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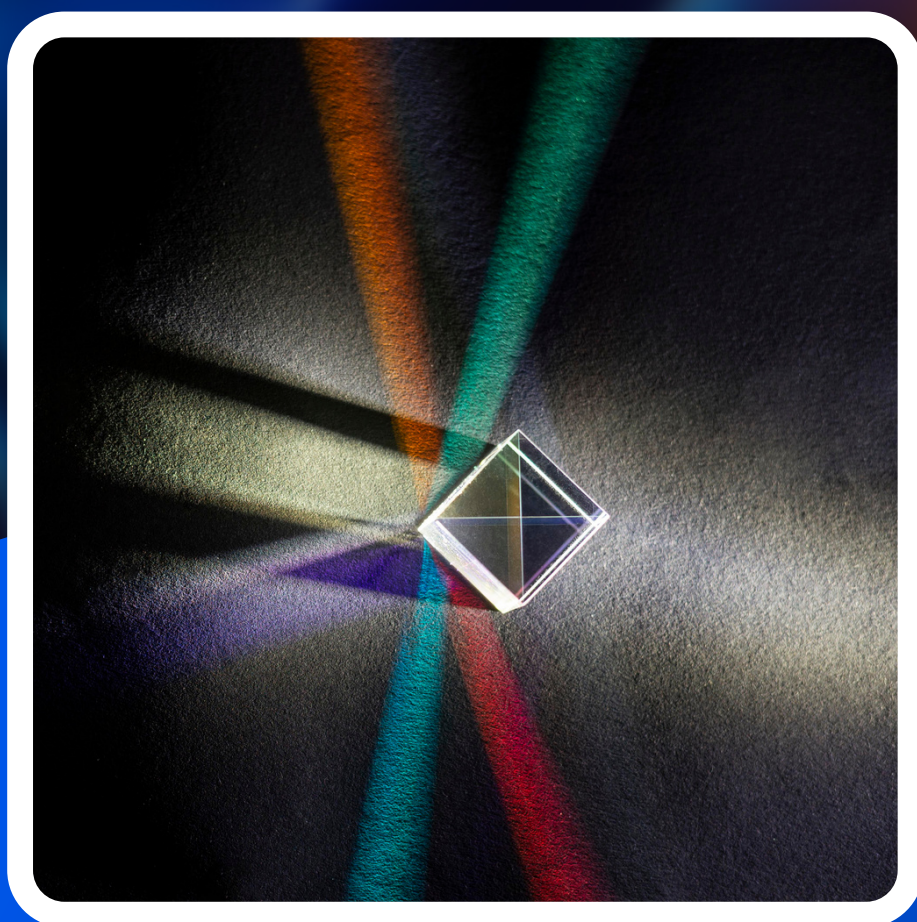
STUDY MATERIAL

Wave Optics

ENGLISH MEDIUM

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CHRISTIAAN HUYGENS (1629 – 1695) DUTCH

Physicist, astronomer, mathematician and the founder of the wave theory of light. His book, Treatise on light, makes fascinating reading even today. He brilliantly explained the double refraction shown by the mineral calcite in this work in addition to reflection and refraction. He was the first to analyse circular and simple harmonic motion and designed and built improved clocks and telescopes. He discovered the true geometry of Saturn's rings.

THOMAS YOUNG (1773 – 1829) ENGLISH

Physicist, physician and Egyptologist. Young worked on a wide variety of scientific problems, ranging from the structure of the eye and the mechanism of vision to the decipherment of the Rosetta stone. He revived the wave theory of light and recognised that interference phenomena provide proof of the wave properties of light.

WAVE OPTICS
INTERFERENCE OF LIGHT

1. **NATURE OF LIGHT**

   Newton's corpuscular theory of light

   This theory was given by Newton.

● **Characteristics of the theory**

   (i) Extremely minute, very light and elastic particles are being constantly emitted by all luminous bodies (light sources) in all directions which are known as corpuscles.

   (ii) These corpuscles travel with the speed of light..

   (iii) When these corpuscles strike the retina of our eye then they produce the sensation of vision.

   (iv) The velocity of these corpuscles in vacuum is 3×10^8 m/s.

   (v) The different colours of light are due to different size of these corpuscles.

   (vi) The rest mass of these corpuscles is zero.

   (vii) The velocity of these corpuscles in an isotropic medium is same in all directions but it changes with the change of medium.

   (viii) These corpuscles travel in straight lines.

   (ix) These corpuscles are invisible.

● **The phenomena explained by this theory**

   (i) Reflection and refraction of light.

   (ii) Rectilinear propagation of light.

   (iii) Existence of energy in light.

● **The phenomena not explained by this theory**

   (i) Interference, diffraction, polarisation, double refraction and total internal reflection.

   (ii) Velocity of light being greater in rarer medium than that in a denser medium.

   (iii) Photoelectric effect

Huygen's Wave theory of light

   This theory was enunciated by Huygen in a hypothetical medium known as luminiferrous ether.

   Ether is that imaginary medium which prevails in all space and is isotropic, perfectly elastic and massless.

   The velocity of light in a medium is constant but changes with change of medium.

   This theory is valid for all types of waves.

   (i) The locus of all ether particles vibrating in same phase is known as wavefront.

   (ii) Light travels in the medium in the form of wavefront.

   (iii) When light travels in a medium then the particles of medium start vibrating and consequently a disturbance is created in the medium.

   (iv) Every point on the wavefront becomes the source of secondary wavelets. It emits secondary wavelets in all directions which travel with the speed of light

   (v) The tangent plane to these secondary wavelets represents the new position of wave front.
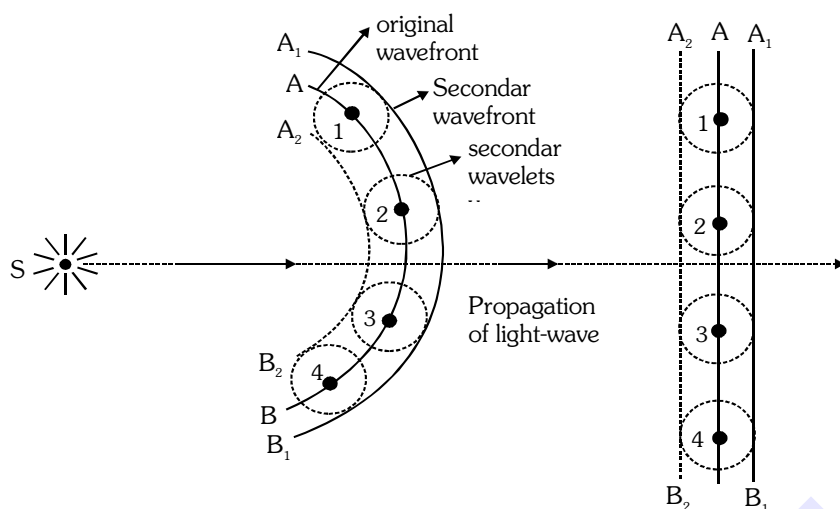
The phenomena explained by this theory

(i)   Reflection, refraction, interference, diffraction.

(ii)   Rectilinear propagation of light.

(iii) Velocity of light in rarer medium being greater than that in denser medium.

Phenomena not explained by this theory

(i)   Photoelectric effect

WAVEFRONT, VARIOUS TYPES OF WAVEFRONT
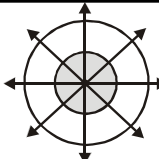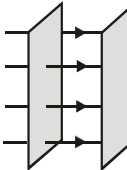
● **Wavefront**

The locus of all the particles vibrating in the same phase is known as wavefront.

● **Types of wavefront**

The shape of wavefront depends upon the shape of the light source where wavefront originates. On this basis there are three types of wavefronts

Comparative study of three types of Wavefront :-

S.No.	Wavefront	Shape of light source	Diagram of shape of wavefront	Variation of amplitude (A) with distance	Variation of Intensity (I) with distance
1.	Spherical	Point source		$A \propto \dfrac{1}{d}$ or $A \propto \dfrac{1}{r}$	$I \propto \dfrac{1}{r^2}$
2.	Cylindrical	Linear source or slit		$A \propto \dfrac{1}{\sqrt{d}}$ or $A \propto \dfrac{1}{\sqrt{r}}$	$I \propto \dfrac{1}{r}$
3.	Plane	Extended large source or point source situated at very large distance		$A =$ constant	$I =$ constant

CHARACTERISTIC OF WAVEFRONT

(a) The phase difference between various particles on the wavefront is zero.

(b) These wavefronts travel with the speed of light in all directions in an isotropic medium.

(c) A point source of light always gives rise to a spherical wavefront in an isotropic medium.

(d) In anisotropic medium it travels with different velocities in different directions.

(e) Normal to the wavefront represents a ray of light.

(f) It always travels in the forward direction in the medium.

2. INTERFERENCE OF LIGHT

When two light waves of same frequency with constant phase difference superimpose over each other, then the resultant amplitude (or intensity) in the region of superimposition is different from the amplitude (or intensity) of individual waves.

This modification in intensity in the region of superposition is called interference.

(a) Constructive interference

When resultant intensity is greater than the sum of two individual wave intensities $[I > (I_1 + I_2)]$, then the interference is said to be constructive.

(b) Destructive interference

When the resultant intensity is less than the sum of two individual wave intensities $[I < (I_1 + I_2)]$, then the interference is said to destructive.

There is no violation of the law of conservation of energy in interference. Here, the energy from the points of minimum energy is shifted to the points of maximum energy.

TYPES OF SOURCES

● **Coherent sources**

Two sources are said to be coherent if they emit light waves of the same frequency and start with same phase or have a constant phase difference. They are obtained from a single source.

Note : Laser is a source of monochromatic light waves of high degree of coherence.

● **Incoherent sources**

Two independent monochromatic sources, emit waves of same frequency.

But the waves are not in phase. So they are incoherent. This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources. By using two independent laser beams it has been possible to record the interference pattern.

Independent Source
(non-coherent source)
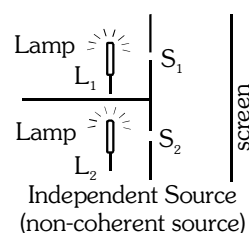
METHODS OF OBTAINING COHERENT SOURCE

● **Division of wave front**

In this method, the wavefront is divided into two or more parts by reflection or refraction using mirrors, lenses or prisms.

Illustration : Young's double slit experiment.
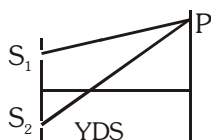
● **Division of amplitude**

The amplitude of incoming beam is divided into two or more parts by partial reflection or refraction. These divided parts travel different paths and are finally brought together to produce interference.

Illustration : The brilliant colour seen in a thin film of transparent material like soap film, oil film.

Condition for sustained interference

To obtain the stationary interference pattern, the following conditions must be fulfilled :

(a) The two sources should be coherent, i.e., they should vibrate in the same phase or there should be a constant phase difference between them.

(b) The two sources must emit continuous waves of same wavelength and frequency.

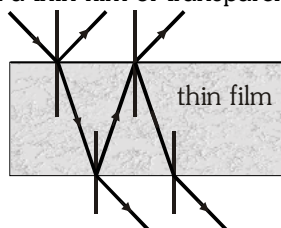(c) The separation between two coherent sources should be small.

(d) The distance of the screen from the two sources should be large.

(e) For good contrast between maxima and minima, the amplitude of two interfering waves should be as nearly equal as possible and the background should be dark.

(f) For a large number of fringes in the field of view, the sources should be narrow and monochromatic.

ANALYSIS OF INTERFERENCE OF LIGHT

When two light waves having same frequency and equal or nearly equal amplitude are moving in the same direction superimpose then different points have diffeent light intensities. At some point the intensity of light is maximum and at some point it is minimum this phenomenon is known as interference of light.

Let two waves having amplitude a_1 and a_2 and same frequency, and constant phase difference ϕ superpose. Let their displacement are :

$$y_1 = a_1\sin\omega t \text{ and } y_2 = a_2\sin(\omega t + \phi)$$

$$y = y_1 + y_2 = A\sin(\omega t + \theta)$$

Where   A = Amplitude of resultant wave

θ = New initial phase angle

Phasor Diagram

By right angle triangle :

$$A^2 = (a_1 + a_2\cos\phi)^2 + (a_2\sin\phi)^2$$

Resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

Phase angle $\theta = \tan^{-1}\left(\dfrac{a_2\sin\phi}{a_1 + a_2\cos\phi}\right)$
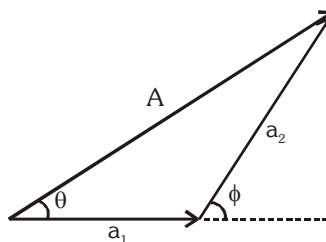
Intensity $\propto$ (Amplitude)^2 $\Rightarrow$ $I \propto A^2$ $\Rightarrow$ $I = KA^2$ so $I_1 = Ka_1^2$ & $I_2 = Ka_2^2$

$$\therefore \ I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\ \cos\phi$$

here, $2\sqrt{I_1}\sqrt{I_2}\ \cos\phi$ is known as interference factor.

● If the distance of a source from two points A and B is x_1 and x_2 then

Path difference $\delta = x_2 - x_1$

Phase difference $\quad \phi = \dfrac{2\pi}{\lambda}\,(x_2 - x_1) \Rightarrow \phi = \dfrac{2\pi}{\lambda}\,\delta$

Time difference $\quad \Delta t = \dfrac{\phi}{2\pi}T$

$$\frac{\text{Phase difference}}{2\pi} = \frac{\text{Path difference}}{\lambda} = \frac{\text{Time difference}}{T} \Rightarrow \frac{\phi}{2\pi} = \frac{\delta}{\lambda} = \frac{\Delta t}{T}$$

S ———————————— A ———— B

x_1

x_2

TYPES OF INTERFERENCE

Constructive Interference

When both waves are in same phase then phase difference is an even multiple of $\pi \Rightarrow \phi = 2n\pi$; n = 0,1,2..

- Path difference is an even multiple of $\dfrac{\lambda}{2}$

  $\therefore \ \dfrac{\phi}{2\pi} = \dfrac{\delta}{\lambda} \Rightarrow \dfrac{2n\pi}{2\pi} = \dfrac{\delta}{\lambda} \Rightarrow \delta = 2n\left(\dfrac{\lambda}{2}\right) \Rightarrow \delta = n\lambda$ (where n = 0,1,2...)

- When time difference is an even multiple of $\dfrac{T}{2}$ $\quad \therefore \ \Delta t = 2n\left(\dfrac{T}{2}\right)$

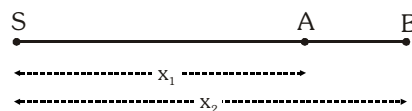- In this condition the resultant amplitude and intensity will be maximum.

  $A_{max} = (a_1 + a_2) \Rightarrow I_{max} = I_1 + I_2 + 2\sqrt{I_1}\,\sqrt{I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

Destructive Interference

When both the waves are in opposite phase. So phase difference is an odd multiple of π .

$$\phi = (2n-1)\,\pi \ ; \ n = 1, 2 \ ...$$

- When path difference is an odd multiple of $\dfrac{\lambda}{2}$, $\delta = (2n-1)\dfrac{\lambda}{2}$, n = 1, 2 ...

- When time difference is an odd multiple of $\dfrac{T}{2}$, $\Delta t = (2n-1)\dfrac{T}{2}$, (n=1,2...)

  In this condition the resultant amplitude and intensity of wave will be minimum.

  $$A_{min} = (a_1 - a_2) \Rightarrow I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

GOLDEN KEY POINTS

- Interference follows law of conservation of energy.

- Average Intensity $I_{av} = \dfrac{I_{max} + I_{min}}{2} = I_1 + I_2 \propto a_1^2 + a_2^2$

- Intensity $\propto$ width of slit $\propto$ (amplitude)$^2 \Rightarrow I \propto w \propto a^2 \Rightarrow \dfrac{I_1}{I_2} = \dfrac{w_1}{w_2} = \dfrac{a_1^2}{a_2^2}$

- $\dfrac{I_{max}}{I_{min}} = \left[\dfrac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right]^2 = \left[\dfrac{a_1 + a_2}{a_1 - a_2}\right]^2 = \left[\dfrac{a_{max}}{a_{min}}\right]^2$

- Fringe visibility $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\%$ when $I_{min} = 0$ then fringe visibility is maximum i.e. when both slits

  are of equal width the fringe visibility is the best and equal to 100%.

Illustrations

Illustration 1

If two waves represented by $y_1 = 4 \sin \omega t$ and $y_2 = 3 \sin\left(\omega t + \dfrac{\pi}{3}\right)$ interfere at a point. Find out the amplitude of the resulting wave.

Solution

Resultant amplitude $A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi} = \sqrt{(4)^2 + (3)^2 + 2.(4)(3)\cos\dfrac{\pi}{3}} \Rightarrow A \simeq 6$

Illustration 2

Two beams of light having intensities I and 4I interferer to produce a fringe pattern on a screen. The phase difference between the beam is $\dfrac{\pi}{2}$ at point A and 2π at point B. Then find out the difference between the resultant intensities at A and B.

Solution

Resultant intensity $I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2} \cos\phi$

Resultant intensity at point A is $I_A = I + 4I + 2\sqrt{I}\sqrt{4I}\cos\dfrac{\pi}{2} = 5I$

Resultant intensity at point B, $I_B = I + 4I + 2\sqrt{I_1}\sqrt{I_2}\cos 2\pi = 9I$ $(\because \cos 2\pi = 1)$ $\therefore I_B - I_A = 9I - 5I \Rightarrow 4I$

Illustration 3

In an interference pattern, the slit widths are in the ratio 1:9. Then find out the ratio of minimum and maximum intensity.

Solution

Slit width ratio

$\dfrac{w_1}{w_2} = \dfrac{1}{9}$ $\because$ $\dfrac{I_1}{I_2} = \dfrac{w_1}{w_2} = \dfrac{a_1^2}{a_2^2} = \dfrac{1}{9} \Rightarrow \dfrac{a_1}{a_2} = \dfrac{1}{3} \Rightarrow 3a_1 = a_2$ $\therefore \dfrac{I_{min}}{I_{max}} = \dfrac{(a_1 - a_2)^2}{(a_1 + a_2)^2} = \dfrac{(a_1 - 3a_1)^2}{(a_1 + 3a_1)^2} = \dfrac{4}{16} = 1:4$

Illustration 4

The intensity variation in the interference pattern obtained with the help of two coherent sources is 5% of the average intensity. Find out the ratio of intensities of two sources.

Solution

Let $I_{avg} = 100$ units

$\dfrac{I_{max}}{I_{min}} = \dfrac{105}{95} = \dfrac{21}{19} \Rightarrow \dfrac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \dfrac{21}{19} \Rightarrow \dfrac{a_1 + a_2}{a_1 - a_2} = \sqrt{\dfrac{21}{19}} = 1.05 \Rightarrow a_1 + a_2 = 1.05\,a_1 - 1.05\,a_2$

$0.05\,a_1 = 2.05\,a_2 \Rightarrow \dfrac{a_1}{a_2} = \dfrac{2.05}{0.05} = \dfrac{41}{1}$ $\qquad \therefore \dfrac{I_1}{I_2} = \dfrac{a_1^2}{a_2^2} = \dfrac{1680}{1}$

Illustration 5

Can two different bulbs, similar in all respect act as coherent sources ? Give reasons for your answer.

Solution.

No, because the light waves emitted by two independent bulbs will not have stable constant phase difference.

Illustration 6

$y = A\sin(\omega t + \pi/3)$

Maxima $(\Delta\phi = 0)$

$y = A\sin(\omega t + \pi/4)$

What is difference between x_1 and x_2 ?

Solution.

$$\Delta\phi = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12} \Rightarrow \Delta x = \frac{\lambda}{2\pi} \times \frac{\pi}{12} = \frac{\lambda}{24}$$

BEGINNER'S BOX-1

1. Consider interference between two sources of intensities I and 4I. Obtain intensity at a point where phase difference is

(a) $\dfrac{\pi}{2}$ (b) $\dfrac{\pi}{4}$ (c) π (d) 4π

2. Two coherent sources whose intensity ratio is 81 : 1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system.

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3. **YOUNG'S DOUBLE SLIT EXPERIMENT (YDSE)**

According to Huygen, light is a wave. It is proved experimentally by YDSE.

S is a narrow slit illuminated by a monochromatic source of light sends wave fronts in all directions. Slits S_1 and S_2 become the source of secondary wavelets which are in phase and of same frequency. These waves are superimposed on each other which give rise to interference. Alternate dark and bright bands are obtained on a screen (called interference fringes) placed certain distance from the plane of slit S_1 and S_2. Central fringe is always bright (due to path length S_1O and S_2O to centre of screen are equal) and is called central maxima.
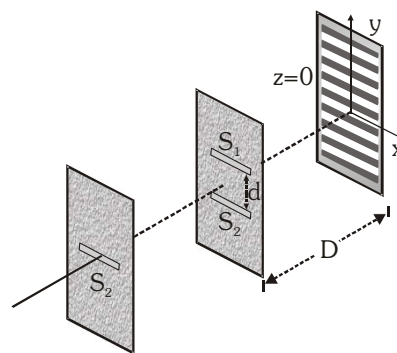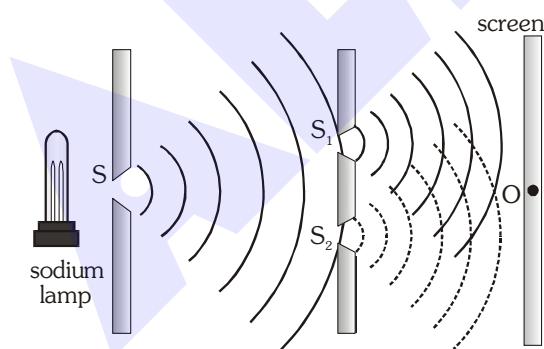
- In YDSE division of wavefront takes place.
- If one of the two slit is closed, the interference pattern disappears. It shows that two coherent sources are required to produce interference pattern.
- If white light is used as parent source, then the fringes will be coloured and of unequal width.
    (i) Central fringe will be white.
    (ii) The fringe closest on either side of the central white fringe is red and the farthest will appear blue. After a few fringes, no clear fringe pattern is seen.

CONDITION FOR BRIGHT AND DARK FRINGES

Bright Fringe

D = distance between slit and screen,   d = distance between slit S_1 and S_2
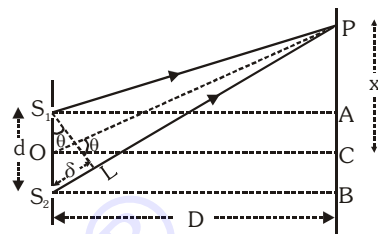
Bright fringe occurs due to constructive interference.

$\because$ For constructive interference path difference should be even multiple of $\dfrac{\lambda}{2}$

$\therefore$ Path difference $\delta = PS_2 - PS_1 = S_2L = (2n)\dfrac{\lambda}{2}$

In $\triangle PCO$ $\tan\theta = \dfrac{x_n}{D}$; In $\triangle S_1S_2L$ $\sin\theta = \dfrac{\delta}{d}$

$\delta = n\lambda$ for bright fringes

If θ is small then $\tan\theta \simeq \sin\theta \Rightarrow \dfrac{x_n}{D} = \dfrac{\delta}{d}$

The distance of nth bright fringe from the central bright fringe $x_n = n\dfrac{D\lambda}{d}$

Dark Fringe

Dark fringe occurs due to destructive interference.

$\because$ For destructive interference path difference should be odd multiple of $\dfrac{\lambda}{2}$.

$\therefore$ Path difference $\delta = (2m-1)\dfrac{\lambda}{2}$

The distance of the mth dark fringe from the central bright fringe $x_m = \dfrac{(2m-1)D\lambda}{2d}$

FRINGE WIDTH

The distance between two successive bright or dark fringe is known as fringe width.

$\beta = x_{n+1} - x_n = \dfrac{(n+1)D\lambda}{d} - \dfrac{nD\lambda}{d}$

Fringe Width $\beta = \dfrac{D\lambda}{d}$
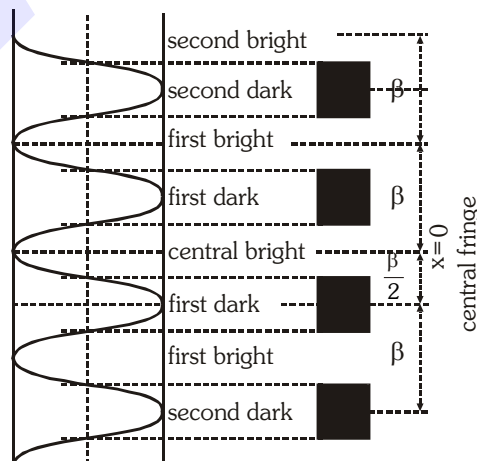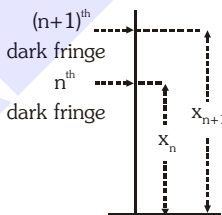
ANGULAR FRINGE WIDTH

Angular fringe width $\alpha = \dfrac{\beta}{D}$, $\alpha = \dfrac{\lambda}{d}$ $\left[\because \dfrac{\beta}{D} = \dfrac{\lambda}{d}\right]$

- The distance of nth bright fringe from the central bright fringe $x_n = \dfrac{n\lambda D}{d} = n\beta$

- The distance between n_1 and n_2 bright fringe $x_{n_2} - x_{n_1} = n_2\dfrac{\lambda D}{d} - n_1\dfrac{\lambda D}{d} = (n_2 - n_1)\beta$

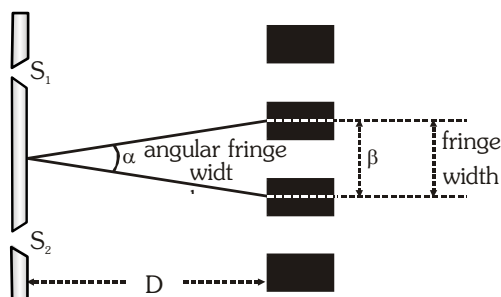- The distance of mth dark fringe from central fringe $x_m = \dfrac{(2m-1)D\lambda}{2d} = \dfrac{(2m-1)\beta}{2}$

- The distance of nth bright fringe from m^{th} dark fringe $x_n - x_m = n\dfrac{D\lambda}{d} - \dfrac{(2m-1)D\lambda}{2d} = n\beta - \dfrac{(2m-1)\beta}{2}$

$$x_n - x_m = \left[n - \dfrac{(2m-1)}{2}\right]\beta$$

GOLDEN KEY POINTS

- If the whole apparatus is immersed in a liquid of refractive index μ, then wavelength of light

  $\lambda' = \dfrac{\lambda}{\mu}$ since $\mu > 1$ so $\lambda' < \lambda \Rightarrow$ wavelength will decrease. Hence fringe width ($\beta \propto \lambda$) will decrease

  $\Rightarrow$ fringe width in liquid $\beta' = \beta/\mu$. Angular width will also decrease.

- With increase in distance between slit and screen D, angular width of maxima does not change, fringe width β increases linearly with D but the intensity of fringes decreases.

- If an additional phase difference of π is created in one of the wave then the central fringe becomes dark.

- When wavelength λ_1 is used to obtain a fringe n_1. At the same point wavelength λ_2 is required to obtain a fringe n_2 then $n_1\lambda_1 = n_2\lambda_2$

- When waves from two coherent sources S_1 and S_2 interfere in space the shape of the fringe is hyperbolic with foci at S_1 and S_2, if the distance D is very large compared to the fringe width, the fringes will be very nearly straight lines.

─────────────── **Illustrations** ───────────────

Illustration 7

Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the fringes are separated by 8.1 mm. A second laser light produces an interference pattern in which the fringes are separated by 7.2 mm. Calculate the wavelength of the second light.

Solution

Fringe separation is given by $\beta = \dfrac{\lambda D}{d}$ i.e. $\dfrac{\beta_2}{\beta_1} = \dfrac{\lambda_2}{\lambda_1} \Rightarrow \lambda_2 = \dfrac{\beta_2}{\beta_1} \times \lambda_1 = \dfrac{7.2}{8.1} \times 630 = 560$ nm

Illustration 8

A double slit is illuminated by light of wavelength 6000 Å. The slits are 0.1 cm apart and the screen is placed one metre away. Calculate :

(i) The angular position of the 10th maximum in radians and

(ii) separation between the two adjacent minima.

Solution

(i) $\lambda = 6000\ \text{Å} = 6 \times 10^{-7}$ m, d = 0.1 cm = 1×10^{-3} m, D = 1m, n = 10

Angular position $\theta_n = \dfrac{n\lambda}{d} = \dfrac{10 \times 6 \times 10^{-7}}{10^{-3}} = 6 \times 10^{-3}$ rad.

(ii)   Separation between two adjacent minima = fringe width β

$$\beta = \dfrac{\lambda D}{d} = \dfrac{6 \times 10^{-7} \times 1}{1 \times 10^{-3}} = 6 \times 10^{-4}\ \text{m} = 0.6\ \text{mm}$$

Illustration 9

In Young's double slit experiment the fringes are formed at a distance of 1m from double slits of separation 0.12 mm. Calculate

(i)   The distance of 3rd dark band from the centre of the screen.

(ii)   The distance of 3rd bright band from the centre of the screen, given $\lambda = 6000\text{Å}$

Solution

(i)   For m^{th} dark fringe $x'_m = (2m - 1)\dfrac{D\lambda}{2d}$ given, D = 1m = 100 cm, d = 0.12 mm = 0.012 cm

$$x'_m = \dfrac{(2 \times 3 - 1) \times 100 \times 6 \times 10^{-5}}{2 \times 0.012} = 1.25\ \text{cm} \qquad [\because m = 3 \text{ and } \lambda = 6 \times 10^{-5}\ \text{cm}]$$

(ii)   For n^{th} bright fringe $x_n = \dfrac{nD\lambda}{d} \Rightarrow x_n = \dfrac{3 \times 100 \times 6 \times 10^{-5}}{0.012} = 1.5\ \text{cm} \qquad [\because n = 3]$

Illustration 10

In Young's double slit experiment the two slits are illuminated by light of wavelength 5890Å and the distance between the fringes obtained on the screen is 0.2°. The whole apparatus is immersed in water, then find out angular fringe width, (refractive index of water = $\dfrac{4}{3}$).

Solution

$$\alpha_{air} = \dfrac{\lambda}{d} \Rightarrow \alpha_{air} = 0.2° \Rightarrow \alpha \propto \lambda \Rightarrow \dfrac{\alpha_w}{\alpha_{air}} = \dfrac{\lambda_w}{\lambda_{air}} \Rightarrow \lambda_w = \dfrac{\lambda_{air}}{\mu} \Rightarrow \alpha_w = \dfrac{\alpha_{air}\ \lambda}{\mu.\lambda} = \dfrac{0.2 \times 3}{4} = 0.15°$$

Illustration 11

The path difference between two interfering waves at a point on screen is 171.5 times the wavelength. If the path difference is 0.01029 cm. Find the wavelength.

Solution

Path difference = 171.5 $\lambda = \dfrac{343}{2}\lambda$ = odd multiple of half wavelength . It means dark finge is observed

According to question $0.01029 = \dfrac{343}{2}\lambda \Rightarrow \lambda = \dfrac{0.01029 \times 2}{343} = 6 \times 10^{-5}\ \text{cm} \Rightarrow \lambda = 6000\ \text{Å}$

Illustration 12

In young's double slit interference experiment, the distance between two sources is $0.1/\pi$ mm. The distance of the screen from the source is 25 cm. Wavelength of light used is 5000 Å. Then what is the angular position of the first dark fringe ?

Solution

The angular position $\theta = \dfrac{\beta}{D} = \dfrac{\lambda}{d}$ ($\because$ $\beta = \dfrac{\lambda D}{d}$) The first dark fringe will be at half the fringe width from the mid point of central maximum. Thus the angular position of first dark fringe will be-

$$\alpha = \dfrac{\theta}{2} = \dfrac{1}{2}\left[\dfrac{\lambda}{d}\right] = \dfrac{1}{2}\left[\dfrac{5000 \times \pi}{0.1 \times 10^{-3}} \times 10^{-10}\right]\dfrac{180}{\pi} = 0.45°.$$

BEGINNER'S BOX-2

1. In Young's double slit experiment, the slits are 2mm apart and are illuminated with a mixture of two wavelength, $\lambda = 7500$ Å and $\lambda' = 9000$ Å. At what minimum distance from the common central bright fringe on a screen 2 m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other ?

2. In a Young's double slit experiment the angular width of fringe formed on a distant screen is 0·1 radian. Find the distance between the two slits if wavelength of light used is 6000Å.

3. Two slits in Youngs experiment are 0·02 cm apart. The interference fringes for light of wavelength 6000Å are formed on a screen 80cm away. Calculate the distance of the fifth bright fringe.

4. In Young's double slit experiment, two slits are separated by 3mm distance and illuminated by light of wavelength 480 nm. The screen is at 2m from the plane of the slits. Calculate the separation between the 8th bright fringe and the 3rd dark fringe obtained with respect to central Bright fringe.

5. In a double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If the distance between slits is 10^{-3} m. Calculate the wavelength of light used.

6. In young's experiment for interference of light the slits 0.2 cm apart are illuminated by yellow light ($\lambda = 5896$ Å). What would be the fringe width on a screen placed 1m from the plane of slits. What will be the fringe width if the system is immersed in water. ($\mu_w = 4/3$) ?

7. The distance between the coherent source is 0.3 mm and the screen is 90 cm from the sources. The second dark band is 0.3 cm away from central bright fringe. Find the wavelength and the distance of the fourth bright fringe from central bright fringe.

8. In Young's double slit experiment, using light of wavelength 400 nm, interference fringes of width 'X' are obtained. The wavelength of light is increased to 600 nm and the separation between the slits is halved. If one wants the observed fringe width on the screen to be the same in the two cases, find the ratio of the distance between the screen and the plane of the slits in the two arrangements.

9. In a Young's double slit experiment, light has a frequency of 6×10^{14} Hz. The distance between the centres of adjacent bright fringes is 0.75 mm. If the screen is 1.5 m away then find the distance between the slits.

10. In a Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2.0 mm. What will be the fringe width, if the entire apparatus is immersed in a liquid of refractive index 1.33 ?

4. OPTICAL PATH LENGTH

Optical path length (OPL) also known as optical length or optical distance is the product of the geometric length of the optical path followed by light and the refractive index of homogeneous medium through which a light ray propagates.
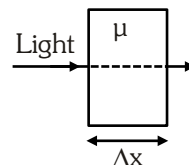
time taken by light to travel in the given medium

$$t = \frac{\Delta x}{v} = \frac{\Delta x \times \mu}{c}$$

In same time distance travelled in vacuum is

$$d = c \times t$$

$$d = c \times \Delta x \times \frac{\mu}{c}$$

$$d = \mu \Delta x \quad \text{optical path length}$$

5. EFFECT OF THIN FILMS

When a glass plate of thickness t and refractive index μ is placed in front of one of the slits in YDSE then the central fringe shifts towards that side in which glass plate is placed because extra path difference is introduced by the glass plate. In the path S_1P distance travelled by wave in air = $(S_1P - t)$
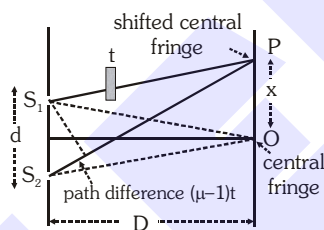
Distance travelled by wave in the sheet = t

Time taken by light to reach up to point P will be same from S_1 and S_2

$$\frac{S_2P}{c} = \frac{S_1P - t}{c} + \frac{t}{c/\mu} \Rightarrow \frac{S_2P}{c} = \frac{S_1P + (\mu - 1)t}{c} \Rightarrow S_2P = S_1P + (\mu - 1)t \Rightarrow S_2P - S_1P = (\mu - 1)t$$

Path difference = $(\mu - 1)t \Rightarrow$ Phase difference $\phi = \frac{2\pi}{\lambda}(\mu - 1)t$; Number of fringes displaced = $\frac{(\mu - 1)t}{\lambda}$

Distance of shifted fringe from central fringe $x = \frac{D(\mu - 1)t}{d}$ $\left[\because \frac{xd}{D} = (\mu - 1)t \right]$

$$\therefore \ x = \frac{\beta(\mu - 1)t}{\lambda} \quad \text{and} \ \beta = \frac{D\lambda}{d}$$

COLOURS IN THIN FILMS

When white light is incident on a thin film (like oil film on the surface of water or a soap bubble) then interference takes place between the waves reflected from its two surfaces and waves refracted through it. The intensity becomes maximum and minimum as a result of interference and colours are seen.

(i)   The source of light must be an extended source.

(ii)   The colours obtained in reflected and transmitted light are mutually complementary.

(iii) The colours obtained in thin films are due to interference whereas those obtained in prism are due to dispersion.

INTERFERENCE DUE TO THIN FILMS

Consider a thin transparent film of thickness t and refractive index μ. Let a ray of light AB be incident on the film at B. At B, a part of light is reflected along BR_1, and a part of light refracted along BC. At C a part of light is reflected along CD and a part of light transmitted along CT_1. At D, a part of light is refracted along DR_2 and a part of light is reflected along DE. Thus interference in this film takes place due to reflected light in between BR_1 and DR_2 also in transmitted light in between CT_1 and ET_2. Here coherent sources are obtained by division of amplitude.
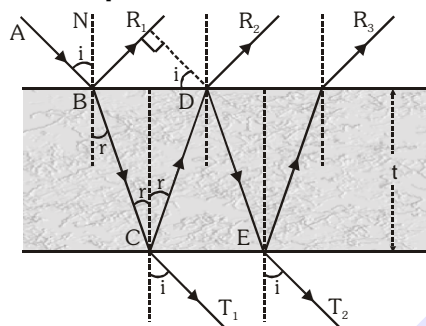
- ### Reflected System

The path difference between BR_1 and DR_2 is x = 2μt cos r. Reflection from the surface of denser medium involves an additional phase difference of π or path difference $\lambda/2$. Therefore the effective path difference between BR_1 and DR_2 is. $\Rightarrow x' = 2\mu t \cos r - \lambda/2$

Maximum or constructive Interference occurs when path difference between the light waves is nλ.

$2\mu t \cos r - \lambda/2 = n\lambda \Rightarrow 2\mu t \cos r = n\lambda + \lambda/2$

So the film will appear bright if $\quad 2\mu t \cos r = (2n + 1)\lambda/2 \qquad$ (n = 0, 1, 2, 3 .....)

- ### For minima or destructive interference :

When path difference is odd multiple of $\quad \dfrac{\lambda}{2} \Rightarrow 2\mu t \cos r - \dfrac{\lambda}{2} = (2n - 1)\dfrac{\lambda}{2}$

So the film will appear dark if $\qquad 2\mu t \cos r = n\lambda$

- ### For transmitted system

Since no additional path difference between transmitted rays CT_1 and ET_2.

So the net path difference between them is x = 2μt cos r

For maxima $\qquad 2\mu t \cos r = n\lambda$, n = 0, 1, 2.............

For minima $\qquad 2\mu t \cos r = (2n +1)\dfrac{\lambda}{2}$, n = 0, 1, 2............

USES OF INTERFERENCE EFFECT

Thin layer of oil on water and soap bubbles show different colours due to interference of waves reflected from two surfaces of their films. Here we get two coherent beams by division of amplitude making use of partial reflection and partial refraction

Uses :

- Used to determine the wavelength of light precisely.
- Used to determine refractive index or thickness of transparent sheet.
- Used in holography to produce 3-D images.

GOLDEN KEY POINTS

- If a glass plate of refractive index μ_1 and μ_2 having same thickness t is placed in the path of rays coming from S_1 and S_2 then path difference $x = \dfrac{D}{d}(\mu_1 - \mu_2)t$

- Distance of displaced fringe from central fringe $x = \dfrac{\beta(\mu_1 - \mu_2)t}{\lambda}$ $\because \dfrac{\beta}{\lambda} = \dfrac{D}{d}$

——————— **Illustrations** ———————

Illustration 13

Light of wavelength 6000Å is incident on a thin glass plate of refractive index 1.5 such that angle of refraction into the plate is 60°. Calculate the smallest thickness of plate which will make it appear dark by reflection.

Solution

$2\mu t \cos r = n\lambda$

$t = \dfrac{n\lambda}{2\mu \cos r} = \dfrac{1 \times 6 \times 10^{-7}}{2 \times 1.5 \times \cos 60}$

$= \dfrac{6 \times 10^{-7}}{1.5} = 4 \times 10^{-7} m$

Illustration 14

Light is incident on a glass plate ($\mu = 1.5$) such that angle of refraction is 60°. Dark band is observed corresponding to the wavelength of 6000Å. If the thickness of glass plate is 1.2×10^{-3} mm. calculate the order of the interference band for reflected system.

Solution

$\mu = 1.5$, r = 60°, $\lambda = 6000$Å $= 6 \times 10^{-7} m$

$t = 1.2 \times 10^{-3}$ mm $= 1.2 \times 10^{-6} m$

For dark band in the reflected light $2\mu t \cos r = n\lambda$

$n = \dfrac{2\mu t \cos r}{\lambda} = \dfrac{2 \times 1.5 \times 1.2 \times 10^{-6} \times \cos 60°}{6 \times 10^{-7}}$

$= \dfrac{2 \times 1.5 \times 1.2 \times 10^{-6} \times \dfrac{1}{2}}{6 \times 10^{-7}} = 3$

Thus third dark band is observed.

BEGINNER'S BOX-3

1. On placing a thin sheet of mica of thickness 12×10^{-7} m in the path of one of interferring beams in a Young's experiment, it is found that the central bright band shifts a distance equal to the width of a bright fringe. If the wavelength of light used is 6×10^{-7} m. then find refractive index of mica.

2. A central fringe of the interference produced by light of wavelength 6000Å is shifted to the position of 5th bright fringes by introducing a thin glass plate of refractive index 1.5. Calculate the thickness of the plate.
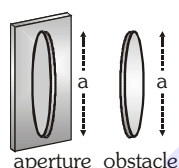
DIFFRACTION OF LIGHT

Bending of light rays from sharp edges of an opaque obstacle or aperture and its spreading in the geometrical shadow region is defined as diffraction of light or deviation of light from its rectilinear propogation tendency is defined as diffraction of light.

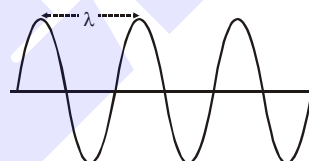diffraction from obstacle   diffraction from aperture
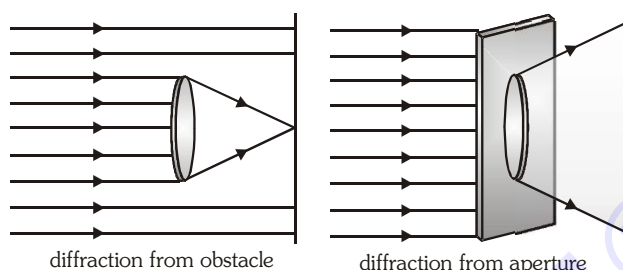
- Diffraction was discovered by Grimaldi.

- Theoretically explained by Fresnel

- Diffraction is possible in all type of waves means mechanical or electromagnetic waves shows diffraction.

    Diffraction depends on two factors :

    (i)   Size of obstacles or aperture   (ii)   Wavelength of the wave


aperture obstacle

Condition of diffraction : Size of obstacle or aperture should be nearly equal to the wavelength of light

i.e. $\lambda \simeq a$ $\dfrac{a}{\lambda} \simeq 1$

If size of obstacle is much greater than wavelength of light, then rectilinear motion of light is observed.

- It is practically observed when size of aperture or obstacle is greater than 50 λ. The obstacle or aperature does not shows diffraction.

- Wavelength of light is of the order 10^{-7} m. In general obstacle of this wavelength is not present so light rays does not show diffraction and it appears to travel in straight line Sound wave shows more diffraction as compared to light rays because wavelength of sound is large (16 mm to 16m). So it is generally diffracted by the objects of our daily life.

- Diffraction of ultrasonic waves is also not observed easily because their wavelength is of the order of about 1 cm. Diffraction of radio waves is very conveniently observed because of its very large wavelength (2.5 m to 250 m). X-rays can be diffracted easily by crystals. It was discovered by Lave.


diffraction of sound from a window
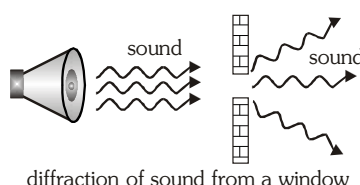
FRAUNHOFER DIFFRACTION

In fraunhofer diffraction both source and screen are effectively at infinite distance from the diffracting device and pattern is the image of source modified by diffraction effects.

Diffraction at single slit, double slit are the example of Fraunhofer diffraction.

1.   FRAUNHOFER DIFFRACTION DUE TO SINGLE SLIT

AB is single slit of width a, Plane wavefront is incident on a slit AB. Secondary wavelets coming from every part of AB reach the axial point O in same phase forming the central maxima. The intensity of central maxima is maximum in this diffraction. where θ_n represents direction of nth minima, Path difference AB' = a sin θ_n
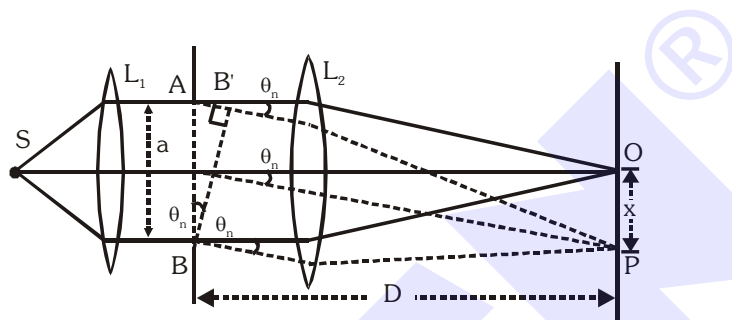
for n^{th} minima a $\sin\theta_n = n\lambda$ $\therefore$ $\sin\theta_n \approx \theta_n = \dfrac{n\lambda}{a}$ (if θ_n is small)

- When path difference between the secondary wavelets coming from A and B is $n\lambda$ or $2n\left[\dfrac{\lambda}{2}\right]$ or even multiple of $\dfrac{\lambda}{2}$ then minima occurs

  For minima $a\sin\theta_n = 2n\left[\dfrac{\lambda}{2}\right]$ where n = 1, 2, 3 ...

- When path difference between the secondary wavelets coming from A and B is $(2n+1)\dfrac{\lambda}{2}$ or odd multiple of $\dfrac{\lambda}{2}$ then maxima occurs

  For maxima $a\sin\theta_n = (2n + 1)\dfrac{\lambda}{2}$ where n = 1, 2, 3 ...

  n = 1 $\rightarrow$ first maxima   and   n = 2 $\rightarrow$ second maxima

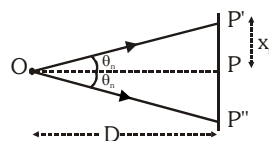- Alternate ordered minima and maxima occurs on both sides of central maxima.

  For n^{th} minima

  If distance of nth minima from central maxima = x_n

  distance of slit from screen = D , width of slit = a

  Path difference δ = a $\sin\theta_n = \dfrac{2n\lambda}{2}$ $\Rightarrow$ $\sin\theta_n = \dfrac{n\lambda}{a}$

  In Δ POP' $\tan\theta_n = \dfrac{x_n}{D}$ If θ_n is small $\Rightarrow$ $\sin\theta_n \approx \tan\theta_n \approx \theta_n$

$$x_n = \frac{n\lambda D}{a} \Rightarrow \theta_n = \frac{x_n}{D} = \frac{n\lambda}{a}$$ First minima occurs on both sides of central maxima.
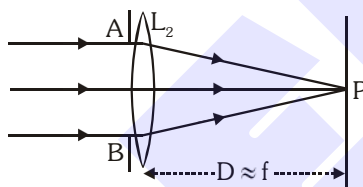
For first minima $x = \dfrac{D\lambda}{a}$ and $\theta = \dfrac{x}{D} = \dfrac{\lambda}{a}$

- Linear width of central maxima $w_x = 2x \Rightarrow w_x = \dfrac{2D\lambda}{a}$

- Angular width of central maxima $w_\theta = 2\theta = \dfrac{2\lambda}{a}$

SPECIAL CASE

Lens L_2 is shifted very near to slit AB. In this case distance between slit and screen will be nearly equal to the

focal length of lens L_2 (i.e. $D \approx f$) $\theta_n = \dfrac{x_n}{f} = \dfrac{n\lambda}{a} \Rightarrow x_n = \dfrac{n\lambda f}{a}$
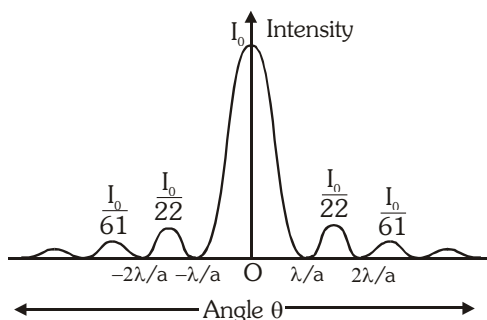

$w_x = \dfrac{2\lambda f}{a}$ and angular width of central maxima $w_\theta = \dfrac{2x}{f} = \dfrac{2\lambda}{a}$

Fringe width : Distance between two consecutive maxima (bright fringe) or minima (dark fringe) is known as fringe width. Fringe width of central maxima is doubled then the width of other maxima i.e.,

$$\beta = x_{n+1} - x_n = (n+1)\frac{\lambda D}{a} - \frac{n\lambda D}{a} = \frac{\lambda D}{a} \text{ (other than central maxima)}$$

Intensity curve of Fraunhofer's diffraction

DIFFERENCE BETWEEN INTERFERENCE AND DIFFRACTION (FOR FRAUNHOFER SINGLE SLIT) :

	Interference		Fraunhofer diffraction
(1)	It is the phenomenon of superposition of two waves coming from two different coherent sources.	(1)	It is the phenomenon of superposition of two waves coming from two different parts of the same wave front.
(2)	In interference pattern, all bright lines are equally bright and equally spaced.	(2)	All bright lines are not equally bright and equally wide. Brightness and width goes on decreasing with the angle of diffraction.
(3)	All dark lines are totally dark	(3)	Dark lines are perfectly dark. Their contrast with bright lines and width goes on decreasing with angle of diffraction.
(4)	In interference bands are large in number	(4)	In diffraction bands are few in number.

GOLDEN KEY POINTS

● The width of central maxima $\propto \lambda$, that is, more for red colour and less for blue.

   i.e., $w_x \propto \lambda$ as $\lambda_{blue} < \lambda_{red}$

   $\Rightarrow$ $w_{blue} < w_{red}$

● For obtaining the fraunhofer diffraction, focal length of second lens (L_2) is used.

   $w_x \propto \lambda \propto f \propto 1/a$

   width will be more for narrow slit

● By decreasing linear width of slit, the width of central maxima increase.

Illustrations

Illustration 15

Light of wavelength 6000Å is incident normally on a slit of width 24×10^{-5} cm. Find out the angular position of second minimum from central maximum ?

Solution

a sinθ = 2λ given $\lambda = 6 \times 10^{-7}$ m, a = $24 \times 10^{-5} \times 10^{-2}$ m

$\sin\theta = \dfrac{2\lambda}{a} = \dfrac{2 \times 6 \times 10^{-7}}{24 \times 10^{-7}} = \dfrac{1}{2}$

$\therefore$ $\theta = 30°$

Illustration 16

Light of wavelength 6328Å is incident normally on a slit of width 0.2 mm. Calculate the angular width of central maximum on a screen distance 9 m ?

Solution

given $\lambda = 6.328 \times 10^{-7}$ m, a = 0.2×10^{-3} m

$w_\theta = \dfrac{2\lambda}{a} = \dfrac{2 \times 6.328 \times 10^{-7}}{2 \times 10^{-4}}$ radian $= \dfrac{6.328 \times 10^{-3} \times 180}{3.14} = 0.36°$

Illustration 17

Light of wavelength 5000Å is incident on a slit of width 0.1 mm. Find out the width of the central bright line on a screen distance 2m from the slit ?

Solution

$w_x = \dfrac{2f\lambda}{a} = \dfrac{2 \times 2 \times 5 \times 10^{-7}}{10^{-4}} = 20$ mm

Illustration 18

The Fraunhofer diffraction pattern of a single slit is formed at the focal plane of a lens of focal length 1m. The width of the slit is 0.3 mm. If the third minimum is formed at a distance of 5 mm from the central maximum then calculate the wavelength of light.

Solution

$x_n = \dfrac{nf\lambda}{a} \Rightarrow \qquad \lambda = \dfrac{ax_n}{fn} = \dfrac{3 \times 10^{-4} \times 5 \times 10^{-3}}{3 \times 1} = 5000$ Å $\qquad [\because\ n = 3]$

Illustration 19

Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000 Å.

Solution

$\because\ \sin\theta = \dfrac{\lambda}{a} \qquad\qquad\qquad \theta$ = half angular width of the central maximum.

a = 12×10^{-5} cm, $\lambda = 6000$ Å = 6×10^{-5} cm $\qquad \therefore \qquad \sin\theta = \dfrac{\lambda}{a} = \dfrac{6 \times 10^{-5}}{12 \times 10^{-5}} = 0.50 \Rightarrow\ \theta = 30°$

Illustration 20

Light of wavelength 6000 Å is incident on a slit of width 0.30 mm. The screen is placed 2 m from the slit. Find (a) the position of the first dark fringe and (b) the width of the central bright fringe.

Solution

The first fringe is on either side of the central bright fringe.

here $\quad$ n = ±1, D = 2 m, λ = 6000 Å = 6×10^{-7} m

$\therefore \qquad \sin\theta = \dfrac{x}{D} \qquad \Rightarrow$ a = 0.30 mm = 3×10^{-4} m $\Rightarrow$ a sin θ = nλ $\Rightarrow \dfrac{ax}{D} = n\lambda$

(a) $\qquad x = \dfrac{n\lambda D}{a} \qquad \Rightarrow\ x = \pm\left[\dfrac{1 \times 6 \times 10^{-7} \times 2}{3 \times 10^{-4}}\right] = \pm 4 \times 10^{-3}$ m

The positive and negative signs corresponds to the dark fringes on either side of the central bright fringe.

(b) $\qquad$ The width of the central bright fringe y = 2x = 2 × 4 × 10^{-3} = 8 × 10^{-3} m = 8 mm

Illustration 21

A Slit of width a is illuminated by monochromatic light of wavelength 650nm at normal incidence. Calculate the value of a when -

(a) the first minimum falls at an angle of diffraction of 30°

(b) the first maximum falls at an angle of diffraction of 30°.

Solution

(a) for first minimum $\sin\theta_1 \dfrac{\lambda}{a}$ $\therefore a = \dfrac{\lambda}{\sin\theta_1} = \dfrac{650\times10^{-9}}{\sin 30°} = \dfrac{650\times10^{-9}}{0.5} = 1.3\times10^{-6}$ m

(b) For first maximum $\sin\theta_1 = \dfrac{3\lambda}{2a}$ $\therefore a = \dfrac{3\lambda}{2\sin\theta} = \dfrac{3\times650\times10^{-9}}{2\times0.5} = 1.95\times10^{-6}$ m

Illustration 22

Red light of wavelength 6500Å from a distant source falls on a slit 0.50 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.8 m. from the slit.

Solution

Given $\lambda = 6500$Å $= 65\times10^{-8}$ m, a = 0.5 mm $= 0.5\times10^{-3}$ m., D = 1.8 m.

Required distance between first two dark bands will be equal to width of central maxima.

$W_x \quad = \dfrac{2\lambda D}{a} = \dfrac{2\times6500\times10^{-10}\times1.8}{0.5\times10^{-3}} = 468\times10^{-5}$ m = 4.68 mm

BEGINNER'S BOX-4

1. A slit of width 0·15 cm is illuminated by light of wavelength 5×10^{-5} cm and a diffraction pattern is obtained on a screen 2·1m away. Calculate the width of central maxima.

2. The light of wavelength 600nm is incident normally on a slit of width 3mm. Calculate the angular width of central maximum on a screen kept 3m away from the slit.

3. Red light of wavelength 6500Å from a distant source falls on a slit 0.50 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.8 m from the slit.

4. In a single slit diffraction experiment first minimum for λ_1 = 660 nm coincides with first maxima for wavelength λ_2. Calculate λ_2.

2. DIFFRACTION FROM CIRCULAR APERTURE :

The diffraction pattern resulting from a uniformly illuminated, circular aperature has a bright central region, known as airy disc, which is surrounded by much fainter concentric circular rings.
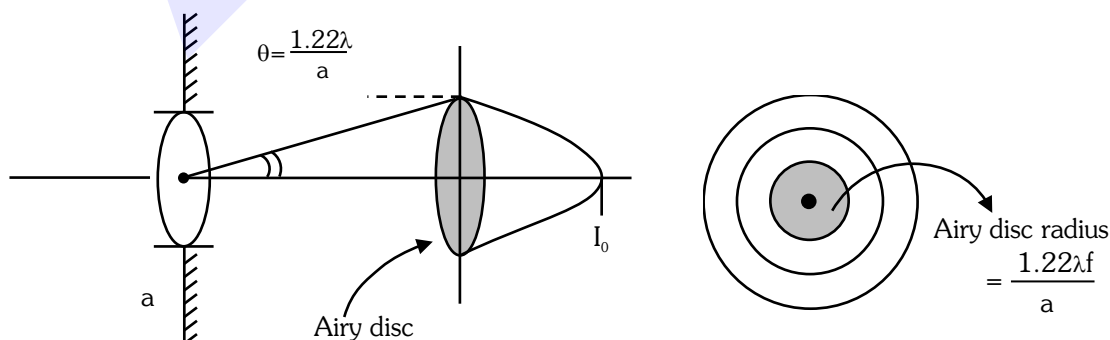
When a beam of light passes through lens, the lens acts like a circular aperature and produce a diffraction pattern (Airy disc) instead of a point image
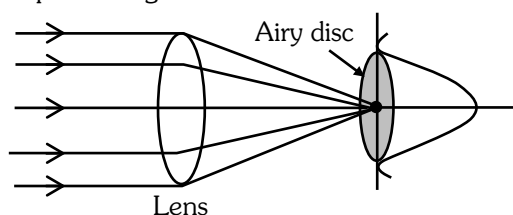

If there are two point objects lying close to each other, the two diffraction pattern are produced, which may overlap on each other and it may be difficult to distinguish them as separate.
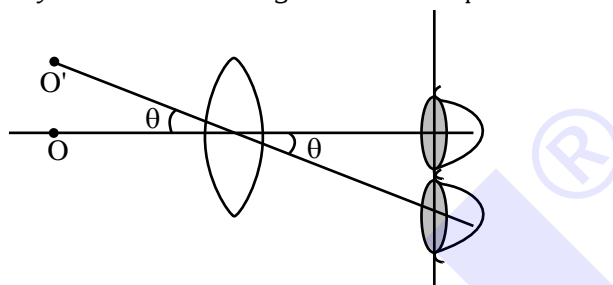
Rayleigh's criterion for resolution

Two points are just resolved by an optical system when the central maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.
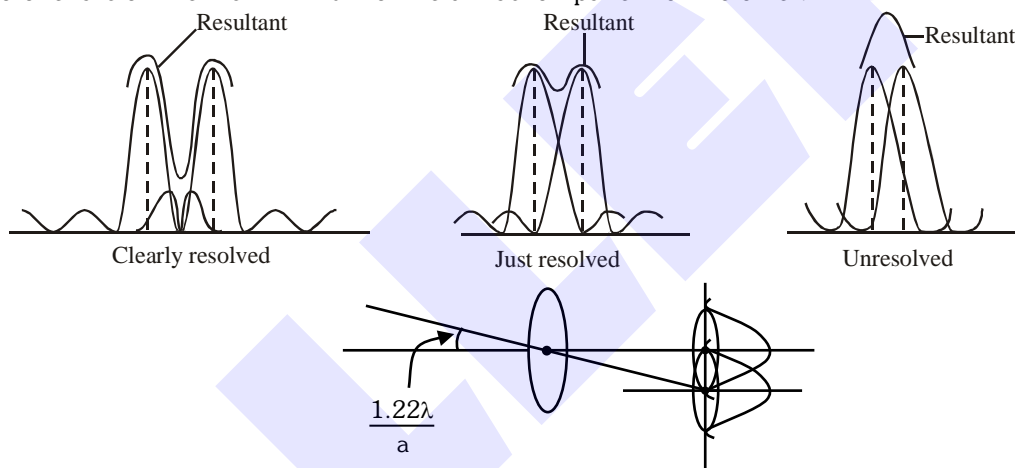
$a \rightarrow$ Diameter of lens

Angular limit of resolution (L.R.) = $\dfrac{1.22\lambda}{a}$

Resolving power = $\dfrac{a}{1.22\lambda}$

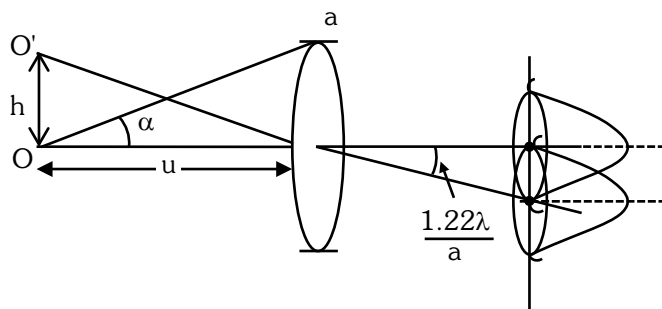Limit of resolution of eye
$a = 2.5$ mm, $\lambda = 5000\text{Å}$

$$\text{L.R.} = \frac{1.22 \times 5 \times 10^{-7}}{2.5 \times 10^{-3}} = 2.44 \times 10^{-4}\text{rad.}$$

$$\text{L.R.} = \frac{2.44 \times 10^{-4} \times 180 \times 60}{\pi} = 0.84 \approx 1'$$

By increasing aperture we can decrease limit of resolution and hence increase resolving power as in telescope.

Same formula can be used for **telescope**

$$\boxed{\text{L.R.} = \frac{1.22\lambda}{a}} \quad , \quad \boxed{\text{R.P.} = \frac{a}{1.22\lambda}}$$

93

Limit of resolution of microscope.

$$\frac{h}{u} = \frac{1.22\lambda}{a}$$

$$h = 1.22\frac{\lambda u}{a}$$

$$\frac{\frac{a}{2}}{u} = \tan\alpha = \sin\alpha$$

$$a = 2u\sin\alpha$$

$$h = \frac{1.22\lambda}{2\sin\alpha}$$

$$h = \frac{0.61\lambda}{\sin\alpha}$$

In presence of medium

$$\boxed{h = \frac{0.61\lambda}{\mu\sin\alpha}}$$ Here 'h' is linear limit of resolution

The resolving power of microscope is given by the reciprocal of the minimum separation of two points seen as distinct.

$\mu\sin\alpha$ is numerical aperture. Oil immersed microscope is better as it's numerical aperture is more giving less limit and more resolving power of microscope.

By taking small wavelength we can increase resolving power as in UV microscope, electron microscope.

─── **Illustrations** ───

Illustration 23

Light of $\lambda = 560$ nm passes through a pinhole of diameter 0.20 mm and fall on a wall at distance of 2.0 m what will be radius of airy disc.

Solution

$$r = \frac{1.22\lambda f}{a} = \frac{1.22\times560\times10^{-9}\times2}{0.2\times10^{-3}} = 6.83 \text{ mm.}$$

Illustration 24

The Hale telescope of Mount Palomar has a diameter of 200 inch. What is its limiting angle of resolution for 600 nm light?

Solution

Here; a = 200 in = 200×2.54 cm = 508 cm = 5.08 m
$\lambda = 600$ nm = 600×10^{-9} m = 6.00×10^{-7} m

$$\Delta\theta = 1.22\left(\frac{\lambda}{a}\right) = 1.22\left(\frac{6.00\times10^{-7}\,m}{5.08\,m}\right) = 1.44\times10^{-7} \text{ rad}$$

Illustration 25

What is limit of resolution of eye, if diameter of pupil is 2mm and $\lambda = 500$ nm.

Solution

$$\theta = \frac{1.22\lambda}{a} = \frac{1.22\times5\times10^{-7}}{2\times10^{-3}} = 3.05\times10^{-4} \text{ rad}$$

$$= \frac{180\times60}{\pi} \times3.05\times10^{-4} \text{ minute} = 10.4\times10^{-1} = 1 \text{ minute.}$$

Illustration 26

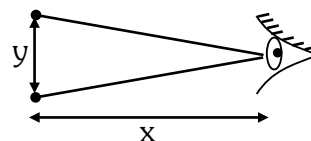What must be minimum distance between two pillar placed at a distance of 11 km from human eye so that they are distinctly visible.

Solution

Resolving limit for eye = 1' = $\dfrac{\pi}{180 \times 60}$ rad.

$\dfrac{y}{x} = \dfrac{\pi}{180 \times 60}$

$y = \dfrac{\pi \times 11 \times 10^3}{180 \times 60} = 3.1$ m.

Illustration 27

The aperture of telescope is 5 cm. Distance between moon and earth is 4×10^5 km. Find the minimum separation between the objects on the surface of moon which can be resolved ($\lambda = 5 \times 10^{-7}$ m).

Solution

$\dfrac{y}{x} = \dfrac{1.22\lambda}{a}$

$y = \dfrac{1.22 \times 5 \times 10^{-7} \times 4 \times 10^8}{5 \times 10^{-2}}$

$= 4.88 \times 10^3$

$= 4880$ m.

BEGINNER'S BOX-5

1. Calculate the resolving power of a telescope, assuming the diameter of the objective lens to be 6 cm and the wavelength of light used to be 540 nm.

2. Calculate the limit of resolution of a microscope if an object of numerical aperture 0.12 is viewed by using light of wavelength 6×10^{-7} m.

3. Assuming human pupil to have diameter 0.25 cm and a comfortable viewing distance of 25 cm. Minimum separation between two lines that human eye can resolve at 500 nm wave length.
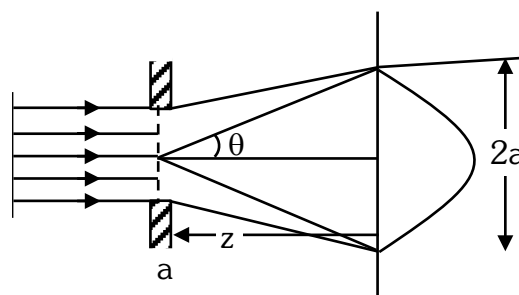
3. FRESNEL DISTANCE & VALIDITY OF RAY OPTICS

It is a distance upto which ray optics can be used. Beyond this distance spreading due to diffraction will be so large that we will not get clear image.

It is a distance in which beam width becomes double due to diffraction.

$\dfrac{2\lambda Z}{a} = 2a$

$\boxed{Z = \dfrac{a^2}{\lambda}}$

This relation shows that for distance much smaller than Z, the spreading due to diffraction is smaller

compared to the size of the beam. It becomes comparable when the distance is approximately Z.

For distance much greater than Z, the spreading due to diffraction dominates over. Ray optics is not valid in

this reion.

Relation also shows that straight line propagation is valid in the limit of λ tending to zero.

Illustration 28

Upto what distance we should apply ray optics (Fresnel distance) if beam width is 2mm and wave length of light 500 nm.

Solution

$$z = \frac{2\lambda z}{a} = 2a$$

$$z = \frac{a^2}{\lambda} = \frac{4 \times 10^{-6}}{5 \times 10^{-7}}$$

$$z = 8m.$$

4.   DIFFRACTION DUE TO DOUBLE SLIT :

In the double slit experiment, we must note that the pattern on the screen is actually a superposition of single slit diffraction from each slit or hole, and the double slit interference pattern as shown in figure. It show a diffraction peak in which there appear several fringes of smaller width due to double – slit interference.

Resultant pattern
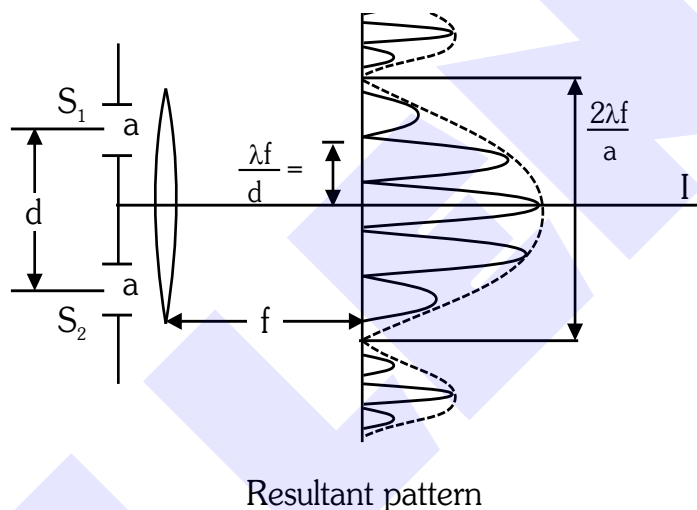
$a \rightarrow$ width of slit

Number of interference fringes inside diffraction pattern (central maxima) :

$$N = \frac{2\lambda f / a}{\frac{\lambda f}{d}} = \frac{2d}{a}$$

It is not depending upon distance of screen.

$a \rightarrow 0$ (when a is very small, diffraction pattern becomes very flat)

$n \rightarrow \infty$

This will be the case of pure interference pattern.

Illustration 29

In double slit experiment if width of each slit is 0.1mm and separation between the slits is 1mm. How many interference fringes we will get inside the central maxima diffraction pattern.

Solution

$$n = \frac{2\lambda f / a}{\lambda f / d} = \frac{2d}{a} = \frac{2 \times 10^{-3}}{0.1 \times 10^{-3}} = 20$$
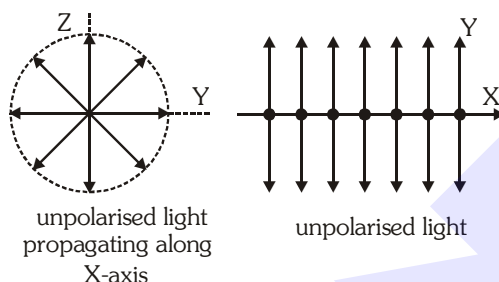
POLARISATION

Experiments on interference and diffraction have shown that light is a form of wave motion. These effects do not tell us about the type of wave motion i.e., whether the light waves are longitudinal or transverse.

The phenomenon of polarization has helped to establish beyond doubt that light waves are transverse waves.

UNPOLARISED LIGHT

An ordinary beam of light consists of a large number of waves emitted by the atoms of the light source. Each atom produces a wave with its own orientation of electric vector $\vec{E}$ so all direction of vibration of are equally probable.

unpolarised light
propagating along
X-axis

unpolarised light

The resultant electromagnetic wave is a superposition of waves produced by the individual atomic sources and it is called unpolarised light. In ordinary or unpolarised light, the vibrations of the electric vector occur symmetrically in all possible directions in a plane perpendicular to the direction of propagation of light.

POLARISATION

The phenomenon of restricting the vibration of light (electric vector) in a particular direction perpendicular to the direction of propagation of wave is called polarisation of light. In polarised light, the vibration of the electric vector occur in a plane perpendicular to the direction of propagation of light and are confined to a single direction in the plane (do not occur symmetrically in all possible directions). After polarisation the vibrations become asymmetrical about the direction of propagation of light.
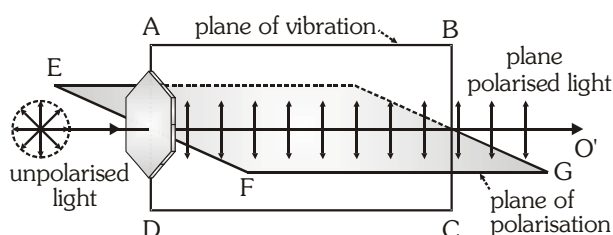
POLARISER

Tourmaline crystal : When light is passed through a tourmaline crystal cut parallel to its optic axis, the vibrations of the light carrying out of the tourmaline crystal are confined only to one direction in a plane perpendicular to the direction of propagation of light. The emergent light from the crystal is said to be plane polarised light.

Nicol Prism : A nicol prism is an optical device which can be used for the production and detection of plane polarised light. It was invented by William Nicol in 1828.

Polaroid : A polaroid is a thin commercial sheet in the form of circular disc which makes use of the property of selective absorption to produce an intense beam of plane polarised light.

PLANE OF POLARISATION AND PLANE OF VIBRATION :

The plane in which vibrations of light vector   and the direction of propogation lie is known as plane of vibration A plane normal to the plane of vibration and in which no vibration takes place is known as plane of polarisation

EXPERIMENTAL DEMONSTRATION OF POLARISATION OF LIGHT

Take two tourmaline crystals cut parallel to their crystallographic axis (optic axis).
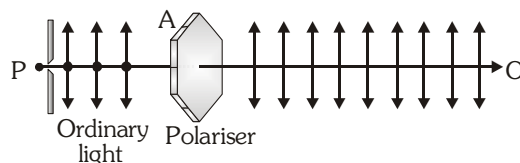

First hold the crystal A normally to the path of a beam of colour light. The emergent beam will be slightly coloured. Rotate the crystal A about PO. No change in the intensity or the colour of the emergent beam of light.

Take another crystal B and hold it in the path of the emergent beam of so that its axis is parallel to the axis of the crystal A. The beam of light passes through both the crystals and outcoming light appears coloured.
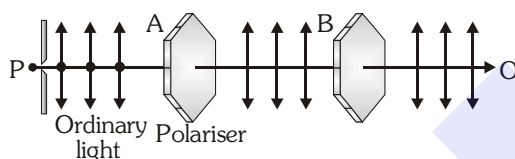

Now, rotate the crystal B about the axis PO. It will be seen that the intensity of the emergent beam decreases and when the axes of both the crystals are at right angles to each other no light comes out of the crystal B.
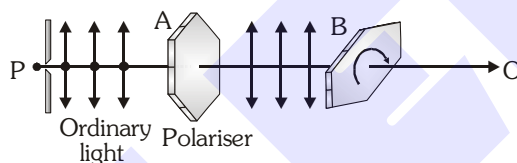
If the crystal B is further rotated light reappears and intensity becomes maximum again when their axis are parallel. This occurs after a further rotation of B through 90°. This experiment confirms that the light waves are transverse in nature. The vibrations in light waves are perpendicular to the direction of propogation of the the wave. First crystal A polarises the light so it is called polariser. Second crystal B, analyses the light whether it is polarised or not, so it is called analyser.

1. METHODS OF OBTAINING PLANE POLARISED LIGHT

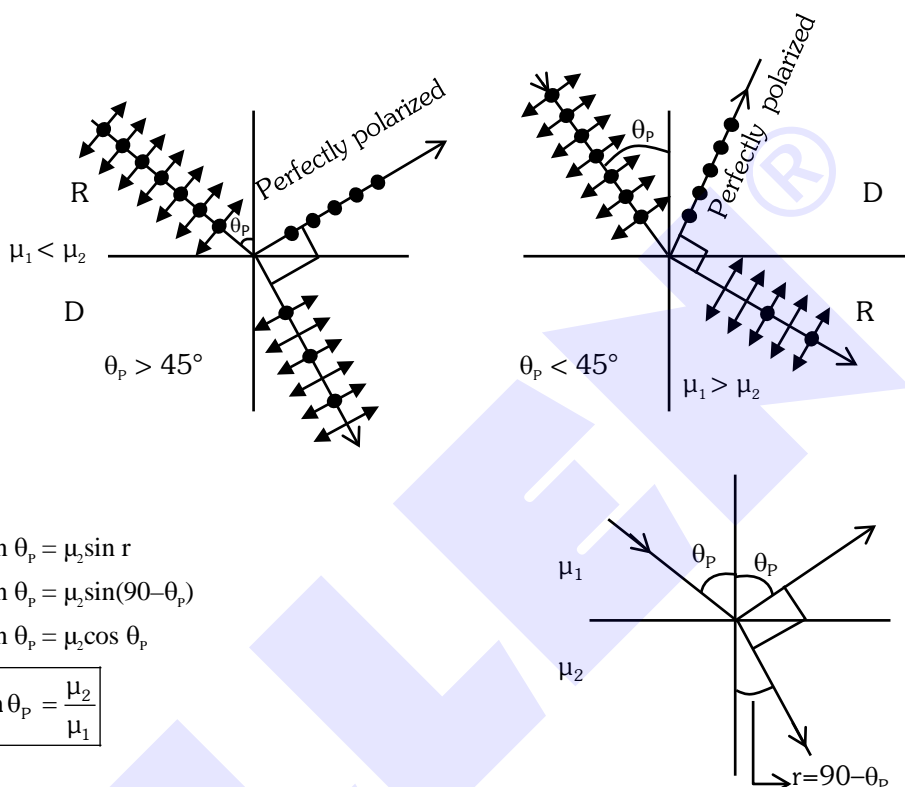● **Polarisation by reflection :** The simplest method to produce plane polarised light is by reflection. This method was discovered by Malus in 1808. When a beam of ordinary light is reflected from a surface, the reflected light is partially polarised. (The degree of polarisation of the polarised light in the reflected beam is greatest when it is incident at an angle called polarising angle or Brewster's angle).

TG: @Chalnaayaaar
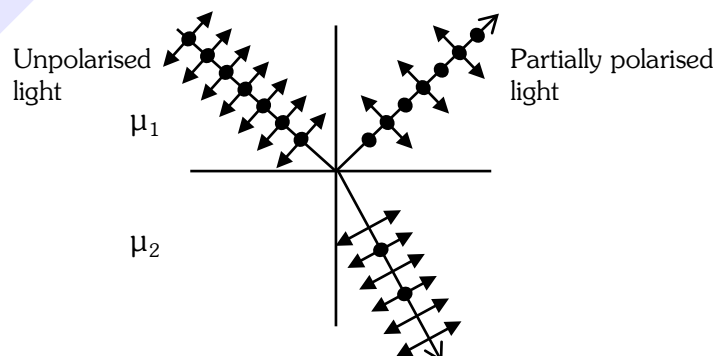
- **Polarising angle** : Polarising angle is that angle of incidence at which the reflected light is completely plane polarised light.

- **Brewster's Law** : When unpolarised light strikes at polarising angle θ_P on an interface separating two medium then the reflected light (light in rare medium) is completely polarised. Also reflected and refracted rays are normal to each other. This is known as Brewster's law.

  In case of polarisation by reflection :

  (i)   For $i = \theta_P$ refracted light is partially polarised.

  (ii)   For $i = \theta_P$ reflected and refracted rays are perpendicular to each other.

  (iii) For $i < \theta_P$ or $i > \theta_P$ both reflected and refracted light are partially polarised.

$\mu_1 \sin\theta_P = \mu_2 \sin r$

$\mu_1 \sin\theta_P = \mu_2 \sin(90-\theta_P)$

$\mu_1 \sin\theta_P = \mu_2 \cos\theta_P$

$$\boxed{\tan\theta_P = \frac{\mu_2}{\mu_1}}$$

By Refraction

In this method, a pile of glass plates is formed by taking 20 to 30 microscope slides and light is made to be incident at polarising angle 57°. According Brewster's law, the reflected light will be plane polarised with vibrations perpendicular to the plane of incidence and the transmitted light will be partially polarised. Since in one reflection about 15% of the light with vibration perpendicular to plane of paper is reflected therefore after passing through a number of plates emerging light will become plane polarised with vibrations in the plane of paper.
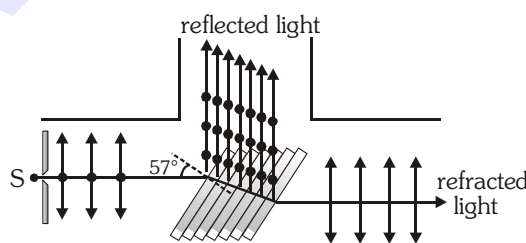

By Dichroism

Some crystals such as tourmaline and sheets of iodosulphate of quinone have the property of strongly absorbing the light with vibrations perpendicular of a specific direction (called transmission axis) and transmitting the light with vibration parallel to it. This selective absorption of light is called dichroism. So if unpolarised light passes through proper thickness of these crystals, the transmitted light will plane polarised with vibrations parallel to transmission axis. Polaroids work on this principle.
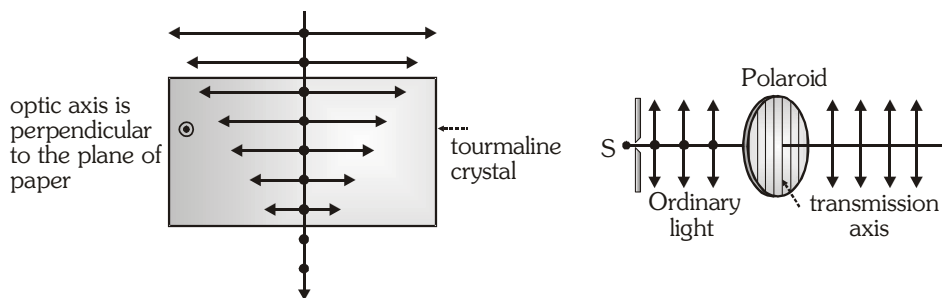
By scattering :

When light is incident on small particles of dust, air molecules etc. s(having smaller size as compared to the wavelength of light), it is absorbed by the electrons and is re-radiated in all directions. The phenomenon is called as scattering. Light scattered in a direction at right angles to the incident light is always plane-polarised.
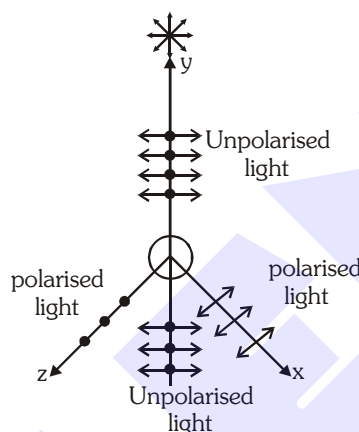

By Double Refraction :

It was found that in certain crystals such as calcite, quartz and tourmaline, etc., incident unpolarised light splits in into two light beams of equal intensities with perpendicular polarisations. One of the rays behaves as ordinary light and is called O-ray (ordinary ray) while the other does not obey laws of refraction and is called E-ray (extra-ordinary ray). This is why when an object is seen through these crystal is rotated, one image (due to E-ray) rotates around the other (due to O-ray).
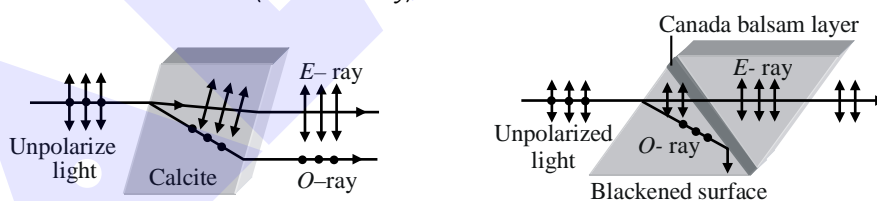
By using the phenomenon of double refraction and isolating one ray from the other we can obtain plane polarised light which actually happens in a Nicol-prism. Nicol-prism is made up of calcite crystal and in it E-ray is isolated from O-ray through total internal reflection of O-ray at canada balsam layer and then absorbing it at the blackened surface as shown in figure.

Law of Malus

● When a completely plane polarised light beam is incident on analyser, then intensity of emergent light varies as the square of cosine of the angle between the planes of transmission axis of the analyser and the polarizer. $I \propto \cos^2\theta \Rightarrow I = I_0 \cos^2\theta$
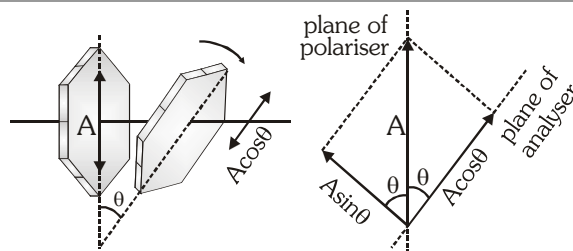
(i)   If $\theta = 0°$ then $I = I_0$ maximum value (Parallel arrangement)

(ii)   If $\theta = 90°$ then $I = 0$ minimum value (Crossed arrangement)

● If plane polarised light of intensity $I_0 (= KA^2)$ is incident on a polaroid and its vibrations of amplitude A make angle θ with transmission axis, then the component of vibrations parallel to transmission axis will be $A\cos\theta$ while perpendicular to it will be $A\sin\theta$.

Polaroid will pass only those vibrations which are parallel to transmission axis i.e. $A\cos\theta$, $\because$ $I_0 \propto A^2$

So the intensity of emergent light $I = K(A\cos\theta)^2 = KA^2\cos^2\theta$

● If an unpolarised light is converted into plane polarised light its intensity becomes half.

● If light of intensity I_1, emerging from one polaroid called polariser is incident on a second polaroid (called analyser) the intensity of light emerging from the second polaroid is

$I_2 = I_1 \cos^2\theta$ θ = angle between the transmission axis of the two polaroids.

GOLDEN KEY POINTS

● Our eyes are nearly insensitive to polarisation of light, but this is not universally true for animals.

● If the angle of incidence is $0°$ or $90°$ the reflected beam is unpolarised.

● A Nicol prism is made by cutting a calcite crystal in a certain way.

Illustrations

Illustration 30

For a given medium, the polarising angle is $60°$. What will be the critical angle for this medium?

Solution

Here $i_p = 60°$

Thus, $\mu = \tan i_p = \tan 60° = \sqrt{3}$

If i_c is the critical angle for the medium.

$\mu = \dfrac{1}{\sin i_c}$ or $\sin i_c = \dfrac{1}{\mu} = \dfrac{1}{\sqrt{3}}$ or $\sin i_c = 0.5774$ or $i_c = 35°16'$

Illustration 31

A ray of light is incident on the surface of glass plate of refracive index 1.5 at polarising angle. What is the angle of refraction? ($\tan^{-1}(1.5) = 56° 19'$)

Solution

As $\mu = \tan i_p$, $\tan i_p = 1.5$ or $i_p = \tan^{-1}(1.5)$ or $i_p = 56°19'$

As $r + i_p = 90°$, $r = 90° - i_p = 33°41'$

BEGINNER'S BOX-6

1. Refractive index of water is 1.33. Calculate the angle of polarisation for light reflected from the surface of a lake. ($\tan^{-1}$ 1.33 = 53°.4')

2. A ray of light strikes a glass plate at an angle of 60°. If the reflected and the refracted rays are perpendicular to each other, find the index of refraction of glass.

3. A parallel beam of light is incident at an angle of 60° on a plane glass surface and the reflected beam is completely polarised.

(a) What is the angle of refraction in glass?

(b) What is the refraction index of glass?

4. Light reflected from the surface of a glass plate of refractive index 1.732 is linearly polarised. Calculate the angle of refraction in glass.

5. Critical angle for a certain wavelength of light in case of glass is 40°. Find the polarising angle and angle of refraction in glass corresponding to this. ($\sin 40° = \dfrac{2}{3}$)

ANSWER'S

BEGINNER'S BOX-1		**BEGINNER'S BOX-3**	
1. (a) 5I (b) 7.8I, (c) I (d) 9I		**1.** 1·5	**2.** 6×10^{-6} m
2. $\dfrac{25}{16}$		**BEGINNER'S BOX-4**	
		1. 1·4 mm	**2.** 0.023°
BEGINNER'S BOX-2		**3.** 4.68 mm	**4.** 440 nm.
1. 4.5 mm		**BEGINNER'S BOX-5**	
2. 6μm		**1.** 9.1×10^{4}	**2.** 3.05×10^{-6} m.
3. 1·2 cm		**3.** 61 μm	
4. 1.76×10^{-3} m		**BEGINNER'S BOX-6**	
5. 6×10^{-7} m		**1.** 53°4'	**2.** 1.732
6. 0.2948 mm, 0.2211 mm		**3.** (a) 30° (b) 1.732	**4.** 30°
7. 8×10^{-3} m, $\lambda = 0.66 \times 10^{-6}$ m		**5.** 57.3°, 32.7°	
8. $\dfrac{3}{1}$	**9.** 10^{-3} m.	**10.** 1.5 mm			