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PHYSICS

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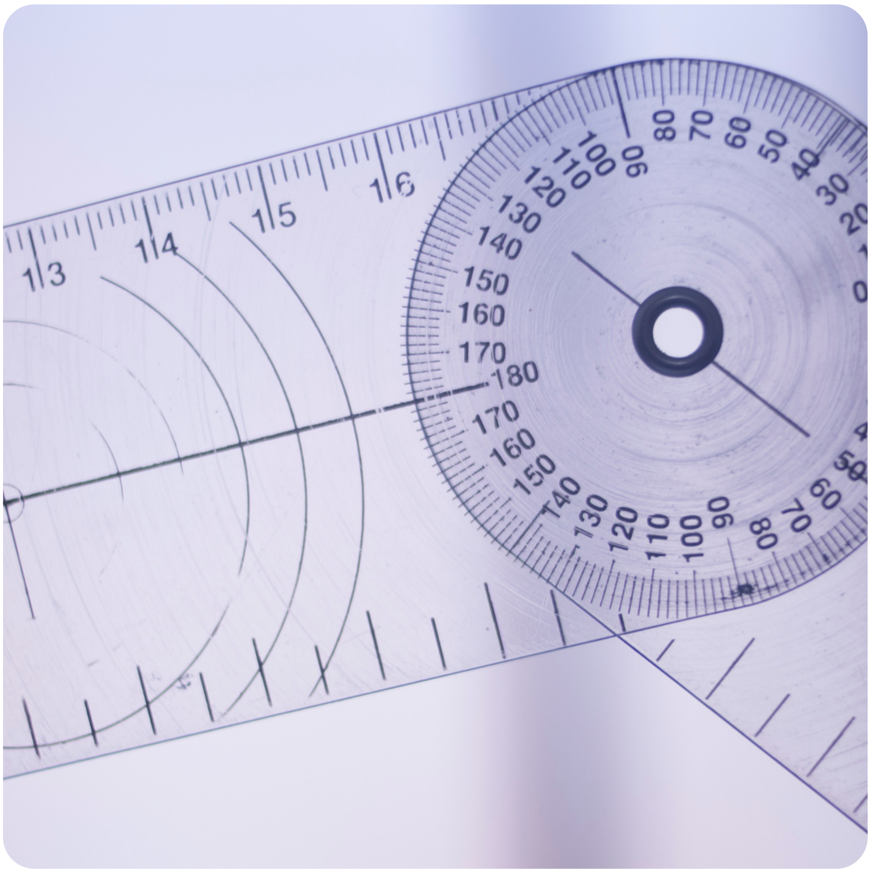


STUDY MATERIAL

Physical world, Units, Dimensions and errors in measurement

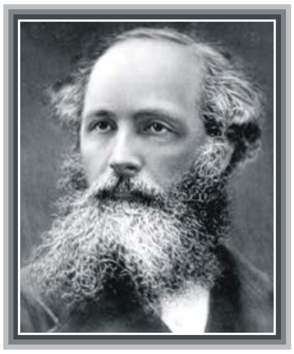
ENGLISH MEDIUM

Copyright Statement

Sir C.V. Raman (1888-1970)

Chandrashekhara Venkata Raman was born on 07 Nov 1888 in Thiruvanaikkaval. He finished his schooling by the age of eleven. He graduated from Presidency College, Madras. After finishing his education he joined financial services of the Indian Government. While in Kolkata, he started working on his area of interest at Indian Association for Cultivation of Science founded by Dr. Mahendra Lal Sirkar, during his evening hours. His area of interest included vibrations, variety of musical instruments, ultrasonics, diffraction and so on. In 1917 he was offered Professorship at Calcutta University. In 1924 he was elected 'Fellow' of the Royal Society of London and received Nobel prize in Physics in 1930 for his discovery, now known as Raman Effect. The Raman Effect deals with scattering of light by molecules of a medium when they are excited to vibrational energy levels. This work opened totally new avenues for research for years to come. He spent his later years at Bangalore, first at Indian Institute of Science and then at Raman Research Institute. His work has inspired generation of young students.

James Clerk Maxwell (1831 – 1879)



Born in Edinburgh, Scotland, was among the greatest physicists of the nineteenth century. He derived the thermal velocity distribution of molecules in a gas and was among the first to obtain reliable estimates of molecular parameters from measurable quantities like viscosity, etc.

Maxwell's greatest acheivement was the unification of the laws of electricity and magnetism (discovered by Coulomb, Oersted, Ampere and Faraday) into a consistent set of equations now called Maxwell's equations. From these he arrived at the most important conclusion that light is an electromagnetic wave. Interestingly, Maxwell did not agree with the idea (strongly suggested by the Faraday's laws of electrolysis) that electricity was particulate in nature.

PHYSICAL WORLD, UNITS & DIMENSIONS AND ERRORS IN MEASUREMENTS

1. PHYSICAL WORLD

- The Word science originates from the Latin verb scientia meaning "to know"
- The Sanskrit word 'Vijnan' and Arabic word 'Ilm' convey similar meaning namely 'knowledge'.

1.1 Scientific Method

A systematic attempt to understand natural phenomena in as much detail and depth as possible and use the knowledge so gained to predict, modify and control phenomena.

The scientific method involves several inter connected steps :-

(I) Systematic observations

(ii) Controlled experiments

(iii) Qualitative and quantitative reasoning

(iv) Mathematical modelling

(v) Prediction and

(vi) Verification or falsification of theories

- Physics comes from a Greek word "Fusis" meaning nature.

1.2 Unification

To explain diverse physical phenomena in terms of a few concepts and laws. The effort to see the physical world as manifestation of some universal laws in different domains and conditions is called unification.

Example :- The attempts to unify fundamental forces in nature

Progress in unification of different forces/ domains in nature

Name of the physicist	Year	Achievement in unification
Isaac Newton	1687	Unified celestial and terrestrial mechanics: showed that the same laws of motion and the law of gravitation apply to both the domains.
Hans Christian Oersted	1820	Showed that electric and magnetic phenomena are
Michael Faraday	1830	Inseparable aspects of a unified domain : electromagnetism
James Clerk Maxwell	1873	Unified electricity, magnetism and optics : showed that light is an electromagnetic wave
Sheldon Glashow, Abdus Salam, Steven Weinberg	1979	Showed that the 'weak' nuclear force and the electromagnetic force could be viewed as different aspects of a single electro-weak force.
Carlo Rubia, Simon Vander Meer	1984	Verified experimentally the predictions of the theory of electro-weak force

1.3 Reductionism

A related effort is to derive the properties of bigger, more complex, system from properties and interaction of its constituent simpler part is called reductionism.

1.4 Scope and Excitement of Physics

(i) Macroscopic : Macroscopic domain includes phenomena at the laboratory, terrestrial and astronomical scales.
These phenomena are studied in "classical Physics" which includes mechanics, thermodynamics, optics and electrodynamics.

(ii) Microscopic : The microscopic domain includes atomic, molecular and nuclear phenomena. These phenomena are governed by "Quantum Physics".

GOLDEN KEY POINTS

- Range of length $\rightarrow 10^{-14}$m to 10^{26}m
- Range of mass $\rightarrow 10^{-30}$ kg to 10^{55} kg
- Range of time $\rightarrow 10^{-22}$ s to 10^{18} s
- Terrestrial phenomena lie somewhere in middle of the above range.

Excitement

The basic laws are simple and universal. It is a source of wonder that such vast realms of experience can be summarized in a single sentence or equation. Einstein put it well when he remarked that
"The most incomprehensible thing about the universe is that it is comprehensible"
"No number of experiments can prove me right, a single experiment can prove me wrong"

SOME PHYSICISTS FROM DIFFERENT COUNTRIES OF THE WORLD AND THEIR MAJOR CONTRIBUTIONS

Name	Major contribution/discovery	Country of Origin
Archimedes	Principle of buoyancy,	Greece
Galileo Galilei	Law of inertia	Italy
Christian Huygens	Wave theory of light	Holland
Isaac Newton	Universal law of gravitation,Laws of motion, Reflecting telescope	U.K.
Michael Faraday	Laws of electromagnetic induction	U.K.
James Clerk Maxwell	Electromagnetic theory, Light-an electromagnetic wave	U.K.
Heinrich Rudolf Hertz	Generation of electromagnetic waves	Germany
J.C. Bose	Ultra short radio waves	India
W.K. Roentgen	X-rays	Germany
J.J. Thomson	Electron	U.K.
Maric Sklodowska Curie	Discovery of radium and polonium, Studies on natural radioactivity	Poland
Albert Einstein	Explanation of photoelectric effect: Theory of relativity	Germany
Victor Francis Hess	Cosmic radiation	Austria
R.A. Millikan	Measurement of electronic charge	U.S.A.
Ernest Rutherford	Nuclear model of atom	New Zealand
Niels Bohr	Quantum model of hydrogen atom	Denmark
C.V. Raman	Inelastic scattering of light by molecules	India
Louis Victor de Broglie	Wave nature of matter	France
M.N. Saha	Thermal ionisation	India
S.N. Bose	Quantum statistics	India
Wolfgang Pauli	Exclusion principle	Austria
Enrico Fermi	Controlled nuclear fission	Italy
Warner Heisenberg	Quantum mechanics, Uncertainty principle	Germany
Paul Dirac	Relativistic theory of electron: Quantum statistics	U.K.
Edwin Hubble	Expanding universe	U.S.A.
Ernest Orlando Lawrence	Cyclotron	U.S.A.
James Chadwick	Neutron	U.K.
Hideki Yukawa	Theory of nuclear forces	Japan
Homi Jehangir Bhabha	Cascade process of cosmic radiation	India
Lev Davidovich Landau	Theory of condensed matter: Liquid helium	Russia
S. Chandrasekhar	Chandrasekhar limit, structure and evolution of stars	India
John Bardeen	Transistors: Theory of super conductivity	U.S.A.
C.H. Townes	Maser, Laser	U.S.A.
Abdus Salam	Unification of weak and electromagnetic interactions	Pakistan

LINK BETWEEN TECHNOLOGY AND PHYSICS

Technology	Scientific principle(s)
Steam engine	Laws of thermodynamics
Nuclear reactor	Controlled nuclear fission
Radio and Television	Generation, propagation and detection of electromagnetic waves
Computers	Digital logic
Lasers	Light amplification by stimulated emission of radiation
Production of ultra high magnetic fields	Superconductivity
Rocket propulsion	Newton's laws of motion
Electric generator	Faraday's laws of electromagnetic induction
Hydroelectric power	Conversion of gravitational potential energy into electrical energy
Aeroplane	Bernoulli's principle in fluid dynamics
Particle accelerators	Motion of charged particles in electromagnetic fields
Sonar	Reflection of ultrasonic waves
Optical fibres	Total internal reflection of light
Non-reflecting coatings	Thin film optical interference
Electron microscope	Wave nature of electrons
Photocell	Photoelectric effect
Fusion test reactor (Tokamak)	Magnetic confinement of plasma
Giant Metrewave Radio Telescope (GMRT)	Detection of cosmic radio waves
Bose-Einstein condensate	Trapping and cooling of atoms by laser beams and magnetic fields

1.5 Fundamental forces in Nature

Few fundamental forces in nature are :-

Gravitational Force

- Gravitational force is weakest force in nature
- It is the force of mutual attraction between any two objects by virtue of their masses
- It is a universal force.
- It plays a key role in the large scale phenomena of universe such as formation and evolution of stars, galaxies and galactic clusters
- The gravitational force is appreciable only when at least one of the two bodies has a large mass.
- They are always attractive in nature.

Electromagnetic Force

- Electromagnetic force is the force between charge particles.
- When charges are at rest, the force is given by coulomb's law.
- When charges are in motion, they produce magnetic field giving rise to a force on a moving charge.
- Electric and magnetic effects are in general inseparable; hence the name electromagnetic force.
- Like the gravitational force, electromagnetic force act over large distances and does not need any intervening medium.
- It is quite strong compared to gravity.
- For example electric force between two protons is 10^{36} times the gravitational force between them, for a certain distance.
- They are attractive as well as repulsive in nature.

Strong Nuclear Force

- The strong nuclear force binds protons and neutrons in a nucleus. It is evident that without some attractive force, a nucleus will be unstable due to electric repulsion between protons.
- The strong nuclear force is the strongest of all fundamental forces.
- It is charge independent.
- It is equal for protons and neutrons.

- It's range is extremly small of the order nuclear dimensions (10^{-15} m)
- It is responsible for the stability of nuclei.
- Recent developments have however indicated that protons and neutrons are composed of still more elementry constituents called quarks.

Weak Nuclear Force

- The weak nuclear force appears only in certain nuclear processe β-decay of a nucleus.
- In β-decay the nucleus emits an electron and an uncharged particle called anti-neutrino.
- The weak nuclear force is not as weak as the gravitational force but much weaker than strong nuclear force.
- The range of weak nuclear fore is exceedingly small of the order 10^{-16} m

Fundamental force of nature

Name	***Relative strength***	***Range***	***Operates among***	***Mediating particle***
Gravitational Force	10^{-39}	Infinite	All objects in the universe	Graviton
Weak Nuclear Force	10^{-13}	Very short, Sub-nuclear size ($\sim 10^{-16}$ m)	Some elementary particles, particularly electron and antineutrino	BOSON
Electromagnetic force	10^{-2}	Infinite	Charged particles	Photon
Strong Nuclear Force	1	Short nuclear size ($\sim 10^{-15}$ m)	Nucleons, heavier elementary particles	Gluon

1.6 Nature of Physical Laws

Law of Physics is a statement in word form or in equation form that summarises the result of experiments and observation for a certain range of physical phenomena.

- A law can not be proved.
- A new development in Physics may extend the range of validity of a law.
- They exists in simple form.

1.7 Conservation laws

Some physical quantities that remain conserved (constant), are called conserved quantities. The law governing the conservation quantity in a process is called conservation law.

Some basic conservation laws are as follows :

- **Law of conservation of energy :** Total energy of a system remains conserved.
- **Law of conservation of charge :** Total charge of an isolated system remains conserved
- **Law of conservation of linear momentum :** In absence of external force, Linear momentum of a system remains conserved.
- **Law of conservation of angular momentum :** In absence of external Torque, Angular momentum of a system remains conserved.

2. PHYSICAL QUANTITIES

All the quantities which are used to describe the laws of physics are known as *physical quantities.*

Classification : Physical quantities can be classified on the following basis :

1. Based on their directional properties

I. Scalars : The physical quantities which have only magnitude but no direction are called *scalar quantities*.

e.g. mass, density, volume, time, etc.

II. Vectors : The physical quantities which have both magnitude and direction and obey laws of vector algebra are called *vector quantities*.

e.g. displacement, force, velocity, etc.

2. Based on their dependency

I. Fundamental or base quantities : The quantities which do not depend upon other quantities for their complete definition are known as *fundamental or base quantities*.

e.g. length, mass, time, etc.

II. Derived quantities : The quantities which can be expressed in terms of the fundamental quantities are known as *derived quantities.*

e.g. Speed (=distance/time), volume, accelaration, force, pressure, etc.

GOLDEN KEY POINTS

- Physical quantities can also be classified as dimensional or dimensionless and constant or variable.
- Some physical quantities can not be completely specified even by specifying their magnitude, unit and direction. These quantities are called *tensors.* e.g. Moment of Inertia.

Illustrations

Illustration 1.

Classify the quantities displacement, mass, force, time, speed, velocity, accelaration, pressure and work under the following categories :

(a) base and scalar
(b) base and vector
(c) derived and scalar
(d) derived and vector

Solution.

(a) mass, time
(b) displacement
(c) speed, pressure, work
(d) force, velocity, acceleration

3. UNITS OF PHYSICAL QUANTITIES

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity.

System of Units :

(i) FPS or British Engineering system – In this system length, mass and time are taken as fundamental quantities and their base units are foot (ft), pound (lb) and second (s) respectively.

(ii) CGS or Gaussian system : In this system the fundamental quantities are length, mass and time and their respective units are centimetre (cm), gram (g) and second (s).

(iii) MKS system : In this system also the fundamental quantities are length, mass and time and their fundamental units are metre (m), kilogram (kg) and second (s) respectively.

(iv) International system (SI) of units : This system is modification of the MKS system and so it is also known as *Rationalised MKS system*. Besides the three base units of MKS system four fundamental and two supplementary units are also included in this system.

SI BASE QUANTITIES AND THEIR UNITS

S. No.	Physical quantity	Unit	Symbol
1	Length	metre	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	K
5	Electric current	ampere	A
6	Luminous intensity	candela	cd
7	Amount of substance	mole	mol

While defining a base unit or standard for a physical quantity the following characteristics must be considered :

(i) Well defined
(ii) Invariability (constancy)
(iii) Accessibility (easy availability)
(iv) Reproducibility
(v) Convenience in use

4. CLASSIFICATION OF UNITS

The units of physical quantities can be classified as follows :

(i) Fundamental or base units : The units of fundamental quantities are called *base units*. In SI there are seven base units.

(ii) Derived units : The units of derived quantities or the units that can be expressed in terms of the base units are called *derived units*.

e.g. unit of speed $= \dfrac{\text{unit of distance}}{\text{unit of time}} = \dfrac{\text{metre}}{\text{second}} = \text{m/s}$

Some derived units are named in honour of great scientists.
e.g. unit of force - newton (N), unit of frequency - hertz (Hz), etc.

(iii) Supplementary units : In International System (SI) of units two *supplementary units* are also defined viz. radian (rad) for plane angle and steradian (sr) for solid angle.

- **radian :** 1 radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle.
- **steradian :** 1 steradian is the solid angle subtended at the centre of a sphere, by that surface of the sphere which is equal in area to the square of the radius of the sphere.

(iv) Practical units : Due to the fixed sizes of SI units, some *practical units* are also defined for both fundamental and derived quantities. e.g. light year (ly) is a practical unit of distance (a fundamental quantity) and horse power (hp) is a practical unit of power (a derived quantity).
Practical units may or may not belong to a particular system of units but can be expressed in any system of units.
e.g. 1 mile = 1.6 km = 1.6×10^3 m = 1.6×10^5 cm.

(v) Improper units : These are the units which are not of the same nature as that of the physical quantities for which they are used. e.g. kg - wt is an improper unit of weight. Here kg is a unit of mass but it is used to measure the weight (force).

UNITS OF SOME PHYSICAL QUANTITIES IN DIFFERENT SYSTEMS

Type of Physical Quantity	Physical Quantity	CGS (Originated in France)	MKS (Originated in France)	FPS (Originated in Britain)
Fundamental	Length	cm	m	ft
	Mass	g	kg	lb
	Time	s	s	s
Derived	Force	dyne	newton (N)	poundal
	Work or Energy	erg	joule (J)	ft - poundal
	Power	erg/s	watt (W)	ft - poundal/s

Conversion factors

To convert a physical quantity from one set of units to the other, the required multiplication factor is called *conversion factor*.
Magnitude of a physical quantity = numeric value (n) × unit (u)
While converting from one set of units to other, the magnitude of the quantity must remain same. Therefore

$$n_1u_1 = n_2u_2 \quad \text{or} \quad nu = \text{constant} \quad \text{or} \quad n \propto \frac{1}{u}$$

That is *the numeric value of a physical quantity is inversely proportional to the unit.*

e.g. 1m = 100 cm = 3.28 ft = 39.4 inch
(SI) (CGS) (FPS)

Illustrations

Illustration 2.

The acceleration due to gravity is $9.8\ m/s^2$. Give its value in ft/s^2

Solution

As 1m = 3.2 ft

$\therefore\quad 9.8\ m/s^2 = 9.8 \times 3.28\ ft/s^2 = 32.14\ ft/s^2 \approx 32\ ft/s^2$

BEGINNER'S BOX-1

1. The value of Gravitational constant G in MKS system is $6.67 \times 10^{-11}\ N\text{-}m^2/kg^2$. What will be its value in CGS system ?

2. Match the type of unit (column A) with its corresponding example (column B)

(A)	(B)
(a) Base unit	(i) N
(b) Derived SI unit	(ii) hp
(c) Improper unit	(iii) kg-wt
(d) Practical unit	(iv) rad
(e) Supplementary unit	(v) kg

5. DIMENSIONS

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to express that quantity.

Dimensional formula : The *dimensional formula* of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity.
It is written by enclosing the symbols for base quantities with appropriate powers in square brackets i.e. [ ]
e. g. Dimensional formula of mass is $[M^1L^0\ T^0]$ and that of speed (= distance/time) is $[M^0L^1T^{-1}]$

Dimensional equation : The equation obtained by equating a physical quantity with its dimensional formula is called a *dimensional equation.* e.g. $[v] = [M^0L^1T^{-1}]$
For example $[F] = [MLT^{-2}]$ is a dimensional equation, $[MLT^{-2}]$ is the dimensional formula of the force and the dimensions of force are 1 in mass, 1 in length and –2 in time

6. APPLICATIONS OF DIMENSIONAL ANALYSIS :

(i) To convert a physical quantity from one system of units to the other :

This is based on the fact that *magnitude of a physical quantity remains same whatever system is used for measurement* i.e. magnitude = numeric value (n) × unit (u) = constant or $n_1u_1 = n_2u_2$

So if a quantity is represented by $[M^aL^bT^c]$ then $\boxed{n_2 = n_1\left(\frac{u_1}{u_2}\right) = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c}$

Here n_2 = numerical value in II system; n_1 = numerical value in I system
M_1 = unit of mass in I system; M_2 = unit of mass in II system
L_1 = unit of length in I system; L_2 = unit of length in II system
T_1 = unit of time in I system; T_2 = unit of time in II system

Illustrations

Illustration 3.

Convert 1 newton (SI unit of force) into dyne (CGS unit of force)

Solution

The dimensional equation of force is $[F] = [M^1\ L^1\ T^{-2}]$

Therefore if n_1, u_1, and n_2, u_2 corresponds to SI & CGS units respectively, then

$$n_2 = n_1\left[\frac{M_1}{M_2}\right]^1\left[\frac{L_1}{L_2}\right]^1\left[\frac{T_1}{T_2}\right]^{-2} = 1\left[\frac{kg}{g}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1 \times 1000 \times 100 \times 1 = 10^5 \quad \therefore 1 \text{ newton} = 10^5 \text{ dyne.}$$

(ii) To check the dimensional correctness of a given physical relation :

If in a given relation, the terms on both the sides have the same dimensions, then the relation is dimensionally correct. This is known as the *principle of homogeneity of dimensions.*

Illustrations

Illustration 4.

Check the accuracy of the relation $T = 2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum using dimensional analysis.

Solution

The dimensions of LHS = the dimension of $T = [M^0 L^0 T^1]$

The dimensions of RHS = $\left(\frac{\text{dimensions of length}}{\text{dimensions of acceleration}}\right)^{1/2}$ ($\because$ 2π is a dimensionless constant)

$$= \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T^2]^{1/2} = [T] = [M^0 L^0 T^1]$$

Since the dimensions are same on both the sides, the relation is correct.

(iii) To derive relationship between different physical quantities :

Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Illustrations

Illustration 5.

It is known that the time of revolution T of a satellite around the earth depends on the universal gravitational constant G, the mass of the earth M, and the radius of the circular orbit R. Obtain an expression for T using dimensional analysis.

Solution

We have $[T] \propto [G]^a [M]^b [R]^c$

$\Rightarrow [M]^0 [L]^0 [T]^1 = [M]^{-a} [L]^{3a} [T]^{-2a} \times [M]^b \times [L]^c = [M]^{b-a} [L]^{c+3a} [T]^{-2a}$

Comparing the exponents

For [T] : $1 = -2a \Rightarrow a = -\frac{1}{2}$ ***For [M] :*** $0 = b - a \Rightarrow b = a = -\frac{1}{2}$

For [L] : $0 = c + 3a \Rightarrow c = -3a = \frac{3}{2}$

Putting the values we get $T \propto G^{-1/2} M^{-1/2} R^{3/2} \Rightarrow T \propto \sqrt{\frac{R^3}{GM}}$

The actual expression is $T = 2\pi\sqrt{\frac{R^3}{GM}}$

7. DIMENSIONS OF SOME MATHEMATICAL FUNCTIONS

Dimensions of differential coefficients and integrals

In General $\left[\frac{d^n y}{dx^n}\right] = \left[\frac{y}{x^n}\right]$ and $[\int y dx] = [yx]$

Illustrations

Illustration 6.

Find dimensional formula :

(i) $\frac{dx}{dt}$ (ii) $m\frac{d^2x}{dt^2}$ (iii) $\int vdt$ (iv) $\int adt$

where x→ displacement, t→ time, v → velocity and a→ acceleration

Solution

(i) $\left[\frac{dx}{dt}\right]=\left[\frac{x}{t}\right]=\left[\frac{L}{T}\right]=[M^0L^1T^{-1}]$ (ii) $\left[m\frac{d^2x}{dt^2}\right]=\left[m\frac{x}{t^2}\right]=\left[\frac{ML}{T^2}\right]=[M^1L^1T^{-2}]$

(iii) $\left[\int vdt\right]=[vt]=[LT^{-1}\times T]=[M^0L^1T^0]$ (iv) $\left[\int adt\right]=[at]=[LT^{-2}\times T]=[M^0L^1T^{-1}]$

Dimensions of trigonometric, exponential, logarithmic functions etc.

All trigonometric, exponential and logarithmic functions and their arguments are dimensionless.

Note : Trigonometric function like $\sin\theta$ and its argument θ are dimensionless.

Illustrations

Illustration 7.

If $\alpha=\frac{F}{v^2}\sin\beta t$, find dimensions of α and β. Here v = velocity, F = force and t = time.

Solution

Here sin βt and βt must be dimensionless

So $[\beta t]=1\Rightarrow[\beta]=\left[\frac{1}{t}\right]=[T^{-1}]$; $[\alpha]=\left[\frac{F}{v^2}\sin\beta t\right]=\left[\frac{F}{v^2}\right]=\left[\frac{MLT^{-2}}{L^2T^{-2}}\right]=[ML^{-1}]$

8. LIMITATIONS OF DIMENSIONAL ANALYSIS

- In Mechanics the formula for a physical quantity depending on more than three physical quantities cannot be derived. It can only be checked.
- This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions cannot be derived using this method. Formulae containing more than one term which are added or subtracted like $s=ut+\frac{1}{2}at^2$ also cannot be derived.
- The relation derived from this method gives no information about the dimensionless constants.
- If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.
- It gives no information whether a physical quantity is a scalar or a vector.

BEGINNER'S BOX-2

1. Match the following :

(i) Dimensional variable	(a) π
(ii) Dimensionless variable	(b) Force
(iii) Dimensional constant	(c) Angle
(iv) Dimensionless constant	(d) Gravitational constant

2. Find the dimensions of the following quantities :

(a) Temperature (b) Kinetic energy (c) Pressure (d) Angular speed

3. Find the dimensions of Planck's constant (h).

4. Centripetal force (F) on a body of mass (m) moving with uniform speed (v) in a circle of radius (r) depends upon m, v and r. Derive a formula for the centripetal force using theory of dimensions.

9. SIGNIFICANT FIGURES OR DIGITS

Significant figures (SF) in a measurement are the figures or digits that are known with certainity plus one that is uncertain (i.e. Last digit).

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

Rules to find out the number of significant figures

I Rule : All the non-zero digits are significant e.g. 1984 has 4 SF.

II Rule : All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF.

III Rule : All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

IV Rule : If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

V Rule : The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. e.g. 01.080 has 4 SF.

VI Rule : The trailing zeros in a number without a decimal point may not be significant e.g. 010100 has 3 SF.

VII Rule : When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example in $x = 12.3 = 1.23 \times 10^1 = 0.123 \times 10^2$ $= 0.0123 \times 10^3 = 123 \times 10^{-1}$ each term has 3 SF only.

Rules for arithmetical operations with significant figures

I Rule : In addition or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. e.g. $12.587 - 12.5 = 0.087 = 0.1$ ($\because$ second term contain lesser i.e. one decimal place)

II Rule : In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. e.g. $2.4 \times 3.65 = 8.8$

GOLDEN KEY POINTS

- To avoid confusion regarding the trailing zeros of the numbers without the decimal point the best way is to report every measurement in scientific notation (in the power of 10). In this notation every number is expressed in the form $a \times 10^b$, where a is the base number between 1 and 10 and b is any positive or negative exponent of 10. The base number (a) is written in decimal form with the decimal after the first digit. While counting the number of SF only base number is considered (Rule VII).
- The change in the unit of measurement of a quantity does not affect the number of SF. For example in 2.308 cm = 23.08 mm = 0.02308 m = 2.308×10^4 μm each term has 4 SF.

Illustrations

Illustration 8.

Write down the number of significant figures in the following.

(a) 165 (b) 2.05 (c) 34.000 m (d) 0.005
(e) 0.02340 N m^{-1} (f) 26900 (g) 26900 kg

Solution

(a) 165	3SF (following rule I)
(b) 2.05	3 SF (following rules I & II)
(c) 34.000 m	5 SF (following rules I & V)
(d) 0.005	1 SF (following rules I & IV)
(e) 0.02340 N m^{-1}	4 SF (following rules I, IV & V)
(f) 26900	3 SF (see rule VI)
(g) 26900 kg	5 SF (see rule VI)

BEGINNER'S BOX-3

1. Write the following in scientific notation :
(a) 3256 g (b) 0.0010 g (c) 50000 g (d) 0.3204

2. Give the number of significant figures in the following :
(a) 0.165 (b) 4.0026 (c) 0.0256 (d) 201
(e) 0.050 (f) 2.653×10^4 (g) 6.02×10^{23} (h) 0.0006032

3. From the point of view of significant figures which of the following statements are correct ?
(i) 10.2 cm + 8 cm = 18.2 cm (ii) 2.53 m – 1.2 m = 1.33 m
(iii) 4.2 m × 1.4 m = 5.88 m^2 (iv) 3.6 m / 1.75 sec = 2.1 m/s
(1) (i) & (iv) only (2) (ii) & (iii) only (3) (iv) only (4) (ii) & (iv) only

10. ROUNDING OFF

To represent the result of any computation containing more than one uncertain digit, it is *rounded off* to appropriate number of significant figures.

Rules for rounding off the numbers :

I Rule : If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. $6.87 \approx 6.9$

II Rule : If the digit to be rounded off is less than 5, then the preceding digit is left unchanged. e.g. $3.94 \approx 3.9$

III Rule : If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. $14.35 \approx 14.4$ and $14.45 \approx 14.4$

Ex. The following values can be rounded off to four significant figures as follows :

(a) $36.879 \approx 36.88$ ($\because$ 9 > 5 $\therefore$ 7 is increased by one i.e.I Rule)

(b) $1.0084 \approx 1.008$ ($\because$ 4 < 5 $\therefore$ 8 is left unchanged i.e. II Rule)

(c) $11.115 \approx 11.12$ ($\because$ last 1 is odd it is increased by one i.e.III Rule)

(d) $11.1250 \approx 11.12$ ($\because$ 2 is even it is left unchanged i.e. III Rule)

(e) $11.1251 \approx 11.13$ ($\because$ 51 > 50 $\therefore$ 2 is increased by 1 i.e. I Rule)

Illustrations

Illustration 9.

The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Solution

length (ℓ) = 4.234 m breadth (b) = 1.005 m thickness (t) = 2.01 cm = 2.01×10^{-2} m

Therefore area of the sheet $= 2(\ell \times b + b \times t + t \times \ell)$

$= 2(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ m^2$

$= 2(4.3604739)\ m^2 = 8.720978\ m^2$

Since area can contain a maximum of 3 SF therefore, rounding off, we get: Area = 8.72 m^2

Likewise volume $= \ell \times b \times t = 4.234 \times 1.005 \times 0.0201\ m^3 = 0.0855289\ m^3$

Since volume can contain 3 SF, therefore after rounding off, we get : Volume = 0.0855 m^3

BEGINNER'S BOX-4

1. Round off the following numbers as indicated:
(a) 25.653 to 3 digits (b) 4.996×10^5 to 3 digits (c) 0.6995 to 1digit
(d) 3.350 to 2 digits (e) 0.03927 kg to 3 digits (f) 4.085×10^8 s to 3 digits

2. Calculate area enclosed by a circle of diameter 1.06 m to correct number of significant figures.

3. Subtract 2.5×10^4 from 3.9×10^5 and give the answer to correct number of significant figures.

4. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) total mass of the box (b) the difference in masses of gold pieces to correct significant figures.

11. ORDER OF MAGNITUDE

Order of magnitude of a quantity is the power of 10 required to represent that quantity. This power is determined after rounding off the value of the quantity properly. For rounding off, the last digit is simply ignored if it is less than 5 and, is increased by one if it is 5 or more than 5.

- When a number is divided by 10^x (where x is the order of the number) the result will always lie between 0.5 and 5 i.e. $0.5 \leq N/10^x < 5$

Ex. Order of magnitude of the following values can be determined as follows :

(a) $49 = 4.9 \times 10^1 \approx 10^1$ $\therefore$ Order of magnitude = 1

(b) $51 = 5.1 \times 10^1 \approx 10^2$ $\therefore$ Order of magnitude = 2

(c) $0.049 = 4.9 \times 10^{-2} \approx 10^{-2}$ $\therefore$ Order of magnitude = –2

(d) $0.050 = 5.0 \times 10^{-2} \approx 10^{-1}$ $\therefore$ Order of magnitude = –1

(e) $0.051 = 5.1 \times 10^{-2} \approx 10^{-1}$ $\therefore$ Order of magnitude = –1

12. ACCURACY AND PRECISION

The result of every measurement by any measuring instrument contains some uncertainty. This uncertainty is called error. Every calculated quantity which is based on measured value, also has an error. Every measurement is limited by the reliability of the measuring instrument and skill of the person making the measurement. If we repeat a particular measurement, we usually do not get precisely the same result as each result is subjected to some experimental error. This imperfection in measurement can be described in terms of accuracy and precision. The accuracy of a measurement is a measure of how close the measured value is to the true value of the quantity. Precision tells us to what resolution or limit the quantity is measured, we can illustrate the difference between accuracy and precision with help of a example. Suppose the true value of a certain length is 1.234 cm. In one experiment , using a measuring instrument of resolution 0.1 cm, the measured value is found to be 1.1cm, while in another experiment using a measuring device of greater resolution of 0.01 cm, the length is determined to be 1.53cm. The first measurement has more accuracy (as it is closer to the true value) but less precision (as resolution is only 0.1 cm), while the second measurement is less accurate but more precise.

Illustrations

Illustration 10.

Two clocks are being tested against a standard clock located in the national laboratory. At 10:00:00 AM by the standard clock, the readings of the two clocks are :

Day	Clock A	Clock B
1^{st}	10 : 00 : 05	8 : 15 : 00
2^{nd}	10 : 01 : 12	8 : 15 : 01
3^{rd}	9 : 59 : 08	8 : 15 : 04
4^{th}	10 : 01 : 13	8 : 14 : 58
5^{th}	9 : 58 : 10	8 : 15 : 03

If you are doing an experiment that requires precision time interval measurments, which of the two clocks will you prefer ?

(1) Clock A
(2) Clock B
(3) Either Clock A or B
(4) Neither A nor B

Solution

The average reading of clock A is much closure to the standard time than the average reading of clock B and the range of variation over the 5 days of observation is much smaller for clock B. As here clock's zero error is not significant for precision work, because a zero error can always be easily corrected. Hence, clock B is to be preferred to clock A.

BEGINNER'S BOX-5

1. Give the order of the following :

(a) 1
(b) 1000
(c) 499
(d) 500
(e) 501
(f) 1 AU (1.496×10^{11} m)
(g) 1 Å (10^{-10} m)
(h) Speed of light (3.00×10^{8} m/s)
(i) Gravitational constant (6.67×10^{-11} N – m^2/kg^2)
(j) Avogadro constant (6.02×10^{23} mol^{-1})
(k) Planck's constant (6.63×10^{-34} J-s)
(l) Charge on electron (1.60×10^{-19} C)
(m) Radius of H– atom (5.29×10^{-11} m)
(n) Atmospheric pressure (1.01×10^{5} Pa)
(o) Mass of earth (5.98×10^{24} kg)
(p) Mean radius of earth (6.37×10^{6} m)

13. ERRORS

The difference between the true value and the measured value of a quantity is known as the error in measurement.

Errors may result from different sources and are usually classified as follows :-

Systematic or Controllable Errors

Systematic errors are the errors whose causes are known. They can be either positive or negative. Due to the known causes these errors can be minimised. Systematic errors can further be classified into three categories :

(i) **Instrumental errors :-** These errors are due to imperfect design or erroneous manufacture or misuse of the measuring instrument. These can be reduced by using more accurate instruments.

(ii) **Environmental errors :-** These errors are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust, vibrations or magnetic and electrostatic fields.

(iii) **Observational errors :-** These errors arise due to improper setting of the apparatus or carelessness in taking observations.

Random Errors :-

These errors are due to unknown causes. Therefore they occur irregularly and are variable in magnitude and sign. Since the causes of these errors are not known precisely, they can not be eliminated completely. For example, when the same person repeats the same observation in the same conditions, he may get different readings different times.

Random erros can be reduced by repeating the observations a large number of times and taking the arithmetic mean of all the observations. This mean value would be very close to the most accurate reading.

Note :- If the number of observations is made n times then the random error reduces to $\left(\frac{1}{n}\right)$ times.

Example :- If the random error in the arithmetic mean of 100 observations is 'x' then the random error in the arithmetic mean of 500 observations will be $\frac{x}{5}$

Gross Errors : Gross errors arise due to human carelessness and mistakes in taking reading or calculating and recording the measurement results.

For example :-

(i) Reading instrument without proper initial settings.

(ii) Taking the observations wrongly without taking necessary precautions.

(iii) Committing mistakes in recording the observations.

(iv) Putting improper values of the observations in calculations.

These errors can be minimised by increasing the sincerity and alertness of the observer.

14. REPRESENTATION OF ERRORS

Errors can be expressed in the following ways :-

Absolute Error (Δa) : The difference between the true value and the individual measured value of the quantity is called the absolute error of the measurement.

Suppose a physical quantity is measured *n* times and the measured values are a_1, a_2, a_3 ....a_n. The arithmetic mean (a_m) of these values is $a_m = \frac{a_1 + a_2 + a_3 + ...a_n}{n} = \frac{1}{n}\sum_{i=1}^{n} a_i$

If the true value of the quantity is not given then mean value (a_m) *can be taken as the true value.* Then the absolute errors in the individual measured values are –

$\Delta a_1 = a_m - a_1$

$\Delta a_2 = a_m - a_2$

....

....

$\Delta a_n = a_m - a_n$

The arithmetic mean of all the absolute errors is defined as the final or mean absolute error $(\Delta a)_m$ or $\overline{\Delta a}$ of the value of the physical quantity a $(\Delta a)_m = \frac{|\Delta a_1| + |\Delta a_2| + ...+ |\Delta a_n|}{n} = \frac{1}{n}\sum_{i=1}^{n} |\Delta a_i|$

So if the measured value of a quantity be 'a' and the error in measurement be Δa, then the true value (a_t) can be written as $a_t = a \pm \Delta a$

Relative or Fractional Error : It is defined as the ratio of the mean absolute error ($(\Delta a)_m$ or $\overline{\Delta a}$) to the true value or the mean value (a_m or $\bar{a}$) of the quantity measured.

Relative or fractional error = $\frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{(\Delta a)_m}{a_m}$ or $\frac{\overline{\Delta a}}{\bar{a}}$

When the relative error is expressed in percentage, it is known as percentage error,

percentage error = relative error $\times$ 100

or percentage error $= \frac{\text{mean absolute error}}{\text{true value}} \times 100\% = \frac{\overline{\Delta a}}{a} \times 100\%$

15. PROPAGATION OF ERRORS IN MATHEMATICAL OPERATIONS

Rule I : The maximum absolute error in the sum or difference of the two quantities is equal to the sum of the absolute errors in the individual quantities.

If X = A + B or X = A – B and if $\pm\Delta A$ and $\pm\Delta B$ represent the absolute errors in A and B respectively, then the maximum absolute error in X is $\Delta X = \Delta A + \Delta B$ and Maximum percentage error = $\frac{\Delta X}{X} \times 100$

The result will be written as $X \pm \Delta X$ (in terms of absolute error) or $X \pm \frac{\Delta X}{X} \times 100\,\%$ (in terms of percentage error)

Rule II : The maximum fractional or relative error in the product or quotient of quantities is equal to the sum of the fractional or relative errors in the individual quantities.

If $X = AB$ *or* $X = \frac{A}{B}$ then $\frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Rule III : The maximum fractional error in a quantity raised to a power (n) is n times the fractional error in the quantity itself, i.e.

If $X = A^n$ then $\frac{\Delta X}{X} = n\left(\frac{\Delta A}{A}\right)$

If $X = A^pB^qC^r$ then $\frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$

If $X = \frac{A^pB^q}{C^r}$ then $\frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$

GOLDEN KEY POINTS

- Systematic errors are repeated consistently with the repetition of the experiment and are produced due to improper conditions or procedures that are consistent in action whereas random errors are accidental and their magnitude and sign cannot be predicted from the knowledge of the measuring system and conditions of measurement.

  Systematic errors can therefore be minimised by improving experimental techniques, selecting better instruments and improving personal skills whereas random errors can be minimised by repeating the observations several times.
- Mean absolute error has the units and dimensions of the quantity itself whereas fractional or relative error is unitless and dimensionless.
- Absolute errors may be positive in certain cases and negative in other cases.

Illustrations

Illustration 11.

Following observations were taken with a vernier callipers while measuring the length of a cylinder.

3.29 cm, 3.28 cm, 3.29 cm, 3.31 cm, 3.28 cm, 3.27 cm, 3.29 cm, 3.30 cm

Then find

(a) Most accurate length of the cylinder.
(b) Absolute error in each observation.
(c) Mean absolute error
(d) Relative error
(e) Percentage error

Express the result in terms of absolute error and percentage error.

Solution

(a) Most accurate length of the cylinder will be the mean length $\left(\overline{\ell}\right)$

$$\overline{\ell} = \frac{3.29+3.28+3.29+3.31+3.28+3.27+3.29+3.30}{8} = 3.28875 \text{ cm or } \overline{\ell} = 3.29 \text{ cm}$$

(b) Absolute error in the first reading = 3.29 – 3.29 = 0.00 cm

Absolute error in the second reading = 3.29 – 3.28 = 0.01 cm

Absolute error in the third reading = 3.29 – 3.29 = 0.00 cm

Absolute error in the forth reading = 3.29 – 3.31 = –0.02 cm

Absolute error in the fifth reading = 3.29 – 3.28 = 0.01 cm

Absolute error in the sixth reading = 3.29 – 3.27 = 0.02 cm

Absolute error in the seventh reading = 3.29 – 3.29 = 0.00 cm

Absolute error in the last reading = 3.29 – 3.30 = –0.01 cm

(c) Mean absolute error $= \overline{\Delta\ell} = \frac{0.00+0.01+0.00+0.02+0.01+0.02+0.00+0.01}{8} = 0.01$ cm

(d) Relative error in length $= \frac{\overline{\Delta\ell}}{\overline{\ell}} = \frac{0.01}{3.29} = 0.0030395 = 0.003$

(e) Percentage error $= \frac{\overline{\Delta\ell}}{\overline{\ell}} \times 100 = 0.003 \times 100 = 0.3\%$

So length $\ell = 3.29$ cm ± 0.01 cm (in terms of absolute error)

or $\ell = 3.29$ cm ± 0.30% (in terms of percentage error)

Illustration 12.

The inital and final temperatures of water as recorded by an observer are (40.6 ± 0.2)°C and (78.3 ± 0.3)°C. Calculate the rise in temperature with proper error limits.

Solution

Given $\theta_1 = (40.6 \pm 0.2)$°C and $\theta_2 = (78.3 \pm 0.3)$°C

Rise in temp. $\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7$°C.

$\Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2) = \pm(0.2 + 0.3) = \pm 0.5$°C $\qquad \therefore$ rise in temperature = (37.7 ± 0.5)°C

Illustration 13.

If a = 8 ± 0.08 and b = 6 ± 0.06, let x = a + b, y = a–b, z = ab.

The correct order of % error in x, y and z is

(1) x = y < z (2) x = y > z (3) x < z < y (4) x > z < y

Solution **Ans. (3)**

$$x = a+b = 14 \pm 0.14 \Rightarrow \%\text{ error} = \frac{0.14}{14}\times 100 = 1\%\,;\ y = a-b = 2 \pm 0.14 \Rightarrow \%\text{ error} = \frac{0.14}{2}\times 100 = 7\%$$

$z = ab = 48 \pm 0.96 \Rightarrow \%\text{ error} = \frac{0.96}{48}\times 100 = 2\%$. Therefore order of % error is x < z < y

Illustration 14.

The side of a cube is (2.00 ± 0.01) cm. The volume and surface area of cube are respectively

(1) (8.00 ± 0.12) cm^3, (24.0 ± 0.24) cm^2 (2) (8.00 ± 0.01) cm^3 (24.0 ± 0.01) cm^2

(3) (8.00 ± 0.04) cm^3 (24.0 ± 0.06) cm^2 (4) (8.00 ± 0.03) cm^3 (24.0 ± 0.02) cm^2

Solution **Ans. (1)**

$$\text{Volume, } V = a^3 = 8\text{cm}^3\text{, Also } \frac{\Delta V}{V} = 3\frac{\Delta a}{a} \Rightarrow \Delta V = 3V\left(\frac{\Delta a}{a}\right) = (3)(8)\left(\frac{0.01}{2.00}\right) = 0.12\text{cm}^3$$

Therefore V = (8.00 ± 0.12)cm^3

Surface Area A = $6a^2$ = 6 $(2.00)^2$ = 24.0 cm^2

$$\text{Also } \frac{\Delta A}{A} = 2\frac{\Delta a}{a} \Rightarrow \Delta A = 2A\left(\frac{\Delta a}{a}\right) = 2(24.0)\left(\frac{0.01}{2.00}\right) = 0.24$$

Therefore A = (24.0 ± 0.24) cm^2

Illustration 15.

A thin copper wire of length L increases in length by 2% when heated from T_1 to T_2. If a copper cube having side 10 L is heated from T_1 to T_2 what will be the percentage change in

(i) Area of one face of the cube and.

(ii) Volume of the cube.

Solution

(i) Area A = 10 L × 10 L = 100 L^2 $\Rightarrow$ $A \propto L^2$

$$\%\text{ change in area} = \frac{\Delta A}{A} \times 100 = 2 \times \frac{\Delta L}{L} \times 100 = 2 \times 2\% = 4\%$$

(ii) Volume V = 10 L × 10 L × 10 L = 1000 L^3 $\Rightarrow V \propto L^3$

$$\%\text{ change in volume} = \frac{\Delta V}{V} \times 100 = 3\frac{\Delta L}{L} = 3 \times 2\% = 6\%$$

Conclusion – The maximum percentage change will be observed in volume, lesser in area and the least (minimum) change will be observed in length or radius.

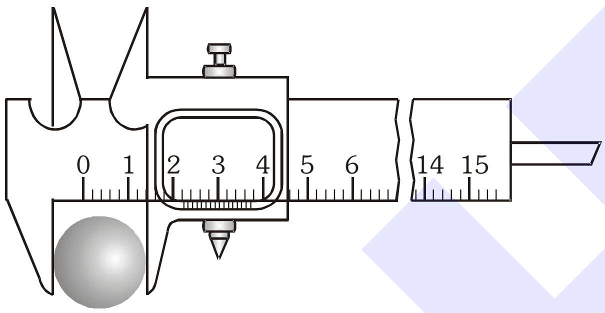
BEGINNER'S BOX-6

1. Two rods have lengths measured as (1.8 ± 0.2)m and (2.3 ± 0.1)m. Calculate their combined length with error limits.
2. The original length of wire is (153.7 ± 0.6) cm . It is stretched to (155.3 ± 0.2) cm. Calculate the elongation in the wire with error limits.
3. Measure of two quantities along with the precision of respective measuring instrument is :-
   A = 2.5 m/s ± 0.5 m/s B = 0.10s ± 0.01 s
   The value of AB will be :-
   (1) (0.25 ± 0.08)m (2) (0.25 ± 0.5) m (3) (0.25 ± 0.05) m (4) (0.25 ± 0.135) m
4. The radius of a sphere is measured to be (2.1 ± 0.5) cm. Calculate its surface area with absolute error limits.
5. A physical quantity x is calculated from the relation $x = a^3b^2/\sqrt{cd}$. Calculate percentage error in x, if a, b, c and d are measured respectively with an error of 1%, 3%, 4% and 2%.
6. An object covers (16.0 ± 0.4) m distance in (4.0 ± 0.2)s. Find out its speed.

16. LEAST COUNT

The smallest value of a physical quantity which can be measured accurately with an instrument is called its *least count* (L. C.).

Least Count of Vernier Callipers : Suppose the size of one main scale division (M.S.D.) is M units and that of one vernier scale division (V. S. D.) is V units. Also let the length of 'a' main scale divisions is equal to the length of 'b' vernier scale divisions.



$aM = bV \Rightarrow V = \frac{a}{b}M$

$\therefore M - V = M - \frac{a}{b}M = \left(\frac{b-a}{b}\right)M$

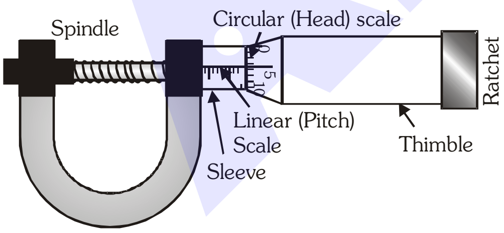
[M → MSD, V → VSD]

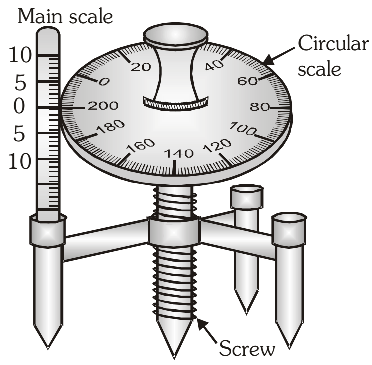
$$\boxed{L.C. = M - V = \left(\frac{b-a}{b}\right)M}$$

The quantity (M– V) is called *vernier constant* (V. C.) or *least count* (L. C.) of the vernier callipers.

Reading = MSR + VSR × LC

Least Count of screw gauge or spherometer





$$\text{Least Count} = \frac{\text{Pitch}}{\text{Total no. of divisions on the circular scale}}$$

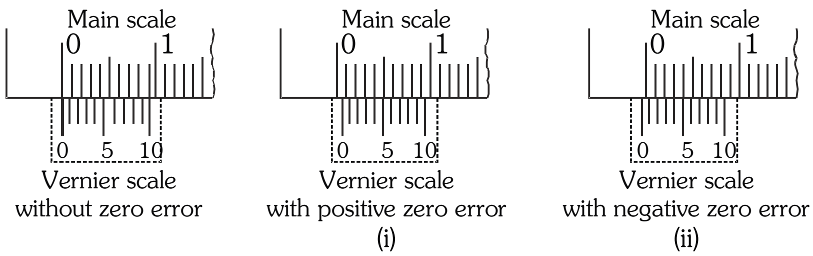
where pitch is defined as the distance moved by the screw head when the circular scale is given one complete rotation. i.e. $\text{Pitch} = \frac{\text{Distance moved by the screw on the linear scale}}{\text{Number of full rotations given}}$

Reading = LSR + CSR × LC

Note : With the decrease in the least count of the measuring instrument, the accuracy of the measurement increases and the error in the measurement decreases.

17. ZERO ERROR

17.1 Zero Error in Vernier Callipers :



<u>Calculation of zero error for vernier callipers :-</u>

Positive zero error = (No. of Division of VS coincided with MS).LC

Negative zero error = (Total division in VS – No. of division of VS coincided with MS).LC

<u>Correct reading with zero error</u>

Correct reading = (Reading) – (Zero error)

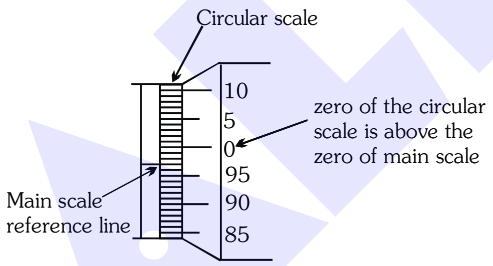
The zero error is always subtracted from the reading to get the corrected value.

17.2 Zero Error in Screw Gauge

If there is no object between the jaws (i.e. jaws are in contact), the screwgauge should give zero reading. But due to extra material on jaws, even if there is no object, it gives some excess reading. This excess reading is called Zero error.

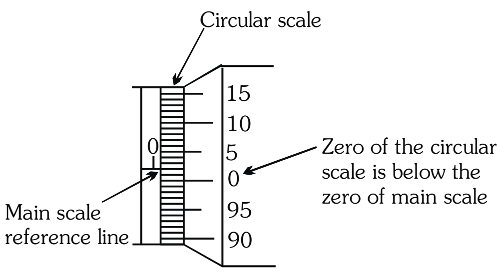
Negative Zero Error

(3 division error) i.e., –0.003 cm



Positive Zero Error

(2 division error) i.e., +0.002 cm



<u>Calculation of zero error for screw gauge :-</u>

Positive zero error = (No. of division of CS on MS).LC

Negative zero error = (Total division on CS – No. of division of CS on MS).LC

Correct reading = (Reading) – (zero error)

Remember :-

To get correct reading take zero error with their sign.

Positive zero error = + (Numerical value of zero error)

Negative zero error = – (Numerical value of zero error)

Illustrations

Illustration. 16.

One cm on the main scale of vernier callipers is divided into ten equal parts. If 20 divisions of vernier scale coincide with 18 small divisions of the main scale. What will be the least count of callipers ?

Solution

20 division of vernier scale = 18 division of main scale $\Rightarrow 1\text{ VSD}=\left(\frac{18}{20}\right)\text{MSD} = 0.9\text{ MSD}$

Least count = 1 MSD – 1 VSD = 1 MSD – 0.9 MSD = 0.1 MSD

$= 0.1 \times 0.1\text{ cm} = 0.01\text{ cm}\ \ (\because 1\text{ MSD}=\frac{1}{10}\text{cm} = 0.1\text{ cm})$

Illustration 17.

The n^{th} division of main scale coincides with $(n + 1)^{th}$ division of vernier scale. Given one main scale division is equal to 'a' units. Find the least count of the vernier.

Solution

(n + 1) divisions of vernier scale = n divisions of main scale

$\therefore$ 1 vernier division = $\frac{n}{n+1}$ main scale division

Least count = 1 MSD – 1VSD = $(1-\frac{n}{n+1})$ MSD = $(\frac{1}{n+1})$ MSD = $\frac{a}{n+1}$

Illustration 18.

A spherometer has 100 equal divisions marked along the periphery of its disc, and one full rotation of the disc advances on the main scale by 0.01 cm. Find the least count of the system.

Solution

Given Pitch = 0.01 cm

$\therefore$ Least count = $\frac{\text{Pitch}}{\text{Total no. of divisions on the circular scale}} = \frac{0.01}{100}\text{cm} = 10^{-4}\text{ cm.}$

Illustration 19.

The least count of a stop watch is $\frac{1}{5}$ second. The time of 20 oscillations of a pendulum is measured to be 25 seconds. What is the percentage error in the measurement of time ?

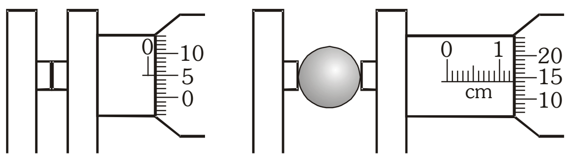
Solution

Error in measuring 25 s = $\frac{1}{5}$ s = 0.2 sec. $\therefore$ percentage error = $\frac{0.2}{25}\times 100 = 0.8\%$

Note : The final absolute error in this type of questions is taken to be equal to the least count of the measuring instrument.

Illustration 20.

What is the diameter of sphere shown in figure. Pitch of screwgauge is 1 mm and number of divisions in circular scale are 50 :-



(1) 1.238 cm (2) 1.218 cm (3) 1.228 cm (4) 1.215 cm

Solution

Zero error :

Least count of circular scale $= \frac{1}{50} = 0.02$ mm

Reading of main scale = 0.0 cm

Number of division coincied = 5

reading of circular scale = 5×0.02

Zero error = 0.10 mm = +0.010 cm

For diameter sphere :

Reading of main scale = 1.2 cm

No. of division coincided = 14

Reading of circular scale = $14 \times 0.02 = 0.28$ mm

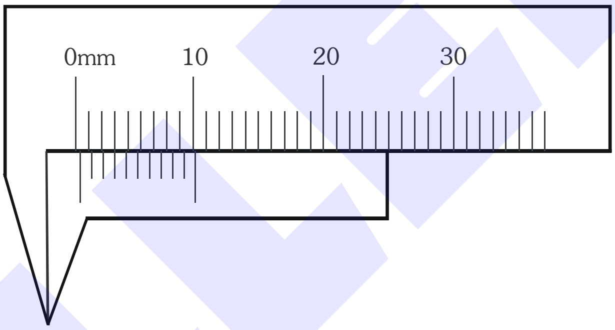
= 0.028 cm

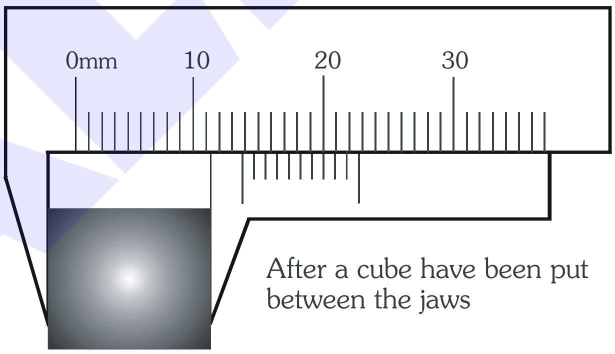
Reading = 1.228 cm

Diameter = 1.228 – 0.010 = 1.218 cm

Illustration 21.

Find the thickness of the cubical object using a defective vernier calliper. Main scale has mm marks and 10 divisions of vernier scale coincide with 9 divisions of main scale.





(1) 13.8 mm (2) 13.4 mm (3) 14.1 mm (4) 13.0 mm

Solution

Least count of vernier callipers = $\left(1-\frac{9}{10}\right) \times 1\text{mm} = 0.1\text{mm}$

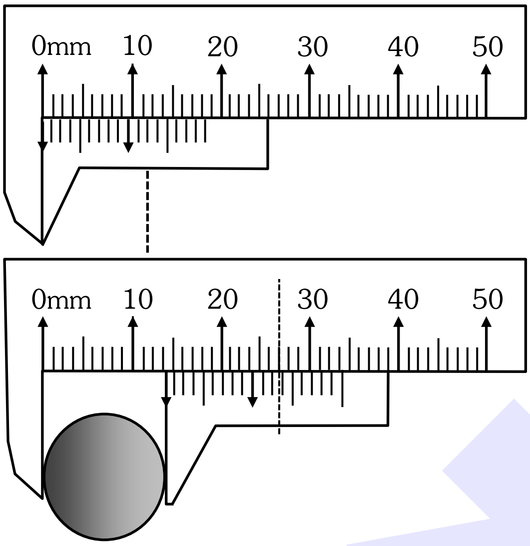
Zero error = VSR $\times$ LC = 3×0.1 = 0.3 mm

Reading = MSR + VSR $\times$ LC = 13mm + 7×0.1 mm = 13.7 mm

Correct reading = Reading – Zero error = 13.7 – 0.3 mm = 13.4 mm

Illustration 22.

Read the special type of vernier 20 division of vernier scale are matching with 19 divisions of main scale.



Solution

20 vernier scale divisions = 19 mm

1 vernier scale division = $\frac{19}{20}$

where least count = (Main scale division – vernier Scale division)

=1 mm – 19/20 mm (from fig.)

= 0.05 mm

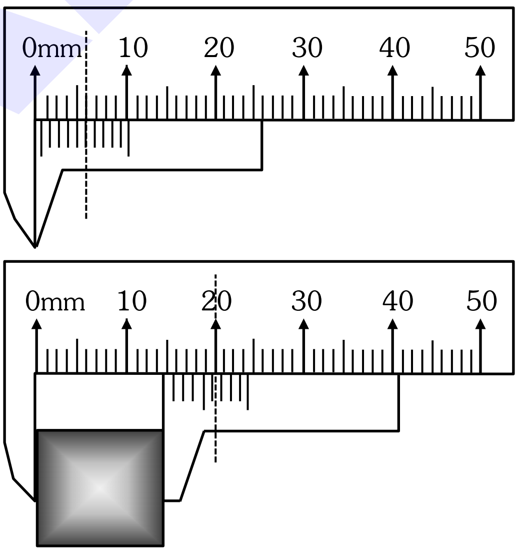
Thickness of the object = (main scale reading) + (vernier scale Reading) (least count)

So thickness of the object = 13 mm + (12) (0.05 mm)

= 13.60 mm **Ans.**

Illustration 23.

In the vernier caliperse, 9 main scale divisions matches with 10 vernier scale divisions. The thickness of the object using the defected vernier calliperse will be



Solution

From first figure, Excess reading (zero error) = 0.6 mm

If an object is p[laced, vernier gives 14.6 mm in which there is 0.6 mm excess reading, which has to be subtracted. So actual thickness = 14.6 – 0.6 = 14.0 mm we can also do it using the formula

Actual reading = observed – excess reading

reading (Zero error)

= 14.6 – 0.6 = 14.0 mm **Ans.**

BEGINNER'S BOX-7

1. One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 20 divisions of vernier scale coincide with 19 small divisions of the main scale then what will be the least count of the callipers.

2. If the number of divisions on the circular scale is 100 and number of full rotations given to screw is 8 and distance moved by the screw is 4 mm, then what will be least count of the screw gauge ?

3. A spherometer has 250 equal divisions marked along the periphery of its disc, and one full rotation of the disc advances by 0.0625 cm. on the main scale. What is the least count of the spherometer ?

ANSWERS

BEGINNER'S BOX-1

1. $(6.67 \times 10^{-8}\ cm^3/g\ s^2)$

2. (a)→ (v); (b) → (i); (c) → (iii); (d) → (ii); (e) → (iv).

BEGINNER'S BOX-2

1. (i) (b), (ii) (c), (iii) (d), (iv) (a)

2. (a) $[M^0\ L^0\ T^0K^1]$ (b) $[M\ L^2\ T^{-2}]$ (c) $[M\ L^{-1}\ T^{-2}]$ (d) $[M^0\ L^0\ T^{-1}]$

3. $[M\ L^2\ T^{-1}]$ **4.** $F = K\frac{mv^2}{r}$

BEGINNER'S BOX-3

1. (a) 3.256×10^3 g (b) 1.0×10^{-3} g (c) 5.0000×10^4 g (d) 3.204×10^{-1}

2. (a) 3 (b) 5 (c) 3 (d) 3 (e) 2 (f) 4 (g) 3 (h) 4

3. (3)

BEGINNER'S BOX-4

1. (a) 25.7 (b) 5.00×10^5 (c) 0.7 (d) 3.4 (e) 0.0393 kg (f) 4.08×10^8 s

2. 0.882 m^2 (3 SF)

3. 3.6×10^5

4. (a) Total mass = 2.3 kg (b) Difference in masses = 0.02g

BEGINNER'S BOX-5

1. (a) 0 (b) 3 (c) 2 (d) 3 (e) 3 (f) 11 (g) –10 (h) 8
(i) –10 (j) 24 (k) –33 (l) –19 (m) –10 (n) 5 (o) 25 (p) 7

BEGINNER'S BOX-6

1. (4.1 ± 0.3) m **2.** (1.6 ± 0.8) cm **3.** (1)

4. (55.4 ± 26.4) cm^2 **5.** ±12% **6.** (4.0 ± 0.3) m/s

BEGINNER'S BOX-7

1. 0.005 cm **2.** 0.005 mm **3.** 2.5×10^{-4} cm

APPENDIX

SI PREFIXES

The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small so as to be expressed more compactly in certain powers of 10.

Table 3 : Prefixes used for different powers of 10

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

General Guidelines for using Symbols for SI Units, Some other Units, and SI prefixes

(i) Symbols for units of physical quantities are printed/written in Roman (upright type), and not in italics

For Example : 1 N is correct but 1 *N* is incorrect.

(ii) (i) Unit is never written with capital initial letter if it is named after a scientist.

For example :

SI unit of force is newton (correct) not Newton (incorrect)

(ii) For a unit named after a scientist, the symbol is a capital letter. But for other units, the symbol is NOT a capital letter.

For example :
force → newton (N)
energy → joule (J)
electric current → ampere (A)
temperature → kelvin (K)
frequency → hertz (Hz)

For example :
length → metre (m)
mass → kilogram (kg)
luminous intensity → candela (cd)
time → second (s)

Note : The single exception is L, for the unit litre.

(iii) Symbols for units do not contain any final full stop at the end of recommended letter and remain unaltered in the plural, using only singular form of the unit.

For example :

Quantity	Correct	Incorrect
25 centimetres	25 cm	25 cm. 25 cms

(iv) Use of solidus (/) is recommended only for indicating a division of one letter unit symbol by another unit symbol. Not more than one solidus is used.

For example :

Correct	Incorrect
m / s^2	m / s / s
N s / m^2	N s / m / m
J / K mol	J / K / mol
kg / m s	kg / m / s

(v) Prefix symbols are printed in roman (upright) type without spacing between the prefix symbol and the unit symbol. Thus certain approved prefixes written very close to the unit symbol are used to indicate decimal fractions or multiples of a SI unit, when it is inconveniently small or large.

For example :

megawatt	1 MW = 10^6 W
centrimetre	1 cm = 10^{-2} m
kilometre	1 km = 10^3 m
millivolt	1 mV = 10^{-3} V
kilowatt-hour	1 kW h = 10^3 W h = 3.6 MJ = 3.6×10^6 J
microampere	1 μA = 10^{-6} A
angstrom	1 Å = 0.1 nm = = 10^{-10} m
nanosecond	1 ns = 10^{-9} s
picofarad`	1 pF = 10^{-12} F
microsecond	1 μs = 10^{-6} s
gigahertz	1 GHz = 10^9 Hz
micron	1 μm = 10^{-6} m

The unit 'fermi', equal to a femtometre or 10^{-15} m has been used as the convenient length unit in nuclear studies.

(vi) When a prefix is placed before the symbol of a unit, the combination of prefix and symbol is considered as a new symbol, for the unit, which can be raised to a positive or negative power without using brackets. These can be combined with other unit symbols to form compound unit.

For example :

Quantity	Correct	Incorrect
cm^3	$(cm)^3 = (0.01\ m)^3 = (10^{-2}\ m)^3 = 10^{-6} m^3$	0.01 m^3 or 10^{-2} m^3 or 1 cm^3
mA^2	$(mA)^2 = (0.001\ A)^2 = (10^{-3}\ A)^2 = 10^{-6}\ A^2$	0.001 A^2 or mA^2

(a) A prefix is never used alone. It is always attached to a unit symbol and written or fixed before the unit symbol.

For example : 10^3 / m^3 = 1000 /m^3 or 1000 m^{-3}, but not k/m^3 or k m^{-3}.

(vii) Prefix symbol is written very close to the unit symbol without space between them, while unit symbols are written separately with spacing when units are multiplied together.

For example :

Quantity	Correct	Incorrect
1 ms^{-1}	1 metre per second	1 milli per second
1 ms	1 millisecond	1 metre second.
1 Cm	1 coulomb metre	1 centimetre
1 cm	1 centimetre	1 coulomb metre

(viii) The use of double prefixes is avoided when single prefix is available.

For example :

Quantity	Correct	Incorrect
10^{-9} m	1 nm (nanometre)	1 mμm (millimicrometre)
10^{-6} m	1 μm (micron)	1 mmm (millimillimetre)
10^{-12} F	1 pF (picofarad)	1 μμF (micromicrofarad)
10^9 W	1 GW (giga watt)	1 kMW (kilomegawatt)

(ix) The use of a combination of unit and the symbols for units is avoided when the physical quantity is expressed by combining two or more units.

Quantity	Correct	Incorrect
joule per mole Kelvin	J/mol K or J mol^{-1} K^{-1}	joule / mole K or J /mol Kelvin or J/mole K
newton metre second	N m s	newton m second or N m second or N metre s or newton metre s

DIMENSIONAL FORMULAE OF PHYSICAL QUANTITIES

Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
Area	Length × breadth	$[L^2]$	$[M^0 L^2 T^0]$
Volume	Length × breadth × height	$[L^3]$	$[M^0L^3T^0]$
Mass density	Mass/volume	$[M]/[L^3]$ or $[M L^{-3}]$	$[ML^{-3}T^0]$
Frequency	1/time period	$1/[T]$	$[M^0 L^0 T^{-1}]$
Velocity, speed	Displacement/time	$[L]/[T]$	$[M^0LT^{-1}]$
Acceleration	Velocity /time	$[LT^{-1}]/[T]$	$[M^0LT^{-2}]$
Force	Mass × acceleration	$[M][LT^{-2}]$	$[M LT^{-2}]$
Impulse	Force × time	$[M LT^{-2}][T]$	$[M LT^{-1}]$
Work, Energy	Force × distance	$[MLT^{-2}][L]$	$[M L^2 T^{-2}]$
Power	Work/time	$[ML^2 T^{-2}]/ [T]$	$[ML^2 T^{-3}]$
Momentum	Mass × velocity	$[M] [LT^{-1}]$	$[MLT^{-1}]$
Pressure, stress	Force/area	$[MLT^{-2}]/[L^2]$	$[ML^{-1}T^{-2}]$
Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$[L] / [L]$ or $[L^3]/[L^3]$	$[M^0L^0 T^0]$
Surface tension	Force/length	$[MLT^{-2}]/[L]$	$[ML^0 T^{-2}]$
Modulus of elasticity	Stress/strain	$\frac{[ML^{-1}T^{-2}]}{[M^0L^0T^0]}$	$[ML^{-1} T^{-2}]$
Surface energy	Energy/area	$[ML^2T^{-2}]/[L^2]$	$[ML^0T^{-2}]$
Velocity gradient	Velocity/distance	$[LT^{-1}] / [L]$	$[M^0L^0T^{-1}]$
Pressure gradient	Pressure/distance	$[ML^{-1}T^{-2}]/[L]$	$[ML^{-2}T^{-2}]$
Pressure energy	Pressure × volume	$[ML^{-1}T^{-2}] [L^3]$	$[ML^2 T^{-2}]$
Coefficient of viscosity	Force/(area × velocity gradient)	$\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	$[ML^{-1}T^{-1}]$
Angle, Angular displacement	Arc/radius	$[L]/[L]$	$[M^0L^0T^0]$
Trigonometric ratio	Length/length	$[L]/[L]$	$[M^0L^0T^0]$
Angular velocity	Angle/time	$[L^0]/[T]$	$[M^0L^0T^{-1}]$
Angular acceleration	Angular velocity/time	$[T^{-1}]/[T]$	$[M^0L^0T^{-2}]$
Radius of gyration	Distance	$[L]$	$[M^0LT^0]$
Moment of inertia	Mass ×(radius of gyration)^2	$[M] [L^2]$	$[ML^2 T^0]$
Angular momentum	Moment of inertia × angular velocity	$[ML^2] [T^{-1}]$	$[ML^2 T^{-1}]$
Moment of force (Couple)	Force ×distance	$[MLT^{-2}] [L]$	$[ML^2 T^{-2}]$
Torque	Angular momentum/time, Or Force × distance	$[ML^2 T^{-1}]/[T]$ or $[MLT^{-2}] [L]$	$[ML^2 T^{-2}]$
Angular frequency	2π × Frequency	$[T^{-1}]$	$[M^0L^0T^{-1}]$
Wavelength	Distance	$[L]$	$[M^0LT^0]$
Hubble constant	Recession speed/distance	$[LT^{-1}]/[L]$	$[M^0L^0T^{-1}]$
Intensity of wave	(Energy/time)/area	$[ML^2 T^{-2}/T]/[L^2]$	$[ML^0T^{-3}]$

Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
Radiation pressure	$\frac{\text{Intensity of wave}}{\text{Speed of light}}$	$[MT^{-3}]/[LT^{-1}]$	$[ML^{-1}T^{-2}]$
Energy density	Energy/volume	$[ML^2\,T^{-2}]/[L^3]$	$[ML^{-1}\,T^{-2}]$
Critical velocity	$\frac{\text{Reynold's number} \times \text{coefficient of viscosity}}{\text{Mass density} \times \text{radius}}$	$\frac{[M^0L^0T^0][ML^{-1}\,T^{-1}]}{[ML^{-3}][L]}$	$[M^0LT^{-1}]$
Escape velocity	(2 × acceleration due to gravity × earth's radius)$^{1/2}$	$[LT^{-2}]^{1/2} \times [L]^{1/2}$	$[M^0LT^{-1}]$
Heat energy, internal energy	Work (= Force × distance)	$[MLT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
Kinetic energy	(1/2) mass × (velocity)^2	$[M]\,[LT^{-1}]^2$	$[ML^2T^{-2}]$
Potential energy	Mass × acceleration due to gravity × height	$[M]\,[LT^{-2}]\,[L]$	$[ML^2\,T^{-2}]$
Rotational kinetic energy	1/2 × moment of inertia × (angular velocity)^2	$[M^0L^0T^0]\,[ML^2] \times [T^{-1}]^2$	$[ML^2\,T^{-2}]$
Efficiency	$\frac{\text{Output work or energy}}{\text{Input work or energy}}$	$\frac{[ML^2T^{-2}]}{[ML^2T^{-2}]}$	$[M^0L^0T^0]$
Angular impulse	Torque × time	$[ML^2T^{-2}]\,[T]$	$[ML^2T^{-1}]$
Gravitational constant	$\frac{\text{Force} \times (\text{distance})^2}{\text{mass} \times \text{mass}}$	$\frac{[MLT^{-2}][L^2]}{[M][M]}$	$[M^{-1}L^3T^{-2}]$
Planck constant	Energy/frequency	$[ML^2\,T^{-2}]\,/[T^{-1}]$	$[ML^2T^{-1}]$
Heat capacity, entropy	Heat energy /temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$
Specific heat capacity	$\frac{\text{Heat Energy}}{\text{Mass} \times \text{temperature}}$	$[ML^2T^{-2}]/[M][K]$	$[M^0L^2T^{-2}K^{-1}]$
Latent heat	Heat energy/mass	$[ML^2\,T^{-2}]/[M]$	$[M^0L^2T^{-2}]$
Thermal expansion coefficient or Thermal expansivity	$\frac{\text{Change in dimension}}{\text{Original dimension} \times \text{temperature}}$	$[L]/[L][K]$	$[M^0L^0K^{-1}]$
Thermal conductivity	$\frac{\text{Heat Energy} \times \text{thickness}}{\text{Area} \times \text{temperature} \times \text{time}}$	$\frac{[ML^2T^{-2}][L]}{[L^2][K][T]}$	$[MLT^{-3}\,K^{-1}]$
Bulk modulus or (compressibility)$^{-1}$	$\frac{\text{Volume} \times (\text{Change in pressure})}{\text{Change in volume}}$	$\frac{[L^3][ML^{-1}T^{-2}]}{[L^3]}$	$[ML^{-1}T^{-2}]$
Centripetal acceleration	(Velocity)^2/radius	$[LT^{-1}]^2/[L]$	$[M^0LT^{-2}]$
Stefan constant	$\frac{(\text{Energy / area} \times \text{time})}{(\text{Temperature})^4}$	$\frac{[ML^2T^{-2}]}{[L^2][T][K]^4}$	$[ML^0T^{-3}K^{-4}]$
Wien constant	Wavelength × temperature	$[L]\,[K]$	$[M^0LT^0K]$
Boltzmann constant	Energy/temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$

Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
Universal gas constant	$\frac{\text{Pressure} \times \text{volume}}{\text{mole} \times \text{temperature}}$	$\frac{[ML^{-1}T^{-2}][L^3]}{[mol][K]}$	$[ML^2T^{-2}K^{-1}mol^{-1}]$
Charge	Current × time	[A][T]	$[M^0L^0TA]$
Current density	Current/area	$[A]/[L^2]$	$[M^0L^{-2}T^0A]$
Voltage, electric potential, electromotive force	Work/charge	$[ML^2T^{-2}]/[AT]$	$[ML^2T^{-3}A^{-1}]$
Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{[ML^2T^{-3}A^{-1}]}{[A]}$	$[ML^2T^{-3}A^{-2}]$
Capacitance	Charge/potential difference	$\frac{[AT]}{[ML^2T^{-3}A^{-1}]}$	$[M^{-1}L^{-2}T^4A^2]$
Electrical resistivity or (electrical conductivity)$^{-1}$	$\frac{\text{Resistance} \times \text{area}}{\text{length}}$	$[ML^2T^{-3}A^{-2}][L^2]/[L]$	$[ML^3T^{-3}A^{-2}]$
Electric field	Electrical force/charge	$[MLT^{-2}]/[AT]$	$[MLT^{-3}A^{-1}]$
Electric flux	Electric field × area	$[MLT^{-3}A^{-1}][L^2]$	$[ML^3T^{-3}A^{-1}]$
Electric dipole moment	Torque/electric field	$\frac{[ML^2T^{-2}]}{[MLT^{-3}A^{-1}]}$	$[M^0LTA]$
Electric field strength or electric field intensity	$\frac{\text{Potential difference}}{\text{distance}}$	$\frac{[ML^2T^{-3}A^{-1}]}{[L]}$	$[MLT^{-3}A^{-1}]$
Magnetic field, magnetic flux density, magnetic induction	$\frac{\text{Force}}{\text{Current} \times \text{length}}$	$[MLT^{-2}]/[A][L]$	$[ML^0T^{-2}A^{-1}]$
Magnetic flux	Magnetic field × area	$[MT^{-2}A^{-1}][L^2]$	$[ML^2T^{-2}A^{-1}]$
Inductance	$\frac{\text{Magnetic flux}}{\text{Current}}$	$\frac{[ML^2T^{-2}A^{-1}]}{[A]}$	$[ML^2T^{-2}A^{-2}]$
Magnetic dipole moment	Torque/magnetic field or current × area	$[ML^2T^{-2}]/[MT^{-2}A^{-1}]$ or $[A][L^2]$	$[M^0L^2T^0A]$
Magnetic field strength, magnetic intensity or magnetic moment density	$\frac{\text{Magnetic moment}}{\text{Volume}}$	$\frac{[L^2A]}{[L^3]}$	$[M^0L^{-1}T^0A]$
Permittivity constant (of free space)	$\frac{\text{Charge} \times \text{charge}}{4\pi \times \text{electricforce} \times (\text{distance})^2}$	$\frac{[AT][AT]}{[MLT^{-2}][L]^2}$	$[M^{-1}L^{-3}T^4A^2]$
Permeability constant (of free space)	$\frac{2\pi \times \text{force} \times \text{distance}}{\text{current} \times \text{current} \times \text{length}}$	$\frac{[M^0L^0T^0][MLT^{-2}][L]}{[A][A][L]}$	$[MLT^{-2}A^{-2}]$
Refractive index	$\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$	$[LT^{-1}]/[LT^{-1}]$	$[M^0L^0T^0]$
Faraday constant	Avogadro constant × elementary charge	[AT]/[mol]	$[M^0L^0TA\ mol^{-1}]$

Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
Wave number	2π/ wavelength	$[M^0L^0T^0]/[L]$	$[M^0L^{-1}T^0]$
Radiant flux, Radiant power	Energy emitted/time	$[ML^2T^{-2}]/[T]$	$[ML^2T^{-3}]$
Luminosity of radiant flux or radiant intensity	$\frac{\text{Radiant power or radiant flux of source}}{\text{Solid angle}}$	$[ML^2T^{-3}]/[M^0L^0T^0]$	$[ML^2T^{-3}]$
Luminous power or luminous flux of source	$\frac{\text{Luminous energy emitted}}{\text{time}}$	$[ML^2 T^{-2}]/[T]$	$[ML^2 T^{-3}]$
Luminous intensity or illuminating power of source	$\frac{\text{Luminous flux}}{\text{Soild angle}}$	$\frac{[ML^2T^{-3}]}{[M^0L^0T^0]}$	$[ML^2T^{-3}]$
Intensity of illumination or luminance	$\frac{\text{Luminous intensity}}{(\text{distance})^2}$	$[ML^2T^{-3}]/[L^2]$	$[ML^0 T^{-3}]$
Relative luminosity	$\frac{\text{Luminous flux of a source of given wavelength and intensity}}{\text{luminous flux of peak sensitivity wavelength (555 nm) source of same power}}$	$\frac{[ML^2T^{-3}]}{[ML^2T^{-3}]}$	$[M^0L^0T^0]$
Luminous efficiency	$\frac{\text{Total luminous flux}}{\text{Total radiant flux}}$	$[ML^2T^{-3}]/[ML^2T^{-3}]$	$[M^0L^0T^0]$
Illuminance or illumination	$\frac{\text{Luminous flux incident}}{\text{area}}$	$[ML^2T^{-3}]/[L^2]$	$[ML^0T^{-3}]$
Mass defect	(sum of masses of nucleons)- (mass of the nucleus)	$[M]$	$[ML^0T^0]$
Binding energy of nucleus	Mass defect $\times$(speed of light in vacuum)^2	$[M]\,[LT^{-1}]^2$	$[ML^2T^{-2}]$
Decay constant	0.693/half life	$[T^{-1}]$	$[M^0L^0T^{-1}]$
Resonant frequency	$(\text{Inductance} \times \text{capacitance})^{-1/2}$	$[ML^2T^{-2}A^{-2}]^{-\frac{1}{2}}[M^{-1}L^{-2}T^4A^2]^{-\frac{1}{2}}$	$[M^0L^0A^0T^{-1}]$
Quality factor or Q- factor of coil	$\frac{\text{Resonant frequency} \times \text{inducatance}}{\text{Resistance}}$	$\frac{[T^{-1}][ML^2T^{-2}A^{-2}]}{[ML^2T^{-3}A^{-2}]}$	$[M^0L^0T^0]$
Power of lens	(Focal length)$^{-1}$	$[L^{-1}]$	$[M^0L^{-1}T^0]$
Magnification	$\frac{\text{Image distance}}{\text{Object distance}}$	$[L]/[L]$	$[M^0L^0T^0]$
Fluid flow rate	$\frac{(\pi/8)\,(\text{pressure}) \times (\text{radius})^4}{(\text{viscosity coefficient}) \times \text{length}}$	$\frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]}$	$[M^0L^3T^{-1}]$
Capacitive reactance	(Angular frequency × capacitance)$^{-1}$	$[T^{-1}]^{-1}\,[M^{-1}L^{-2}\,T^4A^2]^{-1}$	$[ML^2T^{-3}A^{-2}]$
Inductive reactance	(Angular frequency × inductance)	$[T^{-1}]\,[ML^2\,T^{-2}A^{-2}]$	$[ML^2T^{-3}A^{-2}]$

SOME IMPORTANT CONVERSION FACTORS

LENGTH

- 1 m = 100 cm = 1000 mm = 3.28 ft. = 39.37 in = 1.0936 yd (yard)
- 1 km = 0.6215 mi (mile)
- 1 mi = 1609 m
- 1 n mi (nautical mile) = 1852 m
- 1 in = 2.54 cm
- 1 ft = 12 in = 30.48 cm.
- 1 bohr radius = 0.529 Å
- 1 AU (Astronomical unit) = 1.49×10^{11} m (Average distance between sun and earth)
- 1 ly (light year) = 9.461×10^{15} m (Distance travelled by light in vacuum in one year)
- 1 parsec or parallactic second = 3.08×10^{16}m = 3.26 ly (Distance at which an arc of length 1AU subtends an angle of one second at a point)

MASS

- 1 kg = 1000 g = 2.2 lb (pound)
- 1 quintal = 100 kg
- 1 ton = 907.2 kg
- 1 metric tonne = 1000 kg = 10^6 g
- 1 lb = 454 g
- 1 slug = 14.59 kg
- 1 ounce = 28.35 g
- 1 amu = 1.6606×10^{-27} kg = 931.5 MeV/c^2
- 1 Chandra Shekhar Limit = 1.4 M_{sun}

TIME

- 1 h = 60 min = 3600 s
- 1 d = 24 h = 1440 min = 86.4×10^3 s
- 1 y = 365.24 d = 31.56×10^6 s
- 1 shake = 10^{-8} s

AREA

- 1 m^2 = 10^4 cm^2
- 1 km^2 = 0.386 mi^2 = 247 acres
- 1 acre = 43,560 ft^2 = 4047m^2 = 0.4047 hectare
- 1 hectare = 10^4 m^2 = 2.47 acres
- 1 barn = 10^{-28} m^2 (for measuring cross-sectional areas in sub-atomic particle collisions)

VOLUME

- 1 m^3 = 10^6 cm^3 = 10^6 cc = 10^3 L = 35.31 ft^3
- 1 gal (gallon) = 3.786 L (in U.S.A.) or 4.54 L (in U.K.)

DENSITY

- 1 kg/m^3 = 10^{-3} g/ cm^3 = 10^{-3} kg/L

SPEED

- 1 km h^{-1} = 5/18 m/s or 0.2778 m/s = 0.6215 mi/h
- 1 mi h^{-1} = 0.4470 m/s = 1.609 km/h = 1.467 ft/s
- 1 m s^{-1} = 18/5 km/h or 3.6 km/h = 2.24 mi/h

ACCELERATION

- g = 9.8 m/s^2 (*MKS unit*) = 980 cm/s^2 (*CGS unit*) = 32 ft/s^2 (*FPS unit*)

ANGLE AND ANGULAR SPEED

- π rad = $180°$
- 1 rad = $180°/\pi$ or 57.30°
- $1°$ = 1.745×10^{-2} rad = 60' = 1/360 revolution
- 1 rev = $360° = 2\pi$ rad
- 1' (min) = 60'' (second)
- 1 rev/min = 0.1047 rad/s ≈ 0.1 rad/s
- 1 rad/s = 9.549 rev/min

FORCE

- 1 N = 10^5 dyne = 7.23 poundal
- 1 kg–wt = 1 kg–f = 9.8 N
- 1 g–wt = 1 g–f = 980 dyne
- 1 lb–wt = 1 lb–f = 32 poundal

PRESSURE

- 1 Pa = 1 N/m^2 = 10 dyne/cm^2
- 1 bar = 10^5 Pa = 10^6 dyne/cm^2
- 1 atm = 1.01325 bar = 1.01×10^5 Pa = 1.01×10^6 dyne/cm^2 = 760 mm of Hg column
- 1 torr = 1 mm of Hg column = 153.32 Pa

WORK ENERGY

- 1 J = 10^7 erg = 0.239 cal
- 1 eV = 1.6×10^{-19} J
- 1 amu = 931 MeV = 1.492×10^{-10} J
- 1 cal = 4.186 J
- 1 kWh = 3.6 MJ = 860 kcal
- 1 Btu (British thermal unit) = 1055 J

POWER

- 1 hp (horse power) = 745.7 W ≈ 746 W
- 1 kW = 1000 W = 1.34 hp
- 1 W (watt) = 1 J/s
- 1 cal/s = 4.186 W

TEMPERATURE

- K (kelvin) = [°C + 273°] = [°F + 459.67]/1.8 = °R/1.8
- °F = °C × 9/5+ 32

ELECTRIC CHARGE

- 1 C (coulomb) = 3×10^9 stat coulomb = 0.1 ab coulomb
- 1 esu = 1 stat coulomb = 3.33×10^{-10} coulomb
- 1 emu = 1 ab coulomb = 10 coulomb
- 1 A-h = 3600 C (coulomb)

ELECTRIC CURRENT

- 1 A (ampere) = 3×10^9 stat ampere (*esu of current*) = 0.1 ab ampere (*emu of current*)

RADIOACTIVITY

- 1 Bq (bacquerel) = 1 dps (disintegration per second)
- 1 Ci (curie) = 3.7×10^{10} dps = 3.7×10^{10} Bq = 3.7×10^4 Rd
- 1 Rd (rutherford) = 10^6 dps = 10^6 Bq

OTHERS

- 1 weber = 10^8 maxwell (for *Magnetic flux*)
- 1 T (tesla) = 1 weber/m^2 = 10^4 G (gauss) (for *Magnetic flux density*)
- 1 orested = 79.554 A/m (for *Intensity of Magnetic field*)
- 1 poiseuille (N-s/m^2 or Pa-s) = 10 poise (Dyne-s/cm^2) (for *Viscosity*)

SETS OF QUANTITIES HAVING SAME DIMENSIONS

S.No.	Quantities	Dimensions
1.	Strain, refractive index, relative density, angle, solid angle, phase, distance gradient, relative permeability, relative permittivity, angle of contact, Reynolds number, coefficient of friction, mechanical equivalent of heat, electric susceptibility, etc.	$[M^0 L^0 T^0]$
2.	Mass	$[M^1 L^0 T^0]$
3.	Momentum and impulse.	$[M^1 L^1 T^{-1}]$
4.	Thrust, force, weight, tension, energy gradient.	$[M^1 L^1 T^{-2}]$
5.	Pressure, stress, Young's modulus, bulk modulus, shear modulus, modulus of rigidity, energy density.	$[M^1 L^{-1} T^{-2}]$
6.	Angular momentum and Planck's constant (h).	$[M^1 L^2 T^{-1}]$
7.	Acceleration, g and gravitational field intensity.	$[M^0 L^1 T^{-2}]$
8.	Surface tension, free surface energy (energy per unit area), force gradient, spring constant.	$[M^1 L^0 T^{-2}]$
9.	Latent heat and gravitational potential.	$[M^0 L^2 T^{-2}]$
10.	Thermal capacity, Boltzmann constant, entropy.	$[ML^2 T^{-2}K^{-1}]$
11.	Work, torque, internal energy, potential energy, kinetic energy, moment of force, (q^2/C), (LI^2), (qV), (V^2C), (I^2rt), $(\frac{V^2}{r}t)$, (VIt), (RT) $q \rightarrow$ charge, $C \rightarrow$ capacitance, $L \rightarrow$ inductance, $V \rightarrow$ potential, $r \rightarrow$ resistance, $I \rightarrow$ current $T \rightarrow$ temperature, $t \rightarrow$ time, $R \rightarrow$ gas constant	$[M^1 L^2 T^{-2}]$
12.	Frequency, angular frequency, angular velocity, velocity gradient, radioactivity of a sample, $\left(\frac{R}{L}\right), \left(\frac{1}{RC}\right), \left(\frac{1}{\sqrt{LC}}\right)$. $L \rightarrow$ inductance, $R \rightarrow$ resistance, $C \rightarrow$ capacitance	$[M^0 L^0 T^{-1}]$
13.	$\left(\frac{\ell}{g}\right)^{1/2}, \left(\frac{m}{k}\right)^{1/2}, \left(\frac{L}{R}\right), (RC), (\sqrt{LC})$, time $\ell \rightarrow$ length, $g \rightarrow$ gravitational acceleration, $k \rightarrow$ spring constant	$[M^0 L^0 T^1]$
14.	(VI), (I^2r), (V^2/r), Power (r= resistance)	$[ML^2 T^{-3}]$

NUMERICAL CONSTANTS

I. FUNDAMENTAL PHYSICAL CONSTANTS

Name	Symbol	Value	Computational Value
Speed of light	**c**	2.99792458×10^{8} m/s	3.00×10^{8} m/s
Elementary charge	**e**	$1.60217653 \times 10^{-19}$ C	1.60×10^{-19} C
Gravitational constant	**G**	6.6742×10^{-11} N-m²/kg²	6.67×10^{-11} N-m²/kg²
Universal gas constant	**R**	8.314472 J/mol-K	8.31 J/mol-K
Avogadro's constant	N_A	6.0221415×10^{23} molecules/mol	6.02×10^{23} molecules/mol
Boltzmann constant	**k**	$1.3806505 \times 10^{-23}$ J/K	1.38×10^{-23} J/K
Stefan-Boltzmann constant	σ	5.670400×10^{-8} W/m²-K⁴	5.67×10^{-8} W/m²-K⁴
Molar volume of ideal gas at STP*	V_m	22.413996 litre/mol	22.4 litre/mol
Planck's constant	**h**	$6.6260693 \times 10^{-34}$ J-s	6.62×10^{-34} J-s
Mass of electron	m_e	$9.1093826 \times 10^{-31}$ kg	9.11×10^{-31} kg
Mass of proton	m_p	$1.67262171 \times 10^{-27}$ kg	1.67×10^{-27} kg
Mass of neutron	m_n	$1.67492728 \times 10^{-27}$ kg	1.68×10^{-27} kg
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ Wb/A-m	1.27×10^{-6} Wb/A-m
Permittivity of free space	ε_0	$8.85418781762 \times 10^{-12}$ C²/N-m²	8.85×10^{-12} C²/N-m²
	$\frac{1}{4\pi\varepsilon_0}$	$8.987551787 \times 10^{9}$ N-m²/C²	9.0×10^{9} N-m²/C²

* STP means standard temperature and pressure : 0°C and 1.0 atm

II. OTHER USEFUL PHYSICAL CONSTANTS

Name	Symbol	Value	Computational Value
Mechanical equivalent of heat	J	4.186 J/cal	4.2 J/cal
Standard atmospheric pressure	1 atm	1.01325×10^{5} Pa	1.013×10^{5} Pa
Absolute zero	0 K	–273.15° C	–273° C
Electron volt	1 eV	$1.60217653 \times 10^{-19}$ J	1.60×10^{-19} J
Atomic mass unit	1 u	$1.66053886 \times 10^{-27}$ kg	1.66×10^{-27} kg
Electron rest energy	$m_e c^2$	0.510998918 MeV	0.511 MeV
Ratio of proton mass to electron mass	$\frac{m_p}{m_e}$	1836.1526675	1840
Electron charge to mass ratio	$\frac{e}{m_e}$	$1.758820174 \times 10^{11}$ C/kg	1.76×10^{11} C/kg
Bohr magneton	μ_B	$9.27400899 \times 10^{-24}$ J/T	9.2×10^{-24} J/T
Bohr radius	a_0	$5.291772083 \times 10^{-11}$ m	5.29×10^{-11} m
Rydberg constant	R_H	$1.097373156 \times 10^{7}$ m⁻¹	1.10×10^{7} m⁻¹
Energy equivalent of 1 u	mc^2	931.49404 MeV	931.5 MeV
Acceleration due to gravity (standard)	g	9.80665 m/s²	9.81 m/s²

SI Base Quantities and Units

Base Quantity	SI Units		
	Name	**Symbol**	**Definition**
Length	meter	m	The meter is the length of the path traveled by light in vacuum during a time interval of 1/(299, 792, 458) of a second (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric Current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 x 10^{-7} Newton per metre of length. (1948)
Thermodynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of Substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	candela	Cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979).

Note :- On November 16, 2018 at the General Conference on Weights and Measure (GCWM) the 130 years old definition of kilogram was changed forever. It will now defined in terms of plank's constant. It will adopted on 20 May, 2019 (World Metrology Day - 20 May). The new definition of kg involves accurate weighing machine called "Kibble balance".

IMPORTANT NOTES