

$$\mathbf{a} = a_t \hat{\mathbf{u}}_t + a_n \hat{\mathbf{u}}_n$$

$$\hat{\mathbf{u}}_b = \hat{\mathbf{u}}_t \times \hat{\mathbf{u}}_n$$

Centro de curvatura respecto a P

$$\mathbf{r}_{C/P} = \varrho \hat{\mathbf{u}}_n$$

$$\mathbf{I} = \int_{t_1}^{t_2} \mathbf{F} dt$$

$$\Delta \mathbf{v} = \frac{\mathbf{I}_{\text{net}}}{m}$$

$$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$$

$$\mathbf{M}_{O_{\text{net}}} = \mathbf{r} \times \mathbf{F}_{\text{net}}$$

$$\int_{t_1}^{t_2} \mathbf{M}_{O_{\text{net}}} dt = \mathbf{H}_{O_2} - \mathbf{H}_{O_1}$$

$$\mathbf{M}_{O_{\text{net}}} = \frac{d\mathbf{H}_O}{dt}$$

Vector rotante

$$\frac{d\mathbf{A}}{dt} = \boldsymbol{\omega} \times \mathbf{A} \quad \frac{d^2\mathbf{A}}{dt^2} = \boldsymbol{\alpha} \times \mathbf{A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{A}) \quad \frac{d^3\mathbf{A}}{dt^3} = \frac{d\boldsymbol{\alpha}}{dt} \times \mathbf{A} + 2\boldsymbol{\alpha} \times (\boldsymbol{\omega} \times \mathbf{A}) + \boldsymbol{\omega} \times [\boldsymbol{\alpha} \times \mathbf{A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{A})]$$

$$\frac{d\hat{\mathbf{i}}}{dt} = \boldsymbol{\Omega} \times \hat{\mathbf{i}} \quad \frac{d\hat{\mathbf{j}}}{dt} = \boldsymbol{\Omega} \times \hat{\mathbf{j}} \quad \frac{d\hat{\mathbf{k}}}{dt} = \boldsymbol{\Omega} \times \hat{\mathbf{k}}$$

$$\left. \frac{d\mathbf{Q}}{dt} = \frac{d\mathbf{Q}}{dt} \right)_{\text{rel}} + \boldsymbol{\Omega} \times \mathbf{Q}$$

$$\frac{d\mathbf{Q}}{dt} = \frac{dQ_X}{dt} \hat{\mathbf{i}} + \frac{dQ_Y}{dt} \hat{\mathbf{j}} + \frac{dQ_Z}{dt} \hat{\mathbf{k}}$$

$$\left. \frac{d\mathbf{Q}}{dt} \right)_{\text{rel}} = \frac{dQ_x}{dt} \hat{\mathbf{i}} + \frac{dQ_y}{dt} \hat{\mathbf{j}} + \frac{dQ_z}{dt} \hat{\mathbf{k}}$$

Movimiento relativo

$$\mathbf{r} = \mathbf{r}_O + \mathbf{r}_{\text{rel}}$$

$$\mathbf{v} = \mathbf{v}_O + \boldsymbol{\Omega} \times \mathbf{r}_{\text{rel}} + \mathbf{v}_{\text{rel}}$$

$$\mathbf{a} = \mathbf{a}_O + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{\text{rel}} + \underbrace{\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{\text{rel}})}_{\text{Centrípeta}} + \underbrace{2\boldsymbol{\Omega} \times \mathbf{v}_{\text{rel}}}_{\text{Coriolis}} + \mathbf{a}_{\text{rel}}$$

Movimiento en el centro de la tierra

$$\boldsymbol{\omega} = -\dot{\phi} \hat{\mathbf{i}} + \dot{\Lambda} \cos \phi \hat{\mathbf{j}} + \dot{\Lambda} \sin \phi \hat{\mathbf{k}}$$

$$\mathbf{v}_{\text{rel}} = \dot{x} \hat{\mathbf{i}} + \dot{y} \hat{\mathbf{j}} + \dot{z} \hat{\mathbf{k}}$$

$$\mathbf{a}_{\text{rel}} = \left[\ddot{x} + \frac{\dot{x}(\dot{z} - \dot{y} \tan \phi)}{R_E + z} \right] \hat{\mathbf{i}} + \left(\ddot{y} + \frac{\dot{y}\dot{z} + \dot{x}^2 \tan \phi}{R_E + z} \right) \hat{\mathbf{j}} + \left(\ddot{z} - \frac{\dot{x}^2 + \dot{y}^2}{R_E + z} \right) \hat{\mathbf{k}}$$

$$\dot{x} = (R_E + z) \dot{\Lambda} \cos \phi \quad \dot{y} = (R_E + z) \dot{\phi}$$

$$\mathbf{a}_{\text{rel}})_{\text{neglecting earth's curvature}} = \ddot{x} \hat{\mathbf{i}} + \ddot{y} \hat{\mathbf{j}} + \ddot{z} \hat{\mathbf{k}}$$

$$\dot{\phi} = \frac{\dot{y}}{R_E + z} \quad \dot{\Lambda} = \frac{\dot{x}}{(R_E + z) \cos \phi}$$

$$\ddot{\phi} = \frac{(R_E + z) \ddot{y} - \dot{y} \dot{z}}{(R_E + z)^2} \quad \ddot{\Lambda} = \frac{(R_E + z) \ddot{x} \cos \phi - (\dot{z} \cos \phi - \dot{y} \sin \phi) \dot{x}}{(R_E + z)^2 \cos^2 \phi}$$

$$\boldsymbol{\Omega} = \Omega \hat{\mathbf{K}} = \Omega \cos \phi \hat{\mathbf{j}} + \Omega \sin \phi \hat{\mathbf{k}}$$

$$\mathbf{v} = [\dot{x} + \Omega(R_E + z) \cos \phi] \hat{\mathbf{i}} + \dot{y} \hat{\mathbf{j}} + \dot{z} \hat{\mathbf{k}}$$

$$\begin{aligned} \mathbf{a} = & \left[\ddot{x} + \frac{\dot{x}(\dot{z} - \dot{y} \tan \phi)}{R_E + z} + 2\Omega(\dot{z} \cos \phi - \dot{y} \sin \phi) \right] \hat{\mathbf{i}} \\ & + \left\{ \ddot{y} + \frac{\dot{y}\dot{z} + \dot{x}^2 \tan \phi}{R_E + z} + \Omega \sin \phi [\Omega(R_E + z) \cos \phi + 2\dot{x}] \right\} \hat{\mathbf{j}} \\ & + \left\{ \ddot{z} - \frac{\dot{x}^2 + \dot{y}^2}{R_E + z} - \Omega \cos \phi [\Omega(R_E + z) \cos \phi + 2\dot{x}] \right\} \hat{\mathbf{k}} \end{aligned}$$

Casos especiales

Straight and level, unaccelerated flight: $\dot{z} = \ddot{z} = \ddot{x} = \ddot{y} = 0$

$$\begin{aligned} \mathbf{v} &= [\dot{x} + \Omega(R_E + z) \cos \phi] \hat{\mathbf{i}} + \dot{y} \hat{\mathbf{j}} \\ \mathbf{a} &= - \left[\frac{\dot{x} \dot{y} \tan \phi}{R_E + z} + 2\Omega \dot{y} \sin \phi \right] \hat{\mathbf{i}} \\ &\quad + \left\{ \frac{\dot{x}^2 \tan \phi}{R_E + z} + \Omega \sin \phi [\Omega(R_E + z) \cos \phi + 2\dot{x}] \right\} \hat{\mathbf{j}} \\ &\quad - \left\{ \frac{\dot{x}^2 + \dot{y}^2}{R_E + z} + \Omega \cos \phi [\Omega(R_E + z) \cos \phi + 2\dot{x}] \right\} \hat{\mathbf{k}} \end{aligned}$$

Flight due east (x) at constant speed and altitude: $\dot{z} = \ddot{z} = \ddot{x} = \dot{y} = \ddot{y} = 0$

$$\begin{aligned}\mathbf{v} &= [\dot{x} + \Omega(R_E + z) \cos \phi] \hat{\mathbf{i}} \\ \mathbf{a} &= \left\{ \frac{\dot{x}^2 \tan \phi}{R_E + z} + \Omega \sin \phi [\Omega(R_E + z) \cos \phi + 2\dot{x}] \right\} \hat{\mathbf{j}} \\ &\quad - \left\{ \frac{\dot{x}^2}{R_E + z} + \Omega \cos \phi [\Omega(R_E + z) \cos \phi + 2\dot{x}] \right\} \hat{\mathbf{k}}\end{aligned}$$

Flight straight up (z): $\dot{x} = \ddot{x} = \dot{y} = \ddot{y} = 0$

$$\begin{aligned}\mathbf{v} &= \Omega(R_E + z) \cos \phi \hat{\mathbf{i}} + \dot{z} \hat{\mathbf{k}} \\ \mathbf{a} &= 2\Omega(\dot{z} \cos \phi) \hat{\mathbf{i}} + \Omega^2(R_E + z) \sin \phi \cos \phi \hat{\mathbf{j}} \\ &\quad + [\ddot{z} - \Omega^2(R_E + z) \cos^2 \phi] \hat{\mathbf{k}}\end{aligned}$$

$$\Omega = \frac{2\pi \text{ rad}}{\text{sidereal day}} = \frac{2\pi \text{ rad}}{23.93 \text{ hr}} = \frac{2\pi \text{ rad}}{86\,160 \text{ s}} = 7.292 \times 10^{-5} \text{ rad/s}$$

Lagrangiano

$$\mathcal{L} = T - U. \quad \frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

$$(\text{generalized force}) = (\text{rate of change of generalized momentum})$$

Flight due north (y) at constant speed and altitude: $\dot{z} = \ddot{z} = \dot{x} = \ddot{x} = \ddot{y} = 0$

$$\begin{aligned}\mathbf{v} &= \Omega(R_E + z) \cos \phi \hat{\mathbf{i}} + \dot{y} \hat{\mathbf{j}} \\ \mathbf{a} &= -2\Omega \dot{y} \sin \phi \hat{\mathbf{i}} + \Omega^2(R_E + z) \sin \phi \cos \phi \hat{\mathbf{j}} \\ &\quad - \left[\frac{\dot{y}^2}{R_E + z} + \Omega^2(R_E + z) \cos^2 \phi \right] \hat{\mathbf{k}}\end{aligned}$$

Stationary: $\dot{x} = \ddot{x} = \dot{y} = \ddot{y} = \dot{z} = \ddot{z} = 0$

$$\begin{aligned}\mathbf{v} &= \Omega(R_E + z) \cos \phi \hat{\mathbf{i}} \\ \mathbf{a} &= \Omega^2(R_E + z) \sin \phi \cos \phi \hat{\mathbf{j}} - \Omega^2(R_E + z) \cos^2 \phi \hat{\mathbf{k}}\end{aligned}$$

