PRE-MEDICAL

PHYSICS

ENTHUSIAST | LEADER | ACHIEVER

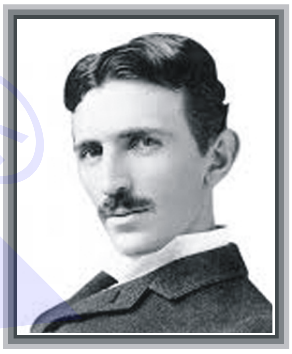
STUDY MATERIAL

Alternating Current

ENGLISH MEDIUM

Copyright Statement

NICOLA TESLA (1836 –1943)



Yugoslov scientist, inventor and genius. He conceived the idea of the rotating magnetic field, which is the basis of practically all alternating current machinery, and which helped usher in the age of electric power. He also invented among other things the induction motor, the polyphase system of ac power, and the high frequency induction coil (the Tesla coil) used in radio and television sets and other electronic equipment. The SI unit of magnetic field is named in his honour.

GEORGE WESTINGHOUSE (1846 – 1914)

A leading proponent of the use of alternating current over direct current. Thus, he came into conflict with Thomas Alva Edison, an advocate of direct current. Westinghouse was convinced that the technology of alternating current was the key to the electrical future. He founded the famous Company named after him and enlisted the services of Nicola Tesla and other inventors in the development of alternating current motors and apparatus for the transmission of high tension current, pioneering in large scale lighting.

ALTERNATING CURRENT

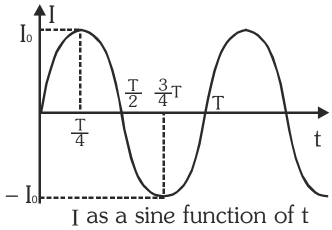
1. ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it changes continuously in magnitude and periodically in direction. It can be represented by a sine curve or cosine curve

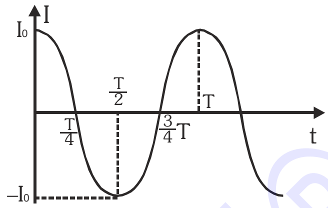
$I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$

where I = Instantaneous value of current at time t, I_0 = Amplitude or peak value

ω = Angular frequency $\omega = \dfrac{2\pi}{T} = 2\pi f$ T = time period   f = frequency



I as a sine function of t



I as a cosine function of t

1.1 Amplitude of AC

The maximum value of current in either direction is called peak value or the amplitude of current. It is represented by I_0. Peak to peak value = $2I_0$

1.2 Periodic Time

The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

1.3 Frequency

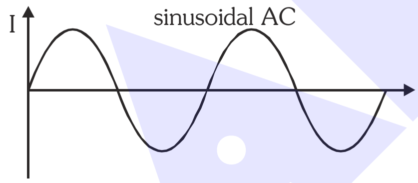
The number of cycle completed by an alternating current in one second is called the frequency of the current.

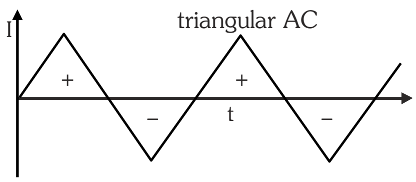
UNIT : (cycle/s) or (Hz)

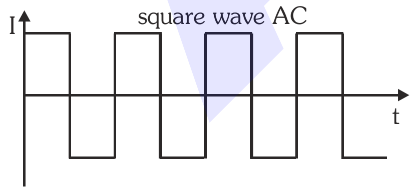
In India : f = 50 Hz , supply voltage = 220 volt **In USA :** f = 60 Hz ,supply voltage = 110 volt

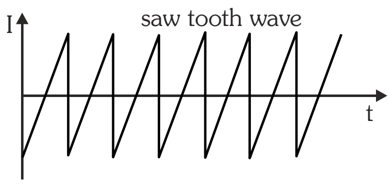
1.4 Condition required for current/ voltage to be Alternating

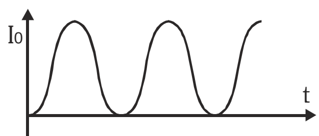
- Alternate half cycle is positive and half negative.
- The alternating current continuously varies in magnitude and periodically reverses its direction.



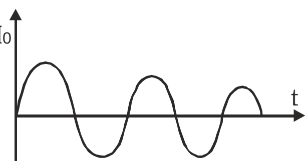




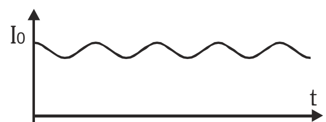




Not AC (direction not change)



Not AC (not periodic)



mixture of AC and DC

I = a + b sin wt (a > b)

1.5 Average Value or Mean Value

The mean value of A.C. over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

average value of current for half cycle $\langle I \rangle = \dfrac{\int_0^{T/2} I dt}{\int_0^{T/2} dt}$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{\frac{T}{2}} I_0 \sin \omega t \, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_0}{\omega T}\left[-\cos \omega t\right]_0^{\frac{T}{2}} = \frac{2I_0}{\pi}$$

$< \sin \theta > = < \sin 2\theta > = 0$
$< \cos \theta > = < \cos 2\theta > = 0$
$< \sin \theta \cos \theta > = 0$
$< \sin^2 \theta > = < \cos^2 \theta > = \frac{1}{2}$

- For symmetric AC, average value over full cycle = 0,
  Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(–ve) half cycle
0	$\frac{2I_0}{\pi}$	$\frac{-2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

1.6 Maximum Value

- $I = a \sin\theta \Rightarrow I_{Max.} = a$
- $I = a + b \sin\theta \Rightarrow I_{Max.} = a + b$ (if a and b > 0)
- $I = a \sin\theta + b \cos \theta \Rightarrow I_{Max.} = \sqrt{a^2 + b^2}$
- $I = a \sin^2 \theta \Rightarrow I_{Max.} = a$ (a > 0)

1.7 Root Mean square (rms) Value

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$

rms value = virtual value = apparent value

rms value of $I = I_0 \sin \omega t$:

$$I_{rms} = \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2 \omega t \, dt} = I_0 \sqrt{\frac{1}{T}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt} = I_0 \sqrt{\frac{1}{T}\left[\frac{t}{2} - \frac{\sin 2\omega t}{2 \times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

- If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	Average	Peak	RMS	Angular frequency
$I_1 = I_0 \sin \omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2}\sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin\omega t + I_0 \cos\omega t$	0	$\sqrt{2} I_0$	I_0	ω

- For above varieties of current

$$\boxed{\text{rms} = \frac{\text{Peak value}}{\sqrt{2}}}$$

1.8 Measurement of A.C.

Alternating current and voltages are measured by a.c. ammeter and a.c. voltmeter respectively. Working of these instruments is based on heating effect of current, hence they are also called hot wire instruments.

Terms	D.C. meter	A.C. meter
Name	moving coil instrument	hot wire instrument
Based on	magnetic effect of current	heating effect of current
Reads	average value	r.m.s. value
If used in	A.C. circuit then they reads zero $\because$ average value of A.C. = zero	A.C. or D.C. then meter works properly as it measures rms value
Deflection	deflection $\propto$ current $\phi \propto I$ (linear)	deflection $\propto$ heat $\phi \propto I^2_{rms}$ (non linear)
Scale ϕ = Number of divisions	Uniform Separation I - 1 2 3 4 5 ϕ - 1 2 3 4 5	Non uniform separation I - 1 2 3 4 5 ϕ - 1 4 9 16 25

1.9 Phase and phase difference

(a) Phase

$I = I_0 \sin(\omega t \pm \phi)$

Initial phase = ϕ (it does not change with time)

Instantaneous phase = $\omega t \pm \phi$ (it changes with time)

- Phase decides both value and sign.
- **UNIT:** radian

(b) Phase difference

Voltage $V = V_0 \sin(\omega t + \phi_1)$

Current $I = I_0 \sin(\omega t + \phi_2)$

- Phase difference of I w.r.t. V $\quad \phi = \phi_2 - \phi_1$
- Phase difference of V w.r.t. I $\quad \phi = \phi_1 - \phi_2$

1.10 Lagging and leading Concept

(a) V leads I or I lags V $\rightarrow$ It means, V reach maximum before I

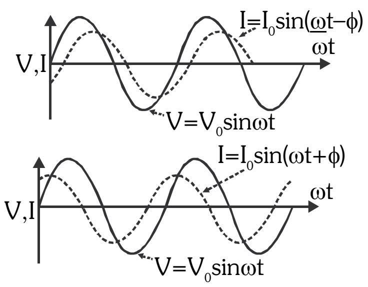
Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t - \phi)$

and if $V = V_0 \sin(\omega t + \phi)$ then $I = I_0 \sin \omega t$

(b) V lags I or I leads V $\rightarrow$ It means V reach maximum after I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t + \phi)$

and if $V = V_0 \sin(\omega t - \phi)$ then $I = I_0 \sin \omega t$



1.11 Phasor and Phasor diagram

A diagram representing alternating current and voltage (of same frequency) as vectors (phasor) with the phase angle between them is called phasor diagram.

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$

In figure (a) two arrows represents phasors. The length of phasors represents the maximum value of quantity. The projection of a phasor on y-axis represents the instantaneous value of quantity. In figure (b) two arrows represents phasor. Their length represents maximum value.

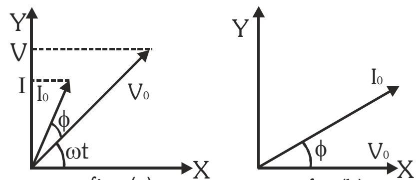


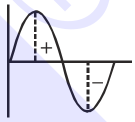
fig. (a) fig (b)

1.12 Advantages of AC

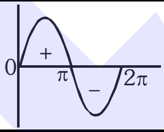
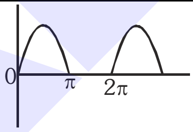
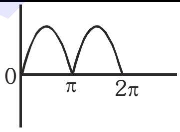
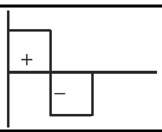
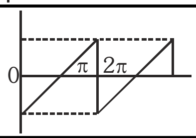
- A.C. is cheaper than D.C
- It can be easily converted into D.C. (by rectifier)
- It can be controlled easily (choke coil)
- It can be transmitted over long distance at low power loss.
- It can be stepped up or stepped down with the help of transformer.

GOLDEN KEY POINTS

- AC can't be used in
  (a) Charging of battery or capacitor (as its average value = 0)
  (b) Electrolysis and electroplating (Due to large inertia, ions can not follow frequency of A.C)
- The rate of change of A.C. →
  - Minimum, at that instant when they are near their peak values
  - Maximum, at that instant when they change their direction.
- For alternating current $I_0 > I_{rms} > I_{av.}$
- Average value over half cycle is zero if one quarter is positive and the other quarter is negative.



- Average value of symmetrical AC for a cycle is zero that's why average potential difference on any element in A.C circuit is zero.
- The instrument based on heating effect of current are works on both A.C and D.C supply and also provides same heating for same value of A.C (rms) and D.C. that's why a bulb bright equally in D.C. and A.C. of same value.
- If the frequency of AC is f then it becomes zero, 2f times in one second and the direction of current changes 2f times in one second. Also it become maximum 2f times in one second.
- Some Important wave forms and their RMS and Average Value

Nature of wave form	Wave–form	RMS Value	Average or mean Value
Sinusoidal		$\frac{I_0}{\sqrt{2}} = 0.707\ I_0$	$\frac{2I_0}{\pi} = 0.637\ I_0$ (Half)
Half wave rectifier		$\frac{I_0}{2} = 0.5\ I_0$	$\frac{I_0}{\pi} = 0.318\ I_0$ (Full)
Full wave rectifier		$\frac{I_0}{\sqrt{2}} = 0.707\ I_0$	$\frac{2I_0}{\pi} = 0.637\ I_0$ (Half and Full)
Square or Rectangular		I_0	I_0 (Half)
Saw Tooth wave		$\frac{I_0}{\sqrt{3}}$	$\frac{I_0}{2}$ (Half)

- D.C meter in AC circuit reads zero because < AC > = 0 (for complete cycle)
- AC meter works in both AC and DC

Illustrations

Illustration 1.

If $I = 2\sqrt{t}$ ampere then calculate average and rms values over t = 2 to 4 s

Solution

$$\langle I\rangle = \frac{\int_2^4 2\sqrt{t}.dt}{\int_2^4 dt} = \frac{4}{3}\frac{(t^{\frac{3}{2}})_2^4}{(t)_2^4} = \frac{2}{3}\left[8-2\sqrt{2}\right] \text{ and } I_{rms} = \sqrt{\frac{\int_2^4 (2\sqrt{t})^2 dt}{\int_2^4 dt}} = \sqrt{\frac{\int_2^4 4t\,dt}{2}} = \sqrt{2\left[\frac{t^2}{2}\right]_2^4} = 2\sqrt{3}\text{ A}$$

Illustration 2.

If E = 20 sin (100π t) volt then calculate value of E at $t = \frac{1}{600}$ s

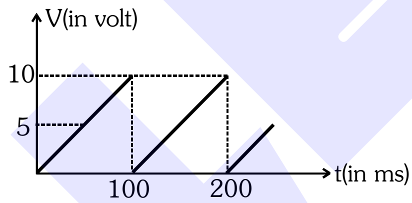
Solution

$$\text{At } t = \frac{1}{600}\text{ s } E = 20 \text{ Sin}\left[100\pi \times \frac{1}{600}\right] = 20 \sin\left[\frac{\pi}{6}\right] = 20 \times \frac{1}{2} = 10\text{V}$$

Illustration 3.

A periodic voltage wave form has been shown in figure. Determine.

(a) Frequency of the wave form. (b) Average value.



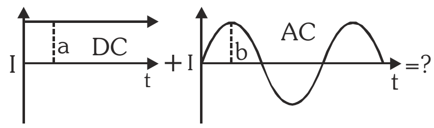
Solution

(a) After 100 ms wave is repeated so time period is T = 100 ms. ⇒ $f = \frac{1}{T}$ = 10 Hz

(b) Average value = Area/time period $= \frac{(1/2)\times 100 \times 10}{(100)}$ = 5 volt

Illustration 4.

If a direct current of value a ampere is superimposed on an alternating current I = b sinωt flowing through a wire, what is the effective value of the resulting current in the circuit ?



Solution

As current at any instant in the circuit will be $I = I_{DC} + I_{AC} = a + b\sin\omega t$

$$\therefore I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a+b\sin\omega t)^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2\sin^2\omega t)dt}$$

$$\text{but as } \frac{1}{T}\int_0^T \sin\omega t\,dt = 0 \text{ and } \frac{1}{T}\int_0^T \sin^2\omega t\,dt = \frac{1}{2} \quad \therefore I_{eff} = \sqrt{a^2 + \frac{1}{2}b^2}$$

Illustration 5.

The Equation of current in AC circuit is $I = 4\sin\left[100\pi t + \frac{\pi}{3}\right]$A. Calculate.

(i) RMS Value (ii) Peak Value (iii) Frequency (iv) Initial phase (v) Current at t = 0

Solution

(i) $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\,A$

(ii) Peak value $I_0 = 4A$

(iii) $\because$ $\omega = 100\,\pi$ rad/s $\therefore$ frequency $f = \frac{100\pi}{2\pi} = 50$ Hz

(iv) Initial phase $= \frac{\pi}{3}$

(v) At t = 0, $I = 4\sin\left[100\pi \times 0 + \frac{\pi}{3}\right] = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}\,A$

Illustration 6.

If $I = I_0 \sin \omega t$, $E = E_0 \cos\left[\omega t + \frac{\pi}{3}\right]$. Calculate phase difference between E and I

Solution

$I = I_0 \sin \omega t$ and $E = E_0 \sin\left[\frac{\pi}{2} + \omega t + \frac{\pi}{3}\right]$ $\therefore$ phase difference $= \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

Illustration 7.

If E = 500 sin (100 πt) volt then calculate time taken to reach from zero to maximum.

Solution

$\because \omega = 100\,\pi \Rightarrow T = \frac{2\pi}{100\pi} = \frac{1}{50}$ s, time taken to reach from zero to maximum $= \frac{T}{4} = \frac{1}{200}$ s

Illustration 8.

If Phase Difference between E and I is $\frac{\pi}{4}$ and f = 50 Hz then calculate time difference.

Solution

$\because$ $2\pi \equiv T \therefore \frac{\text{Phase difference}}{2\pi} = \frac{\text{time difference}}{T}$

$\Rightarrow$ Time difference $= \frac{T}{2\pi} \times \frac{\pi}{4} = \frac{T}{8} = \frac{1}{50 \times 8} = 2.5$ ms

BEGINNER'S BOX-1

1. Explain why A.C. is more dangerous than D.C. ?

2. Show that average heat produced during a cycle of AC is same as produced by DC with $I = I_{rms}$.

3. An ordinary moving coil ammeter used for d.c., cannot be used to measure a.c. even if its frequency is low why ?

4. Find the time required for a 50Hz alternating current to change its value from zero to rms value.

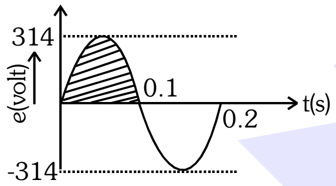
5. The current and voltage in a circuit is given by

$i = 3.5 \sin(628t + 30°)$ A , $V = 28 \sin(628t - 30°)$ volt. Find

(a) time period of current

(b) phase difference between voltage and current.

6. The figure given below shows the variation of an alternating emf with time. What is the average value of the emf for the shaded part of the graph?



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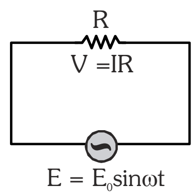
2. DIFFERENT TYPES OF AC CIRCUITS

In order to study the behaviour of A.C. circuits we classify them into two categories :

(a) Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.

(b) Complicated circuit containing any two of the three circuit elements R, L and C or all of the three elements.

2.1 AC circuit containing pure resistance



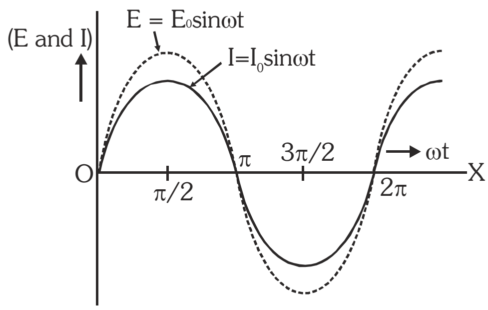
Let at any instant t, the current in the circuit = I.

Potential difference across the resistance = I R

with the help of kirchoff's circuital law $E - IR = 0 \;\; \Rightarrow E_0 \sin \omega t = IR$

$\Rightarrow I = \dfrac{E_0}{R}\sin\omega t = I_0 \sin\omega t$ ($I_0 = \dfrac{E_0}{R}$ = peak or maximum value of current)

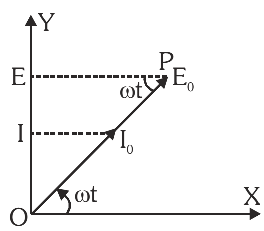
Alternating current developed in a pure resistance is also of the sinusoidal nature. In a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. are as shown in figure.



In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent the instantaneous values of alternating current and voltage.

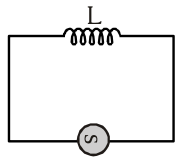
i.e. $\quad I = I_0 \sin \omega t$ and $\quad E = E_0 \sin \omega t$.

Since $I_0 = \dfrac{E_0}{R}$, hence $\dfrac{I_0}{\sqrt{2}} = \dfrac{E_0}{R\sqrt{2}} \Rightarrow \quad I_{rms} = \dfrac{E_{rms}}{R}$



2.2 AC circuit containing pure inductance

A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf. Let the alternating e.m.f. $E = E_0 \sin \omega t$



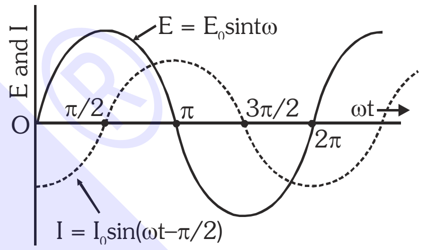
$E = E_0 \sin \omega t$

When a.c. flows through the circuit, emf induced across inductance $= -L\dfrac{dI}{dt}$

Note : Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other than inductance so with the help of Kirchoff's circuital law $E + \left(-L\dfrac{dI}{dt}\right) = 0 \Rightarrow E = L\dfrac{dI}{dt}$ so we get $I = \dfrac{E_0}{\omega L}\sin\left(\omega t - \dfrac{\pi}{2}\right)$

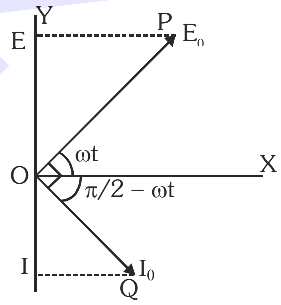
Maximum current $I_0 = \dfrac{E_0}{\omega L} \times 1 = \dfrac{E_0}{\omega L}$, Hence, $I = I_0 \sin\left(\omega t - \dfrac{\pi}{2}\right)$



In a pure inductive circuit current always lags behind the emf by $\dfrac{\pi}{2}$

or alternating emf leads the a. c. by a phase angle of $\dfrac{\pi}{2}$.

Expression $I_0 = \dfrac{E_0}{\omega L}$ resembles the expression $\dfrac{E}{I} = R$.



This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circuit.

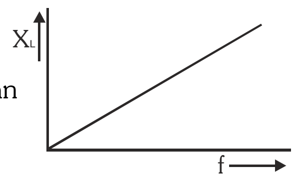
$X_L = \omega L = 2\pi f L$ where f = frequency of A.C.

Unit of X_L : ohm

(ωL) = Unit of L $\times$ Unit of $(\omega = 2\pi f)$ = henry $\times \sec^{-1}$

$= \dfrac{\text{volt}}{\text{ampere / sec}} \times \sec^{-1} = \dfrac{\text{volt}}{\text{ampere}} = \text{ohm}$

Inductive reactance $X_L \propto f$



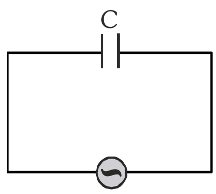
Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.

For d.c. circuit, f = 0 $\therefore X_L = \omega L = 2\pi f L = 0$

Hence, inductor offers no opposition to the flow of d.c. where as a resistive path to a.c.

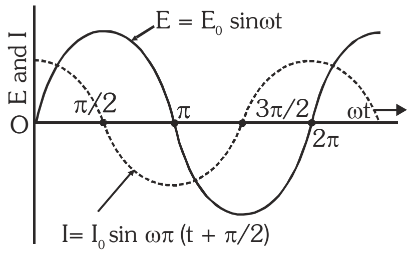
2.3 AC circuit containing pure capacitance

A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E = E_0 \sin \omega t$. When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.



$E = E_0 \sin \omega t$

The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time. Let at any instant t charge on the capacitor = q

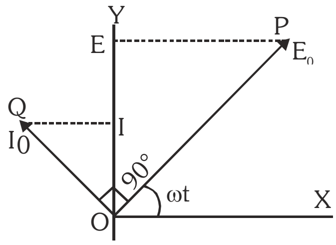


Instantaneous potential difference across the capacitor $E = q/C$

$\Rightarrow q = CE \quad \Rightarrow q = CE_0 \sin \omega t$

The instantaneous value of current $I = \frac{dq}{dt} = \frac{d}{dt}\left(CE_0 \sin\omega t\right) = CE_0\omega\cos\omega t$

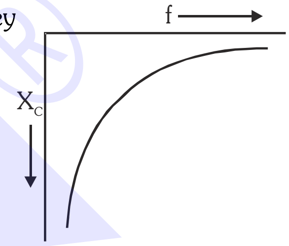
$\Rightarrow I = \frac{E_0}{(1/\omega C)}\sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ where $I_0 = \omega CE_0$



In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of π/2. The alternating emf lags behinds the alternating current by a phase angle of π/2.

IMPORTANT POINTS

E/I is the resistance R when both E and I are in phase, in present case they differ in phase by $\frac{\pi}{2}$, hence $\frac{1}{\omega C}$ is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C. This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

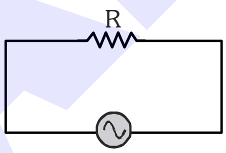
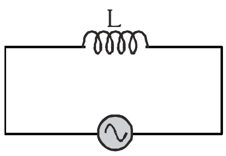
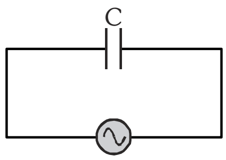


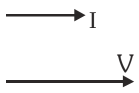
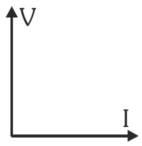
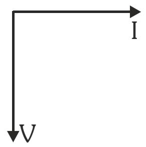
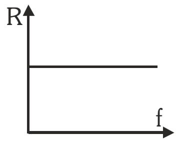
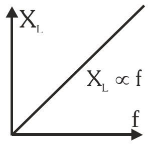
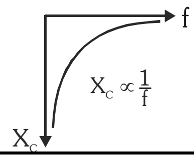
Unit of X_C : ohm

Capacitive reactance X_C is inversely proportional to frequency of A.C. X_C decreases as the frequency increases.

For d.c. circuit f = 0 $\therefore X_C = \frac{1}{2\pi fC} = \infty$ but has a very small value for a.c.

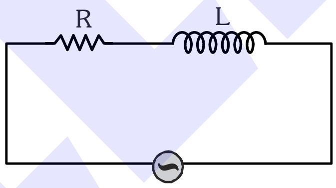
This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c.

INDIVIDUAL COMPONENTS (R or L or C)			
TERM	**R**	**L**	**C**
Circuit			
Supply Voltage	$V = V_0 \sin\omega t$	$V = V_0 \sin\omega t$	$V = V_0 \sin\omega t$
Current	$I = I_0 \sin\omega t$	$I = I_0 \sin(\omega t - \frac{\pi}{2})$	$I = I_0 \sin(\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0\omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	R = Resistance	X_L=Inductive reactance.	X_C=Capacitive reactance.

Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with f			
G, S_L, S_C (mho, seiman)	G=1/R conductance.	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
Ohm's law	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

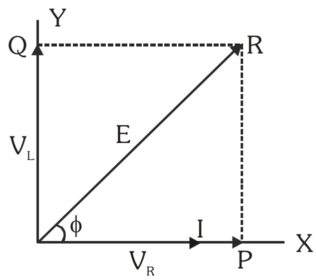
2.4 Resistance and inductance in series (R-L circuit)

A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.



$E = E_0 \sin \omega t$

- **phasor diagram For L-R circuit**



Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin \omega t$. As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.

Then $V_L = IX_L$ and $V_R = IR$

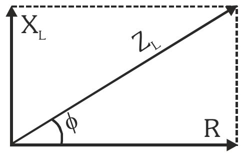
Now, V_R is in phase with the current while V_L leads the current by $\frac{\pi}{2}$.

So V_R and V_L are mutually perpendicular (Note : $E \neq V_R + V_L$)

The vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90°).

The resultant of V_R and V_L = the magnitude of vector OR $E = \sqrt{V_R^2 + V_L^2}$.

Thus $E^2 = V_R^2 + V_L^2 = I^2(R^2 + X_L^2) \Rightarrow I = \dfrac{E}{\sqrt{R^2 + X_L^2}}$



The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f. E by a phase angle ϕ

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R} \quad \Rightarrow \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

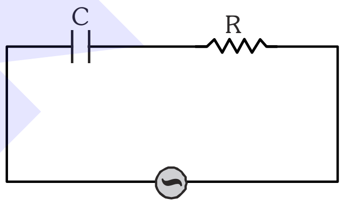
- **Inductive Impedance Z_L :**

In L-R circuit the maximum value of current $I_0 = \dfrac{E_0}{\sqrt{R^2 + \omega^2 L^2}}$ Here $\sqrt{R^2 + \omega^2 L^2}$ represents the effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L. $Z_L = \sqrt{R^2 + \omega^2 L^2} = \sqrt{R^2 + (2\pi f L)^2}$

The reciprocal of impedance is called admittance $Y_L = \dfrac{1}{Z_L} = \dfrac{1}{\sqrt{R^2 + \omega^2 L^2}}$

2.5 Resistance and capacitor in series (R-C circuit)

A circuit containing a series combination of a resistance R and a capacitor C, connected with a source of e.m.f. of peak value E_0 as shown in fig.



- **phasor diagram For R-C circuit**

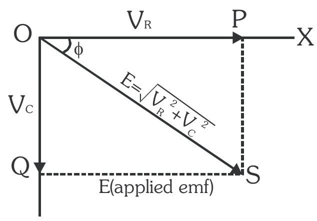
Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$$V_C = I X_C \text{ and } V_R = I R$$

where X_C = capacitive reactance and I = instantaneous current.

Now, V_R is in phase with I, while V_C lags behind I by 90°.

The phasor diagram is shown in fig.



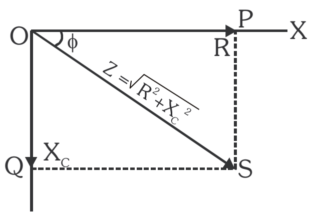
The vector OP represents V_R (which is in phase with I)

and the vector OQ represents V_C (which lags behind I by $\dfrac{\pi}{2}$).

The vector OS represents the resultant of V_R and V_C = the applied e.m.f. E.

Hence $V_R^2 + V_C^2 = E^2 \Rightarrow E = \sqrt{V_R^2 + V_C^2} \Rightarrow E^2 = I^2(R^2 + X_C^2) \Rightarrow I = \dfrac{E}{\sqrt{R^2 + X_C^2}}$

The term $\sqrt{(R^2 + X_C^2)}$ represents the effective resistance of the R-C circuit and called the capacitive impedance Z_C of the circuit.



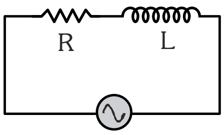
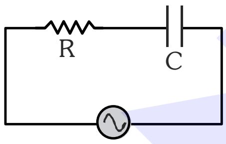
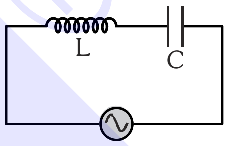
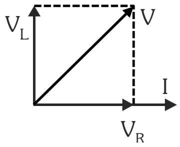
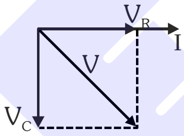
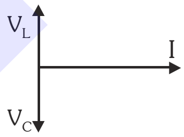
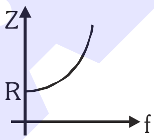
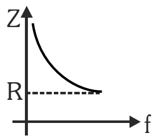
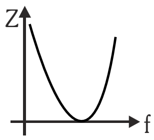
Hence, in C-R circuit $Z_C = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\dfrac{1}{\omega C}\right)^2}$

- **Capacitive Impedance Z_C :**

In R-C circuit the term $\sqrt{R^2+X_C^2}$ effective opposition offered by R-C circuit to the flow of a.c. through it. It is known as impedance of R-C circuit and is represented by Z_C.The phasor diagram also shows that in R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega CR}, \quad \tan\ \phi = \frac{X_C}{R} = \frac{1}{\omega CR} \quad \Rightarrow \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

2.6 Combination of components (R-L or R-C or L-C)

TERM	R-L	R-C	L-C
Circuit	 I is same in R & L	 I is same in R & C	 I is same in L & C
Phasor diagram	 $V^2 = V_R^2 + V_L^2$	 $V^2 = V_R^2 + V_C^2$	 $V = V_L - V_C (V_L > V_C)$ $V = V_C - V_L\ (V_C > V_L)$
Phase difference in between V & I	V leads I (ϕ = 0 to $\frac{\pi}{2}$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0)	V lags I ($\phi = \frac{\pi}{2}$,if $X_C > X_L$) V leads I ($\phi = +\frac{\pi}{2}$,if $X_L > X_C$)
Impedance Variation of Z with f	$Z = \sqrt{R^2+X_L^2}$ as f $\uparrow$, Z $\uparrow$ 	$Z = \sqrt{R^2+(X_C)^2}$ as f $\uparrow$, Z $\downarrow$ 	$Z = \lvert X_L - X_c \rvert$ as f $\uparrow$, Z first $\downarrow$ then $\uparrow$ 
At very low f	$Z \simeq R\ (X_L \to 0)$	$Z \simeq X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R\ (X_C \to 0)$	$Z \simeq X_L$

GOLDEN KEY POINTS

- Phase difference between capacitive and inductive reactance is π
- Inductor is called Low pass filter because it allows low frequency signal to pass.
- Capacitor is called high pass filter because it allows high frequency signal to pass.

Illustrations

Illustration 9.

What is the inductive reactance of a coil if the current through it is 20 mA and voltage across it is 100 V.

Solution

$\because\ V_L = IX_L \qquad \therefore X_L = \dfrac{V_L}{I} = \dfrac{100}{20 \times 10^{-3}} = 5\ k\Omega$

Illustration 10.

The reactance of capacitor is 20 ohm. What does it mean? What will be its reactance if frequency of AC is doubled? What will be its, reactance when connected in DC circuit? What is its consequence?

Solution

The reactance of capacitor is 20 ohm. It means that the hinderance offered by it to the flow of AC at a specific frequency is equivalent to a resistance of 20 ohm. The reactance of capacitance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$

Therefore by doubling frequency, the reactance is halved i.e., it becomes 10 ohm. In DC circuit f = 0. Therefore reactance of capacitor = ∞ (infinite). Hence the capacitor can not be used to control DC.

Illustration 11.

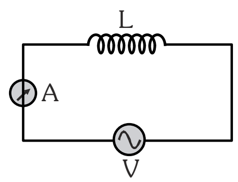
A capacitor of 50 pF is connected to an a.c. source of frequency 1kHz. Calculate its reactance.

Solution

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 10^3 \times 50 \times 10^{-12}} = \frac{10^7}{\pi}\ \Omega$$

Illustration 12.

In given circuit applied voltage $V = 50\sqrt{2}\ \sin(100\pi t)$ volt and ammeter reading is 2A then calculate value of L



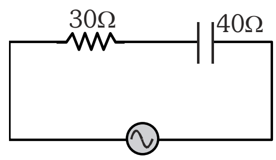
Solution

$V_{rms} = I_{rms} X_L \quad \because$ Reading of ammeter $= I_{rms}$

$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{V_0}{\sqrt{2}\, I_{rms}} = \frac{50\sqrt{2}}{\sqrt{2} \times 2} = 25\ \Omega \Rightarrow L = \frac{X_L}{\omega} = \frac{25}{100\pi} = \frac{1}{4\pi}\ H$$

Illustration 13.

Calculate the impedance of the circuit shown in the figure.



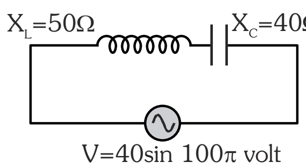
Solution

$$Z = \sqrt{R^2 + (X_c)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50\ \Omega$$

Illustration 14.

If $X_L = 50\ \Omega$ and $X_C = 40\ \Omega$. Calculate effective value of current in given circuit.

Solution

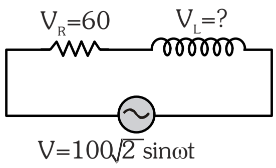


$Z = X_L - X_C = 10\ \Omega$

$$I_0 = \frac{V_0}{Z} = \frac{40}{10} = 4\ A \Rightarrow \quad I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ A$$

Illustration 15.

In given circuit calculate, voltage across inductor



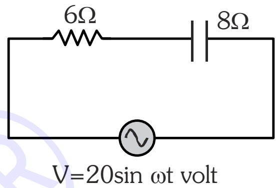
Solution

$\because\ V^2 = V_R^2 + V_L^2 \qquad \therefore\ V_L^{\ 2} = V^2 - V_R^{\ 2}$

$$V_L = \sqrt{V^2 - V_R^2} = \sqrt{(100)^2 - (60)^2} = \sqrt{6400} = 80\text{ V}$$

Illustration 16.

In given circuit find out (i) impedance of circuit (ii) current in circuit



Solution

(i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{(6)^2 + (8)^2} = 10\ \Omega$

(ii) $V = IZ \Rightarrow I = \dfrac{V_0}{Z} = \dfrac{20}{10} = 2\text{A}$, so $I_{rms} = \dfrac{2}{\sqrt{2}} = \sqrt{2}\text{ A}$

Illustration 17.

When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduces to 2 A. Calculate reactance of the coil.

Solution

For 10 V D.C. $\because V = IR$

$\therefore$ Resistance of coil $R = \dfrac{10}{2.5} = 4\Omega$

For 10 V A.C. $\leftarrow V = I\,Z \Rightarrow Z = \dfrac{V}{I} = \dfrac{10}{2} = 5\Omega$

$\because\ Z = \sqrt{R^2 + X_L^2} = 5 \Rightarrow R^2 + X_L^2 = 25 \Rightarrow X_L^2 = 5^2 - 4^2 \Rightarrow X_L = 3\ \Omega$

Illustration 18.

When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by $\pi/2$ radians.

(a) Name the devices X and Y.

(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

Solution

(a) X is resistor and Y is a capacitor

(b) Since the current in the two devices is the same (0.5A at 220 volt)

When R and C are in series across the same voltage then

$$R = X_C = \frac{220}{0.5} = 440\ \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{220}{\sqrt{(440)^2 + (440)^2}} = \frac{220}{440\sqrt{2}} = 0.35\text{A}$$

BEGINNER'S BOX-2

1. A voltage V = 60 sin πt volt is applied across a 20 Ω resistor. What will an ac ammeter in series with the resistor read ?

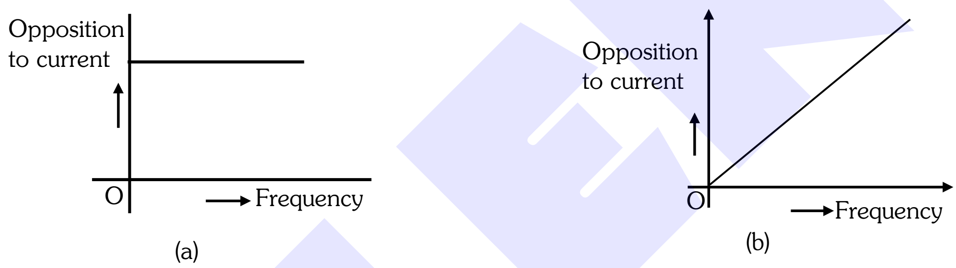
2. An alternating current source E = 100 sin (1000t) volt is connected through a inductor of 10 μH then write down the equation of current.

3. An alternating voltage E = 200$\sqrt{2}$ sin (100t) volt is applied to 2H inductor through an a.c. ammeter. What will be reading of the ammeter ?

4. A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

5. A 60 μF capacitor is connected to a 110 V. 60 Hz a.c. supply. Determine the rms value of the current in the circuit.

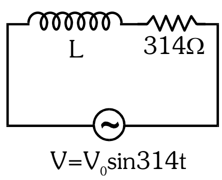
6. The given graphs (a) and (b) represent the variation of the opposition offered by the circuit element to the flow of alternating current, with frequency of the applied emf. Identify the circuit element corresponding to each graph.



7. When a series combination of inductance and resistance are connected with a 10V, 50 Hz a.c. source, a current of 1A flows in the circuit. The voltage leads the current by a phase angle of $\frac{\pi}{3}$ radian. Calculate the values of resistance and inductive reactance.

8. The current in the shown circuit is found to be $4\sin\left(314t-\frac{\pi}{4}\right)$ A.

Find the value of inductance.



9. A current of 4A flows in a coil when connected to a 12V dc source. If the same coil is connected to a 12V, 50 rad/s a.c. source, a current of 2.4A flows in the circuit. Determine the inductance of the coil.

10. An alternating voltage E=200 sin (300t) volt is applied across a series combination of R=10Ω and inductance of 800mH. Calculate the impedance of the circuit.

11. A coil of reactance 100 Ω and resistance 100 Ω is connected to a 240 V, 50 Hz a.c. supply.

(a) What is the maximum current in the coil ?

(b) What is the time lag between the voltage maximum and the current maximum ?

12. An inductance has a resistance of 100 Ω. When a.c. signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. Calculate the self inductance of the coil.

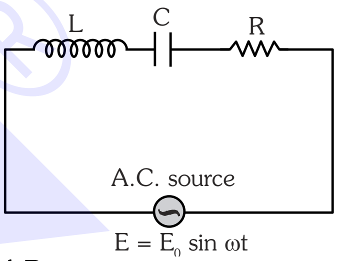
13. Match the following options –

Circuit component across an ac source (ω = 200 rad/sec)		Phase difference between current and source voltage	
(A)	10Ω 500μF	(p)	$\frac{\pi}{2}$
(B)	5H	(q)	$\frac{\pi}{6}$
(C)	500μF	(r)	$\frac{\pi}{4}$
(D)	4H 3μF	(s)	$\frac{\pi}{3}$
(E)	1kΩ 5H	(t)	None of the above

3. INDUCTANCE, CAPACITANCE AND RESISTANCE IN SERIES

3.1 L-C-R series circuit

A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of E_0, as shown in figure.



- **Phasor Diagram For Series L-C-R circuit**

Let in series LCR circuit applied alternating emf is $E = E_0 \sin \omega t$. As L,C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

However voltage across each element bears a different phase relationship with the current.

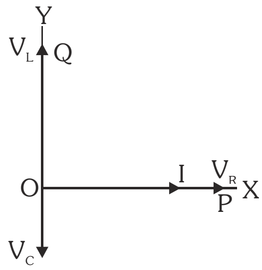
Let at any instant of time t the current in the circuit is I.

Let at this time t the potential differences across L, C, and R

$V_L = I X_L$, $V_C = I X_C$ and $V_R = IR$

Now, V_R is in phase with current I but V_L leads I by 90°

While V_C lags behind I by 90°.

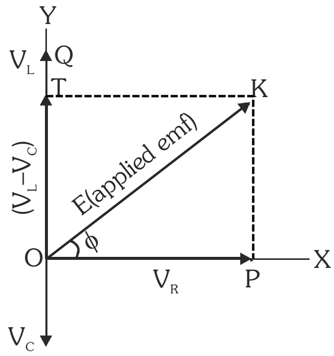


The vector OP represents V_R (which is in phase with I) the vector OQ represent V_L (which leads I by 90°)

and the vector OS represents V_C (which legs behind I by 90°)

V_L and V_C are opposite to each other.

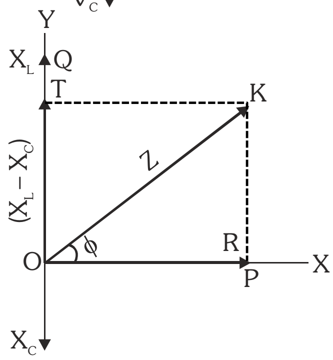
If $V_L > V_C$ (as shown in figure) the their resultant will be $(V_L - V_C)$ which is represented by OT. Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f.



Thus $E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2} \Rightarrow$

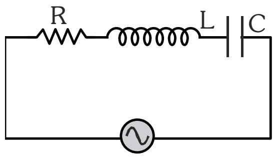
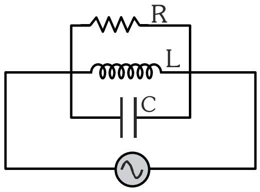
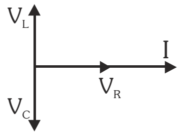
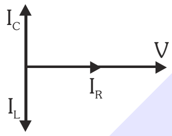
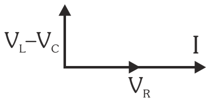
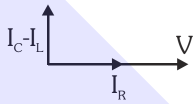
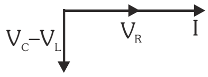
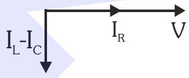
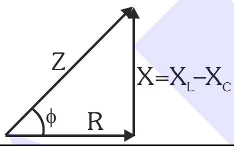
$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$



The phasor diagram also shown that in LCR circuit the applied e.m.f. leads the current I by a phase angle ϕ $\quad \tan \phi = \frac{X_L - X_C}{R}$

3.2 Series LCR and parallel LCR combination

SERIES L-C-R CIRCUIT	PARALLEL L-C-R CIRCUIT
1. Circuit diagram  I same for R, L & C	 V same for R, L and C
2. Phasor diagram  	
(i) If $V_L > V_C$ then 	(i) if $I_C > I_L$ then 
(ii) If $V_C > V_L$ then 	(ii) if $I_L > I_C$ then 
(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$ Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$ $\tan\phi = \dfrac{X_L - X_C}{R} = \dfrac{V_L - V_C}{V_R}$	
(iv) Impedance triangle  	

3.3 Resonance

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance : (i) Series Resonance (ii) Parallel Resonance

3.4 Series Resonance

(a) At Resonance

(i) $X_L = X_C$ (ii) $V_L = V_C$ (iii) $\phi = 0$ (V and I in same phase)

(iv) $Z_{min} = R$ (impedance minimum) (v) $I_{max} = \dfrac{V}{R}$ (current maximum)

(b) Resonance frequency

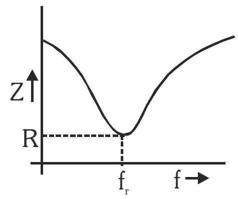
$$\because\ X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) Variation of Z with f

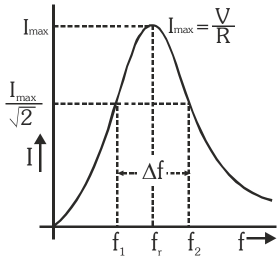
(i) If $f < f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)

(ii) At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, ϕ = zero

(iii) If $f > f_r$ then $X_L > X_C$ circuit nature is inductive, ϕ (positive)



(d) Variation of I with f as f increase, Z first decreases then increase



as f increase, I first increase then decreases

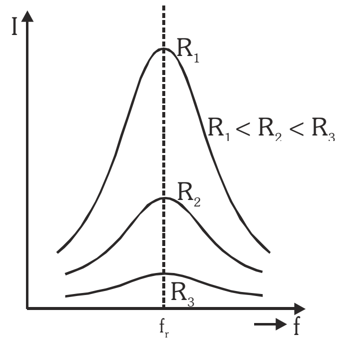
- At resonance, impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.
- **Half power frequencies**

  The frequencies at which, power become half of its maximum value is called half power frequencies
- **Band width** $\Rightarrow \Delta f = f_2 - f_1$
- **Quality factor Q** : Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

$$Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$$

(i) It represents the sharpness of resonance. (ii) It is unit less and dimension less quantity

(iii) $Q = \dfrac{(X_L)_r}{R} = \dfrac{(X_C)_r}{R} = \dfrac{2\pi f_r L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{f_r}{\Delta f} = \dfrac{f_r}{\text{band width}}$



Magnification

At resonance V_L or V_C = QE (where E = supplied voltage)

So at resonance Magnification factor = Q-factor

Sharpness

Sharpness $\propto$ Quality factor $\propto$ Magnification factor

R *decrease* $\Rightarrow$ Q increases $\Rightarrow$ Sharpness increases

GOLDEN KEY POINTS

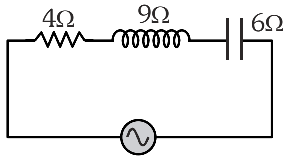
- In A.C. circuit voltage for L or C may be greater than source voltage or current but it happens only when circuit contains L and C both and in R it is never greater than source voltage or current.
- In parallel A.C. circuit phase difference between I_L and I_C is π
- Series resonance circuit gives voltage amplification while parallel resonance circuit gives current amplification.
- At resonance current does not depend on L and C, it depends only on R and V.
- At half power frequencies : net reactance = net resistance.
- As R increases, bandwidth increases
- To obtain resonance in a circuit following parameter can be altered :

  (i) L (ii) C (iii) frequency of source.
- Two series LCR circuit of same resonance frequency f are joined in series then resonance frequency of series combination is also f
- The series resonance circuit called acceptor whereas parallel resonance circuit called rejector circuit.
- Unit of $\sqrt{LC}$ is second

Illustrations

Illustration 19.

Find out the impedance of given circuit.



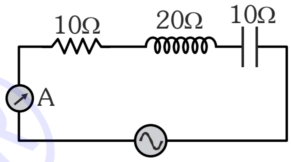
Solution

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{4^2+(9-6)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5\Omega \quad (\because X_L > X_C \therefore \text{Inductive})$$

Illustration 20.

Find out reading of A C ammeter and also calculate the potential difference across, resistance and capacitor.

Solution



E=100sinωt100πt volt

$$Z=\sqrt{R^2+(X_L-X_C)^2}=10\sqrt{2}\ \Omega \qquad \Rightarrow \qquad I_0=\frac{V_0}{Z}=\frac{100}{10\sqrt{2}}=\frac{10}{\sqrt{2}}A$$

$\because$ ammeter reads RMS value, so its reading $=\dfrac{10}{\sqrt{2}\,\sqrt{2}}=5A$

so $V_R = 5\times 10 = 50$ V and $V_C = 5\times 10 = 50$ V

Illustration 21.

In LCR circuit with an AC source R = 300 Ω, C = 20 μF, L = 1.0 H, E_{rms} = 50V and f = 50/π Hz. Find RMS current in the circuit.

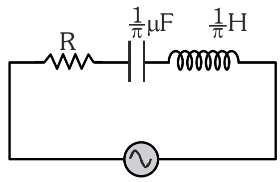
Solution

$$I_{rms}=\frac{E_{rms}}{Z}=\frac{E_{rms}}{\sqrt{R^2+\left[\omega L-\frac{1}{\omega C}\right]^2}}=\frac{50}{\sqrt{300^2+\left[2\pi\times\frac{50}{\pi}\times 1-\frac{1}{20\times10^{-6}\times2\pi\times\frac{50}{\pi}}\right]^2}}$$

$$\Rightarrow \quad I_{rms}=\frac{50}{\sqrt{(300)^2+\left[100-\frac{10^3}{2}\right]^2}}=\frac{50}{100\sqrt{9+16}}=\frac{1}{10}=0.1A$$

Illustration 22.

For what frequency the voltage across the resistance R will be maximum.



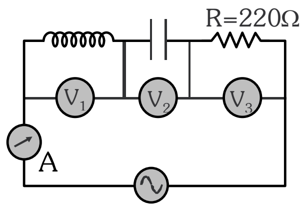
Solution

It happens at resonance

$$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{\frac{1}{\pi}\times10^{-6}\times\frac{1}{\pi}}}=500\ \text{Hz}$$

Illustration 23.

A capacitor, a resistor and a 40 mH inductor are connected in series to an AC source of frequency 60Hz, calculate the capacitance of the capacitor, if the current is in phase with the voltage. Also find the reading of voltmeter V_3 and Ammeter.



110V, 60Hz

Solution

$$\text{At resonance } \omega L=\frac{1}{\omega C},\ C=\frac{1}{\omega^2 L}=\frac{1}{4\pi^2 f^2 L}=\frac{1}{4\pi^2\times(60)^2\times40\times10^{-3}}=176\mu F$$

$$V_3=V_R \Rightarrow V_3=110\ V \quad \text{and } I=\frac{V}{R}=\frac{110}{220}=0.5A$$

Illustration 24.

A coil, a capacitor and an A.C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a battery of emf 12 V, and internal resistance 4Ω , then calculate the current through the coil.

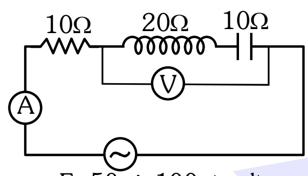
Solution

At resonance current is maximum. $I = \frac{V}{R} \Rightarrow$ Resistance of coil $R = \frac{V}{I} = \frac{24}{6} = 4\ \Omega$

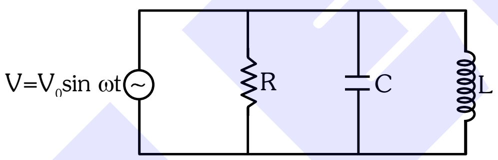
When coil is connected to battery, suppose I current flow through it then $I = \frac{E}{R+r} = \frac{12}{4+4} = 1.5$ A

BEGINNER'S BOX-3

1. In given circuit find the reading of ammeter and voltmeter.



2. For the circuit shown in figure, write down the instantaneous current through each element.



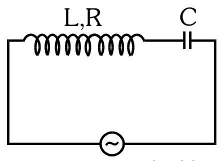
3. A variable frequency 230V alternating voltage source is connected across a series combination of L = 5.0H C = 80μF and R = 40Ω. Calculate

(a) The angular frequency of the source at resonance.

(b) Amplitude of current at resonance frequency

4. A coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.

Find out relation between ω_0, L and C



5. Find the phase difference between voltage and current in series LCR circuit at half power frequencies.

6. A series LCR circuit with L = 0.12H, C = 480 nF, R = 23Ω is connected to a 230 V variable frequency supply Find –

(a) Source frequency for which current is maximum.

(b) Q–factor of the given circuit.

4. POWER IN AC CIRCUIT

4.1 The average power dissipation in LCR AC circuit

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin (\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi) = V_0 I_0 \sin\omega t (\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

Average power $<P> = \frac{1}{T}\int_0^T (V_0 I_0 \sin^2 \omega t \cos\phi - V_0 I_0 \sin\omega t \cos\omega t \sin\phi)dt$

$$= V_0 I_0 \left[\frac{1}{T}\int_0^T \sin^2 \omega t \cos\phi dt - \frac{1}{T}\int_0^T \sin\omega t \cos\omega t \sin\phi dt\right] = V_0 I_0 \left[\frac{1}{2}\cos\phi - 0 \times \sin\phi\right]$$

$$\Rightarrow \quad <P> = \frac{V_0 I_0 \cos\phi}{2} = V_{rms} I_{rm,s} \cos\phi$$

Instantaneous power	Average power/actual power/ dissipated power/power loss	Virtual power/ apparent Power/rms Power	Peak power
P = VI	$P = V_{rms} I_{rms} \cos\phi$	$P = V_{rms} I_{rms}$	$P = V_0 I_0$

- $I_{rms} \cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms} \sin\phi$ is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

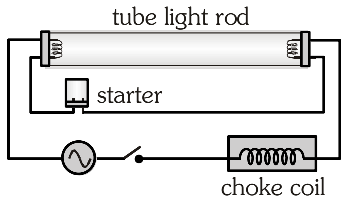
4.2 Power factor :

Average power $\overline{P} = E_{rms} I_{rms} \cos\phi = r\,m\,s\,\text{power} \times \cos\phi$

Power factor $(\cos\phi) = \frac{\text{Average power}}{r\,m\,s\,\text{Power}}$ and $\cos\phi = \frac{R}{Z}$

4.3 Choke Coil

In a direct current circuit, current is reduced with the help of a resistance. Hence there is a loss of electrical energy $I^2 R$ per sec in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced.



Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)

The current in the circuit $I = \frac{E}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$ So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r,

The power loss in the choke $P_{av} = V_{rms} I_{rms} \cos\phi \to 0$

$$\because \quad \cos\phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \to 0$$

GOLDEN KEY POINTS

- $P_{av} \le P_{rms}$.
- Power factor varies from 0 to 1
-

Pure/Ideal	V	Power factor = cos ϕ	Average power
R	V, I same Phase	1 (maximum)	$V_{rms}.\ I_{rms}$
L	V leads I by $\frac{\pi}{2}$	0, lagging	0
C	V lags I by $\frac{\pi}{2}$	0, leading	0
Choke coil	V leads I by $\frac{\pi}{2}$	0, lagging	0

- At resonance power factor is maximum ($\phi = 0$ so $\cos\phi = 1$) and $P_{av} = V_{rms}\, I_{rms}$
- Choke coil is an inductor having high inductance and negligible resistance.
- Choke coil is used to control current in A.C. circuit at negligible power loss
- Choke coil used only in A.C. and not in D.C. circuit
- Choke coil is based on the principle of wattless current.
- Iron cored choke coil is used generally at low frequency and air cored at high frequency.
- Resistance of ideal choke coil is zero.

Illustrations

Illustration 25.

A voltage of 10 V and frequency 10^3 Hz is applied to $\frac{1}{\pi}\mu F$ capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the power dissipated.

Solution

$\because \quad X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^3 \times \frac{10^{-6}}{\pi}} = 500\Omega \qquad \therefore Z = \sqrt{R^2 + X_C^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2}\,\Omega$

Power factor $\cos\phi = \frac{R}{Z} = \frac{500}{500\sqrt{2}} = \frac{1}{\sqrt{2}}$, Power dissipated $= V_{rms} I_{rms} \cos\phi = \frac{V_{rms}^2}{Z}\cos\phi = \frac{(10)^2}{500\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{10}$ W

Illustration 26.

If V = 100 sin 100 t volt and I = 100 sin (100 t + $\frac{\pi}{3}$) mA for an A.C. circuit then find out

(a) phase difference between V and I

(b) total impedance, reactance, resistance

(c) power factor and power dissipated

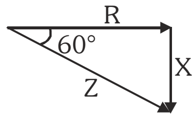
(d) components contains by circuits

Solution

(a) Phase difference $\phi = -\dfrac{\pi}{3}$ (I leads V)

(b) Total impedance $Z = \dfrac{V_0}{I_0} = \dfrac{100}{100 \times 10^{-3}} = 1k\Omega$ Now resistance $R = Z\cos 60° = 1000 \times \dfrac{1}{2} = 500\Omega$

reactance $X = Z\sin 60° = 1000 \times \dfrac{\sqrt{3}}{2} = 500\sqrt{3}\Omega$



(c) $\phi = -60° \Rightarrow$ Power factor = $\cos\phi = \cos(-60°) = 0.5$ (leading)

Power dissipated $P = V_{rms} I_{rms} \cos\phi = \dfrac{100}{\sqrt{2}} \times \dfrac{0.1}{\sqrt{2}} \times \dfrac{1}{2} = 2.5\,W$

(d) Circuit must contains R as $\phi \neq \dfrac{\pi}{2}$ and as ϕ is negative so C must be there, (L may exist but $X_C > X_L$)

Illustration 27.

If power factor of a R-L series circuit is $\dfrac{1}{2}$ when applied voltage is $V = 100 \sin 100\pi t$ volt and resistance of circuit is 200Ω then calculate the inductance of the circuit.

Solution

$\cos\phi = \dfrac{R}{Z} \Rightarrow \dfrac{1}{2} = \dfrac{R}{Z} \Rightarrow Z = 2R \Rightarrow \sqrt{R^2 + X_L^2} = 2R$

$\Rightarrow X_L = \sqrt{3}\,R$

$\omega L = \sqrt{3}\,R \Rightarrow L = \dfrac{\sqrt{3}R}{\omega} = \dfrac{\sqrt{3} \times 200}{100\pi} = \dfrac{2\sqrt{3}}{\pi} H$

Illustration 28.

A circuit consisting of an inductance and a resistance joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current.

Solution

Apparent power = $200 \times 10 = 2000$ W

$\therefore$ Power factor $\cos\phi = \dfrac{\text{True power}}{\text{Apparent power}} = \dfrac{1500}{2000} = \dfrac{3}{4}$

Wattless current = $I_{rms} \sin\phi = 10\sqrt{1 - \left(\dfrac{3}{4}\right)^2} = \dfrac{10\sqrt{7}}{4} A$

Illustration 29.

A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50 V. What would be the potential difference across the choke coil.

Solution

$V = \sqrt{V_R^2 + V_L^2} \Rightarrow V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120$ V

Illustration 30.

An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

Solution

Resistance of lamp $R = \frac{V}{I} = \frac{80}{10} = 8\Omega$

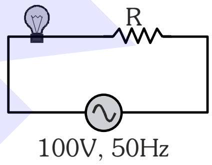
Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run on 100 Volt a.c. then. $Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$ but $Z = \sqrt{R^2 + (\omega L)^2}$

$\Rightarrow (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36 \Rightarrow \omega L = 6 \Rightarrow L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02H$

Illustration 31.

Calculate the resistance or inductance required to operate a lamp (60V, 10W) from a source of (100 V, 50 Hz)

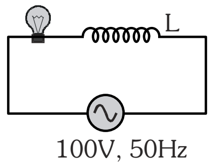
Solution



(a) Maximum voltage across lamp = 60V

$\because V_{Lamp} + V_R = 100 \qquad \therefore V_R = 40V$

Now current throught Lamp is = $\frac{\text{Wattage}}{\text{voltage}} = \frac{10}{60} = \frac{1}{6}A$



But $V_R = IR \Rightarrow 40 = \frac{1}{6}(R) \qquad R = 240\ \Omega$

(b) Now in this case $(V_{Lamp})^2 + (V_L)^2 = (V)^2$

$(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80\ V$

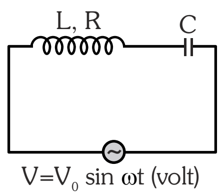
Also $V_L = IX_L = \frac{1}{6} X_L$ so $X_L = 80 \times 6 = 480\ \Omega = L(2\pi f) \Rightarrow L = 1.5\ H$

A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation.

Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

Illustration 32.

A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.



(a) Find out relation between ω_0, L and C

(b) What is phase difference between V and I at resonance, is it changes when resistance of choke coil is zero.

Solution

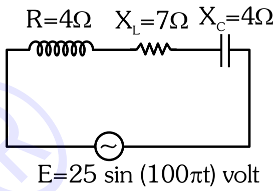
(a) At resonance condition $X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

(b) $\because \cos\phi = \frac{R}{Z} = \frac{R}{R} = 1 \quad \therefore \phi = 0°$ No, It is always zero.

BEGINNER'S BOX-4

1. What is the power factor of a circuit that draws 5A at 160 V and whose power consumption is 600W?

2. In a series LCR circuit as shown in fig.



(a) Find heat developed in 80 seconds

(b) Find wattless current

3. For a series LCR circuit

$I = 100\sin(100\pi t - \pi/3)$mA and $V = 100\sin(100\pi t)$volt, then

(a) Calculate resistance and reactance of circuit.

(b) Find average power loss.

4. The source voltage and current in the circuit are represented by the following equations –

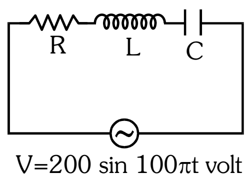
$E = 110 \sin(\omega t + \frac{\pi}{6})$ volt, $I = 5 \sin(\omega t - \frac{\pi}{6})$ ampere

Find :–

(a) Impedance of circuit.

(b) Power factor with nature

5. In given circuit R = 100Ω. If voltage leads current by 60° then find –



(a) Current supply by source.

(b) Average power

6. An inductor of reactance 4Ω and a resistor of resistance 3Ω are connected in series with 100V ac supply, calculate wattless current in circuit.

7. A 100 Ω resistor is connected to a 220 V, 50 Hz a.c. supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

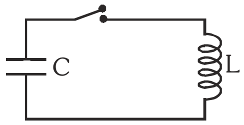
8. A choke coil and a resistance are connected in series in an a.c. circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50V. What would be the potential difference across the choke coil.

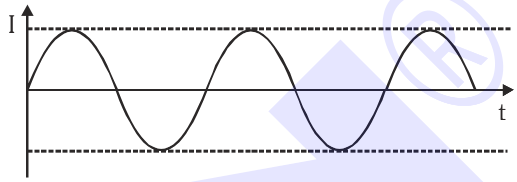
5. LC OSCILLATION

The oscillation of energy between capacitor (electric field energy) and inductor (magnetic field energy) is called LC Oscillation.

5.1 Undamped oscillation

When the circuit has no resistance, the energy taken once from the source and given to capacitor keeps on oscillating between C and L then the oscillation produced will be of constant amplitude. These are called undamped oscillation.





After switch is closed

$$\frac{Q}{C}+L\frac{di}{dt}=0 \Rightarrow \frac{Q}{C}+L\frac{d^2Q}{dt^2}=0 \Rightarrow \frac{d^2Q}{dt^2}+\frac{1}{LC}Q=0$$

By comparing with standard equation of free oscillation $\left[\frac{d^2x}{dt^2}+\omega^2x=0\right]$

$$\omega^2=\frac{1}{LC} \quad \text{Frequency of oscillation } f=\frac{1}{2\pi\sqrt{LC}}$$

Charge varies sinusoidally with time $q = q_m \cos \omega t$

current also varies periodically with t $\quad I=\frac{dq}{dt}=q_m\omega\cos\left(\omega t+\frac{\pi}{2}\right)$

If initial charge on capacitor is q_m then electrical energy stored in capacitor is $U_E=\frac{1}{2}\frac{q_m^2}{C}$

At t = 0 switch is closed, capacitor starts to discharge.

As the capacitor is fully discharged, the total electrical energy is stored in the inductor in the form of magnetic energy.

$$U_B=\frac{1}{2}LI_m^2 \quad \text{where } I_m = \text{max. current}$$

$$(U_{max})_{EPE}=(U_{max})_{MPE} \qquad \Rightarrow \frac{1}{2}\frac{q_m^2}{C}=\frac{1}{2}LI_m^2$$

GOLDEN KEY POINTS

- In damped oscillation amplitude of oscillation decreases exponentially with time.
- At $t=\frac{T}{4},\frac{3T}{4},\frac{5T}{4}$..... energy stored is completely magnetic.
- At $t=\frac{T}{8},\frac{3T}{8},\frac{5T}{8}$..... energy is shared equally between L and C
- Phase difference between charge and current is $\frac{\pi}{2}$ $\left[\begin{array}{l}\text{when charge is maximum, current minimum}\\ \text{when charge is minimum, current maximum}\end{array}\right]$

Illustration 33.

An LC circuit contains a 20mH inductor and a 50μF capacitor with an initial charge of 10mC. The resistance of the circuit is negligible. Let the instant the circuit is closed to be t = 0.

(a) What is the total energy stored initially.

(b) What is the natural frequency of the circuit.

(c) At what minimum time is the energy stored is completely magnetic.

(d) At what minimum time is the total energy shared equally between inductor and the capacitor.

Solution

(a) $U_E = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\times\frac{(10\times10^{-3})^2}{50\times10^{-6}} = 1.0\text{J}$

(b) $\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times10^{-3}\times50\times10^{-6}}} = 10^3 \text{ rad/sec} \Rightarrow \quad f = 159 \text{ Hz}$

(c) $\because \quad q = q_0 \cos \omega t$

Energy stored is completely magnetic (i.e. electrical energy is zero, q = 0)

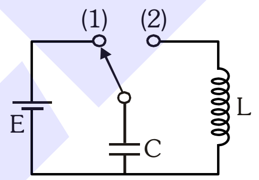
at $\quad t = \frac{T}{4}$, where $T = \frac{1}{f} = 6.3$ ms

(d) Energy is shared equally between L and C when charge on capacitor become $\frac{q_0}{\sqrt{2}}$

so, $\quad$ at $t = \frac{T}{8}$, energy is shared equally between L and C

BEGINNER'S BOX-5

1. Initially key was placed on (1) till the capacitor got fully charged. Key is placed on (2) at t=0. The minimum time when the energy in both capacitor and inductor will be same-



2. An inductor of inductance 2.0 mH is connected across a charged capacitor of capacitance 5.0 μF and the resulting L–C circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor and I the current in the circuit. It is found that the maximum value of Q is 200 μC. **[IIT-JEE 2006]**

(i) Find the maximum value of I.

(ii) When Q = 200 μC, what is the value of I?

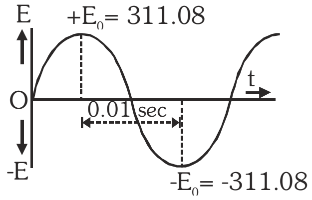
(iii) When Q = 100 μC, what is the value of $|dI/dt|$?

(iv) When I is equal to one–half its maximum value, what is the value of $|Q|$?

ANSWER KEY

BEGINNER'S BOX-1

1. There are two reasons for it :



2. For AC, $I = I_0 \sin\omega t$, the instantaneous value of heat produced (per second) in a resistance R is,

$H = I^2R = I_0^2\sin^2\omega t \times R$ the average value of heat produced during a cycle is :

$$H_{av} = \frac{\int_0^T H\,dt}{\int_0^T dt} = \frac{\int_0^T (I_0^2 \sin^2 \omega t \times R)dt}{\int_0^T dt} = \frac{1}{2}I_0^2 R$$

$$\left[\because \int_0^T I_0^2 \sin^2 \omega t\,dt = \frac{1}{2}I_0^2 T\right]$$

$$\Rightarrow H_{av} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = I_{rms}^2 R \quad .....(i)$$

However, in case of DC, $H_{DC} = I^2 R$...(ii)

$\because I = I_{rms}$ so from equation (i) and (ii) $H_{DC} = H_{av}$

AC produces same heating effects as DC of value $I = I_{rms}$. This is also why AC instruments which are based on heating effect of current give rms value.

3. The average value of a.c. for a cycle is zero. So a d.c. ammeter will always read zero in a.c. circuit.

4. 2.5 m s **5.** (a) 0.01 s, (b) 60°

6. 200 V

BEGINNER'S BOX-2

1. 2.1 A **2.** $10000 \sin\left(1000t - \frac{\pi}{2}\right)$ A

3. 1A

4. The capacitive reactance $X_C = 212\ \Omega$

$I_{rms} = 1.03$ A

The peak current $I_0 = 1.46$ A

If the frequency is doubled the capacitive reactances is halved, the consequently, the current is doubled.

5. 2.49 A **6.** (a) resistor (b) inductor

7. $R = 5\Omega$ and $X_L = 5\sqrt{3}\,\Omega$

8. 1H **9.** 0.08 henry

10. 240.2 Ω **11.** (a) 2.4 A (b) 2.5 ms

12. 15.9 mH

13. (A) – r, (B) – p, (C) – p, (D) – p, (E) – r

BEGINNER'S BOX-3

1. Reading of ammeter = 2·5A

Reading of voltmeter = 25V

2. The three current equations are,

$V = i_R R,\quad V = L\frac{di_L}{dt}$ and $\frac{dV}{dt} = \frac{1}{C}i_C$

so $i_R = \frac{V_0}{R}\sin\omega t$,

$i_L = -\frac{V_0}{\omega L}\cos\omega t$ and $i_C = V_0\omega C\cos\omega t$

3. (a) angular frequancy at resonance $\omega_r = 50$ rad/s

(b) amplitude of current at resonance $I_m = 8.13$ A

4. $\omega_0 = \frac{1}{\sqrt{LC}}$

5. $\phi = \frac{\pi}{4}$

6. (a) 6.63×10^2 Hz (b) Quality factor Q = 21.7

BEGINNER'S BOX-4

1. 0.75

2. (a) 4000 joule (b) 2.12 A

3. (a) R = 500 ohm, $X = 500\sqrt{3}$ ohm (b) 2.5 watts

4. (a) Impedance Z = 22Ω (b) Power factor$=\frac{1}{2}$(lagging)

5. (a) $\frac{1}{\sqrt{2}}$A, (b) 50W

6. Wattless current = 16A

7. (a) 2.2 A (b) 484 watt

8. 120V

BEGINNER'S BOX-5

1. $t = \frac{\pi\sqrt{LC}}{4}$

2. (i) 2.0 A, (ii) Zero, (iii) 10^4 A/s, (iv) 1.732×10^{-4} C