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PRE-MEDICAL

PHYSICS

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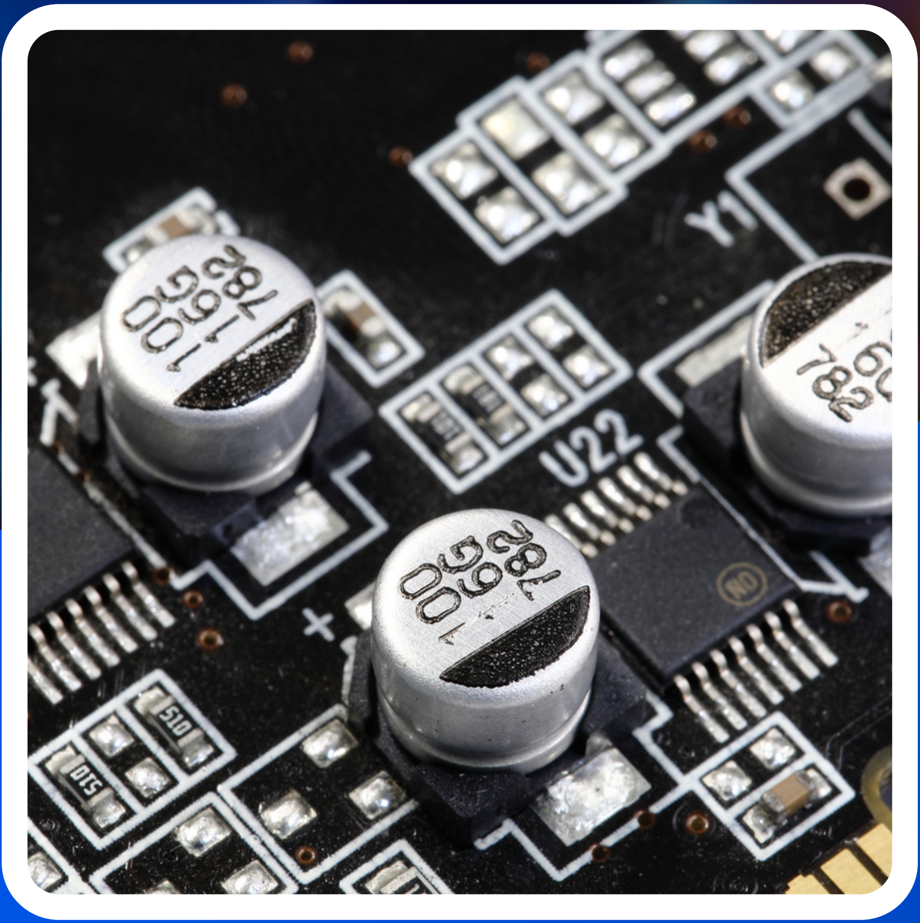

STUDY MATERIAL

Capacitor

ENGLISH MEDIUM

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Count Alessandro Volta (1745 – 1827)

Italian physicist, professor at Pavia. Volta established that the animal electricity observed by Luigi Galvani, 1737–1798, in experiments with frog muscle tissue placed in contact with dissimilar metals, was not due to any exceptional property of animal tissues but was also generated whenever any wet body was sandwiched between dissimilar metals. This led him to develop the first voltaic pile, or battery, consisting of a large stack of moist disks of cardboard (electrolyte) sandwiched between disks of metal (electrodes).

Dr. Bidhan Chandra Roy (1882-1962)

Dr. Bidhan Chandra Roy, was born on 1 July 1882 at Patna, Bihar. One of the very few people who are talented enough to acquire both the M.R.C.P. and F.R.C.S. degrees, was an eminent physician, one of the most important freedom fighters for India and also the second Chief Minister of West Bengal. Bidhan Chandra Roy led a very eventful life during which he excelled in each profession he had taken up. In addition, Dr Bidhan Chandra Roy also laid the foundation stone of cities Bidhannagar and Kalyani in West Bengal. After his flourishing terms as a part of the alumni of the Calcutta Mediacal College and as the Vice Chancellor of Calcutta University, Bidhan Chandra Roy entered into active politics and subsequently was elected the Chief Minister of West Bengal, a post that he held till his death. Dr Bidhan Chandra Roy is fondly remembered through the celebration of the **National Doctor's Day** on July 1 (his birth and death day) every year.

CAPACITANCE

1. CAPACITANCE

1.1 Concept of Capacitance

Capacitance of a conductor is a measure of its ability to store charge. When a conductor is charged, its potential changes. The increase in potential is directly proportional to the charge given to the conductor.

$Q \propto V \Rightarrow Q = CV$

The constant C is known as the capacity or capacitance of the conductor.

Capacitance is a scalar quantity with dimensions $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$

Unit :- farad, coulomb/volt

The capacitance of a conductor is independent of the charge given or rise in its potential. It is also independent

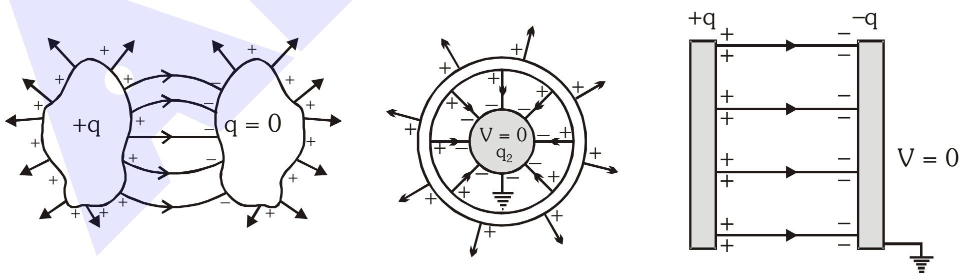
of the nature of material and thickness of the conductor. Theoretically, infinite amount of charge can be given to a conductor.

However, practically the electric field becomes so intense that it causes ionisation of the medium surrounding it. Consequently, the charge on the conductor leaks, reducing its potential.

It is clear that every conductor has a capacity to store charge which is numerically equal to the ratio of charge given to it to the rise in potential, it depends on its shape, size and surrounding medium. However, the capacity of a conductor is small and limited.

It has been found that if a conductor is placed near a charged conductor, then the potential of the charged conductor (relative to the other) decreases and hence it can store more charge, i.e., vicinity of another conductor increases the capacity of a charged conductor.

In case of two conductors (close to each other), if the conductors (called plates) carry equal and opposite charges, the system is called a capacitor or condenser [Figure]. The capacity or capacitance of a capacitor is defined as $C = \frac{\text{Magnitude of charge on either plate}}{\text{PD between the plates}}$



$q_1 = +q$ $q_2 = 0$
(A)

$q_1 = +q$ $q_2 < |-q|$
(B)

$q_1 = +q$ $q_2 = -q$
(C)

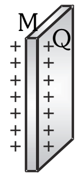
The capacity of a capacitor is found to depend on the geometry, i.e., shape and size of the conductors also on, their relative separation and the intervenning medium (called dielectric) between them.

Note : Here also on, it must be noted that the charges on the plates of a capacitor are equal and opposite, hence total charge on it is zero and all the electric lines of force which originate from one plate terminate on the other plate. If the charges on the two plates are not equal and opposite, system will still have a capacity but will not be called as a capacitor [e.g. figure (A) and (B)].

1.2 Condenser/Capacitor

A pair of conductors having opposite charges of equal magnitude is defined as condenser.

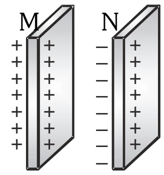
1.3 Principle of a Condenser



It is based on the fact that capacitance can be increased by reducing the potential keeping the charge constant. Consider a conducting plate M which is given a charge Q such that its potential rises to V then

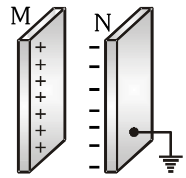
$$C = \frac{Q}{V}$$

Let us place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure). If V_- is the potential at M due to induced negative charge on N and V_+ is the potential at M due to induced positive charge on N, then



$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$

Since V' < V (as the induced negative charge lies closer to the plate M in comparison to induced positive charge). $\Rightarrow$ C' > C Further, if N is earthed from the outer side (see figure) then $V'' = V_+ - V_-$ ($\because$ the entire positive charge flows to the earth)



$$C'' = \frac{Q}{V''} = \frac{Q}{V - V_-} \Rightarrow C'' >> C$$

If an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciably. This is the principle of a parallel plate capacitor.

2. ENERGY STORED IN A CHARGED CAPACITOR

Let C be the capacitance of a capacitor. On being connected to a battery, it charges to a potential V from zero potential. If q is charge on the capacitor at that time then q = CV. Let the battery supply a small amount of charge dq to the capacitor at constant potential V. Then the corresponding small amount of energy stored in capacitor is given by -

$$dU = Vdq = \frac{q}{C}dq \Rightarrow U = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow U = \frac{Q^2}{2C}$$

where Q is the final charge acquired by the conductor.

$$\text{So, } U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV$$

$$\therefore \quad \boxed{U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

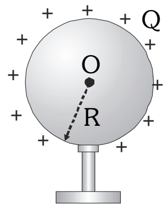
Note :- Work done by battery = Charge supplied $\times$ potential

$$\boxed{W_{\text{Battery}} = QV = CV^2 = \frac{Q^2}{C}}$$

3. THE CAPACITANCE OF A SPHERICAL CONDUCTOR / CAPACITOR

3.1 Isolated Sphere

When a charge Q is given to an isolated spherical conductor, its potential rises.



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R}$$

or, $$C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

If the conductor is placed in a certain medium then,

$C_{medium} = 4\pi\varepsilon R$ or $C_{medium} = 4\pi\varepsilon_0\varepsilon_r R$

Capacitance depends upon :

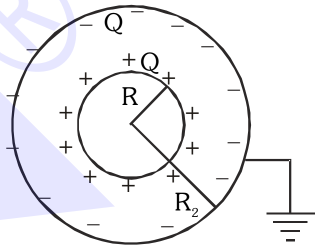
(a) size and Shape of Conductor

(b) surrounding medium

(c) presence of other conductors nearby.

3.2 Outer sphere is earthed (Spherical Capacitor)

When a charge Q is given to the inner sphere it is uniformly distributed on its surface. A charge –Q is induced on the inner surface of outer sphere. The charge +Q induced on the outer surface of outer sphere flows to earth as it is grounded.



$E = 0$ for $r < R_1$ and $E = 0$ for $r > R_2$

Potential of inner sphere $V_1 = \dfrac{Q}{4\pi\varepsilon_0 R_1} + \dfrac{-Q}{4\pi\varepsilon_0 R_2} \Rightarrow \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{R_2 - R_1}{R_1 R_2}\right)$

As outer surface is earthed so potential $V_2 = 0$

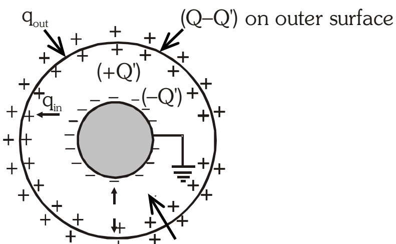
Potential difference between the plates $V = V_1 - V_2 = \dfrac{Q}{4\pi\varepsilon_0}\dfrac{(R_2 - R_1)}{R_1 R_2}$

So $C = \dfrac{Q}{V} = 4\pi\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1}$ (in air or vacuum)

In presence of medium between plate $\boxed{C = 4\pi\varepsilon_r\varepsilon_0 \dfrac{R_1 R_2}{R_2 - R_1}}$

3.3 Inner sphere is earthed

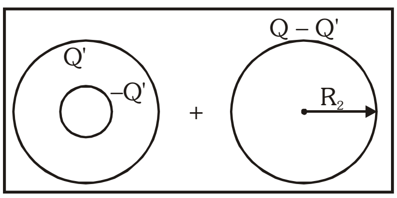
Here, the system is equivalent to a spherical capacitor of inner and outer radii R_1 and R_2 respectively and a spherical conductor of radius R_2 in parallel. This is because charge Q given to outer sphere gets distributed in such a way that for the outer sphere :



Inside +ve and –ve charges are equal

Charge on the inner side $q_{in} = \dfrac{R_1}{R_2}Q$ and

Charge on the outer side $q_{out} = Q - \dfrac{R_1}{R_2}Q = \dfrac{(R_2 - R_1)}{R_2}Q$



So total capacity of the system is $C = 4\pi\varepsilon_0 \dfrac{R_1 R_2}{(R_2 - R_1)} + 4\pi\varepsilon_0 R_2 = \dfrac{4\pi\varepsilon_0 R_2^2}{(R_2 - R_1)}$

$$\boxed{C = \frac{4\pi\varepsilon_0 R_2^2}{(R_2 - R_1)}}$$

GOLDEN KEY POINTS

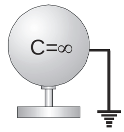
- Work done by a battery W_b = (charge delivered by battery) × (emf) = QV but

  $$\text{Energy stored in conductor} = \frac{1}{2}QV$$

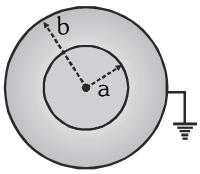
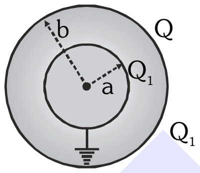
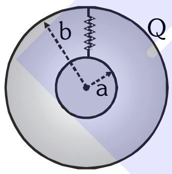
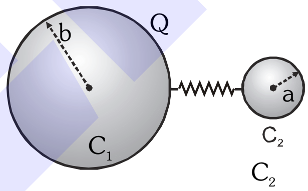
  so 50% of the energy supplied by the battery is lost in the form of heat.
- The amount of energy stored depends on the size of the conductor.
- When a capacitor C charged upto a voltage V is discharged by means of any resistance

  then heat loss $= \dfrac{CV^2}{2}$ (independent of R)
- As the potential of the Earth is assumed to be zero, practically, capacity of earth or a conductor

  connected to earth will be infinite $\quad C = \dfrac{q}{V} = \dfrac{q}{0} = \infty$



- Theoretically, capacity of the Earth $C = 4\pi\varepsilon_0 R = \dfrac{1}{9\times10^9}\times 64\times10^5 = 711\ \mu F$
-

Spherical capacitor outer plate is earthed	Inner plate is earthed and outer plate is given a charge	Connected and outer plate is given a charge	Connected spheres
			 C_2
$C = \dfrac{4\pi\varepsilon_0 ab}{b-a}$ (b > a)	$C = \dfrac{4\pi\varepsilon_0 b^2}{b-a}$ (b > a)	$C = 4\pi\varepsilon_0 b$	$C = C_1 + C_2$ $C = 4\pi\varepsilon_0(a+b)$

Illustrations

Illustration 1.

A capacitor gets a charge of 50 μC when it is connected to a battery of emf 5 V. Calculate the capacity of the capacitor.

Solution

Capacity of the capacitor $C = \dfrac{Q}{V} = \dfrac{50\times10^{-6}}{5} = 10\mu F$

Illustration 2.

The plates of a capacitor are charged to a potential difference of 100 V and then connected across a resistor. The potential difference across the capacitor decays exponentially with respect to time. After one second. the potential difference between the plates of the capacitor is 80 V. What is the fraction of the stored energy which has been dissipated ?

Solution

Energy losses $\quad \Delta U = \dfrac{1}{2}CV_0^2 - \dfrac{1}{2}CV^2$

Fractional energy loss $\quad \dfrac{\Delta U}{U_0} = \dfrac{\frac{1}{2}CV_0^2 - \frac{1}{2}CV^2}{\frac{1}{2}CV_0^2} = \dfrac{V_0^2 - V^2}{V_0^2} = \dfrac{(100)^2-(80)^2}{(100)^2} = \dfrac{20\times180}{(100)^2} = \dfrac{9}{25}$

Illustration 3.

Two uniformly charged spherical drops each at a potential V coalesce to form a larger drop. If the capacity of each smaller drop is C then find the capacity and potential of larger drop.

Solution

When drops coalesce to form a larger drop then total charge and volume remains conserved. If r is the radius and q is the charge on smaller drop then $C = 4\pi\varepsilon_0 r$ and $q = CV$

Equating volume we get $\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3} r$

Capacitance of larger drop $C' = 4\pi\varepsilon_0 R = 2^{1/3} C$

Charge on larger drop $Q = 2q = 2CV$

Potential of larger drop $V' = \frac{Q}{C'} = \frac{2CV}{2^{1/3}C} = 2^{2/3}\,V.$

Illustration 4.

The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between the stratosphere and earth's surface. Take radius of earth as 6400 km.

Solution

The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0 \left(\frac{ab}{b-a}\right)$

b = radius of the stratosphere layer $= 6400\text{ km} + 50\text{ km} = 6450\text{ km} = 6.45 \times 10^6\text{ m}$

a = radius of earth $= 6400\text{ km} = 6.4 \times 10^6\text{ m}$

$$\therefore\ C = \frac{1}{9 \times 10^9} \times \frac{6.45 \times 10^6 \times 6.4 \times 10^6}{(6.45 \times 10^6 - 6.4 \times 10^6)} = 0.092\text{ F}$$

Illustration 5.

Calculate the energy of a sphere of radius 2 cm if it is charged to 300 volts.

Solution

Stored energy $U = \frac{1}{2}CV^2 = \frac{1}{2}(4\pi\epsilon_0 R)V^2 = \frac{RV^2}{2k} = \frac{2 \times 10^{-2} \times 300 \times 300}{2 \times 9 \times 10^9} = 10^{-7}\text{ J}.$

Illustration 6.

Two insulated conductors are charged by transferring electrons from one conductor to another. A potential difference of 100 V is produced by transferring 6.25×10^{15} electrons from one conductor to the other. The capacity of the system will be.

Solution

$Q = CV \Rightarrow C = \frac{Q}{V} = \frac{ne}{V}$

Given $V = 100$ volts ; $n = 6.25 \times 10^{15}$

$$\therefore\quad C = \frac{6.25 \times 10^{15} \times 1.6 \times 10^{-19}}{100} = 10\ \mu\text{F}$$

BEGINNER'S BOX-1

1. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the capacitor is ....

2. A capacitor of capacitance C has a charge Q. The net charge on a capacitor is always ......It stores...... energy.

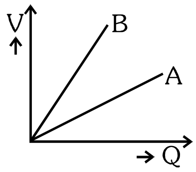
3. A capacitor of capacity C has charge Q and stored energy is W. If the charge is increased to 2Q then what will be the stored energy ?

4. Eight drops of mercury of equal radii and possessing equal charges combine to form a big drop. Then the capacitance of the bigger drop compared to each individual drop is ....

5. The capacitance of a spherical condenser whose inner sphere is grounded is 1μF. If the spacing between the two spheres is 1 mm then what is the radius of the outer sphere ?

6. When 2×10^{16} electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Calculate the capacitance of the two conductors system.

7. The graph shows the variation of voltage V across the plates of two capacitors A & B with charge Q. Which of the two capacitors has larger capacitance ?



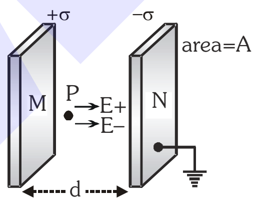
8. For flash pictures, a photographer uses a 30 μF capacitor and a charger that supplies 3×10^3 volt. Calculate the charge and the energy spent for each flash.

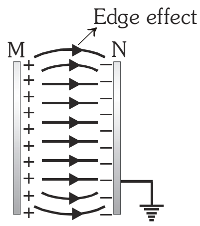
9. Two capacitors C_1 and C_2 have equal amount of energy stored in them. What is the ratio of potential differences across their plates ?

4. PARALLEL PLATE CAPACITOR

- **Capacitance**

It consists of two metallic plates M and N each of area A at a separation d. Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then





Step - I : Finding electric field $E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$ $[\varepsilon = \varepsilon_0\varepsilon_r]$

Step - II : Finding potential difference $V = Ed = \frac{\sigma}{\varepsilon_0\varepsilon_r}d = \frac{qd}{A\varepsilon_0\varepsilon_r}$ $\left(\because E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A}\right)$

Step - III : Finding capacitance $C = \frac{q}{V} = \frac{\varepsilon_r\varepsilon_0 A}{d}$

If the medium between the plates is air or vacuum, then $\varepsilon_r = 1 \Rightarrow C_0 = \frac{\varepsilon_0 A}{d}$

so $C = \varepsilon_r C_0 = KC_0$. (where $\varepsilon_r = K$ = dielectric constant)

- **Force between the plates**

The two plates of a capacitor attract each other because they are oppositely charged.

Electric field due to positive plate $E_1 = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$

Force on negative charge –Q is $F = -Q E_1 = -\frac{Q^2}{2\varepsilon_0 A}$

Magnitude of force $\boxed{F = \frac{Q^2}{2\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 AE^2}$ E = Net electric field between the plates of capacitor.

Force per unit area or energy density or electrostatic pressure = $\boxed{\frac{F}{A} = u = P = \frac{1}{2}\epsilon_0 E^2}$

5. EFFECT OF DIELECTRIC

- The insulators in which microscopic local displacement of charges take place in the presence of electric field are known as dielectrics.
- Dielectrics are non conductors upto a certain value of field depending on its nature. If the field exceeds this limiting value called dielectric strength, they lose their insulating property and begin to conduct.
- Dielectric strength is defined as the maximum value of electric field that a dielectric can withstand without breakdown. Unit is volt/metre. Dimensions : $[M^1L^1T^{-3}A^{-1}]$

Polar dielectrics

- In the absence of external field the centres of positive and negative charges do not coincide in these atoms or molecules due to asymmetric shapes of molecules.
- Each molecule has a permanent dipole moment.
- The dipoles are randomly oriented ; so average dipole moment per unit volume of polar dielectric in the absence of external field is zero.
- In the presence of external field dipoles tend to align in the direction of field.

Ex. Water, Alcohol, HCl, NH_3 etc.

Non-polar dielectrics

- In the absence of external field the centres of positive and negative charges coincide in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In the presence of external field they acquire induced dipole moment.

Ex. Nitrogen, Oxygen, Benzene, Methane etc.

Polarisation :

The alignment of dipole moments of permanent or induced dipoles in the direction of applied electric field is called polarisation.

- **Polarisation vector ($\vec{P}$)**

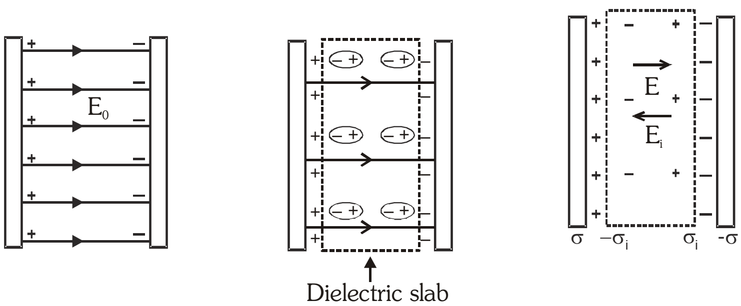
It is a vector quantity which describes the extent to which molecules of dielectric of become polarized by an electric field or oriented in the direction of field.

$\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

$$\vec{P} = n\vec{p} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_i = \text{induced surface charge density.}$$

Unit of $\vec{P}$ is C/m^2 Dimension : $[L^{-2}T^1A^1]$



Let E_0, V_0, C_0 be the electric field, potential difference and capacitance in the absence of dielectric. Let E, V, C be the corresponding quantities in the presence of the dielectric respectively.

Electric field in the absence of dielectric $\quad E_0 = \dfrac{V_o}{d} = \dfrac{\sigma}{\varepsilon_o} = \dfrac{Q}{\varepsilon_o A}$

Electric field in the presence of dielectric $\quad E = E_0 - E_i = \dfrac{\sigma - \sigma_i}{\varepsilon_o} = \dfrac{Q - Q_i}{A\varepsilon_o} = \dfrac{V}{d}$

Capacitance in the absence of dielectric $\quad C_0 = \dfrac{Q}{V_o}$

Capacitance in the presence of dielectric $\quad C = \dfrac{Q - Q_i}{V}$

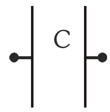
Dielectric constant or relative permittivity K or $\varepsilon_r = \dfrac{E_o}{E} = \dfrac{V_o}{V} = \dfrac{C}{C_o} = \dfrac{Q}{Q - Q_i} = \dfrac{\sigma}{\sigma - \sigma_i} = \dfrac{\varepsilon}{\varepsilon_o}$

From $K = \dfrac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \dfrac{1}{K}\right)$ and $K = \dfrac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \dfrac{1}{K}\right)$

Note :- Above relation is applicable only for PPC.

6. DIELECTRIC SLAB INSIDE A PARALLEL PLATE CAPACITOR

In case of a parallel plate capacitor $C = \dfrac{\varepsilon_0 A}{d}$



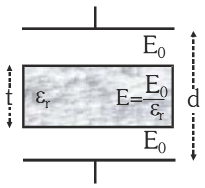
If capacitor is partially filled with dielectric

When the capacitor is filled partially with dielectric between plates, the thickness of dielectric slab is t (t < d) :

For an capacitor, the field E_0 is given by $E_0 = \dfrac{\sigma}{\varepsilon_0}$, exists in the space d.

On inserting a slab of thickness t, a field $E = \dfrac{E_0}{\varepsilon_r}$ appears inside the slab and a field E_0 exists in the remaining space (d – t). If V is the potential difference between the plates then $V = E_0(d - t) + Et$

$$\Rightarrow V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right] \quad \because \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

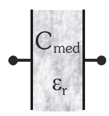


$$\Rightarrow V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right]$$

$$\Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} \qquad \text{...(i)}$$

If the dielectric medium is present between the entire space.

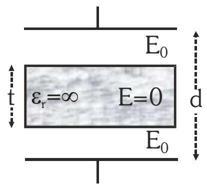
then $\quad t = d$



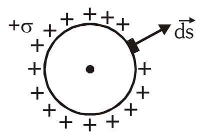
Now from equation (i) $\quad C_{medium} = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

If capacitor is partially filled with a conducting slab of thickness t (t < d).

$\because \quad \varepsilon_r = \infty$ for conductor, so $\quad C = \dfrac{\varepsilon_0 A}{d - t\left(1 - \dfrac{1}{\infty}\right)} = \dfrac{\varepsilon_0 A}{(d - t)}$

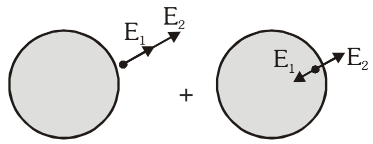


7. ELECTROSTATIC PRESSURE



Force due to electrostatic pressure is directed outwards normal to the surface.
Force on a small element ds of a charged conductor

$$dF = (\text{Charge on ds}) \times \text{Electric field} = (\sigma\, ds)\ \frac{\sigma}{2\epsilon_o}\quad dF = \frac{\sigma^2}{2\epsilon_o} ds$$



Inside $E_1 - E_2 = 0 \Rightarrow E_1 = E_2$

Just outside $E = E_1 + E_2 = 2E_2 \quad \Rightarrow \quad E_2 = \frac{\sigma}{2\epsilon_o}$

(E_1 is field due to charge on the element ds of the surface and E_2 is field due to rest of the sphere).

The electric force acting per unit area of charged surface is defined as electrostatic pressure.

$$P_{electrostatic} = \frac{dF}{dS} = \frac{\sigma^2}{2\epsilon_o}$$

7.1 Equilibrium of charged liquid surfaces (Soap bubble)

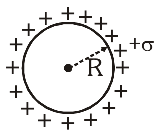
Pressures (forces) act on a charged soap bubble , due to

(i) Surface tension P_T (inward)

(ii) Air outside the bubble P_o (inward)

(iii) Electrostatic pressure P_e (outward)

(iv) Air inside the bubble P_i (outward)

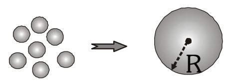


In the state of equilibrium, inward pressure = outward pressure $P_T + P_o = P_i + P_e$

Excess pressure of air inside the bubble $(P_{ex}) = P_i - P_o = P_T - P_e$

but $P_T = \frac{4T}{r}$ and $P_e = \frac{\sigma^2}{2\epsilon_o} \Rightarrow P_{ex} = \frac{4T}{r} - \frac{\sigma^2}{2\epsilon_o}$ if $P_i = P_o$ then $\frac{4T}{r} = \frac{\sigma^2}{2\epsilon_o}$

7.2 Combination of Identical Charged Tiny Drops



N tiny drops Big drops

Let , number of tiny drops = N

for each **tiny** drop	for **Big** drop
(r, q, C, σ, E, V)	$(R, Q, C_B, \sigma_B, E_B, V_B)$

(i) Charge conservation $Q = Nq$

(ii) Volume conservation $\frac{4}{3}\pi N.r^3 = \frac{4}{3}\pi R^3$.

Hence $R = N^{1/3} r$, $Q = Nq$ $\quad \therefore \quad$ $C_B = N^{1/3} C$, $\sigma_B = N^{1/3}\sigma$, $E_B = N^{1/3} E$, $V_B = N^{2/3} V$

- When a soap bubble is charged (either positive or negative) then the size (radius) increases some what and Positive charge $\Rightarrow$ mass $\downarrow$; Negative charge $\Rightarrow$ mass$\uparrow$

7.3 Energy Density (u)

Energy associated per unit volume of electric field is defined as energy density.

$$u = \frac{dW}{dV} = \frac{\epsilon_o E^2}{2} = \frac{\sigma^2}{2\epsilon_o}\ J/m^3$$

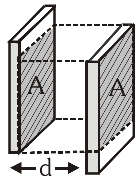
$$U = \int u.dV = \frac{\epsilon_o}{2}\int E^2\, dV$$; V is the volume of electric field.

GOLDEN KEY POINTS

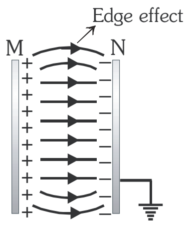
- If one of the plates of a parallel plate capacitor slides parallel to the other then C decreases (As overlapping area decreases).

  $C = \frac{\varepsilon_0 A}{d}$, where

  A = overlapping area



- If both the plates of a parallel plate capacitor are touched with each other then the resultant charge and potential difference becomes zero.
- Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



$\vec{E}$ = uniform between the plates

$\vec{E}$ = non-uniform at the edges

- Capacitance of a parallel plate capacitor does not depend on thickness and nature of metal of plates.
- For a parallel plate capacitor :

  (i) Intensity of electric field between the plates $E = \frac{\sigma}{\varepsilon_0} = \frac{V}{d}$ (uniform)

  (ii) Force between the plates $= \frac{CV^2}{2d} = \frac{QE}{2}$ (E → Electric field)

  (iii) Pressure on the plates $= \frac{\sigma^2}{2\varepsilon_0}$

- If nothing is mentioned then assume the battery to be disconnected and Q is constant.
- A parallel plate capacitor is connected to a battery (V = const.) and a slab of dielectric constant ε_r is inserted between the plates then the total energy delivered by the battery is divided into two parts :

  (i) Half is used to insert the slab (work is done by field)

  (ii) Half is stored in the form of electrostatic potential energy.

Battery disconnected (Q is constant)

Change executed	Q	$V = \frac{Q}{C}$ $V \propto \frac{1}{C}$	$E = \frac{Q}{\varepsilon_0 \varepsilon_r A}$	$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$	$U = \frac{Q^2}{2C}$ $U \propto \frac{1}{C}$
Filled with medium	Unchanged	Decreases	Decreases	Increases	Decreases
Distance is Decreased	Unchanged	Decreases	Unchanged	Increases	Decreases
Area is Increased	Unchanged	Decreases	Decreases	Increases	Decreases

Battery still connected (V is constant)

Change executed	$Q = CV$ $Q \propto C$	V constant	$E = \frac{V}{d}$ $E \propto \frac{1}{d}$	$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$	$U = \frac{1}{2}CV^2$ $U \propto C$
Filled with medium	Increases	Unchanged	Unchanged	Increases	Increases
Distance is Decreased	Increases	Unchanged	Increases	Increases	Increases
Area is Increased	Increases	Unchanged	Unchanged	Increases	Increases

- If a small charge q is moved along a closed path in the field between the plates of a parallel-plate capacitor, no work will be done by the agent.
- Two positively charged identical plates placed parallel to each other form a parallel plate capacitor.
- If a capacitor is connected across a battery, then the charges will be equal in magnitude even if the plates are of different sizes.

Illustrations

Illustration 7.

If the distance between the plates of a capacitor of capacitance C_1 is halved and the area of plates is doubled then what will be the capacitance ?

Solution

$$C = \frac{\varepsilon_0 A}{d} \quad \Rightarrow \quad \frac{C_1}{C_2} = \frac{A_1}{A_2}\frac{d_2}{d_1} = \frac{A_1}{2A_1} \times \left(\frac{1}{d_1}\right)\left(\frac{d_1}{2}\right) = \frac{1}{4} \Rightarrow C_2 = 4C_1$$

Illustration 8.

A capacitor has two circular plates whose radii are 8 cm each and distance between them is 1mm. When mica slab (dielectric constant = 6) is placed between the plates, calculate the capacitance and the energy stored when it is given a potential of 150 volts.

Solution

Area of each plate = $\pi r^2 = \pi \times (8 \times 10^{-2})^2 = 0.0201\ m^2$ and $d = 1$ mm $= 1 \times 10^{-3}$ m

Capacity of capacitor $C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 0.0201}{1 \times 10^{-3}} = 1.068 \times 10^{-9}$ F

Potential difference $V = 150$ volt

Energy stored $U = \frac{1}{2}CV^2 = \frac{1}{2} \times (1.068 \times 10^{-9}) \times (150)^2 = 1.2 \times 10^{-5}$ J

Illustration 9.

A parallel-plate capacitor is formed of two plates, each of area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V/m is introduced between the plates. Find the maximum charge that can be stored in the capacitor without causing any dielectric breakdown.

Solution

If the charge on the capacitor = Q

the surface charge density $\sigma = \frac{Q}{A}$ and the electric field = $\frac{Q}{KA\varepsilon_0}$.

This electric field should not exceed the dielectric strength 1.9×10^7 V/m.

$\therefore$ if the maximum charge which can be given is Q then $\frac{Q}{KA\varepsilon_0} = 1.9 \times 10^7$ V/m

$\because A = 100\ cm^2 = 10^{-2}\ m^2 \Rightarrow Q = (5.0) \times (10^{-2}) \times (8.85 \times 10^{-12}) \times (1.9 \times 10^7) = 8.4 \times 10^{-6}$ C.

Illustration 10.

The distance between the plates of a parallel-plate capacitor is 0.05 m. A field of 3×10^4 V/m is established between the plates. It is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted into the gap between the plates. Find the potential difference of the capacitor (i) before the introduction of the metal plate and (ii) after its introduction. What would be the potential difference if a plate of dielectric constant K = 2 is introduced in place of metal plate ?

Solution

(i) In case of a capacitor as E = (V/d), the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5 \text{ kV}$$

(ii) Now as battery is removed after charging, capacitor is isolated so q = constant. If C' and V' are the capacity and potential after the introduction of plate then, q = CV = C'V' i.e., $V' = \frac{C}{C'}V$

And as $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{(d-t)+(t/K)}$, $V' = \frac{(d-t)+(t/K)}{d}V$

So in case of metal plate as K = ∞, $V_M = \left[\frac{d-t}{d}\right]V = \left[\frac{0.05-0.01}{0.05}\right]1.5 = 1.2 \text{ kV}$

And if instead of metal plate a dielectric with K = 2 is

introduced then $V_D = \left[\frac{(0.05-0.01)+(0.01/2)}{0.05}\right] \times 1.5 = 1.35 \text{ kV}$.

Illustration 11.

A parallel plate capacitor has a potential 20 kV and capacitance 2×10^{-4} μF. If area of each plate is 0.01 m² and distance between them is 2 mm then find the -

(a) potential gradient (b) dielectric constant of medium (c) energy

Solution

(a) potential gradient $= \frac{V}{d} = \frac{20000}{0.002} = 1 \times 10^7$ V/m

(b) $C = \frac{\varepsilon_0 \varepsilon_r A}{d} \Rightarrow \varepsilon_r = \frac{Cd}{\varepsilon_0 A} = \frac{2 \times 10^{-10} \times 2 \times 10^{-3}}{8.85 \times 10^{-12} \times 0.01} = 4.52$

(c) $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 2 \times 10^{-10} \times (20000)^2 = 4 \times 10^{-2}$ J

Illustration 12.

Twenty seven charged water droplets, each of radius 10^{-3} m and having a charge of 10^{-12} C, coalesce to form a single drop. Calculate the potential of the bigger drop.

Solution

Volume of bigger drop=N × volume of smaller drop $\Rightarrow \frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3 \Rightarrow R^3 = Nr^3 \Rightarrow R = N^{1/3} r$

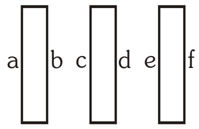
$R = (27)^{1/3} r = 3r$

Potential of bigger drop $V = \frac{kQ}{R} = \frac{kNq}{3r} = \frac{9 \times 10^9 \times 27 \times 10^{-12}}{3 \times 10^{-3}} = 81$ volts

BEGINNER'S BOX-2

1. The plate separation in a parallel plate capacitor is d and plate area is A. If it is charged to V volts then calculate the work done in increasing the plate separation to 2d.

2. Three parallel metallic plates, each of area A are kept as shown in the figure and charges Q_1, Q_2 and Q_3 are given to them. Edge effects are negligible. Calculate the charges on the two outermost surfaces 'a' and 'f'.



(A) $\frac{Q_1+Q_2+Q_3}{2}$ (B) $\frac{Q_1+Q_2+Q_3}{3}$ (C) $\frac{Q_1-Q_2+Q_3}{3}$ (D) $\frac{Q_1-Q_2+Q_3}{2}$

3. A capacitor has a capacitance of 50 pF, which increases to 175 pF with a dielectric material between its plates. What is the dielectric constant of the material ?

4. A parallel plate capacitor has rectangular plates with dimensions 6.0 cm × 8.0 cm. If the plates are separated by a sheet of teflon (K = 2.1) 1.5 mm thick, how much energy is stored in the capacitor when it is connected to a 12 V battery ?

5. The distance between the plates of a parallel plate capacitor is 'd'. Another thick metal plate of thickness d/2 and area same as that of plates is so placed between the plates, that it does not touch them. The capacity of the resulting capacitor :–

(A) remains the same (B) becomes double (C) becomes half (D) becomes one fourth

6. The capacity and the energy stored in a parallel plate condenser with air between its plates are respectively C_0 and W_0. If the air between the plates is replaced by glass (dielectric constant = 5) find the capacitance of the condenser and the energy stored in it.

7. A parallel plate capacitor is to be designed with a voltage rating 1 kV using a material of dielectric constant 10 and dielectric strength 10^6 Vm^{-1}. What minimum area of the plates is required to have a capacitance of 88.5 pF ?

8. A capacitor of capacitance 10 μF is connected to battery of emf 20 V. Without disconnecting the source a dielectric (K=4) is introduced to fill the space between the two plates of the capacitor. Calculate the –

(a) charge before the dielectric was introduced.

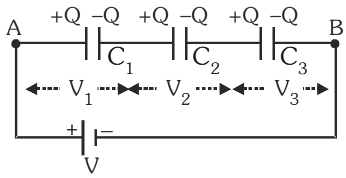
(b) charge after the dielectric is introduced.

9. An air capacitor of capacity C = 10 μF is connected to a constant voltage battery of 10 V. Now the space between the plates is filled with a liquid of dielectric constant 5 . Calculate additional charge which flows from the battery to the capacitor.

10. If the distance between the plates of a capacitor is d and potential difference is V then what is the energy density between the plates ?

11. 64 droplets of mercury each of radius r and carrying charge q, coalesce to form a big drop. Compare the surface density of charge of each drop with that of the big drop.

8. COMBINATION OF CAPACITORS



- **Capacitors in series**:

In this arrangement of capacitors, charge has no alternative path(s) to flow.

(i) The charge on each capacitor is equal

i.e. $Q = C_1V_1 = C_2V_2 = C_3V_3$

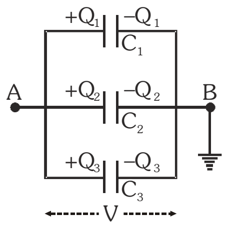
(ii) The total potential difference across AB is shared by the capacitors in the inverse ratio of their respective capacitances $V = V_1 + V_2 + V_3$

If C_S is the net capacitance of the series combination, then

$$\frac{Q}{C_S}=\frac{Q}{C_1}+\frac{Q}{C_2}+\frac{Q}{C_3} \Rightarrow \boxed{\frac{1}{C_S}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}}$$

- **Capacitors in parallel**

In such an arrangement of capacitors, charge has an alternative path(s) to flow.



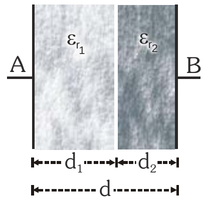
(i) The potential difference across each capacitor is same and equal to the total potential difference applied. i.e. $V=V_1=V_2=V_3 \Rightarrow V=\frac{Q_1}{C_1}=\frac{Q_2}{C_2}=\frac{Q_3}{C_3}$

(ii) The total charge Q is shared by the capacitor in the direct ratio of their respective capacitances. $Q = Q_1 + Q_2 + Q_3$

If C_P is the net capacitance for the parallel combination of capacitors, then :

$$C_PV = C_1V + C_2V + C_3V \Rightarrow \boxed{C_P = C_1 + C_2 + C_3}$$

- **Combination of Dielectric Slabs**
- **Plate Separation Division**



(i) Plate Separation gets divided and area remains same.

(ii) Capacitors are in series.

(iii) Individual capacitances are $C_1=\frac{\varepsilon_0\varepsilon_{r_1}A}{d_1}$, $C_2=\frac{\varepsilon_0\varepsilon_{r_2}A}{d_2}$

These two are in series $\therefore \frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2} \Rightarrow \frac{1}{C}=\frac{d_1}{\varepsilon_0\varepsilon_{r_1}A}+\frac{d_2}{\varepsilon_0\varepsilon_{r_2}A} \Rightarrow \frac{1}{C}=\frac{1}{\varepsilon_0 A}\left[\frac{d_1\varepsilon_{r_2}+d_2\varepsilon_{r_1}}{\varepsilon_{r_1}\varepsilon_{r_2}}\right]$

$$\Rightarrow C=\varepsilon_0 A\left[\frac{\varepsilon_{r_1}\varepsilon_{r_2}}{d_1\varepsilon_{r_2}+d_2\varepsilon_{r_1}}\right]$$

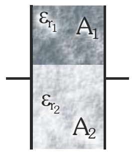
Special case : If $d_1=d_2=\frac{d}{2} \Rightarrow \boxed{C=\frac{\varepsilon_0 A}{d}\left[\frac{2\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1}+\varepsilon_{r_2}}\right]}$

- **Plate Area Division**

(i) Plate area gets divided and distance between them remains same.

(ii) Capacitors are in parallel.

(iii) Individual capacitances are $C_1=\frac{\varepsilon_0\varepsilon_{r_1}A_1}{d}$, $C_2=\frac{\varepsilon_0\varepsilon_{r_2}A_2}{d}$



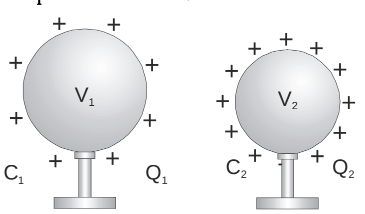
These two are in parallel so $C = C_1 + C_2 = \frac{\varepsilon_0\varepsilon_{r_1}A_1}{d}+\frac{\varepsilon_0\varepsilon_{r_2}A_2}{d}=\frac{\varepsilon_0}{d}(\varepsilon_{r_1}A_1+\varepsilon_{r_2}A_2)$

Special case : If $A_1 = A_2 = \frac{A}{2}$ Then $\boxed{C=\frac{\varepsilon_0 A}{d}\left(\frac{\varepsilon_{r_1}+\varepsilon_{r_2}}{2}\right)}$

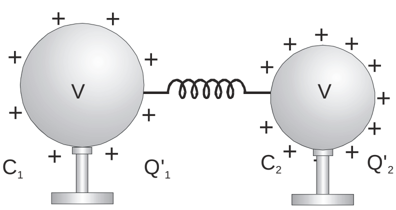
9. SHARING OF CHARGES

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of both conductors become equal.

Let the amounts of charges after the conductors are connected be Q_1' and Q_2' respectively and their common potential be V then



(Before connection)



(After connection)

- **Common potential**

  According to the law of conservation of charge $Q_{\text{before connection}} = Q_{\text{after connection}}$

$$C_1V_1 + C_2V_2 = C_1V + C_2V$$

  Common potential after connection $\boxed{V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}}$

- **Charges after connection**

  $Q_1' = C_1V = C_1\left(\dfrac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\dfrac{C_1}{C_1 + C_2}\right)Q$ (Q = Total charge on the system)

  $Q_2' = C_2V = C_2\left(\dfrac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\dfrac{C_2}{C_1 + C_2}\right)Q$

  Ratio of the charges after redistribution $\boxed{\dfrac{Q_1'}{Q_2'} = \dfrac{C_1V}{C_2V} = \dfrac{R_1}{R_2} \text{ (in case of spherical conductors)}}$

- **Loss of energy in charge redistribution**

  When charge flows through the conducting wire certain **energy is lost** and electrical energy is converted into heat energy, so change in energy of this system is,

$$\Delta U = U_f - U_i \Rightarrow \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) \Rightarrow \Delta U = -\frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$$

  Here negative sign indicates that energy of the system decreases in the process.

GOLDEN KEY POINTS

- If space between the plates is divided equally into two parts

  (i) Distance-wise division C_e = (Harmonic mean of ε_{r_1} & ε_{r_2}) × C = $\left(\dfrac{2\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right)C$

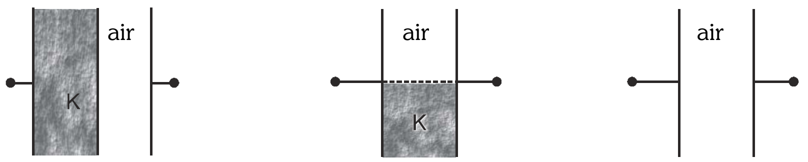
  (ii) Area-wise division C_e = (Arithmetic mean of ε_{r_1} & ε_{r_2}) × C = = $\left(\dfrac{\varepsilon_{r_1} + \varepsilon_{r_2}}{2}\right)C$

  Where C = capacity of PPC without any dielectric

- If $V_1 = V_2$ then neither charge flows nor energy is lost when two charged conductors are connected.

- A charged capacitor of energy U is connected to an identical uncharged capacitor.

  Then electrostatic potential energy of the system = $\dfrac{U}{2}$, Heat loss = $\dfrac{U}{2}$ and energy of each capacitor = $\dfrac{U}{4}$

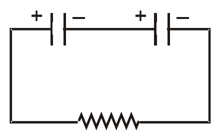
- 

  $C_1 = \left[\dfrac{2K}{K+1}\right]C$ $C_2 = \left[\dfrac{K+1}{2}\right]C$ $C_3 = C$ When no dielectric is used

  $C_2 > C_1 > C_3$

- For a given voltage, in order to store maximum energy capacitors should be connected in parallel.
- If N identical capacitors each having breakdown voltage V are joined in (i) series then the break down voltage of the combination = NV (ii) parallel then the breakdown voltage of the combination = V
- Two capacitors are connected in series with a battery.

  Now, the battery is removed and loose wires ends are connected together then the final charge on each capacitor is zero.



- If N identical capacitors are connected then $C_{series} = \frac{C}{N}$, $C_{parallel} = NC$

Illustrations

Illustration 13.

An infinite number of capacitors of capacitance C, 4C, 16C ... ∞ are connected in series then what will be their resultant capacitance ?

Solution

Let the equivalent capacitance of the combination = C_{eq}

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{4C} + \frac{1}{16C} + \ldots\infty$$

$$= \left[1 + \frac{1}{4} + \frac{1}{16} + \ldots\infty\right]\frac{1}{C}$$

$$\frac{1}{C_{eq}} = \frac{1}{1-\frac{1}{4}} \times \frac{1}{C} \Rightarrow C_{eq} = \frac{3}{4}C$$

Illustration 14.

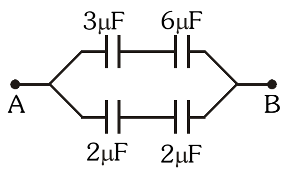
Three identical capacitors are connected together differently. For the same voltage applied across each combination, which one stores maximum energy ?

Solution

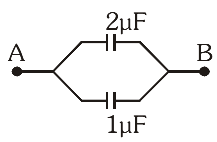
When all three are connected in parallel combination then this combination stores maximum energy because C_{eq} will be maximum in parallel combination and $U = \frac{1}{2}C_{eq}V^2$.

Illustration 15.

What is the effective capacity between points A and B of the network of capacitors shown in figure?



Solution

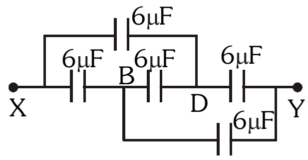


$C_1 = \frac{3 \times 6}{3+6} = 2\ \mu F$, $C_2 = \frac{2}{2} = 1\ \mu F$

$\Rightarrow C_{eq} = 2 + 1 = 3\ \mu F$

Illustration 16.

What is the effective capacitance between the points X and Y ?



Solution

This is a balanced Wheatstone bridge, so BD can be removed.

6 μF and 6 μF are in series so equivalent capacitance = 3 μF

then 3 μF and 3 μF are in parallel so equivalent capacitance between X and Y is 6μF.

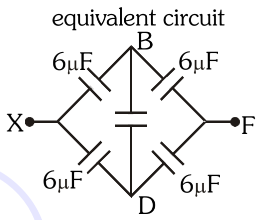
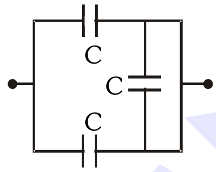


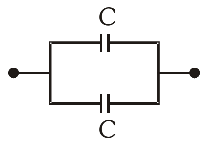
Illustration 17.

What is the the equivalent capacitance of the combination ?



Solution

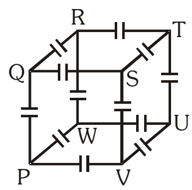
The capacitor shown as a vertical element is shorted, so it can be removed.



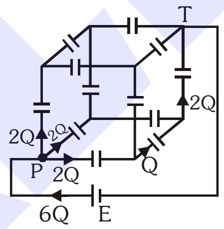
$\Rightarrow C_{eq} = C + C = 2C$

Illustration 18.

Twelve identical capacitors each of capacitance C are connected as shown in figure. Find the effective capacitance between P and T.



Solution



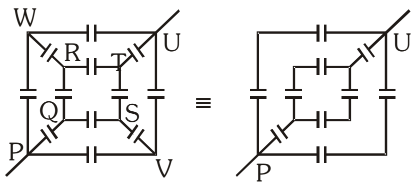
$E = \frac{2Q}{C} + \frac{Q}{C} + \frac{2Q}{C} = \frac{5Q}{C}$, $C_{eff} = \frac{6Q}{E} = \frac{6C}{5}$.

Illustration 19.

In Illustration 18, find the effective capacitance between P and U.

Solution

Given circuit can be redrawn as

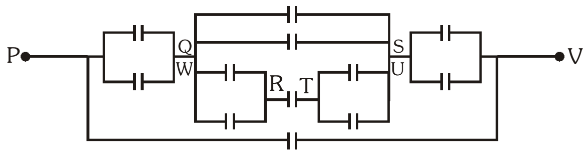


Equivalent capacitance between P and U $= \frac{C}{3} + \frac{C}{2} + \frac{C}{2} = \frac{4C}{3}$.

Illustration 20.

In Illustration 18, find the effective capacitance between P and V.

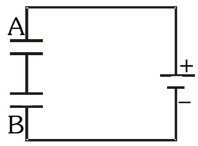
Solution



If a battery be connected across the terminals P and V, from symmetry $V_Q = V_W$ and $V_S = V_U$

$$\Rightarrow \text{ Equivalent capacitance } = \frac{\left(\frac{5}{2}C\right)(C)}{\frac{5}{2}C+C}+C=\frac{12C}{7}.$$

Illustration 21.



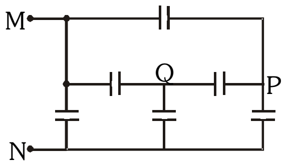
Two identical capacitors A and B shown in the given circuit are joined in series with a battery. A dielectric slab of dielectric constant K is slipped between the plates of capacitor B with battery remaining connected. Then state wheter the energy of capacitor A will increase or decrease ?

Solution

Capacitance of capacitor B will increase so equivalent capacitance and charge on each capacitor will also increase.

We know that $U=\frac{Q^2}{2C}$, So energy of capacitor A will increase.

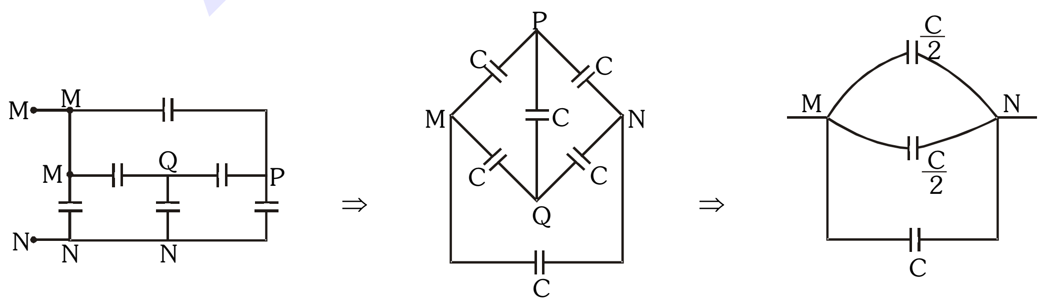
Illustration 22.



Six equal capacitors each of value 4 μF are connected as shown in figure.

Calculate the equivalent capacitance between the points M and N.

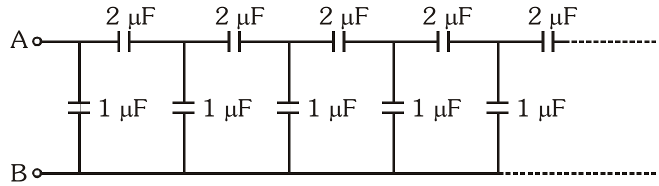
Solution



$\Rightarrow \quad C_{eq} = 2C = 2 \times 4 = 8\ \mu F.$

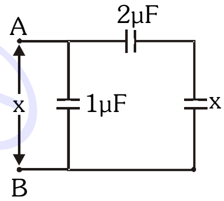
Illustration 23.

Find the equivalent capacitance of the infinite ladder shown in figure between the points A & B.



Solution

Let equivalent capacitance be x.



$$x = \frac{2x}{2+x} + 1$$

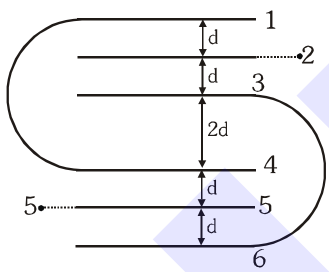
$$x = \frac{2x+2+x}{2+x} \Rightarrow x(2 + x) = 3x + 2 \Rightarrow 2x + x^2 = 3x + 2$$

$$\Rightarrow x^2 - x - 2 = 0$$

Using $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{1 \pm \sqrt{1+8}}{2} = \dfrac{1 \pm 3}{2} = 2$ and -1

$x = 2$, $C_{eq} = 2\ \mu F$ $\quad (\because C \neq -ve)$

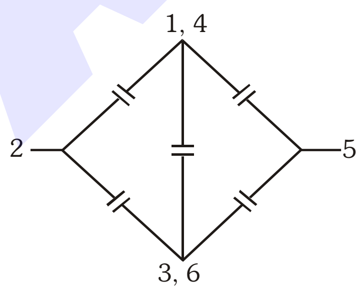
Illustration 24.



There are six plates of equal area A and separation between the adjacent plates is d or 2d (d<<A). They are arranged as shown in figure. Find the equivalent capacitance between points 2 and 5.

Solution

Redrawing the circuit

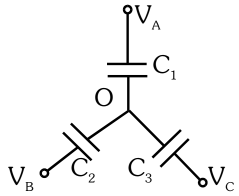


It is a wheatstone bridge with points (3, 6) and (1, 4) being equipotential. So, the capacitance C/2 can be removed.

$$\therefore \quad C_{eq} = C = \frac{\epsilon_0 A}{d}$$

Illustration 25.

Calculate the potential of point O in terms of C_1, C_2, C_3, V_A, V_B, & V_C in the following curcuit.

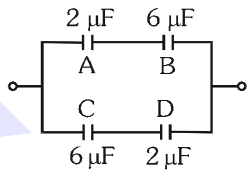


Solution

Let the potential of the junction O be V_0. Now apply Kirchhoff's current law at a junction.

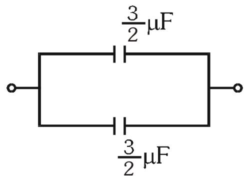
$$C_1 (V_A - V_0) + C_2 (V_B - V_0) + C_3 (V_C - V_0) = 0 \Rightarrow V_0 = \frac{C_1V_A + C_2V_B + C_3V_C}{C_1 + C_2 + C_3}.$$

Illustration 26.



Four capacitors are arranged to form the given circuit. If this arrangement is connected across a voltage source then charge supplied by the source is 24 μC. Calculate the charge on capacitor A.

Solution

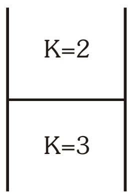


Given circuit can be redrawn as shown in figure.

As capacitance of both the branches are same so 24 μC charge will be equally divided.

∴ charge on capacitor A = 12 μC

Illustration 27.



A parallel plate capacitor with no dielectric has a capacitance of 0.5 μF.

Half of the space between the plates is filled with a medium of dielectric constant 2 and remaining half is filled with a medium of dielectric constant of 3 as shown in figure. Find its net capacity.

Solution

Given that original capacitance $C = \frac{\epsilon_0 A}{d} = 0.5\ \mu F$

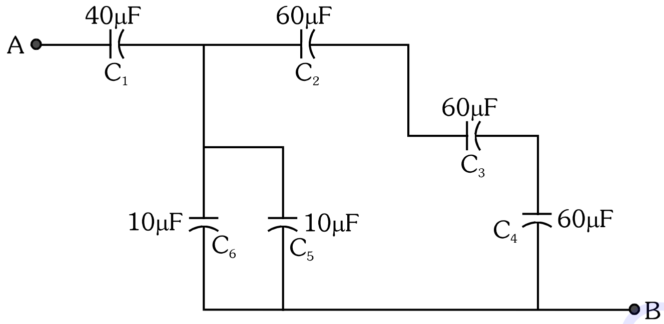
Capacitance of part with dielectric constant 2 is $C_1 = \frac{2\epsilon_0 A/2}{d} = \frac{\epsilon_0 A}{d} = 0.5\ \mu F$

Capacitance of part with dielectric constant 3 is $C_2 = \frac{3\epsilon_0 A/2}{d} = \frac{3\epsilon_0 A}{2d} = 0.75\ \mu F$

As both the capacitors are connected in parallel so $C_{eq} = C_1 + C_2 = 0.5 + 0.75 = 1.25\ \mu F$

BEGINNER'S BOX-3

1. Find the equivalent capacitance of the combination of capacitors between the points A and B as shown in figure. Also calculate the total charge that flows in the circuit when a 100 V battery is connected between the points A and B

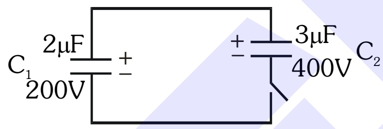


2. Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination ?

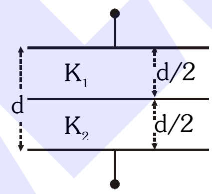
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

3. Two capacitors of capacity C_1 and C_2 are connected as shown in figure.

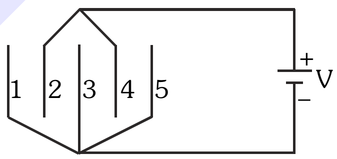


Now the switch is closed. Calculate the charge on each capacitor.

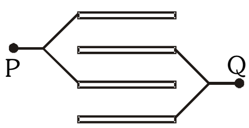
4. Two dielectric slabs of dielectric constants K_1 and K_2 have been inserted in between the plates of a capacitor as shown below. What will be the capacitance of the capacitor inserted ?
(Plate area = A)



5. Five identical plates each of area A are joined as shown in the figure. The distance between successive plates is d. The plates are connected to potential difference of V volt. Find the charges of plates 1 and 4



6. Four plates of the same area A are joined as shown in the figure. The distance between successive plates is d. Find the equivalent capacity across PQ will be



7. Two identical capacitors each of capacity C are charged upto same potential V. Now their oppositely charged plates are connected together then calculate the –

(a) energy of each capacitor before connection.

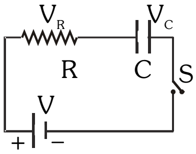
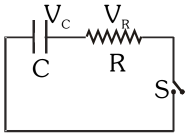
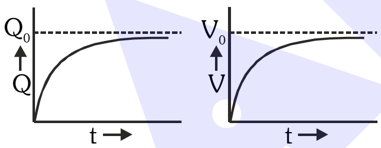
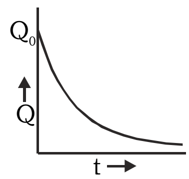
(b) potential of each capacitor after connection.

(c) charge of each capacitor after connection.

(d) energy stored in each capacitor after connection.

(e) energy loss in the form of heat.

10. CHARGING & DISCHARGING OF A CONDENSER

Charging	Discharging
• When a capacitor, resistance and battery are are connected in series and key is closed.  	• When a charged capacitor, resistance and key connected in series and key is closed, then energy stored in the capacitor is used to drive current in the circuit. 
$V = V_C + V_R$ $V = \frac{Q}{C} + IR = \frac{Q}{C} + \frac{dQ}{dt}R$ $R\frac{dQ}{dt} = \frac{CV-Q}{C} \Rightarrow \int_0^Q \frac{dQ}{CV-Q} = \int_0^t \frac{dt}{CR}$ $-\ell n\frac{CV-Q}{CV} = \frac{t}{CV} \Rightarrow 1-\frac{Q}{CV} = e^{-t/RC}$ $\boxed{Q = CV\left[1-e^{-t/RC}\right]}$ $\boxed{Q = Q_0\left[1-e^{-t/RC}\right]}$ This is the charge at any instant t.	$V_C + V_R = 0$ $\boxed{V = V_0 e^{-t/RC}}$ $\boxed{Q = Q_0 e^{-t/RC}}$ This is the quantity of charge & voltage at any instant t.
• t = RC is known as time constant. At the end of one time constant the charge on the capacitor rises to 63% of its maximum value.	• At t = RC the charge falls to 37% of its maximum initial value.
• Potential diff. across the condenser plates at any instant is $V = V_0(1-e^{-t/RC})$  	• 

11. VAN DE GRAAFF GENERATOR (Only for 12th Board)

(i) It uses a moving belt to buildup very high amounts of electrical potential (of the order of ten million volts) on a hollow metal globe on the top of a stand.

(ii) This machine acts on the principle of corona discharge.

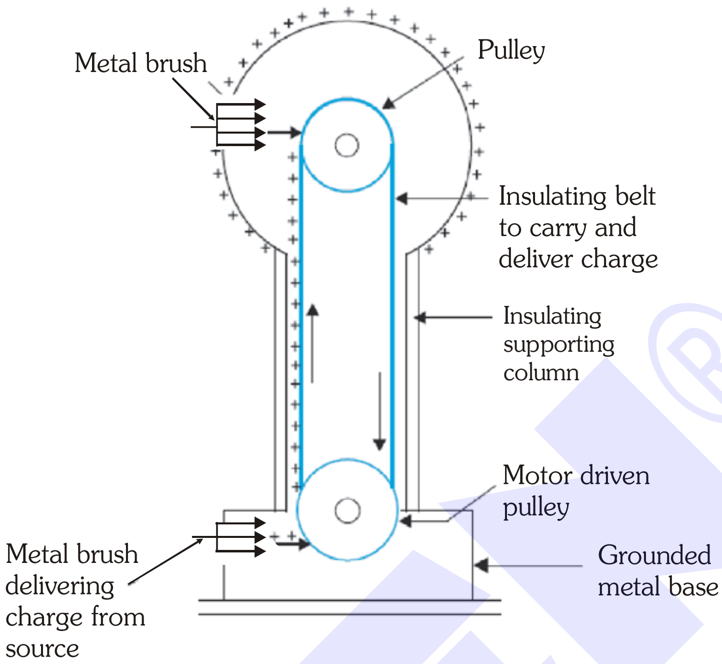
(iii) It is based on the following principles

- Action of sharp points : charges are leaked from pointed ends of charged conductors. This creates on electric wind (as moving air is ionized) which moves away from the conductor.
- The property that the charge given to a hollow conductor is transferred to the outer surface and is distributed uniformly on it.

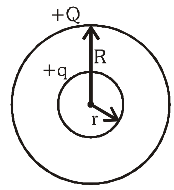
(iv) A Van de Graaff generator operates by transferring electric charge from a moving belt to a terminal

(v) The potential at the outer surface would also keep rising, at least until we reach the break down field of air, which is about 3×10^6 V/m.

(vi) It is a source of high voltage for accelerating charge particles to a high speed.



(vii) Consider a shell of radius R and having a charge Q enclosing a smaller sphere of radius r and having a charge q. The potential of the two spheres are



$$V(R) = \frac{1}{4\pi \epsilon_0}\left(\frac{Q}{R} + \frac{q}{R}\right)$$

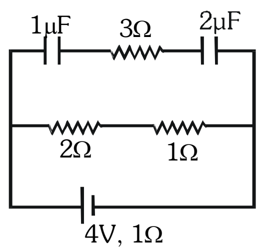
and $$V(r) = \frac{1}{4\pi \epsilon_0}\left(\frac{Q}{R} + \frac{q}{r}\right)$$

The potential difference between the inner and outer sphere is $V(r) - V(R) = \frac{q}{4\pi \epsilon_0}\left[\frac{1}{r} - \frac{1}{R}\right]$ thus for positive q, whatever be the magnitude and sign of Q, the small sphere is at a higher potential than the shell. When an electrical contact is established, charge would flow from the small sphere to the shell.

Illustrations

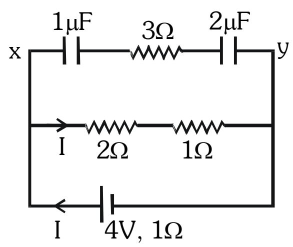
Illustration 28.

In the given circuit calculate the potential difference across the 2 μF capacitor in the steady state condition, if internal resistance of battery is 1 ohm.



Solution

Current in the steady state condition = $\frac{4}{2+1+1} = 1$ A



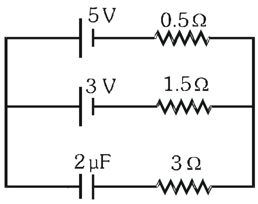
Potential difference between x and y = 3 V.

Divide this 3V in inverse ratio of capacity

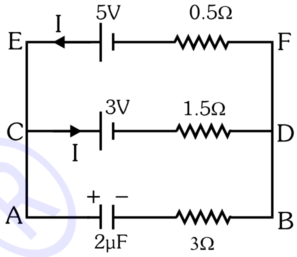
Voltage on 2 μF capacitor = $\frac{1}{2+1} \times 3 = 1$V.

Illustration 29.

In the given circuit find the charge on the capacitor.



Solution



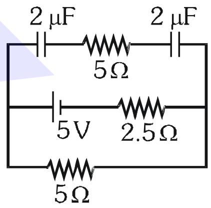
In the steady state no current is there in capacitors's branch.

So current $I = \dfrac{2}{0.5+1.5} = 1A$

Voltage across capacitor $V_C = 3 + 1.5 \times 1 = 4.5$ V

$\Rightarrow \quad Q = CV_C = 2 \times 10^{-6} \times 4.5 = 9 \times 10^{-6}$ C

Illustration 30.



In the given circuit find the :-

(a) initial current supplied by the battery.

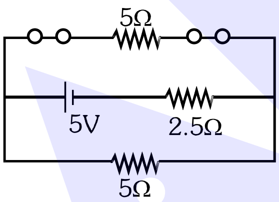
(b) final current supplied by the battery.

Solution

(a) Initially capacitor behaves just like a zero resistance wire

Total resistance R = 5Ω

$I = \dfrac{5}{5} = 1A$



(b) Finally it behaves as open circuit resistance wire

Total resistance R = 7.5 Ω

$I = \dfrac{5}{7.5} = \dfrac{2}{3}A$

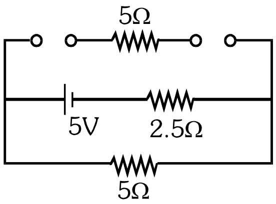
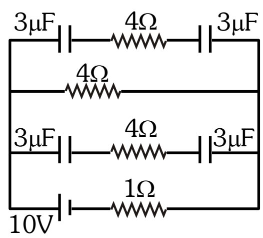


Illustration 31.

Calculate the charge on each capacitor in the steady state .

Solution



When capacitor is fully charged, no current will flow through it

$\therefore$ current in the circuit will be $I = \dfrac{10}{1+4} = 2$ A

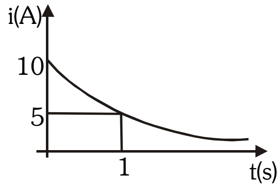
$\therefore$ voltage drop across 4 Ω resistance is V = IR = 2 × 4 = 8 V.

This voltage will get divided between the two capacitors. So voltage across each capacitor is $V_C = 4V$

$\therefore$ charge on each capacitor $Q = CV = 4 \times 3 \times 10^{-6}$ C = 12 μC

BEGINNER'S BOX-4

1. Figure shows, the graph of the current in a discharging circuit of a capacitor through a resistor of resistance $10\ \Omega$:



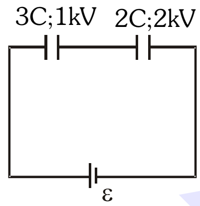
(i) Find the initial potential difference across the capacitor.

(ii) Find the capacitance of the capacitor.

(iii) Find the total heat produced in the circuit.

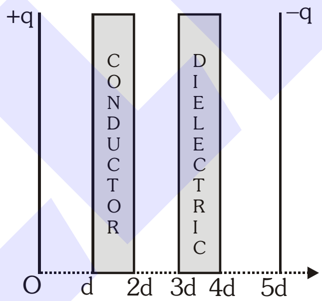
(iv) Find the time constant of the circuit.

2. The diagram shows two capacitors with capacitance and breakdown voltages as mentioned. What should be the maximum value of the external emf source such that no capacitor undergoes breaks down?

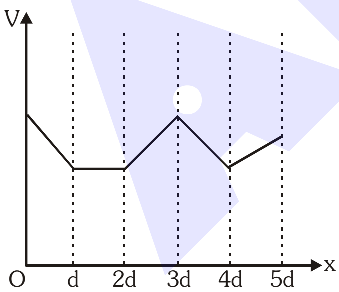


(A) 2.5 kV (B) 10/3 kV (C) 3 kV (D) 1 kV

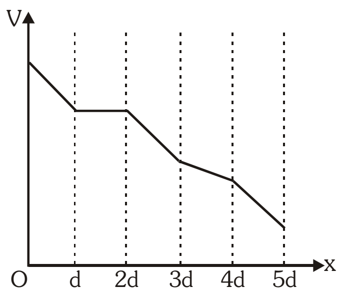
3. The distance between plates of a parallel plate capacitor is 5d. The positively charged plate is at x=0 and negatively charged plates is at x=5d. Two slabs – one of conductor and the other of a dielectric, both of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance x graph will be



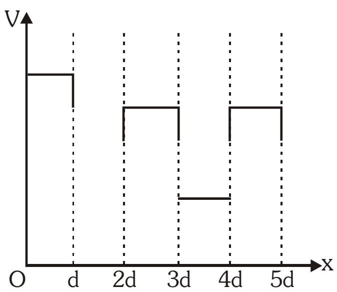
(A)



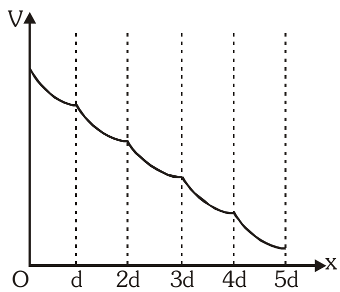
(B)



(C)

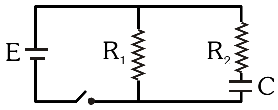


(D)



4. A 400 μF condenser is charged at the steady rate of 100 μC per second. Calculate the time required to establish a potential difference of 100 volts between its plates.

5. For the circuit shown in figure, find



(a) the initial currents through each resistor after the switch is closed.

(b) steady state currents through each resistor after the switch is closed.

(c) final energy stored in the capacitor after the switch is closed.

(d) time constant of the circuit when switch is opened.

(e) time constant of the circuit when switch is closed.

6. A condenser of capacitance 2 μF has been charged to 200 V. It is now discharged through a resistance; the heat produced in the wire is ....

ANSWERS

BEGINNER'S BOX-1

1. Zero

2. Zero, $\frac{1}{2}\frac{Q^2}{C}$

3. 4W

4. 2 times

5. 3 m

6. 3.2×10^{-4} F

7. $C_A > C_B$

8. 0.09 C, 135 J

9. $\sqrt{\frac{C_2}{C_1}}$

BEGINNER'S BOX-2

1. $\frac{\epsilon_0 AV^2}{2d}$

2. A

3. 3.5

4. 4.3×10^{-9} J

5. B

6. $5C_0$, $W_0/5$

7. 10^{-3} m²

8. 200 μC; 800 μC

9. 400 μC

10. $\frac{1}{2}\varepsilon_0\frac{V^2}{d^2}$

11. $\sigma_{Big} = 4\sigma_{Small}$

BEGINNER'S BOX-3

1. 20 μF ; 2×10^{-3} C

2. (a) 3 pF; (b) 40 V

3. 640 μC ; 960 μC

4. $\frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$

5. $\frac{-\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$

6. $\frac{3\varepsilon_0 A}{d}$

7. (a) $\frac{1}{2}CV^2$ (b) Zero; (c) Zero & Zero;

(d) Zero & Zero; (e) CV^2

BEGINNER'S BOX-4

1. (i) 100 volts ; (ii) $\frac{1}{10\ell n2}$ F;

(iii) $\frac{500}{\ell n2}$ joules; (iv) $\frac{1}{\ell n2}$ seconds

2. (A) **3.** (B) **4.** 400 s

5. (a) $i_1 = \frac{E}{R_1}$ and $i_2 = \frac{E}{R_2}$; (b) $i_1 = \frac{E}{R_1}$, $i_2 = 0$;

(c) $U = \frac{1}{2}CE^2$; (d) $C(R_1+R_2)$; (e) $\tau = R_2C$

6. 0.04 J