TG: @Chalnaayaaar

PRE-MEDICAL

# PHYSICS

ENTHUSIAST | LEADER | ACHIEVER

**STUDY MATERIAL**

## Kinematics (Motion along a straight line and motion in a plane)

ENGLISH MEDIUM

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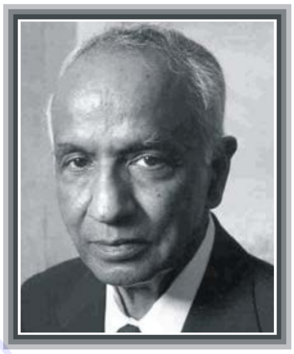
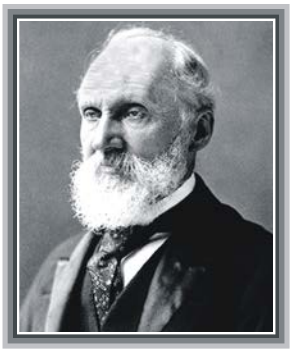
### Subrahmanyan Chandrasekhar



Subrahmanyan Chandrasekhar was born on October 19, 1910 in Lahore. Subrahmanyan Chandrasekhar was one of the greatest scientists of the 20th century. He did commendable work in astrophysics, physics and applied mathematics. Chandrasekhar was awarded the Nobel Prize in Physics in 1983. In July 1930, he was awarded a Government of India scholarship for graduate studies in Cambridge, England. In October 1933, Chandrasekhar was elected to a Prize fellowship at Trinity College for the period 1933-37. Subrahmanyan Chandrasekhar, was offered a position as a Research Associate at the University of Chicago and remained there ever since. He showed that there is a maximum mass which can be supported against gravity by pressure made up of electrons and atomic nuclei. The value of this limit is about 1.44 times a solar mass. The formulation of the Chandrasekhar Limit led to the discovery of neutron starts and black holes. Apart from discovery of Chandrasekhar Limit, Major work done by Subrahmanyan Chandrasekhar includes : theory of brownian motion (1938-1943) ; theory of the illumination and the polarization of the sunlit sky (1943-1950); theory of the illumination and the polarization of the sunlit sky (1943-1950). He died on August 21, 1995.

### Lord Kelvin (William Thomson) (1824-1907)



Lord Kelvin (William Thomson) born in Belfast, Ireland in 1824, is among the foremost British scientists of the nineteenth century. Thomson played a key role in the development of the law of conservation of energy suggested by the work of James Joule (1818-1889), Julius Mayer (1814- 1878) and Hermann Helmholtz (1821-1894). He collaborated with Joule on the so-called Joule-Thomson effect : cooling of a gas when it expands into vacuum. He introduced the notion of the absolute zero of temperature and proposed the absolute temperature scale, now called the Kelvin scale in his honour. From the work of Sadi Carnot (1796-1832), Thomson arrived at a form of the Second Law of Thermodynamics. Thomson was a versatile physicist, with notable contributions to electromagnetic theory and hydrodynamics.

# KINEMATICS

## KINEMATICS

Study of motion of objects without taking into account the factors which cause the motion (i.e. nature of force).

## 1. FRAME OF REFERENCE

Motion of a body can be observed only if it changes its position with respect to some other body. Therefore, for motion to be observed there must be a body, which is changing its position with respect to another body and a person who is observing motion. The person observing motion is known as observer. The observer for the purpose of investigation must have its own clock to measure time and a point in the space attached with the other body as origin and a set of coordinate axes. These two things (the time measured by the clock and the coordinate system) are collectively known as reference frame.

In this way, motion of the moving body is expressed in terms of its position coordinates changing with time.

## 2. MOTION & REST

If a body changes its position with time, it is said to be moving otherwise it is at rest. Motion/rest is always relative to the observer.

Motion/rest is a combined property of the object under study and the observer. There is no meaning of rest or motion without the observer or frame of reference.

- To locate the position of a particle we need a reference frame. A commonly used reference frame is cartesian coordinate system or x-y-z coordinate system.

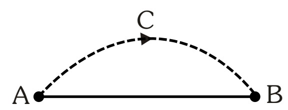
  The coordinates (x, y, z) of the particle specify the position of the particle with respect to origin of that frame. If all the three coordinates of the particle remain unchanged as time passes it means the particle is at rest w.r.t. this frame.
- If only one coordinate changes with time, motion is one dimensional (1 - D) or straight line motion.

  If only two coordinates change with time, motion is two dimensional (2 - D) or motion in a plane. If all three coordinates change with time, motion is three dimensional (3 - D) or motion in space.
- The reference frame is chosen according to problem.
- If frame is not mentioned, then ground is taken as reference frame.

## 3. DISTANCE & DISPLACEMENT

### Distance

Distance is total length of path covered by the particle, in definite time interval.
Let a body moves from A to B via C. The length of path ACB is called the distance travelled by the body.

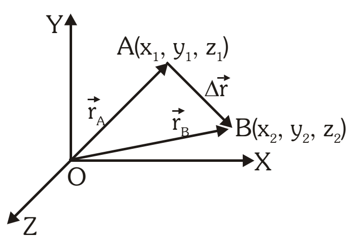



But overall, body is displaced from A to B. A vector from A to B, i.e. $\vec{AB}$ is its displacement vector or displacement that is the minimum distance and directed from initial position to final position.

### Displacement in terms of position vector

Let a body be displaced from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$

then its displacement is given by vector AB.




From $\Delta OAB$ $\quad \vec{r}_A + \Delta\vec{r} = \vec{r}_B \qquad$ or $\qquad \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$\because \qquad \vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\therefore \qquad \Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$ or $\Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

## GOLDEN KEY POINTS

- Distance is a scalar while displacement is a vector.
- Distance depends on path while displacement is independent of path but depends only on final and initial positions.
- For a moving body, distance cannot have zero or negative values but displacement may be positive, negative or zero.
- Infinite distances are possible between two fixed points because infinite paths are possible between two fixed points.
- Only single value of displacement is possible between two fixed points.
- If motion is in straight line without change in direction then

  distance = | displacement | = magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance.

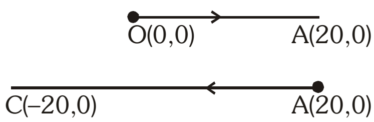
  i.e., distance $\geq$ | displacement |

## Illustrations

**Illustration 1.**

A particle starts from the origin, goes along the X-axis upto the point (20m, 0) and then returns along the same line to the point (–20m, 0). Find the distance and displacement of the particle during the trip.

**Solution**




Distance = | OA | + | AC |

= 20 + 40 = 60m

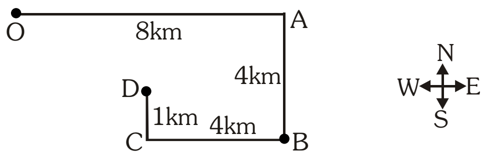
Displacement = OA + AC

$= 20\hat{i} + (-40\hat{i}) = (-20\hat{i})\text{m}$

**Illustration 2.**

A car moves from O to D along the path OABCD shown in fig.

What is distance travelled and its net displacement ?




**Solution**

Distance $= \left|\vec{OA}\right| + \left|\vec{AB}\right| + \left|\vec{BC}\right| + \left|\vec{CD}\right|$

= 8 + 4 + 4 + 1 = 17 km

Displacement $= \vec{OA} + \vec{AB} + \vec{BC} + \vec{CD}$

$= 8\hat{i} + (-4\hat{j}) + (-4\hat{i}) + \hat{j} = 4\hat{i} - 3\hat{j}$

$\Rightarrow$ | displacement | $= \sqrt{(4)^2 + (3)^2} = 5$

So, Displacement = 5 km, 37° S of E

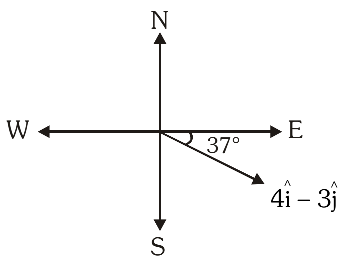
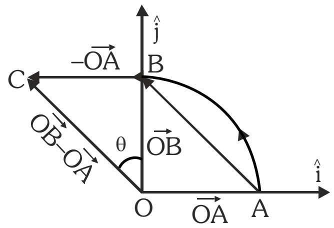


**Illustration 3.**

A particle goes along a quadrant of a circle of radius 10m from A to B as shown in fig. Find the magnitude of displacement and distance along the path AB, and angle between displacement vector and x-axis ?




**Solution**

Displacement $\vec{AB} = \vec{OB} - \vec{OA} = (10\hat{j} - 10\hat{i})$m

$|\vec{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$m

From ΔOBC $\quad \tan\theta = \dfrac{OA}{OB} = \dfrac{10}{10} = 1 \quad \Rightarrow \quad \theta = 45°$

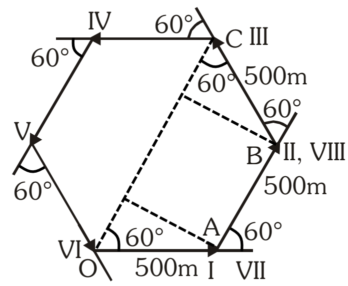
Angle between displacement vector $\vec{OC}$ and x-axis = 90° + 45° = 135°

Distance of path AB = $\dfrac{1}{4}$ (circumference) = $\dfrac{1}{4}$ (2πR) m = (5π) m

**Illustration 4.**

On an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

**Solution**




At III turn

$$|\text{Displacement}| = |\vec{OA} + \vec{AB} + \vec{BC}| = |\vec{OC}|$$
$$= 500 \cos 60° + 500 + 500 \cos 60°$$
$$= 500 \times \frac{1}{2} + 500 + 500 \times \frac{1}{2} = 1000 \text{ m}$$

So $|\text{Displacement}|$ = 1000 m from O to C

Distance = 500 + 500 + 500 = 1500 m $\quad \therefore \quad \dfrac{|\text{Displacement}|}{\text{Distance}} = \dfrac{1000}{1500} = \dfrac{2}{3}$

At VI turn

$\because$ initial and final positions are same so $|\text{displacement}|$ = 0 and distance = 500×6 = 3000 m

$$\therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{0}{3000} = 0$$

At VIII turn

$$|\text{Displacement}| = 2(500)\cos\left(\frac{60°}{2}\right) = 1000 \times \cos 30° = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3} \text{ m}$$

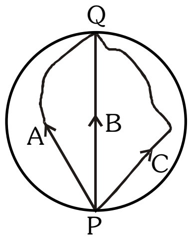
Distance = 500 × 8 = 4000 m

$$\therefore \frac{|\text{Displacement}|}{\text{Distance}} = \frac{500\sqrt{3}}{4000} = \frac{\sqrt{3}}{8}$$

## BEGINNER'S BOX-1

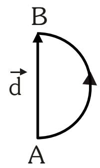
**1.** A particle moves on a circular path of radius 'r', It completes one revolution in 40 s. Calculate distance and displacement in 2 min 20 s.

**2.** Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement for each ? For which girl is this equal to the actual length of path skate?




**3.** A man moves 4 m along east direction, then 3m along north direction, after that he climbs up a pole to a height 12m. Find the distance covered by him and his displacement.

**4.** A person moves on a semicircular track of radius 40 m. If he starts at one end of the track and reaches the other end, find the distance covered and magnitude of displacement of the person.




**5.** A man has to go 50m due north, 40m due east and 20m due south to reach a cafe from his home. (A) What distance he has to walk to reach the cafe ? (B) What is his displacement from his home to the cafe?

## 4. SPEED & VELOCITY

### 4.1 Speed

The rate at which distance is covered with respect to time is called speed. It is a scalar quantity

Dimension : $[M^0L^1T^{-1}]$

Unit : m/s (S.I.), cm/s (C.G.S.)

**Note :** For a moving particle speed can never be negative or zero, it is always positive.

**Uniform speed**

When a particle covers equal distances in equal intervals of time, (no matter how small the intervals are) then it is said to be moving with uniform speed.

$$\text{Uniform speed} = \frac{\text{Distance}}{\text{Time}}$$

**Non-uniform (variable) speed**

In non-uniform speed particle covers unequal distances in equal intervals of time.

**Average speed :** The average speed of a particle for a given 'interval of time' is defined as the ratio of total distance travelled to the time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Time taken}} \quad \text{i.e. } v_{av} = \frac{\Delta s}{\Delta t}$$

## GOLDEN KEY POINTS

- When a particle moves with different uniform speeds $v_1, v_2, v_3 ......v_n$ in different time intervals $t_1, t_2, t_3, ....t_n$ respectively, its average speed over the total time of journey is given as

$$v_{av}=\frac{\text{Total distance covered}}{\text{Total time elapsed}}=\frac{s_1+s_2+s_3+......+s_n}{t_1+t_2+t_3+......+t_n}=\frac{v_1t_1+v_2t_2+v_3t_3+......}{t_1+t_2+t_3+......}$$

If $t_1=t_2=t_3=......=t_n$ then

$$v_{av}=\frac{v_1+v_2+v_3+......+v_n}{n}$$ (Arithmetic mean of speeds)

- When a particle describes different distances $s_1, s_2, s_3, ........ s_n$ with speeds $v_1, v_2, v_3 ........ v_n$ respectively then the average speed of particle over the total distance will be given as

$$v_{av}=\frac{\text{Total distance covered}}{\text{Total time elapsed}}=\frac{s_1+s_2+s_3+......+s_n}{t_1+t_2+t_3+......+t_n}=\frac{s_1+s_2+s_3+......+s_n}{\frac{s_1}{v_1}+\frac{s_2}{v_2}+\frac{s_3}{v_3}+......+\frac{s_n}{v_n}}$$

If $s_1=s_2=s_3=......=s_n$ then

$$v_{av}=\frac{n}{\frac{1}{v_1}+\frac{1}{v_2}+\frac{1}{v_3}+......+\frac{1}{v_n}}$$ (Harmonic mean of speeds)

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**Instantaneous speed**

It is the speed of a particle at a particular instant of time.

Instantaneous speed $v=\lim\limits_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=\frac{ds}{dt}$

### 4.2 Velocity

The rate of change of position is called velocity.

It is a vector quantity

Dimension : $[M^0L^1T^{-1}]$

Unit : m/s (S.I.), cm/s (C.G.S.)

## GOLDEN KEY POINTS

- Velocity may be positive, negative or zero.
- Direction of average velocity is always in the direction of change in position.
- Speedometer measures the instantaneous speed of a vehicle.

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**Uniform velocity**

A particle is said to have uniform velocity, if magnitude as well as direction of its velocity remain same. This is possible only when it moves in a straight line without reversing its direction.

**Non-uniform velocity**

A particle is said to have non-uniform velocity, if both either magnitude or direction of velocity change.

**Average velocity**

It is defined as the ratio of displacement to time taken by the body

Average velocity $=\frac{\text{Displacement}}{\text{Time taken}}$; $\quad \vec{v}_{av}=\frac{\Delta\vec{r}}{\Delta t}$

Its direction is along the displacement.

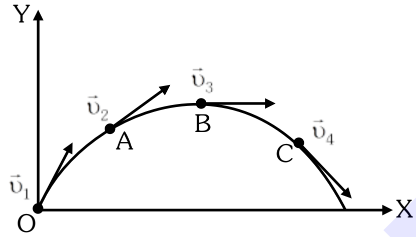
**Instantaneous velocity**

It is the velocity of a particle at a particular instant of time.

Instantaneous velocity $\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$

## GOLDEN KEY POINTS

- Average speed ≥ | Average velocity |
- The direction of instantaneous velocity is always tangential to the path followed by the particle.




- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.
- A particle may have constant speed but variable velocity.

  *Example* : When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.
- When particle moves with uniform velocity then its instantaneous speed ,magnitude of instantaneous velocity, average speed and magnitude of average velocity are all equal.

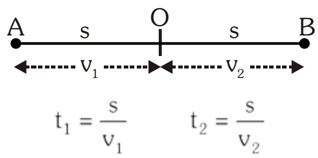
## Illustrations

**Illustration 5.**

If a particle travels the first half distance with speed $v_1$ and second half distance with speed $v_2$.Find its average speed during the journey.

**Solution**

$$v_{av} = \frac{s+s}{t_1+t_2} = \frac{2s}{\frac{s}{v_1}+\frac{s}{v_2}} = \frac{2v_1v_2}{v_1+v_2}$$




**Note :-** Here $v_{av}$ is the harmonic mean of two speeds.

**Illustration 6.**

If a particle travels with speed $v_1$ during first half time interval and with speed $v_2$ during second half time interval. Find its average speed during its journey.

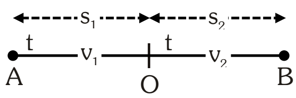
**Solution**

$s_1 = v_1t$ and $s_2 = v_2t$

Total distance = $s_1 + s_2 = (v_1 + v_2)t$

total time = t + t = 2t

then $v_{av} = \frac{s_1+s_2}{t+t} = \frac{(v_1+v_2)t}{2t} = \frac{v_1+v_2}{2}$




**Note :-** here $v_{av}$ is arithmetic mean of two speeds.

**Illustration 7.**

A car moves with a velocity 2.24 km/h in first minute, with 3.60 km/h in the second minute and with 5.18 km/h in the third minute. Calculate the average velocity in these three minutes.

**Solution**

Distance travelled in first minute $s_1 = v_1 \times t_1 = 2.24 \times \frac{1}{60}$ km

Distance travelled in second minute $s_2 = v_2 \times t_2 = 3.60 \times \frac{1}{60}$ km

Distance travelled in third minute $s_3 = v_3 \times t_3 = 5.18 \times \frac{1}{60}$ km

Total distance travelled $s = s_1 + s_2 + s_3 = \frac{2.24}{60} + \frac{3.60}{60} + \frac{5.18}{60} = \frac{11.02}{60}$ km

Total time taken, $t = 1 + 1 + 1 = 3 \text{ min} = \frac{1}{20}$ h

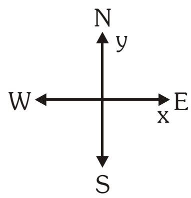
$\therefore$ average velocity $= \frac{s}{t} = \frac{11.02}{60} \times \frac{20}{1} = 3.67$ km/h

**Illustration 8.**

A bird flies due north with velocity 20 m/s for 15 s it rests for 5 s and then flies due south with velocity 24m/s for 10 s. Find the average speed and magnitude of average velocity. For the whole trip.

**Solution**

Average speed $= \frac{\text{Total Distance}}{\text{Total Time}} = \frac{20 \times 15 + 24 \times 10}{15 + 5 + 10} = \frac{540}{30} = 18$ m/s




Average velocity $= \frac{\text{Displacement}}{\text{Total Time}} = \frac{(20 \times 15)\hat{j} + (24 \times 10)(-\hat{j})}{15 + 5 + 10} = \frac{60\hat{j}}{30} = 2\hat{j}$

Magnitude of average velocity $= |2\hat{j}| = 2$ m/s

**Illustration 9.**

The displacement of a point moving along a straight line is given by

$s = 4t^2 + 5t - 6$

Here s is in cm and t is in seconds calculate

(i) Initial speed of particle

(ii) Speed at t = 4s

**Solution**

(i) Speed, $v = \frac{ds}{dt} = 8t + 5$ Initial speed (i.e at t = 0), v = 5 cm/s

(ii) At t = 4s, v = 8(4) + 5 = 37 cm/s

**Illustration 10.**

(a) If $s = 2t^3 + 3t^2 + 2t + 8$ then find time at which acceleration is zero.

(b) Velocity of a particle (starting at t = 0) varies with time as v = 4t. Calculate the displacement of particle between t = 2 to t = 4 s **[AIPMT Mains - 2004]**

**Solution**

(a) $v = \frac{ds}{dt} = 6t^2 + 6t + 2 \Rightarrow a = \frac{dv}{dt} = 12t + 6 = 0 \Rightarrow t = -\frac{1}{2}$

which is impossible. Therefore acceleration can never be zero.

(b) $\because \frac{dx}{dt} = v \therefore x = \int v\,dt = \int_2^4 4t\,dt = \left[2t^2\right]_2^4 = 2(4)^2 - 2(2)^2 = 32 - 8 = 24$ m

## BEGINNER'S BOX-2

**1.** Air distance between Kota to Jaipur is 260 km and road distance is 320 km. A deluxe bus which moves from Jaipur to Kota takes 8 h while an aeroplane reaches in just 15 min. Find

(i) average speed of bus in km/h

(ii) average velocity of bus in km/h

(iii) average speed of aeroplane in km/h

(iv) average velocity of aeroplane in km/h

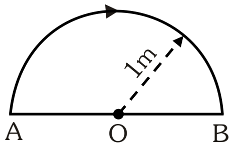
**2.** A particle moves on a straight line in such way that it covers 1st half distance with speed 3 m/s and next half distance in 2 equal time intervals with speeds 4.5 m/s and 7.5 m/s respectively. Find average speed of the particle.

**3.** Length of a minute hand of a clock is 4.5 cm. Find the average velocity of the tip of minute's hand between 6 A.M. to 6.30 A.M. & 6 A.M. to 6.30 P.M.

**4.** A particle of mass 2 kg moves on a circular path with constant speed 10 m/s. Find change in speed and magnitude of change in velocity. When particle completes half revolution.

**5.** The distance travelled by a particle in time t is given by $s = (2.5\ t^2)$ m. Find (a) the average speed of the particle during time 0 to 5.0s and (b) the instantaneous speed at t = 5.0 s.

**6.** A particle goes from point A to point B, moving in a semicircle of radius 1m in 1 second. Find the magnitude of its average velocity.




**7.** Straight distance between a hotel and a railway station is 10 km, but circular route is followed by a taxi covering 23 km in 28 minute. What is average speed and magnitude of average velocity? Are they equal ?

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## 5. ACCELERATION

The rate of change of velocity of an object is called acceleration of the object.

It is a vector quantity. It's direction is same as that of change in velocity (Not in the direction of the velocity).

Dimension : $[M^0L^1T^{-2}]$

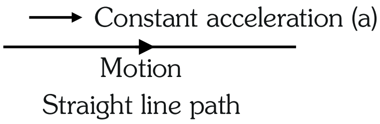
Unit : $m/s^2$ (S.I.); $cm/s^2$ (C.G.S.)

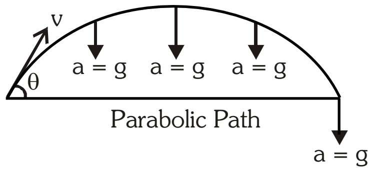
**Uniform acceleration**

A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during motion of particle .

## GOLDEN KEY POINTS

- When a particle moves with constant acceleration, then its path may be straight line or parabolic.




- When a particle starts from rest and moves with constant acceleration then its path must be a straight line.


- When a particle moves with variable velocity then acceleration must be present.

- When a particle moves continuously on a same straight line with uniform speed then acceleration of the particle is zero.
- When a particle moves continuously on a curved path with uniform speed then acceleration of the particle is non zero. For example uniform circular motion is an accelerated motion
- For a particle moving with uniform velocity acceleration must be zero.

---

**Non-uniform acceleration**

A body is said to have non-uniform acceleration, if either magnitude or direction or both change during motion.

**Average acceleration**

It is the ratio of total change in velocity to the total time taken by the particle

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

**Instantaneous acceleration**

It is the acceleration of a particle at a particular instant of time.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

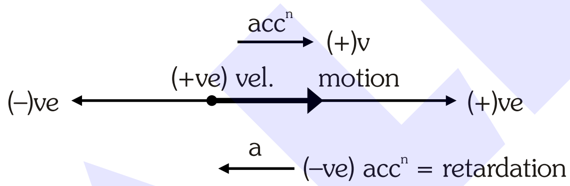
i.e. first derivative of velocity is called instantaneous acceleration

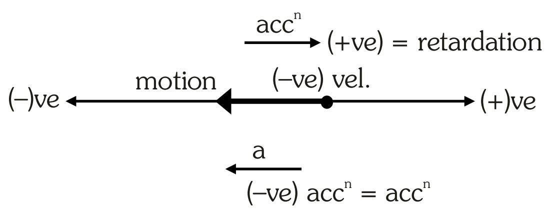
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \qquad \left[\text{As } \vec{v} = \frac{d\vec{r}}{dt}\right]$$

i.e. second derivative of position vector is called instantaneous acceleration

## GOLDEN KEY POINTS

- When a particle moves with non-uniform speed then acceleration of the particle must be non zero.
- The direction of average acceleration vector is the direction of the change in velocity vector as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$
- Acceleration which opposes the motion of body is called retardation.




- Sign of velocity (+ve or -ve) represents the direction of motion but sign of acceleration indicates the direction of acceleration
- If velocity and acceleration both are having same sign, then magnitude of velocity (i.e speed) is increasing and if both have opposite signs, then magnitude of velocity (i.e. speed) is decreasing.

## Illustrations

**Illustration 11.**

The velocity of a particle is given by $v = (2t^2 - 4t + 3)$ m/s where t is time in seconds. Find its acceleration at t = 2 second.

**Solution**

Acceleration (a) $= \frac{dv}{dt} = \frac{d}{dt}(2t^2 - 4t + 3) = 4t - 4$

Therefore acceleration at t = 2s is equal to, $a = (4 \times 2) - 4 = 4\ m/s^2$

**Illustration 12.**

A particle is moving along a straight line OX. At a time t (in seconds) the distance x (in metres) of particle from point O is given by $x = 10 + 6t - 3t^2$. How long would the particle travel before coming to rest ?

**Solution**

Initial value of x, at t = 0, $x_1 = 10m$

Velocity $v = \frac{dx}{dt} = 6 - 6t$ When $v = 0$, $t = 1s$

Final value of x, at t = 1s, $x_2 = 10 + 6 \times 1 - 3(1^2) = 13$ m

Distance travelled = $x_2 - x_1 = 13 - 10 = 3m$

**Illustration 13.**

The acceleration of a particle moving in a straight line varies with its displacement as, a = 2s+1 velocity of the particle is zero at zero displacement. Find the corresponding velocity - displacement equation.

**Solution**

$$a = 2s + 1 \Rightarrow \frac{dv}{dt} = 2s+1 \Rightarrow \frac{dv}{ds}.\frac{ds}{dt} = 2s+1 \Rightarrow \frac{dv}{ds}.v = 2s+1$$

$$\Rightarrow \int_0^v vdv = 2\int_0^s sds + \int_0^s ds$$

$$\Rightarrow \left(\frac{v^2}{2}\right)_0^v = 2\left(\frac{s^2}{2}\right)_0^s + [s]_0^s \Rightarrow \frac{v^2}{2} = s^2 + s \Rightarrow v = \sqrt{2s(s+1)}$$

**BEGINNER'S BOX-3**

**1.** A particle moves on circular path of radius 5 m with constant speed 5 m/s. Find the magnitude of its average acceleration when it completes half revolution.

**2.** The position of a particle moving on X-axis is given by $x = At^2 + Bt + C$ The numerical values of A, B and C are 7, –2 and 5 respectively and SI units are used. Find

(a) The velocity of the particle at t = 5

(b) The acceleration of the particle at t = 5

(c) The average velocity during the interval t = 0 to t = 5

(d) The average acceleration during the interval t = 0 to t = 5

## 6. EQUATIONS OF MOTION

Equations of motion are valid when acceleration is constant.

- $v = u + at$
- $s = ut + \frac{1}{2}at^2$
- $v^2 - u^2 = 2as$
- $s_{n^{th}} = u + \frac{1}{2}a(2n - 1)$
- $s = v_{av}t = \frac{(u+v)}{2}t$
- $s = vt - \frac{1}{2}at^2$

| | |
|---|---|
| a | = acceleration = constant |
| u | = Initial velocity |
| v | = Final velocity |
| s | = Displacement |
| $s_{n^{th}}$ | = Displacement in the $n^{th}$ second |

## Illustrations

**Illustration 14.**

For a particle moving with constant acceleration, prove that the displacement in the $n^{th}$ second is given by

$$s_{n^{th}} = u + \frac{a}{2}(2n-1).$$

**Solution**

From $s = ut + \frac{1}{2}at^2$

$$s_n = un + \frac{1}{2}an^2 \;\ldots\ldots\; (1)$$

$$s_{n-1} = u(n-1) + \frac{1}{2}a(n-1)^2 \;\ldots\ldots\; (2)$$

By equation (1) & (2)

$$\boxed{s_n - s_{n-1} = s_{n^{th}} = u + \frac{a}{2}(2n-1)}$$

### GOLDEN KEY POINTS

- Identification of equation of motion

  (i) If t = given and v = ? then use

  $v = u + at$

  (ii) If t = given and s = ? then use

  $s = ut + \frac{1}{2}at^2$

  (iii) If s = given and v = ? then use

  $v^2 = u^2 + 2as$

- All the equations of motion can be used in 2–D motion in vector form.

**Vector Form of Equations of motion**

$\vec{v} = \vec{u} + \vec{a}t$

$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$

$v^2 = u^2 + 2\vec{a}.\vec{s}$

$\vec{s}_{n^{th}} = \vec{u} + \frac{1}{2}\vec{a}(2n-1)$

$\vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t$

$\vec{s} = \vec{v}t - \frac{1}{2}\vec{a}t^2$

**Concept of stopping distance and stopping time**

A body moving with a velocity *u* is stopped by application of brakes after covering a distance *s*. If the same body moves with velocity *nu* and same braking force is applied on it then it will come to rest after covering a distance of $n^2s$.

As $v^2 = u^2 - 2as$

$\Rightarrow$ $0 = u^2 - 2as$

$\Rightarrow$ $s = \frac{u^2}{2a}$

$\Rightarrow$ $s \propto u^2$ [since *a* is constant]

So we can say that if *u* becomes *n* times then *s* becomes $n^2$ times that of previous value.

**Stopping time :**

$v = u - at$

$\Rightarrow$ $0 = u - at$

$\Rightarrow$ $t = \frac{u}{a}$ $\Rightarrow$ $t \propto u$ [since a is constant]

So we can say that if u becomes n times then t becomes n times that of previous value.

## Illustrations

**Illustration 15.**

Two cars start off a race with velocities 2m/s and 4m/s travel in straight line with uniform accelerations $2 m/s^2$ and $1 m/s^2$ respectively. What is the length of the path if they reach the final point at the same time ? **[AIPMT (Main) -2008]**

**Solution**

Let both particles reach at same position in same time t then from $s = ut + \frac{1}{2}at^2$

For $1^{st}$ particle : $s = 4(t) + \frac{1}{2}(1)t^2 = 4t + \frac{t^2}{2}$, For $2^{nd}$ particle : $s = 2(t) + \frac{1}{2}(2)t^2 = 2t + t^2$

Equating above equation we get $4t + \frac{t^2}{2} = 2t + t^2 \Rightarrow t = 4\ s$

Subsituting value of t in above equation $s = 4(4) + \frac{1}{2}(1)(4)^2 = 16 + 8 = 24\ m$

**Illustration 16.**

A particle moves in a straight line with a uniform acceleration a. Initial velocity of the particle is zero. Find the average velocity of the particle in first 's' distance.

**Solution**

$\because \quad s = \frac{1}{2}at^2 \qquad \therefore \quad \frac{s^2}{t^2} = \frac{1}{2}as \qquad$ Average velocity $= \frac{s}{t} = \sqrt{\frac{as}{2}}$

**Illustration 17.**

A train, travelling at 20 km/hr is approaching a platform. A bird is sitting on a pole on the platform. When the train is at a distance of 2 km from pole, brakes are applied which produce a uniform deceleration in it. At that instant the bird flies towards the train at 60 km/hr and after touching the nearest point on the train flies back to the pole and then flies towards the train and continues repeating itself. Calculate how much distance the bird covers before the train stops ?

**Solution**

For retardation of train $v^2 = u^2 + 2as \Rightarrow 0 = (20)^2 + 2(a)(2) \Rightarrow a = -100\ km/hr^2$

Time required to stop the train $v = u + at \Rightarrow 0 = 20 - 100t \Rightarrow t = \frac{1}{5}$ hr

For Bird, speed $= \frac{\text{Distance}}{\text{time}} \Rightarrow s_B = v_B \times t = 60 \times \frac{1}{5} = 12$ km.

## BEGINNER'S BOX-4

**1.** A particle starts from rest, moves with constant acceleration for 15s. If it covers $s_1$ distance in first 5s then distance $s_2$ in next 10s, then find the relation between $s_1$ & $s_2$.

**2.** The engine of a train passes an electric pole with a velocity 'u' and the last compartment of the train crosses the same pole with a velocity v. Then find the velocity with which the mid-point of the train passes the pole. Assume acceleration to be uniform.

**3.** A bullet losses 1/n of its velocity in passing through a plank. What is the least number of planks required to stop the bullet ? (Assuming constant retardation)

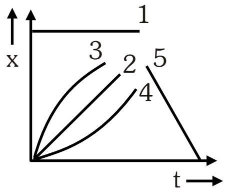
**4.** A car moving along a straight highway with speed 126 km $h^{-1}$ is brought to a halt within a distance of 200m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop?

**5.** A car is moving with speed u. Driver of the car sees red traffic light. His reaction time is t, then find out the distance travelled by the car before coming at rest after the instant he saw red signal. Assume uniform retardation 'a' after applying brakes.

**6.** If a body starts from rest and travels 120cm in the $6^{th}$ second then what is the acceleration ?

## 7. GRAPHICAL SECTION

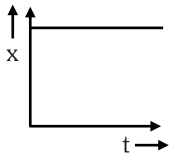
### Position-time graph




Slope of this graph represents instantaneous velocity.

$$\because \ \tan\theta = \frac{\text{displacement}}{\text{time}} = \text{velocity}$$

**(i)**



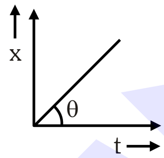

$\theta = 0°$

$\tan\theta = \tan 0° = 0$

velocity = 0

i.e. body is at rest.

**(ii)**



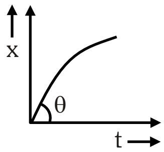

$\theta$ = constant

$\tan\theta$ = constant

velocity = constant

i.e. the body is in uniform motion

**(iii)**



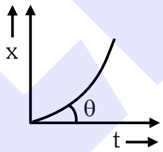

$\theta$ is decreasing with time

$\therefore$ $\tan\theta$ is decreasing with time

$\therefore$ velocity is decreasing with time

i.e. non uniform motion

**(iv)**



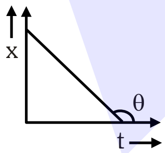

$\theta$ is increasing with time

$\therefore$ $\tan\theta$ is increasing with time

$\therefore$ velocity is increasing with time

i.e. non uniform motion

**(v)**




$\theta > 90°$

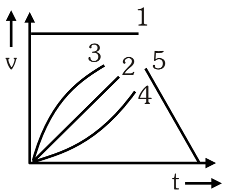
$\tan\theta$ = –ve

velocity = –ve but constant

i.e. uniform motion

Area of x-t graph = $\int x\,dt$ = No physical significance

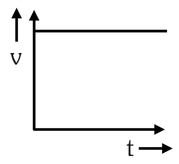
### Velocity-time graph




Slope of this graph represents acceleration.

$$\because \ \tan\theta = \frac{\text{change in velocity}}{\text{time}} = \text{acceleration}$$

**(i)**



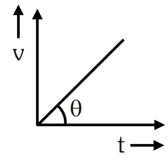

$\theta = 0°$

$\tan\theta = \tan 0° = 0$

acceleration = 0

i.e. v = constant or uniform motion

**(ii)**



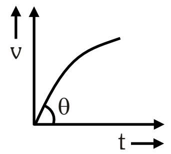

$\theta$ = constant

$\tan\theta$ = constant

acceleration = constant

i.e. uniformly accelerated motion

**(iii)**



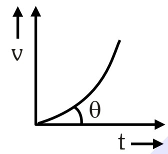

$\theta$ is decreasing with time

$\therefore$ $\tan\theta$ is decreasing with time

$\therefore$ acceleration is decreasing with time

i.e. acceleration goes on decreasing with time but it is not retardation

**(iv)**



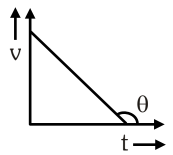

$\theta$ is increasing with time

$\therefore$ $\tan\theta$ is increasing with time

$\therefore$ acceleration is increasing with time

i.e. acceleration goes on increasing with time

**(v)**




$\theta > 90°$

$\tan\theta = -ve$

acceleration = –ve but constant

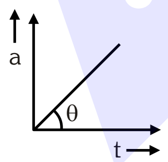
i.e. constant or uniform retardation is acting on the body

Area of v-t graph = $\int v\,dt$ = displacement = change in position

**Acceleration-time graph**

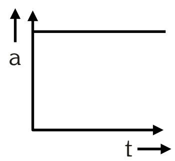
Area of a–t graph = $\int a\,dt = \int dv = v_2 - v_1$ = change in velocity

**(i)**




i.e. uniformly increasing acceleration.

**(ii)**

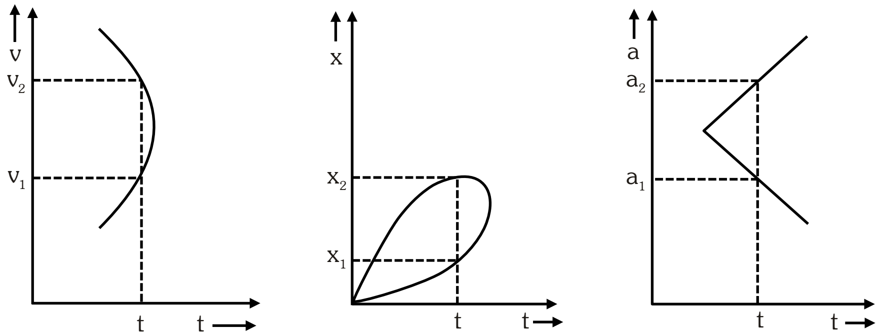



$a \propto t^0$ i.e. uniform or constant acceleration

## GOLDEN KEY POINTS

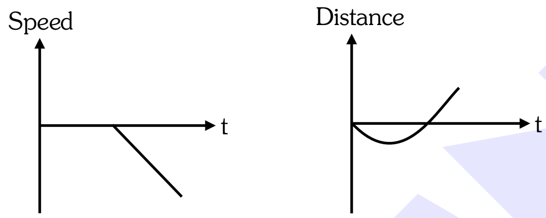
- Total area enclosed between speed-time (v-t) graph and time axis represent distance.
- Vector sum of total area enclosed between v-t graph and time axis represent displacement.
- Following graphs do not exist in practice :

**Case-I**



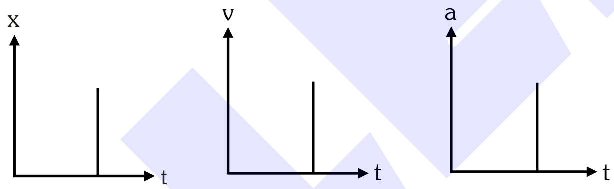

**Explanation :** In practice, at any instant body can not have two velocities or displacements or accelerations simultaneously.

**Case-II**




**Explanation :** Speed or distance can never be negative.

**Case-III**




**Explanation :** It is not possible to change any quantity without consuming time i.e. time can't be constant.

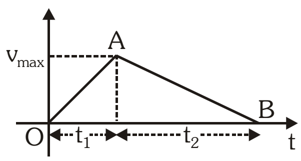
## Illustrations

**Illustration 18.**

A car accelerates from rest at a constant rate $\alpha$ for some time, after which it decelerates at a constant rate $\beta$, to come to rest. If the total time elapsed is t, evaluate (a) the maximum velocity attained (b) the total distance travelled.

**Solution**

(a) Let the car accelerate for time $t_1$ and decelerate




for time $t_2$ then

$t = t_1 + t_2$ ......(i)

and corresponding velocity-time graph will be as shown in. fig.

From the graph $\alpha$ = slope of line OA = $\dfrac{v_{max}}{t_1}$ or $t_1 = \dfrac{v_{max}}{\alpha}$ ...(ii)

and $\beta$ = – slope of line AB = $\dfrac{v_{max}}{t_2}$ or $t_2 = \dfrac{v_{max}}{\beta}$ ...(iii)

From Eqs. (i),(ii) and (iii) $\frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t$ or $v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t$ or $v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$

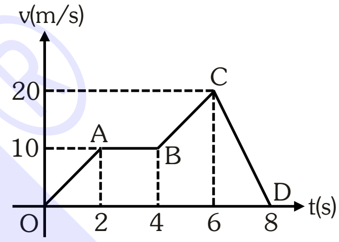
(b) Total distance = area under v-t graph = $\frac{1}{2} \times t \times v_{max} = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta}$

Distance = $\frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$.

**Note:** This problem can also be solved by using equations of motion (v = u + at, etc.).

**Illustration 19.**

Velocity-time graph of a particle moving in a straight line is shown. Plot the corresponding displacement-time graph of the particle.




**Solution**

Displacement = area under velocity-time graph.

Hence, $s_{OA} = \frac{1}{2} \times 2 \times 10 = 10$ m

$s_{AB} = 2 \times 10 = 20$ m

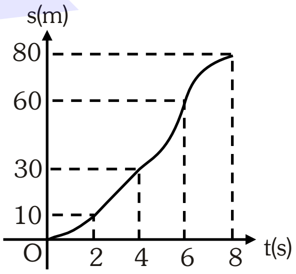
or $s_{OAB} = 10+20 = 30$ m

$s_{BC} = 2 \times \left(\frac{10+20}{2}\right) = 30$ m

or $s_{OABC} = 30+30 = 60$ m

and $s_{CD} = \frac{1}{2}(2 \times 20) = 20$m

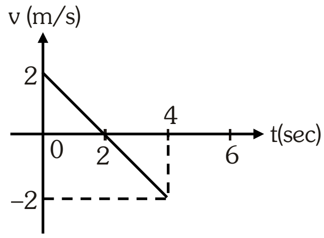
or $s_{OABCD} = 60 + 20 = 80$ m




Between 0 to 2 s and 4 to 6 s motion is accelerated, hence displacement-time graph is a parabola. Between 2 to 4 s motion is uniform, so displacement-time graph will be a straight line. Between 6 to 8 s motion is decelerated hence displacement-time graph is again a parabola but inverted in shape. At the end of 8 s velocity is zero, therefore, slope of displacement-time graph should be zero. The corresponding graph is shown in the figure.

**Illustration 20.**

Velocity-time graph for a particle moving in a straight line is given Calculate the displacement of the particle and distance travelled in first 4 seconds.




**Solution**

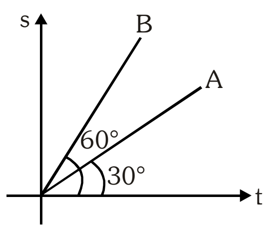
Take the area above time axis as positive and area below time axis negative then displacement =(2 – 2)m = 0

while for distance take all areas as positive the distance covered

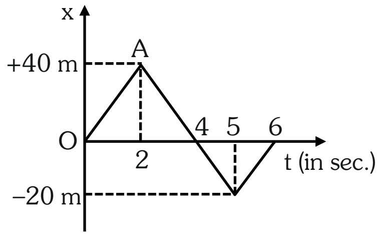
s = (2 + 2)m = 4m

**BEGINNER'S BOX-5**

**1.** s-t graph of two particles A and B are shown in fig. Find the ratio of velocity of A to velocity of B.



**2.** Position-time graph of a particle in motion is shown in fig.



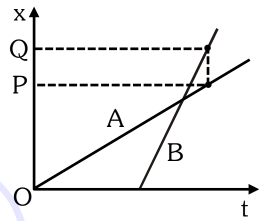

Calculate –

(i) Total distance covered

(ii) Displacement

(iii) Average speed (iv) Average velocity.

**3.** The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in fig. Choose correct entries in the brackets below :




(a) (A/B) lives closer to the school than (B/A)

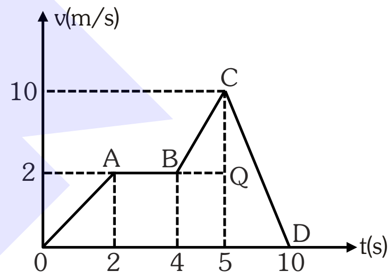
(b) (A/B) starts from the school earlier than (B/A)

(c) (A/B) walks faster than (B/A)

(d) A and B reach home at the (same / different) time

(e) (A/B) overtakes (B/A) on the road (once/ twice).

**4.** A particle moves on straight line according to the velocity–time graph shown in fig. Calculate –




(i) Total distance covered

(ii) Average speed

(iii) In which part of the graph the acceleration is maximum and also find its value.

(iv) Retardation

**5.** A body starts from rest and moves with a uniform acceleration of 10 $ms^{-2}$ for 5 seconds. During the next 10 seconds it moves with uniform velocity. Find the total distance travelled by the body (Using graphical analysis).

---

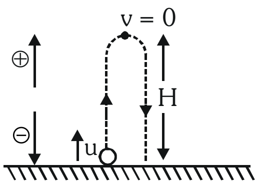
## 8. MOTION UNDER GRAVITY (FREE FALL)

Acceleration produced in a body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

Value of $g = 9.8\ m/s^2 = 980\ cm/s^2 = 32\ ft/s^2$

In the absence of air, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude
(h << earth's radius) is called motion under gravity. Free fall means acceleration of body is equal to acceleration due to gravity.

### 8.1 If a Body is Projected Vertically Upward




Positive / Negative directions are a matter of choice. You may take another choice.

(i) **Equations of motion :** Taking initial position as origin and direction of motion (i.e. vertically up) as positive a = – g [As acceleration is downwards while motion upwards]

So, if a body is projected with velocity u and after time t it reaches a height h then

$$v = u - gt,\ h = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gh,\ h_{n^{th}} = u - \frac{g}{2}(2n - 1)$$

(ii) **For maximum height** v = 0

So from above equation u = g t

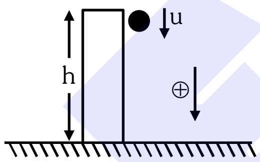
it is called time of ascent ($t_1$) = u/g

In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent ($t_2$) = time of ascent ($t_1$) = u/g

$\therefore$ Total time of flight $T = t_1 + t_2 = \frac{2u}{g}$

and $u^2 = 2gH \Rightarrow H = \frac{u^2}{2g}$

## 8.2 If a Body is Projected Vertically Downward With Some Initial Velocity From Some Height

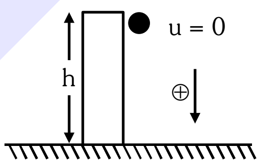



**Equations of motion :** Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, we have

$$v = u + gt \qquad h = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gh \qquad h_n = u + \frac{g}{2}(2n - 1)$$

## 8.3 If a body is dropped from some height (initial velocity zero)




**Equations of motion :** Taking initial position as origin and direction of motion (*i.e.*, downward direction) as a positive, here we have

u = 0 [As body starts from rest]

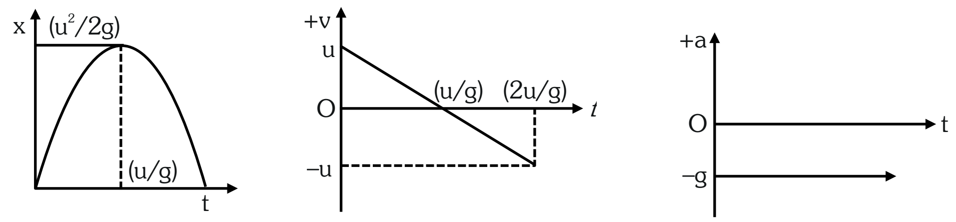
a = +g [As acceleration is in the direction of motion]

so $v = gt,\ h = \frac{1}{2}gt^2$

## GOLDEN KEY POINTS

- In case of motion under gravity, the speed with which a body is projected up is equal to the speed with which it comes back to the point of projection.
- The magnitude of velocity at any point on the path is same whether the body is moving in upward or downward direction.

- Graph of displacement, velocity and acceleration with respect to time :
  (For a body projected vertically upward)

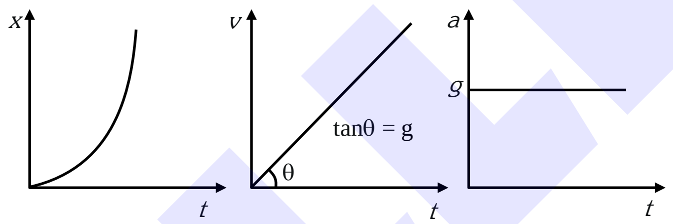



- As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of consequtive integers. (in case of free fall, from rest)
- A particle at rest, is dropped vertically from a height. The time taken by it to fall through successive distance of 1 m each will then be in the ratio of the difference in the square roots of the integers *i.e.*

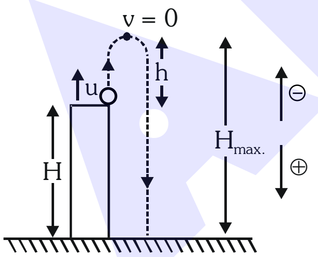
$$\sqrt{1},(\sqrt{2}-\sqrt{1}),(\sqrt{3}-\sqrt{2}),(\sqrt{4}-\sqrt{3}),......$$

- The motion is independent of the mass of body, as mass is not involved in any equation of motion. It is due to this reason that a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity *i.e.*, $t=\sqrt{(2h/g)}$ and $v=\sqrt{2gh}$ .
- The distance covered in the $n^{th}$ second, $h_n = \frac{1}{2}g(2n-1)$

  So distance covered in $1^{st}$, $2^{nd}$, $3^{rd}$ second, etc., will be in the ratio of 1 : 3 : 5, *i.e.*, odd integers only.
- Graph of distance, velocity and acceleration with respect to time :
  (For a body dropped from some height)




## 8.4. If a Body is Projected Vertically Upward With Some Initial Velocity From a Certain Height




**Equations of motion :** Taking initial position as origin and direction of motion (*i.e.*, upward direction) as negative, here we have

$v = -u + gt$; $\qquad H = -ut + \frac{1}{2}gt^2$

$v^2 = u^2 + 2gh$; $\qquad h_{n^{th}} = -u + \frac{g}{2}(2n-1)$

- Maximum height attained by the body

$$H_{max} = H + h = H + \frac{u^2}{2g}$$

- Distance travelled by the body

$$H + 2h = H + \frac{u^2}{g}$$

Time taken by the body to reach the ground

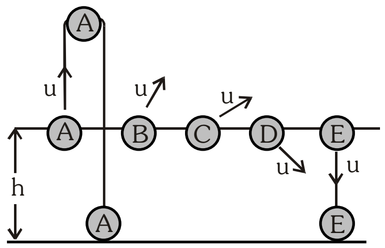
$$H = -ut + \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - ut - H = 0$$

$\Rightarrow$ $gt^2 - 2ut - 2H = 0$

After solving this equation we get the result.

## GOLDEN KEY POINTS

- If various particles thrown with same initial speed but in different directions then



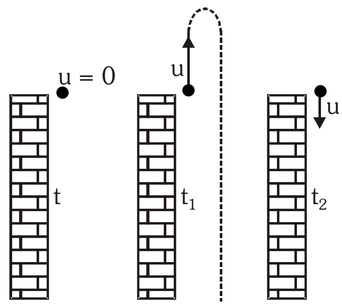

(i) They strike the ground with same speed at different times irrespective of their initial direction of velocities.

(ii) Time would be least for particle E which was thrown vertically downward.

(iii) Time would be maximum for particle A which was thrown vertically upward.

- A ball is dropped from a building of height h and it reaches ground after time t. From the same building if two balls are thrown (one upwards and other downwards) with the same speed u and they reach the ground after $t_1$ and $t_2$ seconds respectively then

$$t = \sqrt{t_1 t_2}$$



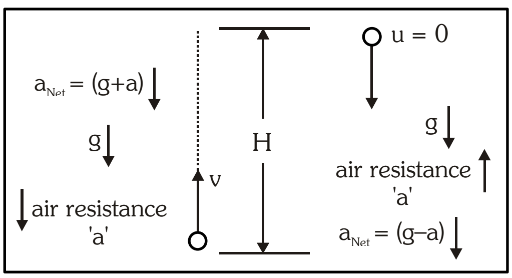

### 8.5 A body is thrown vertically upwards, if Constant Air resistance is to be taken into account:

For upward motion :-

Net acceleration $a_{Net} = g + a$ (downwards)

If maximum height attained by the particle is 'H' then

$$t_{ascent} = \sqrt{\frac{2H}{a_{Net}}} \Rightarrow t_{ascent} = \sqrt{\frac{2H}{g+a}}$$




For downward motion :-

Net acceleration $a_{Net} = g - a$ (downwards)

So $t_{descent} = \sqrt{\frac{2H}{g-a}}$

Also $t_{desent} > t_{ascent}$

## GOLDEN KEY POINTS

- For downward motion a and g will work in opposite directions because a always acts in direction opposite to motion and g always acts vertically downwards.

## Illustrations

**Illustration 21.**

A body is dropped from a height h above the ground. Find the ratio of distances fallen in first one second, first two seconds, first three seconds, also find the ratio of distances fallen in $1^{st}$ second, in $2^{nd}$ second, in $3^{rd}$ second etc.

**Solution**

From second equation of motion, i.e. $h = \frac{1}{2}gt^2$ $\quad$ ($h = ut + \frac{1}{2}gt^2$ and $u = 0$)

$s_1 : s_2 : s_3 \ldots\ldots = \frac{1}{2}g(1)^2 : \frac{1}{2}g(2)^2 : \frac{1}{2}g(3)^2 = 1^2 : 2^2 : 3^2 \ldots\ldots = 1 : 4 : 9 : \ldots\ldots$ .

Now from the expression of distance travelled in $n^{th}$ second $S_n = u + \frac{1}{2}a(2n - 1)$

here $u = 0$, $a = g$ $\quad$ So $\quad S_n = \frac{1}{2}g(2n - 1)$ therefore

$S_{1st} : S_{2nd} : S_{3rd} \ldots\ldots = \frac{1}{2}g(2 \times 1 - 1) : \frac{1}{2}g(2 \times 2 - 1) : \frac{1}{2}g(2 \times 3 - 1) = 1 : 3 : 5 \ldots\ldots$

**Illustration 22.**

A rocket is fired vertically up from the ground with a resultant vertical acceleration of $10 m/s^2$. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached?

(b) After the fuel is finished, calculate the time for which it continues its upwards motion.

(Take $g = 10 m/s^2$)

**Solution**

(a) The distance travelled by the rocket during burning interval (1minute= 60s) in which resultant acceleration is vertically upwards and $10 m/s^2$ will be $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000$ m = 18 km and velocity acquired by it will be $v = 0 + 10 \times 60 = 600$ m/s

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to gravity opposes its motion. So, it will go to a height $h_2$ from this point, till its velocity becomes zero such that $0 = (600)^2 - 2gh_2$ $\quad$ or $\quad h_2 = 18000$ m = 18 km $\quad [g = 10 m/s^2]$

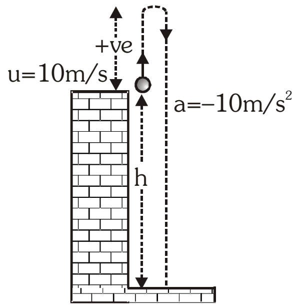
So the maximum height reached by the rocket from the ground, $H = h_1 + h_2 = 18 + 18 = 36$ km

(b) As after burning of fuel the initial velocity 600m/s and gravity opposes the motion of rocket, so from $1^{st}$ equation of motion time taken by it till its velocity v =0

$0 = 600 - gt \quad \Rightarrow \quad t = 60$ s

**Illustration 23.**

A ball is thrown upwards from the top of a tower 40 m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10\ m/s^2$)




**Solution**

In the problem $u = +\ 10\ m/s$, $a = -\ 10\ m/s^2$ and $s = -40m$ (at the point where ball strikes the ground)

Substituting in $s = ut + \frac{1}{2}at^2$

$$-\ 40 = 10t - 5t^2 \text{ or } 5t^2 - 10t - 40 = 0 \quad \text{or} \quad t^2 - 2t - 8 = 0$$

Solving this we have t = 4 s and – 2s. Taking the positive value t = 4s.

**Illustration 24.**

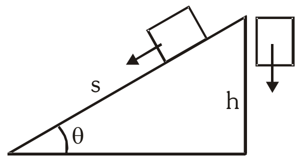
A block slides down a smooth inclined plane when released from the top, while another falls freely from the same point. Which one of them will strike the ground: (a)earlier (b) with greater speed?

**Solution**

In case of sliding motion on the inclined plane.

$$\frac{h}{s} = \sin\theta \quad \Rightarrow \quad s = \frac{h}{\sin\theta}, \quad a = g\sin\theta$$

$$t_s = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2}{g\sin\theta} \times \frac{h}{\sin\theta}} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}} = \frac{t_F}{\sin\theta}$$




$$v_s = \sqrt{2as} = \sqrt{2g\sin\theta \times \frac{h}{\sin\theta}} = \sqrt{2gh}$$

In case of free fall $t_F = \sqrt{\frac{2h}{g}}$ and $v_F = \sqrt{2gh} = v_s$

(a) $\because$ $\sin\theta < 1$, $t_F < t_s$, i.e.,falling body reaches the ground first.

(b) $v_F = v_s$, i.e.,both reach the ground with same speed.

**Special Note :** (not same velocity, as for falling body direction is vertical while for sliding body along the plane downwards).

**Illustration 25.**

A Juggler throws balls into air. He throws one ball whenever the previous one is at its highest point. How high do the balls rise if he throws n balls each second ? Acceleration due to gravity is g.

**Solution**

Juggler throws n balls in one second so time interval between two consecutive throws is $t = \frac{1}{n}s$

each ball takes $\frac{1}{n}$ s to reach maximum height

$$\text{So } h_{max} = \frac{1}{2} \times gt^2 = \frac{1}{2} \times g\left(\frac{1}{n}\right)^2$$

$$h_{max} = \frac{g}{2n^2}$$

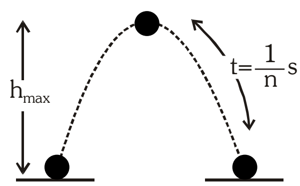
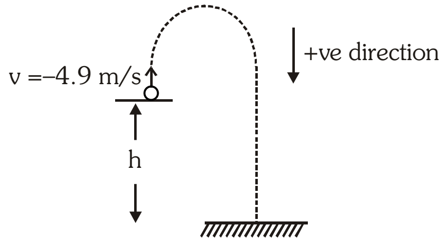


**Illustration 26.**

A pebble is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2 s. If acceleration due to gravity is 9.8m/s² (a) what is the height of the bridge? (b) with what velocity does the pebble strike the water ?

**Solution**




Let height of the bridge be h then

$$h = -4.9 \times 2 + \frac{1}{2} \times 9.8 \times (2)^2$$

$\Rightarrow$ h = 9.8 m

velocity with which the ball strikes the water surface

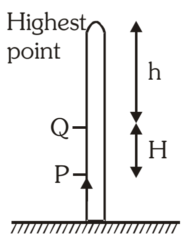
v = u + at

$\Rightarrow$ v = –4.9 + 9.8 × 2 = 14.7 m/s

**Illustration 27.**

A particle is thrown vertically upwards from the surface of the earth. Let $T_P$ be the time taken by the particle to travel from a point P above the earth to its highest point and back to the point P. Similarly, let $T_Q$ be the time taken by the particle to travel from another point Q above the earth to its highest point and back to the same point Q. If the distance between the points P and Q is H, find the expression for acceleration due to gravity in terms of $T_P$, $T_Q$ and H. **[AIPMT (Mains) - 2007]**

**Solution**




Time taken from point P to point P $\quad T_P = 2\sqrt{\frac{2(h+H)}{g}}$

Time taken from point Q to point Q $\quad T_Q = 2\sqrt{\frac{2h}{g}}$

$$\Rightarrow T_P^2 = \frac{8(h+H)}{g} \;\&\; T_Q^2 = \frac{8h}{g} \Rightarrow T_P^2 = T_Q^2 + \frac{8H}{g}$$

$$\Rightarrow g = \frac{8H}{T_P^2 - T_Q^2}$$

**BEGINNER'S BOX-6**

**1.** A particle is projected vertically upwards from ground with velocity 10 m/s. Find the time taken by it to reach at the highest point ?

**2.** A particle is projected vertically up from the top of a tower with velocity 10 m/s. It reaches the ground in 5s. Find–

(a) Height of tower.

(b) Striking velocity of particle at ground

(c) Distance traversed by particle.

(d) Average speed & average velocity of particle.

**3.** A balloon starts rising from the ground with an acceleration of 1.25 $m/s^2$. A stone is released from the balloon after 10s. Determine

(1) maximum height of stone from ground

(2) time taken by stone to reach the ground

**4.** A rocket is fired vertically up from the ground with an acceleration 10 $m/s^2$. If its fuel is finished after 1 minute then calculate –

(a) Maximum velocity attained by rocket in ascending motion.

(b) Height attained by rocket before fuel is finished.

(c) Time taken by the rocket in the whole motion.

(d) Maximum height attained by rocket.

**5.** A particle is dropped from the top of a tower. During its motion it covers $\frac{9}{25}$ part of height of tower in the last 1 second Then find the height of tower.

**6.** A particle is dropped from the top of a tower. It covers 40 m in last 2s. Find the height of the tower.

**7.** A player throws a ball upwards with an initial speed of 29.4 m/s.

(a) What is the direction of acceleration during the upward motion of the ball?

(b) What are the velocity and acceleration of the ball at the highest point of its motion?

(c) Choose x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.

(d) To what height does the ball rise and how long does the ball take to return to the player's hands ?

(Take g = 9.8 $m/s^2$ and neglect air resistance).

**8.** A particle is dropped from the top of a tower. The distance covered by it in the last one second is equal to that covered by it in the first three seconds. Find the height of the tower.

**9.** Water drops are falling in regular intervals of time from top of a tower of height 9 m. If $4^{th}$ drop begins to fall when $1^{st}$ drop reaches the ground, find the positions of $2^{nd}$ & $3^{rd}$ drops from the top of the tower.

## 9. PROJECTILE MOTION : Introduction

When a body is projected such that velocity of projection is not parallel to the force, then it moves along a curved path. This motion is called two dimensional motion. If force on the body is constant then curved path of the body is parabolic. This motion is studied under projectile motion.

(i) It is an example of two dimensional motion.

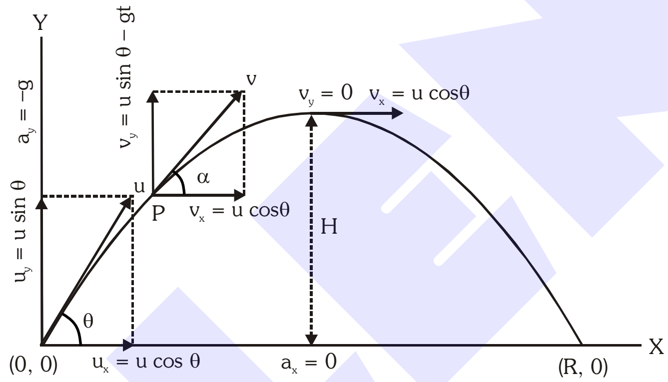
(ii) It is an example of motion with constant (or uniform) acceleration. Thus equations of motion can be used to analyse projectile motion.

(iii) A particle thrown in the space which moves under the effect of gravity only is called a "**projectile**". The motion of this particle is referred to as projectile motion.

(iv) If a particle possesses a uniform acceleration in a directions oblique to its initial velocity, the resultant path will be parabolic.

## 10. GROUND TO GROUND PROJECTION

Let X-axis is along the ground and Y-axis is along the vertical then path of projectile projected at an angle θ from the ground is as shown.




Projectile motion can be considered as two mutually perpendicular motions, which are independent of each other. i.e. Projectile motion = Horizontal motion + Vertical motion

**Horizontal Motion**

- Initial velocity in horizontal direction = $u \cos\theta = u_x$
- Acceleration along horizontal direction = $a_x = 0$. (Neglect air resistance)
- Therefore, Horizontal velocity remains unchanged.
- At any instant horizontal velocity $u_x = u \cos\theta$
- At time t, x co-ordinate or displacement along X-direction is

$$x = u_x t \qquad \text{or} \qquad x = (u \cos\theta)t$$

**Vertical Motion :** It is motion under the effect of gravity so that as particle moves upwards the magnitude of its vertical velocity decreases.

- Initial velocity in vertical direction = $u \sin\theta = u_y$
- Acceleration along vertical direction = $a_y = -g$
- At time t, vertical speed $v_y = u_y - gt = u \sin\theta - gt$
- In time t, displacement in vertical direction or "height" of the particle above the ground

$$y = u_y t - \frac{1}{2}gt^2 = u\sin\theta t - \frac{1}{2}gt^2$$

**Net Motion :** Net initial velocity = $\vec{u} = u_x\hat{i} + u_y\hat{j} = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

Direction of u can be explained in terms of angle θ it makes with the ground

Net acceleration = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$ (direction of g is downwards)

**Coordinates of particle at time t :** (x, y) $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$

Net displacement in t time $= \sqrt{x^2 + y^2}$

**Velocity of particle at time t**

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

Magnitude of velocity $|\vec{v}| = \sqrt{v_x^2 + v_y^2}$

If angle made by velocity with the ground is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x}$$

$$\Rightarrow \quad \tan\alpha = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

**Change in velocity and momentum ($\vec{p} = m\vec{v}$) of projectile**

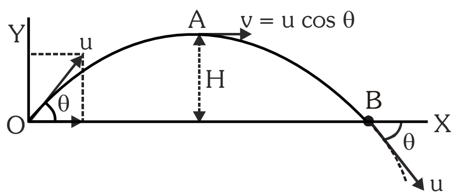
When particle returns to ground again at B point, its y coordinate is zero and the magnitude of its velocity is u at angle θ with ground. Total angular change = 2θ

Initial velocity $\vec{u}_i = u\cos\theta\hat{i} + u\sin\theta\hat{j}$

Final velocity $\vec{u}_f = u\cos\theta\hat{i} - u\sin\theta\hat{j}$

Total change in its velocity, $|\Delta\vec{v}| = 2u\sin\theta$

Total change in momentum, $|\Delta\vec{p}| = m|\Delta\vec{v}| = 2mu\sin\theta$




**Time of flight (T)**

At time T particle will be at ground again, i.e. displacement along Y-axis becomes zero.

$\because \quad y = u_y t - \frac{1}{2}gt^2 \qquad \therefore \qquad 0 = u_y T - \frac{1}{2}gT^2$

Time of flight is the time for which projectile remains in air.

or $\quad T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$ (neglecting T = 0)

Time of ascent = Time of descent = $\frac{T}{2} = \frac{u_y}{g} = \frac{u\sin\theta}{g}$

at time $\frac{T}{2}$ particle attains maximum height of its trajectory.

**Maximum height attained H**

At maximum height vertical component of velocity becomes zero. At this instant y coordinate is, its maximum height.

$\because \quad v_y^2 = u_y^2 - 2gy \qquad \therefore \quad 0 = u_y^2 - 2gH \qquad \{\because \; v_y = 0, \quad y = H\}$

$$H = \frac{u_y^2}{2g} = \frac{u^2 \sin^2\theta}{2g}$$

**Horizontal range or Range (R)**

It is the displacement of particle along X-direction during its complete flight.

$\because \; x = u_x t \quad \therefore \; R = u_x T = u_x \dfrac{2u_y}{g}; \qquad R = \dfrac{2u_x u_y}{g}$

$$R = \frac{2(u\cos\theta)(u\sin\theta)}{g} \Rightarrow R = \frac{u^2 \sin 2\theta}{g} \quad (\because \; 2\sin\theta\cos\theta = \sin 2\theta)$$

**Maximum horizontal range ($R_{max}$)**

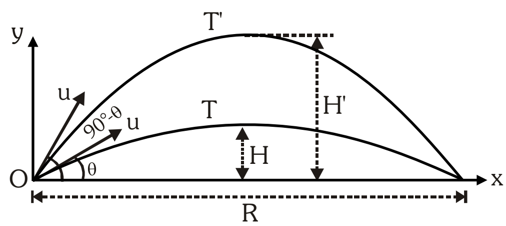
If value of θ is increased from θ = 0° to 90°, then range increases from θ = 0° to 45° but it decreases beyond 45°. Thus range is maximum at θ = 45°

For maximum range, θ =45° and $\quad R_{max} = \dfrac{u^2 \sin 2(45^\circ)}{g} = \dfrac{u^2 \sin 90^\circ}{g}$

$$\Rightarrow \qquad R_{max} = \frac{u^2}{g}$$

**Comparison of two projectiles of equal range**

When two projectiles are thrown with equal speeds at angles θ and (90° – θ) then their ranges are equal but maximum heights attained are different and time of flights are also different.




At angle θ, $\quad R = \dfrac{u^2 \sin 2\theta}{g}$

At angle (90° – θ), $\; R' = \dfrac{u^2 \sin 2(90^\circ - \theta)}{g} = \dfrac{u^2 \sin(180^\circ - 2\theta)}{g} = \dfrac{u^2 \sin 2\theta}{g}$

Thus, R' = R

Maximum heights of projectiles

$$H = \frac{u^2 \sin^2\theta}{2g} \quad \text{and} \quad H' = \frac{u^2 \sin^2(90^\circ - \theta)}{2g} = \frac{u^2 \cos^2\theta}{2g}$$

- $\dfrac{H}{H'} = \dfrac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$
- $HH' = \dfrac{u^4 \sin^2\theta \cos^2\theta}{4g^2} = \dfrac{R^2}{16} \Rightarrow R = 4\sqrt{HH'}$
- $H + H' = \dfrac{u^2 \sin^2\theta}{2g} + \dfrac{u^2 \cos^2\theta}{2g} \Rightarrow H + H' = \dfrac{u^2}{2g}$

Time of flight of projectiles

$$T = \frac{2u\sin\theta}{g} \quad ; \qquad T' = \frac{2u\sin(90-\theta)}{g} = \frac{2u\cos\theta}{g}$$

- $\frac{T}{T'} = \frac{\sin\theta}{\cos\theta} = \tan\theta$
- $TT' = \frac{4u^2\sin\theta\cos\theta}{g^2} = \frac{2R}{g} \quad \Rightarrow \quad TT' \propto R$

**Equation of Trajectory**

Along horizontal direction $x = u_x t$ or $x = u\cos\theta t$

Along vertical direction $y = u_y t - \frac{1}{2}gt^2$ or $y = u\sin\theta t - \frac{1}{2}gt^2$

On eliminating t from these two equations

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

$$\Rightarrow \qquad y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$$

This is an equation of a parabola so it can be stated that projectile follows a parabolic path.

Again $\quad y = x\tan\theta\left[1 - \frac{gx}{2u^2\sin\theta\cos\theta}\right] = x\tan\theta\left[1 - \frac{x}{R}\right]$

**Kinetic Energy of a Projectile**

Kinetic energy = $\frac{1}{2} \times \text{Mass} \times (\text{Speed})^2$

Let a body is projected with velocity u at an angle $\theta$.

Thus initial kinetic energy of projectile, $K_0 = \frac{1}{2}mu^2$

Since velocity of projectile at maximum height is $u\cos\theta$.

Kinetic energy at highest point, $K = \frac{1}{2}m(u\cos\theta)^2 = K_0\cos^2\theta$

which is the minimum kinetic energy during whole motion.

## GOLDEN KEY POINTS

- At maximum height, $v_y = 0$ and $v_x = u_x = u\cos\theta$ so that at maximum height $v = \sqrt{v_x^2 + v_y^2} = u\cos\theta$
- At maximum height angle between velocity and acceleration is 90°.
- Magnitude of velocity at height 'h' .

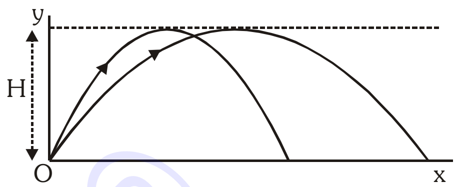
$$v_y^2 = u_y^2 - 2gh$$

$$v_y^2 = (u\sin\theta)^2 - 2gh$$

$$v_x = u\cos\theta$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta)^2 - 2gh}$$

$$|\vec{v}| = \sqrt{u^2 - 2gh}$$



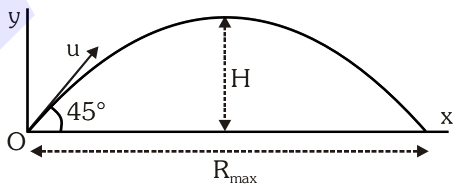

- $T = \dfrac{2u_y}{g}$, $H = \dfrac{u_y^2}{2g}$, $R = \dfrac{2u_x u_y}{g}$

T and H depend only upon initial vertical speed $u_y$

If two projectiles thrown in different directions, have equal times of flight then their initial vertical speeds are same so that their maximum height are is also same.

If $H_A = H_B$ then $(u_y)_A = (u_y)_B$ and $T_A = T_B$

- For situation shown in figure for $\theta = 45°$




here $R_{max} = \dfrac{u^2}{g}$ and $H = \dfrac{u^2\sin^2 45°}{2g} = \dfrac{u^2}{4g}$

$\therefore$ $R_{max} = 4H = 4\times$(maximum height attained)

- When $R = H$

$R = \dfrac{u^2(2\sin\theta\cos\theta)}{g}$ and $H = \dfrac{u^2\sin^2\theta}{2g}$ $\Rightarrow$ $\dfrac{R}{H} = 4\cot\theta = 1$

$\Rightarrow$ $4\cot\theta = 1$ $\Rightarrow$ $\tan\theta = 4$ $\Rightarrow$ $\theta = \tan^{-1}(4) \simeq 76°$

## Illustrations

**Illustration 28.**

A projectile is thrown with speed u making angle θ with horizontal at t =0. It just crosses two points of equal height, at time t = 1s and t = 3s respectively. Calculate the maximum height attained by it? (g=10m/s²)

**Solution**

Displacement in y direction $y = u_y \times 1 - \dfrac{1}{2}g\times(1)^2 = u_y \times 3 - \dfrac{1}{2}g(3)^2 \Rightarrow u_y = 2g = 20$ m/s

Maximum height attained $h_{max} = \dfrac{u_y^2}{2g} = 20$m.

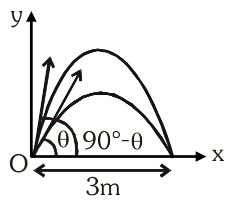
**Illustration 29.**

A stone is to be thrown so as to cover a horizontal distance of 3m. If the velocity of the projectile is 7 m/s, find :

(a) the angle at which is must be thrown.

(b) the largest horizontal displacement that is possible with the projection speed of 7 m/s.

**Solution**




(a) Range $R = \frac{u^2}{g}\sin 2\theta$

$\Rightarrow \quad \sin 2\theta = \frac{gR}{u^2} = \frac{9.8\times 3}{(7)^2} = 0.6 = \sin 37° \Rightarrow 2\theta = 37° \Rightarrow \theta = 18.5°$

angle of projection may also be $= 90° - \theta = 90° - 18.5° = 71.5°$

(b) For largest horizontal displacement $\theta = 45°$

maximum range $R_{max.} = \frac{u^2}{g} = \frac{(7)^2}{9.8} = \frac{49}{98}\times 10 = 5$ m.

**Illustration 30.**

Two projectiles are projected at angles $(\theta)$ and $\left(\frac{\pi}{2}-\theta\right)$ to the horizontal respectively with same speed 20 m/s. One of them rises 10 m higher than the other. Find the angles of projection. (Take g=10 m/s²)

**Solution**

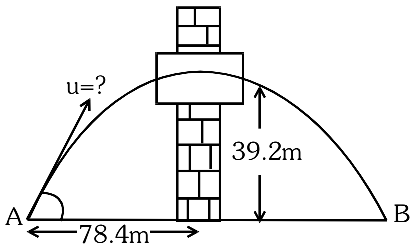
Maximum height $H = \frac{u^2\sin^2\theta}{2g} \Rightarrow h_1 = \frac{(20)^2\sin^2\theta}{2g} = 20\sin^2\theta$ & $h_2 = \frac{(20)^2\sin^2(\pi/2-\theta)}{2g} = 20\cos^2\theta$

$h_2 - h_1 = 20[\cos^2\theta - \sin^2\theta] = 10 \Rightarrow 20\cos 2\theta = 10 \Rightarrow \cos 2\theta = \frac{1}{2} \Rightarrow 2\theta = 60° \Rightarrow \theta = 30°$

and $\theta' = 90° - \theta = 60°$

**Illustration 31.**

A boy stands 78.4 m away from a building and throws a ball which just enters a window at maximum height 39.2m above the ground. Calculate the velocity of projection of the ball.




**Solution**

Maximum height $= \frac{u^2\sin^2\theta}{2g} = 39.2$ m ... (i) Range $= \frac{u^2\sin 2\theta}{g} = \frac{2u^2\sin\theta\cos\theta}{g} = 2\times 78.4$ ... (ii)

from equation (i) divided by equation (ii) $\tan\theta = 1 \Rightarrow \theta = 45°$

from equation (ii) range $= \frac{u^2\sin 90°}{g} = 2\times 78.4 \Rightarrow u = \sqrt{2\times 78.4\times 9.8} = 39.2$ m/s

**Illustration 32.**

A particle thrown over a triangle from one end of a horizontal base falls on the other end of the base after grazing the vertex. If $\alpha$ and $\beta$ are the base angles of triangle and angle of projection is $\theta$, then prove that $\tan\theta = \tan\alpha + \tan\beta$.

**Solution**

From triangle $y = x\tan\alpha$ and $y = (R-x)\tan\beta$

$\tan\alpha + \tan\beta = \frac{y}{x} + \frac{y}{R-x} = \frac{yR}{x(R-x)}$

$\because \quad y = x\tan\theta\left[1-\frac{x}{R}\right] \Rightarrow \tan\theta = \frac{yR}{x(R-x)}$

$\therefore \quad \tan\theta = \tan\alpha + \tan\beta$

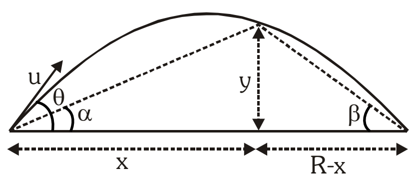


**Illustration 33.**

A particle is projected from the ground at an angle such that it just clears the top of a pole after $t_1$ time in its path. It takes further $t_2$ time to reach the ground. What is the height of the pole ?

**Solution**

Height of the pole is equal to the vertical displacement of the particle at time $t_1$

Vertical displacement $y = u_y t_1 + \frac{1}{2}a_y t_1^2 = u_y t_1 - \frac{1}{2}gt_1^2$ ...........(i)

and total flight time $t_1 + t_2 = \frac{2u_y}{g} \Rightarrow u_y = \frac{g}{2}(t_1 + t_2)$

put value $u_y$ in equation (i) $y = \frac{g}{2}(t_1 + t_2)t_1 - \frac{1}{2}g(t_1)^2 = \frac{1}{2}g t_1 t_2$, so height of the pole $= \frac{1}{2}gt_1t_2$

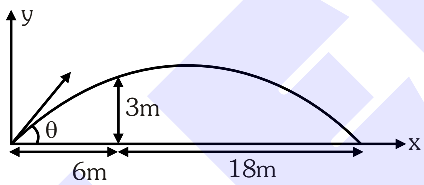
**Illustration 34.**

A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is :-

(A) $\tan^{-1}\left(\frac{3}{2}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$ (C) $\tan^{-1}\left(\frac{1}{2}\right)$ (D) $\tan^{-1}\left(\frac{3}{4}\right)$

**Solution** **Ans. (B)**

From equation of trajectory, $y = x\tan\theta\left[1 - \frac{x}{R}\right] \Rightarrow 3 = 6\tan\theta\left[1 - \frac{1}{4}\right] \Rightarrow \tan\theta = \frac{2}{3}$




**BEGINNER'S BOX-7**

**1.** A football player kicks a ball at an angle of 30° to the horizontal with an initial speed of 20 m/s. Assuming that the ball travels in a vertical plane, calculate (a) the time at which the ball reaches the highest point (b) the maximum height reached (c) the horizontal range of the ball (d) the time for which the ball is in the air. ($g = 10$ m/s$^2$)

**2.** A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the ball, with the same speed ?

**3** Two bodies are thrown with the same initial speed at angles $\alpha$ and $(90° - \alpha)$ with the horizontal. What will be the ratio of (a) maximum heights attained by them and (b) horizontal ranges ?

**4.** A ball is thrown at angle $\theta$ and another ball is thrown at angle $(90° - \theta)$ with the horizontal direction from the same point each with speeds of 40 m/s. The second ball reaches 50m higher than the first ball. Find their individual heights. $g = 10$ m/s$^2$.

**5.** The range of a particle when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle 45° to the horizontal.

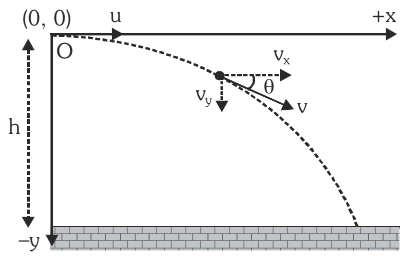
**6.** Show that the projection angle $\theta_0$ for a projectile launched from the origin is given by :

$\theta_0 = \tan^{-1}\left[\frac{4H_m}{R}\right]$ Where $H_m$ = Maximum height, R = Horizontal range

**7.** A ball of mass m is thrown vertically up. Another ball of mass 2m is thrown at an angle $\theta$ with the vertical. Both of them stay in air for the same periods of time. What is the ratio of the height attained by the two balls ?

**8.** The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m/s can go without hitting the ceiling of the hall ? ($g = 10$ m/s$^2$)

## 11. HORIZONTAL PROJECTION FROM HEIGHT




Consider a projectile thrown from point O at some height h from the ground with a velocity u in horizontal direction.

Now we shall deal the characteristics of projectile motion separately along horizontal and vertical directions i.e.

| Horizontal direction : | | Vertical direction : |
|---|---|---|
| (i) | Initial velocity $u_x = u$ | Initial velocity $u_y = 0$ |
| (ii) | Acceleration $a_x = 0$ | Acceleration $a_y = -g$ (downward) |

**Trajectory Equation**

The path traced by projectile is called its trajectory. After time t,

$x = ut$ and $y = -\frac{1}{2}gt^2$ negative sign indicates that the direction of vertical displacement is downward.

so $\quad y = -\frac{1}{2}g\frac{x^2}{u^2} \quad (\because t = \frac{x}{u})$ This is equation of a parabola

Above equation is called trajectory equation

**Velocity at a general point P(x, y)**

$$v = \sqrt{v_x^2 + v_y^2}$$

Horizontal velocity of the projectile after time t is $v_x = u$ (remains constant)

Velocity of projectile in vertical direction after time t is

$v_y = 0 - (g)t = -gt$ (downward) $\quad \because v = \sqrt{u^2 + g^2t^2}$

and $\tan\theta = \frac{v_y}{v_x}$ or $\tan\theta = -\frac{gt}{u}$ (negative sign indicates clockwise direction)

**Displacement**

The displacement of the particle is expressed by

$$\vec{s} = x\hat{i} + y\hat{j} \text{ Where } |\vec{s}| = \sqrt{x^2 + y^2} = \left|(ut)\hat{i} - (\frac{1}{2}gt^2)\hat{j}\right|$$

**Time of flight**

From equation of motion for vertical direction.

$$h = u_y t + \frac{1}{2}gt^2$$

At highest point $u_y = 0 \Rightarrow h = \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$

**Horizontal range**

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground. $\quad R = u_x t = u\sqrt{\frac{2h}{g}}$

**Velocity after falling a height $h_1$ :**

Along vertical direction $v_y^2 = 0^2 + 2(h_1)(g)$

$$v_y = \sqrt{2gh_1}$$

Along horizontal direction $v_x = u_x = u$

So velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh_1}$

## 12. OBLIQUE PROJECTION FROM A CERTAIN HEIGHT

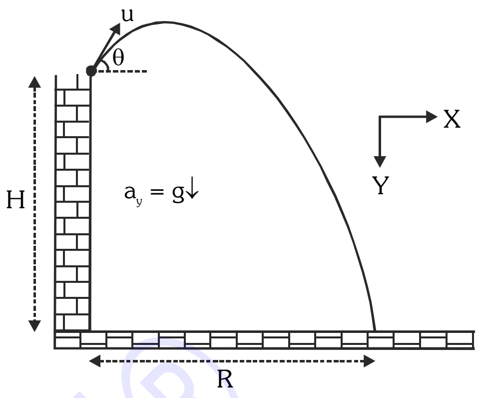
**(i) Projection from a height at an angle θ above horizontal :**

$u_x = u \cos\theta \qquad u_y = -u \sin\theta$

$x = (u \cos\theta)\, t \qquad a_y = g$

$H = (-u \sin\theta)\, t + \frac{1}{2} gt^2$

$gt^2 - (2u \sin\theta)\, t - 2H = 0$




After solving the above equation we get the result

**Velocity after falling height h :**

Along vertical direction; $v_y^2 = (-u\sin\theta)^2 + 2(h)(g)$

Along horizontal direction, $v_x = u_x = u\cos\theta$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$$

**(ii) Projection from a height at an angle θ below horizontal :**

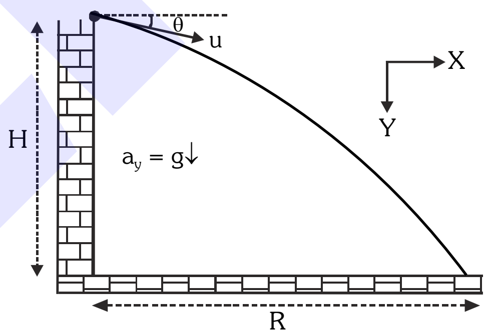
$u_x = u \cos\theta \qquad u_y = u \sin\theta$

$x = (u \cos\theta)\, t \qquad a_y = g$

$H = (u \sin\theta)\, t + \frac{1}{2} gt^2$

$gt^2 + (2u \sin\theta)\, t - 2H = 0$

After solving the above equation we get the result.




**Velocity after falling height h :**

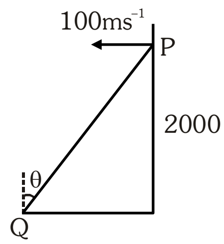
Along vertical direction, $v_y^2 = (u\sin\theta)^2 + 2(h)(g)$

Along horizontal direction, $v_x = u_x = u\cos\theta$; So velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

## Illustrations

**Illustration 35.**

An aeroplane is travelling horizontally at a height of 2000 m from the ground. The aeroplane, when at a point P, drops a bomb to hit a stationary target Q on the ground. In order that the bomb hits the target, what angle θ must the line PQ make with the vertical ? [$g = 10\ m/s^2$] **[AIPMT (Mains) 2007]**




**Solution**

Let t be the time taken by bomb to hit the target.

$h = 2000 = \frac{1}{2} gt^2 \Rightarrow t = 20s$

$R = ut = (100)(20) = 2000m$

$\because \tan\theta = \frac{R}{h} = \frac{2000}{2000} = 1 \Rightarrow \theta = 45°$

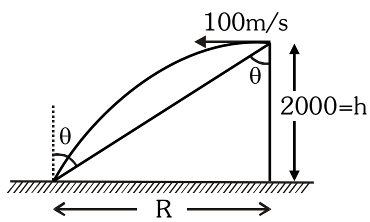


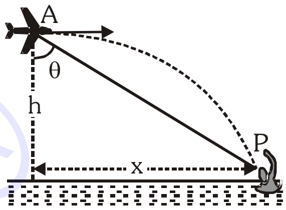
**Illustration 36.**

A relief aeroplane is flying at a constant height of 1960 m with 600 km/hr speed above the ground towards a point directly over a person struggling in flood water. At what angle of sight with the vertical should the pilot release a survival kit if it is to reach the person in water? ($g = 9.8 m/s^2$)

**Solution**

Plane is flying at a speed = $600 \times \frac{5}{18} = \frac{500}{3}$ m/s horizontally (at a height 1960m)

time taken by the kit to reach the ground $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20s$




in this time the kit will move horizontally by $x = ut = \frac{500}{3} \times 20 = \frac{10,000}{3}$ m

So $\tan\theta = \frac{x}{h} = \frac{10,000}{3 \times 1960} = \frac{10}{5.88} = 1.7 \simeq \sqrt{3}$ or $\theta = 60°$

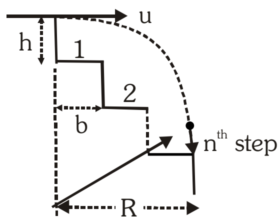
**Illustration 37.**

A ball rolls off the top of a stair way with a horizontal velocity u. If each step has height h and width b the ball will just hit the edge of $n^{th}$ step. Find the value of n.

**Solution**

If the ball hits the $n^{th}$ step, the horizontal and vertical distances traversed are nb and nh respectively. Let t be the time taken by the ball for these horizontal and vertical displacements. Velocity along horizontal direction = u (remains constant) and initial vertical velocity = zero.

$\therefore$ $nb = ut$ and $nh = 0 + \frac{1}{2}gt^2$




Eliminating t from the equation

$$nh = \frac{1}{2}g\left(\frac{nb}{u}\right)^2 \Rightarrow n = \frac{2hu^2}{gb^2}$$

**Illustration 38.**

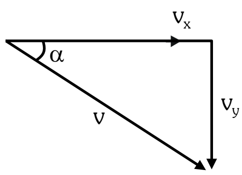
A particle is projected horizontally with a speed 20 m/s from the top of a tower. After what time will the velocity of particle be at 45° angle from the initial direction of projection ? [Let $g = 10 m/s^2$]

**Solution**

Let x and y axes be adopted along horizontal and vertically downward directions respectively.

After time t, $v_x = u_x = 20$ m/s, velocity in y direction $v_y = u_y + a_y t = 0 + g\,t = gt$

$\tan\alpha = \frac{v_y}{v_x} = \frac{gt}{20}$ if $\alpha = 45°$ than $1 = \frac{10 \times t}{20} \Rightarrow t = 2s$



## BEGINNER'S BOX-8

**1.** A projectile is fired horizontally with a velocity of 98 $ms^{-1}$ from the top of a hill 490m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the angle at which the projectile hits the ground. (g = 9.8 $m/s^2$)

**2.** Two tall buildings face each other and are at a distance of 180m from each other. With what velocity must a ball be thrown horizontally from a window 55m above the ground in one building, so that it enters a window 10m above the ground in the second building ? ( g = 10 $m/s^2$)

**3.** Two paper screens A and B are separated by a distance of 100m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. (g = 9.8 $m/s^2$)

**4.** A ball is thrown up from the top of a tower with an initial velocity of 10 m/s at an angle of 30° with the horizontal. It hits the ground at a distance of 17.3 m from the base of tower. Calculate the height of the tower. (g = 10 $m/s^2$)

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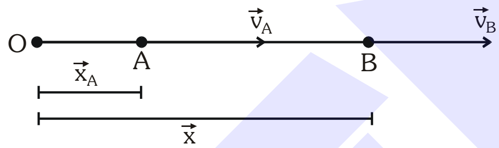
## 13. RELATIVE VELOCITY IN ONE DIMENSION




Displacement of B with respect to A = Displacement of B as measured from A

$\Rightarrow \quad \vec{x}_{BA} = \vec{x}_B - \vec{x}_A$

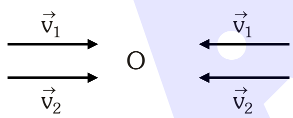
$\Rightarrow \quad \dfrac{d\vec{x}_{BA}}{dt} = \dfrac{d\vec{x}_B}{dt} - \dfrac{d\vec{x}_A}{dt}$

$\Rightarrow \quad \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

Relative = Actual – Reference

**For same direction**

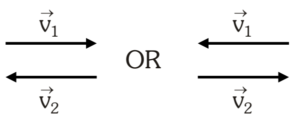
When two particles are moving in the same direction, then magnitude of their relative velocity is equal to the difference between their individual speeds.




$|\vec{v}_{12}|$ or $|\vec{v}_{21}| = v_1 \sim v_2$

**For opposite directions**

When two particles are moving in the opposite directions, then magnitude of their relative velocity is always equal sum of thier individual speeds.



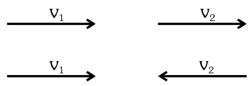

$|\vec{v}_{12}|$ or $|\vec{v}_{21}| = v_1 + v_2$

**Note :-** When two particles move simultaneously then the concept of relative motion becomes applicable conveneintly.

## Numerical Applications

**(i)** When two particles are moving along a straight line with constant speeds then their relative acceleration must be zero and in this condition relative velocity is the ratio of relative displacement to time.

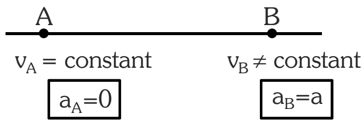
$v_1$ = const. $v_2$ = const.




when $a_{rel.} = 0$

$$\boxed{v_{rel.} = \frac{s_{Relative}}{time}}$$

**(ii)** When two particles move in such a way that their relative acceleration non zero but constant then we apply equations of motion in the relative form.




$a_{AB} = a_A - a_B$

$= 0 - a = -a \neq 0 =$ constant

**Equations of Motion (Relative)**

- $v_{rel.} = u_{rel.} + a_{rel.}t$
- $s_{rel.} = u_{rel.}t + \frac{1}{2} a_{rel.}t^2$
- $v^2_{rel.} = u^2_{rel.} + 2.a_{rel.}.s_{rel.}$
- $s_{rel.} = \frac{1}{2}(u_{rel.} + v_{rel.})t$

## Illustrations

**Illustration 39.**

Buses A and B are moving in the same direction with speeds 20 m/s and 15 m/s respectively. Find the relative velocity of A w.r.t. B and relative velocity of B w.r.t A.

**Solution**

Let their direction of motion be along + x-axis then $\vec{v}_A = (20\text{m/s})\hat{i}$ and $\vec{v}_B = (15\text{m/s})\hat{i}$

(a) Relative velocity of A w.r.t. B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ = (actual velocity of A) – (velocity of B)

$$= (20 \text{ m/s})\hat{i} - (15 \text{ m/s})\hat{i} = 5\text{m/s } \hat{i}$$

i.e. A is moving with speed 5 m/s w.r.t B in the same direction.

(b) Relative velocity of B w.r.t. A is $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$ = (actual velocity of B) – (velocity of A)

$$= (15 \text{ m/s})\hat{i} - (20 \text{ m/s})\hat{i} = (-5\text{m/s})\hat{i} = (5 \text{ m/s})(-)\hat{i}$$

i.e. B is moving in opposite direction w.r.t. A, at a speed 5 m/s

**Illustration 40.**

A police van moving on a highway with a speed of 30 km/hr fires a bullet at a thief's car which is speeding away in the same direction with a speed of 190 km/hr. If the muzzle speed of the bullet is 150 m/s, find the speed of the bullet with respect to the thief's car.

**Solution** $v_b \rightarrow$ velocity of bullet

$v_p \rightarrow$ velocity of police van

$v_t \rightarrow$ velocity of thief's car

$v_{bp} = v_b - v_p$

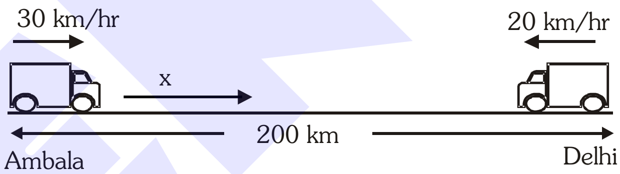
$v_b = v_{bp} + v_p = 150 \times \frac{18}{5}$ km/hr + 30 km/hr =570 km/hr

$v_{bt} = v_b - v_t$ = 570 km/hr – 190 km/hr = 380 km/hr.

**Illustration 41.**

Delhi is at a distance of 200 km from Ambala. Car A sets out from Ambala at a speed of 30 km/hr. and car B set out at the same time from Delhi at a speed of 20 km/hr. When will they cross each other ? What is the distance of that crossing point from Ambala?

**Solution**




$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

= 30 – (–20)

= 50 km/hr

They will meet after time t

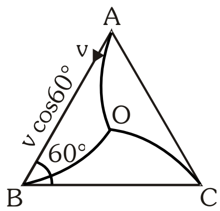
given by $t = \frac{s}{v_{AB}} = \frac{200}{50} = 4$ hr

Distance from Ambla where they will meet is x = 30 × 4 = 120 km

**Illustration 42.**

Three boys A, B and C are situated at the vertices of an equilateral triangle of side d at t = 0. Each of the boys move with constant speed v. A always moves towards B , B towards C and C towards A. When and where will they meet each other ?

**Solution**




By symmetry they will meet at the centroid of the triangle. Approaching velocity of A and B towards each other is v + v cos 60° and they cover distance d when they meet.

So that time taken, is given by

$$\therefore \quad t = \frac{d}{v + v\cos 60^\circ} = \frac{d}{v + \frac{v}{2}} = \frac{2d}{3v}$$

**Illustration 43.**

Two cars approach each other on a straight road with velocities 10 m/s and 12 m/s respectively. When they are 150 metres apart, both drivers apply their brakes and each car decelerates at 2 m/s² until they stops. How far apart will they be when both come to a halt ?

**Solution**

Let $x_1$ and $x_2$ be the distances travelled by the cars before they stop under deceleration.

From III[rd] equation of motion $v^2 = u^2 + 2as$, $\Rightarrow 0 = (10)^2 - 2 \times 2\, x_1 \Rightarrow x_1 = 25$ m

and $0 = (12)^2 - 2 \times 2\, x_2 \Rightarrow x_2 = 36$ m

Total distance covered by the two cars = $x_1 + x_2$ = 25 + 36 = 61 m

Distance between the two cars when they stop = 150 – 61 = 89 m.

**Illustration 44.**

Two trains A and B which are 100 m and 60 m long are moving in opposite directions on parallel tracks. Velocity of the shorter train is 3 times that of the longer one. If the trains take 4 seconds to cross each other find the velocities of the trains ?

**Solution**

Given that $v_B = 3v_A$

Trains move in opposite directions then

relative velocity $v_{rel} = \frac{d_{rel}}{time} \Rightarrow v_A + v_B = \frac{100+60}{4} \Rightarrow 4v_A = 40 \Rightarrow v_A = 10$ m/s, $v_B = 3v_A = 30$ m/s

**BEGINNER'S BOX-9**

**1.** Two trains A and B each of length 50 m, are moving with constant speeds. If one train A overtakes the other in 40s, while crosses the other in 20s. Find the speeds of each train.

**2.** Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km/h in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m/s$^2$. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between the guard of B & driver of A ?

**3.** A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m/s. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m/s and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands ?

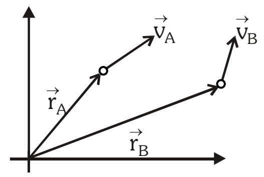
## 14. RELATIVE VELOCITY IN A PLANE

- For 2-dimensional motion :

  Relative velocity of A with respect to 'B' can be calculated as

  $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

  $|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A \cdot v_B \cos\theta}$




**Note :**

- For two particles to collide :

  (i) their combined relative displacement becomes zero.

  (ii) their combined vertical velocities will be same : if they are projected from same level (in case of projectiles)

  (iii) their combined motion can be converted into two 1D motions.

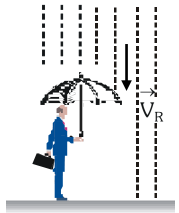
- **Relative path of a projectiles w.r.t. another projectile**

  Two projectiles are thrown from ground with different velocities at different angles. Since both projectiles have equal accelerations so their relative acceleration is zero. Thus path of one projectile w.r.t. other is a straight line and motion of one projectile w.r.t. other is uniform.

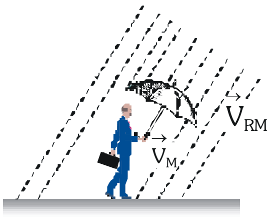
  - If $u_1 \cos\theta_1 = u_2 \cos\theta_2$ then relative path is a vertical line.
  - If $u_1 \sin\theta_1 = u_2 \sin\theta_2$ then relative path is a horizontal line.

## 15. RAIN - MAN PROBLEM

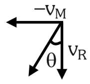
(i)

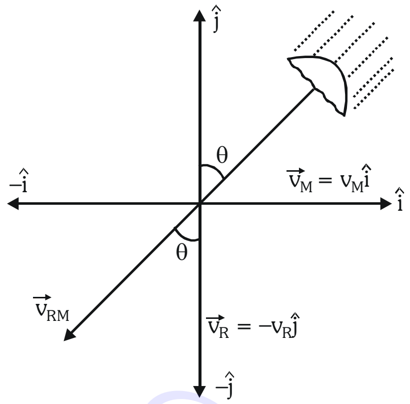



When man is at rest




As observed by the man when moving




If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ the velocity of rain relative to observer will be

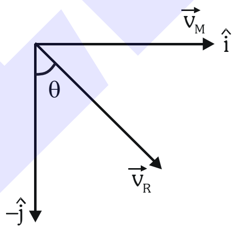
$$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M \Rightarrow \vec{v}_{RM} = -v_R\hat{j} - v_M\hat{i}$$

which by law of vector addition has magnitude

$$v_{RM} = \sqrt{v_R^2 + v_M^2}$$

The direction of $\vec{v}_{RM}$ is such that it makes an angle θ with the vertical given by $\theta = \tan^{-1}(v_m/v_R)$ as shown in figure.

(ii) If rain is already falling at an angle θ with the vertical with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ finds that the rain drops are hitting on his head vertically downwards




Here $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

$$\vec{v}_{RM} = (v_R\sin\theta - v_M)i - v_R\cos\theta\hat{j}$$

Now for rain to appear falling vertically, the horizontal component of $\vec{v}_{RM}$ should be zero, i.e.

$$v_R\sin\theta - v_M = 0 \Rightarrow \sin\theta = \frac{v_M}{v_R}$$

and $$|\vec{v}_{RM}| = v_R\cos\theta = v_R\sqrt{1-\sin^2\theta} = v_R\sqrt{1-\frac{v_M^2}{v_R^2}} = v_{RM} = \sqrt{v_R^2 - v_M^2}$$

## Illustrations

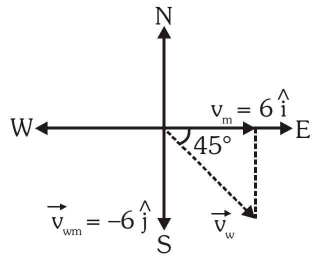
**Illustration 45.**

A person moves due east at a speed 6m/s and feels the wind is blowing towards south at a speed 6m/s.

(a) Find actual velocity of wind blow.

(b) If person doubles his velocity then find the relative velocity of wind blow w.r.t. man.

**Solution**




(a) $\vec{v}_{act} = \vec{v}_{rel} + \vec{v}_{ref}$

$\vec{v}_w = \vec{v}_{wm} + \vec{v}_m = -6j + 6\hat{i}$

$\vec{v}_w = 6\hat{i} - 6\hat{j}$

$|\vec{v}| = 6\sqrt{2}$ m/s and it is blowing along S – E

(b) Person doubles its velocity then $\vec{v}_m = 12\hat{i}$

but actual wind velocity remains unchanged.

$\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (6\hat{i} - 6\hat{j}) - 12\hat{i}$

$\vec{v}_{wm} = -6\hat{i} - 6\hat{j}$

Now relative velocity of wind is $6\sqrt{2}$m/s along S – W.

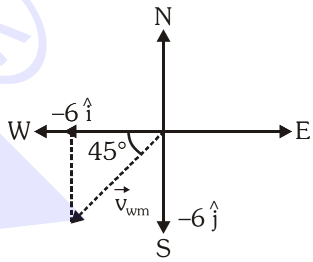



**Illustration 46.**

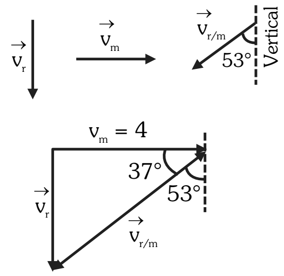
A man at rest observes the rain falling vertically. When he walks at 4 km/h, he has to hold his umbrella at an angle of 53° from the vertical. Find the velocity of raindrops.

**Solution**

Assigning usual symbols $\vec{v}_m$, $\vec{v}_r$ and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following equation

$$\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$$

The above equation suggests that a standstill man observes velocity $\vec{v}_r$ of rain relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes velocity of rain relative to himself $\vec{v}_{r/m}$.



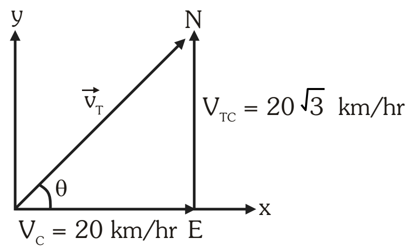

It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.

Therefore $v_r = v_m \tan 37° = 3$ km/h

**Illustration 47.**

A man is going east in a car with a velocity of 20 km/hr, a train appears to move towards north to him with a velocity of $20\sqrt{3}$ km/hr. What is the actual velocity and direction of motion of train ?

**Solution.**




$\vec{v}_{TC} = \vec{v}_T - \vec{v}_C$

$\vec{v}_T = \vec{v}_{TC} + \vec{v}_C = 20\sqrt{3}\hat{j} + 20\hat{i}$

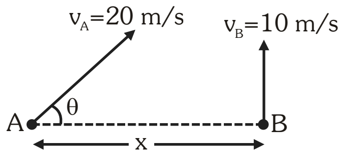
$|\vec{v}_T| = \sqrt{(20\sqrt{3})^2 + (20)^2} = \sqrt{1600} = 40$ km / hr

$\tan\theta = \dfrac{20\sqrt{3}}{20} \Rightarrow \theta = 60°$

So direction of motion of train is 60° N of E or E-60°-N

**Illustration 48.**

Two particles A and B are projected from the ground simultaneously in the directions shown in the figure with initial velocities $v_A$ = 20 m/s and $v_B$ =10 m/s respectively. They collide after 0.5 s. Find out the angle θ and the distance x.




**Solution.**

Both particle will collide if they are at same height in same time.

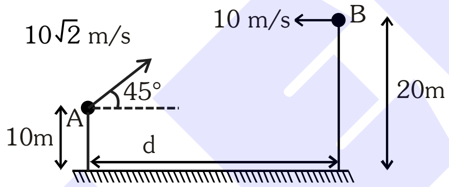
$$y_A = y_B \Rightarrow (u_y)_A t - \frac{1}{2}gt^2 = (u_y)_B t - \frac{1}{2}gt^2 \Rightarrow (u_y)_A = (u_y)_B$$

$$\Rightarrow (v_A \sin\theta) = v_B \Rightarrow 20\sin\theta = 10 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$$

In 0.5s horizontal distance covered by A is $x = (u_x)_A t = (20\cos 30°)0.5 = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$ m

**Illustration 49.**

Two particles are projected from the two towers simultaneously, as shown in the figure.




What should be the value of 'd' for their collision ?

**Solution.**

Their is no relative acceleration of between A and B.

so time of collision $t = \frac{y_{BA}}{(v_y)_{BA}}$ where $y_{BA}$ = vertical displacement of B w.r.t. A = 10 m.

$(v_y)_{BA}$ = vertical velocity of B w.r.t. A = $0 - (-10\sqrt{2}\sin 45°) = 10$ m/s $\Rightarrow t = \frac{10}{10} = 1$s

d = horizontal distance travelled by B w.r.t. A = $(v_x)_{BA} \times t = (10+10\sqrt{2}\cos 45°) \times 1 = 20$ m.

**Illustration 50.**

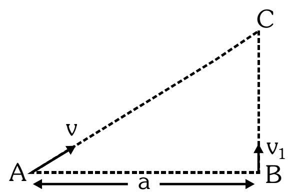
Two boys are standing at the ends A and B of a ground where AB = a. The boy at B starts running in a direction perpendicular to AB, with velocity $v_1$. The boy at A starts running similtaneously with velocity v and catches the other boy in a time t, then find t.

**Solution**

Let the two boys meet at point C after time 't'

Then AC = vt, BC = $v_1$t but $(AC)^2 = (AB)^2 + (BC)^2 \Rightarrow v^2t^2 = a^2 + v_1^2t^2$

$$\Rightarrow t^2(v^2 - v_1^2) = a^2 \Rightarrow t = \sqrt{\frac{a^2}{v^2 - v_1^2}}$$

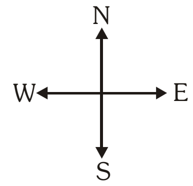


## BEGINNER'S BOX-10

**1.** A man 'A' moves in the north direction with a speed 10 m/s and another man B moves in E–30°–N with 10 m/s. Find the relative velocity of B w.r.t. A.

**2.** A and B are moving with the same speed 10 m/s in the directions E–30°–N and E–30°–S respectively. Find the relative velocity of A w.r.t. B.

**3.** Two bodies A and B are 10 km apart such that B is to the south of A. A and B start moving with the same speed 20 km/hr eastward and northward respectively then find.




(a) relative velocity of A w.r.t. B.

(b) minimum separation attained during motion

(c) time elapsed from starting, to attain minimum separation.

**4.** Rain is falling vertically with a speed of 30 m/s. A woman rides a bicycle with a speed of 10 m/s in the north to south direction. What is the direction in which she should hold her umbrella?

**5.** A man is running up hill with a velocity $(2\hat{i}+3\hat{j})\ m/s$ w.r.t. ground. He feels that the rain drops are falling vertically with velocity 4 m/s. If he runs down hill with same speed, find $v_{rm}$.

**6.** A body is projected with velocity $u_1$ from point A as shown in figure. At the same time another body is projected vertically upwards with a velocity $u_2$ from point B. What should be the value of $\frac{u_1}{u_2}$ for both bodies to collide.

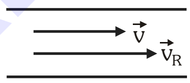

## 16. RIVER-BOAT (OR MAN) PROBLEM

A man can swim with velocity $\vec{v}$, i.e. it is the velocity of man w.r.t. still water.

If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$

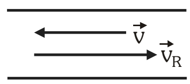
(i) If the swimming is in the direction of flow of water or along the downstream then

$$v_m = v + v_R$$



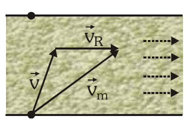

(ii) If the swimming is in the direction opposite to the flow of water or along the upstream then

$$v_m = v - v_R$$




(iii) If the man is crossing the river i.e. $\vec{v}$ and $\vec{v}_R$ are non collinear then use vector algebra.

$$\vec{v}_m = \vec{v} + \vec{v}_R$$



**To Cross a River**

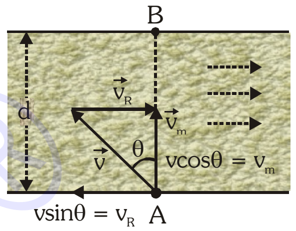
Minimum distance of approach
OR
Cross the river along shortest possible path
OR
Cross the river and reach a point just opposite to the starting path.

Cross the river in shortest possible time

**For shortest path :**

If a man wants to cross the river such that his "displacement should be minimum", it means he intends to reach just opposite point across the river. He should start swimming at an angle $\theta$ with the perpendicular to the flow of river towards upstream.

such that its resultant velocity $\vec{v}_m = (\vec{v} + \vec{v}_R)$ It is in the direction of displacement AB.



(for minimum displacement)

To reach at B $\qquad v \sin\theta = v_R \qquad \Rightarrow \qquad \sin\theta = \dfrac{v_R}{v}$

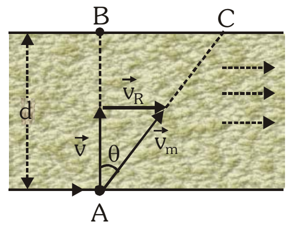
component of velocity of man along AB is $v\cos\theta$ $\qquad$ so time taken $\boxed{T = \dfrac{d}{v\cos\theta} = \dfrac{d}{\sqrt{v^2 - v_R^2}}}$

**For minimum time**

To cross the river in minimum time, the velocity along AB ($v\cos\theta$) should be maximum.

It is possible if $\theta = 0$, i.e. swimming should start perpendicular to water current.

Due to effect of river velocity man will reach at point C along resultant velocity, i.e. his displacement will not be minimum but time taken to cross the river will be minimum,



(for minimum time)

$$\boxed{t_{min} = \frac{d}{v}}$$

In time $t_{min}$ swimmer travels distance BC along the river with speed of river $v_R$ $\therefore$ $BC = t_{min}\, v_R$

distance travelled along river flow = drift of man = $t_{min}\, v_R = \dfrac{d}{v} v_R$

## Illustrations

**Illustration 51.**

A boat moves along the flow of river between two fixed points A and B. It takes $t_1$ time when going downstream and takes $t_2$ time when going upstream between these two points. What time it will take in still water to cover the distance equal to AB.

**Solution**

$$t_1 = \frac{AB}{v_b + v_R},\ t_2 = \frac{AB}{v_b - v_R} \qquad \text{or} \qquad v_b + v_R = \frac{AB}{t_1} \qquad \text{and} \qquad v_b - v_R = \frac{AB}{t_2}$$

$$\Rightarrow \quad 2v_b = \frac{AB}{t_1} + \frac{AB}{t_2} = AB\left(\frac{t_1 + t_2}{t_1 t_2}\right)$$

$$\text{or} \quad \left(\frac{2t_1t_2}{t_1 + t_2}\right) = \frac{AB}{v_b} = \text{time taken by the boat to cover AB}$$

**Illustration 52.**

A boat can be rowed at 5 m/s in still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

(a) In which direction must it be steered to cross the river perpendicular to current?

(b) How long will it take to cross the river in a direction perpendicular to the river flow?

(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

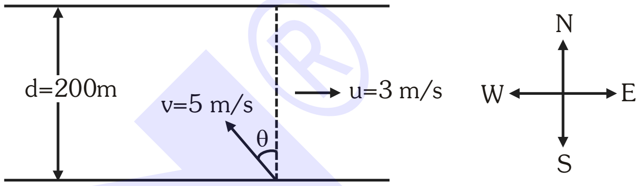
**Solution**

(a) To cross the river perpendicular to current i.e. along shortest path

$$\sin\theta = \frac{u}{v} = \frac{3}{5} \Rightarrow \theta = 37°$$

(b) Time taken by boat,

$$t = \frac{d}{v\cos\theta} = \frac{200}{5\times\frac{4}{5}} = 50s$$




(c) To cross the river in minimum time, θ = 0°

Therefore $t_{min} = \frac{d}{v} = \frac{200}{5}s = 40s$

Drift = $u(t_{min})$ = 3(40)m = 120 m

## BEGINNER'S BOX-11

**1.** A man can swim at a speed 2 m/s in still water. He starts swimming in a river at an angle 150° to the direction of water flow and reaches the directly opposite point on the opposite bank.

(a) Find the speed of flowing water.

(b) If width of river is 1 km then calculate the time taken to cross the river.

**2.** 2 km wide river flowing at the rate of 5 km/hr. A man can swim in still water with 10 km/hr. He wants to cross the river along the shortest path. Find –

(a) in which direction should the person swim.

(b) crossing time.

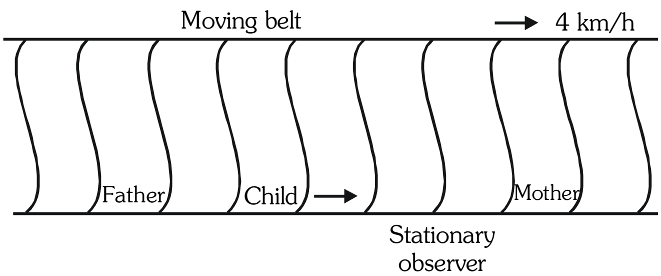
**3.** A child runs to and fro with a speed 9 km/h (with respect to the belt) on a long horizontally moving belt (fig.) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km/h. For an observer on stationary platform outside, what is the

(a) speed of the child while running in the direction of motion of the belt?

(b) speed of the child while running opposite to the direction of motion of the belt?

(c) time taken by the child in (a) and (b)?

Which of the answers will alter if motion is viewed by one of the parents?



# ANSWER'S KEY

## BEGINNER'S BOX-1

**1.** $7\pi r$, 2r **2.** 400 m for each, B.

**3.** 19m, 13m **4.** $(40\pi)$m, 80m from A to B

**5.** (A) 110m, (B) 50m, 37°N of E

## BEGINNER'S BOX-2

**1.** (i) 40 km/hr (ii) 32.5 km/hr
(iii) 1040 km/hr (iv) 1040 km/hr

**2.** 4 m/s **3.** $5\times10^{-3}$ cm/s, $2\times10^{-4}$ cm/s

**4.** 0 m/s, 20 m/s **5.** (a) 12.5 m/s (b) 25 m/s

**6.** 2 m/s **7.** 49.3 km/h, 21.4 km/h

## BEGINNER'S BOX-3

**1.** $\frac{10}{\pi}$ m/s$^2$

**2.** (a) 68, (b) 14, (c) 33, (d) 14

## BEGINNER'S BOX-4

**1.** $s_2 = 8s_1$ **2.** $\sqrt{\frac{u^2+v^2}{2}}$

**3.** $\frac{n^2}{2n-1}$ **4.** 3.06 ms$^{-2}$ ; 11.4 s

**5.** $ut+\frac{u^2}{2a}$ **6.** 0.218 m/s$^2$

## BEGINNER'S BOX-5

**1.** 1 : 3

**2.** (i) 120m (ii) 0 (iii) 20m/s (iv) 0

**3.** (a) A, B, (b) A,B, (c) B, A, (d) Same, (e) B, A, once

**4.** (i) 37 m (ii) 3.7 m/s (iii) Part BC, a = 8 m/s$^2$
(iv) 2 m/s$^2$

**5.** 625 m

## BEGINNER'S BOX-6

**1.** 1 s

**2.** (a) 75m, (b) 40m/s, (c) 85m, (d) 17m/s, 15m/s

**3.** (i) 70.3m (ii) 5 s

**4.** (a) 600 m/s, (b) 18 km, (c) $(2+\sqrt{2})$ min, (d) 36 km

**5.** 125 m

**6.** 45 m

**7.** (a) Vertically downwards;
(b) zero velocity, acceleration of 9.8 m/s$^2$ downwards.
(c) x > 0 (upward and downward motion) v < 0 (upward), v > 0 (downward), a > 0 throughout;
(d) 44.1 m, 6s.

**8.** 125m

**9.** 4m & 1m from top

## BEGINNER'S BOX-7

**1.** (a) 1s, (b) 5m, (c) 34.64 m, (d) 2s

**2.** 50 m. **3.** (a) $\tan^2\alpha$ (b)1

**4.** 15 m & 65 m **5.** 3 km

**7.** 1 **8.** 148.32 m

## BEGINNER'S BOX-8

**1.** (i) 10s, (ii) 980m, (iii) $\beta = 45°$

**2.** 60 m/s

**3.** 700 m/s

**4.** 10 meter.

## BEGINNER'S BOX-9

**1.** 3.75 m/s & 1.25 m/s

**2.** 1250 m.

**3.** 10 sec, 10 sec

## BEGINNER'S BOX-10

**1.** $5\sqrt{3}\hat{i}-5\hat{j}$, E– 30° - S

**2.** In north direction 10m/s

**3.** $20\sqrt{2}$ km/hr S – E, $5\sqrt{2}$ km, 15 min.

**4.** $\alpha = \tan^{-1}\left(\frac{1}{3}\right)$ =18.4° from vertical towards south

**5.** $\sqrt{20}$ m/s

**6.** $\frac{2}{\sqrt{3}}$

## BEGINNER'S BOX-11

**1.** (a) $\sqrt{3}$ m/s
(b) 1000s.

**2.** (a) Direction from Down Stream = 120°
(b) $\frac{2}{5\sqrt{3}}$ hr.

**3.** (a) 13 km/h, (b) 5 km/h, (c) 20s
If the motion is viewed by one of the parents answers to (a) and (b) are altered while answer to (c) remain unchanged.