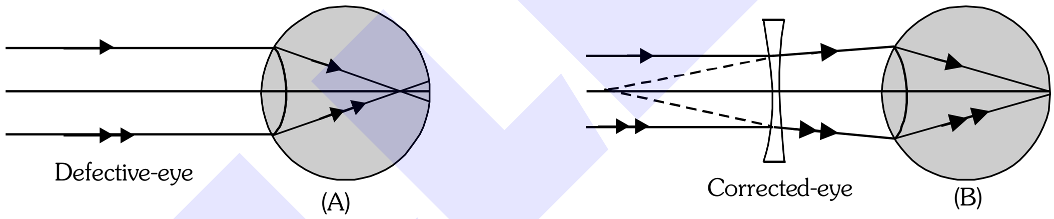
3. The focal lengths of objective and eye piece of a Galilean telescope are respectively 30 cm and 5 cm. Calculate its magnifying power and length when used to view distant objects.

4. A telescope consisting of an objective of focal length 60 cm and an eyepiece of focal length 5 cm is focussed to a distant object in such a way that parallel rays emerge from the eye piece. If the object subtends an angle of 2° at the objective, then find the angular width of the image.

5. The focal lengths of the objective and the eye piece of an astronomical telescope are 60 cm and 5 cm respectively. Calculate the magnifying power and the length of the telescope when the final image is formed at (i) infinity, (ii) least distance of distinct vision (25 cm).

13. DEFECTS OF VISION

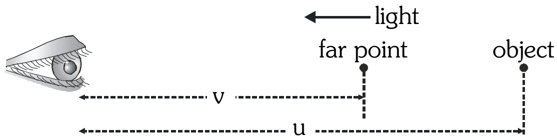
MYOPIA [or Short–sightedness or Near – sightedness]



(i) Distant objects are not clearly visible, but nearby objects are clearly visible because image is formed before the retina.

(ii) To rectify the defect concave lens is used.

- The maximum distance which a person can see without the help of spectacles is known as far point of distinct vision.



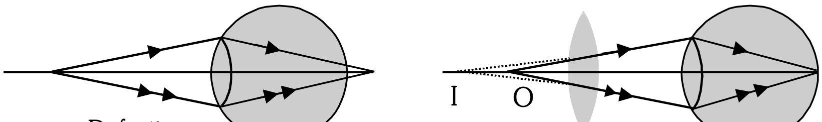
- If the reference of object is not given then it is taken as infinity.
- In this case image of the object is formed at the far point of the person.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=P \Rightarrow \frac{1}{\text{distance of far point (in m)}}-\frac{1}{\text{distance of object (in m)}}=\frac{1}{f}=P$$

$$\frac{100}{\text{distance of far point (in cm)}}-\frac{100}{\text{distance of object (in cm)}}=P$$

distance of far point = –ve, distance of object = –ve, P = –ve

LONG–SIGHTEDNESS OR HYPERMETROPIA



Defective-eye (A) Corrected-eye (B)

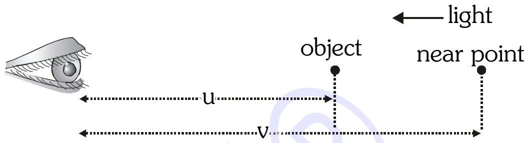
(i) Nearby objects are not clearly visible.

(ii) The image of nearby objects is formed behind the retina.

(iii) To remove this defect convex lens is used.

Near Point :–

The minimum distance which a person can see without the help of spectacles.



- In this case the image of the object is formed at the near point.
- If reference of object is not given it is taken as 25 cm.

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}=P \Rightarrow \frac{1}{\text{distance of near point (in m)}}-\frac{1}{\text{distance of object (in m)}}=\frac{1}{f}=P$$

distance of near point = –ve, distance of object = –ve, P = +ve

PRESBYOPIA

In this case, both nearby and distant objects are not clearly visible. To remove this defect, two separate lenses one for myopia and other for hypermetropia are used or bifocal lenses are used.

ASTIGMATISM

In this defect a person cannot see object in two orthogonal directions clearly. It can be removed by using cylindrical lens in a certain orientation.

Illustrations

Illustration 40.

A person cannot see clearly an object kept at a distance beyond of 100 cm. Find the nature and the power of lens to be used for seeing clearly the object at infinity.

Solution

For lens u = – ∞ and and v = – 100 cm

$$\therefore \frac{1}{v}-\frac{1}{u}=\frac{1}{f}=\frac{1}{v}=\frac{1}{f} \Rightarrow f=v=-100\,\text{cm}(\text{concave})$$

$$\therefore \text{ Power of lens } P=\frac{1}{f}=-\frac{1}{1}=-1D$$

Illustration 41.

A far sighted person has a near point 60 cm away. What should be the power of a lens he should use for eye glasses so that he can read a book at a distance of 25 cm ?

Solution

Here v = – 60 cm, u = –25 cm

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=-\frac{1}{60}+\frac{1}{25} \Rightarrow f=\frac{300}{7}\text{cm} \therefore \text{Power} =\frac{1}{f(\text{in m})}=\frac{1}{(3/7)}=+2.33D$$

BEGINNER'S BOX-11

1. What should be the focal length of the reading spectacles for a person for whom near point is 50 cm ?

2. A person can see objects placed beyond distance of 1 m. Find the optical power of the spectacles compensating the defect of vision for this eye.

3. A near sighted man can see objects clearly up to a distance of 1·5m. Calculate the power of the lens of the spectacles necessary for the remedy of this defect.

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14. SOME NATURAL PHENOMENON DUE TO SUNLIGHT

- **Rainbow**

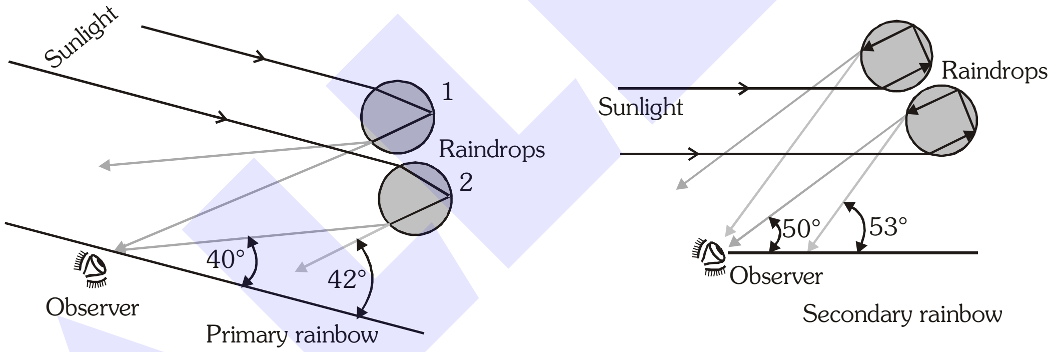
After a light drizzle, an observer with the sun facing his back, sees a number of concentric coloured arcs looming in the sky, with the common centre of these arcs lying on the line joining the sun and the observer. These arcs constitute the primary rainbow.

The inner edge of the primary rainbow is violet and the outer edge is red. Besides the primary rainbow, a bigger but a fainter rainbow is also seen. This is called the secondary rainbow. The sequence of colours in the secondary rainbow is the reverse of that in the primary rainbow, i.e., the inner edge is red and the outer edge is violet.

Both these rainbows are formed by :

(i) Dispersion and

(ii) Internal reflection of the Sun's rays in the rain drops suspended in the atmosphere.



- **Scattering of light**

The deflection of light energy by fine particles of solid, liquid or gaseous matter from the main direction of the beam is called the scattering of light.

The basic process involved in scattering is the absorption of light by the molecules followed by its re-radiation in different directions. The intensity of the scattered light depends on :

(i) the wavelength (λ) of light (ii) the size of the particles causing scattering.

Depending upon the size of the scatterers, the following two situations arise :

(a) If the scattering particles (air molecules) are of size smaller than the wavelength of light, the intensity of the scattered light (I) varies inversely as the fourth power of the wavelength of light,

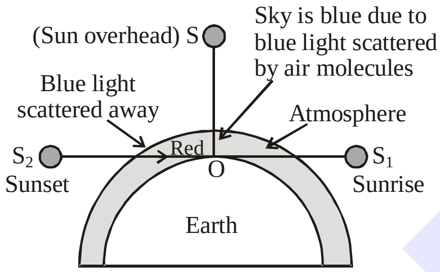
This statement, which holds for elastic scattering, is known as the Rayleigh's law of scattering. Since the wavelength of blue light is less than that of red light, blue light is scattered most while the red light least. Blue colour of sky, reddish appearance of Sun during sunrise and sunset are due to this phenomenon as discussed below.

Further, it is due to this reason that red signals are used to indicate danger. Such signals go to large distances without an appreciable loss due to scattering.

(b) If the scattering particles are of sizes greater than the wavelength of light (e.g., dust particles, water droplets), Rayleigh's law of scattering is not applicable and all colours are scattered equally. It is due to this reason that clouds generally appear white.

Blue colour of the sky

If an observer (O) looks at the sky when the Sun is overhead at noon as shown by position S in figure it is the scattered light that is received by the observer. Since blue light is scattered more (almost six times) than red, the sky appears blue to the observer.



- **Reddish appearance of the Sun during sunrise and sunset**

During sunrise (S_1) and sunset (S_2), the light coming from the sun has to travel a larger distance through the atmosphere (than it does at noon) before entering the observers eye. As a result of this, most of the blue light is scattered on its way to the observer. The transmitted light (sunlight minus the scattered light), which reaches the observer, is rich in red and orange colour and makes the Sun appear reddish orange.

ANSWERS KEY

BEGINNER'S BOX-1

1. 0.9 m **2.** 4 **3.** 60 m **4.** 60°

5. 6.18 m **6.** 3

BEGINNER'S BOX-2

1. (A)P (B)R,S (C)S (D)Q,S **2.** u = –0.1 m

3. $\frac{f}{3}$, m = $\frac{2}{3}$ so image will be virtual, erect and smaller than the object

4. 19.35 cm behind the mirror. **5.** u = –12 cm

6. – 40 cm

7. – 54 cm, h_I = –5 cm,

m = –2 real, inverted, magnified

BEGINNER'S BOX-3

1. 30 cm **2.** $2\cos^{-1}\left(\frac{\mu}{2}\right)$

3. Frequency is a characteristic of source, so it will not change with change in medium.

BEGINNER'S BOX-4

1. $\sin^{-1}\left(\frac{10t_1}{t_2}\right)$ **2.** $C = \sin^{-1}\frac{1}{\sqrt{2}} = 45^0$ **3.** i < 45°

BEGINNER'S BOX-5

1. 3.63 cm **2.** –3.07 cm

BEGINNER'S BOX-6

1. 15 cm, real, inverted, samll in size

2. + 15 cm **3.** 1.6 **4.**7.5 D

5. 7.5 D **6.** 2 **7.** – f (1+n)

BEGINNER'S BOX-7

1. 20.5 cm **2.** f_1 = –45 cm and f_2 = 30 cm

3. 1.6 cm. **4**. f_1 = –20 cm, f_2 = +15 cm

5. 0.2 cm

BEGINNER'S BOX-8

1. 30° **2.** $\mu > \frac{2}{\sqrt{3}}$ **3.** $\sqrt{3}$ **4.** 90°

5. $\sqrt{2}$ **6.** 70°

BEGINNER'S BOX-9

1. 0·090° **2.** (a) 6.59°, 6.41° (b) 0.18° (c) 0.0276

3. 0.2

BEGINNER'S BOX-10

1. 5 **2.** 204.17 cm

3. 6, 25 cm **4.** 24°

5. (i) –12, 65 cm (ii) –14.4, 64.17 cm

BEGINNER'S BOX-11

1. + 50 cm **2.** +3 D **3.** – 0.66 D