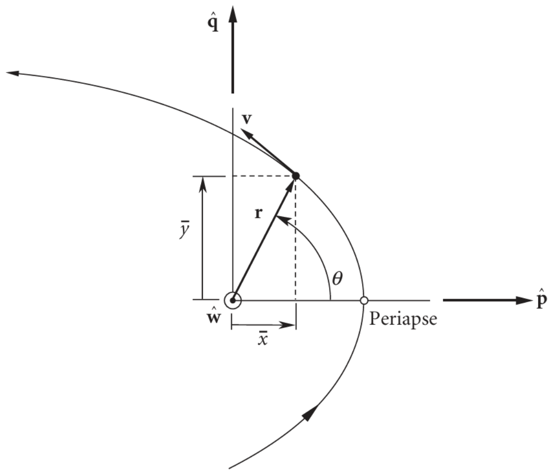
Plano Perifocal



$$\hat{\mathbf{w}} = \frac{\mathbf{h}}{h}$$

$$\mathbf{v} = \frac{\mu}{h}[-\sin\theta\hat{\mathbf{p}} + (e+\cos\theta)\hat{\mathbf{q}}]$$

$$\mathbf{r} = \bar{x}\hat{\mathbf{p}} + \bar{y}\hat{\mathbf{q}}$$

$$\bar{x} = r\cos\theta \qquad \bar{y} = r\sin\theta$$

$$\dot{\bar{x}} = -\frac{\mu}{h}\sin\theta \qquad \dot{\bar{y}} = \frac{\mu}{h}(e+\cos\theta)$$

+Para cualquier órbita

$$v = \frac{\mu}{h}\sqrt{1+2e\cos\theta+e^2}$$

Momento angular

$$h = \bar{x}_0\dot{\bar{y}}_0 - \bar{y}_0\dot{\bar{x}}_0$$

Coeficientes de Lagrange

$$\mathbf{r} = f\mathbf{r}_0 + g\mathbf{v}_0$$

$$\mathbf{v} = \dot{f}\mathbf{r}_0 + \dot{g}\mathbf{v}_0$$

$$r = \frac{h^2}{\mu}\frac{1}{1+e\cos(\theta_0+\Delta\theta)} = \frac{h^2}{\mu}\frac{1}{1+\left(\dfrac{h^2}{\mu r_0}-1\right)\cos\Delta\theta - \dfrac{h v_{r0}}{\mu}\sin\Delta\theta}$$

$$f = \frac{\bar{x}\dot{\bar{y}}_0 - \bar{y}\dot{\bar{x}}_0}{h} = \frac{\mu r}{h^2}(e\cos\theta + \cos\Delta\theta) = 1 - \frac{\mu r}{h^2}(1-\cos\Delta\theta)$$

$$g = \frac{-\bar{x}\bar{y}_0 + \bar{y}\bar{x}_0}{h} = \frac{r r_0}{h}\sin(\Delta\theta)$$

$$\dot{g} = \frac{-\dot{\bar{x}}\bar{y}_0 + \dot{\bar{y}}\bar{x}_0}{h} = 1 - \frac{\mu r_0}{h^2}(1-\cos\Delta\theta)$$

$$\dot{f} = \frac{1}{g}(f\dot{g}-1) = \frac{\mu}{h}\frac{1-\cos\Delta\theta}{\sin\Delta\theta}\left[\frac{\mu}{h^2}(1-\cos\Delta\theta) - \frac{1}{r_0} - \frac{1}{r}\right]$$

$$r_0 = \frac{h^2}{\mu}\frac{1}{1+e\cos\theta_0}$$

$$v_{r0} = \mathbf{v}_0 \cdot \frac{\mathbf{r}_0}{r_0} = \frac{\mu}{h}e\sin\theta_0$$

$$f\dot{g} - \dot{f}g = 1 \quad \text{(conservation of angular momentum)}$$

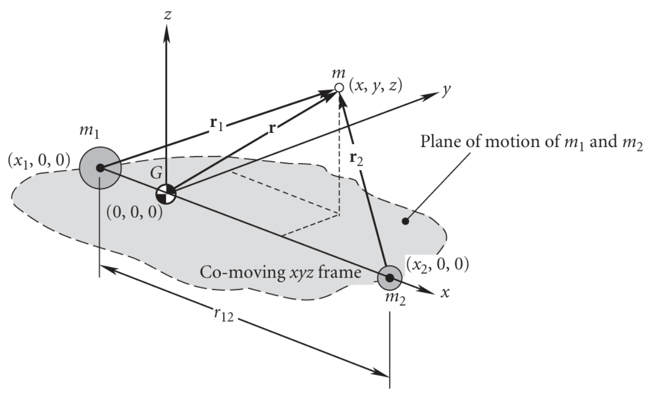
Posición en función del tiempo (Para tiempos pequeños)

$$\mathbf{r}(t) = \left\{1 - \frac{\mu}{2r_0^3}\Delta t^2 + \frac{\mu}{2}\frac{\mathbf{r}_0\cdot\mathbf{v}_0}{r_0^5}\Delta t^3 + \frac{\mu}{24}\left[-2\frac{\mu}{r_0^6} + 3\frac{v_0^2}{r_0^5} - 15\frac{(\mathbf{r}_0\cdot\mathbf{v}_0)^2}{r_0^7}\right]\Delta t^4\right\}\mathbf{r}_0 + \left[\Delta t - \frac{1}{6}\frac{\mu}{r_0^3}\Delta t^3 + \frac{\mu}{4}\frac{(\mathbf{r}_0\cdot\mathbf{v}_0)}{r_0^5}\Delta t^4\right]\mathbf{v}_0$$

$$f = 1 - \frac{\mu}{2r_0^3}\Delta t^2 + \frac{\mu}{2}\frac{\mathbf{r}_0\cdot\mathbf{v}_0}{r_0^5}\Delta t^3 + \frac{\mu}{24}\left[-2\frac{\mu}{r_0^6} + 3\frac{v_0^2}{r_0^5} - 15\frac{(\mathbf{r}_0\cdot\mathbf{v}_0)^2}{r_0^7}\right]\Delta t^4$$

$$g = \Delta t - \frac{1}{6}\frac{\mu}{r_0^3}\Delta t^3 + \frac{\mu}{4}\frac{(\mathbf{r}_0\cdot\mathbf{v}_0)}{r_0^5}\Delta t^4$$

Problema de los tres cuerpos restringido



$$\Omega = \frac{2\pi}{T} = \sqrt{\frac{\mu}{r_{12}^3}}$$

$$T = 2\pi\frac{r_{12}^{\frac{3}{2}}}{\sqrt{\mu}}$$

$$\mu = GM = G(m_1+m_2)$$

Ecuaciones que rigen el movimiento de la masa m respecto al marco de referencia móvil

$$\ddot{x} - 2\Omega\dot{y} - \Omega^2 x = -\frac{\mu_1}{r_1^3}(x + \pi_2 r_{12}) - \frac{\mu_2}{r_2^3}(x - \pi_1 r_{12})$$

$$\ddot{y} + 2\Omega\dot{x} - \Omega^2 y = -\frac{\mu_1}{r_1^3}y - \frac{\mu_2}{r_2^3}y$$

$$\ddot{z} = -\frac{\mu_1}{r_1^3}z - \frac{\mu_2}{r_2^3}z$$

$$\pi_1 = \frac{m_1}{m_1 + m_2}$$

$$\pi_2 = \frac{m_2}{m_1 + m_2}$$

Puntos de Lagrange

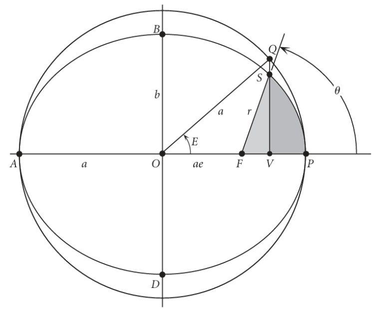
$\dot{x} = \dot{y} = \dot{z} = 0 \quad \text{and} \quad \ddot{x} = \ddot{y} = \ddot{z} = 0$

$$L_4, L_5\colon x = \frac{r_{12}}{2} - \pi_2 r_{12}, \quad y = \pm\frac{\sqrt{3}}{2}r_{12}, \quad z = 0$$

Puntos L1, L2 y L3 con $y = 0$

$$f(\xi) = \frac{1 - \pi_2}{|\xi + \pi_2|^3}(\xi + \pi_2) + \frac{\pi_2}{|\xi + \pi_2 - 1|^3}(\xi + \pi_2 - 1) - \xi \qquad \xi = \frac{x}{r_{12}}$$

Las raíces de la función nos permiten encontrar los puntos de Lagrange restantes



Constante de Jacobi

$$\frac{1}{2}v^2 - \frac{1}{2}\Omega^2(x^2 + y^2) - \frac{\mu_1}{r_1} - \frac{\mu_2}{r_2} = C$$

$$r_1 = \sqrt{(x + \pi_2 r_{12})^2 + y^2} \quad r_2 = \sqrt{(x - \pi_1 r_{12})^2 + y^2}$$

CAPÍTULO 3

$$\frac{\mu^2}{h^3}t = \int_0^\theta \frac{d\vartheta}{(1 + e\cos\vartheta)^2}$$

Órbitas Circulares

$$t = \frac{h^3}{\mu^2}\theta = \frac{r^{\frac{3}{2}}}{\sqrt{\mu}}\theta$$

$$t = \frac{\theta}{2\pi}T$$

Órbitas elípticas

1. Calcular t dada θ

$$\cos E = \frac{e + \cos\theta}{1 + e\cos\theta} \longrightarrow M_e = E - e\sin E \longrightarrow t = \frac{M_e}{2\pi}T$$

$$\sin E = \frac{\sqrt{1 - e^2}\sin\theta}{1 + e\cos\theta}$$

$$\tan\frac{E}{2} = \sqrt{\frac{1 - e}{1 + e}}\tan\frac{\theta}{2}$$

2. Calcular θ dado t

$$M_e = nt = \frac{2\pi}{T}t \longrightarrow M_e = E - e\sin E \longrightarrow \tan\frac{E}{2} = \sqrt{\frac{1 - e}{1 + e}}\tan\frac{\theta}{2}$$

$$\cos\theta = \frac{e - \cos E}{e\cos E - 1}$$

Órbitas parabólicas

$$M_p = \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{6}\tan^3\frac{\theta}{2}$$

$$M_p = \frac{\mu^2 t}{h^3}$$

$$\tan\frac{\theta}{2} = \left[3M_p + \sqrt{(3M_p)^2 + 1}\right]^{\frac{1}{3}} - \left[(3M_p + \sqrt{(3M_p)^2 + 1})\right]^{-\frac{1}{3}}$$

Órbitas hiperbólicas

1. Calcular t dada θ

$$\tanh\frac{F}{2} = \sqrt{\frac{e - 1}{e + 1}}\tan\frac{\theta}{2} \longrightarrow M_h = e\sinh F - F \longrightarrow M_h = \frac{\mu^2}{h^3}(e^2 - 1)^{\frac{3}{2}}t$$

2. Calcular θ dado t

$$M_h = \frac{\mu^2}{h^3}(e^2 - 1)^{\frac{3}{2}}t \longrightarrow M_h = e\sinh F - F \longrightarrow$$

$$\cos\theta = \frac{\cosh F - e}{1 - e\cosh F}$$

$$\tan\frac{\theta}{2} = \sqrt{\frac{e + 1}{e - 1}}\tanh\frac{F}{2}$$

ALGORITHM 3.1

Solve Kepler's equation for the eccentric anomaly E given the eccentricity e and the mean anomaly M_e. See Appendix D.2 for the implementation of this algorithm in MATLAB®.

1. Choose an initial estimate of the root E as follows (Prussing and Conway, 1993). If $M_e < \pi$, then $E = M_e + e/2$. If $M_e > \pi$, then $E = M_e - e/2$. Remember that the angles E and M_e are in radians. (When using a hand-held calculator, be sure it is in radian mode.)
2. At any given step, having obtained E_i from the previous step, calculate $f(E_i) = E_i - e \sin E_i - M_e$ and $f'(E_i) = 1 - e \cos E_i$.
3. Calculate $\text{ratio}_i = f(E_i)/f'(E_i)$.
4. If $|\text{ratio}_i|$ exceeds the chosen tolerance (e.g., 10^{-8}), then calculate an updated value of E

$$E_{i+1} = E_i - \text{ratio}_i$$

Return to step 2.

5. If $|\text{ratio}_i|$ is less than the tolerance, then accept E_i as the solution to within the chosen accuracy.

ALGORITHM 3.2

Solve Kepler's equation for the hyperbola for the hyperbolic eccentric anomaly F given the eccentricity e and the hyperbolic mean anomaly M_h. See Appendix D.3 for the implementation of this algorithm in MATLAB.

1. Choose an initial estimate of the root F.
   (a) For hand computations read a rough value of F_0 (no more than two significant figures) from Figure 3.17 in order to keep the number of iterations to a minimum.
   (b) In computer software let $F_0 = M_h$, an inelegant choice which may result in many iterations but will nevertheless rapidly converge on today's high speed desktop and laptop computers.
2. At any given step, having obtained F_i from the previous step, calculate $f(F_i) = e \sinh F_i - F_i - M_h$ and $f'(F_i) = e \cosh F_i - 1$.
3. Calculate $\text{ratio}_i = f(F_i)/f'(F_i)$.
4. If $|\text{ratio}_i|$ exceeds the chosen tolerance (e.g., 10^{-8}), then calculate an updated value of F,

$$F_{i+1} = F_i - \text{ratio}_i$$

Return to step 2.

5. If $|\text{ratio}_i|$ is less than the tolerance, then accept F_i as the solution to within the desired accuracy.

$$\bar{r}_t = \frac{1}{T}\int_0^T r\,dt = a\left(1 + \frac{e^2}{2}\right)$$ Time-averaged radius of an elliptical orbit.

$$\bar{r}_\theta = a\sqrt{3 - 2\frac{\bar{r}_t}{a}}$$