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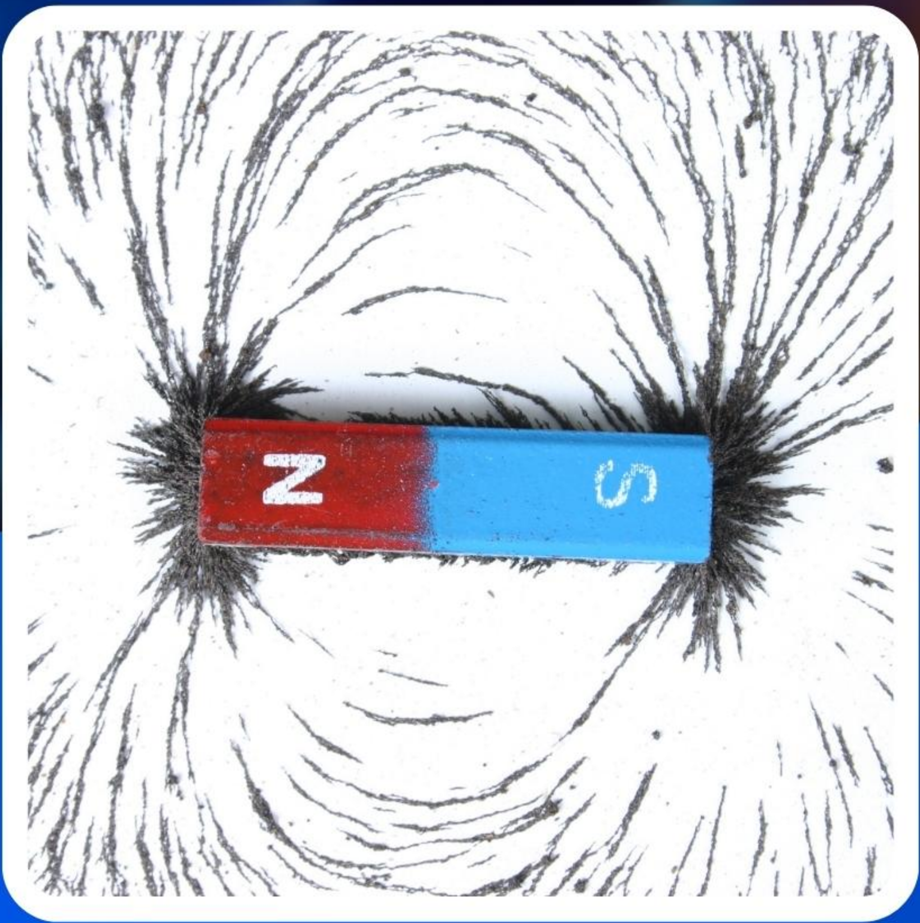

STUDY MATERIAL

Magnetic effect of current and Magnetism

HINDI MEDIUM

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HANS CHRISTIAN OERSTED (1777–1851)

Danish physicist and chemist, professor at Copenhagen. He observed that a compass needle suffers a deflection when placed near a wire carrying an electric current. This discovery gave the first empirical evidence of a connection between electric and magnetic phenomena.

ANDRE AMPERE (1775 – 1836)

Andre Marie Ampere was a French physicist, mathematician and chemist who founded the science of electrodynamics. Ampere was a child prodigy who mastered advanced mathematics by the age of 12. Ampere grasped the significance of Oersted's discovery. He carried out a large series of experiments to explore the relationship between current electricity and magnetism. These investigations culminated in 1827 with the publication of the 'Mathematical Theory of Electrodynamic Phenomena Deduced Solely from Experiments'. He hypothesised that all magnetic phenomena are due to circulating electric currents. Ampere was humble and absentminded. He once forgot an invitation to dine with the Emperor Napoleon. He died of pneumonia at the age of 61. His gravestone bears the epitaph: Tandem Felix (Happy at last).

MAGNETIC EFFECT OF CURRENT AND MAGNETISM

The branch of physics which deals with the magnetism due to electric current or moving charge (i.e. electric current is equivalent to the charges or electrons in motion) is called electromagnetism.

Magnetic Field

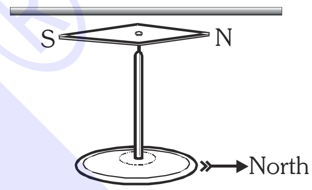
A region of space near a magnet, electric current or moving charged particle in which magnetic effects are exerted on any other magnet, electric current, or moving charged particle. It is also known as magnetic flux density or magnetic induction or magnetic field.

Unit : SI : weber/(meter)^2 or tesla (T) **C.G.S. :** gauss (1 Tesla = 10^4 gauss) **Dimensions :** $[MT^{-2}A^{-1}]$

1. OERSTED'S DISCOVERY AND BIOT-SAVART LAW

- **Oersted Discovery**

The relation between electricity and magnetism was discovered by Oersted in 1820. Oersted showed that the electric current through the conducting wire deflects the magnetic needle held near the wire.



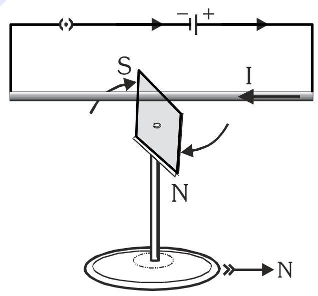
- When the direction of current in conductor is reversed then deflection of magnetic needle is also reversed.
- On increasing the current in conductor or bringing the needle closer to the conductor, the deflection of magnetic needle increases.

Oersted discovered a magnetic field around a conductor carrying electric current. Other related facts are as follows:

(a) A magnet at rest produces a magnetic field around it while an electric charge at rest produce an electric field around it.

(b) A current carrying conductor has a magnetic field and not an electric field around it. On the other hand, a charge moving with a uniform velocity has an electric as well as a magnetic field around it.

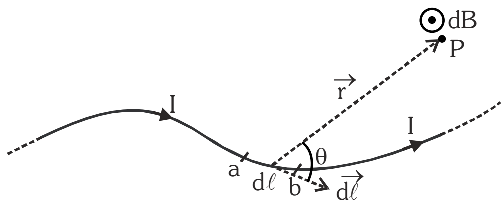
(c) All oscillating or an accelerated charge produces E.M. waves also in additions to electric and magnetic fields.



Oersted's experiment. Current in the wire deflects the compass needle

- **Current Element**

A very small element ab of length $d\ell$ of a thin conductor carrying current is called current element. Current element is a vector quantity whose magnitude is equal to the product of current and length of small element having the direction of the flow of current.



- **Biot – Savart's Law**

With the help of experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field. That expression is based on the following experimental observations for the magnetic field $d\vec{B}$ at a point P associated with a length element $d\vec{\ell}$ of a wire carrying a steady current I.

$$dB \propto I,\ dB \propto d\ell,\ dB \propto \sin\theta \text{ and } dB \propto \frac{1}{r^2} \Rightarrow dB \propto \frac{Id\ell\sin\theta}{r^2} \quad dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$$

where $\mu_0/4\pi$ = constant = $10^{-7}\,\frac{T\times m}{A}$, μ_0 is permeability constant of free space.

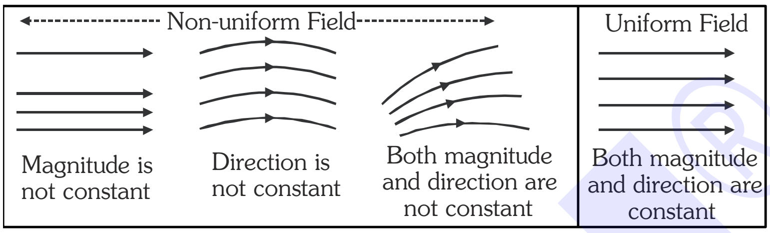
Vector form of Biot-Savart's law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}\hat{n} \qquad \hat{n} = \text{unit vector perpendicular to the plane of } (Id\vec{\ell}) \text{ and } (\vec{r})$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3} \qquad [\because\ Id\vec{\ell}\times\vec{r} = (Id\ell)\,(r)\sin\theta\ \hat{n}]$$

Magnetic field lines (By Michael Faraday)

In order to visualise a magnetic field graphically, Michael faraday introduced the concept of field lines. Field lines of magnetic field are imaginary lines which represents direction of magnetic field continuously.



- Magnetic field lines emanate from or enters in the surface of a magnetic material at any angle.
- Magnetic field lines exist inside every magnetised material.
- Magnetic field lines can be mapped by using iron dust or using compass needle.

2. SPECIAL THUMB RULES

2.1 Right Hand Thumb Rule

This rule gives the pattern of magnetic field lines due to current carrying straight and circular wire.

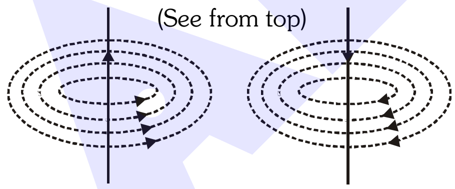
(i) Straight current :

Thumb $\rightarrow$ In the direction of current

Curling fingers $\rightarrow$ Gives field lines pattern

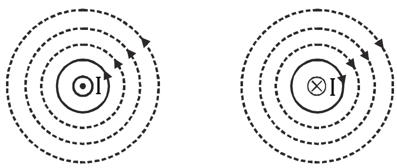
Case I : wire in the plane of the paper:-

(See from top)



ACW, concentric & circular field lines | CW, concentric & circular field lines

Case II : Wire is $\perp$ to the plane of the paper:-



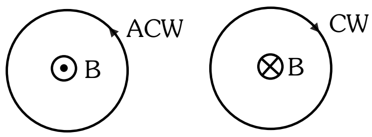
ACW, concentric & circular field lines | CW, concentric & circular field lines

(ii) Circular current :

Curling fingers $\rightarrow$ In the direction of current

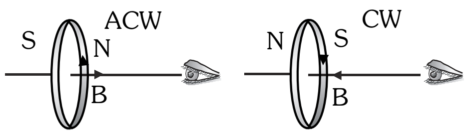
Thumb $\rightarrow$ Gives field line pattern

Case I : wire in the plane of the paper



Towards observer or perpendicular out-wards | Away from the observer or perpendicular inwards

Case II : Wire is $\perp$ to the plane of the paper

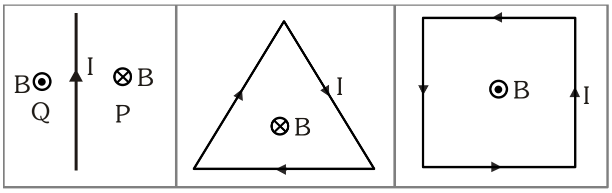


Towards observer | Away from the observer

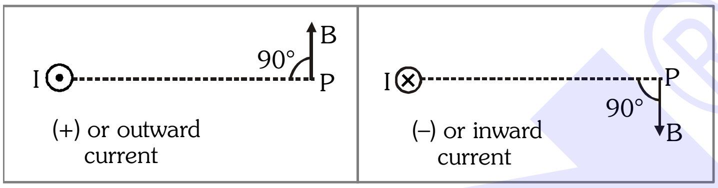
2.2 Right Hand Palm Rule (By Flemming)

This rule gives the direction of magnetic field vector at a point due to current carrying straight wire.

Case I :– Wire is in the plane of the paper :- **(knock–knock rule)**

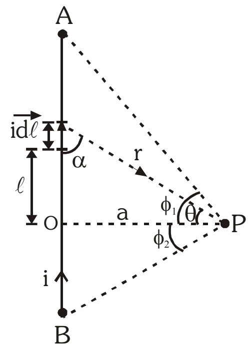


Case II :– Wire is perpendicular to the plane of the paper :- **(Sliding rule)**



3. APPLICATION OF BIOT-SAVART LAW :

3.1 Magnetic field surrounding a thin straight current carrying conductor (Derivation is Only for Board Purpose)



AB is a straight conductor carrying current i from B to A. At a point P, whose perpendicular distance from AB is OP =a, the direction of field is perpendicular to the plane of paper, inwards (represented by a cross)

$\ell = a\tan\theta \Rightarrow d\ell = a\sec^2\theta d\theta$ ...(i)

$\alpha = 90° - \theta$ & $r = a\sec\theta$

By Biot-savart's Law

$dB = \frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2}\otimes$ (due to a current element $id\ell$ at point P)

$\Rightarrow B = \int dB = \int\frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2}$ (due to wire AB) $\therefore B = \frac{\mu_0 i}{4\pi a}\int\cos\theta d\theta$

Taking limits of integration as $-\phi_2$ to ϕ_1

$$B = \frac{\mu_0 i}{4\pi a}\int_{-\phi_2}^{\phi_1}\cos\theta d\theta = \frac{\mu_0 i}{4\pi a}[\sin\theta]_{-\phi_2}^{\phi_1} = \frac{\mu_0 i}{4\pi a}[\sin\phi_1 + \sin\phi_2] \text{ (inwards)}$$

GOLDEN KEY POINTS

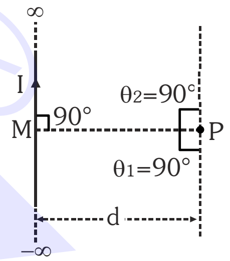
- According to $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$, direction of magnetic field vector $(d\vec{B})$ is always perpendicular to the plane of vectors $(Id\vec{\ell})$ and $(\vec{r})$, where plane of $(Id\vec{\ell})$ and $(\vec{r})$ is the plane of wire.
- Magnetic field on the axis of current carrying conductor is always zero ($\theta=0°$ or $\theta = 180°$)
- Magnetic field lines are closed curves.
- Tangent drawn at any point on field line represents direction of the field at that point.

- Field lines never intersects to each other.
- At any place crowded lines represent stronger field while distant lines represents weaker field.
- In any region, if field lines are **equidistant and straight** the field is uniform otherwise not.
- When current is straight, field is circular.
- When current is circular, field is straight (along axis).
- When wire is in the plane of paper, the field is perpendicular to the plane of the paper.
- When wire is perpendicular to the plane of paper, the field is in the plane of the paper.

Illustrations

Illustration 1.

Find magnetic field due to infinite length wire at point 'P'.



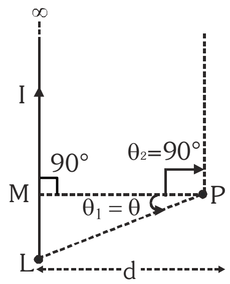
Solution

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin 90° + \sin 90°]$$

$$B_P = \frac{\mu_0 I}{2\pi d}$$

Illustration 2.

Find magnetic field due to given wire at point 'P'.



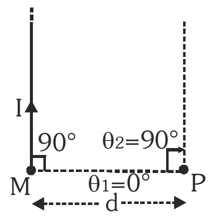
Solution

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + \sin 90°]$$

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + 1]$$

Illustration 3.

Find magnetic field due to semi infinite length wire at point 'P'.



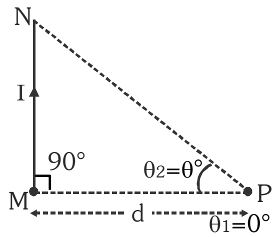
Solution

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin 0° + \sin 90°]$$

$$B_P = \frac{\mu_0 I}{4\pi d}$$

Illustration 4.

Find magnetic field due to finite length wire at point 'P'.

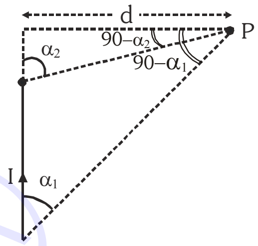


Solution

$$B_P = \frac{\mu_0 I}{4\pi d}\left[\sin 0^\circ + \sin\theta\right] \quad ; B_P = \frac{\mu_0 I}{4\pi d}\sin\theta$$

Illustration 5.

If point 'P' lies out side the line of wire then magnetic field at point 'P' will be :



Solution

$$B_P = \frac{\mu_0 I}{4\pi d}\left[\sin(90-\alpha_1)-\sin(90-\alpha_2)\right] = \frac{\mu_0 I}{4\pi d}(\cos\alpha_1 - \cos\alpha_2)$$

Illustration 6.

Same current i is flowing in three infinitely long wires along positive x, y and z directions. The magnetic field at a point (0, 0, –a) would be

Solution :

Point (0, 0, –a) lies on z-axis, therefore magnetic field due to current along z-axis is zero and due to rest two wires is $\frac{\mu_0 I}{2\pi a}$ in mutually perpendicular directions along positive y direction and negative x direction.

$$\therefore\ \vec{B} = \frac{\mu_0}{2\pi}\frac{i}{a}(\hat{j}-\hat{i})$$

BEGINNER'S BOX-1

1. A proton moves along horizontal line and towards observer, the pattern of concentric circular field lines of magnetic field which produced due to its motion :-

(1) ACW, In horizontal plane (2) ACW, In vertical plane
(3) CW, In horizontal plane (4) CW, In vertical plane

2. An electron moves along vertical line and towards observer, the pattern of concentric circular field lines of magnetic field which is produced due to its motion :-

(1) ACW, In horizontal plane (2) ACW, In vertical plane
(3) CW, In horizontal plane (4) CW, In vertical plane

3. A current is flowing in electricity line towards north, the direction of magnetic field at a point which is just below the line :-

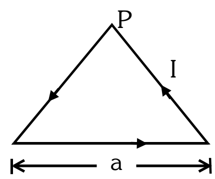
(1) towards north (2) towards south (3) towards east (4) towards west

4. A current is flowing in a vertical wire in downward direction, the direction of magnetic field of a point which is just right side to the wire :-

(1) towards north (2) towards south (3) towards east (4) towards west

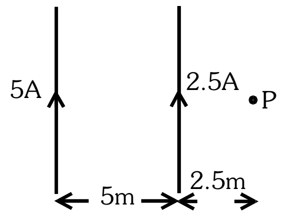
5. An equilateral triangle of side 'a' carries a current 'I'.

Magnetic field at point 'P' which is vertex of triangle. :-



(1) $\frac{\mu_0 I}{2\sqrt{3}\pi a}\odot$ (2) $\frac{\mu_0 I}{2\sqrt{3}\pi a}\otimes$ (3) $\frac{9}{2}\left(\frac{\mu_0 I}{\pi a}\right)\odot$ (4) $\frac{9}{2}\left(\frac{\mu_0 I}{\pi a}\right)\otimes$

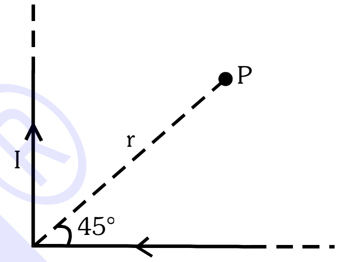
6. Magnetic field at point 'P' due to both infinite long current carrying wires is :-



(1) $\frac{\mu_0}{2\pi}\otimes$ (2) $\frac{5\mu_0}{6\pi}\otimes$

(3) $\frac{5\mu_0}{6\pi}\odot$ (4) $\frac{\mu_0}{2\pi}\odot$

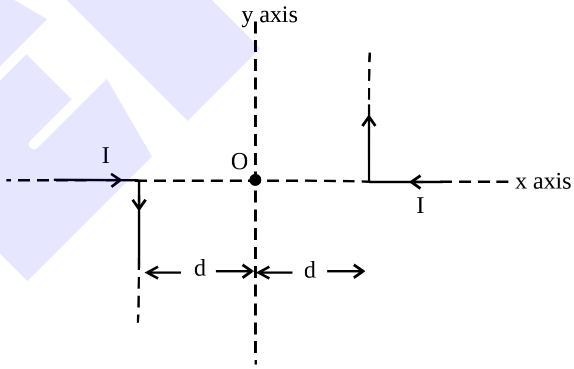
7. Magnetic field at point 'P' due to given current distribution is :-



(1) $\frac{\mu_0 I}{4\pi r}\left(1+\sqrt{2}\right)\odot$ (2) $\frac{\mu_0 I}{2\pi r}\left(1+\sqrt{2}\right)\odot$

(3) $\frac{\mu_0 I}{4\pi r}\left(1+\sqrt{2}\right)\otimes$ (4) $\frac{\mu_0 I}{2\pi r}\left(1+\sqrt{2}\right)\otimes$

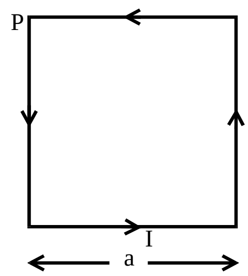
8. Magnetic field at origin 'O' due to given current distribution is :-



(1) $\frac{\mu_0 I}{2\pi d}\odot$ (2) $\frac{\mu_0 I}{2\pi d}\otimes$

(3) $\frac{\mu_0 I}{4\pi d}\odot$ (4) $\frac{\mu_0 I}{4\pi d}\otimes$

9. A square loop of side 'a' is made by a current carrying wire. Magnetic field at its vertex 'P' is :-



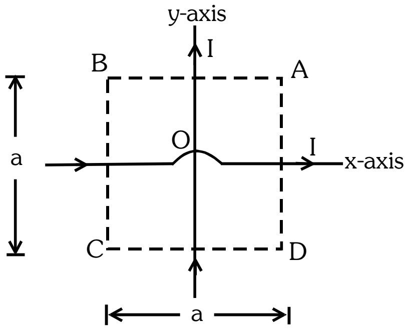
(1) $\frac{\mu_0 I}{2\sqrt{2}\pi a}\odot$ (2) $\frac{\mu_0 I}{2\sqrt{2}\pi a}\otimes$

(3) $2\sqrt{2}\frac{\mu_0 I}{\pi a}\otimes$ (4) $2\sqrt{2}\frac{\mu_0 I}{\pi a}\odot$

10. Two long parallel wires carry i and 2i current in same direction. Magnetic field just between the wires is 'B'. If 2i current is switched off then magnetic field at the same point is :-

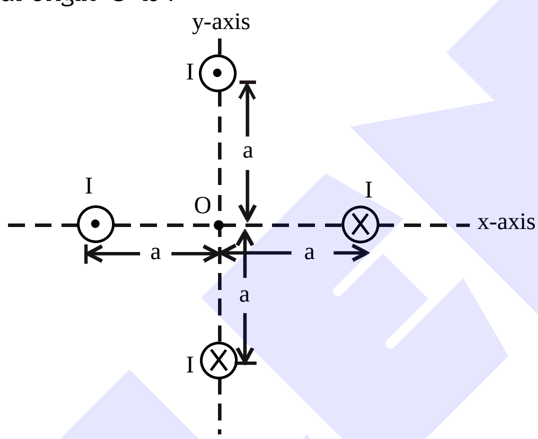
(1) 2B (2) B (3) B/2 (4) $\sqrt{2}$ B

11. Two infinite length wires carry equal current and placed along x and y axis respectively. At which points the resultant magnetic field is zero ?



(1) A, B (2) B, D (3) A, C (4) C, D

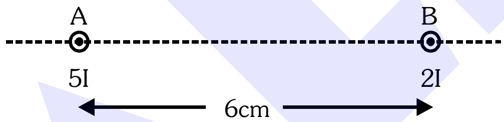
12. For given current distribution, each infinite length wire produces magnetic field 'B' at origin then resultant magnetic field at origin 'O' is :-



(1) 4B (2) $\sqrt{2}$ B (3) $2\sqrt{2}$ B (4) zero

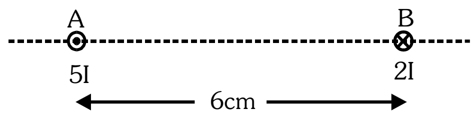
13. The position of point from wire 'B', where net magnetic field is zero due to following current distribution.

(a)



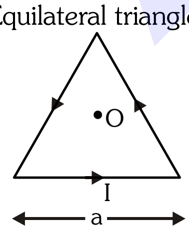
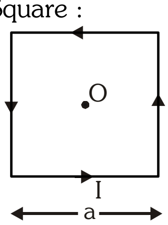
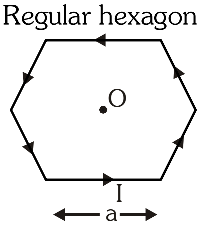
(1) 6/7 cm (2) 12/7 cm
(3) 18/17 cm (4) 16/7 cm

(b)



(1) 4 cm (2) 2 cm
(3) 8 cm (4) 12 cm

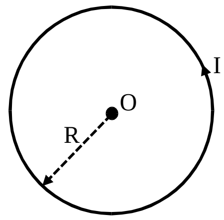
14. Magnetic field at the centre of various **regular polygons,** which are formed by current carrying wires.

(a) Equilateral triangle :	(b) Square :	(c) Regular hexagon
		
(1) $\frac{\mu_0 I}{2\sqrt{3}\pi a}$ (2) $\frac{27\mu_0 I}{2\pi a}$	(1) $\frac{2\sqrt{2}\mu_0 I}{\pi a}$ (2) $\frac{\mu_0 I}{2\sqrt{2}\pi a}$	(1) $\frac{6\mu_o I}{\pi a}$ (2) $\frac{\sqrt{3}\mu_0 I}{\pi a}$
(3) $\frac{9\mu_0 I}{2\pi a}$ (4) $\sqrt{3}\frac{\mu_0 I}{\pi a}$	(3) $8\sqrt{2}\frac{\mu_0 I}{\pi a}$ (4) $\sqrt{2}\frac{\mu_o I}{\pi a}$	(3) $6\sqrt{3}\frac{\mu_0 I}{\pi a}$ (4) $\frac{\mu_o I}{\sqrt{3}\pi a}$

3.2 Magnetic Field at the Centre of Current Carrying Circular Loop and Coil

(a) **Circular loop :-** (N = 1)

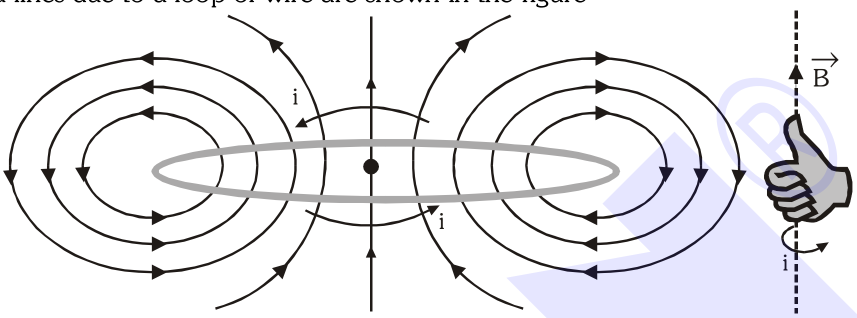
$$B_0 = \frac{\mu_0 I}{2R}$$



(b) **Circular coil :-** (N > 1)

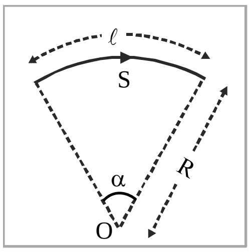
$$B_0 = \frac{\mu_0 NI}{2R}$$, Where N number of turns in coil

- **Magnetic field due to a loop of current**
  Magnetic field lines due to a loop of wire are shown in the figure



The direction of magnetic field on the axis of current loop can be determined by right hand thumb rule. If fingers of right hand are curled in the direction of current, the stretched thumb is in the direction of magnetic field.

3.3 Magnetic field at the centre of current carrying circular arc



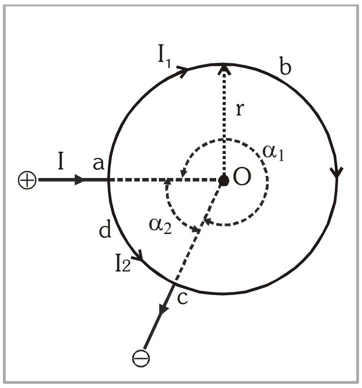
$$B_{o\alpha} = \frac{\mu_0 I\alpha}{4\pi R}$$

$$B_{o\alpha} = \left(\frac{B_o}{2\pi}\right)\alpha \qquad \text{Where } \alpha \text{ is always in radian and } B_0 = \frac{\mu_0 I}{2R}$$

3.4 Magnetic Field at the Centre of Current Carrying Loop

(i) Wire of circuit loop is uniform :

Magnetic field at the centre of arc abc and adc is



$$\left.\begin{aligned} B_{abc} &= \frac{\mu_0 I_1 \alpha_1}{4\pi r} \\ B_{adc} &= \frac{\mu_0 I_2 \alpha_2}{4\pi r} \end{aligned}\right\} \Rightarrow \frac{B_{abc}}{B_{adc}} = \frac{I_1\alpha_1}{I_2\alpha_2} \quad \text{........ (1)}$$

$$\because \text{angle} = \frac{\text{arc length}}{\text{Radius}} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\ell_1}{\ell_2} \quad \text{.........(2)}$$

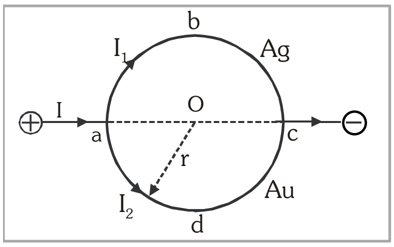
$$\text{and} \quad \left.\begin{aligned} V = IR &\Rightarrow I \propto \frac{1}{R} \\ R = \frac{\rho\ell}{A} &\Rightarrow R \propto \ell \end{aligned}\right\} \Rightarrow \frac{I_1}{I_2} = \frac{\ell_2}{\ell_1} \quad \text{.........(3)}$$

put the value of (2) & (3) in (1)

$$\frac{B_{abc}}{B_{adc}} = \left(\frac{\ell_2}{\ell_1}\right)\left(\frac{\ell_1}{\ell_2}\right) \Rightarrow \boxed{\frac{B_{abc}}{B_{adc}} = \frac{1}{1}}$$

- Magnetic field produced by both the arc is at the centre of the circuit loop is equal in magnitude and opposite in direction, So $(B_0)_{net} = 0$ (always and it is free from angle of connection of terminals)

(ii) Wire of circuit loop is non uniform :



Case I : Thickness $\longrightarrow$ Same, Material $\longrightarrow$ different :-

Magnetic field at the centre of arc abc and adc is –

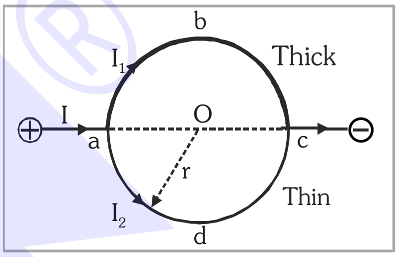
$$B_{abc} = \frac{\mu_0 I_1}{4r} \otimes,\ B_{adc} = \frac{\mu_0 I_2}{4r} \odot$$

Both arc connected in parallel with source, so V = IR = constant

$$I \propto \frac{1}{R} \Rightarrow R_{A_u} > R_{A_g} \Rightarrow I_{A_u} < I_{A_g}$$

Net magnetic field at centre

$$\boxed{(B_0)_{net} = \frac{\mu_0}{4r}(I_1 - I_2)} \otimes \text{ (where } I_1 > I_2)$$



Case II : Thickness $\longrightarrow$ different, Material $\longrightarrow$ same :–

Magnetic field at the centre of arc abc and adc

$$B_{abc} = \frac{\mu_0 I_1}{4r} \otimes,\ B_{adc} = \frac{\mu_0 I_2}{4r} \odot$$

Both arc are connected in parallel with source, so

V = IR = constant and resistance of wire $R = \frac{\rho \ell}{A}$

$$\boxed{I \propto \frac{1}{R}} \ ....(1) \qquad \boxed{R \propto \frac{1}{A}} \ ....(2) \qquad \text{From (1) \& (2)} \ \boxed{I \propto A} \Rightarrow I_{thick} > I_{thin}$$

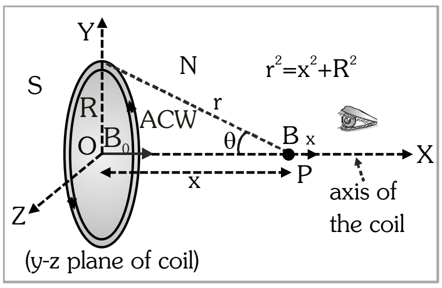
Net magnetic field at centre

$$\boxed{(B_0)_{net} = \frac{\mu_0}{4r}(I_1 - I_2)} \otimes \text{ (where } I_1 > I_2)$$

3.5 Magnetic Field at axial point of the current carrying circular coil

magnetic field at axial point 'P' given by :-

$$B_x = \frac{\mu_0 N I R^2}{2\left(x^2 + R^2\right)^{3/2}} = B_{centre}\sin^3\theta \text{ where } \sin\theta = \frac{R}{\sqrt{R^2 + x^2}}$$



(a) At the centre, x=0, $B_{centre} = \frac{\mu_0 N i}{2R}$

(b) At points very close to centre,

$$x << R \Rightarrow B = \frac{\mu_0 N i}{2R}\left(1 + \frac{x^2}{R^2}\right)^{-3/2} = \frac{\mu_0 N i}{2R}\left(1 - \frac{3x^2}{2R^2}\right)$$

(c) At points far off from the centre, $x >> R \Rightarrow B \simeq \frac{\mu_0 N i}{4\pi}\frac{2\pi R^2}{x^3}$

- **B–x curve :-**

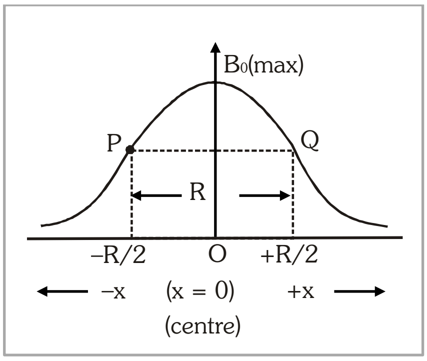
  (a) It is symmetrical about the centre of coil (x = 0)

  (b) (P, Q) points → Points of inflexion; → Points of curvature change; → Points of zero curvature

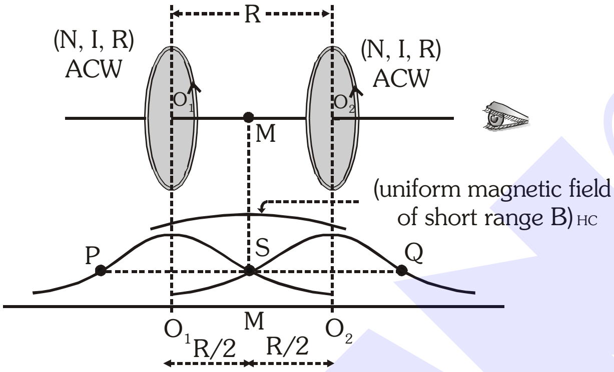
  (c) Points of inflexion are R/2 distance from the centre of the coil.

  (d) Separation between points of inflexion is equal to radius of coil (R).

  (e) Application of point of inflexion is **"Helmholtz coils arrangement".**



Helmholt'z coils arrangement



This arrangement is used to produce **uniform** magnetic field of **short range.**

It consists :-

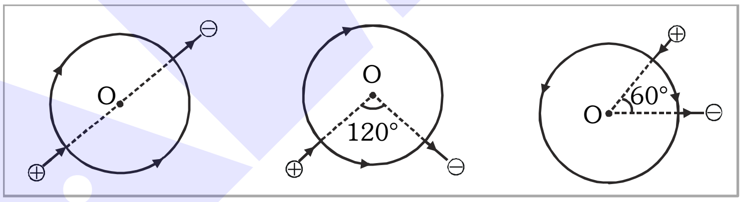
- Two identical co–axial coils (N, I, R same)
- Placed at distance (center to center) equal to radius ('R') of coils
- Planes of both coils are parallel to each other.
- Current direction is same in both coils (observed from same side) otherwise this arrangement is not called "Helmholtz coil arrangement".

GOLDEN KEY POINTS

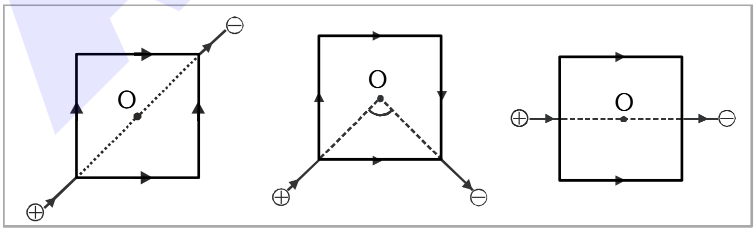
- Magnetic field at the centre of any **geometrically symmetrical circuit loop** (which is made by uniform wire) is always zero

  Examples :

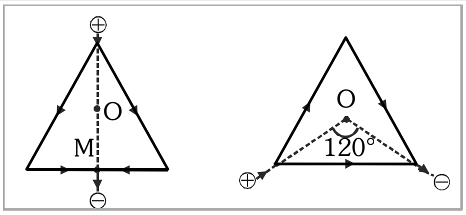
  (a) Circle



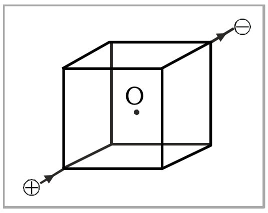
  (b) Square



  (c) Equilateral triangle



  (d) Cube



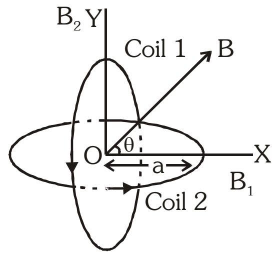
Illustrations

Illustration 7.

For the arrangement made up of two identical coils as shown in figure, determine the magnetic field at the centre O.

Solution :

The two coils are perpendicular to each other. Coil 1 produce field along X axis and coil 2 produce field along y axis. thus the resultant field will be



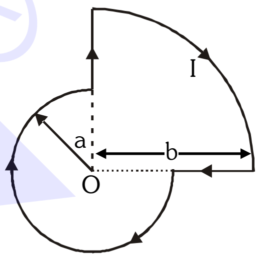
$$B = \sqrt{B_1^2 + B_2^2}$$

making an angle θ from x-axis $\tan\theta = \dfrac{B_2}{B_1}$

For identical coils, $B_1 = B_2$; $B = \sqrt{2}\left(\dfrac{\mu_0 NI}{2a}\right)$ and hence θ = 45°

Illustration 8.

The magnetic induction at the centre O is ?



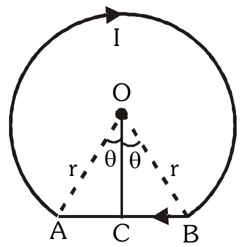
Solution :

$$B = \frac{3}{4}\left[\frac{\mu_0 I}{2a}\right] + \frac{1}{4}\left[\frac{\mu_0 I}{2b}\right] \otimes \qquad B = \frac{3\mu_0 I}{8a} + \frac{\mu_0 I}{8b} \otimes$$

Illustration 9.

A wire is bent in the form of a circular arc with a straight portion AB. Magnetic induction at O when current I flowing in the wire, is ?

Solution :



$$B_{AB} = \frac{\mu_0 I}{4\pi(OC)}[2\sin\theta]$$

But OC = rcosθ or $B_{AB} = \dfrac{\mu_0 I}{2\pi r}\tan\theta$

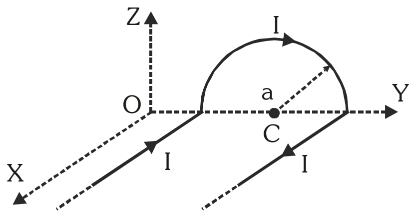
Magnetic field due to circular portion,

$$B_{AB} = \frac{\mu_0 I}{2r}\left(\frac{2\pi - 2\theta}{2\pi}\right) = \frac{\mu_0 I}{2\pi r}(\pi - \theta)$$

Total magnetic field $= \dfrac{\mu_0 I}{2\pi r}\tan\theta + \dfrac{\mu_0 I}{2\pi r}(\pi - \theta) = \dfrac{\mu_0 I}{2\pi r}[\tan\theta + \pi - \theta]$

Illustration 10.

A long wire bent as shown in the figure carries current I. If the radius of the semi-circular portion is "a" then find the magnetic induction at the centre C.



Solution

Due to semi circular part $\vec{B}_1 = \dfrac{\mu_0 I}{4a}\left(-\hat{i}\right)$

due to parallel parts of currents $\vec{B}_2 = 2 \times \dfrac{\mu_0 I}{4\pi a}(-\hat{k})$, $\vec{B}_{net} = \vec{B}_C = \vec{B}_1 + \vec{B}_2 = \dfrac{\mu_0 I}{4a}(-\hat{i}) + \dfrac{\mu_0 I}{2\pi a}(-\hat{k})$

magnitude of resultant field $B_{net} = \sqrt{B_1^2 + B_2^2} = \dfrac{\mu_0 I}{4\pi a}\sqrt{\pi^2 + 4}$

BEGINNER'S BOX-2

1. The magnetic field at the centre of a circular coil of radius r carrying current is B_1. The field at the centre of another coil of radius $\frac{r}{2}$ carrying same current is B_2 then ratio of B_1/B_2 is :-

(1) 1 : 2 (2) 2 : 1 (3) 1 : 1 (4) 4 : 1

2. A circular coil of one turn in formed by a 6.28m length wire, which carries a current of 3.14A. The magnetic field at the centre of coil is :-

(1) 1×10^{-6} T (2) 4×10^{-6} T (3) 0.5×10^{-6} T (4) 2×10^{-6} T

3. A length of wire carries a steady current, is bent first to form a plane circular coil of one turn, same length now bent more sharply to give three turns of smaller radius. Magnetic field becomes :-

(1) 3 times (2) 1/3 times (3) 9 times (4) unchange

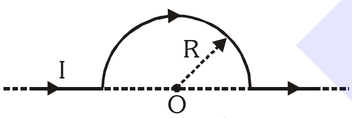
4. Two concentric coplanner coils of equal turns have radii 10 cm and 30 cm respectively. Same current flowing in both the coils in same direction. Now direction of current is reversed in one coil then ratio of magnetic field at their common centre in two conditions respectively :-

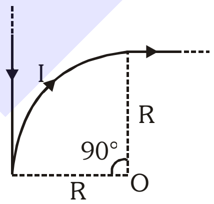
(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 4 : 1

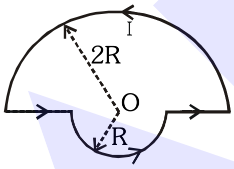
5. Two concentric coplanner coils of turns n_1 and n_2 have radii ratio 2 : 1 respectively. Equal current in both the coils flows in opposite direction. If net magnetic field is zero at their common centre then $n_1 : n_2$ is :-

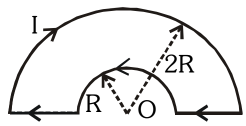
(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 4 : 1

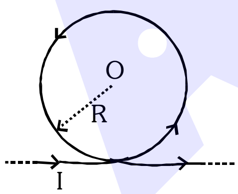
6. Find out magnetic field at point 'O' for the following current distributions.

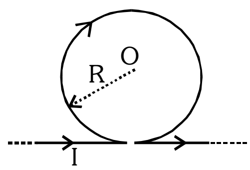
(i) **(a)**


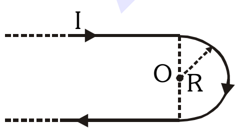
(b)


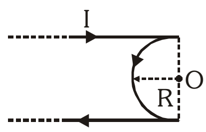
(ii) **(a)**


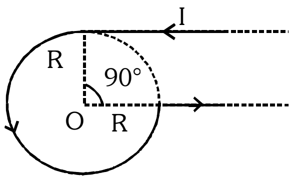
(b)


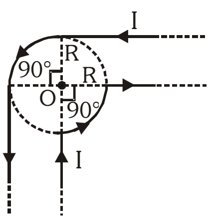
(iii) **(a)**


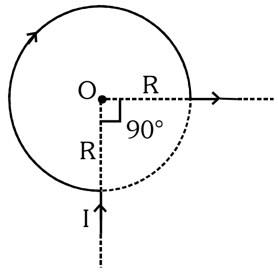
(b)


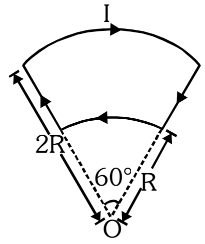
(iv) **(a)**


(b)


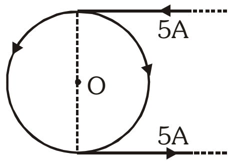
(v) **(a)**


(b)


(vi) **(a)** 

(b) 

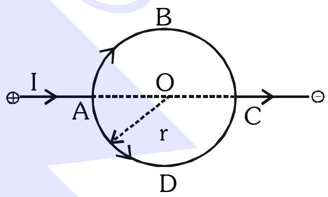
7. Magnetic field at point 'O' due to given current distribution. If 5A current is flowing in this system and the diameter of the loop is 10cm.



(1) 2×10^{-5} T, $\otimes$ (2) 10^{-5}T, $\odot$

(3) 10^{-5} T, $\otimes$ (4) 2×10^{-5} T, $\odot$

8. Figure shows a circular loop with radius 'r'. The resistance of arc ABC is 5Ω and that of ADC is 10Ω. Magnetic field at the centre of the loop is :-



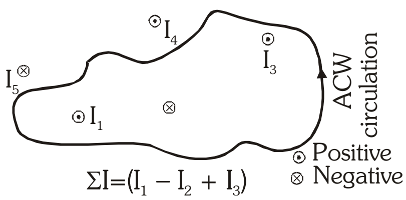
(1) $\frac{\mu_0 I}{6r} \otimes$ (2) $\frac{\mu_0 I}{12r} \otimes$ (3) $\frac{\mu_0 I}{12r} \odot$ (4) $\frac{\mu_0 I}{6r} \odot$

9. Radius of current carrying coil is 'R'. If fractional decreases in field value with respect to centre of the coil for a near by axial point is 1% then find axial position of that point.

---

4. AMPERE'S CIRCUITAL LAW

Ampere's circuital law states that line integral of the magnetic field around any closed path in free space or vacuum is equal to μ_0 times of the net current or the total current which is crossing through the area bounded by the closed path. Mathematically $\boxed{\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I}$



This law is independent of size and shape of the closed path.

Any current outside the closed path is not included in writing the right hand side of the law.

Note : This law is suitable for infinite long and symmetrical current distribution.

Application of Ampere's circuital law

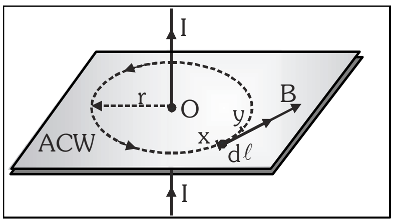
4.1 Magnetic field due to infinite long thin current carrying straight conductor

Consider a circle of radius 'r'. Let XY be the small element of length $d\ell$. $\vec{B}$ and $d\vec{\ell}$ are in the same direction because direction of magnetic field is along the tangent of the circle. By A.C.L.

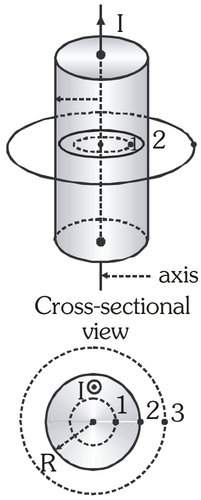
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I, \ \oint B d\ell \cos\theta = \mu_0 I \quad (\text{where } \theta = 0°)$$

$$\oint B \ d\ell \cos 0° = \mu_0 I \ \Rightarrow B \oint d\ell = \mu_0 I \quad (\text{where } \oint d\ell = 2\pi r)$$

$$B\,(2\pi r) = \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi r}$$



4.2 Magnetic field due to infinite long solid cylindrical conductor



- For a point inside the cylinder $r < R$, Current from area $\pi R^2 = I$

  so current from area $\pi r^2 = \frac{I}{\pi R^2}(\pi r^2) = \frac{Ir^2}{R^2}$

  By Ampere circuital law for circular path 1 of radius r

  $B_{in}(2\pi r) = \mu_0 I' = \mu_0 \frac{Ir^2}{R^2} \Rightarrow B_{in} = \frac{\mu_0 Ir}{2\pi R^2} \Rightarrow B_{in} \propto r$

- For a point on the axis of the cylinder ($r = 0$); $B_{axis} = 0$
- For a point on the surface of cylinder ($r = R$)

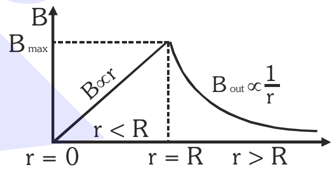
  By Ampere circuital law for circular path 2 of radius R

  $B_s(2\pi R) = \mu_0 I \Rightarrow B_s = \frac{\mu_0 I}{2\pi R}$ (it is maximum)

- For a point outside the cylinder ($r > R$) :-

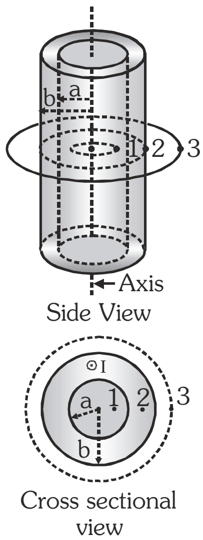
  By Ampere circuital law for circular path 3 of radius r

  $B_{out}(2\pi r) = \mu_0 I \Rightarrow B_{out} = \frac{\mu_0 I}{2\pi r} \Rightarrow B_{out} \propto \frac{1}{r}$



Magnetic field outside the cylindrical conductor does not depend upon nature (thick/thin or solid/hollow) of the conductor as well as its radius of cross section.

4.3 Magnetic field due to infinite long hollow cylindrical conductor



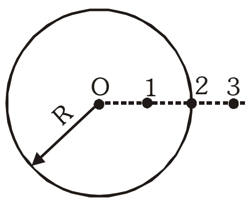
- For a point at a distance r such that $r < a < b$ $B_1 = 0$
- For a point at a distance r such that $a < r < b$

  $B_2(2\pi r) = \mu_0 I' \Rightarrow$ $B_2(2\pi r) = \mu_0 I\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$

  $\Rightarrow B_2 = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$
  - $r = a$ (inner surface) $\Rightarrow B_{IS} = 0$
  - $r = b$ (outer surface) $\Rightarrow B_{OS} = \frac{\mu_0 I}{2\pi b}$ (maximum)

- For a point at a distance r such that $r > b > a$, $B_3(2\pi r) = \mu_0 I \Rightarrow B_3 = \frac{\mu_0 I}{2\pi r}$
- For a point at the axis of cylinder $r = 0$ $B_{axis} = 0$

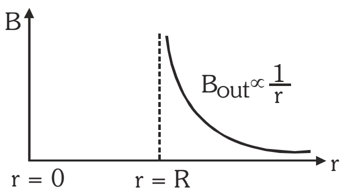
4.4 Magnetic field at specific positions for thin hollow cylindrical conductor



At point 1 $B_1 = 0$ (for the point on axis $B_{axis} = 0$)

At point 2 $B_2 = \frac{\mu_0 I}{2\pi R}$

At point 3 $B_3 = \frac{\mu_0 I}{2\pi r}$



GOLDEN KEY POINTS

Magnetomotive force (M.M.F.)

As $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I$, where $\vec{B} = \mu_0 \vec{H}$, $\oint \mu_0 \vec{H} \cdot d\vec{\ell} = \mu_0 \Sigma I \Rightarrow \oint \vec{H} \cdot d\vec{\ell} = \Sigma I$

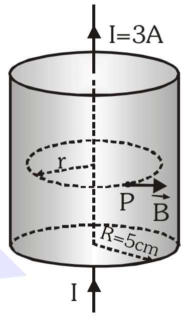
The line integral of magnetising field around any closed path is equal to net current crossing through the area bounded by the closed path, also called 'magnetomotive force'. Magnetomotive force (M.M.F.) = $\oint \vec{H} . d\vec{\ell}$

Illustrations

Illustration 11.

A long straight solid conductor of radius 5 cm carries a current of 3A, which is uniformly distributed over its circular cross-section. Find the magnetic field induction at a distance 4 cm from the axis of the conductor. (Relative permeability of the conductor = 1000).

Solution



$B = \dfrac{\mu_0 \mu_r I r}{2\pi R^2}$ (inside the solid conductor)

$\therefore B = \dfrac{4\pi \times 10^{-7} \times 1000 \times 3 \times 0.04}{2\pi \times (0.05)^2} = 9.6 \times 10^{-3}$ T.

BEGINNER'S BOX-3

1. A solid cylinderical wire of radius 'R' Carries a current 'I'. The ratio of magnetic fields at points which are located at R/2 and 2R distance away from the axis of the wire :-

(1) 1 : 1 (2) 1 : 2
(3) 2 : 1 (4) 1 : 4

2. A solid cylinderical wire of radius 'R' carries a current 'I'. The magnetic field is 5μT at a point, which is '2R' distance away from the axis of wire. Magnetic field at a point which is R/3 distance inside from the surface of the wire is :-

(1) $\frac{10}{3}\mu T$ (2) $\frac{20}{3}\mu T$ (3) $\frac{5}{3}\mu T$ (4) $\frac{40}{3}\mu T$

3. A hollow cylinderical wire carries a current I, having inner and outer radii 'R' and 2R respectively. Magnetic field at a point which 3R/2 distance away from its axis is :-

(1) $\frac{5\mu_0 I}{18\pi R}$ (2) $\frac{\mu_0 I}{36\pi R}$ (3) $\frac{5\mu_0 I}{36\pi R}$ (4) $\frac{5\mu_0 I}{9\pi R}$

4. A long, straight and solid metal wire of radius 2mm carries a current uniformly distributed over its circular cross section. The magnetic field at a distance 2 mm from its axis is B. Magnetic field at a distance 1 mm from the axis of the wire is :-

(1) B (2) 4B (3) 2B (4) B/2

5. A long straight wire (radius = 3.0 mm) carries a constant current distributed uniformly over a cross section perpendicular to the axis of the wire. If the current density is 100 A/m^2. The magnitudes of the magnetic field at (a) 2.0 mm from the axis of the wire and (b) 4.0 mm from the axis of the wire is :-

(1) $2\pi \times 10^{-8}$ T, $\frac{9\pi}{2} \times 10^{-8}$ T (2) $4\pi \times 10^{-8}$ T, $\frac{\pi}{2} \times 10^{-8}$ T

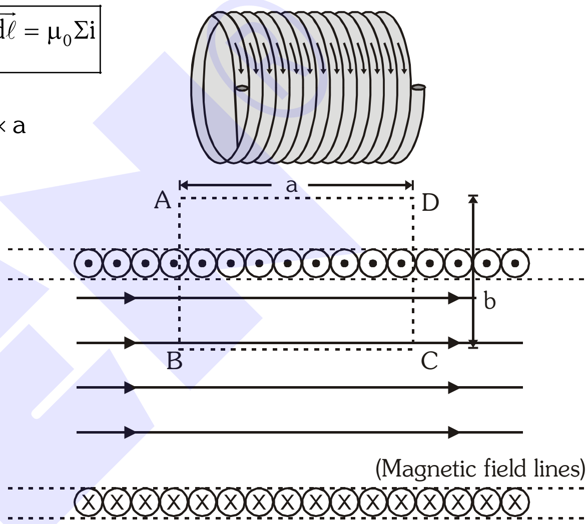
(3) $4\pi \times 10^{-8}$ T, $\frac{9\pi}{2} \times 10^{-8}$ T (4) $\pi \times 10^{-4}$ T, $\frac{9\pi}{2} \times 10^{-8}$ T

4.5 Magnetic field due to solenoid

It is a coil which has length and used to produce uniform magnetic field of long range. It consists a conducting wire which is tightly wound over a cylinderical frame in the form of helix. All the adjacent turns are electrically insulated to each other. The magnetic field at a point on the axis of a solenoid can be obtained by superposition of field due to large number of identical circular turns having their centres on the axis of solenoid.

- **Magnetic field due to a long solenoid (Derivation Only for Board)**

A solenoid is a tightly wound helical coil of wire. If length of solenoid is large, as compared to its radius, then in the central region of the solenoid, a reasonably uniform magnetic field is present. Figure shows a part of long solenoid with number of turns/length n. We can find the field by using Ampere circuital law.



Consider a rectangular loop ABCD. For this loop $\boxed{\oint_{ABCD} \vec{B}.\vec{d\ell} = \mu_0 \Sigma i}$

Now $\oint_{ABCD} \vec{B}.\vec{d\ell} = \oint_{AB} \vec{B}.\vec{d\ell} + \oint_{BC} \vec{B}.\vec{d\ell} + \oint_{CD} \vec{B}.\vec{d\ell} + \oint_{DA} \vec{B}.\vec{d\ell} = B \times a$

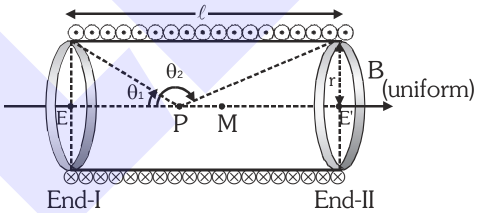
This is because $\oint_{AB} \vec{B}.\vec{d\ell} = \oint_{CD} \vec{B}.\vec{d\ell} = 0,\ \vec{B} \perp \vec{d\ell}$.

And, $\oint_{DA} \vec{B}.\vec{d\ell} = 0$ ($\because$ $\vec{B}$ outside the solenoid is negligible)

Now, $\Sigma i = (n \times a) \times i \Rightarrow B \times a = \mu_0 (n \times a \times i) \Rightarrow B = \mu_0 ni$

Finite length solenoid :

Its length and diameter are comparable.



By the concept of BSL magnetic field at the axial point 'P' obtained as : $B_P = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$

Angle θ_1 and θ_2 both measured in same sense from the axis of the solenoid to end vectors.

Infinite length solenoid :

Its length very large as compared to its diameter i.e. ends of solenoid tends to infinity.

(a) Magnetic field at axial point which is well inside the solenoid

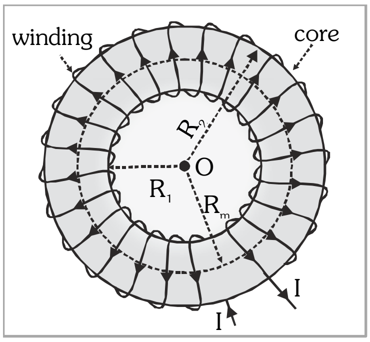
$$\theta_1 \simeq 0° \text{ and } \theta_2 \simeq 180° \Rightarrow B \simeq \frac{\mu_0 nI}{2}[\cos 0° - \cos 180°] \simeq \frac{\mu_0 nI}{2}[(1) - (-1)] \simeq \mu_0 nI$$

(b) Magnetic field at both axial end points of solenoid

$$\theta_1 = 90° \text{ and } \theta_2 \simeq 180° \Rightarrow B \simeq \frac{\mu_0 nI}{2}[\cos 90° - \cos 180°] \simeq \frac{\mu_0 nI}{2}[(0) - (-1)] \simeq \frac{\mu_0 nI}{2}$$

4.6 Magnetic Field Due to Toroid

A toroid can be considered as a ring shaped closed solenoid also called end less solenoid. Magnetic field inside a toroid by A.C.L. given as :–



$$\boxed{B = \mu_0 nI}$$

Where $n \Rightarrow$ turn density $= \left(\dfrac{N}{2\pi R_m}\right)$

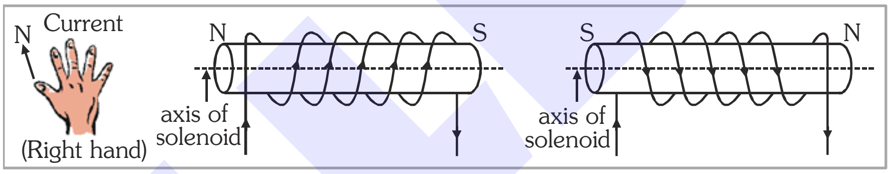
$N \Rightarrow$ total number of turns.

$R_m \Rightarrow$ mean radius of toroid $= \left(\dfrac{R_1 + R_2}{2}\right)$

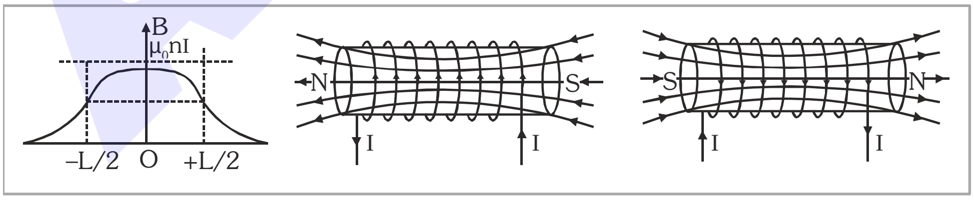
R_1 & $R_2 \Rightarrow$ internal and external radius of toroid respectively.

GOLDEN KEY POINTS

- Solenoid normally taken as infinite long (ℓ >>>> R)
- Magnetic field produced by solenoid does not depend on its radius of cross section (or average diameter)
- **Maxwell's solenoid rule :–** This rule is used to find out polarity of solenoid.



- Magnetic field produced by solenoid is directed along its axis.
- Magnetic field inside the solenoid is uniform.
- Magnetic field outside the volume of the solenoid approaches to zero.
- Magnetic field and its variation with distance along the axis of finite length solenoid shown in figure below.



- Magnetic field produced by a toroid is directed along its circular axis and constant in magnitude over its entire cross section.
- Magnetic field outside the volume of toroid is always zero.
- Magnetic field at the centre of toroid is always zero.

Illustrations

Illustration 12.

The length of solenoid is 0.1m. and its diameter is very small. A wire is wound over it in two layers. The number of turns in inner layer is 50 and that of outer layer is 40. The strength of current flowing in two layers in opposite direction is 3A. Then find magnetic induction at the middle of the solenoid.

Solution

Direction of magnetic field due to both layers is opposite, as direction of current is opposite, so

$$B_{net} = B_1 - B_2 = \mu_0 n_1 I_1 - \mu_0 n_2 I_2 = \mu_0 \frac{N_1}{\ell} I - \mu_0 \frac{N_2}{\ell} I \quad (\because I_1 = I_2 = I)$$

$$= \frac{\mu_0 I}{\ell}(N_1 - N_2) = \frac{4\pi \times 10^{-7} \times 3}{0\cdot 1}(50 - 40) = 12\pi \times 10^{-5}\ T$$

Illustration 13.

A closely wound, solenoid 80 cm. long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0A. Estimate the magnetic field

(a) Inside the solenoid (b) Axial end points of the solenoid

Solution

(a) Magnetic field inside the solenoid

$$B_{in} = \mu_0 nI = \mu_0 \frac{N}{\ell} I,\ (N = 400 \times 5 = 2000) = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{(80 \times 10^{-2})} = 8\pi \times 10^{-3}\ T$$

(b) Magnetic field at axial end points of solenoid $B_{ends} = \frac{\mu_0 nI}{2} = \frac{8\pi \times 10^{-3}}{2} = 4\pi \times 10^{-3}\ T$

Illustration 14.

A straight long solenoid produces magnetic field 'B' at its centre. If it is cut into two equal parts and same number of turns are wound on one part in double layer. Find magnetic field produced by new solenoid at its centre.

Solution

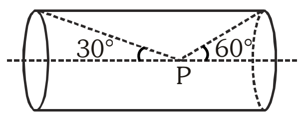
Magnetic field produced by a long solenoid is $B = \mu_0 nI$, where $n = N/\ell$

$\because$ Same number of turns wound over half length

$\therefore$ Magnetic field produced by new solenoid is $B' = \mu_0 \left(\frac{N}{\ell/2}\right) I = 2\left(\frac{\mu_0 NI}{\ell}\right) = 2B$

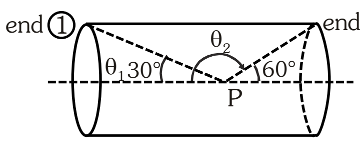
Illustration 15.

Find out magnetic field at axial point 'P' of solenoid shown in figure (where turn density 'n' and current through it is I)



Solution

Magnetic field at point 'P' due to finite length solenoid



$B_P = \frac{\mu_0 nI}{2}[\cos\theta_1 - \cos\theta_2]$, where $\theta_1 = 30°$ (CW),

$\theta_2 = (180° - 60°) = 120°$ (CW) $= \frac{\mu_0 nI}{2}[\cos 30° - \cos 120°]$

$$= \frac{\mu_0 nI}{2}\left[\frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right)\right] = \frac{\mu_0 nI}{4}(\sqrt{3} + 1)$$

Illustration 16.

A toroid has a core (nonferromagnetic) of inner radius 25 cm and outer radius 26 cm around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field : (a) Outside the toroid (b) Inside the core of the toroid (c) In the empty space surrounded by the toroid.

Solution :

We are given that, mean radius of the toroid, i.e.,

$$r = \frac{25\text{cm} + 26\text{cm}}{2} = 25.5 \text{ cm} = 0.255 \text{ m}$$

total number of turns, N = 3500

current through the toroid, I = 11 A

Clearly number of turns per unit length of the toroid,

$$n = \frac{N}{2\pi r} = \frac{3500}{2\pi \times 0.255}$$

(a) Magnetic field outside the toroid is **zero** as it non-zero inside its core.

(b) Magnetic field inside the core of the toroid, i.e.,

$$B = \mu_0 nI = \left[(4\pi \times 10^{-7})\left(\frac{3500}{2\pi \times 0.255}\right) \times 11\right] \text{T} = 3.0 \times 10^{-2} \text{ T}$$

Note : Inside the toroid, $B \propto \frac{1}{r}$. The above value of B corresponds to mean radius 25.5 cm.

(c) In the empty space surrounded by the toroid, B = 0

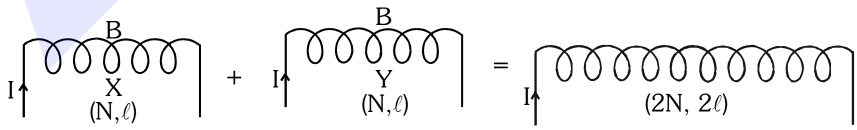
BEGINNER'S BOX-4

1. State the following statements are true or false :

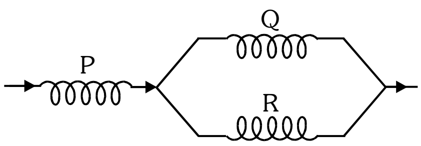
(a) The magnetic field inside straight solenoid of finite length is always less than $\mu_0 nI$.

(b) The magnetic field of an infinitely long ideal solenoid of radius 'R' carrying current I is constant inside and zero outside.

2. Two identical long solenoids X and Y carries equal current. The magnetic field produced at their axial mid point is B. If both the solenoids joined to each other according to figure. Find magnetic field at axial mid point of new solenoid.

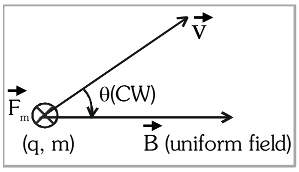


3. Three identical long solenoids P, Q and R connected to each other as shown in figure. If the magnetic field at the centre of P is 2.0 T, what would be the field at the centre of Q ?



5. MOTION OF CHARGE IN MAGNETIC FIELD

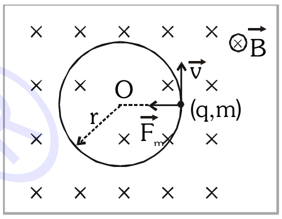
5.1 Magnetic Force on Moving Charge



Consider a charge q moving with velocity $\vec{v}$ in uniform magnetic field $\vec{B}$ such that $\vec{v}$ makes an angle θ with $\vec{B}$, then magnetic force experienced by the charge is given by $\boxed{\vec{F}_m = q(\vec{v} \times \vec{B})}$

5.2 Motion of a Charge Particle in Uniform Transverse Magnetic Field

When a charge +q projected in uniform transverse magnetic field ($\theta = 90°, \vec{v} \perp \vec{B}$) then maximum magnetic force of constant magnitude always acts perpendicular to its direction of motion so the charge moves along circular path and required centripetal force provided by the magnetic force.



Magnetic force on moving charge given as

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$

$$F_m = qvB \sin 90° \;(\theta = 90°, \vec{v} \perp \vec{B}) \Rightarrow F_m = qvB \;\ldots\ldots (1)$$

Centripetal force on moving charge given as $F_{cp} = \dfrac{mv^2}{r}$ .... (2)

Where 'm' be the mass of charge and 'r' be the radius of circular path from (1) & (2)

$$F_m = F_{cp} \qquad \boxed{qvB = \frac{mv^2}{r}}$$

- **Radius of circular path (r) :–** Radius of circular path of charge in uniform transverse magnetic field given as

$$\boxed{r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mE_K}}{qB} = \frac{\sqrt{2mq\,V_{acc.}}}{qB} = \frac{1}{B}\sqrt{\frac{2mV_{ac}}{q}}}$$

- **Time period (T) :–** Time period of charge particle of a circular motion in uniform transverse magnetic field given as

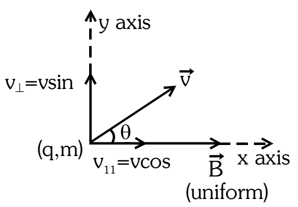
$$T = \frac{2\pi r}{v} \text{ and } r = \frac{mv}{qB} \text{ So } \qquad T = \frac{2\pi}{v}\left(\frac{mv}{qB}\right) \Rightarrow \boxed{T = \frac{2\pi m}{qB}} \;\&\; \boxed{\omega = \frac{qB}{m}}$$

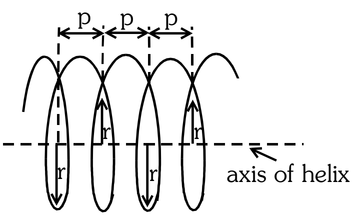
- Kinetic energy of charge :-

$$E_K = \frac{1}{2}mv^2, \text{ where } v = \frac{qBr}{m} \Rightarrow E_K = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 \Rightarrow \boxed{E_K = \frac{q^2B^2r^2}{2m}}$$

5.3 Motion of Charge in Uniform Magnetic Field At an Acute / obtuse Angle

Consider a charge 'q' of mass 'm' moving with velocity $\vec{v}$ at an angle θ with the directon of magnetic field $\vec{B}$. In this situation resolving the velocity of charge along and perpendicular to the field, we find that the particle moves with constant velocity vcosθ along the field (along x-axis) and at the same time it is also moving with velocity vsinθ perpendicular to the field due to which it will describe a circle in a plane perpendicular to the field (y–z plane) so in the combined effect of both the component of velocity, charge will move along '**helical path**' and the curve is called '**helix**'.





- **Axis of helix** :- It is a straight line along external magnetic field which joins all the centres of circular turns, called axis of helix.

- **Radius of circular path :-** Centripetal force required to move the charge in a circle is provided by the magnetic force $F_m = F_{cp}$

$$q\,(v\sin\theta)\,B\sin 90° = \frac{m(v\sin\theta)^2}{r} \Rightarrow \boxed{r = \frac{mv\sin\theta}{qB}}$$

- **Time period of circular motion :-**

$$T = \frac{2\pi r}{v\sin\theta}, \text{ where } r = \frac{mv\sin\theta}{qB}$$

$$T = \frac{2\pi}{v\sin\theta}\left(\frac{mv\sin\theta}{qB}\right) \Rightarrow \boxed{T = \frac{2\pi m}{qB}}$$

- **Pitch of helix (p)** :- The linear distance travelled by the charge particle in one revolution or in one time period along external magnetic field direction is called 'pitch of helix'.

$$p = (v\cos\theta)T\,, \quad \text{where } T = \frac{2\pi m}{qB} \Rightarrow \boxed{p = \frac{2\pi mv\cos\theta}{qB}}$$

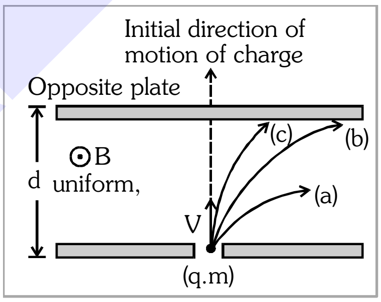
5.4 Lorentz Force on Moving Charge (F_L)

When a charge is moving in a region, where both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, then electric and magnetic forces are acting on it. The resultant of these forces called electromagnetic or Lorentz force on charge. (zero gravity)

$$\vec{F}_L = \vec{F}_e + \vec{F}_m \Rightarrow \vec{F}_L = q\vec{E} + q(\vec{v}\times\vec{B})$$

5.5 Motion of Charge in Uniform Transverse Limited Magnetic Field ($\vec{v} \perp \vec{B}$)

A uniform magnetic field of width 'd' shown in figure.



A charge (q,m) is projected from **field free region to field region** in such a way that its velocity (v) is perpendicular to the magnetic field (B). The charge deflected from its initial direction of motion and trace a part of circle according to following conditions :-

(a) Charge does not strike to the opposite plate and completes the semi circle $\Rightarrow r < d$

(b) Charge does not strike to the opposite plate and just completes the semi circle $\Rightarrow r = d$

(c) Charge strikes to the opposite plate and does not completes the semi circle $\Rightarrow r > d$

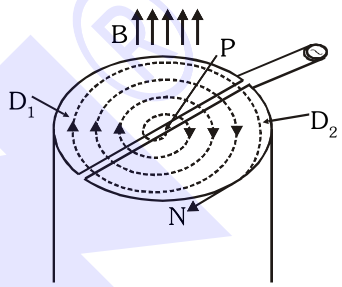
5.6 Cyclotron

The cyclotron is a device which is used to accelerate the charge particles or ions to high energies. It was invented by **E.O. Lawrence** and **M.S. Livingston** in 1934.

Principle :– When a charged particle or ion is made to move again and again in a high frequency electric field and strong magnetic field, it gets accelerated and aquires sufficiently large amount of energy.

Construction :– It consists of two hollow D–shaped metallic chambers D_1 and D_2 called dees. These dees are separated by a small gap, where a source (P) of charged particles or ions is placed. Dees are connected to high frequency oscillator, which provides high frequency electric field across the gap of the dees. This arrangement is placed between the two poles of a strong electromagnet. The magnetic field due to this electromagnet is perpendicular to the plane of the dees.

Working : A charge particle of mass 'm' and charge 'q' moves at right angle to the magnetic field 'B' inside the dees.



(i) Radius of circular path traversed by the charge :–

$$\boxed{r = \frac{mv}{qB}}$$

(ii) Time taken by the charge to complete the semi circle :–

$$\boxed{t = \frac{\pi m}{qB}}$$

This shows that time taken by the charge to complete any semi circle is same.

(iii) The time period of alternating electric field is 'T'. The polarities of dee will change after time 'T/2'. The charge will be accelerated if :–

$$\frac{T}{2} = t = \frac{\pi m}{qB} = t = \text{ (Resonance condition)} \Rightarrow \boxed{T = \frac{2\pi m}{qB}}$$

(iv) The frequency of alternating electric field :-

$$\nu = \frac{1}{T} \Rightarrow \boxed{\nu = \frac{qB}{2\pi m}}$$

(v) Maximum kinetic energy gained by the charge :

$$\boxed{(E_K)_{max.} = \left(\frac{q^2B^2}{2m}\right) r^2_{max.}}$$ where $r_{max.}$ is radius of the longest semi circle described by the ion.

(vi) Total energy aquired by the charge during 'N' complete revolution :

$\boxed{E_T = 2NqV}$, where V is potential difference to which the charge is subjected every time it enters a dee. This energy must be equal to the maximum kinetic energy of charge :

$$2NqV = \frac{B^2q^2r^2_{max}}{2m} \Rightarrow \boxed{N = \frac{B^2q\,r^2_{max}}{4Vm}}$$

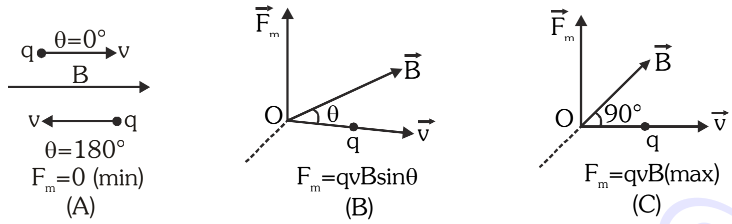
Limitations of Cyclotron :–

(i) Cyclotron is not used to accelerate uncharged particles like neutron.

(ii) Cyclotron is not used to accelerates electrons because they have very small mass.

GOLDEN KEY POINTS

- The force $\vec{F}_m$ is always perpendicular to both the velocity $\vec{v}$ and the field $\vec{B}$
- A charged particle at rest in a steady magnetic field does not experience any force.
  If the charged particle is at rest then $\vec{v} = \vec{0}$, so $\vec{v} \times \vec{B} = \vec{0}$
- A moving charged particle does not experience any force in a magnetic field if its motion is parallel or antiparallel to the field.



- If the particle is moving perpendicular to the field. In this situation all the three vectors $\vec{F}, \vec{v}$ and $\vec{B}$ are mutually perpendicular to each other. Then sin θ = max = 1, i.e., θ = 90°,
  The force will be maximum $F_m = q\,v\,B$
- Work done by force due to magnetic field in motion of a charged particle is always zero.
  When a charged particle move in a magnetic field, then force acts on it is always perpendicular to displacement, so the work done, $W = \int \vec{F}.d\vec{s} = \int Fds \cos 90° = 0$ (as θ = 90°),
  And as by work-energy theorem W = Δ KE, the kinetic energy $\left(=\frac{1}{2}mv^2\right)$ remains unchanged and hence speed of charged particle v remains constant.
  However, in this situation the force changes the direction of motion, so the direction of velocity $\vec{v}$ of the charged particle changes continuously.

Illustrations

Illustration 17.

A charge q = –4 μC has an instantaneous velocity $\vec{v} = (2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6\,ms^{-1}$ in a uniform magnetic field $\vec{B} = (2\hat{i} + 5\hat{j} - 3\hat{k}) \times 10^{-2}\,T$. What is the force on the charge ?

Solution We know $\vec{F} = q(\vec{v} \times \vec{B})$

$$\Rightarrow \quad \vec{F} = (-4 \times 10^{-6})[(2\hat{i} - 3\hat{j} + \hat{k}) \times 10^6 \times (2\hat{i} + 5\hat{j} - 3\hat{k}) \times 10^{-2}]$$

$$\Rightarrow \quad \vec{F} = -(-16\hat{i} + 32\hat{j} + 64\hat{k}) \times 10^{-2}N$$

$$\Rightarrow \quad \vec{F} = -16(\hat{i} + 2\hat{j} + 4\hat{k}) \times 10^{-2}N$$

Illustration 18.

A beam of protons is deflected sideways. Could this deflection be caused by
(i) a magnetic field (ii) an electric field? If either possible, what would be the difference?

Solution

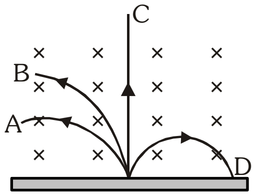
Yes, the moving charged particle (e.g. proton, α-particles etc.) may be deflected sideway either by an electric or by a magnetic field.

(i) The force exerted by a magnetic field on the moving charged particle is always perpendicular to direction of motion, so that no work is done on the particle by this magnetic force. That is the magnetic field simply deflects the particle and does not increases its kinetic energy.

(ii) The force exerted by electric field on the charged particle at rest or in motion is always along the direction of field and the kinetic energy of the particle changes.

Illustration 19.

A neutron, a proton, an electron an α-particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inwards normal to the plane of the paper. The tracks of the particles are shown in fig. Relate the tracks to the particles.



Solution

Force on a charged particle in magnetic field $\vec{F} = q(\vec{v}\times\vec{B})$

For neutron q=0, F=0 hence it will pass undeflected i.e., tracks C corresponds to neutron.

If the particle is negatively charged, i.e. electron. $\vec{F} = -e(\vec{v}\times\vec{B})$

It will experience a force to the right; so track D corresponds to electron.

If the charge on particle is positive. It will experience a force to the left; so both tracks A and B corresponds to positively charged particles (i.e., protons and α-particles). When motion of charged particle perpendicular to the magnetic field the path is a circle with radius

$r = \dfrac{mv}{qB}$ i.e. $r \propto \dfrac{m}{q}$ and as $\left(\dfrac{m}{q}\right)_\alpha = \left(\dfrac{4m}{2e}\right)$ while $\left(\dfrac{m}{q}\right)_p = \dfrac{m}{e} \Rightarrow \left(\dfrac{m}{q}\right)_\alpha > \left(\dfrac{m}{q}\right)_p$

So $r_\alpha > r_p \Rightarrow$ track B to α-particle and A corresponds to proton.

Illustration 20.

An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV enters a region with uniform magnetic field of 0.15 T. Determine the radius of the trajectory of the electron if the field is –

(a) Transverse to its initial velocity(b) Makes an angle of 30° with the initial velocity

[Given : $m_e = 9\times10^{-31}$ kg]

Solution

$$\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2\times10^3}{9\times10^{-31}}} = \frac{8}{3}\times10^7 \text{ m/s}$$

(a) Radius $r_1 = \dfrac{mv}{qB} = \dfrac{9\times10^{-31}\times(8/3)\times10^7}{1.6\times10^{-19}\times0.15} = 10^{-3}\text{m} = 1\text{mm}$

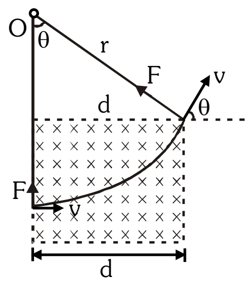
(b) Radius $r_2 = \dfrac{mv\sin\theta}{qB} = r_1\sin\theta = 1\times\sin30° = 1\times\dfrac{1}{2} = 0.5$ mm

Illustration 21.

A particle of mass m and charge q is projected into a region having a perpendicular uniform magnetic field B of width d. Find the angle of deviation θ of the particle as it comes out of the magnetic field.

Solution

The radius of the circular orbit is $r = \dfrac{mv}{qB}$



The deviation θ may be obtained from the fig. as

$$\sin\theta = \frac{d}{r} = \frac{dBq}{mv} \quad \text{or} \quad \theta = \sin^{-1}\left(\frac{dBq}{mv}\right)$$

BEGINNER'S BOX-5

1. A test charge $1{\cdot}6 \times 10^{-19}$ Cb is moving with velocity $\vec{v} = (2\hat{i} + 3\hat{j})$ m/sec is a magnetic field $\vec{B} = (2\hat{i} + 3\hat{j})$ Wb/m^2. The magnetic force on the test charge :-

(1) $6\hat{k}$T (2) $(4\hat{i}+6\hat{j})$T (3) $(4\hat{i}+6\hat{j}) \times 10^{-19}$T (4) zero

2. A charge with 10^{-11} Cb and 10^{-7} kg mass moving with a velocity of 10^8 m/sec along x–axis. A uniform static magnetic field of $0{\cdot}5$T is acting along the y–axis. The magnetic force (magnitude and direction) on charge :-

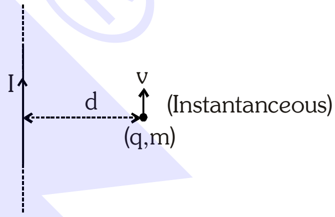
(1) zero (2) 5×10^{-4}N, along z-axis

(3) 5×10^{-4} N, along x-axis (4) 5×10^{-4} N, along y-axis

3. A 2MeV proton is moving perpendicular to uniform magnetic field of $2{\cdot}5$T. The magnetic force on the proton :-

(1) 8×10^{-12} N (2) 4×10^{-12} N (3) 16×10^{-12} N (4) 2×10^{-12} N

4. An electron is moving at 10^6 m/sec in a direction parallel to a current of 5A flowing through an infinite long straight wire separated by a perpendicular distance of 10cm in air. Magnetic force experienced by the electron :-



(1) 1.6×10^{-19} N (2) 1.6×10^{-20} N

(3) 1.6×10^{-18} N (4) 1.6×10^{-21} N

5. Three charge proton, deutron and α–particle are projected in uniform transverse magnetic field, then find ratio of their radii of circular tracks respectively, if their –

(a) v $\rightarrow$ same (b) P $\rightarrow$ same (c) $E_K \rightarrow$ same (d) $V_{acc.} \rightarrow$ same

6. Three charge proton, deutron and α–particle are projected in uniform transverse magnetic field in such a way that their radii of circular tracks are equal then find their following ratio respectively :-

(a) v $\rightarrow$ ratio (b) P $\rightarrow$ ratio (c) $E_K \rightarrow$ ratio (d) $V_{acc.} \rightarrow$ ratio

7. A proton and an electron are projected in uniform transverse magnetic field, then trajectory of which charge is more curved or deflection of which charge is more from its initial direction of motion, if their

(a) v $\rightarrow$ same (b) P $\rightarrow$ same (c) $E_K \rightarrow$ same (d) $V_{acc.} \rightarrow$ same

8. Three charge proton, deutron and α–particle are projected in uniform transverse magnetic field then find ratio of following parameters of circular motion respectively.

(a) Time period (b) Frequency (c) Angular frequency

9. A beam of proton moving with velocity 4×10^5 m/sec enters in a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Calculate radius of helical path and pitch of helix.

10. A proton is projected with a speed of 2×10^6 m/sec at an angle of 60° to the x-axis. If a uniform magnetic field of 0.104T is applied along y axis. Calculate radius of helix and time period of proton.

11. A charge projected in uniform magnetic field at angle 30° to the field direction. The time period of circular motion of helical path is 'T'. If angle of projection becomes 60° then what is the new time period.

12. If a charge projected in a zero gravity region. Find possible cases of electric and magnetic field in that region so no net force exerts on charge.

(a) E = 0, B = 0 (b) E = 0, B $\neq$ 0 (c) E $\neq$ 0, B $\neq$ 0 (d) E $\neq$ 0, B = 0

13. If a projected charge passes through a zero gravity region without change in its velocity, then find possible cases of electric and magnetic field in that region

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$ (c) $E \neq 0, B \neq 0$ (d) $E \neq 0, B = 0$

14. If a projected charge passes through a zero gravity region, remains undeviated or undeflected, then find possible cases of electric field and magnetic field in that region

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$ (c) $E \neq 0, B \neq 0$ (d) $E \neq 0, B = 0$

15. If a projected charge passes through a zero gravity region, its speed may be constant, then find possible cases of electric and magnetic field in that region

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$ (c) $E \neq 0, B \neq 0$ (d) $E \neq 0, B = 0$

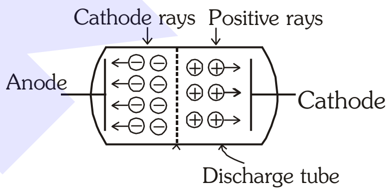
16. In a region a uniform magnetic field acts in horizontal plane towards north. If cosmic particles (80% protons) falling vertically downwards, then they are deflected towards–

(1) North (2) South (3) East (4) West

17. An electron is moving along +x direction. To get it moving along an anticlockwise circular path in x–y plane, magnetic field applied along :-

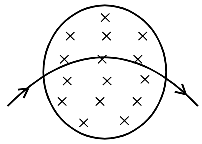
(1) +y–direction (2) +z–direction
(3) –y–direction (4) –z–direction

18. A magnetic field is applied at perpendicular to the axis of cylinderical discharge tube then cathode and positive rays deflected in –



(1) Same direction.
(2) Opposite direction.
(3) Mutually perpendicular direction.
(4) No deflection for both.

19. There is a magnetic field acting in a plane perpendicular downwards. A particle in vacuum moves in the plane of paper from left to right as shown in figure. The path indicated by the arrow could be due to –



(1) Proton (2) Neutron
(3) Electron (4) Alpha particle

20. A proton is moving towards north along horizontal line passes through a zero gravity region, where electric and magnetic field are mutually perpendicular to each other. The path of particle remains undeviated or undeflected. If the direction of electric field is towards east then direction of magnetic field is

(1) Towards east (2) Vertically downwards
(3) Towards west (4) Vertically upwards

21. A proton moving along z-axis with constant velocity. If a magnetic field is applied along x-axis then direction of magnetic force on proton

(1) along z-axis (2) along y-axis (3) along x-axis (4) zero force

22. A vertical wire carries a current in upward directon. If an electron beam sent horizontally towards the wire, then it will deflected

(1) vertically downwards and perpendicular to the plane of the paper
(2) vertically upwards and perpendicular to the plane of the paper
(3) In the plane of the paper
(4) No deflection

23. A charge is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The charge will moves along which path :-

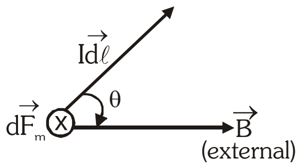
(1) Circular (2) Helical (3) Parabola (4) Straight line

6. CURRENT CARRYING CONDUCTOR IN MAGNETIC FIELD

When a current carrying conductor placed in magnetic field, a magnetic force exerts on each free electron which are present inside the conductor. The resultant of these forces on all the free electrons is called magnetic force on conductor.

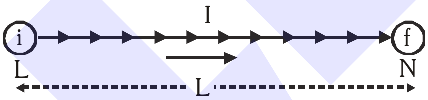
- **Magnetic force on current element**

Through experiments Ampere established that when current element $I d\vec{\ell}$ is placed in magnetic field $\vec{B}$, it experiences a magnetic force $d\vec{F}_m = I(d\vec{\ell} \times \vec{B})$



- Current element in a magnetic field does not experience any force if the current in it is parallel or anti–parallel with the field $\theta = 0°$ or $180°$ $dF_m = 0$ (min.)
- Current element in a magnetic field experiences maximum force if the current in it is perpendicular with the field $\theta = 90°$ $dF_m = BId\ell$ (max.)
- Magnetic force on current element is always perpendicular to the current element vector and magnetic field vector. $d\vec{F}_m \perp Id\vec{\ell}$ and $d\vec{F}_m \perp \vec{B}$ (always)
- Total magnetic force on straight current carrying conductor in uniform magnetic field is given as

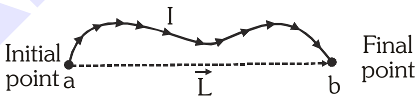
$$\vec{F}_m = \int d\vec{F}_m = \int I(d\vec{\ell} \times \vec{B}) = I\left[\int_i^f d\vec{\ell}\right] \times \vec{B} \Rightarrow \vec{F}_m = I(\vec{L} \times \vec{B})$$



Where $\vec{L} = \int_i^f d\vec{\ell}$, vector sum of all length elements from initial to final point.

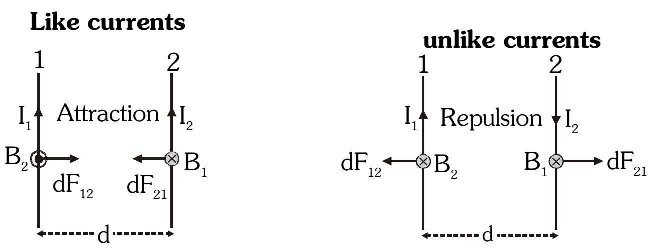
- Total magnetic force on arbitrary shape current carrying conductor in uniform magnetic field $\vec{B}$ is

$$\vec{F}_m = \int_i^f d\vec{F}_m = I\left[\int_i^f d\vec{\ell}\right] \times \vec{B} = I(\vec{L} \times \vec{B})$$



Where $\vec{L} = \int_i^f d\vec{\ell}$, vector sum of all length elements from initial to final point or displacement between free ends of an arbitrary conductor from initial to final point.

Magnetic force between two parallel current carrying conductors



The net magnetic force acting on a current carrying conductor due to its own field is zero. So consider two infinite long parallel conductors separated by distance 'd' carrying currents I_1 and I_2.

Magnetic field at each point on conductor (ii) due to current I_1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d} \text{ [uniform field for conductor (2)]}$$

Magnetic field at each point on conductor (i) due to current I_2 is

$$B_2 = \frac{\mu_0 I_2}{2\pi d} \text{ [Uniform field for conductor (1)]}$$

consider a small element of length '$d\ell$' on each conductor. These elements are right angle to the external magnetic field, so magnetic force experienced by elements of each conductor given as

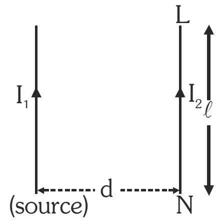
$$dF_{12} = B_2 I_1 d\ell = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1 d\ell \quad \text{...(i)} \quad (\text{Where } I_1 d\ell \perp B_2)$$

$$dF_{21} = B_1 I_2 d\ell = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 d\ell \quad \text{... (ii)} \quad (\text{Where } I_2 d\ell \perp B_1)$$

Where dF_{12} is magnetic force on element of conductor (i), due field of conductor (i) and dF_{21} is magnetic force on element of conductor (ii), due to field of conductor (i).

Magnetic force per unit length of each conductor is $\frac{dF_{12}}{d\ell} = \frac{dF_{21}}{d\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$

$$f = \frac{\mu_0 I_1 I_2}{2\pi d} \text{ N/m (in S.I.)} \quad f = \frac{2 I_1 I_2}{d} \text{ dyne/cm (In C.G.S.)}$$

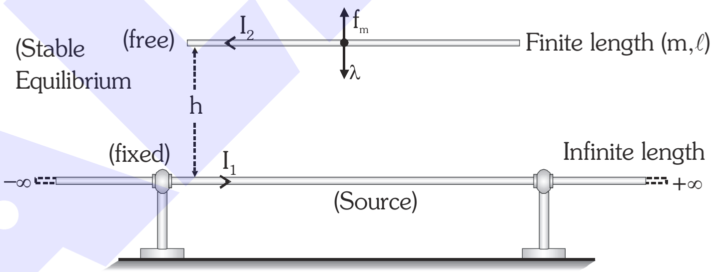


- Force scale $f = \frac{\mu_0 I_1 I_2}{2\pi d}$ is applicable when at least one conductor must be of infinite length so it behaves like source of uniform magnetic field for other conductor.

  Magnetic force on conductor 'LN' is $F_{LN} = f \times \ell \Rightarrow \quad F_{LN} = \left(\frac{\mu_0 I_1 I_2}{2\pi d}\right)\ell$

- **Equilibrium of free wire**

  Case I : Upper wire is free : Consider a long horizontal wire which is rigidly fixed another wire is placed directly above and parallel to fixed wire.



Magnetic force per unit length of free wire $f_m = \frac{\mu_0 I_1 I_2}{2\pi h}$, and it is repulsive in nature because currents are unlike.

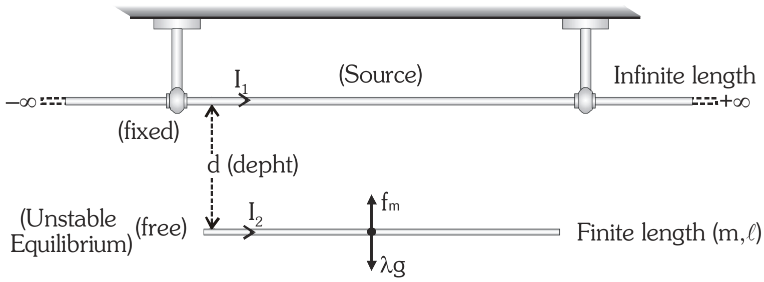
Free wire may remains suspended if the magnetic force per unit length is equal to weight of its unit length

At balanced condition f_m = W'. Weight per unit length of free wire = $\frac{\mu_0 I_1 I_2}{2\pi h} = \frac{m}{\ell} g$ (stable equilibrium condition)

If free wire is slightly displaced and released then it will executes S.H.M. in vertical plane.

The time period of motion is $T = 2\pi\sqrt{\frac{h}{g}}$

Case II : Lower wire is free : Consider a long horizontal wire which is rigidly fixed. Another wire is placed directly below and parallel to the fixed wire.



Magnetic force per unit length of free wire is $f_m = \dfrac{\mu_0 I_1 I_2}{2\pi d}$, and it is attractive in nature because currents are like.

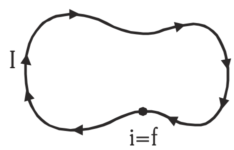
Free wire may remains suspended if the magnetic force per unit length is equal to weight of its unit length

At balanced condition $\quad f_m = W'$

Weight per unit length of free wire $\quad \dfrac{\mu_0 I_1 I_2}{2\pi d} = \dfrac{m}{\ell} g$ (unstable equilibrium condition)

GOLDEN KEY POINTS

- A current carrying closed loop (or coil) of any shape placed in uniform magnetic field then no net magnetic force act on it (Torque may or may not be zero)

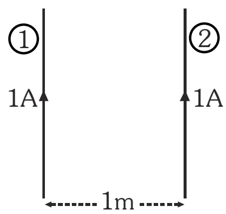


$$\vec{L} = \int_i^f d\vec{\ell} = 0 \text{ or } \oint d\vec{\ell} = 0$$

So net magnetic force acting on a current carrying closed loop $\left|\vec{F}_m\right| = 0$ (always)

- When a current carrying closed loop (or coil) of any shape placed in non uniform magnetic field then net magnetic force and torque may or may not be zero.

- **Definition of ampere** :

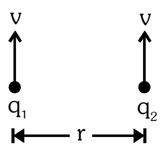


Magnetic force/unit length for both infinite length conductors given as

$$f = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{(4\pi \times 10^{-7})(1)(1)}{2\pi(1)} = 2 \times 10^{-7} \text{ N/m}$$

'Ampere' is the current which, when passed through each of two parallel infinite long straight conductors placed in free space at a distance of 1 m from each other, produces between them a force of 2×10^{-7} N/m

- Forces between two parallel moving charges



Electric force between charges :–

$$F_e = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} \qquad \text{....(1)}$$

Magnetic force between charges :–

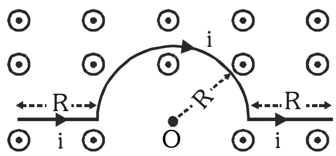
$$F_m = q_1 v B_2 = q_1 v \left[\frac{\mu_0 q_2 v}{4\pi r^2}\right] \Rightarrow \frac{F_m}{F_e} = \mu_0 \epsilon_0 v^2, \quad \text{where } \mu_0 \epsilon_0 = \frac{1}{c_0^2} \Rightarrow \frac{F_m}{F_e} = \frac{v^2}{c_0^2}$$

where c_0 is velocity of light in vacuum

Illustrations

Illustration 22.

A wire bent as shown in fig carries a current i and is placed in a uniform field of magnetic induction $\vec{B}$ that emerges from the plane of the figure. Calculate the force acting on the wire.

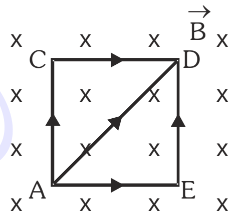


Solution

The total force on the whole wire is $F_m = I\,|\vec{L}|\,B = I(R + 2R + R)B = 4RIB$

Illustration 23.

A square of side 2.0 m is placed in a uniform magnetic field $\vec{B} = 2.0\text{ T}$ in a direction perpendicular to the plane of the square inwards. Equal current i = 3.0 A is flowing in the directions shown in figure. Find the magnitude of magnetic force on the loop.



Solution

Net force on the loop = $3\,(\vec{F}_{AD})$ $\quad\because$ Force on wire ACD = Force on AD = Force on AED

$\Rightarrow F_{net} = 3(i)\,(AD)\,(B) = (3)\,(3.0)\,(2\sqrt{2})(2.0)\text{ N} = 36\sqrt{2}\text{ N}$. Direction of this force is towards EC.

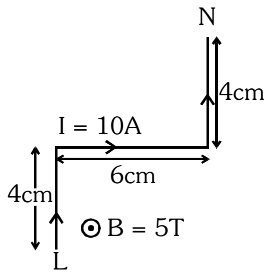
BEGINNER'S BOX-6

1. A wire of length 5 cm. is placed inside the solenoid near its centre such that it makes an angle of 30° with the axis of solenoid. The wire carries a current of 5A and the magnetic field due to solenoid is 2.5×10^{-2} T. Magnetic force on the wire is :-

(1) 3.12×10^{-4} N (2) 31.2×10^{-4} N

(3) 312×10^{-4} N (4) 0.312×10^{-4} N

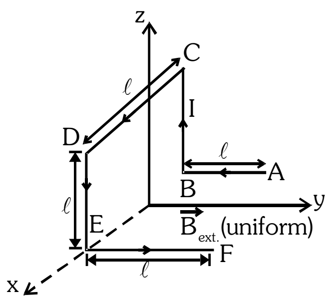
2. A wire 'LN' bent as shown in figure is placed in uniform perpendicular magnetic field of 5T. A 10A current flows through the wire. Magnetic force experienced by the wire is :-



(1) 5N (2) 10N

(3) 2.5 N (4) 1.25 N

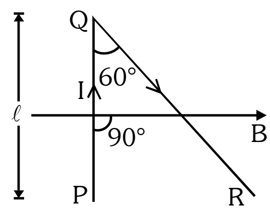
3. A wire ABCDEF with each side of length 'ℓ' bent as shown in figure and carrying a current I. If it is placed in a uniform magnetic field B which is parallel to +y direction. Magnetic force experienced by the wire is:-



(1) $BI\ell$, along + z direction (2) $BI\ell$, along –z direction

(3) $2BI\ell$, along + z direction (4) $2BI\ell$, along –z direction

4. A current 'I' is flowing through wire PQR. This wire is bent in form of an angle and placed is uniform magnetic field 'B' according to figure. If PQ = ℓ and $\angle PQR = 60°$. The ratio of magnetic forces on PQ to QR respectively:-



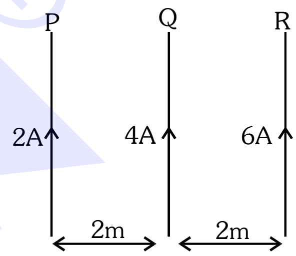
(1) 1 : 2 (2) $\sqrt{3}:2$ (3) $2:\sqrt{3}$ (4) 1 : 1

5. A current of 10A flows through two long parallel wires. The magnetic force on each wire is 2×10^{-3}N/m. If their currents makes doubled and separation between them makes half then magnetic force per unit length of each wire becomes :-

(1) 16×10^{-3} N/m (2) 8×10^{-3} N/m

(3) 4×10^{-3} N/m (4) 32×10^{-3} N/m

6. Three long straight wires, carrying currents are arranged according to figure. Magnetic force on 10cm part of the wire Q is :-



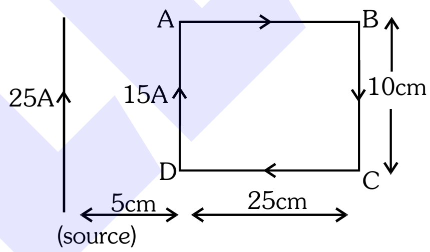
(1) 16×10^{-9} N, towards right

(2) 16×10^{-8} N, towards right

(3) 16×10^{-8} N, towards left

(4) 16×10^{-9} N, towards left

7. A rectangular loop ABCD is placed near on infinite length current carrying wire. Magnetic force on the loop is :-



(1) 1.25×10^{-4}N, Attraction (2) 1.25×10^{-4} N, Repulsion

(3) 12.5×10^{-4} N, Repulsion (4) 12.5×10^{-4}N, Attraction

8. A long horizontal wire 'P' which is rigidly fixed, carries a current 50A. An another wire 'Q' is placed directly above and parallel to 'P'. The weight of wire Q is 0.075 N/m and carries a current of 25A. The position of wire 'Q' from P so that the wire 'Q' remains suspended due to magnetic repulsion :-

(1) 33.3 mm (2) 0.33 mm (3) 333 mm (4) 3.33 mm

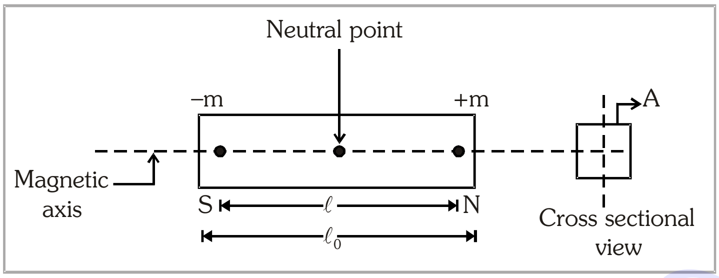
9. A long horizontal wire is rigidly fixed and carries 100A current. An another wire of linear mass density 2×10^{-3} kg/m placed below and parallel to the fixed wire. If the free wire kept 2cm below and hangs in air, then current in free wire is :-

(1) 19.6 A (2) 9.8 A (3) 4.9 A (4) 100 A

7 MAGNETIC DIPOLE MOMENT

A magnetic dipole consists of a pair of magnetic poles of equal and opposite strength separated by small distance. Ex. magnetic needle, bar magnet, current carrying solenoid, current carrying coil or loop.

- **Magnetic moment of Bar magnet :–**

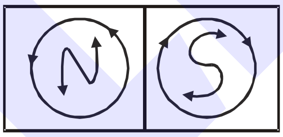


The magnetic moment of a bar magnet is defined as a vector quantity having magnitude equal to the product of pole strength (m) with effective length (ℓ) and directed along the axis of the magnet from south pole to north pole.

$\boxed{\vec{M} = m\vec{\ell}}$ — It is an axial vector; S.I. unit :– A-m^2

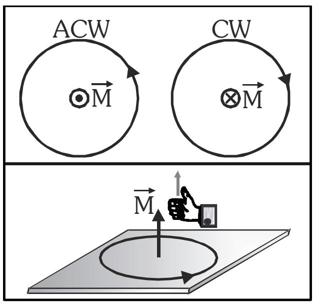
Magnetic moment of current carrying coil (loop)

Current carrying coil (or loop) behaves like magnetic dipole. The face of coil in which current appears to flow anticlockwise acts as north pole while face of coil in which current appears to flow clock wise acts as south pole.



- A loop of geometrical area 'A', carries a current 'I' then magnetic moment of coil $M = IA$
- A coil of turns 'N', geometrical area 'A', carries a current 'I' then magnetic moment $M = NIA$

Magnetic moment of current carrying coil is an axial vector $\vec{M} = NI\vec{A}$ where $\vec{A}$ is a area vector perpendicular to the plane of the coil and along its axis. SI unit : A-m^2 or J/T



Direction of $\vec{M}$ find out by right hand thumb rule

- Curling fingers $\Rightarrow$ In the direction of current
- Thumb $\Rightarrow$ Gives the direction of $\vec{M}$

For a current carrying coil, its magnetic moment and magnetic field vectors both are parallel axial vectors.

GOLDEN KEY POINTS

- Attractive property : A bar magnet attracts certain magnetic substances (eg. Iron dust). The attracting power of the bar magnet is maximum at two points near the ends called poles. So the attracting power of a bar magnet at its poles called 'pole strength'
- The 'pole strength' of north and south pole of a bar magnet is conventionally represented by +m and –m respectively.
- The 'pole strength' is a scalar quantity with S.I. unit A–m.
- The 'pole strength' of bar magnet is directly proportional to its area of cross section. $m \propto A$
- The attracting power of a bar magnet at its centre point is zero, so it is called 'neutral point'.
- Magnetic poles are always exists in pairs i.e. mono pole does not exist in magnetism. So Gauss law in magnetism given as $\oint \vec{B} \cdot d\vec{s} = 0$
- Effective length or magnetic length :– It is distance between two poles along the axis of a bar magnet. As pole are not exactly at the ends, the effective length (ℓ) is less than the geometrical length (ℓ_0) of the bar magnet. $\ell \simeq 0 \cdot 91 \ell_0$
- **Inverse square law (Coulomb law) :** The magnetic force between two isolated magnetic poles of strength m_1 and m_2 lying at a distance 'r' is directly proportional to the product of pole strength and inversely proportional to the square of distance between their centres. The magnetic force between the poles can be attractive or repulsive according to the nature of the poles.

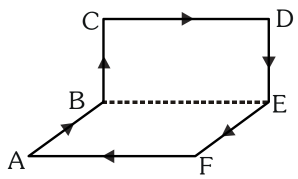
$$\left.\begin{array}{l} F_m \propto m_1 m_2 \\ F_m \propto \dfrac{1}{r^2} \end{array}\right] F_m = k\,\frac{m_1 m_2}{r^2} \qquad \text{where } k \begin{cases} \dfrac{\mu_0}{4\pi} & \text{(S.I.)} \\ 1 & \text{(C.G.S.)} \end{cases}$$

Inverse square law of Coulomb in magnetism is applicable only for two long bar magnets because isolated poles cannot exist.

Illustrations

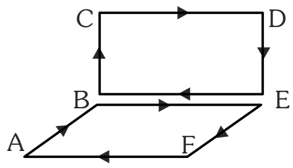
Illustration 24.

Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is 10 cm long and current in the loop is i = 2.0 A



Solution

By assuming two equal and opposite currents in BE, two current carrying loops (ABEFA and BCDEB) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other.



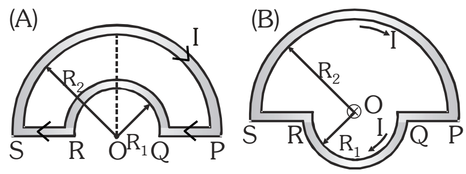
Hence, $M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$

where $M = iA = (2.0)(0.1)(0.1) = 0.02$ A-m^2

$\Rightarrow \quad M_{net} = (\sqrt{2})(0.02)$ A-m^2 = 0.028 A-m^2

Illustration 25.

The wire loop PQRSP formed by joining two semicircular wires of radii R_1 and R_2 carries a current I as shown in fig. What is the magnetic induction at the centre O and magnetic moment of the loop in cases (A) and (B) ?



Solution

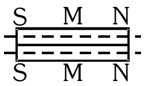
As the point O is along the length of the straight wires, so the field at O due to them will be zero and hence.

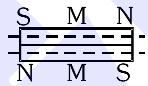
(A) $\vec{B} = \frac{\mu_0}{4\pi}\pi I\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \odot$ & $\vec{M} = NI\vec{A} = 1 \times I\left[\frac{1}{2}\pi R_2^2 \otimes + \frac{1}{2}\pi R_1^2 \odot\right] = \frac{1}{2}\pi I\left[R_2^2 - R_1^2\right]\otimes$

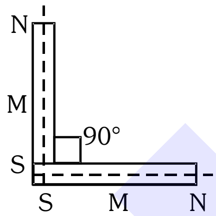
(B) Following as in case (A), in this situation, $\vec{B} = \frac{\mu_0}{4\pi}\pi I\left[\frac{1}{R_1} + \frac{1}{R_2}\right] \otimes$ and, $\vec{M} = \frac{1}{2}\pi I\left[R_2^2 + R_1^2\right]\otimes$

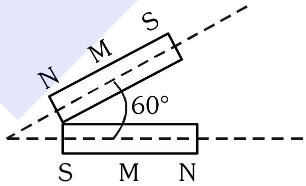
BEGINNER'S BOX-7

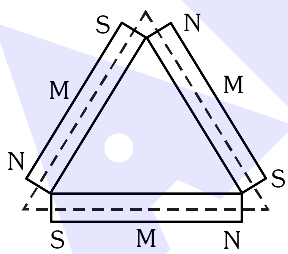
1. Two/three/four identical bar magnets of magnetic moment 'M' are combined according to figure. Find net magnetic moment of the system.

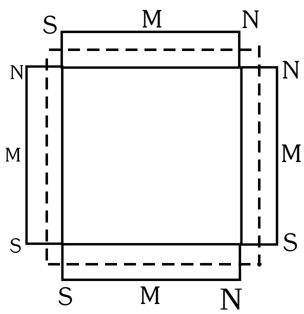
(a) 

(b) 

(c) 

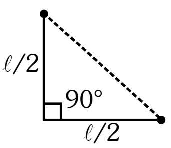
(d) 

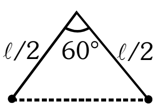
(e) 

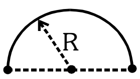
(f) 

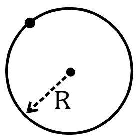
2. A bar magnet of magnetic moment 'M' is cuts into two equal parts in following two fashions (a) Parallel to its length (b) Perpendicular to its length, then what happens to the magnetic moment of each part.

3. A straight steel wire of length 'ℓ' has a magnetic moment 'M'. It is bent into various shapes shown in figure. Find out new magnetic moment of each shape.

(a) 

(b) 

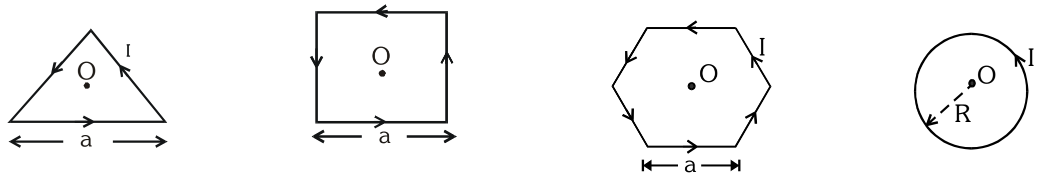
(c) 

(d) 

4. A square current carrying loop of side 'a' produced magnetic field at its centre is 'B'. Find magnetic moment of this loop in terms of 'B'.

5. A wire of length 'ℓ' carries a current bent into various regular polygons shown below. Find magnetic moment in terms of length of the wire.

(a) Equilateral triangle (b) Square (c) Regular hexagon (d) Circular loop



6. A current carrying wire of length 'ℓ' is bent to form a circular coil of one turn. After that it is again bent to form a circular coil of two turns. Find ratio of magnetic moment of coils respectively.

7. Circular coils of different turns are made by a constant length current carrying wire. Find number of turns in coil for its maximum magnetic moment.

8. Two identical current carrying coils are concentric and perpendicular to each other. If magnetic moment of each coil is 'M'. Find resultant magnetic moment of two coils.

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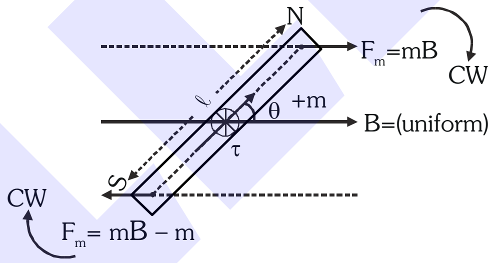
8. MAGNETIC DIPOLE IN MAGNETIC FIELD

8.1 Torque on magnetic dipole



(a) Bar magnet

τ = force $\times$ perpendicular distance between force couple

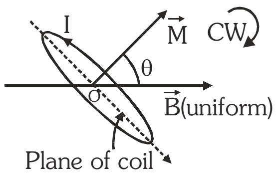
$\tau = (mB)(\ell\sin\theta)$, where $M = m\ell$

$\boxed{\tau = MB\sin\theta}$
- $\theta = 90° \Rightarrow \tau = MB$ (maximum)
- $\theta = 0°$ or $180° \Rightarrow \tau = 0$ (minimum)

Vector form $\vec{\tau} = \vec{M} \times \vec{B}$

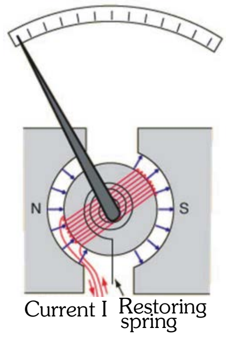
(b) Coil or Loop

$\vec{\tau} = \vec{M} \times \vec{B}$ $\qquad$ $\vec{\tau} = NI(\vec{A} \times \vec{B})$



$\boxed{\tau = BINA\sin\theta}$
- $\theta = 90° \Rightarrow \tau = BINA$ (maximum)
- $\theta = 0°$ or $180° \Rightarrow \tau = 0$ (minimum)

8.2 Moving Coil Galvanometer



This is an instrument used for detection and measurement of small electric current.

Principle : Its working in based on the fact that when a current carrying coil is placed in a magnetic field it experience a torque.

$\vec{\tau} = NI(\vec{A} \times \vec{B})$

τ = NIAB sinα [in this case I = current through coil,

A = Area of coil, B = Magnetic field

N = no. of turns, α= 90°]

τ = NIAB

The restoring torque produced = Cθ [θ = twist produced]

deflecting torque = restoring torque; NIAB = Cθ or $I = \frac{C}{NBA}\theta$

or $I \propto \theta$ it means, the deflection produced is proportional to the current flowing through the galvanometer.

- **Current Sensitivity (C.S.)**

Current sensitivity is defined as the deflection produced in the galvanometer when a unit current flows through it.

C.S. = $\frac{\theta}{I} = \frac{NBA}{C}$ (in radian/Ampere).

- **Voltage Sensitivity (V.S.)**

Voltage sensitivity is equal to the deflection per unit voltage applied across voltmeter.

V.S. = $\frac{\theta}{V} = \frac{\theta}{IR} = \frac{NBA}{CR}$ (in radian/volt).

- **Advantage of a Moving Coil Galvanometer**

1. As the deflection of the coil is proportional to the current passed through it, so a linear scale can be used to measure the deflection.
2. A moving coil galvanometer can be made highly sensitive by increasing N, B, A and decreasing C.
3. As the coil is placed in a strong magnetic field of a powerful magnet, its deflection is not affected by external magnetic fields. This enables us to use the galvanometer in any position.
4. As the coil is wound over a metallic frame, the eddy currents produced in the frame bring the coil to rest quickly.

- **Disadvantage of a Moving Coil Galvanometer**

1. The main disadvantage is that its sensitivity cannot be changed at will.
2. All types of moving coil glavanometer are easily damaged by overloading. A current greater than that which the instrument is intended to measure will burn out its hair springs or suspension.

8.3 Work done in rotating a magnetic dipole

Work done in rotating a dipole in a uniform magnetic field through small angle 'dθ'

$$dW = \tau.d\theta = MB \sin\theta\, d\theta$$

So work done in rotating a dipole from angular position θ_1 to θ_2 with respect to the Magnetic field direction

$$W = \int_{\theta_1}^{\theta_2} dW = \int_{\theta_1}^{\theta_2} MB\sin\theta d\theta = MB(\cos\theta_1 - \cos\theta_2)$$

- If magnetic dipole is rotated from field direction i.e. $\theta_1 = 0°$ to position $\theta_2 = \theta$ then work done is $W_\theta = MB\,(1 - \cos\theta) = 2MB\sin^2\theta/2$
- Work done to rotate a dipole in a magnetic field is stored in the form of potential energy of magnetic dipole.

8.4 Potential energy of magnetic dipole

The potential energy of dipole defined as work done in rotating the dipole from a direction perpendicular to the given direction. $U = W_\theta - W_{90°} \Rightarrow U = MB\,(1 - \cos\theta) - MB = -MB\cos\theta$,

In vector form $\boxed{U = -\vec{M}\,.\,\vec{B}}$

GOLDEN KEY POINTS

- Torque on dipole is an axial vector and it is directed along axis of rotation of dipole.
- Tendency of torque on dipole is try to align the $\vec{M}$ in the direction of $\vec{B}$ or tries to makes the axis of dipole parallel to $\vec{B}$ or makes the plane of coil (or loop) perpendicular to $\vec{B}$.
- Dipole in uniform magnetic field
  - $F_{net} = 0$ (no translatory motion)
  - τ may or may not be zero (decides by θ)
- Dipole in non uniform magnetic field
  - F_{net} may or may not be zero
  - τ may or may not be zero (decides by θ)
- When a current carrying coil (or loop) is placed in longitudinal magnetic field then maximum torque acts on it. $\theta = 90°\ (\vec{M} \perp \vec{B}) \Rightarrow \tau_{max} = MB = BINA$
- When a current carrying coil (or loop) is placed in transverse magnetic field the no torque acts on it.

$$\theta = 0°\ (\vec{M}\|\vec{B}) \text{ or } \theta = 180°\ (\vec{M} \text{ anti}\|\vec{B}) \Rightarrow \tau_{min} = 0$$

- When $\vec{M}$ and $\vec{B}$ are parallel (θ = 0°), the dipole has minimum potential energy and it is in stable equilibrium.

$$U = -MB \quad \text{(minimum)}$$

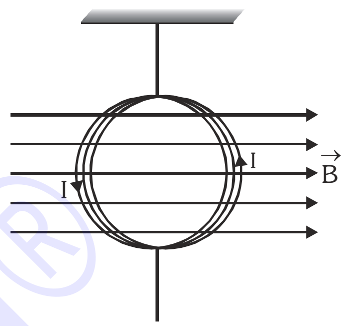
- When $\vec{M}$ and $\vec{B}$ are antiparallel (θ = 180°), the dipole has maximum potential energy and it is in unstable equilibrium. $U = MB$ (maximum)
- When $\vec{M}$ and $\vec{B}$ are perpendicular to each other (θ = 90°), the dipole has potential energy U = 0 and in this situation maximum torque acts on it hence no equilibrium.

Illustrations

Illustration 26.

A circular coil of 25 turns and radius 6.0 cm, carrying a current of 10 A, is suspended vertically in a uniform magnetic field of magnitude 1.2 T. The field lines run horizontally in the plane of the coil. Calculate the force and torque on coil due to the magnetic field. In which direction should a balancing torque be applied to prevent the coil from turning ?

Solution



Magnetic force $F_m = I\left(\oint d\vec{\ell} \times \vec{B}\right)$

For coil or close loop $\oint d\vec{\ell} = 0$ so $\left|\vec{F}_m\right| = 0$

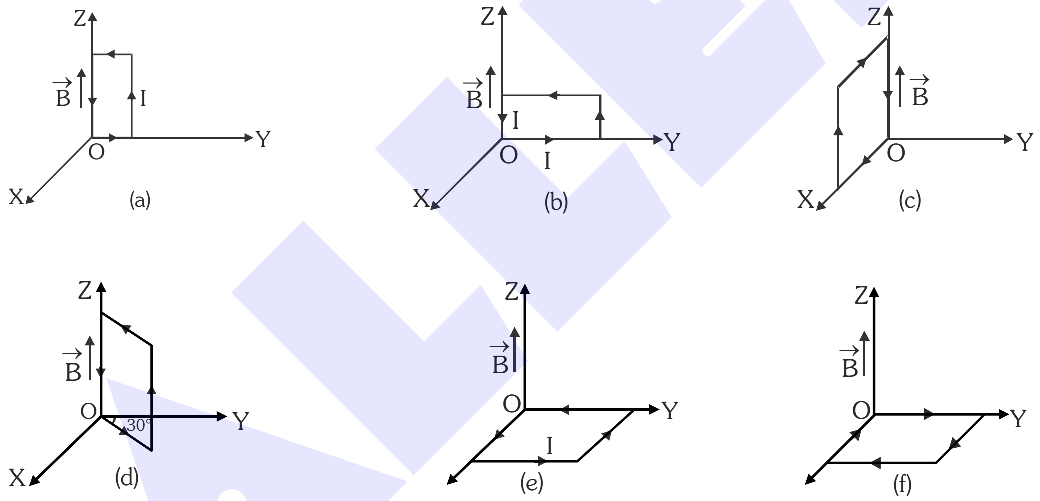
The torque on a coil of any shape having N turns and current I in a magnetic field B is given by τ = NIABsinθ

$\tau = 25 \times 10 \times \pi \times 6 \times 6 \times 10^{-4} \times 1.2 \times \sin 90° = 3.39$ N

The direction of $\vec{\tau}$ is vertically upwards. To prevent the coil from turning, an equal and opposite torque must be applied.

Illustration 27.

A uniform magnetic field of 5000 gauss is established along the positive z-direction. A rectangular loop of side 20 cm and 5 cm carries a current of 10 A is suspended in this magnetic field. What is the torque on the loop in the different cases shown in the following figures ? What is the force in each case ? Which case corresponds to stable equilibrium ?



Solution

(a) Torque on loop, τ = BIA sinθ

Here, θ=90°; B = 5000 gauss = 5000×10^{-4} tesla = 0.5 tesla

I = 10 ampere, A = $20 \times 5\text{cm}^2 = 100 \times 10^{-4} = 10^{-2}$ m^2

Now, $\tau = 0.5 \times 10 \times 10^{-2} = 5 \times 10^{-2}$ Nm   It is directed along –y-axis

(b) Same as (a).

(c) $\tau = 5 \times 10^{-2}$ Nm along –x-direction

(d) $\tau = 5\times10^{-2}$ N m at an angle of 240° with +x direction.

(e) τ is zero. [$\because$ Angle between plane of loop and direction of magnetic field is 90°]

(f) τ is zero.

Resultant force is zero in each case. Case (e) corresponds to stable equilibrium.

Illustration 28.

A circular coil of 100 turns and having a radius of 0.05 m carries a current of 0.1 A. Calculate the work required to turn the coil in an external field of 1.5 T through 180° about an axis perpendicular to the magnetic field ? The plane of coil is initially at right angles to magnetic field.

Solution

Work done $W = MB(\cos\theta_1 - \cos\theta_2) = NIAB(\cos\theta_1 - \cos\theta_2)$

$\Rightarrow W = NI\pi r^2 B(\cos\theta_1 - \cos\theta_2) = 100 \times 0.1 \times 3.14 \times (0.05)^2 \times 1.5(\cos 0° - \cos\pi) = 0.2355\text{J}$

Illustration 29.

A bar magnet of magnetic moment 1.5 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

(a) What is the amount of work required to turn the magnet so as to align its magnetic moment.

(i) Normal to the field direction?

(ii) Opposite to the field direction?

(b) What is the torque on the magnet in case (i) and (ii)?

Solution

Here, $M = 1.5\ JT^{-1}$, $B = 0.22$ T.

(a) P.E. with magnetic moment aligned to field = – MB

P.E. with magnetic moment normal to field = 0

P.E. with magnetic moment antiparallel to field = + MB

(i) Work done = increase in P.E. = $0 - (-MB) = MB = 1.5 \times 0.22 = 0.33$ J.

(ii) Work done = increase in P.E. = $MB - (-MB) = 2MB = 2 \times 1.5 \times 0.22 = 0.66$ J.

(b) We have $\tau = MB\sin\theta$

(i) $\tau = MB\sin\theta = 1.5 \times 0.22 \times 1 = 0.33$ J. $(\theta=90° \Rightarrow \sin\theta=1)$

This torque will tend to align M with B.

(ii) $\tau = MB\sin\theta = 1.5 \times 0.22 \times 0 = 0 (\theta = 180° \Rightarrow \sin\theta = 0)$

Illustration 30.

A short bar magnet of magnetic moment 0.32 J/T is placed in uniform field of 0.15 T. If the bar is free to rotate in plane of field then which orientation would correspond to its (i) stable and (ii) unstable equilibrium? What is potential energy of magnet in each case?

Solution

(i) If M is parallel to B then $\theta=0°$. So potential energy $U = U_{min} = -MB$

$U_{min} = -MB = -0.32 \times 0.15\ J = -4.8 \times 10^{-2}$ J (stable equilibrium)

(ii) If M is antiparallel to B then $\theta = \pi$ So potential energy

$U = U_{max} = +MB = +0.32 \times 0.15 = 4.8 \times 10^{-2}$ J (unstable equilibrium.)

BEGINNER'S BOX-8

1. When a magnetic needle placed in non uniform field the needle may experierces a :-

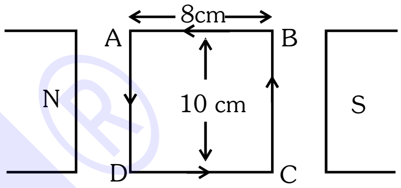
(1) Force (2) Torque
(3) Force and torque (4) No force and no torque

2. A magnet of magnetic moment $50\hat{i}$ A–m^2 placed along x–axis. Where magnetic field is $\vec{B} = (0.5\hat{i} + 3.0\hat{j})$ tesla. The torque acting on magnet is :–

(1) 175 $\hat{k}$ N–m (2) 150 $\hat{k}$ N–m (3) 75 $\hat{k}$ N–m (4) $25\sqrt{37}\ \hat{k}$ N–m

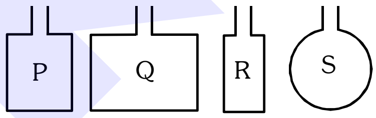
3. A coil of 100 turns kept in magnetic field B=0.2T, carries a current of 2A as shown in figure. Torque on coil and which side of coil comes out from the plane of the paper :-

(1) 0.16 N-m, BC side comes out from plane of paper
(2) 0.16 N–m, AD side comes out from plane of paper
(3) 0.32 N, BC side comes out from plane of paper
(4) 0.32 N–m, AD side comes out from plane of paper



4. Four wires each of length 2.0 m, are bent in four loop P,Q,R and S then suspended in uniform magnetic field. All loop carries equal currents, which statement of the following is true :

(1) Couple acting on the loop P is greatest.
(2) Couple acting on the loop Q is greatest.
(3) Couple acting on the loop R is greatest
(4) Couple acting on the loop S is greatest.



5. A current carrying wire is bent to form a circular coil. If this coil is placed in any other magnetic field then for maximum torque on the coil, the number of turns will be -

(1) 1 (2) 2 (3) 4 (4) 8

6. A magnet of magnetic moment 4A–m^2 is held in a uniform magnetic field 5×10^{-4}T with the magnetic moment vector makes an angle 30° with the field. Work done in increasing the angle from 30° to 45°:-

(1) 3.2×10^{-4} J (2) 1.6×10^{-4} J (3) 1.6×10^{-3} J (4) 3.2×10^{-3} J

7. A magnetic needle laying parallel to the magnetic field requires W units of work to turn it through 60°. Torque needed to maintain the needle in this position :-

(1) W (2) $\sqrt{3}$ W (3) $\frac{W}{2}$ (4) $\frac{\sqrt{3}W}{2}$

8. A bar magnet has a magnetic moment 2.5 A–m^2 and it is placed in a magnetic field of 0.2T. Calculate the Work done in turning the magnet from parallel to antiparallel position relative to the field direction:-

(1) 0.1 J (2) 1J (3) 2J (4) 5J

9. A circular coil of 'N' turns, 'R' radius carries a 'i' current. Work done in rotating this coil in an external magnetic field from $\theta_1 = 0°$ to $\theta_2 = 90°$:-

(1) $2\pi NiR^2B$ (2) $\frac{\pi NiR^2B}{2}$ (3) πNiR^2B (4) $\frac{\sqrt{3}\pi NiR^2B}{2}$

10. A short bar magnet placed with its axis at 30° with uniform magnetic field of 0.16T, experiences a torque of magnitude 0.032 N–m. The potential energy of bar magnet in stable equillibrium and in unstable equillibrium respectively :-

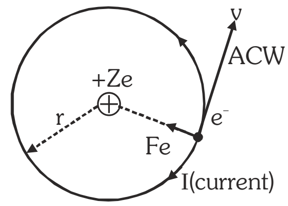
(1) –0.064 J, + 0.064 J (2) –0.032 J, + 0.032 J
(3) –0.016 J, + 0.016 J (4) zero, zero

9. ATOMIC MAGNETISM

An atomic orbital electron, which is performing bounded uniform circular motion around nucleus. A current constitues with this orbital motion and hence orbit behaves like current carrying loop. Due to this, magnetic field is produced at nucleus position. This phenomenon called as 'atomic magnetism'.

Bohr's postulates :

(i) $\frac{mv^2}{r} = \frac{kze^2}{r^2}$ (ii) $L = mvr = n\left(\frac{h}{2\pi}\right)$, where n = 1, 2,3 ....



Basic elements of atomic magnetism :

(a) **Orbital current :-** $I = ef = \frac{e}{T} = \frac{ev}{2\pi r} = \frac{e\omega}{2\pi}$

(b) **Magnetic induction at nucleus position :-** As circular orbit behaves like current carrying loop, so magnetic induction at nucleus position $B_N = \frac{\mu_0 I}{2r}$

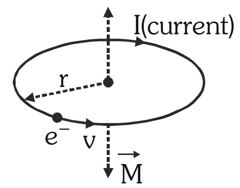
$$B_N = \frac{\mu_0 ef}{2r} = \frac{\mu_0 e}{2Tr} = \frac{\mu_0 ev}{4\pi r^2} = \frac{\mu_0 e\omega}{4\pi r}$$

(c) **Magnetic moment of circular orbit :-** Magnetic dipole moment of circular orbit

M = IA where A is area of circular orbit. $M = ef\left(\pi r^2\right) = \frac{\pi e r^2}{T} = \frac{evr}{2} = \frac{e\omega r^2}{2}$

- **Relation between magnetic moment and angular momentum of orbital electron**

Magnetic moment $M = \frac{evr}{2} \times \frac{m}{m} = \frac{eL}{2m}$ ($\because$ angular momentum L = mvr)



Vector form $\vec{M} = \frac{-e\vec{L}}{2m}$

For orbital electron its $\vec{M}$ and $\vec{L}$ both are antiparallel axial vectors.

Bohr magneton (μ_B)

According of Bohr's theory, angular momentum of orbital electron is given by

$L = \frac{nh}{2\pi}$, where n = 1, 2, 3 .... and h is plank's constant.

Magnetic moment of orbital electron is given by $M = \frac{eL}{2m} = n\frac{eh}{4\pi m}$

- If n = 1 then $M = \frac{eh}{4\pi m}$, which is Bohr magneton denoted by μ_B

- **Definition of μ_B :**

  Bohr magneton can be defined as the magnetic moment of orbital electron which revolves in first orbit of an atom.

- $\mu_B = \frac{eh}{4\pi m} = \frac{1.6\times10^{-19}\times6.6\times10^{-34}}{4\times3.14\times9.1\times10^{-31}} = 0.923\times10^{-23}\ Am^2$

- **Basic elements of atomic magnetism for first orbit of H-atom (n=1, z = 1)**

(a) Accurate form :- ($v = 2.18 \times 10^6$ m/sec, $f = 6.6 \times 10^{15}$ cy/sec. $r = 0.529$Å)

- Orbital current I = 0.96 mA
- Magnetic induction at nucleus position $B_N = 12.8$ T
- Magnetic moment of orbital electron $M = 0.923 \times 10^{-23}$ Am^2

(b) Simple form :- ($v \simeq 2 \times 10^6$ m/sec, $f \simeq 6 \times 10^{15}$cy/sec, $r \simeq 0.5$Å)

- Orbital current $I \simeq 1$mA
- Magnetic induction at nucleus position $B_N \simeq 4\pi$T
- Magnetic moment of orbital electron $M = \mu_B$ Am^2

A nonconducting charged body is rotated with some angular speed :-

In this case the ratio of magnetic moment and angular momentum is constant which is equal to $\frac{q}{2m}$

here q = charge and m = the mass of the body.

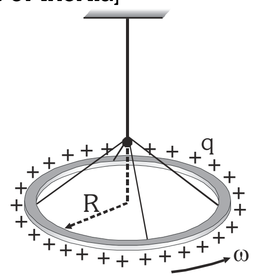
Example

In case of a ring, of mass m, radius R and charge q distributed on it circumference.

Angular momentum $L = I^*\omega = (mR^2)(\omega)$ ... (i) [here I* = Moment of Inertia]

Magnetic moment $M = iA = (qf)(\pi R^2)$

$$M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\frac{\omega R^2}{2} \quad \text{...(ii)}$$



$\because f = \frac{\omega}{2\pi}$ From Eqs. (i) and (ii) $\frac{M}{L} = \frac{q}{2m}$

Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disc or a sphere. $M = \frac{qL}{2m} \Rightarrow M = \frac{q(I^*\omega)}{2m}$, where $L = I^*\omega$

R.B.Name	**Moment of Inertia (I*)**	**$M = \frac{qI^*\omega}{2m}$**
Ring	mR^2	$\frac{1}{2}q\omega R^2$
Disc	$\frac{mR^2}{2}$	$\frac{1}{4}q\omega R^2$
Solid sphere	$\frac{2}{5}mR^2$	$\frac{1}{5}q\omega R^2$
sp. shell	$\frac{2}{3}mR^2$	$\frac{1}{3}q\omega R^2$

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1. Magnetic moment and angular momentum of an orbital electron are 'M' and L respectively. Specific charge of orbital electron :-

(1) $\frac{M}{2L}$ (2) $\frac{2M}{L}$ (3) $\frac{L}{2M}$ (4) $\frac{2L}{M}$

2. A particle of charge 'q' and mass 'm' is moving in a circular orbit with angular speed 'ω'. The ratio of its magnetic moment to that of its angular momentum depends on :-

(1) ω, q (2) ω,q,m (3) q,m (4) ω,m

3. Magnetic moment of orbital electron in first orbit is μ_B then magnetic moment of that electron in third orbit :-

(1) μ_B (2) $\frac{\mu_B}{3}$ (3) $3\mu_B$ (4) $9\mu_B$

4. A helium nucleus is moving in a circular path of radius 0.8m. If it takes 2 sec to complete one revolution. Magnetic field produced at the centre of the circle :-

(1) $\mu_0 \times 10^{-19}$ T (2) $\frac{10^{-19}}{\mu_0}$ T (3) 2×10^{-19} T (4) $\frac{2\times 10^{-19}}{\mu_0}$ T

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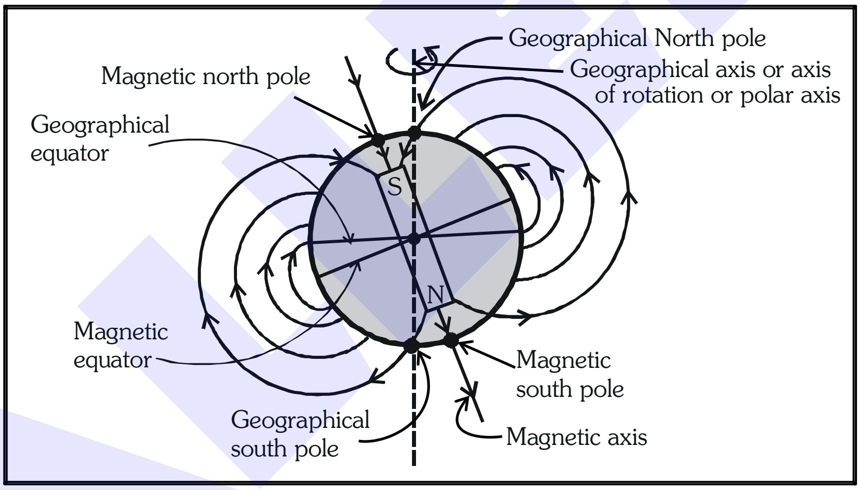
10. GEO-MAGNETISM (By Dr. William Gilbert)

The branch of physics which deals with the study of earth's magnetic field is called geomagnetism.



10.1 Important Definitions

(a) Geographic axis : It is a straight line which passes through the geographical poles of the earth. It is also called axis of rotation or polar axis of the earth.

(b) Geographic Meridian (GM) : It is a vertical plane which passes through geographical axis of the earth.

(c) Geographic equator : It is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distance from the geographic poles.

A great plane which passes through geographic equator and perpendicular to the geographic axis called **geographic equatorial plane**. This plane cuts the earth in two equal parts, a part has geographic north called **northen hemisphere** (NHS) and another part has geographic south called **southern hemisphere** (SHS).

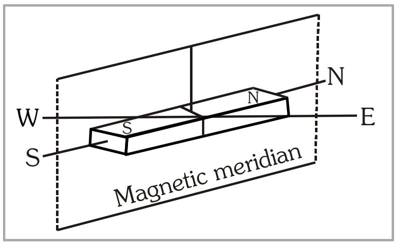
(d) Magnetic axis : It is a straight line which passes through magnetic poles of the earth. It is inclined to the geographic axis at nearly 11.3°.

(e) Magnetic Meridian (MM) :

(i) It is a vertical plane which passes through magnetic axis of the earth.

(ii) At a particular place it is a vertical plane, which passes through axis of free suspended bar magnet or magetic needle.

(iii) At a particular place it is a vertical plane, which contains all the field lines of earth's magnetic field.

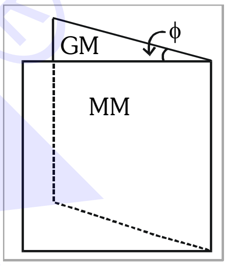


(f) Magnetic equator : It is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distance from the magnetic poles.

10.2 Main Elements of Earth's Magnetic Field

- **Angle of declination (ϕ)**

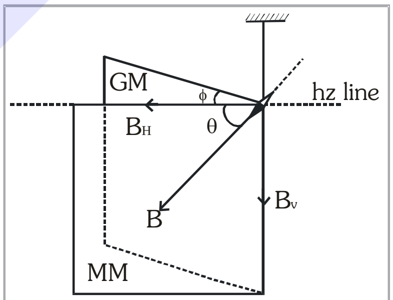
At a given place the acute angle between geographic meridian and the magnetic meridian is called angle of declination, i.e. at a given place it is the angle between the geographical north-south direction and the direction indicated by a magnetic compass needle in its equillibrium.



- **Angle of dip (θ)**

(i) At a given place it is the angle which the direction of resultant magnetic field of the earth subtends with respect to horizontal in magnetic meridian.

(ii) At a given place it is the angle which the axis of freely suspended magnetic needle subtends with the horizontal in magnetic meridian.

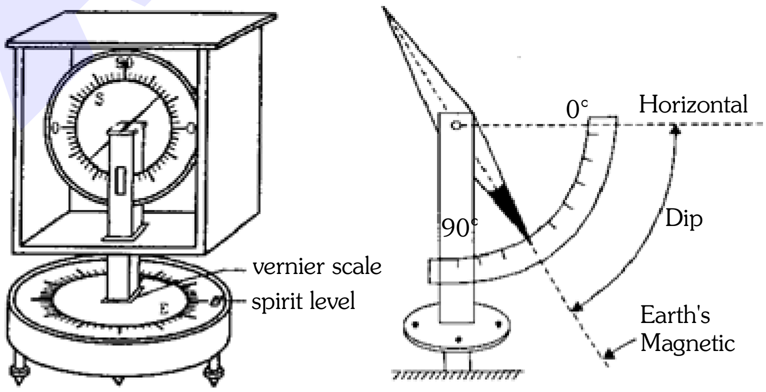


(This figure is for NHS)

Note :-

In NHS, north pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface.
In SHS, south pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface.

Dip circle : Angle of dip is measured by the instrument called 'Dip-circle' in which a magnetic needle is free to rotate in vertical plane about horizontal axis. The ends of the needle move over a vertical scale graduated in degree.



- **Horizontal component of earth's magnetic field (B_H)**

  At a given place horizontal component of earth magnetic field is the component of resultant magnetic field of the earth along the horizontal line in magnetic meridian.

$$B_H = B\cos\theta \quad \text{and } B_v = B\sin\theta \qquad \text{.......(1)}$$

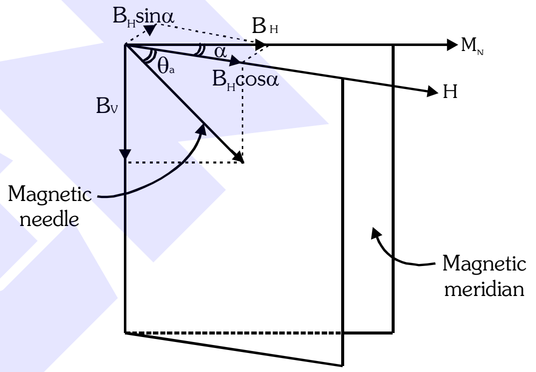
so that $\quad \tan\theta = \dfrac{B_V}{B_H}$ and $B = \sqrt{B_H^2 + B_V^2}$ ....(2)

Note 1 : At magnetic poles $\theta = 90°$ $\begin{cases} B_H = 0 \text{ (min)} \\ B_V = B \text{ (max) (fully)} \end{cases}$

At magnetic equator $\Rightarrow \theta = 0°$ $\begin{cases} B_H = B \text{ (max) (fully)} \\ B_V = 0 \text{ (min)} \end{cases}$

Note 2 : ϕ decides the plane in which magnetic field lies at any place, (ϕ) and (θ) decides the direction of magnetic field and (θ) and (B_H) decides the magnitude of the field.

Apparent angle of Dip (θ_a)



- If the dip circle is not kept in the magnetic meridian, the needle will not show the correct direction of earth's magnetic field.
  The angle made by the needle with horizontal is called the apparent dip for this plane.

- If dip circle is at an angle α to the meridian, the effective horizontal component in this plane is $B_H\cos\alpha$.

- Thus $\tan\theta_a = \dfrac{B_V}{B_H\cos\alpha} = \dfrac{\tan\theta}{\cos\alpha}$ ...(1)

  where θ_a represent apparent dip angle.

- Now if the dip circle is rotated through an angle 90° for this position. It will make angle (90 - α) with the meridian, lets this time apparent dip is θ'_a

  Thus $\tan\theta'_a = \dfrac{\tan\theta}{\cos(90-\alpha)} = \dfrac{\tan\theta}{\sin\alpha}$ ...(2)

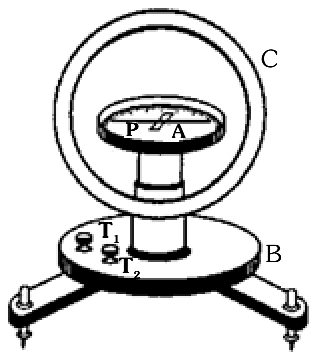
  as $\quad \sin^2\alpha + \cos^2\alpha = 1$

  $\Rightarrow \quad \cot^2\theta_a + \cot^2\theta'_a = \cot^2\theta$

  Thus, one can get the true dip angle θ, without locating the magnetic meridian.

11. APPLICATIONS OF GEO-MAGNETISM (BASED ON B_H)

11.1 Tangent Galvanometer



It is an instrument which can detect/measure electric d.c. currents.

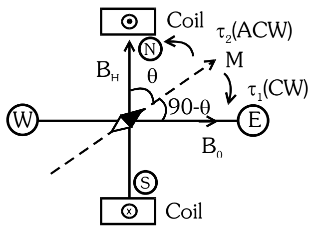
It is also called moving magnet type galvanometer.

Principle :- It is based on 'tangent law'.

Construction :- (i) It consists of a circular coil of a large number of turns of insulated copper wire wound over a vertical circular frame.

(ii) A small magnetic compass needle is pivoted at the centre of vertical circular coil. This needle can rotate freely in a horizontal plane.

Tangent law :- If a current is passed through the vertical coil, then magnetic field produced at its centre is perpendicular to the horizontal component of earth's magnetic field since coil is in magnetic meridian. So in the effect of two crossed fields ($B_H \perp B_0$) compass needle comes in equilibrium according to tangent law.



Torque on needle due to (B_0) :- $\tau_1 = MB_0\sin(90-\theta)$

Torque on needle due to (B_H) :- $\tau_2 = MB_H\sin\theta$

At equilibrium condition of needle net torque on it is zero

$\vec{\tau}_1 + \vec{\tau}_2 = 0$; $|\vec{\tau}_1| = |\vec{\tau}_2|$

$MB_0\sin(90-\theta) = MB_H\sin\theta \quad \Rightarrow \quad B_0\cos\theta = B_H\sin\theta$

$B_0 = B_H\dfrac{\sin\theta}{\cos\theta} \Rightarrow \boxed{B_0 = B_H \tan\theta}$, where $B_H = B\cos\theta^*$, $\theta^* \rightarrow$ angle of dip.

- The electric current is proportional to the tangent of the angle of deflection.

$$B_0 = B_H \tan\theta \Rightarrow \quad \frac{\mu_0 NI}{2R} = B_H \tan\theta \quad \Rightarrow \quad I = \left(\frac{2B_H R}{\mu_0 N}\right)\tan\theta$$

$I = K\tan\theta$, where $K = \dfrac{2B_H R}{\mu_0 N}$ called reduction factor of tangent galvanometer. So for this galvanometer I tanθ

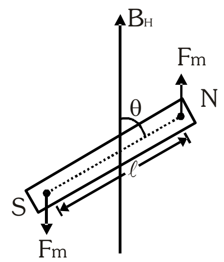
- The reduction factor of a tangent galvanometer is numerically equal to the current required to produce a deflection of 45° in it.

  $\theta = 45° \Rightarrow I = K\tan(45°) \Rightarrow I = K$

- SI unit of 'K' ⇒ ampere
- Sensitivity and accuracy of tangent galvanometer is maximum when the deflection is ⇒ θ = 45°

11.2 Vibration Magnetometer

It is an instrument used to compare the horizontal component of magnetic field of earth of two different places, to compare magnetic fields and magnetic moments of two bar magnets. It is also called oscillation magnetometer.



Principle : This device works on the principle, that whenever a freely suspended bar magnet in earth horizontal magnetic field component (B_H) is slightly disturbed from its equilibrium position then, it will experience a torque and execute angular S.H.M. (Rotation is possible only in horizontal plane.)

Angular S.H.M of magnetic dipole :- When a dipole is suspended in a uniform magnetic field it will align itself parallel to field. Now if it is given a small angular displacement θ with respect to its equilibrium position. The restoring torque acts on it :-

$\tau = -MB_H\sin\theta \quad \Rightarrow \quad I\alpha = -MB_H\sin\theta$

$I\alpha = -MB_H\,\theta$, $(\because \sin\theta \simeq \theta)$

$$\left.\begin{array}{l}\alpha = \dfrac{MB_H}{I}(-\theta) \\ \alpha = \omega^2(-\theta)\end{array}\right] \Rightarrow \omega^2 = \frac{MB_H}{I}$$

The time period of angular S.H.M. is given by :–

$$T = \frac{2\pi}{\omega} \quad \Rightarrow \quad \boxed{T = 2\pi\sqrt{\frac{I}{MB_H}}}$$

where :

M : magnetic moment of bar magnet

I : moment of inertia of bar magnet about its geomagnetic axis

- **Comparision of magnetic moments of magnets of the same size**

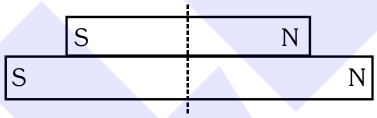
Let the two magnets of same size have moment of inertia I and magnetic moments M_1 and M_2. Suspend these two given magnets one by one in the metal stirrup of the vibration magnetometer and note the time period in each case.

Then $T_1 = 2\pi\sqrt{\frac{I}{M_1B_H}}$ and $T_2 = 2\pi\sqrt{\frac{I}{M_2B_H}}$ Dividing, $\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}$ or $\boxed{\frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}}$

By knowing T_1 and T_2 the ratio $\frac{M_1}{M_2}$ can be determined.

- **Comparision of magnetic moments of magnets of different sizes**

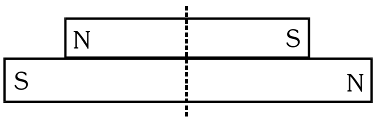
Let the two magnets have moments of inertia I_1 and I_2 and magnetic moments M_1 and M_2 respectively. Place these two given magnets one upon the other as shown in Fig. (a). This combination is called sum combination. It has moment of inertia $(I_1 + I_2)$ and magnetic moment $(M_1 + M_2)$. Put this combination in the magnetometer and set it into oscillations. The time period of combination T_1 is determined.



(a) Sum combination

$$T_1 = 2\pi\sqrt{\frac{(I_1 + I_2)}{(M_1 + M_2)B_H}} \qquad ....(1)$$

Now, the two magnets are placed as shown in Fig. (b). This combination is called 'difference combination'. It has moment of inertia $(I_1 + I_2)$ and magnetic moment $(M_1 - M_2)$. This combination is put in the magnetometer and its time period T_2 is determined.



(b) Difference combination

$$T_2 = 2\pi\sqrt{\frac{(I_1 + I_2)}{(M_1 - M_2)B_H}} \qquad ....(2)$$

Dividing, $\boxed{\frac{T_1}{T_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}}$ [from equation (1) and (2)]

By knowing T_1 and T_2, we can determine $\frac{M_1}{M_2}$.

- **Comparision of earth's magnetic field at two different places**

Let the vibrating magnet have moment of inertia I and magnetic moment M. Let it be vibrated at places, where earth's magnetic field is B_{H_1} and B_{H_2}

Then, $T_1 = 2\pi\sqrt{\dfrac{I}{MB_{H_1}}}$ and $T_2 = 2\pi\sqrt{\dfrac{I}{MB_{H_2}}}$

T_1 and T_2 are determined by placing magnetometer at two different places, one by one.

Dividing, $\dfrac{T_1}{T_2} = \sqrt{\dfrac{B_{H_2}}{B_{H_1}}} \Rightarrow \dfrac{T_1^2}{T_2^2} = \dfrac{B_{H_2}}{B_{H_1}} \Rightarrow \dfrac{T_1^2}{T_2^2} = \dfrac{B_2\cos\theta_2}{B_1\cos\theta_1}$

$$\boxed{\dfrac{B_1}{B_2} = \dfrac{T_2^2\cos\theta_2}{T_1^2\cos\theta_1}}$$

By knowing T_1, T_2 and θ_1, θ_2 the ratio $\dfrac{B_1}{B_2}$ can be determined.

Illustrations

Illustration 31.

Answer the following questions regarding earth's magnetism :

(a) The angle of dip at a location in southern India is about 18°. Would you expect a greater or smaller dip angle in Britain ?

(b) If you made a map of magnetic field lines at Melbourne in Australia. would the lines seem to go into the ground or come out of the ground ?

(c) A compass needle free to move in horizontal plane. In which direction it would stay, if located right on the geomagnetic north or south pole ?

(d) The earth's magnetic field varies from point to point in space. Does it also change with time? If so, on what time scale does it change appreciably?

(e) The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

Solution

(a) Greater in Britain (it is about 70°), because Britain is closer to the magnetic north pole.

(b) Field lines of B due to the earth's magnetism would seem to come out of the ground.

(c) It will remain in any direction in horizontal plane.

(d) Yes, it does change with time. Time scale for appreciable change is roughly a few hundred years. But even on a much smaller scale of a few years, its variations are not completely negligible.

(e) Earth's magnetic field gets weakly 'recorded' in certain rocks during solidification. Analysis of this rock magnetism offers clues to geomagnetic history.

Illustration 32.

The horizontal component of the earth's magnetic field at a certain place is 3×10^{-5} T and the direction of the field is from the geographic south to the geographic north. A very long straight conductor is carrying a steady current of 1 A. What is the force per unit length on it when it is placed on a horizontal table and the direction of the current is

(a) east to west; (b) south to north ?

Solution :

$\vec{F} = I\vec{\ell} \times \vec{B}$ or $F = I\ell B\sin\theta$

(a) When the current is flowing from east to west, $\theta = 90°$

Hence, $F = I\ell B = (1A)(1m)(3\times10^{-5}T)$

The direction of the force is downwards. This direction may be obtained either by Fleming's left hand rule or the directional property of cross product of vectors.

(b) When the current is flowing from south to north, $\theta = 0° = \Rightarrow F = 0$

Hence no force per unit length is exerted on the conductor.

Illustration 33.

A magnetic needle, pivoted about the horizontal axis, and free to move in the magnetic meridian, is observed to point along the

(a) vertical direction at a place A (b) horizontal direction at a place B

Give the value of the angle of dip at these two places.

Solution

(a) Angle of dip at A is 90° (b) Angle of dip at B is 0°

Illustration 34.

A tangent galvanometer has a coil of 50 turns and a radius of 20 cm, the horizontal component of the earth's magnetic field is $B_H = 3 \times 10^{-5}$ T. Find the current which gives a deflection of 45°.

Solution :

We have $i = K\tan\theta = \dfrac{2RB_H}{\mu_0 N}\tan\theta = \dfrac{2\times(0.20m)\times(3\times10^{-5}T)}{4\pi\times10^{-7}TmA^{-1}\times50}\tan45° = 0.19A$.

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1. A cell of an emf of 2V and internal resistance of 0.5 Ω is sending current through a tangent galvanometer of resistance 4.5Ω. If another external resistance of 95 Ω is introduced, the deflection of galvanometer is 45°. Raduction factor of galvanometer is :-

(1) 0.02 A (2) 0.01 A (3) 0.04 A (4) 0.05 A

2. Two tangent galvanometers A and B connected in series to a d. c. source have their number of turns in the ratio 1 : 3 and diameters in the ratio 1 : 2 of their coils which is correct for reduction factor and deflection of both galvanometers.

(1) $K_A < K_B, \theta_A > \theta_B$ (2) $K_A = K_B, \theta_A = \theta_B$ (3) $K_A > K_B, \theta_A < \theta_B$ (4) $K_A > K_B, \theta_A > \theta_B$

3. The time period of a freely suspended bar magnet does not depends on :-

(1) The length of the magnet (2) The pole strength of the magnet

(3) The horizontal component of the earth's magnetic field (4) The length of the suspension fibre

4. For a vibration magnetometer, the time period of freely suspended bar magnet can be reduced by:-

(1) Moving it towards south pole (2) Moving it towards north pole

(3) Moving it towards equator (4) None

5. A bar magnet is freely suspended in such a way that, when it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place, where dip angle is 30° and 15 oscillations per minute at a place, where dip angle is 60°. Ratio of total earth's magnetic field at these two places :-

(1) $9\sqrt{3}:16$ (2) $9:\sqrt{3}$ (3) $\sqrt{3}:16$ (4) $16:9\sqrt{3}$

6. Magnetic moments of two identical bar magnets are M and 2M respectively. Both are combined in such a way that their similar poles are together. The time period in this situation is 'T_1'. If polarity of one of the magnet is reversed then its period becomes 'T_2' then :-

(1) $T_1 > T_2$ (2) $T_1 < T_2$ (3) $T_1 = T_2$ (4) None of these

7. Time period of thin bar magnet of vibration magnetometer is 'T'. If it cuts into two equal parts in then the time period of each part fashions.

(a) Along or parallel to its length (b) Perpendicular to its length,

(1) T, T (2) T,T/2 (3) T/2, T (4) T/2, T/2

11.3 Neutral Point

It is a point where net magnetic field is zero.

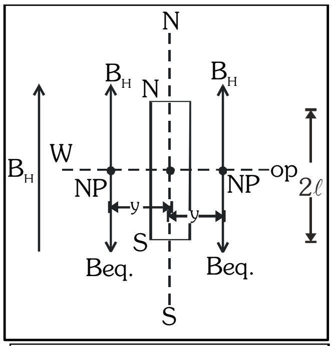
At this point magnetic field of bar magnet or current carrying coil or current carrying wire is just neutralised by horizontal component of earth magnetic field (B_H).

A compass needle placed at neutral point then it may set itself in any direction.

Location of Neutral Points :

(a) **When N–pole of magnet directed towards North :-**

Two neutral points symmetrically located on equatorial line of magnet. Let distance of each neutral point from centre of magnet is 'y' then

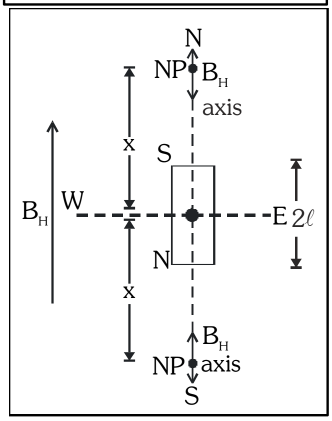


$B_{eq} = B_H$

$$\Rightarrow \frac{\mu_0}{4\pi}\cdot\frac{M}{y^3} = B_H \quad (\text{If } y >>> \ell)$$

(b) **When S–pole of magnet directed towards North :-**

Two neutral points symmetrically located on the axial line of magnet. Let distance of each neutral point from centre of the magnet is x, then



$B_{axis} = B_H$

$$\Rightarrow \frac{\mu_0}{4\pi}\cdot\frac{2M}{x^3} = B_H \quad (\text{If } x >>> \ell)$$

Illustrations

Illustration 35.

The earth's magnetic field at the equator is approximately 0.4 G. Estimate the earth's dipole moment.

Solution

Equatorial magnetic field $B_E = \dfrac{\mu_0 M}{4\pi r^3}$

Magnetic moment $M = \dfrac{B_E(4\pi r^3)}{\mu_0}$

where $B_E = 0.4G = 4\times10^{-5}T$, $r = 6.4\times10^{6}$ m

$= \dfrac{4\times10^{-5}\times(6.4\times10^{6})^3}{\mu_0/4\pi}$

$= 4\times10^{2}\times(6.4\times10^{6})^3$

$= 1.04\times10^{23}\,A\text{–}m^2$

BEGINNER'S BOX-11

1. The ratio of the magnetic field due to a small bar magnet in end on position to broad side on position is :-

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

2. The magnetic field at a point X on the axis of a small bar magnet is equal to the field at a point Y on the equator of the same magnet. The ratio of the distances of point X and Y from the centre of the magnet is :-

(1) 2^{-3} (2) $2^{-1/3}$ (3) 2^{3} (4) $2^{1/3}$

3. If a wire in the earth's magnetic field carries a current vertically downwards , it is possible to obtain a neutral point :-

(1) North of the wire (2) South of the wire (3) East of the wire (4) West of the wire

4. A short bar magnet placed on the table along the north south line. Its north pole is directed towards geographic north then location of neutral points is :-

(A) North (B) South (C) East (D) West

(1) A and B (2) C and D (3) A and D (4) B and C

5. A short magnet of moment 6.75 $A\text{–}m^2$ produces neutral points on its axis. If the horizontal component of earth's magnetic field 5×10^{-5} Wb/m^2, then the distance of the neutral points from the centre of magnet:-

(1) 10 cm (2) 20 cm (3) 30 cm (4) 40 cm

6. A straight wire carries current in vertical upward direction. A point 'P' lies just east of it at a small distance and another point 'Q' lies to the west just same distance. The magnetic field at these points :–
(consider earth magnetic field)

(1) $B_P > B_Q$ (2) $B_P < B_Q$ (3) $B_P = B_Q$ (4) None

12. MAGNETIC MATERIALS

12.1 Important definations and Relations

Magnetising field or Magnetic intensity $(\vec{H})$

Field in which a material is placed for magnetisation, called as magnetising field.

Magnetising field $\vec{H} = \dfrac{\vec{B}_0}{\mu_0} = \dfrac{\text{Magnetic field}}{\text{Permeability of free space}}$

Unit of $\vec{H}$: ampere/meter

Intensity of magnetisation $(\vec{I})$

When a magnetic material is placed in magnetising field then induced dipole moment per unit volume of that material is known as intensity of magnetisation $(\vec{I})$ $\vec{I} = \dfrac{\vec{M}}{V}$

Unit of $\vec{I}$: Ampere/meter $\left[\because \dfrac{\vec{M}}{V} = \dfrac{I\vec{A}}{V} = \dfrac{\text{ampere} \times \text{meter}^2}{\text{meter}^3}\right]$

Magnetic susceptibility (χ_m)

$\chi_m = \dfrac{I}{H}$ [It is a scalar with no units & dimensions]

Physically it represent the ease with which a magnetic material can be magnetised

Note:- A material with more χ_m, can be change into magnet easily.

Magnetic permeability μ

$$\mu = \frac{B_m}{H} = \frac{\text{Total magnetic field inside the material}}{\text{Magnetising field}}$$

It measures the degree to which a magnetic material can be penetrated (or permeated) by the magnetic field lines

Unit of μ : $\mu = \dfrac{B_m}{H} \equiv \dfrac{Wb/m^2}{A/m} \equiv \dfrac{Weber}{A-m} \equiv \dfrac{H-A}{A-m} = \dfrac{H}{m}$

$[\because \phi = LI \therefore \text{weber} \equiv \text{henry} - \text{ampere}]$

Relative permeability $\mu_r = \dfrac{\mu}{\mu_0}$ (It has no units and dimensions.)

Relation between permeability & susceptibility

When a magnetic material is placed in magnetic field $\vec{B}_0$ for magnetisation then total magnetic field in material

$\vec{B}_m = \vec{B}_0 + \vec{B}_i$, where $\vec{B}_i$ = induced field.

$$\because \vec{B}_0 = \mu_0\vec{H};\ \vec{B}_i = \mu_0\vec{I} \ \therefore \vec{B}_m = \mu_0\vec{H} + \mu_0\vec{I} \Rightarrow \vec{B}_m = \mu_0(\vec{H} + \vec{I}) = \mu_0\vec{H}(1 + I/H)$$

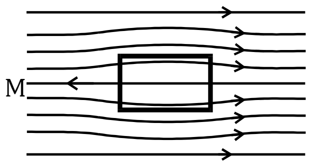
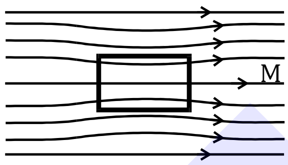
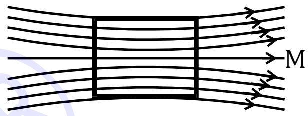
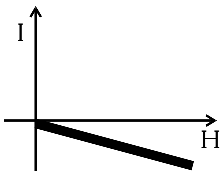
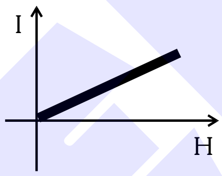
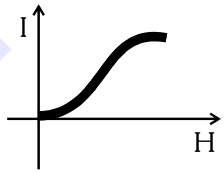
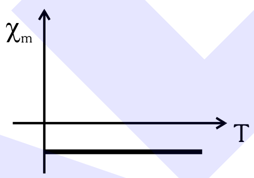
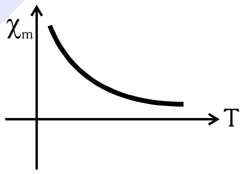
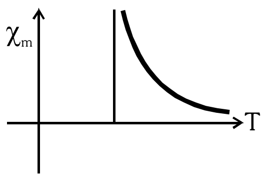
$$\Rightarrow \frac{B}{H} = \mu_0\left(1 + \frac{I}{H}\right) \Rightarrow \boxed{\mu = \mu_0(1 + \chi_m)} \Rightarrow \boxed{\mu_r = (1 + \chi_m)}$$

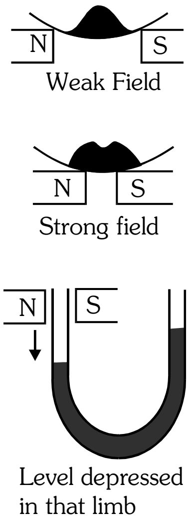
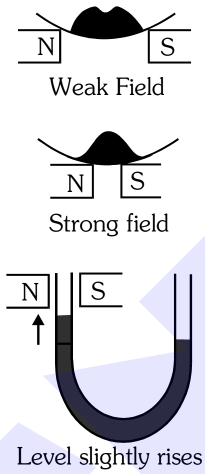
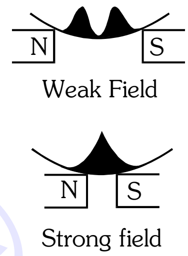
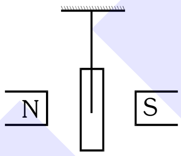
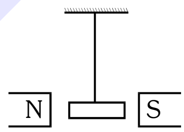
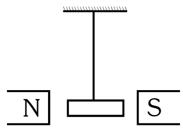
for vacuum $\chi_m = 0$, $(\because \mu_r = 1)$

at STP for Air $\chi_m = 0.04$ $(\because$ at S.T.P. for Air $\mu_r = 1.04)$

12.2 Classification of Magnetic Materials

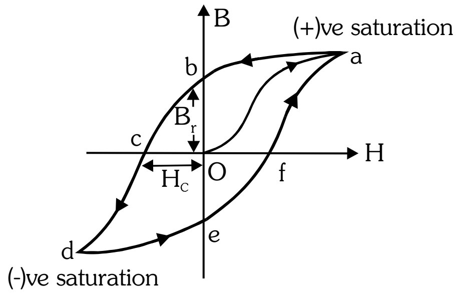
On the basis of magnetic properties of the materials [as magnetisation intensity (I), Susceptibility (χ_m) and relative permeability (μ_r)] Faraday devide these materials in three classes –

PROPERTIES	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Substance placed in uniform magnetic field.	Poor magnetisation in opposite direction. Here $B_m < B_0$ 	Poor magnetisation in same direction. Here $B_m > B_0$ 	Strong magnetisation in same direction. Here $B_m >>> B_0$ 
I – H curve	$I \rightarrow$ Small, negative, varies linearly with field 	$I \rightarrow$ Small, positive, varies linearly with field 	$I \rightarrow$ very large, positive & varies non–linearly with field 
χ_m – T curve	$\chi_m \rightarrow$ small, negative & temperature independent $\chi_m \propto T^{\circ}$ 	$\chi_m \rightarrow$ small, positive & varies inversely with temp. $\chi_m \propto \frac{1}{T}$ (Curie law) 	$\chi_m \rightarrow$ very large, positive & temp. dependent $\chi_m \propto \frac{1}{T-T_C}$ (Curie Weiss law) (for $T > T_C$) (T_C = Curie temperature)  $T_C(I_{iron})$=770°C or 1043K
μ_r	$(\mu < \mu_0)$ $1 > \mu_r > 0$	$2 > \mu_r > 1$ $(\mu > \mu_0)$	$\mu_r >>> 1$ $(\mu >>> \mu_0)$
Magnetic moment of single atom	Atoms do not have any permanent magnetic moment	Atoms have permanent megnetic moment which are randomly oriented. (i.e. in absence of external magnetic field the magnetic moment of whole material is zero)	Atoms have permanent megnetic moment which are organised in domains.

PROPERTIES	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
Behaviour of substance in Nonuniform magnetic field	It moves from stronger to weaker magnetic field.  	It moves with weak force from weaker magnetic field to stronger magnetic field.  	Strongly attract from weaker magnetic field to stronger magnetic field. 
When rod of material is suspended between poles of magnet.	It becomes perpendicular to the direction of external magnetic field.  	If there is strong magnetic field in between the poles then rod becomes parallel to the magnetic field.  	Weak magnetic field between magnetic poles can made rod parallel to field direction. 
Magnetic moment of substance in presence of external magnetic field	Value $\vec{M}$ is very less and opposite to $\vec{H}$.	Value $\vec{M}$ is low but in direction of $\vec{H}$.	$\vec{M}$ is very high and in direction of $\vec{H}$.
Examples	Bi, Cu, Ag, Pb, H_2O, Hg, H_2, He, Ne, Au, Zn, Sb, NaCl, Diamond.(May be found in solid, liquid or gas).	Na, K, Mg, Mn, Sn, Pt, Al, O_2 (May be found in solid, liquid or gas.)	Fe,Co, Ni all their alloys, Fe_3O_4 Gd, Alnico, etc. (Normally found only in solids) (crystalline solids)

12.3 Magnetic Hysteresis

Only Ferromagnetic materials show magnetic hysteresis, when Ferromagnetic material is placed in external magnetic field for magnetisation then B increases with H non-linearly along Oa. If H is again bring to zero then it decreases along path ab. Due to lagging behind of B with H this curve is known as hysteresis curve. [Lagging of B behind H is called hysteresis]



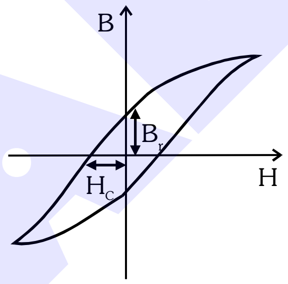
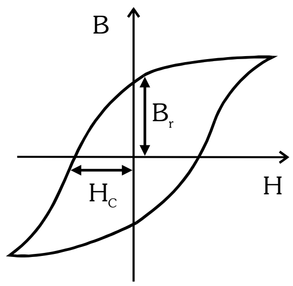
Cause of hysteresis :

By removing external magnetising field (H = 0), the magnetic moment of some domains remains aligned in the applied direction of previous magnetising field which results into a residual magnetism.

Residual magnetism (ob) = $\mathbf{B_r}$ ≡ retentivity ≡ remanence

Retentivity of a specimen is a measure of the magnetic field remaining in the ferromagnetic specimen when the magnetising field is removed.

Coercivity (oc) : Coercivity is a measure of magnetising field required to destroy the residual magnetism of the ferromagnetic specimen.

Ferromagnetic materials	
Soft magnetic materials	**Hard magnetic materials**
Low retentivity, low coercivity & small hysteresis loss. suitable for making electromagnets, cores of transformers etc. Ex. Soft iron, (used in magnetic shielding) 	High retentivity, high coercivity and large hysteresis loss suitable for permanent magnet Ex. Steel, Alnico 

12.4 Hysteresis loss

(i) The area of hystersis loop is equal to the energy loss per cycle per unit volume.

Area of hysteresis loop = $\oint B.dH = \mu_0 \oint I.dH$

(ii) Its value is different for different materials.

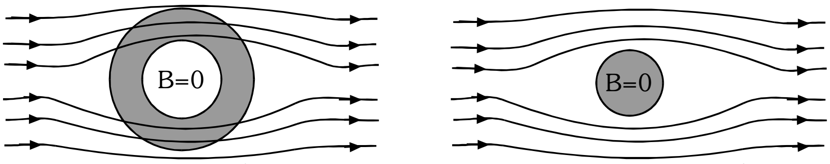
(iii) The work done per cycle per unit volume of material is equal to the area of hysteresis loop.

$\therefore$ Total energy loss in material = W_H = VAnt Joule $= \dfrac{VAnt}{J}$ calorie

i.e W_H = volume of material × area of hysteresis curve × frequency × time.

12.5 Magnetic shielding

If a soft iron ring is placed in magnetic field, most of the lines are found to pass through the ring and no lines pass through the space inside the ring. The inside of the ring is thus protected against any external magnetic effect. This phenomenon is called magnetic screening or shielding and is used to protect costly wrist–watches and other instruments from external magnetic fields by enclosing them in a soft–iron case or box.



Iron ring in a field   Super conductor in a field

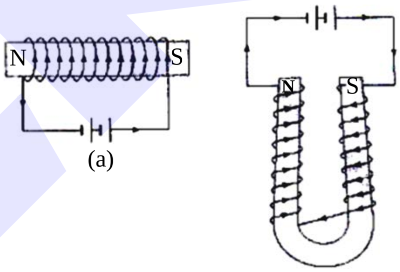
Special Note :–

(i) Super conductors also provide perfect magnetic screening due to exclusion of lines of force. This effect is called **'Meissner effect'**

(ii) Relative magnetic premeability of super conductor is zero. So we can say that super conductors behave like perfect dia-magnetic.

12.6 Electromagnet

As we know that a current carrying solenoid behaves like a bar magnet. If we place a soft iron rod in the solenoid, the magnetism of the solenoid increases hundreds of times and the solenoid is called an 'electromagnet'. It is a temporary magnet. An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft iron straight rod or a U shaped rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.



(a)   (b)

Application of electromagnets:

(i) Electromagnets are used in electric bell, electric lock, electric water valve, telephone diaphragms etc.

(ii) In medical field they are used in extracting bullets from the human body.

(iii) Large electromagnets are used in cranes for lifting and transferring big machines and parts.

Illustrations

Illustration 36.

A solenoid of 500 turns/m is carrying a current of 3A. Relative permeability of core material of solenoid is 5000. Determine the magnitudes of the magnetic intensity, magnetization and the magnetic field inside the core.

Solution :

magnetic intensity $H = ni$

$= 500\ m^{-1} \times 3A = 1500\ Am^{-1}$

$\mu_r = 1 + \chi$, i.e., $\chi = 5000 - 1 = 4999$

Hence, the magnetisation $I = \chi H\ (\chi \approx 5000)$

$= 7.5 \times 10^6\ Am^{-1}$

The magnetic field $B = \mu_0 \mu_r H$

$B = 5000\ \mu_0 H$

$= 5000 \times 4\pi \times 10^{-7} \times 1500 = 9.4\ T$

Illustration 37.

Relation between permeability μ and magnetising field H for a sample of iron is $\mu = \left(\frac{0.4}{H} + 12 \times 10^{-4}\right)$ henery/meter, where unit of H is A/m. Find value of H for which magnetic induction of 1.0 Wb/m^2 can be produce.

Solution :

Magnetic induction $B = \mu H$ (For medium)

$$1 = \left(\frac{0.4}{H} + 12 \times 10^{-4}\right) H$$

$$1 = 0.4 + 12 \times 10^{-4} H \Rightarrow H = \frac{1 - 0.4}{12 \times 10^{-4}} = 500 \text{ A/m}$$

Illustration 38.

When a rod of magnetic material of size 10 cm $\times$ 0.5 cm $\times$ 0.2 cm is located in magnetising field of 0.5×10^4 A/m then a magnetic moment of 5 A-m^2 is induced in it. Find out magnetic induction in rod.

Solition :

Total magnetic induction $B = \mu_0 (I + H)$

$$= \mu_0 \left(\frac{M}{V} + H\right) \qquad \left(\because I = \frac{M}{V}\right)$$

$$= 4\pi \times 10^{-7} \left(\frac{5}{10^{-6}} + 0.5 \times 10^4\right) = 6.28 \text{ Wb/m}^2$$

Illustration 39.

A rod of magnetic material of cross section 0.25 cm^2 is located in 4000 A/m magnetising field. Magnetic flux passes through the rod is 25×10^{-6} Wb. Find out for rod

(i) permeability (ii) magnetic susceptibility (iii) magnetisation

Solution :

(i) Magnetic flux $\phi = BA \Rightarrow B = \phi/A = \frac{25 \times 10^{-6}}{0.25 \times 10^{-4}} = 1 \text{ Wb/m}^2$

$$B = \mu H \Rightarrow \mu = \frac{B}{H} = \frac{1}{4000} = 2.25 \times 10^{-4} \text{ Wb/A-m}$$

(ii) $\mu_r = 1 + \chi_m \qquad \chi_m = \mu_r - 1 \left(\because \mu_r = \frac{\mu}{\mu_0}\right)$

$$\chi_m = \frac{\mu}{\mu_0} - 1 = \left(\frac{2.5 \times 10^{-4}}{4\pi \times 10^{-7}} - 1\right) = 199 - 1 = 198$$

(iii) Magnetization $I = \chi_m H = 198 \times 4000 = 7.92 \times 10^5$ A/m

Illustration 40.

As the earth's core has NiFe. Yet geologists do not regard this as a source of the earth's magnetism. Why?

Solution

Temperature of earth's core is very high, at this temperature NiFe is found in molten state only, which has no magnetic property.

Illustration 41.

Obtain the earth's magnetisation. Assume that the earth's field can be approximated by a giant bar magnet of magnetic moment 8.0×10^{22} Am^2. The earth's radius is 6400 km.

Solution :

The earth's radius R = 6400 km. = 6.4×10^6 m.

Magnetisation is the magnetic moment per unit volume. Hence,

$$I = \frac{M}{\frac{4\pi}{3}R^3} = \frac{8.0 \times 10^{22} \times 3}{4 \times \pi \times (6.4 \times 10^6)^3} = \frac{24.0 \times 10^4}{4 \times 262.1} = 72.9\ Am^{-1}$$

BEGINNER'S BOX-12

1. **1.** Fill in the blanks

(i) Diamagnetism is the .... property of every material and it is generated due to mutual interaction between the applied magnetic field and .... motion of electrons.

2. Match the column I with column II -

Column-I		Column-II	
(A)	Paramagnetic substance	(p)	$\chi_m = -1$
(B)	Diamagnetic substance	(q)	$\chi_m < 0$
(C)	Super conductor	(r)	$\chi_m >> 1$
(D)	Ferromagnetic substance	(s)	$\mu_r > 1$

3. How the magnetic susceptibility (χ_m) of diamagnetic, paramagnetic and feromagnetic substances depends on temperature (T).

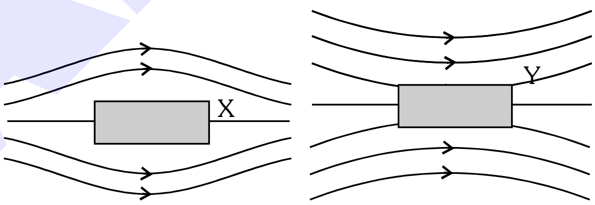
4. A solenoid has 10^3 turns per unit length. On passing a current of 2A, magnetic induction is measured to be 4π Wb/m^2. Calculate magnetic susceptibility of core.

5. A magnetising field of 1600 $A\text{-}m^{-1}$ produces a magnetic flux of 3.14×10^{-5} weber in a bar of iron of cross-section 0.2 cm^2 . Calculate permeability and susceptibility of the bar.

6. A uniform magnetic field gets modified as shown below, when the specimans X and Y are placed in it.

(a) Identify the two specimans X and Y

(b) State the reason for the behaviour of the field lines in X and Y.



7. How will a dia., para. and a ferromagnetic material behave when kept in a non–uniform external magnetic field ? Give two examples of each of these materials. Name two main characteristics of a ferromagnetic material which help us to decide its suitability for making

(a) a permanent magnet (b) an electromagnet.

Which of these two characteristics should have high or low values for each of these two types of magnets?

8. An iron rod of cross sectional area 4 cm^2 is placed with its length parallel to a magnetising field 1600 Am^{-1}. The flux through the rod is 4×10^{-4} weber. Find out permeability of the material of the rod ?

ANSWER KEY

BEGINNER'S BOX-1

1. (2) **2.** (3) **3.** (4) **4.** (2)

5. (1) **6.** (2) **7.** (4) **8.** (1)

9. (1) **10.** (2) **11.** (3) **12.** (3)

13.(a) 2 **(b)** 1

14. (a) 3 **(b)** 1 **(c)** 2

BEGINNER'S BOX-2

1. (1) **2.** (4) **3.** (3)

4. (1) **5.** (1)

6. **i.** (a) $\frac{\mu_0 I}{4R}\otimes$ (b) $\frac{\mu_0 I}{8R}\otimes$ **ii.** (a) $\frac{3\mu_0 I}{8R}\odot$ (b) $\frac{\mu_0 I}{8R}\odot$

iii. (a) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)\odot$ (b) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)\otimes$

iv. (a) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right)\otimes$ (b) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}-\frac{1}{\pi}\right)\odot$

v. (a) $\frac{\mu_0 I}{8R}\left(3+\frac{2}{\pi}\right)\odot$ (b) $\frac{\mu_0 I}{4R}\left(1+\frac{2}{\pi}\right)\odot$

vi. (a) $\frac{3\mu_0 I}{8R}\otimes$ (b) $\frac{\mu_0 I}{24R}\odot$

7. (4) **8.** (2) **9.** $\frac{R}{\sqrt{150}}$

BEGINNER'S BOX-3

1. (1) **2.** (2) **3.** (3) **4.** (4)

5. (3)

BEGINNER'S BOX-4

1. (a) True (b) True

2. B

3. 1T

BEGINNER'S BOX-5

1. (4) **2.** (2) **3.** (1) **4.** (3)

5. (a) 1 : 2 : 2 (b) 2 : 2 : 1 (c) $1:\sqrt{2}:1$ (d) $1:\sqrt{2}:\sqrt{2}$

6. (a) 2 : 1 : 1 (b) 1 : 1 : 2 (c) 2 : 1 : 2 (d) 2 : 1 : 1

7. (a) $C_p < C_e$ (b) $C_p = C_e$ (c) $C_p < C_e$ (d) $C_p < C_e$
(C = curvature)

8. (a) 1 : 2 : 2 (b) 2 : 1 : 1 (c) 2 : 1 : 1

9. 1.2 cm, 4.35 cm

10. 0.1m, 6.28×10^{-7} sec

11. T

12. (a), (b) and (c)

13. (a), (b) and (c)

14. (a), (b), (c) and (d)

15. (a), (b) and (c)

16. (3) **17.** (2)

18. (1) **19.** (3) **20.** (2) **21.** (2)

22. (2) **23.** (4)

BEGINNER'S BOX-6

1. (2) **2.** (1) **3.** (1) **4.** (4)

5. (1) **6.** (2) **7.** (1) **8.** (4)

9. (1)

BEGINNER'S BOX-7

1. (a) 2M, (b) Zero, (c) $\sqrt{2}$ M, (d) M,(e) Zero, (f) $2\sqrt{2}$M

2. (a) $\frac{M}{2}$, (b) $\frac{M}{2}$

3. (a) $\frac{M}{\sqrt{2}}$, (b) $\frac{M}{2}$, (c) $\frac{2M}{\pi}$, (d) Zero

4. $\frac{\pi a^3 B}{2\sqrt{2}\mu_0}$

5. (a) $\sqrt{3}\frac{I\ell^2}{36}$, (b) $\frac{I\ell^2}{16}$, (c) $\frac{\sqrt{3}}{24}I\ell^2$, (d) $\frac{I\ell^2}{4\pi}$

6. 2 : 1 **7.** One turn **8.** $\sqrt{2}$M

BEGINNER'S BOX-8

1. (3)	**2.** (2)	**3.** (4)	**4.** (4)
5. (1)	**6.** (1)	**7.** (2)	**8.** (2)
9. (3)	**10.** (1)		

BEGINNER'S BOX-9

1. (2)	**2.** (3)	**3.** (3)	**4.** (1)

BEGINNER'S BOX-10

1. (1)	**2.** (3)	**3.** (4)	**4.** (3)
5. (4)	**6.** (2)	**7.** (2)	

BEGINNER'S BOX-11

1. (4)	**2.** (4)	**3.** (3)	**4.** (2)
5. (3)	**6.** (1)		

BEGINNER'S BOX-12

1. Intrinsic, orbital

2. A – s, B – q, C – p, D – r

3. For diamagnetic $\chi_m \propto T^0$

For paramagnetic $\chi_m \propto \frac{1}{T}$

For ferromagnetic $\chi_m \propto \frac{1}{T - T_c}$ here $T_c \rightarrow$ Curie temperature

4. 4999

5. Permeability (μ) = $9.8 \times 10^{-4} \frac{Wb}{A-m}$

Susceptibility (χ_m) = 779.25

6. (a) X → Dia-magnetic Y → Para-magnetic

(b) Magnetic permeability of dia-magentic speciman is slightly less than vacuum and magnetic permeability of para–magnetic speciman is slightly more than vacuum.

7. **Diamagnetic :** Moves (very weakly) away from strong field region towards the weak field region.

Para magnetic : Moves (weakly) towards the strong field region.

Ferromagnetic : Moves (strongly) towards the strong field region.

Two examples : Diamagnetic (Bismuth, Copper)

Paramagnetic (Aluminium, Oxygen)

Ferromagnetic (Iron, Nickel)

Two charatertics of ferromagnetic materials : Coercivity and Retentivity

For Permanent Magnets : High Coercivity and High Retentivity

For Electromagnets : Low Coercivity and Low Retentivity

8. $\mu = 0.625 \times 10^{-3}$ Wb /A-m