Taylor

Energía potencial gravitatoria

$$U(\mathbf{r}_1, \mathbf{r}_2) = -\frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|}.$$

Lagrangiano de dos masas atraídas gravitacionalmente

$$\mathcal{L} = \tfrac{1}{2}m_1\dot{\mathbf{r}}_1^2 + \tfrac{1}{2}m_2\dot{\mathbf{r}}_2^2 - U(r).$$

Vector Centro de Masa

$$\mathbf{R}_G = \frac{m_1\mathbf{R}_1 + m_2\mathbf{R}_2}{m_1 + m_2}$$

$$\mathbf{R}_G = \mathbf{R}_{G_0} + \mathbf{v}_G t$$

Problema de los dos cuerpos

$$-\frac{Gm_1m_2}{r^2}\hat{\mathbf{u}}_r = m_2\ddot{\mathbf{R}}_2$$

$$\frac{Gm_1m_2}{r^2}\hat{\mathbf{u}}_r = m_1\ddot{\mathbf{R}}_1$$

Reducción del problema a un solo cuerpo de masa μ

$$\mathcal{L} = T - U = \tfrac{1}{2}M\dot{\mathbf{R}}^2 + \left(\tfrac{1}{2}\mu\dot{\mathbf{r}}^2 - U(r)\right)$$

$$= \mathcal{L}_{\text{cm}} + \mathcal{L}_{\text{rel}}.$$

$$\mu\ddot{r} = -\frac{dU}{dr} + \mu r\dot{\phi}^2 = -\frac{dU}{dr} + F_{\text{cf}}$$

Energía potencial efectiva

$$\mu\ddot{r} = -\frac{d}{dr}[U(r) + U_{\text{cf}}(r)] = -\frac{d}{dr}U_{\text{eff}}(r),$$

$$U_{\text{eff}}(r) = U(r) + U_{\text{cf}}(r) = U(r) + \frac{\ell^2}{2\mu r^2}.$$

Ley de conservación de la energía

$$\tfrac{1}{2}\mu\dot{r}^2 + U_{\text{eff}}(r) = \tfrac{1}{2}\mu\dot{r}^2 + \tfrac{1}{2}\mu r^2\dot{\phi}^2 + U(r)$$

$$= E.$$

$$\mu = \frac{m_1m_2}{M} \equiv \frac{m_1m_2}{m_1 + m_2} \qquad \text{[reduced mass]}$$

Ecuación de la órbita

$$u''(\phi) = -u(\phi) - \frac{\mu}{\ell^2 u(\phi)^2}F.$$

$$u = \frac{1}{r} \qquad u(\phi) = A\cos(\phi - \delta),$$

Las órbitas de Kepler

$$F(r) = -\frac{\gamma}{r^2} = -\gamma u^2,$$

$$\gamma = Gm_1m_2$$

$$u''(\phi) = -u(\phi) + \gamma\mu/\ell^2.$$

$$c = \frac{\ell^2}{\gamma\mu}$$

Parámetro de la órbita

$$w(\phi) = u(\phi) - \gamma\mu/\ell^2,$$

$$w''(\phi) = -w(\phi),$$

$$w(\phi) = A\cos(\phi - \delta),$$

$$u(\phi) = \frac{\gamma\mu}{\ell^2} + A\cos\phi = \frac{\gamma\mu}{\ell^2}(1 + \epsilon\cos\phi)$$

Solución del Problema de Kepler $\quad r(\phi) = \dfrac{c}{1 + \epsilon\cos\phi}.$

Leyes de Kepler

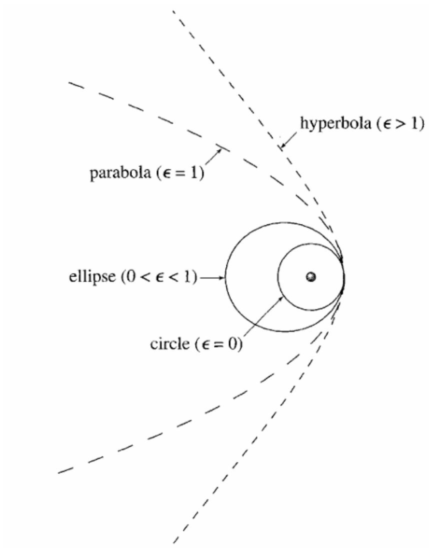
1.) $$r(\phi) = \frac{c}{1 + \epsilon\cos\phi}.$$

2.) $$\frac{dA}{dt} = \frac{\ell}{2\mu}.$$

3.) $$\tau^2 = \frac{4\pi^2}{GM_s}a^3.$$

Relación energía-excentricidad

$$E = U_{\text{eff}}(r_{\min}) = \frac{\gamma^2\mu}{2\ell^2}(\epsilon^2 - 1).$$



eccentricity	energy	orbit
$\epsilon = 0$	$E < 0$	circle
$0 < \epsilon < 1$	$E < 0$	ellipse
$\epsilon = 1$	$E = 0$	parabola
$\epsilon > 1$	$E > 0$	hyperbola

Algunas fórmulas extras

Ley del seno

$$\frac{a}{\operatorname{sen}\alpha} = \frac{b}{\operatorname{sen}\beta} = \frac{c}{\operatorname{sen}\gamma}$$

Ley del coseno

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

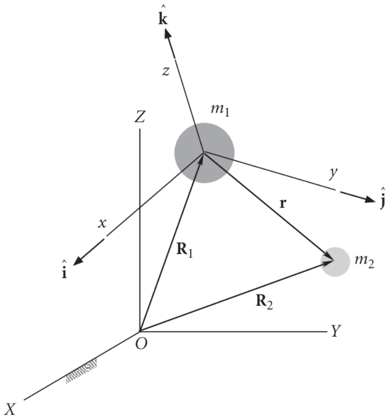
Fuerza como gradiente de la energía potencial

$$\mathbf{F} = -\nabla V$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$$

$$\mathbf{A}\cdot(\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B})\cdot\mathbf{C}$$

Howard



Ecuación de m_2 respecto a m_1

$$\ddot{\mathbf{r}} = -\frac{\mu}{r^3}\mathbf{r}$$

$$\mu = G(m_1 + m_2)$$

El momento angular de m_2 respecto a m_1

$$\mathbf{H}_{2/1} = \mathbf{r} \times m_2\dot{\mathbf{r}}$$

Solución al problema de Kepler

$$r = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta}$$

$p = \dfrac{h^2}{\mu}$, parámetro de la órbita

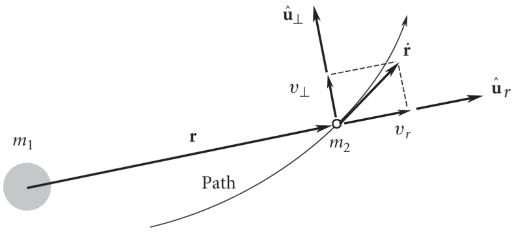
Momento angular por unidad de masa

$$\mathbf{h} = \mathbf{r} \times \dot{\mathbf{r}} \qquad h = r v_\perp \qquad \frac{dA}{dt} = \frac{h}{2}$$

$$h = r^2\dot{\theta} \qquad v_\perp = r\dot{\theta}$$

$$v_\perp = \frac{\mu}{h}(1 + e\cos\theta) \qquad v_r = \frac{\mu}{h}e\sin\theta$$

$\theta \rightarrow$ Anomalía verdadera

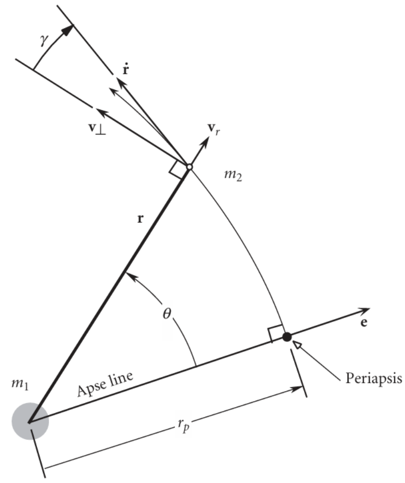


Nueva base: $(\hat{u}_\perp, \hat{u}_r)$

Ángulo de vuelo γ

$$\tan\gamma = \frac{v_r}{v_\perp}$$

$$\tan\gamma = \frac{e\sin\theta}{1 + e\cos\theta}$$



Ley de la energía

En general, la energía por unidad de masa es

$$\frac{v^2}{2} - \frac{\mu}{r} = \varepsilon = -\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2)$$

TIPOS DE ÓRBITAS

1. Órbitas Circulares $(e = 0)$

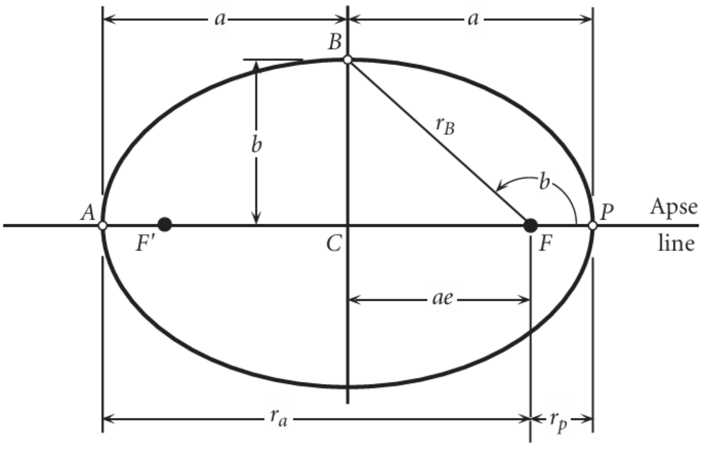
$$r = \frac{h^2}{\mu} \qquad T_{\text{circular}} = \frac{2\pi}{\sqrt{\mu}}r^{\frac{3}{2}}$$

$$\mu_{\text{earth}} = 398\,600\ \text{km}^3/\text{s}^2$$

$$\omega_E = 72.9217 \times 10^{-6}\ \text{rad/s}$$

$$v_{\text{circular}} = \sqrt{\frac{\mu}{r}} \qquad \varepsilon_{\text{circular}} = -\frac{1}{2}\frac{\mu^2}{h^2} = -\frac{\mu}{2r}$$

2. Órbitas Elípticas $(0 < e < 1)$



$$T = \frac{2\pi ab}{h}$$

$$r_a = \frac{h^2}{\mu}\frac{1}{1 - e}$$

$$r_p = a(1 - e)$$

$$r_B = a\frac{1 - e^2}{1 + e\cos\beta} = a$$

$$\frac{r_p}{r_a} = \frac{1 - e}{1 + e}$$

$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a}$$

$$CF = a - FP = a - r_p$$

$$2a = r_p + r_a$$

$$a = \frac{h^2}{\mu}\frac{1}{1 - e^2}$$

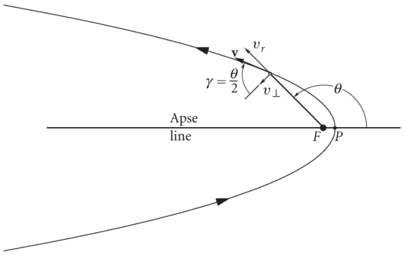
$$e = -\cos\beta$$

$$b = a\sqrt{1 - e^2}$$

$$T = \frac{2\pi}{\sqrt{\mu}}a^{\frac{3}{2}}$$

$$e = \frac{r_a - r_p}{r_a + r_p} \qquad eccentricity = \frac{\text{distance between the foci}}{\text{length of the major axis}} \qquad \bar{r}_\theta = \sqrt{r_p r_a}$$

3. Órbitas Parabólicas (e = 1)



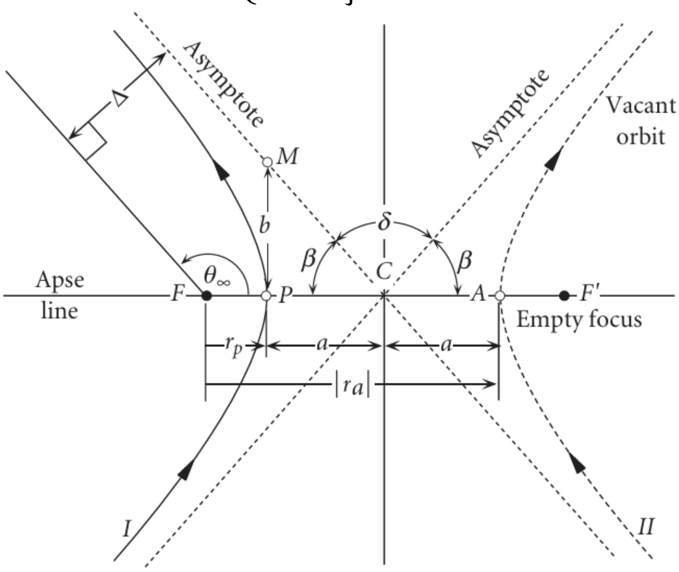
$$r = \frac{h^2}{\mu}\frac{1}{1+\cos\theta} \qquad v_{\text{esc}} = v = \sqrt{\frac{2\mu}{r}} \qquad \gamma = \frac{\theta}{2} \qquad \tan\gamma = \frac{\sin\theta}{1+\cos\theta}$$

$$x = r\cos\theta = p\frac{\cos\theta}{1+\cos\theta}$$

$$y = r\sin\theta = p\frac{\sin\theta}{1+\cos\theta}$$

$$x = \frac{p}{2} - \frac{y^2}{2p}$$

4. Órbitas Hiperbólicas (e > 1)



$$\theta_\infty = \cos^{-1}(-1/e) \qquad \beta = 180^\circ - \theta_\infty$$

$$\beta = \cos^{-1}(1/e)$$

$$\sin\theta_\infty = \frac{\sqrt{e^2-1}}{e}$$

$$2a = |r_a| - r_p = -r_a - r_p$$

$$\delta = 2\sin^{-1}(1/e)$$

$$r_p = a(e-1)$$

$$r_p = \frac{h^2}{\mu}\frac{1}{1+e}$$

$$r_a = -a(e+1)$$

$$b = a\sqrt{e^2-1}$$

$$r_a = \frac{h^2}{\mu}\frac{1}{1-e}$$

$$\begin{aligned}
\Delta &= (r_p + a)\sin\beta \\
&= ae\sin\beta && \text{(Equation 2.95a)} \\
&= ae\frac{\sqrt{e^2-1}}{e} && \text{(Equation 2.89)} \\
&= ae\sin\theta_\infty && \text{(Equation 2.88)} \\
&= ae\sqrt{1-\cos^2\theta_\infty} && \text{(trig identity)} \\
&= ae\sqrt{1-\frac{1}{e^2}} && \text{(Equation 2.87)} \\
\Delta &= a\sqrt{e^2-1}
\end{aligned}$$

$$v_\infty = \sqrt{\frac{\mu}{a}}$$

$$v^2 = v_{\text{esc}}^2 + v_\infty^2$$

$$\frac{v^2}{2} - \frac{\mu}{r} = \frac{\mu}{2a} \quad = \frac{v_\infty^2}{2}$$

$$C_3 = v_\infty^2$$

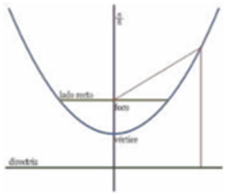
$$v_\infty = \frac{\mu}{h}e\sin\theta_\infty = \frac{\mu}{h}\sqrt{e^2-1}$$

CIRCUNFERENCIA

$$x^2 + y^2 + Dx + Ey + F = 0$$

$$(x-h)^2 + (y-k)^2 = r^2$$

PARÁBOLA

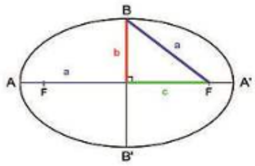


$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

$$y = ax^2 + bx + c$$

$$x = ay^2 + by + c$$

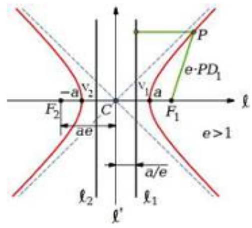
ELIPSE



$$Ax^2 + By^2 + Cx + Dy + E = 0$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

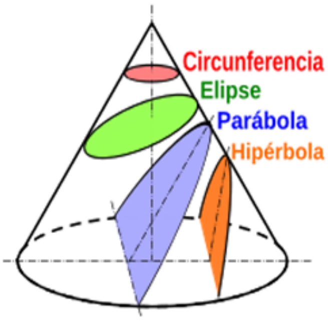
HIPERBOLA



$$Ax^2 - By^2 + Cx + Dy - E = 0$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

CLASIFICACIÓN



β < α : Hipérbola (naranja)
β = α : Parábola (azulado)
β > α : Elipse (verde)
β = 90º: Circunferencia (rojo)

IDENTIFICACIÓN DE CONICAS

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

En función de los valores de los parámetros se tendrá:

- $h^2 > ab$: Hipérbola.
- $h^2 = ab$: Parábola.
- $h^2 < ab$: Elipse.
- a = b y $h = 0$: Circunferencia.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

- (Negativo) $B^2 - 4AC < 0$: Elipse.
- (Cero) $B^2 - 4AC = 0$: Parábola.
- (Positivo) $B^2 - 4AC > 0$: Hipérbola.
- A = C y x.y = 0 : Circunferencia