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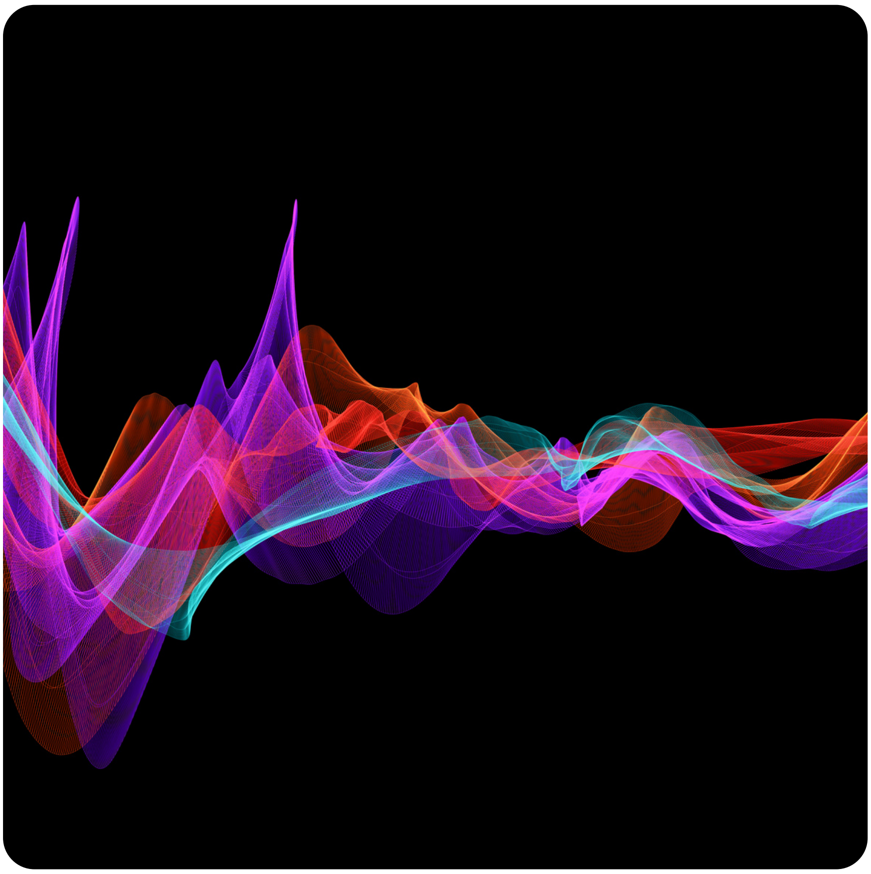

STUDY MATERIAL

Wave Motion & Doppler's Effect

ENGLISH MEDIUM

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CHRISTIAN DOPPLER (1803 - 1853)

Christian Doppler was an Austrian mathematician and physicist best known for articulating a principle known as the 'Doppler effect'. Christian Andreas Doppler was born on November 29, 1803, in Salzburg, Austria. Doppler studied philosophy in Salzburg, and mathematics and physics at the Vienna University of Technology and the University of Vienna. In 1842, Doppler gave a presentation at the Royal Bohemian Society of Sciences. The paper theorized that since the pitch of sound from a moving source varies for a stationary observer, the color of the light from a star should alter according to the star's velocity relative to Earth. This principle came to be known as the "Doppler effect". The Doppler effect has been used to support the Big Bang Theory and is often referenced in weather forecasting, radar and navigation.

PIERRE-SIMON LAPLACE (1749 - 1827)

He was born on March 23, 1749 at Beaumont-en-Auge, Normandy, France. Pierre-Simon Laplace was a prominent French mathematical physicist and astronomer of the 19th century, who made crucial contributions in the arena of planetary motion by applying Sir Isaac Newton's theory of gravitation to the entire solar system. His work regarding the theory of probability and statistics is considered pioneering and has influenced a whole new generation of mathematicians. Laplace in 1816 was the first to point out that the speed of sound in air depends on the heat capacity ratio. Newton's original theory gave too low a value, because it does not take account of the adiabatic compression of the air which results in a local rise in temperature and pressure.

WAVE THEORY, SOUND WAVES AND DOPPLER'S EFFECTS

1. WAVE AND ITS CHARACTERISTICS

1.1 Introduction of Waves

What is wave motion ?

- When a particle moves through space, it carries KE with itself. Wherever the particle goes, the energy goes with it. (One way of transport energy from one place to another place).
- There is another way (wave motion) to transport energy from one part of space to other without any bulk motion of material together with it. Sound is transmitted in air in this manner.

Ex. You (in Kota) want to communicate your friend (in Delhi)

Write a letter — Use telephone

1^{st} option involves the concept of particle & the second choice involves the concept of wave.

Ex. When you say "Namaste" to your friend no material particle is ejected from your lips to fall on your friends ear. Basically you create some disturbance in the part of the air close to your lips. Energy is transferred to these air particles either by pushing them ahead or pulling them back. The density of the air in this part temporarily increases or decreases. These disturbed particles exert force on the next layer of air, transferring the disturbance to that layer. In this way, the disturbance proceeds in air and finally the air near the ear of the listener gets disturbed.

Note :- In the above example air itself does not move.

A **wave** is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of

Few examples of waves :

The ripples on a pond (water waves), the sound we hear, visible light, radio and TV signals etc.

1.2 Classification of waves

Wave classification According to

- Necessity of medium
  - (i) Elastic or mechanical wave
  - (ii) Electro magnetic wave (E.M. wave) or non-mech.
- Propagation of energy
  - (i) Progressive
  - (ii) Stationary
- Dimension
  - (i) One dimensional
  - (ii) Two dimensional
  - (iii) Three dimentional
- Vibration of particle
  - (i) Transverse
  - (ii) Longitudinal

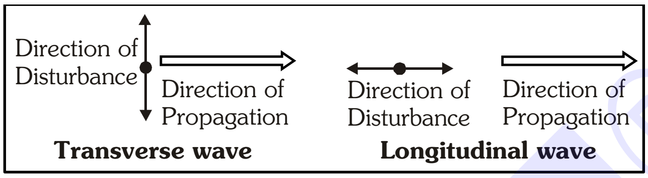
(i) Based on medium necessity :- A wave may or may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as **elastic waves**.

Example : Sound waves in water, seismic waves in earth's crust.

(ii) **Based on energy propagation :-** Waves can be divided into two parts on the basis of energy propagation (i) Progressive wave (ii) Stationary waves. The progressive wave propagates with constant velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.

(iii) ***Based on direction of propagation*** :- Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two dimensional , while sound or light waves from a point source are three dimensional.

(iv) ***Based on the motion of particles of medium*** :



Waves are of two types on the basis of motion of particles of the medium.

(i) Longitudinal waves

(ii) Transverse waves

In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

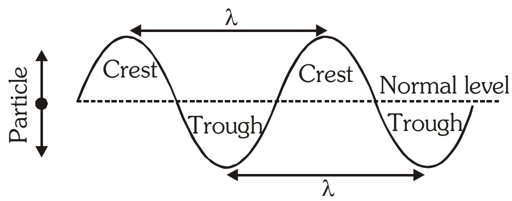
1.3 Transverse Wave Motion

Mechanical transverse waves are produced in such type of medium which have shearing property, so they are known as shear wave or S-wave

Note :- Shearing is the property of a body by which it changes its shape on application of force.

⇒ Mechanical transverse waves are generated only in solids and surface of liquid.

Individual particles of the medium execute SHM about their mean position in direction perpendicular to the direction of propagation of wave.

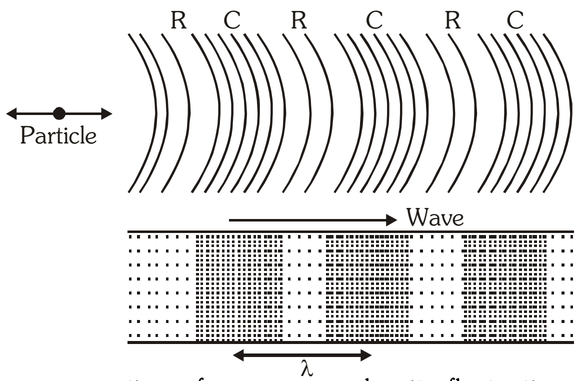


A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

1.4 Longitudinal Wave Motion

In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).

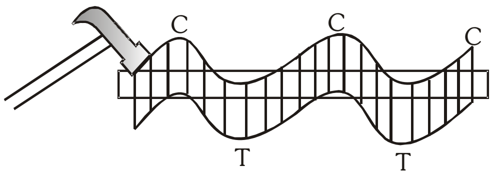


Note : The regions of high particle density are called compressions and regions of low particle density are called rarefactions.

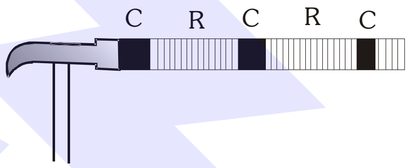
The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

1.5 Mechanical Waves in Different Media

- A mechanical wave will be transverse or longitudinal depending on the nature of medium and mode of excitation.
- In strings, mechanical waves are always transverse when string is under a tension. In the bulk of gases and liquids mechanical waves are always longitudinal e.g. sound waves in air or water. This is because fluids cannot sustain shear.
- In solids, mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle as shown in fig. (A) the waves in the rod will be transverse while if the rod is struck at the side as shown in fig. (B) or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal.



Transverse-wave
(A)



Longitudinal-wave
(B)

Further more in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds

[$v_S \cong 5$ km/s while $v_P \cong 9$ km/s] S–waves are transverse while P–waves are longitudinal.

Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane.

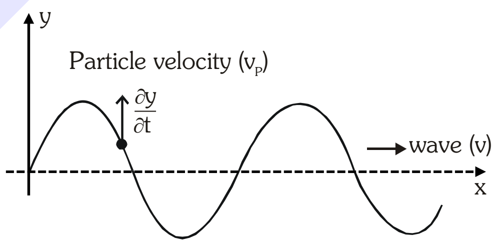
Ripple

1.6 Characteristics of wave motion

- In wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.
- The energy is transferred from one place to another without any actual transfer of the particles of the medium.
- Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.
- The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.
- The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at mean position and zero at extreme position.
- For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction among its particles.

1.7 Some important terms connected with wave motion

- **Wavelength (λ) [length of one wave]**
  Distance travelled by the wave during the time interval in which any one particle of the medium completes one cycle about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.
- **Frequency (n)** : Number of cycle (number of complete wavelengths) completed by a particle in unit time.
- **Time period (T)** : Time taken by wave to travel a distance equal to one wavelength.
- **Amplitude (A)** : Maximum displacement of vibrating particle from its equilibrium position.
- **Angular frequency (ω) :** It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$
- **Phase :** Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation $y = A \sin(\omega t - kx)$; $(\omega t - kx)$ = phase.
- **Angular wave number or propagation constant (k) :** It is defined as $k = \frac{2\pi}{\lambda}$
- **Wave number ($\bar{v}$) :** It is defined as $\bar{v} = \frac{1}{\lambda} = \frac{k}{2\pi}$ = number of waves in unit length of the wave pattern.
- **Particle velocity, wave velocity and particle's acceleration :** In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae that we studied in SHM apply to the particles here also. For example, maximum particle velocity is $\pm A\omega$ at mean position and it is zero at extreme positions. Similarly maximum particle acceleration is $\pm\omega^2 A$ at extreme positions and zero at mean position. However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between $+A\omega$ and $-A\omega$) the wave velocity is constant for given characteristics of the medium.
- **Particle velocity in wave motion :**
  The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "particle velocity".



- **Wave velocity :** The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.
- **Relation between particle velocity and wave velocity :**

  Wave equation :- $y = A \sin(\omega t - kx)$, Particle velocity $v_p = \frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx)$.

  Wave velocity $= V = \frac{\lambda}{T} = \lambda\frac{\omega}{2\pi} = \frac{\omega}{k}$, $\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) = -\frac{A}{\omega}\omega k \cos(\omega t - kx) = -\frac{1}{V}\frac{\partial y}{\partial t} \Rightarrow \boxed{\frac{\partial y}{\partial x} = -\frac{1}{V}\frac{\partial y}{\partial t}}$

  Note : $\frac{\partial y}{\partial x}$ represents the slope of the string (wave) at the point x.

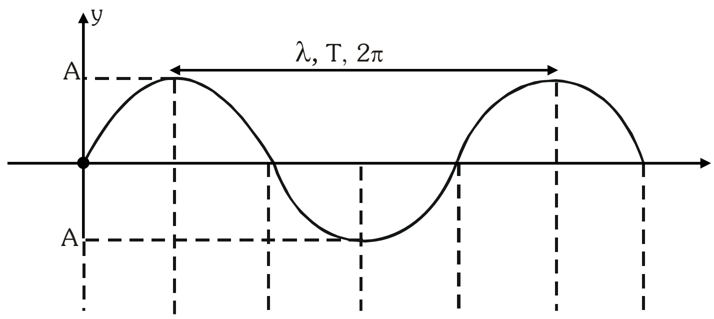
  Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.
- **Particle velocity (v_p) and acceleration (a_p) in a sinusoidal wave :**

  The acceleration of the particle is the second partial derivative of y (x, t) with respect to t,

$$\therefore\ a_P = \frac{\partial^2 y(x,t)}{\partial t^2} = -\omega^2 A \sin(\omega t - kx) = -\omega^2 y(x,t)$$

  i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the same result we obtained for SHM. Thus, $a_p = -\omega^2$ (displacement)

- **Relation between Phase difference, Path difference & Time difference**



Phase difference (Δϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Path difference (Δλ)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3\lambda}{2}$
Time difference (Δt)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

$\Rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta t}{T} \Rightarrow$ Path difference $= \left(\frac{\lambda}{2\pi}\right)$ Phase difference

GOLDEN KEY POINTS

- A wave motion is a kind of disturbance which travels through a medium due to repeated periodic motion of the particles of the medium about their mean position, the motion being transferred continuously from particle to particle.
- Mechanical waves transmit energy but not the matter.
- The transmission of energy is possible due to two properties of the medium, the elasticity and the inertia.
- Material medium is necessary for the propagation of mechanical waves :
- If we generate waves in a medium continuosly, the particles of the medium oscillate continuosly. In this situation, the disturbance produced in the medium is called a progressive wave.
- When a progressive waves propagates in a medium, then, at any instant all the particles of the medium oscillate in the same way but the phase of oscillation changes from particle to particle.
- Equation of a plane progressive simple harmonic wave : $y = a\sin\frac{2\pi}{\lambda}(Vt - x) = a\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

  Where V is the velocity of wave, λ is the wavelength of wave, a is the amplitude of the oscillation of particles of the medium and y is the instantaneous displacement of the particle located at x at instant t.
- Phase difference between two medium particle having a path difference Δλ is :

  $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta\lambda$ If $\Delta\lambda = \lambda$ Then $\Delta\phi = 2\pi$
- Also $\Delta\phi = \frac{2\pi}{T} \times \Delta t$ If $\Delta t = T$ Then $\Delta\phi = 2\pi$

  That is after one time period the phase of oscillation of a particle becomes the same as in the begining.

Illustrations

Illustration 1.

Given below are some examples of wave motion. State in each case the wave motion is transverse, longitudinal or a combination of both :

(a) Motion of kink in a long coil spring produced by displacing one end of the spring side ways.
(b) Waves produced in a cylinder containing a liquid by moving its piston back and forth.
(c) Waves produced by a motorboat sailing in water.
(d) Ultrasonic waves in air produced by a vibrating quartz crystal.

Solution

(a) Transverse and longitudinal (b) Longitudinal
(c) Transverse and longitudinal (d) Longitudinal

Illustration 2.

A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase.

Solution

We know that for a wave $v = f\lambda$ So $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

Phase difference $\Delta\phi = 60^\circ = (\pi/180) \times 60 = (\pi/3)$ rad,

so path difference $\Delta\lambda = \frac{\lambda}{2\pi}(\Delta\phi) = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

Illustration 3.

On the average a human heart is found to beat 75 times in a minute. Calculate its beat freqeuncy of heart and period.

Solution

The beat frequency of heart $= \frac{75}{60} = 1.25 = 1.25$ Hz

The time period $T = \frac{1}{1.25} = 0.8$ s.

BEGINNER'S BOX-1

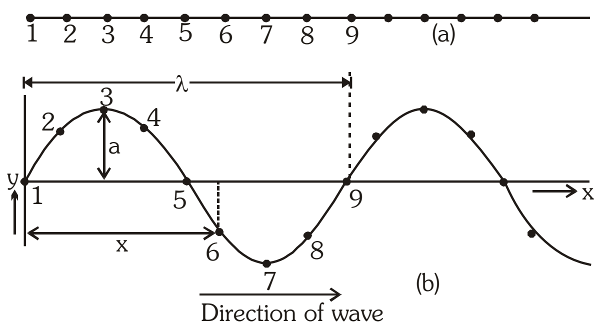
1. A wave has a speed of 300 m/sec and frequency 500 Hz. The phase difference between two adjacent points is $\pi/3$ radian. What will be the path difference between them ?

2. A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top, given that the speed of sound in air is 340 m s^{-1} ? ($g = 9.8\ ms^{-2}$)

2. PROGRESSIVE WAVE ON STRING

2.1 Equation of a Plane Progressive Wave

If, during propagation of wave in a medium, the particles of the medium perform simple harmonic motion then the wave is called a 'simple harmonic progressive wave'. Suppose, a simple harmonic progressive wave is propagating in a medium along the positive direction of the x-axis (from left to right). In fig. (a) the equilibrium positions of the particles 1, 2, 3 .... are shown.



When the wave propagates, these particles oscillate about their equilibrium positions. In Fig. (b) the instantaneous positions of these particles at a particular instant are shown. The curve joining these positions represents the wave. Let the time be counted from the instant when the particle 1 situated at the origin starts oscillating. If y be the displacement of this particle after t seconds, then $y = a \sin \omega t$...(i)

where a is the amplitude of oscillation and $\omega = 2\pi n$, where n is the frequency. As the wave reaches the particles beyond the particle 1, the particles start oscillating. If the speed of the wave be v, then it will reach particle 6, distant x from the particle 1, in x/v sec. Therefore, the particle 6 will start oscillating x/v sec after the particle 1. It means that the displacement of the particle 6 at a time t will be the same as that of the particle 1 at a time x/v sec earlier i.e. at time t – (x/v). The displacement of particle 1 at time t – (x/v) is equal to particle 6, distant x from the origin (particle 1), at time t which is given by

$y = a \sin \omega \left(t - \frac{x}{V}\right)$ But $\omega = 2\pi n$, $y = a \sin (\omega t - kx)$ $\left(k = \frac{\omega}{V}\right)$ ...(ii)

$y = a \sin \left[\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right]$ Also $k = \frac{2\pi}{\lambda}$ ...(iii) $y = a \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$ ...(iv)

This is the equation of a simple harmonic wave travelling along +x direction. If the wave is travelling along the –x direction, then there will be plus sign instead of minus sign inside the brackets in the above equations. For example, equation (iv) will be of the following form : $y = a \sin 2\pi \left(\frac{t}{T} + \frac{x}{\lambda}\right)$. If ϕ be the initial phase difference between the above wave travelling along +x direction and an other wave, then the equation of other wave will be

$$y = a \sin \left\{2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right) \pm \phi\right\}$$

2.2 Velocity of Transverse Mechanical Waves

Velocity of a transverse wave propagating along a string having tension T and mass per unit length μ, is given by $V = \sqrt{\frac{T}{\mu}}$

If D is the diameter of the string, ℓ is its length and ρ is its density, then

$$\Rightarrow \quad \mu = \frac{\pi\left(\frac{D}{2}\right)^2 \ell\rho}{\ell} = \frac{\pi D^2 \rho}{4} \quad \Rightarrow \quad V = \sqrt{\frac{T}{\frac{\pi D^2}{4}\rho}} = \frac{2}{D}\sqrt{\frac{T}{\pi\rho}}$$

Wave always travel with respect to medium (not w.r.t. ground). Therefore any expression derived for the velocity of wave always gives velocity w.r.t. medium not w.r.t. ground.

2.3 Intensity of Waves :

When waves incident perpendicularly on an imaginary surface (wavefront) then the max. energy of waves per unit area, per unit time is known as intensity of waves.

$$I = \frac{1}{2}\rho\omega^2 A^2 v \qquad \frac{W}{m^2} \text{ or } \frac{J}{s - m^2}$$

when medium is fixed then $\boxed{I \propto \omega^2 A^2}$

GOLDEN KEY POINTS

- Equation y = a sin ($\omega t - kx$) represents displacement of all medium particles at any time 't', in other words it can give us displacement of any specific particle at each and every time so it is called as equation of progressive wave.
- Negative sign between ωt and kx indicates that direction of propagation is positive.
- Positive sign between ωt and kx indicates that direction of propagation is negative.
- For a plane progressive wave y = a sin($\omega t - kx$) producing transverse vibration show that $\frac{\partial y}{\partial x} = -\frac{1}{V} \times \frac{\partial y}{\partial t}$

  i.e. velocity of particle at a given point = (–) wave velocity×slope of wave at that point. (Slope of wave is also called as wave strain) and $\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 y}{\partial t^2}$ is known as differential equation of wave motion.
- **Differential equation of harmonic progressive waves :**

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 y}{\partial t^2}$$

- **Relations :-** $\omega = 2\pi n$, $V = n\lambda$, $k = \frac{2\pi}{\lambda}$, $\omega = Vk$
- **Intensity of wave (I) :-**

  $I = 2\pi^2 n^2 a^2 V \rho$

  $\rho \rightarrow$ density of medium

  For Interference $\rightarrow I \propto a^2$

  For Beats $\rightarrow I \propto n^2a^2$
- **Energy density (U) :-** Maximum wave energy per unit volume of medium

$$U = \frac{E_{max}}{vol} = \frac{\frac{1}{2}Mv_{max}^2}{vol} = \frac{\frac{1}{2}vol \times \rho \times v_{max}^2}{vol} = 2\pi^2 n^2 a^2 \rho \Rightarrow I = V \times U$$

Illustrations

Illustration 4.

The equation of a wave is, $y(x,t) = 0.05\sin\left[\frac{\pi}{2}(10x - 40t) - \frac{\pi}{4}\right]$m

Find : (a) The wavelength, the frequency and the wave velocity

(b) The particle velocity and acceleration at x=0.5 m and t=0.05 s.

Solution

(a) The equation may be rewritten as, $y(x,t) = 0.05\sin\left(5\pi x - 20\pi t - \frac{\pi}{4}\right)$m

Comparing this with equation of plane progressive harmonic wave,

$y(x,t) = A\sin(kx - \omega t + \phi)$ we have,

wave number $k = \frac{2\pi}{\lambda} = 5\pi \text{ rad/m}$ $\therefore \lambda = 0.4$m

The angular frequency is, $\omega = 2\pi f = 20\pi \text{ rad/s}$ $\therefore f = 10$Hz

The wave velocity is,

$$V = f\lambda = \frac{\omega}{k} = 4ms^{-1} \text{ in } + x \text{ direction}$$

(b) The particle velocity and acceleration are,

$$v_p = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right) = 2.22 m/s$$

$$a_p = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2} - \pi - \frac{\pi}{4}\right) = 140 \text{ m/s}^2$$

Illustration 5.

For the travelling harmonic wave $y(x, t) = 2\cos 2\pi(10t - 0.0080x + 0.35)$

where x and y are in cm and t in s. Calculate the phase difference between oscillatory motion of two points separated by a distance of

(a) 4m (b) 0.5m

(c) half of wavelength (d) Third fourth of the wavelength

Solution

Given equation $y = 2\cos 2\pi(10t - 0.008x + 0.35) = 2\cos(20\pi t - 0.016\pi x + 0.7\pi)$

compare with standard equation $y = a\cos(\omega t - kx + \phi)$

$k = 0.016\pi \text{ cm}^{-1}$, $\omega = 20\pi \text{ s}^{-1}$,

(a) When x = 4m = 400 cm then phase difference = $0.016\pi \times 400 = 6.4\pi$ rad.

(b) When x = 0.5m = 50 cm then phase difference = $0.016\pi \times 50 = 0.8\pi$ rad.

(c) When $x = \frac{\lambda}{2}$ then phase difference = $kx = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$ rad.

(d) When $x = \frac{3\lambda}{4}$ then phase difference = $kx = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}$ rad.

Illustration 6.

Which of the following functions represent a travelling wave ?

(a) $(x - vt)^2$ (b) $\ell n(x + vt)$ (c) $e^{-(x-vt)^2}$ (d) $\frac{1}{x + vt}$

Solution

Although all the four functions are written in the form f (ax ± bt), only (c) among the four functions is finite everywhere at all times. Hence only (c) represents a travelling wave.

Illustration 7.

Two mechanical waves, $y_1 = 2\sin 2\pi(50t - 2x)$ & $y_2 = 4\sin 2\pi(ax + 100t)$ propagate in a medium with same speed.

(1) The ratio of their intensities is 1 : 16

(2) The ratio of their intensities is 1 : 4

(3) The value of 'a' is 4 units

(4) The value of 'a' is 2 units

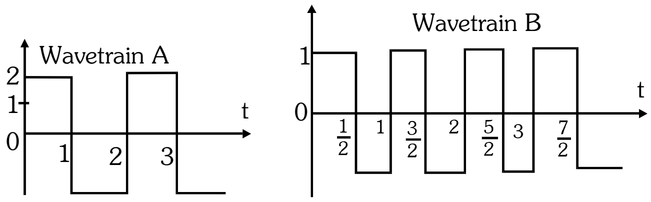
Ans. (1,3)

Solution

$I = \frac{1}{2}\rho v\omega^2 A^2$ and velocity $= \frac{\omega}{k}$

Illustration 8.

Calculate the ratio of intensity of wavetrain A to wavetrain B.



Solution

$$\because I \propto a^2n^2 \therefore \frac{I_A}{I_B} = \frac{a_A^2 n_A^2}{a_B^2 n_B^2} = \left(\frac{2}{1}\right)^2 \times \left(\frac{1}{2}\right)^2 = 1$$

Illustration 9.

A steel wire has a length of 12.0 m and a mass of 2.10 kg. What should be the tension in the wire so that speed of a transverse wave on the wire equals the speed of sound in dry air at 20°C = 343 ms^{-1}.

Solution

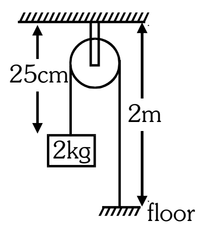
$$\mu = \frac{2.10}{12} = 0.175 \text{ kgm}^{-1} \qquad \because \sqrt{\frac{T}{\mu}} = 343 \text{ ms}^{-1} \qquad \Rightarrow T = (343)^2 \times 0.175 = 2.06 \times 10^4 \text{ N}$$

BEGINNER'S BOX-2

1. A travelling wave in stretched string is given by the equation : $y = 40 \cos(3x - 5t)$ cm where t is in sec. Determine the maximum speed of particles of medium.

2. The displacement produced by a simple harmonic wave is $y = \frac{10}{\pi}\sin\left(200\pi t - \frac{\pi x}{17}\right)$. Then find the time period and maximum velocity of the particle. ($x \to$ m, $y \to$ cm, $t \to$ sec.)

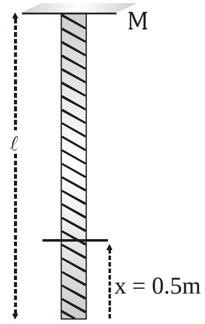
3. In the given figure the string has mass 4.5 g. Find the time taken by a transverse pulse produced at the floor to reach the pulley. ($g = 10 \text{ ms}^{-2}$).



4. A wave moves with speed 300 ms^{-1} on a wire which is under a tension of 500 N. Find how much the tension must be changed to increase the speed to 360 ms^{-1}.

5. A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

6. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling as shown in diagram.



(a) find speed of transverse wave in the rope at a point 0.5 m distance from the lower end.

(b) Calculate time taken by a transverse wave to travel the full length of rope.

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3. SOUND WAVES & ITS CHARACTERISTICS

3.1 Ultrasonic, Infrasonic and Audible Sound

Sound waves can be classified in three groups according to their range of frequencies.

Infrasonic Waves

Longitudinal waves having frequencies below 20 Hz are called infrasonic waves. They cannot be heard by human beings. They are produced during earthquakes. Infrasonic waves can be heard by snakes.

Audible Waves

Longitudinal waves having frequencies lying between 20-20,000 Hz are called audible waves.

Ultrasonic Waves

Longitudinal waves having frequencies above 20,000 Hz are called ultrasonic waves. They are produced and heard by bats. They have a large energy content.

Applications of Ultrasonic Waves

Ultrasonic waves have a large range of applications. Ultrasonic waves can be used –

(i) to detect fine internal cracks in metal.

(ii) for determining the depth of the sea, lakes etc.

(iii) to detect submarines, icebergs etc.

(iv) to clean clothes, fine machinery parts etc.

(v) to kill smaller animals like rats, fish and frogs etc.

- **Equations of sound wave**

A longitudinal mechanical wave can be described in two ways :

(A) Displacement wave form (B) Pressure wave form

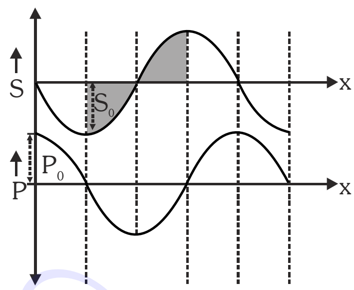
(A) Displacement wave form :- When sound wave is described in term of longitudinal displacement suffered by particles of the medium, it is called displacement wave.

Which can be given by $y = A \sin(\omega t - kx)$

(B) Pressure wave form :- When sound wave is described in term of excess pressure generated due to compression and rarefaction called pressure wave.

Which can be given by $\Delta P = P_0 \cos(\omega t - kx)$

$P_0 = ABK$ where A = displacement of amplitude, B = Bulk modulus,
K = propagation constant, P_0 = Amplitude of pressure wave
P = Pressure in sound wave



- The phase difference between pressure wave form and displacement wave form is 90° and path diff is $\lambda/4$. Displacement will be maximum when pressure is minimum and vice versa.
- When we consider the interference of sound as pressure wave then there is no change in phase when reflected from a "rigid boundary" but have a phase change of π when reflected from free end.

  This is in contrast to reflection of displacement wave which have a phase change of π from a "rigid-end" and no change in phase from "free-end".
- A sound sensor-eg. ear, mike or listener, observer detects change in pressure. So in this case we prefer pressure wave.
- If detector is displacement sensor, we will prefer displacement wave.
- In stationary wave at the place of displacement node, pressure antinode will form and vice versa.

3.2 Speed of longitudinal (Sound) waves

Newton Formula : $v_{medium} = \sqrt{\dfrac{E}{\rho}}$ (Use for every medium)

where E = Elasticity coefficient of medium & ρ = Density of medium

- **For solid medium :** $v_{solid} = \sqrt{\dfrac{Y}{\rho}}$ where E = Y = Young's modulus of elasticity
- **For liquid medium :** $v_{liquid} = \sqrt{\dfrac{B}{\rho}}$ where E = B, where B = volume elasticity coefficient of liquid
- **For gas medium :**

  The formula for velocity of sound in air was first obtained by Newton. He assumed that when sound propagates through air temperature remains constant (i.e. the process is isothermal).

  so $E_{IT} = P \therefore v_{air} = \sqrt{(P/\rho)}$

  At NTP for air $P = 1.01 \times 10^5$ N/m^2 and $\rho = 1.3$ kg/m^3

  so $v_{air} = \sqrt{\dfrac{1.01 \times 10^5}{1.3}} = 279$ m/s

  However, the experimental value of sound in air is 332 m/s which is much higher than that given by Newton's formula.

- **Laplace Correction**

In order to remove the discrepancy between theoretical and experimental values of velocity of sound, Laplace modified Newton's formula assuming that propagation of sound in air is adiabatic process, i.e.

$E_{AD} = \gamma p$ so that $v = \sqrt{\dfrac{\gamma P}{\rho}}$ i.e. $v = \sqrt{1.41} \times 279 = 331.3$ m/s [as $\gamma_{air} = 1.41$]

Which is in good agreement with the experimental value (332 m/s). This establishes that sound propagates adiabatically through gases.

The velocity of sound in air at NTP is 332 m/s which is much lesser than that of light and radio–waves ($= 3 \times 10^8$ m/s). This implies that –

(a) If we set our watch by the sound of a distant siren it will be slow.

(b) If we record the time in a race by hearing sound from starting point it will be lesser than actual.

(c) In a cloud–lightening, though light and sound are produced simultaneously but as c > v, light preceds thunder.

In case of gases –

$$v_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{\text{mass}}} \quad \left[\text{as } \rho = \frac{\text{mass}}{\text{volume}} = \frac{M}{V}\right]$$

or $v_s = \sqrt{\dfrac{\gamma\, \mu RT}{M}}$ [as $PV = \mu RT$] or $v_s = \sqrt{\dfrac{\gamma\, RT}{M_w}}$

$$\left[\text{as } \mu = \frac{\text{mass}}{M_w} = \frac{M}{M_w} \text{ where } M_w = \text{Molecular weight}\right]$$

And from kinetic-theory of gases $v_{rms} = \sqrt{(3RT / M_w)}$ So $\dfrac{v_s}{v_{rms}} = \sqrt{\dfrac{\gamma}{3}}$

3.3 Effect of Various Quantities

(a) Effect of temperature

For a gas γ & M_W is constant $v \propto \sqrt{T} \Rightarrow \dfrac{v_2}{v_1} = \sqrt{\dfrac{T_2}{T_1}} \Rightarrow \dfrac{v_t}{v_0} = \sqrt{\dfrac{t+273}{273}} \Rightarrow v_t = v_0\left[1 + \dfrac{t}{273}\right]^{\frac{1}{2}}$

By applying Binomial theorem.

(i) For any gas medium $v_t = v_0\left[1 + \dfrac{t}{546}\right]$

(ii) For air : $v_t = v_0 + 0.61t$ m/sec. ($v_0 = 332$ m/sec.)

(b) Effect of Relative Humidity

With increase in humidity, density decreases so from $v = \sqrt{\gamma\,(P/\rho)}$ we conclude that with rise in humidity velocity of sound increases. This is why sound travels faster in humid air (rainy season) than in dry air (summer) at same temperature. Due to this in rainy season the sound of factories siren and whistle of train can be heard more than summer.

(c) Effect of Pressure

As velocity of sound $v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M_w}}$

So pressure has no effect on velocity of sound in a gas as long as temperature remain constant. This is why while going up in the atmosphere, though both pressure and density decrease, velocity of sound remains constant as long as temperature remains constant. Further more it has also been established that all other factors such as amplitude, frequency, phase, loudness pitch, quality etc. has apparently no effect on velocity of sound.

Velocity of sound in air is measured by resonance tube or Hebb's method while in gases by Quinke's tube. Kundt's tube is used to determine velocity of sound in any medium solid, liquid or gas.

(d) Effect of Motion of Air

If air is blowing then the speed of sound changes. If the actual speed of sound is v and the speed of air is w, then the speed of sound in the direction in which air is blowing will be (v+ w), and in the opposite direction it will be (v – w).

(e) Effect of Frequency

There is no effect of frequency on the speed of sound. Sound waves of different frequencies travel with the same speed in air although their wavelength in air are different. If the speed of sound were dependent on the frequency, then we could not have enjoyed orchestra.

3.4 Characteristics of Sound

Sound is characterised by the following three parameters :

Loudness :- The quality of sound on the basis of which, sound is said to be high or low. It depends on :

(1) Shape & size of the source

(2) Intensity of sound

⇒ According to **Weber - Fechner** the loudness of a sound of intensity I is given by: $L \propto \log_{10} I$

Which is called Weber-Fechner law [unit of L is 'phon' It also measured in decible]

So $\Delta L = L_2 - L_1 = 10 \log_{10} \frac{I_2}{I_1}$

For zero decibel Loudness, Initensity is called threshold of hearing.

In decibel the loudness of a sound of intensity I is given by $L = 10 \log_{10} (I/I_0)$

Where I_0 represents the threshold of hearing at 0 dB loudness level.

The loudness of a roaring lion is more than the sound produced by a mosquito.

Range :

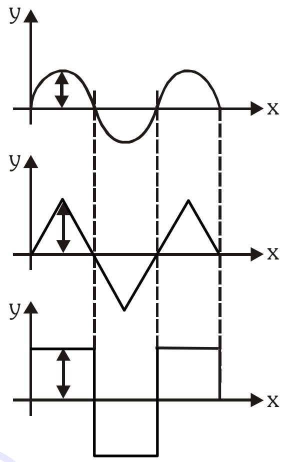
	Intensity	**Loudness level**
Threshold of hearing	$\simeq 10^{-12}$	$\simeq$ 0 dB
Whispering	10^{-11}	10 dB
Normal Talk	10^{-6}	60 dB
Shout	10^{-5}	70 dB
Non tolerable	1	above 120 dB

Pitch : It is the sensation received by the ear due to frequency and is the characteristic which distinguishes a shrill (or sharp) sound from a grave (or flat) sound. As pitch depends on frequency, higher the frequency higher will be the pitch and shriller will be the sound. For example

(1) The buzzing of a bee or humming of a mosquito has high pitch but low loudness while the roar of a lion has large loudness but low pitch.

(2) Due to more harmonics usually the pitch of female voice is higher than male.

Quality (or timbre) : It is the sensation received by the ear due to 'waveform'.



Two sounds of same intensity and frequency as shown in figure will produce different sensation on the ear if their waveforms are different. Now as waveform depends on overtones present, quality of sound depends on number of overtones, i.e., harmonics present and their relative intensities.

GOLDEN KEY POINTS

- The density of a solid is much larger than that of a gas but the elasticity is larger by a greater factor. Hence longitudinal waves in a solid travel much faster than that in a gas.
- In a liquid the speed lies in between the two i.e. $v_{solid} > v_{liquid} > v_{gas}$
- Speed of longitudinal waves in a solid rod : $v = \sqrt{\frac{Y}{\rho}}$
- The property that determines the extent to which an element of a medium changes in volume when the pressure on it changes is the bulk modules (B)

$$B = \frac{-\Delta P}{\Delta V/V}$$

$\frac{\Delta V}{V}$ = Fractional change in volume produced by a change in pressure (ΔP)

- Speed of longitudinal waves in a liquid $v = \sqrt{\frac{B}{\rho}}$ Where B is the bulk modulus of elasticity.
- The velocity of mechanical waves or sound waves in gases is given by :

$v = \sqrt{\frac{\gamma P}{\rho}}$ where P = Pressure ρ = Density

γ = ratio of specific heats of the gas at constant pressure and constant volume. This is known as Laplace's formula.

Illustrations

Illustration 10.

Determine the change in volume of 6 liters of alcohol if the pressure is decreased from 200 cm of Hg to 75 cm. [velocity of sound in alcohol is 1280 m/s, density of alcohol = 0.81 gm/cc, density of Hg = 13.6 gm/cc and g = 9.81 m/s^2]

Solution

For propagation of sound in liquid $v = \sqrt{(B/\rho)}$ i.e., $B = v^2\rho$

But by definition $B = -V\frac{\Delta P}{\Delta V}$ So $-V\frac{\Delta P}{\Delta V} = v^2\rho$, i.e. $\Delta V = \frac{V(-\Delta P)}{\rho v^2}$

Here $\Delta P = H_2\rho g - H_1\rho g = (75 - 200) \times 13.6 \times 981 = -1.667 \times 10^6$ dynes/cm^2

So $\Delta V = \dfrac{(6\times10^3)(1.667\times10^6)}{0.81\times(1.280\times10^5)^2} = 0.75$ cc

Illustration 11.

(a) Speed of sound in air is 332 m/s at NTP. What will be the speed of sound in hydrogen at NTP if the density of hydrogen at NTP is (1/16) that of air.

(b) Calculate the ratio of the speed of sound in neon to that in water vapour at any temperature.

[Molecular weight of neon = 2.02×10^{-2} kg/mol and for water vapours = 1.8×10^{-2} kg/mol]

Solution

The velocity of sound in air is given by $v = \sqrt{\dfrac{E}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M_w}}$

(a) In terms of density and pressure $\dfrac{v_H}{v_{air}} = \sqrt{\dfrac{P_H}{\rho_H} \times \dfrac{\rho_{air}}{P_{air}}} = \sqrt{\dfrac{\rho_{air}}{\rho_H}}$ [as $P_{air} = P_H$]

$$\Rightarrow \quad v_H = v_{air} \times \sqrt{\frac{\rho_{air}}{\rho_H}} = 332 \times \sqrt{\frac{16}{1}} = 1328 \text{ m/s}$$

(b) In terms of temperature and molecular weight

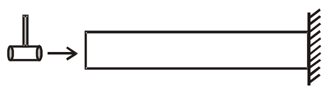
$$\frac{v_{Ne}}{v_W} = \sqrt{\frac{\gamma_{Ne}}{M_{Ne}} \times \frac{M_W}{\gamma_W}} \text{ [as } T_{Ne} = T_W]$$

Now as neon is mono atomic ($\gamma = 5/3$) while water vapours poly atomic ($\gamma = 4/3$) so

$$\frac{v_{Ne}}{v_W} = \sqrt{\frac{(5/3)\times1.8\times10^{-2}}{(4/3)\times2.02\times10^{-2}}} = \sqrt{\frac{5}{4}\times\frac{1.8}{2.02}} = 1.055$$

BEGINNER'S BOX-3

1. A 1000 m long rod of density 10.0×10^4 kg/m^3 and having young's modulus Y = 10^{11} Pa, is clamped at one end. It is hammered at the other free end as shown in the figure. The longitudinal pulse goes to right end, gets reflected and again returns to the left end. How much time (in sec) the pulse will take to go back to initial point?



2. If the bulk modulus of water is 4000 MPa, what is the speed of sound in water?

3. The minimum intensity of audibility of sound is 10^{-12} watt/m^2. If the intensity of sound is 10^{-9} watt/m^2, then calculate the intensity level of this sound in decibels.

4. Sound can be heard at great distances after rainfall. Explain.

5. A sound wave is propagating in given gaseous medium then plot graph between

(a) v_{gas} & T

(b) v_{gas} & P

(c) v_{gas} & $\sqrt{T}$

[v_{gas} = Speed of sound , P = Pressure , T = Temperature] Also write down proper units on axis.

6. Why are temple bells made of large size?

7. Explain why sound travels faster in warm air than in cool air ?

8. Explosion on other planets are not heard on earth. Explain why ?

9. Calculate velocity of sound in mixture of 1 mole He and 1 mole O_2 at 127°C.

10. Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air

(a) is independent of pressure. (b) increases with temperature. (c) increases with humidity.

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4. PRINCIPLE OF SUPERPOSITION OF WAVES

4.1 Superposition of Waves

Two or more waves can traverse the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements produced by individual wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore according to superposition principle.

> The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacements caused to the particle by the individual waves.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, \ldots$ are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \ldots$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

Due to superposition of waves the following phenomenon can be seen

1. **Interference :** Superposition of two waves having equal frequency and nearly equal amplitude.
2. **Beats :** Superposition of two waves of nearly equal frequency in same direction.
3. **Stationary waves :** Superposition of equal waves from opposite direction.
4. **Lissajous' figure :** Superposition of perpendicular waves.

4.2 Interference

When two coherent waves of same frequency propagate in same direction and superimpose on one another then the intensity of resultant wave becomes maximum at some points and at some points it becomes minimum. This phenomena of intensity variation w.r.t. position is known as interference.

Coherent Waves :

Two waves are said to be coherent if their phase difference does not depend on time.

(a) Mathematcial Analysis

First wave $\Rightarrow$ $Y_1 = a_1 \sin(\omega t + \phi_1)$

Second wave $\Rightarrow$ $Y_2 = a_2 \sin(\omega t + \phi_2)$

$\Delta\phi$= phase difference $= \phi_1 - \phi_2$

For Ist $I_1 \propto a_1^2$

For IInd $I_2 \propto a_2^2$

From superposition

Resultant wave = y = $y_1 + y_2$, $y = A \sin(\omega t + \theta)$

$I \rightarrow I_1 \propto a_1^2 \quad \Rightarrow \quad I_1 = Ka_1^2$

$I_2 \propto a_2^2 \quad \Rightarrow \quad I_2 = Ka_2^2$

$I \propto A^2 \quad \Rightarrow \quad I = KA^2$

$\boxed{A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\Delta\phi} \quad \Rightarrow \quad \boxed{I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi}$

(b) Types of Interference

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\Delta\phi, \quad I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$$

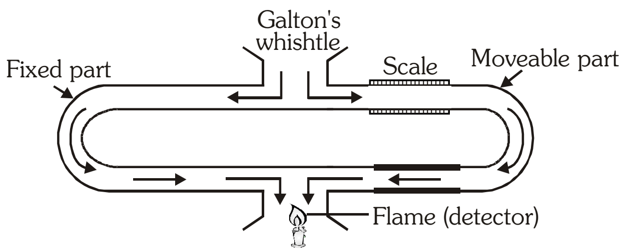
Constructive Interference : Same Phase [Maxima]	**Destructive Interference :** Opposite Phase [Minima]
If $\cos\Delta\phi = +1$ $\Rightarrow$ then I_{max} & A_{max} therefore phase difference $\Delta\phi = 0, 2\pi, 4\pi, 6\pi, \ldots 2N\pi$ (N = 0, 1, 2,...) and Path difference $\Delta\lambda = 0, \lambda, 2\lambda, 3\lambda \ldots N\lambda$ (N= 0, 1, 2, ...) So $A_{max} = a_1 + a_2$ and $I_{max} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}$ or $I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$ and obviously $I_{max} > I_1 + I_2$ or $I_{max} \propto (a_1+a_2)^2$	If $\cos\Delta\phi = -1$ $\Rightarrow$ then I_{min} & A_{min} therefore phase difference $\Delta\phi = \pi, 3\pi, 5\pi, 7\pi \ldots (2N+1)\pi$ (N = 0, 1, 2...) and path difference $\Delta\lambda = \lambda/2, 3\lambda/2, 5\lambda/2, \ldots (2N+1)\lambda/2$ (N= 0, 1, 2...) Put the value of $\Delta\phi$ in the formula $A_{min} = a_1 - a_2$ and $I_{min} = I_1 + I_2 - 2\sqrt{I_1}\sqrt{I_2}$ or $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ obviously $I_{min} < I_1 + I_2$, $I_{min} \propto (a_1 - a_2)^2$

If path difference is zero then maxima is obtained, if path difference is $\lambda/2$ then minima and if path difference is λ then maxima. If path difference is $3\lambda/2$ then minima. So the distance between to consecutive maximum or to consecutive minima is λ.

(c) Quincke's Tube :-

(1) It gives practical proof of interference of sound.

(2) Quincke's tube is practical method for finding the speed of sound in gaseous medium.



(3) (a) As we slide movable part of tube by ℓ unit, path difference will become 2ℓ so $\Delta x = 2\ell$.

(b) If vibration in flame becomes **maximum** to **minimum** or **minimum** to **maximum** then $\Delta x = \frac{\lambda}{2} = 2\ell$

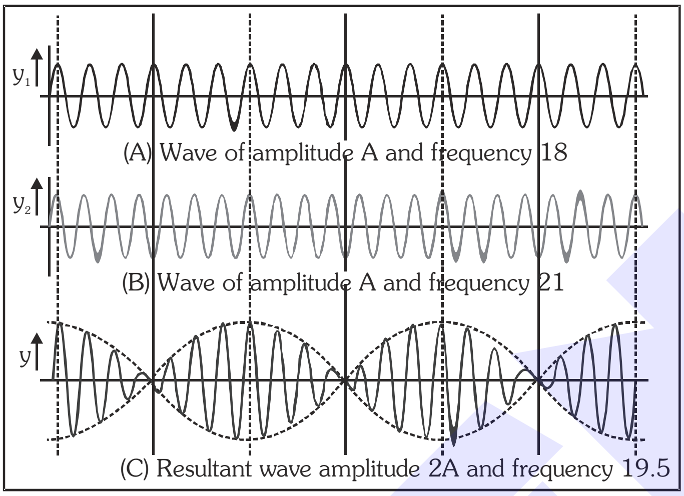
(c) If vibration in flame becomes **minimum** to **minimum** or **maximum** to **maximum** then $\Delta x = \lambda = 2\ell$

(4) Quincke tube is used as a sound filter.

(5) Galton's whistle produces ultrasonic waves.

4.3 Beats

When two sound waves of nearly equal (but not exactly equal) frequencies travel in same direction, at a given point due to their super position, intensity alternatively increases and decreases periodically. This periodic waxing and waning of sound at a given position is called beats.



1. Condition: Two waves of approximatly equal frequency propagate in same direction

Time	n_1 = 12 Hz	n_2= 10 Hz	Diff. of vibration	phase diff.	Intensity	Remarks
0 sec.	0	0	0	0	Max.	
¼ sec.	3	2.5	0.5	π	Min.	
½ sec.	6	5	1	2π	Max.	Ist Beat
¾ sec.	9	7.5	1.5	3π	Min.	
1 sec.	12	10	2	4π	Max.	IInd Beat
Beat frequency (b) = $n_1 \sim n_2$				(~ = positive difference)		

2. Mathematical analysis :

I $\longrightarrow y_1 = a \sin \omega_1 t = a\sin 2\pi n_1 t$

II $\longrightarrow y_2 = a \sin \omega_2 t = a\sin 2\pi n_2 t$

For surperposition

$$y = y_1 + y_2 = a\{\sin 2\pi n_1 t + \sin 2\pi n_2 t\} = a\left\{2\sin 2\pi\frac{(n_1+n_2)t}{2}\cos 2\pi\frac{(n_1-n_2)t}{2}\right\}$$

$$y = \left[2a\cos 2\pi\frac{(n_1-n_2)t}{2}\right]\sin 2\pi\frac{(n_1+n_2)t}{2}$$

$y = A \sin 2\pi n't$ Resultant wave = $y = A \sin 2\pi n't$ Amplitude $A = 2a\cos 2\pi\frac{(n_1-n_2)t}{2}$

Frequency of resultant wave $\Rightarrow n' = (n_1 + n_2)/2$

GOLDEN KEY POINTS

- Superposition phenomena is applicable for all types of waves.
- For the resultant wave in interference phenomenon frequency, wavelength & velocity is identical to intial or super-posing waves but, its amplitude and intial phase is changed.
- Superposition phenomena applicable for all vector quantities.
- $\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$ and $\frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2$
- Average intensity of interference pattern

  I_{av} or $< I > = \frac{I_{max} + I_{min}}{2}$ or $I_{av} = I_1 + I_2$

  I_{av} = Sum of intensities of individual waves
- **Degree of Interference pattern (f)** = f represents contrast effect (clarity of interference pattern)

  $f = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\%$
- Conditions for perfect interference

  If $a_1 = a_2 = a$ and $I_1 = I_2 = I$ $\quad I_{max} = 4I \quad I_{min} = 0$

  $\frac{I_{max}}{I_{av}} = \frac{2}{1} \Rightarrow I_{av} = 2I \quad f_{max} = 1$ (unity) $= 100\%$
- Above formulae and Results also apply to interference of light.
- Frequency of resultant wave $= \frac{(n_1+n_2)}{2}$, beats frequency or freq of intensity variation $= n_1 \sim n_2$

  Frequency of amplitude variation $= \frac{n_1 \sim n_2}{2}$
- When source of higher frequency execute one vibration more than source of lower frequency, one beat is formed.

  $2\pi \equiv T$ (one oscillation difference)
- No. of Beats formed per sec is known as Beat frequency and time taken to complete one beat is known as beat time period.
- **Interference :-** Permanent intensity pattern and it is the function of position [I = f(x).]

  Beats :- Temporary intensity pattern and it is the function of time [I = f(t)]
- Periodic vibration in the intensity (or amplitude) of sound due to superposition of two sound waves of slightly different frequencies are called beats.
- To hear beats, the number of beats per second should not be more than ten beats per sec. ($n_1 \sim n_2 < 10$)
- Filing the prongs of tuning fork raises the frequency and loading decreases the frequency.
- **Vibration of tuning fork :** when tuning fork is sounded by striking its one end on rubber pad, then the ends of prongs vibrate in and out while the stem vibrates up and down or vibration of the prongs are transverse and that of the stem is longitudinal. Generally tuning fork produces fundamental tone.

Illustrations

Illustration 12.

If $\dfrac{I_{max}}{I_{min}} = \dfrac{25}{1}$ then $\dfrac{I_1}{I_2}$?

Solution. If $\dfrac{I_{max}}{I_{min}} = \dfrac{25}{1} \Rightarrow \dfrac{a_1+a_2}{a_1-a_2} = \dfrac{5}{1}$ (by C & D) $\dfrac{a_1}{a_2} = \dfrac{6}{4} = \dfrac{3}{2}$ Thus $\dfrac{I_1}{I_2} = \dfrac{9}{4}$

Illustration 13.

In interference phenomena if the degree of interference pattern in interference is 60% then find the ratio of intensity & amplitudes of interferring wave form.

Solution.

$\dfrac{I_{max}-I_{min}}{I_{max}+I_{min}} = \dfrac{60}{100} = \dfrac{3}{5}$ By C & D $\dfrac{I_{max}}{I_{min}} = \dfrac{5+3}{5-3} = \dfrac{4}{1}$

Thus $\dfrac{a_1+a_2}{a_1-a_2} = \dfrac{2}{1}$ & $\dfrac{a_1}{a_2} = \dfrac{2+1}{2-1} = \dfrac{3}{1}$ thus $\dfrac{I_1}{I_2} = \dfrac{9}{1}$ Ans.

Illustration 14.

T.F. having n = 300 Hz produces 5 beats/sec. with another T.F. If impurity is added on the arm of known tuning fork number of beats decreases then find frequency of unknown T.F. ?

Solution. 295 $\quad \overset{300}{\longleftarrow}$ wax is added $\quad$ 305

If it would be 305 Hz, beats would have increased but with 295 Hz beats decreases so answer is 295 Hz.

Illustration 15.

A T. F. having n = 158 Hz, produce 3 beats/sec. with another T. F. As we file the arm of unknown, beats become 7 then find frequency of unknown.

Solution. before filling $\quad 158 \pm 3 \quad$ so 155 $\quad$ or $\quad$ 161

after filling $\quad$ b = 7

155 $\quad$ 158 $\quad$ 161 $\quad$ 165 (after filling)

165 (after filling)

filling $\quad$ filling

Both T.F. give 7 beat/sec. after filling. So answer is both.

Illustration 16.

41 tuning forks are arranged in a series in such a way that each T.F. produce 3 beats with its neighbouring T.F. If the frequency of last is 3 times of first then find the frequency of 1^{st} 11^{th} 16^{th} 21^{st} & last T.F.

Solution. $n_1 = n$ (let) $\quad$ So $\quad n_{41} = 3n$ (according to Que.)

$n_2 = n + b$

$n_3 = n + 2b$ $\quad$ So $\quad n_{41} = n + 40 \times 3$

$n_4 = n + 3b$ $\quad 3n = n + 120$

$n_{41} = n + 40\,b$ $\quad n = 60$ Hz

$n_{11} = n + 10\,b = 90$ Hz, $\quad n_{16} = n + 15\,b = 105$ Hz

$n_{21} = n + 20b = 120$ Hz

Illustration 17.

Three simple harmonic waves, identical in frequency n and amplitude A moving in the same direction are superimposed in air in such a way, that the first, second and the third wave have the phase angles $\phi, \phi+\frac{\pi}{2}$ and $(\phi+\pi)$ respectively at a given point P in the superpositon.

Then as the waves progress, the superposition will result in

(1) a periodic, non-simple harmonic wave of amplitude 3A

(2) a stationary simple harmonic wave of amplitude 3A

(3) a simple harmonic progressive wave of amplitude A

(4) the velocity of the superposed resultant wave will be the same as the velocity of each wave

Solution **Ans. (3,4)**

Since the first wave and the third wave moving in the same direction have the phase angles ϕ and $(\phi+\pi)$, they superpose with opposite phase at every point of the vibrating medium and thus cancel out each other, in displacement, velocity and acceleration. They, in effect, destroy each other out. Hence we are left with only the second wave which progresses as a simple harmonic wave of amplitude A. The velocity of this wave is the same

Illustration 18.

Two vibrating tuning forks produce progressive waves given by $y_1 = 4\sin(500\pi t)$ and $y_2 = 2\sin(506\pi t)$. These tuning forks are held near the ear of a person. The person will hear α beats/s with intensity ratio between maxima and minima equal to β. Find the value of $\beta-\alpha$.

Solution **Ans. 6**

$y_1 = 4\sin(500\,\pi t)$ $\quad$ $y_2 = 2\sin(506\,\pi t)$

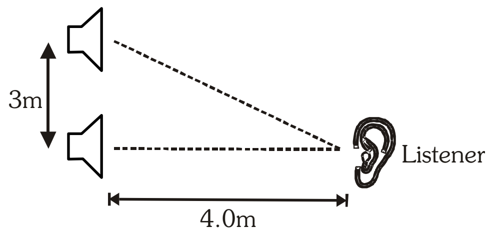
Number of beats = 253 – 250 = 3 beat/sec $\Rightarrow \alpha = 3$ beats/sec

As $I_1 \propto (16)$ and $I_2 \propto 4 \Rightarrow \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2} \Rightarrow \left(\frac{4+2}{4-2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9 \Rightarrow \beta = 9$

So $\beta - \alpha = 9 - 3 = 6$

Illustration 19.

Two loudspeakers as shown in fig. below separated by a distance 3 m, are in phase. Assume that the amplitudes of the sound from the speakers is approximately same at the position of a listener, Who is at a distance 4.0 m in front of one of the speakers. For what frequencies does the listener hear minimum signal? Given that the speed of sound in air is 330 ms^{-1}.



Solution

The distance of the listener from the second speaker = $\sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$ m

path difference = (5 – 4.0) m = 1 m

For fully destructive interference 1 m = $(2m + 1)\lambda/2$

Hence $\quad \lambda = 2/(2m + 1)$ m

The corresponding frequencies are given by

$n = [330 \times (2m + 1)]/2\ s^{-1}$, for m = 0, 1, 2, 3, 4, ...

$= 165\ (2m + 1)\ s^{-1}$, for m = 0, 1, 2, 3, 4, ...

Therefore the frequencies for which the listener would hear a minimum intensity 165 Hz. 495 Hz, 825 Hz,......

Illustration 20.

Two plane harmonic sound waves are expressed by the following equations

$$y_1(x,t) = A \sin (0.5\ \pi x - 100\ \pi t)$$

$$y_2(x,t) = A \sin (0.48\pi x - 96\ \pi t)$$

(All parameters are in MKS system)

(a) How many times does an observer hear maximum intensity in 1 second ?

(b) What is the speed of the sound ?

(c) What is the amplitude of $y_1 + y_2$ at $x = 0$ and $t = 0.25$ s ?

Solution

(a) Beat frequency $= f_1 - f_2 = \frac{1}{2\pi}\ [100\ \pi - 96\ \pi] = 2$

Therefore observer heard maximum intensity twice in one second.

(b) Speed of the sound wave $= \frac{\omega_1}{k_1} = \frac{100\pi}{0.5\pi} = 200\ ms^{-1}$

(c) At $x = 0$ & $t = 0.25$ s $y_1 = 0$ & $y_2 = 0$ so $y_1 + y_2 = 0$

Illustration 21.

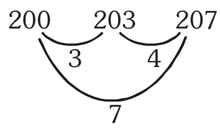
Three tuning forks of frequencies 200, 203 and 207 Hz are sounded together. Find out the beat frequency.

Solution

$\frac{1}{3}$ $\quad \frac{2}{3}$ $\quad \left(\frac{3}{3}\right)$

$\frac{1}{4}$ $\quad \frac{2}{4}$ $\quad \frac{3}{4}$ $\quad \left(\frac{4}{4}\right)$

$\frac{1}{7}$ $\quad \frac{2}{7}$ $\quad \frac{3}{7}$ $\quad \frac{4}{7}$ $\quad \frac{5}{7}$ $\quad \frac{6}{7}$ $\quad \left(\frac{7}{7}\right)$



Divide 1 second into 3, 4 or 7 equal divisions

Ans. 12

Eliminate common time instants. Total Maxima in one second 3 + 3 + 6 = 12

BEGINNER'S BOX-4

1. Two waves having same frequency and same amplitude are superimposed at a point. Find the phase difference between two waves if resultant amplitude is :

(i) 2a (ii) $\sqrt{2}$a (iii) a (iv) Zero

2. During interference phenomenon of two wave it is observed that maximum amplitude to minimum amplitude ratio is 9 : 7. Find the intensity ratio of waves.

3. During interference phenomenon of two wave it is observed that amplitude ratio of waves is 5 : 4. Find the maximum to minimum intensity ratio of resulting wave.

4. Two sound waves emitted by one sound source is approaching at a point through two path. When path difference is 36 cm or 60 cm, then points are silence point if speed of sound in air 330 m/sec then find the frequency of sound source.

5. In a Quincke's tube there are two positions where sound becomes minimum, the sliding distance between them is 16.6 cm. Find the freq. of sound source. (Sound velocity in air = 332 m/sec.)

6. A tuning fork having frequency 300 Hz produce, four beats per sec with x. If we file arm of unknown and again vibrate, number of beats decreases. Determine x.

7. 81 TF are arranged in the increasing order of their frequency and each TF produce 4 beats per sec. with its near by TF. If the frequency of last TF is octave of frequency of first TF then find the frequency of last TF.

8. Two sound waves $y_1 = 3 \sin 400\pi t$; $y_2 = 4 \sin 402\pi t$
(a) Find frequency of resultant wave
(b) Beat frequency
(c) Intensity ratio of waves

9. A Standard tuning fork contains frequency n_0, another TF n_A have 20% more compared to n_0, n_B 30% more compared to n_0, n_A & n_B have beat relation of 6 beat/sec. Then determine n_0, n_A, n_B

10. Two tuning fork having frequency 320 Hz & 324 Hz produce beat phenomena. Determine –
(a) Beat time period
(b) Minimum time interval in which maximum intensity become minimum.
(c) Number of beat per three seconds.

11. Two audio speakers are kept some distance apart and are driven by the same amplifier system. A person is sitting at a place 6.0 m from one of the speakers and 6.4 m from the other. If the sound signal is continuously varied from 500 Hz to 5000 Hz, what are the frequencies for which there is a destructive interference at the place of the listener ? Speed of sound in air = 320 m/s.

12. 51 tuning fork are arranged in a series in such a way that each fork produce five beats/sec with neighbouring tunning fork. If frequency of last is six time of first then determine frequency of first, 11th, 17th, 27th , 33th and last tunning fork.

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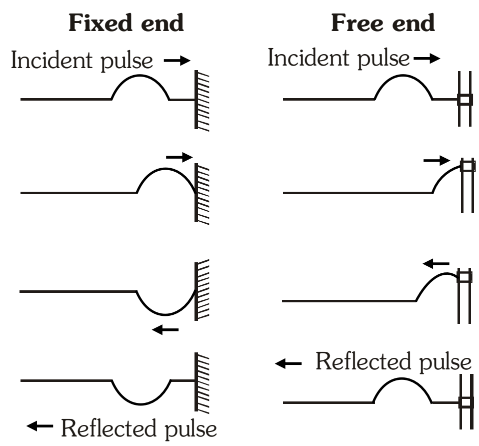
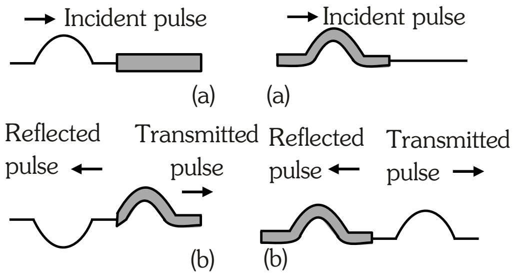
5. REFLECTION OF WAVES, STATIONARY WAVES, STANDING WAVES IN STRINGS

5.1 Reflection and transmission of waves

Whenever a travelling wave reaches a boundary, part or all of the wave will be reflected. For example, consider a pulse travelling on a string fixed at one end (figure). When the pulse reaches the fixed wall, it will be reflected. Since the support attaching the string to the wall is assumed to be rigid, it does not transmit any part of the disturbance to the wall. Note that the reflected pulse is inverted. This can be explained by Newton's third law, the support must exert an equal and opposite (downward) reaction force on the string. This downward force causes the pulse to invert upon reflection.



Now consider another case where the pulse arrives at the end of a string that is free to move vertically. The tension at the free end is maintained by tying the string to a ring of negligible mass that to slide vertically on a smooth post. Again, the pulse will be reflected, but this time its displacement is not inverted. As the pulse reaches the post, it exerts a force on the free end, causing the ring to accelerate upward.



In the process, the ring "overshoots" the height of the incoming pulse and is then returned to its original position by the downward component of the tension.

Finally, we may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as shown in figure. When a pulse travelling on the light reaches the junction, part of it is reflected and inverted, and part of it is transmitted to the heavier string. As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behavior of a pulse meeting a rigid boundary, when it is totally reflected.

When a pulse travelling on a heavy string strikes the boundary of a lighter string, as shown in figure, again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative densities of the two string.

Thus, the speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on a light string, if both are under the same tension. The following general rules apply to reflected waves :

(1) When a wave pulse travels from medium A to medium B and $v_A > v_B$ (B is denser than A), the pulse will be inverted upon reflection.

(2) When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

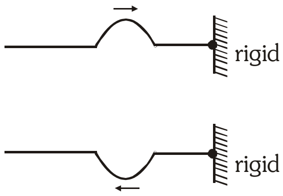
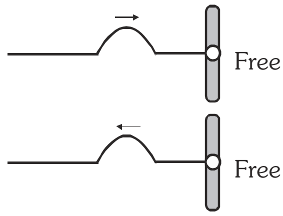
5.2 Stationary Waves

(1) When two indentical progressive waves (transverse or longitudinal) propagating in opposite direction superimpose in a bounded medium (having boundaries) the resultant wave is called **stationary wave or standing wave**.

(2) Stationary wave pattern may be obtained only and only in limited region.

(3) We can obtain two same type of progressive waves, only & only by method of reflection.

(4) According to the nature of reflected surface, reflection are of two types –

(a) Rigid End	**(b) Free End**
In such type of reflection incident and reflected waves have phase difference of π and direction of propagation are opposite.  incident wave $y_1 = a \sin(\omega t - kx)$ reflected wave $y_2 = a \sin(\omega t + kx + \pi)$ or $y_2 = -a \sin(\omega t + kx)$ $y = y_1 + y_2$ $y = a\{\sin(\omega t - kx) - \sin(\omega t + kx)\}$ after solving $y = -2a \sin kx \cos \omega t$ $y = -A \cos \omega t$ where $A = 2a \sin kx$ at $x = 0$ and $A = 0$	In such type of reflection incident and reflected waves are in phase and direction of propagation are opposite.  incident wave $y_1 = a \sin(\omega t - kx)$ reflected wave $y_2 = a \sin(\omega t + kx)$ From superposition of wave $y = y_1 + y_2$ $y = a\{\sin(\omega t - kx) + \sin(\omega t + kx)\}$ after solving $y = 2a \cos kx \sin \omega t$ $y = A \sin \omega t$ where $A = 2a \cos kx$ so $x = 0$ and $A = 2a$

Special properties of stationary wave pattern

- **Zero wave velocity :** No transfer of energy between two points, particle velocity is non zero but wave velocity is zero.
- Position of antinodes & nodes in this pattern remains fix.
- The particles between two consecutive nodes vibrate in same phase while medium particles nearby of any node on both sides always vibrate in opposite phase.
- All medium particles doing simple harmonic vibrations have identical time period but different vibration Amplitude and because of this their maximum velocity at mean position is different
- All medium particles pass through their mean position simultaneously but with different maximum velocity.
- All medium particles pass their mean position in their one complete vibration two times hence stationary wave pattern is obtained as straight line twice in its one complete cycle.
- In this pattern, at antinode position, displacement and velocity is maximum, but wave strain is minimum.

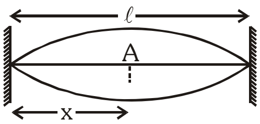
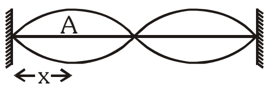
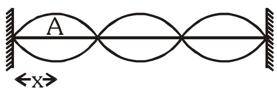
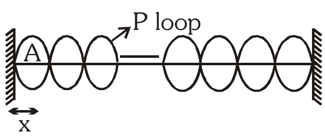
  Strain = slope of stationary wave pattern $\left(\dfrac{dy}{dx}\right)$

  At node position displacement and velocity is minimum but wave strain is maximum.
- Amplitude of incident wave > Amplitude of reflected wave

  For node $a_1 - a_2 \Rightarrow$ minima

  For antinode $a_1 + a_2 \Rightarrow$ maxima
- For any wave each and every reflecting surface have some absorptive power and due to this the energy, intensity & amplitude of reflected wave is always less compared to that of incident wave. Two waves differ in their amplitude having same frequency and wavelength and propagate in reverse or opposite direction always give stationary wave pattern by their superposition.

5.3 Transverse Stationary wave

Sonometer wire	Plucking distance is x	Overtone	Harmonic	Node	Antinode
	$x = \frac{\ell}{2\times 1}$, $\ell = \frac{\lambda_1}{2}$ $\Rightarrow \lambda_1 = 2\ell$, $n_1 = \frac{v}{2\ell}$	0	1	2	1
	$x = \frac{\ell}{2\times 2}$, $\ell = \frac{2\lambda_2}{2}$ $\Rightarrow \lambda_2 = \frac{2\ell}{2}$, $n_2 = \frac{2v}{2\ell}$	1	2	3	2
	$x=\frac{\ell}{2\times 3}$, $\ell = 3\left(\frac{\lambda_3}{2}\right)$ $\Rightarrow \lambda_3 = \frac{2\ell}{3}$, $n_3 = \frac{3v}{2\ell}$	2	3	4	3
	$x=\frac{\ell}{2\times P}$, $\ell = P\left(\frac{\lambda_P}{2}\right)$ $\Rightarrow \lambda_P = \frac{2\ell}{P}$, $n_P = \frac{PV}{2\ell}$	P–1	P	P + 1	P

Sonometer practical -

(i) If a vibrating Tuning fork is pressed against a sonometer wire then forced vibrations are produced in table of hollow box & these vibrations are transferred to air coloum filled in hollow box which results into increase in vibration amplitude of sound & intensity of sound increases. Air filled hollow box is called sound box.

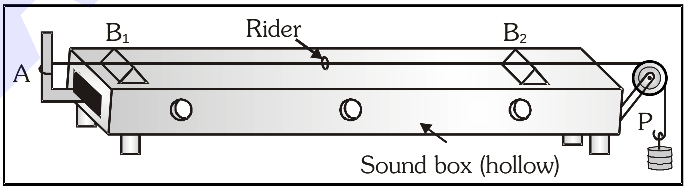
(ii) During contact with table some energy is transferred to table so TF can not do vibrations for longer duration.

(iii) At resonance maximum energy is transferred to table so TF can do vibrations not for longer duration.

(iv) At resonance maximum energy is transferred from TF to vibrating wire and sound intensity is maximum.

Frequency corresponding to 'P' loop

$$\boxed{n = \frac{P}{2\ell}\sqrt{\frac{T}{\pi r^2 d}} = \frac{P}{2\ell}\sqrt{\frac{T}{Ad}} = \frac{P}{2\ell}\sqrt{\frac{T}{m}} = \frac{P}{2}\sqrt{\frac{T}{M\ell}}} \quad \text{where } m=\frac{M}{\ell}$$



$$n \propto \frac{1}{\ell}\ ;\ n \propto \sqrt{T}\ ;\ n \propto \frac{1}{r}$$

$$n \propto \frac{1}{\sqrt{A}}\ \&\ n \propto \frac{1}{\sqrt{d}}$$

at resonace vibration in rider will be maximum

Illustrations

Illustration 22.

A transverse wave, travelling along the positive x-axis, given by y=Asin(kx–ωt) is superposed with another wave travelling along the negative x-axis given by y=–Asin(kx+ωt). The point x=0 is

(1) a node
(2) an anitnode
(3) neither a node nor an antinode
(4) a node or antinode depending on t.

Solution **Ans. (2)**

At x =0, y_1 = Asin (–ωt) and y_2 = –Asinωt; $y_1 + y_2 = -2A\sin\omega t$ (antinode)

Illustration 23.

A standing wave is created on a string of length 120 m and it is vibrating in 6th harmonic. Maximum possible amplitude of any particle is 10 cm and maximum possible velocity will be 10 cm/s. Choose the correct statement.

(1) Angular wave number of two waves will be $\frac{\pi}{20}$.
(2) Time period of any particle's SHM will be 4π sec.
(3) Any particle will have same kinetic energy as potential energy.
(4) Amplitude of interfering waves are 10 cm each.

Solution **Ans. (1)**

$$6\left(\frac{\lambda}{2}\right) = 120 \Rightarrow \lambda = 40 \Rightarrow k = \frac{\pi}{20} \Rightarrow A\omega = v_{max} \Rightarrow \omega = 1 \Rightarrow T = 2\pi$$

Illustration 24.

Two identical wires under the same tension have a fundamental frequency of 500 Hz. What fractional increase in the tension of one wire will give 5 beats per second ? **[AIPMT 2007]**

Solution

Let n_1 be the frequency of the wire having tension T + ΔT and n_2 be the frequency of the wire having tension T, then $\frac{n_1}{n_2} = \sqrt{\frac{T+\Delta T}{T}} \Rightarrow \frac{n+5}{n} = \sqrt{\frac{T+\Delta T}{T}} \Rightarrow \frac{500+5}{500} = \sqrt{\frac{T+\Delta T}{T}}$

$$\Rightarrow \frac{T+\Delta T}{T} = \left(1+\frac{5}{500}\right)^2 \approx 1 + \frac{2}{100} \approx 1.02 \Rightarrow \frac{\Delta T}{T} = 0.02$$

OR

For stretched wire frequency $n \propto \sqrt{T} \Rightarrow \frac{\Delta n}{n} = \frac{1}{2}\frac{\Delta T}{T}$

$$\Rightarrow \frac{\Delta T}{T} = 2\left(\frac{\Delta n}{n}\right) = 2\left(\frac{5}{500}\right) = 0.02$$

Illustration 25.

A string with a mass density of 4×10^{-3} kg/m is under tension of 360 N and is fixed at both ends. One of its resonance frequencies is 375 Hz. The next higher resonance frequency is 450 Hz. Find the mass of the string. **[AIPMT 2007]**

Solution

$n_1 = 375 = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$ and $n_2 = 450 = \frac{p+1}{2\ell}\sqrt{\frac{T}{m}}$ where p is number of loops

$$\Rightarrow \frac{450}{375} = \frac{p+1}{p} \Rightarrow p = 5$$

so $$\ell = \frac{p}{2 \times n_1}\sqrt{\frac{T}{m}} = \frac{5}{2 \times 375}\sqrt{\frac{360}{4 \times 10^{-3}}} = 2m$$

$\Rightarrow$ Mass of wire = (m) (ℓ) = (4×10^{-3}) (2) = 8×10^{-3} kg

OR

Difference between two consecutive resonating frequency $n_2 - n_1 = \dfrac{1}{2\ell}\sqrt{\dfrac{T}{m}}$

$$\Rightarrow \quad 450 - 375 = \frac{1}{2\ell}\sqrt{\frac{360}{4\times10^{-3}}}$$

$$\Rightarrow \quad \ell = \frac{1}{2\times75}\sqrt{\frac{360}{4\times10^{-3}}} = \frac{1}{150}\times\frac{6\times10^2}{2} = 2\text{ m}$$

$\Rightarrow$ Mass of wire = (m) (ℓ) = (4×10^{-3}) (2) = 8×10^{-3} kg

Illustration 26.

A steel wire of length 1 m and mass 0.1 kg and having a uniform cross-sectional area of 10^{-6} m² is rigidly fixed at both ends. The temperature of the wire is lowered by 20°C. If the wire is vibrating in fundamental mode, find the frequency (in Hz).($Y_{steel} = 2\times10^{11}$ N/m², $\alpha_{steel} = 1.21\times10^{-5}$/°C)

(1) 11 (2) 20 (3) 15 (4) 10

Solution **Ans. (1)**

$$\Delta\ell = \alpha\ell\Delta\theta \Rightarrow Y = \frac{T/A}{\Delta\ell/\ell} \Rightarrow T = YA\frac{\Delta\ell}{\ell} \Rightarrow T = \alpha YA\Delta\theta = 48.4\text{N}\,; \quad v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{48.4}{\left(\frac{0.1}{1}\right)}} = 22\text{m/s}$$

$$\because \text{ for fundamental note } \ell = \frac{\lambda}{2} \Rightarrow \lambda = 2\text{m} \Rightarrow f = \frac{v}{\lambda} = \frac{22}{2} = 11\text{Hz}$$

Illustration 27.

A piezo electric quartz plate of thickness 0.005 m is vibrating in resonant conditions. Calculate its fundamental frequency if for quartz $Y = 8\times10^{10}$ N/m² and $\rho = 2.65\times10^3$ kg/m³

Solution

We know that for longitudinal waves in solids $v = \sqrt{\dfrac{Y}{\rho}}$, So $v = \sqrt{\dfrac{8\times10^{10}}{2.65\times10^3}} = 5.5\times10^3$ m/s

Further more for fundamental mode of plate $\Rightarrow (\lambda/2) = L$ So $\lambda = 2\times5\times10^{-3} = 10^{-2}$ m

But as $v = f\lambda$, i.e., $f = (v/\lambda)$ so $f = [5.5\times10^3/10^{-2}] = 5.5\times10^5$ Hz = 550 kHz

Illustration 28.

A stretched wire of length 114 cm is divide into three segment whose fundamental frequencies are in the ratio 4 : 3 : 1, the length of the segments must be in the ratio

Solution

$$n_1 : n_2 : n_3 = 4 : 3 : 1 \quad \left[\because n \propto \frac{1}{\ell}\right] \Rightarrow \quad \ell_1 : \ell_2 : \ell_3 = \frac{1}{4} : \frac{1}{3} : 1 = 3 : 4 : 12$$

BEGINNER'S BOX-5

1. The equation of progressive wave is y = 0.09 sin8π [t – (x/20)] after reflection of this wave from rigid end the amplitude of reflected wave becomes 2/3 of initial then (i) Find the equation of reflected wave (ii) Find the displacement of that particle which is situated at x =0, in reflected wave.

2. The transverse displacement of a string is given by

$$y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

where x and y are in m and t in s. The length of the string is 1.5m and its mass is 3.0×10^{-2} kg. Answer the following :

(a) Does the function represent a travelling wave or a stationary wave ?

(b) What are the wavelength, frequency, and speed of each wave ?

(c) Determine the tension in the string.

3. For a plane progressive wave : $y = 0.02 \sin 2\pi (330t - x)$

(i) If this wave is reflected from rigid end and amplitude becomes 60% of initial then equation of wave will be ?

(ii) If this wave is reflected from free end and amplitude becomes 75% of initial then equation of wave will be ?

4. A progressive wave the equation is $y = 0.08 \sin 2\pi (200t - x)$ it is reflected from rigid end and amplitude becomes half then equation of reflected wave and if it is reflected from free end then equation of refelcted wave is .

5. If a stretched string which is fixed at both ends has m nodes, then calculate its length.

6. A sonometer wire emits a fundamental note of frequency 150 Hz. Calculate the frequency of the note emitted when the tension is changed in the ratio of 9 : 16 and length in the ratio of 1 : 2.

7. A tuning fork is found to give 20 beats in 12 seconds when sounded in conjuction with a stretched string vibrating under a tension of 14.4 or 10 kgf. Calculate the frequency of the fork.

8. If we increase the tension of streached wire by 5 kg. wt. then fundamental freq increase with ratio of 2 : 3 find the initial tension.

9. **Fill in the blanks for Sonometer.**

(i) In sonometer if tension of wire and length of wire becomes double and mass per unit length of wire remain constant then fundamental frequency of wire will becomes..........

(ii) In sonometer if tension of wire and length becomes double and mass of wire remain constant then fundamental frequency of wire will becomes..........

(iii) In sonometer if tension of wire and length becomes double and mass per unit length remain constant then wave velocity will becomes..........

(iv) In a stretched wire calculate speed of wave if tension in wire is 40 N and mass for unit length is 4×10^{-3} kg/m. ....

(v) In a stretched wire calculate speed of wave in wire if tension in wire is 80 N and mass is 4×10^{3} kg. and length of wire is 2 m. ....

(vi) In sonometer, if tensions and length both becomes four times, taking mass per unit length constant then, for vibration of same frequency Tunning fork, Number of loop will becomes...........

---

6. STATIONARY WAVES IN ORGAN PIPE

According to nature of superposing waves stationary waves are of two types –

Transverse stationary waves $\rightarrow$ Musical instruments based on wire (sonometer).

Longitudinal stationary waves $\rightarrow$ Musical instruments based on air (resonance tube).

Only applied to longitudinal stationary wave

$\rightarrow \; \rightarrow$ N $\leftarrow \; \leftarrow$

Pressure $\uparrow$ (compression) density $\uparrow$

If medium particles move in this way

$\leftarrow \; \leftarrow$ N $\rightarrow \; \rightarrow$

Pressure $\downarrow$ (Rarefaction) density $\downarrow$

at antinode $\rightarrow$ Pressure & density constant so variations min.

at node $\rightarrow$ Pressure & density variations maximum.

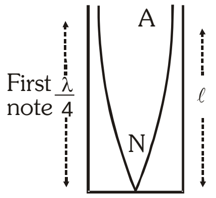
$$E_{gas} = \frac{\text{change in pressure}}{\text{volumetric strain}} = \frac{dP}{\left(\frac{dy}{dx}\right)} = \text{const.} \quad \text{then } dP \propto \text{strain} \left(\frac{dy}{dx}\right)$$

So strain is maximum at node positions and minimum at antinode positions.

Types of Stationary Waves

- Longitudinal Stationary waves [Organ pipe (Ex. Resonance Tube)]
- Transverse Stationary waves [Vibration in stretched string (Ex.Sonometer)]

Closed Organ Pipe

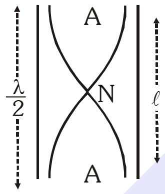


ℓ = Length of air coloum

$\boxed{n = \frac{V}{4\ell}}$ Fundamental Frequency

$\mathbf{n_1 : n_2 : n_3 : n_4 :: 1 : 3 : 5 : 7}$

Open organ pipe



$\boxed{n = \frac{V}{2\ell}}$ Fundamental Frequency

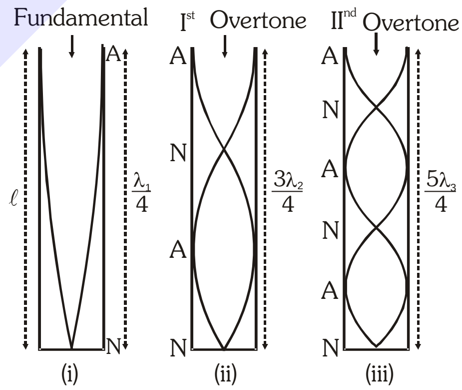
$\mathbf{n_1 : n_2 : n_3 : n_4 :: 1 : 2 : 3 : 4}$

6.1 Vibration of Air Columns

When two longitudinal waves of same frequency and amplitude travel in a medium in opposite directions then by superposition, standing waves are produced. These waves are produced in air columns in cylindrical tube of uniform diameter. These sound producing tubes are called organ pipes.

Vibration of air column in closed organ pipe :

The tube which is closed at one end and open at the other end is called closed organ pipe. On blowing air at the open end, a wave travels towards closed end from where it is reflected towards open end. As the wave reaches open end, it is reflected again. So two longitudinal waves travel in opposite directions to superpose and produce stationary waves. At the closed end there is a node since particles does not have freedom to vibrate whereas at open end there is an antinode because particles have greatest freedom to vibrate.



Hence on blowing air at the open end, the column vibrates forming antinode at free end and node at closed end. If ℓ is length of pipe and λ be the wavelength and v be the velocity of sound in organ pipe then

Case (i) $\ell = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4\ell \Rightarrow n_1 = \frac{v}{\lambda_1} = \frac{v}{4\ell}$ Fundamental frequency.

Case (ii) $\ell = \frac{3\lambda_2}{4} \Rightarrow \lambda_2 = \frac{4\ell}{3} \Rightarrow n_2 = \frac{v}{\lambda_2} = \frac{3v}{4\ell} = 3n_1$ First overtone frequency.

Case (iii) $\ell = \frac{5\lambda_3}{4} \Rightarrow \lambda_3 = \frac{4\ell}{5} \Rightarrow n_3 = \frac{v}{\lambda_3} = \frac{5v}{4\ell} = 5n_1$ Second overtone frequency.

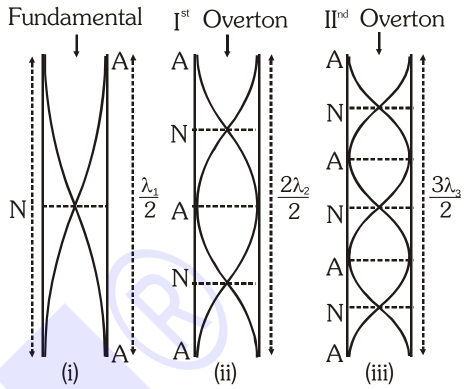
When closed organ pipe vibrate in m[th] overtone then

$$\ell = (2m+1)\frac{\lambda}{4} \qquad \text{so} \qquad \lambda = \frac{4\ell}{(2m+1)} \Rightarrow \qquad n = (2m+1)\frac{v}{4\ell}$$

Hence frequency of overtones is given by $n_1 : n_2 : n_3$ .... = 1 : 3 : 5 ....

Vibration of air columns in open organ pipe :

The tube which is open at both ends is called an open organ pipe. On blowing air at the open end, a wave travel towards the other end from which waves travel in opposite direction to superpose and produce stationary wave. Now the pipe is open at both ends by which an antinode is formed at open end. Hence on blowing air at the open end antinodes are formed at each end and nodes in the middle. If ℓ is length of the pipe and λ be the wavelength and v is velocity of sound in organ pipe.



Case (i) $\ell = \frac{\lambda_1}{2} \quad \Rightarrow \quad \lambda_1 = 2\ell \quad \Rightarrow \quad n_1 = \frac{v}{\lambda_1} = \frac{v}{2\ell}$. Fundamental frequency.

Case (ii) $\ell = \frac{2\lambda_2}{2} \quad \Rightarrow \quad \lambda_2 = \frac{2\ell}{2} \quad \Rightarrow \quad n_2 = \frac{v}{\lambda_2} = \frac{2v}{2\ell} = 2n_1$ First overtone frequency.

Case (iii) $\ell = \frac{3\lambda_3}{2} \quad \Rightarrow \quad \lambda_3 = \frac{2\ell}{3} \quad \Rightarrow \quad n_3 = \frac{v}{\lambda_3} = \frac{3v}{2\ell} = 3n_1$ Second overtone frequency.

Hence frequency of overtones are given by the relation

$n_1 : n_2 : n_3$ .... = 1 : 2 : 3 ....

If an open pipe and a closed pipe have same length $n_{open} = 2\ n_{Closed}$

When open organ pipe vibrate in m[th] overtone then

$$\ell = (m+1)\frac{\lambda}{2} \quad \text{so} \qquad \lambda = \frac{2\ell}{(m+1)} \qquad \Rightarrow \qquad n = (m+1)\frac{v}{2\ell}$$

6.2 End correction :

Due to finite motion of air molecules in organ pipes reflection takes place not exactly at open end but little what above it so in an organ pipe antinode is not formed exactly at free–end but above it at a distance e = 0.6r (called end correction or Rayleigh's correction) with r being the radius of pipe. So for closed organ pipe L → L + 0.6r while for open L → L + 2 × 0.6r (as both ends are open)

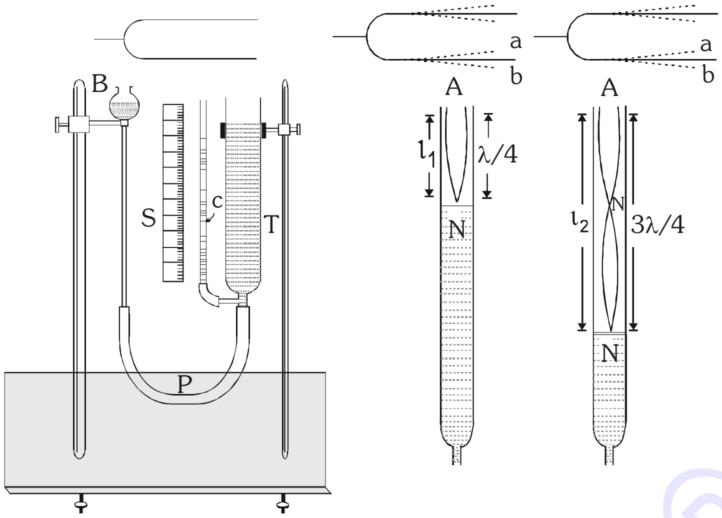
so that – $f_C = \dfrac{v}{4(L+0.6r)}$ while $f_0 = \dfrac{v}{2(L+1.2r)}$

This is why for a given v and L narrower the pipe higher will the frequency or pitch and shriller will be the sound.

- For an organ pipe (closed or open) if v = constant. $f \propto (1/L)$

Resonance Tube

Construction : The resonance tube is a tube T (figure) made of brass or glass, about 1 meter long and 5 cm in diameter and fixed on a vertical stand. Its lower end is connected to a water reservoir B by means of a flexible rubber tube. The rubber tube carries a pinch-cock P. The level of water in T can be raised or lowered by water adjusting the height of the reservoir B and controlling the flow of water from B to T or from T to B by means of the pinch-cock P. Thus the length of the air–column in T can be changed. The position of the water level in T can be read by means of a side tube C and a scale S.



Determination of the speed of sound in air by resonance tube

First of all the water reservoir B is raised until the water level in the tube T rises almost to the top of the tube. Then the pinch–cock P is tightened and the reservoir B is lowered. The water level in T stays at the top. Now a tuning fork is sounded and held over the mouth of tube .The pinch–cock P is opened slowly so that the water level in T falls and the length of the air–column increases. At a particular length of air–column in T, a loud sound is heard. This is the first state of resonance. In this position the following phenomenon takes place inside the tube.

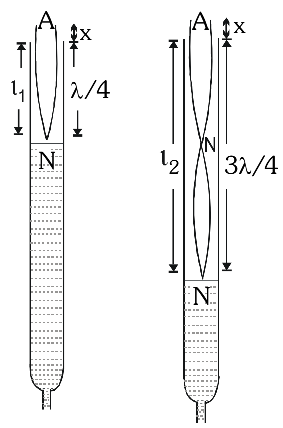
(i) For first resonance $\ell_1 = \lambda/4$

(ii) For second resonance $\ell_2 = 3\lambda/4 \Rightarrow \ell_2 - \ell_1 = \lambda/2 \Rightarrow \lambda = 2(\ell_2 - \ell_1)$

If the frequency of the fork be n and the temperature of the air-column be t°C, then the speed of sound at t°C is given by $v_t = n\lambda = 2n(\ell_2 - \ell_1)$

The speed of sound wave at 0°C $v_0 = (v_t - 0.61\,t)$ m/s.

Calculation of end Correction : In the resonance tube, the antinode is not formed exactly at the open end but slightly outside at a distance e. Hence the length of the air -column in the first and second states of resonance are $(\ell_1 + e)$ and $(\ell_2 + e)$ then



(i) For first resonance $\ell_1 + e = \lambda/4$ ....(i)

(ii) For second resonance $\ell_2 + e = 3\lambda/4$ ....(ii)

Subtract Equation (ii) from Equation (i)

$$\ell_2 - \ell_1 = \lambda/2$$

$$\lambda = 2(\ell_2 - \ell_1)$$

Put the value of λ in Equation (i) $\ell_1 + e = \dfrac{2(\ell_2 - \ell_1)}{4}$

$$\Rightarrow \ell_1 + e = \frac{\ell_2 - \ell_1}{2} \quad \Rightarrow e = \frac{\ell_2 - 3\ell_1}{2}$$

6.3 Comparison of progressive and stationary waves

	Progressive waves	**Stationary waves**
1.	These waves travels in a medium with definite velocity.	These waves do not travel and remain confined between two boundaries in the medium.
2.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.
3.	The phase of vibration varies continuously from particle to particle.	The phase of all the particles in between two nodes is always same. But particles on adjacent side of a node differ in phase by 180°
4.	No particle of medium is permanently at rest.	Particles at nodes are permanently at rest.
5.	All particles of the medium vibrate and amplitude of vibration is same.	The amplitude of vibration changes from particle to particle. The amplitude is zero at nodes and maximum at antinodes.
6.	All the particles do not attain the maximum displacement position simultaneously.	All the particles attain the maximum displacement position simultaneously.

GOLDEN KEY POINTS

Wave property	**Reflection**	**Transmission (Refraction)**
v	does not change	changes
n, T, ω	do not change	do not change
λ,k	do not change	change
A, I	change	change
Phase difference ($\Delta\phi$)	$\Delta\phi = 0$, from a rarer medium $\Delta\phi = \pi$, from a denser medium	does not change

- A pulse undergoes a phase change of π on reflection from a rigid boundary.
- A pulse does not suffer any phase change on reflection from a free boundary.
- The transmitted wave is never inverted.
- A rod clamped at one end or a string fixed at one end is similar to vibration of closed end organ pipe.
- A rod clamped in the middle is similar to the vibration of open end organ pipe.
- If an open pipe is half submerged in water, it becomes a closed organ pipe of length half that of open pipe i.e. frequency remains same.

  So with decrease in length of vibrating air column wavelength decrease ($\lambda \propto$ L), frequency or pitch will increase and vice–versa. This is why the pitch increases gradually as an empty vessel fills slowly.
- For an organ pipe if f = constant. $v \propto \lambda$ or $v \propto L$, $f = \frac{v}{\lambda}$ = constant i.e. the frequency of an organ pipe will remain unchanged if the ratio of speed of sound in to its wave length remains constant.

- As for a given length of organ pipe ℓ = constant $f \propto v$ So

  (a) With rise in temperature as velocity will increase ($v \propto \sqrt{T}$), the pitch will increase.
  (Change in length with temperature is not considered unless stated)

  (b) With change in gas in the pipe as v will change and so f will change ($v \propto \sqrt{\gamma / M_w}$)

  (c) With increase in moisture as v will increase and so the pitch will also increase.

- The equation of stationary wave is -

  (a) When the wave is reflected from a free boundary $y = 2a\cos\frac{2\pi x}{\lambda}\sin\frac{2\pi t}{T}$

  (b) When the wave is reflected from a rigid boundary $y = -2a\sin\frac{2\pi x}{\lambda}\cos\frac{2\pi t}{T}$

- Nodes are the points on a stationary wave, where the amplitude of oscillating particle is always minimum.
- Antinodes are the points on a stationary wave, where the amplitude of oscillating particle is maximum.
- Only odd harmonics are produced in a closed organ pipe. Moreover, first overtone is the third harmonic, second overtone is the fifth harmonic and so on.
- Both of the odd and even harmonic are produced in open organ pipe. Moreover, here first overtone is the second harmonic, second overtone is the third harmonic and so on.
- If the weight which produces tension in wire is immersed in any liquid, the frequency of vibrating wire decreases.
- If a wire fixed between two ends heat up from its one end, its frequency will increase because of thermal tension produced in it.

  $T = YA\alpha\Delta\theta$ where Y = Young's module, α = coeff. of linear expansion, A = cross-section area
  $\Delta\theta$ = change in temp.

- **Echo (Based on reflection) :**

  Multiple reflection of sound is called an echo. If the distance of reflector from the source is d then, $2d = vt$

  Hence, v = speed of sound and t = the time of echo. $\therefore$ $d = \frac{vt}{2}$

  Since, the effect of ordinary sound remains on our ear for 1\10 second, therefore, if the sound returns to the starting point before $\frac{1}{10}$ second, then it will not be distinguished from the original sound and no echo will be heard. Therefore, the minimum distance of the reflector is,

  $$d_{min} = \frac{v \times t}{2} = \left(\frac{330}{2}\right)\left(\frac{1}{10}\right) = 16.5\text{m}$$

- **Musical Interval**

  The ratio between the frequencies of two notes is called the musical interval. Following are the names of some musical intervals.

  (i) Unison $\frac{n_2}{n_1} = 1$

  (ii) Octave $\frac{n_2}{n_1} = 2$

Illustrations

Illustration 29.

Two tuning forks A and B produce 8 beats/s when sounded together. A gas column 37.5 cm long in a pipe closed at one end resonate to its fundamental mode with fork A whereas a column of length 38.5 cm of the same gas in a similar pipe is required for a similar resonance with fork B. Calculate the frequency of these two tuning forks. **[AIPMT 2006]**

Solution

$$\left.\begin{array}{l} \text{For tuning fork 'A'} \ \dfrac{\lambda_1}{4} = 37.5 \quad \text{so } n_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{4 \times 37.5} \\ \text{For tuning fork 'B'} \ \dfrac{\lambda_2}{4} = 38.5 \quad \therefore n_2 = \dfrac{v}{\lambda_2} = \dfrac{v}{4 \times 38.5} \end{array}\right\}$$

$$\therefore \quad n_1 - n_2 = 8 \Rightarrow \frac{v}{4 \times 37.5} - \frac{v}{4 \times 38.5} = 8 \quad \therefore \quad v = (8 \times 4 \times 37.5 \times 38.5)$$

$$n_1 = \frac{8 \times 4 \times 37.5 \times 38.5}{4 \times 37.5} = 308 \text{ Hz} \qquad \text{and} \qquad n_2 = 308 - 8 = 300 \text{ Hz}$$

Illustration 30.

Column I represents the standing waves in air columns and string. Column II represents frequency of the note. Match the column-I with column-II. [v = velocity of the sound in the medium]

	Column-I		**Column-II**
(A)	Second harmonic for the tube open at both ends	(P)	$\frac{v}{4\ell}$
(B)	Fundamental frequency for the tube closed at one end	(Q)	$\frac{v}{2\ell}$
(C)	First overtone for the tube closed at one end	(R)	$\frac{3v}{4\ell}$
(D)	Fundamental frequency for the string fixed at both ends	(S)	$\frac{v}{\ell}$
		(T)	$\frac{5v}{4\ell}$

Ans. (A) S (B) P (C) R (D) Q

Solution

For (A) : For open organ pipe 2nd harmonic = $2\left(\dfrac{v}{2\ell}\right)$

For (B) : For closed organ pipe fundamental frequency = $\dfrac{v}{4\ell}$

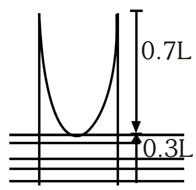
For (C) : For closed organ pipe, first overtone frequency = $\dfrac{3v}{4\ell}$

For (D) : For string fixed at both ends, fundamental frequency = $\dfrac{v}{2\ell}$

Illustration 31.

If an OOP of fundamental frequency 1400 Hz dipped 30% in water then calculate produced frequency.

Solution



$$\frac{v}{2L} = 1400 \text{ Hz} \qquad \Rightarrow \qquad \frac{v}{L} = 2800 \text{ Hz}$$

$$n' = \frac{v}{4L'} = \frac{v}{4[0 \cdot 7L]} = \frac{v}{2 \cdot 8L} = \frac{2800}{2 \cdot 8} = 1000 \text{ Hz}$$

Illustration 32.

If a 'COP' is cut into two equal parts then determine frequency of both part. if initially it was 'n'.

Solution $n = \dfrac{V}{4\ell}$

$$n_1' = \frac{V}{2\left(\frac{\ell}{2}\right)} = \frac{V}{\ell} = 4\left(\frac{V}{4\ell}\right) = 4n\,; \qquad n_2 = \frac{V}{4\left(\frac{\ell}{2}\right)} = \frac{V}{2\ell} = 2\left(\frac{V}{4\ell}\right) = 2n$$

Illustration 33.

For given C.O.P. (closed organ pipe) if 9^{th} O.T. (over tone) has frequency 1900 Hz. then fundamental frequency of same length O.O.P. is ?

Solution

$19n = 1900\,;\qquad n = 100$

$OOP = 2n = 200$ Hz

For same length OOP have double freq. than COP.

Illustration 34.

Two C.O.P. having length 20 cm & 20.5 cm produce 5 beat/sec determine the freg of both C.O.P.

Solution

$\dfrac{n}{n+5} = \dfrac{20}{20.5}$; $\quad n = 200 \qquad$ For 20.5 cm

$n + 5 = 205 \qquad$ For 20 cm

Illustration 35.

An organ pipe closed at one end vibrating in its first overtone and another pipe, open at both ends vibrating in its third overtone are in resonance with a given tuning fork. The ratio of their lengths are given by

Solution

First Over tone of COP = $3\dfrac{V}{4\ell_C}$ and Third Over tone of OOP = $\dfrac{4V}{2\ell_0}$

according to question boath are equal $\quad\therefore\quad 3\dfrac{V}{4\ell_C} = \dfrac{4V}{2\ell_0} \quad\Rightarrow\quad \dfrac{\ell_c}{\ell_0} = \dfrac{3}{8}$

Illustration 36.

A source emits ultrasonic sound of frequency 200 kHz. This sound falls on the water surface. Calculate the wavelength.

(a) in the reflected sound

(b) in the transmitted sound

velocity of sound in air = 340 ms^{-1} , velocity of sound in water = 1480 ms^{-1}

Solution

Wave length $\lambda = \dfrac{\text{Velocity}}{\text{Frequency}}$

(a) Reflected sound travels in air so $\lambda = \dfrac{340}{200\times10^3} = 1.7\times10^{-3}$ m

(b) Transmitted sound travels in water so $\lambda = \dfrac{1480}{200\times10^3} = 7.4\times10^{-3}$ m

BEGINNER'S BOX-6

1. An organ pipe produces a sound of frequency 400 Hz. If it is blown a little harder then it produces a sound of frequency 800 Hz. It is an open pipe or a closed pipe ?

2. A 20 cm long pipe is closed at one end. Which harmonic mode of the pipe is resonantly excited by 425Hz source ? Will the same source be in resonance with the pipe if both ends are open ? (Speed of sound in air = 340 m/s)

3. If frequency of third overtone of closed organ pipe is equal to frequency of sixth overtone of an open organ pipe, then determine their length ratio $\frac{\ell_c}{\ell_o}$.

4. Third overtone of a closed organ pipe is in unison with fourth harmonic of an open organ pipe. find the ratio of lengths of the pipes.

5. Two successive resonant frequencies in an open organ pipe are 1944 and 2592 Hz. If the speed of sound in air 324 ms^{-1}, then find the length of tube.

6. If OOP have 10 antinodes and fundamental freq. 300 Hz. then freq at 10^{th} Antinode.

7. The sum of frequencies of COP's first overtone and OOP's second overtone is 180 find fundamental frequencies of COP and OOP. (length is same)

8. A tuning fork produce four beats per second with two OOP's having length 30 & 31 cm find the freq. of T.F.

9. Two tuning fork n_1, n_2 produce two beats per sec. n_1 resonant with 15 cm COP and n_2 resonant with 30.5 cm OOP. Find the freq. of n_1 & n_2.

10. A tube 1 m long is closed at one end. A stretched wire is placed near the open end. The wire is 0.3m long and has a mass of 0.01kg. It is held fixed at both ends and vibrates in its fundamental mode. It sets the air column in the tube into vibration at its fundamental frequency by resonance. Find –

(a) the frequency of oscillation of the air column.

(b) the tension in the wire, If speed of sound in air is 330 m/s.

11. In resonance tube experiment if V = 300 m/s , n = 500 Hz., L = 125 cm

(i) Find out maximum order of resonance that can be established ?

(ii) Maximum number of resonance ?

(iii) Maximum & minimum water level kept at resonance condition ?

12. Fill in the blanks for COP and OOP.

(i) In COP if the freq of 7^{th} overtone is 600 Hz then fundamental frequency of COP is..........

(ii) In COP if the frequency of 3^{rd} overtone is 1400 Hz then fundamental frequency of same length OOP is..........

(iii) In OOP if the frequency of 7^{th} overtone is 800 Hz then fundamental frequency of OOP is..........

(iv) In COP if the frequency of 7^{th} overtone is 600 Hz then frequency of 3^{rd} overtone is..........

(v) In OOP if the frequency of 7^{th} overtone is 1600 Hz then frequency of 3^{rd} overtone is..........

(vi) In OOP if the frequency of 7^{th} overtone is 1600 Hz then fundamental frequency of same length COP is..........

(vii) In OOP if the frequency of 7^{th} overtone is 800 Hz then frequency of OOP corresponding to first overtone is..........

(viii) Length of OOP is 44 cm speed of sound 340 m/s fundamental frequency is 340 Hz then value of end correction is..........

(ix) Length of OOP is 38 cm speed of sound 340 m/s fundamental frequency is 340 Hz then value of radius of pipe is..........

13. Fill in the blanks for Resonance tube.

(i) In resonance tube first resonanting length is 25 cm then its second resonanting length will be (if $e = 0$) ....

(ii) In resonance tube first resonanting length is 25 cm then its second resonanting length will be (if $e \neq 0$) ....

(iii) Length of resonance tube is 140 cm. How many resonance are possible for wave have wave length 40 cm. ....

(iv) Length of resonance tube is 150 cm. How many resonance are possible for wave having frequency 400 Hz and speed of sound 320 m/s. ....

(v) Length of resonance tube is 150 cm. Maximum level of liquid , in resonance candition for wave having wavelength 80 cm. is.........

(vi) Length of resonance tube is 150 cm Minimum level of liquid , in resonance candition for wave having wavelength 80 cm. is.........

(vii) In resonance tube first resonanting length is 17 cm, second resonanting length is 55 cm then wavelength of wave is..........

(viii) In resonance tube first resonanting length is 17 cm, second resonanting length is 55 cm then radius of tube is..........

7. DOPPLER EFFECT IN SOUND WAVES AND LIGHT WAVES

7.1 Acoustic Doppler Effect (Doppler Effect for Sound Waves)

The apparent change in frequency or pitch due to relative motion of source and observer along the line of sight is called Doppler Effect.

While deriving these expressions, we make the following assumptions :

(i) The velocity of the source, the observer and the medium are along the line joining the positions of the source and the observer.

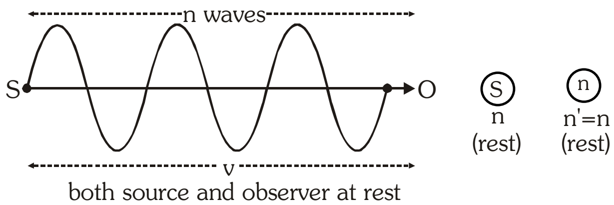
(ii) The velocity of the source and the observer is less than velocity of sound.

Doppler effect takes place both in sound and light. In sound it depends on whether the source or observer or both are in motion while in light it depends on whether the distance between source and observer is increasing or decreasing.

NOTATIONS

$n \rightarrow$ actual frequency	$n' \rightarrow$ observed frequency (apparent frequency)
$\lambda \rightarrow$ actual wave length	$\lambda' \rightarrow$ observed (apparent) wave length
$v \rightarrow$ velocity of sound	$v_s \rightarrow$ velocity of source
$v_0 \rightarrow$ velocity of observer	$v_w \rightarrow$ wind velocity

Case I : Source in motion, observer at rest, medium at rest :

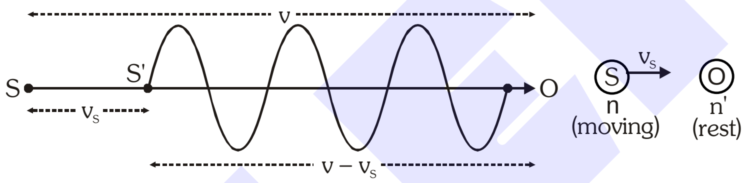


Suppose the source S and observer O are separated by distance v. Where v is the velocity of sound. Let n be the frequency of sound emitted by the source. Then n waves will be emitted by the source in one second. These n waves will be accommodated in distance v.

So, wave length $\lambda = \dfrac{\text{total distance}}{\text{total number of waves}} = \dfrac{v}{n}$

(1) Source moving towards stationary observer :

Let the source start moving towards the observer with velocity v_s. After one second, the n waves will be crowded in distance $(v - v_s)$. Now the observer shall feel that he is listening to sound of wavelength λ' and frequency n'



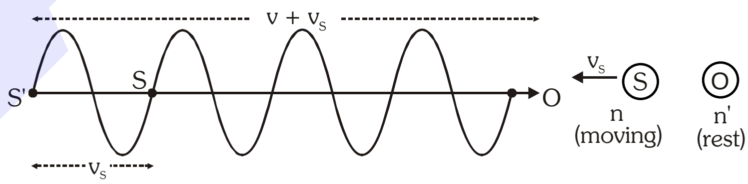
Now apparent wavelength $\lambda' = \dfrac{\text{total distance}}{\text{total number of waves}} = \dfrac{v - v_s}{n}$

and changed frequency, $n' = \dfrac{v}{\lambda'} = \dfrac{v}{\left(\dfrac{v - v_s}{n}\right)} = n\left(\dfrac{v}{v - v_s}\right)$

So, as the source of sound approaches the observer the apparent frequency n' becomes greater than the true frequency n.

(2) When source move away from stationary observer :-

For this situation n waves will be crowded in distance $v + v_s$.



So, apparent wavelength $\lambda' = \dfrac{v + v_s}{n}$

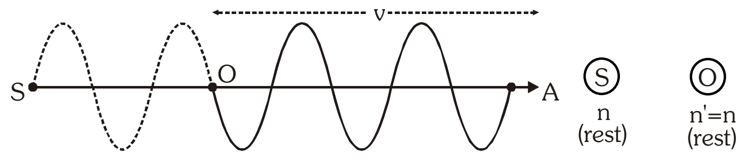
and apparent frequency, $n' = \dfrac{v}{\lambda'} = \dfrac{v}{\left(\dfrac{v + v_s}{n}\right)} = n\left(\dfrac{v}{v + v_s}\right)$

So, n' becomes less than n. (n' < n)

Case II :

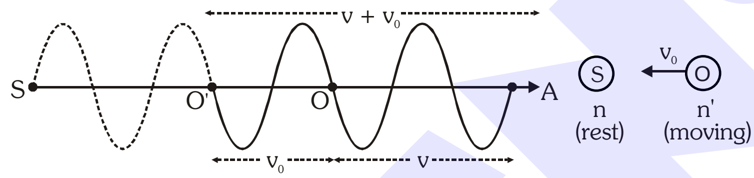
Observer in motion, source at rest, medium at rest :-

Let the source (S) and observer (O) are in rest at their respective places. Then n waves given by source 'S' would be crossing observer 'O' in one second and fill the space OA (=v)



both source S and O at rest

(1) Observer move towards stationary source :-



When observer 'O' moves towards 'S' with velocity v_o, it will cover v_o distance in one second. So the observer has received not only the n waves occupying OA but also received additional number of Δn waves occupying the distance OO' (= v_o).

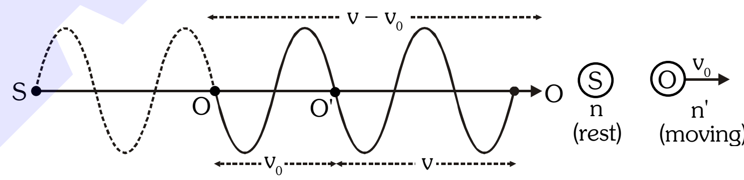
So, total waves received by observer in one second

i.e., apparent frequency (n') = Actual waves (n) + Additional waves (Δn)

$$n' = \frac{v}{\lambda} + \frac{v_o}{\lambda} = \frac{v + v_o}{(v/n)} = n\left(\frac{v + v_o}{v}\right) \quad \left(\because \quad \lambda = \frac{v}{n}\right) \qquad (\text{so, } n' > n)$$

(2) Observer move away from stationary source :-

For this situation n waves will be crowded in distance v – v_o.



When observer move away from source with v_o velocity then he will get Δn waves less than real number of waves. So, total number of waves received by observer i.e.,

Apparent frequency (n') = Actual waves (n) – reduction in number of waves (Δn)

$$n' = \frac{v}{\lambda} - \frac{v_o}{\lambda} = \frac{v - v_o}{\lambda} = \frac{v - v_o}{(v/n)} = \left(\frac{v - v_o}{v}\right)n \quad \left(\because \quad \lambda = \frac{v}{n}\right) \qquad (\text{so } n' < n)$$

Case III :

Effect of motion of medium :-

General formula for doppler effect = $n' = n\left[\frac{v \pm v_o}{v \mp v_s}\right]$ ...(i)

If medium (air) is also moving with v_m velocity in direction of source and observer. Then velocity of sound relative to observer will be $v \pm v_m$ (–ve sign, if v_m is opposite to sound velocity). So,

$$n' = n\left(\frac{v \pm v_m \pm v_o}{v \pm v_m \mp v_s}\right) \text{ [On replacing v by } v \pm v_m \text{ in equation (i)]}$$

Note :- When both 'S' and 'O' are in rest (i.e. $v_s = v_o = 0$) then there is no effect on frequency due to motion of air.

SPECIAL CASES

Case-I If medium moves in a direction opposite to the direction of propagation of sound, then

$$n' = \left(\frac{v - v_m \pm v_O}{v - v_m \pm v_S}\right)n$$

Case-II Source in motion towards the observer. Both medium and observer are at rest.

$$n' = \left(\frac{v}{v - v_S}\right)n \text{ ; Clearly } n' > n$$

So, when a source of sound approaches a stationary observer, the apparent frequency is more than the actual frequency.

Case-III Source in motion away from the observer. Both medium and observer are at rest.

$$n' = \left(\frac{v}{v + v_S}\right)n \text{ ; Clearly } n' < n$$

So, when a source of sound moves away from a stationary observer, the apparent frequency is less than actual frequency.

Case-IV Observer in motion towards the source. Both medium and source are at rest.

$$n' = \left(\frac{v + v_O}{v}\right)n \text{ ; Clearly } n' > n$$

So, when observer is in motion towards the source, the apparent frequency is more than the actual frequency.

Case-V Observer in motion away from the source. Both medium and source are at rest.

$$n' = \left(\frac{v - v_O}{v}\right)n \text{ ; Clearly } n' < n$$

So, when observer is in motion away from the source, the apparent frequency is less than the actual frequency.

Case-VI Both source and observer are moving away from each other. Medium at rest.

$$n' = \left(\frac{v - v_O}{v + v_S}\right)n \text{ ; Clearly } n' > n$$

7.2 Doppler's effect in reflection of sound (echo)

When the sound is reflected from the reflector the observer receives two notes one directly from the source and other from the reflector. If the two frequencies are different then superposition of these waves result in beats and by the beat frequency we can calculate speed of the source.

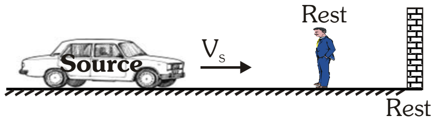
Case-I

- When source moves towards stationary target.

  n'_D = direct apparent frequency

  n' = apparent frequency at reflector

  n'_R = reflected apparent frequency

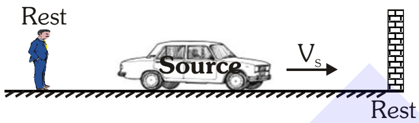


$$n'_D = \left(\frac{V}{V-V_S}\right)n$$

$$n' = \left(\frac{V}{V-V_S}\right)n$$

$$n'_R = n'$$

$$b = \Delta n = n'_R - n'_D = 0$$

- 

$$n'_D = \left(\frac{V}{V+V_S}\right)n$$

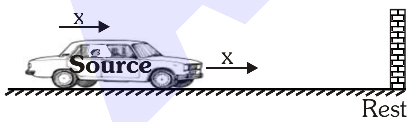
$$n' = \left(\frac{V}{V-V_S}\right)n$$

$$b = \Delta n = \left(\frac{V}{V-V_S} - \frac{V}{V+V_S}\right)n = \frac{2V_S V n}{V^2 - V_S^2} \cong \left(\frac{2V_S}{V}\right)n$$

Note Stationary target behave as an observer for incident sound and behave as a source for reflected sound.

Case-II

When source and observer both move towards statonary target.



$$n'_D = n,$$

$$n' = \left(\frac{V}{V-x}\right)n$$

$$n'_R = \left(\frac{V+x}{V}\right)n' = \left[\frac{V+x}{V}\right]\left[\frac{V}{V-x}\right]n$$

$$\boxed{n'_R = \left(\frac{V+x}{V-x}\right)n}$$

$$b = n'_R - n'_D = \left(\frac{V+x}{V-x}\right)n - n \Rightarrow \boxed{b = \frac{2x}{V-x}n}$$

CONDITIONS WHEN DOPPLER'S EFFECT IS NOT OBSERVED FOR SOUND WAVES

1. When the source of sound and observer both are at rest then doppler effect is not observed.
2. When the source and observer both are moving with same velocity in same direction.
3. When the source and observer are moving mutually in perpendicular directions.
4. When the medium only is moving.
5. When the distance between the source and observer is constant.

7.3 Doppler effect in light :

Doppler effect holds also for EM waves. As speed of light is independent of relative motion between source and observer, the formulae are different from that of sound. Here when either source or observer (detector) or both are in motion, only two cases are possible (approach or recession)

Doppler's effect in light is symmetrical (unlike sound). It means the observer moving towards the source at a particular speed will produce the same frequency change as the source moving towards the observer at the same speed.

In case of approach $n' = n\left(\dfrac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{\frac{1}{2}}$ and $\lambda' = \lambda\left(\dfrac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{\frac{1}{2}}$

In case of recession $n' = n\left(\dfrac{1-\frac{v}{c}}{1+\frac{v}{c}}\right)^{\frac{1}{2}}$ and $\lambda' = \lambda\left(\dfrac{1+\frac{v}{c}}{1-\frac{v}{c}}\right)^{\frac{1}{2}}$

If v << c then in case of approach $n' \approx n\left(1+\frac{v}{c}\right)$ in case of recession $n' \approx n\left(1-\frac{v}{c}\right)$

So at low speeds doppler effect in light and sound is governed by the same formula.

DOPPLER'S SHIFT :

When radiation coming from distant stars are analysed by radio telescopes and compared with their natural radiation wavelength focussed on mean wavelength on a visible spectrum, it is observed that coming radiation has a shift towards red or violet end.

Red shift $\Delta\lambda = \lambda' - \lambda = \left(\frac{v}{c}\right)\lambda \quad \Rightarrow \quad \Delta\lambda = \frac{v}{c}\lambda$

Violet shift (or blue shift) $\Delta\lambda = \lambda' - \lambda = -\left(\frac{v}{c}\right)\lambda \quad \Rightarrow \quad \Delta\lambda = -\frac{v}{c}\lambda$

In case of approach frequency increases while wavelength decreases i.e. shift $\Delta\lambda$ is towards violet end of the spectrum while in case of recession frequency decreases and wavelength increases i.e. shift $\Delta\lambda$ is towards red end.

GOLDEN KEY POINTS

SONAR System	RADAR System
To find out velocity and position of submarines If V_S = vel. of submarine w.r.t. SONAR system V = Vel. of sound n_0 = Original frequnecy Δn = change in frequency then $\Delta n = \frac{2V_S}{V}(n_0)$	To find out velocity and position of planes If V_R = vel. of fighter plane w.r.t. RADAR System ν_0 = Material frequency of Radio wave transmitted from RADAR $\Delta\nu$ = Change in frequency in light $\Delta\nu = \frac{2V_R}{C}(\nu_o)$

Illustrations

Illustration 37.

Two tuning forks A and B lying on opposite sides of observer 'O' and of natural frequency 85 Hz move with velocity 10 m/s relative to stationary observer O. Fork A moves away from the observer while the fork B moves towards him. A wind with a speed 10 m/s is blowing in the direction of motion of fork A. Find the beat frequency measured by the observer in Hz. [Take speed of sound in air as 340 m/s]

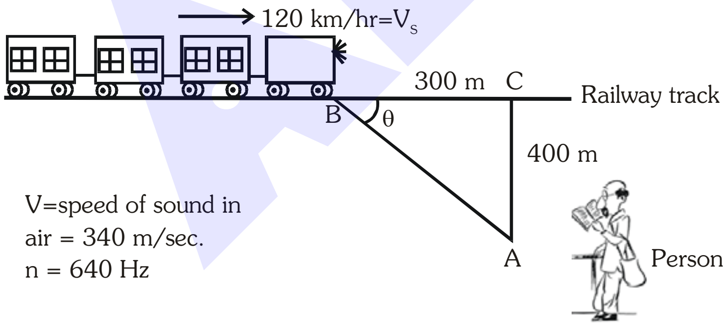
(1) 5 (2) 6 (3) 7 (4) 8

Solution **Ans. (1)**

$$f_{\text{observer for source 'A'}} = f_0\left[\frac{v_{\text{sound}} - v_{\text{medium}}}{v_{\text{sound}} - v_{\text{medium}} + v_{\text{source}}}\right] = \frac{33}{34}f_0\ ;\ f_{\text{observer for source 'B'}} = f_0\left[\frac{v_{\text{sound}} + v_{\text{medium}}}{v_{\text{sound}} + v_{\text{medium}} - v_{\text{source}}}\right] = \frac{35}{34}f_0$$

$\therefore$ Beat frequency = $f_1 - f_2 = \left(\frac{35-33}{34}\right)f_0 = 5$

Illustration 38.



If engine of train produce horn at B point then find apparent frequency observed by observer at A point.

Solution

$$n' = \left(\frac{v}{v - v_s\cos\theta}\right)n\ ;\ \text{Direction in AB velocity} = v_s\cos\theta\left[120\times\frac{5}{18}\right]\times\cos\theta = 120\times\frac{5}{18}\times\frac{3}{5} = 20\ \text{m/sec.}$$

$$\Rightarrow n' = \left(\frac{340}{340-20}\right)\times 640 = 680\ \text{Hz.}$$

Illustration 39.

A train, standing in a station-yard, blows a whistle of frequency 400 Hz in still air. The wind starts blowing in the direction from the yard to the station with at a speed of 10 ms^{-1}. What are the frequency, wavelength, and speed of sound for an observer standing on the station's platform ? Is the situation exactly identical to the case when the air is still and the observer runs towards the yard at a speed of 10 ms^{-1} ? The speed of sound in still air can be taken as 340 ms^{-1}.

Solution

Here source and observer both are at rest when wind starts blowing from source to observer then effecting velocity of sound, $v_{sound} = v + v_m = 340 + 10 = 350\ ms^{-1}$

Since there is no relative motion between source and observer therefore frequency remains the same i.e. 400 Hz.

$$\text{Now wave length } \lambda = \frac{v_{sound}}{n} = \frac{350}{400} = 0.875 \text{ m}$$

The two situations are not exactly identical, because in second case medium is at rest, while observer in motion.

Illustration 40.

A SONAR system fixed in a submarine operates at a frequency 40.0 kHz. An enemy submarine moves towards the SONAR with a speed of 360 km h^{-1}. What is the frequency of sound reflected by the submarine ? Take the speed of sound in water to be 1450 ms^{-1}.

Solution

As the sound is observed by enemy subsmarine.

Here observer (enemy submarine) is moving towards the source (SONAR)

$$\therefore \quad \text{Apparent frequency } n' = \left(\frac{v+v_0}{v-v_s}\right)n = \left(\frac{1450+100}{1450}\right)\times 40\times 10^3 = \frac{1550}{1450}\times 40\times 10^3 \text{ Hz}$$

After the sound is reflected, enemy submarine acts. as a source of frequency n'. This source moves with a speed of 100 ms^{-1} towards the observer (SONAR)

$\therefore$ Apparent frequency of sound reflected by the enemy submarine

$$n'' = \left(\frac{v-v_0}{v-v_s}\right)n' = \left(\frac{1450-0}{1450-100}\right)\times\left(\frac{1550}{1450}\times 40\times 10^3\right) = 45.93 \text{ kHz}$$

Illustration 41.

Two trains travelling in opposite directions at 126 km/hr each, cross each other while one of them is whistling. If the frequency of the note is 2.22 kHz find the apparent frequency as heard by an observer in the other train :

(a) Before the trains cross each other, (b) After the trains have crossed each other. (v_{sound} = 335 m/sec)

Solution

$$\text{Here } v_1 = 126 \times \frac{5}{18} = 35 \text{ m/s}$$

(i) In this situation $\circ\!\longrightarrow v_1 \qquad v_1 \longleftarrow\!\circ$

Observed freq $\quad n' = \left(\frac{v+v_1}{v-v_1}\right)\times n = \left(\frac{335+35}{335-35}\right)\times 2220 = 2738 \text{ Hz}$

(ii) In this situation $v_1 \longleftarrow\!\circ \qquad \circ\!\longrightarrow v_1$

Observed freq $\quad n' = \left(\frac{v-v_1}{v+v_1}\right)\times n = \left(\frac{335-35}{335+35}\right)\times 2220 = 1800 \text{ Hz}$

Illustration 42.

A star which is emitting radiation at a wavelength of 5000 Å, is approaching the earth with a velocity of $1{\cdot}5 \times 10^3$ m/s. Calculate the change in wavelength of the radiation as received by the earth.

Solution

$$\Delta\lambda = \frac{v}{c}\lambda = \frac{1.5\times10^3}{3\times10^8}\times5000 = 0.025\text{Å}$$

Illustration 43.

A person going away from a factory on his scooter at a speed of 36 km/hr listens to the siren of the factory. If the actual frequency of the siren is 700Hz and a wind is blowing along the direction of the scooter at 36 km/hr, find the observed frequency heard by the person. (Given speed of sound = 340 m/s)

Solution

In this situation $n' = \left(\frac{v+w-v_0}{v+w}\right)n$

where $w = v_0 = 36$ km/hr = 10m/s

$$n' = \left(\frac{340+10-10}{340+10}\right)\times700 = \frac{340}{350}\times700 = 680\text{Hz}$$

BEGINNER'S BOX-7

1. A bat is flitting about in a cave, navigating via ultrasonic bleeps. Assume that the sound emission frequency of the bat is 40 kHz. During one fast swoop directly toward a flat wall surface, the bat is moving at 0.03 times the speed of sound in air. What frequency does the bat hear reflected off the wall ?

2. A railway engine moving with a speed of 60 m/s passes a stationary listener. The real frequency of its whistle is 400 Hz. Calculate the apparent frequency heard by the listener.

(a) When the engine is approching the listener.

(b) When the engine is moving away from the listener. (velocity of sound = 340 m/s)

3. A stationary source emits sound of frequency 1200 Hz. If wind blows at the speed of 0.1v, deduce

(a) the change in the frequency for a stationary observer on the wind side of the source.

(b) Report the calculations for the case when there is no wind but the observer moves at 0.1v speed towards the source. (given : velocity of sound = v)

4. A car has two horns having a difference in frequency of 180 Hz. The car is approaching a stationary observer with a speed of 60 ms^{-1}. Calculate the difference in frequencies of the notes as heard by the observer, if velocity of sound in air is 330 ms^{-1}.

5. When both source and observer approach each other with a speed equal to the half the speed of sound, then determine the percentage change in frequency of sound as detected by the listener.

ANSWER KEY

BEGINNER'S BOX-1

1. 10 cm. **2.** 8.707 s

BEGINNER'S BOX-2

1. 200 cm/s **2.** $\frac{1}{100}$ s, 20 m/s **3.** 0.02 s

4. 220 N **5.** 0.5 s

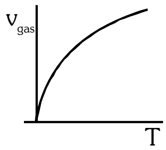
6. **(a)** 2.21 m/sec; **(b)** 1 sec.

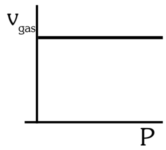
BEGINNER'S BOX-3

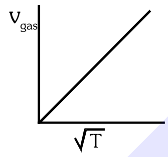
1. 2 s **2.** 2000 ms^{-1} **3.** 30dB

4. After rain fall, the humidity of air increases. This lowers the density of air hence there is increase in velocity of sound $\left(\because v \propto \frac{1}{\sqrt{\rho}}\right)$

5. We know $v_{gas} = \sqrt{\frac{\gamma RT}{M_W}}$

(a) 

(b) 

(c) 

6. Due to this loudness of sound increases.

7. Velocity of sound wave in air $v_{air} = \sqrt{\frac{\gamma RT}{M_w}} \Rightarrow v_{air} \propto \sqrt{T}$

so sound travels faster in warm air than in cool air.

8. Sound wave require material medium for their propagation.

9. $\sqrt{27 \cdot 7} \times 100$ m/s

10. (a) Independent of pressure.

(b) Increases as $\sqrt{T}$ (c) Increase

BEGINNER'S BOX-4

1. 0°, 90°, 120°, 180° **2.** 64 : 1 **3.** 81 : 1

4. 1375 Hz **5.** 1000 Hz **6.** 296 Hz

7. 640 Hz

8. (a) 200.5, (b) 1, (c) nearly 9/16

9. 60, 72, 78

10. (a) $\frac{1}{4}$ s, (b) $\frac{1}{8}$ s, (c) 12 beats

11. 1200 Hz, 2000 Hz, 2800 Hz, 3600 Hz, 4400 Hz

12. n_1 = 50 Hz n_{11} = 100 Hz

n_{17} = 130 Hz n_{27} = 180 Hz

n_{33} = 210 Hz n_{51} = 300 Hz

BEGINNER'S BOX-5

1. (i) $y = -0.06 \sin 8\pi [t + (x/20)]$,

(ii) $y = -0.06 \sin 8\pi t$

2. (a) The function represents a stationary wave.

(b) 3m, 60 Hz, 180 ms^{-1}

(c) 648 N

3. (i) $y = -0.012 \sin 2\pi (330 t + x)$

(ii) $y = 0.015 \sin 2\pi (330 t + x)$

4. $\pm 0.04 \sin 2\pi (200 t + x)$; + = free, – = rigid

5. $(m-1)\frac{\lambda}{2}$ **6.** 100 Hz

7. $\frac{55}{3}$ Hz. **8.** 4 kg wt.

9. (i) $\frac{1}{\sqrt{2}}$ times (ii) No change

(iii) $\sqrt{2}$ times (iv) 100 m/s

(v) 200 m/s (vi) 2 times

BEGINNER'S BOX-6

1. Open organ pipe

2. For closed organ pipe $f_0 = \frac{v}{4\ell} = \frac{340}{4 \times 20 \times 10^{-2}}$

= 425 Hz $\Rightarrow$ first harmonic

If pipe is open, its fundamental frequency = $\frac{v}{2\ell}$

= 850 Hz so it will not resonant with the given source

3. $\frac{1}{2}$ **4.** $\frac{7}{8}$ **5.** 0.25m

6. 2700 **7.** 20, 40 **8.** 244

9. 122,120 **10.** (a) 82.5 Hz; (b) 81.675 N

11. (i) forth order; (ii) 4; (iii) 110 cm, 20 cm

12. (i) 40 Hz (ii) 400 Hz (iii) 100 Hz (iv) 280 Hz

(v) 800 Hz (vi) 100 Hz (vii) 200 Hz

(viii) 3 cm (ix) 10 cm

13. (i) 75 cm

(ii) Slightly more than 75 cm

(iii) 7 (iv) 4

(v) 130 cm (vi) 10 cm

(vii) 76 cm (viii) 3.33 cm

BEGINNER'S BOX-7

1. 42.47 kHz

2. (a) 485.7 Hz; (b) 340 Hz

3. (a) no change in frequency; (b) 1320 Hz

4. 220 Hz

5. 200%