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PRE-MEDICAL

PHYSICS

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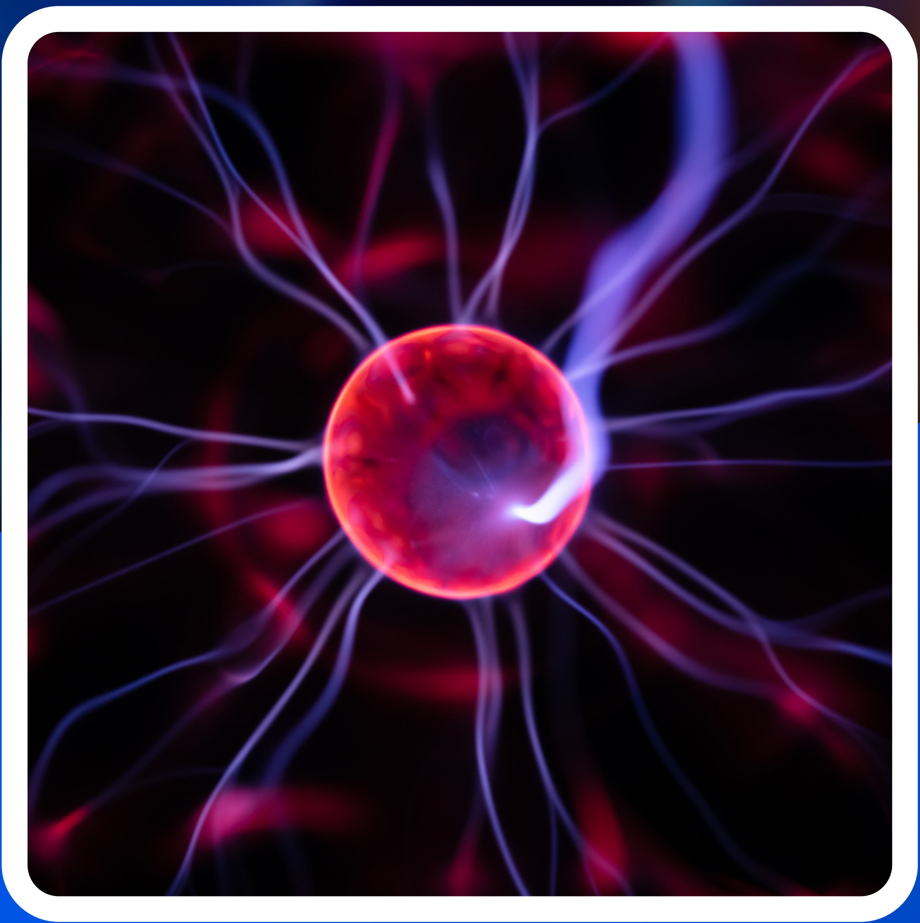

STUDY MATERIAL

Electrostatics

ENGLISH MEDIUM

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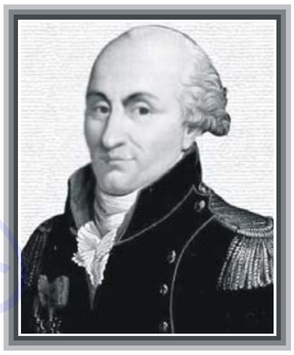
Charles Augustin de Coulomb (1736 – 1806)



Coulomb, a French physicist, began his career as a military engineer in the west Indies. In 1776, he returned to Paris and retired to a small estate to do his scientific research. He invented a torsion balance to measure the quantity of a force and used it for determination of forces of electric attraction or repulsion between small charged spheres. He thus arrived in 1785 at the inverse square law relation, now known as Coulomb's law. The law had been anticipated by Priestley and also by Cavendish earlier, though Cavendish never published his results. Coulomb also found the inverse square law of force between unlike and like magnetic poles.

Karl Friedrich Gauss (1777-1855)

He was a child prodigy and was gifted in mathematics, physics, enginerring, astronomy and even land surveying. The properties of numbers fascinated him, and in his work he anticipated major mathematical development of later times. Along with Wilhelm Welser, he built the first electric telegraph in 1833. His mathematical theory of curved surface laid the foundation for the later work of Riemann. In physics, his (Gauss') law is formulated in 1835, but was not published until 1867.

ELECTROSTATICS

1. ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body is the causes of net charge on a body.

Types of charge :

(i) Positive charge : It is the deficiency of electrons as compared to protons.

(ii) Negative charge : It is the excess of electrons as compared to protons.

SI unit of charge : ampere × second i.e. coulomb; Dimension : [A T]

Practical units of charge are ampere × hour (=3600 C) and faraday (=96500 C)

- Millikan determined quanta of charge and estimated it to be equal to charge of electron.
- 1 C = 3×10^9 stat coulomb, 1 absolute - coulomb =10 C, 1 Faraday = 96500 C.

Note : Charge of fundamental particles (i.e. electron, proton etc.) is their internal characteristic while charge on a body depends on the number of protons & electrons inside the body.

1.1 Specific Properties of Charge

- **Charge is a scalar quantity :** It represents excess or deficiency of electrons.
- **Charge is transferable :** If a charged body is put in contact with an another body, then charge can be transferred to another body.
- **Charge is always associated with mass**

  Charge cannot exist without mass though mass can exist without charge.
  - So the presence of charge itself is a convincing proof of existence of mass.
  - The mass of a body changes after being charged.
  - When a body is given a positive charge, its mass decreases.
  - When a body is given a negative charge, its mass increases.
- **Charge is quantised**

  The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e. Thus charge q of a body is always given by

  $$\mathbf{q = ne} \qquad n = \text{positive or negative integer}$$

  The quantum of charge is the charge that an electron or proton carries.

  Note : Charge on a proton = (–) charge on an electron = 1.6×10^{-19} C
- **Charge is conserved**

  In an isolated system, total charge does not change with time though individual charge may change, i.e. charge can neither be created nor destroyed. Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear. No exceptions to the rule have ever been found.
- **Charge is invariant**

  Charge is independent of frame of reference. i.e. charge on a body does not change whatever be its speed.

Accelerated charge radiates energy

v = 0 (i.e. at rest) $\oplus$ Q produces only $\vec{E}$ (electric field)	v = constant $\oplus\rightarrow$ Q produces both $\vec{E}$ and $\vec{B}$ (magnetic field) but no radiation	v ≠ constant (i.e. time varying) $\oplus\rightarrow$ Q produces $\vec{E}$, $\vec{B}$ and radiates energy

1.2 Conductors & Insulators

Conductors : Materials in which the outer electrons of each atom or molecule are weakly bound and these electrons are almost free to move throughout the body of the material are known as conductors.

Insulators : Materials in which all the electrons are tightly bound to their respective atoms or molecules are known as insulators. In insulators there are very few free electrons. Such materials are also called dielectrics.

1.3 Methods of Charging

- **Friction**

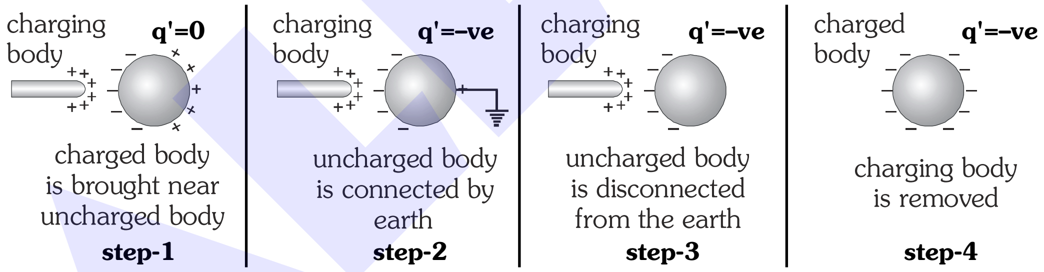
If we rub one body with another body, electrons are transferred from one body to the other.

Positive charge	Negative charge
Glass rod	Silk cloth
Woollen cloth	Rubber shoes, Amber, Plastic object
Dry hair	Comb
Cat skin	Ebonite rod
Note : Clouds get charged due to friction	

- **Electrostatic induction**

If a charged body is brought near a neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'. Hence induction is a phenomena of **redistribution** of charge on a body when any other charged body is brought near it.

Charging a body by induction (in four successive steps)



In case of induction it is worth noting that :

(i) Inducing body neither gains nor loses charge.

(ii) The nature of induced charge is always opposite to that of inducing charge.

(iii) Induced charge can be lesser or equal to inducing charge (but never greater).

(iv) Induction takes place only in bodies (either conducting or non conducting) and not in particles.

- **Conduction**

The process of transfer of charge by contact of two conducting bodies is known as conduction.

If a charged conducting body is put in contact with uncharged conducting body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

The charged body loses some of its charge (which is equal to the charge gained by the uncharged body).

GOLDEN KEY POINTS

- Charge differs from mass in the following aspects :
  - (i) In SI units, charge is a derived physical quantity while mass is a fundamental quantity.
  - (ii) Charge is always conserved but mass is not.
  - (iii) Charge cannot exist without mass but mass can exist without charge.
  - (iv) Charges are of two type (positive and negative) but mass is of only one type (positive).
  - (v) For a moving charged body, mass increases while charge remains constant.
- True test of electrification is repulsion and not attraction as attraction can take place between a charged and an uncharged body or between two similarly charged bodies.
- For a non relativistic (i.e. $v << c$) charged particle, specific charge $\frac{q}{m}$ = constant.
- Charge can be detected and/or measured with the help of gold-leaf electroscope, electrometer, voltameter etc.

Illustrations

Illustration 1.

When a piece of polythene is rubbed with wool, a charge of -2×10^{-7} C is developed on polythene. What is the amount of mass, which is transferred to the polythene ?

Solution

From Q = ne, the number of electrons $n = \frac{Q}{e} = \frac{2\times10^{-7}}{1.6\times10^{-19}} = 1.25 \times 10^{12}$

$\therefore$ mass of transferred electrons $= n \times$ mass of one electron

$= 1.25 \times 10^{12} \times 9.1 \times 10^{-31} = 11.38 \times 10^{-19}$ kg

Illustration 2.

10^{12} α–particles (nuclei of helium) fall per second on a neutral sphere; calculate the time in which the sphere gets charged by 2 μC.

Solution

Number of α–particles falling in t seconds = $10^{12}t$

Charge on α–particle = $+2e$, So charge incident in time t = $(10^{12}t).(2e)$

$\because$ Given charge is 2 μC

$\therefore$ $2 \times 10^{-6} = (10^{12}t)(2e)$

$\Rightarrow t = \frac{10^{-18}}{1.6\times10^{-19}} = 6.25$ s

BEGINNER'S BOX-1

1. In a neutral sphere, 5×10^{21} electrons are present. If 10 percent electrons are removed, then calculate the charge on the sphere.

2. Calculate the number of electrons in 100 grams of CO_2.

3. Can a body have a charge of (a) 0.32×10^{-18} C (b) 0.64×10^{-20} C (c) 4.8×10^{-21} C ?

4. A glass tumbler contains 1 billion amoeba and two sodium ions are there on the body of each amoeba. Find out the charge contained in the glass.

5. How many electrons should be removed from a conductor so that it acquires a positive charge of 3.5 nC ?

6. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves made up of more elementary units called "quarks". A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge +(2/3)e, and the 'down' quark (denoted by d) of charge (–1/3)e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

7. A polythene piece rubbed with wool is found to have a negative charge of 3.2×10^{-7} C. Find the

(a) number of electrons transferred.

(b) mass gained by the polythene.

8. Two identical metal spheres A and B placed in contact are supported on insulating stand. What kinds of charge will A and B develop when a negatively charged ebonite rod is brought near A ?

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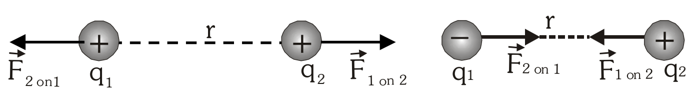
2. COULOMB'S LAW

The electrostatic force of interaction between two static point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

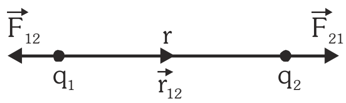
Consider two points charges q_1 and q_2 separated by a distance r. Let F be the electrostatic force between these two charges. According to Coulomb's law,

$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$



$F = \frac{kq_1q_2}{r^2}$ where $\left[k = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \,N-m^2/C^2\right]$ k = Coulomb's constant or electrostatic force constant

2.1 Coulomb's law in vector form

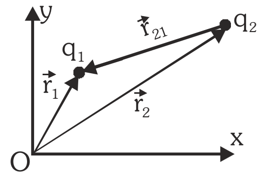


$\vec{F}_{12}$ = force on q_1 due to q_2 = $\frac{kq_1q_2}{r^2}\hat{r}_{21}$

$\vec{F}_{21} = \frac{kq_1q_2}{r^2}\hat{r}_{12}$ (here $\hat{r}_{12}$ is unit vector from q_1 to q_2)

2.2 Coulomb's law in terms of position vector

$$\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$



2.3 Principle of superposition

Coulomb's force is a two body interaction, implying that electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on a charged particle due to a number of point charges is the resultant of forces due to individual point charges, i.e.,

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots \vec{F}_{1N}$$

When a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges individually,

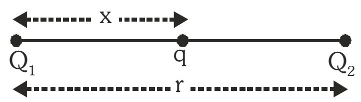
$$\vec{F} = \frac{kq_0q_1}{r_1^2}\hat{r}_1 + \frac{kq_0q_2}{r_2^2}\hat{r}_2 + \cdots + \frac{kq_0q_i}{r_i^2}\hat{r}_i + \ldots \frac{kq_0q_n}{r_n^2}\hat{r}_n$$

In vector form $\vec{F} = kq_0 \sum_{i=1}^{n} \frac{q_i}{r_i^2}\hat{r}_i$.

2.4 Equilibrium of charged particles

In equilibrium, net electric force on every charged particle is zero. The equilibrium of a charged particle, under the action of Coulombian forces alone can never be stable.

- **Equilibrium of three point charges**



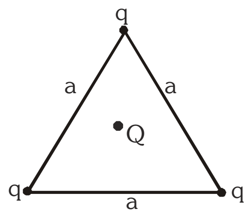
(i) Two charges must be like in nature as $F_q = \frac{KQ_1q}{x^2} - \frac{KQ_2q}{(r-x)^2} = 0$

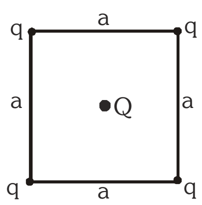
(ii) Third charge should be unlike in nature as $F_{Q_1} = \frac{KQ_1Q_2}{r^2} + \frac{KQ_1q}{x^2} = 0$

Therefore $x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}}r$ and $q = \frac{-Q_1Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2}$

- **Equilibrium of symmetric geometrical point charged system**

Value of Q at centre for which system to be in state of equilibrium

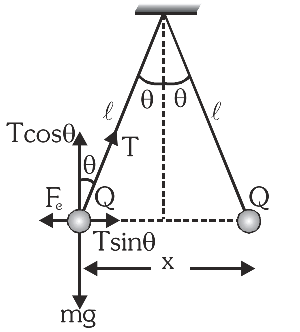
(a)


(b)


(i) For situation (a) $Q = \frac{-q}{\sqrt{3}}$ (ii) For situation (b) $Q = \frac{-q(2\sqrt{2}+1)}{4}$

- **Equilibrium of suspended point charge system**

For equilibrium position

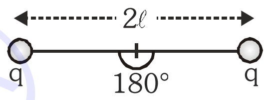


$$T\cos\theta = mg \quad \text{and} \qquad T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

- If θ is small then $\tan\theta \approx \sin\theta = \dfrac{x}{2\ell}$

$$\Rightarrow \frac{x}{2\ell} = \frac{kQ^2}{x^2 mg} \Rightarrow x^3 = \frac{2kQ^2\ell}{mg} \Rightarrow x = \left[\frac{Q^2\ell}{2\pi\epsilon_0\, mg}\right]^{\frac{1}{3}}$$

- If the whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$) then $T = F_e = \dfrac{kq^2}{4\ell^2}$



GOLDEN KEY POINTS

- Coulomb's law is based on physical observations and is not logically derivable from any other concept. Experiments reveal its universal nature till today.
- The law is analogous to Newton's law of gravitation : $F = G\dfrac{m_1m_2}{r^2}$ with the difference that :
  - (a) Electric force between charged particles is much stronger than gravitational force, i.e., $F_E >> F_G$. Consequently F_G is neglected when both F_E and F_G are present.
  - (b) Electric force can be attractive or repulsive while gravitational force is always attractive.
  - (c) Electric force depends on the nature of medium between the charges while gravitational force does not.
- The force is conservative, i.e., work done in moving a point charge round a closed path under the action of Coulomb's force is zero.
- The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its 'centre' as this is true only for spherically charged bodies, that too for external points.
- Electric force between two charges does not depend on neighbouring charges.
- The net Coulomb's force between two charged particles in free space and in an infinitely extending medium are

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \text{ and } F' = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}. \text{ So } \frac{F}{F'} = \frac{\varepsilon}{\varepsilon_0}$$

Dielectric constant (K) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium extending infinitely.

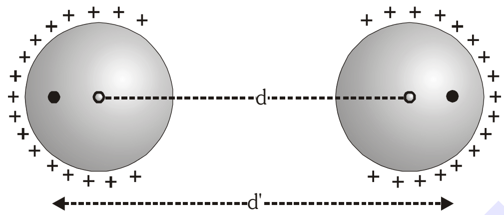
- Although the net electric force on both particles change in the presence of dielectric but force due to one charged particle on another charged particle does not depend on the medium between them.

Illustrations

Illustration 3.

The force of repulsion between two point charges is F, when these are at a distance of 1 m apart. Now they are replaced by conducting spheres each of radii 25 cm having the same charge as that of point charges. The distance between their centres is 1 m, then compare the force of repulsion in the two cases.

Solution



In 2nd case due to mutual repulsion the effective distance between their respactive centres of charge will be increased (d' > d), so force of repulsion decreases as $F \propto \frac{1}{d^2}$

Illustration 4.

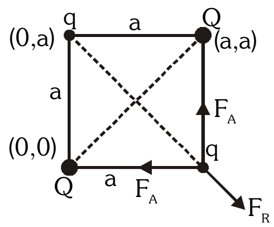
For the system shown in figure, find Q for which the resultant force on q is zero.

Solution

For force on q to be zero, charges q and Q must be of opposite nature.

Net attractive force on q due to both charges each of magnitude

Q = Repulsive force due to q



$$\sqrt{2}\ F_A = F_R \quad \Rightarrow \quad \sqrt{2}\ \frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2} \quad \Rightarrow \quad q = 2\sqrt{2}\ Q$$

$$\text{Hence} \quad q = -2\sqrt{2}\,Q \quad \Rightarrow \quad Q = \frac{-q}{2\sqrt{2}}$$

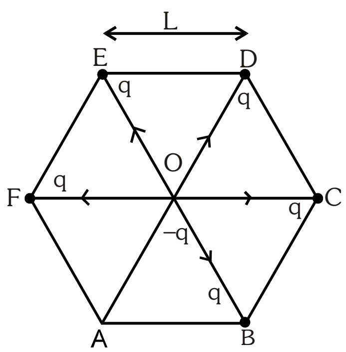
Illustration 5.

Five point charges, each of value +q are placed on five vertices of a regular hexagon of side L. What is the magnitude of the force on a point charge of value –q placed at the centre of the hexagon?

Solution

If there had been a sixth charge +q at the vacant vertex of hexagon then force due to all the six charges on –q at O would have been zero (as the forces forming a pair due to charges lying opposite to each other will balance each other).

If $\vec{f}$ is the force due to sixth charge and $\vec{F}$ due to remaining five charges.

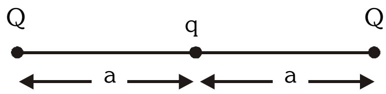


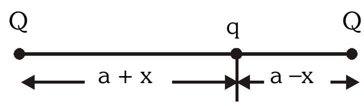
$$\vec{F} + \vec{f} = \vec{0} \Rightarrow \vec{F} = -\vec{f} \Rightarrow F = f = \frac{1}{4\pi\varepsilon_0}\frac{q \times q}{L^2} = \frac{q^2}{4\pi\varepsilon_0 L^2}$$

llustration 6.

Two charges each of magnitude Q are fixed at 2a distance apart and a third charge q of mass m is placed at the mid point of the line joining the two charges; all charges being of the same sign. If q charge is slightly displaced from its position along the line and released then determine its time period.

Solution





If q is displaced by x from equilibrium position then net force on it

$$F = \frac{kQq}{(a+x)^2} + \frac{-kQq}{(a-x)^2} = \frac{Qq}{4\pi\epsilon_0}\frac{(-4ax)}{(a^2-x^2)^2} = \frac{-Qqa}{\pi\epsilon_0\ (a^2-x^2)^2}x$$

If x << a , then $F = \frac{-Qqx}{a^3\pi\ \epsilon_0}$

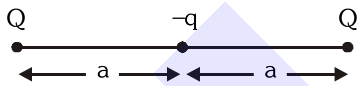
$\Rightarrow$ $F \propto -x$, so the particle will undergo S.H.M. and hence

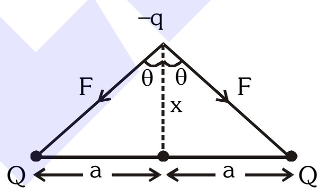
$$\omega^2 = \frac{Qq}{ma^3\pi\varepsilon_0} \Rightarrow T = 2\pi\sqrt{\frac{ma^3\pi\varepsilon_0}{Qq}}$$

Illustration 7.

Two charges each of magnitude Q are fixed at 2a distance apart. A third charge (–q of mass 'm') is placed at the mid point of the two charges; now –q charge is slightly displaced perpendicular to the line joining the charges then find its time period.

Solution





$$F_{net} = -2F\cos\theta = -\frac{2kQq}{(a^2+x^2)}\frac{x}{\sqrt{a^2+x^2}}$$

$$\Rightarrow F_{net} = \frac{-2Qqx}{4\pi\ \epsilon_0\ (a^2+x^2)^{3/2}}$$ if x << a, then

$$F_{net} = \frac{-Qqx}{2\pi\ \epsilon_0\ a^3}$$

Comparing with $F_{net} = -kx$

Here, $k = \frac{Qq}{2\pi\epsilon_0 a^3}$

Now, $T = 2\pi\sqrt{\frac{m}{k}}$

Hence, $T = 2\pi\sqrt{\frac{m2\pi\ \epsilon_0\ a^3}{Qq}}$

BEGINNER'S BOX-2

1. Two identical metal spheres carry charges of + q and – 2q respectively. When the spheres are separated by a large distance r, the force between them is F. Now the spheres are allowed to touch and then moved back to the same separation. Find the new force of repulsion between them.

2. The electrostatic force of repulsion between two positive ions carrying equal charges is approximately 3.7×10^{-9} N, when their separation is 5 Å. How many electrons are missing from each ?

3. Two identical particles each of mass M and charge Q are placed a certain distance apart. If they are in equilibrium under mutual gravitational and electric force then calculate the order of $\frac{Q}{M}$ in SI units.

4. The force between two point charges is 100 N in air. Calculate the force if the distance between them is increased by 50%.

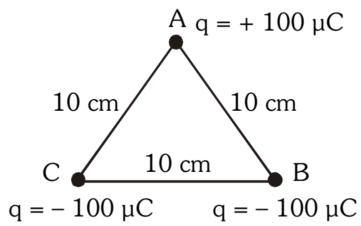
5. Two neutral insulating small spheres are rubbed against each other and are then kept 4 m apart. If they attract each other with a force of 3.6 N, then

(i) calculate the charge on each sphere, and

(ii) calculate the number of electrons transferred from one sphere to the other during rubbing.

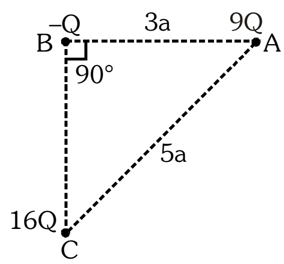
6. Two equal point charges $Q = +\sqrt{2}\ \mu C$ are placed at each of the two opposite corners of a square and equal point charges q at each of the other two corners. What must be the value of q so that the resultant force on Q is zero ?

7. In the given figure three point charges are situated at the corners of an equilateral triangle of side 10 cm. Calculate the resultant force on the charge at B. What is its direction?

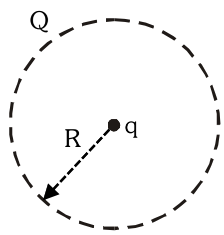


8. Two positively charged particles, each of mass 1.7×10^{-27} kg and carrying a charge of 1.6×10^{-19} C are placed at a distance d apart. If each experiences a repulsive force equal to its weight, find the value of d.

9. ABC is a right angled triangle. Calculate the magnitude of force on charge –Q.



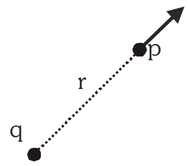
10. Charge Q of mass m revolves around a point charge q due to electrostatic attraction. Show that its period of revolution is given by $T^2 = \frac{16\pi^3 \epsilon_0 m R^3}{Qq}$.



3. ELECTRIC FIELD

In order to explain 'action at a distance', i.e., 'force without contact' between charges it is assumed that a charge or charge distribution produces a field in space surrounding it. So the region surrounding a charge (or charge distribution) in which its electrical effects are perceptible is called the electric field of the given charge. Electric field at a point is characterized either by a vector function of position $\vec{E}$ called 'electric intensity' or by a scalar function of position V called 'electric potential'. The electric field in a certain space is also visualized graphically in terms of 'lines of force.' So electric field intensity, potential and lines of force are different ways of describing the same field.

3.1 Intensity of electric field due to point charge



Electric field intensity is defined as force on unit test charge.

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{kq}{r^3}\vec{r}$$

Here $\lim\limits_{q_0 \to 0}$ represent that q_0 is very small charge

Note : Test charge (q_0) is a fictitious charge that exerts no force on nearby charges but experiences forces due to them.

3.2 Properties of electric field intensity

(i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.

(ii) Electric field due to positive charge is always away from it while due to negative charge it is always towards it.

(iii) Its unit is newton/coulomb.

(iv) Its dimensional formula is $[MLT^{-3}A^{-1}]$

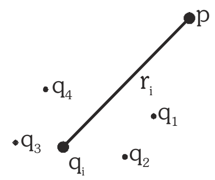
(v) Force on a point charge is in the same direction as that of electric field; on positive charge and in opposite direction on a negative charge.

$$\vec{F} = q\vec{E}$$

(vi) It obeys the superposition principle i.e., the field intensity due to a charge distribution is the vector sum of the field intensities due to individual charges at any given point.

3.3 Electric field intensities due to various charge distributions

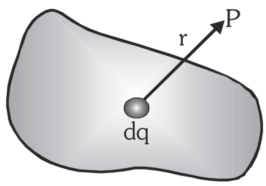
Discrete distribution of charge



By principle of superposition, intensity of electric field due to i[th] charge $\vec{E}_{ip} = \frac{kq}{r_i^3}\vec{r}_i$

$\therefore$ Net electric field due to whole distribution of charge $\vec{E}_p = \sum\limits_{i=1} \vec{E}_i$

Continuous distribution of charge



Treating a small element as a point charge $\vec{E} = \frac{1}{4\pi \epsilon_0}\int \frac{dq}{r^3}\vec{r}$

Due to linear charge distribution $E = k\int_{\ell} \frac{\lambda d\ell}{r^2}$ [λ = charge per unit length]

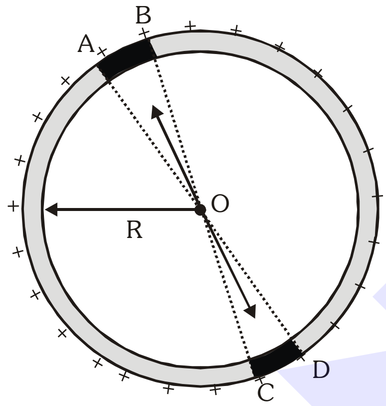
$k\int_{s} \frac{\sigma ds}{r^2}$ [σ = charge per unit area]

Due to volume charge distribution $E = k\int_{V} \frac{\rho dV}{r^2}$ [ρ =charge per unit volume]

3.4 Electric field due to a uniformly charged ring

- **At its centre :**

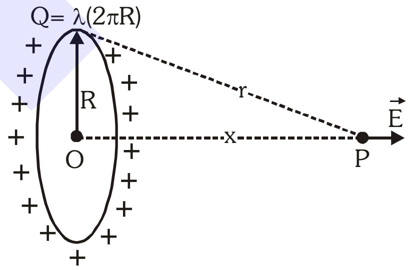
  Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field due to an identical segment diametrically opposite to it. The electric field strength at the centre due to segment AB is cancelled by that due to segment CD. Thus net electric field strength at the centre of a uniformly charged ring is $E_0=0$



- **At an axial point :**

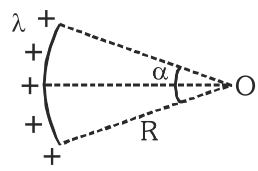
$$E_P = \frac{\lambda R\ x}{2\in_0\left[R^2+x^2\right]^{3/2}} = \frac{kQx}{r^3},\ \text{ where } r=\sqrt{R^2+x^2}$$

or $$E_P = \frac{kQx}{\left(R^2+x^2\right)^{3/2}}$$; At centre of ring x = 0 so $E_0 = 0$



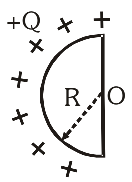
- **Segment of ring : (charged arc)**

$$E_0 = \frac{2k\lambda}{R}\sin\frac{\alpha}{2}\ \left[\text{Where } \lambda = \frac{Q}{R\alpha}\right]$$

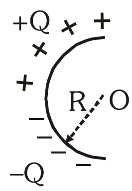


- **Uniformly charged semicircular ring :**

(i) $E_0 = \frac{2k\lambda}{R} = \frac{Q}{2\pi^2\in_0 R^2}$ $\because$ $\lambda = \frac{Q}{\pi R}$



(ii) $E_0 = \frac{Q}{\pi^2\in_0 R^2}$



GOLDEN KEY POINTS

- Charged particle always experiences a force in an electric field whether it is at rest or in motion.
- Electric field decreases in the presence of a dielectric and becomes $\frac{1}{\in_r}$ times of its value in free space.
- Test charge is an infinitely small (+ ve) charge. $\vec{E} = \frac{\vec{F}_{test}}{\text{test charge}}$
- If identical charges are placed on each vertices of a regular polygon, then $\vec{E}$ = zero at centre.

Illustrations

Illustration 8.

A point charge of 0.009 μC is placed at the origin. Calculate the intensity of electric field due to this point charge at point $(\sqrt{2},\sqrt{7},0)$ m.

Solution

$$\vec{E} = \frac{q\vec{r}}{4\pi\epsilon_0 r^3} \;;\; \text{where } \vec{r} = x\hat{i} + y\hat{j} = \sqrt{2}\,\hat{i} + \sqrt{7}\,\hat{j}$$

$$\vec{E} = \frac{9\times10^{9}\times9\times10^{-9}(\sqrt{2}\,\hat{i}+\sqrt{7}\,\hat{j})}{(3)^3} = (3\sqrt{2}\,\hat{i}+3\sqrt{7}\,\hat{j})\ \text{NC}^{-1}.$$

Illustration 9.

Calculate the electric field at origin due to infinite number of charges as shown in figures (a) and (b) below.

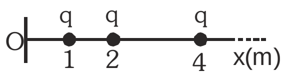


figure (a)

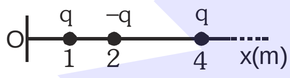


figure (b)

Solution

(a) $$E_0 = kq\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+-----\right] = \frac{kq.1}{(1-1/4)} = \frac{4kq}{3} \quad [\because S_\infty = \frac{a}{1-r},\ a = 1 \text{ and } r = \frac{1}{4}]$$

(b) $$E_0 = kq\left[\frac{1}{1}-\frac{1}{4}+\frac{1}{16}------\right] = \frac{kq.1}{(1-(-1/4))} = \frac{4kq}{5}$$

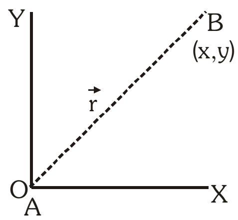
Illustration 10.

A point charge Q is placed at a point A (0,0). Calculate the electric field at B(x,y) in vector form.

Electric field is

$$E = \frac{kQ}{r^2}$$

In vector form, $\vec{E} = \left(\frac{kQ}{r^2}\right)\hat{r} = \frac{kQ\vec{r}}{r^3}\left(\because \hat{r} = \frac{\vec{r}}{r}\right)$



Now, $\vec{r} = \vec{r}_B - \vec{r}_A = x\hat{i} + y\hat{j}$ and $r^2 = x^2 + y^2$

so $$\vec{E} = \frac{kQ(x\hat{i}+y\hat{j})}{(x^2+y^2)^{3/2}}$$

Illustration 11.

A charged particle is kept in equilibrium in an electric field between the plates in the millikan oil drop experiment. If the direction of the electric field between the plates is reversed, then calculate the acceleration of the charged particle.

Solution

Let mass of the particle = m, charge on particle = q

intensity of electric field between the plates = E

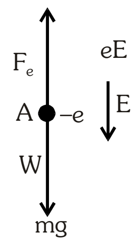
initially mg = qE (for equillibrium)

After reversing the field ma = mg + qE $\Rightarrow$ ma = 2mg $\therefore$ acceleration of particle $\Rightarrow$ a = 2g

Illustration 12.

Calculate the electric field intensity E which would be just sufficient to balance the weight of an electron. If this electric field is produced by a second electron located below the first one what would be the distance between them?

[Given : $e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg and $g = 9.8$ m/s^2]



(A)

Solution

As force on a charge e in an electric field E is

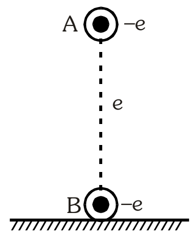
$$F_e = eE$$

so according to the given problem,

$$F_e = W \Rightarrow eE = mg$$

$$\Rightarrow \quad E = \frac{mg}{e} = \frac{9.1 \times 10^{-31} \times 9.8}{1.6 \times 10^{-19}} = 5.57 \times 10^{-11}\ \text{Vm}^{-1}$$

As this intensity E is produced by another electron B located at a distance r below A



(B)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{e}{r^2} \Rightarrow r = \sqrt{\frac{e}{4\pi\varepsilon_0 E}} \quad \text{So,} \quad r = \left[\frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{5.57 \times 10^{-11}}\right]^{\frac{1}{2}} \simeq 5\ \text{m}.$$

BEGINNER'S BOX-3

1. Two charges of value 2 μC and – 50 μC are placed 80 cm apart. Calculate the distance of the point from the smaller charge where the intensity is zero.

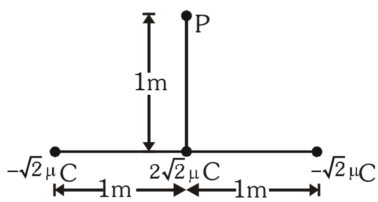
2. A charged particle of mass 2 mili gram remains freely in air in an electric field of strength 4 N/C directed upward. Calculate the charge and determine its nature (g = 10 m/s^2).

3. How many electrons should be added or removed from a neutral body of mass 10 mili gram so that it may remain stationary in air in an electric field of strength 100 N/C directed upwards (g = 10 m/s^2) ?

4. Work out the magnitude and direction of field at point P, when a charge of 2 μC experiences an electrical force of $5 \times 10^{-2}\ \hat{j}$ N at point P.

5. Two charges 4 μC and 36 μC are placed 60 cm apart. At what distance from the larger charge is the electric field intensity is zero ?

6. Three charges of respective values $-\sqrt{2}\,\mu C$, $2\sqrt{2}\,\mu C$ and $-\sqrt{2}\,\mu C$ are arranged along a straight line as shown in the figure. Calculate the total electric field intensity due to all three charges at the point P.



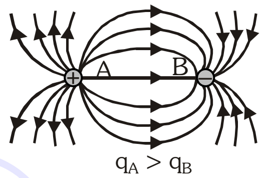
4. ELECTRIC FIELD LINES AND ELECTRIC FLUX

4.1 Field Lines

A field line (in an electric field) is an imaginary line drawn in such a way that its direction at any point (i.e. the direction of its tangent) is the same as the direction of the field at that point.

Field lines can also be used for comparing the magnitude of electric field intensity. The intensity of the field is proportional to number of field lines crossing unit area at right angles.

Electrostatic lines of force have the following properties :



(i) Imaginary and smooth curves.

(ii) Can never cross each other.

(iii) Can never form closed loops.

(iv) The number of lines originating from or terminating on a charge is proportional to the magnitude of charge.

(v) Electric field lines end or start normally, at the surface of a conductor.

(vi) If there is no electric field there will be no electric field lines.

(vii) Electric field lines normal to unit area is proportional to magnitude of intensity; crowded lines represent strong field while distant lines represent weak field.

(viii) Tangent to the field line at a point in an electric field gives the direction of electric field.

4.2 Electric flux (ϕ) :

The electric flux ϕ of any electric field $\vec{E}$ through a surface $\vec{S}$ is defined by the equation :

$$\boxed{\phi = \oint \vec{E}\cdot\vec{S}}$$

The flux linked with any surface is proportional to the number of field lines crossing any area at right angles.

For open surface $\phi_o = \int d\phi = \int \vec{E}.d\vec{s}$

For closed surface:
- $\phi_c = \oint \vec{E}.d\vec{s}$ (by definition of flux)
- $\phi_c = \frac{q_{net}}{\epsilon_o}$ or $4\pi k q_{net}$ (Gauss' law)

$\vec{S}$ is always normal to the surface and points outwards.

(i) Electric flux is a scalar quantity

(ii) Units : (V-m) and N-m^2/C, Dimensions : $[ML^3T^{-3}A^{-1}]$

(iii) The value of ϕ corresponding to a closed surface does not depend upon the distribution of charges and the distance between them inside the closed surface.

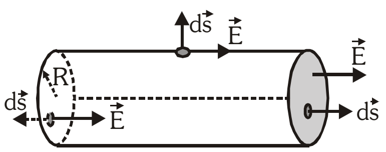
The value of ϕ is zero in the following circumstances :

(a) If a dipole is (or many dipoles are) enclosed by a closed surface.

(b) Magnitude of (+ve) and (-ve) charges are equal inside a closed surface.

(c) If no charge is enclosed by the closed surface.

(d) Incoming flux (- ve) = outgoing flux (+ve)



$\phi_{in} = -\pi R^2 E$ and $\phi_{out} = \pi R^2 E$ $\Rightarrow$ $\phi_{total} = 0$

4.3. Gauss' Theorem

The total flux linked with a closed surface is $\frac{1}{\epsilon_0}$ times the charge enclosed by the closed surface (gaussian surface) i.e. $\boxed{\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}}$

This law is suitable for symmetrical charge distribution and valid for all vector fields obeying inverse square law.

4.4 Characteristics of Gaussian surface & Important points regarding Gauss' law

(i) Flux through Gaussian surface is independent of its shape.

(ii) Flux through Gaussian surface depends only on charges present inside it.

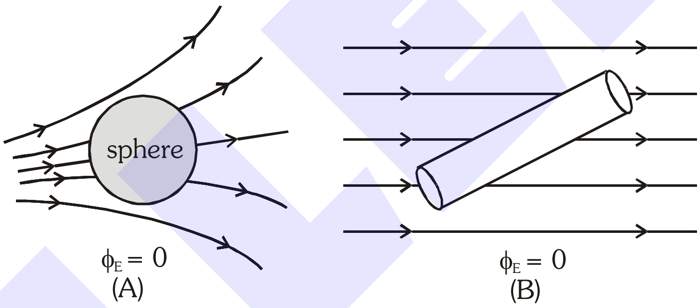
(iii) Flux through Gaussian surface is independent of position of charges inside it.

(iv) Electric field intensity at the Gaussian surface is due to all the charges present (inside as well as outside)

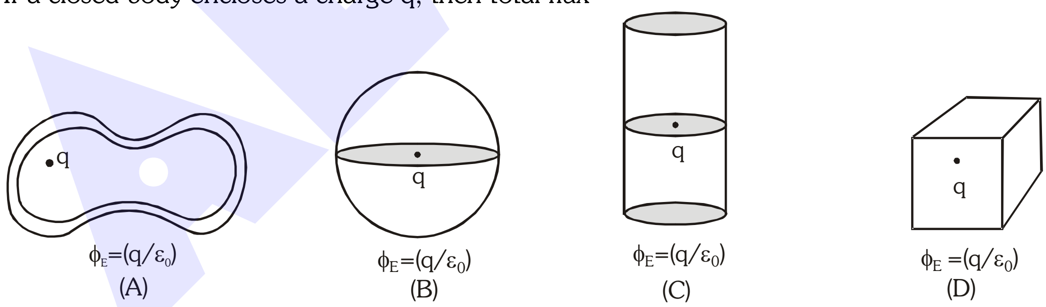
(v) In a closed surface incoming flux is taken negative while outgoing flux is taken positive.

(vi) In a Gaussian surface $\phi = 0$ does not imply $E = 0$ but $E = 0$ implies $\phi = 0$.

(vii) (a) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non–uniform) total flux linked with it will be zero.



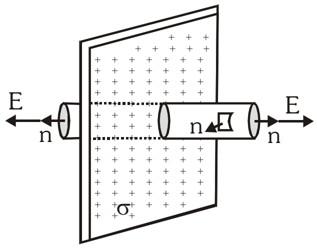
(b) If a closed body encloses a charge q, then total flux



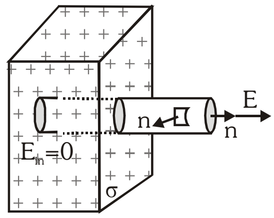
linked with the body will be $\oint_S \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$. From this expression it is clear that the flux linked with a closed body is independent of the shape and size of the body and position of charge inside it.[See figures A, B, C and D]

Note : So in case of symmetrically closed body with charge q at its centre, flux linked with each half will be $\frac{1}{2}(\phi_E) = \left(\frac{q}{2\varepsilon_0}\right)$ and the symmetrical closed body has n identical faces with point charge q at its centre, flux linked with each face will be $\left(\frac{\phi_E}{n}\right) = \left(\frac{q}{n\varepsilon_0}\right)$

(viii) Gauss' law is a powerful tool for calculating electric field intensity in case of symmetrical charge distribution by choosing a Gaussian– surface in such a way that $\vec{E}$ is either parallel or perpendicular to its various faces. As an example, consider the case of a plane sheet of charge having charge density σ. To calculate E at a point P close to it consider a Gaussian surface in the form of a 'pill box' of cross–section S as shown in figure.



Sheet of charge
Figure (A)



Conductor
Figure (B)

The charge enclosed by the Gaussian–surface = σS and the flux linked with the pill box = ES + 0 + ES = 2ES (as E is parallel to curved surface and perpendicular to planend faces)

So from Gauss' law, $\phi_E = \frac{1}{\varepsilon_0}(q)$, $2ES = \frac{1}{\varepsilon_0}(\sigma S) \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

(ix) If $\vec{E} = \vec{0}$, $\phi = \oint \vec{E}.\vec{ds} = 0$, so q = 0 but if q=0, $\oint \vec{E}.\vec{ds} = 0$. So, $\vec{E}$ may or may not be zero.

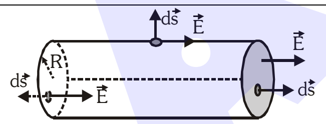
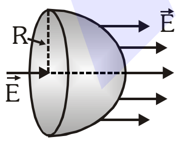
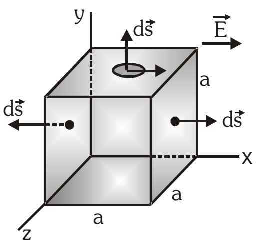
If a dipole is enclosed by a closed surface then, q =0, so $\oint \vec{E}.\vec{ds} = 0$, but $|\vec{E}| \neq 0$

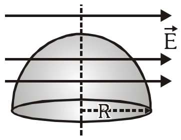
Note : If instead of a plane sheet of charge, we have a charged conductor, then as shown in figure (B) $E_{in}=0$.

So ϕ_E = ES and hence in this case $E = \frac{\sigma}{\epsilon_0}$. This result can be verified from the fact that intensity at the surface of a charged spherical conductor of radius R is, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ with $q = 4\pi R^2\sigma$

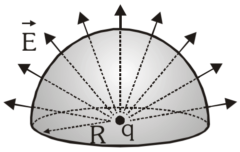
So for a point close to the surface of a conductor, $E = \frac{1}{4\pi\varepsilon_0 R^2} \times \left(4\pi R^2\sigma\right) = \frac{\sigma}{\varepsilon_0}$

4.5 FLUX CALCULATION USING GAUSS' LAW

	$\phi_{in} = -\pi R^2 E$ and $\phi_{out} = \pi R^2 E$ $\Rightarrow \phi_{total} = 0$
	$\phi_{in} = \phi_{circular} = -\pi R^2 E$ and $\phi_{out} = \phi_{curved} = \pi R^2 E \Rightarrow \phi_{total} = 0$
	$\phi_{in} = -a^2 E$ and $\phi_{out} = a^2 E \Rightarrow \phi_{total} = 0$



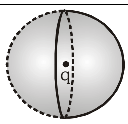
$$\phi_{in} = -\frac{1}{2}\pi R^2 E \text{ and } \phi_{out} = \frac{1}{2}\pi R^2 E \Rightarrow \phi_{total} = 0$$

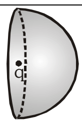


$$E = \frac{kq}{R^2}$$

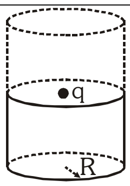
$$\phi = 2\pi R^2 \times \frac{q}{4\pi \in_0 R^2} = \frac{q}{2 \in_0}$$

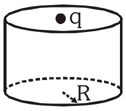
Note : Electric field is radial



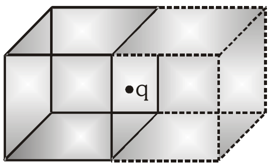


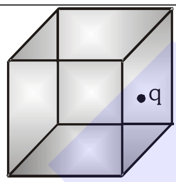
$$\phi_{\text{hemisphere}} = \frac{q}{2 \in_0}$$



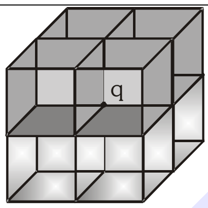


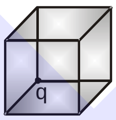
$$\phi_{\text{cylinder}} = \frac{q}{2 \in_0}$$



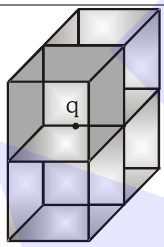


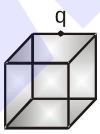
$$\phi_{\text{cube}} = \frac{q}{2 \in_0}$$





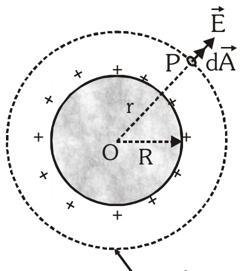
$$\phi = \frac{q}{8 \in_0}$$





$$\phi = \frac{q}{4 \in_0}$$

4.6 Applications of Gauss' law



Gaussian surface

- **Electric field due to solid or hollow conducting sphere or shell**
  - **For outside point (r > R)**

Using Gauss' theorem $\oint \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0}$

$\because$ At every point on the Gaussian surface $\vec{E} \parallel d\vec{s}$ $\quad \therefore \vec{E}.d\vec{s} = E\,ds \cos 0° = E\,ds$

$\therefore \oint E.ds = \frac{\Sigma q}{\in_0}$ [E is constant over the Gaussian surface] $\Rightarrow E \times 4\pi r^2 = \frac{q}{\in_0} \Rightarrow E_p = \frac{q}{4\pi \in_0 r^2}$

- **For a point on the surface (r = R) :**

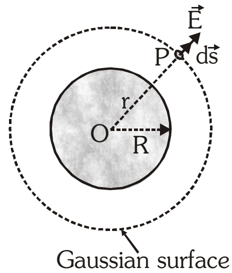
  $$E_S = \frac{q}{4\pi \in_0 R^2}$$

- **For Inside point (r < R)** :

  Since charge inside the conducting sphere or shell is zero, i.e. $\Sigma q = 0$

  so $\oint \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0} = 0 \Rightarrow E_{in} = 0$

- **Electric field due to solid non conducting sphere**



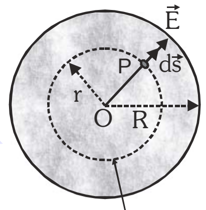
- **For outside point (r > R)**

  From Gauss' theorem

  $$\oint_s \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0} \Rightarrow E \times 4\pi r^2 = \frac{q}{\in_0} \Rightarrow E_P = \frac{q}{4\pi \in_0 r^2}$$

- **At a point on the surface (r = R) :**

  $$E_S = \frac{q}{4\pi \in_0 R^2} \quad \text{Put } r = R$$



- **For inside point (r < R) :**

  From Gauss' theorem $\oint_s \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0}$

  where Σq charge contained within the Gaussian surface of radius r

  $$E(4\pi r^2) = \frac{\Sigma q}{\in_0} \Rightarrow E = \frac{\Sigma q}{4\pi r^2 \in_0} \quad \ldots(i)$$

As the sphere is uniformly charged, the volume charge density (charge/volume) ρ is constant throughout the sphere $\rho = \frac{q}{\frac{4}{3}\pi R^3} \Rightarrow$ charge enclosed in Gaussian surface $\Sigma q = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{q}{(4/3)\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) \Rightarrow \frac{qr^3}{R^3}$

putting this value in equation (i) we get $E_{in} = \frac{1}{4\pi \in_0} \frac{qr}{R^3} = \frac{\rho r}{3\in_0}$

- **ELECTRIC FIELD DUE TO AN INFINITELY LONG LINEAR DISTRIBUTION OF CHARGE**

Let a wire of infinite length be uniformly charged having a uniform linear charge density λ.
P is the point where electric field is to be calculated.

Let us draw a Gaussian cylindrical surface of length ℓ coaxial with the wire. (Figure a)

From Gauss' theorem

$$\int_{s_1} \vec{E}.d\vec{S}_1 + \int_{s_2} \vec{E}.d\vec{S}_2 + \int_{s_3} \vec{E}.d\vec{S}_3 = \frac{q}{\in_0}$$

$\vec{E} \perp d\vec{S}_1$ so $\vec{E}.d\vec{S}_1 = 0$ and $\vec{E} \perp d\vec{S}_2$ so $\vec{E}.d\vec{S}_2 = 0$

$$E \times 2\pi r\ell = \frac{q}{\in_0} \qquad [\because \vec{E} \parallel d\vec{S}_3]$$

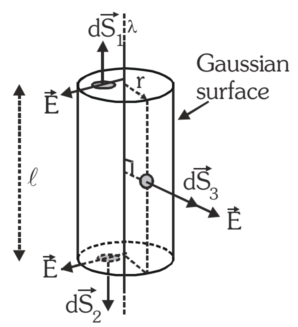


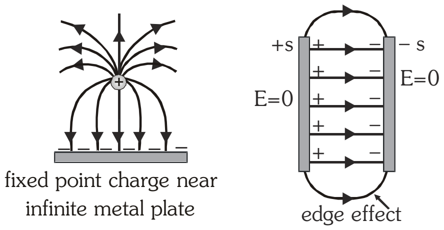
Figure (a)

Charge enclosed in the Gaussian surface $q = \lambda\ell$.

So, $E \times 2\pi r\ell = \frac{\lambda\ell}{\in_0} \Rightarrow E = \frac{\lambda}{2\pi \in_0 r} = \frac{2k\lambda}{r}$ where $k = \frac{1}{4\pi \in_0}$.

GOLDEN KEY POINTS

- Lines of force starts from (+ve) charge and end on (–ve) charge.
- Lines of force start and end normally on the surface of a conductor.



- Lines of force never intersect because, the field at a point (point of intersection) cannot have two distinct directions.
- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero ($E \neq 0$ if net charge is present inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. $x >> R$.

Illustrations

Illustration 13.

Consider an electric field $\vec{E} = (3 \times 10^3)\ \hat{i}$ (N/C). What is the flux through the square of 10 cm side, if the normal to its plane makes 60° angle with the X axis ?

Solution

$$\phi = ES\cos\theta = 3 \times 10^3 \times [10 \times 10^{-2}]^2 \times \cos 60° = 3 \times 10^3 \times 10^{-2} \times \frac{1}{2} = 15\ \text{Nm}^2/\text{C}.$$

Illustration 14.

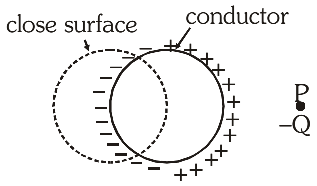
The electric field in a region is given by $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$ where $E_0 = 2 \times 10^3\ \text{NC}^{-1}$. Find the flux due to this field through a rectangular surface of area $0.2\ \text{m}^2$ parallel to the Y–Z plane.

Solution

$$\text{Flux } \phi = \vec{E}\cdot\vec{s} = \left(\frac{6}{5}\times 10^3\hat{i} + \frac{8}{5}\times 10^3\hat{j}\right).(0.2\,\hat{i}) = \frac{1.2}{5}\times 10^3 = 240\ \text{Nm}^2\text{C}^{-1}.$$

Illustration 15.

As shown in figure a closed surface intersects a spherical conductor. If a negative charge Q is placed at point P, what is the nature of the electric flux coming out of the closed surface ?



Solution

Point charge Q induces charge on conductor as shown in figure. Net charge enclosed by closed surface is (-ve) so flux is negative.

Illustration 16.

If a point charge q is placed at the centre of a cube, what is the flux linked (a) with the cube? (b) with each face of the cube?

Solution

(a) According to Gauss' law flux linked with a closed body is $(1/\varepsilon_0)$ times the charge enclosed and here the closed body (cube) is enclosing a charge q so, $\phi_T = \dfrac{q}{\varepsilon_0}$

(b) Now as cube is a symmetrical body with 6–faces and the point charge is at its centre, so electric flux linked with each face will be $\phi_F = \dfrac{\phi_T}{6} = \dfrac{q}{6\varepsilon_0}$

Note: (i) Here flux linked with the cube or any of its faces is independent of the size of cube.

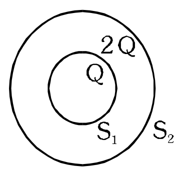
(ii) If charge is not at the centre of cube (but anywhere inside it), total flux will not change, but the flux linked with different faces will be different.

Illustration 17.

S_1 and S_2 are two concentric shells enclosing charges Q and 2Q respectively as shown in figure.

(a) What is the ratio of the electric flux through S_1 and S_2 ?

(b) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant ϵ_r is introduced in the space within S_1 in place of air ?



Solution

(a) Flux through sphere S_1 $\phi_1 = \dfrac{\text{charge}}{\epsilon_0} = \dfrac{Q}{\epsilon_0}$ and flux through sphere S_2 $\phi_2 = \dfrac{Q+2Q}{\epsilon_0} = \dfrac{3Q}{\epsilon_0}$

Ratio $\dfrac{\phi_1}{\phi_2} = \dfrac{Q/\epsilon_0}{3Q/\epsilon_0}$ $\dfrac{1}{3}$

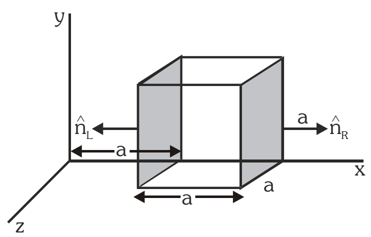
(b) If a medium of dielectric constant ϵ_r is introduced then flux through S_1 will become $\dfrac{1}{\epsilon_r}$ time of flux in air.

Illustration 18.

The electric field components of the field shown in figure are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$, in which $\alpha = 5$ N/C $m^{1/2}$. Calculate

(a) the flux ϕ_E through the cube, and

(b) the charge within the cube. Assume a = 1m. **[AIPMT (Mains) 2008]**



Solution

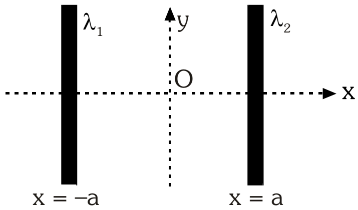
(a) $\phi_{net} = \phi_{right} + \phi_{left} = \left(\alpha\sqrt{2a}\right)a^2\cos 0° + \left(\alpha\sqrt{a}\right)a^2\cos 180°$

$= \alpha(a)^{5/2}\left(\sqrt{2}-1\right) = 5\times(1)^{5/2}\left(\sqrt{2}-1\right) = 2.0\ \text{Nm}^2/\text{C}$

(b) $Q_{in} = \phi_{net}\,\varepsilon_0 = 2\times 8.85\times 10^{-12} = 1.77\times 10^{-11}$ C.

Illustration 19.

Two positively charged infinitely long wires ($\lambda_1 > \lambda_2$) are placed parallel to each other at x = –a and x=+a as in figure. Find the x – coordinate of a point at which electric field is zero.



Solution

Let coordinates of P are (x, 0)

the distance of P from wire 1 = (a + x) and from wire 2 = (a – x)

$\because \quad E_1 = E_2 \quad \therefore \frac{2k\lambda_1}{(a+x)} = \frac{2k\lambda_2}{(a-x)}$

$\Rightarrow \quad \lambda_1(a - x) = \lambda_2(a + x) \Rightarrow \lambda_1 a - \lambda_1 x = \lambda_2 a + \lambda_2 x$

$\Rightarrow \quad (\lambda_1 - \lambda_2)a = (\lambda_1 + \lambda_2)x \Rightarrow x = \frac{(\lambda_1 - \lambda_2)a}{(\lambda_1 + \lambda_2)}$

BEGINNER'S BOX-4

1. Two point charges Q and 4Q are 12 cm apart. Sketch the lines of force and calculate the distance of neutral point from 4Q charge.

2. A charge Q is uniformly distributed over a large plastic (non-conducting) sheet. The electric field at a point close to the centre of the plate near the surface is 20 V/m. If the plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge Q, then what is the electric field at that point?

3. What is the net flux of a uniform electric field $\vec{E} = 3\times 10\,\hat{i}\,NC^{-1}$ through a cube of side 20 cm oriented such that its faces are parallel to the coordinate planes ?

4. Charges Q_1 and Q_2 lie inside and outside a closed surface S respectively. Let E be the field at any point on S and ϕ be the flux of E over S. Which statement is wrong?

(A) If Q_1 changes, both E and ϕ will change.

(B) If Q_2 changes, E will change but ϕ will not change.

(C) If Q_1=0 and $Q_2 \neq 0$ then $E \neq 0$ but ϕ =0

(D) If $Q_1 \neq 0$ and Q_2=0 then E=0 but $\phi \neq 0$

5. A charge 'q' is placed at the centre of a cube whose top face is open (it has only 5 faces). Calculate the total electric flux passing through the cube.

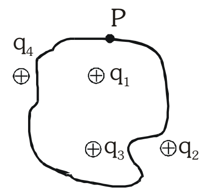
6. A point charge of 2.0 μC is at the centre of a cubic Gaussian surface of edge 9.0 cm. What is the net electric flux through the surface ?

7. An electric flux of -6×10^{-3} Nm²/C passes normally through a spherical Gaussian surface of radius 10 cm, due to a point charge placed at its centre.

(a) What is the charge enclosed by the Gaussian surface ?

(b) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface ?

8. A Gaussian surface encloses two of the four positively charged particles as shown in figure. Which of the particles contribute to the electric field at a point P on the surface ?



9. Is it possible to have flux associated with an imaginary closed surface to be zero even when electric field on this surface is non–zero. If yes, then give one example.

10. Two large, thin metal plates are parallel and close to each other. The plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-12} C/m^2 on their inner faces. What is electric field,
   (a) in the outer region of the first plate ?
   (b) between the plates ?

11. Plot the following graphs –
   (a) Electric field inside a conducting sphere with distance from its centre.
   (b) E versus (1/r) where E is electric field due to a point charge and r is the distance from the charge.

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5. ELECTROSTATIC POTENTIAL ENERGY & ELECTRIC POTENTIAL

5.1 Electrostatic potential energy

Potential energy of a system of particles is defined only for conservative fields. As electric field is conservative, so we define potential energy in relation to it. Potential energy of a system of particles is defined as the work done in assembling the system in a given configuration against the interaction forces between them. Electrostatic potential energy is defined in two ways.

(i) Interaction energy of charged particles of a system.

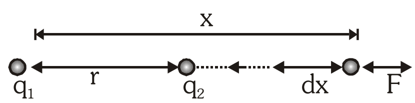
(ii) Self energy of a charged object.

- **Electrostatic Interaction Energy**

Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to the given configuration. When charged particles are at infinite separation, their potential energy is taken to be zero as no interaction is there between them. When these charges are moved to form a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system, hence final potential energy will be positive. If the force between the particles is attractive, work will be done by the system and final potential energy will be negative.

- **Interaction Energy of a system of two charged particles**

Figure shows two + ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be expressed as work done in bringing charge q_2 from infinity to the given separation from q_1.



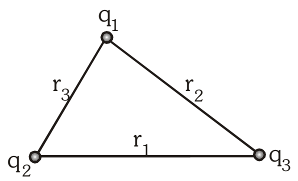
It can be calculated as $W = \int_{\infty}^{r} \vec{F}.\vec{dx} = -\int_{\infty}^{r} \frac{kq_1q_2}{x^2}dx$ [– ve sign shows that x is decreasing]

$$W = \frac{kq_1q_2}{r} = U \text{ [interaction energy]}$$

If the two charges are of opposite signs, then potential energy will be negative as $U = -\frac{kq_1q_2}{r}$

- **Interaction Energy for a system of charged particles**

When more than two charged particles are there in a system, the interaction energy can be given as the sum of interaction energies of all the different possible pairs of particles. For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure.



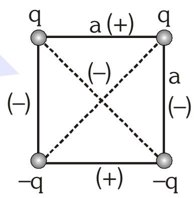
The total interaction energy of this system can be given as $U = \frac{kq_1q_2}{r_3} + \frac{kq_1q_3}{r_2} + \frac{kq_2q_3}{r_1}$

Note : Number of pairs corresponding to n charges = $\frac{n(n-1)}{2}$

Example : Square (4 charges)

Number of pairs = $\frac{4x3}{2} = 6$ Sides $\rightarrow 4$

Diagonals $\rightarrow$ 2

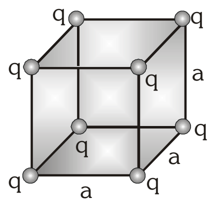


$$U_{system} = kq^2\left[\left(\frac{1-1+1-1}{a}\right)+\left(\frac{-1-1}{a\sqrt{2}}\right)\right] = \frac{-2kq^2}{\sqrt{2}a}$$

Example : Cube (8 charges)

Variety	**distance**	**Number of pairs**
side	$\sqrt{1}a$	12
Face diagonal	$\sqrt{2}a$	12
Main diagonal	$\sqrt{3}a$	4

$N_P = \frac{8x7}{2} = 28$



so $U_{system} = \frac{12kq^2}{a} + \frac{12kq^2}{\sqrt{2}a} + \frac{4kq^2}{\sqrt{3}a}$

5.2 Electric Potential

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field potential is defined as the interaction energy of a unit positive charge. If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as

$$\boxed{V = \frac{U}{q_0}} \text{ joule/coulomb}$$

Potential energy of a charge in an electric field is defined as the work done in bringing the charge from infinity to the given point in the electric field. Similarly we can define electric potential as work done in bringing a unit positive charge from infinity to the given point against the electric forces.

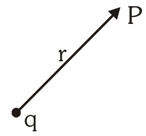
Potential at a point can be physically interpreted as the work done within the conservative field in displacing a unit (+ve) charge from infinity to that point.

$$\boxed{V = \frac{W}{q_0}} \text{ joule/coulomb}$$

Regarding potential it is worth noting that :

(i) It is a scalar; Dimensions : $[ML^2 T^{-3} A^{-1}]$, Unit : volt (V) or J/C

(ii) $V = -\int_{\infty}^{r} \vec{E}.d\vec{r}$

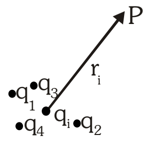


(iii) Potential produced by a point charge is $V = \dfrac{q}{4\pi \in_0 r}$

(iv) Potential due to multiple charges ;

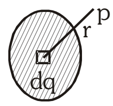
(a) For discrete distribution of charge

$$V = V_1 + V_2 + V_3 + \ldots.. + V_n = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$$



(b) For continuous charge distribution (treating each element as a point)

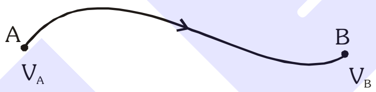
$$dV = \frac{1}{4\pi \in_0}\frac{dq}{r} \quad ; \quad V = \frac{1}{4\pi \in_0}\int\frac{dq}{r}$$



(v) In the presence of a dielectric medium potential decreases and becomes $\dfrac{1}{\in_r}$ times of its free space value.

5.3 Potential Difference Between Two points in an Electric Field

Potential difference between two points in electric field can be defined as work done in displacing a unit positive charge from one point to another.



If a unit +ve charge is displaced from point A to point B as shown, work required can be given as $V_B - V_A = -\int_A^B \vec{E}.d\vec{r}$

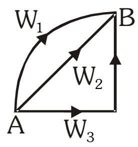
If a charge q is shifted from point A to point B, work done against electric forces can be given as $W = q\,(V_B - V_A)$

If in a situation work done by electric forces is asked, we use $W = q\,(V_A - V_B)$

If $V_B < V_A$, then charges have a tendency to move towards B (low potential point), it implies that electric forces carry the charge from high potential to low potential points. Hence it can be said that electric potential decreases in the direction of electric fields.

As electric field is conservative, work done and hence potential difference between two points is path independent and depends only on the position of initial and final points

$$W_1 = W_2 = W_3$$



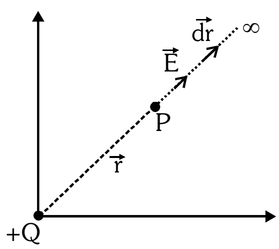
5.4 Electric Potential due to a Point Charge

Electric potential due to +Q charge at point P (with position vector $\vec{r}$) is given by

$$V = -\int_{\infty}^{r}\vec{E}\cdot d\vec{r} = -\int_{\infty}^{r}\frac{Q}{4\pi \in_0 r^2}dr$$

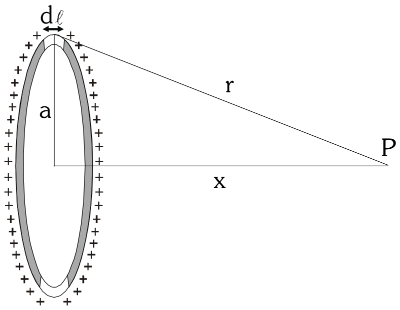
$$\Rightarrow \quad V = -\frac{Q}{4\pi \in_0}\left[-\frac{1}{r}\right]_{\infty}^{r} \Rightarrow V = \frac{Q}{4\pi \in_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\Rightarrow \quad V = \frac{Q}{4\pi \in_0 r}.$$



5.5 Electric Potential due to a Charged Ring

Let electric potential at point P due to small element of length $d\ell$ be dV.



$$dV = \frac{dq}{4\pi\varepsilon_0 r}$$ (distance between small element and point P is equal to r)

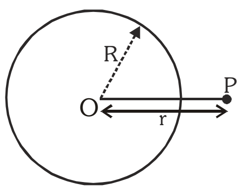
Electric potential due to whole ring $V = \sum dV = \sum \frac{dq}{4\pi\varepsilon_0 r} = \frac{1}{4\pi\varepsilon_0 r}\sum dq$

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{q}{4\pi\varepsilon_0 (a^2 + x^2)^{1/2}}$$

5.6 Electric Potential due to a Conducting Sphere or a Shell

- **Outside the sphere**

According to definition of electric potential, at point P



(r>R)

$$V = -\int_{\infty}^{r} \vec{E}.\vec{dr}$$

$$= -\int_{\infty}^{r} \frac{q}{4\pi\varepsilon_0 r^2} dr \qquad \left[\because \ E_{out} = \frac{q}{4\pi\varepsilon_0 r^2}\right]$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r} \frac{1}{r^2} dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{\infty}^{r} = \frac{q}{4\pi\varepsilon_0 r}$$

- **On the surface**

$$V = -\int_{\infty}^{R} \vec{E}.\vec{dr} = -\int_{\infty}^{R} \frac{q}{4\pi\varepsilon_0 r^2} dr \qquad \left[\because \ E_{out} = \frac{q}{4\pi\varepsilon_0 r^2}\right]$$

$$V = -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{R} \left(\frac{1}{r^2}\right) dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_{\infty}^{R} \Rightarrow V = \frac{q}{4\pi\varepsilon_0 R}$$

- **Inside the surface**

$\because$ Inside the surface E = 0 , $\frac{dV}{dr} = 0$ or V = constant $\left[\because E = -\frac{dV}{dr}\right]$ so $V = \frac{q}{4\pi\varepsilon_0 R}$

5.7 Electric Potential due to Solid Non-Conducting Sphere

- **Outside the sphere**

  Same as conducting sphere. (See article 5.6)

- **On the Surface**

  Same as conducting sphere.

- **Inside the sphere**

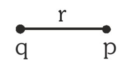
$$V = -\int_{\infty}^{r} \vec{E}.\vec{dr} \Rightarrow V = -\left[\int_{\infty}^{R} E_1 dr + \int_{R}^{r} E_2 dr\right] = -\left[\int_{\infty}^{R}\left(\frac{kq}{r^2}\right)dr + \int_{R}^{r}\left(\frac{kqr}{R^3}\right)dr\right]$$

$$\Rightarrow V = -\left[kq\left(-\frac{1}{r}\right)_{\infty}^{R} + \frac{kq}{R^3}\left(\frac{r^2}{2}\right)_{R}^{r}\right] \Rightarrow V = -kq\left[-\frac{1}{R} + \frac{r^2}{2R^3} - \frac{R^2}{2R^3}\right] \Rightarrow V = \frac{kq}{2R^3}\left[3R^2 - r^2\right]$$

$$\Rightarrow \text{At the centre } r = 0,\ V_{centrre} = \frac{3}{2}\frac{kq}{R} = \frac{3}{2}V_S\ .$$

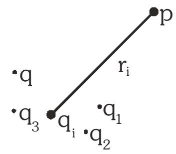
5.8 Potential Due to Special Charge Distribution

(i) Point charge ; $V = \dfrac{kq}{r}$



(ii) Group of point charges;

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \ldots\ldots\frac{kq_n}{r_n} \Rightarrow V = \sum_{i=1}^{n}\frac{kq_i}{r_i}$$

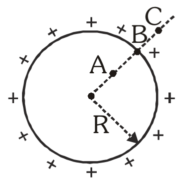


- **Charged conducting sphere**

(i) $V_c = \dfrac{kq}{r}$; $r > R$

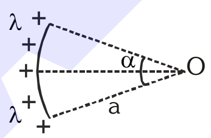
$\dfrac{kq}{R}$; $r = R$

(iii) $V_A = \dfrac{kq}{R}$; $r < R$



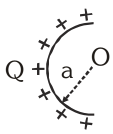
Segment of ring

$V_0 = k\lambda.\alpha$

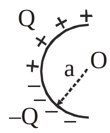


Uniform charged semicircular ring

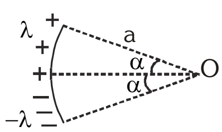
(i) $V_0 = \dfrac{kQ}{a}$



(ii) $V_0 = \text{zero}$



(iii) $V_0 = \text{zero}$

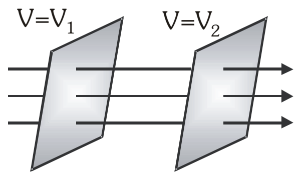


5.9 Equipotential Surface

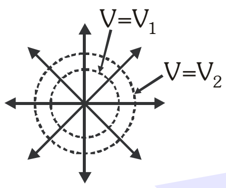
- For a given charge distribution , the plane or surface correspoinding to the locus of all points having same potential is called 'equipotential surface'.
- Equipotential surfaces can never cross each other (otherwise potential at a point will have two values which is absurd).
- Equipotential surfaces are always perpendicular to the direction of electric field.
- If a charge is moved from one point to an other over an equipotential surface

  then work done $W_{AB} = -U_{AB} = q(V_B - V_A) = 0 \quad [\because V_B = V_A]$

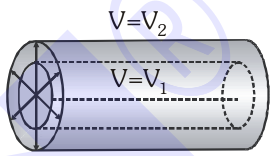
- Shapes of equipotential surfaces



for uniform electric field equipotential surfaces are parallel planes



for a point charge and spherical conductors equipotential surfaces are concentric spheres



for a line distribution of charge equipotential surfaces are coaxial cylinders

- The intensity of electric field along an equipotential surface is always zero.

5.10 Relation between Electric field & Electric Potential

$$\vec{E} = -\vec{\nabla}V = -\text{grad } V = -\frac{dV}{dr}\hat{r} \quad \text{(Note :- Here } \vec{E} \text{ is radial field)}$$

Note : Potential is a scalar quantity but the gradient of potential is a vector quantity

In cartesian co-ordinates $\vec{\nabla}V = \left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$

(i) Direction of $\vec{E}$ is from high potential to low potential.

(ii) If V = constant over a region, then E = 0 (in that region)

GOLDEN KEY POINTS

- If potential energy at infinity is zero, external work done in changing the configuration of a system from 'i' to 'f' is $W = U_f - U_i$.

  In assembling a given charge system $(U_i = 0) \Rightarrow W = U_f$ and in disassembling $(U_f = 0) \Rightarrow W = -U_i$

- eV is the smallest practical unit of energy used in atomic and nuclear physics. ($1 \text{ eV} = 1.6 \times 10^{-19}$ J)
- Potential is theoretically zero at infinity; practically we consider it zero on the earth surface.
- Potential on earth is assumed to be zero as it is a large conductor and its potential is more or less constant in relation to finite charge given to or taken from it.
- The direction of electric field is from regions of higher electric potential to regions of lower potential irrespective of the charge configuration.

- If one moves in the direction of $\vec{E}$, electric potential V decreases ; If one moves in the direction opposite to $\vec{E}$ potential increases.
- Electric potential is the same at all points in a region in which E = 0.
- Equipotential surfaces need not be physical surfaces. Any imaginary surface over which the electric field is perpendicular to it every where is called an equipotential surface.
- Equipotential surfaces are closely spaced where electric field intensity is large and widely spaced where electric field intensity is small.
- Electric field must be zero within an equipotential volume.
- Different equipotential surfaces due to a point charge whose potentials differ by a constant amount (say 1 volt) are not equispaced.
- Two equipotential surfaces never intersect.
- When we place a charged conductor inside a hollow conductor and connect them by a conducting wire, the charge will be completely transferred from the inner conductor to the other conductor.

Illustrations

Illustration 20.

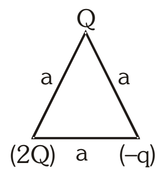
If separation between two point charges decreases then electric potential energy of the system increases; whatever be the sign of charges. Yes or No ?

Solution

The potential energy may increase or decrease. [$\because$ It depends on sign of charges.]

Illustration 21.

If the electric potential energy of the given system (shown in figure) is positive then prove that 2Q > 3q



Solution

U(system) = Sum of potential energies of all possible pairs

$$= \frac{k(Q)(2Q)}{a} - \frac{k2Qq}{a} - \frac{kQq}{a} = \frac{kQ}{a}(2Q - 3q)$$

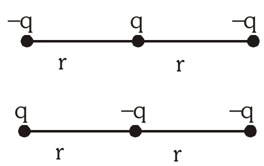
Given that U is positive $\Rightarrow$ U > 0

$$\Rightarrow \frac{kQ}{a}(2Q - 3q) > 0 \quad \Rightarrow \quad 2Q > 3q$$

Illustration 22.

Two negative charges, each of magnitude q are 2r distance apart. A positive charge q is lying at the middle them. The potential energy of the system is U_1. If the two nearest charges are mutually interchanged and the potential energy becomes U_2, then $\frac{U_1}{U_2}$ will be :-

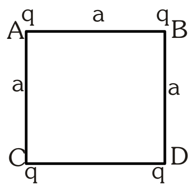
Solution



$$U_1 = \frac{-kq^2}{r} + \frac{-kq^2}{r} + \frac{kq^2}{2r} = \frac{-3kq^2}{2r}$$

$$U_2 = \frac{-kq^2}{r} + \frac{kq^2}{r} - \frac{kq^2}{2r} = \frac{-kq^2}{2r} \qquad \frac{U_1}{U_2} = 3$$

llustration 23.



Four charges q each are placed at the four corners of a square of side a. Find the potential energy of one of the charges.

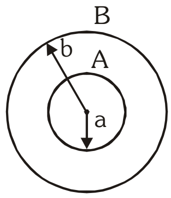
Solution

The electric potential at corner A due to charges at corners B, C and D is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{2}a} + \frac{1}{4\pi\varepsilon_0}\frac{q}{a} = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q}{a}$$

$\therefore$ Potential energy of the charge at A is $u = qV = \frac{1}{4\pi\varepsilon_0}\left(2+\frac{1}{\sqrt{2}}\right)\frac{q^2}{a}$.

Illustration 24.



A total charge Q is given to (Two concentric shells) so that their surface charge densities are equal. Deduce an expresion for potential at their common centre.

Solution

Let surface charge density of each shell be σ then

Charge on A + charge on B = Q $\Rightarrow$ $\sigma 4\pi a^2 + \sigma 4\pi b^2 = Q$

or $\sigma = \frac{Q}{4\pi(a^2+b^2)}$ ...(1)

Now potential at common centre = potential due to A + potential due to B

$V_0 = \frac{\sigma}{\epsilon_0}a + \frac{\sigma}{\epsilon_0}b = \frac{\sigma}{\epsilon_0}(a+b)$. Putting expression for σ from eq^n (1)

$$V_0 = \frac{(a+b)Q}{4\pi\epsilon_0(a^2+b^2)}$$

Infinitely large number of point charges each equal to q are placed at positions x = 1,2,4,8....... . Calculate the electrostatic potential at the origin.

Solution

Electrostatic potential at a point distant r from a charge q is given by $\frac{1}{4\pi\epsilon_0}\frac{q}{r}$

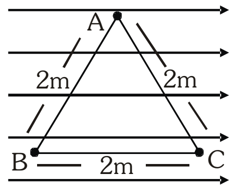
The net electrostatic potential at origin (x = 0) due to the infinite array is,

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+........\right) = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{2^0}+\frac{1}{2^1}+\frac{1}{2^2}+\frac{1}{2^3}+........\right)$$

$$= \frac{q}{4\pi\epsilon_0}\cdot\frac{1}{(1-\frac{1}{2})} = \frac{2q}{4\pi\epsilon_0} = \frac{q}{2\pi\epsilon_0}$$

Illustration 26.

In a uniform electric field $\vec{E} = 10\,N/C$ as shown in figure, find :



(i) $V_A - V_B$ (ii) $V_B - V_C$

Solution

(i) $V_B > V_A$, so, $V_A - V_B$ will be negative

If d denotes effective displacement between two points along the field, then

$d_{AB} = 2 \cos 60° = 1m$

$\therefore\ V_A - V_B = -Ed_{AB} = (-10)(1) = -10V$

(ii) $V_B > V_C$, So $V_B - V_C$ will be positive

Now $d_{BC} = 2.0$ m

$\therefore\ V_B - V_C = 10(2) = 20$ V

Illustration 27.

When a 2 μC charge is carried from point A to point B, the amount of work done by the electric field is 50 μJ. What is the potential difference between them and which is at a higher potential ?

Solution

$W_{ef} = q(V_A - V_B)$

$\Rightarrow\quad 50 \times 10^{-6} = 2 \times 10^{-6}(V_A - V_B)$

$\Rightarrow\quad V_A - V_B = 25$ volt

$\because\quad V_A - V_B > 0$

$\therefore\quad V_A > V_B$

Illustration 28.

The potential function of an electrostatic field is given by $V = 2x^2$. Determine the electric field strength at the point (2m, 0, 3m).

Solution

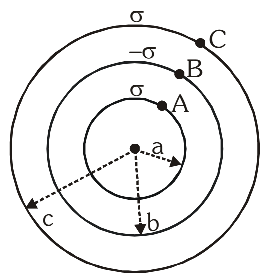
$$E_x = -\frac{dV}{dx} \Rightarrow E_x = -4x$$

Electric field at x = 2m

$\Rightarrow\quad E_x = -8\ NC^{-1} \Rightarrow \vec{E} = -8\hat{i}\ NC^{-1}$

llustration 29.

Find V_A, V_B, V_C, and If $V_A = V_C$ then what is the required condition ?



Solution

For conducting sphere or shell

$$V_{surface} = V_{in} = \frac{\sigma}{\varepsilon_0}R$$

$$V_{out} = \frac{\sigma}{\varepsilon_0}\frac{R^2}{r}$$ (Where R = radius of sphere or shell and r = distance from the centre)

$$V_A = \frac{\sigma a}{\epsilon_0} - \frac{\sigma b}{\epsilon_0} + \frac{\sigma c}{\epsilon_0} = \frac{\sigma}{\epsilon_0}(a - b + c)$$

$$\frac{\sigma a^2}{\epsilon_0 b} - \frac{\sigma b}{\epsilon_0} + \frac{\sigma c}{\epsilon_0} = \frac{\sigma}{\epsilon_0}\left(\frac{a^2}{b} - b + c\right);$$

$$V_C = \frac{\sigma a^2}{\epsilon_0 c} - \frac{\sigma b^2}{\epsilon_0 c} + \frac{\sigma c}{\epsilon_0} = \frac{\sigma}{\epsilon_0}\left(\frac{a^2 - b^2 + c^2}{c}\right)$$

$\Rightarrow$ Now if $V_A = V_C \Rightarrow (a - b + c) = \dfrac{a^2 - b^2 + c^2}{c} \Rightarrow c\,(a - b) = a^2 - b^2 \Rightarrow c = a + b$

BEGINNER'S BOX-5

1. Three charges –q, Q and –q are placed at equal distances on a straight line. If the potential energy of the system of three charges is zero, then what is the ratio Q : q ?

2. Work done in moving a charge q coulombs on the surface of a given charged conductor of potential V volts is ....

3. Three point charges q, –2q and –2q are placed at the vertices of an equilateral triangle of side a. Find the work done by external forces to increase their separation to 2a ?

4. Two electrons lying 10 cm apart are released. What will be their speed when they are 20 cm apart ?

5. (i) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and –2 μC (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively.

(ii) How much work is required to separate them to an infinitely large distance ?

6. (i) Calculate the potential at point P due to a charge of 4×10^{-7} C located 9 cm away.

(ii) Hence obtain the value of work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought ?

7. A regular hexagon of 10 cm side has charges of 5 μC at each of its vertices. Calculate the potential at its centre.

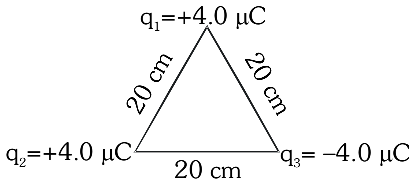
8. (i) What is the difference in potentials between two points, one at 10 cm, and the other at 20 cm distance from a charge of –5.5 μC ?

(ii) Which point is at a higher potential ?

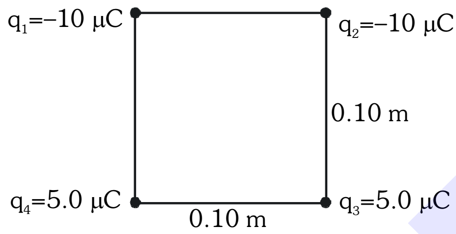
9. (i) How far would a point be from a +0.60 μC charge if it had a value of electric potential equal to 10 kV ?

(ii) If the point were moved to four times of its initial distance from the charge then what potential change would occur ? Is it an increase or a decrease ?

10. What is the electric potential at the center of the triangle shown in figure ?



11. Calculate the electric potential at the center of the square shown in figure.



12. If potential at a point P (1,1,1) is given by the relation $V_P = x^2 - y^2 + 2z$, then calculate the electric field at P.

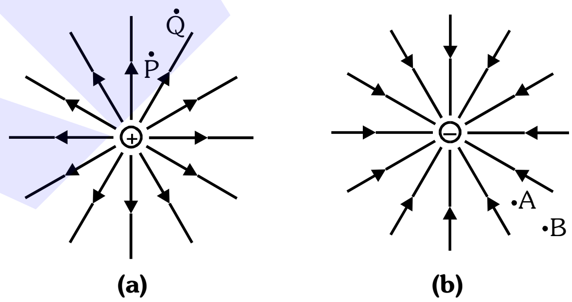
13. For a point charge of +3.50 μC what is the radius of the equipotential surface which is at a potential of 2.50 kV ?

14. A proton is moved 15 cm on a path parallel to the field lines of a uniform electric field of 2.0×10^5 V/m.

(i) What are the possible changes in potential ? Consider both cases of moving the proton along and against the field.

(ii) How much work would be done if the proton were moved perpendicular to the electric field ?

15. Figure (a) and (b) show the field lines of positive and negative point charge respectively.



(a) Give the signs of the potential difference : $V_P - V_Q$; $V_B - V_A$

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P ; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

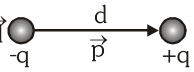
(d) Give the sign of the work done by an external agency in moving a small negative charged from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A ?

6. ELECTRIC DIPOLE :-

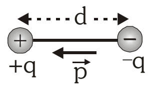
A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between them charges, given as

$\vec{p} = q\vec{d}$



In certain molecules, the centres of positive and negative charges do not coincide. This results in the formation of electric dipoles. Atom is non – polar because the centres of positive and negative charges in it coincide. Polarity can be induced in an atom by the application of electric field. in that case it is called as induced dipole.

- **Dipole Moment** : Dipole moment $\vec{p} = q\vec{d}$



(i) It is a vector quantity directed from negative to positive charge.

(ii) **Dimensions** : [LTA], **Units** : coulomb × metre (or C-m)

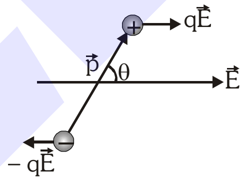
(iii) Practical unit is "debye" ≡ Two equal and opposite point charges each having charge 10^{-10} franklin and separated by 1Å has dipole moment $(\vec{p})$ of 1 debye.

$$1 \text{ debye} = 10^{-10} \times 10^{-10} \text{ Fr} \times \text{m} = 10^{-20} \times \frac{\text{C} \times \text{m}}{3 \times 10^{9}} \simeq 3.3 \times 10^{-30} \text{ C-m}$$

- **Dipole Placed in Uniform Electric Field**

Figure shows a dipole of dipole moment $\vec{p}$ placed at an angle θ to the direction of electric field. Here the charges constituting the dipole experience forces qE each in opposite directions as shown.

$$\vec{F}_{net} = \left[q\vec{E} + (-q)\vec{E}\right] = \vec{0}$$



Thus we can state that when a dipole is placed in a uniform electric field, net force on the dipole is zero. But as equal and opposite forces act with a separation in their lines of action, they produce a couple which tend to align the dipole along the direction of electric field. The torque due to this couple can be given as

τ = Force × separation between the lines of action of forces = qE × d sin θ = pE sin θ

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{d} \times q\vec{E} = q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

- **Work done during Rotation of a Dipole in Electric field**

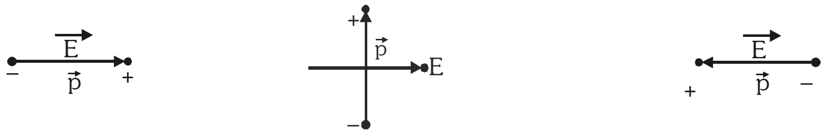
When a dipole is placed at an angle θ to an electric field the torque due to electric field on it is $\tau = pE\sin\theta$

Work done in rotating the dipole from θ_1 to θ_2 [field being uniform]

$$dW = \tau\, d\theta \quad \text{so } W = \int dW = \int \tau\, d\theta \text{ and } W_{\theta_1 \to \theta_2} = W = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2)$$

e.g. $W_{0 \to 180} = pE[1-(-1)] = 2pE$; $W_{0 \to 90} = pE(1-0) = pE$

If a dipole is rotated from field direction ($\theta = 0°$) to θ then $W = pE\,(1 - \cos\theta)$



$\theta = 0$	$\theta = 90°$	$\theta = 180°$
τ = minimum = 0	τ = maximum = pE	τ = minimum = 0
W = minimum = 0	W = pE	W = maximum = 2pE

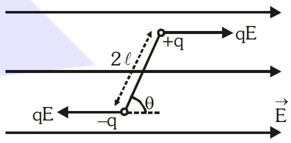
- **Electrostatic potential energy :**

Electrostatic potential energy of a dipole placed in a uniform field is defined as the work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e.,

$$W_{90°\to\theta} = \int_{90°}^{\theta} pE\sin\theta\, d\theta = -pE\cos\theta = -\vec{p}.\vec{E}$$

$\vec{E}$ is a conservative field so what ever work is done in rotating a dipole from θ_1 to θ_2 it is just equal to change in the electrostatic potential energy

$$W_{\theta_1\to\theta_2} = U_{\theta_2} - U_{\theta_1} = pE\,(\cos\theta_1 - \cos\theta_2) \quad (\because U = -pE\cos\theta)$$



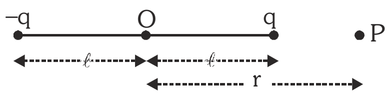
- **Force on an electric dipole in Non–uniform electric field :**

If dipole is placed in a non–uniform electric field then at a point where electric field is E, the interaction energy of the dipole at this point is $U = -\vec{p}.\vec{E}$. Now the force on dipole due to electric field is $F = -\dfrac{\Delta U}{\Delta r}$

If the dipole is placed in the direction of electric field then $\vec{F} = -\vec{p}.\dfrac{d\vec{E}}{dr}$

- **ELECTRIC POTENTIAL DUE TO DIPOLE**

- **At axial point**



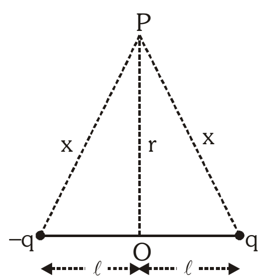
Electric potential due to +q charge $\quad V_1 = \dfrac{kq}{(r-\ell)}$

Electric potential due to –q charge $\quad V_2 = \dfrac{-kq}{(r+\ell)}$

Net electric potential $\quad V = V_1 + V_2 = \dfrac{kq}{(r-\ell)} + \dfrac{-kq}{(r+\ell)} = \dfrac{kq\times 2\ell}{(r^2-\ell^2)} = \dfrac{kp}{r^2-\ell^2}$

If $r >> \ell$ then $V = \dfrac{kp}{r^2}$

- **At equatorial point**



Electric potential at P due to +q charge $\quad V_1 = \dfrac{kq}{x}$

Electric potential at P due to –q charge $\quad V_2 = -\dfrac{kq}{x}$

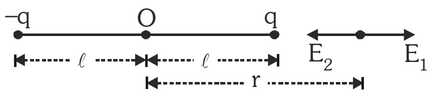
Net potential $\quad V = V_1 + V_2 = \dfrac{kq}{x} - \dfrac{kq}{x} = 0 \;\therefore\; V = 0$

- **At a general point**

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} \text{ where } \vec{p} = q\vec{d} \text{ (electric dipole moment)}$$

ELECTRIC FIELD DUE TO DIPOLE

- **At a point on the axis of a dipole :**

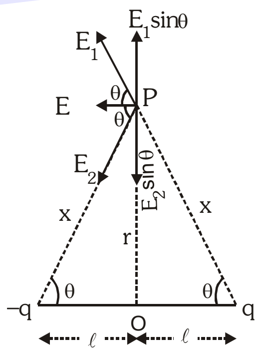


Electric field due to +q charge $E_1 = \frac{kq}{(r-\ell)^2}$ Electric field due to –q charge $E_2 = \frac{kq}{(r+\ell)^2}$

Net electric field $E = E_1 - E_2 = \frac{kq}{(r-\ell)^2} - \frac{kq}{(r+\ell)^2} = \frac{kq\times 4r\ell}{(r^2-\ell^2)^2}$ [$\because$ p = q × 2ℓ = Dipole moment]

$$E = \frac{2kpr}{(r^2-\ell^2)^2} \text{ If } r >> \ell \text{ then } E = \frac{2kp}{r^3} \text{ or } \vec{E} = \frac{2k\vec{p}}{r^3}$$

- **At a point on the equatorial line of dipole :**



Electric field due to +q charge $E_1 = \frac{kq}{x^2}$; Electric field due to –q charge $E_2 = \frac{kq}{x^2}$

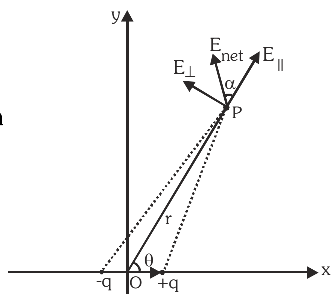
Vertical component of E_1 and E_2 will cancel each other and horizontal components will get added, so net electric field at P is

$$E = E_1\cos\theta + E_2\cos\theta \; [\because E_1 = E_2]$$

$$E = 2E_1\cos\theta = \frac{2kq}{x^2}\cos\theta \because \cos\theta = \frac{\ell}{x} \text{ and } x = \sqrt{r^2+\ell^2}$$

$$E = \frac{2kq\ell}{x^3} = \frac{2kq\ell}{(r^2+\ell^2)^{3/2}} = \frac{kp}{(r^2+\ell^2)^{3/2}} \text{ If } r >> \ell \text{ then } E = \frac{kp}{r^3} \text{ or}$$

- **At any general point :**

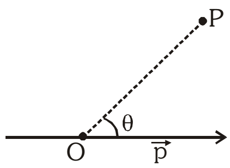


Consider a dipole having dipole moment p placed at the origin pointing along x-axis. We have to find the electric field at point P at a separation r from origin O forming angle θ with dipole moment.

Draw two components of p one along OP and the other perpendicular to OP.

Component along OP = p cosθ

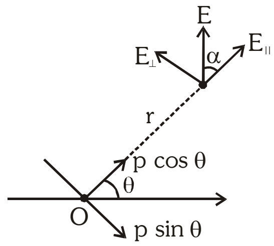
Component perpenducular to OP = p sinθ



Now, electric field $(E_{||})$ due to p cosθ = $\frac{2kp\cos\theta}{r^3}$ (axial)

Electric field $(E_{\perp})$ due to p sinθ = $\frac{kp\sin\theta}{r^3}$ (equatorial)

$\therefore$ Net field $E = \sqrt{E_{\parallel}^2 + E_{\perp}^2} = \frac{kp}{r^3}\sqrt{1+3\cos^2\theta}$

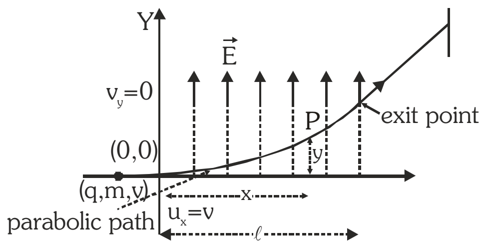


Now $\tan\alpha = \frac{E_{\perp}}{E_{\parallel}} = \frac{1}{2}\tan\theta$

Where α is the angle made by E with OP.

7. MOTION OF A CHARGED PARTICLE IN UNIFORM ELECTRIC FIELD

ℓ = extension of electic field, normal to its direction



Position of particle at time t : for x direction $x = vt$

and for y direction $y = u_y t + \frac{1}{2} a_y t^2$

$\because \quad u_y = 0 ; t = \frac{x}{v} ; a_y = \frac{qE}{m}$

$\therefore \quad y = 0 + \frac{1}{2}\frac{qE}{m}\left(\frac{x}{v}\right)^2$

or $\quad y = \left(\frac{qE}{2mv^2}\right)x^2$ this is an equation of a parabola.

Special Results :

(1) Time taken by the particle to cover the entire length i.e., extension of electric field, in other words, time for which particle moves under the influence of electric field is $T = \frac{\ell}{v}$

(2) Total deviation in the trajectory within electric field is

$$y = \frac{1}{2}\frac{qE}{m}\left(\frac{\ell}{v}\right)^2$$

GOLDEN KEY POINTS

- For a dipole, potential is zero at equator, while at any finite point $E \neq 0$
- In a uniform $\vec{E}$, dipole may experience a torque but not a force.
- If a dipole is placed in a field due a point charge, then torque on dipole may be zero.
-

Distribution	Point charge	Dipole	Quadrapole	Octapole
V proportional to	r^{-1}	r^{-2}	r^{-3}	r^{-4}
E proportional to	r^{-2}	r^{-3}	r^{-4}	r^{-5}

-

Force between	Two Point charges	Dipole and point charge	Dipole-dipole
Proportional to	r^{-2}	r^{-3}	r^{-4}

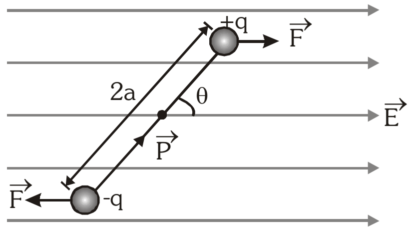
Illustrations

Illustration 30.

Prove that the frequency of oscillation of an electric dipole of moment p and rotational inertia I is $\frac{1}{2\pi}\sqrt{\left(\frac{pE}{I}\right)}$ for small amplitudes about its equilibrium position in a uniform electric field strength E

Solution

Let an electric dipole (with charges q and –q 2a distance apart) be placed in a uniform electric field of strength E.



Restoring torque on dipole $\tau = -pE\sin\theta \simeq -pE\,\theta$ (as θ is small)

Here – ve sign shows the restoring tendency of torque. $\because \tau = I\alpha$ $\therefore$ angular acceleration $= \alpha = \frac{\tau}{I} = \frac{PE}{I}\theta$

For SHM $\alpha = -\omega^2\theta$; Comparing we get $\omega = \sqrt{\frac{pE}{I}}$

Thus, frequency of oscillation of dipole is $n = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{pE}{I}\right)}$

Illustration 31.

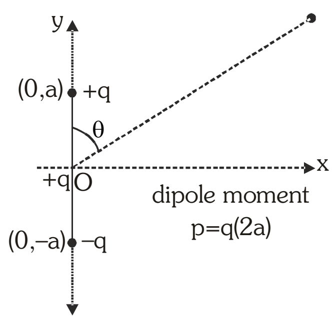
Three charges -q, +q and +q are situated in X-Y plane at points (0, -a), (0, 0) and (0, a) respectively. Find the potential at a distant point r (r >> a) in a direction making an angle θ from the Y-axis :-

Solution

Potential at point P due to charge q (placed at origin) is

$V_0 = \frac{kq}{r}$

potential due to remaining charge system (dipole) can be written as



$V_d = \frac{kp}{r^2}\cos\theta = \frac{kq.2a\cos\theta}{r^2}$

Hence $V_P = V_0 + V_d = \frac{kq}{r}\left[1+\frac{2a\cos\theta}{r}\right]$

Illustration 32.

A region consists of uniform electric field E and uniform gravitational field g perpendicular to each other. A particle of charge Q and mass m begins to move in this region.Deduce an equation for its trajectory.

Solution

If E is in positive x–direction

then $a_x = \frac{QE}{m}$; $x = \frac{1}{2}\frac{QE}{m}t^2$ --------------------------- (1)

$a_y = g$; $y = \frac{1}{2}gt^2$ --------------------------- (2)

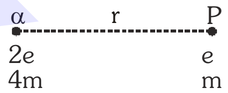
Solving (1) & (2) we get equation of trajectory

$$y = \left(\frac{mg}{QE}\right)x \qquad \text{(Straight line)}$$

Illustration 33.

A proton and an α - particle are initially at a distance 'r' apart. Find the KE of α-particle at a large separation from proton after being released.

Solution



Total energy of system initially $E_i = \frac{ke.2e}{r} = \frac{2ke^2}{r}$

Momentum of the two charges system remains constant throughout Hence $E \propto \frac{1}{m}$

So divide total energy in inverse ratio of their respective masses $E_p = \frac{4}{5}E_i = \frac{4}{5}\left(\frac{2ke^2}{r}\right)$, $E_\alpha = \frac{1}{5}E_i = \frac{1}{5}\left(\frac{2ke^2}{r}\right)$

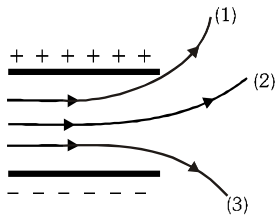
BEGINNER'S BOX-6

1. A sample of HCl gas is placed in an electric field of strength 2.5×10^4 NC^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} Cm. Find the maximum torque that can act on a molecule.

2. An electric dipole when placed in a uniform electric field E will have minimum potential energy, when the angle made by the dipole moment with the field E is ....

3. An electric dipole is placed making at an angle 60° with an electric field of strength 4×10^5 N/C. It experiences a torque equal to $8\sqrt{3}$ N-m. Calculate the charge on the dipole, if it is of length 4 cm.

4. Figure shows the tracks of three charged particles in a uniform electric field.



Give the signs of the three charges.

Which particle has the largest charge to mass ratio ?

6. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$C located at points $A \equiv (0,0,-15)$ cm and $B \equiv (0, 0, 15)$ cm respectively. What are the total charge and electric dipole moment vector of the system ?

8. CONDUCTOR AND IT'S PROPERTIES [FOR ELECTROSTATIC CONDITION ONLY]

(i) Conductors are materials which contain large number of free electrons which can move freely inside the conductor.

(ii) In electrostatics, conductors are always equipotential surfaces.

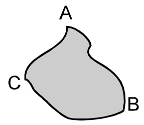
(iii) Charge always resides on the outer surface of a conductor.

(iv) If there is a cavity inside a charged conductor with the cavity devoid of any charge then charge will always reside only on the outer surface of the conductor.

(v) Electric field is always perpendicular to conducting surface.

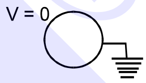
(vi) Electrostatic field lines never exist within conducting materials

(vii) Electric field intensity near a conducting surface is given by the formula $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$



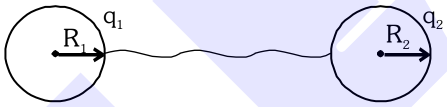
$$\vec{E}_A = \frac{\sigma_A}{\varepsilon_0}\hat{n} \; ; \; \vec{E}_B = \frac{\sigma_B}{\varepsilon_0}\hat{n} \text{ and } \vec{E}_C = \frac{\sigma_C}{\varepsilon_0}\hat{n}$$

(viii) When a conductor is grounded its potential becomes zero.



8.1 Sharing of Charges :

Two conducting spherical shells of radii R_1 and R_2 having charges Q_1 and Q_2 respectively and separated by a large distance, are joined by a conducting wire. Let the final charges on the two spheres be q_1 and q_2 respectively.



Potentials of both shells become equal after joining, therefore

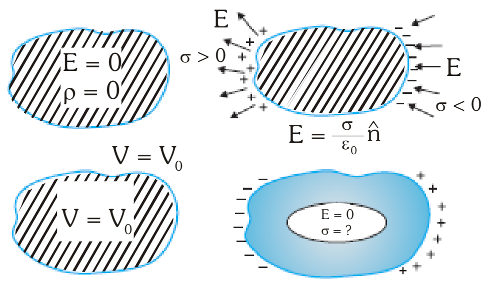
$V_1 = V_2$ $\quad\therefore\quad q \propto R \quad\Rightarrow\quad \frac{q_1}{q_2} = \frac{R_1}{R_2}$

Since $q_1 + q_2 = Q_1 + Q_2$ $\quad\therefore\quad q_1 \frac{R_1}{(R_1+R_2)} = Q$ and $q_2 = \frac{R_2}{(R_1+R_2)} Q$

$\Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$ $\qquad \Rightarrow \frac{E_1}{E_2} = \frac{R_2}{R_1}$

8.2 Electrostatic Shielding

- Electrostatic shielding is the method of protecting a certain region from the effect of electric field.
- A cavity surrounded by conducting walls is a field free region as long as there are no charges inside the cavity.
- Whatever be the charge and field configuration the field inside the cavity is always zero. This is known as electrostatic shielding.
- Electrostatic shielding can be achieved by enclosing sensitive instruments in a hollow conductor.
- Figure gives a summary of the important electrostatic properties of a conductor.



GOLDEN KEY POINTS

- Electrostatic experiments do not work well on humid days because moist air conducts electricity.
- A charged matellic body has same potential but the distribution of charge may or may not be uniform.
- A small metal ball is suspended in a uniform electric field with the help of an insulated thread. If a high energy X-ray beam falls on the ball, it will be deflected in the direction of the field.
- Sensitive electronic instruments are shielded from external electrical influence by enclosing them in metal boxes.

Illustrations

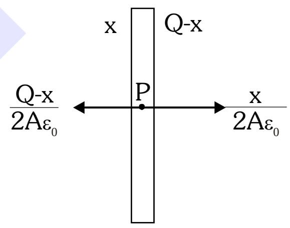
Illustration 34.

Prove that if an isolated (isolated means no charges are present near by) large conducting sheet is given a charge then the charge distributes equally on its two surfaces.

Solution

Let there be x charge on left side of the sheet and Q–x charge on the right side of the sheet.

Since point P lies inside the conductor so E_P = zero



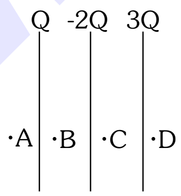
$$\frac{x}{2A\varepsilon_0} - \frac{Q-x}{2A\varepsilon_0} = 0 \Rightarrow \frac{2x}{2A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} \Rightarrow x = \frac{Q}{2}$$

$$Q - x = \frac{Q}{2}$$

So charge is equally distributed on both faces.

Illustration 35.

Three large conducting sheets placed close and parallel to each other contain charges Q, –2Q and 3Q respectively. Find the electric field at points A, B , C, and D.



Sol. $E_A = E_Q + E_{-2Q} + E_{3Q}$. (i) Here E_Q means electric field due to charge 'Q'.

$$E_A = \frac{Q}{2A\varepsilon_0} - \frac{2Q}{2A\varepsilon_0} + \frac{3Q}{2A\varepsilon_0} = \frac{2Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}, \text{ towards left}$$

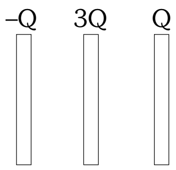
(ii) $$E_B = \frac{Q-(-2Q+3Q)}{2A\varepsilon_0} = 0$$

(iii) $$E_C = \frac{(Q-2Q)-(3Q)}{2A\varepsilon_0} = \frac{-2Q}{A\varepsilon_0}, \text{ towards right} \Rightarrow \frac{2Q}{A\,\varepsilon_0} \text{ towards left}$$

$$\frac{Q}{2A\varepsilon_0} - \frac{2Q}{2A\varepsilon_0} + \frac{3Q}{2A\varepsilon_0} = \frac{Q}{A\varepsilon_0}, \text{ towards right}$$

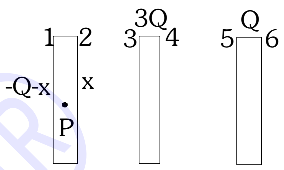
Illustration 36.

Figure shows three large metallic plates with charges – Q, 3Q and Q respectively. Determine the final charges on all the surfaces.



Solution

We assume that charge on surface 2 is x. From conservation of charge, we see that surface 1 has charge (– Q – x). The electric field inside the metal plate is zero so field at P is zero.



Resultant field at P : $E_P = 0 \Rightarrow \frac{-Q-x}{2A\varepsilon_0} = \frac{x+3Q+Q}{2A\varepsilon_0}$

$\Rightarrow -Q - x = x + 4Q \Rightarrow x = \frac{-5Q}{2}$

We know that charges on the facing surfaces of the plates are of equal magnitude and opposite signs. This can be in general proved by Gauss' theorem also. Remember this it is an important result.

Thus the final charge distribution on all the surfaces is :

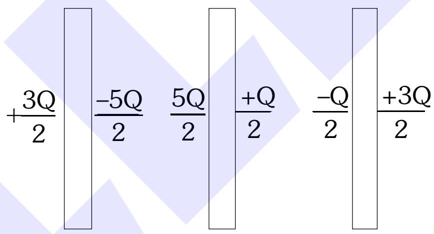
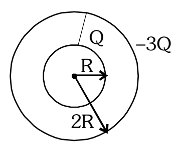


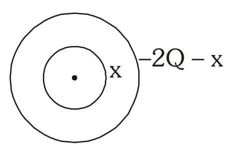
Illustration 37.

The two conducting spherical shells shown in figure are joined by a conducting wire and disconnected after the charge stops flowing. Find out the charges on each sphere after that.



Solution

After disconnecting, the potentials of both the shells are equal



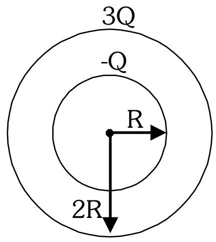
Thus, potential of inner shell $V_{in} = \frac{Kx}{R} + \frac{K(-2Q-x)}{2R} = \frac{K(x-2Q)}{2R}$

and potential of outer shell $V_{out} = \frac{Kx}{2R} + \frac{K(-2Q-x)}{2R} = \frac{-KQ}{R}$

$$\frac{-KR}{R} = \frac{K(x-2Q)}{2R} \Rightarrow -2Q = x - 2Q \qquad \Rightarrow x = 0$$

So charge on inner spherical shell = 0 and outer spherical shell = – 2Q.

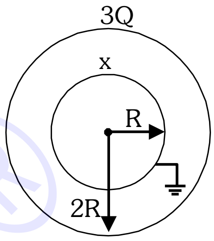
Illustration 38.



Two conducting spherical shells of radii R and 2R carry charges – Q and 3Q respectively. How much charge will flow into the earth if the inner shell is grounded ?

Solution

When inner shell is grounded the potential of inner shell becomes zero because potential of the Earth is taken to be zero.



$$\frac{Kx}{R} + \frac{K3Q}{2R} = 0$$

$x = \frac{-3Q}{2}$, the charge that has increased

$$= \frac{-3Q}{2} - (-Q) = \frac{-Q}{2} .$$

Hence the amount of charge that flows into the Earth = $\frac{Q}{2}$

BEGINNER'S BOX-7

1. A cube of metal is given a charge (+ Q); which of the following statements is true ?

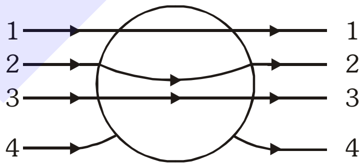
(A) Potential on the surface of cube is zero

(B) Potential within the cube is zero

(C) Electric field is normal to the surface of the cube

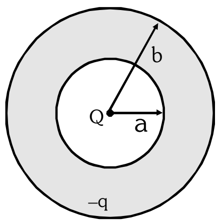
(D) Electric field varies within the cube

2. A solid metallic sphere is placed in a uniform electric field. The lines of force follow the path(s) (shown in figure) :-



(A) 1 (B) 2 (C) 3 (D) 4

3. Shown in the figure is a spherical shell with inner radius 'a' and outer radius 'b' which is made of conducting material. A point charge +Q is placed at the centre of the shell and a total charge –q is placed on the shell. Charge –q is distributed on the surfaces as :-



(A) –Q on the inner surface, –q on the outer surface

(B) –Q on the inner surface, –q+Q on the outer surface

(C) +Q on the inner surface, –q–Q on the outer surface

(D) The charge –q is spread uniformly between the inner and outer surfaces.