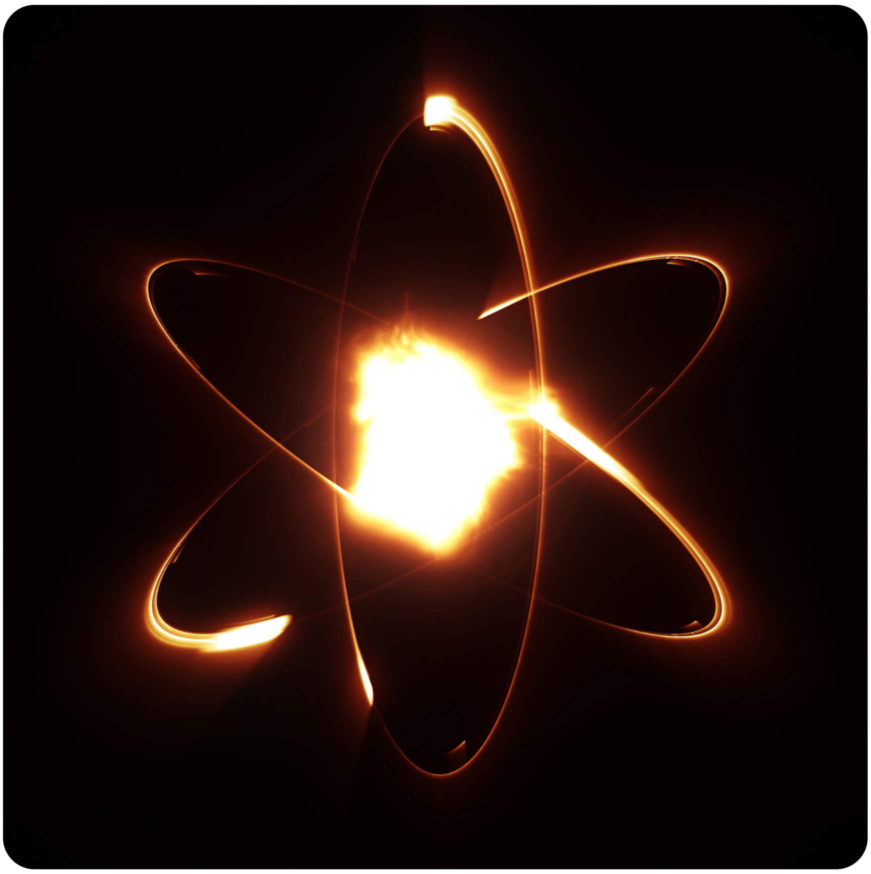
ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
path to success

PRE-MEDICAL

PHYSICS

ENTHUSIAST | LEADER | ACHIEVER



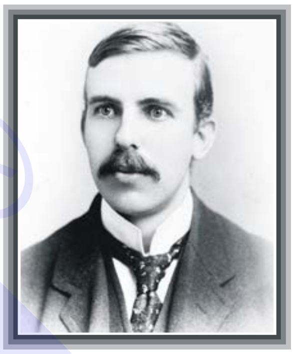
STUDY MATERIAL

Modern Phlysics-II

ENGLISH MEDIUM

Copyright Statement

ERNST RUTHERFORD (1871 –1937)



British physicist who did pioneering work on radioactive radiation. He discovered alpha-rays and beta-rays. Along with Federick Soddy, he created the modern theory of radioactivity. He studied the ‘emanation’ of thorium and discovered a new noble gas, an isotope of radon, now known as thoron. By scattering alpha-rays from the metal foils, he discovered the atomic nucleus and proposed the plenatery model of the atom. He also estimated the approximate size of the nucleus.

MARIE SKLODOWSKA CURIE (1867-1934)

Born in Poland. She is recognised both as a physicist and as a chemist. The discovery of radioactivity by Henri Becquerel in 1996 inspired Marie and her husband Pierre Curie in their researches and analyses which led to the isolation of radium and polonium elements. She was the first person to be awarded two Nobel Prizes - for Physics in 1903 and for chemistry in 1911.

NUCLEAR PHYSICS

1. NUCLEUS

(i) Central core of every atom.

(ii) Discovered by Rutherford in α-scattering experiment.

(iii) The order of nuclear size = 10^{-15} m or fm while the order of atomic size = 10^{-10} m or Å

(iv) Protons and neutrons, together referred as nucleons.

(v) A nuclide is represented by ${}^{A}_{Z}X$

Z = atomic number = p (no. of protons)

A = mass number = total no. of nucleons = n + p

(vi) Atomic masses are generally represented by atomic mass unit (u)

$$1u = \frac{\text{mass of } C^{12} \text{ atom}}{12} = 1.66 \times 10^{-27} \text{ kg}$$

$m_p = 1.6726 \times 10^{-27}$ kg = 1.00727 u

$m_n = 1.6749 \times 10^{-27}$ kg = 1.00866 u

$m_e = 9.1 \times 10^{-31}$ kg = 0.00055 u

1.1 Types of Nuclei

(i) Isotope : same Z

(ii) Isobar : same A

(iii) Isotone : same (A – Z)

1.2 Properties of Nuclei

Size of Nucleus : (Order is fermi)

As the number of nucleons in nucleus increases its size also increases and relation between its radius and mass number is $R \propto A^{1/3}$

$R = R_0 A^{1/3}$

Here R_0 is a constant and its value $R_0 \approx 1.2$ fm.

Volume of Nucleus

Volume $\propto R^3$ (But $R \propto A^{1/3}$) or volume $\propto$ A

Mass of Nucleus

Its mass is quite small compare to gm or kg. Therefore it is measured in another unit - amu (Atomic Mass Unit)

Mass of an nucleus of mass number A is $\simeq Am_p \simeq A$ amu or mass of an nucleus, $m \propto A$

Density of Nucleus (ρ)

$$\rho = \frac{\text{mass}}{\text{volume}} \cong \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi R_0^3 A} = \frac{3m_p}{4\pi R_0^3} \simeq 2.3 \times 10^{17} \text{kg/m}^3$$

It means ρ is independent of A. Density of nuclei of all types of element is same and its order is 10^{17} kg/m^3 or 10^{14} g/cm^3

Illustrations

Illustration 1.

Calculate mass no. of that nucleus whose radius is half of Ge^{72}.

Solution.

$r \propto A^{1/3}$

$$\frac{\frac{r}{2}}{r} = \left(\frac{A}{72}\right)^{1/3} \Rightarrow \frac{1}{8} = \frac{A}{72} \Rightarrow A = 9$$

Illustration 2.

Find the density of ${}_{6}^{12}C$

Solution.

Mass of nucleus $\approx 12m_P = 12 \times 1.66 \times 10^{-27}$ kg $\quad \{m_P$ = mass of proton$\}$

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{12 \times 1.66 \times 10^{-27}\,\text{kg}}{\frac{4}{3}\pi\left[1.2 \times 10^{-15}(12)^{1/3}\right]^3 \text{m}^3} = 2.4 \times 10^{17} \text{ kg m}^{-3}$$

1.3 Forces acting inside the nucleus

There are three forces interacting between nucleons, these are

(i) Gravitational force - weakest force of nature

(ii) Electrostatic repulsive (coulombian) force $\rightarrow$ only works between proton proton. This is stronger than gravitational force.

(iii) Nuclear force $\rightarrow$ strongest intraction that holds nucleons together to form nuclei and it is powerful enough to overcome the electric repulsion of proton and proton.

1.4 Features of Nuclear Force (F_n) :–

1. The strongest force in the universe.
2. Works only between the nucleons.
3. **Very short range :** only upto size of nucleus (3 or 4 fermi). More than this distance, nuclear force is almost zero.
4. **Very much depends upon distance :–** Small variation in distance may cause of large change in nuclear force while electrostatic force remains almost unaffected.
5. **Independent of charge :–** Interacts between n–n as well as between p–p and also between n–p.
6. **Spin dependent :–** It is stronger between nucleons having same sense of spin than between nucleons having opposite sense of spin.
7. **It is not a central force :–** Definition of central force (F_c) : Whose line of action always passes through a fixed point and its magnitude depends only on distance, if medium is same.

$$\vec{F}_c = \frac{K}{r^n}(\pm\hat{r}) \text{ is central force.}$$

Electrostatic and gravitational forces are central forces.

8. **Nature :–** **(i) Attractive –** If distance is greater than 0.8 fm or above. **(ii) Repulsive –** If distance is lesser than 0.8 fm.

2. EINSTEIN'S MASS ENERGY EQUIVALENCE

According to Einstein, mass can be converted into energy and energy into mass. This relation is given by -

$$E = mc^2$$

Here E = total energy associated with mass m ; c^2 = used as a conversion coefficient

2.1 Mass defect

(i) Mass of a nucleus is always less than the sum of masses of its constituent nucleons. This difference is called mass defect.

(ii) If observed mass of nucleus $_zX^A$ be M, mass of proton is M_p and mass of neutron is M_n then

mass defect = $\Delta m = [ZM_p + (A - Z)M_n] - M$.

(iii) If M is taken as mass of atom of $_zX^A$ instead of mass of nucleus then

$$\Delta m = [Z(M_p + M_e) + (A - Z)M_n] - M_{atom}$$

2.2 Binding energy (E_b)

(i) Binding energy of a nucleus is the energy required to split it into its nucleons (free).

(ii) $\Delta E_b = \Delta m.c^2$

(iii) It is always positive and numerically equal to the energy equivalent of mass defect (or equal to the energy liberated when it was formed)

2.3 Binding Energy per Nucleon $\left[\frac{\Delta E_b}{A}\right]$

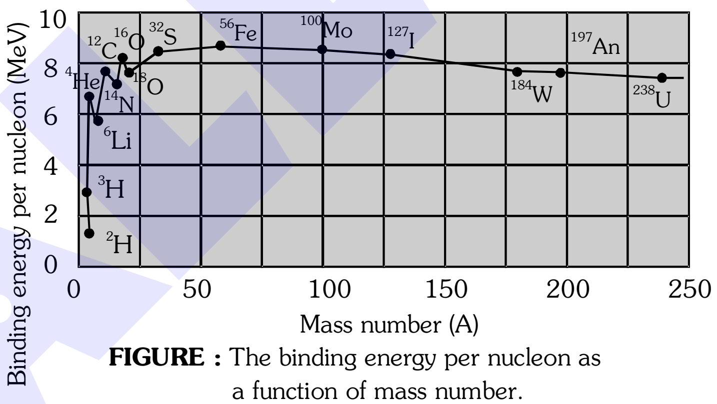


FIGURE : The binding energy per nucleon as a function of mass number.

(i) The value of binding energy per nucleon decides the stability of a nucleus. It is obtained by dividing binding energy by the mass number of given nucleus.

(ii) The following figure shows the binding energy per nucleon plotted against the mass number of various atoms nuclei

Greater the binding energy per nucleon, the more stable the nucleus.

(iii) It is maximum for isotope of iron $-^{56}_{26}Fe$ and is 8.8 MeV/nucleon. It is the most stable nucleus.

(iv) For Uranium, binding energy per nucleon is about 7.7 MeV/nucleon and it is unstable.

(v) The medium size nuclei are more stable than light or heavy nuclei.

3. NUCLEAR FISSION

Splitting of a heavy nucleus (A > 230) into two or more lighter nuclei when struck by a neutron.

In this process certain mass disapears which is obtained in the form of energy (enormous amount)

$A + n \rightarrow \text{excited nucleus} \rightarrow B + C + Q$

Hahn and Strassmann done the first fission (fission of nucleus of U^{235}).

When U^{235} is bombarded by a neutron it splits into two fragments and 2 or 3 secondary neutrons and releases about 200 MeV energy per fission (or from single nucleus)

Fragments are uncertain but each time energy released is almost same.

Possible reactions are -

$U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3{}_0n^1 + 200\text{ MeV}$

or $U^{235} + {}_0n^1 \rightarrow Xe + Sr + 2{}_0n^1 + 200\text{ MeV}$

and many other reactions are possible.

(i) The average number of secondary neutrons is 2.5.

(ii) Nuclear fission can be explained by using " liquid drop model" also.

(iii) The mass defect Δm is about 0.1% of mass of fissioned nucleus

(iv) About 93% of released energy (Q) is appear in the form of kinetic energies of products and about 7% part in the form of γ - rays.

4. NUCLEAR CHAIN REACTION

The equation of fission of U^{235} is $\quad {}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3{}_0n^1 + Q$

These three secondary neutrons produced in the reaction may cause of fission of three more U^{235} and give 9 neutrons, which in turn, may cause of nine more fission of U^{235} and so on.

Thus a continuous 'Nuclear Chain reacion' would start.

If there is no control on chain reaction then in a short time (10^{-6} sec.) a huge amount of energy will be released. (This is the principle of 'Atom bomb')

If chain reaction is controlled then produced energy can be used for peaceful purposes. For example nuclear reactor (Based on fission) generats electricity.

4.1 Natural Uranium

It is mixture of U^{235} (0.7%) and U^{238} (99.3%)

U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV.

To improve the quality, percentage of U^{235} is increased to 3%. The improved uranium is called 'Enriched Uranium' (97% U^{238} and 3% U^{235})

4.2 Losses of Secondary Neutrons

Leakage of neutrons from the system

Due to their high K.E. some neutrons escape from the system.

Absorption of neutrons by U^{238}

U^{238} is not fissionable by these secondary fast neutrons. But U^{238} absorbs some fast neutrons.

4.3 Critical Size (or mass)

In order to sustain chain reaction in a sample of enriched uranium, it is required that the number of lost neutrons should be much smaller than the number of neutrons produced in a fission process. For it the size of uranium block should be equal or greater than a certain size called **critical size**.

4.4 **Multiplication factor** (K) = $\dfrac{\text{No. of neutrons in a stage}}{\text{No. of neutrons in just previous stage}}$

(i) If size of Uranium used is 'Critical' then K = 1 and the chain reaction will be steady or sustained (As in nuclear reaction)

(ii) If size of Uranium used is 'Super critical' then K > 1 and chain reaction will accelerate resulting in a explosion (As in atom bomb)

(iii) If size of Uranium used is 'Sub Critical' then K < 1 and chain reaction will retard and ultimately stop.

5. NUCLEAR REACTOR (K = 1)

Its main constituents are -

5.1 Nuclear Fuel : Commonly used are U^{235} , Pu^{239} .

Pu^{239} is the best.

But Pu^{239} is not naturally available and U^{235} is used in most of the reactors.

5.2 Moderator

Its function is to slow down the fast secondary neutrons. Because only slow neutrons are capable for the fission of U^{235}. The moderator should be light and it should not absorb neutrons. Commonly, Heavy water (D_2O, molecular weight 20 gm.) are used.

5.3 Control rods

They have the ability to capture the slow neutrons and can control the rate of chain reaction at any stage. Boron and Cadmium are best absorber of neutrons.

5.4 Coolant

A substance which absorb the produced heat and transfers it to water for further use. Generaly coolant is water at high pressure

6. FAST BREADER REACTORS

The atomic reactor in which fresh fissionable fuel (Pu^{239}) is produced along with energy.

Fuel : Natural Uranium.

During fission of U^{235}, energy and secondary neutrons are produced. These secondary neutrons are absorbed by U^{238} and U^{239} is formed. This U^{239} converts into Pu^{239} after two beta decay. This Pu^{239} can be separated, its half life is 2400 years.

${}_{92}U^{238} + {}_0n^1 \rightarrow {}_{92}U^{239} \xrightarrow{2\beta^-} {}_{94}Pu^{239}$ (best fuel of fission)

This Pu^{239} can be used in nuclear weapons because of its small critical size than U^{235}.

7. NUCLEAR FUSION

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

$A + B \rightarrow C + Q$ (Fusion)

The product (C) is more stable then reactants (A and B).

and $m_c < (m_a + m_b)$

and mass defect $\Delta m = [(m_a + m_b) - m_c]$ amu

Energy released is $E = \Delta m \times 931$ MeV/amu

The total binding energy and binding energy per nucleon C both are more than of A and B.

$$\Delta E = E_c - (E_a + E_b)$$

Fusion of four hydrogen nuclei into helium nucleus -

$$4({}_1H^1) \rightarrow {}_2He^4 + 2\,{}_{+}\beta^0 + 2\,\nu + 26 \text{ MeV}$$

(i) Energy released per fission >> Energy released per fusion

(ii) Energy per nucleon in fission $\left[=\dfrac{200}{235} \simeq 0.85\text{MeV}\right]$ << energy per nucleon in fusion $\left[=\dfrac{26}{4} \simeq 6.5\text{MeV}\right]$

7.1 Required condition for nuclear fusion

(a) High Temperature

Which provide kinetic energy to nuclei to overcome the repulsive electrostatic force between them.

(b) High Pressure (or density)

Which ensure frequent collision and increases the probability of fusion.

These condition exist in the sun and in many other stars. The source of energy in the sun is nuclear fusion, where hydrogen is in plasma state and protons fuse to form helium nuclei.

8. PAIR PRODUCTION AND PAIR ANNIHILATION

PAIR PRODUCTION	**PAIR ANNIHILATION**
A γ-photon of energy more than $\geq$ 1.02 MeV, when interact with a nucleus produces pair of electron (e^-) and positron (e^+). The energy equivalent to rest mass of e^- (or e^+)=0.51 MeV. The energy equivalent to rest mass of pair $(e^- + e^+)$ = 1.02 MeV. For pair production Energy of photon1.02 MeV. If energy of photon is more than 1.02 MeV, the extra energy (E-1.02) MeV devides approximately in equal amount to each particle as the kinetic energy. $(K.E.)_{e^- \text{ or } e^+} = \left[\dfrac{E_{Ph} - 1.02}{2}\right]$MeV If E < 1.02 MeV, pair will not produce.	When electron and positron combines they annihilates to each other and only energy is released in the form of two gama photons.

GOLDEN KEY POINTS

- The density of nuclear matter is independent of the size of the nucleus. The mass density of the atom does not follow this rule.
- The radius of a nucleus determined by electron scattering is found to be slightly different from that determined by alpha-particle scattering. This is because electron scattering senses the charge distribution of the nucleus, whereas alpha and similar particles sense the nuclear matter.
- The nature of the binding energy (per nucleon) curve shows that exothermic nuclear reactions are possible, when two light nuclei fuse or when a heavy nucleus undergoes fission into nuclei with intermediate mass.
- For fusion, the light nuclei must have sufficient initial energy to overcome the coulomb potential barrier. That is why fusion requires very high temperatures.
- Although the binding energy (per nucleon) curve is smooth and slowly varying, it shows peaks at nuclides like ^{4}He, ^{16}O etc. This is considered as evidence of atom-like shell structure in nuclei.
- A free neutron is unstable ($n \rightarrow p + e^- + \bar{\nu}$). But a similar free proton decay is not possible, since a proton is (slightly) lighter than a neutron.
- Gamma emission usually follows alpha or beta emission. A nucleus in an excited (higher) state goes to a lower state by emitting a gamma photon. A nucleus may be left in an excited state after alpha or beta emission. Successive emission of gamma rays from the same nucleus is a clear proof that nuclei also have discrete energy levels as do the atoms.
- Hydrogen bomb is based on fusion.

Illustrations

Illustration 3.

The mass defect in a nuclear fusion reaction is 0.05%. What amount of energy will be liberated in one kg fusion reaction ?

Solution.

Mass defect = Δm = 0.05% of 1 kg = $\dfrac{0.05}{100}$ kg = 5×10^{-4} kg

Energy liberated = $(\Delta m)c^2 = (5 \times 10^{-4})(9 \times 10^{16})\text{J} = 45 \times 10^{12}$ J

Illustration 4.

What is energy released by fission of 1 gm U^{235} ?

Solution.

Number of atom in 1 gm of $U^{235} = \dfrac{N_A}{235}$

$$\text{Energy released} = \frac{N_A}{235} \times 200 \text{ MeV} = \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV} = 5 \times 10^{23} \text{ MeV}$$

$$= (5 \times 10^{23})(1.6 \times 10^{-13} \text{ J}) = 8 \times 10^{10} \text{ J}$$

$$= \frac{8 \times 10^{10}}{3.6 \times 10^6} \text{ kWH} = 2.22 \times 10^4 \text{ kWH}$$

Illustration 5.

What is the power output of $^{235}_{92}U$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy ?

Solution.

Number of atoms in 2 kg of $^{235}_{92}U$ = $\dfrac{6.02\times10^{23}\times2\times10^{3}}{235}$ = 5.12×10^{24}

Therefore, energy released in 30 day = $5.12\times10^{24}\times185$ MeV = 1.51×10^{14} J

$\therefore$ Energy released per second = $\dfrac{1\cdot51\times10^{14}}{30\times24\times60\times60}$ = 58.4 MW

Illustration 6.

Obtain the binding energy of a nitrogen nucleus $\left(^{14}_{7}N\right)$ in MeV from the following data.

$m_H = 1\cdot00783u$, $m_n = 1\cdot00867$ u, $m_N = 14\cdot00307$ u

Solution.

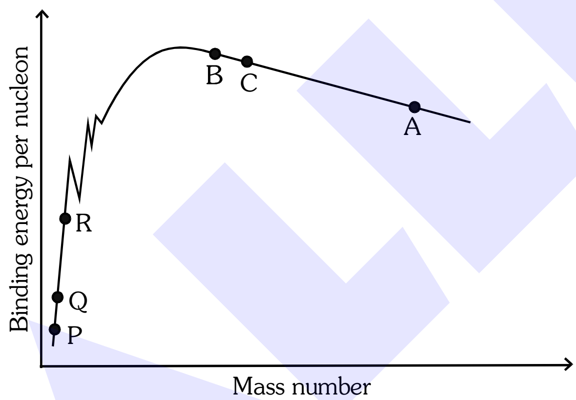
Mass defect $\Delta m = 7m_p + 7m_n - m_N = 7\times1\cdot00783 + 7\times1\cdot00867 - 14\cdot00307 = 0\cdot11243$ amu

Binding energy = $\Delta m\times931$ MeV = $0\cdot11243\times931$ MeV = $104\cdot67$ MeV

Illustration 7.

Explain nuclear fission & fusion on the basis of binding energy of nucleus. **[AIPMT 2004]**

Solution.



In fission : nucleus A breaks into B & C
In fussion : P & Q fuse to result in nucleus R

In both cases the net B.E. increases resulting in energy release.

Illustration 8.

Show that the nucleus $^{238}_{92}U$ emitting a proton through the decay process $^{238}_{92}U \rightarrow {}^{237}_{91}Pa + {}^{1}_{1}H$ can not proceed spontaneously . **[AIPMT 2006]**

Mass of uranium = 238.05079 a.m.u.

Mass of paladium = 237.05121 a.m.u.

Mass of proton = 1.00783 a.m.u.

Solution.

Here Q = $(238.05079 - 237.05121 - 1.00783)c^2 = (-0.00825u)c^2$

As the Q for this process is negative, the decay can not proceed spontaneously

Illustration 9.

Write three characteristic features which distinguish nuclear force from coulomb force. **[AIPMT 2007]**

Solution.

	Nuclear force	**Coulomb force** (Write any three)
(i)	Short range	Long range
(ii)	Not a central force	Central force
(iii)	Spin dependent	Spin independent
(iv)	Charge independent	Charge dependent
(v)	Strong force	Comparitively weak force

Illustration 10.

The radionuclide ${}_{6}^{11}C$ decays by β^{+} emission. Write symbolically this decay process.

Given that $m({}_{6}^{11}C) = 11.011434$ u **[AIPMT 2008]**

$m({}_{5}^{11}B) = 11.009305$ u

$m_e = 0.000548$ u, $1u = 931.5$ MeV/c^2

Calculate the Q-value.

Solution.

Equation of β^{+}-decay of ${}_{6}C^{11}$; ${}_{6}C^{11} \longrightarrow {}_{5}B^{11} + {}_{+1}\beta^{0} + \nu + Q$

Q-value of reaction = Δmc^2

$$= \left[m\left({}_{6}C^{11}\right) - 6m_e - m\left({}_{5}B^{11}\right) + 5m_e - m_e\right]c^2 = \left[m\left({}_{6}C^{11}\right) - m\left({}_{5}B^{11}\right) - 2m_e\right]c^2$$

$$= [11.011434 - 11.009305 - 2 \times 0.000548]uc^2$$

$$= [0.001033]\ uc^2 = 0.001033 \times 931.5 \text{ MeV} = 0.962 \text{ MeV}$$

Illustration 11.

Calculate the percent increase in mass of an electron accelerated by a potential difference of 500 kV.

[AIPMT 2004]

Solution.

Kinetic energy of electron = 500 keV & Rest mass energy of electron = 511 keV

Total energy = $mc^2 = m_0c^2 + KE = (511 + 500)$ keV

$$\text{Percent increase in mass} = \frac{m - m_0}{m_0} \times 100 = \frac{500}{511} \times 100 = 97.8\ \%$$

BEGINNER'S BOX-1

1. The Q value of a nuclear reaction A + b → C + d is defined by

$Q = [m_A + m_b - m_C - m_d]c^2$

where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}_1^1H + {}_1^3H \rightarrow {}_1^2H + {}_1^2H$

(ii) ${}_6^{12}C + {}_6^{12}C \rightarrow {}_{10}^{20}Ne + {}_2^4He$

Atomic masses are given to be

$m\left({}_1^1H\right) = 1.007825$ u

$m\left({}_1^2H\right) = 2.014102$ u

$m\left({}_1^3H\right) = 3.016049$ u

$m\left({}_6^{12}C\right) = 12.000000$ u

$m\left({}_{10}^{20}Ne\right) = 19.992439$ u

mass of He atom is 4.0015 amu

2. Calculate the binding energy of ${}_{17}Cl^{35}$ if mass of ${}_{17}Cl^{35}$ nucleus is 34.98 amu, mass of neutron is 1.008665 amu and mass of proton is 1.007277 amu.

3. Calculate the energy released by the fission of 2 g of ${}_{92}U^{235}$ in kWh. Given that the energy released per fission is 200 MeV.

4. If the energy released in the fission of one nucleus is $3 \cdot 2 \times 10^{-11}$J, then find number of nuclei required per second in a power plant of 16 kW.

5. Find out the mass of Uranium required per day to generate 10 MW power from the fission of ${}_{92}U^{235}$.

6. The mass defect in a nuclear fusion reaction is 0.3 percent. What amount of energy will be liberated in one kg fusion reaction ?

7. Two nuclei have their mass numbers in ratio 1 : 3. What is the ratio of nuclear densities ?**[AIPMT 2006]**

RADIOACTIVITY

1. RADIOACTIVITY :

(i) Spontaneous emission of radiations from the nucleus is known as **radioactivity** and substances showing this property are called **radioactive substances**.

(ii) Only unstable nuclei exhibit this property.

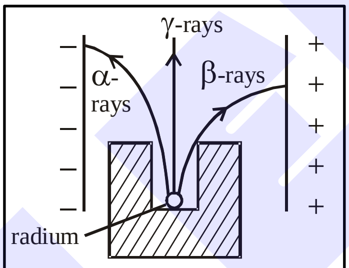
(iii) A particular nuclide (element) can radiate only a particular type of radiations at a time, according to its requirement of stability.

(iv) This phenomenon (Radioactivity) was discovered by ***Becquerel*** therefore the radioactive radiations are also called Becquerel radiations.

(v) Later on Curie couple (Merie Curie and Pierre Curie) discovered many other radioactive substances.

2. NATURE OF RADIOACTIVE RADIATIONS :

2.1 Rutherford's Experiment :–



Pb box (best absorber of nuclear radiations)

He put a sample of radioactive substance in a lead box and allow the emission of radiations through a small hole only. When the radiation enter into the external electric field, they split into three parts.

Radiations which deflect towards negative plate are called α–rays.

Radiations which deflect towards positive plate are called β–rays.

Radiations which are undeflected, called γ–rays.

(i) **Alpha rays :–** These are stream of positive charged particles i.e. particle nature.

(ii) **Beta rays :–** These are stream of negative charged particles i.e. particle nature.

(iii) **Gamma rays :–** These are electromagnetic waves.

2.2 Properties of α, β and γ rays :–

	Features	**α–particles**	**β–particles**	**γ–rays**
1.	Identity	Helium nucleus or doubly ionised helium ion ($_2He^4$)	Fast moving electrons ($_{-}\beta^{0}$ or β^{-})	Electromagnetic wave (photons)
2.	Charge	Twice of proton (+ 2e)	Electronic (– e)	Neutral

3.	Mass	$\approx 4m_p$, m_p–mass of proton	(rest mass of β) = (rest mass of ele.)	rest mass = 0
4.	Speed	$\approx 10^7$ m/s Their speed depends on nature of the nucleus. So, it is a characteristic speed.	$\approx 10^7$ m/s β-particles come out with different speeds from the same type of nucleus. Therefore can not be a characteristic speed.	Only $c = 3 \times 10^8$ m/s γ-photons come out with same speed from all types of nucleus. So, can not be a characteristic speed.
5.	K.E.	≈ MeV	≈ MeV	≈ MeV
6.	Energy spectrum	Line and discrete	Continuous	Line and discrete
7.	Ionization power (α>β>γ)	10,000 times of γ-rays	100 times of γ-rays (or $\frac{1}{100}$ times of α)	1 (or $\frac{1}{100}$ times of β)
8.	Penetration power (γ>β>α)	$\frac{1}{10000}$ times of γ-rays	$\frac{1}{100}$ times of γ-rays (100 times of α)	1 (100 times of β)
9.	Effect of electric or magnetic field	Deflection	Deflection (More than α)	No deflection

3. TYPE OF RADIOACTIVE DECAY

3.1 α–decay

In this decay, mass number decreases by 4 and atomic number decreases by 2. Its decay equation is

$$\underset{\text{(Parent)}}{{}^{A}_{Z}X} \longrightarrow \underset{\text{(Daughter)}}{{}^{A-4}_{Z-2}Y} + \underset{\text{Alpha}}{{}^{4}_{2}He}$$

In this decay total mass of product is less than the mass of parent. This difference in mass appears as kinetic energy of the products.

The disintegration energy or Q value for α-decay.

$$Q = (m_X - m_Y - m_{He})c^2$$

3.2 β–decay

(A) The basic nuclear process underlying β^--decay is the conversion of neutron to proton.

$$n \longrightarrow p + e^- + \bar{\nu}$$

(i) Its decay equation

$$^{A}_{Z}X \longrightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta + \bar{\nu}$$

(ii) After $_{-}\beta^{\circ}$ decay, n/p ratio decreases.

(iii) $_{-}\beta^{\circ}$ always comes out from the nucleus along with antineutrino.

(B) The basic nuclear process underlying β^+-decay is the conversion of proton into neutron.

$$p \longrightarrow n + e^+ + \nu$$

(i) Its decay equation

$${}^{A}_{Z}X \longrightarrow {}^{A}_{Z-1}Y + {}^{o}_{+1}\beta + \nu$$

(ii) After $_{+}\beta^{\circ}$ decay, n/p ratio increases.

(iii) $_{+}\beta^{\circ}$ comes out from the nucleus along with neutrino.

3.3 γ–decay :–

Similar to an atom, nucleus also have certain energy levels and nucleons occupy them. After α-decay (or β decay), daughter nucleus may be in excited state and return to ground state by emitting photons of high energy (MeV order) called γ - photons.

Equation of γ–decay :– $({}_{Z}X^{A})^{*} \rightarrow {}_{Z}X^{A} + \gamma$ – photons (or hν)

∗Shows excited nucleus

(i) γ emission don't change the structure of nucleus

(ii) No change in Z and A

GOLDEN KEY POINTS

- In β-decay either an electron or a positron is emitted by a nucleus, along with an antineutrino or a neutrino. The emitted particles share the available disintegration energy. The electrons and positrons emitted in β-decay have a continuous spectrum of energies from zero to a limit [$Q = (\Delta m)\, c^2$]

- Properties of neutrino & antineutrino
  - (i) Both are chargeless
  - (ii) Have almost zero rest mass (very light particles)
  - (iii) Have spin quantum number $\pm\ 1/2$ and spin angular momentum $\pm\ h/2\pi$ similar to electron.
  - (iv) These are suggested by Pauli to explain the problems of energy conservation, linear momentum conservation, spin conservation and spin angular momentum conservation in β–decay.

- In β-decay parent & daughter are isobar.

- **The K-electron capture :** In K-capture a nucleus captures one of the inner orbital electrons and a proton transforms into a neutron. Hence K capture is like positron decay, in both n/p ratio increases. In this event a vacancy is created in K-shell to fill up the vacancy, electron transition takes place and X-rays are emitted.

$${}_{Z}X^{A} + {}_{-1}e^{0}\,(\text{K-capture}) \longrightarrow {}_{Z-1}Y^{A} + \text{X-rays} + \upsilon$$

Illustrations

Illustration 1.

$${}_{90}Th^{234} \xrightarrow{\alpha} {}_{a}X^{b} \xrightarrow{{}_{-}\beta^{0}} {}_{c}Y^{d} \xrightarrow{(p)} {}_{90}Th^{230}$$

Find a, b, c and d and identify particle p

Solution.

$${}_{90}Th^{234} \xrightarrow{\alpha} {}_{88}X^{230} \xrightarrow{{}_{-}\beta^{0}} {}_{89}Y^{230} \xrightarrow{P=({}_{-}\beta^{0})} {}_{90}Th^{230}$$

Illustration 2.

$_{90}Th^{234} \xrightarrow{n\alpha,\, n'_{-}\beta^0} {}_{83}Bi^{214}$, Find n and n'

Solution.

Number of α particle, $n = N_\alpha = \dfrac{234-214}{4} = \dfrac{20}{4} = 5$

Number of $_{-}\beta$ particle, $n' = N_{-\beta} = Z_f - [Z_i - 2 \times \text{No. of } \alpha] = 83 - (90 - 2 \times 5) = 83 - 80 = 3$

BEGINNER'S BOX-2

1. $_{92}U^{238} \xrightarrow{\alpha} \xrightarrow{_{+}\beta^0} {}_aX^b$, find a & b.

2. $_aX^b \xrightarrow{_{-}\beta^0} \xrightarrow{\alpha} {}_CY^{215} \xrightarrow{_{-}\beta^0} {}_{110}Y^d$ Find a, b, c and d.

3. $_{92}U^{238} \xrightarrow{n\alpha,n'_{-}\beta^0} {}_{82}Pb^{206}$. Find n & n'

4. Thorium isotope $_{90}Th^{232}$ emits some α–particles and some β–particles and gets transformed into lead isotope $_{82}Pb^{200}$. Find the number of α and β particles emitted.

5. A radioactive nucleus undergoes a series of decays according to the following scheme :

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72 respectively, what are these numbers for A_4?

6. Write nuclear reaction equations for

(i) α-decay of $^{226}_{88}Ra$

(ii) α-decay of $^{242}_{94}Pu$

(iii) β^--decay of $^{32}_{15}P$

(iv) β^--decay of $^{210}_{83}Bi$

(v) β^+-decay of $^{11}_{6}C$

(vi) β^+-decay of $^{97}_{43}Tc$

(vii) Electron capture of $^{120}_{54}Xe$

4. NUCLEAR REACTIONS

- It can be written as X (a, b) Y i. e. $X + a \rightarrow Y + b$
- All nuclear reactions follow conservation of number of nucleons and charge (i.e.Z) conservation as well as energy + mass conservations, linear and angular momentum.

5. MATHEMATICAL DERIVATION OF EXPONENTIAL DECAY

Rutherford and Soddy's law

At an instant rate of decay of active nuclei is directly proportional to the number of active nuclei at that instant

$-\dfrac{dN}{dt}$ = rate of decay of nuclei at time t

$$N = \text{active nuclei at time t} \quad -\frac{dN}{dt} \propto N \quad \text{or} \quad -\frac{dN}{dt} = \lambda N \qquad \ldots\text{(i)}$$

Here λ is the decay constant which depends only on the nature of substance.

equation (i) can be written as $\frac{dN}{N} = -\lambda\, dt$

Integrate it $\int \frac{dN}{N} = -\lambda \int dt$

$\log_e N = -\lambda t + C$ ...(ii)

Let at t = 0 number of active nuclei were N_0

(by putting t = 0 and $N = N_0$ in equation (ii))

$\log_e N_0 = C$

Now equation (ii) is $\log_e N = -\lambda t + \log_e N_0$

$\log_e N - \log_e N_0 = -\lambda t \Rightarrow \log_e = \frac{N}{N_0} - \lambda t$ i.e. $\frac{N}{N_0} = e^{-\lambda t}$

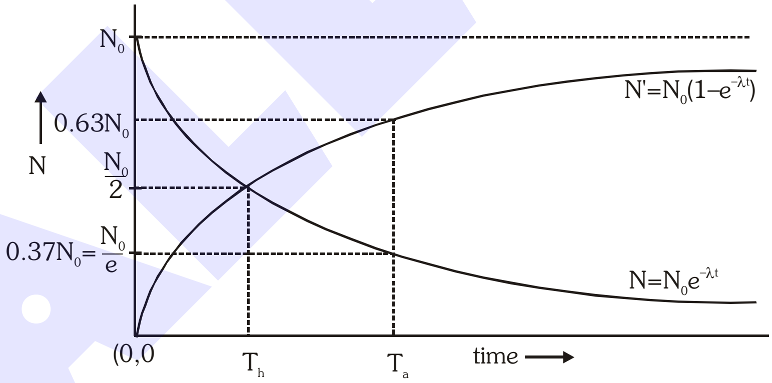
$N = N_0 e^{-\lambda t}$ ...(iii)

equation (iii) gives number of active nuclei in a sample at desire instant t.

GOLDEN KEY POINTS

- Number of nuclei, which has been decayed in duration t ⇒ $N' = N_0 - N = N_0(1 - e^{-\lambda t})$
- λ is independent of amount of active substance (N or m) and time and any physical or chemical changes.
- λ is called decay constant or disintegration constant or radioactivity constant or Rutherford Soddy's constant or the probability of decay per unit time of a nucleus.

Graph : Time versus N (or N')



5.1 Half life (T_h)

It is the time during which number of active nuclei reduce to half of initial value.

If at t = 0 no. of active nuclei N_0 then at $t = T_h$ number of active nuclei will be $\frac{N_0}{2}$

from decay equation $N = N_0 e^{-\lambda t}$

$$\frac{N_0}{2} = N_0 e^{-\lambda T_h} \Rightarrow T_h = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} \approx \frac{0.7}{\lambda}$$

5.2 Mean or Average Life (T_a)

It is the average of age of all active nuclei i.e.

$$T_a = \frac{\text{sum of times of existance of all nuclei in a sample}}{\text{initial number of active nuclei in that sample}} = \frac{1}{\lambda}$$

(i) At t = 0, number of active nuclei = N_0 then

number of active nuclei at $t = T_a$ is $N = N_0 e^{-\lambda T_a} = N_0 e^{-1} = \frac{N_0}{e} = 0.37N_0 = 37\%$ of N_0

(ii) Number of nuclei which have been disintegrated within duration T_a is

$N' = N_0 - N = N_0 - 0.37\ N_0 = 0.63\ N_0 = 63\%$ of N_0

(iii) $T_a = \frac{1}{\lambda} = \frac{T_h}{\ln 2} = \frac{T_h}{0.693} = 1.44\ T_h$

(iv) Within duration $T_h \Rightarrow 50\%$ of N_0 decayed and 50% of N_0 remains active

(v) Within duration $T_a \Rightarrow 63\%$ of N_0 decayed and 37% of N_0 remains active

5.3 Activity of a sample (A or R) (or decay rate)

(i) It is the rate of decay of a radioactive sample $R = -\frac{dN}{dt} = N\lambda$ or $R = R_0 e^{-\lambda t}$

(ii) Activity of a sample at any instant depends upon number of active nuclei at that instant.

$R \propto N$ (or active mass) , $R \propto m$

(iii) R also decreases exponentially w.r.t. time same as the number of active nuclei decreases.

(iv) R is not a constant with N, m and time while λ, T_h and T_a are constant

(v) At t = 0, $R = R_0$ then at $t = T_h \Rightarrow R = \frac{R_0}{2}$ and at $t = T_a \Rightarrow R = \frac{R_0}{e}$ or $0.37\ R_0$

(vi) Similarly active mass of radioactive sample decreases exponentially. $m = m_0 e^{-\lambda t}$

(vii) Activity of m gm active sample (molecular weight M_w) is $R = \lambda N \frac{0.693}{T_h} m \frac{N_{AV}}{M_W}$

here N_{AV} = Avogadro number = 6.023×10^{23}

SI UNIT of R : 1 becquerel (1 Bq)= 1 decay/sec

Other Unit is curie : 1 Ci = 3.70×10^{10} decays/sec

GOLDEN KEY POINTS

- In radioactive sample a radioactive substance (A) converts into rather stable substance (B), B converts into more stable substance (C). This process ends into stable element. $A \to B \to C \to D \to ....$
  - Stability order $D > C > B > A$
  - Radioactivity order $A > B > C > D$
  - For stable element $\lambda = 0$
  - For stable element $T_h = \infty$
- At equilibrium, rate of production of B = rate of decay of B (and also for C,D, etc.)

  Therefore number of active nuclei of B becomes constant (or activity $N\lambda$ also becomes constant)

  At equilibrium radio activities of all daughter nuclides are equal i.e.

  at radioactive equilibrium $\to \begin{cases} R_1 = R_2 = R_3 = R_4 ... \\ N_1\lambda_1 = N_2\lambda_2 = N_3\lambda_3 \end{cases}$

  At equilibrium $N_2 = N_1 \dfrac{\lambda_1}{\lambda_2} = N_1 \dfrac{T_{h_2}}{T_{h_1}} = \dfrac{m_2}{M_{W_2}} N_A = \dfrac{m_1}{M_{W_1}} N_A \dfrac{T_{h_2}}{T_{h_1}} \Rightarrow m_2 = m_1 \left[\dfrac{M_{W_2}}{M_{W_1}}\right]\left[\dfrac{T_{h_2}}{T_{h_1}}\right]$
- Radiation dozes is measured in **sieverts** (Sv) or **Rontgen**.
- **Uses of radioactive isotopes in human life**

  (a) In medicine

  (i) Testing of blood circulation – Cr^{57}

  (ii) Brain tumer detecting – Hg^{203}

  (iii) Thyroid testing (cancer) – I^{131}

  (iv) Cancer cure – Co^{60}

  (v) Blood cancer cure – Au^{189}/Na^{24}

  (b) In Archaeology

  (i) For determining age of archaeological sample ($\approx 30,000$ yr old) – C^{14} (carbon dating)

  (ii) For determining age of earth or meteorites (very old) – K^{40} and Uranium

  (c) In Agriculture

  (i) For protacting potato from earthworm – Co^{60}

  (ii) Artificial rains by – AgI

  (iii) As fertilizers – P^{32}
- **Geiger – Muller counter** is used for detecting (or counting) the α particles and β–particles.

Illustrations

Illustration 3.

In a old rock, ratio of nuclei of uranium and lead is 1 : 1. Half life of uranium is 4.5×10^9 yrs. Let initially it contains only uranium nuclei. How old is the rock ?

Solution.

Let present active nuclei of uranium is N then intial active nuclei is 2N.

Present active fraction of uranium = $\dfrac{1}{2} \Rightarrow \dfrac{1}{2} = \dfrac{1}{2^{t/T_{1/2}}}$

or $\dfrac{t}{T_{1/2}} = 1$ or $t = T_{1/2} = 4.5 \times 10^9$ yr

Illustration 4.

The mean lives of a radioactive substance are T_1 and T_2 for α–emission and β–emission respectively. If it is decaying by both α–emission and β–emission simultaneously then find its mean life and decay constant ?

Solution.

$$\because \quad \lambda = \lambda_1 + \lambda_2 \quad \Rightarrow \quad \lambda = \frac{1}{T_1} + \frac{1}{T_2} \Rightarrow \quad \tau = \frac{T_1 T_2}{T_1 + T_2} \quad [\because \tau = \frac{1}{\lambda}]$$

Illustration 5.

The half lives of X and Y are 3 minutes and 27 minutes respectively. At some instant activity of both are same, then the ratio of active nuclei of X and Y at that instant is ?

Solution.

$$A_1 = \lambda_1 N_1 \quad \text{and} \quad A_2 = \lambda_2 N_2$$

$$A_1 = A_2 \quad \Rightarrow \quad \frac{0.693}{T_1} N_1 = \frac{0.693}{T_2} N_2$$

$$\Rightarrow \quad \frac{N_1}{T_1} = \frac{N_2}{T_2} \quad \Rightarrow \quad \frac{N_1}{N_2} = \frac{3}{27} = \frac{1}{9} \quad \Rightarrow \quad N_1 : N_2 = 1 : 9$$

Illustration 6.

Decay constant of two radioactive samples is λ and 3λ respectively. At $t = 0$, they have equal number of active nuclei. Calculate when will be the ratio of active nuclei becomes $e : 1$.

Solution.

Number of active nuclei of two radioactive sample is

$$N_1 = N_{01}e^{-\lambda t} \text{ and } N_2 = N_{02}e^{-3\lambda t} \quad \therefore \frac{N_1}{N_2} = \frac{e}{1} = \frac{N_{01}e^{-\lambda t}}{N_{02}e^{-3\lambda t}} = e^{2\lambda t} \quad [\because N_{01} = N_{02}]$$

$$\therefore \ 1 = 2\lambda t \quad \Rightarrow \quad t = \frac{1}{2\lambda}$$

Illustration 7.

The fraction of a radioactive sample which remains active after time t is . What fraction remains active after $\frac{t}{2}$ time.

Solution.

$$\text{Active fraction} = \frac{N}{N_0} = e^{-\lambda t} \qquad \text{At time t, } \frac{9}{16} = e^{-\lambda t}$$

$$\text{At time t/2 active fraction} = x = e^{-\lambda t/2} = \left(e^{-\lambda t}\right)^{\frac{1}{2}} \quad \text{So } x = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$$

Illustration 8.

Calculate the radioactive disintegration constant if 3.7×10^{10} alpha particles are emitted by 1 gram of radium per second. Avogadro's number is 6.03×10^{23} and the mass number of radium is 226.

Solution.

$$\text{Activity} = N\lambda = \left(\frac{N_A}{M_w} \times m\right) \lambda \Rightarrow 3.7 \times 10^{10} = \left(\frac{6.03 \times 10^{23}}{226} \times 1\right)\lambda$$

$$\lambda = \frac{3.7 \times 10^{10} \times 226}{6.03 \times 10^{23}} = 1.38 \times 10^{-11} \text{ per second}$$

Illustration 9.

A free neutron is unstable against β–decay with a half life of about 600 seconds.–

(i) Write the expression of this decay process.

(ii) If there are 600 free neutrons initially, calculate the time by which 450 of them have decayed. Also determine the initial decay rate of the sample. **[AIPMT 2005]**

Solution.

(i) $n \rightarrow p + e^{-} + \bar{\nu}$

(ii) The number of undecayed neutron would be 150 by using $N = N_0 e^{-\lambda t}$

$150 = 600e^{-\lambda t} \Rightarrow t = 2T_{1/2} = 1200$ sec

Decay rate (initial) $R = \lambda N_0 = 0.693$ Bq

Illustration 10.

Obtain the amount of polonium ($_{84}Po^{210}$) necessary to provide a radioactivity source of 5.0 mili curie strength. The half life of polonium is 138 days. (given : 1 curie = 3.7×10^{10} disintregration/sec., Avogadro number = 6.02×10^{26} per k-mole). **[AIPMT 2006]**

Solution.

$$\frac{dN}{dt} = -\lambda N \quad \Rightarrow \quad N = -\frac{1}{\lambda}\frac{dN}{dt}$$

Given : $\frac{dN}{dt} = 5 \times 10^{-3} \times 3.7 \times 10^{10}$ disint./sec. & $T_{1/2} = 138 \times 24 \times 3600$ sec.

$$\Rightarrow N = \frac{138 \times 24 \times 3600 \times 5 \times 3.7 \times 10^{7}}{0.693} = 3.18 \times 10^{15} \text{ atoms}$$

But mass of one $_{84}Po^{210}$ atoms $= \frac{210}{6.02 \times 10^{23}}$

Amount of $_{84}Po^{210}$ in grams required $= \frac{210 \times 3.18 \times 10^{15}}{6.02 \times 10^{23}} = 1.11 \times 10^{-6}$

BEGINNER'S BOX-3

1. The half lives of radioactive elements x and y are 3 minute and 27 minute respectively. If the activities of both are same, then calculate the ratio of number of atoms of x and y.

2. Carbon has two stable isotopes. Natural carbon has 98.9% carbon–12 and 1.1% carbon–13, calculate the average atomic weight of carbon.

3. A radioactive isotope has a half life of T. After how much time is its activity reduced to 6.25% of its original activity ?

4. $\frac{2}{3}$ fraction of a sample disintegrates in 7 days. How much fraction of it will decay in 21 days ?

5. The half life of radium is 1600 years. After how many years 25% of radium block remains undecayed ?

6. A radioactive isotope has a half-life of T years. How long will it take the activity to reduce to a) 3.125%, b) 1% of its original value?

7. Obtain the amount of $^{60}_{27}Co$ necessary to provide a radioactive source of 8.0 mCi strength. The half-life of $^{60}_{27}Co$ is 5.3 years.

ANSWERS KEY

BEGINNER BOX-1

1. (i) –4.031 MeV, endothermic

(ii) 5.64 MeV, exothermic

2. 278.92 MeV

3. 4.55×10^4 kWh.

4. 5×10^{14}

5. 10.5 g

6. 2.7×10^{14} J

7. 1 : 1

BEGINNER BOX-2

1. a = 89, b = 234

2. a = 110, b = 219, c = 109, d = 215

3. n = 8, n' = 6

4. $N_\alpha = 8$, $N_\beta = 8$

5. Mass number = 172 and Atomic number = 69

6. (i) ${}^{226}_{88}Ra \xrightarrow{-\alpha} {}^{222}_{86}Rn + {}^{4}_{2}He$

(ii) ${}^{242}_{94}Pu \xrightarrow{-\alpha} {}^{238}_{92}U + {}^{4}_{2}He$

(iii) ${}^{32}_{15}P \xrightarrow{-\beta^-} {}^{32}_{16}S + {}^{0}_{-1}\beta + \overline{\upsilon}$

(iv) ${}^{210}_{83}Bi \xrightarrow{-\beta^-} {}^{210}_{84}Po + {}^{0}_{-1}\beta + \overline{\upsilon}$

(v) ${}^{11}_{6}C \xrightarrow{-\beta^+} {}^{11}_{5}B + {}^{0}_{+1}\beta + \upsilon$

(vi) ${}^{97}_{43}Tc \xrightarrow{-\beta^+} {}^{97}_{42}Mo + {}^{0}_{+1}\beta + \upsilon$

(vii) ${}^{120}_{54}Xe + {}_{-1}e^0 \xrightarrow{\text{electron capture}} {}^{120}_{53}I + \text{X-Ray} + \upsilon$

BEGINNER BOX-3

1. $\frac{1}{9}$

2. 12.011 amu

3. 4T

4. $\frac{26}{27}$

5. 3200 years

6. (a) 5T, (b) 6.64T

7. 7 μg

APPENDIX

Elementary particles

- Bosons (s = 0, ±1, ±2....)
  - Photon
  - Higgs boson (s = 0)
  - Gluons
  - W boson & Z boson
  - Graviton (Hypothetical)
  - π-mesons
- Fermions (s = ±1/2, ±3/2....)
  - Baryons → n, $\overline{n}$, p, $\overline{p}$ etc.
  - Leptons → e, e^+, ν, $\overline{\nu}$ etc.
  - Quarks → up and down etc.

Note : Fermions obey pauli's exclusion principle but bosons do not obey it.